Computer algebra independent integration tests

6-Hyperbolic-functions/6.4-Hyperbolic-cotangent/6.4.2-Hyperbolic-cotangent-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [224]. This is test number [175].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 81.7 (183)	% 18.3 (41)
Mathematica	% 100. (224)	% 0. (0)
Maple	% 73.21 (164)	% 26.79 (60)
Maxima	% 46.88 (105)	% 53.12 (119)
Fricas	% 79.02 (177)	% 20.98 (47)
Sympy	% 13.84 (31)	% 86.16 (193)
Giac	% 56.25 (126)	% 43.75 (98)

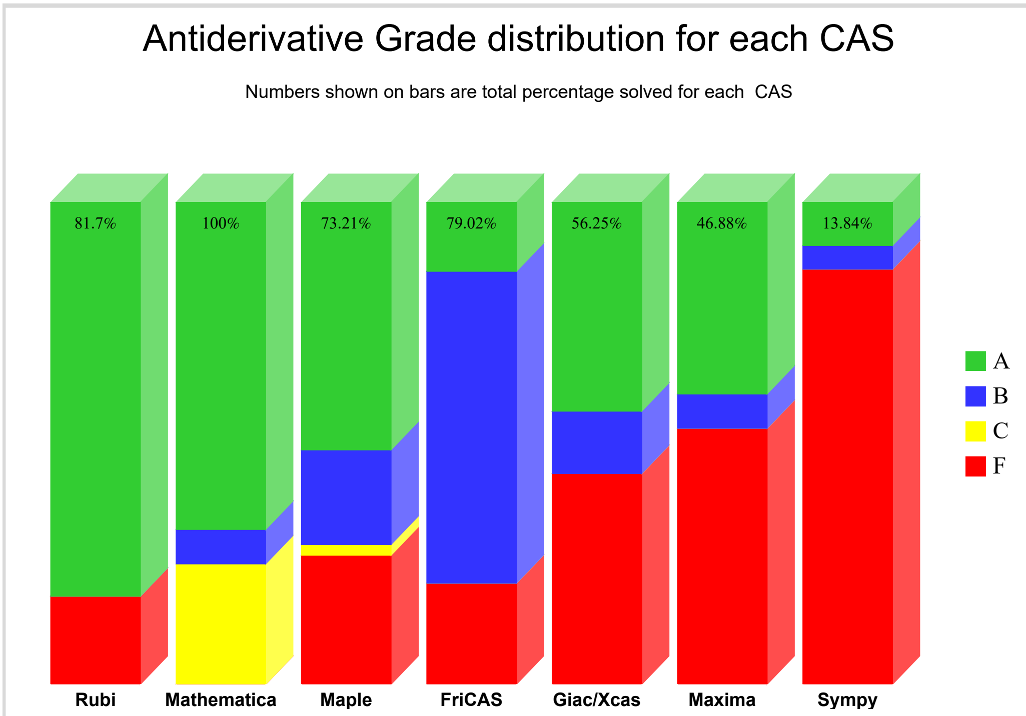
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

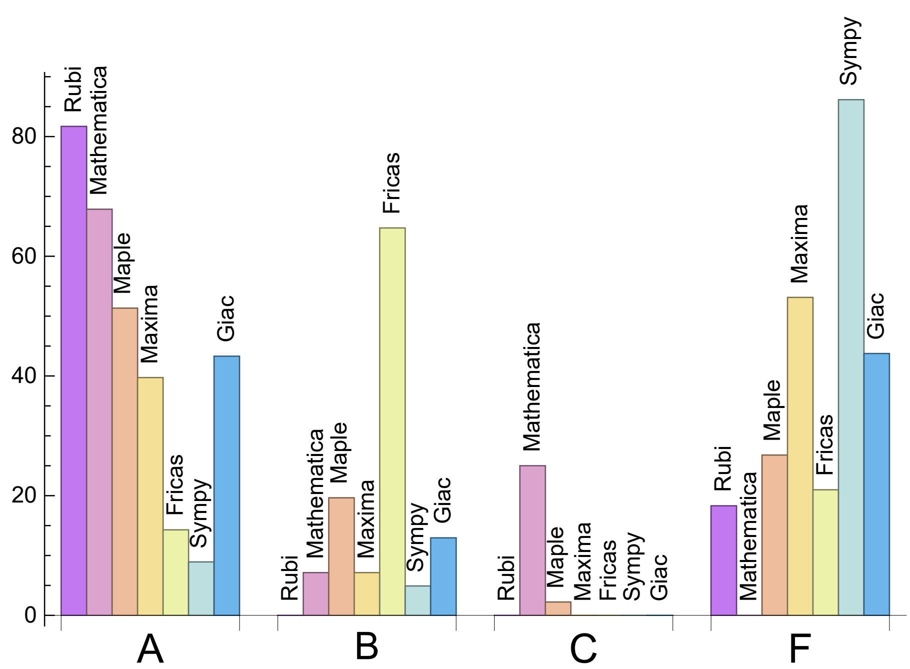
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	81.7	0.	0.	18.3
Mathematica	67.86	7.14	25.	0.
Maple	51.34	19.64	2.23	26.79
Maxima	39.73	7.14	0.	53.12
Fricas	14.29	64.73	0.	20.98
Sympy	8.93	4.91	0.	86.16
Giac	43.3	12.95	0.	43.75

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	84.35	0.99	60.	1.
Mathematica	1.36	96.6	1.31	60.	1.05
Maple	0.04	93.86	1.5	70.5	1.28
Maxima	1.33	111.23	2.05	63.	1.5
Fricas	3.04	2321.98	23.59	828.	14.48
Sympy	3.76	196.35	3.74	104.	2.54
Giac	1.16	107.52	2.01	81.	1.66

1.4 list of integrals that has no closed form antiderivative

{220, 224}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {159, 160, 161, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 196, 197, 198, 209, 212}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

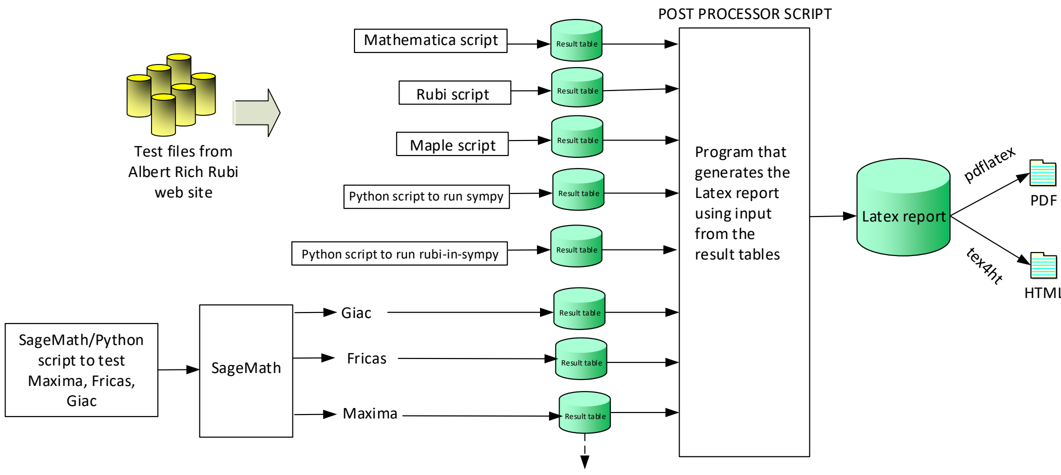
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 162, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { }

C grade: { }

F grade: { 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 10, 12, 15, 16, 17, 18, 19, 20, 21, 22, 24, 28, 29, 30, 35, 36, 37, 38, 39, 42, 43, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 153, 155, 157, 158, 165, 167, 169, 170, 171, 172, 173, 181, 184, 185, 186, 187, 189, 190, 191, 193, 194, 195, 196, 198, 199, 200, 201, 202, 205, 206, 208, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { 151, 166, 168, 174, 175, 176, 177, 178, 179, 180, 182, 183, 197, 207, 209, 210 }

C grade: { 6, 7, 8, 9, 11, 13, 14, 23, 25, 26, 27, 31, 32, 33, 34, 40, 41, 44, 46, 47, 48, 49, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 87, 113, 132, 133, 134, 135, 136, 137, 138, 139, 152, 154, 156, 159, 160, 161, 162, 163, 164, 188, 192, 203, 204, 212 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 29, 30, 31, 32, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 98, 100, 101, 102, 109, 115, 116, 117, 118, 119, 122, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 157, 158, 159, 160, 161, 164, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { 33, 34, 35, 36, 37, 38, 77, 78, 79, 80, 89, 90, 91, 92, 95, 96, 97, 99, 103, 104, 105, 106, 107, 108, 110, 111, 112, 114, 120, 121, 123, 124, 125, 126, 140, 141, 152, 154, 155, 156, 162, 181, 188, 213 }

C grade: { 163, 210, 214, 215, 216 }

F grade: { 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 113, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 208, 209 }

2.1.4 Maxima

A grade: { 18, 19, 20, 21, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 64, 65, 66, 67, 68, 69, 80, 81, 82, 85, 86, 89, 90, 91, 92, 93, 94, 97, 99, 102, 105, 106, 107, 108, 109, 110, 114, 116, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 181, 188, 211, 212, 213, 214, 215, 216, 220, 224

B grade: { 61, 62, 63, 77, 78, 79, 83, 84, 95, 96, 104, 111, 112, 191, 192, 193 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 87, 88, 98, 100, 101, 103, 113, 115, 117, 118, 120, 132, 133, 134, 135, 136, 137, 138, 139, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 218, 219, 221, 222, 223 }

2.1.5 FriCAS

A grade: { 9, 10, 46, 81, 85, 86, 91, 92, 93, 101, 107, 108, 118, 122, 143, 144, 145, 146, 151, 152, 153, 155, 156, 158, 163, 164, 213, 214, 218, 220, 222, 224 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 47, 48, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 147, 148, 149, 150, 154, 157, 159, 160, 161, 162, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 216, 217, 219, 221, 223 }

C grade: { }

F grade: { 15, 16, 17, 27, 28, 39, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198 }

2.1.6 Sympy

A grade: { 61, 62, 63, 64, 77, 78, 79, 80, 81, 85, 86, 144, 145, 146, 147, 148, 149, 162, 220, 224 }

B grade: { 65, 66, 67, 68, 69, 127, 128, 129, 130, 131, 155 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223 }

2.1.7 Giac

A grade: { 18, 19, 21, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 188, 192, 211, 212, 213, 214, 215, 216, 220, 224 }

B grade: { 6, 11, 12, 20, 70, 71, 72, 73, 74, 75, 76, 95, 102, 104, 113, 119, 121, 132, 133, 134, 135, 136, 137, 138, 139, 150, 181, 191, 193 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 87, 88, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 218, 219, 221, 222, 223 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	83	80	0	4135	0	0
normalized size	1	1.	0.86	0.82	0.	42.63	0.	0.
time (sec)	N/A	0.07	0.232	0.026	0.	2.507	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	63	0	2654	0	0
normalized size	1	1.	0.87	0.81	0.	34.03	0.	0.
time (sec)	N/A	0.049	0.213	0.01	0.	2.388	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	61	62	0	1754	0	0
normalized size	1	1.	0.81	0.83	0.	23.39	0.	0.
time (sec)	N/A	0.05	0.095	0.011	0.	2.272	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	47	0	1636	0	0
normalized size	1	1.	0.88	0.81	0.	28.21	0.	0.
time (sec)	N/A	0.036	0.042	0.023	0.	2.214	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	0	1646	0	0
normalized size	1	1.	0.86	0.81	0.	28.88	0.	0.
time (sec)	N/A	0.031	0.034	0.033	0.	2.433	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	36	65	0	2576	0	277
normalized size	1	1.	0.46	0.83	0.	33.03	0.	3.55
time (sec)	N/A	0.051	0.074	0.013	0.	2.716	0.	1.335

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	38	64	0	3933	0	0
normalized size	1	1.	0.48	0.81	0.	49.78	0.	0.
time (sec)	N/A	0.05	0.065	0.011	0.	2.854	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	38	83	0	5797	0	0
normalized size	1	1.	0.38	0.83	0.	57.97	0.	0.
time (sec)	N/A	0.071	0.096	0.013	0.	2.599	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	36	209	0	865	0	0
normalized size	1	1.	0.15	0.89	0.	3.67	0.	0.
time (sec)	N/A	0.282	0.026	0.02	0.	1.961	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	149	193	0	871	0	0
normalized size	1	1.	0.68	0.89	0.	4.	0.	0.
time (sec)	N/A	0.294	0.177	0.016	0.	2.047	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	38	115	0	828	0	293
normalized size	1	1.	0.29	0.87	0.	6.27	0.	2.22
time (sec)	N/A	0.109	0.041	0.015	0.	2.001	0.	1.687

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	98	115	0	4398	0	292
normalized size	1	1.	0.74	0.87	0.	33.32	0.	2.21
time (sec)	N/A	0.104	0.121	0.012	0.	2.23	0.	1.642

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	36	193	0	991	0	0
normalized size	1	1.	0.17	0.89	0.	4.55	0.	0.
time (sec)	N/A	0.235	0.029	0.017	0.	2.01	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	36	211	0	9677	0	0
normalized size	1	1.	0.15	0.89	0.	40.66	0.	0.
time (sec)	N/A	0.316	0.062	0.016	0.	2.449	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	45	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.063	0.165	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.041	0.191	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.048	0.247	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	53	131	2068	0	122
normalized size	1	1.	0.92	0.87	2.15	33.9	0.	2.
time (sec)	N/A	0.037	0.118	0.026	1.604	1.981	0.	1.203

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	45	73	304	0	73
normalized size	1	1.	1.26	1.45	2.35	9.81	0.	2.35
time (sec)	N/A	0.02	0.045	0.036	1.59	1.934	0.	1.159

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	46	309	0	81
normalized size	1	1.	1.	1.68	1.48	9.97	0.	2.61
time (sec)	N/A	0.02	0.059	0.04	1.624	1.92	0.	1.2

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	48	79	113	2082	0	149
normalized size	1	1.	0.74	1.22	1.74	32.03	0.	2.29
time (sec)	N/A	0.036	0.143	0.02	1.589	1.963	0.	1.242

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	166	0	0	5673	0	0
normalized size	1	1.	0.56	0.	0.	19.1	0.	0.
time (sec)	N/A	0.284	0.338	0.082	0.	2.207	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	43	0	0	5751	0	0
normalized size	1	1.	0.15	0.	0.	19.9	0.	0.
time (sec)	N/A	0.18	0.039	0.079	0.	2.145	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	151	0	0	4625	0	0
normalized size	1	1.	0.57	0.	0.	17.52	0.	0.
time (sec)	N/A	0.212	0.101	0.106	0.	2.082	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	41	0	0	23547	0	0
normalized size	1	1.	0.16	0.	0.	89.19	0.	0.
time (sec)	N/A	0.169	0.054	0.105	0.	3.482	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	41	0	0	5739	0	0
normalized size	1	1.	0.14	0.	0.	19.86	0.	0.
time (sec)	N/A	0.222	0.063	0.079	0.	2.328	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	43	0	0	0	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.154	0.079	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.041	0.24	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	82	107	0	5846	0	0
normalized size	1	1.	0.61	0.8	0.	43.63	0.	0.
time (sec)	N/A	0.06	0.519	0.04	0.	3.087	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	63	86	0	1743	0	0
normalized size	1	1.	0.61	0.83	0.	16.76	0.	0.
time (sec)	N/A	0.047	0.096	0.045	0.	2.188	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	41	91	0	2554	0	0
normalized size	1	1.	0.39	0.87	0.	24.32	0.	0.
time (sec)	N/A	0.048	0.029	0.041	0.	2.383	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	43	106	0	8204	0	0
normalized size	1	1.	0.3	0.75	0.	58.18	0.	0.
time (sec)	N/A	0.061	0.065	0.024	0.	2.947	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	43	145	117	2689	0	0
normalized size	1	1.	0.58	1.96	1.58	36.34	0.	0.
time (sec)	N/A	0.034	0.061	0.098	1.757	2.272	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	119	46	992	0	0
normalized size	1	1.	0.82	2.38	0.92	19.84	0.	0.
time (sec)	N/A	0.024	0.028	0.097	1.617	2.189	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	192	69	363	0	0
normalized size	1	1.	1.26	6.19	2.23	11.71	0.	0.
time (sec)	N/A	0.02	0.026	0.092	1.741	2.134	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	192	43	460	0	0
normalized size	1	1.	1.	6.19	1.39	14.84	0.	0.
time (sec)	N/A	0.02	0.028	0.093	1.711	2.444	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	40	119	50	724	0	0
normalized size	1	1.	0.8	2.38	1.	14.48	0.	0.
time (sec)	N/A	0.024	0.06	0.093	1.75	2.237	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	51	149	120	4035	0	0
normalized size	1	1.	0.64	1.86	1.5	50.44	0.	0.
time (sec)	N/A	0.035	0.088	0.099	1.77	2.372	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.044	0.264	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	43	77	185	9090	0	108
normalized size	1	1.	0.39	0.7	1.68	82.64	0.	0.98
time (sec)	N/A	0.045	0.063	0.036	1.735	2.65	0.	1.183

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	55	46	1046	0	43
normalized size	1	1.	0.82	1.1	0.92	20.92	0.	0.86
time (sec)	N/A	0.023	0.031	0.041	1.82	2.158	0.	1.145

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	40	59	49	1079	0	46
normalized size	1	1.	0.8	1.18	0.98	21.58	0.	0.92
time (sec)	N/A	0.023	0.073	0.037	1.736	2.193	0.	1.169

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	68	84	209	9179	0	138
normalized size	1	1.	0.58	0.71	1.77	77.79	0.	1.17
time (sec)	N/A	0.046	0.233	0.022	1.801	2.677	0.	1.251

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	68	0	0	7973	0	0
normalized size	1	1.	0.19	0.	0.	22.59	0.	0.
time (sec)	N/A	0.196	0.139	0.099	0.	2.938	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	166	0	0	1783	0	0
normalized size	1	1.	0.57	0.	0.	6.13	0.	0.
time (sec)	N/A	0.215	0.409	0.094	0.	2.264	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	43	0	0	855	0	0
normalized size	1	1.	0.15	0.	0.	2.96	0.	0.
time (sec)	N/A	0.176	0.036	0.099	0.	2.267	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	41	0	0	9634	0	0
normalized size	1	1.	0.14	0.	0.	33.34	0.	0.
time (sec)	N/A	0.218	0.053	0.093	0.	2.911	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	43	0	0	3221	0	0
normalized size	1	1.	0.15	0.	0.	11.07	0.	0.
time (sec)	N/A	0.185	0.051	0.094	0.	2.487	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	43	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	0.042	0.101	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.048	2.588	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.075	0.341	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.04	0.186	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.047	0.195	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.066	0.168	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.07	0.112	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.042	0.113	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.039	0.114	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.039	0.112	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.04	0.142	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.066	0.112	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	94	31	189	1481	48	55
normalized size	1	1.	2.29	0.76	4.61	36.12	1.17	1.34
time (sec)	N/A	0.039	0.238	0.007	1.061	2.049	2.589	1.183

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	84	25	128	915	37	47
normalized size	1	1.	2.71	0.81	4.13	29.52	1.19	1.52
time (sec)	N/A	0.031	0.18	0.004	1.101	2.069	1.884	1.195

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	61	19	74	479	31	39
normalized size	1	1.	2.65	0.83	3.22	20.83	1.35	1.7
time (sec)	N/A	0.021	0.149	0.002	1.04	2.039	1.18	1.17

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	26	189	22	28
normalized size	1	1.	1.	1.	2.	14.54	1.69	2.15
time (sec)	N/A	0.013	0.004	0.003	1.13	1.957	0.746	1.187

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	14	88	27	14
normalized size	1	1.	1.12	1.5	0.88	5.5	1.69	0.88
time (sec)	N/A	0.009	0.031	0.016	1.014	2.012	0.577	1.147

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	32	22	173	88	24
normalized size	1	1.	1.15	1.23	0.85	6.65	3.38	0.92
time (sec)	N/A	0.017	0.058	0.017	1.029	2.395	0.99	1.156

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	44	40	30	278	182	32
normalized size	1	1.	1.22	1.11	0.83	7.72	5.06	0.89
time (sec)	N/A	0.027	0.087	0.017	1.05	2.269	1.472	1.148

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	53	48	38	394	299	41
normalized size	1	1.	1.15	1.04	0.83	8.57	6.5	0.89
time (sec)	N/A	0.036	0.121	0.017	1.179	2.377	2.555	1.152

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	62	56	46	540	444	49
normalized size	1	1.	1.11	1.	0.82	9.64	7.93	0.88
time (sec)	N/A	0.046	0.138	0.024	1.038	2.272	3.234	1.142

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	101	43	0	1469	0	216
normalized size	1	1.	1.77	0.75	0.	25.77	0.	3.79
time (sec)	N/A	0.042	0.259	0.016	0.	2.418	0.	1.17

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	92	35	0	869	0	151
normalized size	1	1.	2.04	0.78	0.	19.31	0.	3.36
time (sec)	N/A	0.032	0.153	0.013	0.	2.39	0.	1.174

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	69	27	0	450	0	85
normalized size	1	1.	2.09	0.82	0.	13.64	0.	2.58
time (sec)	N/A	0.022	0.106	0.013	0.	2.487	0.	1.164

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	45	17	0	181	0	50
normalized size	1	1.	2.14	0.81	0.	8.62	0.	2.38
time (sec)	N/A	0.012	0.074	0.036	0.	2.316	0.	1.177

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	51	27	0	300	0	89
normalized size	1	1.	1.59	0.84	0.	9.38	0.	2.78
time (sec)	N/A	0.023	0.292	0.036	0.	2.361	0.	1.154

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	35	0	578	0	177
normalized size	1	1.	1.76	0.71	0.	11.8	0.	3.61
time (sec)	N/A	0.031	0.308	0.011	0.	2.367	0.	1.191

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	94	43	0	919	0	242
normalized size	1	1.	1.54	0.7	0.	15.07	0.	3.97
time (sec)	N/A	0.042	0.758	0.013	0.	2.456	0.	1.223

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	141	322	470	6413	325	308
normalized size	1	1.	0.99	2.27	3.31	45.16	2.29	2.17
time (sec)	N/A	0.209	0.799	0.006	1.081	2.763	22.938	1.155

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	109	246	296	3318	233	211
normalized size	1	1.	1.08	2.44	2.93	32.85	2.31	2.09
time (sec)	N/A	0.123	0.905	0.004	1.114	2.535	10.181	1.137

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	86	173	184	1563	175	140
normalized size	1	1.	1.25	2.51	2.67	22.65	2.54	2.03
time (sec)	N/A	0.064	0.402	0.004	1.111	2.644	4.431	1.143

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	65	116	66	535	104	84
normalized size	1	1.	1.71	3.05	1.74	14.08	2.74	2.21
time (sec)	N/A	0.024	0.128	0.005	1.013	2.854	1.866	1.124

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	64	76	70	149	236	85
normalized size	1	1.	1.28	1.52	1.4	2.98	4.72	1.7
time (sec)	N/A	0.054	0.079	0.019	1.122	2.886	3.149	1.143

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	100	101	167	980	0	185
normalized size	1	1.	1.18	1.19	1.96	11.53	0.	2.18
time (sec)	N/A	0.095	1.42	0.025	1.165	2.676	0.	1.155

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	134	166	435	3182	0	285
normalized size	1	1.	1.04	1.29	3.37	24.67	0.	2.21
time (sec)	N/A	0.18	3.432	0.031	1.184	3.204	0.	1.18

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	214	230	705	8128	0	424
normalized size	1	1.	1.27	1.36	4.17	48.09	0.	2.51
time (sec)	N/A	0.264	6.208	0.034	1.345	3.468	0.	1.216

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	46	38	130	42	41
normalized size	1	1.	1.71	1.48	1.23	4.19	1.35	1.32
time (sec)	N/A	0.044	0.039	0.017	1.101	2.554	1.362	1.179

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	46	39	126	42	38
normalized size	1	1.	1.71	1.48	1.26	4.06	1.35	1.23
time (sec)	N/A	0.042	0.037	0.016	1.058	2.498	1.183	1.192

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	128	63	0	5682	0	0
normalized size	1	1.	1.73	0.85	0.	76.78	0.	0.
time (sec)	N/A	0.07	3.216	0.053	0.	3.212	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	62	0	5844	0	0
normalized size	1	1.	0.99	0.84	0.	78.97	0.	0.
time (sec)	N/A	0.068	0.142	0.046	0.	3.265	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	110	49	321	0	57
normalized size	1	1.	0.7	1.83	0.82	5.35	0.	0.95
time (sec)	N/A	0.063	0.103	0.033	1.069	2.543	0.	1.199

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	36	80	45	221	0	42
normalized size	1	1.	1.24	2.76	1.55	7.62	0.	1.45
time (sec)	N/A	0.047	0.076	0.03	1.059	2.402	0.	1.154

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	30	70	30	176	0	41
normalized size	1	1.	0.79	1.84	0.79	4.63	0.	1.08
time (sec)	N/A	0.052	0.052	0.03	1.083	2.454	0.	1.128

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	40	23	99	0	26
normalized size	1	1.	1.11	2.11	1.21	5.21	0.	1.37
time (sec)	N/A	0.036	0.048	0.027	1.102	2.478	0.	1.125

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	7	11	8	32	0	8
normalized size	1	1.	0.7	1.1	0.8	3.2	0.	0.8
time (sec)	N/A	0.022	0.003	0.001	1.063	2.515	0.	1.153

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	58	0	16
normalized size	1	1.	1.	1.14	1.29	8.29	0.	2.29
time (sec)	N/A	0.036	0.003	0.017	1.051	2.592	0.	1.158

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	14	23	42	306	0	35
normalized size	1	1.	1.75	2.88	5.25	38.25	0.	4.38
time (sec)	N/A	0.04	0.037	0.019	1.088	2.592	0.	1.144

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	32	55	181	0	14
normalized size	1	1.	1.	2.91	5.	16.45	0.	1.27
time (sec)	N/A	0.034	0.025	0.023	1.102	2.372	0.	1.151

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	354	224	2882	0	309
normalized size	1	1.	1.01	2.28	1.45	18.59	0.	1.99
time (sec)	N/A	0.24	0.266	0.049	1.108	2.847	0.	1.175

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	171	197	0	4251	0	220
normalized size	1	1.	1.28	1.47	0.	31.72	0.	1.64
time (sec)	N/A	0.237	0.788	0.044	0.	2.964	0.	1.135

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	75	175	112	818	0	154
normalized size	1	1.	0.82	1.9	1.22	8.89	0.	1.67
time (sec)	N/A	0.142	0.152	0.039	1.177	2.695	0.	1.173

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	93	0	1103	0	97
normalized size	1	1.	1.1	1.27	0.	15.11	0.	1.33
time (sec)	N/A	0.107	0.436	0.032	0.	2.66	0.	1.154

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	39	0	420	0	47
normalized size	1	1.	1.21	1.03	0.	11.05	0.	1.24
time (sec)	N/A	0.036	0.031	0.017	0.	2.652	0.	1.18

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	16	127	0	62
normalized size	1	1.	1.67	1.08	1.33	10.58	0.	5.17
time (sec)	N/A	0.044	0.05	0.019	1.175	2.702	0.	1.19

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	65	115	0	1230	0	115
normalized size	1	1.	1.14	2.02	0.	21.58	0.	2.02
time (sec)	N/A	0.109	0.105	0.028	0.	2.926	0.	1.149

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	50	116	149	1087	0	143
normalized size	1	1.	1.25	2.9	3.72	27.18	0.	3.58
time (sec)	N/A	0.067	0.137	0.031	1.033	2.723	0.	1.176

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	118	49	319	0	57
normalized size	1	1.	0.7	1.97	0.82	5.32	0.	0.95
time (sec)	N/A	0.067	0.088	0.032	0.998	2.506	0.	1.177

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	34	82	36	200	0	34
normalized size	1	1.	1.36	3.28	1.44	8.	0.	1.36
time (sec)	N/A	0.177	0.058	0.028	1.031	2.536	0.	1.187

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	24	78	30	176	0	41
normalized size	1	1.	0.63	2.05	0.79	4.63	0.	1.08
time (sec)	N/A	0.059	0.047	0.029	1.03	2.536	0.	1.113

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	42	15	90	0	15
normalized size	1	1.	1.12	2.47	0.88	5.29	0.	0.88
time (sec)	N/A	0.113	0.017	0.024	1.015	2.486	0.	1.13

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	16	19	16	101	0	14
normalized size	1	1.	1.6	1.9	1.6	10.1	0.	1.4
time (sec)	N/A	0.114	0.023	0.023	1.534	2.58	0.	1.163

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	9	36	24	269	0	36
normalized size	1	1.	0.6	2.4	1.6	17.93	0.	2.4
time (sec)	N/A	0.041	0.03	0.028	1.564	2.698	0.	1.188

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	45	45	508	0	34
normalized size	1	1.	1.	2.25	2.25	25.4	0.	1.7
time (sec)	N/A	0.167	0.039	0.029	1.523	2.507	0.	1.149

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	38	101	286	0	24
normalized size	1	1.	1.	2.24	5.94	16.82	0.	1.41
time (sec)	N/A	0.045	0.05	0.029	1.035	2.523	0.	1.144

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	160	0	0	813	0	201
normalized size	1	1.	7.62	0.	0.	38.71	0.	9.57
time (sec)	N/A	0.046	5.006	0.177	0.	2.552	0.	1.162

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	144	319	208	2747	0	292
normalized size	1	1.	0.98	2.17	1.41	18.69	0.	1.99
time (sec)	N/A	0.339	0.522	0.047	1.199	2.856	0.	1.16

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	167	200	0	4208	0	221
normalized size	1	1.	1.24	1.48	0.	31.17	0.	1.64
time (sec)	N/A	0.251	1.403	0.041	0.	3.019	0.	1.164

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	73	146	108	826	0	140
normalized size	1	1.	0.86	1.72	1.27	9.72	0.	1.65
time (sec)	N/A	0.164	0.229	0.041	1.156	2.651	0.	1.149

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	1106	0	96
normalized size	1	1.	1.1	1.28	0.	15.36	0.	1.33
time (sec)	N/A	0.112	0.301	0.033	0.	2.871	0.	1.13

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	54	0	564	0	65
normalized size	1	1.	1.2	1.08	0.	11.28	0.	1.3
time (sec)	N/A	0.138	0.113	0.033	0.	2.863	0.	1.145

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	59	62	362	0	103
normalized size	1	1.	0.93	2.03	2.14	12.48	0.	3.55
time (sec)	N/A	0.055	0.081	0.04	1.766	2.617	0.	1.147

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	85	187	0	2423	0	138
normalized size	1	1.	1.02	2.25	0.	29.19	0.	1.66
time (sec)	N/A	0.237	0.182	0.047	0.	3.13	0.	1.154

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	68	257	180	2187	0	271
normalized size	1	1.	0.86	3.25	2.28	27.68	0.	3.43
time (sec)	N/A	0.1	0.258	0.053	1.705	2.768	0.	1.148

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	38	41	57	169	0	35
normalized size	1	1.	1.23	1.32	1.84	5.45	0.	1.13
time (sec)	N/A	0.103	0.051	0.052	1.708	2.796	0.	1.134

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	96	74	1897	0	63
normalized size	1	1.	0.93	2.23	1.72	44.12	0.	1.47
time (sec)	N/A	0.115	0.096	0.043	1.701	2.654	0.	1.11

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	80	58	1168	0	53
normalized size	1	1.	0.89	2.16	1.57	31.57	0.	1.43
time (sec)	N/A	0.099	0.053	0.037	1.54	2.648	0.	1.117

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	65	39	632	0	47
normalized size	1	1.	0.93	2.24	1.34	21.79	0.	1.62
time (sec)	N/A	0.074	0.046	0.035	1.675	2.697	0.	1.145

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	47	23	259	0	23
normalized size	1	1.	1.21	2.47	1.21	13.63	0.	1.21
time (sec)	N/A	0.042	0.028	0.029	1.672	2.959	0.	1.158

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	14	88	27	14
normalized size	1	1.	1.12	1.5	0.88	5.5	1.69	0.88
time (sec)	N/A	0.008	0.025	0.	1.126	2.769	0.614	1.14

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	14	88	27	14
normalized size	1	1.	1.12	1.5	0.88	5.5	1.69	0.88
time (sec)	N/A	0.022	0.015	0.018	1.136	2.805	0.597	1.112

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	24	32	259	92	24
normalized size	1	1.	1.21	1.26	1.68	13.63	4.84	1.26
time (sec)	N/A	0.038	0.029	0.017	1.109	3.092	0.906	1.124

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	28	51	632	162	49
normalized size	1	1.	0.87	0.9	1.65	20.39	5.23	1.58
time (sec)	N/A	0.055	0.041	0.017	1.093	3.02	1.381	1.133

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	73	1168	197	54
normalized size	1	1.	0.89	0.86	1.97	31.57	5.32	1.46
time (sec)	N/A	0.068	0.058	0.02	1.112	3.043	1.859	1.114

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	90	35	0	869	0	182
normalized size	1	1.	2.	0.78	0.	19.31	0.	4.04
time (sec)	N/A	0.052	0.136	0.014	0.	3.052	0.	1.185

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	53	26	0	455	0	96
normalized size	1	1.	1.66	0.81	0.	14.22	0.	3.
time (sec)	N/A	0.038	0.119	0.036	0.	2.891	0.	1.166

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	97	25	0	300	0	119
normalized size	1	1.	3.23	0.83	0.	10.	0.	3.97
time (sec)	N/A	0.039	0.195	0.039	0.	2.99	0.	1.162

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	84	35	0	578	0	144
normalized size	1	1.	1.71	0.71	0.	11.8	0.	2.94
time (sec)	N/A	0.051	0.332	0.013	0.	2.957	0.	1.184

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	70	35	0	1449	0	266
normalized size	1	1.	1.56	0.78	0.	32.2	0.	5.91
time (sec)	N/A	0.063	0.254	0.019	0.	2.953	0.	1.2

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	61	26	0	817	0	180
normalized size	1	1.	1.79	0.76	0.	24.03	0.	5.29
time (sec)	N/A	0.048	0.169	0.039	0.	3.169	0.	1.181

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	81	35	0	637	0	119
normalized size	1	1.	1.93	0.83	0.	15.17	0.	2.83
time (sec)	N/A	0.059	0.359	0.041	0.	3.119	0.	1.206

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	35	0	578	0	182
normalized size	1	1.	1.76	0.71	0.	11.8	0.	3.71
time (sec)	N/A	0.082	0.349	0.017	0.	2.929	0.	1.209

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	105	283	197	3087	0	190
normalized size	1	1.	1.08	2.92	2.03	31.82	0.	1.96
time (sec)	N/A	0.525	0.328	0.056	1.752	3.182	0.	1.131

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	88	167	127	1539	0	131
normalized size	1	1.	1.16	2.2	1.67	20.25	0.	1.72
time (sec)	N/A	0.326	0.303	0.053	1.733	3.245	0.	1.176

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	64	110	90	660	0	100
normalized size	1	1.	1.07	1.83	1.5	11.	0.	1.67
time (sec)	N/A	0.193	0.132	0.046	1.785	2.986	0.	1.134

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	88	68	185	0	77
normalized size	1	1.	0.9	1.73	1.33	3.63	0.	1.51
time (sec)	N/A	0.082	0.076	0.039	1.742	2.817	0.	1.199

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	50	108	148	58
normalized size	1	1.	0.74	1.41	1.28	2.77	3.79	1.49
time (sec)	N/A	0.046	0.053	0.013	1.269	2.539	1.227	1.13

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	49	109	134	58
normalized size	1	1.	0.74	1.41	1.26	2.79	3.44	1.49
time (sec)	N/A	0.058	0.045	0.017	1.238	2.573	1.342	1.127

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	60	85	186	372	80
normalized size	1	1.	0.78	0.95	1.35	2.95	5.9	1.27
time (sec)	N/A	0.092	0.076	0.019	1.276	2.803	2.94	1.148

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	67	111	660	639	103
normalized size	1	1.	1.	1.05	1.73	10.31	9.98	1.61
time (sec)	N/A	0.13	0.12	0.022	1.171	2.83	5.45	1.115

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	88	76	161	1538	882	135
normalized size	1	1.	1.16	1.	2.12	20.24	11.61	1.78
time (sec)	N/A	0.221	0.201	0.021	1.195	2.93	7.993	1.179

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	108	96	228	3089	1013	193
normalized size	1	1.	1.15	1.02	2.43	32.86	10.78	2.05
time (sec)	N/A	0.392	0.287	0.023	1.264	3.096	13.074	1.176

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	73	92	491	0	228
normalized size	1	1.	0.91	1.35	1.7	9.09	0.	4.22
time (sec)	N/A	0.085	0.159	0.112	1.952	2.684	0.	1.174

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	30	0	64	24	49	72	0	32
normalized size	1	0.	2.13	0.8	1.63	2.4	0.	1.07
time (sec)	N/A	0.029	0.024	0.026	1.215	2.535	0.	1.143

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	B	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	64	83	65	171	0	73
normalized size	1	0.	1.42	1.84	1.44	3.8	0.	1.62
time (sec)	N/A	0.021	0.232	0.048	1.757	2.685	0.	1.141

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	23	0	26	37	49	81	0	50
normalized size	1	0.	1.13	1.61	2.13	3.52	0.	2.17
time (sec)	N/A	0.016	0.182	0.024	1.044	2.594	0.	1.117

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	B	A	B	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	40	0	58	71	61	159	0	69
normalized size	1	0.	1.45	1.78	1.52	3.98	0.	1.72
time (sec)	N/A	0.007	0.179	0.045	1.609	2.582	0.	1.121

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	21	26	14	47	27	28
normalized size	1	1.	1.75	2.17	1.17	3.92	2.25	2.33
time (sec)	N/A	0.014	0.027	0.003	1.051	2.535	1.689	1.095

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	B	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	41	0	62	93	63	149	0	70
normalized size	1	0.	1.51	2.27	1.54	3.63	0.	1.71
time (sec)	N/A	0.022	0.162	0.042	1.733	2.656	0.	1.158

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	B	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	21	0	27	35	41	92	0	45
normalized size	1	0.	1.29	1.67	1.95	4.38	0.	2.14
time (sec)	N/A	0.021	0.156	0.026	1.03	2.584	0.	1.108

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	47	0	86	41	72	140	0	54
normalized size	1	0.	1.83	0.87	1.53	2.98	0.	1.15
time (sec)	N/A	0.066	0.103	0.018	1.051	2.468	0.	1.122

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	A	B	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	68	0	154	100	89	266	0	97
normalized size	1	0.	2.26	1.47	1.31	3.91	0.	1.43
time (sec)	N/A	0.048	2.899	0.046	1.624	2.596	0.	1.144

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	A	B	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	41	0	163	54	72	169	0	73
normalized size	1	0.	3.98	1.32	1.76	4.12	0.	1.78
time (sec)	N/A	0.03	2.933	0.02	1.061	2.572	0.	1.111

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	A	B	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	60	0	153	86	81	246	0	89
normalized size	1	0.	2.55	1.43	1.35	4.1	0.	1.48
time (sec)	N/A	0.01	1.951	0.043	1.57	2.641	0.	1.119

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	28	35	26	68	32	28
normalized size	1	1.	2.	2.5	1.86	4.86	2.29	2.
time (sec)	N/A	0.024	0.054	0.003	1.032	2.521	17.227	1.116

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	A	A	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	86	0	153	101	93	236	0	104
normalized size	1	0.	1.78	1.17	1.08	2.74	0.	1.21
time (sec)	N/A	0.045	3.127	0.033	1.564	2.604	0.	1.121

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	A	A	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	60	0	155	55	68	178	0	77
normalized size	1	0.	2.58	0.92	1.13	2.97	0.	1.28
time (sec)	N/A	0.046	2.863	0.023	1.078	2.581	0.	1.152

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	59	0	46	0	0	0	0	0
normalized size	1	0.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.093	0.038	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	79	0	165	0	0	0	0	0
normalized size	1	0.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.369	0.027	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	177	0	215	0	0	0	0	0
normalized size	1	0.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.825	0.037	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	79	0	259	0	0	0	0	0
normalized size	1	0.	3.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	1.889	0.106	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	99	0	126	0	0	0	0	0
normalized size	1	0.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	2.977	0.03	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	52	0	83	0	0	0	0	0
normalized size	1	0.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.391	0.036	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	108	0	125	0	0	0	0	0
normalized size	1	0.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.527	0.027	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	162	0	142	0	0	0	0	0
normalized size	1	0.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.561	0.027	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	194	0	223	0	0	0	0	0
normalized size	1	0.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.907	0.024	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0
normalized size	1	0.	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	1.548	0.05	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0
normalized size	1	0.	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	1.734	0.052	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0
normalized size	1	0.	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	1.814	0.05	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	58	0	198	0	0	0	0	0
normalized size	1	0.	3.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	7.015	1.095	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	62	0	207	0	0	0	0	0
normalized size	1	0.	3.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	4.896	1.046	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	54	0	193	0	0	0	0	0
normalized size	1	0.	3.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	6.939	1.	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	52	0	198	0	0	0	0	0
normalized size	1	0.	3.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	8.329	0.915	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	40	56	32	205	0	99
normalized size	1	1.	1.6	2.24	1.28	8.2	0.	3.96
time (sec)	N/A	0.021	0.057	0.004	1.158	2.914	0.	1.241

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	58	0	197	0	0	0	0	0
normalized size	1	0.	3.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	3.83	1.026	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	55	0	191	0	0	0	0	0
normalized size	1	0.	3.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	3.678	1.026	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	132	0	155	0	0	0	0	0
normalized size	1	0.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	6.606	0.365	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	136	0	165	0	0	0	0	0
normalized size	1	0.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	4.532	0.113	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	130	0	151	0	0	0	0	0
normalized size	1	0.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	6.481	0.11	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	126	0	160	0	0	0	0	0
normalized size	1	0.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	7.686	0.099	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	49	80	50	197	0	50
normalized size	1	1.	1.75	2.86	1.79	7.04	0.	1.79
time (sec)	N/A	0.029	0.104	0.005	1.3	2.512	0.	1.33

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	134	0	158	0	0	0	0	0
normalized size	1	0.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	3.563	0.12	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	135	0	156	0	0	0	0	0
normalized size	1	0.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	3.554	0.127	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	52	67	446	1841	0	170
normalized size	1	1.	1.21	1.56	10.37	42.81	0.	3.95
time (sec)	N/A	0.04	0.207	0.009	1.372	2.673	0.	1.429

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	86	674	541	0	90
normalized size	1	1.	0.98	1.91	14.98	12.02	0.	2.
time (sec)	N/A	0.037	0.114	0.003	1.511	2.508	0.	1.434

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	88	1154	5060	0	216
normalized size	1	1.	1.02	1.33	17.48	76.67	0.	3.27
time (sec)	N/A	0.057	0.28	0.003	1.639	2.687	0.	1.344

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	87	0	158	0	0	0	0	0
normalized size	1	0.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	13.231	1.436	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	168	0	312	0	0	0	0	0
normalized size	1	0.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	14.437	0.221	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	306	0	600	0	0	0	0	0
normalized size	1	0.	1.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	16.586	0.24	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	115	0	387	0	0	0	0	0
normalized size	1	0.	3.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	3.629	0.081	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	135	0	174	0	0	0	0	0
normalized size	1	0.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	5.072	0.069	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	64	93	0	2090	0	0
normalized size	1	1.	0.88	1.27	0.	28.63	0.	0.
time (sec)	N/A	0.051	0.286	0.028	0.	2.436	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	57	92	0	1100	0	0
normalized size	1	1.	0.81	1.31	0.	15.71	0.	0.
time (sec)	N/A	0.05	0.146	0.015	0.	2.512	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	72	0	999	0	0
normalized size	1	1.	1.	1.5	0.	20.81	0.	0.
time (sec)	N/A	0.039	0.075	0.013	0.	2.441	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	0	1000	0	0
normalized size	1	1.	1.	0.94	0.	21.28	0.	0.
time (sec)	N/A	0.04	0.111	0.015	0.	2.348	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	44	93	0	2087	0	0
normalized size	1	1.	0.62	1.31	0.	29.39	0.	0.
time (sec)	N/A	0.052	0.144	0.015	0.	2.842	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	92	0	3644	0	0
normalized size	1	1.	0.64	1.28	0.	50.61	0.	0.
time (sec)	N/A	0.051	0.2	0.013	0.	2.889	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	266	149	0	24521	0	0
normalized size	1	1.	1.97	1.1	0.	181.64	0.	0.
time (sec)	N/A	0.353	9.015	0.104	0.	18.619	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	199	90	0	18436	0	0
normalized size	1	1.	1.9	0.86	0.	175.58	0.	0.
time (sec)	N/A	0.217	25.823	0.063	0.	14.573	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	141	52	0	4811	0	0
normalized size	1	1.	2.43	0.9	0.	82.95	0.	0.
time (sec)	N/A	0.117	18.196	0.065	0.	9.668	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	203	0	0	18421	0	0
normalized size	1	1.	1.92	0.	0.	173.78	0.	0.
time (sec)	N/A	0.24	7.83	0.304	0.	13.848	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	2574	0	0	24602	0	0
normalized size	1	1.	14.07	0.	0.	134.44	0.	0.
time (sec)	N/A	0.332	20.687	0.274	0.	19.634	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	304	559	0	22316	0	0
normalized size	1	1.	2.3	4.23	0.	169.06	0.	0.
time (sec)	N/A	0.231	7.991	0.061	0.	24.07	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	164	320	225	4176	0	244
normalized size	1	1.	0.51	1.	0.71	13.09	0.	0.76
time (sec)	N/A	0.912	10.266	0.255	1.609	2.677	0.	1.432

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	334	298	151	1574	0	209
normalized size	1	1.	1.7	1.51	0.77	7.99	0.	1.06
time (sec)	N/A	0.284	3.77	0.199	1.826	2.422	0.	1.27

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	213	76	196	0	127
normalized size	1	1.	0.61	2.57	0.92	2.36	0.	1.53
time (sec)	N/A	0.144	0.058	0.238	1.705	2.373	0.	1.192

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	218	47	132	0	81
normalized size	1	1.	0.61	2.63	0.57	1.59	0.	0.98
time (sec)	N/A	0.194	0.118	0.231	1.72	2.26	0.	1.183

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	104	301	122	1175	0	176
normalized size	1	1.	0.54	1.56	0.63	6.09	0.	0.91
time (sec)	N/A	0.864	0.297	0.209	1.666	2.542	0.	1.218

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	133	324	196	3186	0	250
normalized size	1	1.	0.43	1.04	0.63	10.24	0.	0.8
time (sec)	N/A	1.743	0.461	0.22	1.762	2.616	0.	1.253

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	929	0	0
normalized size	1	1.	0.79	0.75	0.	5.92	0.	0.
time (sec)	N/A	0.375	0.242	0.025	0.	2.75	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	102	0	474	0	0
normalized size	1	1.	0.77	0.89	0.	4.12	0.	0.
time (sec)	N/A	0.262	0.167	0.02	0.	2.526	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	58	0	464	0	0
normalized size	1	1.	0.77	0.75	0.	6.03	0.	0.
time (sec)	N/A	0.144	0.128	0.016	0.	2.438	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	2.68	0.165	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	929	0	0
normalized size	1	1.	0.79	0.75	0.	5.92	0.	0.
time (sec)	N/A	0.373	0.244	0.023	0.	2.968	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	102	0	474	0	0
normalized size	1	1.	0.77	0.89	0.	4.12	0.	0.
time (sec)	N/A	0.251	0.166	0.021	0.	2.78	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	463	0	0
normalized size	1	1.	0.81	0.75	0.	6.01	0.	0.
time (sec)	N/A	0.14	0.106	0.022	0.	2.698	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	5.484	0.067	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [109] had the largest ratio of [0.7778]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.	12	0.5
2	A	6	6	1.	12	0.5
3	A	6	6	1.	12	0.5
4	A	5	5	1.	12	0.417
5	A	5	5	1.	12	0.417
6	A	6	6	1.	12	0.5
7	A	6	6	1.	12	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	7	6	1.	12	0.5
9	A	13	9	1.	12	0.75
10	A	12	8	1.	12	0.667
11	A	9	9	1.	12	0.75
12	A	9	9	1.	12	0.75
13	A	12	8	1.	12	0.667
14	A	13	9	1.	12	0.75
15	A	2	2	1.	8	0.25
16	A	2	2	1.	10	0.2
17	A	3	3	1.	12	0.25
18	A	3	3	1.	14	0.214
19	A	2	2	1.	14	0.143
20	A	2	2	1.	14	0.143
21	A	3	3	1.	14	0.214
22	A	14	10	1.	14	0.714
23	A	14	10	1.	14	0.714
24	A	13	9	1.	14	0.643
25	A	13	9	1.	14	0.643
26	A	14	10	1.	14	0.714
27	A	14	10	1.	14	0.714
28	A	3	3	1.	12	0.25
29	A	8	7	1.	14	0.5
30	A	7	7	1.	14	0.5
31	A	7	7	1.	14	0.5
32	A	8	7	1.	14	0.5
33	A	4	3	1.	14	0.214
34	A	3	3	1.	14	0.214
35	A	2	2	1.	14	0.143
36	A	2	2	1.	14	0.143
37	A	3	3	1.	14	0.214
38	A	4	3	1.	14	0.214
39	A	3	3	1.	12	0.25
40	A	5	3	1.	14	0.214
41	A	3	3	1.	14	0.214
42	A	3	3	1.	14	0.214
43	A	5	3	1.	14	0.214
44	A	16	10	1.	14	0.714
45	A	14	10	1.	14	0.714
46	A	14	10	1.	14	0.714
47	A	14	10	1.	14	0.714
48	A	14	10	1.	14	0.714
49	A	16	10	1.	14	0.714
50	A	3	3	1.	12	0.25
51	A	3	3	1.	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	3	3	1.	14	0.214
53	A	3	3	1.	14	0.214
54	A	3	3	1.	14	0.214
55	A	3	3	1.	14	0.214
56	A	3	3	1.	14	0.214
57	A	3	3	1.	14	0.214
58	A	3	3	1.	14	0.214
59	A	3	3	1.	14	0.214
60	A	3	3	1.	14	0.214
61	A	5	3	1.	6	0.5
62	A	4	3	1.	6	0.5
63	A	3	3	1.	6	0.5
64	A	2	2	1.	6	0.333
65	A	2	2	1.	6	0.333
66	A	3	2	1.	6	0.333
67	A	4	2	1.	6	0.333
68	A	5	2	1.	6	0.333
69	A	6	2	1.	6	0.333
70	A	5	3	1.	8	0.375
71	A	4	3	1.	8	0.375
72	A	3	3	1.	8	0.375
73	A	2	2	1.	8	0.25
74	A	3	3	1.	8	0.375
75	A	4	3	1.	8	0.375
76	A	5	3	1.	8	0.375
77	A	5	4	1.	12	0.333
78	A	4	4	1.	12	0.333
79	A	3	3	1.	12	0.25
80	A	2	2	1.	12	0.167
81	A	2	2	1.	12	0.167
82	A	3	3	1.	12	0.25
83	A	4	4	1.	12	0.333
84	A	5	4	1.	12	0.333
85	A	2	2	1.	12	0.167
86	A	2	2	1.	12	0.167
87	A	5	4	1.	14	0.286
88	A	5	4	1.	14	0.286
89	A	4	3	1.	11	0.273
90	A	3	2	1.	11	0.182
91	A	4	3	1.	11	0.273
92	A	2	2	1.	9	0.222
93	A	1	1	1.	9	0.111
94	A	2	2	1.	11	0.182
95	A	2	2	1.	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
96	A	2	1	1.	11	0.091
97	A	5	4	1.	13	0.308
98	A	9	6	1.	13	0.462
99	A	4	3	1.	13	0.231
100	A	5	5	1.	11	0.454
101	A	2	2	1.	11	0.182
102	A	2	2	1.	13	0.154
103	A	5	5	1.	13	0.385
104	A	3	2	1.	13	0.154
105	A	5	4	1.	11	0.364
106	A	9	7	1.	11	0.636
107	A	5	4	1.	11	0.364
108	A	8	6	1.	9	0.667
109	A	8	7	1.	9	0.778
110	A	3	2	1.	11	0.182
111	A	8	7	1.	11	0.636
112	A	4	3	1.	11	0.273
113	A	4	4	1.	13	0.308
114	A	5	3	1.	13	0.231
115	A	10	9	1.	13	0.692
116	A	4	3	1.	13	0.231
117	A	6	6	1.	11	0.546
118	A	6	5	1.	11	0.454
119	A	3	2	1.	13	0.154
120	A	9	7	1.	13	0.538
121	A	3	2	1.	13	0.154
122	A	6	5	1.	13	0.385
123	A	6	4	1.	11	0.364
124	A	5	4	1.	11	0.364
125	A	4	4	1.	11	0.364
126	A	4	4	1.	9	0.444
127	A	2	2	1.	6	0.333
128	A	2	2	1.	9	0.222
129	A	3	2	1.	11	0.182
130	A	3	3	1.	11	0.273
131	A	4	4	1.	11	0.364
132	A	4	4	1.	11	0.364
133	A	3	3	1.	11	0.273
134	A	3	3	1.	11	0.273
135	A	4	4	1.	11	0.364
136	A	4	4	1.	13	0.308
137	A	3	3	1.	13	0.231
138	A	4	4	1.	13	0.308
139	A	4	4	1.	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	6	6	1.	13	0.462
141	A	5	5	1.	13	0.385
142	A	4	4	1.	13	0.308
143	A	3	3	1.	11	0.273
144	A	2	2	1.	8	0.25
145	A	2	2	1.	11	0.182
146	A	4	4	1.	13	0.308
147	A	5	5	1.	13	0.385
148	A	6	6	1.	13	0.462
149	A	7	7	1.	13	0.538
150	A	3	3	1.	14	0.214
151	F	0	0	N/A	0	N/A
152	F	0	0	N/A	0	N/A
153	F	0	0	N/A	0	N/A
154	F	0	0	N/A	0	N/A
155	A	2	1	1.	11	0.091
156	F	0	0	N/A	0	N/A
157	F	0	0	N/A	0	N/A
158	F	0	0	N/A	0	N/A
159	F	0	0	N/A	0	N/A
160	F	0	0	N/A	0	N/A
161	F	0	0	N/A	0	N/A
162	A	3	2	1.	13	0.154
163	F	0	0	N/A	0	N/A
164	F	0	0	N/A	0	N/A
165	F	0	0	N/A	0	N/A
166	F	0	0	N/A	0	N/A
167	F	0	0	N/A	0	N/A
168	F	0	0	N/A	0	N/A
169	F	0	0	N/A	0	N/A
170	F	0	0	N/A	0	N/A
171	F	0	0	N/A	0	N/A
172	F	0	0	N/A	0	N/A
173	F	0	0	N/A	0	N/A
174	F	0	0	N/A	0	N/A
175	F	0	0	N/A	0	N/A
176	F	0	0	N/A	0	N/A
177	F	0	0	N/A	0	N/A
178	F	0	0	N/A	0	N/A
179	F	0	0	N/A	0	N/A
180	F	0	0	N/A	0	N/A
181	A	2	1	1.	17	0.059
182	F	0	0	N/A	0	N/A
183	F	0	0	N/A	0	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	F	0	0	N/A	0	N/A
185	F	0	0	N/A	0	N/A
186	F	0	0	N/A	0	N/A
187	F	0	0	N/A	0	N/A
188	A	3	2	1.	19	0.105
189	F	0	0	N/A	0	N/A
190	F	0	0	N/A	0	N/A
191	A	3	2	1.	17	0.118
192	A	4	2	1.	17	0.118
193	A	4	2	1.	17	0.118
194	F	0	0	N/A	0	N/A
195	F	0	0	N/A	0	N/A
196	F	0	0	N/A	0	N/A
197	F	0	0	N/A	0	N/A
198	F	0	0	N/A	0	N/A
199	A	7	6	1.	19	0.316
200	A	7	6	1.	19	0.316
201	A	6	5	1.	19	0.263
202	A	6	5	1.	19	0.263
203	A	7	6	1.	19	0.316
204	A	7	6	1.	19	0.316
205	A	8	7	1.	23	0.304
206	A	7	6	1.	23	0.261
207	A	4	4	1.	21	0.19
208	A	8	5	1.	21	0.238
209	A	11	6	1.	23	0.261
210	A	8	7	1.	21	0.333
211	A	9	7	1.	25	0.28
212	A	8	7	1.	25	0.28
213	A	4	4	1.	25	0.16
214	A	4	4	1.	25	0.16
215	A	8	7	1.	25	0.28
216	A	9	7	1.	25	0.28
217	A	19	5	1.	9	0.556
218	A	13	5	1.	9	0.556
219	A	9	4	1.	7	0.571
220	A	0	0	0.	0	0.
221	A	19	5	1.	9	0.556
222	A	13	5	1.	9	0.556
223	A	9	4	1.	7	0.571
224	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int (b\coth(c+dx))^{7/2}\,dx$

Optimal. Leaf size=97

$$-\frac{2b^3\sqrt{b\coth(c+dx)}}{d}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

```
[Out] (b^(7/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(7/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b^3*Sqrt[b*Coth[c + d*x]])/d - (2*b*(b*Cot
h[c + d*x])^(5/2))/(5*d)
```

Rubi [A] time = 0.07041, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3476, 329, 212, 206, 203}

$$-\frac{2b^3\sqrt{b\coth(c+dx)}}{d}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(7/2),x]
```

```
[Out] (b^(7/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(7/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b^3*Sqrt[b*Coth[c + d*x]])/d - (2*b*(b*Cot
h[c + d*x])^(5/2))/(5*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (b\coth(c+dx))^{7/2}\,dx &= -\frac{2b(b\coth(c+dx))^{5/2}}{5d}+b^2\int (b\coth(c+dx))^{3/2}\,dx\\
&= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}+b^4\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx\\
&= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}-\frac{b^5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx,x,b\coth(c+dx)\right)}{d}\\
&= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}-\frac{\left(2b^5\right)\mathrm{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\coth(c+dx)}\right.}{d}\\
&= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}+\frac{b^4\,\mathrm{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{d}+\frac{b}{}\\
&= \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+}{5d}
\end{aligned}$$

Mathematica [A] time = 0.232407, size = 83, normalized size = 0.86

$$\frac{b^3\sqrt{b\coth(c+dx)}\left(-2\coth^{\frac{5}{2}}(c+dx)-10\sqrt{\coth(c+dx)}+5\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)+5\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{5d\sqrt{\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(7/2),x]
```

```
[Out] (b^3*Sqrt[b*Coth[c + d*x]]*(5*ArcTan[Sqrt[Coth[c + d*x]]] + 5*ArcTanh[Sqrt[
Coth[c + d*x]]] - 10*Sqrt[Coth[c + d*x]] - 2*Coth[c + d*x]^(5/2)))/(5*d*Sqr
t[Coth[c + d*x]])
```

Maple [A] time = 0.026, size = 80, normalized size = 0.8

$$\frac{1}{d}b^{\frac{7}{2}}\arctan\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)+\frac{1}{d}b^{\frac{7}{2}}\mathrm{Artanh}\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)-\frac{2b}{5d}(b\coth(dx+c))^{\frac{5}{2}}-2\frac{b^3\sqrt{b\coth(dx+}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(7/2),x)
```

```
[Out] b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2/5*b*(b*coth(d*x+c))^(5/2)/d-2*b^3*(b*coth(d*x+c))^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\coth(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(7/2), x)
```

Fricas [B] time = 2.50709, size = 4135, normalized size = 42.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3
*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 -
 b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 5*(b^3*
cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4
 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x +
 c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b)
*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x
 + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4
+ 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*c
osh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(3*b^3*cosh(d*x + c)^
4 + 12*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 - 4*b^3*co
sh(d*x + c)^2 + 3*b^3 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*sinh(d*x + c)^2 +
 4*(3*b^3*cosh(d*x + c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh
(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
 c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 -
d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)
```

```
+ d), 1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 +
 b^3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)
^2 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b))
+ 5*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d
*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*s
inh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))
*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b
*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*
sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + si
nh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 +
 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d
*x + c)/sinh(d*x + c)) - b) - 16*(3*b^3*cosh(d*x + c)^4 + 12*b^3*cosh(d*x +
 c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 - 4*b^3*cosh(d*x + c)^2 + 3*b^3
 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*sinh(d*x + c)^2 + 4*(3*b^3*cosh(d*x +
c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh(d*x + c)/sinh(d*x +
c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x +
c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 +
4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\coth(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c))^(7/2), x)
```

3.2 $\int (b\coth(c+dx))^{5/2}\,dx$

Optimal. Leaf size=78

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (b^(5/2)*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*(b*Coth[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.0490394, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3476, 329, 298, 203, 206}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(5/2),x]
```


```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (b^(5/2)*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*(b*Coth[c + d*x])^(3/2))/(3*d)
```


Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (b\coth(c+dx))^{5/2}\,dx &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} + b^2\int \sqrt{b\coth(c+dx)}\,dx\\ &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} - \frac{b^3\,\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}\\ &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\coth(c+dx))^{3/2}}{3d}\end{aligned}$$

Mathematica [A] time = 0.212902, size = 68, normalized size = 0.87

$$-\frac{(b\coth(c+dx))^{5/2}\left(2\coth^{\frac{3}{2}}(c+dx)+3\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)-3\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{3d\coth^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(5/2),x]
```

```
[Out] -((b*Coth[c + d*x])^(5/2)*(3*ArcTan[Sqrt[Coth[c + d*x]]] - 3*ArcTanh[Sqrt[C
oth[c + d*x]]] + 2*Coth[c + d*x]^(3/2)))/(3*d*Coth[c + d*x]^(5/2))
```

Maple [A] time = 0.01, size = 63, normalized size = 0.8

$$-\frac{1}{d}b^{\frac{5}{2}}\arctan\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)+\frac{1}{d}b^{\frac{5}{2}}\text{Artanh}\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)-\frac{2b}{3d}(b\coth(dx+c))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(5/2),x)
```

```
[Out] -b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*coth(d*x+c))^(3/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\coth(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(5/2), x)
```

Fricas [B] time = 2.38775, size = 2654, normalized size = 34.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*si
nh(d*x + c)^2 - b^2)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/
(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 +
b)) - 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 - b^2)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*s
inh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c
)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4))
 + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*
x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2
*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d), -1/12*(6*(b^2*cosh
(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2
)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 3*(b^2*c
osh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 -
b^2)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) +
12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 +
2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)
^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*co
sh(d*x + c)/sinh(d*x + c)) - b) + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x +
 c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*
x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
+ c)^2 - d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth (dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c))^(5/2), x)
```

3.3 $\int (b\coth(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

```
[Out] (b^(3/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Coth[c + d*x]])/d
```

Rubi [A] time = 0.0500011, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3476, 329, 212, 206, 203}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Coth[c + d*x]])/d
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (b\coth(c+dx))^{3/2}\,dx &= -\frac{2b\sqrt{b\coth(c+dx)}}{d} + b^2\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx \\
&= -\frac{2b\sqrt{b\coth(c+dx)}}{d} - \frac{b^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d} \\
&= -\frac{2b\sqrt{b\coth(c+dx)}}{d} - \frac{(2b^3)\,\mathrm{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} \\
&= -\frac{2b\sqrt{b\coth(c+dx)}}{d} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b}\right.}{d} \\
&= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\coth(c+dx)}}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0947051, size = 61, normalized size = 0.81

$$
\frac{(b\coth(c+dx))^{3/2}\left(-2\sqrt{\coth(c+dx)}+\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{d\coth^{\frac{3}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(3/2),x]
```

```
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth[
c + d*x]])*(b*Coth[c + d*x])^(3/2))/(d*Coth[c + d*x]^(3/2))
```

Maple [A] time = 0.011, size = 62, normalized size = 0.8

$$
\frac{1}{d}b^{\frac{3}{2}}\arctan\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)+\frac{1}{d}b^{\frac{3}{2}}\mathrm{Artanh}\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)-2\frac{b\sqrt{b\coth(dx+c)}}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(3/2),x)
```

```
[Out] b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2*b*(b*coth(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int (b\coth(dx+c))^{\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(3/2), x)
```

Fricas [B] time = 2.27195, size = 1754, normalized size = 23.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*b*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt
(-b)*b*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*b*sqrt(b*cosh(d*
x + c)/sinh(d*x + c)))/d, 1/4*(2*b^(3/2)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c
)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + b)) + b^(3/2)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^
3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*
sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))
*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*b*sqrt(b*cosh(d*x + c
)/sinh(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\coth(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\coth(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c))^(3/2), x)
```

3.4 $\int \sqrt{b \coth(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

```
[Out] -((Sqrt[b]*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (Sqrt[b]*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d
```

Rubi [A] time = 0.0356433, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3476, 329, 298, 203, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] -((Sqrt[b]*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (Sqrt[b]*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\coth(c+dx)}\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}\\ &= -\frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= \frac{b\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0419602, size = 51, normalized size = 0.88

$$\frac{\sqrt{b\coth(c+dx)}\left(\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)-\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{d\sqrt{\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ((-ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[b*Coth[
c + d*x]])/(d*Sqrt[Coth[c + d*x]])
```

Maple [A] time = 0.023, size = 47, normalized size = 0.8

$$-\frac{1}{d}\arctan\left(\sqrt{b\coth\left(dx+c\right)}\frac{1}{\sqrt{b}}\right)\sqrt{b}+\frac{1}{d}\text{Artanh}\left(\sqrt{b\coth\left(dx+c\right)}\frac{1}{\sqrt{b}}\right)\sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(1/2),x)
```

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))*b^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\coth\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)), x)
```

Fricas [B] time = 2.21435, size = 1636, normalized size = 28.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(b)*arct
an(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(b)*log(2*b*cosh(d
*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d
*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(co
sh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh
(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 -
 cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))
- b))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)), x)
```

3.5 $\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(Sqrt[b]*d)
```

Rubi [A] time = 0.0308044, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3476, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(Sqrt[b]*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}\\ &= -\frac{(2b)\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}+\frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}\end{aligned}$$

Mathematica [A] time = 0.0340077, size = 49, normalized size = 0.86

$$\frac{\sqrt{\coth(c+dx)}\left(\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{d\sqrt{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[Coth[c +
 d*x]])/(d*Sqrt[b*Coth[c + d*x]])
```

Maple [A] time = 0.033, size = 46, normalized size = 0.8

$$\frac{1}{d}\arctan\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)\frac{1}{\sqrt{b}}+\frac{1}{d}\text{Artanh}\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(1/2),x)
```

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/d/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)), x)
```

Fricas [B] time = 2.43283, size = 1646, normalized size = 28.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/(b*d), 1/4*(2*sqrt(b)*a
rctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(b)*log(2*b*cos
h(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*
(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*c
osh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^
3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c
)) - b))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)), x)
```

3.6 $\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(3/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(3/2)*d) - 2/(b*d*Sqrt[b*Coth[c + d*x]])
```

Rubi [A] time = 0.0512037, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3474, 3476, 329, 298, 203, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-3/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(3/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(3/2)*d) - 2/(b*d*Sqrt[b*Coth[c + d*x]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx &= -\frac{2}{bd\sqrt{b\coth(c+dx)}}+\frac{\int \sqrt{b\coth(c+dx)}\,dx}{b^2}\\
&= -\frac{2}{bd\sqrt{b\coth(c+dx)}}-\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx,x,b\coth(c+dx)\right)}{bd}\\
&= -\frac{2}{bd\sqrt{b\coth(c+dx)}}-\frac{2\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{bd}\\
&= -\frac{2}{bd\sqrt{b\coth(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{bd}-\frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx,x,\sqrt{b\,c}\right.}{bd}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\coth(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.0737085, size = 36, normalized size = 0.46

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\coth^2(c+dx)\right)}{bd\sqrt{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Coth[c + d*x]^2])/(b*d*Sqrt[b*Coth[c +
d*x]])
```

Maple [A] time = 0.013, size = 65, normalized size = 0.8

$$-\frac{1}{d}\arctan\left(\sqrt{b\coth\left(dx+c\right)}\frac{1}{\sqrt{b}}\right)b^{-\frac{3}{2}}+\frac{1}{d}\text{Artanh}\left(\sqrt{b\coth\left(dx+c\right)}\frac{1}{\sqrt{b}}\right)b^{-\frac{3}{2}}-2\,\frac{1}{bd\sqrt{b\coth\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(3/2),x)
```

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*coth(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\coth(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-3/2), x)
```

Fricas [B] time = 2.71562, size = 2576, normalized size = 33.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1
)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x
 + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*
d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c
)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*s
inh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2
- cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sq
rt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^
2*d*sinh(d*x + c)^2 + b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\coth(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-3/2), x)
```

Giac [B] time = 1.33478, size = 277, normalized size = 3.55

$$\frac{\frac{(\pi+\log(|b|)+8)\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}{\sqrt{b}d}+\frac{4\,\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}d\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}-\frac{2\,\log\left(\left|-\sqrt{b}e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)}{\sqrt{b}d\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}-\frac{16}{\left(\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}+\cdots\right.}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + log(abs(b)) + 8)*sgn(e^(2*d*x + 2*c) - 1)/(sqrt(b)*d) + 4*arctan
(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt(b))/(sqrt(b)
*d*sgn(e^(2*d*x + 2*c) - 1)) - 2*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*
e^(4*d*x + 4*c) - b)))/(sqrt(b)*d*sgn(e^(2*d*x + 2*c) - 1)) - 16/((sqrt(b)*
e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))*d*sgn(e^(2*d*x + 2
*c) - 1)))/b
```

3.7 $\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Coth[c + d*x])^(3/2))
```

Rubi [A] time = 0.0495235, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3474, 3476, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-5/2),x]
```

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Coth[c + d*x])^(3/2))
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx &= -\frac{2}{3bd(b\coth(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx}{b^2}\\ &= -\frac{2}{3bd(b\coth(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx,x,b\coth(c+dx)\right)}{bd}\\ &= -\frac{2}{3bd(b\coth(c+dx))^{3/2}}-\frac{2\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{bd}\\ &= -\frac{2}{3bd(b\coth(c+dx))^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{b^2d}+\frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx,x,\right.}{b^2d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0654699, size = 38, normalized size = 0.48

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\coth^2(c+dx)\right)}{3bd(b\coth(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Coth[c + d*x]^2])/(3*b*d*(b*Coth[c + d*
x])^(3/2))
```

Maple [A] time = 0.011, size = 64, normalized size = 0.8

$$\frac{1}{d}\arctan\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{5}{2}}+\frac{1}{d}\text{Artanh}\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{5}{2}}-\frac{2}{3\,bd}(b\coth(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(5/2),x)
```

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*coth(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-5/2), x)
```

Fricas [B] time = 2.85404, size = 3933, normalized size = 49.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*
cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 + b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sin
h(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2
+ 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1
)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*s
inh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x +
c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x +
c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d
*x + c))*sinh(d*x + c) + 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^3*d*cos
h(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4 + 2*b^3*d*cosh(d*x + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 + b^3*d)*s
inh(d*x + c)^2 + 4*(b^3*d*cosh(d*x + c)^3 + b^3*d*cosh(d*x + c))*sinh(d*x +
 c)), 1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4
*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(b
)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*
x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
 + c) + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x +
 c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c
)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x +
 c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*
x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqr
t(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x +
c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c
) + 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*
d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4 + 2*b^3*d*cosh(d*x
+ c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 + b^3*d)*sinh(d*x + c)^2 + 4*(b
```

```
^3*d*cosh(d*x + c)^3 + b^3*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.8 $\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(7/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(7/2)*d) - 2/(5*b*d*(b*Coth[c + d*x])^(5/2)) - 2/(b^3
*d*Sqrt[b*Coth[c + d*x]])
```

Rubi [A] time = 0.0705722, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3474, 3476, 329, 298, 203, 206}

$$-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-7/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(7/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(7/2)*d) - 2/(5*b*d*(b*Coth[c + d*x])^(5/2)) - 2/(b^3
*d*Sqrt[b*Coth[c + d*x]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx &= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}+\frac{\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx}{b^2}\\
&= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}+\frac{\int \sqrt{b\coth(c+dx)}\,dx}{b^4}\\
&= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}-\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx,x,b\coth(c+dx)\right)}{b^3d}\\
&= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}-\frac{2\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{b^3d}\\
&= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{b^3d}-\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c}
\end{aligned}$$

Mathematica [C] time = 0.0964058, size = 38, normalized size = 0.38

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};\coth^2(c+dx)\right)}{5bd(b\coth(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, Coth[c + d*x]^2])/(5*b*d*(b*Coth[c + d
*x])^(5/2))
```

Maple [A] time = 0.013, size = 83, normalized size = 0.8

$$-\frac{1}{d}\arctan\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{7}{2}}+\frac{1}{d}\text{Artanh}\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{7}{2}}-\frac{2}{5\,bd}\left(b\coth(dx+c)\right)^{-\frac{5}{2}}-2\,\frac{1}{b^3d\sqrt{b\coth}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(7/2),x)
```

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(7/2)/d-2/5/b/d/(b*coth(d*x+c))^(5/2)-2/b^3/d/(b*coth(d*x+c)
```

```
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\coth(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-7/2), x)
```

Fricas [B] time = 2.59864, size = 5797, normalized size = 57.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x +
c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5
*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4
+ 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x
+ c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arc
tan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqr
t(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 5*(cosh(d*x + c)^6 + 6*cosh
(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sin
h(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x +
c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*
x + c))*sinh(d*x + c) + 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x
+ c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh
(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4)) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sin
h(d*x + c)^6 + (45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + cosh(d*x + c)^4 +
 4*(15*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)
^4 + 6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d
*x + c)^5 + 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt(b*co
sh(d*x + c)/sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*
sinh(d*x + c)^5 + b^4*d*sinh(d*x + c)^6 + 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d
*cosh(d*x + c)^2 + b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c
)^4 + 4*(5*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 3*(5*b^4*d*cosh(d*x + c)^4 + 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c
)^2 + 6*(b^4*d*cosh(d*x + c)^5 + 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x +
 c))*sinh(d*x + c)), -1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x
+ c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*co
sh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3
*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x +
```

```
c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x +
c) + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 5*
(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5
*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x +
 c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*
x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*
cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*log(2*b*cosh(d*
x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cos
h(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(
d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 -
cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) -
 b) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sinh(d*x
 + c)^6 + (45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + cosh(d*x + c)^4 + 4*(1
5*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)^4 +
6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d*x +
c)^5 + 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt(b*cosh(d*
x + c)/sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*sinh(
d*x + c)^5 + b^4*d*sinh(d*x + c)^6 + 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d*cosh
(d*x + c)^2 + b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^4 +
 4*(5*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5
*b^4*d*cosh(d*x + c)^4 + 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 +
 6*(b^4*d*cosh(d*x + c)^5 + 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x + c))*
sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.9 $\int (b\coth(c+dx))^{4/3}\,dx$

Optimal. Leaf size=236

$$-\frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{4/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}-$$

```
[Out] -(Sqrt[3]*b^(4/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]]
)/(2*d) + (Sqrt[3]*b^(4/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))
/Sqrt[3]])/(2*d) + (b^(4/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (
3*b*(b*Coth[c + d*x])^(1/3))/d - (b^(4/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c +
 d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d) + (b^(4/3)*Log[b^(2/3) + b^(
1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d)
```

Rubi [A] time = 0.281901, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{4/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(4/3),x]
```

```
[Out] -(Sqrt[3]*b^(4/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]]
)/(2*d) + (Sqrt[3]*b^(4/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))
/Sqrt[3]])/(2*d) + (b^(4/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (
3*b*(b*Coth[c + d*x])^(1/3))/d - (b^(4/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c +
 d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d) + (b^(4/3)*Log[b^(2/3) + b^(
1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (b\coth(c+dx))^{4/3}\,dx = -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} + b^2\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{(3b^3)\,\text{Subst}\left(\int \frac{1}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} + \frac{b^{4/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} + \frac{b^{4/3}\,\text{Subst}\left(\int \frac{\ldots}{b^{2/3}+\ldots}\right.}{\ldots}$$

$$= \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c\ldots}\right.}{4d}$$

$$= \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\,\text{co}\ldots\right.}{4d}$$

$$= -\frac{\sqrt{3}b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{\sqrt{3}b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d}$$

Mathematica [C] time = 0.0259547, size = 36, normalized size = 0.15

$$\frac{3b\sqrt[3]{b\coth(c+dx)}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(4/3),x]
```

```
[Out] (3*b*(b*Coth[c + d*x])^(1/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c +
d*x]^2]))/d
```

Maple [A] time = 0.02, size = 209, normalized size = 0.9

$$-3\,\frac{b\sqrt[3]{b\coth(dx+c)}}{d} + \frac{1}{2d}b^{\frac{4}{3}}\ln\left(\sqrt[3]{b\coth(dx+c)}+\sqrt[3]{b}\right) - \frac{1}{4d}b^{\frac{4}{3}}\ln\left(b^{\frac{2}{3}}-\sqrt[3]{b}\sqrt[3]{b\coth(dx+c)}+(b\coth(dx+c))^{\frac{2}{3}}\right) + \frac{\sqrt{\ldots}}{2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(4/3),x)
```

```
[Out] -3*b*(b*coth(d*x+c))^(1/3)/d+1/2*b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)+b^(1/3)
)-1/4*b^(4/3)*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3
))/d+1/2*b^(4/3)/d*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1
/3)-1))-1/2*b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)-b^(1/3))+1/4*b^(4/3)*ln(b^(2
/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d+1/2*b^(4/3)*arct
an(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth (dx + c))^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(4/3), x)
```

Fricas [A] time = 1.96127, size = 865, normalized size = 3.67

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\, b \arctan\left(\frac{\sqrt{3}b+2\sqrt{3}(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) - 2\sqrt{3}b^{\frac{4}{3}} \arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) + (-b)^{\frac{1}{3}}\, b \log\left((-b)^{\frac{2}{3}} - (\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*b*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(4/3)*arctan(-1/3*(s
qrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-
b)^(1/3)*b*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(4/3)*log(b^(2/3) - b^(1/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/
3)) - 2*(-b)^(1/3)*b*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
) - 2*b^(4/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*b*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.10 $\int (b\coth(c+dx))^{2/3}\,dx$

Optimal. Leaf size=218

$$-\frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{2/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+$$

```
[Out] (Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])
/(2*d) - (Sqrt[3]*b^(2/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/
Sqrt[3]])/(2*d) + (b^(2/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (b
^(2/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2
/3)])/(4*d) + (b^(2/3)*Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*C
oth[c + d*x])^(2/3)])/(4*d)
```

Rubi [A] time = 0.293867, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{2/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])
/(2*d) - (Sqrt[3]*b^(2/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/
Sqrt[3]])/(2*d) + (b^(2/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (b
^(2/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2
/3)])/(4*d) + (b^(2/3)*Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*C
oth[c + d*x])^(2/3)])/(4*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
```

```
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (b\coth(c+dx))^{2/3}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{x^{2/3}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(3b)\,\text{Subst}\left(\int \frac{x^4}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\text{cot}}\right.}{d}$$

$$= \frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d} + \frac{b^{2/3}\,\text{Subs}}{}$$

$$= \frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{b}{}$$

$$= \frac{\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d}$$

Mathematica [A] time = 0.177229, size = 149, normalized size = 0.68

$$\frac{(b\coth(c+dx))^{2/3}\left(-\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)+\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)+2\sqrt{3}\tan^{-}\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(2/3),x]
```

```
[Out] ((b*Coth[c + d*x])^(2/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt
[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[Co
th[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] + L
og[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2/3
))
```

Maple [A] time = 0.016, size = 193, normalized size = 0.9

$$\frac{1}{2d}b^{\frac{2}{3}}\ln\left(\sqrt[3]{b\coth(dx+c)}+\sqrt[3]{b}\right)-\frac{1}{4d}b^{\frac{2}{3}}\ln\left(b^{\frac{2}{3}}-\sqrt[3]{b}\sqrt[3]{b\coth(dx+c)}+(b\coth(dx+c))^{\frac{2}{3}}\right)-\frac{\sqrt{3}}{2d}b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{bc}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(2/3),x)
```

```
[Out] 1/2*b^(2/3)/d*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/4*b^(2/3)*ln(b^(2/3)-b^(1
/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d-1/2*b^(2/3)/d*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1))-1/2*b^(2/3)/d*ln((b*c
oth(d*x+c))^(1/3)-b^(1/3))+1/4*b^(2/3)*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(
1/3)+(b*coth(d*x+c))^(2/3))/d-1/2*b^(2/3)*arctan(1/3*(1+2*(b*coth(d*x+c))^(
1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(b\coth(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(2/3), x)
```

Fricas [A] time = 2.0473, size = 871, normalized size = 4.

$$-\frac{2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+2\sqrt{3}\left(b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}\left(b^2\right)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+\left(-b^2\right)^{\frac{1}{3}}\log\left(b\left(\frac{bc}{si}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b^2)^(1/3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 2*sqrt(3)*(b^2)^(1/3)*arctan(
-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/
3))/b) + (-b^2)^(1/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^
(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + (b^2)^(1/3)
*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(1/3)*log(b*(b*cosh(d*x +
 c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) - 2*(b^2)^(1/3)*log(b*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\coth(c+dx))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(2/3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.11 $\int \sqrt[3]{b\coth(c+dx)}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4d}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{b^{2/3}+2}{}\right.}{2d}$$

```
[Out] -(Sqrt[3]*b^(1/3)*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(
2/3))])/(2*d) - (b^(1/3)*Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)])/(2*d) + (b
^(1/3)*Log[b^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4
/3)])/(4*d)
```

Rubi [A] time = 0.109114, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3476, 329, 275, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4d}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{b^{2/3}+2}{}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(1/3),x]
```

```
[Out] -(Sqrt[3]*b^(1/3)*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(
2/3))])/(2*d) - (b^(1/3)*Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)])/(2*d) + (b
^(1/3)*Log[b^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4
/3)])/(4*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt[3]{b\coth(c+dx)}\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{\sqrt[3]{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}\\
&= -\frac{(3b)\,\text{Subst}\left(\int \frac{x^3}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}\\
&= -\frac{(3b)\,\text{Subst}\left(\int \frac{x}{-b^2+x^3}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}\\
&= -\frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{-b^{2/3}+x}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-b^{2/3}+x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{4d}-\\
&= -\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+d\right.}{4d}\\
&= -\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2d}-\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(b^{4/3}+b^{2/3}(\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.040844, size = 38, normalized size = 0.29

$$
\frac{3(b\coth(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\coth^2(c+dx)\right)}{4bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(1/3),x]
```

```
[Out] (3*(b*Coth[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 1, 5/3, Coth[c + d*x]^2])
/(4*b*d)
```

Maple [A] time = 0.015, size = 115, normalized size = 0.9

$$-\frac{b}{2d}\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\sqrt[3]{b^2}\right)\frac{1}{\sqrt[3]{b^2}}+\frac{b}{4d}\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\sqrt[3]{b^2}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{b^2}}-\frac{b\sqrt{3}}{2d}\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(1/3),x)
```

```
[Out] -1/2*b/d/(b^2)^(1/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))+1/4*b/d/(b^2)^(1
/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3))
-1/2*b/d*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+
c))^(2/3)+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(b\coth(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(1/3), x)
```

Fricas [B] time = 2.00133, size = 828, normalized size = 6.27

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(-b)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}}{3b}\right)-2(-b)^{\frac{1}{3}}\log\left(-(-b)^{\frac{2}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)+(-b)^{\frac{1}{3}}\log\left(\frac{(\cosh(dx+c)^2+2\cosh}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b)^(1/3)*(b
*cosh(d*x + c)/sinh(d*x + c))^(2/3))/b) - 2*(-b)^(1/3)*log(-(-b)^(2/3) + (b
*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(1/3)*log(((cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x
+ c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c)/sinh(d*x +
c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
```

```
^2 - 1)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \coth(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(1/3), x)
```

Giac [B] time = 1.68703, size = 293, normalized size = 2.22

$$-\frac{b\left(\frac{2\sqrt{3}|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3|b|^{\frac{2}{3}}}\right)}{b^2}-\frac{|b|^{\frac{4}{3}}\log\left(\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2dx+2c)}+b\right)\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{1}{3}}}{e^{(2dx+2c)}-1}\right)}{b^2}+\frac{2|b|^{\frac{4}{3}}\log\left(\left|\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right|\right)}{b^2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] -1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) +
b)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 - abs(b)^
(4/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2/3
) + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(2*d
*x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 + 2*abs(b)^(4/3)*log(abs((
(b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/b^2)/
d
```

3.12 $\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx$

Optimal. Leaf size=132

$$-\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}+\frac{\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}+\frac{\sqrt{3}\tan^{-1}\left(\frac{b^{2/3}+2(b\coth(c+dx}{\sqrt{3}b^{2/3}}\right.}{2\sqrt[3]{b}d}$$

```
[Out] (Sqrt[3]*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))])/(
2*b^(1/3)*d) - Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)]/(2*b^(1/3)*d) + Log[b
^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4/3)]/(4*b^(1
/3)*d)
```

Rubi [A] time = 0.10391, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3476, 329, 275, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}+\frac{\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}+\frac{\sqrt{3}\tan^{-1}\left(\frac{b^{2/3}+2(b\coth(c+dx}{\sqrt{3}b^{2/3}}\right.}{2\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))])/(
2*b^(1/3)*d) - Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)]/(2*b^(1/3)*d) + Log[b
^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4/3)]/(4*b^(1
/3)*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\left(-b^2+x^2\right)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(3b)\,\text{Subst}\left(\int \frac{x}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= -\frac{(3b)\,\text{Subst}\left(\int \frac{1}{-b^2+x^3}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-b^{2/3}+x}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} - \frac{\text{Subst}\left(\int \frac{-2b^{2/3}-x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2}\right.}{2\sqrt[3]{b}d}$$

$$= -\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{4\sqrt[3]{b}d} + \frac{\left(3\sqrt[3]{b}\right.}{}$$

$$= -\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{4/3}+b^{2/3}(b\coth(c+\right.}{4}$$

Mathematica [A] time = 0.120959, size = 98, normalized size = 0.74

$$\frac{\sqrt[3]{\coth(c+dx)}\left(-2\log\left(1-\coth^{\frac{2}{3}}(c+dx)\right)+\log\left(\coth^{\frac{4}{3}}(c+dx)+\coth^{\frac{2}{3}}(c+dx)+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\coth^{\frac{2}{3}}(c+dx)+1}{\sqrt{3}}\right)\right)}{4d\sqrt[3]{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-1/3),x]
```

```
[Out] (Coth[c + d*x]^(1/3)*(2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(2/3))/Sqrt[3]]
 - 2*Log[1 - Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(2/3) + Coth[c +
d*x]^(4/3)]))/(4*d*(b*Coth[c + d*x])^(1/3))
```

Maple [A] time = 0.012, size = 115, normalized size = 0.9

$$-\frac{b}{2d}\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\sqrt[3]{b^2}\right)\left(b^2\right)^{-\frac{2}{3}}+\frac{b}{4d}\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\sqrt[3]{b^2}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)\left(b^2\right)^{-\frac{2}{3}}+\frac{b\sqrt{3}}{2d}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(1/3),x)
```

```
[Out] -1/2*b/d/(b^2)^(2/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))+1/4*b/d/(b^2)^(2
/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3))
+1/2*b/d/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+
c))^(2/3)+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(b\coth(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-1/3), x)
```

Fricas [B] time = 2.23023, size = 4398, normalized size = 33.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt((-b)^(1/3)/b)*log((3*b*cosh(d*x + c)^4 + 12*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(9*b*
cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)
```

```
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c
)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)
 + 1)*(-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - sqrt(3)*((cosh(d*x
 + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x
 + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cos
h(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(
2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - 2
*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)
^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
- b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)/b) + 4*(3*b*cos
h(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 3*b)/(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(-b)^(2/3)*log(-(-b)^(2/
3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(2/3)*log(((cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*co
sh(d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*s
inh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c)/sinh
(d*x + c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2 - 1)))/(b*d), 1/4*(2*sqrt(3)*b*sqrt(-(-b)^(1/3)/b)*arctan((2*sqrt
(3)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x
 + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(2/3)*sqrt(-(-b)^(1/3)/b) + sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*co
sh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*
x + c))*sinh(d*x + c) + b)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) - 4*sqrt(3)*(b*co
sh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b))/(9*b*cosh(d*x + c
)^4 + 36*b*cosh(d*x + c)*sinh(d*x + c)^3 + 9*b*sinh(d*x + c)^4 + 14*b*cosh(
d*x + c)^2 + 2*(27*b*cosh(d*x + c)^2 + 7*b)*sinh(d*x + c)^2 + 4*(9*b*cosh(d
*x + c)^3 + 7*b*cosh(d*x + c))*sinh(d*x + c) + 9*b)) - 2*(-b)^(2/3)*log(-(-
b)^(2/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(2/3)*log(((cosh(d
*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)
*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c
)/sinh(d*x + c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b \coth(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-1/3), x)
```

Giac [B] time = 1.64217, size = 292, normalized size = 2.21

$$\frac{b\left(\frac{2\sqrt{3}|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3|b|^{\frac{2}{3}}}\right)}{b^2}+\frac{|b|^{\frac{2}{3}}\log\left(\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2dx+2c)}+b\right)\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{1}{3}}}{e^{(2dx+2c)}-1}\right)}{b^2}-\frac{2|b|^{\frac{2}{3}}\log\left(\left|\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right|\right)}{b^2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) + b
)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 + abs(b)^(
2/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2/3)
 + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(2*d*
x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 - 2*abs(b)^(2/3)*log(abs(((
b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/b^2)/d
```

3.13 $\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}-\frac{\sqrt{3}}{}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(
2/3)*d) + (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]
])/(2*b^(2/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(2/3)*d) - L
og[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*
b^(2/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*
x])^(2/3)]/(4*b^(2/3)*d)
```

Rubi [A] time = 0.23531, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}-\frac{\sqrt{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-2/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(
2/3)*d) + (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]
])/(2*b^(2/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(2/3)*d) - L
og[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*
b^(2/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*
x])^(2/3)]/(4*b^(2/3)*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
```

```
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(3b)\,\text{Subst}\left(\int \frac{1}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{2/3}d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+2x}{b^{2/3}+\sqrt[3]{b}x+x}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\log\left(b^{2/3}+\right.}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\log\left(b^{2/3}\right.}{}$$

Mathematica [C] time = 0.0286256, size = 36, normalized size = 0.17

$$\frac{3\sqrt[3]{b\coth(c+dx)}\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-2/3),x]
```

```
[Out] (3*(b*Coth[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])
/(b*d)
```

Maple [A] time = 0.017, size = 193, normalized size = 0.9

$$\frac{1}{2d}\ln\left(\sqrt[3]{b\coth(dx+c)}+\sqrt[3]{b}\right)b^{-\frac{2}{3}}-\frac{1}{4d}\ln\left(b^{\frac{2}{3}}-\sqrt[3]{b}\sqrt[3]{b\coth(dx+c)}+(b\coth(dx+c))^{\frac{2}{3}}\right)b^{-\frac{2}{3}}+\frac{\sqrt{3}}{2d}\arctan\left(\frac{\sqrt{3}}{3}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(2/3),x)
```

```
[Out] 1/2/b^(2/3)/d*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/4*ln(b^(2/3)-b^(1/3)*(b*c
oth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(2/3)/d+1/2/b^(2/3)/d*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1))-1/2/b^(2/3)/d*ln((b*c
oth(d*x+c))^(1/3)-b^(1/3))+1/4*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*
coth(d*x+c))^(2/3))/b^(2/3)/d+1/2*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(
1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(b\coth(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-2/3), x)
```

Fricas [B] time = 2.01017, size = 991, normalized size = 4.55

$$2\sqrt{3}b\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}b\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}-2\sqrt{3}\left(-b^2\right)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3b^2}\right)+2\sqrt{3}\left(b^2\right)^{\frac{1}{6}}b\arctan\left(-\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\left(\left(b^2\right)^{\frac{1}{3}}b-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*b*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sq
rt(-(-b^2)^(1/3)) - 2*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3)*sqrt(-(-b^2)^(1/3)))/b^2) + 2*sqrt(3)*(b^2)^(1/6)*b*arctan(-1/3*sqrt(
3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c
))^(1/3))/b^2) + (-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) -
 (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - (b^
2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2
)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(2/3)*log(b*(b*co
sh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 2*(b^2)^(2/3)*log(b*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-2/3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.14 $\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=238

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\sqrt{3}}{}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(4
/3)*d) - (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]]
)/(2*b^(4/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d) - 3/
(b*d*(b*Coth[c + d*x])^(1/3)) - Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/
3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth
[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d)
```

Rubi [A] time = 0.31572, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\sqrt{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-4/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(4
/3)*d) - (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]]
)/(2*b^(4/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d) - 3/
(b*d*(b*Coth[c + d*x])^(1/3)) - Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/
3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth
[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d)
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} + \frac{\int (b\coth(c+dx))^{2/3}\,dx}{b^2}$$

$$= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{x^{2/3}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{bd}$$

$$= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{3\,\text{Subst}\left(\int \frac{x^4}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{bd}$$

$$= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{4/3}d} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\ldots}{b^{2/3}+\sqrt[3]{b}x\ldots}\right.}{\ldots}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx\ldots}\right.}{4b^{4/3}d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c\ldots\right.}{4b^{4/3}d}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{\ldots}{bd\sqrt[3]{b\ldots}}$$

Mathematica [C] time = 0.0619372, size = 36, normalized size = 0.15

$$-\frac{3\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};\coth^2(c+dx)\right)}{bd\sqrt[3]{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(b*d*(b*Coth[c + d*x]
)^(1/3))
```

Maple [A] time = 0.016, size = 211, normalized size = 0.9

$$-3\frac{1}{bd\sqrt[3]{b\coth(dx+c)}} + \frac{1}{2d}\ln\left(\sqrt[3]{b\coth(dx+c)}+\sqrt[3]{b}\right)b^{-\frac{4}{3}} - \frac{1}{4d}\ln\left(b^{\frac{2}{3}}-\sqrt[3]{b}\sqrt[3]{b\coth(dx+c)}+(b\coth(dx+c))^{\frac{2}{3}}\right)b\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(4/3),x)
```

```
[Out] -3/b/d/(b*coth(d*x+c))^(1/3)+1/2/b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)+b^(1/3)
)-1/4*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/
3)/d-1/2/b^(4/3)/d*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1
/3)-1))-1/2/b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)-b^(1/3))+1/4*ln(b^(2/3)+b^(1
/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/3)/d-1/2*arctan(1/3*(
1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\coth(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-4/3), x)
```

Fricas [B] time = 2.44882, size = 9677, normalized size = 40.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh
(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*cosh(d*x
+ c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(
d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)
+ (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2
- b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)
/b) + b) + sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + b)*sqrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c
)/sinh(d*x + c))^(2/3)*sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x
+ c)*sinh(d*x + c) - b*sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)
) + (sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3) - 3*b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^
(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
+ 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 12*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x
 + c)/sinh(d*x + c))^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*
sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*sqrt(3)*(b*cosh(d*x
 + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-
b)^(1/3)/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - sqrt(3)*(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*s
qrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
```

```
 + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)*
sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) - b*
sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)) + (sqrt(3)*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1
/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-
b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x +
 c)/sinh(d*x + c))^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x +
c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log(
(-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 +
2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) +
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2
/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*s
inh(d*x + c)^2 + b^2*d), 1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*
x + c)^2 + 6*b*cosh(d*x + c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(
d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x +
c)/sinh(d*x + c))^(2/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))*sqrt((-b)^(1/3)/b) + b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3))
- (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b
^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cos
h(d*x + c)/sinh(d*x + c))^(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x +
 c)/sinh(d*x + c))^(1/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x
+ c))^(1/3)) - 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x +
c)/sinh(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 12*(cosh(d*x + c)^2 + 2*cosh(d*
x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))
^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*
d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)/b)*arctan(-
1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cosh(d*x + c)/s
inh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - (cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1
/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))
^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
+ (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*co
sh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/s
inh(d*x + c))^(1/3)) + 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cos
h(d*x + c)/sinh(d*x + c))^(1/3))/b^(1/3))/b^(1/3) + 12*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d
*x + c))^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c
```

```
) + b^2*d*sinh(d*x + c)^2 + b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\coth{(c+dx)})^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-4/3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.15 $\int \coth^n(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\coth^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(a+bx)\right)}{b(n+1)}$$

```
[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a +
b*x]^2])/(b*(1 + n))
```

Rubi [A] time = 0.0223006, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3476, 364}

$$\frac{\coth^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^n,x]
```

```
[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a +
b*x]^2])/(b*(1 + n))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \coth^n(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^n}{-1+x^2}\,dx,x,\coth(a+bx)\right)}{b}\\ &= \frac{\coth^{1+n}(a+bx)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\coth^2(a+bx)\right)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0625768, size = 45, normalized size = 1.05

$$\frac{\coth^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;\coth^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^n,x]
```

```
[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, Coth[
a + b*x]^2])/(b*(1 + n))
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int (\coth (bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^n,x)
```

```
[Out] int(coth(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth (bx + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(coth(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth (bx + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(coth(b*x + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^n (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**n,x)
```

```
[Out] Integral(coth(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(bx+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(coth(b*x + a)^n, x)
```

3.16 $\int (b\coth(c+dx))^n\,dx$

Optimal. Leaf size=48

$$\frac{(b\coth(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(c+dx)\right)}{bd(n+1)}$$

```
[Out] ((b*Coth[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[
c + d*x]^2])/(b*d*(1 + n))
```

Rubi [A] time = 0.0270079, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3476, 364}

$$\frac{(b\coth(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(c+dx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^n,x]
```

```
[Out] ((b*Coth[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[
c + d*x]^2])/(b*d*(1 + n))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (b\coth(c+dx))^n\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{x^n}{-b^2+x^2}\,dx,x,b\coth(c+dx)\right)}{d}\\ &= \frac{(b\coth(c+dx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\coth^2(c+dx)\right)}{bd(1+n)}\end{aligned}$$

Mathematica [A] time = 0.041099, size = 51, normalized size = 1.06

$$\frac{\coth(c+dx)(b\coth(c+dx))^n\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;\coth^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x])^n*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 +
 n)/2, Coth[c + d*x]^2])/(d*(1 + n))
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int (b\coth(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^n,x)
```

```
[Out] int((b*coth(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\coth(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\coth(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\coth(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**n,x)
```

```
[Out] Integral((b*coth(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c))^n, x)
```

3.17 $\int \left(b \coth^2(c+dx)\right)^n dx$

Optimal. Leaf size=57

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(2n+1);\frac{1}{2}(2n+3);\coth^2(c+dx)\right)}{d(2n+1)}$$

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, (1 + 2*n)/2, (3 +
 2*n)/2, Coth[c + d*x]^2])/(d*(1 + 2*n))
```

Rubi [A] time = 0.040331, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3658, 3476, 364}

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(2n+1);\frac{1}{2}(2n+3);\coth^2(c+dx)\right)}{d(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, (1 + 2*n)/2, (3 +
 2*n)/2, Coth[c + d*x]^2])/(d*(1 + 2*n))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\coth^2(c+dx)\right)^n dx = \left(\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\right)\int \coth^{2n}(c+dx)\,dx$$
$$= -\frac{\left(\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{2n}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1, \frac{1}{2}(1+2n); \frac{1}{2}(3+2n); \coth^2(c+dx)\right)}{d(1+2n)}$$

Mathematica [A] time = 0.0478515, size = 47, normalized size = 0.82

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1, n+\frac{1}{2}; n+\frac{3}{2}; \coth^2(c+dx)\right)}{2dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, 1/2 + n, 3/2 + n,
 Coth[c + d*x]^2])/(d + 2*d*n)
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^2\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^n,x)
```

```
[Out] int((b*coth(d*x+c)^2)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth(dx+c)^2\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\coth(dx+c)^2\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^2)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth^2\left(c+dx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^n, x)
```

3.18 $\int \left(b \coth^2(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=61

$$\frac{b \tanh(c+dx)\sqrt{b \coth^2(c+dx)} \log(\sinh(c+dx))}{d} - \frac{b \coth(c+dx)\sqrt{b \coth^2(c+dx)}}{2d}$$

```
[Out] -(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^2])/(2*d) + (b*Sqrt[b*Coth[c + d*x]^
2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0368725, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{b \tanh(c+dx)\sqrt{b \coth^2(c+dx)} \log(\sinh(c+dx))}{d} - \frac{b \coth(c+dx)\sqrt{b \coth^2(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(3/2),x]
```

```
[Out] -(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^2])/(2*d) + (b*Sqrt[b*Coth[c + d*x]^
2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(b \coth^2(c+dx)\right)^{3/2} dx &= \left(b\sqrt{b \coth^2(c+dx)} \tanh(c+dx)\right) \int \coth^3(c+dx)\, dx \\
&= -\frac{b \coth(c+dx)\sqrt{b \coth^2(c+dx)}}{2d} + \left(b\sqrt{b \coth^2(c+dx)} \tanh(c+dx)\right) \int \coth(c+dx)\, dx \\
&= -\frac{b \coth(c+dx)\sqrt{b \coth^2(c+dx)}}{2d} + \frac{b\sqrt{b \coth^2(c+dx)} \log(\sinh(c+dx)) \tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.118465, size = 56, normalized size = 0.92

$$-\frac{\tanh^3(c+dx)\left(b\coth^2(c+dx)\right)^{3/2}\left(\coth^2(c+dx)-2\log(\tanh(c+dx))-2\log(\cosh(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(3/2),x]
```

```
[Out] -((b*Coth[c + d*x]^2)^(3/2)*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] - 2*Log
[Tanh[c + d*x]])*Tanh[c + d*x]^3)/(2*d)
```

Maple [A] time = 0.026, size = 53, normalized size = 0.9

$$-\frac{(\coth(dx+c))^2+\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)}{2\,d\,(\coth(dx+c))^3}\left(b\,(\coth(dx+c))^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(3/2),x)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^2)^(3/2)*(coth(d*x+c)^2+ln(coth(d*x+c)-1)+ln(coth(d*x
+c)+1))/coth(d*x+c)^3
```

Maxima [A] time = 1.60448, size = 131, normalized size = 2.15

$$-\frac{(dx+c)b^{\frac{3}{2}}}{d}-\frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*b^(3/2)/d - b^(3/2)*log(e^(-d*x - c) + 1)/d - b^(3/2)*log(e^(-d*
x - c) - 1)/d - 2*b^(3/2)*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d
*x - 4*c) - 1))
```

Fricas [B] time = 1.98088, size = 2068, normalized size = 33.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d*x*cosh(d*x + c)^4 - (b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x + c)^4 -
4*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*sinh(d*x + c)
^3 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^2 + 2*(3*b*d*x*cosh(d*x + c)^2 - b
*d*x - (3*b*d*x*cosh(d*x + c)^2 - b*d*x + b)*e^(2*d*x + 2*c) + b)*sinh(d*x
+ c)^2 - (b*d*x*cosh(d*x + c)^4 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^2)*e^
```

```
(2*d*x + 2*c) - (b*cosh(d*x + c)^4 - (b*e^(2*d*x + 2*c) - b)*sinh(d*x + c)^
4 - 4*(b*cosh(d*x + c)*e^(2*d*x + 2*c) - b*cosh(d*x + c))*sinh(d*x + c)^3 -
 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - (3*b*cosh(d*x + c)^2 - b)*e
^(2*d*x + 2*c) - b)*sinh(d*x + c)^2 - (b*cosh(d*x + c)^4 - 2*b*cosh(d*x + c
)^2 + b)*e^(2*d*x + 2*c) + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c) - (b*cosh
(d*x + c)^3 - b*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + b)*log(2*si
nh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b*d*x*cosh(d*x + c)^3 - (
b*d*x - b)*cosh(d*x + c) - (b*d*x*cosh(d*x + c)^3 - (b*d*x - b)*cosh(d*x +
c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x
+ 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^4 +
 (d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^4 + 4*(d*cosh(d*x + c)*e^(2*d*x + 2*
c) + d*cosh(d*x + c))*sinh(d*x + c)^3 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d
*x + c)^2 + (3*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d)*sinh(d*x + c)^2
+ (d*cosh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) + 4*(d*cosh
(d*x + c)^3 - d*cosh(d*x + c) + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth^2(c+dx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(3/2), x)
```

Giac [A] time = 1.20253, size = 122, normalized size = 2.

$$-\frac{\left((dx+c)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)-\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)+\frac{2\,e^{(2\,dx+2\,c)}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}\right)b^{\frac{3}{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^
(4*d*x + 4*c) - 1) + 2*e^(2*d*x + 2*c)*sgn(e^(4*d*x + 4*c) - 1)/(e^(2*d*x +
 2*c) - 1)^2)*b^(3/2)/d
```

3.19 $\int \sqrt{b \coth^2(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\log(\sinh(c+dx))}{d}$$

```
[Out] (Sqrt[b*Coth[c + d*x]^2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0196248, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Sqrt[b*Coth[c + d*x]^2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\coth^2(c+dx)}\,dx &= \left(\sqrt{b\coth^2(c+dx)}\tanh(c+dx)\right)\int \coth(c+dx)\,dx\\ &= \frac{\sqrt{b\coth^2(c+dx)}\log(\sinh(c+dx))\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0452803, size = 39, normalized size = 1.26

$$\frac{\tanh(c+dx)\sqrt{b\coth^2(c+dx)}(\log(\tanh(c+dx))+\log(\cosh(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Sqrt[b*Coth[c + d*x]^2]*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c +
 d*x])/d
```

Maple [A] time = 0.036, size = 45, normalized size = 1.5

$$-\frac{\ln\left(\coth\left(dx+c\right)-1\right)+\ln\left(\coth\left(dx+c\right)+1\right)}{2\,d\coth\left(dx+c\right)}\sqrt{b\left(\coth\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^2)^(1/2)*(ln(coth(d*x+c)-1)+ln(coth(d*x+c)+1))/coth(d
*x+c)
```

Maxima [A] time = 1.59022, size = 73, normalized size = 2.35

$$-\frac{(dx+c)\sqrt{b}}{d}-\frac{\sqrt{b}\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\sqrt{b}\log\left(e^{(-dx-c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*sqrt(b)/d - sqrt(b)*log(e^(-d*x - c) + 1)/d - sqrt(b)*log(e^(-d*
x - c) - 1)/d
```

Fricas [B] time = 1.93362, size = 304, normalized size = 9.81

$$-\frac{\left(dxe^{(2\,dx+2\,c)}-dx-\left(e^{(2\,dx+2\,c)}-1\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{de^{(2\,dx+2\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c
) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*e^(2*d*x + 2*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\coth^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**2), x)
```

Giac [A] time = 1.15947, size = 73, normalized size = 2.35

$$-\frac{\left((dx+c)\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)-\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)\right)\sqrt{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^
(4*d*x + 4*c) - 1))*sqrt(b)/d
```

3.20 $\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Rubi [A] time = 0.0204262, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx &= \frac{\coth(c+dx)\int \tanh(c+dx)\,dx}{\sqrt{b\coth^2(c+dx)}}\\ &= \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0594719, size = 31, normalized size = 1.

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Maple [A] time = 0.04, size = 52, normalized size = 1.7

$$-\frac{\coth(dx+c)\left(\ln(\coth(dx+c)+1)-2\ln(\coth(dx+c))+\ln(\coth(dx+c)-1)\right)}{2d}\frac{1}{\sqrt{b(\coth(dx+c))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)+1)-2*ln(coth(d*x+c))+ln(coth(d*x+c)-1))/
(b*coth(d*x+c)^2)^(1/2)
```

Maxima [A] time = 1.62442, size = 46, normalized size = 1.48

$$-\frac{dx+c}{\sqrt{b}d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(d*x + c)/(sqrt(b)*d) - log(e^(-2*d*x - 2*c) + 1)/(sqrt(b)*d)
```

Fricas [B] time = 1.92024, size = 309, normalized size = 9.97

$$-\frac{\left(dxe^{(2\,dx+2\,c)}-dx-\left(e^{(2\,dx+2\,c)}-1\right)\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{bde^{(2\,dx+2\,c)}+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*cosh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c
) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b*d*e^(2*d*x + 2*c) + b*
d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**2), x)
```

Giac [B] time = 1.19993, size = 81, normalized size = 2.61

$$-\frac{\frac{dx+c}{\sqrt{b}\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{\sqrt{b}\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/(
sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)))/d
```

3.21 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(b*d*Sqrt[b*Coth[c + d*x]^2]) - Tanh[c +
 d*x]/(2*b*d*Sqrt[b*Coth[c + d*x]^2])
```

Rubi [A] time = 0.0356045, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-3/2),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(b*d*Sqrt[b*Coth[c + d*x]^2]) - Tanh[c +
 d*x]/(2*b*d*Sqrt[b*Coth[c + d*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx = \frac{\coth(c+dx)\int \tanh^3(c+dx)\,dx}{b\sqrt{b\coth^2(c+dx)}}$$
$$= -\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}} + \frac{\coth(c+dx)\int \tanh(c+dx)\,dx}{b\sqrt{b\coth^2(c+dx)}}$$
$$= \frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}} - \frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

Mathematica [A] time = 0.143311, size = 48, normalized size = 0.74

$$\frac{2\coth(c+dx)\log(\cosh(c+dx)) - \tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-3/2),x]
```

```
[Out] (2*Coth[c + d*x]*Log[Cosh[c + d*x]] - Tanh[c + d*x])/(2*b*d*Sqrt[b*Coth[c +
 d*x]^2])
```

Maple [A] time = 0.02, size = 79, normalized size = 1.2

$$-\frac{\coth(dx+c)\left(\ln(\coth(dx+c)+1)(\coth(dx+c))^2 - 2\ln(\coth(dx+c))(\coth(dx+c))^2 + \ln(\coth(dx+c)-1)(\mathrm{co}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(3/2),x)
```

```
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)+1)*coth(d*x+c)^2-2*ln(coth(d*x+c))*coth(
d*x+c)^2+ln(coth(d*x+c)-1)*coth(d*x+c)^2+1)/(b*coth(d*x+c)^2)^(3/2)
```

Maxima [A] time = 1.58899, size = 113, normalized size = 1.74

$$-\frac{2\sqrt{b}e^{(-2\,dx-2\,c)}}{\left(2\,b^2e^{(-2\,dx-2\,c)} + b^2e^{(-4\,dx-4\,c)} + b^2\right)d} - \frac{dx+c}{b^{\frac{3}{2}}d} - \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b)*e^(-2*d*x - 2*c)/((2*b^2*e^(-2*d*x - 2*c) + b^2*e^(-4*d*x - 4*c)
 + b^2)*d) - (d*x + c)/(b^(3/2)*d) - log(e^(-2*d*x - 2*c) + 1)/(b^(3/2)*d)
```

Fricas [B] time = 1.96297, size = 2082, normalized size = 32.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^4 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^4 - 4*(d*x
*cosh(d*x + c)*e^(2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(d*
x - 1)*cosh(d*x + c)^2 + 2*(3*d*x*cosh(d*x + c)^2 + d*x - (3*d*x*cosh(d*x +
 c)^2 + d*x - 1)*e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^2 + d*x - (d*x*cosh(d*x
 + c)^4 + 2*(d*x - 1)*cosh(d*x + c)^2 + d*x)*e^(2*d*x + 2*c) + ((e^(2*d*x +
 2*c) - 1)*sinh(d*x + c)^4 - cosh(d*x + c)^4 + 4*(cosh(d*x + c)*e^(2*d*x +
2*c) - cosh(d*x + c))*sinh(d*x + c)^3 - 2*(3*cosh(d*x + c)^2 - (3*cosh(d*x
+ c)^2 + 1)*e^(2*d*x + 2*c) + 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + (cos
h(d*x + c)^4 + 2*cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) - 4*(cosh(d*x + c)^3
- (cosh(d*x + c)^3 + cosh(d*x + c))*e^(2*d*x + 2*c) + cosh(d*x + c))*sinh(d
*x + c) - 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(d*x*
cosh(d*x + c)^3 + (d*x - 1)*cosh(d*x + c) - (d*x*cosh(d*x + c)^3 + (d*x - 1
)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*d*x + 4*c) +
2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b^2*d*
cosh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + (b^2*d*e^(2*d*x + 2*c) + b^2*d)
*sinh(d*x + c)^4 + 4*(b^2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x
+ c))*sinh(d*x + c)^3 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d + (3*b^2
*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(
d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c) + 4*(b^2*d*co
sh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d
*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(-3/2), x)
```

Giac [A] time = 1.2425, size = 149, normalized size = 2.29

$$-\frac{\frac{dx+c}{\sqrt{b}d\,\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{\sqrt{b}d\,\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{2\,e^{(2\,dx+2\,c)}}{\sqrt{b}d\left(e^{(2\,dx+2\,c)}+1\right)^2\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/(sqrt(b)*d*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)
/(sqrt(b)*d*sgn(e^(4*d*x + 4*c) - 1)) - 2*e^(2*d*x + 2*c)/(sqrt(b)*d*(e^(2*
d*x + 2*c) + 1)^2*sgn(e^(4*d*x + 4*c) - 1)))/b
```

3.22 $\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx$

Optimal. Leaf size=297

$$-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}-\frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{b\sqrt[3]{b\coth^2(c+dx)}}{}$$

```
[Out] (Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^
(1/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^(
1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (b*Ar
cTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3
)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^2)^(1/3))/(5*d) - (b*(b*Coth[c + d
*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[
c + d*x]^(2/3)) + (b*(b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c + d*x]^(1/3)
+ Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rubi [A] time = 0.284479, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}-\frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{b\sqrt[3]{b\coth^2(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(4/3),x]
```

```
[Out] (Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^
(1/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^(
1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (b*Ar
cTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3
)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^2)^(1/3))/(5*d) - (b*(b*Coth[c + d
*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[
c + d*x]^(2/3)) + (b*(b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c + d*x]^(1/3)
+ Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx = \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\int \coth^{\frac{8}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+ \right.}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \frac{\left(3b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c \right.}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+ \right.}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \left(b\sqrt[3]{b\,c\right.$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - b\sqrt[3]{b\,co$$

$$= \frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+d}}{2d\coth^{\frac{2}{3}}(c+dx)}$$

Mathematica [A] time = 0.338393, size = 166, normalized size = 0.56

$$-\frac{\left(b\coth^2(c+dx)\right)^{4/3}\left(12\coth^{\frac{5}{3}}(c+dx) - 20\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) - 5\left(-\log\left(\coth^{\frac{2}{3}}(c+dx) - \sqrt[3]{\coth(c+dx)}+1\right)+\right.\right.}{20d\coth^{\frac{8}{3}}(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(4/3),x]
```

```
[Out] -((b*Coth[c + d*x]^2)^(4/3)*(-20*ArcTanh[Coth[c + d*x]^(1/3)] + 12*Coth[c +
 d*x]^(5/3) - 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*
Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c + d*x]
^(1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]
^(2/3)])))/(20*d*Coth[c + d*x]^(8/3))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^2\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(4/3),x)
```

```
[Out] int((b*coth(d*x+c)^2)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth \left(dx+c\right)^2\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)
```

Fricas [B] time = 2.20668, size = 5673, normalized size = 19.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/20*(10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x +
 c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*(-b)^(1/3)*arctan(1/3*(sqrt(3)
*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*si
nh(d*x + c)^2 + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d
*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) +
sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 + b)) - 10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*
sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*b^(1/3)*arctan(-1/3*
(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt
(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c
)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*b^(2/3)*((b*cosh(d*x +
 c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/
3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 + b)) + 5*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x +
 c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x +
c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6
*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d
*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cos
h(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x
+ c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x
+ c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + s
inh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(
d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) + 5*(b
*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)
*b^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4
*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(
```

```
d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1)
)^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*s
inh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d
*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + c
osh(d*x + c))*sinh(d*x + c) + 1)) - 10*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3)*log(((cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d
*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d
*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))
^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
+ 1)) - 10*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 - b)*b^(1/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)
/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 12*(b*cosh(d*x + c)^2 + 2*
b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*((b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(d
*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^{2}\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)
```

3.23 $\int \left(b\coth^2(c+dx)\right)^{2/3} dx$

Optimal. Leaf size=289

$$-\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c}\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(
2/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3
))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(2/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTanh
[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(2/3))/(d*Coth[c + d*x]^(4/3)) -
((b*Coth[c + d*x]^2)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3
)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^2)^(2/3)*Log[1 + Coth[c +
 d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Coth[
c + d*x]^2)^(2/3)*Tanh[c + d*x])/d
```

Rubi [A] time = 0.180361, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c}\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(2/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(
2/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3
))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(2/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTanh
[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(2/3))/(d*Coth[c + d*x]^(4/3)) -
((b*Coth[c + d*x]^2)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3
)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^2)^(2/3)*Log[1 + Coth[c +
 d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Coth[
c + d*x]^2)^(2/3)*Tanh[c + d*x])/d
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(b\coth^2(c+dx)\right)^{2/3} dx = \frac{\left(b\coth^2(c+dx)\right)^{2/3} \int \coth^{\frac{4}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + \frac{\left(b\coth^2(c+dx)\right)^{2/3} \int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} - \frac{\left(b\coth^2(c+dx)\right)^{2/3} \text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}dx, x, \text{co}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} - \frac{\left(3\left(b\coth^2(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-1+x^6}dx, x, \sqrt[3]{\text{c}}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + \frac{\left(b\coth^2(c+dx)\right)^{2/3} \text{Subst}\left(\int \frac{1}{1-x^2}dx, x, \sqrt[3]{\coth(}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} - \frac{(b}{}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth(c+}\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c\right.}{2d\coth^{\frac{4}{3}}(c+dx)}$$

Mathematica [C] time = 0.0389619, size = 43, normalized size = 0.15

$$\frac{3\tanh(c+dx)\left(b\coth^2(c+dx)\right)^{2/3}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(2/3),x]
```

```
[Out] (3*(b*Coth[c + d*x]^2)^(2/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c +
d*x]^2])*Tanh[c + d*x])/d
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \left(b(\coth(dx+c))^2\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(2/3),x)
```

```
[Out] int((b*coth(d*x+c)^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth \left(dx+c\right)^2\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)
```

Fricas [B] time = 2.14525, size = 5751, normalized size = 19.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) +
sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*cosh
(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x
+ c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c)
 + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*((b*cosh(d*x + c)^2 + b*
sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(
3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 + b)) + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x +
 c) + sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b
*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh
(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x
 + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*((b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + s
qrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + b)) + (-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*
x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 1)*(-b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c
)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 -
b*cosh(d*x + c))*sinh(d*x + c) + b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^
2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*
cosh(d*x + c))*sinh(d*x + c) + b)*(-b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d
*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(
d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d
*x + c) + 1)) + (b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*log(-((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*
x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 1)*(b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^
2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^4
+ 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b
*cosh(d*x + c))*sinh(d*x + c) + b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 +
4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2
```

```
 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*c
osh(d*x + c))*sinh(d*x + c) + b)*(b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*
x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
 + c) + 1)) - 2*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)*log(-((-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1) - (b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh
(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*(b^2)^(1
/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*log(((b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2 + 1) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x
+ c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(d*cosh(d*x + c)^2
+ 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth^2\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)
```

3.24 $\int \sqrt[3]{b \coth^2(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1
/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3)) - (
(b*Coth[c + d*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(2/3)) + ((b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rubi [A] time = 0.211663, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3658, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1
/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3)) - (
(b*Coth[c + d*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(2/3)) + ((b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \frac{\sqrt[3]{b\coth^2(c+dx)}\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}$$
$$= -\frac{\sqrt[3]{b\coth^2(c+dx)}\,\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\coth^{\frac{2}{3}}(c+dx)}$$
$$= -\frac{\left(3\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}$$
$$= \frac{\sqrt[3]{b\coth^2(c+dx)}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)} + \frac{\sqrt[3]{b\coth^2(c+dx)}\,\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,d\right.}{d\coth^{\frac{2}{3}}(c+dx)}$$
$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^2(c+dx)}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$
$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} +$$

Mathematica [A] time = 0.101159, size = 151, normalized size = 0.57

$$\frac{\sqrt[3]{b\coth^2(c+dx)}\left(-\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)+\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)+2\sqrt{3}\tan^{-1}\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(1/3),x]
```

```
[Out] ((b*Coth[c + d*x]^2)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sq
rt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[
Coth[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] +
 Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2
/3))
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\left(\coth(dx+c)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(1/3),x)
```

```
[Out] int((b*coth(d*x+c)^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^2\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(1/3), x)
```

Fricas [B] time = 2.08247, size = 4625, normalized size = 17.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3
)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 + 2*(sqrt(3)*co
sh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x +
c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(co
sh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2*sqrt(3)*b^
(1/3)*arctan(-1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*si
nh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sq
rt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*b^(2
/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*
x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*si
nh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (-b)^(1/3)*log(((cosh(d*x + c)^4 +
4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 +
1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c)
)*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4
+ 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2
- 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x +
c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d
*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c
)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
 + 1)) + b^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 +
sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x +
c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (co
sh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*(
(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c
)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh
(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2
 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)
^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))
/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(
3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x +
```

```
c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b)^(1/3)*log(((cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 -
 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c
)^2 + 1)) - 2*b^(1/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)
/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\coth^2{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth{(dx+c)}^2\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(1/3), x)
```

3.25 $\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx$

Optimal. Leaf size=264

$$-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/
(2*d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3
))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*d*(b*Coth[c + d*x]^2)^(1/3)) + (ArcTanh
[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(d*(b*Coth[c + d*x]^2)^(1/3)) -
(Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4
*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth[c + d*x]^
(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(1/3))
```

Rubi [A] time = 0.169139, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3658, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/
(2*d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3
))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*d*(b*Coth[c + d*x]^2)^(1/3)) + (ArcTanh
[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(d*(b*Coth[c + d*x]^2)^(1/3)) -
(Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4
*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth[c + d*x]^
(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(1/3))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\operatorname{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x\right.}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+\right.}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(\right.}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \cdots$$

Mathematica [C] time = 0.0540409, size = 41, normalized size = 0.16

$$\frac{3\coth(c+dx)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-1/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])/(d*(b*Cot
h[c + d*x]^2)^(1/3))
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\left(\coth(dx+c)\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(1/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \coth (dx+c)^2\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

Fricas [B] time = 3.48159, size = 23547, normalized size = 89.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)
*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*sqrt(3)*(cosh(d*x + c)^4 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh
(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(
d*x + c) + 1)*b^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*
x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3)*sqrt(-1/b^(2/3)) - 2*b*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + sqrt(3)*(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x +
 c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
+ b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*sqrt(-1/b^(2/3)) + 4*(b*cosh(
d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) - (sqrt(3)*(b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 +
4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*sqrt(-1/b^(2/3))
 + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b
^(2/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sin
h(d*x + c)^2 - 1))^(1/3) - 3*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
 + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(
d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))) + sqrt(3
)*b*sqrt((-b)^(1/3)/b)*log(-(3*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)*sinh(
d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(9*b*cosh(d*x +
c)^2 + b)*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x
 + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3
 + sinh(d*x + c)^4 - 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - sqrt(3)*(2*(cosh(d*x
+ c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x
+ c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh
(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) + (b*cosh(d*x + c
)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
 + b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - (b*cosh(d*x + c)^4 + 4*
b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b
*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))*s
qrt((-b)^(1/3)/b) + 4*(3*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)
 - b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
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1)) + (-b)^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh (d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d *x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d* x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^ (2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - b^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d *x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3* sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d* x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*si nh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*c osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 2*b^(2/3)*log(((cosh(d *x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b *cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^ 2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(b*d), -1/4*(2*sqrt(3)*b*sqrt(-(-b)^(1/3)/b)*arctan(-1/3*(sqr t(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) - 2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3)*sqrt(-(-b)^(1/3) /b))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - sqrt(3)*b*sqrt(-1/b^(2/3))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)* sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*sqrt(3)*(cosh(d*x + c)^4 + 4*cosh(d *x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d *x + c) + 1)*b^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3)*sqrt(-1/b^(2/3)) - 2*b*cosh(d*x + c)^ 2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + sqrt(3)*(b*cosh(d*x + c)^ 4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*sqrt(-1/b^(2/3)) + 4*(b*cosh(d *x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) - (sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4 *b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2* sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^ (2/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh (d*x + c)^2 - 1))^(1/3) - 3*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d *x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))) - (-b)^(2

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/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)
^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cos
h(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c
)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 +
 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh
(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1
))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x +
c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(c
osh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*
sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(
d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(
d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 +
cosh(d*x + c))*sinh(d*x + c) + 1)) + b^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh
(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sin
h(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh
(d*x + c) + 1)*b^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c)
 + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + si
nh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(co
sh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*
x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x
 + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 +
sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh
(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) + 2*(
-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*
x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*
sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*b^(2/3)*log(((cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c
)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3)
)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))
/(b*d), 1/4*(sqrt(3)*b*sqrt((-b)^(1/3)/b)*log(-(3*b*cosh(d*x + c)^4 + 12*b*
cosh(d*x + c)*sinh(d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 +
 2*(9*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^4 + 4*cosh(
d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x +
 c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 +
 b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - sq
rt(3)*(2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)
^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cos
h(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*((b*cosh(d*x +
c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3
) + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4
*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - (b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*si
nh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*
((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x +
c)^2 - 1))^(1/3))*sqrt((-b)^(1/3)/b) + 4*(3*b*cosh(d*x + c)^3 + b*cosh(d*x
+ c))*sinh(d*x + c) - b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)) - 2*sqrt(3)*b^(2/3)*arctan(-1/3*sqrt(3)*((cosh(d*x +
 c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3) - 2*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*c
osh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2
- 1))^(1/3))/((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*b^(1/3))) + (-b)^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^
```

2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - b^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 2*b^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(b*d), -1/4*(2*sqrt(3)*b*sqrt(-(-b)^(1/3)/b)*arctan(-1/3*(sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) - 2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3)*sqrt(-(-b)^(1/3)/b))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 2*sqrt(3)*b^(2/3)*arctan(-1/3*sqrt(3)*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3))) - (-b)^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) + b^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x + c)^4 + 4*co

```
sh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*
x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(d*x + c)^2 +
 b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (c
osh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*c
osh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 +
4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 +
1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c)
)*sinh(d*x + c) + 1)) + 2*(-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c
)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2*co
sh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*si
nh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*b^(2/3
)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 +
sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\coth^2{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

3.26 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=289

$$-\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+d\right.}{4d\left(b\coth^2(c+\right.}$$

```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^2)^(2/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^2)^
(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^2)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^2)^(2/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(2
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^2)^(2/3))
```

Rubi [A] time = 0.221971, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+d\right.}{4d\left(b\coth^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^2)^(2/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^2)^
(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^2)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^2)^(2/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(2
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^2)^(2/3))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(}{}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{Sub}}{4d\,(}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c+dx)\log(}{4d}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\text{co}}}{\sqrt{}}\right.}{2d\,(b\,\text{cot}}$$

Mathematica [C] time = 0.0634714, size = 41, normalized size = 0.14

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};\coth^2(c+dx)\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(d*(b*C
oth[c + d*x]^2)^(2/3))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \left(b\left(\coth\left(dx+c\right)\right)^2\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(2/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

Fricas [B] time = 2.32815, size = 5739, normalized size = 19.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sin
h(d*x + c)^2 + b)*sqrt(-(-b^2)^(1/3))*arctan(1/3*(2*sqrt(3)*(-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(
-(-b^2)^(1/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^
2 + sinh(d*x + c)^2 - 1))^(1/3) - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(-b^2)^(1/3)*sqrt(-(-b^2)^(1/3
)))/(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x
 + c)^2 + b^2)) + 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/6)*arctan(1/3*sqrt(3)*(b^2)^(1/6)*(
2*(b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 +
sinh(d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d
*x + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/3))/(b^2*cosh(d*x + c)^2 + 2*b^2*
cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)) + (-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log((
(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b^2)
^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh
(d*x + c)^2 - 1))^(1/3) + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
 c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)
+ b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d
*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) +
b)*(-b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(
d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2
+ 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - (b^2)^(2/3)*(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(-(
(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(b^2)^
(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(
d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b
)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) +
 b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*
```

```
x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*
sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b
)*(b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*
x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 +
4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(-
((-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)
^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 + 1)) + 2*(b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(((b^2)^(2/3)*(cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1) + (b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*((b*cosh(d*
x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^
(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
 1)) - 12*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2
+ sinh(d*x + c)^2 - 1))^(1/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x +
c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth^2\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

3.27 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=309

$$-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}-$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/
(2*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1
/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Arc
Tanh[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(b*d*(b*Coth[c + d*x]^2)^(1/
3)) - (Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3
)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth[c
 + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) - (
3*Tanh[c + d*x])/(5*b*d*(b*Coth[c + d*x]^2)^(1/3))
```

Rubi [A] time = 0.182234, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-4/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/
(2*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1
/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Arc
Tanh[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(b*d*(b*Coth[c + d*x]^2)^(1/
3)) - (Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3
)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth[c
 + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) - (
3*Tanh[c + d*x])/(5*b*d*(b*Coth[c + d*x]^2)^(1/3))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{8}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx)\right)}{bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+}{}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\,\text{Subst}\left(\int\right.}{4bd\sqrt[3]{b\,c}}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c\right.}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\text{ta}}{}$$

Mathematica [C] time = 0.153863, size = 43, normalized size = 0.14

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{5}{6},1;\frac{1}{6};\coth^2(c+dx)\right)}{5d\left(b\coth^2(c+dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-5/6, 1, 1/6, Coth[c + d*x]^2])/(5*d*(b
*Coth[c + d*x]^2)^(4/3))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \left(b\left(\coth\left(dx+c\right)\right)^{2}\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(4/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^2)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^2\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \coth^2 \left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^2\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-4/3), x)
```

3.28 $\int \left(b \coth^3(c+dx)\right)^n \, dx$

Optimal. Leaf size=55

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};\coth^2(c+dx)\right)}{d(3n+1)}$$

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))
```

Rubi [A] time = 0.0394953, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3658, 3476, 364}

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};\coth^2(c+dx)\right)}{d(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\coth^3(c+dx)\right)^n dx = \left(\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\right)\int \coth^{3n}(c+dx)\,dx$$
$$= -\frac{\left(\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{3n}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(1+3n);\frac{3(1+n)}{2};\coth^2(c+dx)\right)}{d(1+3n)}$$

Mathematica [A] time = 0.0411529, size = 53, normalized size = 0.96

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};\coth^2(c+dx)\right)}{3dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/(d + 3*d*n)
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^3\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^n,x)
```

```
[Out] int((b*coth(d*x+c)^3)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth(dx+c)^3\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^3)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\coth(dx+c)^3\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^3)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth^3(c+dx)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth(dx+c)^3\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^n, x)
```

3.29 $\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=134

$$-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}-\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{b\sqrt{b\coth^3(c+dx)}\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{b\sqrt{b\coth}}{}$$

```
[Out] (-2*b*Sqrt[b*Coth[c + d*x]^3])/(3*d) - (b*ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[
b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) + (b*ArcTanh[Sqrt[Coth[c + d*x]
]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) - (2*b*Coth[c + d*x]^2*
Sqrt[b*Coth[c + d*x]^3])/(7*d)
```

Rubi [A] time = 0.0602026, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3658, 3473, 3476, 329, 298, 203, 206}

$$-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}-\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{b\sqrt{b\coth^3(c+dx)}\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{b\sqrt{b\coth}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Coth[c + d*x]^3])/(3*d) - (b*ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[
b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) + (b*ArcTanh[Sqrt[Coth[c + d*x]
]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) - (2*b*Coth[c + d*x]^2*
Sqrt[b*Coth[c + d*x]^3])/(7*d)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(b\coth^3(c+dx)\right)^{3/2} dx = \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int \coth^{\frac{9}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int \coth^{\frac{5}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int \sqrt{\text{cot}}}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} - \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\text{Subst}\left(\right.}{d\coth^{\frac{3}{2}}(}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} - \frac{\left(2b\sqrt{b\coth^3(c+dx)}\right)\text{Subst}}{d\coth}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\text{Subst}\left(\right.}{d\coth^{\frac{3}{2}}}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{b\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{b\tanh^{-1}\left(\sqrt{\coth(c+}\right.}{d\coth}$$

Mathematica [A] time = 0.518629, size = 82, normalized size = 0.61

$$-\frac{\left(b\coth^3(c+dx)\right)^{3/2}\left(6\coth^{\frac{7}{2}}(c+dx)+14\coth^{\frac{3}{2}}(c+dx)+21\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)-21\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{21d\coth^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(3/2),x]
```

```
[Out] -((b*Coth[c + d*x]^3)^(3/2)*(21*ArcTan[Sqrt[Coth[c + d*x]]] - 21*ArcTanh[Sq
rt[Coth[c + d*x]]] + 14*Coth[c + d*x]^(3/2) + 6*Coth[c + d*x]^(7/2)))/(21*d
*Coth[c + d*x]^(9/2))
```

Maple [A] time = 0.04, size = 107, normalized size = 0.8

$$\frac{1}{21\,d\left(\coth\left(dx+c\right)\right)^{3}b^{2}}\left(b\left(\coth\left(dx+c\right)\right)^{3}\right)^{\frac{3}{2}}\left(21\,b^{7/2}\mathrm{Artanh}\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)-21\,b^{7/2}\arctan\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(3/2),x)
```

```
[Out] 1/21/d*(b*coth(d*x+c)^3)^(3/2)*(21*b^(7/2)*arctanh((b*coth(d*x+c))^(1/2)/b^
(1/2))-21*b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))-6*(b*coth(d*x+c))^(
7/2)-14*b^2*(b*coth(d*x+c))^(3/2))/coth(d*x+c)^3/(b*coth(d*x+c))^(3/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\coth\left(dx+c\right)^{3}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(3/2), x)
```

Fricas [B] time = 3.08659, size = 5846, normalized size = 43.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(
d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c
)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cos
h(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x +
 c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(
d*x + c) - b)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh
(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2
1*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^
6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(
5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c
)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6
*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)
```

```
- b)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) +
 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 +
b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cos
h(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x
 + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(5*b*cos
h(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d*x + c)^6 + b
*cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 4*(25*b*cos
h(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (75*b
*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(15*b*cosh(
d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 5*b)*sq
rt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*s
inh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x
+ c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c) - d), -1/84*(42*(b*cosh(d*x + c)^6 + 6*b*c
osh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*
(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*co
sh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4
 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*co
sh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*arctan(sqrt(b)*
sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*
sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 21*(b*cosh(d*x + c)^6 + 6*b*cosh(
d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b
*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d
*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6
*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d
*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*log(2*b*cosh(d*x +
c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*
x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x
+ c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh
(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b)
+ 16*(5*b*cosh(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d
*x + c)^6 + b*cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4
+ 4*(25*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x +
 c)^2 + (75*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 +
2*(15*b*cosh(d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x +
 c) + 5*b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^6 + 6*d*co
sh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(
5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cos
h(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4
- 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cos
h(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth^3(c+dx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth \left(dx+c\right)^{3}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(3/2), x)
```

3.30 $\int \sqrt{b\coth^3(c+dx)}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{b\coth^3(c+dx)}\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\coth^3(c+dx)}\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}-\frac{2\tanh(c+dx)\sqrt{b\coth^3(c+d}}{d}$$

```
[Out] (ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2
)) + (ArcTanh[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x
]^(3/2)) - (2*Sqrt[b*Coth[c + d*x]^3]*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0474185, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3658, 3473, 3476, 329, 212, 206, 203}

$$\frac{\sqrt{b\coth^3(c+dx)}\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\coth^3(c+dx)}\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}-\frac{2\tanh(c+dx)\sqrt{b\coth^3(c+d}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2
)) + (ArcTanh[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x
]^(3/2)) - (2*Sqrt[b*Coth[c + d*x]^3]*Tanh[c + d*x])/d
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \frac{\sqrt{b\coth^3(c+dx)}\int \coth^{\frac{3}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt{b\coth^3(c+dx)}\int \frac{1}{\sqrt{\coth(c+dx)}}\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} - \frac{\sqrt{b\coth^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \coth(c+ \ldots}{d\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} - \frac{\left(2\sqrt{b\coth^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth(c+ \ldots}{d\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt{b\coth^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(c+dx)}\right) \ldots}{d\coth^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} - \ldots$$

Mathematica [A] time = 0.0962737, size = 63, normalized size = 0.61

$$\frac{\sqrt{b\coth^3(c+dx)}\left(-2\sqrt{\coth(c+dx)} + \tan^{-1}\left(\sqrt{\coth(c+dx)}\right) + \tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{d\coth^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth[
c + d*x]])*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2))
```

Maple [A] time = 0.045, size = 86, normalized size = 0.8

$$\frac{1}{d\coth(dx+c)}\sqrt{b(\coth(dx+c))^3}\left(-2\sqrt{b\coth(dx+c)}+\sqrt{b}\,\mathrm{Artanh}\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)+\sqrt{b}\arctan\left(\sqrt{b\coth(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(1/2),x)
```

```
[Out] 1/d*(b*coth(d*x+c)^3)^(1/2)/coth(d*x+c)/(b*coth(d*x+c))^(1/2)*(-2*(b*coth(d
*x+c))^(1/2)+b^(1/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))+b^(1/2)*arctan(
(b*coth(d*x+c))^(1/2)/b^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\coth(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^3), x)
```

Fricas [B] time = 2.18796, size = 1743, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*sqrt(b*cosh(d*x + c)
/sinh(d*x + c)))/d, 1/4*(2*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh
(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 + b)) + sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh
(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d
*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh
(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - c
osh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(
b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\coth^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\coth(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^3), x)
```

3.31 $\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}-\frac{\coth^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}$$

```
[Out] (-2*Coth[c + d*x])/(d*Sqrt[b*Coth[c + d*x]^3]) - (ArcTan[Sqrt[Coth[c + d*x]
]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c
+ d*x]]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3])
```

Rubi [A] time = 0.0480527, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3658, 3474, 3476, 329, 298, 203, 206}

$$-\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}-\frac{\coth^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (-2*Coth[c + d*x])/(d*Sqrt[b*Coth[c + d*x]^3]) - (ArcTan[Sqrt[Coth[c + d*x]
]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c
+ d*x]]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{3}{2}}(c+dx)}\,dx}{\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \sqrt{\coth(c+dx)}\,dx}{\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\left(2\coth^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+d}{}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\mathrm{c}}{d\sqrt{b\coth^3(c+}}$$

Mathematica [C] time = 0.0294052, size = 41, normalized size = 0.39

$$-\frac{2\coth(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\coth^2(c+dx)\right)}{d\sqrt{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, Coth[c + d*x]^2])/(d*Sqrt
[b*Coth[c + d*x]^3])
```

Maple [A] time = 0.041, size = 91, normalized size = 0.9

$$\frac{\coth(dx+c)}{d}\left(-2\,b^{5/2}+\mathrm{Artanh}\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)b^2\sqrt{b\coth(dx+c)}-\arctan\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)b^2\sqrt{b\coth(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(1/2),x)
```

```
[Out] 1/d*coth(d*x+c)*(-2*b^(5/2)+arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*c
oth(d*x+c))^(1/2)-arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*coth(d*x+c))
^(1/2))/(b*coth(d*x+c)^3)^(1/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth(dx+c)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)^3), x)
```

Fricas [B] time = 2.38283, size = 2554, normalized size = 24.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1
)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x
 + c)))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh
(d*x + c)^2 + b*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
```

```
+ c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x
+ c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x +
c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d
*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sq
rt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))
)/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x +
 c)^2 + b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth^3\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.32 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{2}{3bd\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}}-\frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}\left(\sqrt{\cot\right.}{bd\sqrt{b\coth^3(c+dx}}$$

```
[Out] -2/(3*b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTan[Sqrt[Coth[c + d*x]]]*Coth[c +
d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c + d*x]]]*C
oth[c + d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) - (2*Tanh[c + d*x]^2)/(7*
b*d*Sqrt[b*Coth[c + d*x]^3])
```

Rubi [A] time = 0.0608829, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3658, 3474, 3476, 329, 212, 206, 203}

$$-\frac{2}{3bd\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}}-\frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}\left(\sqrt{\cot\right.}{bd\sqrt{b\coth^3(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-3/2),x]
```

```
[Out] -2/(3*b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTan[Sqrt[Coth[c + d*x]]]*Coth[c +
d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c + d*x]]]*C
oth[c + d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) - (2*Tanh[c + d*x]^2)/(7*
b*d*Sqrt[b*Coth[c + d*x]^3])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{9}{2}}(c+dx)}\,dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{5}{2}}(c+dx)}\,dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\coth(c+dx)}}\,dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx,\right.}{bd\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} - \frac{\left(2\coth^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx,\right.}{bd\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\right.}{bd\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} + \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt{\coth(c+\right.}{bd\sqrt{b\coth}}$$

Mathematica [C] time = 0.0653703, size = 43, normalized size = 0.3

$$-\frac{2\coth(c+dx)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};\coth^2(c+dx)\right)}{7d\left(b\coth^3(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[-7/4, 1, -3/4, Coth[c + d*x]^2])/(7*d*(
b*Coth[c + d*x]^3)^(3/2))
```

Maple [A] time = 0.024, size = 106, normalized size = 0.8

$$\frac{\coth(dx+c)}{21\,d}\left(-14\,b^{15/2}\left(\coth(dx+c)\right)^{2}-6\,b^{15/2}+21\,\mathrm{Artanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{4}\left(b\coth(dx+c)\right)^{7/2}+21\,\arctan\left(\frac{\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(3/2),x)
```

```
[Out] 1/21/d*coth(d*x+c)/b^(15/2)*(-14*b^(15/2)*coth(d*x+c)^2-6*b^(15/2)+21*arcta
nh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2)+21*arctan((b*co
th(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2))/(b*coth(d*x+c)^3)^(3/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth(dx+c)^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-3/2), x)
```

Fricas [B] time = 2.94701, size = 8204, normalized size = 58.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 + sinh(d*x +
c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7
*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4
 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(
d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*c
osh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^
2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x
 + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d
*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
 + b*sinh(d*x + c)^2 + b)) + 21*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x
```

```
 + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*c
osh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 +
2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*
x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^
2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*
x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*l
og(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 +
4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cos
h(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(5*cosh(d*x + c)^8 + 40
*cosh(d*x + c)*sinh(d*x + c)^7 + 5*sinh(d*x + c)^8 + 2*(70*cosh(d*x + c)^2
- 3)*sinh(d*x + c)^6 - 6*cosh(d*x + c)^6 + 4*(70*cosh(d*x + c)^3 - 9*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(175*cosh(d*x + c)^4 - 45*cosh(d*x + c)^2 + 1)
*sinh(d*x + c)^4 + 2*cosh(d*x + c)^4 + 8*(35*cosh(d*x + c)^5 - 15*cosh(d*x
+ c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + 2*(70*cosh(d*x + c)^6 - 45*cosh(d
*x + c)^4 + 6*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*cosh(d*x + c)^2 + 4*
(10*cosh(d*x + c)^7 - 9*cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 - 3*cosh(d*x +
c))*sinh(d*x + c) + 5)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x
 + c)^8 + 8*b^2*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*d*sinh(d*x + c)^8 + 4
*b^2*d*cosh(d*x + c)^6 + 6*b^2*d*cosh(d*x + c)^4 + 4*(7*b^2*d*cosh(d*x + c)
^2 + b^2*d)*sinh(d*x + c)^6 + 8*(7*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^2 + 2*(35*b^2*d*cosh(d*x + c
)^4 + 30*b^2*d*cosh(d*x + c)^2 + 3*b^2*d)*sinh(d*x + c)^4 + 8*(7*b^2*d*cosh
(d*x + c)^5 + 10*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*x +
c)^3 + b^2*d + 4*(7*b^2*d*cosh(d*x + c)^6 + 15*b^2*d*cosh(d*x + c)^4 + 9*b^
2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 8*(b^2*d*cosh(d*x + c)^7 + 3
*b^2*d*cosh(d*x + c)^5 + 3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sin
h(d*x + c)), 1/84*(42*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 +
sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x +
c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh
(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 +
 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^2 + 1)*sin
h(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*x + c)^5 +
 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(
b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 21*(cosh(d*x + c)^8 + 8*cosh(d
*x + c)*sinh(d*x + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(
d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x +
c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 +
9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x +
c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) +
 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c
)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*c
osh(d*x + c)/sinh(d*x + c)) - b) - 16*(5*cosh(d*x + c)^8 + 40*cosh(d*x + c)
*sinh(d*x + c)^7 + 5*sinh(d*x + c)^8 + 2*(70*cosh(d*x + c)^2 - 3)*sinh(d*x
+ c)^6 - 6*cosh(d*x + c)^6 + 4*(70*cosh(d*x + c)^3 - 9*cosh(d*x + c))*sinh(
d*x + c)^5 + 2*(175*cosh(d*x + c)^4 - 45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)
^4 + 2*cosh(d*x + c)^4 + 8*(35*cosh(d*x + c)^5 - 15*cosh(d*x + c)^3 + cosh(
d*x + c))*sinh(d*x + c)^3 + 2*(70*cosh(d*x + c)^6 - 45*cosh(d*x + c)^4 + 6*
cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*cosh(d*x + c)^2 + 4*(10*cosh(d*x +
```

```
c)^7 - 9*cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x +
c) + 5)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x + c)^8 + 8*b^
2*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*d*sinh(d*x + c)^8 + 4*b^2*d*cosh(d*
x + c)^6 + 6*b^2*d*cosh(d*x + c)^4 + 4*(7*b^2*d*cosh(d*x + c)^2 + b^2*d)*si
nh(d*x + c)^6 + 8*(7*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*
x + c)^5 + 4*b^2*d*cosh(d*x + c)^2 + 2*(35*b^2*d*cosh(d*x + c)^4 + 30*b^2*d
*cosh(d*x + c)^2 + 3*b^2*d)*sinh(d*x + c)^4 + 8*(7*b^2*d*cosh(d*x + c)^5 +
10*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*d +
4*(7*b^2*d*cosh(d*x + c)^6 + 15*b^2*d*cosh(d*x + c)^4 + 9*b^2*d*cosh(d*x +
c)^2 + b^2*d)*sinh(d*x + c)^2 + 8*(b^2*d*cosh(d*x + c)^7 + 3*b^2*d*cosh(d*
x + c)^5 + 3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.33 $\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx$

Optimal. Leaf size=74

$$-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}-\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}+bx\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}$$

```
[Out] -((b*(b*Coth[c + d*x]^3)^(1/3))/d) - (b*Coth[c + d*x]^2*(b*Coth[c + d*x]^3)
^(1/3))/(3*d) + b*x*(b*Coth[c + d*x]^3)^(1/3)*Tanh[c + d*x]
```

Rubi [A] time = 0.0335459, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}-\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}+bx\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(4/3),x]
```

```
[Out] -((b*(b*Coth[c + d*x]^3)^(1/3))/d) - (b*Coth[c + d*x]^2*(b*Coth[c + d*x]^3)
^(1/3))/(3*d) + b*x*(b*Coth[c + d*x]^3)^(1/3)*Tanh[c + d*x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(b\coth^3(c+dx)\right)^{4/3}dx = \left(b\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)\right)\int \coth^4(c+dx)\,dx$$
$$= -\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d} + \left(b\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)\right)\int \coth^2(c+dx)\,d$$
$$= -\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d} - \frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d} + \left(b\sqrt[3]{b\coth^3(c+dx)}\tanh(c+d\right.$$
$$= -\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d} - \frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d} + bx\sqrt[3]{b\coth^3(c+dx)}\tanh(c+d$$

Mathematica [C] time = 0.0607881, size = 43, normalized size = 0.58

$$-\frac{\tanh(c+dx)\left(b\coth^3(c+dx)\right)^{4/3}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(4/3),x]
```

```
[Out] -((b*Coth[c + d*x]^3)^(4/3)*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^
2]*Tanh[c + d*x])/(3*d)
```

Maple [B] time = 0.098, size = 145, normalized size = 2.

$$\frac{b\left(e^{2dx+2c}-1\right)x}{1+e^{2dx+2c}}\sqrt[3]{\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}} - \frac{4b\left(3e^{4dx+4c}-3e^{2dx+2c}+2\right)}{\left(3+3e^{2dx+2c}\right)\left(e^{2dx+2c}-1\right)^2 d}\sqrt[3]{\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(4/3),x)
```

```
[Out] b/(1+exp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+
2*c)-1)^3)^(1/3)*x-4/3*b/(1+exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)^2*(b*(1+exp(
2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)*(3*exp(4*d*x+4*c)-3*exp(2*d*x+2*c
)+2)/d
```

Maxima [A] time = 1.75654, size = 117, normalized size = 1.58

$$\frac{(dx+c)b^{\frac{4}{3}}}{d} - \frac{4\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)} - 3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)} - 2\,b^{\frac{4}{3}}\right)}{3\,d\left(3\,e^{(-2\,dx-2\,c)} - 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(4/3)/d - 4/3*(3*b^(4/3)*e^(-2*d*x - 2*c) - 3*b^(4/3)*e^(-4*d*x
 - 4*c) - 2*b^(4/3))/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
```

```
x - 6*c) - 1))
```

Fricas [B] time = 2.27201, size = 2689, normalized size = 36.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*d*x*cosh(d*x + c)^6 - 3*(b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x
+ c)^6 - 18*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*sin
h(d*x + c)^5 - 3*(3*b*d*x + 4*b)*cosh(d*x + c)^4 + 3*(15*b*d*x*cosh(d*x + c
)^2 - 3*b*d*x - (15*b*d*x*cosh(d*x + c)^2 - 3*b*d*x - 4*b)*e^(2*d*x + 2*c)
- 4*b)*sinh(d*x + c)^4 + 12*(5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh
(d*x + c) - (5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c)^3 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c)^2 +
 3*(15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2
- (15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2 +
 4*b)*e^(2*d*x + 2*c) + 4*b)*sinh(d*x + c)^2 - (3*b*d*x*cosh(d*x + c)^6 - 3
*(3*b*d*x + 4*b)*cosh(d*x + c)^4 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c
)^2 - 8*b)*e^(2*d*x + 2*c) + 6*(3*b*d*x*cosh(d*x + c)^5 - 2*(3*b*d*x + 4*b)
*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c) - (3*b*d*x*cosh(d*x + c)^5
 - 2*(3*b*d*x + 4*b)*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c) - 8*b)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c)
+ 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*
x + 2*c) - 1))^(1/3)/(d*cosh(d*x + c)^6 + (d*e^(2*d*x + 2*c) + d)*sinh(d*x
+ c)^6 + 6*(d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c
)^5 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + (5*d*cosh(d*x + c)^2 -
 d)*e^(2*d*x + 2*c) - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cos
h(d*x + c) + (5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x
+ c)^2 + (5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) +
d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*d*cosh(d*
x + c)^2 - d)*e^(2*d*x + 2*c) + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c) + (d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + d*cosh(d*x +
 c))*e^(2*d*x + 2*c))*sinh(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth (dx + c)^3\right)^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(4/3), x)
```

3.34 $\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx$

Optimal. Leaf size=50

$$x\tanh^2(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}-\frac{\tanh(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}}{d}$$

```
[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Tanh[c + d*x])/d) + x*(b*Coth[c + d*x]^3)^(2/3
)*Tanh[c + d*x]^2
```

Rubi [A] time = 0.0244887, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$x\tanh^2(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}-\frac{\tanh(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(2/3),x]
```

```
[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Tanh[c + d*x])/d) + x*(b*Coth[c + d*x]^3)^(2/3
)*Tanh[c + d*x]^2
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx &= \left(\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\right)\int \coth^2(c+dx)\,dx\\ &= -\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d}+\left(\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\right)\int 1\,dx\\ &= -\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d}+x\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\end{aligned}$$

Mathematica [C] time = 0.0275691, size = 41, normalized size = 0.82

$$-\frac{\tanh(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(2/3),x]
```

```
[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^
2]*Tanh[c + d*x])/d)
```

Maple [B] time = 0.097, size = 119, normalized size = 2.4

$$\frac{\left(e^{2dx+2c}-1\right)^2 x}{\left(1+e^{2dx+2c}\right)^2}\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}-2\frac{e^{2dx+2c}-1}{\left(1+e^{2dx+2c}\right)^2 d}\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(2/3),x)
```

```
[Out] (b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(1+exp(2*d*x+2*c))^2*(e
xp(2*d*x+2*c)-1)^2*x-2*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/
(1+exp(2*d*x+2*c))^2*(exp(2*d*x+2*c)-1)/d
```

Maxima [A] time = 1.61666, size = 46, normalized size = 0.92

$$\frac{(dx+c)b^{\frac{2}{3}}}{d}+\frac{2\,b^{\frac{2}{3}}}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(2/3)/d + 2*b^(2/3)/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 2.18871, size = 992, normalized size = 19.84

$$\frac{\left(dx\cosh(dx+c)^2+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh(dx+c)^2-dx+\left(dx\cosh(dx+c)^2-dx-2\right)e^{(4\,dx+4\,c)}-2\,(d}{d\cosh(dx+c)^2+\left(de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d\right)\sinh(dx+c)^2+\left(d\cosh(dx+c)^2-d\right)e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2
```

```
*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) - 2)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x +
2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2
/3)/(d*cosh(d*x + c)^2 + (d*e^(4*d*x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh
(d*x + c)^2 + (d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^
2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x
+ c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^3\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(2/3), x)
```

3.35 $\int \sqrt[3]{b \coth^3(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\log(\sinh(c+dx))}{d}$$

```
[Out] ((b*Coth[c + d*x]^3)^(1/3)*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0202028, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(1/3),x]
```

```
[Out] ((b*Coth[c + d*x]^3)^(1/3)*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \left(\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)\right)\int \coth(c+dx)\,dx$$
$$= \frac{\sqrt[3]{b\coth^3(c+dx)}\log(\sinh(c+dx))\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.0258951, size = 39, normalized size = 1.26

$$\frac{\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}(\log(\tanh(c+dx))+\log(\cosh(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(1/3),x]
```

```
[Out] ((b*Coth[c + d*x]^3)^(1/3)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c
 + d*x])/d
```

Maple [B] time = 0.092, size = 192, normalized size = 6.2

$$\frac{\left(e^{2dx+2c}-1\right)x}{1+e^{2dx+2c}}\sqrt[3]{\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}}-2\frac{\left(e^{2dx+2c}-1\right)(dx+c)}{\left(1+e^{2dx+2c}\right)d}\sqrt[3]{\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}}+\frac{\left(e^{2dx+2c}-1\right)\ln\left(e^{2dx+2c}-1\right)}{\left(1+e^{2dx+2c}\right)d}\sqrt[3]{b\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(1/3),x)
```

```
[Out] (b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(1+exp(2*d*x+2*c))*(exp
(2*d*x+2*c)-1)*x-2*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(1+e
xp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)/d*(d*x+c)+(b*(1+exp(2*d*x+2*c))^3/(exp(2*
d*x+2*c)-1)^3)^(1/3)/(1+exp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)/d*ln(exp(2*d*x+2
*c)-1)
```

Maxima [A] time = 1.74058, size = 69, normalized size = 2.23

$$\frac{(dx+c)b^{\frac{1}{3}}}{d}+\frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(1/3)/d + b^(1/3)*log(e^(-d*x - c) + 1)/d + b^(1/3)*log(e^(-d*x
 - c) - 1)/d
```

Fricas [B] time = 2.13365, size = 363, normalized size = 11.71

$$-\frac{\left(dxe^{(2dx+2c)}-dx-\left(e^{(2dx+2c)}-1\right)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6dx+6c)}+3be^{(4dx+4c)}+3be^{(2dx+2c)}+b}{e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1}\right)^{\frac{1}{3}}}{de^{(2dx+2c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) +
3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x
+ 2*c) - 1))^(1/3)/(d*e^(2*d*x + 2*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt[3]{b\coth^{3}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth (dx + c)^3\right)^{\frac{1}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(1/3), x)
```

3.36 $\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Rubi [A] time = 0.0202842, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-1/3),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx &= \frac{\coth(c+dx)\int \tanh(c+dx)\,dx}{\sqrt[3]{b\coth^3(c+dx)}}\\ &= \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0284736, size = 31, normalized size = 1.

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-1/3),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Maple [B] time = 0.093, size = 192, normalized size = 6.2

$$\frac{\left(1+e^{2dx+2c}\right)x}{e^{2dx+2c}-1}\frac{1}{\sqrt[3]{\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}}}-2\frac{\left(1+e^{2dx+2c}\right)(dx+c)}{\left(e^{2dx+2c}-1\right)d}\frac{1}{\sqrt[3]{\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}}}+\frac{\left(1+e^{2dx+2c}\right)\ln\left(1+e^{2dx+2c}\right)}{\left(e^{2dx+2c}-1\right)d}\frac{1}{\sqrt[3]{\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(1/3),x)
```

```
[Out] 1/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(1
+exp(2*d*x+2*c))*x-2/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(e
xp(2*d*x+2*c)-1)*(1+exp(2*d*x+2*c))/d*(d*x+c)+1/(b*(1+exp(2*d*x+2*c))^3/(ex
p(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(1+exp(2*d*x+2*c))/d*ln(1+exp(2
*d*x+2*c))
```

Maxima [A] time = 1.71109, size = 43, normalized size = 1.39

$$\frac{dx+c}{b^{\frac{1}{3}}d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{b^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(b^(1/3)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^(1/3)*d)
```

Fricas [B] time = 2.44438, size = 460, normalized size = 14.84

$$-\frac{\left(dxe^{(4dx+4c)}-2dxe^{(2dx+2c)}+dx-\left(e^{(4dx+4c)}-2e^{(2dx+2c)}+1\right)\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6dx+6c)}+3be^{(4dx+4c)}+3be^{(2dx+2c)}+}{e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1}\right.}{bde^{(4dx+4c)}+2bde^{(2dx+2c)}+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x - (e^(4*d*x + 4*c) - 2*
e^(2*d*x + 2*c) + 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))*
((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*
d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(b*d*e^(4*d*
x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\coth^3{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{3}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-1/3), x)
```

3.37 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=50

$$\frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}-\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}}$$

```
[Out] -(Coth[c + d*x]/(d*(b*Coth[c + d*x]^3)^(2/3))) + (x*Coth[c + d*x]^2)/(b*Cot
h[c + d*x]^3)^(2/3)
```

Rubi [A] time = 0.0242453, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}-\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-2/3),x]
```

```
[Out] -(Coth[c + d*x]/(d*(b*Coth[c + d*x]^3)^(2/3))) + (x*Coth[c + d*x]^2)/(b*Cot
h[c + d*x]^3)^(2/3)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{\left(b\coth^3(c+dx)\right)^{2/3}}$$
$$= -\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}} + \frac{\coth^2(c+dx)\int 1\,dx}{\left(b\coth^3(c+dx)\right)^{2/3}}$$
$$= -\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}} + \frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}$$

Mathematica [A] time = 0.0599557, size = 40, normalized size = 0.8

$$\frac{\coth(c+dx)\left(\tanh^{-1}(\tanh(c+dx))\coth(c+dx)-1\right)}{d\left(b\coth^3(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-2/3),x]
```

```
[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*(b*Coth[c + 
d*x]^3)^(2/3))
```

Maple [B] time = 0.093, size = 119, normalized size = 2.4

$$\frac{\left(1+e^{2dx+2c}\right)^2 x}{\left(e^{2dx+2c}-1\right)^2}\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{-\frac{2}{3}} + 2\,\frac{1+e^{2dx+2c}}{\left(e^{2dx+2c}-1\right)^2 d}\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{-2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(2/3),x)
```

```
[Out] 1/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(exp(2*d*x+2*c)-1)^2*
(1+exp(2*d*x+2*c))^2*x+2/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3
)/(exp(2*d*x+2*c)-1)^2*(1+exp(2*d*x+2*c))/d
```

Maxima [A] time = 1.74955, size = 50, normalized size = 1.

$$\frac{dx+c}{b^{\frac{2}{3}}d} - \frac{2}{\left(b^{\frac{2}{3}}e^{(-2dx-2c)} + b^{\frac{2}{3}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(b^(2/3)*d) - 2/((b^(2/3)*e^(-2*d*x - 2*c) + b^(2/3))*d)
```

Fricas [B] time = 2.23736, size = 724, normalized size = 14.48

$$-\frac{\left(dx\cosh\left(dx+c\right)^{2}-\left(dxe^{(2\,dx+2\,c)}-dx\right)\sinh\left(dx+c\right)^{2}+dx-\left(dx\cosh\left(dx+c\right)^{2}+dx+2\right)e^{(2\,dx+2\,c)}-2\left(dx\cosh\left(dx+c\right.\right.}{bd\cosh\left(dx+c\right)^{2}+\left(bde^{(2\,dx+2\,c)}+bd\right)\sinh\left(dx+c\right)^{2}+bd+\left(bd\cosh\left(dx+c\right)^{2}+bd\right)e^{(2\,dx+2\,c)}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -(d*x*cosh(d*x + c)^2 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^2 + d*x -
 (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) - 2*(d*x*cosh(d*x + c)*e^(
2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c) + 2)*((b*e^(6*d*x + 6*c) +
3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*
x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(1/3)/(b*d*cosh(d*x + c)^2 + (b*d*e^(2*d
*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d + (b*d*cosh(d*x + c)^2 + b*d)*e^(2*d
*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b*d*cosh(d*x + c))*sinh(
d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\coth^{3}\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\coth\left(dx+c\right)^{3}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-2/3), x)
```

3.38 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=80

$$\frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}}-\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}}-\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

```
[Out] -(1/(b*d*(b*Coth[c + d*x]^3)^(1/3))) + (x*Coth[c + d*x])/(b*(b*Coth[c + d*x
]^3)^(1/3)) - Tanh[c + d*x]^2/(3*b*d*(b*Coth[c + d*x]^3)^(1/3))
```

Rubi [A] time = 0.0353556, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}}-\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}}-\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-4/3),x]
```

```
[Out] -(1/(b*d*(b*Coth[c + d*x]^3)^(1/3))) + (x*Coth[c + d*x])/(b*(b*Coth[c + d*x
]^3)^(1/3)) - Tanh[c + d*x]^2/(3*b*d*(b*Coth[c + d*x]^3)^(1/3))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \frac{\coth(c+dx)\int \tanh^4(c+dx)\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$
$$= -\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{\coth(c+dx)\int \tanh^2(c+dx)\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$
$$= -\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{\coth(c+dx)\int 1\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$
$$= -\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

Mathematica [A] time = 0.0878472, size = 51, normalized size = 0.64

$$\frac{-\tanh^2(c+dx) + 3\tanh^{-1}(\tanh(c+dx))\coth(c+dx) - 3}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-4/3),x]
```

```
[Out] (-3 + 3*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x] - Tanh[c + d*x]^2)/(3*b*d*(b*C
oth[c + d*x]^3)^(1/3))
```

Maple [B] time = 0.099, size = 149, normalized size = 1.9

$$\frac{\left(1+e^{2dx+2c}\right)x}{b\left(e^{2dx+2c}-1\right)}\frac{1}{\sqrt[3]{\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}}} + \frac{12\,e^{4dx+4c}+12\,e^{2dx+2c}+8}{3b\left(1+e^{2dx+2c}\right)^2\left(e^{2dx+2c}-1\right)d}\frac{1}{\sqrt[3]{\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(4/3),x)
```

```
[Out] 1/b*(1+exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*
x+2*c)-1)^3)^(1/3)*x+4/3/b/(1+exp(2*d*x+2*c))^2/(exp(2*d*x+2*c)-1)/(b*(1+ex
p(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)*(3*exp(4*d*x+4*c)+3*exp(2*d*x+2
*c)+2)/d
```

Maxima [A] time = 1.77013, size = 120, normalized size = 1.5

$$-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{3\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)}+3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)}+b^{\frac{4}{3}}e^{(-6\,dx-6\,c)}+b^{\frac{4}{3}}\right)d}+\frac{dx+c}{b^{\frac{4}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
```

```
[Out] -4/3*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/((3*b^(4/3)*e^(-2*d*x -
2*c) + 3*b^(4/3)*e^(-4*d*x - 4*c) + b^(4/3)*e^(-6*d*x - 6*c) + b^(4/3))*d)
+ (d*x + c)/(b^(4/3)*d)
```

Fricas [B] time = 2.37196, size = 4035, normalized size = 50.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cosh(d*x + c)^6 + 3*(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c)
 + d*x)*sinh(d*x + c)^6 + 18*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d*x*cos
h(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(3*d*x
+ 4)*cosh(d*x + c)^4 + 3*(15*d*x*cosh(d*x + c)^2 + 3*d*x + (15*d*x*cosh(d*x
 + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^2 + 3*d*x +
4)*e^(2*d*x + 2*c) + 4)*sinh(d*x + c)^4 + 12*(5*d*x*cosh(d*x + c)^3 + (3*d*
x + 4)*cosh(d*x + c) + (5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*
e^(4*d*x + 4*c) - 2*(5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*e^(
2*d*x + 2*c))*sinh(d*x + c)^3 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*(15*d*x*c
osh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + (15*d*x*cosh(d*x +
 c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*c) - 2*(15*
d*x*cosh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4)*e^(2*d*x +
 2*c) + 4)*sinh(d*x + c)^2 + 3*d*x + (3*d*x*cosh(d*x + c)^6 + 3*(3*d*x + 4)
*cosh(d*x + c)^4 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 8)*e^(4*d*x + 4*
c) - 2*(3*d*x*cosh(d*x + c)^6 + 3*(3*d*x + 4)*cosh(d*x + c)^4 + 3*(3*d*x +
4)*cosh(d*x + c)^2 + 3*d*x + 8)*e^(2*d*x + 2*c) + 6*(3*d*x*cosh(d*x + c)^5
+ 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c) + (3*d*x*cosh(d
*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*e^(4
*d*x + 4*c) - 2*(3*d*x*cosh(d*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3
*d*x + 4)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + 8)*((b*e^(6*d*x +
 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3
*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(b^2*d*cosh(d*x + c)^6 + 3
*b^2*d*cosh(d*x + c)^4 + (b^2*d*e^(4*d*x + 4*c) + 2*b^2*d*e^(2*d*x + 2*c) +
 b^2*d)*sinh(d*x + c)^6 + 6*(b^2*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b^2*d*
cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^5 + 3*b^
2*d*cosh(d*x + c)^2 + 3*(5*b^2*d*cosh(d*x + c)^2 + b^2*d + (5*b^2*d*cosh(d*
x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(
2*d*x + 2*c))*sinh(d*x + c)^4 + 4*(5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d
*x + c) + (5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*e^(4*d*x + 4*c)
 + 2*(5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^3 + b^2*d + 3*(5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2
 + b^2*d + (5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4
*d*x + 4*c) + 2*(5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2 + b^2*d)
*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^6 + 3*b^2*d*cosh(d
*x + c)^4 + 3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(b^2*d*cos
h(d*x + c)^6 + 3*b^2*d*cosh(d*x + c)^4 + 3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e
^(2*d*x + 2*c) + 6*(b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x + c)^3 + b^2*d
*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x + c)^3 + b^2*d*c
osh(d*x + c))*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x
 + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth^3\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-4/3), x)
```

3.39 $\int \left(b \coth^4(c+dx)\right)^n \, dx$

Optimal. Leaf size=57

$$\frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(4n+1);\frac{1}{2}(4n+3);\coth^2(c+dx)\right)}{d(4n+1)}$$

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, (1 + 4*n)/2, (3 +
 4*n)/2, Coth[c + d*x]^2])/(d*(1 + 4*n))
```

Rubi [A] time = 0.0388827, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3658, 3476, 364}

$$\frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(4n+1);\frac{1}{2}(4n+3);\coth^2(c+dx)\right)}{d(4n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, (1 + 4*n)/2, (3 +
 4*n)/2, Coth[c + d*x]^2])/(d*(1 + 4*n))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(b\coth^4(c+dx)\right)^n dx &= \left(\coth^{-4n}(c+dx)\left(b\coth^4(c+dx)\right)^n\right)\int \coth^{4n}(c+dx)\,dx \\
&= -\frac{\left(\coth^{-4n}(c+dx)\left(b\coth^4(c+dx)\right)^n\right)\operatorname{Subst}\left(\int \frac{x^{4n}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d} \\
&= \frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n {}_2F_1\left(1, \frac{1}{2}(1+4n); \frac{1}{2}(3+4n); \coth^2(c+dx)\right)}{d(1+4n)}
\end{aligned}
$$

Mathematica [A] time = 0.0437062, size = 51, normalized size = 0.89

$$
\frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n {}_2F_1\left(1, 2n+\frac{1}{2}; 2n+\frac{3}{2}; \coth^2(c+dx)\right)}{4dn+d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, 1/2 + 2*n, 3/2 +
2*n, Coth[c + d*x]^2])/(d + 4*d*n)
```

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$
\int \left(b\left(\coth(dx+c)\right)^4\right)^n dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^n,x)
```

```
[Out] int((b*coth(d*x+c)^4)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(b\coth(dx+c)^4\right)^n dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$
\operatorname{integral}\left(\left(b\coth(dx+c)^4\right)^n, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^4)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth^4\left(c+dx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^4\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^n, x)
```

3.40 $\int \left(b\coth^4(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=110

$$-\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d}-\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d}+bx\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}-\frac{b\tanh(c+}{}$$

```
[Out] -(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^4])/(3*d) - (b*Coth[c + d*x]^3*Sqrt[
b*Coth[c + d*x]^4])/(5*d) - (b*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d + b
*x*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x]^2
```

Rubi [A] time = 0.0452879, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d}-\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d}+bx\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}-\frac{b\tanh(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(3/2),x]
```

```
[Out] -(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^4])/(3*d) - (b*Coth[c + d*x]^3*Sqrt[
b*Coth[c + d*x]^4])/(5*d) - (b*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d + b
*x*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x]^2
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = \left(b\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)\right)\int \coth^6(c+dx)\,dx$$
$$= -\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} + \left(b\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)\right)\int \coth^4(c+d$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} + \left(b\sqrt{b\coth^4(c+\right.$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} - \frac{b\sqrt{b\coth^4(c+}{}$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} - \frac{b\sqrt{b\coth^4(c+}{}$$

Mathematica [C] time = 0.0633555, size = 43, normalized size = 0.39

$$-\frac{\tanh(c+dx)\left(b\coth^4(c+dx)\right)^{3/2}\ {}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\tanh^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(3/2),x]
```

```
[Out] -((b*Coth[c + d*x]^4)^(3/2)*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d*x]^
2]*Tanh[c + d*x])/(5*d)
```

Maple [A] time = 0.036, size = 77, normalized size = 0.7

$$-\frac{6\left(\coth(dx+c)\right)^5+10\left(\coth(dx+c)\right)^3+15\ln\left(\coth(dx+c)-1\right)-15\ln\left(\coth(dx+c)+1\right)+30\coth(dx+c)}{30\,d\left(\coth(dx+c)\right)^6}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(3/2),x)
```

```
[Out] -1/30/d*(b*coth(d*x+c)^4)^(3/2)*(6*coth(d*x+c)^5+10*coth(d*x+c)^3+15*ln(cot
h(d*x+c)-1)-15*ln(coth(d*x+c)+1)+30*coth(d*x+c))/coth(d*x+c)^6
```

Maxima [A] time = 1.73484, size = 185, normalized size = 1.68

$$\frac{(dx+c)b^{\frac{3}{2}}}{d} - \frac{2\left(70\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)} - 140\,b^{\frac{3}{2}}e^{(-4\,dx-4\,c)} + 90\,b^{\frac{3}{2}}e^{(-6\,dx-6\,c)} - 45\,b^{\frac{3}{2}}e^{(-8\,dx-8\,c)} - 23\,b^{\frac{3}{2}}\right)}{15\,d\left(5\,e^{(-2\,dx-2\,c)} - 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} - 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(3/2)/d - 2/15*(70*b^(3/2)*e^(-2*d*x - 2*c) - 140*b^(3/2)*e^(-4
*d*x - 4*c) + 90*b^(3/2)*e^(-6*d*x - 6*c) - 45*b^(3/2)*e^(-8*d*x - 8*c) - 2
```

```
3*b^(3/2))/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))
```

Fricas [B] time = 2.65019, size = 9090, normalized size = 82.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*b*d*x*cosh(d*x + c)^10 + 15*(b*d*x*e^(4*d*x + 4*c) - 2*b*d*x*e^(2*
d*x + 2*c) + b*d*x)*sinh(d*x + c)^10 + 150*(b*d*x*cosh(d*x + c)*e^(4*d*x +
4*c) - 2*b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + b*d*x*cosh(d*x + c))*sinh(d*
x + c)^9 - 15*(5*b*d*x + 6*b)*cosh(d*x + c)^8 + 15*(45*b*d*x*cosh(d*x + c)^
2 - 5*b*d*x + (45*b*d*x*cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^(4*d*x + 4*c) -
2*(45*b*d*x*cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^(2*d*x + 2*c) - 6*b)*sinh(d*
x + c)^8 + 120*(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c) +
(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c))*e^(4*d*x + 4*c)
- 2*(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c))*e^(2*d*x + 2
*c))*sinh(d*x + c)^7 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 30*(105*b*d*x*c
osh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + (105*b*d*x*
cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + 6*b)*e^(4*
d*x + 4*c) - 2*(105*b*d*x*cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*co
sh(d*x + c)^2 + 6*b)*e^(2*d*x + 2*c) + 6*b)*sinh(d*x + c)^6 + 60*(63*b*d*x*
cosh(d*x + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^3 + 3*(5*b*d*x + 6*b)*co
sh(d*x + c) + (63*b*d*x*cosh(d*x + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^
3 + 3*(5*b*d*x + 6*b)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(63*b*d*x*cosh(d*x
 + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^3 + 3*(5*b*d*x + 6*b)*cosh(d*x +
 c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 - 10*(15*b*d*x + 28*b)*cosh(d*x + c)^
4 + 10*(315*b*d*x*cosh(d*x + c)^6 - 105*(5*b*d*x + 6*b)*cosh(d*x + c)^4 - 1
5*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + (315*b*d*x*cosh(d*x + c)^6 -
 105*(5*b*d*x + 6*b)*cosh(d*x + c)^4 - 15*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d
*x + c)^2 - 28*b)*e^(4*d*x + 4*c) - 2*(315*b*d*x*cosh(d*x + c)^6 - 105*(5*b
*d*x + 6*b)*cosh(d*x + c)^4 - 15*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d*x + c)^2
 - 28*b)*e^(2*d*x + 2*c) - 28*b)*sinh(d*x + c)^4 + 40*(45*b*d*x*cosh(d*x +
c)^7 - 21*(5*b*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c
)^3 - (15*b*d*x + 28*b)*cosh(d*x + c) + (45*b*d*x*cosh(d*x + c)^7 - 21*(5*b
*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c)^3 - (15*b*d*
x + 28*b)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(45*b*d*x*cosh(d*x + c)^7 - 21
*(5*b*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c)^3 - (15
*b*d*x + 28*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^3 - 15*b*d*x +
 5*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + 5*(135*b*d*x*cosh(d*x + c)^8 - 84*(5
*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x + 6*b)*cosh(d*x + c)^4 + 15*b*d
*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + (135*b*d*x*cosh(d*x + c)^8 - 84
*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x + 6*b)*cosh(d*x + c)^4 + 15*
b*d*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + 28*b)*e^(4*d*x + 4*c) - 2*(1
35*b*d*x*cosh(d*x + c)^8 - 84*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x
 + 6*b)*cosh(d*x + c)^4 + 15*b*d*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 +
 28*b)*e^(2*d*x + 2*c) + 28*b)*sinh(d*x + c)^2 + (15*b*d*x*cosh(d*x + c)^10
 - 15*(5*b*d*x + 6*b)*cosh(d*x + c)^8 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6
- 10*(15*b*d*x + 28*b)*cosh(d*x + c)^4 - 15*b*d*x + 5*(15*b*d*x + 28*b)*cos
h(d*x + c)^2 - 46*b)*e^(4*d*x + 4*c) - 2*(15*b*d*x*cosh(d*x + c)^10 - 15*(5
*b*d*x + 6*b)*cosh(d*x + c)^8 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6 - 10*(15
*b*d*x + 28*b)*cosh(d*x + c)^4 - 15*b*d*x + 5*(15*b*d*x + 28*b)*cosh(d*x +
c)^2 - 46*b)*e^(2*d*x + 2*c) + 10*(15*b*d*x*cosh(d*x + c)^9 - 12*(5*b*d*x +
 6*b)*cosh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x + c)^5 - 4*(15*b*d*x +
```

```
28*b)*cosh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x + c) + (15*b*d*x*cosh(d*
x + c)^9 - 12*(5*b*d*x + 6*b)*cosh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x
 + c)^5 - 4*(15*b*d*x + 28*b)*cosh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x
+ c))*e^(4*d*x + 4*c) - 2*(15*b*d*x*cosh(d*x + c)^9 - 12*(5*b*d*x + 6*b)*co
sh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x + c)^5 - 4*(15*b*d*x + 28*b)*co
sh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x
+ c) - 46*b)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*x +
 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) + 6*e
^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^10 + (d*e^(4*d*x
+ 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^10 + 10*(d*cosh(d*x + c)*e^
(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d
*x + c)^9 - 5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + (9*d*cosh(d*x +
c)^2 - d)*e^(4*d*x + 4*c) + 2*(9*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d
)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d
*x + c)^3 - d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*d*cosh(d*x + c)^3 - d*c
osh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*
(21*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 + (21*d*cosh(d*x + c)^4 - 14*d
*cosh(d*x + c)^2 + d)*e^(4*d*x + 4*c) + 2*(21*d*cosh(d*x + c)^4 - 14*d*cosh
(d*x + c)^2 + d)*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x +
c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c) + (63*d*cosh(d*x + c)^5 -
70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(63*d*cosh(d
*x + c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^5 - 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 - 35*d*cosh(
d*x + c)^4 + 15*d*cosh(d*x + c)^2 + (21*d*cosh(d*x + c)^6 - 35*d*cosh(d*x +
 c)^4 + 15*d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(21*d*cosh(d*x + c)^6
 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d)*si
nh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*
x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5
*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*d*cosh(d*x + c
)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(2*d*x
 + 2*c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 - 2
8*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + (9*d*co
sh(d*x + c)^8 - 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x
 + c)^2 + d)*e^(4*d*x + 4*c) + 2*(9*d*cosh(d*x + c)^8 - 28*d*cosh(d*x + c)^
6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) + d)*s
inh(d*x + c)^2 + (d*cosh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 10*d*cosh(d*x
+ c)^6 - 10*d*cosh(d*x + c)^4 + 5*d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) +
2*(d*cosh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 10*d*cosh(d*x + c)^6 - 10*d*c
osh(d*x + c)^4 + 5*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 10*(d*cosh(d*x
+ c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 +
d*cosh(d*x + c) + (d*cosh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x +
 c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(d*cosh(
d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth^4(c+dx)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(3/2), x)
```

Giac [A] time = 1.18266, size = 108, normalized size = 0.98

$$\frac{1}{15}\, b^{\frac{3}{2}} \left(\frac{15\,(dx + c)}{d} - \frac{2\left(45\, e^{(8\,dx+8\,c)} - 90\, e^{(6\,dx+6\,c)} + 140\, e^{(4\,dx+4\,c)} - 70\, e^{(2\,dx+2\,c)} + 23\right)}{d\left(e^{(2\,dx+2\,c)} - 1\right)^{5}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*b^(3/2)*(15*(d*x + c)/d - 2*(45*e^(8*d*x + 8*c) - 90*e^(6*d*x + 6*c) +
 140*e^(4*d*x + 4*c) - 70*e^(2*d*x + 2*c) + 23)/(d*(e^(2*d*x + 2*c) - 1)^5)
)
```

3.41 $\int \sqrt{b \coth^4(c+dx)}\, dx$

Optimal. Leaf size=50

$$x \tanh^2(c+dx)\sqrt{b \coth^4(c+dx)} - \frac{\tanh(c+dx)\sqrt{b \coth^4(c+dx)}}{d}$$

```
[Out] -((Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d) + x*Sqrt[b*Coth[c + d*x]^4]*Ta
nh[c + d*x]^2
```

Rubi [A] time = 0.0228098, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$x \tanh^2(c+dx)\sqrt{b \coth^4(c+dx)} - \frac{\tanh(c+dx)\sqrt{b \coth^4(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -((Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d) + x*Sqrt[b*Coth[c + d*x]^4]*Ta
nh[c + d*x]^2
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b \coth^4(c+dx)}\, dx &= \left(\sqrt{b \coth^4(c+dx)} \tanh^2(c+dx)\right) \int \coth^2(c+dx)\, dx \\
&= -\frac{\sqrt{b \coth^4(c+dx)} \tanh(c+dx)}{d} + \left(\sqrt{b \coth^4(c+dx)} \tanh^2(c+dx)\right) \int 1\, dx \\
&= -\frac{\sqrt{b \coth^4(c+dx)} \tanh(c+dx)}{d} + x\sqrt{b \coth^4(c+dx)} \tanh^2(c+dx)
\end{aligned}$$

Mathematica [C] time = 0.0306745, size = 41, normalized size = 0.82

$$-\frac{\tanh(c+dx)\sqrt{b\coth^4(c+dx)}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -((Sqrt[b*Coth[c + d*x]^4]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2]
*Tanh[c + d*x])/d)
```

Maple [A] time = 0.041, size = 55, normalized size = 1.1

$$-\frac{2\coth(dx+c)+\ln(\coth(dx+c)-1)-\ln(\coth(dx+c)+1)}{2d(\coth(dx+c))^2}\sqrt{b(\coth(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^4)^(1/2)*(2*coth(d*x+c)+ln(coth(d*x+c)-1)-ln(coth(d*x
+c)+1))/coth(d*x+c)^2
```

Maxima [A] time = 1.81973, size = 46, normalized size = 0.92

$$\frac{(dx+c)\sqrt{b}}{d}+\frac{2\sqrt{b}}{d\left(e^{(-2dx-2c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] (d*x + c)*sqrt(b)/d + 2*sqrt(b)/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 2.15774, size = 1046, normalized size = 20.92

$$\frac{\left(dx\cosh(dx+c)^2+\left(dxe^{(4dx+4c)}-2dxe^{(2dx+2c)}+dx\right)\sinh(dx+c)^2-dx+\left(dx\cosh(dx+c)^2-dx-2\right)e^{(4dx+4c)}-2\left(d\right.}{d\cosh(dx+c)^2+\left(de^{(4dx+4c)}+2de^{(2dx+2c)}+d\right)\sinh(dx+c)^2+\left(d\cosh(dx+c)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) - 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*
```

```
x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) +
6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^2 + (d*e^(4*d*
x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^2 -
d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(
d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x +
 c))*sinh(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**4), x)
```

Giac [A] time = 1.14477, size = 43, normalized size = 0.86

$$\sqrt{b}\left(\frac{dx+c}{d} - \frac{2}{d\left(e^{(2\,dx+2\,c)} - 1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b)*((d*x + c)/d - 2/(d*(e^(2*d*x + 2*c) - 1)))
```

3.42 $\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx$

Optimal. Leaf size=50

$$\frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}-\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}$$

```
[Out] -(Coth[c + d*x]/(d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/Sqrt[b*C
oth[c + d*x]^4]
```

Rubi [A] time = 0.0226638, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}-\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -(Coth[c + d*x]/(d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/Sqrt[b*C
oth[c + d*x]^4]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}dx = \frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+dx)\int 1\,dx}{\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}} + \frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}$$

Mathematica [A] time = 0.072736, size = 40, normalized size = 0.8

$$\frac{\coth(c+dx)\left(\tanh^{-1}(\tanh(c+dx))\coth(c+dx)-1\right)}{d\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*Sqrt[b*Coth[
c + d*x]^4])
```

Maple [A] time = 0.037, size = 59, normalized size = 1.2

$$\frac{\coth(dx+c)(\ln(\coth(dx+c)+1)\coth(dx+c)-\ln(\coth(dx+c)-1)\coth(dx+c)-2)}{2d}\frac{1}{\sqrt{b(\coth(dx+c))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(1/2),x)
```

```
[Out] 1/2/d*coth(d*x+c)*(ln(coth(d*x+c)+1)*coth(d*x+c)-ln(coth(d*x+c)-1)*coth(d*x
+c)-2)/(b*coth(d*x+c)^4)^(1/2)
```

Maxima [A] time = 1.73557, size = 49, normalized size = 0.98

$$\frac{dx+c}{\sqrt{b}d} - \frac{2\sqrt{b}}{\left(be^{(-2\,dx-2\,c)}+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(sqrt(b)*d) - 2*sqrt(b)/((b*e^(-2*d*x - 2*c) + b)*d)
```

Fricas [B] time = 2.19262, size = 1079, normalized size = 21.58

$$\frac{\left(dx\cosh\left(dx+c\right)^{2}+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^{2}+dx+\left(dx\cosh\left(dx+c\right)^{2}+dx+2\right)e^{(4\,dx+4\,c)}-2\,(d}{bd\cosh\left(dx+c\right)^{2}+\left(bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd\right)\sinh\left(dx+c\right)^{2}+bd+\left(bd\cosh\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 + d*x + (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) + 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*
x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) +
6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b*d*cosh(d*x + c)^2 + (b*d*e^(
4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d + (b*d*co
sh(d*x + c)^2 + b*d)*e^(4*d*x + 4*c) + 2*(b*d*cosh(d*x + c)^2 + b*d)*e^(2*d
*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b*d*cosh(d*x + c)*e^(2
*d*x + 2*c) + b*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\coth^{4}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**4), x)
```

Giac [A] time = 1.16942, size = 46, normalized size = 0.92

$$\frac{dx+c}{\sqrt{b}d}+\frac{2}{\sqrt{b}d\left(e^{(2\,dx+2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] (d*x + c)/(sqrt(b)*d) + 2/(sqrt(b)*d*(e^(2*d*x + 2*c) + 1))
```

3.43 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}}-\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}$$

```
[Out] -(Coth[c + d*x]/(b*d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/(b*Sqr
t[b*Coth[c + d*x]^4]) - Tanh[c + d*x]/(3*b*d*Sqrt[b*Coth[c + d*x]^4]) - Tan
h[c + d*x]^3/(5*b*d*Sqrt[b*Coth[c + d*x]^4])
```

Rubi [A] time = 0.0459973, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}}-\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-3/2),x]
```

```
[Out] -(Coth[c + d*x]/(b*d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/(b*Sqr
t[b*Coth[c + d*x]^4]) - Tanh[c + d*x]/(3*b*d*Sqrt[b*Coth[c + d*x]^4]) - Tan
h[c + d*x]^3/(5*b*d*Sqrt[b*Coth[c + d*x]^4])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \frac{\coth^2(c+dx)\int \tanh^6(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+dx)\int \tanh^4(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+dx)\int 1}{b\sqrt{b\coth^4(c+d}}$$
$$= -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}} + \frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx}}$$

Mathematica [A] time = 0.232707, size = 68, normalized size = 0.58

$$\frac{-3\tanh^3(c+dx) - 5\tanh(c+dx) - 15\coth(c+dx) + 15\tanh^{-1}(\tanh(c+dx))\coth^2(c+dx)}{15bd\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-3/2),x]
```

```
[Out] (-15*Coth[c + d*x] + 15*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]^2 - 5*Tanh[c +
 d*x] - 3*Tanh[c + d*x]^3)/(15*b*d*Sqrt[b*Coth[c + d*x]^4])
```

Maple [A] time = 0.022, size = 84, normalized size = 0.7

$$\frac{\coth(dx+c)\left(15\ln(\coth(dx+c)+1)(\coth(dx+c))^5 - 15\ln(\coth(dx+c)-1)(\coth(dx+c))^5 - 30(\coth(dx+c))^4}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(3/2),x)
```

```
[Out] 1/30/d*coth(d*x+c)*(15*ln(coth(d*x+c)+1)*coth(d*x+c)^5-15*ln(coth(d*x+c)-1)
*coth(d*x+c)^5-30*coth(d*x+c)^4-10*coth(d*x+c)^2-6)/(b*coth(d*x+c)^4)^(3/2)
```

Maxima [A] time = 1.80108, size = 209, normalized size = 1.77

$$-\frac{2\left(70\sqrt{b}e^{(-2dx-2c)} + 140\sqrt{b}e^{(-4dx-4c)} + 90\sqrt{b}e^{(-6dx-6c)} + 45\sqrt{b}e^{(-8dx-8c)} + 23\sqrt{b}\right)}{15\left(5b^2e^{(-2dx-2c)} + 10b^2e^{(-4dx-4c)} + 10b^2e^{(-6dx-6c)} + 5b^2e^{(-8dx-8c)} + b^2e^{(-10dx-10c)} + b^2\right)d} + \frac{dx+c}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15*(70*sqrt(b)*e^(-2*d*x - 2*c) + 140*sqrt(b)*e^(-4*d*x - 4*c) + 90*sqrt
(b)*e^(-6*d*x - 6*c) + 45*sqrt(b)*e^(-8*d*x - 8*c) + 23*sqrt(b))/((5*b^2*e^
(-2*d*x - 2*c) + 10*b^2*e^(-4*d*x - 4*c) + 10*b^2*e^(-6*d*x - 6*c) + 5*b^2*
e^(-8*d*x - 8*c) + b^2*e^(-10*d*x - 10*c) + b^2)*d) + (d*x + c)/(b^(3/2)*d)
```

Fricas [B] time = 2.67664, size = 9179, normalized size = 77.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cosh(d*x + c)^10 + 15*(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x +
2*c) + d*x)*sinh(d*x + c)^10 + 150*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d
*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c)^9 + 15*
(5*d*x + 6)*cosh(d*x + c)^8 + 15*(45*d*x*cosh(d*x + c)^2 + 5*d*x + (45*d*x*
cosh(d*x + c)^2 + 5*d*x + 6)*e^(4*d*x + 4*c) - 2*(45*d*x*cosh(d*x + c)^2 +
5*d*x + 6)*e^(2*d*x + 2*c) + 6)*sinh(d*x + c)^8 + 120*(15*d*x*cosh(d*x + c)
^3 + (5*d*x + 6)*cosh(d*x + c) + (15*d*x*cosh(d*x + c)^3 + (5*d*x + 6)*cosh
(d*x + c))*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^3 + (5*d*x + 6)*cosh(d
*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 30*(5*d*x + 6)*cosh(d*x + c)^6
+ 30*(105*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + (1
05*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + 6)*e^(4*d
*x + 4*c) - 2*(105*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5
*d*x + 6)*e^(2*d*x + 2*c) + 6)*sinh(d*x + c)^6 + 60*(63*d*x*cosh(d*x + c)^5
 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x + c) + (63*d*x*c
osh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x +
c))*e^(4*d*x + 4*c) - 2*(63*d*x*cosh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x +
 c)^3 + 3*(5*d*x + 6)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 10*
(15*d*x + 28)*cosh(d*x + c)^4 + 10*(315*d*x*cosh(d*x + c)^6 + 105*(5*d*x +
6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + (315*d*x*cos
h(d*x + c)^6 + 105*(5*d*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x +
c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(315*d*x*cosh(d*x + c)^6 + 105*(5*d
*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(
2*d*x + 2*c) + 28)*sinh(d*x + c)^4 + 40*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x
 + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh
(d*x + c) + (45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(
5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) -
 2*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6
)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x
+ c)^3 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 5*(135*d*x*cosh(d*x + c)^8 + 84*
(5*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x +
 28)*cosh(d*x + c)^2 + 15*d*x + (135*d*x*cosh(d*x + c)^8 + 84*(5*d*x + 6)*c
osh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 28)*cosh(d*x
 + c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(135*d*x*cosh(d*x + c)^8 + 84*(5
*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 2
8)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(2*d*x + 2*c) + 28)*sinh(d*x + c)^2 + 1
5*d*x + (15*d*x*cosh(d*x + c)^10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d
*x + 6)*cosh(d*x + c)^6 + 10*(15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28
)*cosh(d*x + c)^2 + 15*d*x + 46)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^
10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d*x + 6)*cosh(d*x + c)^6 + 10*(
15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 4
6)*e^(2*d*x + 2*c) + 10*(15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh(d*x +
 c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c)^3 +
(15*d*x + 28)*cosh(d*x + c) + (15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh
(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c
```

```
)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c
)^9 + 12*(5*d*x + 6)*cosh(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(
15*d*x + 28)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c)
)*sinh(d*x + c) + 46)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e
^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6
*c) + 6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b^2*d*cosh(d*x + c)^10 +
 5*b^2*d*cosh(d*x + c)^8 + (b^2*d*e^(4*d*x + 4*c) + 2*b^2*d*e^(2*d*x + 2*c)
 + b^2*d)*sinh(d*x + c)^10 + 10*(b^2*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b^
2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^9 +
10*b^2*d*cosh(d*x + c)^6 + 5*(9*b^2*d*cosh(d*x + c)^2 + b^2*d + (9*b^2*d*co
sh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(9*b^2*d*cosh(d*x + c)^2 + b^2*d
)*e^(2*d*x + 2*c))*sinh(d*x + c)^8 + 40*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*co
sh(d*x + c) + (3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(4*d*x + 4*
c) + 2*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^7 + 10*b^2*d*cosh(d*x + c)^4 + 10*(21*b^2*d*cosh(d*x + c)^4 + 14
*b^2*d*cosh(d*x + c)^2 + b^2*d + (21*b^2*d*cosh(d*x + c)^4 + 14*b^2*d*cosh(
d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(21*b^2*d*cosh(d*x + c)^4 + 14*b^2*
d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^6 + 4*(63*b^2*d*c
osh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c) + (63*b^
2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c))*e^
(4*d*x + 4*c) + 2*(63*b^2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15
*b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 5*b^2*d*cosh(d*x +
 c)^2 + 10*(21*b^2*d*cosh(d*x + c)^6 + 35*b^2*d*cosh(d*x + c)^4 + 15*b^2*d*
cosh(d*x + c)^2 + b^2*d + (21*b^2*d*cosh(d*x + c)^6 + 35*b^2*d*cosh(d*x + c
)^4 + 15*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(21*b^2*d*cosh(
d*x + c)^6 + 35*b^2*d*cosh(d*x + c)^4 + 15*b^2*d*cosh(d*x + c)^2 + b^2*d)*e
^(2*d*x + 2*c))*sinh(d*x + c)^4 + 40*(3*b^2*d*cosh(d*x + c)^7 + 7*b^2*d*cos
h(d*x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (3*b^2*d*cos
h(d*x + c)^7 + 7*b^2*d*cosh(d*x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*co
sh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*b^2*d*cosh(d*x + c)^7 + 7*b^2*d*cosh(d*
x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*
sinh(d*x + c)^3 + b^2*d + 5*(9*b^2*d*cosh(d*x + c)^8 + 28*b^2*d*cosh(d*x +
c)^6 + 30*b^2*d*cosh(d*x + c)^4 + 12*b^2*d*cosh(d*x + c)^2 + b^2*d + (9*b^2
*d*cosh(d*x + c)^8 + 28*b^2*d*cosh(d*x + c)^6 + 30*b^2*d*cosh(d*x + c)^4 +
12*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(9*b^2*d*cosh(d*x + c
)^8 + 28*b^2*d*cosh(d*x + c)^6 + 30*b^2*d*cosh(d*x + c)^4 + 12*b^2*d*cosh(d
*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^
10 + 5*b^2*d*cosh(d*x + c)^8 + 10*b^2*d*cosh(d*x + c)^6 + 10*b^2*d*cosh(d*x
 + c)^4 + 5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(
d*x + c)^10 + 5*b^2*d*cosh(d*x + c)^8 + 10*b^2*d*cosh(d*x + c)^6 + 10*b^2*d
*cosh(d*x + c)^4 + 5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c) + 10*(b
^2*d*cosh(d*x + c)^9 + 4*b^2*d*cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 +
4*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^9 + 4*
b^2*d*cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^3 +
 b^2*d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(d*x + c)^9 + 4*b^2*d*
cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^3 + b^2*d
*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-3/2), x)
```

Giac [A] time = 1.25118, size = 138, normalized size = 1.17

$$\frac{\frac{15\,(dx+c)}{\sqrt{b}d}+\frac{2\left(45\,\sqrt{b}e^{(8\,dx+8\,c)}+90\,\sqrt{b}e^{(6\,dx+6\,c)}+140\,\sqrt{b}e^{(4\,dx+4\,c)}+70\,\sqrt{b}e^{(2\,dx+2\,c)}+23\,\sqrt{b}\right)}{bd\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)/(sqrt(b)*d) + 2*(45*sqrt(b)*e^(8*d*x + 8*c) + 90*sqrt(b)
*e^(6*d*x + 6*c) + 140*sqrt(b)*e^(4*d*x + 4*c) + 70*sqrt(b)*e^(2*d*x + 2*c)
 + 23*sqrt(b))/(b*d*(e^(2*d*x + 2*c) + 1)^5))/b
```

3.44 $\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx$

Optimal. Leaf size=353

$$-\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d}-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d}-\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\mathrm{co}}\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

```
[Out] -(Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)
^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^
(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (b*A
rcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(4/
3)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^4)^(1/3))/(7*d) - (3*b*Coth[c + d
*x]^3*(b*Coth[c + d*x]^4)^(1/3))/(13*d) - (b*(b*Coth[c + d*x]^4)^(1/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) +
 (b*(b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*b*(b*Coth[c + d*x]^4)^(1/3)*Tanh[c +
d*x])/d
```

Rubi [A] time = 0.196008, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d}-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d}-\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\mathrm{co}}\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(4/3),x]
```

```
[Out] -(Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)
^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^
(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (b*A
rcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(4/
3)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^4)^(1/3))/(7*d) - (3*b*Coth[c + d
*x]^3*(b*Coth[c + d*x]^4)^(1/3))/(13*d) - (b*(b*Coth[c + d*x]^4)^(1/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) +
 (b*(b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*b*(b*Coth[c + d*x]^4)^(1/3)*Tanh[c +
d*x])/d
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(b\coth^4(c+dx)\right)^{4/3} dx = \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \coth^{\frac{16}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} + \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \coth^{\frac{10}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} + \frac{\left(b\sqrt[3]{b\coth^4(c+\right.}{\text{co}}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\coth^4(c+}{}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\coth^4(c+}{}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\coth^4(c+}{}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\coth^4(c+}{}$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth}{}$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth}{}$$

$$= -\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+}{2d\coth^{\frac{4}{3}}(c+dx)}$$

Mathematica [C] time = 0.139246, size = 68, normalized size = 0.19

$$-\frac{3b\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}\left(-91\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)+7\coth^4(c+dx)+13\coth^2(c+dx)+91\right)}{91d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(4/3),x]
```

```
[Out] (-3*b*(b*Coth[c + d*x]^4)^(1/3)*(91 + 13*Coth[c + d*x]^2 + 7*Coth[c + d*x]^
4 - 91*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])*Tanh[c + d*x])/(91*
d)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \left(b\left(\coth\left(dx+c\right)\right)^4\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(4/3),x)
```

```
[Out] int((b*coth(d*x+c)^4)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^4\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)
```

Fricas [B] time = 2.93785, size = 7973, normalized size = 22.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/364*(182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*sinh(d*x
 + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4*(7*sq
rt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b*cosh(
d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b*cosh(
d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x + c)^2
 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10*sqrt(
3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*sqrt(
3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b*cosh(d
*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2 + 8*(s
qrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b*cosh(d
*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b)*(-b)^(1/3)*
arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b) - 182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*si
nh(d*x + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4
*(7*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b
*cosh(d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b
*cosh(d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x
+ c)^2 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10
*sqrt(3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4
*sqrt(3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b*
cosh(d*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2
+ 8*(sqrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b*
cosh(d*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b)*b^(1/
3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c
))^(1/3))/b) + 91*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 +
b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(
d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 +
6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)
*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh
```

```
(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 -
 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cos
h(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))
*sinh(d*x + c) + b)*(-b)^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)
/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 91*(b*cosh
(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*c
osh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(
d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(
35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b
*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^
3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9
*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d
*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/
3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(2/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
 b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(1/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
 b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)
/sinh(d*x + c))^(1/3)) + 12*(111*b*cosh(d*x + c)^8 + 888*b*cosh(d*x + c)*si
nh(d*x + c)^7 + 111*b*sinh(d*x + c)^8 - 336*b*cosh(d*x + c)^6 + 84*(37*b*co
sh(d*x + c)^2 - 4*b)*sinh(d*x + c)^6 + 168*(37*b*cosh(d*x + c)^3 - 12*b*cos
h(d*x + c))*sinh(d*x + c)^5 + 562*b*cosh(d*x + c)^4 + 2*(3885*b*cosh(d*x +
c)^4 - 2520*b*cosh(d*x + c)^2 + 281*b)*sinh(d*x + c)^4 + 8*(777*b*cosh(d*x
+ c)^5 - 840*b*cosh(d*x + c)^3 + 281*b*cosh(d*x + c))*sinh(d*x + c)^3 - 336
*b*cosh(d*x + c)^2 + 12*(259*b*cosh(d*x + c)^6 - 420*b*cosh(d*x + c)^4 + 28
1*b*cosh(d*x + c)^2 - 28*b)*sinh(d*x + c)^2 + 8*(111*b*cosh(d*x + c)^7 - 25
2*b*cosh(d*x + c)^5 + 281*b*cosh(d*x + c)^3 - 84*b*cosh(d*x + c))*sinh(d*x
+ c) + 111*b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/(d*cosh(d*x + c)^8 + 8
*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6
+ 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x +
 c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^
5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*
x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c
)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d
*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^{4}\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)
```

3.45 $\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx$

Optimal. Leaf size=291

$$-\frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{8}{3}}(c+dx)}+\frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+}\right.}{4d\coth^{\frac{8}{3}}(c+dx)}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2
/3))/(2*d*Coth[c + d*x]^(8/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2/3))/(2*d*Coth[c + d*x]^(8/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(2/3))/(d*Coth[c + d*x]^(8/3)) - (
(b*Coth[c + d*x]^4)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(8/3)) + ((b*Coth[c + d*x]^4)^(2/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(8/3)) - (3*(b*Coth[c
 + d*x]^4)^(2/3)*Tanh[c + d*x])/(5*d)
```

Rubi [A] time = 0.215087, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{8}{3}}(c+dx)}+\frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+}\right.}{4d\coth^{\frac{8}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(2/3),x]
```


```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2
/3))/(2*d*Coth[c + d*x]^(8/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2/3))/(2*d*Coth[c + d*x]^(8/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(2/3))/(d*Coth[c + d*x]^(8/3)) - (
(b*Coth[c + d*x]^4)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(8/3)) + ((b*Coth[c + d*x]^4)^(2/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(8/3)) - (3*(b*Coth[c
 + d*x]^4)^(2/3)*Tanh[c + d*x])/(5*d)
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(b\coth^4(c+dx)\right)^{2/3} dx = \frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \coth^{\frac{8}{3}}(c+dx)\,dx}{\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}dx, x, \coth(c+$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} - \frac{\left(3\left(b\coth^4(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}dx, x, \sqrt[3]{\coth}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1}{1-x^2}dx, x, \sqrt[3]{\coth(c+}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} - \frac{(b\,\text{co}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth(c+dx)}\right.}{4d\coth^{\frac{8}{3}}(c+dx)}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx\right.}{2d\coth^{\frac{8}{3}}(c+dx)}$$

Mathematica [A] time = 0.408708, size = 166, normalized size = 0.57

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(-12\coth^{\frac{5}{3}}(c+dx)+20\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)+5\left(-\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)+\right.\right.}{20d\coth^{\frac{8}{3}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(2/3),x]
```

```
[Out] ((b*Coth[c + d*x]^4)^(2/3)*(20*ArcTanh[Coth[c + d*x]^(1/3)] - 12*Coth[c + d
*x]^(5/3) + 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*Sq
rt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c + d*x]^(
1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])))/(20*d*Coth[c + d*x]^(8/3))
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^4\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(2/3),x)
```

```
[Out] int((b*coth(d*x+c)^4)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth (dx+c)^4\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)
```

Fricas [B] time = 2.26442, size = 1783, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/20*(10*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c)
+ sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b -
2*sqrt(3)*(-b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 10*(sqrt
(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(
d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^
(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 5*(-b^2)^(1/3)*(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh
(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3)) + 5*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c
))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
)) - 10*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3
)) - 10*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3))
 + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x
 + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth^4 (c+dx)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth (dx+c)^4\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)
```

3.46 $\int \sqrt[3]{b\coth^4(c+dx)}\,dx$

Optimal. Leaf size=289

$$-\frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+d}\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(
1/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3
))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTanh
[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(4/3)) -
((b*Coth[c + d*x]^4)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3
)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c +
 d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Coth[
c + d*x]^4)^(1/3)*Tanh[c + d*x])/d
```

Rubi [A] time = 0.175738, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+d}\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(
1/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3
))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTanh
[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(4/3)) -
((b*Coth[c + d*x]^4)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3
)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c +
 d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Coth[
c + d*x]^4)^(1/3)*Tanh[c + d*x])/d
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \frac{\sqrt[3]{b\coth^4(c+dx)}\int \coth^{\frac{4}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt[3]{b\coth^4(c+dx)}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} - \frac{\sqrt[3]{b\coth^4(c+dx)}\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} - \frac{\left(3\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+\right.}}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt[3]{b\coth^4(c+dx)}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} - \frac{\sqrt[3]{b\coth^4}}{}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+c\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+d}}{2d\coth^{\frac{4}{3}}(c+dx)}$$

Mathematica [C] time = 0.0356893, size = 43, normalized size = 0.15

$$\frac{3\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(1/3),x]
```

```
[Out] (3*(b*Coth[c + d*x]^4)^(1/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c +
d*x]^2])*Tanh[c + d*x])/d
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\left(\coth(dx+c)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(1/3),x)
```

```
[Out] int((b*coth(d*x+c)^4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth (dx + c)^{4}\right)^{\frac{1}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)
```

Fricas [A] time = 2.26736, size = 855, normalized size = 2.96

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}b+2\sqrt{3}(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) - 2\sqrt{3}b^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) + (-b)^{\frac{1}{3}}\log\left((-b)^{\frac{2}{3}} - (-b)^{\frac{1}{3}}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*
cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(1/3)*arctan(-1/3*(sqr
t(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-b)
^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) +
(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(1/3)*log(b^(2/3) - b^(1/3)*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3))
- 2*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*
b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(b*cosh(d
*x + c)/sinh(d*x + c))^(1/3))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \coth^{4}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth (dx + c)^{4}\right)^{\frac{1}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)
```

3.47 $\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx$

Optimal. Leaf size=289

$$-\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)\right.}{4d\sqrt[3]{b\coth^4(c+}}$$

```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^4)^
(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^(1
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^4)^(1/3))
```

Rubi [A] time = 0.217884, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)\right.}{4d\sqrt[3]{b\coth^4(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-1/3),x]
```


```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^4)^
(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^(1
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^4)^(1/3))
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\mathrm{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+d}{}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\mathrm{Subst}\left(\int\right.}{4d\sqrt[3]{b\,\mathrm{c}}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\right.}{4d\sqrt[3]{b}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+}}{\sqrt{3}}\right.}{2d\sqrt[3]{b\coth^4(}}$$

Mathematica [C] time = 0.0532765, size = 41, normalized size = 0.14

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};\coth^2(c+dx)\right)}{d\sqrt[3]{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-1/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(d*(b*C
oth[c + d*x]^4)^(1/3))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\left(\coth(dx+c)\right)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \coth (dx+c)^4\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

Fricas [B] time = 2.91105, size = 9634, normalized size = 33.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh
(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*cosh(d*x
+ c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(
d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)
+ (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2
- b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)
/b) + b) + sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + b)*sqrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c
)/sinh(d*x + c))^(2/3)*sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x
+ c)*sinh(d*x + c) - b*sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)
) + (sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3) - 3*b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^
(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
+ 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 12*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x
 + c)/sinh(d*x + c))^(2/3))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh
(d*x + c) + b*d*sinh(d*x + c)^2 + b*d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2
+ 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)
/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cos
```

```
h(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - sqrt(3)*(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-1/b
^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)*sqrt(-1/
b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) - b*sinh(d*x
 + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)) + (sqrt(3)*(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1/b^(2/3)
) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - (cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3)
 - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh
(d*x + c))^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x +
c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/
3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(
d*x + c)/sinh(d*x + c))^(1/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*
d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2
 + b*d), 1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
+ b*sinh(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*c
osh(d*x + c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x +
c)/sinh(d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(2/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 - b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-
b)^(1/3)/b) + b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/
3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(
d*x + c))^(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c
))^(1/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2
*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b^(1/3))/b^(1/3) - 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*d*cos
h(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2 + b*
d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) +
b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*s
qrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt
(-(-b)^(1/3)/b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/
3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(
d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c
))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 2
*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x + c)/sinh(d*x + c)
```

```
)^(1/3))/b^(1/3))/b^(1/3) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*d*cos
h(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2 + b*
d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth(dx+c)^4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

3.48 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=291

$$-\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+\right.}{4d\left(b\coth^4(c\right.}$$

```
[Out] (-3*Coth[c + d*x])/(5*d*(b*Coth[c + d*x]^4)^(2/3)) - (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(8/3))/(2*d*(b*Coth[c + d*x]^4
)^(2/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*
x]^(8/3))/(2*d*(b*Coth[c + d*x]^4)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*C
oth[c + d*x]^(8/3))/(d*(b*Coth[c + d*x]^4)^(2/3)) - (Coth[c + d*x]^(8/3)*Lo
g[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^
(2/3)) + (Coth[c + d*x]^(8/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])/(4*d*(b*Coth[c + d*x]^4)^(2/3))
```

Rubi [A] time = 0.184945, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+\right.}{4d\left(b\coth^4(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(5*d*(b*Coth[c + d*x]^4)^(2/3)) - (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(8/3))/(2*d*(b*Coth[c + d*x]^4
)^(2/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*
x]^(8/3))/(2*d*(b*Coth[c + d*x]^4)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*C
oth[c + d*x]^(8/3))/(d*(b*Coth[c + d*x]^4)^(2/3)) - (Coth[c + d*x]^(8/3)*Lo
g[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^
(2/3)) + (Coth[c + d*x]^(8/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])/(4*d*(b*Coth[c + d*x]^4)^(2/3))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\coth^{\frac{8}{3}}(c+dx)}\,dx}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx)\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\left(3\coth^{\frac{8}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{8}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{8}{3}}}{}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}(c+dx)\,\text{Sul}}{4d}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}(c+dx)\log}{4}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}}{}\right.}{2d\left(b\,\text{co}\right.}$$

Mathematica [C] time = 0.0508721, size = 43, normalized size = 0.15

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{5}{6},1;\frac{1}{6};\coth^2(c+dx)\right)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-5/6, 1, 1/6, Coth[c + d*x]^2])/(5*d*(b
*Coth[c + d*x]^4)^(2/3))
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^4\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{4}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

Fricas [B] time = 2.48705, size = 3221, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/20*(10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d
*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt
(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sqrt(-(-b^2)^(1/3)) - 2
*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b^2)^(1
/3)))/b^2) + 10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) +
b)*(b^2)^(1/6)*arctan(-1/3*sqrt(3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b^2) + 5*(cosh(d*x + c)^4 + 4*cos
h(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*si
nh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sin
h(d*x + c) + 1)*(-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) -
(-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 5*(c
osh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*c
osh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/si
nh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x
+ c))^(1/3)) - 10*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh
(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2
 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b^2)^(2/3)*log(
b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 10*(cosh(d*x + c)
^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)
^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x
 + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))
^(1/3) + (b^2)^(2/3)) - 12*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)
 + b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/(b^2*d*cosh(d*x + c)^4 + 4*b^2
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x
 + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(
b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth^4\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

3.49 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=369

$$-\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\ldots\right.}{4bd\sqrt[3]{b\coth^4(c+\ldots}}$$

```
[Out] (-3*Coth[c + d*x])/(b*d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]
^4)^(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c +
d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3
)]*Coth[c + d*x]^(4/3))/(b*d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4
/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c +
d*x]^4)^(1/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c
+ d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x])/(7*b*d
*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x]^3)/(13*b*d*(b*Coth[c + d*x]^
4)^(1/3))
```

Rubi [A] time = 0.244683, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\ldots\right.}{4bd\sqrt[3]{b\coth^4(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-4/3),x]
```


```
[Out] (-3*Coth[c + d*x])/(b*d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]
^4)^(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c +
d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3
)]*Coth[c + d*x]^(4/3))/(b*d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4
/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c +
d*x]^4)^(1/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c
+ d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x])/(7*b*d
*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x]^3)/(13*b*d*(b*Coth[c + d*x]^
4)^(1/3))
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{16}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{10}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int}{b\sqrt[3]{b\coth}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\mathrm{S}}{b}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx\right.}{}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\,\mathrm{S}}{b}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} -$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\right.}{4bd\sqrt[3]{b}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+d}}{\sqrt{3}}\right.}{2bd\sqrt[3]{b\coth^4}}$$

Mathematica [C] time = 0.0421127, size = 43, normalized size = 0.12

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{13}{6},1;-\frac{7}{6};\coth^2(c+dx)\right)}{13d\left(b\coth^4(c+dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-13/6, 1, -7/6, Coth[c + d*x]^2])/(13*d
*(b*Coth[c + d*x]^4)^(4/3))
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^4\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(4/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^4)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{4}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth^{4}\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{4}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

3.50 $\int \left(b\coth^m(c+dx)\right)^n\,dx$

Optimal. Leaf size=57

$$\frac{\coth(c+dx)\left(b\coth^m(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\coth^2(c+dx)\right)}{d(mn+1)}$$

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
 m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))
```

Rubi [A] time = 0.0423468, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3659, 3476, 364}

$$\frac{\coth(c+dx)\left(b\coth^m(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\coth^2(c+dx)\right)}{d(mn+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
 m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\coth^m(c+dx)\right)^n\,dx = \left(\coth^{-mn}(c+dx)\left(b\coth^m(c+dx)\right)^n\right)\int \coth^{mn}(c+dx)\,dx$$
$$= -\frac{\left(\coth^{-mn}(c+dx)\left(b\coth^m(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{mn}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}$$
$$= \frac{\coth(c+dx)\left(b\coth^m(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(1+mn);\frac{1}{2}(3+mn);\coth^2(c+dx)\right)}{d(1+mn)}$$

Mathematica [A] time = 0.0481855, size = 55, normalized size = 0.96

$$\frac{\coth(c+dx)\left(b\coth^m(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\coth^2(c+dx)\right)}{dmn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
 m*n)/2, Coth[c + d*x]^2])/(d + d*m*n)
```

Maple [F] time = 2.588, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^m\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^n,x)
```

```
[Out] int((b*coth(d*x+c)^m)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth(dx+c)^m\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\coth(dx+c)^m\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^m)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth^{m}\left(c+dx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^{m}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^n, x)
```

3.51 $\int \left(b \coth^m(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=63

$$\frac{2b\coth^{m+1}(c+dx)\sqrt{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\coth^2(c+dx)\right)}{d(3m+2)}$$

```
[Out] (2*b*Coth[c + d*x]^(1 + m)*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2
+ 3*m)/4, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

Rubi [A] time = 0.0454474, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2b\coth^{m+1}(c+dx)\sqrt{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\coth^2(c+dx)\right)}{d(3m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(3/2),x]
```

```
[Out] (2*b*Coth[c + d*x]^(1 + m)*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2
+ 3*m)/4, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\coth^m(c+dx)\right)^{3/2} dx = \left(b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\int \coth^{\frac{3m}{2}}(c+dx)\,dx$$
$$= -\frac{\left(b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\mathrm{Subst}\left(\int \frac{x^{3m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{2b\coth^{1+m}(c+dx)\sqrt{b\coth^m(c+dx)}\,{}_2F_1\left(1, \frac{1}{4}(2+3m); \frac{3(2+m)}{4}; \coth^2(c+dx)\right)}{d(2+3m)}$$

Mathematica [A] time = 0.0747188, size = 58, normalized size = 0.92

$$\frac{2\coth(c+dx)\left(b\coth^m(c+dx)\right)^{3/2}\,{}_2F_1\left(1, \frac{1}{4}(3m+2); \frac{3(m+2)}{4}; \coth^2(c+dx)\right)}{d(3m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(3/2),x]
```

```
[Out] (2*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(3/2)*Hypergeometric2F1[1, (2 + 3*m)/4
, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^m\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(3/2),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth(dx+c)^m\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth \left(dx+c\right)^{m}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)
```

3.52 $\int \sqrt{b \coth^m(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+2}{4};\frac{m+6}{4};\coth^2(c+dx)\right)}{d(m+2)}$$

```
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Rubi [A] time = 0.0407944, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+2}{4};\frac{m+6}{4};\coth^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \left(\coth^{-\frac{m}{2}}(c+dx)\sqrt{b \coth^m(c+dx)}\right)\int \coth^{\frac{m}{2}}(c+dx)\,dx$$
$$= -\frac{\left(\coth^{-\frac{m}{2}}(c+dx)\sqrt{b \coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{2\coth(c+dx)\sqrt{b \coth^m(c+dx)}\,{}_2F_1\left(1, \frac{2+m}{4}; \frac{6+m}{4}; \coth^2(c+dx)\right)}{d(2+m)}$$

Mathematica [A] time = 0.0397064, size = 54, normalized size = 1.

$$\frac{2\coth(c+dx)\sqrt{b \coth^m(c+dx)}\,{}_2F_1\left(1, \frac{m+2}{4}; \frac{m+6}{4}; \coth^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Maple [F] time = 0.186, size = 0, normalized size = 0.

$$\int \sqrt{b \left(\coth(dx+c)\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(1/2),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^m), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth^{m}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**m), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx + c)^{m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^m), x)
```

3.53 $\int \frac{1}{\sqrt{b \coth^m(c+dx)}} \, dx$

Optimal. Leaf size=60

$$\frac{2 \coth(c+dx)\,{}_2F_1\left(1, \frac{2-m}{4}; \frac{6-m}{4}; \coth^2(c+dx)\right)}{d(2-m)\sqrt{b \coth^m(c+dx)}}$$

```
[Out] (2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^2
])/(d*(2 - m)*Sqrt[b*Coth[c + d*x]^m])
```

Rubi [A] time = 0.047278, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2 \coth(c+dx)\,{}_2F_1\left(1, \frac{2-m}{4}; \frac{6-m}{4}; \coth^2(c+dx)\right)}{d(2-m)\sqrt{b \coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^2
])/(d*(2 - m)*Sqrt[b*Coth[c + d*x]^m])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \frac{\coth^{\frac{m}{2}}(c+dx)\int\coth^{-\frac{m}{2}}(c+dx)\,dx}{\sqrt{b\coth^m(c+dx)}}$$
$$= -\frac{\coth^{\frac{m}{2}}(c+dx)\,\text{Subst}\left(\int\frac{x^{-m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt{b\coth^m(c+dx)}}$$
$$= \frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

Mathematica [A] time = 0.0473323, size = 58, normalized size = 0.97

$$-\frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(m-2)\sqrt{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^
2])/(d*(-2 + m)*Sqrt[b*Coth[c + d*x]^m])
```

Maple [F] time = 0.195, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\left(\coth\left(dx+c\right)\right)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth\left(dx+c\right)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth^{m}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**m), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth(dx+c)^{m}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

3.54 $\int \frac{1}{\left(b \coth^m(c+dx)\right)^{3/2}} \, dx$

Optimal. Leaf size=69

$$\frac{2 \coth^{1-m}(c+dx)\, {}_2F_1\left(1,\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

```
[Out] (2*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (2 - 3*m)/4, (3*(2 - m))/4, C
oth[c + d*x]^2])/(b*d*(2 - 3*m)*Sqrt[b*Coth[c + d*x]^m])
```

Rubi [A] time = 0.0481878, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2 \coth^{1-m}(c+dx)\, {}_2F_1\left(1,\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-3/2),x]
```

```
[Out] (2*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (2 - 3*m)/4, (3*(2 - m))/4, C
oth[c + d*x]^2])/(b*d*(2 - 3*m)*Sqrt[b*Coth[c + d*x]^m])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^m(c+dx)\right)^{3/2}}\,dx = \frac{\coth^{\frac{m}{2}}(c+dx)\int \coth^{-\frac{3m}{2}}(c+dx)\,dx}{b\sqrt{b\coth^m(c+dx)}}$$
$$= -\frac{\coth^{\frac{m}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{x^{-3m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{bd\sqrt{b\coth^m(c+dx)}}$$
$$= \frac{2\coth^{1-m}(c+dx)\,{}_2F_1\left(1, \frac{1}{4}(2-3m); \frac{3(2-m)}{4}; \coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

Mathematica [A] time = 0.0661005, size = 58, normalized size = 0.84

$$-\frac{2\coth(c+dx)\,{}_2F_1\left(1, \frac{1}{4}(2-3m); -\frac{3}{4}(m-2); \coth^2(c+dx)\right)}{d(3m-2)\left(b\coth^m(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - 3*m)/4, (-3*(-2 + m))/4, Coth[c
 + d*x]^2])/(d*(-2 + 3*m)*(b*Coth[c + d*x]^m)^(3/2))
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^m\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth(dx+c)^m\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{m}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

3.55 $\int \left(b\coth^m(c+dx)\right)^{4/3}\,dx$

Optimal. Leaf size=65

$$\frac{3b\coth^{m+1}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{1}{6}(4m+3);\frac{1}{6}(4m+9);\coth^2(c+dx)\right)}{d(4m+3)}$$

```
[Out] (3*b*Coth[c + d*x]^(1 + m)*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (
3 + 4*m)/6, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

Rubi [A] time = 0.0453596, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3b\coth^{m+1}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{1}{6}(4m+3);\frac{1}{6}(4m+9);\coth^2(c+dx)\right)}{d(4m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(4/3),x]
```

```
[Out] (3*b*Coth[c + d*x]^(1 + m)*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (
3 + 4*m)/6, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\coth^m(c+dx)\right)^{4/3}\,dx = \left(b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\int \coth^{\frac{4m}{3}}(c+dx)\,dx$$
$$= -\frac{\left(b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{4m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{3b\coth^{1+m}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1, \frac{1}{6}(3+4m); \frac{1}{6}(9+4m); \coth^2(c+dx)\right)}{d(3+4m)}$$

Mathematica [A] time = 0.0703297, size = 60, normalized size = 0.92

$$\frac{3\coth(c+dx)\left(b\coth^m(c+dx)\right)^{4/3}\,{}_2F_1\left(1, \frac{1}{6}(4m+3); \frac{1}{6}(4m+9); \coth^2(c+dx)\right)}{d(4m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(4/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(4/3)*Hypergeometric2F1[1, (3 + 4*m)/6
, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^m\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(4/3),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth(dx+c)^m\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth \left(dx + c\right)^{m}\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)
```

3.56 $\int \left(b \coth^m(c + dx)\right)^{2/3} \, dx$

Optimal. Leaf size=60

$$\frac{3 \coth(c + dx) \left(b \coth^m(c + dx)\right)^{2/3} {}_2F_1\left(1, \frac{1}{6}(2m + 3); \frac{1}{6}(2m + 9); \coth^2(c + dx)\right)}{d(2m + 3)}$$

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6
, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

Rubi [A] time = 0.0430382, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3 \coth(c + dx) \left(b \coth^m(c + dx)\right)^{2/3} {}_2F_1\left(1, \frac{1}{6}(2m + 3); \frac{1}{6}(2m + 9); \coth^2(c + dx)\right)}{d(2m + 3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(2/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6
, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\coth^m(c+dx)\right)^{2/3} dx = \left(\coth^{-\frac{2m}{3}}(c+dx)\left(b\coth^m(c+dx)\right)^{2/3}\right)\int \coth^{\frac{2m}{3}}(c+dx)\,dx$$
$$= -\frac{\left(\coth^{-\frac{2m}{3}}(c+dx)\left(b\coth^m(c+dx)\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{x^{2m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{3\coth(c+dx)\left(b\coth^m(c+dx)\right)^{2/3} {}_2F_1\left(1, \frac{1}{6}(3+2m); \frac{1}{6}(9+2m); \coth^2(c+dx)\right)}{d(3+2m)}$$

Mathematica [A] time = 0.0417521, size = 60, normalized size = 1.

$$\frac{3\coth(c+dx)\left(b\coth^m(c+dx)\right)^{2/3} {}_2F_1\left(1, \frac{1}{6}(2m+3); \frac{1}{6}(2m+9); \coth^2(c+dx)\right)}{d(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(2/3),x]

[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6
, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^m\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(2/3),x)

[Out] int((b*coth(d*x+c)^m)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth(dx+c)^m\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \coth \left(dx+c\right)^{m}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)
```

3.57 $\int \sqrt[3]{b \coth^m(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+3}{6};\frac{m+9}{6};\coth^2(c+dx)\right)}{d(m+3)}$$

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Rubi [A] time = 0.0437366, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+3}{6};\frac{m+9}{6};\coth^2(c+dx)\right)}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(1/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \left(\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\int \coth^{\frac{m}{3}}(c+dx)\,dx$$
$$= -\frac{\left(\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\mathrm{Subst}\left(\int \frac{x^{m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{3+m}{6};\frac{9+m}{6};\coth^2(c+dx)\right)}{d(3+m)}$$

Mathematica [A] time = 0.0386344, size = 54, normalized size = 1.

$$\frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+3}{6};\frac{m+9}{6};\coth^2(c+dx)\right)}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(1/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\left(\coth\left(dx+c\right)\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(1/3),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth\left(dx+c\right)^m\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\coth^{m}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\coth(dx+c)^{m}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)
```

3.58 $\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}} \, dx$

Optimal. Leaf size=60

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{3-m}{6};\frac{9-m}{6};\coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^2
])/(d*(3 - m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rubi [A] time = 0.0426262, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{3-m}{6};\frac{9-m}{6};\coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-1/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^2
])/(d*(3 - m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx = \frac{\coth^{\frac{m}{3}}(c+dx)\int \coth^{-\frac{m}{3}}(c+dx)\,dx}{\sqrt[3]{b\coth^m(c+dx)}}$$
$$= -\frac{\coth^{\frac{m}{3}}(c+dx)\,\text{Subst}\left(\int \frac{x^{-m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^m(c+dx)}}$$
$$= \frac{3\coth(c+dx)\,{}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}$$

Mathematica [A] time = 0.0390112, size = 58, normalized size = 0.97

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c+dx)\right)}{d(m-3)\sqrt[3]{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-1/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^
2])/(d*(-3 + m)*(b*Coth[c + d*x]^m)^(1/3))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\,(\coth(dx+c))^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth(dx+c)^m\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b \coth^{m}(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \coth(dx+c)^{m}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

3.59 $\int \frac{1}{\left(b\coth^m(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=60

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-2m);\frac{1}{6}(9-2m);\coth^2(c+dx)\right)}{d(3-2m)\left(b\coth^m(c+dx)\right)^{2/3}}$$

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*
x]^2])/(d*(3 - 2*m)*(b*Coth[c + d*x]^m)^(2/3))
```

Rubi [A] time = 0.0445252, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-2m);\frac{1}{6}(9-2m);\coth^2(c+dx)\right)}{d(3-2m)\left(b\coth^m(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-2/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*
x]^2])/(d*(3 - 2*m)*(b*Coth[c + d*x]^m)^(2/3))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^m(c+dx)\right)^{2/3}}\,dx = \frac{\coth^{\frac{2m}{3}}(c+dx)\int \coth^{-\frac{2m}{3}}(c+dx)\,dx}{\left(b\coth^m(c+dx)\right)^{2/3}}$$
$$= -\frac{\coth^{\frac{2m}{3}}(c+dx)\,\text{Subst}\left(\int \frac{x^{-2m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\left(b\coth^m(c+dx)\right)^{2/3}}$$
$$= \frac{3\coth(c+dx)\,{}_2F_1\left(1, \frac{1}{6}(3-2m); \frac{1}{6}(9-2m); \coth^2(c+dx)\right)}{d(3-2m)\left(b\coth^m(c+dx)\right)^{2/3}}$$

Mathematica [A] time = 0.0398727, size = 60, normalized size = 1.

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(1, \frac{1}{6}(3-2m); \frac{1}{6}(9-2m); \coth^2(c+dx)\right)}{d(2m-3)\left(b\coth^m(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d
*x]^2])/(d*(-3 + 2*m)*(b*Coth[c + d*x]^m)^(2/3))
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^m\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth(dx+c)^m\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth^{m}\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{m}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

3.60 $\int \frac{1}{\left(b\coth^m(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=69

$$\frac{3\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[Out] (3*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Cot
h[c + d*x]^2])/(b*d*(3 - 4*m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rubi [A] time = 0.0472706, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-4/3),x]
```

```
[Out] (3*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Cot
h[c + d*x]^2])/(b*d*(3 - 4*m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^m(c+dx)\right)^{4/3}}\,dx = \frac{\coth^{\frac{m}{3}}(c+dx)\int \coth^{-\frac{4m}{3}}(c+dx)\,dx}{b\sqrt[3]{b\coth^m(c+dx)}}$$
$$= -\frac{\coth^{\frac{m}{3}}(c+dx)\,\text{Subst}\left(\int \frac{x^{-4m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{bd\sqrt[3]{b\coth^m(c+dx)}}$$
$$= \frac{3\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

Mathematica [A] time = 0.0659447, size = 60, normalized size = 0.87

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{d(4m-3)\left(b\coth^m(c+dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Coth[c + d
*x]^2])/(d*(-3 + 4*m)*(b*Coth[c + d*x]^m)^(4/3))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \left(b\left(\coth(dx+c)\right)^m\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(4/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\coth(dx+c)^m\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \coth \left(dx + c\right)^{m}\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

3.61 $\int (1+\coth(x))^5 \, dx$

Optimal. Leaf size=41

$$16x - \frac{1}{4}(\coth(x)+1)^4 - \frac{2}{3}(\coth(x)+1)^3 - 2(\coth(x)+1)^2 - 8\coth(x) + 16\log(\sinh(x))$$

```
[Out] 16*x - 8*Coth[x] - 2*(1 + Coth[x])^2 - (2*(1 + Coth[x])^3)/3 - (1 + Coth[x]
)^4/4 + 16*Log[Sinh[x]]
```

Rubi [A] time = 0.0394656, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3478, 3477, 3475}

$$16x - \frac{1}{4}(\coth(x)+1)^4 - \frac{2}{3}(\coth(x)+1)^3 - 2(\coth(x)+1)^2 - 8\coth(x) + 16\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^5,x]
```

```
[Out] 16*x - 8*Coth[x] - 2*(1 + Coth[x])^2 - (2*(1 + Coth[x])^3)/3 - (1 + Coth[x]
)^4/4 + 16*Log[Sinh[x]]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (1+\coth(x))^5 \, dx &= -\frac{1}{4}(1+\coth(x))^4 + 2\int (1+\coth(x))^4 \, dx \\
&= -\frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 4\int (1+\coth(x))^3 \, dx \\
&= -2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 8\int (1+\coth(x))^2 \, dx \\
&= 16x - 8\coth(x) - 2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 16\int \coth(x) \, dx \\
&= 16x - 8\coth(x) - 2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 16\log(\sinh(x))
\end{aligned}$$

Mathematica [C] time = 0.238116, size = 94, normalized size = 2.29

$$\frac{\sinh(x)(\coth(x)+1)^5\left(-20\sinh(x)\cosh^3(x)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(x)\right)-120\sinh^3(x)\cosh(x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(x)\right)-\right.}{12(\sinh(x)+\cosh(x))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^5,x]
```

```
[Out] ((1 + Coth[x])^5*Sinh[x]*(-3*Cosh[x]^4 - 20*Cosh[x]^3*Hypergeometric2F1[-3/
2, 1, -1/2, Tanh[x]^2]*Sinh[x] - 66*Cosh[x]^2*Sinh[x]^2 - 120*Cosh[x]*Hyper
geometric2F1[-1/2, 1, 1/2, Tanh[x]^2]*Sinh[x]^3 + 12*(x + 16*Log[Cosh[x]] +
 16*Log[Tanh[x]])*Sinh[x]^4))/(12*(Cosh[x] + Sinh[x])^5)
```

Maple [A] time = 0.007, size = 31, normalized size = 0.8

$$-\frac{(\coth(x))^4}{4}-\frac{5\,(\coth(x))^3}{3}-\frac{11\,(\coth(x))^2}{2}-15\coth(x)-16\ln(\coth(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^5,x)
```

```
[Out] -1/4*coth(x)^4-5/3*coth(x)^3-11/2*coth(x)^2-15*coth(x)-16*ln(coth(x)-1)
```

Maxima [B] time = 1.0613, size = 189, normalized size = 4.61

$$27\,x-\frac{20\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}-2\right)}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}+\frac{4\left(e^{(-2\,x)}-e^{(-4\,x)}+e^{(-6\,x)}\right)}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}+\frac{20\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\frac{20}{e^{(-2\,x)}-1}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^5,x, algorithm="maxima")
```

```
[Out] 27*x - 20/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*
x) - 1) + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e
^(-6*x) - e^(-8*x) - 1) + 20*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 20/(e^(
-2*x) - 1) + 11*log(e^(-x) + 1) + 11*log(e^(-x) - 1) + 5*log(sinh(x))
```

Fricas [B] time = 2.04947, size = 1481, normalized size = 36.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^5,x, algorithm="fricas")
```

```
[Out] -4/3*(48*cosh(x)^6 + 288*cosh(x)*sinh(x)^5 + 48*sinh(x)^6 + 36*(20*cosh(x)^
2 - 3)*sinh(x)^4 - 108*cosh(x)^4 + 48*(20*cosh(x)^3 - 9*cosh(x))*sinh(x)^3
+ 8*(90*cosh(x)^4 - 81*cosh(x)^2 + 11)*sinh(x)^2 + 88*cosh(x)^2 - 12*(cosh(
```

```
x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*
cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*co
sh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*co
sh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)
^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh
(x) + 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 16*(18*cosh(x)^5 - 27*cosh(x)
^3 + 11*cosh(x))*sinh(x) - 25)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8
 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x)
)*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 +
 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15
*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*co
sh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [A] time = 2.58923, size = 48, normalized size = 1.17

$$32x - 16\log(\tanh(x) + 1) + 16\log(\tanh(x)) - \frac{15}{\tanh(x)} - \frac{11}{2\tanh^2(x)} - \frac{5}{3\tanh^3(x)} - \frac{1}{4\tanh^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**5,x)
```

```
[Out] 32*x - 16*log(tanh(x) + 1) + 16*log(tanh(x)) - 15/tanh(x) - 11/(2*tanh(x)**
2) - 5/(3*tanh(x)**3) - 1/(4*tanh(x)**4)
```

Giac [A] time = 1.18271, size = 55, normalized size = 1.34

$$-\frac{4\left(48\,e^{(6\,x)} - 108\,e^{(4\,x)} + 88\,e^{(2\,x)} - 25\right)}{3\left(e^{(2\,x)} - 1\right)^4} + 16\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^5,x, algorithm="giac")
```

```
[Out] -4/3*(48*e^(6*x) - 108*e^(4*x) + 88*e^(2*x) - 25)/(e^(2*x) - 1)^4 + 16*log(
abs(e^(2*x) - 1))
```

3.62 $\int (1 + \coth(x))^4\,dx$

Optimal. Leaf size=31

$$8x-\frac{1}{3}(\coth(x)+1)^3-(\coth(x)+1)^2-4\coth(x)+8\log(\sinh(x))$$

```
[Out] 8*x - 4*Coth[x] - (1 + Coth[x])^2 - (1 + Coth[x])^3/3 + 8*Log[Sinh[x]]
```

Rubi [A] time = 0.0309066, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3478, 3477, 3475}

$$8x-\frac{1}{3}(\coth(x)+1)^3-(\coth(x)+1)^2-4\coth(x)+8\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^4,x]
```

```
[Out] 8*x - 4*Coth[x] - (1 + Coth[x])^2 - (1 + Coth[x])^3/3 + 8*Log[Sinh[x]]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (1+\coth(x))^4\,dx &= -\frac{1}{3}(1+\coth(x))^3+2\int (1+\coth(x))^3\,dx\\
&= -(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+4\int (1+\coth(x))^2\,dx\\
&= 8x-4\coth(x)-(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+8\int \coth(x)\,dx\\
&= 8x-4\coth(x)-(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+8\log(\sinh(x))
\end{aligned}$$

Mathematica [C] time = 0.179641, size = 84, normalized size = 2.71

$$\frac{\sinh(x)(\coth(x)+1)^4\left(3\sinh(x)\left(-6\sinh(x)\cosh(x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(x)\right)-2\cosh^2(x)+\sinh^2(x)(x+8\log(\tanh(x)\right.\right.}{3(\sinh(x)+\cosh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^4,x]
```

```
[Out] ((1 + Coth[x])^4*Sinh[x]*(-(Cosh[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh
[x]^2]) + 3*Sinh[x]*(-2*Cosh[x]^2 - 6*Cosh[x]*Hypergeometric2F1[-1/2, 1, 1/
2, Tanh[x]^2]*Sinh[x] + (x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]])*Sinh[x]^2)))/
(3*(Cosh[x] + Sinh[x])^4)
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$-\frac{(\coth(x))^3}{3} - 2(\coth(x))^2 - 7\coth(x) - 8\ln(\coth(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^4,x)
```

```
[Out] -1/3*coth(x)^3-2*coth(x)^2-7*coth(x)-8*ln(coth(x)-1)
```

Maxima [B] time = 1.10124, size = 128, normalized size = 4.13

$$12x - \frac{4\left(3e^{(-2x)} - 3e^{(-4x)} - 2\right)}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)} + \frac{8e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{12}{e^{(-2x)} - 1} + 4\log\left(e^{(-x)} + 1\right) + 4\log\left(e^{(-x)} - 1\right) + 4\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^4,x, algorithm="maxima")
```

```
[Out] 12*x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x
) - 1) + 8*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 12/(e^(-2*x) - 1) + 4*log
(e^(-x) + 1) + 4*log(e^(-x) - 1) + 4*log(sinh(x))
```

Fricas [B] time = 2.06893, size = 915, normalized size = 29.52

$$-\frac{4\left(18\cosh(x)^4 + 72\cosh(x)\sinh(x)^3 + 18\sinh(x)^4 + 27\left(4\cosh(x)^2 - 1\right)\sinh(x)^2 - 27\cosh(x)^2 - 6\left(\cosh(x)^6\right.\right.}{3\left(\cosh(x)^6 + 6\cosh(x)\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^4,x, algorithm="fricas")
```

```
[Out] -4/3*(18*cosh(x)^4 + 72*cosh(x)*sinh(x)^3 + 18*sinh(x)^4 + 27*(4*cosh(x)^2
- 1)*sinh(x)^2 - 27*cosh(x)^2 - 6*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh
(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2
+ 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)*log(2*sinh(x)/(cosh(x)
 - sinh(x))) + 18*(4*cosh(x)^3 - 3*cosh(x))*sinh(x) + 11)/(cosh(x)^6 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 +
 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*
```

```
sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1
)
```

Sympy [A] time = 1.88352, size = 37, normalized size = 1.19

$$16x - 8\log(\tanh(x) + 1) + 8\log(\tanh(x)) - \frac{7}{\tanh(x)} - \frac{2}{\tanh^2(x)} - \frac{1}{3\tanh^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**4,x)
```

```
[Out] 16*x - 8*log(tanh(x) + 1) + 8*log(tanh(x)) - 7/tanh(x) - 2/tanh(x)**2 - 1/(
3*tanh(x)**3)
```

Giac [A] time = 1.1945, size = 47, normalized size = 1.52

$$-\frac{4\left(18\,e^{(4\,x)} - 27\,e^{(2\,x)} + 11\right)}{3\left(e^{(2\,x)} - 1\right)^3} + 8\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^4,x, algorithm="giac")
```

```
[Out] -4/3*(18*e^(4*x) - 27*e^(2*x) + 11)/(e^(2*x) - 1)^3 + 8*log(abs(e^(2*x) - 1
))
```

3.63 $\int (1+\coth(x))^3\,dx$

Optimal. Leaf size=23

$$4x-\frac{1}{2}(\coth(x)+1)^2-2\coth(x)+4\log(\sinh(x))$$

```
[Out] 4*x - 2*Coth[x] - (1 + Coth[x])^2/2 + 4*Log[Sinh[x]]
```

Rubi [A] time = 0.0214076, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3478, 3477, 3475}

$$4x-\frac{1}{2}(\coth(x)+1)^2-2\coth(x)+4\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^3,x]
```

```
[Out] 4*x - 2*Coth[x] - (1 + Coth[x])^2/2 + 4*Log[Sinh[x]]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d, x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^3\,dx &= -\frac{1}{2}(1+\coth(x))^2+2\int (1+\coth(x))^2\,dx\\ &= 4x-2\coth(x)-\frac{1}{2}(1+\coth(x))^2+4\int \coth(x)\,dx\\ &= 4x-2\coth(x)-\frac{1}{2}(1+\coth(x))^2+4\log(\sinh(x))\end{aligned}$$

Mathematica [C] time = 0.149008, size = 61, normalized size = 2.65

$$\frac{1}{4}\text{csch}^2(x)\left(-6\sinh(2x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(x)\right)-2x-8\log(\tanh(x))-8\log(\cosh(x))+\cosh(2x)(2x+8\log(\tanh$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^3,x]
```

```
[Out] (Csch[x]^2*(-1 - 2*x - 8*Log[Cosh[x]] - 8*Log[Tanh[x]] + Cosh[2*x]*(-1 + 2*
x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]]) - 6*Hypergeometric2F1[-1/2, 1, 1/2, Ta
nh[x]^2]*Sinh[2*x]))/4
```

Maple [A] time = 0.002, size = 19, normalized size = 0.8

$$-\frac{(\coth(x))^2}{2} - 3\coth(x) - 4\ln(\coth(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^3,x)
```

```
[Out] -1/2*coth(x)^2-3*coth(x)-4*ln(coth(x)-1)
```

Maxima [B] time = 1.03956, size = 74, normalized size = 3.22

$$5x + \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{6}{e^{(-2x)} - 1} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right) + 3\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="maxima")
```

```
[Out] 5*x + 2*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 6/(e^(-2*x) - 1) + log(e^(-x
) + 1) + log(e^(-x) - 1) + 3*log(sinh(x))
```

Fricas [B] time = 2.03912, size = 479, normalized size = 20.83

$$-\frac{2\left(4\cosh(x)^2 - 2\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 4\left(\cosh\right.\right.\right.}{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="fricas")
```

```
[Out] -2*(4*cosh(x)^2 - 2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*
log(2*sinh(x)/(cosh(x) - sinh(x))) + 8*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2
 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [A] time = 1.17971, size = 31, normalized size = 1.35

$$8x - 4\log(\tanh(x) + 1) + 4\log(\tanh(x)) - \frac{3}{\tanh(x)} - \frac{1}{2\tanh^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**3,x)
```

```
[Out] 8*x - 4*log(tanh(x) + 1) + 4*log(tanh(x)) - 3/tanh(x) - 1/(2*tanh(x)**2)
```

Giac [A] time = 1.1699, size = 39, normalized size = 1.7

$$-\frac{2\left(4\,e^{(2\,x)}-3\right)}{\left(e^{(2\,x)}-1\right)^{2}}+4\,\log\left(\left|e^{(2\,x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="giac")
```

```
[Out] -2*(4*e^(2*x) - 3)/(e^(2*x) - 1)^2 + 4*log(abs(e^(2*x) - 1))
```

3.64 $\int (1+\coth(x))^2\,dx$

Optimal. Leaf size=13

$$2x-\coth(x)+2\log(\sinh(x))$$

```
[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]
```

Rubi [A] time = 0.0133208, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3477, 3475}

$$2x-\coth(x)+2\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^2,x]
```

```
[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^2\,dx &= 2x-\coth(x)+2\int \coth(x)\,dx\\ &= 2x-\coth(x)+2\log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0043895, size = 13, normalized size = 1.

$$2x-\coth(x)+2\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^2,x]
```

```
[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]
```

Maple [A] time = 0.003, size = 13, normalized size = 1.

$$-\coth(x)-2\ln(\coth(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^2,x)
```

```
[Out] -coth(x)-2*ln(coth(x)-1)
```

Maxima [A] time = 1.13003, size = 26, normalized size = 2.

$$2\,x + \frac{2}{e^{(-2\,x)} - 1} + 2\,\log\left(\sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x + 2/(e^(-2*x) - 1) + 2*log(sinh(x))
```

Fricas [B] time = 1.95735, size = 189, normalized size = 14.54

$$\frac{2\left(\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 1\right)\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right) - 1\right)}{\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^2,x, algorithm="fricas")
```

```
[Out] 2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) -
 sinh(x))) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [A] time = 0.745568, size = 22, normalized size = 1.69

$$4x - 2\log\left(\tanh\left(x\right) + 1\right) + 2\log\left(\tanh\left(x\right)\right) - \frac{1}{\tanh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**2,x)
```

```
[Out] 4*x - 2*log(tanh(x) + 1) + 2*log(tanh(x)) - 1/tanh(x)
```

Giac [A] time = 1.18667, size = 28, normalized size = 2.15

$$-\frac{2}{e^{(2\,x)} - 1} + 2\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^2,x, algorithm="giac")
```

```
[Out] -2/(e^(2*x) - 1) + 2*log(abs(e^(2*x) - 1))
```

3.65 $\int \frac{1}{1+\coth(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rubi [A] time = 0.0086758, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\coth(x)}\,dx &= -\frac{1}{2(1+\coth(x))}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0309119, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x-\sinh(2x)+\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-1),x]
```

```
[Out] (2*x + Cosh[2*x] - Sinh[2*x])/4
```

Maple [A] time = 0.016, size = 24, normalized size = 1.5

$$-\frac{1}{2+2\coth(x)}+\frac{\ln(1+\coth(x))}{4}-\frac{\ln(\coth(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x)),x)
```

```
[Out] -1/2/(1+coth(x))+1/4*ln(1+coth(x))-1/4*ln(coth(x)-1)
```

Maxima [A] time = 1.01401, size = 14, normalized size = 0.88

$$\frac{1}{2}x+\frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

Fricas [B] time = 2.01175, size = 88, normalized size = 5.5

$$\frac{(2x+1)\cosh(x)+(2x-1)\sinh(x)}{4(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.57725, size = 27, normalized size = 1.69

$$\frac{x\tanh(x)}{2\tanh(x)+2}+\frac{x}{2\tanh(x)+2}+\frac{1}{2\tanh(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Giac [A] time = 1.14728, size = 14, normalized size = 0.88

$$\frac{1}{2}x+\frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

3.66 $\int \frac{1}{(1+\coth(x))^2}\,dx$

Optimal. Leaf size=26

$$\frac{x}{4}-\frac{1}{4(\coth(x)+1)}-\frac{1}{4(\coth(x)+1)^2}$$

```
[Out] x/4 - 1/(4*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rubi [A] time = 0.0169901, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{4}-\frac{1}{4(\coth(x)+1)}-\frac{1}{4(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-2),x]
```

```
[Out] x/4 - 1/(4*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\coth(x))^2}\,dx &= -\frac{1}{4(1+\coth(x))^2}+\frac{1}{2}\int \frac{1}{1+\coth(x)}\,dx\\ &= -\frac{1}{4(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}+\frac{\int 1\,dx}{4}\\ &= \frac{x}{4}-\frac{1}{4(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0576583, size = 30, normalized size = 1.15

$$\frac{1}{16}(4x-4\sinh(2x)+\sinh(4x)+4\cosh(2x)-\cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-2),x]
```

```
[Out] (4*x + 4*Cosh[2*x] - Cosh[4*x] - 4*Sinh[2*x] + Sinh[4*x])/16
```

Maple [A] time = 0.017, size = 32, normalized size = 1.2

$$-\frac{1}{4\left(1+\coth\left(x\right)\right)^{2}}-\frac{1}{4+4\coth\left(x\right)}+\frac{\ln\left(1+\coth\left(x\right)\right)}{8}-\frac{\ln\left(\coth\left(x\right)-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^2,x)
```

```
[Out] -1/4/(1+coth(x))^2-1/4/(1+coth(x))+1/8*ln(1+coth(x))-1/8*ln(coth(x)-1)
```

Maxima [A] time = 1.02873, size = 22, normalized size = 0.85

$$\frac{1}{4}\,x+\frac{1}{4}\,e^{\left(-2\,x\right)}-\frac{1}{16}\,e^{\left(-4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/4*e^(-2*x) - 1/16*e^(-4*x)
```

Fricas [B] time = 2.39471, size = 173, normalized size = 6.65

$$\frac{\left(4\,x-1\right)\cosh\left(x\right)^{2}+2\left(4\,x+1\right)\cosh\left(x\right)\sinh\left(x\right)+\left(4\,x-1\right)\sinh\left(x\right)^{2}+4}{16\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="fricas")
```

```
[Out] 1/16*((4*x - 1)*cosh(x)^2 + 2*(4*x + 1)*cosh(x)*sinh(x) + (4*x - 1)*sinh(x)
^2 + 4)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] time = 0.990182, size = 88, normalized size = 3.38

$$\frac{x\tanh^{2}\left(x\right)}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{2x\tanh\left(x\right)}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{x}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{3\tanh\left(x\right)}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**2,x)
```

```
[Out] x*tanh(x)**2/(4*tanh(x)**2 + 8*tanh(x) + 4) + 2*x*tanh(x)/(4*tanh(x)**2 + 8
*tanh(x) + 4) + x/(4*tanh(x)**2 + 8*tanh(x) + 4) + 3*tanh(x)/(4*tanh(x)**2
+ 8*tanh(x) + 4) + 2/(4*tanh(x)**2 + 8*tanh(x) + 4)
```

Giac [A] time = 1.15641, size = 24, normalized size = 0.92

$$\frac{1}{16}\left(4\,e^{\left(2\,x\right)}-1\right)e^{\left(-4\,x\right)}+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*e^(2*x) - 1)*e^(-4*x) + 1/4*x
```

3.67 $\int \frac{1}{(1+\coth(x))^3}\,dx$

Optimal. Leaf size=36

$$\frac{x}{8}-\frac{1}{8(\coth(x)+1)}-\frac{1}{8(\coth(x)+1)^2}-\frac{1}{6(\coth(x)+1)^3}$$

```
[Out] x/8 - 1/(6*(1 + Coth[x])^3) - 1/(8*(1 + Coth[x])^2) - 1/(8*(1 + Coth[x]))
```

Rubi [A] time = 0.0274045, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{8}-\frac{1}{8(\coth(x)+1)}-\frac{1}{8(\coth(x)+1)^2}-\frac{1}{6(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-3),x]
```

```
[Out] x/8 - 1/(6*(1 + Coth[x])^3) - 1/(8*(1 + Coth[x])^2) - 1/(8*(1 + Coth[x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\coth(x))^3}\,dx &= -\frac{1}{6(1+\coth(x))^3}+\frac{1}{2}\int \frac{1}{(1+\coth(x))^2}\,dx\\ &= -\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}+\frac{1}{4}\int \frac{1}{1+\coth(x)}\,dx\\ &= -\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}-\frac{1}{8(1+\coth(x))}+\frac{\int 1\,dx}{8}\\ &= \frac{x}{8}-\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}-\frac{1}{8(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0870341, size = 44, normalized size = 1.22

$$\frac{1}{96}(12x-18\sinh(2x)+9\sinh(4x)-2\sinh(6x)+18\cosh(2x)-9\cosh(4x)+2\cosh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-3),x]
```

```
[Out] (12*x + 18*Cosh[2*x] - 9*Cosh[4*x] + 2*Cosh[6*x] - 18*Sinh[2*x] + 9*Sinh[4*
x] - 2*Sinh[6*x])/96
```

Maple [A] time = 0.017, size = 40, normalized size = 1.1

$$-\frac{1}{6\,(1+\coth(x))^3}-\frac{1}{8\,(1+\coth(x))^2}-\frac{1}{8+8\coth(x)}+\frac{\ln(1+\coth(x))}{16}-\frac{\ln(\coth(x)-1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^3,x)
```

```
[Out] -1/6/(1+coth(x))^3-1/8/(1+coth(x))^2-1/8/(1+coth(x))+1/16*ln(1+coth(x))-1/1
6*ln(coth(x)-1)
```

Maxima [A] time = 1.05014, size = 30, normalized size = 0.83

$$\frac{1}{8}\,x+\frac{3}{16}\,e^{(-2\,x)}-\frac{3}{32}\,e^{(-4\,x)}+\frac{1}{48}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="maxima")
```

```
[Out] 1/8*x + 3/16*e^(-2*x) - 3/32*e^(-4*x) + 1/48*e^(-6*x)
```

Fricas [B] time = 2.26864, size = 278, normalized size = 7.72

$$\frac{2\,(6\,x+1)\cosh(x)^3+6\,(6\,x+1)\cosh(x)\sinh(x)^2+2\,(6\,x-1)\sinh(x)^3+3\left(2\,(6\,x-1)\cosh(x)^2+9\right)\sinh(x)+9\ \mathrm{c}}{96\left(\cosh(x)^3+3\,\cosh(x)^2\sinh(x)+3\,\cosh(x)\sinh(x)^2+\sinh(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(2*(6*x + 1)*cosh(x)^3 + 6*(6*x + 1)*cosh(x)*sinh(x)^2 + 2*(6*x - 1)*s
inh(x)^3 + 3*(2*(6*x - 1)*cosh(x)^2 + 9)*sinh(x) + 9*cosh(x))/(cosh(x)^3 +
3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [B] time = 1.47227, size = 182, normalized size = 5.06

$$\frac{3x\tanh^3(x)}{24\tanh^3(x)+72\tanh^2(x)+72\tanh(x)+24}+\frac{9x\tanh^2(x)}{24\tanh^3(x)+72\tanh^2(x)+72\tanh(x)+24}+\frac{}{24\tanh^3(x)+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**3,x)
```

```
[Out] 3*x*tanh(x)**3/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh
(x)**2/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh(x)/(24*
tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 3*x/(24*tanh(x)**3 + 72*tan
h(x)**2 + 72*tanh(x) + 24) - 7*tanh(x)**3/(24*tanh(x)**3 + 72*tanh(x)**2 +
72*tanh(x) + 24) + 6*tanh(x)/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) +
24) + 3/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24)
```

Giac [A] time = 1.14846, size = 32, normalized size = 0.89

$$\frac{1}{96}\left(18\,e^{(4\,x)} - 9\,e^{(2\,x)} + 2\right)e^{(-6\,x)} + \frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="giac")
```

```
[Out] 1/96*(18*e^(4*x) - 9*e^(2*x) + 2)*e^(-6*x) + 1/8*x
```

3.68 $\int \frac{1}{(1+\coth(x))^4} dx$

Optimal. Leaf size=46

$$\frac{x}{16}-\frac{1}{16(\coth(x)+1)}-\frac{1}{16(\coth(x)+1)^2}-\frac{1}{12(\coth(x)+1)^3}-\frac{1}{8(\coth(x)+1)^4}$$

```
[Out] x/16 - 1/(8*(1 + Coth[x])^4) - 1/(12*(1 + Coth[x])^3) - 1/(16*(1 + Coth[x])
^2) - 1/(16*(1 + Coth[x]))
```

Rubi [A] time = 0.0359077, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{16}-\frac{1}{16(\coth(x)+1)}-\frac{1}{16(\coth(x)+1)^2}-\frac{1}{12(\coth(x)+1)^3}-\frac{1}{8(\coth(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-4),x]
```

```
[Out] x/16 - 1/(8*(1 + Coth[x])^4) - 1/(12*(1 + Coth[x])^3) - 1/(16*(1 + Coth[x])
^2) - 1/(16*(1 + Coth[x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^4} dx &= -\frac{1}{8(1+\coth(x))^4}+\frac{1}{2}\int \frac{1}{(1+\coth(x))^3} dx \\
&= -\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}+\frac{1}{4}\int \frac{1}{(1+\coth(x))^2} dx \\
&= -\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}+\frac{1}{8}\int \frac{1}{1+\coth(x)} dx \\
&= -\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}+\frac{\int 1\,dx}{16} \\
&= \frac{x}{16}-\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.121232, size = 53, normalized size = 1.15

$$\frac{1}{384}(\cosh(4x)-\sinh(4x))(32\sinh(2x)+24x\sinh(4x)+3\sinh(4x)+64\cosh(2x)+3(8x-1)\cosh(4x)-36)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-4),x]
```

```
[Out] ((Cosh[4*x] - Sinh[4*x])*(-36 + 64*Cosh[2*x] + 3*(-1 + 8*x)*Cosh[4*x] + 32*
Sinh[2*x] + 3*Sinh[4*x] + 24*x*Sinh[4*x]))/384
```

Maple [A] time = 0.017, size = 48, normalized size = 1.

$$-\frac{1}{8\,(1+\coth(x))^4}-\frac{1}{12\,(1+\coth(x))^3}-\frac{1}{16\,(1+\coth(x))^2}-\frac{1}{16+16\coth(x)}+\frac{\ln(1+\coth(x))}{32}-\frac{\ln(\coth(x)-1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^4,x)
```

```
[Out] -1/8/(1+coth(x))^4-1/12/(1+coth(x))^3-1/16/(1+coth(x))^2-1/16/(1+coth(x))+1
/32*ln(1+coth(x))-1/32*ln(coth(x)-1)
```

Maxima [A] time = 1.17884, size = 38, normalized size = 0.83

$$\frac{1}{16}\,x+\frac{1}{8}\,e^{(-2\,x)}-\frac{3}{32}\,e^{(-4\,x)}+\frac{1}{24}\,e^{(-6\,x)}-\frac{1}{128}\,e^{(-8\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="maxima")
```

```
[Out] 1/16*x + 1/8*e^(-2*x) - 3/32*e^(-4*x) + 1/24*e^(-6*x) - 1/128*e^(-8*x)
```

Fricas [B] time = 2.37733, size = 394, normalized size = 8.57

$$\frac{3\,(8\,x-1)\cosh(x)^4+12\,(8\,x+1)\cosh(x)\sinh(x)^3+3\,(8\,x-1)\sinh(x)^4+2\left(9\,(8\,x-1)\cosh(x)^2+32\right)\sinh(x)^2+64}{384\left(\cosh(x)^4+4\,\cosh(x)^3\sinh(x)+6\,\cosh(x)^2\sinh(x)^2+4\,\cosh(x)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(3*(8*x - 1)*cosh(x)^4 + 12*(8*x + 1)*cosh(x)*sinh(x)^3 + 3*(8*x - 1)
*sinh(x)^4 + 2*(9*(8*x - 1)*cosh(x)^2 + 32)*sinh(x)^2 + 64*cosh(x)^2 + 4*(3
*(8*x + 1)*cosh(x)^3 + 16*cosh(x))*sinh(x) - 36)/(cosh(x)^4 + 4*cosh(x)^3*s
inh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [B] time = 2.55462, size = 299, normalized size = 6.5

$$\frac{3x\tanh^4(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{12x\tanh^3(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**4,x)
```

```
[Out] 3*x*tanh(x)**4/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(
x) + 48) + 12*x*tanh(x)**3/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2
 + 192*tanh(x) + 48) + 18*x*tanh(x)**2/(48*tanh(x)**4 + 192*tanh(x)**3 + 28
8*tanh(x)**2 + 192*tanh(x) + 48) + 12*x*tanh(x)/(48*tanh(x)**4 + 192*tanh(x
)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 3*x/(48*tanh(x)**4 + 192*tanh(x
)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 45*tanh(x)**3/(48*tanh(x)**4 +
192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 84*tanh(x)**2/(48*tan
h(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 61*tanh(x)/
(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 16/(
48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48)
```

Giac [A] time = 1.15168, size = 41, normalized size = 0.89

$$\frac{1}{384}\left(48\,e^{(6\,x)} - 36\,e^{(4\,x)} + 16\,e^{(2\,x)} - 3\right)e^{(-8\,x)} + \frac{1}{16}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="giac")
```

```
[Out] 1/384*(48*e^(6*x) - 36*e^(4*x) + 16*e^(2*x) - 3)*e^(-8*x) + 1/16*x
```

3.69 $\int \frac{1}{(1+\coth(x))^5} dx$

Optimal. Leaf size=56

$$\frac{x}{32} - \frac{1}{32(\coth(x)+1)} - \frac{1}{32(\coth(x)+1)^2} - \frac{1}{24(\coth(x)+1)^3} - \frac{1}{16(\coth(x)+1)^4} - \frac{1}{10(\coth(x)+1)^5}$$

```
[Out] x/32 - 1/(10*(1 + Coth[x])^5) - 1/(16*(1 + Coth[x])^4) - 1/(24*(1 + Coth[x]
)^3) - 1/(32*(1 + Coth[x])^2) - 1/(32*(1 + Coth[x]))
```

Rubi [A] time = 0.0462366, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{32} - \frac{1}{32(\coth(x)+1)} - \frac{1}{32(\coth(x)+1)^2} - \frac{1}{24(\coth(x)+1)^3} - \frac{1}{16(\coth(x)+1)^4} - \frac{1}{10(\coth(x)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-5),x]
```

```
[Out] x/32 - 1/(10*(1 + Coth[x])^5) - 1/(16*(1 + Coth[x])^4) - 1/(24*(1 + Coth[x]
)^3) - 1/(32*(1 + Coth[x])^2) - 1/(32*(1 + Coth[x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^5} dx &= -\frac{1}{10(1+\coth(x))^5} + \frac{1}{2}\int \frac{1}{(1+\coth(x))^4} dx \\
&= -\frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} + \frac{1}{4}\int \frac{1}{(1+\coth(x))^3} dx \\
&= -\frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} + \frac{1}{8}\int \frac{1}{(1+\coth(x))^2} dx \\
&= -\frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} - \frac{1}{32(1+\coth(x))^2} + \frac{1}{16}\int \frac{1}{1+\coth} \\
&= -\frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} - \frac{1}{32(1+\coth(x))^2} - \frac{1}{32(1+\coth(x)} \\
&= \frac{x}{32} - \frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} - \frac{1}{32(1+\coth(x))^2} - \frac{1}{32(1+\coth}
\end{aligned}$$

Mathematica [A] time = 0.138169, size = 62, normalized size = 1.11

$$\frac{(\cosh(5x)-\sinh(5x))(-500\sinh(x)+375\sinh(3x)+120x\sinh(5x)-12\sinh(5x)-100\cosh(x)+225\cosh(3x)+120x}{3840}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-5),x]
```

```
[Out] ((Cosh[5*x] - Sinh[5*x])*(-100*Cosh[x] + 225*Cosh[3*x] + 12*Cosh[5*x] + 120
*x*Cosh[5*x] - 500*Sinh[x] + 375*Sinh[3*x] - 12*Sinh[5*x] + 120*x*Sinh[5*x]
))/3840
```

Maple [A] time = 0.024, size = 56, normalized size = 1.

$$-\frac{1}{10\,(1+\coth(x))^5}-\frac{1}{16\,(1+\coth(x))^4}-\frac{1}{24\,(1+\coth(x))^3}-\frac{1}{32\,(1+\coth(x))^2}-\frac{1}{32+32\coth(x)}+\frac{\ln(1+\mathrm{cot}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^5,x)
```

```
[Out] -1/10/(1+coth(x))^5-1/16/(1+coth(x))^4-1/24/(1+coth(x))^3-1/32/(1+coth(x))^
2-1/32/(1+coth(x))+1/64*ln(1+coth(x))-1/64*ln(coth(x)-1)
```

Maxima [A] time = 1.03789, size = 46, normalized size = 0.82

$$\frac{1}{32}\,x+\frac{5}{64}\,e^{(-2\,x)}-\frac{5}{64}\,e^{(-4\,x)}+\frac{5}{96}\,e^{(-6\,x)}-\frac{5}{256}\,e^{(-8\,x)}+\frac{1}{320}\,e^{(-10\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="maxima")
```

```
[Out] 1/32*x + 5/64*e^(-2*x) - 5/64*e^(-4*x) + 5/96*e^(-6*x) - 5/256*e^(-8*x) + 1
/320*e^(-10*x)
```

Fricas [B] time = 2.2716, size = 540, normalized size = 9.64

$$\frac{12\,(10\,x+1)\cosh(x)^5+60\,(10\,x+1)\cosh(x)\sinh(x)^4+12\,(10\,x-1)\sinh(x)^5+15\left(8\,(10\,x-1)\cosh(x)^2+25\right)\mathrm{sinl}}{3840\left(\cosh(x)^5+5\,\cosh(x)^4\sinh(x)+10\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="fricas")
```

```
[Out] 1/3840*(12*(10*x + 1)*cosh(x)^5 + 60*(10*x + 1)*cosh(x)*sinh(x)^4 + 12*(10*
x - 1)*sinh(x)^5 + 15*(8*(10*x - 1)*cosh(x)^2 + 25)*sinh(x)^3 + 225*cosh(x)
^3 + 15*(8*(10*x + 1)*cosh(x)^3 + 45*cosh(x))*sinh(x)^2 + 5*(12*(10*x - 1)*
cosh(x)^4 + 225*cosh(x)^2 - 100)*sinh(x) - 100*cosh(x))/(cosh(x)^5 + 5*cosh
(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)
*sinh(x)^4 + sinh(x)^5)
```

Sympy [B] time = 3.23362, size = 444, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**5,x)
```

```
[Out] 15*x*tanh(x)**5/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*
tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tanh(x)**4/(480*tanh(x)**5 + 2400*t
anh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 150*x
*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh
(x)**2 + 2400*tanh(x) + 480) + 150*x*tanh(x)**2/(480*tanh(x)**5 + 2400*tanh
(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tan
h(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2
+ 2400*tanh(x) + 480) + 15*x/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(
x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) - 93*tanh(x)**5/(480*tanh(x)*
*5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 4
80) + 195*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 +
4800*tanh(x)**2 + 2400*tanh(x) + 480) + 275*tanh(x)**2/(480*tanh(x)**5 + 24
00*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 1
60*tanh(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(
x)**2 + 2400*tanh(x) + 480) + 35/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*t
anh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480)
```

Giac [A] time = 1.14209, size = 49, normalized size = 0.88

$$\frac{1}{3840}\left(300\,e^{(8\,x)} - 300\,e^{(6\,x)} + 200\,e^{(4\,x)} - 75\,e^{(2\,x)} + 12\right)e^{(-10\,x)} + \frac{1}{32}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="giac")
```

```
[Out] 1/3840*(300*e^(8*x) - 300*e^(6*x) + 200*e^(4*x) - 75*e^(2*x) + 12)*e^(-10*x
) + 1/32*x
```

3.70 $\int (1+\coth(x))^{7/2}\,dx$

Optimal. Leaf size=57

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-\frac{4}{3}(\coth(x)+1)^{3/2}-8\sqrt{\coth(x)+1}+8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 8*Sqrt[1 + Coth[x]] - (4*(1
+ Coth[x])^(3/2))/3 - (2*(1 + Coth[x])^(5/2))/5
```

Rubi [A] time = 0.0419785, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3478, 3480, 206}

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-\frac{4}{3}(\coth(x)+1)^{3/2}-8\sqrt{\coth(x)+1}+8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(7/2),x]
```

```
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 8*Sqrt[1 + Coth[x]] - (4*(1
+ Coth[x])^(3/2))/3 - (2*(1 + Coth[x])^(5/2))/5
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (1+\coth(x))^{7/2}\,dx &= -\frac{2}{5}(1+\coth(x))^{5/2}+2\int (1+\coth(x))^{5/2}\,dx\\
&= -\frac{4}{3}(1+\coth(x))^{3/2}-\frac{2}{5}(1+\coth(x))^{5/2}+4\int (1+\coth(x))^{3/2}\,dx\\
&= -8\sqrt{1+\coth(x)}-\frac{4}{3}(1+\coth(x))^{3/2}-\frac{2}{5}(1+\coth(x))^{5/2}+8\int \sqrt{1+\coth(x)}\,dx\\
&= -8\sqrt{1+\coth(x)}-\frac{4}{3}(1+\coth(x))^{3/2}-\frac{2}{5}(1+\coth(x))^{5/2}+16\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\text{c}}\right.\\
&= 8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-8\sqrt{1+\coth(x)}-\frac{4}{3}(1+\coth(x))^{3/2}-\frac{2}{5}(1+\coth(x))^{5/2}
\end{aligned}$$

Mathematica [C] time = 0.258514, size = 101, normalized size = 1.77

$$-\frac{2(\coth(x)+1)^{7/2}\left((8\sinh(2x)+3)\sinh(x)\sqrt{i(\coth(x)+1)}+4\sinh^3(x)\left(19\sqrt{i(\coth(x)+1)}-(15-15i)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\right.\right.\right.}{15\sqrt{i(\coth(x)+1)}(\sinh(x)+\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(7/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(7/2)*(4*((-15 + 15*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Cot
h[x])]] + 19*Sqrt[I*(1 + Coth[x])])*Sinh[x]^3 + Sqrt[I*(1 + Coth[x])]*Sinh[
x]*(3 + 8*Sinh[2*x])))/(15*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x])^3)
```

Maple [A] time = 0.016, size = 43, normalized size = 0.8

$$-\frac{4}{3}(1+\coth(x))^{\frac{3}{2}}-\frac{2}{5}(1+\coth(x))^{\frac{5}{2}}+8\,\mathrm{Artanh}\left(1/2\sqrt{1+\coth(x)}\sqrt{2}\right)\sqrt{2}-8\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(7/2),x)
```

```
[Out] -4/3*(1+coth(x))^(3/2)-2/5*(1+coth(x))^(5/2)+8*arctanh(1/2*(1+coth(x))^(1/2
)*2^(1/2))*2^(1/2)-8*(1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(\coth(x)+1)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(7/2), x)
```

Fricas [B] time = 2.41786, size = 1469, normalized size = 25.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/15*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^5 + 115*sqrt(2)*cosh(x)*sinh(x)^4 + 23
*sqrt(2)*sinh(x)^5 + 5*(46*sqrt(2)*cosh(x)^2 - 7*sqrt(2))*sinh(x)^3 - 35*sq
rt(2)*cosh(x)^3 + 5*(46*sqrt(2)*cosh(x)^3 - 21*sqrt(2)*cosh(x))*sinh(x)^2 +
 5*(23*sqrt(2)*cosh(x)^4 - 21*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x) + 15*s
qrt(2)*cosh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^6 +
 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 -
 sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt
```

```
(2)*cosh(x))*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqr
t(2))*sinh(x)^2 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*co
sh(x)^3 + sqrt(2)*cosh(x))*sinh(x) - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(c
osh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) +
2*sinh(x)^2 - 1))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(
x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 +
 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 -
 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16966, size = 216, normalized size = 3.79

$$-\frac{4}{15}\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4+135\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3+170\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+100\sqrt{e^{(4x)}-e^{(2}}\cdots}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="giac")
```

```
[Out] -4/15*sqrt(2)*(2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 135*(sqrt(e^(4
*x) - e^(2*x)) - e^(2*x))^3 + 170*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 1
00*sqrt(e^(4*x) - e^(2*x)) - 100*e^(2*x) + 23)/(sqrt(e^(4*x) - e^(2*x)) - e
^(2*x) + 1)^5 + 15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn
(e^(2*x) - 1)
```

3.71 $\int (1+\coth(x))^{5/2}\,dx$

Optimal. Leaf size=45

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-4\sqrt{\coth(x)+1}+4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 4*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rubi [A] time = 0.031965, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3478, 3480, 206}

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-4\sqrt{\coth(x)+1}+4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(5/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 4*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (1+\coth(x))^{5/2}\,dx &= -\frac{2}{3}(1+\coth(x))^{3/2}+2\int (1+\coth(x))^{3/2}\,dx\\
&= -4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+4\int \sqrt{1+\coth(x)}\,dx\\
&= -4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+8\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\
&= 4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.153248, size = 92, normalized size = 2.04

$$-\frac{2\sinh(x)(\coth(x)+1)^{5/2}\left(\cosh(x)\sqrt{i(\coth(x)+1)}+\sinh(x)\left(7\sqrt{i(\coth(x)+1)}-(6-6i)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)\right)\right)}{3\sqrt{i(\coth(x)+1)}(\sinh(x)+\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(5/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(5/2)*Sinh[x]*(Cosh[x]*Sqrt[I*(1 + Coth[x])] + ((-6 + 6*I
)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]] + 7*Sqrt[I*(1 + Coth[x])])*Sinh
[x]))/(3*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x])^2)
```

Maple [A] time = 0.013, size = 35, normalized size = 0.8

$$-\frac{2}{3}\left(1+\coth\left(x\right)\right)^{\frac{3}{2}}+4\,\mathrm{Artanh}\left(1/2\,\sqrt{1+\coth\left(x\right)}\sqrt{2}\right)\sqrt{2}-4\,\sqrt{1+\coth\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(5/2),x)
```

```
[Out] -2/3*(1+coth(x))^(3/2)+4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-4*(
1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\coth\left(x\right)+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(5/2), x)
```

Fricas [B] time = 2.39008, size = 869, normalized size = 19.31

$$-\frac{2\left(2\,\sqrt{2}\left(4\,\sqrt{2}\cosh\left(x\right)^{3}+12\,\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{2}+4\,\sqrt{2}\sinh\left(x\right)^{3}+3\left(4\,\sqrt{2}\cosh\left(x\right)^{2}-\sqrt{2}\right)\sinh\left(x\right)-3\,\sqrt{2}\cosh\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^3 + 12*sqrt(2)*cosh(x)*sinh(x)^2 + 4*sqr
t(2)*sinh(x)^3 + 3*(4*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - 3*sqrt(2)*cosh
(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sin
```

```
h(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x
)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17387, size = 151, normalized size = 3.36

$$-\frac{2}{3}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+9\sqrt{e^{(4x)}-e^{(2x)}}-9e^{(2x)}+4\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^3}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)\mathrm{sgn}\left(e^{(2x)}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 9*sqrt(e^(4*x) -
 e^(2*x)) - 9*e^(2*x) + 4)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3 + 3*lo
g(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

3.72 $\int (1+\coth(x))^{3/2}\,dx$

Optimal. Leaf size=33

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rubi [A] time = 0.0219855, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3478, 3480, 206}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^{3/2}\,dx &= -2\sqrt{1+\coth(x)}+2\int \sqrt{1+\coth(x)}\,dx\\ &= -2\sqrt{1+\coth(x)}+4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}\end{aligned}$$

Mathematica [C] time = 0.106497, size = 69, normalized size = 2.09

$$-\frac{2\sinh(x)(\coth(x)+1)^{3/2}\left(\sqrt{i(\coth(x)+1)}-(1-i)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)\right)}{\sqrt{i(\coth(x)+1)}(\sinh(x)+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(3/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(3/2)*((-1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]
 + Sqrt[I*(1 + Coth[x])])*Sinh[x])/(Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x
]))
```

Maple [A] time = 0.013, size = 27, normalized size = 0.8

$$2\,\mathrm{Artanh}\left(1/2\,\sqrt{1+\coth(x)}\sqrt{2}\right)\sqrt{2}-2\,\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(3/2),x)
```

```
[Out] 2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(\coth(x)+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(3/2), x)
```

Fricas [B] time = 2.48679, size = 450, normalized size = 13.64

$$-\frac{2\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh(x)^{2}+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^{2}-\sqrt{2}\right)\log\left(2\right.}{\cosh(x)^{2}+2\cosh(x)\sinh(x)+\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sin
h(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
 - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(\coth(x)+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(3/2), x)
```

Giac [B] time = 1.16432, size = 85, normalized size = 2.58

$$-\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)} - e^{(2x)}} - e^{(2x)} + 1} + \log\left(\left|2\sqrt{e^{(4x)} - e^{(2x)}} - 2e^{(2x)} + 1\right|\right)\right)\operatorname{sgn}\left(e^{(2x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1) + log(abs(2*sqrt(e^(4*x
) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

3.73 $\int \sqrt{1+\coth(x)}\,dx$

Optimal. Leaf size=21

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]
```

Rubi [A] time = 0.0120375, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3480, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\coth(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.0741134, size = 45, normalized size = 2.14

$$\frac{(1+i)(\coth(x)+1)^{3/2}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{(i(\coth(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Coth[x]],x]
```

```
[Out] ((1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*(1 + Coth[x])^(3/2))/(I*
(1 + Coth[x]))^(3/2)
```

Maple [A] time = 0.036, size = 17, normalized size = 0.8

$$\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(1/2),x)
```

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1), x)
```

Fricas [B] time = 2.31598, size = 181, normalized size = 8.62

$$\frac{1}{2}\sqrt{2}\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1), x)
```

Giac [B] time = 1.17684, size = 50, normalized size = 2.38

$$-\frac{1}{2}\sqrt{2}\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x
) - 1)
```

3.74 $\int \frac{1}{\sqrt{1+\coth(x)}} \, dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}$$

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]]
```

Rubi [A] time = 0.0228691, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3479, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Coth[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+\coth(x)}} \, dx &= -\frac{1}{\sqrt{1+\coth(x)}}+\frac{1}{2} \int \sqrt{1+\coth(x)} \, dx \\
&= -\frac{1}{\sqrt{1+\coth(x)}}+\text{Subst}\left(\int \frac{1}{2-x^2} \, dx, x, \sqrt{1+\coth(x)}\right) \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}
\end{aligned}$$

Mathematica [C] time = 0.291743, size = 51, normalized size = 1.59

$$\frac{-2+(-1-i)\sqrt{i(\coth(x)+1)}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{2\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Coth[x]],x]
```

```
[Out] (-2 - (1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*Sqrt[I*(1 + Coth[x]
)])/(2*Sqrt[1 + Coth[x]])
```

Maple [A] time = 0.036, size = 27, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)-\frac{1}{\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(coth(x) + 1), x)
```

Fricas [B] time = 2.36078, size = 300, normalized size = 9.38

$$\frac{\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh\cdots\right.}{4(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sin
h(x)^2 - 1) - 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(coth(x) + 1), x)
```

Giac [B] time = 1.15375, size = 89, normalized size = 2.78

$$\frac{\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}}-\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2\,e^{(2x)}+1\right|\right)\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x)) - log(abs(2*sqrt(e^(4*x)
 - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

3.75 $\int \frac{1}{(1+\coth(x))^{3/2}} dx$

Optimal. Leaf size=49

$$-\frac{1}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rubi [A] time = 0.0307411, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3479, 3480, 206}

$$-\frac{1}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^{3/2}} dx &= -\frac{1}{3(1+\coth(x))^{3/2}}+\frac{1}{2}\int \frac{1}{\sqrt{1+\coth(x)}} dx\\
&= -\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{4}\int \sqrt{1+\coth(x)}\, dx\\
&= -\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{1+\coth(x)}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}
\end{aligned}$$

Mathematica [C] time = 0.307893, size = 86, normalized size = 1.76

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{\coth(x)+1}\left(\left(\frac{1}{6}-\frac{i}{6}\right)(-5\sinh(2x)+\sinh(4x)+5\cosh(2x)-\cosh(4x)-4)-\frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)}\right.}{\sqrt{i(\coth(x)+1)}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-3/2),x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] + (1/6 - I/6)*(-4 + 5*Cosh[2*x] - Cosh[4*x] - 5*
Sinh[2*x] + Sinh[4*x]))
```

Maple [A] time = 0.011, size = 35, normalized size = 0.7

$$-\frac{1}{3}(1+\coth(x))^{-\frac{3}{2}}+\frac{\sqrt{2}}{4}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)-\frac{1}{2}\frac{1}{\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^(3/2),x)
```

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1
/2/(1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(-3/2), x)
```

Fricas [B] time = 2.36685, size = 578, normalized size = 11.8

$$-\frac{2\sqrt{2}\left(4\sqrt{2}\cosh(x)^2+8\sqrt{2}\cosh(x)\sinh(x)+4\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh\right.}{24\left(\cosh(x)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^2 + 8*sqrt(2)*cosh(x)*sinh(x) + 4*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
```

```
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(-3/2), x)
```

Giac [B] time = 1.19078, size = 177, normalized size = 3.61

$$-\frac{1}{24}\sqrt{2}\left(\frac{3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)}{\mathrm{sgn}\left(e^{(2x)}-1\right)}-\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{2}+3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{3}\mathrm{sgn}\left(e^{(2x)}-1\right)}-8\,\mathrm{sgn}\left(e^{(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))/sgn(e^
(2*x) - 1) - 2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 3*sqrt(e^(4*x) -
e^(2*x)) - 3*e^(2*x) + 1)/((sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x
) - 1)) - 8*sgn(e^(2*x) - 1))
```

3.76 $\int \frac{1}{(1+\coth(x))^{5/2}} \, dx$

Optimal. Leaf size=61

$$-\frac{1}{4\sqrt{\coth(x)+1}}-\frac{1}{6(\coth(x)+1)^{3/2}}-\frac{1}{5(\coth(x)+1)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Coth[x])^(5/2))
- 1/(6*(1 + Coth[x])^(3/2)) - 1/(4*Sqrt[1 + Coth[x]])
```

Rubi [A] time = 0.0419238, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3479, 3480, 206}

$$-\frac{1}{4\sqrt{\coth(x)+1}}-\frac{1}{6(\coth(x)+1)^{3/2}}-\frac{1}{5(\coth(x)+1)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-5/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Coth[x])^(5/2))
- 1/(6*(1 + Coth[x])^(3/2)) - 1/(4*Sqrt[1 + Coth[x]])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\coth(x))^{5/2}}\,dx &= -\frac{1}{5(1+\coth(x))^{5/2}}+\frac{1}{2}\int \frac{1}{(1+\coth(x))^{3/2}}\,dx\\ &= -\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}+\frac{1}{4}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx\\ &= -\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+\coth(x)}}+\frac{1}{8}\int \sqrt{1+\coth(x)}\,dx\\ &= -\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+\coth(x)}}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+\coth(x)}}\end{aligned}$$

Mathematica [C] time = 0.757937, size = 94, normalized size = 1.54

$$-\frac{1}{60}\sqrt{\coth(x)+1}(\cosh(3x)-\sinh(3x))(-24\sinh(x)+13\sinh(3x)-10\cosh(x)+10\cosh(3x))+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(\coth(x)+1)}{(i(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-5/2),x]
```

```
[Out] ((1/8 + I/8)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*(1 + Coth[x])^(3/2))
/(I*(1 + Coth[x]))^(3/2) - (Sqrt[1 + Coth[x]]*(Cosh[3*x] - Sinh[3*x])*(-10*
Cosh[x] + 10*Cosh[3*x] - 24*Sinh[x] + 13*Sinh[3*x]))/60
```

Maple [A] time = 0.013, size = 43, normalized size = 0.7

$$-\frac{1}{5}(1+\coth(x))^{-\frac{5}{2}}-\frac{1}{6}(1+\coth(x))^{-\frac{3}{2}}+\frac{\sqrt{2}}{8}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)-\frac{1}{4}\frac{1}{\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^(5/2),x)
```

```
[Out] -1/5/(1+coth(x))^(5/2)-1/6/(1+coth(x))^(3/2)+1/8*arctanh(1/2*(1+coth(x))^(1
/2)*2^(1/2))*2^(1/2)-1/4/(1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x)+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(-5/2), x)
```

Fricas [B] time = 2.45624, size = 919, normalized size = 15.07

$$-\frac{2\sqrt{2}\left(23\sqrt{2}\cosh(x)^4 + 92\sqrt{2}\cosh(x)\sinh(x)^3 + 23\sqrt{2}\sinh(x)^4 + \left(138\sqrt{2}\cosh(x)^2 - 11\sqrt{2}\right)\sinh(x)^2 - 11\sqrt{2}\cosh\ldots\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/240*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^4 + 92*sqrt(2)*cosh(x)*sinh(x)^3 + 23
*sqrt(2)*sinh(x)^4 + (138*sqrt(2)*cosh(x)^2 - 11*sqrt(2))*sinh(x)^2 - 11*sq
rt(2)*cosh(x)^2 + 2*(46*sqrt(2)*cosh(x)^3 - 11*sqrt(2)*cosh(x))*sinh(x) + 3
*sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^5 + 5*sqr
t(2)*cosh(x)^4*sinh(x) + 10*sqrt(2)*cosh(x)^3*sinh(x)^2 + 10*sqrt(2)*cosh(x
)^2*sinh(x)^3 + 5*sqrt(2)*cosh(x)*sinh(x)^4 + sqrt(2)*sinh(x)^5)*log(2*sqrt
(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4
*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*
cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x
)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x) + 1)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**(5/2),x)
```

```
[Out] Integral((coth(x) + 1)**(-5/2), x)
```

Giac [B] time = 1.22279, size = 242, normalized size = 3.97

$$-\frac{1}{240}\sqrt{2}\left(\frac{15\log\left(\left|2\sqrt{e^{(4x)} - e^{(2x)}} - 2e^{(2x)} + 1\right|\right)}{\mathrm{sgn}\left(e^{(2x)} - 1\right)} - \frac{2\left(45\left(\sqrt{e^{(4x)} - e^{(2x)}} - e^{(2x)}\right)^4 + 45\left(\sqrt{e^{(4x)} - e^{(2x)}} - e^{(2x)}\right)^3 + 35\left(\ldots\right.\right.}{\left(\sqrt{e^{(4x)} - e^{(2x)}} - e^{(2x)}\right)^5 \mathrm{s}\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(2)*(15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))/sgn(
e^(2*x) - 1) - 2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 45*(sqrt(e^(4*
x) - e^(2*x)) - e^(2*x))^3 + 35*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 15*
sqrt(e^(4*x) - e^(2*x)) - 15*e^(2*x) + 3)/((sqrt(e^(4*x) - e^(2*x)) - e^(2*
x))^5*sgn(e^(2*x) - 1)) - 46*sgn(e^(2*x) - 1))
```

3.77 $\int (a+b\coth(c+dx))^5\,dx$

Optimal. Leaf size=142

$$-\frac{b(3a^2+b^2)(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}+\frac{b(10a^2b^2+5a^4+b^4)\log(\sinh(c+dx))}{d}+ax(10a^2b$$

```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x - (4*a*b^2*(a^2 + b^2)*Coth[c + d*x])/d - (b
*(3*a^2 + b^2)*(a + b*Coth[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Coth[c + d*x]
)^3)/(3*d) - (b*(a + b*Coth[c + d*x])^4)/(4*d) + (b*(5*a^4 + 10*a^2*b^2 + b
^4)*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.208768, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3482, 3528, 3525, 3475}

$$-\frac{b(3a^2+b^2)(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}+\frac{b(10a^2b^2+5a^4+b^4)\log(\sinh(c+dx))}{d}+ax(10a^2b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^5,x]
```


```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x - (4*a*b^2*(a^2 + b^2)*Coth[c + d*x])/d - (b
*(3*a^2 + b^2)*(a + b*Coth[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Coth[c + d*x]
)^3)/(3*d) - (b*(a + b*Coth[c + d*x])^4)/(4*d) + (b*(5*a^4 + 10*a^2*b^2 + b
^4)*Log[Sinh[c + d*x]])/d
```


Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\coth(c+dx))^5\,dx = -\frac{b(a+b\coth(c+dx))^4}{4d} + \int (a+b\coth(c+dx))^3 \left(a^2+b^2+2ab\coth(c+dx)\right) dx$$
$$= -\frac{2ab(a+b\coth(c+dx))^3}{3d} - \frac{b(a+b\coth(c+dx))^4}{4d} + \int (a+b\coth(c+dx))^2 \left(a\left(a^2+3\right.\right.$$
$$= -\frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx))^2}{2d} - \frac{2ab(a+b\coth(c+dx))^3}{3d} - \frac{b(a+b\coth(c+dx))^4}{4d}$$
$$= a\left(a^4+10a^2b^2+5b^4\right)x - \frac{4ab^2\left(a^2+b^2\right)\coth(c+dx)}{d} - \frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx)}{2d}$$
$$= a\left(a^4+10a^2b^2+5b^4\right)x - \frac{4ab^2\left(a^2+b^2\right)\coth(c+dx)}{d} - \frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx)}{2d}$$

Mathematica [A] time = 0.798875, size = 141, normalized size = 0.99

$$-\frac{6b^3\left(10a^2+b^2\right)\coth^2(c+dx)+60ab^2\left(2a^2+b^2\right)\coth(c+dx)-12b\left(10a^2b^2+5a^4+b^4\right)\log(\tanh(c+dx))+20ab^4}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^5,x]
```

```
[Out] -(60*a*b^2*(2*a^2 + b^2)*Coth[c + d*x] + 6*b^3*(10*a^2 + b^2)*Coth[c + d*x]
^2 + 20*a*b^4*Coth[c + d*x]^3 + 3*b^5*Coth[c + d*x]^4 + 6*(a + b)^5*Log[1 -
 Tanh[c + d*x]] - 12*b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Tanh[c + d*x]] - 6*(a
 - b)^5*Log[1 + Tanh[c + d*x]])/(12*d)
```

Maple [B] time = 0.006, size = 322, normalized size = 2.3

$$-5\frac{(\coth(dx+c))^2a^2b^3}{d} + \frac{\ln(\coth(dx+c)+1)a^5}{2d} - \frac{5\ln(\coth(dx+c)+1)a^4b}{2d} + 5\frac{\ln(\coth(dx+c)+1)a^3b^2}{d} - 5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^5,x)
```

```
[Out] -5/d*coth(d*x+c)^2*a^2*b^3+1/2/d*ln(coth(d*x+c)+1)*a^5-5/2/d*ln(coth(d*x+c)
+1)*a^4*b+5/d*ln(coth(d*x+c)+1)*a^3*b^2-5/d*ln(coth(d*x+c)+1)*a^2*b^3+5/2/d
*ln(coth(d*x+c)+1)*a*b^4-1/2/d*ln(coth(d*x+c)+1)*b^5-1/2/d*ln(coth(d*x+c)-1
)*a^5-5/2/d*ln(coth(d*x+c)-1)*a^4*b-5/d*ln(coth(d*x+c)-1)*a^3*b^2-5/d*ln(co
th(d*x+c)-1)*a^2*b^3-5/2/d*ln(coth(d*x+c)-1)*a*b^4-1/2/d*ln(coth(d*x+c)-1)*
b^5-1/2/d*coth(d*x+c)^2*b^5-1/4/d*b^5*coth(d*x+c)^4-5/3/d*coth(d*x+c)^3*a*b
^4-5/d*a*b^4*coth(d*x+c)-10/d*a^3*b^2*coth(d*x+c)
```

Maxima [B] time = 1.08076, size = 470, normalized size = 3.31

$$\frac{5}{3}ab^4\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+b^5\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/3*a*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d
*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^5*(
x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^
(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 10*a^2*b^3*
(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
- 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 10*a^3*b^2*(x + c
/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^5*x + 5*a^4*b*log(sinh(d*x + c))/d
```

Fricas [B] time = 2.76279, size = 6413, normalized size = 45.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d
*x + c)^8 + 24*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5)*d*x*sinh(d*x + c)^8 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^
4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*co
sh(d*x + c)^6 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 - 7*(a^5 - 5*a^4*
b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^2 + (a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*sinh(d*x + c)^6 + 24
*(7*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x
+ c)^3 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3
*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 60
*a^3*b^2 + 40*a*b^4 + 6*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^
5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^4
 + 6*(35*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh
(d*x + c)^4 + 30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b
 + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x - 30*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 24*(7*(a^5 - 5*a^4*b + 10*a^3*b^2
 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^5 - 10*(5*a^3*b^2 + 5*a^2*
b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
b^5)*d*x)*cosh(d*x + c)^3 + (30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3
*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
- b^5)*d*x - 4*(45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5 + 3*(a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2 + 4*(21*
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)
^6 - 45*a^3*b^2 - 15*a^2*b^3 - 25*a*b^4 - 3*b^5 - 45*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^4 - 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b
^4 - b^5)*d*x + 9*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*
a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 3*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^8 + 8*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a^4*b + 10*a^2*b^3 +
 b^5)*sinh(d*x + c)^8 - 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^6 - 4*
(5*a^4*b + 10*a^2*b^3 + b^5 - 7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^3 - 3*
(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*a^4*b + 10*
a^2*b^3 + b^5 + 6*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 2*(15*a^4*
b + 30*a^2*b^3 + 3*b^5 + 35*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 -
30*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*
```

```
a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 - 10*(5*a^4*b + 10*a^2*b^3 + b^5)
*cosh(d*x + c)^3 + 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x +
 c)^3 - 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 4*(7*(5*a^4*b + 10
*a^2*b^3 + b^5)*cosh(d*x + c)^6 - 5*a^4*b - 10*a^2*b^3 - b^5 - 15*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 9*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
7 - 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 3*(5*a^4*b + 10*a^2*b^
3 + b^5)*cosh(d*x + c)^3 - (5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh
(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(3*(a^5
 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^7 -
 9*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 1
0*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^5 + 3*(30*a^3*b^2 + 20*a^2*b^
3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5)*d*x)*cosh(d*x + c)^3 - (45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5
+ 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^
7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*s
inh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^
5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 +
3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)
^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d
*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x +
 c))*sinh(d*x + c) + d)
```

Sympy [A] time = 22.9375, size = 325, normalized size = 2.29

$$\begin{cases} x\left(a+b\coth\left(c\right)\right)^{5} \\ a^{5}x+\tilde{\infty}a^{4}bx+\tilde{\infty}a^{3}b^{2}x+\tilde{\infty}a^{2}b^{3}x+\tilde{\infty}ab^{4}x+\tilde{\infty}b^{5}x \\ a^{5}x+5a^{4}bx-\frac{5a^{4}b\log\left(\tanh\left(c+dx\right)+1\right)}{d}+\frac{5a^{4}b\log\left(\tanh\left(c+dx\right)\right)}{d}+10a^{3}b^{2}x-\frac{10a^{3}b^{2}}{d\tanh\left(c+dx\right)}+10a^{2}b^{3}x-\frac{10a^{2}b^{3}\log\left(\tanh\left(c+dx\right)+1\right)}{d}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**5,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**5, Eq(d, 0)), (a**5*x + zoo*a**4*b*x + zoo*a*
*3*b**2*x + zoo*a**2*b**3*x + zoo*a*b**4*x + zoo*b**5*x, Eq(c, log(exp(-d*x
))) | Eq(c, log(-exp(-d*x)))), (a**5*x + 5*a**4*b*x - 5*a**4*b*log(tanh(c +
 d*x) + 1)/d + 5*a**4*b*log(tanh(c + d*x))/d + 10*a**3*b**2*x - 10*a**3*b**
2/(d*tanh(c + d*x)) + 10*a**2*b**3*x - 10*a**2*b**3*log(tanh(c + d*x) + 1)/
d + 10*a**2*b**3*log(tanh(c + d*x))/d - 5*a**2*b**3/(d*tanh(c + d*x)**2) +
5*a*b**4*x - 5*a*b**4/(d*tanh(c + d*x)) - 5*a*b**4/(3*d*tanh(c + d*x)**3) +
 b**5*x - b**5*log(tanh(c + d*x) + 1)/d + b**5*log(tanh(c + d*x))/d - b**5/
(2*d*tanh(c + d*x)**2) - b**5/(4*d*tanh(c + d*x)**4), True))
```

Giac [A] time = 1.15487, size = 308, normalized size = 2.17

$$\frac{\left(a^{5}-5\,a^{4}b+10\,a^{3}b^{2}-10\,a^{2}b^{3}+5\,ab^{4}-b^{5}\right)(dx+c)}{d}+\frac{\left(5\,a^{4}b+10\,a^{2}b^{3}+b^{5}\right)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}+\frac{4\left(15\,a^{3}b^{2}+10\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="giac")
```

```
[Out] (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(d*x + c)/d + (5*
a^4*b + 10*a^2*b^3 + b^5)*log(abs(e^(2*d*x + 2*c) - 1))/d + 4/3*(15*a^3*b^2
 + 10*a*b^4 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5)*e^(6*d*x + 6*c) + 3
*(15*a^3*b^2 + 10*a^2*b^3 + 10*a*b^4 + b^5)*e^(4*d*x + 4*c) - (45*a^3*b^2 +
 15*a^2*b^3 + 25*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/(d*(e^(2*d*x + 2*c) - 1)^4
)
```

3.78 $\int (a+b\coth(c+dx))^4\,dx$

Optimal. Leaf size=101

$$-\frac{b^2\left(3a^2+b^2\right)\coth(c+dx)}{d}+\frac{4ab\left(a^2+b^2\right)\log(\sinh(c+dx))}{d}+x\left(6a^2b^2+a^4+b^4\right)-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a$$

```
[Out] (a^4 + 6*a^2*b^2 + b^4)*x - (b^2*(3*a^2 + b^2)*Coth[c + d*x])/d - (a*b*(a +
 b*Coth[c + d*x])^2)/d - (b*(a + b*Coth[c + d*x])^3)/(3*d) + (4*a*b*(a^2 +
b^2)*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.122539, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3482, 3528, 3525, 3475}

$$-\frac{b^2\left(3a^2+b^2\right)\coth(c+dx)}{d}+\frac{4ab\left(a^2+b^2\right)\log(\sinh(c+dx))}{d}+x\left(6a^2b^2+a^4+b^4\right)-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^4,x]
```

```
[Out] (a^4 + 6*a^2*b^2 + b^4)*x - (b^2*(3*a^2 + b^2)*Coth[c + d*x])/d - (a*b*(a +
 b*Coth[c + d*x])^2)/d - (b*(a + b*Coth[c + d*x])^3)/(3*d) + (4*a*b*(a^2 +
b^2)*Log[Sinh[c + d*x]])/d
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\coth(c+dx))^4\,dx = -\frac{b(a+b\coth(c+dx))^3}{3d} + \int (a+b\coth(c+dx))^2\left(a^2+b^2+2ab\coth(c+dx)\right)dx$$
$$= -\frac{ab(a+b\coth(c+dx))^2}{d} - \frac{b(a+b\coth(c+dx))^3}{3d} + \int (a+b\coth(c+dx))\left(a\left(a^2+3b^2\right)+$$
$$= \left(a^4+6a^2b^2+b^4\right)x - \frac{b^2\left(3a^2+b^2\right)\coth(c+dx)}{d} - \frac{ab(a+b\coth(c+dx))^2}{d} - \frac{b(a+b\coth(}{3d}$$
$$= \left(a^4+6a^2b^2+b^4\right)x - \frac{b^2\left(3a^2+b^2\right)\coth(c+dx)}{d} - \frac{ab(a+b\coth(c+dx))^2}{d} - \frac{b(a+b\coth(}{3d}$$

Mathematica [A] time = 0.905219, size = 109, normalized size = 1.08

$$-\frac{6b^2\left(6a^2+b^2\right)\coth(c+dx) - 24ab\left(a^2+b^2\right)\log(\tanh(c+dx)) + 12ab^3\coth^2(c+dx) - 3(a-b)^4\log(\tanh(c+dx)+1}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^4,x]
```

```
[Out] -(6*b^2*(6*a^2 + b^2)*Coth[c + d*x] + 12*a*b^3*Coth[c + d*x]^2 + 2*b^4*Coth
[c + d*x]^3 + 3*(a + b)^4*Log[1 - Tanh[c + d*x]] - 24*a*b*(a^2 + b^2)*Log[T
anh[c + d*x]] - 3*(a - b)^4*Log[1 + Tanh[c + d*x]])/(6*d)
```

Maple [B] time = 0.004, size = 246, normalized size = 2.4

$$-\frac{b^4(\coth(dx+c))^3}{3d} - 2\frac{(\coth(dx+c))^2ab^3}{d} - 6\frac{a^2\coth(dx+c)b^2}{d} - \frac{\coth(dx+c)b^4}{d} - \frac{\ln(\coth(dx+c)-1)a^4}{2d} - 2\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^4,x)
```

```
[Out] -1/3/d*b^4*coth(d*x+c)^3-2/d*coth(d*x+c)^2*a*b^3-6/d*coth(d*x+c)*a^2*b^2-1/
d*coth(d*x+c)*b^4-1/2/d*ln(coth(d*x+c)-1)*a^4-2/d*ln(coth(d*x+c)-1)*a^3*b-3
/d*ln(coth(d*x+c)-1)*a^2*b^2-2/d*ln(coth(d*x+c)-1)*a*b^3-1/2/d*ln(coth(d*x+
c)-1)*b^4+1/2/d*ln(coth(d*x+c)+1)*a^4-2/d*ln(coth(d*x+c)+1)*a^3*b+3/d*ln(co
th(d*x+c)+1)*a^2*b^2-2/d*ln(coth(d*x+c)+1)*a*b^3+1/2/d*ln(coth(d*x+c)+1)*b^
4
```

Maxima [B] time = 1.11372, size = 296, normalized size = 2.93

$$\frac{1}{3}b^4\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2\right)}{d\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1\right)}\right) + 4ab^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b^3
*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
```

```
- 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 6*a^2*b^2*(x + c
/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^4*x + 4*a^3*b*log(sinh(d*x + c))/d
```

Fricas [B] time = 2.53483, size = 3318, normalized size = 32.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^6 + 18
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c
)^5 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*sinh(d*x + c)^6 - 3
*(12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x)*cosh(d*x + c)^4 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4
)*d*x*cosh(d*x + c)^2 - 12*a^2*b^2 - 8*a*b^3 - 4*b^4 - 3*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*sinh(d*x + c)^4 - 36*a^2*b^2 - 8*b^4 + 12*(5
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 - (12*a^2*
b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x + 3*(24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x*cosh(d*x + c)^4 + 24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x - 6*(12*a^2*b^2 + 8*a*b^3 + 4*b^4
+ 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 12*((a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(a^3*b + a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^3*b + a*b^3)*sinh(d*x + c)^6 - 3*(a^3*b + a*b^
3)*cosh(d*x + c)^4 - 3*(a^3*b + a*b^3 - 5*(a^3*b + a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 - a^3*b - a*b^3 + 4*(5*(a^3*b + a*b^3)*cosh(d*x + c)^3 - 3*
(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3*b + a*b^3)*cosh(d*x
 + c)^2 + 3*(5*(a^3*b + a*b^3)*cosh(d*x + c)^4 + a^3*b + a*b^3 - 6*(a^3*b +
 a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((a^3*b + a*b^3)*cosh(d*x + c)
^5 - 2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*cosh(d*x + c))*sin
h(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^5 - 2*(12*a^2*b^
2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*co
sh(d*x + c)^3 + (24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^
2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4
 + 3*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x +
 c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2
*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [A] time = 10.181, size = 233, normalized size = 2.31

$$\begin{cases} x\left(a + b\coth\left(c\right)\right)^{4} \\ a^{4}x + \tilde{\infty}a^{3}bx + \tilde{\infty}a^{2}b^{2}x + \tilde{\infty}ab^{3}x + \tilde{\infty}b^{4}x \\ a^{4}x + 4a^{3}bx - \frac{4a^{3}b\log\left(\tanh\left(c+dx\right)+1\right)}{d} + \frac{4a^{3}b\log\left(\tanh\left(c+dx\right)\right)}{d} + 6a^{2}b^{2}x - \frac{6a^{2}b^{2}}{d\tanh\left(c+dx\right)} + 4ab^{3}x - \frac{4ab^{3}\log\left(\tanh\left(c+dx\right)+1\right)}{d} + \frac{4ab^{3}}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**4,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**4, Eq(d, 0)), (a**4*x + zoo*a**3*b*x + zoo*a*
*2*b**2*x + zoo*a*b**3*x + zoo*b**4*x, Eq(c, log(exp(-d*x))) | Eq(c, log(-e
xp(-d*x)))), (a**4*x + 4*a**3*b*x - 4*a**3*b*log(tanh(c + d*x) + 1)/d + 4*a
**3*b*log(tanh(c + d*x))/d + 6*a**2*b**2*x - 6*a**2*b**2/(d*tanh(c + d*x))
+ 4*a*b**3*x - 4*a*b**3*log(tanh(c + d*x) + 1)/d + 4*a*b**3*log(tanh(c + d*
x))/d - 2*a*b**3/(d*tanh(c + d*x)**2) + b**4*x - b**4/(d*tanh(c + d*x)) - b
**4/(3*d*tanh(c + d*x)**3), True))
```

Giac [A] time = 1.13745, size = 211, normalized size = 2.09

$$\frac{\left(a^4 - 4\,a^3b + 6\,a^2b^2 - 4\,ab^3 + b^4\right)(dx + c)}{d} + \frac{4\left(a^3b + ab^3\right)\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right)}{d} - \frac{4\left(9\,a^2b^2 + 2\,b^4 + 3\left(3\,a^2b^2 + 2\,ab^3 + b^4\right)\right.}{3\,d\left(e^{(2\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(d*x + c)/d + 4*(a^3*b + a*b^3)
*log(abs(e^(2*d*x + 2*c) - 1))/d - 4/3*(9*a^2*b^2 + 2*b^4 + 3*(3*a^2*b^2 +
2*a*b^3 + b^4)*e^(4*d*x + 4*c) - 3*(6*a^2*b^2 + 2*a*b^3 + b^4)*e^(2*d*x + 2
*c))/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.79 $\int (a+b\coth(c+dx))^3\,dx$

Optimal. Leaf size=69

$$\frac{b(3a^2+b^2)\log(\sinh(c+dx))}{d}+ax(a^2+3b^2)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

```
[Out] a*(a^2 + 3*b^2)*x - (2*a*b^2*Coth[c + d*x])/d - (b*(a + b*Coth[c + d*x])^2)
/(2*d) + (b*(3*a^2 + b^2)*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0638579, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3482, 3525, 3475}

$$\frac{b(3a^2+b^2)\log(\sinh(c+dx))}{d}+ax(a^2+3b^2)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^3,x]
```

```
[Out] a*(a^2 + 3*b^2)*x - (2*a*b^2*Coth[c + d*x])/d - (b*(a + b*Coth[c + d*x])^2)
/(2*d) + (b*(3*a^2 + b^2)*Log[Sinh[c + d*x]])/d
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\coth(c+dx))^3\,dx &= -\frac{b(a+b\coth(c+dx))^2}{2d}+\int (a+b\coth(c+dx))\left(a^2+b^2+2ab\coth(c+dx)\right)dx\\ &= a(a^2+3b^2)x-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}+\left(b(3a^2+b^2)\right)\int \coth(c\\ &= a(a^2+3b^2)x-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}+\frac{b(3a^2+b^2)\log(\sinh(c+}{d}\end{aligned}$$

Mathematica [A] time = 0.402468, size = 86, normalized size = 1.25

$$-\frac{-2b(3a^2+b^2)\log(\tanh(c+dx))+6ab^2\coth(c+dx)+(a-b)^3(-\log(\tanh(c+dx)+1))+(a+b)^3\log(1-\tanh(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^3,x]
```

```
[Out] -(6*a*b^2*Coth[c + d*x] + b^3*Coth[c + d*x]^2 + (a + b)^3*Log[1 - Tanh[c +
d*x]] - 2*b*(3*a^2 + b^2)*Log[Tanh[c + d*x]] - (a - b)^3*Log[1 + Tanh[c + d
*x]])/(2*d)
```

Maple [B] time = 0.004, size = 173, normalized size = 2.5

$$-\frac{b^3 (\coth (dx+c))^2}{2d} - 3\frac{ab^2 \coth (dx+c)}{d} - \frac{\ln (\coth (dx+c)-1)\, a^3}{2d} - \frac{3\ln (\coth (dx+c)-1)\, a^2 b}{2d} - \frac{3\ln (\coth (dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^3,x)
```

```
[Out] -1/2/d*b^3*coth(d*x+c)^2-3*a*b^2*coth(d*x+c)/d-1/2/d*ln(coth(d*x+c)-1)*a^3-
3/2/d*ln(coth(d*x+c)-1)*a^2*b-3/2/d*ln(coth(d*x+c)-1)*a*b^2-1/2/d*ln(coth(d
*x+c)-1)*b^3+1/2/d*ln(coth(d*x+c)+1)*a^3-3/2/d*ln(coth(d*x+c)+1)*a^2*b+3/2/
d*ln(coth(d*x+c)+1)*a*b^2-1/2/d*ln(coth(d*x+c)+1)*b^3
```

Maxima [B] time = 1.11061, size = 184, normalized size = 2.67

$$b^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1\right)}\right) + 3\,ab^2\left(x + \frac{c}{d} + \frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 3*a*b^2*(x +
c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^3*x + 3*a^2*b*log(sinh(d*x + c))/d
```

Fricas [B] time = 2.64428, size = 1563, normalized size = 22.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*x*sinh(d*x + c)^4 + 6*a*b^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x
- 2*(3*a*b^2 + b^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 +
 2*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^2 - 3*a*b^2 - b^3 -
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((3*a^2*b + b^3)*co
sh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b
+ b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3 - 2*(3*a^2*b + b^3)*cosh(d*x + c)^2
```

```
- 2*(3*a^2*b + b^3 - 3*(3*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((3*a^2*b + b^3)*cosh(d*x + c)^3 - (3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x
 + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^3 - (3*a*b^2 + b^3 + (a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4
*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2
+ 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*co
sh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 4.43112, size = 175, normalized size = 2.54

$$\begin{cases} x\left(a+b\coth(c)\right)^3 \\ a^3x+\tilde{\infty}a^2bx+\tilde{\infty}ab^2x+\tilde{\infty}b^3x \\ a^3x+3a^2bx-\frac{3a^2b\log(\tanh(c+dx)+1)}{d}+\frac{3a^2b\log(\tanh(c+dx))}{d}+3ab^2x-\frac{3ab^2}{d\tanh(c+dx)}+b^3x-\frac{b^3\log(\tanh(c+dx)+1)}{d}+\frac{b^3\log(\tanh}{d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**3,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**3, Eq(d, 0)), (a**3*x + zoo*a**2*b*x + zoo*a*
b**2*x + zoo*b**3*x, Eq(c, log(exp(-d*x))) | Eq(c, log(-exp(-d*x)))), (a**3
*x + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d + 3*a**2*b*log(tanh(c +
 d*x))/d + 3*a*b**2*x - 3*a*b**2/(d*tanh(c + d*x)) + b**3*x - b**3*log(tanh
(c + d*x) + 1)/d + b**3*log(tanh(c + d*x))/d - b**3/(2*d*tanh(c + d*x)**2),
 True))
```

Giac [A] time = 1.14267, size = 140, normalized size = 2.03

$$\frac{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)(dx+c)}{d}+\frac{\left(3\,a^2b+b^3\right)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}+\frac{2\left(3\,ab^2-\left(3\,ab^2+b^3\right)e^{(2\,dx+2\,c)}\right)}{d\left(e^{(2\,dx+2\,c)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(d*x + c)/d + (3*a^2*b + b^3)*log(abs(e^(2*
d*x + 2*c) - 1))/d + 2*(3*a*b^2 - (3*a*b^2 + b^3)*e^(2*d*x + 2*c))/(d*(e^(2
*d*x + 2*c) - 1)^2)
```

3.80 $\int (a + b\coth(c + dx))^2\, dx$

Optimal. Leaf size=38

$$x\left(a^2+b^2\right)+\frac{2ab\log(\sinh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

```
[Out] (a^2 + b^2)*x - (b^2*Coth[c + d*x])/d + (2*a*b*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.024292, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3477, 3475}

$$x\left(a^2+b^2\right)+\frac{2ab\log(\sinh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^2,x]
```

```
[Out] (a^2 + b^2)*x - (b^2*Coth[c + d*x])/d + (2*a*b*Log[Sinh[c + d*x]])/d
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\coth(c+dx))^2\,dx &= \left(a^2+b^2\right)x-\frac{b^2\coth(c+dx)}{d}+(2ab)\int \coth(c+dx)\,dx\\ &= \left(a^2+b^2\right)x-\frac{b^2\coth(c+dx)}{d}+\frac{2ab\log(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.127926, size = 65, normalized size = 1.71

$$\frac{(a-b)^2\log(\tanh(c+dx)+1)-(a+b)^2\log(1-\tanh(c+dx))+4ab\log(\tanh(c+dx))-2b^2\coth(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^2,x]
```

```
[Out] (-2*b^2*Coth[c + d*x] - (a + b)^2*Log[1 - Tanh[c + d*x]] + 4*a*b*Log[Tanh[c
 + d*x]] + (a - b)^2*Log[1 + Tanh[c + d*x]])/(2*d)
```

Maple [B] time = 0.005, size = 116, normalized size = 3.1

$$-\frac{b^2\coth(dx+c)}{d}-\frac{\ln(\coth(dx+c)-1)\,a^2}{2d}-\frac{\ln(\coth(dx+c)-1)\,ab}{d}-\frac{\ln(\coth(dx+c)-1)\,b^2}{2d}+\frac{\ln(\coth(dx+c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^2,x)
```

```
[Out] -b^2*coth(d*x+c)/d-1/2/d*ln(coth(d*x+c)-1)*a^2-1/d*ln(coth(d*x+c)-1)*a*b-1/
2/d*ln(coth(d*x+c)-1)*b^2+1/2/d*ln(coth(d*x+c)+1)*a^2-1/d*ln(coth(d*x+c)+1)
*a*b+1/2/d*ln(coth(d*x+c)+1)*b^2
```

Maxima [A] time = 1.01268, size = 66, normalized size = 1.74

$$b^2\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)+a^2x+\frac{2\,ab\log(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^2*x + 2*a*b*log(sinh(d*x +
 c))/d
```

Fricas [B] time = 2.85372, size = 535, normalized size = 14.08

$$\frac{\left(a^2-2\,ab+b^2\right)dx\cosh(dx+c)^2+2\left(a^2-2\,ab+b^2\right)dx\cosh(dx+c)\sinh(dx+c)+\left(a^2-2\,ab+b^2\right)dx\sinh(dx+c)}{d\cosh(dx+c)^2+2\,d\cosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - 2*a*b + b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c) + (a^2 - 2*a*b + b^2)*d*x*sinh(d*x + c)^2 - (a^2 - 2*
a*b + b^2)*d*x - 2*b^2 + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(
d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(2*sinh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*s
inh(d*x + c)^2 - d)
```

Sympy [A] time = 1.86648, size = 104, normalized size = 2.74

$$\begin{cases} x\,(a+b\coth(c))^2 & \text{for } d=0 \\ a^2x+\tilde{\infty}abx+\tilde{\infty}b^2x & \text{for } c=\log\left(-e^{-dx}\right)\vee c=\log\left(e^{-dx}\right) \\ a^2x+2abx-\frac{2ab\log(\tanh(c+dx)+1)}{d}+\frac{2ab\log(\tanh(c+dx))}{d}+b^2x-\frac{b^2}{d\tanh(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**2, Eq(d, 0)), (a**2*x + zoo*a*b*x + zoo*b**2*
x, Eq(c, log(exp(-d*x))) | Eq(c, log(-exp(-d*x)))), (a**2*x + 2*a*b*x - 2*a
*b*log(tanh(c + d*x) + 1)/d + 2*a*b*log(tanh(c + d*x))/d + b**2*x - b**2/(d
*tanh(c + d*x)), True))
```

Giac [A] time = 1.12428, size = 84, normalized size = 2.21

$$\frac{2\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}+\frac{\left(a^{2}-2\,ab+b^{2}\right)(dx+c)}{d}-\frac{2\,b^{2}}{d\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*a*b*log(abs(e^(2*d*x + 2*c) - 1))/d + (a^2 - 2*a*b + b^2)*(d*x + c)/d - 2
*b^2/(d*(e^(2*d*x + 2*c) - 1))
```

3.81 $\int \frac{1}{a+b\coth(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\left(a^2-b^2\right)}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)
*d)
```

Rubi [A] time = 0.0539133, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3484, 3530}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)
*d)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\coth(c+dx)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(b\cosh(c+dx)+a\sinh(c+dx))}{\left(a^2-b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.0792178, size = 64, normalized size = 1.28

$$\frac{(b-a)\log(1-\coth(c+dx))+(a+b)\log(\coth(c+dx)+1)-2b\log(a+b\coth(c+dx))}{2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-1),x]
```

```
[Out] ((-a + b)*Log[1 - Coth[c + d*x]] + (a + b)*Log[1 + Coth[c + d*x]] - 2*b*Log
[a + b*Coth[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A] time = 0.019, size = 76, normalized size = 1.5

$$\frac{\ln(\coth(dx+c)+1)}{d(2a-2b)} - \frac{\ln(\coth(dx+c)-1)}{d(2b+2a)} - \frac{b\ln(a+b\coth(dx+c))}{d(a-b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c)),x)
```

```
[Out] 1/d/(2*a-2*b)*ln(coth(d*x+c)+1)-1/d/(2*b+2*a)*ln(coth(d*x+c)-1)-1/d*b/(a-b)
/(a+b)*ln(a+b*coth(d*x+c))
```

Maxima [A] time = 1.12232, size = 70, normalized size = 1.4

$$-\frac{b\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{\left(a^2-b^2\right)d} + \frac{dx+c}{(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2 - b^2)*d) + (d*x + c)/((a +
 b)*d)
```

Fricas [A] time = 2.88622, size = 149, normalized size = 2.98

$$\frac{(a+b)dx - b\log\left(\frac{2\,(b\cosh(dx+c)+a\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a + b)*d*x - b*log(2*(b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 - b^2)*d)
```

Sympy [A] time = 3.14948, size = 236, normalized size = 4.72

$$\begin{cases} \frac{\tilde{\infty}x}{\coth(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ -\frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)-2bd} + \frac{dx}{2bd\tanh(c+dx)-2bd} - \frac{1}{2bd\tanh(c+dx)-2bd} & \text{for } a=-b \\ \frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)+2bd} + \frac{dx}{2bd\tanh(c+dx)+2bd} + \frac{1}{2bd\tanh(c+dx)+2bd} & \text{for } a=b \\ \frac{x}{a+b\coth(c)} & \text{for } d=0 \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}}{b} & \text{for } a=0 \\ \frac{adx}{a^2d-b^2d} - \frac{bdx}{a^2d-b^2d} + \frac{b\log(\tanh(c+dx)+1)}{a^2d-b^2d} - \frac{b\log\left(\tanh(c+dx)+\frac{b}{a}\right)}{a^2d-b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/coth(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*x*tanh(c + d
*x)/(2*b*d*tanh(c + d*x) - 2*b*d) + d*x/(2*b*d*tanh(c + d*x) - 2*b*d) - 1/(
2*b*d*tanh(c + d*x) - 2*b*d), Eq(a, -b)), (d*x*tanh(c + d*x)/(2*b*d*tanh(c
+ d*x) + 2*b*d) + d*x/(2*b*d*tanh(c + d*x) + 2*b*d) + 1/(2*b*d*tanh(c + d*x
) + 2*b*d), Eq(a, b)), (x/(a + b*coth(c)), Eq(d, 0)), ((x - log(tanh(c + d*
x) + 1)/d)/b, Eq(a, 0)), (a*d*x/(a**2*d - b**2*d) - b*d*x/(a**2*d - b**2*d)
 + b*log(tanh(c + d*x) + 1)/(a**2*d - b**2*d) - b*log(tanh(c + d*x) + b/a)/
(a**2*d - b**2*d), True))
```

Giac [A] time = 1.14268, size = 85, normalized size = 1.7

$$-\frac{b\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^2d-b^2d}+\frac{dx+c}{ad-bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^2*d - b^2*d)
+ (d*x + c)/(a*d - b*d)
```

3.82 $\int \frac{1}{(a+b\coth(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{b}{d\left(a^2-b^2\right)(a+b\coth(c+dx))}-\frac{2ab\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{x\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}$$

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 + b/((a^2 - b^2)*d*(a + b*Coth[c + d*x])) - (
2*a*b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^2*d)
```

Rubi [A] time = 0.0946267, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3483, 3531, 3530}

$$\frac{b}{d\left(a^2-b^2\right)(a+b\coth(c+dx))}-\frac{2ab\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{x\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-2),x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 + b/((a^2 - b^2)*d*(a + b*Coth[c + d*x])) - (
2*a*b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^2*d)
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{b}{\left(a^2-b^2\right)d(a+b\coth(c+dx))} + \frac{\int \frac{a-b\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{a^2-b^2}$$
$$= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2} + \frac{b}{\left(a^2-b^2\right)d(a+b\coth(c+dx))} - \frac{(2iab)\int \frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{\left(a^2-b^2\right)^2}$$
$$= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2} + \frac{b}{\left(a^2-b^2\right)d(a+b\coth(c+dx))} - \frac{2ab\log(b\cosh(c+dx)+a\sinh(c+dx}{\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 1.4201, size = 100, normalized size = 1.18

$$\frac{\frac{2b\left(\frac{b^3-a^2b}{a\tanh(c+dx)+b}-2a^2\log(a\tanh(c+dx)+b)\right)}{a\left(a^2-b^2\right)^2}-\frac{\log(1-\tanh(c+dx))}{(a+b)^2}+\frac{\log(\tanh(c+dx)+1)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-2),x]
```

```
[Out] (-(Log[1 - Tanh[c + d*x]]/(a + b)^2) + Log[1 + Tanh[c + d*x]]/(a - b)^2 + (
2*b*(-2*a^2*Log[b + a*Tanh[c + d*x]] + (-(a^2*b) + b^3)/(b + a*Tanh[c + d*x
])))/(a*(a^2 - b^2)^2))/(2*d)
```

Maple [A] time = 0.025, size = 101, normalized size = 1.2

$$\frac{\ln(\coth(dx+c)+1)}{2d(a-b)^2} - \frac{\ln(\coth(dx+c)-1)}{2d(a+b)^2} + \frac{b}{d(a-b)(a+b)(a+b\coth(dx+c))} - 2\frac{ab\ln(a+b\coth(dx+c))}{d(a+b)^2(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^2,x)
```

```
[Out] 1/2/d/(a-b)^2*ln(coth(d*x+c)+1)-1/2/d/(a+b)^2*ln(coth(d*x+c)-1)+1/d*b/(a-b)
/(a+b)/(a+b*coth(d*x+c))-2/d*a*b/(a+b)^2/(a-b)^2*ln(a+b*coth(d*x+c))
```

Maxima [A] time = 1.16493, size = 167, normalized size = 1.96

$$-\frac{2ab\log\left(-(a-b)e^{(-2dx-2c)}+a+b\right)}{\left(a^4-2a^2b^2+b^4\right)d} - \frac{2b^2}{\left(a^4-2a^2b^2+b^4-\left(a^4-2a^3b+2ab^3-b^4\right)e^{(-2dx-2c)}\right)d} + \frac{dx+c}{\left(a^2+2ab+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^4 - 2*a^2*b^2 + b^4)*d) -
 2*b^2/((a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*d*x
- 2*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

Fricas [B] time = 2.67566, size = 980, normalized size = 11.53

$$\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)dx\cosh\left(dx+c\right)^2+2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a^3+3\,a^2b+3\,ab\right.}{\left(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5\right)d\cosh\left(dx+c\right)^2+2\left(a^5+a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + 2*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x*sinh(d*x + c)^2 - 2*a*b^2 + 2*b^3 - (a^3 + a^2*b - a*b^2 - b^3)*d*
x + 2*(a^2*b - a*b^2 - (a^2*b + a*b^2)*cosh(d*x + c)^2 - 2*(a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c) - (a^2*b + a*b^2)*sinh(d*x + c)^2)*log(2*(b*cos
h(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))))/((a^5 + a^4
*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)^2 + 2*(a^5 + a^4*
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a
^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*sinh(d*x + c)^2 - (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.1549, size = 185, normalized size = 2.18

$$-\frac{2\,ab\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^4d-2\,a^2b^2d+b^4d}+\frac{dx+c}{a^2d-2\,abd+b^2d}-\frac{2\left(ab^2-b^3\right)}{\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right)(a+b)^2(a-b)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^4*d - 2*a
^2*b^2*d + b^4*d) + (d*x + c)/(a^2*d - 2*a*b*d + b^2*d) - 2*(a*b^2 - b^3)/(
(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b)*(a + b)^2*(a - b)^2*d)
```

3.83 $\int \frac{1}{(a+b\coth(c+dx))^3} dx$

Optimal. Leaf size=129

$$\frac{2ab}{d(a^2-b^2)^2(a+b\coth(c+dx))}+\frac{b}{2d(a^2-b^2)(a+b\coth(c+dx))^2}-\frac{b(3a^2+b^2)\log(a\sinh(c+dx)+b\cosh(c+
$$

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*d*(a + b*Coth[c + d*x]
)^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])) - (b*(3*a^2 + b^2)*L
og[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^3*d)
```

Rubi [A] time = 0.179718, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3483, 3529, 3531, 3530}

$$\frac{2ab}{d(a^2-b^2)^2(a+b\coth(c+dx))}+\frac{b}{2d(a^2-b^2)(a+b\coth(c+dx))^2}-\frac{b(3a^2+b^2)\log(a\sinh(c+dx)+b\cosh(c+
$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-3),x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*d*(a + b*Coth[c + d*x]
)^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])) - (b*(3*a^2 + b^2)*L
og[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = \frac{b}{2\left(a^2-b^2\right)d(a+b\coth(c+dx))^2} + \frac{\int \frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^2}\,dx}{a^2-b^2}$$

$$= \frac{b}{2\left(a^2-b^2\right)d(a+b\coth(c+dx))^2} + \frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))} + \frac{\int \frac{a^2+b^2-2ab\coth(c+d}{a+b\coth(c+dx)}}{\left(a^2-b^2\right)^2}$$

$$= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3} + \frac{b}{2\left(a^2-b^2\right)d(a+b\coth(c+dx))^2} + \frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))} - \frac{(i}{}$$

$$= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3} + \frac{b}{2\left(a^2-b^2\right)d(a+b\coth(c+dx))^2} + \frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))} - \frac{b}{}$$

Mathematica [A] time = 3.43228, size = 134, normalized size = 1.04

$$-\frac{\frac{b\left(\frac{b\left(b^2-a^2\right)\left(\left(2ab^2-6a^3\right)\tanh(c+dx)-5a^2b+b^3\right)}{a^2(a\tanh(c+dx)+b)^2}+2\left(3a^2+b^2\right)\log(a\tanh(c+dx)+b)\right)}{\left(a^2-b^2\right)^3}+\frac{\log(1-\tanh(c+dx))}{(a+b)^3}-\frac{\log(\tanh(c+dx)+1)}{(a-b)^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-3),x]
```

```
[Out] -(Log[1 - Tanh[c + d*x]]/(a + b)^3 - Log[1 + Tanh[c + d*x]]/(a - b)^3 + (b*
(2*(3*a^2 + b^2)*Log[b + a*Tanh[c + d*x]] + (b*(-a^2 + b^2)*(-5*a^2*b + b^3
 + (-6*a^3 + 2*a*b^2)*Tanh[c + d*x]))/(a^2*(b + a*Tanh[c + d*x])^2)))/(a^2
- b^2)^3)/(2*d)
```

Maple [A] time = 0.031, size = 166, normalized size = 1.3

$$\frac{\ln(\coth(dx+c)+1)}{2d(a-b)^3} - \frac{\ln(\coth(dx+c)-1)}{2d(a+b)^3} + \frac{b}{2d(a-b)(a+b)(a+b\coth(dx+c))^2} + 2\frac{ab}{d(a+b)^2(a-b)^2(a+b\coth}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^3,x)
```

```
[Out] 1/2/d/(a-b)^3*ln(coth(d*x+c)+1)-1/2/d/(a+b)^3*ln(coth(d*x+c)-1)+1/2/d*b/(a-
b)/(a+b)/(a+b*coth(d*x+c))^2+2/d*a*b/(a+b)^2/(a-b)^2/(a+b*coth(d*x+c))-3/d*
b/(a+b)^3/(a-b)^3*ln(a+b*coth(d*x+c))*a^2-1/d*b^3/(a+b)^3/(a-b)^3*ln(a+b*co
th(d*x+c))
```

Maxima [B] time = 1.18432, size = 435, normalized size = 3.37

$$-\frac{\left(3a^2b+b^3\right)\log\left(-(a-b)e^{(-2dx-2c)}+a+b\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)d} - \frac{}{\left(a^7+a^6b-3a^5b^2-3a^4b^3+3a^3b^4+3a^2b^5-ab^6-b^7-2\left(a^7-a^6b-3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b + b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*d) - 2*(3*a^2*b^2 + 3*a*b^3 - (3*a^2*b^2 - 2*a*b^3 - b^4)
*e^(-2*d*x - 2*c))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7 - 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*e^(-2*d*x - 2*c) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^
4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*e^(-4*d*x - 4*c))*d) + (d*x +
c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] time = 3.20442, size = 3182, normalized size = 24.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c
)^4 + 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*x*sinh(d*x + c)^4 + 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + (a^5 + a
^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*x - 2*(3*a^3*b^2 - a^2*b^3 -
3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x
)*cosh(d*x + c)^2 - 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 - 3*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^2 + (a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*sinh(d*x + c)^2 - (3*
a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
 + 2*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^4*b + 6*a^3*b^2 + 4*
a^2*b^3 + 2*a*b^4 + b^5)*sinh(d*x + c)^4 - 2*(3*a^4*b - 2*a^2*b^3 - b^5)*co
sh(d*x + c)^2 - 2*(3*a^4*b - 2*a^2*b^3 - b^5 - 3*(3*a^4*b + 6*a^3*b^2 + 4*a
^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^3 - (3*a^4*b - 2*a^2*b^3
 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*cosh(d*x + c) + a*sinh(d*x +
 c))/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^3 - (3*a^3*b^2 - a^2*b^3 - 3*a*
b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c))/((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3
*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^4 + 4*(a^8 + 2*a^7*b - 2*
a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c
)*sinh(d*x + c)^3 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*
a^2*b^6 - 2*a*b^7 - b^8)*d*sinh(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*d*cosh(d*x + c)^2 + 2*(3*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6
*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^2 - (a^8
- 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sinh(d*x + c)^2 + (a^8 - 2*a^
7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)*d + 4*
((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 -
 b^8)*d*cosh(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d
*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.18017, size = 285, normalized size = 2.21

$$-\frac{\left(3\,a^{2}b+b^{3}\right)\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^{6}d-3\,a^{4}b^{2}d+3\,a^{2}b^{4}d-b^{6}d}+\frac{dx+c}{a^{3}d-3\,a^{2}bd+3\,ab^{2}d-b^{3}d}-\frac{2\left(\left(3\,a^{2}b^{2}-4\,ab^{3}+b^{4}\right)e^{(2\,dx+2\,c)}-\frac{3}{-}\right.}{\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right)^{2}(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b + b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a
^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d) + (d*x + c)/(a^3*d - 3*a^2*b*d +
3*a*b^2*d - b^3*d) - 2*((3*a^2*b^2 - 4*a*b^3 + b^4)*e^(2*d*x + 2*c) - 3*(a^
3*b^2 - 2*a^2*b^3 + a*b^4)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c)
 - a + b)^2*(a + b)^2*(a - b)^3*d)
```

3.84 $\int \frac{1}{(a+b\coth(c+dx))^4}\,dx$

Optimal. Leaf size=169

$$\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a+b\coth(c+dx))}+\frac{ab}{d\left(a^2-b^2\right)^2(a+b\coth(c+dx))^2}+\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}-\frac{4ab\left(a^2+\right.}{}$$

```
[Out] ((a^4 + 6*a^2*b^2 + b^4)*x)/(a^2 - b^2)^4 + b/(3*(a^2 - b^2)*d*(a + b*Coth[
c + d*x])^3) + (a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])^2) + (b*(3*a^2
+ b^2))/((a^2 - b^2)^3*d*(a + b*Coth[c + d*x])) - (4*a*b*(a^2 + b^2)*Log[b*
Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^4*d)
```

Rubi [A] time = 0.264259, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3483, 3529, 3531, 3530}

$$\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a+b\coth(c+dx))}+\frac{ab}{d\left(a^2-b^2\right)^2(a+b\coth(c+dx))^2}+\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}-\frac{4ab\left(a^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-4),x]
```

```
[Out] ((a^4 + 6*a^2*b^2 + b^4)*x)/(a^2 - b^2)^4 + b/(3*(a^2 - b^2)*d*(a + b*Coth[
c + d*x])^3) + (a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])^2) + (b*(3*a^2
+ b^2))/((a^2 - b^2)^3*d*(a + b*Coth[c + d*x])) - (4*a*b*(a^2 + b^2)*Log[b*
Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
```

```
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{\int \frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^3}\,dx}{a^2-b^2}$$

$$= \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))^2} + \frac{\int \frac{a^2+b^2-2ab\coth(c+}{(a+b\coth(c+dx))}}{\left(a^2-b^2\right)^2}$$

$$= \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))^2} + \frac{b\left(3a^2+\right.}{\left(a^2-b^2\right)^3 d(a+b}$$

$$= \frac{\left(a^4+6a^2b^2+b^4\right)x}{\left(a^2-b^2\right)^4} + \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx)}$$

$$= \frac{\left(a^4+6a^2b^2+b^4\right)x}{\left(a^2-b^2\right)^4} + \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx)}$$

Mathematica [A] time = 6.20822, size = 214, normalized size = 1.27

$$-\frac{b^4}{3a^3d\left(a^2-b^2\right)(a\tanh(c+dx)+b)^3} + \frac{b^3\left(2a^2-b^2\right)}{a^3d\left(a^2-b^2\right)^2(a\tanh(c+dx)+b)^2} - \frac{b^2\left(-3a^2b^2+6a^4+b^4\right)}{a^3d\left(a^2-b^2\right)^3(a\tanh(c+dx)+b)} - \frac{4ab}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-4),x]
```

```
[Out] -Log[1 - Tanh[c + d*x]]/(2*(a + b)^4*d) + Log[1 + Tanh[c + d*x]]/(2*(a - b)
^4*d) - (4*a*b*(a^2 + b^2)*Log[b + a*Tanh[c + d*x]])/((a^2 - b^2)^4*d) - b^
4/(3*a^3*(a^2 - b^2)*d*(b + a*Tanh[c + d*x])^3) + (b^3*(2*a^2 - b^2))/(a^3*
(a^2 - b^2)^2*d*(b + a*Tanh[c + d*x])^2) - (b^2*(6*a^4 - 3*a^2*b^2 + b^4))/
(a^3*(a^2 - b^2)^3*d*(b + a*Tanh[c + d*x]))
```

Maple [A] time = 0.034, size = 230, normalized size = 1.4

$$\frac{\ln(\coth(dx+c)+1)}{2d(a-b)^4} - \frac{\ln(\coth(dx+c)-1)}{2d(a+b)^4} + \frac{b}{3d(a-b)(a+b)(a+b\coth(dx+c))^3} + \frac{ab}{d(a+b)^2(a-b)^2(a+b\coth}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^4,x)
```

```
[Out] 1/2/d/(a-b)^4*ln(coth(d*x+c)+1)-1/2/d/(a+b)^4*ln(coth(d*x+c)-1)+1/3/d*b/(a-
b)/(a+b)/(a+b*coth(d*x+c))^3+1/d*a*b/(a+b)^2/(a-b)^2/(a+b*coth(d*x+c))^2+3/
d*b/(a+b)^3/(a-b)^3/(a+b*coth(d*x+c))*a^2+1/d*b^3/(a+b)^3/(a-b)^3/(a+b*coth
(d*x+c))-4/d*b*a^3/(a+b)^4/(a-b)^4*ln(a+b*coth(d*x+c))-4/d*b^3*a/(a+b)^4/(a
-b)^4*ln(a+b*coth(d*x+c))
```

Maxima [B] time = 1.3454, size = 705, normalized size = 4.17

$$-\frac{4\left(a^{3}b+ab^{3}\right)\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)d}-\frac{}{3\left(a^{10}+2\,a^{9}b-3\,a^{8}b^{2}-8\,a^{7}b^{3}+2\,a^{6}b^{4}+12\,a^{5}b^{5}+2\,a^{4}b^{6}-8\,a^{3}b^{7}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -4*(a^3*b + a*b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d) - 4/3*(9*a^4*b^2 + 18*a^3*b^3 + 11*a^2*b
^4 + 4*a*b^5 + 2*b^6 - 3*(6*a^4*b^2 + 2*a^3*b^3 - 5*a^2*b^4 - 2*a*b^5 - b^6
)*e^(-2*d*x - 2*c) + 3*(3*a^4*b^2 - 4*a^3*b^3 + b^6)*e^(-4*d*x - 4*c))/((a^
10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 -
 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10 - 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4
- 10*a^4*b^6 + 5*a^2*b^8 - b^10)*e^(-2*d*x - 2*c) + 3*(a^10 - 2*a^9*b - 3*a
^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2
*b^8 - 2*a*b^9 + b^10)*e^(-4*d*x - 4*c) - (a^10 - 4*a^9*b + 3*a^8*b^2 + 8*a
^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^10)*
e^(-6*d*x - 6*c))*d) + (d*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d)
```

Fricas [B] time = 3.4678, size = 8128, normalized size = 48.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 +
 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^6 + 18*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a
^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)*sinh(d*
x + c)^5 + 3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2
*b^5 + 7*a*b^6 + b^7)*d*x*sinh(d*x + c)^6 - 36*a^5*b^2 + 108*a^4*b^3 - 116*
a^3*b^4 + 60*a^2*b^5 - 24*a*b^6 + 8*b^7 - 3*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^
3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*
b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^4 - 3*(12*a^5*b^2 + 4*a
^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 - 15*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35
*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^2 + 3
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 -
 b^7)*d*x)*sinh(d*x + c)^4 + 12*(5*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3
 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^3 - (12*a^5*b
^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^
2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^3 - 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7)*d*x + 3*(24*a^5*b^2 - 32*a^4*b^3 - 12*a^3*b^4 + 28*a^2
*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^
4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c)^2 + 3*(24*a^5*b^2 - 32*a^4*
b^3 - 12*a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 15*(a^7 + 7*a^6*b + 21*a
^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x
 + c)^4 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*
a*b^6 + b^7)*d*x - 6*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7
 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b
^6 - b^7)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*(a^6*b - 3*a^5*b^2 + 4
```

```
*a^4*b^3 - 4*a^3*b^4 + 3*a^2*b^5 - a*b^6 - (a^6*b + 3*a^5*b^2 + 4*a^4*b^3 +
 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^6 - 6*(a^6*b + 3*a^5*b^2 + 4*
a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)*sinh(d*x + c)^5 - (a
^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*sinh(d*x + c)
^6 + 3*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^4 + 3*(a^6*b + a^5
*b^2 - a^2*b^5 - a*b^6 - 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a
^2*b^5 + a*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(a^6*b + 3*a^5*b^2
+ 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^3 - 3*(a^6*b + a
^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^6*b - a^5*b
^2 - a^2*b^5 + a*b^6)*cosh(d*x + c)^2 - 3*(a^6*b - a^5*b^2 - a^2*b^5 + a*b^
6 + 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(
d*x + c)^4 - 6*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 - 6*((a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^
6)*cosh(d*x + c)^5 - 2*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^3
+ (a^6*b - a^5*b^2 - a^2*b^5 + a*b^6)*cosh(d*x + c))*sinh(d*x + c))*log(2*(
b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*
(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^
6 + b^7)*d*x*cosh(d*x + c)^5 - 2*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a
*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2
*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^3 + (24*a^5*b^2 - 32*a^4*b^3 - 12*
a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^
4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c))*sinh(d*x +
 c))/((a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14
*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x +
 c)^6 + 6*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5
+ 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b
^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10
+ b^11)*d*sinh(d*x + c)^6 - 3*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a
^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b
^10 - b^11)*d*cosh(d*x + c)^4 + 3*(5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^
3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9
+ 3*a*b^10 + b^11)*d*cosh(d*x + c)^2 - (a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b
^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*
b^9 - a*b^10 - b^11)*d)*sinh(d*x + c)^4 + 3*(a^11 - a^10*b - 5*a^9*b^2 + 5*
a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5
*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^11 + 3*a^10*b - a^9*b
^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*
b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^3 - 3*(a^11 + a^10*b - 5*a
^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*
a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5
*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^
4 - 6*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10
*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x +
 c)^2 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 -
10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d)*sinh(d*
x + c)^2 - (a^11 - 3*a^10*b - a^9*b^2 + 11*a^8*b^3 - 6*a^7*b^4 - 14*a^6*b^5
 + 14*a^5*b^6 + 6*a^4*b^7 - 11*a^3*b^8 + a^2*b^9 + 3*a*b^10 - b^11)*d + 6*(
(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b
^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^5
- 2*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a
^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c
)^3 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10
*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x +
 c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21574, size = 424, normalized size = 2.51

$$-\frac{4\left(a^{3}b+ab^{3}\right)\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^{8}d-4\,a^{6}b^{2}d+6\,a^{4}b^{4}d-4\,a^{2}b^{6}d+b^{8}d}+\frac{dx+c}{a^{4}d-4\,a^{3}bd+6\,a^{2}b^{2}d-4\,ab^{3}d+b^{4}d}-\frac{4\left(3\left(3\,a^{4}b^{2}-2\,a^{3}b^{3}-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4*(a^3*b + a*b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/
(a^8*d - 4*a^6*b^2*d + 6*a^4*b^4*d - 4*a^2*b^6*d + b^8*d) + (d*x + c)/(a^4*
d - 4*a^3*b*d + 6*a^2*b^2*d - 4*a*b^3*d + b^4*d) - 4/3*(3*(3*a^4*b^2 - 2*a^
3*b^3 - 2*a^2*b^4 + 2*a*b^5 - b^6)*e^(4*d*x + 4*c) - 3*(6*a^4*b^2 - 14*a^3*
b^3 + 11*a^2*b^4 - 4*a*b^5 + b^6)*e^(2*d*x + 2*c) + (9*a^5*b^2 - 27*a^4*b^3
 + 29*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6 - 2*b^7)/(a + b))/((a*e^(2*d*x + 2*c)
+ b*e^(2*d*x + 2*c) - a + b)^3*(a + b)^3*(a - b)^4*d)
```

3.85 $\int \frac{1}{4+6\coth(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{3\log(2\sinh(c+dx)+3\cosh(c+dx))}{10d}-\frac{x}{5}$$

```
[Out] -x/5 + (3*Log[3*Cosh[c + d*x] + 2*Sinh[c + d*x]])/(10*d)
```

Rubi [A] time = 0.0438151, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3484, 3530}

$$\frac{3\log(2\sinh(c+dx)+3\cosh(c+dx))}{10d}-\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -x/5 + (3*Log[3*Cosh[c + d*x] + 2*Sinh[c + d*x]])/(10*d)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+6\coth(c+dx)}\,dx &= -\frac{x}{5}+\frac{3}{10}i\int \frac{-6i-4i\coth(c+dx)}{4+6\coth(c+dx)}\,dx\\ &= -\frac{x}{5}+\frac{3\log(3\cosh(c+dx)+2\sinh(c+dx))}{10d}\end{aligned}$$

Mathematica [A] time = 0.0387123, size = 53, normalized size = 1.71

$$-\frac{\log(1-\tanh(c+dx))}{20d}-\frac{\log(\tanh(c+dx)+1)}{4d}+\frac{3\log(2\tanh(c+dx)+3)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -Log[1 - Tanh[c + d*x]]/(20*d) - Log[1 + Tanh[c + d*x]]/(4*d) + (3*Log[3 +
2*Tanh[c + d*x]])/(10*d)
```

Maple [A] time = 0.017, size = 46, normalized size = 1.5

$$-\frac{\ln\left(\coth\left(dx+c\right)+1\right)}{4d}-\frac{\ln\left(\coth\left(dx+c\right)-1\right)}{20d}+\frac{3\ln\left(2+3\coth\left(dx+c\right)\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+6*coth(d*x+c)),x)
```

```
[Out] -1/4/d*ln(coth(d*x+c)+1)-1/20/d*ln(coth(d*x+c)-1)+3/10/d*ln(2+3*coth(d*x+c)
)
```

Maxima [A] time = 1.10128, size = 38, normalized size = 1.23

$$\frac{dx+c}{10d}+\frac{3\log\left(e^{(-2dx-2c)}+5\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/10*(d*x + c)/d + 3/10*log(e^(-2*d*x - 2*c) + 5)/d
```

Fricas [A] time = 2.55375, size = 130, normalized size = 4.19

$$-\frac{5dx-3\log\left(\frac{2(3\cosh(dx+c)+2\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/10*(5*d*x - 3*log(2*(3*cosh(d*x + c) + 2*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/d
```

Sympy [A] time = 1.36237, size = 42, normalized size = 1.35

$$\begin{cases}\frac{x}{10}-\frac{3\log\left(\tanh\left(c+dx\right)+1\right)}{10d}+\frac{3\log\left(\tanh\left(c+dx\right)+\frac{3}{2}\right)}{10d} & \text{for}\ d\neq 0\\ \frac{x}{6\coth\left(c\right)+4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x)
```

```
[Out] Piecewise((x/10 - 3*log(tanh(c + d*x) + 1)/(10*d) + 3*log(tanh(c + d*x) + 3
/2)/(10*d), Ne(d, 0)), (x/(6*coth(c) + 4), True))
```

Giac [A] time = 1.17926, size = 41, normalized size = 1.32

$$-\frac{dx + c}{2\,d} + \frac{3\,\log\left(5\,e^{(2\,dx+2\,c)} + 1\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)/d + 3/10*log(5*e^(2*d*x + 2*c) + 1)/d
```

3.86 $\int \frac{1}{4-6\coth(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}-\frac{x}{5}$$

```
[Out] -x/5 - (3*Log[3*Cosh[c + d*x] - 2*Sinh[c + d*x]])/(10*d)
```

Rubi [A] time = 0.0424359, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3484, 3530}

$$-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}-\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -x/5 - (3*Log[3*Cosh[c + d*x] - 2*Sinh[c + d*x]])/(10*d)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-6\coth(c+dx)}\,dx &= -\frac{x}{5}-\frac{3}{10}i\int \frac{6i-4i\coth(c+dx)}{4-6\coth(c+dx)}\,dx\\ &= -\frac{x}{5}-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}\end{aligned}$$

Mathematica [A] time = 0.0374029, size = 53, normalized size = 1.71

$$-\frac{3\log(3-2\tanh(c+dx))}{10d}+\frac{\log(1-\tanh(c+dx))}{4d}+\frac{\log(\tanh(c+dx)+1)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 6*Coth[c + d*x])^(-1),x]
```

```
[Out] (-3*Log[3 - 2*Tanh[c + d*x]])/(10*d) + Log[1 - Tanh[c + d*x]]/(4*d) + Log[1
 + Tanh[c + d*x]]/(20*d)
```

Maple [A] time = 0.016, size = 46, normalized size = 1.5

$$\frac{\ln(\coth(dx+c)+1)}{20d} - \frac{3\ln(-2+3\coth(dx+c))}{10d} + \frac{\ln(\coth(dx+c)-1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-6*coth(d*x+c)),x)
```

```
[Out] 1/20/d*ln(coth(d*x+c)+1)-3/10/d*ln(-2+3*coth(d*x+c))+1/4/d*ln(coth(d*x+c)-1
)
```

Maxima [A] time = 1.0579, size = 39, normalized size = 1.26

$$-\frac{1}{2}x - \frac{c}{2d} - \frac{3\log\left(5e^{(-2dx-2c)}+1\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*c/d - 3/10*log(5*e^(-2*d*x - 2*c) + 1)/d
```

Fricas [A] time = 2.49815, size = 126, normalized size = 4.06

$$\frac{dx - 3\log\left(\frac{2(3\cosh(dx+c)-2\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/10*(d*x - 3*log(2*(3*cosh(d*x + c) - 2*sinh(d*x + c))/(cosh(d*x + c) - si
nh(d*x + c))))/d
```

Sympy [A] time = 1.18326, size = 42, normalized size = 1.35

$$\begin{cases} -\frac{x}{2} - \frac{3\log\left(\tanh(c+dx)-\frac{3}{2}\right)}{10d} + \frac{3\log(\tanh(c+dx)+1)}{10d} & \text{for } d \neq 0 \\ \frac{x}{4-6\coth(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x)
```

```
[Out] Piecewise((-x/2 - 3*log(tanh(c + d*x) - 3/2)/(10*d) + 3*log(tanh(c + d*x) +
 1)/(10*d), Ne(d, 0)), (x/(4 - 6*coth(c)), True))
```

Giac [A] time = 1.19247, size = 38, normalized size = 1.23

$$\frac{dx+c}{10\,d}-\frac{3\,\log\left(e^{(2\,dx+2\,c)}+5\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10*(d*x + c)/d - 3/10*log(e^(2*d*x + 2*c) + 5)/d
```

3.87 $\int \sqrt{a+b\coth(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]])/d
```

Rubi [A] time = 0.0700508, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3485, 700, 1130, 207}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]])/d
```

Rule 3485

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\coth(c+dx)}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}$$

$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{\right.}}{d}$$

$$= -\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Mathematica [C] time = 3.21564, size = 128, normalized size = 1.73

$$\frac{\sqrt{a+b\coth(c+dx)}\left(\sqrt{i(a+b)}\tanh^{-1}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a+b)}}\right) - \sqrt{i(a-b)}\tanh^{-1}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a-b)}}\right)\right)}{d\sqrt{i(a+b\coth(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] ((-(Sqrt[I*(a - b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a - b)]])
 + Sqrt[I*(a + b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a + b)]])*
Sqrt[a + b*Coth[c + d*x]])/(d*Sqrt[I*(a + b*Coth[c + d*x])])
```

Maple [A] time = 0.053, size = 63, normalized size = 0.9

$$\frac{1}{d}\text{Artanh}\left(\sqrt{a+b\coth(dx+c)}\frac{1}{\sqrt{a+b}}\right)\sqrt{a+b} - \frac{1}{d}\sqrt{-a+b}\arctan\left(\sqrt{a+b\coth(dx+c)}\frac{1}{\sqrt{-a+b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^(1/2),x)
```

```
[Out] arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d-1/d*(-a+b)^(1/2)
*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\coth(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c) + a), x)
```

Fricas [B] time = 3.21213, size = 5682, normalized size = 76.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh(d*x + c
)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*x + c)^4
 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - (2*a
 + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)
^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c)
+ a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) +
8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))*sinh(d*
x + c)) + sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*
b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*si
nh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x
 + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*
a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((b*cosh(d*x +
 c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 -
2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x +
 c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 - a^2 + b^2)) -
sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*
x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c
)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh
(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x
 + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((b*cosh(d*x + c) + a*si
nh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 - 2*(a^2 - a
*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)*sqrt(-a +
b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 - b^2)*cos
h(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh
(d*x + c)^2 - a^2 + 2*a*b - b^2)) - sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a
^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^
2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x +
 c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)
*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*si
nh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2
 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/d, -1/2*(sqrt(-a + b)*arctan(-(a*cos
h(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)
*sqrt(-a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2
 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2
```

```
- b^2)*sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) + sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 - a^2 + b^2)))/d
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \coth(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*coth(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*coth(d*x + c) + a), x)
```

3.88 $\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.0678776, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3485, 708, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 3485

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$
$$= -\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

Mathematica [A] time = 0.141617, size = 73, normalized size = 0.99

$$-\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -((ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b] - ArcTanh[Sqr
t[a + b*Coth[c + d*x]]/Sqrt[a + b]]/Sqrt[a + b])/d)
```

Maple [A] time = 0.046, size = 62, normalized size = 0.8

$$\frac{1}{d}\text{Artanh}\left(\sqrt{a+b\coth(dx+c)}\frac{1}{\sqrt{a+b}}\right)\frac{1}{\sqrt{a+b}} + \frac{1}{d}\arctan\left(\sqrt{a+b\coth(dx+c)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^(1/2),x)
```

```
[Out] arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/d/(-a+b)^(1/2)
*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\coth(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c) + a), x)
```

Fricas [B] time = 3.26532, size = 5844, normalized size = 78.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh
(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(
d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d
*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
+ c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c)) + (a + b)*sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 +
4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)
^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2
*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(
d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(
d*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sq
rt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*co
sh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a^2 - b^2)*d), -1/4*(
2*(a - b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
 + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 - a)*sqrt(-a - b)*sqrt((b*cos
h(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b +
 b^2)*sinh(d*x + c)^2 - a^2 + b^2)) - (a + b)*sqrt(a - b)*log(((2*a^2 - b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2
- b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)
*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 -
 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 - (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x +
 c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a
 - b)*sqrt(a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) +
 4*((2*a^2 - b^2)*cosh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x +
 c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a
^2 - b^2)*d), -1/4*(2*(a + b)*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a
*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)*sqrt(-a + b)*sqrt
((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 - b^2)*cosh(d*x +
 c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x +
c)^2 - a^2 + 2*a*b - b^2)) - sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(
a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b
^2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x
+ c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b
)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*s
inh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/((a^2 - b^2)*d), -1/2*((a + b)*sqrt
(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*s
inh(d*x + c)^2 - a + b)*sqrt(-a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c
```

```
))/sinh(d*x + c))/((a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c
)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) + (a -
b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*s
inh(d*x + c) + (a + b)*sinh(d*x + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x +
 c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^2 - a^2 + b^2)))/((a^2 - b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \coth (c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*coth(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth (dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c) + a), x)
```

3.89 $\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=60

$$\frac{5x}{16}+\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}-\frac{3}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

```
[Out] (5*x)/16 + 1/(32*(1 - Coth[x])^2) + 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x
])^3) - 3/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rubi [A] time = 0.062674, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3487, 44, 207}

$$\frac{5x}{16}+\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}-\frac{3}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (5*x)/16 + 1/(32*(1 - Coth[x])^2) + 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x
])^3) - 3/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x)^3(1+x)^4}\,dx, x, \coth(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}+\frac{1}{8(1+x)^4}+\frac{3}{16(1+x)^3}+\frac{3}{16(1+x)^2}-\frac{5}{16\left(-1+x^2\right)}\right)dx,\right.\\
&= \frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}-\frac{5}{16}\\
&= \frac{5x}{16}+\frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.103101, size = 42, normalized size = 0.7

$$\frac{1}{192}(60x - 45\sinh(2x) + 9\sinh(4x) - \sinh(6x) + 15\cosh(2x) - 6\cosh(4x) + \cosh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (60*x + 15*Cosh[2*x] - 6*Cosh[4*x] + Cosh[6*x] - 45*Sinh[2*x] + 9*Sinh[4*x]
 - Sinh[6*x])/192
```

Maple [B] time = 0.033, size = 110, normalized size = 1.8

$$\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-6} - \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5} + \frac{5}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4} + \frac{5}{12}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} - \frac{3}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{5}{16}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(1+coth(x)),x)
```

```
[Out] 1/3/(tanh(1/2*x)+1)^6-1/(tanh(1/2*x)+1)^5+5/8/(tanh(1/2*x)+1)^4+5/12/(tanh(
1/2*x)+1)^3-3/8/(tanh(1/2*x)+1)+5/16*ln(tanh(1/2*x)+1)+1/8/(tanh(1/2*x)-1)^
4+1/4/(tanh(1/2*x)-1)^3-1/8/(tanh(1/2*x)-1)^2-1/4/(tanh(1/2*x)-1)-5/16*ln(t
anh(1/2*x)-1)
```

Maxima [A] time = 1.06854, size = 49, normalized size = 0.82

$$-\frac{1}{128}\left(10\,e^{(-2\,x)}-1\right)e^{(4\,x)} + \frac{5}{16}\,x + \frac{5}{32}\,e^{(-2\,x)} - \frac{5}{128}\,e^{(-4\,x)} + \frac{1}{192}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -1/128*(10*e^(-2*x) - 1)*e^(4*x) + 5/16*x + 5/32*e^(-2*x) - 5/128*e^(-4*x)
+ 1/192*e^(-6*x)
```

Fricas [B] time = 2.54278, size = 321, normalized size = 5.35

$$\frac{5\cosh(x)^5 + 25\cosh(x)\sinh(x)^4 + \sinh(x)^5 + 5\left(2\cosh(x)^2-3\right)\sinh(x)^3 - 45\cosh(x)^3 + 5\left(10\cosh(x)^3 - 27\right.}{384\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + 5*(2*cosh(x)^2 - 3)
*sinh(x)^3 - 45*cosh(x)^3 + 5*(10*cosh(x)^3 - 27*cosh(x))*sinh(x)^2 + 60*(2
*x + 1)*cosh(x) + 5*(cosh(x)^4 - 9*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh(x)
 + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^4(x)}{\coth(x) + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**4/(coth(x) + 1), x)
```

Giac [A] time = 1.19885, size = 57, normalized size = 0.95

$$-\frac{1}{384} \left(110 e^{(6x)} - 60 e^{(4x)} + 15 e^{(2x)} - 2\right) e^{(-6x)} + \frac{5}{16} x + \frac{1}{128} e^{(4x)} - \frac{5}{64} e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/384*(110*e^(6*x) - 60*e^(4*x) + 15*e^(2*x) - 2)*e^(-6*x) + 5/16*x + 1/12
8*e^(4*x) - 5/64*e^(2*x)
```

3.90 $\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=29

$$\frac{4\cosh^3(x)}{15}-\frac{4\cosh(x)}{5}-\frac{\sinh^3(x)}{5(\coth(x)+1)}$$

```
[Out] (-4*Cosh[x])/5 + (4*Cosh[x]^3)/15 - Sinh[x]^3/(5*(1 + Coth[x]))
```

Rubi [A] time = 0.0474609, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3502, 2633}

$$\frac{4\cosh^3(x)}{15}-\frac{4\cosh(x)}{5}-\frac{\sinh^3(x)}{5(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (-4*Cosh[x])/5 + (4*Cosh[x]^3)/15 - Sinh[x]^3/(5*(1 + Coth[x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx &= -\frac{\sinh^3(x)}{5(1+\coth(x))}+\frac{4}{5}\int \sinh^3(x)\,dx\\ &= -\frac{\sinh^3(x)}{5(1+\coth(x))}-\frac{4}{5}\operatorname{Subst}\left(\int \left(1-x^2\right)dx,x,\cosh(x)\right)\\ &= -\frac{4\cosh(x)}{5}+\frac{4\cosh^3(x)}{15}-\frac{\sinh^3(x)}{5(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0763492, size = 36, normalized size = 1.24

$$\frac{\operatorname{csch}(x)(-40\sinh(2x)+4\sinh(4x)-20\cosh(2x)+\cosh(4x)-45)}{120(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (Csch[x]*(-45 - 20*Cosh[2*x] + Cosh[4*x] - 40*Sinh[2*x] + 4*Sinh[4*x]))/(12
0*(1 + Coth[x]))
```

Maple [B] time = 0.03, size = 80, normalized size = 2.8

$$-\frac{2}{5}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{3}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{6}\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1+coth(x)),x)
```

```
[Out] -2/5/(tanh(1/2*x)+1)^5+1/(tanh(1/2*x)+1)^4-1/3/(tanh(1/2*x)+1)^3-1/2/(tanh(
1/2*x)+1)^2-3/8/(tanh(1/2*x)+1)-1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2
+3/8/(tanh(1/2*x)-1)
```

Maxima [A] time = 1.05927, size = 45, normalized size = 1.55

$$-\frac{1}{48}\left(12\,e^{(-2\,x)}-1\right)e^{(3\,x)}-\frac{3}{8}\,e^{(-x)}+\frac{1}{12}\,e^{(-3\,x)}-\frac{1}{80}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -1/48*(12*e^(-2*x) - 1)*e^(3*x) - 3/8*e^(-x) + 1/12*e^(-3*x) - 1/80*e^(-5*x
)
```

Fricas [B] time = 2.40185, size = 221, normalized size = 7.62

$$\frac{\cosh\left(x\right)^{4}+16\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\,\cosh\left(x\right)^{2}-10\right)\sinh\left(x\right)^{2}-20\,\cosh\left(x\right)^{2}+16\left(\cosh\left(x\right)^{3}-5\,\cosh\left(x\right.\right.}{120\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/120*(cosh(x)^4 + 16*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 10)*
sinh(x)^2 - 20*cosh(x)^2 + 16*(cosh(x)^3 - 5*cosh(x))*sinh(x) - 45)/(cosh(x
) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^{3}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**3/(coth(x) + 1), x)
```

Giac [A] time = 1.15439, size = 42, normalized size = 1.45

$$-\frac{1}{240}\left(90\,e^{(4\,x)}-20\,e^{(2\,x)}+3\right)e^{(-5\,x)}+\frac{1}{48}\,e^{(3\,x)}-\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/240*(90*e^(4*x) - 20*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) - 1/4*e^x
```

3.91 $\int \frac{\sinh^2(x)}{1+\coth(x)} \, dx$

Optimal. Leaf size=38

$$-\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

```
[Out] (-3*x)/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) + 1/(4*(1 + Coth[x])
)
```

Rubi [A] time = 0.0522239, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3487, 44, 207}

$$-\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) + 1/(4*(1 + Coth[x])
)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{1+\coth(x)} \, dx &= -\text{Subst}\left(\int \frac{1}{(1-x)^2(1+x)^3} \, dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}+\frac{1}{4(1+x)^2}-\frac{3}{8\left(-1+x^2\right)}\right) dx, x, \coth(x)\right)\\
&= -\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, \coth(x)\right)\\
&= -\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.0523216, size = 30, normalized size = 0.79

$$\frac{1}{32}(-12x + 8\sinh(2x) - \sinh(4x) - 4\cosh(2x) + \cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (-12*x - 4*Cosh[2*x] + Cosh[4*x] + 8*Sinh[2*x] - Sinh[4*x])/32
```

Maple [B] time = 0.03, size = 70, normalized size = 1.8

$$\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4} - \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} + \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{3}{8}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-2} + \frac{1}{4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1+coth(x)),x)
```

```
[Out] 1/2/(tanh(1/2*x)+1)^4-1/(tanh(1/2*x)+1)^3+1/2/(tanh(1/2*x)+1)-3/8*ln(tanh(1
/2*x)+1)+1/4/(tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)+3/8*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.08251, size = 30, normalized size = 0.79

$$-\frac{3}{8}x + \frac{1}{16}e^{(2x)} - \frac{3}{16}e^{(-2x)} + \frac{1}{32}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -3/8*x + 1/16*e^(2*x) - 3/16*e^(-2*x) + 1/32*e^(-4*x)
```

Fricas [A] time = 2.45374, size = 176, normalized size = 4.63

$$\frac{3\cosh(x)^3 + 9\cosh(x)\sinh(x)^2 + \sinh(x)^3 - 6(2x+1)\cosh(x) + 3\left(\cosh(x)^2 - 4x + 2\right)\sinh(x)}{32(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 - 6*(2*x + 1)*cosh(x) +
 3*(cosh(x)^2 - 4*x + 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**2/(coth(x) + 1), x)
```

Giac [A] time = 1.12842, size = 41, normalized size = 1.08

$$\frac{1}{32}\left(9\,e^{(4\,x)} - 6\,e^{(2\,x)} + 1\right)e^{(-4\,x)} - \frac{3}{8}\,x + \frac{1}{16}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/32*(9*e^(4*x) - 6*e^(2*x) + 1)*e^(-4*x) - 3/8*x + 1/16*e^(2*x)
```

3.92 $\int \frac{\sinh(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=19

$$\frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(\coth(x)+1)}$$

```
[Out] (2*Cosh[x])/3 - Sinh[x]/(3*(1 + Coth[x]))
```

Rubi [A] time = 0.0355857, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3502, 2638}

$$\frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Coth[x]),x]

[Out] (2*Cosh[x])/3 - Sinh[x]/(3*(1 + Coth[x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{1+\coth(x)}\,dx &= -\frac{\sinh(x)}{3(1+\coth(x))}+\frac{2}{3}\int \sinh(x)\,dx\\ &= \frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0476313, size = 21, normalized size = 1.11

$$\frac{1}{12}\left(4\sinh^3(x)+9\cosh(x)-\cosh(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Coth[x]),x]

[Out] (9*Cosh[x] - Cosh[3*x] + 4*Sinh[x]^3)/12
```

Maple [B] time = 0.027, size = 40, normalized size = 2.1

$$-\frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+coth(x)),x)
```

```
[Out] -2/3/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2+1/2/(tanh(1/2*x)+1)-1/2/(tanh(1/
2*x)-1)
```

Maxima [A] time = 1.10248, size = 23, normalized size = 1.21

$$\frac{1}{2}\,e^{(-x)}-\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-x) - 1/12*e^(-3*x) + 1/4*e^x
```

Fricas [A] time = 2.47767, size = 99, normalized size = 5.21

$$\frac{\cosh(x)^2+4\,\cosh(x)\sinh(x)+\sinh(x)^2+3}{6\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(cosh(x)^2 + 4*cosh(x)*sinh(x) + sinh(x)^2 + 3)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)/(coth(x) + 1), x)
```

Giac [A] time = 1.12542, size = 26, normalized size = 1.37

$$\frac{1}{12}\left(6\,e^{(2\,x)}-1\right)e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/12*(6*e^(2*x) - 1)*e^(-3*x) + 1/4*e^x
```

3.93 $\int \frac{\operatorname{csch}(x)}{1+\coth(x)} \, dx$

Optimal. Leaf size=10

$$-\frac{\operatorname{csch}(x)}{\coth(x)+1}$$

[Out] -(Csch[x]/(1 + Coth[x]))

Rubi [A] time = 0.0221637, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3488}

$$-\frac{\operatorname{csch}(x)}{\coth(x)+1}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]/(1 + Coth[x]),x]

[Out] -(Csch[x]/(1 + Coth[x]))

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)} \, dx = -\frac{\operatorname{csch}(x)}{1+\coth(x)}$$

Mathematica [A] time = 0.0031722, size = 7, normalized size = 0.7

$$\sinh(x)-\cosh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]/(1 + Coth[x]),x]

[Out] -Cosh[x] + Sinh[x]

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$-\frac{\operatorname{csch}(x)}{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(1+coth(x)),x)
```

```
[Out] -csch(x)/(1+coth(x))
```

Maxima [A] time = 1.06341, size = 8, normalized size = 0.8

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)
```

Fricas [A] time = 2.51462, size = 32, normalized size = 3.2

$$-\frac{1}{\cosh\left(x\right) + \sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(x\right)}{\coth\left(x\right) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)/(coth(x) + 1), x)
```

Giac [A] time = 1.15261, size = 8, normalized size = 0.8

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -e^(-x)
```

3.94 $\int \frac{\text{csch}^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=7

$$-\log(\coth(x)+1)$$

```
[Out] -Log[1 + Coth[x]]
```

Rubi [A] time = 0.0357077, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3487, 31}

$$-\log(\coth(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(1 + Coth[x]),x]

[Out] -Log[1 + Coth[x]]
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{csch}^2(x)}{1+\coth(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\coth(x)\right)\\ &= -\log(1+\coth(x))\end{aligned}$$

Mathematica [A] time = 0.0029622, size = 7, normalized size = 1.

$$\log(\sinh(x))-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(1 + Coth[x]),x]

[Out] -x + Log[Sinh[x]]
```

Maple [A] time = 0.017, size = 8, normalized size = 1.1

$$-\ln(1+\coth(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(1+coth(x)),x)
```

```
[Out] -ln(1+coth(x))
```

Maxima [A] time = 1.05112, size = 9, normalized size = 1.29

$$-\log\left(\coth\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -log(coth(x) + 1)
```

Fricas [B] time = 2.59185, size = 58, normalized size = 8.29

$$-2\,x+\log\left(\frac{2\,\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -2*x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^{2}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)**2/(coth(x) + 1), x)
```

Giac [A] time = 1.15829, size = 16, normalized size = 2.29

$$-2\,x+\log\left(\left|e^{\left(2\,x\right)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2*x + log(abs(e^(2*x) - 1))
```

3.95 $\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=8

$$\tanh^{-1}(\cosh(x)) - \operatorname{csch}(x)$$

```
[Out] ArcTanh[Cosh[x]] - Csch[x]
```

Rubi [A] time = 0.0400128, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3501, 3770}

$$\tanh^{-1}(\cosh(x)) - \operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(1 + Coth[x]),x]

[Out] ArcTanh[Cosh[x]] - Csch[x]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx &= -\operatorname{csch}(x) - \int \operatorname{csch}(x)\,dx\\ &= \tanh^{-1}(\cosh(x)) - \operatorname{csch}(x)\end{aligned}$$

Mathematica [A] time = 0.0373046, size = 14, normalized size = 1.75

$$-\operatorname{csch}(x) - \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(1 + Coth[x]),x]

[Out] -Csch[x] - Log[Tanh[x/2]]
```

Maple [B] time = 0.019, size = 23, normalized size = 2.9

$$\frac{1}{2}\tanh\left(\frac{x}{2}\right)-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(1+coth(x)),x)
```

```
[Out] 1/2*tanh(1/2*x)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))
```

Maxima [B] time = 1.0878, size = 42, normalized size = 5.25

$$\frac{2\,e^{(-x)}}{e^{(-2\,x)}-1}+\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(e^(-2*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] time = 2.59187, size = 306, normalized size = 38.25

$$\frac{\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sin}{\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] ((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) + 1)
 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) -
1) - 2*cosh(x) - 2*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^{3}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)**3/(coth(x) + 1), x)
```

Giac [B] time = 1.14436, size = 35, normalized size = 4.38

$$-\frac{2\,e^{x}}{e^{(2\,x)}-1}+\log\left(e^{x}+1\right)-\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2*e^x/(e^(2*x) - 1) + log(e^x + 1) - log(abs(e^x - 1))
```

3.96 $\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=11

$$\coth(x) - \frac{\coth^2(x)}{2}$$

```
[Out] Coth[x] - Coth[x]^2/2
```

Rubi [A] time = 0.0343565, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3487}

$$\coth(x) - \frac{\coth^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(1 + Coth[x]),x]
```

```
[Out] Coth[x] - Coth[x]^2/2
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx &= \operatorname{Subst}\left(\int (1-x)\,dx, x, \coth(x)\right)\\ &= \coth(x) - \frac{\coth^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0245922, size = 11, normalized size = 1.

$$\coth(x) - \frac{\operatorname{csch}^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(1 + Coth[x]),x]
```

```
[Out] Coth[x] - Csch[x]^2/2
```

Maple [B] time = 0.023, size = 32, normalized size = 2.9

$$-\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{1}{2}\tanh\left(\frac{x}{2}\right) - \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} + \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(1+coth(x)),x)
```

```
[Out] -1/8*tanh(1/2*x)^2+1/2*tanh(1/2*x)-1/8/tanh(1/2*x)^2+1/2/tanh(1/2*x)
```

Maxima [B] time = 1.10184, size = 55, normalized size = 5.

$$\frac{4e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} - \frac{2}{2e^{(-2x)} - e^{(-4x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 4*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2/(2*e^(-2*x) - e^(-4*x) - 1)
```

Fricas [B] time = 2.37182, size = 181, normalized size = 16.45

$$-\frac{2}{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 4\left(\cosh(x)^3 - \cosh(x)\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -2/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(
x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(x)}{\coth(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)**4/(coth(x) + 1), x)
```

Giac [A] time = 1.15129, size = 14, normalized size = 1.27

$$-\frac{2}{\left(e^{(2x)} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2/(e^(2*x) - 1)^2
```

3.97 $\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=155

$$-\frac{b^5\log(a+b\coth(x))}{\left(a^2-b^2\right)^3}-\frac{\left(3a^2+9ab+8b^2\right)\log(1-\coth(x))}{16(a+b)^3}+\frac{\left(3a^2-9ab+8b^2\right)\log(\coth(x)+1)}{16(a-b)^3}-\frac{\sinh^4(x)(b-a\,c}{4\left(a^2-b^2\right.}$$

```
[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Coth[x]])/(16*(a + b)^3) + ((3*a^2 - 9*a*
b + 8*b^2)*Log[1 + Coth[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Coth[x]])/(a^2
 - b^2)^3 - ((4*b^3 - a*(7 - (3*a^2)/b^2)*b^2*Coth[x])*Sinh[x]^2)/(8*(a^2 -
 b^2)^2) - ((b - a*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rubi [A] time = 0.239974, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3506, 741, 823, 801}

$$-\frac{b^5\log(a+b\coth(x))}{\left(a^2-b^2\right)^3}-\frac{\left(3a^2+9ab+8b^2\right)\log(1-\coth(x))}{16(a+b)^3}+\frac{\left(3a^2-9ab+8b^2\right)\log(\coth(x)+1)}{16(a-b)^3}-\frac{\sinh^4(x)(b-a\,c}{4\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Coth[x]),x]
```


```
[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Coth[x]])/(16*(a + b)^3) + ((3*a^2 - 9*a*
b + 8*b^2)*Log[1 + Coth[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Coth[x]])/(a^2
 - b^2)^3 - ((4*b^3 - a*(7 - (3*a^2)/b^2)*b^2*Coth[x])*Sinh[x]^2)/(8*(a^2 -
 b^2)^2) - ((b - a*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```


Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^3}\,dx, x, b\coth(x)\right)}{b}$$

$$= -\frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} + \frac{b\,\text{Subst}\left(\int \frac{-4+\frac{3a^2}{b^2}+\frac{3ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\coth(x)\right)}{4\left(a^2-b^2\right)}$$

$$= -\frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{b^5\,\text{Subst}\left(\int \frac{-\frac{3a^4-7a^2b^2+8b^4}{b^6}+\frac{a(}{}}{(a+x)\left(1-\frac{x^2}{b^2}\right.}\right.}{8\left(a^2-\right.}$$

$$= -\frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{b^5\,\text{Subst}\left(\int\left(-\frac{(a-b)^2\left(3a^2+9ab\right.}{2b^5(a+b)(b-}\right.\right.}{}$$

$$= -\frac{\left(3a^2+9ab+8b^2\right)\log(1-\coth(x))}{16(a+b)^3} + \frac{\left(3a^2-9ab+8b^2\right)\log(1+\coth(x))}{16(a-b)^3} - \frac{b^5\log(a+b\coth(x)}{\left(a^2-b^2\right)^3}$$

Mathematica [A] time = 0.265987, size = 156, normalized size = 1.01

$$\frac{-40a^3b^2x+24a^3b^2\sinh(2x)-2a^3b^2\sinh(4x)+4b\left(-4a^2b^2+a^4+3b^4\right)\cosh(2x)-b\left(a^2-b^2\right)^2\cosh(4x)+12a^5x-8a^5\,\text{s}}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (12*a^5*x - 40*a^3*b^2*x + 60*a*b^4*x + 4*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Cosh[
2*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*b^5*Log[b*Cosh[x] + a*Sinh[x]] - 8*a^
5*Sinh[2*x] + 24*a^3*b^2*Sinh[2*x] - 16*a*b^4*Sinh[2*x] + a^5*Sinh[4*x] - 2
*a^3*b^2*Sinh[4*x] + a*b^4*Sinh[4*x])/(32*(a - b)^3*(a + b)^3)
```

Maple [B] time = 0.049, size = 354, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*coth(x)),x)
```

```
[Out] -16/(64*a-64*b)/(tanh(1/2*x)+1)^4+64/(128*a-128*b)/(tanh(1/2*x)+1)^3+1/8/(a
-b)^2/(tanh(1/2*x)+1)^2*a-3/8/(a-b)^2/(tanh(1/2*x)+1)^2*b-3/8/(a-b)^2/(tanh
(1/2*x)+1)*a+5/8/(a-b)^2/(tanh(1/2*x)+1)*b+3/8/(a-b)^3*ln(tanh(1/2*x)+1)*a^
2-9/8/(a-b)^3*ln(tanh(1/2*x)+1)*a*b+1/(a-b)^3*ln(tanh(1/2*x)+1)*b^2+16/(64*
a+64*b)/(tanh(1/2*x)-1)^4+64/(128*a+128*b)/(tanh(1/2*x)-1)^3-1/8/(a+b)^2/(t
anh(1/2*x)-1)^2*a-3/8/(a+b)^2/(tanh(1/2*x)-1)^2*b-3/8/(a+b)^2/(tanh(1/2*x)-
1)*a-5/8/(a+b)^2/(tanh(1/2*x)-1)*b-3/8/(a+b)^3*ln(tanh(1/2*x)-1)*a^2-9/8/(a
+b)^3*ln(tanh(1/2*x)-1)*a*b-1/(a+b)^3*ln(tanh(1/2*x)-1)*b^2-b^5/(a-b)^3/(a+
b)^3*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Maxima [A] time = 1.10772, size = 224, normalized size = 1.45

$$-\frac{b^5 \log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{\left(3a^2+9ab+8b^2\right)x}{8\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\left(4(2a+3b)e^{(-2x)}-a-b\right)e^{(4x)}}{64\left(a^2+2ab+b^2\right)}+\frac{4(2a-3b)e^{(-2x)}-}{64\left(a^2-2ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1
/8*(3*a^2 + 9*a*b + 8*b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*(2*a
 + 3*b)*e^(-2*x) - a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*(2*a - 3*b)
*e^(-2*x) - (a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Fricas [B] time = 2.84713, size = 2882, normalized size = 18.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(2*a^5 - a^4*b
- 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^6 - 4*(2*a^5 - a^4*b - 6
*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5 - 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 - 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^
5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 +
 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 30*(
2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^2 + 4*(3*a
^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 10*(2*a^5 - a^4*b - 6*a^3*b^2
+ 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^3 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x))*sinh(x)^3 + 4*(2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 +
 4*a*b^4 + 3*b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^6 + 2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3
*b^5 - 15*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^4 + 12*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^2)*sinh(x)^2 - 64
*(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b
```

```
^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x
) - sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^7 - 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^5 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^3 + (2*a^5 + a^4*b
 - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^
2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^4(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**4/(a + b*coth(x)), x)
```

Giac [A] time = 1.17534, size = 309, normalized size = 1.99

$$-\frac{b^5\log\left(\left|-ae^{(2x)} - be^{(2x)} + a - b\right|\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{\left(3a^2 - 9ab + 8b^2\right)x}{8\left(a^3 - 3a^2b + 3ab^2 - b^3\right)} - \frac{\left(18a^2e^{(4x)} - 54abe^{(4x)} + 48b^2e^{(4x)} - 8a^2e^{(2x)} + 20ab\right.}{64\left(a^3 - 3a^2b + 3ab^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6) + 1/8*(3*a^2 - 9*a*b + 8*b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/
64*(18*a^2*e^(4*x) - 54*a*b*e^(4*x) + 48*b^2*e^(4*x) - 8*a^2*e^(2*x) + 20*a
*b*e^(2*x) - 12*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b +
3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x) - 8*a*e^(2*x) - 12*b*e^(2*x))/
(a^2 + 2*a*b + b^2)
```

3.98 $\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=134

$$-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{b^3\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}+\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a\cosh(x)}{a^2-b^2}-\frac{b^4\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

```
[Out] -((b^4*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
)) + (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3
)/(3*(a^2 - b^2)) - (b^3*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 - b
^2))
```

Rubi [A] time = 0.237369, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3511, 3486, 2633, 2638, 3509, 206}

$$-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{b^3\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}+\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a\cosh(x)}{a^2-b^2}-\frac{b^4\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b^4*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
)) + (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3
)/(3*(a^2 - b^2)) - (b^3*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 - b
^2))
```

Rule 3511

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \frac{\int (a-b\coth(x))\sinh^3(x)\,dx}{a^2-b^2}+\frac{b^2\int \frac{\sinh(x)}{a+b\coth(x)}\,dx}{a^2-b^2}$$
$$= -\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{b^2\int (a-b\coth(x))\sinh(x)\,dx}{\left(a^2-b^2\right)^2}+\frac{b^4\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{\left(a^2-b^2\right)^2}+\frac{a\int \sinh^3(x)\,dx}{a^2-b^2}$$
$$= -\frac{b^3\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{\left(ab^2\right)\int \sinh(x)\,dx}{\left(a^2-b^2\right)^2}-\frac{b^4\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,i(-ib-ia\coth(x))\text{si}\right.}{\left(a^2-b^2\right)^2}$$
$$= -\frac{b^4\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a\cosh(x)}{a^2-b^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{b^3\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\sinh^3(x}{3\left(a^2-b^{\cdot}\right.}$$

Mathematica [A] time = 0.787788, size = 171, normalized size = 1.28

$$\frac{3b\sqrt{b-a}\left(a^2b+a^3-5ab^2-5b^3\right)\sinh(x)-3a\sqrt{b-a}\left(3a^2b+3a^3-7ab^2-7b^3\right)\cosh(x)+24b^4\sqrt{a+b}\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right.}{12(b-a)^{5/2}(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (24*b^4*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] -
3*a*Sqrt[-a + b]*(3*a^3 + 3*a^2*b - 7*a*b^2 - 7*b^3)*Cosh[x] - a*(-a + b)^(
3/2)*(a + b)^2*Cosh[3*x] + 3*b*Sqrt[-a + b]*(a^3 + a^2*b - 5*a*b^2 - 5*b^3)
*Sinh[x] + b*(-a + b)^(3/2)*(a + b)^2*Sinh[3*x])/(12*(-a + b)^(5/2)*(a + b)
^3)
```

Maple [A] time = 0.044, size = 197, normalized size = 1.5

$$-16\,\frac{1}{(32\,a-32\,b)\,(\tanh(x/2)+1)^2}+\frac{32}{96\,a-96\,b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{a}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{b}{(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*coth(x)),x)
```

```
[Out] -16/(32*a-32*b)/(tanh(1/2*x)+1)^2+32/3/(tanh(1/2*x)+1)^3/(32*a-32*b)-1/2/(a
-b)^2/(tanh(1/2*x)+1)*a+1/(a-b)^2/(tanh(1/2*x)+1)*b-32/3/(tanh(1/2*x)-1)^3/
```

```
(32*a+32*b)-16/(32*a+32*b)/(tanh(1/2*x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)*a+
1/(a+b)^2/(tanh(1/2*x)-1)*b+2*b^4/(a-b)^2/(a+b)^2/(-a^2+b^2)^(1/2)*arctan(1
/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.96357, size = 4251, normalized size = 31.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b
^3 + 7*a*b^4 - 5*b^5)*cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3
 + 7*a*b^4 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^
5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4
 + 5*b^5)*cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 +
 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 +
6*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sin
h(x)^2 + 24*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x
)^2 + b^4*sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cos
h(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) +
a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
 a + b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5
 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^3 -
 (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*sinh(x
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b
 - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 -
2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b
^4 - 5*b^5)*cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4
 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)
*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(
x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)
)*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*
cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5 - 5
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 -
```

```
a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sinh(x)^2 + 4
8*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*
sinh(x)^3)*sqrt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a +
 b)*sinh(x))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x
)^3 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*s
inh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*coth(x)), x)
```

Giac [A] time = 1.13538, size = 220, normalized size = 1.64

$$-\frac{2\,b^4\arctan\left(-\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{\left(9\,ae^{(2\,x)}-15\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-9\,a^2e^x-24\,abe^x-15}{24\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*b^4*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 15*b*e^(2*x) - a + b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^
x - 24*a*b*e^x - 15*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.99 $\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=92

$$-\frac{b^3\log(a+b\coth(x))}{(a^2-b^2)^2}-\frac{\sinh^2(x)(b-a\coth(x))}{2(a^2-b^2)}+\frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2}-\frac{(a-2b)\log(\coth(x)+1)}{4(a-b)^2}$$

```
[Out] ((a + 2*b)*Log[1 - Coth[x]])/(4*(a + b)^2) - ((a - 2*b)*Log[1 + Coth[x]])/(
4*(a - b)^2) - (b^3*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Si
nh[x]^2)/(2*(a^2 - b^2))
```

Rubi [A] time = 0.141564, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3506, 741, 801}

$$-\frac{b^3\log(a+b\coth(x))}{(a^2-b^2)^2}-\frac{\sinh^2(x)(b-a\coth(x))}{2(a^2-b^2)}+\frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2}-\frac{(a-2b)\log(\coth(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] ((a + 2*b)*Log[1 - Coth[x]])/(4*(a + b)^2) - ((a - 2*b)*Log[1 + Coth[x]])/(
4*(a - b)^2) - (b^3*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Si
nh[x]^2)/(2*(a^2 - b^2))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\coth(x)\right)}{b}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{b\,\text{Subst}\left(\int \frac{-2+\frac{a^2}{b^2}+\frac{ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\coth(x)\right)}{2\left(a^2-b^2\right)}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{b\,\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right)dx, x, b\coth(x)\right)}{2\left(a^2-b^2\right)}$$

$$= \frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2} - \frac{(a-2b)\log(1+\coth(x))}{4(a-b)^2} - \frac{b^3\log(a+b\coth(x))}{\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.151654, size = 75, normalized size = 0.82

$$\frac{a\left(a^2-b^2\right)\sinh(2x)+\left(b^3-a^2b\right)\cosh(2x)-2a^3x+6ab^2x-4b^3\log(a\sinh(x)+b\cosh(x))}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (-2*a^3*x + 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] - 4*b^3*Log[b*Cosh[x] +
a*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Maple [B] time = 0.039, size = 175, normalized size = 1.9

$$-8\,\frac{1}{(16\,a-16\,b)\left(\tanh(x/2)+1\right)^2}+16\,\frac{1}{(32\,a-32\,b)\left(\tanh(x/2)+1\right)}-\frac{a}{2\,(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{b}{(a-b)^2}\ln\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*coth(x)),x)
```

```
[Out] -8/(16*a-16*b)/(tanh(1/2*x)+1)^2+16/(32*a-32*b)/(tanh(1/2*x)+1)-1/2*a/(a-b)
^2*ln(tanh(1/2*x)+1)+1/(a-b)^2*ln(tanh(1/2*x)+1)*b+8/(16*a+16*b)/(tanh(1/2*
x)-1)^2+16/(32*a+32*b)/(tanh(1/2*x)-1)+1/2*a/(a+b)^2*ln(tanh(1/2*x)-1)+1/(a
+b)^2*ln(tanh(1/2*x)-1)*b-b^3/(a-b)^2/(a+b)^2*ln(tanh(1/2*x)^2*b+2*a*tanh(1
/2*x)+b)
```

Maxima [A] time = 1.17738, size = 112, normalized size = 1.22

$$-\frac{b^3\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^4-2\,a^2b^2+b^4}-\frac{(a+2\,b)x}{2\left(a^2+2\,ab+b^2\right)}+\frac{e^{(2\,x)}}{8\,(a+b)}-\frac{e^{(-2\,x)}}{8\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(a + 2*b)
*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Fricas [B] time = 2.69496, size = 818, normalized size = 8.89

$$\frac{\left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)^4 + 4\left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)\sinh(x)^3 + \left(a^3 - a^2b - ab^2 + b^3\right)\sinh(x)^4 - 4\left(a^3 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 - 3*a*b^
2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 - 8*(b^3*cosh
(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(b*cosh(x) + a*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2*(a^3
- 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*coth(x)), x)
```

Giac [A] time = 1.17309, size = 154, normalized size = 1.67

$$-\frac{b^3\log\left(\left|-ae^{(2x)} - be^{(2x)} + a - b\right|\right)}{a^4 - 2a^2b^2 + b^4} - \frac{(a - 2b)x}{2\left(a^2 - 2ab + b^2\right)} + \frac{\left(2ae^{(2x)} - 4be^{(2x)} - a + b\right)e^{(-2x)}}{8\left(a^2 - 2ab + b^2\right)} + \frac{e^{(2x)}}{8(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/2
*(a - 2*b)*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) - a + b)*
e^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

3.100 $\int \frac{\sinh(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=73

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b^2\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -((b^2*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2
)) + (a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rubi [A] time = 0.106747, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3511, 3486, 2638, 3509, 206}

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b^2\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b^2*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2
)) + (a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rule 3511

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx = \frac{\int (a-b\coth(x))\sinh(x)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{a^2-b^2}$$
$$= -\frac{b\sinh(x)}{a^2-b^2} + \frac{a\int \sinh(x)\,dx}{a^2-b^2} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, i(-ib-ia\coth(x))\sinh(x)\right)}{a^2-b^2}$$
$$= -\frac{b^2\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a\cosh(x)}{a^2-b^2} - \frac{b\sinh(x)}{a^2-b^2}$$

Mathematica [A] time = 0.435651, size = 80, normalized size = 1.1

$$\frac{a\cosh(x)}{a^2-b^2} + b\left(\frac{\sinh(x)}{b^2-a^2} - \frac{2b\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{(b-a)^{3/2}(a+b)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Coth[x]),x]
```

```
[Out] (a*Cosh[x])/(a^2 - b^2) + b*((-2*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*S
qrt[a + b])])/((-a + b)^(3/2)*(a + b)^(3/2)) + Sinh[x]/(-a^2 + b^2))
```

Maple [A] time = 0.032, size = 93, normalized size = 1.3

$$-8\,\frac{1}{(8\,a+8\,b)\,(\tanh(x/2)-1)} + 8\,\frac{1}{(8\,a-8\,b)\,(\tanh(x/2)+1)} + 2\,\frac{b^2}{(a+b)\,(a-b)\,\sqrt{-a^2+b^2}}\arctan\left(1/2\,\frac{2\,\tanh(x/2)\,b}{\sqrt{-a^2+b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*coth(x)),x)
```

```
[Out] -8/(8*a+8*b)/(tanh(1/2*x)-1)+8/(8*a-8*b)/(tanh(1/2*x)+1)+2*b^2/(a+b)/(a-b)/
(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66047, size = 1103, normalized size = 15.11

$$\left[\frac{a^3 + a^2b - ab^2 - b^3 + \left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)^2 + 2\left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)\sinh\left(x\right) + \left(a^3 - a^2b - ab^2 + b^3\right)}{2\left(\left(a^4 - 2\,a^2b^2 + b^4\right)\cosh\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2
*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*
sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cosh
(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(
cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*
a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3
 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2
 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(x\right)}{a + b\coth\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)/(a + b*coth(x)), x)
```

Giac [A] time = 1.15365, size = 97, normalized size = 1.33

$$\frac{2\,b^2\arctan\left(\frac{ae^x + be^x}{\sqrt{-a^2 + b^2}}\right)}{\left(a^2 - b^2\right)\sqrt{-a^2 + b^2}} + \frac{e^{(-x)}}{2\,(a - b)} + \frac{e^x}{2\,(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2
)) + 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

3.101 $\int \frac{\textbf{csch}(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] -(ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rubi [A] time = 0.035892, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3509, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Coth[x]),x]
```

```
[Out] -(ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{csch}(x)}{a+b\coth(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, i(-ib-ia\coth(x))\sinh(x)\right)\\ &= -\frac{i\tan^{-1}\left(\frac{(-ib-ia\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] time = 0.0311623, size = 46, normalized size = 1.21

$$\frac{2\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{\sqrt{b-a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Coth[x]),x]
```

```
[Out] (2*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*Sqrt
[a + b])
```

Maple [A] time = 0.017, size = 39, normalized size = 1.

$$2\,\frac{1}{\sqrt{-a^2+b^2}}\arctan\left(1/2\,\frac{2\,\tanh(x/2)\,b+2\,a}{\sqrt{-a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*coth(x)),x)
```

```
[Out] 2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.65223, size = 420, normalized size = 11.05

$$\left[\frac{\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\,\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)}{\sqrt{a^2-b^2}},\ \frac{2\,\sqrt{-a^2+b^2}\arctan\left(\frac{\sqrt{-a^2+b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)}{a^2-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2
*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b))/sqrt(a^2 - b^2), 2*sqrt(-a^
2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/(a^2
- b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*coth(x)), x)
```

Giac [A] time = 1.17974, size = 47, normalized size = 1.24

$$\frac{2\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

3.102 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\coth(x))}{b}$$

```
[Out] -(Log[a + b*Coth[x]]/b)
```

Rubi [A] time = 0.0438356, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$-\frac{\log(a+b\coth(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -(Log[a + b*Coth[x]]/b)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b}\\ &= -\frac{\log(a+b\coth(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0496965, size = 20, normalized size = 1.67

$$\frac{\log(\sinh(x))-\log(a\sinh(x)+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b
```

Maple [A] time = 0.019, size = 13, normalized size = 1.1

$$-\frac{\ln\left(a+b\coth\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*coth(x)),x)
```

```
[Out] -ln(a+b*coth(x))/b
```

Maxima [A] time = 1.17506, size = 16, normalized size = 1.33

$$-\frac{\log\left(b\coth\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -log(b*coth(x) + a)/b
```

Fricas [B] time = 2.70178, size = 127, normalized size = 10.58

$$-\frac{\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(
x) - sinh(x))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^{2}\left(x\right)}{a+b\coth\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*coth(x)), x)
```

Giac [B] time = 1.19027, size = 62, normalized size = 5.17

$$-\frac{(a+b)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{ab+b^{2}}+\frac{\log\left(\left|e^{(2\,x)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(2
*x) - 1))/b
```

3.103 $\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\operatorname{csch}(x)}{b}$$

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTanh[((b + a*Coth[x])*Sinh[x
])/Sqrt[a^2 - b^2]])/b^2 - Csch[x]/b
```

Rubi [A] time = 0.109173, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3510, 3486, 3770, 3509, 206}

$$-\frac{\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\operatorname{csch}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTanh[((b + a*Coth[x])*Sinh[x
])/Sqrt[a^2 - b^2]])/b^2 - Csch[x]/b
```

Rule 3510

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Dist[d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[(d^2*(a^2 + b^2))/b^2, Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\mathrm{csch}^3(x)}{a+b\coth(x)}\,dx = -\frac{\int (a-b\coth(x))\mathrm{csch}(x)\,dx}{b^2}+\frac{\left(a^2-b^2\right)\int \frac{\mathrm{csch}(x)}{a+b\coth(x)}\,dx}{b^2}$$
$$= -\frac{\mathrm{csch}(x)}{b}-\frac{a\int \mathrm{csch}(x)\,dx}{b^2}-\frac{\left(a^2-b^2\right)\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,i(-ib-ia\coth(x))\sinh(x)\right)}{b^2}$$
$$= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{\mathrm{csch}(x)}{b}$$

Mathematica [A] time = 0.104844, size = 65, normalized size = 1.14

$$-\frac{2\sqrt{b-a}\sqrt{a+b}\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)+a\log\left(\tanh\left(\frac{x}{2}\right)\right)+b\mathrm{csch}(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((2*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a
 + b])] + b*Csch[x] + a*Log[Tanh[x/2]])/b^2)
```

Maple [B] time = 0.028, size = 115, normalized size = 2.

$$\frac{1}{2b}\tanh\left(\frac{x}{2}\right)-\frac{1}{2b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{a}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\frac{a^2}{b^2\sqrt{-a^2+b^2}}\arctan\left(1/2\frac{2\tanh(x/2)b+2a}{\sqrt{-a^2+b^2}}\right)-2\frac{1}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*coth(x)),x)
```

```
[Out] 1/2/b*tanh(1/2*x)-1/2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))+2/b^2/(-a^2+b^2)^
(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))*a^2-2/(-a^2+b^2)^(
1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.92631, size = 1230, normalized size = 21.58

$$\left[\frac{\sqrt{a^2-b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\sqrt{a^2-b^2}(\cos}{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2
- b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*
sinh(x) + (a + b)*sinh(x)^2 - a + b)) - 2*b*cosh(x) + (a*cosh(x)^2 + 2*a*co
sh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (a*cosh(x)^2
+ 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) - 1) - 2*b*s
inh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2), (2*s
qrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(sqrt
(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 2*b*cosh(x) + (a*cosh(x
)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x)
- 1) - 2*b*sinh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
- b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*coth(x)), x)
```

Giac [A] time = 1.14891, size = 115, normalized size = 2.02

$$\frac{a\log(e^x+1)}{b^2}-\frac{a\log(|e^x-1|)}{b^2}+\frac{2\left(a^2-b^2\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2}-\frac{2e^x}{b\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1))/b^2 + 2*(a^2 - b^2)*arctan((a*e^x
+ b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - 2*e^x/(b*(e^(2*x) - 1))
```

3.104 $\int \frac{\text{csch}^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

```
[Out] (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - ((a^2 - b^2)*Log[a + b*Coth[x]])/b^3
```

Rubi [A] time = 0.0674236, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 697}

$$-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - ((a^2 - b^2)*Log[a + b*Coth[x]])/b^3
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{csch}^4(x)}{a+b\coth(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{b^2}}{a+x}\,dx,x,b\coth(x)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{b^2}-\frac{x}{b^2}+\frac{-a^2+b^2}{b^2(a+x)}\right)dx,x,b\coth(x)\right)}{b}\\
&= \frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.137139, size = 50, normalized size = 1.25

$$\frac{2(a^2-b^2)(\log(\sinh(x))-\log(a\sinh(x)+b\cosh(x)))+2ab\coth(x)-b^2\text{csch}^2(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b*Coth[x] - b^2*Csch[x]^2 + 2*(a^2 - b^2)*(Log[Sinh[x]] - Log[b*Cosh[x
] + a*Sinh[x]]))/(2*b^3)
```

Maple [B] time = 0.031, size = 116, normalized size = 2.9

$$-\frac{1}{8b}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\frac{a}{2b^{2}}\tanh\left(\frac{x}{2}\right)-\frac{1}{8b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{a^{2}}{b^{3}}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{a}{2b^{2}}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*coth(x)),x)
```

```
[Out] -1/8/b*tanh(1/2*x)^2+1/2/b^2*a*tanh(1/2*x)-1/8/b/tanh(1/2*x)^2+1/b^3*ln(tan
h(1/2*x))*a^2-1/b*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x)-1/b^3*ln(tanh(1/2*x
)^2*b+2*a*tanh(1/2*x)+b)*a^2+1/b*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Maxima [B] time = 1.0331, size = 149, normalized size = 3.72

$$\frac{2\left((a+b)e^{(-2x)}-a\right)}{2b^{2}e^{(-2x)}-b^{2}e^{(-4x)}-b^{2}}-\frac{\left(a^{2}-b^{2}\right)\log\left(-(a-b)e^{(-2x)}+a+b\right)}{b^{3}}+\frac{\left(a^{2}-b^{2}\right)\log\left(e^{(-x)}+1\right)}{b^{3}}+\frac{\left(a^{2}-b^{2}\right)\log\left(e^{(-x)}-1\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] 2*((a + b)*e^(-2*x) - a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) - (a^2 - b^2
)*log(-(a - b)*e^(-2*x) + a + b)/b^3 + (a^2 - b^2)*log(e^(-x) + 1)/b^3 + (a
^2 - b^2)*log(e^(-x) - 1)/b^3
```

Fricas [B] time = 2.72258, size = 1087, normalized size = 27.18

$$2\left(ab-b^{2}\right)\cosh\left(x\right)^{2}+4\left(ab-b^{2}\right)\cosh\left(x\right)\sinh\left(x\right)+2\left(ab-b^{2}\right)\sinh\left(x\right)^{2}-2\,ab-\left(\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{4}+4\left(a^{2}-b^{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] (2*(a*b - b^2)*cosh(x)^2 + 4*(a*b - b^2)*cosh(x)*sinh(x) + 2*(a*b - b^2)*si
nh(x)^2 - 2*a*b - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3
+ (a^2 - b^2)*sinh(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x
)^2 - a^2 + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 -
b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) +
 ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*sin
h(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 - a^2 + b^2)*
sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 - b^2)*cosh(x))*sin
h(x))*log(2*sinh(x)/(cosh(x) - sinh(x))))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)*si
nh(x)^3 + b^3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)
```

```
*sinh(x)^2 + 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*coth(x)), x)
```

Giac [B] time = 1.17598, size = 143, normalized size = 3.58

$$-\frac{\left(a^3 + a^2 b - ab^2 - b^3\right)\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{ab^3 + b^4} + \frac{\left(a^2 - b^2\right)\log\left(\left|e^{(2x)} - 1\right|\right)}{b^3} - \frac{2\left(ab - \left(ab - b^2\right)e^{(2x)}\right)}{b^3\left(e^{(2x)} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + a^2*b - a*b^2 - b^3)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b^3
 + b^4) + (a^2 - b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2
*x))/(b^3*(e^(2*x) - 1)^2)
```

3.105 $\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=60

$$\frac{x}{16}-\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}+\frac{5}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

```
[Out] x/16 + 1/(32*(1 - Coth[x])^2) - 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x])^3
) + 5/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rubi [A] time = 0.0669269, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3516, 848, 88, 207}

$$\frac{x}{16}-\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}+\frac{5}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(1 + Coth[x]),x]
```

```
[Out] x/16 + 1/(32*(1 - Coth[x])^2) - 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x])^3
) + 5/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = -\text{Subst}\left(\int \frac{x^4}{(1+x)\left(-1+x^2\right)^3}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \frac{x^4}{(-1+x)^3(1+x)^4}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}-\frac{1}{8(1+x)^4}+\frac{5}{16(1+x)^3}-\frac{3}{16(1+x)^2}+\frac{1}{16\left(-1+x^2\right)}\right)dx,$$
$$= \frac{1}{32(1-\coth(x))^2}-\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}+\frac{5}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}-\frac{1}{16}$$
$$= \frac{x}{16}+\frac{1}{32(1-\coth(x))^2}-\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}+\frac{5}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}$$

Mathematica [A] time = 0.088029, size = 42, normalized size = 0.7

$$\frac{1}{192}(12x+3\sinh(2x)-3\sinh(4x)-\sinh(6x)+15\cosh(2x)+6\cosh(4x)+\cosh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (12*x + 15*Cosh[2*x] + 6*Cosh[4*x] + Cosh[6*x] + 3*Sinh[2*x] - 3*Sinh[4*x]
- Sinh[6*x])/192
```

Maple [B] time = 0.032, size = 118, normalized size = 2.

$$\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-6}-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5}+\frac{13}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}-\frac{19}{12}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{3}{8}\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(1+coth(x)),x)
```

```
[Out] 1/3/(tanh(1/2*x)+1)^6-1/(tanh(1/2*x)+1)^5+13/8/(tanh(1/2*x)+1)^4-19/12/(tan
h(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2-3/8/(tanh(1/2*x)+1)+1/16*ln(tanh(1/2*x)+1
)+1/8/(tanh(1/2*x)-1)^4+1/4/(tanh(1/2*x)-1)^3+3/8/(tanh(1/2*x)-1)^2+1/4/(ta
nh(1/2*x)-1)-1/16*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 0.997534, size = 49, normalized size = 0.82

$$\frac{1}{128}\left(6\,e^{(-2\,x)}+1\right)e^{(4\,x)}+\frac{1}{16}\,x+\frac{1}{32}\,e^{(-2\,x)}+\frac{3}{128}\,e^{(-4\,x)}+\frac{1}{192}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/128*(6*e^(-2*x) + 1)*e^(4*x) + 1/16*x + 1/32*e^(-2*x) + 3/128*e^(-4*x) +
1/192*e^(-6*x)
```

Fricas [B] time = 2.50562, size = 319, normalized size = 5.32

$$\frac{5 \cosh(x)^5 + 25 \cosh(x)\sinh(x)^4 + \sinh(x)^5 + \left(10 \cosh(x)^2 + 9\right)\sinh(x)^3 + 27 \cosh(x)^3 + \left(50 \cosh(x)^3 + 81 \cosh(x)\right)\sinh(x)^2 + 12 (2x+1)\cosh(x) + \left(5\cosh(x)^4 + 27\cosh(x)^2 + 24x - 12\right)\sinh(x)}{384 (\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + (10*cosh(x)^2 + 9)*
sinh(x)^3 + 27*cosh(x)^3 + (50*cosh(x)^3 + 81*cosh(x))*sinh(x)^2 + 12*(2*x
+ 1)*cosh(x) + (5*cosh(x)^4 + 27*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh(x) +
 sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^4(x)}{\coth(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(cosh(x)**4/(coth(x) + 1), x)
```

Giac [A] time = 1.17667, size = 57, normalized size = 0.95

$$-\frac{1}{384}\left(22\, e^{(6x)} - 12\, e^{(4x)} - 9\, e^{(2x)} - 2\right)e^{(-6x)} + \frac{1}{16}\, x + \frac{1}{128}\, e^{(4x)} + \frac{3}{64}\, e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/384*(22*e^(6*x) - 12*e^(4*x) - 9*e^(2*x) - 2)*e^(-6*x) + 1/16*x + 1/128*
e^(4*x) + 3/64*e^(2*x)
```

3.106 $\int \frac{\cosh^3(x)}{1+\coth(x)} \, dx$

Optimal. Leaf size=25

$$-\frac{\sinh^5(x)}{5}-\frac{\sinh^3(x)}{3}+\frac{\cosh^5(x)}{5}$$

```
[Out] Cosh[x]^5/5 - Sinh[x]^3/3 - Sinh[x]^5/5
```

Rubi [A] time = 0.17683, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3518, 3108, 3107, 2565, 30, 2564, 14}

$$-\frac{\sinh^5(x)}{5}-\frac{\sinh^3(x)}{3}+\frac{\cosh^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 + Coth[x]),x]
```

```
[Out] Cosh[x]^5/5 - Sinh[x]^3/3 - Sinh[x]^5/5
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\cosh^3(x)\sinh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \cosh^3(x)\sinh(x)(-\cosh(x)+\sinh(x))\,dx\\
&= i\int \left(-i\cosh^4(x)\sinh(x)+i\cosh^3(x)\sinh^2(x)\right)\,dx\\
&= \int \cosh^4(x)\sinh(x)\,dx-\int \cosh^3(x)\sinh^2(x)\,dx\\
&= -\left(i\,\mathrm{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,i\sinh(x)\right)\right)+\mathrm{Subst}\left(\int x^4\,dx,x,\cosh(x)\right)\\
&= \frac{\cosh^5(x)}{5}-i\,\mathrm{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,i\sinh(x)\right)\\
&= \frac{\cosh^5(x)}{5}-\frac{\sinh^3(x)}{3}-\frac{\sinh^5(x)}{5}
\end{aligned}
$$

Mathematica [A] time = 0.0579108, size = 34, normalized size = 1.36

$$\frac{1}{120}(\cosh(x)-\sinh(x))(10\sinh(2x)+\sinh(4x)+20\cosh(2x)+4\cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 + Coth[x]),x]
```

```
[Out] ((Cosh[x] - Sinh[x])*(20*Cosh[2*x] + 4*Cosh[4*x] + 10*Sinh[2*x] + Sinh[4*x]
))/120
```

Maple [B] time = 0.028, size = 82, normalized size = 3.3

$$-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}+\frac{2}{5}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5}+\frac{4}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{3}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{6}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1+coth(x)),x)
```

```
[Out] -1/(tanh(1/2*x)+1)^4+2/5/(tanh(1/2*x)+1)^5+4/3/(tanh(1/2*x)+1)^3-1/(tanh(1/
2*x)+1)^2+3/8/(tanh(1/2*x)+1)-1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2-3
/8/(tanh(1/2*x)-1)
```

Maxima [A] time = 1.03129, size = 36, normalized size = 1.44

$$\frac{1}{48}\left(6e^{(-2x)}+1\right)e^{(3x)}+\frac{1}{24}e^{(-3x)}+\frac{1}{80}e^{(-5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/48*(6*e^(-2*x) + 1)*e^(3*x) + 1/24*e^(-3*x) + 1/80*e^(-5*x)
```

Fricas [B] time = 2.53558, size = 200, normalized size = 8.

$$\frac{\cosh(x)^4+\cosh(x)\sinh(x)^3+\sinh(x)^4+\left(6\cosh(x)^2+5\right)\sinh(x)^2+5\cosh(x)^2+\left(\cosh(x)^3+5\cosh(x)\right)\sinh(x)}{30\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(x)^4 + cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 5)*sinh(x)
^2 + 5*cosh(x)^2 + (cosh(x)^3 + 5*cosh(x))*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh^3(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(cosh(x)**3/(coth(x) + 1), x)
```

Giac [A] time = 1.18732, size = 34, normalized size = 1.36

$$\frac{1}{240}\left(10e^{(2x)}+3\right)e^{(-5x)}+\frac{1}{48}e^{(3x)}+\frac{1}{8}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/240*(10*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) + 1/8*e^x
```

3.107 $\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{8}-\frac{1}{8(1-\coth(x))}-\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

```
[Out] x/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rubi [A] time = 0.0592763, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3516, 848, 88, 207}

$$\frac{x}{8}-\frac{1}{8(1-\coth(x))}-\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 + Coth[x]),x]
```

```
[Out] x/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = -\text{Subst}\left(\int \frac{x^2}{(1+x)\left(-1+x^2\right)^2}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \frac{x^2}{(-1+x)^2(1+x)^3}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}-\frac{1}{4(1+x)^2}+\frac{1}{8\left(-1+x^2\right)}\right)dx, x, \coth(x)\right)$$
$$= -\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \coth(x)\right)$$
$$= \frac{x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}$$

Mathematica [A] time = 0.047073, size = 24, normalized size = 0.63

$$\frac{1}{32}(4x-\sinh(4x)+4\cosh(2x)+\cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (4*x + 4*Cosh[2*x] + Cosh[4*x] - Sinh[4*x])/32
```

Maple [B] time = 0.029, size = 78, normalized size = 2.1

$$\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{1}{8}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1+coth(x)),x)
```

```
[Out] 1/2/(tanh(1/2*x)+1)^4-1/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2
*x)+1)+1/8*ln(tanh(1/2*x)+1)+1/4/(tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)-1/8*
ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.02968, size = 30, normalized size = 0.79

$$\frac{1}{8}x+\frac{1}{16}e^{(2x)}+\frac{1}{16}e^{(-2x)}+\frac{1}{32}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/8*x + 1/16*e^(2*x) + 1/16*e^(-2*x) + 1/32*e^(-4*x)
```

Fricas [A] time = 2.53566, size = 176, normalized size = 4.63

$$\frac{3\cosh(x)^3 + 9\cosh(x)\sinh(x)^2 + \sinh(x)^3 + 2(2x+1)\cosh(x) + \left(3\cosh(x)^2 + 4x - 2\right)\sinh(x)}{32(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 + 2*(2*x + 1)*cosh(x) +
 (3*cosh(x)^2 + 4*x - 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(cosh(x)**2/(coth(x) + 1), x)
```

Giac [A] time = 1.11309, size = 41, normalized size = 1.08

$$-\frac{1}{32}\left(3e^{(4x)} - 2e^{(2x)} - 1\right)e^{(-4x)} + \frac{1}{8}x + \frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/32*(3*e^(4*x) - 2*e^(2*x) - 1)*e^(-4*x) + 1/8*x + 1/16*e^(2*x)
```

3.108 $\int \frac{\cosh(x)}{1+\coth(x)} dx$

Optimal. Leaf size=17

$$\frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

```
[Out] Cosh[x]^3/3 - Sinh[x]^3/3
```

Rubi [A] time = 0.113401, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3518, 3108, 3107, 2565, 30, 2564}

$$\frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(1 + Coth[x]),x]

[Out] Cosh[x]^3/3 - Sinh[x]^3/3
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\cosh(x)\sinh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \cosh(x)\sinh(x)(-\cosh(x)+\sinh(x))\,dx\\
&= i\int \left(-i\cosh^2(x)\sinh(x)+i\cosh(x)\sinh^2(x)\right)\,dx\\
&= \int \cosh^2(x)\sinh(x)\,dx-\int \cosh(x)\sinh^2(x)\,dx\\
&= -\left(i\,\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)\right)+\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)\\
&= \frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0166151, size = 19, normalized size = 1.12

$$\frac{1}{12}\left(-4\sinh^3(x)+3\cosh(x)+\cosh(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(1 + Coth[x]),x]
```

```
[Out] (3*Cosh[x] + Cosh[3*x] - 4*Sinh[x]^3)/12
```

Maple [B] time = 0.024, size = 42, normalized size = 2.5

$$\frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1+coth(x)),x)
```

```
[Out] 2/3/(tanh(1/2*x)+1)^3-1/(tanh(1/2*x)+1)^2+1/2/(tanh(1/2*x)+1)-1/2/(tanh(1/2
*x)-1)
```

Maxima [A] time = 1.01463, size = 15, normalized size = 0.88

$$\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

Fricas [A] time = 2.48556, size = 90, normalized size = 5.29

$$\frac{\cosh(x)^2 + \cosh(x)\sinh(x) + \sinh(x)^2}{3(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x)
```

```
[Out] Integral(cosh(x)/(coth(x) + 1), x)
```

Giac [A] time = 1.13034, size = 15, normalized size = 0.88

$$\frac{1}{12}e^{(-3x)} + \frac{1}{4}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

3.109 $\int \frac{\text{sech}(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=10

$$-\sinh(x)+\cosh(x)+\tan^{-1}(\sinh(x))$$

```
[Out] ArcTan[Sinh[x]] + Cosh[x] - Sinh[x]
```

Rubi [A] time = 0.114155, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3518, 3108, 3107, 2638, 2592, 321, 203}

$$-\sinh(x)+\cosh(x)+\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(1 + Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]] + Cosh[x] - Sinh[x]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\tanh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int (-\cosh(x)+\sinh(x))\tanh(x)\,dx\\
&= i\int (-i\sinh(x)+i\sinh(x)\tanh(x))\,dx\\
&= \int \sinh(x)\,dx - \int \sinh(x)\tanh(x)\,dx\\
&= \cosh(x) - \operatorname{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= \cosh(x) - \sinh(x) + \operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= \tan^{-1}(\sinh(x)) + \cosh(x) - \sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.0233032, size = 16, normalized size = 1.6

$$-\sinh(x)+\cosh(x)+2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(1 + Coth[x]),x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] + Cosh[x] - Sinh[x]
```

Maple [A] time = 0.023, size = 19, normalized size = 1.9

$$2\,\arctan\left(\tanh\left(x/2\right)\right)+2\,\left(\tanh\left(x/2\right)+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(1+coth(x)),x)
```

```
[Out] 2*arctan(tanh(1/2*x))+2/(tanh(1/2*x)+1)
```

Maxima [A] time = 1.53425, size = 16, normalized size = 1.6

$$-2\,\arctan\left(e^{(-x)}\right)+e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-x)) + e^(-x)
```

Fricas [B] time = 2.57969, size = 101, normalized size = 10.1

$$\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] (2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) + 1)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)/(coth(x) + 1), x)
```

Giac [A] time = 1.16349, size = 14, normalized size = 1.4

$$2\arctan\left(e^{x}\right)+e^{\left(-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x) + e^(-x)
```

3.110 $\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=15

$$\tanh(x)-\log(\tanh(x))-\log(\coth(x)+1)$$

```
[Out] -Log[1 + Coth[x]] - Log[Tanh[x]] + Tanh[x]
```

Rubi [A] time = 0.0411416, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3516, 44}

$$\tanh(x)-\log(\tanh(x))-\log(\coth(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(1 + Coth[x]),x]
```

```
[Out] -Log[1 + Coth[x]] - Log[Tanh[x]] + Tanh[x]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, \coth(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx, x, \coth(x)\right)\\
&= -\log(1+\coth(x))-\log(\tanh(x))+\tanh(x)
\end{aligned}$$

Mathematica [A] time = 0.0296206, size = 9, normalized size = 0.6

$$-x+\tanh(x)+\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(1 + Coth[x]),x]
```

```
[Out] -x + Log[Cosh[x]] + Tanh[x]
```

Maple [B] time = 0.028, size = 36, normalized size = 2.4

$$-2\ln\left(\tanh\left(x/2\right)+1\right)+2\,\frac{\tanh\left(x/2\right)}{\left(\tanh\left(x/2\right)\right)^{2}+1}+\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1+coth(x)),x)
```

```
[Out] -2*ln(tanh(1/2*x)+1)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)+ln(tanh(1/2*x)^2+1)
```

Maxima [A] time = 1.56431, size = 24, normalized size = 1.6

$$\frac{2}{e^{(-2\,x)}+1}+\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) + 1) + log(e^(-2*x) + 1)
```

Fricas [B] time = 2.6983, size = 269, normalized size = 17.93

$$-\frac{2\,x\cosh\left(x\right)^{2}+4\,x\cosh\left(x\right)\sinh\left(x\right)+2\,x\sinh\left(x\right)^{2}-\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+2\,x+2}{\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -(2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*x + 2)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{2}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)**2/(coth(x) + 1), x)
```

<u>Giac</u> [A] time = 1.18812, size = 36, normalized size = 2.4

$$-2x - \frac{e^{(2x)} + 3}{e^{(2x)} + 1} + \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2*x - (e^(2*x) + 3)/(e^(2*x) + 1) + log(e^(2*x) + 1)
```

3.111 $\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)} dx$

Optimal. Leaf size=20

$$-\operatorname{sech}(x) - \frac{1}{2}\tan^{-1}(\sinh(x)) + \frac{1}{2}\tanh(x)\operatorname{sech}(x)$$

```
[Out] -ArcTan[Sinh[x]]/2 - Sech[x] + (Sech[x]*Tanh[x])/2
```

Rubi [A] time = 0.167309, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3518, 3108, 3107, 2606, 8, 2611, 3770}

$$-\operatorname{sech}(x) - \frac{1}{2}\tan^{-1}(\sinh(x)) + \frac{1}{2}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(1 + Coth[x]),x]
```

```
[Out] -ArcTan[Sinh[x]]/2 - Sech[x] + (Sech[x]*Tanh[x])/2
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \operatorname{sech}^2(x)(-\cosh(x)+\sinh(x))\tanh(x)\,dx\\
&= i\int \left(-i\operatorname{sech}(x)\tanh(x)+i\operatorname{sech}(x)\tanh^2(x)\right)\,dx\\
&= \int \operatorname{sech}(x)\tanh(x)\,dx-\int \operatorname{sech}(x)\tanh^2(x)\,dx\\
&= \frac{1}{2}\operatorname{sech}(x)\tanh(x)-\frac{1}{2}\int \operatorname{sech}(x)\,dx-\operatorname{Subst}\left(\int 1\,dx,x,\operatorname{sech}(x)\right)\\
&= -\frac{1}{2}\tan^{-1}(\sinh(x))-\operatorname{sech}(x)+\frac{1}{2}\operatorname{sech}(x)\tanh(x)
\end{aligned}
$$

Mathematica [A] time = 0.0391703, size = 20, normalized size = 1.

$$
\frac{1}{2}(\tanh(x)-2)\operatorname{sech}(x)-\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(1 + Coth[x]),x]
```

```
[Out] -ArcTan[Tanh[x/2]] + (Sech[x]*(-2 + Tanh[x]))/2
```

Maple [B] time = 0.029, size = 45, normalized size = 2.3

$$
4\,\frac{-1/4\,(\tanh(x/2))^3-1/2\,(\tanh(x/2))^2+1/4\,\tanh(x/2)-1/2}{\left((\tanh(x/2))^2+1\right)^2}-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(1+coth(x)),x)
```

```
[Out] 4*(-1/4*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2+1/4*tanh(1/2*x)-1/2)/(tanh(1/2*x)^2
+1)^2-arctan(tanh(1/2*x))
```

Maxima [B] time = 1.52306, size = 45, normalized size = 2.25

$$
-\frac{e^{(-x)}+3\,e^{(-3\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+\arctan\left(e^{(-x)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 3*e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + arctan(e^(-x))
```

Fricas [B] time = 2.5068, size = 508, normalized size = 25.4

$$-\frac{\cosh\left(x\right)^{3} + 3\,\cosh\left(x\right)\sinh\left(x\right)^{2} + \sinh\left(x\right)^{3} + \left(\cosh\left(x\right)^{4} + 4\,\cosh\left(x\right)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} + 2\left(3\,\cosh\left(x\right)^{2} + 1\right)\sin}{\cosh\left(x\right)^{4} + 4\,\cosh\left(x\right)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} + 2\left(3\,\cosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x
)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(cosh(x)^2 + 1)*s
inh(x) + 3*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{3}\left(x\right)}{\coth\left(x\right) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)**3/(coth(x) + 1), x)
```

Giac [A] time = 1.14922, size = 34, normalized size = 1.7

$$-\frac{e^{\left(3\,x\right)} + 3\,e^{x}}{\left(e^{\left(2\,x\right)} + 1\right)^{2}} - \arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -(e^(3*x) + 3*e^x)/(e^(2*x) + 1)^2 - arctan(e^x)
```

3.112 $\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\tanh^2(x)}{2}-\frac{\tanh^3(x)}{3}$$

```
[Out] Tanh[x]^2/2 - Tanh[x]^3/3
```

Rubi [A] time = 0.0454175, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3516, 848, 43}

$$\frac{\tanh^2(x)}{2}-\frac{\tanh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(1 + Coth[x]),x]
```

```
[Out] Tanh[x]^2/2 - Tanh[x]^3/3
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{-1+x^2}{x^4(1+x)}\,dx,x,\coth(x)\right)\\
&= -\operatorname{Subst}\left(\int \frac{-1+x}{x^4}\,dx,x,\coth(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x^3}\right)\,dx,x,\coth(x)\right)\\
&= \frac{\tanh^2(x)}{2}-\frac{\tanh^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0498663, size = 17, normalized size = 1.

$$\frac{1}{6}\left(-2\tanh^3(x)-3\text{sech}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(1 + Coth[x]),x]
```

```
[Out] (-3*Sech[x]^2 - 2*Tanh[x]^3)/6
```

Maple [B] time = 0.029, size = 38, normalized size = 2.2

$$-4\,\frac{-1/2\,(\tanh(x/2))^4+2/3\,(\tanh(x/2))^3-1/2\,(\tanh(x/2))^2}{\left((\tanh(x/2))^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(1+coth(x)),x)
```

```
[Out] -4*(-1/2*tanh(1/2*x)^4+2/3*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2)/(tanh(1/2*x)^2+
1)^3
```

Maxima [B] time = 1.03484, size = 101, normalized size = 5.94

$$-\frac{2\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}-\frac{4\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}-\frac{2}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 4*e^(-4*x)/(3*e^(-2*
x) + 3*e^(-4*x) + e^(-6*x) + 1) - 2/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
 1)
```

Fricas [B] time = 2.52306, size = 286, normalized size = 16.82

$$-\frac{4\,(\cosh(x)+2\,\sinh(x))}{3\left(\cosh(x)^5+5\,\cosh(x)\sinh(x)^4+\sinh(x)^5+\left(10\,\cosh(x)^2+3\right)\sinh(x)^3+3\,\cosh(x)^3+\left(10\,\cosh(x)^3+9\,\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(cosh(x) + 2*sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 + 3)*sinh(x)^3 + 3*cosh(x)^3 + (10*cosh(x)^3 + 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 + 9*cosh(x)^2 + 2)*sinh(x) + 4*cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)**4/(coth(x) + 1), x)
```

Giac [A] time = 1.14445, size = 24, normalized size = 1.41

$$-\frac{2\left(3\,e^{(2\,x)}-1\right)}{3\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) - 1)/(e^(2*x) + 1)^3
```

3.113 $\int \sqrt{1+\coth(x)}\,\mathbf{sech}^2(x)\,dx$

Optimal. Leaf size=21

$$\tanh^{-1}\left(\sqrt{\coth(x)+1}\right)+\tanh(x)\sqrt{\coth(x)+1}$$

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]] + Sqrt[1 + Coth[x]]*Tanh[x]
```

Rubi [A] time = 0.045848, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3516, 47, 63, 207}

$$\tanh^{-1}\left(\sqrt{\coth(x)+1}\right)+\tanh(x)\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]] + Sqrt[1 + Coth[x]]*Tanh[x]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{1+\coth(x)}\mathrm{sech}^2(x)\,dx = -\,\mathrm{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \coth(x)\right)$$
$$= \sqrt{1+\coth(x)}\tanh(x) - \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \coth(x)\right)$$
$$= \sqrt{1+\coth(x)}\tanh(x) - \mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\coth(x)}\right)$$
$$= \tanh^{-1}\left(\sqrt{1+\coth(x)}\right) + \sqrt{1+\coth(x)}\tanh(x)$$

Mathematica [C] time = 5.00565, size = 160, normalized size = 7.62

$$\frac{1}{2}\sqrt{\coth(x)+1}\left(2\tanh(x) + \frac{(1-i)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{\sqrt{i(\coth(x)+1)}} + \frac{\sinh\left(\frac{x}{2}\right)\left(4\tanh^{-1}\left(\sqrt{\tanh\left(\frac{x}{2}\right)}\right)+\sqrt{2}\left(\log\left(\tan\right.\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
```

```
[Out] (Sqrt[1 + Coth[x]]*(((1 - I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]])/Sqr
t[I*(1 + Coth[x])] + ((4*ArcTanh[Sqrt[Tanh[x/2]]] + Sqrt[2]*(Log[1 - Sqrt[2
]*Sqrt[Tanh[x/2]] + Tanh[x/2]] - Log[1 + Sqrt[2]*Sqrt[Tanh[x/2]] + Tanh[x/2
]]))*Sinh[x/2]*(-Cosh[x/2] + Sinh[x/2]))/Sqrt[Tanh[x/2]] + 2*Tanh[x]))/2
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int (\mathrm{sech}\,(x))^2\sqrt{1+\coth\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(1+coth(x))^(1/2),x)
```

```
[Out] int(sech(x)^2*(1+coth(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth\,(x)+1}\,\mathrm{sech}\,(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*sech(x)^2, x)
```

Fricas [B] time = 2.55175, size = 813, normalized size = 38.71

$$4\sqrt{2}\left(\sqrt{2}\cosh\,(x)+\sqrt{2}\sinh\,(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} + \left(\cosh\,(x)^2 + 2\,\cosh\,(x)\sinh\,(x) + \sinh\,(x)^2 + 1\right)\log\left(\frac{2\sqrt{2}\left(\sqrt{2}\cosh(x)+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) -
sinh(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((2*sqrt(2)*
(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*c
osh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 1)/(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(-(2
*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x
))) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 1)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(1+coth(x))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16151, size = 201, normalized size = 9.57

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2\sqrt{2}-\log\left(-\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)+\sqrt{2}\log\left(\frac{\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^{2}-2\sqrt{2}+3}{\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^{2}+2\sqrt{2}+3}\right)-\frac{8\left(3\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^{2}+1\right)}{\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^{4}+6\left(\sqrt{e^{(2x)}-1}-e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*(2*sqrt(2) - log(-(sqrt(2) - 1)/(sqrt(2) + 1))) + sqr
t(2)*log(((sqrt(e^(2*x) - 1) - e^x)^2 - 2*sqrt(2) + 3)/((sqrt(e^(2*x) - 1)
- e^x)^2 + 2*sqrt(2) + 3)) - 8*(3*(sqrt(e^(2*x) - 1) - e^x)^2 + 1)/((sqrt(e
^(2*x) - 1) - e^x)^4 + 6*(sqrt(e^(2*x) - 1) - e^x)^2 + 1))*sgn(e^(2*x) - 1)
```

3.114 $\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=147

$$-\frac{a^4 b\log(a+b\coth(x))}{\left(a^2-b^2\right)^3}-\frac{\sinh^4(x)(b-a\coth(x))}{4\left(a^2-b^2\right)}-\frac{\sinh^2(x)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\coth(x)\right)}{8\left(a^2-b^2\right)^2}-\frac{a(3a+b)\log(1-}{16(a+b)}$$

```
[Out] -(a*(3*a + b)*Log[1 - Coth[x]])/(16*(a + b)^3) + (a*(3*a - b)*Log[1 + Coth[
x]])/(16*(a - b)^3) - (a^4*b*Log[a + b*Coth[x]])/(a^2 - b^2)^3 - ((4*b*(2*a
^2 - b^2) - a*(5*a^2 - b^2)*Coth[x])*Sinh[x]^2)/(8*(a^2 - b^2)^2) - ((b - a
*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rubi [A] time = 0.339408, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3516, 1647, 801}

$$-\frac{a^4 b\log(a+b\coth(x))}{\left(a^2-b^2\right)^3}-\frac{\sinh^4(x)(b-a\coth(x))}{4\left(a^2-b^2\right)}-\frac{\sinh^2(x)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\coth(x)\right)}{8\left(a^2-b^2\right)^2}-\frac{a(3a+b)\log(1-}{16(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -(a*(3*a + b)*Log[1 - Coth[x]])/(16*(a + b)^3) + (a*(3*a - b)*Log[1 + Coth[
x]])/(16*(a - b)^3) - (a^4*b*Log[a + b*Coth[x]])/(a^2 - b^2)^3 - ((4*b*(2*a
^2 - b^2) - a*(5*a^2 - b^2)*Coth[x])*Sinh[x]^2)/(8*(a^2 - b^2)^2) - ((b - a
*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = -\left(b\,\text{Subst}\left(\int \frac{x^4}{(a+x)\left(-b^2+x^2\right)^3}\,dx, x, b\coth(x)\right)\right)$$

$$= -\frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2-b^2}-\frac{3ab^4x}{a^2-b^2}+4b^2x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\coth(x)\right)}{4b}$$

$$= -\frac{\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4(3a^2\cdots}{(a^2-b^2\cdots}}{(a\cdots}\right.}{\cdots}$$

$$= -\frac{\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int\left(-\frac{ab^3\cdots}{2(a+\cdots}\right.\right.}{\cdots}$$

$$= -\frac{a(3a+b)\log(1-\coth(x))}{16(a+b)^3} + \frac{a(3a-b)\log(1+\coth(x))}{16(a-b)^3} - \frac{a^4b\log(a+b\coth(x))}{\left(a^2-b^2\right)^3} - \frac{\left(4b\left(2a^2\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 0.521728, size = 144, normalized size = 0.98

$$\frac{24a^3b^2x+8a^3\left(a^2-b^2\right)\sinh(2x)-2a^3b^2\sinh(4x)-4b\left(-4a^2b^2+3a^4+b^4\right)\cosh(2x)-b\left(a^2-b^2\right)^2\cosh(4x)-32a^4b\cdots}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (12*a^5*x + 24*a^3*b^2*x - 4*a*b^4*x - 4*b*(3*a^4 - 4*a^2*b^2 + b^4)*Cosh[2
*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*a^4*b*Log[b*Cosh[x] + a*Sinh[x]] + 8*a
^3*(a^2 - b^2)*Sinh[2*x] + a^5*Sinh[4*x] - 2*a^3*b^2*Sinh[4*x] + a*b^4*Sinh
[4*x])/(32*(a - b)^3*(a + b)^3)
```

Maple [B] time = 0.047, size = 319, normalized size = 2.2

$$-\frac{1}{4a-4b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}+4\frac{1}{(8a-8b)(\tanh(x/2)+1)^3}+\frac{5a}{8(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{3b}{8(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*coth(x)),x)
```

```
[Out] -1/(4*a-4*b)/(tanh(1/2*x)+1)^4+4/(8*a-8*b)/(tanh(1/2*x)+1)^3+5/8/(a-b)^2/(t
anh(1/2*x)+1)*a-3/8/(a-b)^2/(tanh(1/2*x)+1)*b-7/8/(a-b)^2/(tanh(1/2*x)+1)^2
*a+5/8/(a-b)^2/(tanh(1/2*x)+1)^2*b+3/8/(a-b)^3*ln(tanh(1/2*x)+1)*a^2-1/8/(a
-b)^3*ln(tanh(1/2*x)+1)*a*b+1/(4*a+4*b)/(tanh(1/2*x)-1)^4+4/(8*a+8*b)/(tanh
(1/2*x)-1)^3+7/8/(a+b)^2/(tanh(1/2*x)-1)^2*a+5/8/(a+b)^2/(tanh(1/2*x)-1)^2*
b+5/8/(a+b)^2/(tanh(1/2*x)-1)*a+3/8/(a+b)^2/(tanh(1/2*x)-1)*b-3/8/(a+b)^3*l
n(tanh(1/2*x)-1)*a^2-1/8/(a+b)^3*ln(tanh(1/2*x)-1)*a*b-a^4*b/(a-b)^3/(a+b)^
3*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Maxima [A] time = 1.19885, size = 208, normalized size = 1.41

$$-\frac{a^4 b \log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{\left(3a^2+ab\right)x}{8\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{\left(4(2a+b)e^{(-2x)}+a+b\right)e^{(4x)}}{64\left(a^2+2ab+b^2\right)}-\frac{4(2a-b)e^{(-2x)}+(a-}{64\left(a^2-2ab+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^4*b*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
 1/8*(3*a^2 + a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*(2*a + b)*e^
(-2*x) + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*(2*a - b)*e^(-2*x) +
(a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Fricas [B] time = 2.85631, size = 2747, normalized size = 18.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(2*a^5 - 3*a^4*
b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^6 + 4*(2*a^5 - 3*a^4*b - 2*a^3*b^2
 + 4*a^2*b^3 - b^5 + 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^4
+ 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(2
*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^
4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 30*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*
a^2*b^3 - b^5)*cosh(x)^2 + 4*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x)*sinh(
x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 +
 10*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^3 + 4*(3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x))*sinh(x)^3 - 4*(2*a^5 + 3*a^4*b - 2*
a^3*b^2 - 4*a^2*b^3 + b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^6 - 2*a^5 - 3*a^4*b + 2*a^3*b^2 + 4*a^2*b^3 -
b^5 + 15*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^4 + 12*(3*
a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^4*b*cosh(
x)^4 + 4*a^4*b*cosh(x)^3*sinh(x) + 6*a^4*b*cosh(x)^2*sinh(x)^2 + 4*a^4*b*co
sh(x)*sinh(x)^3 + a^4*b*sinh(x)^4)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) -
 sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^7 + 3*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^5 + 4*(3*a^5
 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^3 - (2*a^5 + 3*a^4*b - 2*a^3*b^2
- 4*a^2*b^3 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*si
nh(x)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)**4/(a + b*coth(x)), x)
```

Giac [A] time = 1.15977, size = 292, normalized size = 1.99

$$-\frac{a^4 b \log\left(\left|-ae^{(2x)} - be^{(2x)} + a - b\right|\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{\left(3a^2 - ab\right)x}{8\left(a^3 - 3a^2b + 3ab^2 - b^3\right)} - \frac{\left(18a^2e^{(4x)} - 6abe^{(4x)} + 8a^2e^{(2x)} - 12abe^{(2x)} + 4b\right.}{64\left(a^3 - 3a^2b + 3ab^2 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^4*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) + 1/8*(3*a^2 - a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/64*(18*a
^2*e^(4*x) - 6*a*b*e^(4*x) + 8*a^2*e^(2*x) - 12*a*b*e^(2*x) + 4*b^2*e^(2*x)
 + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^
(4*x) + b*e^(4*x) + 8*a*e^(2*x) + 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

3.115 $\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=135

$$\frac{a\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{a\sinh(x)}{a^2-b^2}+\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

```
[Out] (a^3*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
- (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*b^2*Si
nh[x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^
2))
```

Rubi [A] time = 0.250881, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3518, 3109, 2633, 2565, 30, 3100, 2637, 3074, 206}

$$\frac{a\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{a\sinh(x)}{a^2-b^2}+\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a^3*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
- (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*b^2*Si
nh[x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^
2))
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = -\left(i\int \frac{\cosh^3(x)\sinh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)$$

$$= \frac{a\int \cosh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(iab)\int \frac{\cosh^2(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{\left(ia^3b\right)\int \frac{1}{-ib\cosh(x)-ia\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(ab^2\right)\int \cosh(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{(ia)\operatorname{Subst}\left(\int \left(1-x^2\right) \ldots\right.}{a^2-b^2}$$

$$= -\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{b\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(a^3b\right)\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2} \ldots\right.}{\ldots}$$

$$= \frac{a^3b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{b\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh \ldots}{3\left(a^2 \ldots\right.}$$

Mathematica [A] time = 1.40314, size = 167, normalized size = 1.24

$$\frac{1}{12}\left(\frac{3a(3a^2+b^2)\sinh(x)}{(a-b)^2(a+b)^2}+\frac{3b(b^2-5a^2)\cosh(x)}{(a-b)^2(a+b)^2}+\frac{a^3\sinh(3x)}{(a-b)^2(a+b)^2}-\frac{24a^3b\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{(b-a)^{5/2}(a+b)^{5/2}}-\frac{ab^2\sinh(3x)}{(a-b)^2(a+b)^2}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] ((-24*a^3*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)
^(5/2)*(a + b)^(5/2)) + (3*b*(-5*a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2)
+ (b*Cosh[3*x])/((-a + b)*(a + b)) + (3*a*(3*a^2 + b^2)*Sinh[x])/((a - b)^2
*(a + b)^2) + (a^3*Sinh[3*x])/((a - b)^2*(a + b)^2) - (a*b^2*Sinh[3*x])/((a
 - b)^2*(a + b)^2))/12
```

Maple [A] time = 0.041, size = 200, normalized size = 1.5

$$-\frac{4}{12a-12b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+2\frac{1}{(4a-4b)(\tanh(x/2)+1)^2}-\frac{a}{(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{b}{2(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*coth(x)),x)
```

```
[Out] -4/3/(tanh(1/2*x)+1)^3/(4*a-4*b)+2/(4*a-4*b)/(tanh(1/2*x)+1)^2-1/(a-b)^2/(t
anh(1/2*x)+1)*a+1/2/(a-b)^2/(tanh(1/2*x)+1)*b-4/3/(tanh(1/2*x)-1)^3/(4*a+4*
b)-2/(4*a+4*b)/(tanh(1/2*x)-1)^2-1/(a+b)^2/(tanh(1/2*x)-1)*a-1/2/(a+b)^2/(t
anh(1/2*x)-1)*b-2*a^3*b/(a-b)^2/(a+b)^2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh
(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.01861, size = 4208, normalized size = 31.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
```

```
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*
a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
 a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a
 + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) +
 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^
5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2
+ (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^
4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)
^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 + 5*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 +
5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a^4
*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^
3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cosh(
x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(-a^2 + b^2
)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^5 - 5*a^4*b
 - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(x)}{a + b\coth(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*coth(x)), x)
```

Giac [A] time = 1.1639, size = 221, normalized size = 1.64

$$\frac{2\,a^3 b \arctan\left(-\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\,\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}+9\,a^2e^x+12\,abe^x+3\,b^2e^x}{24\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*a^3*b*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) + a - b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 9*a^2*e^
x + 12*a*b*e^x + 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.116 $\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{a^2 b\log(a+b\coth(x))}{\left(a^2-b^2\right)^2}-\frac{\sinh^2(x)(b-a\coth(x))}{2\left(a^2-b^2\right)}-\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(\coth(x)+1)}{4(a-b)^2}$$

```
[Out] -(a*Log[1 - Coth[x]])/(4*(a + b)^2) + (a*Log[1 + Coth[x]])/(4*(a - b)^2) -
(a^2*b*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Sinh[x]^2)/(2*(
a^2 - b^2))
```

Rubi [A] time = 0.163791, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3516, 1647, 801}

$$-\frac{a^2 b\log(a+b\coth(x))}{\left(a^2-b^2\right)^2}-\frac{\sinh^2(x)(b-a\coth(x))}{2\left(a^2-b^2\right)}-\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(\coth(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -(a*Log[1 - Coth[x]])/(4*(a + b)^2) + (a*Log[1 + Coth[x]])/(4*(a - b)^2) -
(a^2*b*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Sinh[x]^2)/(2*(
a^2 - b^2))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = -\left(b\,\text{Subst}\left(\int \frac{x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\coth(x)\right)\right)$$
$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^2}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}}{(a+x)\left(-b^2+x^2\right)}\,dx, x, b\coth(x)\right)}{2b}$$
$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \left(-\frac{ab}{2(a+b)^2(b-x)} + \frac{2a^2b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{ab}{2(a-b)^2(b+x)}\right)dx, x, b\coth(x\right.}{2b}$$
$$= -\frac{a\log(1-\coth(x))}{4(a+b)^2} + \frac{a\log(1+\coth(x))}{4(a-b)^2} - \frac{a^2b\log(a+b\coth(x))}{\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.228807, size = 73, normalized size = 0.86

$$\frac{\left(b^3-a^2b\right)\cosh(2x) + a\left(2x\left(a^2+b^2\right)+\left(a^2-b^2\right)\sinh(2x) - 4ab\log(a\sinh(x)+b\cosh(x))\right)}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] ((-(a^2*b) + b^3)*Cosh[2*x] + a*(2*(a^2 + b^2)*x - 4*a*b*Log[b*Cosh[x] + a*
Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Maple [A] time = 0.041, size = 146, normalized size = 1.7

$$-2\,\frac{1}{(4\,a-4\,b)\,(\tanh(x/2)+1)^2} + 4\,\frac{1}{(8\,a-8\,b)\,(\tanh(x/2)+1)} + \frac{a}{2\,(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + 2\,\frac{1}{(4\,a+4\,b)\,(\tanh(x/2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*coth(x)),x)
```

```
[Out] -2/(4*a-4*b)/(tanh(1/2*x)+1)^2+4/(8*a-8*b)/(tanh(1/2*x)+1)+1/2*a/(a-b)^2*ln
(tanh(1/2*x)+1)+2/(4*a+4*b)/(tanh(1/2*x)-1)^2+4/(8*a+8*b)/(tanh(1/2*x)-1)-1
/2*a/(a+b)^2*ln(tanh(1/2*x)-1)-a^2*b/(a-b)^2/(a+b)^2*ln(tanh(1/2*x)^2*b+2*a
*tanh(1/2*x)+b)
```

Maxima [A] time = 1.15573, size = 108, normalized size = 1.27

$$-\frac{a^2b\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^4-2\,a^2b^2+b^4} + \frac{ax}{2\left(a^2+2\,ab+b^2\right)} + \frac{e^{(2\,x)}}{8\,(a+b)} - \frac{e^{(-2\,x)}}{8\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*x/(a^
2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Fricas [B] time = 2.65051, size = 826, normalized size = 9.72

$$\frac{\left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)^4 + 4\left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)\sinh\left(x\right)^3 + \left(a^3 - a^2b - ab^2 + b^3\right)\sinh\left(x\right)^4 + 4\left(a^3 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 + 2*a^2*
b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 - 8*(a^2*b*co
sh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(b*cosh(x) + a*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2
*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*coth(x)), x)
```

Giac [A] time = 1.14898, size = 140, normalized size = 1.65

$$-\frac{a^2b\log\left(\left|-ae^{(2x)} - be^{(2x)} + a - b\right|\right)}{a^4 - 2a^2b^2 + b^4} + \frac{ax}{2\left(a^2 - 2ab + b^2\right)} - \frac{\left(2ae^{(2x)} + a - b\right)e^{(-2x)}}{8\left(a^2 - 2ab + b^2\right)} + \frac{e^{(2x)}}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^2*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1
/2*a*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) + a - b)*e^(-2*x)/(a^2 - 2*a*
b + b^2) + 1/8*e^(2*x)/(a + b)
```

3.117 $\int \frac{\cosh(x)}{a+b\coth(x)} dx$

Optimal. Leaf size=72

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}+\frac{ab\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] (a*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
(b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rubi [A] time = 0.112425, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3518, 3109, 2637, 2638, 3074, 206}

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}+\frac{ab\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Coth[x]),x]
```

```
[Out] (a*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
(b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\cosh(x)\sinh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\ &= \frac{a\int \cosh(x)\,dx}{a^2-b^2}-\frac{b\int \sinh(x)\,dx}{a^2-b^2}+\frac{(iab)\int \frac{1}{-ib\cosh(x)-ia\sinh(x)}\,dx}{a^2-b^2}\\ &= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{(ab)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -a\cosh(x)-b\sinh(x)\right)}{a^2-b^2}\\ &= \frac{ab\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.301377, size = 79, normalized size = 1.1

$$\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{b^2-a^2}+\frac{2ab\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{(b-a)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)^(3/2
)*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Maple [A] time = 0.033, size = 92, normalized size = 1.3

$$-4\,\frac{1}{(4\,a+4\,b)\,(\tanh(x/2)-1)}-4\,\frac{1}{(4\,a-4\,b)\,(\tanh(x/2)+1)}-2\,\frac{ab}{(a+b)\,(a-b)\,\sqrt{-a^2+b^2}}\arctan\left(1/2\,\frac{2\,\tanh(x/2)\,b}{\sqrt{-a^2+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*coth(x)),x)
```

```
[Out] -4/(4*a+4*b)/(tanh(1/2*x)-1)-4/(4*a-4*b)/(tanh(1/2*x)+1)-2*a*b/(a+b)/(a-b)/
(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87074, size = 1106, normalized size = 15.36

$$\left[-\frac{a^3+a^2b-ab^2-b^3-\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^2-2\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)-\left(a^3-a^2b-ab^2+b^3\right)}{2\left(\left(a^4-2a^2b^2+b^4\right)\cosh\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cos
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*
(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2
*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b -
a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a
^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a
^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*coth(x)), x)
```

Giac [A] time = 1.13028, size = 96, normalized size = 1.33

$$-\frac{2\,ab\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}-\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^x}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*a*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^
2)) - 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

3.118 $\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\tan^{-1}(\sinh(x))}{a}$$

```
[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a
*Sqrt[a^2 - b^2])
```

Rubi [A] time = 0.138412, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3518, 3110, 3770, 3074, 206}

$$\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a
*Sqrt[a^2 - b^2])
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\tanh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= -\int \left(-\frac{\operatorname{sech}(x)}{a}+\frac{ib}{a(ib\cosh(x)+ia\sinh(x))}\right)dx\\
&= \frac{\int \operatorname{sech}(x)\,dx}{a}-\frac{(ib)\int \frac{1}{ib\cosh(x)+ia\sinh(x)}\,dx}{a}\\
&= \frac{\tan^{-1}(\sinh(x))}{a}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,a\cosh(x)+b\sinh(x)\right)}{a}\\
&= \frac{\tan^{-1}(\sinh(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}
\end{aligned}$$

Mathematica [A] time = 0.112744, size = 60, normalized size = 1.2

$$\frac{2\left(\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{b\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Coth[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] - (b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*Sqrt[a + b])))/a
```

Maple [A] time = 0.033, size = 54, normalized size = 1.1

$$2\,\frac{\arctan\left(\tanh\left(x/2\right)\right)}{a}-2\,\frac{b}{a\sqrt{-a^2+b^2}}\arctan\left(1/2\,\frac{2\,\tanh\left(x/2\right)b+2\,a}{\sqrt{-a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*coth(x)),x)
```

```
[Out] 2/a*arctan(tanh(1/2*x))-2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.863, size = 564, normalized size = 11.28

$$\left[\frac{\sqrt{a^2-b^2}b\log\left(\frac{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)+2\left(a^2-b^2\right)\arctan(\cosh(x)}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(a^2
 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*b*ar
ctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^2)*ar
ctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*coth(x)), x)
```

Giac [A] time = 1.14499, size = 65, normalized size = 1.3

$$-\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a}+\frac{2\,\arctan\left(e^x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arct
an(e^x)/a
```

3.119 $\int \frac{\text{sech}^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=29

$$-\frac{b\log(\tanh(x))}{a^2}-\frac{b\log(a+b\coth(x))}{a^2}+\frac{\tanh(x)}{a}$$

```
[Out] -((b*Log[a + b*Coth[x]])/a^2) - (b*Log[Tanh[x]])/a^2 + Tanh[x]/a
```

Rubi [A] time = 0.0554407, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3516, 44}

$$-\frac{b\log(\tanh(x))}{a^2}-\frac{b\log(a+b\coth(x))}{a^2}+\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -((b*Log[a + b*Coth[x]])/a^2) - (b*Log[Tanh[x]])/a^2 + Tanh[x]/a
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^2(x)}{a+b\coth(x)}\,dx &= -\left(b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx,x,b\coth(x)\right)\right)\\
&= -\left(b\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx,x,b\coth(x)\right)\right)\\
&= -\frac{b\log(a+b\coth(x))}{a^2}-\frac{b\log(\tanh(x))}{a^2}+\frac{\tanh(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0806487, size = 27, normalized size = 0.93

$$\frac{-b\log(a\sinh(x)+b\cosh(x))+a\tanh(x)+b\log(\cosh(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (b*Log[Cosh[x]] - b*Log[b*Cosh[x] + a*Sinh[x]] + a*Tanh[x])/a^2
```

Maple [A] time = 0.04, size = 59, normalized size = 2.

$$2\frac{\tanh(x/2)}{a\left((\tanh(x/2))^2+1\right)}+\frac{b}{a^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)-\frac{b}{a^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2b+2a\tanh(x/2)+b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*coth(x)),x)
```

```
[Out] 2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/a^2*b*ln(tanh(1/2*x)^2+1)-b/a^2*ln(tanh
(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Maxima [A] time = 1.76628, size = 62, normalized size = 2.14

$$-\frac{b\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2}+\frac{b\log\left(e^{(-2x)}+1\right)}{a^2}+\frac{2}{ae^{(-2x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) + a + b)/a^2 + b*log(e^(-2*x) + 1)/a^2 + 2/(a*e^(-
2*x) + a)
```

Fricas [B] time = 2.61689, size = 362, normalized size = 12.48

$$-\frac{\left(b\cosh(x)^2+2b\cosh(x)\sinh(x)+b\sinh(x)^2+b\right)\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\left(b\cosh(x)^2+2b\cosh(x)\sinh(x)+\right.}{a^2\cosh(x)^2+2a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*log(2*(b*cosh(x) +
a*sinh(x))/(cosh(x) - sinh(x))) - (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 + b)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*a)/(a^2*cosh(x)^2 + 2*a
^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*coth(x)), x)
```

Giac [B] time = 1.1468, size = 103, normalized size = 3.55

$$-\frac{\left(ab+b^2\right)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^3+a^2b}+\frac{b\log\left(e^{(2x)}+1\right)}{a^2}-\frac{be^{(2x)}+2a+b}{a^2\left(e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a*b + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 + a^2*b) + b*log(
e^(2*x) + 1)/a^2 - (b*e^(2*x) + 2*a + b)/(a^2*(e^(2*x) + 1))
```

3.120 $\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=83

$$-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}+\frac{b\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

```
[Out] ArcTan[Sinh[x]]/(2*a) - (b^2*ArcTan[Sinh[x]])/a^3 + (b*Sqrt[a^2 - b^2]*ArcT
anh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 - (b*Sech[x])/a^2 + (Sech
[x]*Tanh[x])/(2*a)
```

Rubi [A] time = 0.237053, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3518, 3110, 3768, 3770, 3104, 3074, 206}

$$-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}+\frac{b\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) - (b^2*ArcTan[Sinh[x]])/a^3 + (b*Sqrt[a^2 - b^2]*ArcT
anh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 - (b*Sech[x])/a^2 + (Sech
[x]*Tanh[x])/(2*a)
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= -\int\left(-\frac{\operatorname{sech}^3(x)}{a}+\frac{ib\operatorname{sech}^2(x)}{a(ib\cosh(x)+ia\sinh(x))}\right)dx\\
&= \frac{\int \operatorname{sech}^3(x)\,dx}{a}-\frac{(ib)\int \frac{\operatorname{sech}^2(x)}{ib\cosh(x)+ia\sinh(x)}\,dx}{a}\\
&= -\frac{b\operatorname{sech}(x)}{a^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}+\frac{\int \operatorname{sech}(x)\,dx}{2a}-\frac{b^2\int \operatorname{sech}(x)\,dx}{a^3}-\frac{\left(ib\left(a^2-b^2\right)\right)\int \frac{1}{ib\cosh(x)+ia\sinh}
\end{aligned}$$

$$= \frac{\tan^{-1}(\sinh(x))}{2a}-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}+\frac{\left(b\left(a^2-b^2\right)\right)\operatorname{Subst}\left(\int \frac{}{a^2-}\right.}{}$$

$$= \frac{\tan^{-1}(\sinh(x))}{2a}-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}+\frac{b\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\operatorname{sech}(x$$

Mathematica [A] time = 0.182099, size = 85, normalized size = 1.02

$$\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+4b\sqrt{b-a}\sqrt{a+b}\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)+a\operatorname{sech}(x)(a\tanh(x)-2b)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (2*(a^2 - 2*b^2)*ArcTan[Tanh[x/2]] + 4*b*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a
 + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] + a*Sech[x]*(-2*b + a*Tanh[x]))
/(2*a^3)
```

Maple [B] time = 0.047, size = 187, normalized size = 2.3

$$-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}-2\frac{(\tanh(x/2))^2 b}{a^2\left((\tanh(x/2))^2+1\right)^2}+\frac{1}{a}\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2}-2\frac{b}{a^2\left((\tanh(x/2)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*coth(x)),x)
```

```
[Out] -1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-2/a^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x
)^2*b+1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)-2/a^2/(tanh(1/2*x)^2+1)^2*b+1/a*a
rctan(tanh(1/2*x))-2/a^3*arctan(tanh(1/2*x))*b^2-2/a*b/(-a^2+b^2)^(1/2)*arc
tan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))+2*b^3/a^3/(-a^2+b^2)^(1/2)*
arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.13031, size = 2423, normalized size = 29.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [((a^2 - 2*a*b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*
b)*sinh(x)^3 + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cos
h(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh
(x) + b)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + (
(a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*b^2)
*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 + a^2
 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 - 2*b
^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x) + (
3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*co
sh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^
2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x)), ((a^2 - 2*a*
b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*b)*sinh(x)^3
- 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*
(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sq
rt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))
 + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*
b^2)*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 +
 a^2 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 -
 2*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x)
 + (3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^
3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh
(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*coth(x)), x)
```

Giac [A] time = 1.15398, size = 138, normalized size = 1.66

$$\frac{(a^2-2b^2)\arctan(e^x)}{a^3}-\frac{2(a^2b-b^3)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^3}+\frac{ae^{(3x)}-2be^{(3x)}-ae^x-2be^x}{a^2\left(e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] (a^2 - 2*b^2)*arctan(e^x)/a^3 - 2*(a^2*b - b^3)*arctan((a*e^x + b*e^x)/sqrt
(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a*e^(3*x) - 2*b*e^(3*x) - a*e^x - 2
*b*e^x)/(a^2*(e^(2*x) + 1)^2)
```

3.121 $\int \frac{\text{sech}^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=79

$$\frac{(a^2-b^2)\tanh(x)}{a^3}-\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}-\frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}$$

```
[Out] -((b*(a^2 - b^2)*Log[a + b*Coth[x]])/a^4) - (b*(a^2 - b^2)*Log[Tanh[x]])/a^
4 + ((a^2 - b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/(2*a^2) - Tanh[x]^3/(3*a)
```

Rubi [A] time = 0.099762, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3516, 894}

$$\frac{(a^2-b^2)\tanh(x)}{a^3}-\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}-\frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -((b*(a^2 - b^2)*Log[a + b*Coth[x]])/a^4) - (b*(a^2 - b^2)*Log[Tanh[x]])/a^
4 + ((a^2 - b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/(2*a^2) - Tanh[x]^3/(3*a)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^4(x)}{a+b\coth(x)}\,dx &= -\left(b\,\text{Subst}\left(\int \frac{-b^2+x^2}{x^4(a+x)}\,dx,x,b\coth(x)\right)\right)\\
&= -\left(b\,\text{Subst}\left(\int \left(-\frac{b^2}{ax^4}+\frac{b^2}{a^2x^3}+\frac{a^2-b^2}{a^3x^2}+\frac{-a^2+b^2}{a^4x}+\frac{a^2-b^2}{a^4(a+x)}\right)dx,x,b\coth(x)\right)\right)\\
&= -\frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4}-\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}+\frac{(a^2-b^2)\tanh(x)}{a^3}+\frac{b\tanh^2(x)}{2a^2}-
\end{aligned}$$

Mathematica [A] time = 0.258318, size = 68, normalized size = 0.86

$$\frac{(4a^3-6ab^2)\tanh(x)-6b(b^2-a^2)(\log(\cosh(x))-\log(a\sinh(x)+b\cosh(x)))+a^2\text{sech}^2(x)(2a\tanh(x)-3b)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (-6*b*(-a^2 + b^2)*(Log[Cosh[x]] - Log[b*Cosh[x] + a*Sinh[x]]) + (4*a^3 - 6
*a*b^2)*Tanh[x] + a^2*Sech[x]^2*(-3*b + 2*a*Tanh[x]))/(6*a^4)
```

Maple [B] time = 0.053, size = 257, normalized size = 3.3

$$2\frac{(\tanh(x/2))^5}{a\left((\tanh(x/2))^2+1\right)^3}-2\frac{(\tanh(x/2))^5 b^2}{a^3\left((\tanh(x/2))^2+1\right)^3}+2\frac{b(\tanh(x/2))^4}{a^2\left((\tanh(x/2))^2+1\right)^3}+\frac{4}{3a}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*coth(x)),x)
```

```
[Out] 2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5-2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)
^5*b^2+2/a^2/(tanh(1/2*x)^2+1)^3*b*tanh(1/2*x)^4+4/3/a/(tanh(1/2*x)^2+1)^3*
tanh(1/2*x)^3-4/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^2+2/a^2/(tanh(1/2*x
)^2+1)^3*b*tanh(1/2*x)^2+2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)-2/a^3/(tanh(1/
2*x)^2+1)^3*tanh(1/2*x)*b^2+1/a^2*b*ln(tanh(1/2*x)^2+1)-1/a^4*ln(tanh(1/2*x
)^2+1)*b^3-b/a^2*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)+b^3/a^4*ln(tanh(1/2*
x)^2*b+2*a*tanh(1/2*x)+b)
```

Maxima [A] time = 1.7053, size = 180, normalized size = 2.28

$$\frac{2\left(2a^2-3b^2+3\left(2a^2-ab-2b^2\right)e^{(-2x)}-3\left(ab+b^2\right)e^{(-4x)}\right)}{3\left(3a^3e^{(-2x)}+3a^3e^{(-4x)}+a^3e^{(-6x)}+a^3\right)}-\frac{\left(a^2b-b^3\right)\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4}+\frac{\left(a^2b-b^3\right)\log\left(e\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] 2/3*(2*a^2 - 3*b^2 + 3*(2*a^2 - a*b - 2*b^2)*e^(-2*x) - 3*(a*b + b^2)*e^(-4
*x))/(3*a^3*e^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) - (a^2*b - b^3)
*log(-(a - b)*e^(-2*x) + a + b)/a^4 + (a^2*b - b^3)*log(e^(-2*x) + 1)/a^4
```

Fricas [B] time = 2.76842, size = 2187, normalized size = 27.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^2*b - a*b^2)*cosh(x)^4 + 24*(a^2*b - a*b^2)*cosh(x)*sinh(x)^3 +
6*(a^2*b - a*b^2)*sinh(x)^4 + 4*a^3 - 6*a*b^2 + 6*(2*a^3 + a^2*b - 2*a*b^2)
*cosh(x)^2 + 6*(2*a^3 + a^2*b - 2*a*b^2 + 6*(a^2*b - a*b^2)*cosh(x)^2)*sinh
(x)^2 + 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a
^2*b - b^3)*sinh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2
```

```
*b - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b -
b^3)*cosh(x))*sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a
^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2
+ 6*((a^2*b - b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*co
sh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^
2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^2*b - b^3)*si
nh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2*b - b^3)*cosh
(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*
sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^2*b - b^3)*co
sh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^2*b -
b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x)
)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 12*(2*(a^2*b - a*b^2)*cosh(x)^3 + (2
*a^3 + a^2*b - 2*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*si
nh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh
(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 + 3*a^4*cosh(x))*sinh(x)^
3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^
5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*coth(x)), x)
```

Giac [B] time = 1.14789, size = 271, normalized size = 3.43

$$-\frac{\left(a^3b + a^2b^2 - ab^3 - b^4\right)\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^5 + a^4b} + \frac{\left(a^2b - b^3\right)\log\left(e^{(2x)} + 1\right)}{a^4} - \frac{11\,a^2be^{(6x)} - 11\,b^3e^{(6x)} + 45\,a^2be^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a^3*b + a^2*b^2 - a*b^3 - b^4)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a
^5 + a^4*b) + (a^2*b - b^3)*log(e^(2*x) + 1)/a^4 - 1/6*(11*a^2*b*e^(6*x) -
11*b^3*e^(6*x) + 45*a^2*b*e^(4*x) - 12*a*b^2*e^(4*x) - 33*b^3*e^(4*x) + 24*
a^3*e^(2*x) + 45*a^2*b*e^(2*x) - 24*a*b^2*e^(2*x) - 33*b^3*e^(2*x) + 8*a^3
+ 11*a^2*b - 12*a*b^2 - 11*b^3)/(a^4*(e^(2*x) + 1)^3)
```

3.122 $\int \frac{\mathbf{sech}(x)}{i+2\coth(x)}\,dx$

Optimal. Leaf size=31

$$-i\tan^{-1}(\sinh(x))-\frac{2\tanh^{-1}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] (-I)*ArcTan[Sinh[x]] - (2*ArcTanh[(Cosh[x] - (2*I)*Sinh[x])/Sqrt[5]])/Sqrt[
5]
```

Rubi [A] time = 0.103316, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3518, 3110, 3770, 3074, 206}

$$-i\tan^{-1}(\sinh(x))-\frac{2\tanh^{-1}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + 2*Coth[x]),x]
```

```
[Out] (-I)*ArcTan[Sinh[x]] - (2*ArcTanh[(Cosh[x] - (2*I)*Sinh[x])/Sqrt[5]])/Sqrt[
5]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx &= -\left(i\int \frac{\tanh(x)}{-2i\cosh(x)+\sinh(x)}\,dx\right)\\
&= -\int \left(i\operatorname{sech}(x)-\frac{2i}{2\cosh(x)+i\sinh(x)}\right)dx\\
&= -(i\int \operatorname{sech}(x)\,dx)+2i\int \frac{1}{2\cosh(x)+i\sinh(x)}\,dx\\
&= -i\tan^{-1}(\sinh(x))-2\,\text{Subst}\left(\int \frac{1}{5-x^2}\,dx,x,\cosh(x)-2i\sinh(x)\right)\\
&= -i\tan^{-1}(\sinh(x))-\frac{2\tanh^{-1}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}
$$

Mathematica [A] time = 0.0514349, size = 38, normalized size = 1.23

$$
-\frac{4\tanh^{-1}\left(\frac{1-2i\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}-2i\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + 2*Coth[x]),x]
```

```
[Out] (-2*I)*ArcTan[Tanh[x/2]] - (4*ArcTanh[(1 - (2*I)*Tanh[x/2])/Sqrt[5]])/Sqrt[
5]
```

Maple [A] time = 0.052, size = 41, normalized size = 1.3

$$
-\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\frac{4i}{5}\sqrt{5}\arctan\left(\frac{\sqrt{5}}{5}\left(2\tanh\left(x/2\right)+i\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+2*coth(x)),x)
```

```
[Out] -ln(tanh(1/2*x)-I)+4/5*I*5^(1/2)*arctan(1/5*(2*tanh(1/2*x)+I)*5^(1/2))+ln(t
anh(1/2*x)+I)
```

Maxima [A] time = 1.70802, size = 57, normalized size = 1.84

$$
\frac{2}{5}\sqrt{5}\log\left(-\frac{2\sqrt{5}-(4i+2)\,e^{(-x)}}{2\sqrt{5}+(4i+2)\,e^{(-x)}}\right)+2i\arctan\left(e^{(-x)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*log(-(2*sqrt(5) - (4*I + 2)*e^(-x))/(2*sqrt(5) + (4*I + 2)*e^(-
x))) + 2*I*arctan(e^(-x))
```

Fricas [A] time = 2.79643, size = 169, normalized size = 5.45

$$-\frac{2}{5}\sqrt{5}\log\left(\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^{x}\right)+\frac{2}{5}\sqrt{5}\log\left(-\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^{x}\right)+\log\left(e^{x}+i\right)-\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(5)*log((2/5*I + 1/5)*sqrt(5) + e^x) + 2/5*sqrt(5)*log(-(2/5*I + 1
/5)*sqrt(5) + e^x) + log(e^x + I) - log(e^x - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{2\coth(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x)
```

```
[Out] Integral(sech(x)/(2*coth(x) + I), x)
```

Giac [A] time = 1.13408, size = 35, normalized size = 1.13

$$\frac{4}{5}i\sqrt{5}\arctan\left(\left(\frac{1}{5}i+\frac{2}{5}\right)\sqrt{5}e^{x}\right)+\log\left(e^{x}+i\right)-\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="giac")
```

```
[Out] 4/5*I*sqrt(5)*arctan((1/5*I + 2/5)*sqrt(5)*e^x) + log(e^x + I) - log(e^x -
I)
```

3.123 $\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=43

$$\frac{5x}{2}-\frac{5\tanh^3(x)}{6}+\tanh^2(x)-\frac{5\tanh(x)}{2}-2\log(\cosh(x))+\frac{\tanh^3(x)}{2(\coth(x)+1)}$$

```
[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Ta
nh[x]^3/(2*(1 + Coth[x]))
```

Rubi [A] time = 0.114614, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3552, 3529, 3531, 3475}

$$\frac{5x}{2}-\frac{5\tanh^3(x)}{6}+\tanh^2(x)-\frac{5\tanh(x)}{2}-2\log(\cosh(x))+\frac{\tanh^3(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Ta
nh[x]^3/(2*(1 + Coth[x]))
```

Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c
 - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Ta
n[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \frac{\tanh^3(x)}{2(1+\coth(x))} - \frac{1}{2}\int (-5+4\coth(x))\tanh^4(x)\,dx$$
$$= -\frac{5}{6}\tanh^3(x) + \frac{\tanh^3(x)}{2(1+\coth(x))} - \frac{1}{2}i\int (-4i+5i\coth(x))\tanh^3(x)\,dx$$
$$= \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1+\coth(x))} + \frac{1}{2}\int (5-4\coth(x))\tanh^2(x)\,dx$$
$$= -\frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1+\coth(x))} + \frac{1}{2}i\int (4i-5i\coth(x))\tanh(x)\,dx$$
$$= \frac{5x}{2} - \frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1+\coth(x))} - 2\int \tanh(x)\,dx$$
$$= \frac{5x}{2} - 2\log(\cosh(x)) - \frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1+\coth(x))}$$

Mathematica [A] time = 0.095612, size = 40, normalized size = 0.93

$$\frac{1}{12}\left(30x - 3\sinh(2x) + 3\cosh(2x) - 28\tanh(x) - 24\log(\cosh(x)) + (4\tanh(x)-6)\mathrm{sech}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (30*x + 3*Cosh[2*x] - 24*Log[Cosh[x]] - 3*Sinh[2*x] - 28*Tanh[x] + Sech[x]^
2*(-6 + 4*Tanh[x]))/12
```

Maple [B] time = 0.043, size = 96, normalized size = 2.2

$$\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{9}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - 4\,\frac{(\tanh(x/2))^5 - 1/2\,(\tanh(x/2))^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(1+coth(x)),x)
```

```
[Out] 1/(tanh(1/2*x)+1)^2-1/(tanh(1/2*x)+1)+9/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/2
*x)-1)-4*(tanh(1/2*x)^5-1/2*tanh(1/2*x)^4+8/3*tanh(1/2*x)^3-1/2*tanh(1/2*x)
^2+tanh(1/2*x))/(tanh(1/2*x)^2+1)^3-2*ln(tanh(1/2*x)^2+1)
```

Maxima [A] time = 1.70061, size = 74, normalized size = 1.72

$$\frac{1}{2}x - \frac{2\left(15e^{(-2x)} + 12e^{(-4x)} + 7\right)}{3\left(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1\right)} + \frac{1}{4}e^{(-2x)} - 2\log\left(e^{(-2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/3*(15*e^(-2*x) + 12*e^(-4*x) + 7)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-
6*x) + 1) + 1/4*e^(-2*x) - 2*log(e^(-2*x) + 1)
```

Fricas [B] time = 2.6535, size = 1897, normalized size = 44.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 + 3*(54*x +
 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 + 54*x + 17)*sinh(x)^6 + 18*(168*x*cosh
(x)^3 + (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420*x*
cosh(x)^4 + 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*cosh(
x)^5 + 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 + (54*x +
65)*cosh(x)^2 + (1512*x*cosh(x)^6 + 45*(54*x + 17)*cosh(x)^4 + 486*(2*x + 1
)*cosh(x)^2 + 54*x + 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^3 +
9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cosh(x)
^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh(x)^
7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) -
sinh(x))) + 2*(216*x*cosh(x)^7 + 9*(54*x + 17)*cosh(x)^5 + 162*(2*x + 1)*co
sh(x)^3 + (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^
7 + sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^
3 + 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cos
h(x)^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh
(x)^7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)**4/(coth(x) + 1), x)
```

Giac [A] time = 1.11034, size = 63, normalized size = 1.47

$$\frac{9}{2}\,x+\frac{\left(51\,e^{(6\,x)}+81\,e^{(4\,x)}+65\,e^{(2\,x)}+3\right)e^{(-2\,x)}}{12\left(e^{(2\,x)}+1\right)^{3}}-2\,\log\left(e^{(2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 9/2*x + 1/12*(51*e^(6*x) + 81*e^(4*x) + 65*e^(2*x) + 3)*e^(-2*x)/(e^(2*x) +
 1)^3 - 2*log(e^(2*x) + 1)
```

3.124 $\int \frac{\tanh^3(x)}{1+\coth(x)} \, dx$

Optimal. Leaf size=37

$$-\frac{3x}{2} - \tanh^2(x) + \frac{3\tanh(x)}{2} + 2\log(\cosh(x)) + \frac{\tanh^2(x)}{2(\coth(x)+1)}$$

```
[Out] (-3*x)/2 + 2*Log[Cosh[x]] + (3*Tanh[x])/2 - Tanh[x]^2 + Tanh[x]^2/(2*(1 + C
oth[x]))
```

Rubi [A] time = 0.0991369, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3552, 3529, 3531, 3475}

$$-\frac{3x}{2} - \tanh^2(x) + \frac{3\tanh(x)}{2} + 2\log(\cosh(x)) + \frac{\tanh^2(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(1 + Coth[x]),x]
```


```
[Out] (-3*x)/2 + 2*Log[Cosh[x]] + (3*Tanh[x])/2 - Tanh[x]^2 + Tanh[x]^2/(2*(1 + C
oth[x]))
```


Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c
 - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Ta
n[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx &= \frac{\tanh^2(x)}{2(1+\coth(x))} - \frac{1}{2}\int (-4+3\coth(x))\tanh^3(x)\,dx \\ &= -\tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))} - \frac{1}{2}i\int (-3i+4i\coth(x))\tanh^2(x)\,dx \\ &= \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))} + \frac{1}{2}\int (4-3\coth(x))\tanh(x)\,dx \\ &= -\frac{3x}{2} + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))} + 2\int \tanh(x)\,dx \\ &= -\frac{3x}{2} + 2\log(\cosh(x)) + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0532405, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(-6x + \sinh(2x) - \cosh(2x) + 4\tanh(x) + 2\text{sech}^2(x) + 8\log(\cosh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (-6*x - Cosh[2*x] + 8*Log[Cosh[x]] + 2*Sech[x]^2 + Sinh[2*x] + 4*Tanh[x])/4
```

Maple [B] time = 0.037, size = 80, normalized size = 2.2

$$-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{7}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + 2\,\frac{(\tanh(x/2))^3 - (\tanh(x/2)}{\left((\tanh(x/2))^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(1+coth(x)),x)
```

```
[Out] -1/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)-7/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/
2*x)-1)+2*(tanh(1/2*x)^3-tanh(1/2*x)^2+tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+2*l
n(tanh(1/2*x)^2+1)
```

Maxima [A] time = 1.54025, size = 58, normalized size = 1.57

$$\frac{1}{2}x + \frac{2\left(2e^{(-2x)}+1\right)}{2e^{(-2x)}+e^{(-4x)}+1} - \frac{1}{4}e^{(-2x)} + 2\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) + 1)/(2*e^(-2*x) + e^(-4*x) + 1) - 1/4*e^(-2*x) + 2*l
og(e^(-2*x) + 1)
```

Fricas [B] time = 2.64764, size = 1168, normalized size = 31.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 + (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 + 28*x + 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 + (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 +
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh
(x)^3 + 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 + (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh(x)^3 + 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)**3/(coth(x) + 1), x)
```

Giac [A] time = 1.11696, size = 53, normalized size = 1.43

$$-\frac{7}{2}\,x-\frac{\left(e^{(4\,x)}+10\,e^{(2\,x)}+1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}+1\right)^2}+2\,\log\left(e^{(2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -7/2*x - 1/4*(e^(4*x) + 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1)^2 + 2*log(e^
(2*x) + 1)
```

3.125 $\int \frac{\tanh^2(x)}{1+\coth(x)} \, dx$

Optimal. Leaf size=29

$$\frac{3x}{2} - \frac{3\tanh(x)}{2} - \log(\cosh(x)) + \frac{\tanh(x)}{2(\coth(x)+1)}$$

```
[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]/(2*(1 + Coth[x]))
```

Rubi [A] time = 0.0742931, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3552, 3529, 3531, 3475}

$$\frac{3x}{2} - \frac{3\tanh(x)}{2} - \log(\cosh(x)) + \frac{\tanh(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]/(2*(1 + Coth[x]))
```

Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c
 - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Ta
n[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx &= \frac{\tanh(x)}{2(1+\coth(x))}-\frac{1}{2}\int(-3+2\coth(x))\tanh^2(x)\,dx\\ &= -\frac{3\tanh(x)}{2}+\frac{\tanh(x)}{2(1+\coth(x))}-\frac{1}{2}i\int(-2i+3i\coth(x))\tanh(x)\,dx\\ &= \frac{3x}{2}-\frac{3\tanh(x)}{2}+\frac{\tanh(x)}{2(1+\coth(x))}-\int\tanh(x)\,dx\\ &= \frac{3x}{2}-\log(\cosh(x))-\frac{3\tanh(x)}{2}+\frac{\tanh(x)}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0456504, size = 27, normalized size = 0.93

$$\frac{1}{4}(6x-\sinh(2x)+\cosh(2x)-4\tanh(x)-4\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (6*x + Cosh[2*x] - 4*Log[Cosh[x]] - Sinh[2*x] - 4*Tanh[x])/4
```

Maple [B] time = 0.035, size = 65, normalized size = 2.2

$$\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{5}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-2\frac{\tanh(x/2)}{(\tanh(x/2))^2+1}-\ln\left(\left(\tanh\left(\frac{x}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(1+coth(x)),x)
```

```
[Out] 1/(tanh(1/2*x)+1)^2-1/(tanh(1/2*x)+1)+5/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/2
*x)-1)-2*tanh(1/2*x)/(tanh(1/2*x)^2+1)-ln(tanh(1/2*x)^2+1)
```

Maxima [A] time = 1.67494, size = 39, normalized size = 1.34

$$\frac{1}{2}x-\frac{2}{e^{(-2x)}+1}+\frac{1}{4}e^{(-2x)}-\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/(e^(-2*x) + 1) + 1/4*e^(-2*x) - log(e^(-2*x) + 1)
```

Fricas [B] time = 2.69727, size = 632, normalized size = 21.79

$$\frac{10x\cosh(x)^4+40x\cosh(x)\sinh(x)^3+10x\sinh(x)^4+(10x+9)\cosh(x)^2+\left(60x\cosh(x)^2+10x+9\right)\sinh(x)^2-4\,(}{4\left(\cosh(x)^4+4\cosh(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 + (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 + 10*x + 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*co
sh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 + (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + c
osh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)**2/(coth(x) + 1), x)
```

Giac [A] time = 1.14485, size = 47, normalized size = 1.62

$$\frac{5}{2}\,x + \frac{\left(9\,e^{(2\,x)} + 1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)} + 1\right)} - \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 5/2*x + 1/4*(9*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1) - log(e^(2*x) + 1)
```

3.126 $\int \frac{\tanh(x)}{1+\coth(x)} dx$

Optimal. Leaf size=19

$$-\frac{x}{2}+\frac{1}{2(\coth(x)+1)}+\log(\cosh(x))$$

```
[Out] -x/2 + 1/(2*(1 + Coth[x])) + Log[Cosh[x]]
```

Rubi [A] time = 0.0417203, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3551, 3479, 8, 3475}

$$-\frac{x}{2}+\frac{1}{2(\coth(x)+1)}+\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(1 + Coth[x]),x]
```

```
[Out] -x/2 + 1/(2*(1 + Coth[x])) + Log[Cosh[x]]
```

Rule 3551

```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Tan[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{1+\coth(x)}\,dx &= -\int \frac{1}{1+\coth(x)}\,dx+\int \tanh(x)\,dx\\
&= \frac{1}{2(1+\coth(x))}+\log(\cosh(x))-\frac{\int 1\,dx}{2}\\
&= -\frac{x}{2}+\frac{1}{2(1+\coth(x))}+\log(\cosh(x))
\end{aligned}$$

Mathematica [A] time = 0.0281422, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x+\sinh(2x)-\cosh(2x)+4\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(1 + Coth[x]),x]
```

```
[Out] (-2*x - Cosh[2*x] + 4*Log[Cosh[x]] + Sinh[2*x])/4
```

Maple [B] time = 0.029, size = 47, normalized size = 2.5

$$-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{3}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1+coth(x)),x)
```

```
[Out] -1/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)-3/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/
2*x)-1)+ln(tanh(1/2*x)^2+1)
```

Maxima [A] time = 1.67204, size = 23, normalized size = 1.21

$$\frac{1}{2}x-\frac{1}{4}e^{(-2x)}+\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*e^(-2*x) + log(e^(-2*x) + 1)
```

Fricas [B] time = 2.95861, size = 259, normalized size = 13.63

$$-\frac{6\,x\cosh(x)^{2}+12\,x\cosh(x)\sinh(x)+6\,x\sinh(x)^{2}-4\left(\cosh(x)^{2}+2\,\cosh(x)\sinh(x)+\sinh(x)^{2}\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+1}{4\left(\cosh(x)^{2}+2\,\cosh(x)\sinh(x)+\sinh(x)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)/(coth(x) + 1), x)
```

Giac [A] time = 1.15774, size = 23, normalized size = 1.21

$$-\frac{3}{2}x - \frac{1}{4}e^{(-2x)} + \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -3/2*x - 1/4*e^(-2*x) + log(e^(2*x) + 1)
```

3.127 $\int \frac{1}{1+\coth(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rubi [A] time = 0.0083708, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\coth(x)}\,dx &= -\frac{1}{2(1+\coth(x))}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0252688, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x-\sinh(2x)+\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-1),x]
```

```
[Out] (2*x + Cosh[2*x] - Sinh[2*x])/4
```

Maple [A] time = 0., size = 24, normalized size = 1.5

$$-\frac{1}{2+2\coth(x)}+\frac{\ln(1+\coth(x))}{4}-\frac{\ln(\coth(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x)),x)
```

```
[Out] -1/2/(1+coth(x))+1/4*ln(1+coth(x))-1/4*ln(coth(x)-1)
```

Maxima [A] time = 1.1258, size = 14, normalized size = 0.88

$$\frac{1}{2}x+\frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

Fricas [B] time = 2.76851, size = 88, normalized size = 5.5

$$\frac{(2x+1)\cosh(x)+(2x-1)\sinh(x)}{4(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.613514, size = 27, normalized size = 1.69

$$\frac{x\tanh(x)}{2\tanh(x)+2}+\frac{x}{2\tanh(x)+2}+\frac{1}{2\tanh(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Giac [A] time = 1.1395, size = 14, normalized size = 0.88

$$\frac{1}{2}x+\frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

3.128 $\int \frac{\coth(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}+\frac{1}{2(\coth(x)+1)}$$

[Out] x/2 + 1/(2*(1 + Coth[x]))

Rubi [A] time = 0.0215589, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3526, 8}

$$\frac{x}{2}+\frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/(1 + Coth[x]),x]

[Out] x/2 + 1/(2*(1 + Coth[x]))

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{1+\coth(x)}\,dx &= \frac{1}{2(1+\coth(x))}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0152798, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x+\sinh(2x)-\cosh(2x))$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]/(1 + Coth[x]),x]

[Out] (2*x - Cosh[2*x] + Sinh[2*x])/4

Maple [A] time = 0.018, size = 24, normalized size = 1.5

$$\frac{1}{2+2\coth\left(x\right)}+\frac{\ln\left(1+\coth\left(x\right)\right)}{4}-\frac{\ln\left(\coth\left(x\right)-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+coth(x)),x)
```

```
[Out] 1/2/(1+coth(x))+1/4*ln(1+coth(x))-1/4*ln(coth(x)-1)
```

Maxima [A] time = 1.13635, size = 14, normalized size = 0.88

$$\frac{1}{2}\,x-\frac{1}{4}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*e^(-2*x)
```

Fricas [B] time = 2.80484, size = 88, normalized size = 5.5

$$\frac{\left(2\,x-1\right)\cosh\left(x\right)+\left(2\,x+1\right)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*x - 1)*cosh(x) + (2*x + 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.59691, size = 27, normalized size = 1.69

$$\frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}-\frac{1}{2\tanh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 1/(2*tanh(x) + 2)
```

Giac [A] time = 1.11193, size = 14, normalized size = 0.88

$$\frac{1}{2}\,x-\frac{1}{4}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*e^(-2*x)
```

3.129 $\int \frac{\coth^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{x}{2}-\frac{1}{2(\coth(x)+1)}+\log(\sinh(x))$$

```
[Out] -x/2 - 1/(2*(1 + Coth[x])) + Log[Sinh[x]]
```

Rubi [A] time = 0.0382739, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3540, 3475}

$$-\frac{x}{2}-\frac{1}{2(\coth(x)+1)}+\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(1 + Coth[x]),x]

[Out] -x/2 - 1/(2*(1 + Coth[x])) + Log[Sinh[x]]
```

Rule 3540

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(a*c + b*d)^2*(a + b*Tan[e + f*x])^m)
/(2*a^3*f*m), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*
c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{1+\coth(x)}\,dx &= -\frac{1}{2(1+\coth(x))}-\frac{1}{2}\int (1-2\coth(x))\,dx\\
&= -\frac{x}{2}-\frac{1}{2(1+\coth(x))}+\int \coth(x)\,dx\\
&= -\frac{x}{2}-\frac{1}{2(1+\coth(x))}+\log(\sinh(x))
\end{aligned}$$

Mathematica [A] time = 0.0285523, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x-\sinh(2x)+\cosh(2x)+4\log(\sinh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(1 + Coth[x]),x]
```

[Out] (-2*x + Cosh[2*x] + 4*Log[Sinh[x]] - Sinh[2*x])/4

Maple [A] time = 0.017, size = 24, normalized size = 1.3

$$-\frac{1}{2+2\coth(x)}-\frac{3\ln(1+\coth(x))}{4}-\frac{\ln(\coth(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(1+coth(x)),x)

[Out] -1/2/(1+coth(x))-3/4*ln(1+coth(x))-1/4*ln(coth(x)-1)

Maxima [A] time = 1.10925, size = 32, normalized size = 1.68

$$\frac{1}{2}x+\frac{1}{4}e^{(-2x)}+\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="maxima")

[Out] 1/2*x + 1/4*e^(-2*x) + log(e^(-x) + 1) + log(e^(-x) - 1)

Fricas [B] time = 3.09236, size = 259, normalized size = 13.63

$$-\frac{6x\cosh(x)^2+12x\cosh(x)\sinh(x)+6x\sinh(x)^2-4\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right.}{4\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="fricas")

[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)

Sympy [B] time = 0.905729, size = 92, normalized size = 4.84

$$\frac{x\tanh(x)}{2\tanh(x)+2}+\frac{x}{2\tanh(x)+2}-\frac{2\log(\tanh(x)+1)\tanh(x)}{2\tanh(x)+2}-\frac{2\log(\tanh(x)+1)}{2\tanh(x)+2}+\frac{2\log(\tanh(x))\tanh(x)}{2\tanh(x)+2}+\frac{2\text{lo}}{2\text{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2/(1+coth(x)),x)

[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 2*log(tanh(x) + 1)*tanh(x)/(2*tanh(x) + 2) - 2*log(tanh(x) + 1)/(2*tanh(x) + 2) + 2*log(tanh(x))*tanh(

```
x)/(2*tanh(x) + 2) + 2*log(tanh(x))/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Giac [A] time = 1.12442, size = 24, normalized size = 1.26

$$-\frac{3}{2}x + \frac{1}{4}e^{(-2x)} + \log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -3/2*x + 1/4*e^(-2*x) + log(abs(e^(2*x) - 1))
```

3.130 $\int \frac{\coth^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=31

$$\frac{3x}{2}+\frac{\coth^2(x)}{2(\coth(x)+1)}-\frac{3\coth(x)}{2}-\log(\sinh(x))$$

```
[Out] (3*x)/2 - (3*Coth[x])/2 + Coth[x]^2/(2*(1 + Coth[x])) - Log[Sinh[x]]
```

Rubi [A] time = 0.0553428, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3550, 3525, 3475}

$$\frac{3x}{2}+\frac{\coth^2(x)}{2(\coth(x)+1)}-\frac{3\coth(x)}{2}-\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(1 + Coth[x]),x]
```

```
[Out] (3*x)/2 - (3*Coth[x])/2 + Coth[x]^2/(2*(1 + Coth[x])) - Log[Sinh[x]]
```

Rule 3550

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(c + d*Tan[e + f*x])^(n - 1))/
(2*a*f*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{1+\coth(x)}\,dx &= \frac{\coth^2(x)}{2(1+\coth(x))}-\frac{1}{2}\int (2-3\coth(x))\coth(x)\,dx\\
&= \frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{\coth^2(x)}{2(1+\coth(x))}-\int \coth(x)\,dx\\
&= \frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{\coth^2(x)}{2(1+\coth(x))}-\log(\sinh(x))
\end{aligned}$$

Mathematica [A] time = 0.0413891, size = 27, normalized size = 0.87

$$\frac{1}{4}(6x + \sinh(2x) - \cosh(2x) - 4\coth(x) - 4\log(\sinh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(1 + Coth[x]),x]
```

```
[Out] (6*x - Cosh[2*x] - 4*Coth[x] - 4*Log[Sinh[x]] + Sinh[2*x])/4
```

Maple [A] time = 0.017, size = 28, normalized size = 0.9

$$-\coth(x) + \frac{1}{2 + 2\coth(x)} + \frac{5\ln(1 + \coth(x))}{4} - \frac{\ln(\coth(x) - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(1+coth(x)),x)
```

```
[Out] -coth(x)+1/2/(1+coth(x))+5/4*ln(1+coth(x))-1/4*ln(coth(x)-1)
```

Maxima [A] time = 1.09329, size = 51, normalized size = 1.65

$$\frac{1}{2}x + \frac{2}{e^{(-2x)} - 1} - \frac{1}{4}e^{(-2x)} - \log\left(e^{(-x)} + 1\right) - \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2/(e^(-2*x) - 1) - 1/4*e^(-2*x) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] time = 3.01997, size = 632, normalized size = 20.39

$$\frac{10\,x\cosh(x)^4 + 40\,x\cosh(x)\sinh(x)^3 + 10\,x\sinh(x)^4 - (10\,x + 9)\cosh(x)^2 + \left(60\,x\cosh(x)^2 - 10\,x - 9\right)\sinh(x)^2 -}{4\left(\cosh(x)^4 + 4\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 - (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 - 10*x - 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*co
sh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 - (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - c
osh(x))*sinh(x))
```

Sympy [B] time = 1.3806, size = 162, normalized size = 5.23

$$\frac{x\tanh^2(x)}{2\tanh^2(x)+2\tanh(x)}+\frac{x\tanh(x)}{2\tanh^2(x)+2\tanh(x)}+\frac{2\log(\tanh(x)+1)\tanh^2(x)}{2\tanh^2(x)+2\tanh(x)}+\frac{2\log(\tanh(x)+1)\tanh(x)}{2\tanh^2(x)+2\tanh(x)}-\frac{2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(1+coth(x)),x)
```

```
[Out] x*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + x*tanh(x)/(2*tanh(x)**2 + 2*tanh(
x)) + 2*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + 2*log(tanh
(x) + 1)*tanh(x)/(2*tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/(2*
tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*tanh(x)**2 + 2*tanh(x))
 + 3*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) - 2/(2*tanh(x)**2 + 2*tanh(x))
```

Giac [A] time = 1.13294, size = 49, normalized size = 1.58

$$\frac{5}{2}\,x-\frac{\left(9\,e^{(2\,x)}-1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}-1\right)}-\log\left(\left|e^{(2\,x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 5/2*x - 1/4*(9*e^(2*x) - 1)*e^(-2*x)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

3.131 $\int \frac{\coth^4(x)}{1+\coth(x)} dx$

Optimal. Leaf size=37

$$-\frac{3x}{2} + \frac{\coth^3(x)}{2(\coth(x)+1)} - \coth^2(x) + \frac{3\coth(x)}{2} + 2\log(\sinh(x))$$

```
[Out] (-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + Coth[x]^3/(2*(1 + Coth[x])) + 2*Log[
Sinh[x]]
```

Rubi [A] time = 0.0677348, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3550, 3528, 3525, 3475}

$$-\frac{3x}{2} + \frac{\coth^3(x)}{2(\coth(x)+1)} - \coth^2(x) + \frac{3\coth(x)}{2} + 2\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + Coth[x]^3/(2*(1 + Coth[x])) + 2*Log[
Sinh[x]]
```

Rule 3550

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(c + d*Tan[e + f*x])^(n - 1))/
(2*a*f*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{\coth^3(x)}{2(1+\coth(x))} - \frac{1}{2}\int (3-4\coth(x))\coth^2(x)\,dx$$
$$= -\coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + \frac{1}{2}i\int (-4i+3i\coth(x))\coth(x)\,dx$$
$$= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + 2\int \coth(x)\,dx$$
$$= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + 2\log(\sinh(x))$$

Mathematica [A] time = 0.0575645, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(-6x - \sinh(2x) + \cosh(2x) + 4\coth(x) - 2\text{csch}^2(x) + 8\log(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(1 + Coth[x]),x]
```

```
[Out] (-6*x + Cosh[2*x] + 4*Coth[x] - 2*Csch[x]^2 + 8*Log[Sinh[x]] - Sinh[2*x])/4
```

Maple [A] time = 0.02, size = 32, normalized size = 0.9

$$-\frac{(\coth(x))^2}{2} + \coth(x) - \frac{1}{2+2\coth(x)} - \frac{7\ln(1+\coth(x))}{4} - \frac{\ln(\coth(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(1+coth(x)),x)
```

```
[Out] -1/2*coth(x)^2+coth(x)-1/2/(1+coth(x))-7/4*ln(1+coth(x))-1/4*ln(coth(x)-1)
```

Maxima [A] time = 1.11246, size = 73, normalized size = 1.97

$$\frac{1}{2}x + \frac{2\left(2e^{(-2x)}-1\right)}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{1}{4}e^{(-2x)} + 2\log\left(e^{(-x)}+1\right) + 2\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) - 1)/(2*e^(-2*x) - e^(-4*x) - 1) + 1/4*e^(-2*x) + 2*l
og(e^(-x) + 1) + 2*log(e^(-x) - 1)
```

Fricas [B] time = 3.04278, size = 1168, normalized size = 31.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 - (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 - 28*x - 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 - (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 -
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh
(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*sinh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 - (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) - 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [B] time = 1.85889, size = 197, normalized size = 5.32

$$\frac{x\tanh^{3}(x)}{2\tanh^{3}(x)+2\tanh^{2}(x)}+\frac{x\tanh^{2}(x)}{2\tanh^{3}(x)+2\tanh^{2}(x)}-\frac{4\log(\tanh(x)+1)\tanh^{3}(x)}{2\tanh^{3}(x)+2\tanh^{2}(x)}-\frac{4\log(\tanh(x)+1)\tanh^{2}(x)}{2\tanh^{3}(x)+2\tanh^{2}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(1+coth(x)),x)
```

```
[Out] x*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + x*tanh(x)**2/(2*tanh(x)**3 + 2
*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2)
- 4*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(
x))*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**2/(2
*tanh(x)**3 + 2*tanh(x)**2) + 3*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) +
tanh(x)/(2*tanh(x)**3 + 2*tanh(x)**2) - 1/(2*tanh(x)**3 + 2*tanh(x)**2)
```

Giac [A] time = 1.11409, size = 54, normalized size = 1.46

$$-\frac{7}{2}x+\frac{\left(e^{(4x)}-10e^{(2x)}+1\right)e^{(-2x)}}{4\left(e^{(2x)}-1\right)^{2}}+2\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -7/2*x + 1/4*(e^(4*x) - 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) - 1)^2 + 2*log(ab
s(e^(2*x) - 1))
```

3.132 $\int \coth(x)(1+\coth(x))^{3/2}\,dx$

Optimal. Leaf size=45

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rubi [A] time = 0.051792, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3527, 3478, 3480, 206}

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rule 3527

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \coth(x)(1+\coth(x))^{3/2}\,dx &= -\frac{2}{3}(1+\coth(x))^{3/2}+\int (1+\coth(x))^{3/2}\,dx\\ &= -2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+2\int \sqrt{1+\coth(x)}\,dx\\ &= -2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}\end{aligned}$$

Mathematica [C] time = 0.135816, size = 90, normalized size = 2.

$$-\frac{2(\coth(x)+1)^{3/2}\left(\cosh(x)\sqrt{i(\coth(x)+1)}+4\sinh(x)\sqrt{i(\coth(x)+1)}-(3-3i)\sinh(x)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)}\right.\right.}{3\sqrt{i(\coth(x)+1)}(\sinh(x)+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*(1 + Coth[x])^(3/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(3/2)*(Cosh[x]*Sqrt[I*(1 + Coth[x])] - (3 - 3*I)*ArcTan[(
1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*Sinh[x] + 4*Sqrt[I*(1 + Coth[x])]*Sinh[x]
))/(3*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x]))
```

Maple [A] time = 0.014, size = 35, normalized size = 0.8

$$-\frac{2}{3}(1+\coth(x))^{\frac{3}{2}}+2\,\text{Artanh}\left(1/2\sqrt{1+\coth(x)}\sqrt{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(1+coth(x))^(3/2),x)
```

```
[Out] -2/3*(1+coth(x))^(3/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x)+1)^{\frac{3}{2}}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(3/2)*coth(x), x)
```

Fricas [B] time = 3.052, size = 869, normalized size = 19.31

$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh(x)^3+15\sqrt{2}\cosh(x)\sinh(x)^2+5\sqrt{2}\sinh(x)^3+3\left(5\sqrt{2}\cosh(x)^2-\sqrt{2}\right)\sinh(x)-3\sqrt{2}\cosh(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x)*sinh(x)^2 + 5*sqr
t(2)*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - 3*sqrt(2)*cosh
(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x
)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + 1)^{\frac{3}{2}} \coth(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(3/2)*coth(x), x)
```

Giac [B] time = 1.18529, size = 182, normalized size = 4.04

$$-\frac{1}{3}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)} - e^{(2x)}} - 2e^{(2x)} + 1\right|\right)\operatorname{sgn}\left(e^{(2x)} - 1\right) + \frac{2\left(9\left(\sqrt{e^{(4x)} - e^{(2x)}} - e^{(2x)}\right)^2 \operatorname{sgn}\left(e^{(2x)} - 1\right) + 12\left(\sqrt{e^{(4x)} - e\right.\right.}{\left(\sqrt{e^{(4x)} - e^{(2x)}} - e^{(2\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 2*(9*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 1
2*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + 5*sgn(e^(2*x) - 1)
)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)
```

3.133 $\int \coth(x)\sqrt{1+\coth(x)}\,dx$

Optimal. Leaf size=32

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rubi [A] time = 0.0381682, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3527, 3480, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rule 3527

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \coth(x)\sqrt{1+\coth(x)}\,dx &= -2\sqrt{1+\coth(x)}+\int \sqrt{1+\coth(x)}\,dx\\ &= -2\sqrt{1+\coth(x)}+2\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}\end{aligned}$$

Mathematica [C] time = 0.119345, size = 53, normalized size = 1.66

$$(1+i)\sqrt{\coth(x)+1}\left(-\frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{\sqrt{i(\coth(x)+1)}}-(1-i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[1 + Coth[x]],x]
```

```
[Out] (1 + I)*Sqrt[1 + Coth[x]]*((-1 + I) - (I*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Cot
h[x])]])/Sqrt[I*(1 + Coth[x])])
```

Maple [A] time = 0.036, size = 26, normalized size = 0.8

$$\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(1+coth(x))^(1/2),x)
```

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x)+1}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*coth(x), x)
```

Fricas [B] time = 2.89116, size = 455, normalized size = 14.22

$$-\frac{4\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\log\left(2\right.}{2\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) -
 sinh(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x)+1}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1)*coth(x), x)
```

Giac [B] time = 1.1659, size = 96, normalized size = 3.

$$-\frac{1}{2}\sqrt{2}\left(\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{4\operatorname{sgn}\left(e^{(2x)}-1\right)}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*
x) - 1) + 4*sgn(e^(2*x) - 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1))
```

3.134 $\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal. Leaf size=30

$$\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Coth[x]]
```

Rubi [A] time = 0.0386308, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3526, 3480, 206}

$$\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[1 + Coth[x]],x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Coth[x]]
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx &= \frac{1}{\sqrt{1+\coth(x)}}+\frac{1}{2}\int \sqrt{1+\coth(x)}\,dx\\
&= \frac{1}{\sqrt{1+\coth(x)}}+\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{1+\coth(x)}}
\end{aligned}$$

Mathematica [C] time = 0.194822, size = 97, normalized size = 3.23

$$\frac{\operatorname{csch}(x)\left(\frac{1}{2}\sinh(2x)-\frac{1}{2}\cosh(2x)+\frac{1}{2}\right)(\sinh(x)+\cosh(x))}{\sqrt{\coth(x)+1}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\operatorname{csch}(x)(\sinh(x)+\cosh(x))\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i}\right.}{\sqrt{i\coth(x)+i}\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[1 + Coth[x]],x]
```

```
[Out] ((1/2 - I/2)*ArcTan[(1/2 + I/2)*Sqrt[I + I*Coth[x]]]*Csch[x]*(Cosh[x] + Sin
h[x]))/(Sqrt[I + I*Coth[x]]*Sqrt[1 + Coth[x]]) + (Csch[x]*(Cosh[x] + Sinh[x
])*(1/2 - Cosh[2*x]/2 + Sinh[2*x]/2))/Sqrt[1 + Coth[x]]
```

Maple [A] time = 0.039, size = 25, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)+\frac{1}{\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+coth(x))^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(coth(x) + 1), x)
```

Fricas [B] time = 2.99012, size = 300, normalized size = 10.

$$\frac{\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sin\right.}{4(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sin
h(x)^2 - 1) + 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(coth(x) + 1), x)
```

Giac [B] time = 1.16177, size = 119, normalized size = 3.97

$$-\frac{1}{2}\sqrt{2}\operatorname{sgn}\left(e^{(2x)}-1\right)-\frac{\sqrt{2}\log\left(\left|-2\sqrt{e^{(4x)}-e^{(2x)}}+2e^{(2x)}-1\right|\right)}{4\operatorname{sgn}\left(e^{(2x)}-1\right)}-\frac{\sqrt{2}}{2\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)\operatorname{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*sgn(e^(2*x) - 1) - 1/4*sqrt(2)*log(abs(-2*sqrt(e^(4*x) - e^(2*
x)) + 2*e^(2*x) - 1))/sgn(e^(2*x) - 1) - 1/2*sqrt(2)/((sqrt(e^(4*x) - e^(2*
x)) - e^(2*x))*sgn(e^(2*x) - 1))
```

3.135 $\int \frac{\coth(x)}{(1+\coth(x))^{3/2}} dx$

Optimal. Leaf size=49

$$-\frac{1}{2\sqrt{\coth(x)+1}}+\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rubi [A] time = 0.0508973, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3526, 3479, 3480, 206}

$$-\frac{1}{2\sqrt{\coth(x)+1}}+\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(1 + Coth[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{1}{3(1+\coth(x))^{3/2}} + \frac{1}{2}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx$$
$$= \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}} + \frac{1}{4}\int \sqrt{1+\coth(x)}\,dx$$
$$= \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}}$$

Mathematica [C] time = 0.33222, size = 84, normalized size = 1.71

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{\coth(x)+1}\left(\left(\frac{1}{6}-\frac{i}{6}\right)(-\sinh(2x)-\sinh(4x)+\cosh(2x)+\cosh(4x)-2)-\frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{\sqrt{i(\coth(x)+1)}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 + Coth[x])^(3/2),x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] + (1/6 - I/6)*(-2 + Cosh[2*x] + Cosh[4*x] - Sinh
[2*x] - Sinh[4*x]))
```

Maple [A] time = 0.013, size = 35, normalized size = 0.7

$$\frac{1}{3}\left(1+\coth(x)\right)^{-\frac{3}{2}} + \frac{\sqrt{2}}{4}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right) - \frac{1}{2}\frac{1}{\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+coth(x))^(3/2),x)
```

```
[Out] 1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/
2/(1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(coth(x) + 1)^(3/2), x)
```

Fricas [B] time = 2.95701, size = 578, normalized size = 11.8

$$-\frac{2\sqrt{2}\left(2\sqrt{2}\cosh(x)^2+4\sqrt{2}\cosh(x)\sinh(x)+2\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh\right.}{24\left(\cosh(x)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(2*sqrt(2)*cosh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x) + 2*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))**(3/2),x)
```

```
[Out] Integral(coth(x)/(coth(x) + 1)**(3/2), x)
```

Giac [B] time = 1.18417, size = 144, normalized size = 2.94

$$-\frac{1}{24}\sqrt{2}\left(\frac{3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)}{\operatorname{sgn}\left(e^{(2x)}-1\right)}+\frac{2\left(3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3\operatorname{sgn}\left(e^{(2x)}-1\right)}-4\operatorname{sgn}\left(e^{(2x)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))/sgn(e^
(2*x) - 1) + 2*(3*sqrt(e^(4*x) - e^(2*x)) - 3*e^(2*x) + 1)/((sqrt(e^(4*x) -
 e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1)) - 4*sgn(e^(2*x) - 1))
```

3.136 $\int \coth^2(x)(1+\coth(x))^{3/2}\,dx$

Optimal. Leaf size=45

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(5/2))/5
```

Rubi [A] time = 0.0628274, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3543, 3478, 3480, 206}

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(5/2))/5
```

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \coth^2(x)(1+\coth(x))^{3/2}\,dx &= -\frac{2}{5}(1+\coth(x))^{5/2}+\int (1+\coth(x))^{3/2}\,dx\\
&= -2\sqrt{1+\coth(x)}-\frac{2}{5}(1+\coth(x))^{5/2}+2\int \sqrt{1+\coth(x)}\,dx\\
&= -2\sqrt{1+\coth(x)}-\frac{2}{5}(1+\coth(x))^{5/2}+4\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\
&= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}-\frac{2}{5}(1+\coth(x))^{5/2}
\end{aligned}
$$

Mathematica [C] time = 0.253652, size = 70, normalized size = 1.56

$$
-\frac{2\left(2\coth^2(x)+\operatorname{csch}^2(x)+(5+5i)\sqrt{i(\coth(x)+1)}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)+\coth(x)\left(\operatorname{csch}^2(x)+9\right)+7\right)}{5\sqrt{\coth(x)+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

```
[Out] (-2*(7 + 2*Coth[x]^2 + (5 + 5*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*
Sqrt[I*(1 + Coth[x])] + Csch[x]^2 + Coth[x]*(9 + Csch[x]^2)))/(5*Sqrt[1 + C
oth[x]])
```

Maple [A] time = 0.019, size = 35, normalized size = 0.8

$$
-\frac{2}{5}\left(1+\coth(x)\right)^{\frac{5}{2}}+2\,\mathrm{Artanh}\left(1/2\sqrt{1+\coth(x)}\sqrt{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(1+coth(x))^(3/2),x)
```

```
[Out] -2/5*(1+coth(x))^(5/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int (\coth(x)+1)^{\frac{3}{2}}\coth(x)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(3/2)*coth(x)^2, x)
```

Fricas [B] time = 2.95329, size = 1449, normalized size = 32.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*sqrt(2)*(9*sqrt(2)*cosh(x)^5 + 45*sqrt(2)*cosh(x)*sinh(x)^4 + 9*sqr
t(2)*sinh(x)^5 + 10*(9*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^3 - 10*sqrt(2)*
cosh(x)^3 + 30*(3*sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x)^2 + 5*(9*sqr
t(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 5*sqrt(2)*cosh(x)
)*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 5*(sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh
(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(
x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*si
nh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2
 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*cosh(x)^3 + sqrt(
2)*cosh(x))*sinh(x) - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x
)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1
))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(
x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4
 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 +
cosh(x))*sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x)+1)^{\frac{3}{2}}\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(3/2)*coth(x)**2, x)
```

Giac [B] time = 1.20042, size = 266, normalized size = 5.91

$$-\frac{1}{5}\sqrt{2}\left(5\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\mathrm{sgn}\left(e^{(2x)}-1\right)+\frac{2\left(25\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4\mathrm{sgn}\left(e^{(2x)}-1\right)+60\left(\sqrt{e^{(4x)}-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(2)*(5*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 2*(25*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4*sgn(e^(2*x) - 1) +
60*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1) + 70*(sqrt(e^(4*x
) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 40*(sqrt(e^(4*x) - e^(2*x)) -
e^(2*x))*sgn(e^(2*x) - 1) + 9*sgn(e^(2*x) - 1))/(sqrt(e^(4*x) - e^(2*x)) -
e^(2*x) + 1)^5)
```

3.137 $\int \coth^2(x)\sqrt{1+\coth(x)}\,dx$

Optimal. Leaf size=34

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}$$

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*(1 + Coth[x])^(3/2))/3
```

Rubi [A] time = 0.0478325, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3543, 3480, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*(1 + Coth[x])^(3/2))/3
```

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \coth^2(x)\sqrt{1+\coth(x)}\,dx &= -\frac{2}{3}(1+\coth(x))^{3/2}+\int \sqrt{1+\coth(x)}\,dx\\ &= -\frac{2}{3}(1+\coth(x))^{3/2}+2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\coth(x))^{3/2}\end{aligned}$$

Mathematica [C] time = 0.168769, size = 61, normalized size = 1.79

$$\frac{-2\coth^2(x)-4\coth(x)-(3+3i)\sqrt{i(\coth(x)+1)}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)-2}{3\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Sqrt[1 + Coth[x]],x]
```

```
[Out] (-2 - 4*Coth[x] - 2*Coth[x]^2 - (3 + 3*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Co
th[x])]]*Sqrt[I*(1 + Coth[x])])/(3*Sqrt[1 + Coth[x]])
```

Maple [A] time = 0.039, size = 26, normalized size = 0.8

$$-\frac{2}{3}\left(1+\coth\left(x\right)\right)^{\frac{3}{2}}+\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth\left(x\right)}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(1+coth(x))^(1/2),x)
```

```
[Out] -2/3*(1+coth(x))^(3/2)+arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth\left(x\right)+1}\,\coth\left(x\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*coth(x)^2, x)
```

Fricas [B] time = 3.16877, size = 817, normalized size = 24.03

$$-\frac{8\sqrt{2}\left(\sqrt{2}\cosh\left(x\right)^{3}+3\sqrt{2}\cosh\left(x\right)^{2}\sinh\left(x\right)+3\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{2}+\sqrt{2}\sinh\left(x\right)^{3}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh\left(x\right)\right.}{6(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(8*sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2
)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*sqrt(sinh(x)/(cosh(x) - sinh(x)))
- 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt
(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh
(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh
(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) +
 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x)+1}\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1)*coth(x)**2, x)
```

Giac [B] time = 1.18143, size = 180, normalized size = 5.29

$$-\frac{1}{6}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{8\left(3\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2\operatorname{sgn}\left(e^{(2x)}-1\right)+3\left(\sqrt{e^{(4x)}-\cdots}\right.\right.}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 8*(3*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 3
*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + sgn(e^(2*x) - 1))/(
sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)
```

3.138 $\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal. Leaf size=42

$$-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]] - 2*Sqrt[1
 + Coth[x]]
```

Rubi [A] time = 0.058539, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3543, 3479, 3480, 206}

$$-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/Sqrt[1 + Coth[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]] - 2*Sqrt[1
 + Coth[x]]
```

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx &= -2\sqrt{1+\coth(x)}+\int \frac{1}{\sqrt{1+\coth(x)}}\,dx\\ &= -\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}+\frac{1}{2}\int \sqrt{1+\coth(x)}\,dx\\ &= -\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}+\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}\end{aligned}$$

Mathematica [C] time = 0.359309, size = 81, normalized size = 1.93

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\operatorname{csch}(x)(\sinh(x)+\cosh(x))\left(\left(\frac{1}{2}-\frac{i}{2}\right)(-\sinh(2x)+\cosh(2x)-5)-\frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{\sqrt{i(\coth(x)+1)}}\right)}{\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/Sqrt[1 + Coth[x]],x]
```

```
[Out] ((1/2 + I/2)*Csch[x]*(Cosh[x] + Sinh[x])*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(
1 + Coth[x])]])/Sqrt[I*(1 + Coth[x])] + (1/2 - I/2)*(-5 + Cosh[2*x] - Sinh[
2*x])))/Sqrt[1 + Coth[x]]
```

Maple [A] time = 0.041, size = 35, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1+coth(x))^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)-2*(1
+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/sqrt(coth(x) + 1), x)
```

Fricas [B] time = 3.11865, size = 637, normalized size = 15.17

$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh\left(x\right)^{2}+10\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+5\sqrt{2}\sinh\left(x\right)^{2}-\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh\left(x\right)^{3}+3\sqrt{2}\cosh\left(x\right)\ldots}{4\left(\cosh\left(x\right)^{3}+3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^2 + 10*sqrt(2)*cosh(x)*sinh(x) + 5*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - (sqrt(2)*cosh(x
)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^
2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x)
 - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh
(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1
)*sinh(x) - cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^{2}\left(x\right)}{\sqrt{\coth\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(1+coth(x))**(1/2),x)
```

```
[Out] Integral(coth(x)**2/sqrt(coth(x) + 1), x)
```

Giac [B] time = 1.20583, size = 119, normalized size = 2.83

$$-\frac{\frac{5\sqrt{2}e^{(2x)}}{\operatorname{sgn}\left(e^{(2x)}-1\right)}-\frac{\sqrt{2}}{\operatorname{sgn}\left(e^{(2x)}-1\right)}}{2\sqrt{e^{(4x)}-e^{(2x)}}}-\frac{\sqrt{2}\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)}{4\operatorname{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(5*sqrt(2)*e^(2*x)/sgn(e^(2*x) - 1) - sqrt(2)/sgn(e^(2*x) - 1))/sqrt(e
^(4*x) - e^(2*x)) - 1/4*sqrt(2)*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*
x) + 1))/sgn(e^(2*x) - 1)
```

3.139 $\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{3}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
+ 3/(2*Sqrt[1 + Coth[x]])
```

Rubi [A] time = 0.0821066, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3540, 3526, 3480, 206}

$$\frac{3}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(1 + Coth[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
+ 3/(2*Sqrt[1 + Coth[x]])
```

Rule 3540

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(a*c + b*d)^2*(a + b*Tan[e + f*x])^m)
/(2*a^3*f*m), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*
c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = -\frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2}\int \frac{1-2\coth(x)}{\sqrt{1+\coth(x)}}\,dx$$
$$= -\frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}} + \frac{1}{4}\int \sqrt{1+\coth(x)}\,dx$$
$$= -\frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}}$$

Mathematica [C] time = 0.349202, size = 86, normalized size = 1.76

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{\coth(x)+1}\left(-\left(\frac{1}{6}-\frac{i}{6}\right)(-7\sinh(2x)-\sinh(4x)+7\cosh(2x)+\cosh(4x)-8)-\frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)\cdots}\right.}{\sqrt{i(\coth(x)+1)}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(1 + Coth[x])^(3/2),x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] - (1/6 - I/6)*(-8 + 7*Cosh[2*x] + Cosh[4*x] - 7*
Sinh[2*x] - Sinh[4*x]))
```

Maple [A] time = 0.017, size = 35, normalized size = 0.7

$$-\frac{1}{3}(1+\coth(x))^{-\frac{3}{2}} + \frac{\sqrt{2}}{4}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right) + \frac{3}{2}\frac{1}{\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1+coth(x))^(3/2),x)
```

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+3
/2/(1+coth(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(coth(x) + 1)^(3/2), x)
```

Fricas [B] time = 2.92866, size = 578, normalized size = 11.8

$$\frac{2\sqrt{2}\left(8\sqrt{2}\cosh(x)^2+16\sqrt{2}\cosh(x)\sinh(x)+8\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}+3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh\right.}{24\left(\cosh(x)^3+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(2)*(8*sqrt(2)*cosh(x)^2 + 16*sqrt(2)*cosh(x)*sinh(x) + 8*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(1+coth(x))**(3/2),x)
```

```
[Out] Integral(coth(x)**2/(coth(x) + 1)**(3/2), x)
```

Giac [B] time = 1.20938, size = 182, normalized size = 3.71

$$-\frac{2}{3}\sqrt{2}\operatorname{sgn}\left(e^{(2x)}-1\right)-\frac{\sqrt{2}\log\left(\left|-2\sqrt{e^{(4x)}-e^{(2x)}}+2e^{(2x)}-1\right|\right)}{8\operatorname{sgn}\left(e^{(2x)}-1\right)}-\frac{\sqrt{2}\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2-3\sqrt{e^{(4x)}-e^{(2x)}}+3\right.}{12\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3\operatorname{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(2)*sgn(e^(2*x) - 1) - 1/8*sqrt(2)*log(abs(-2*sqrt(e^(4*x) - e^(2*
x)) + 2*e^(2*x) - 1))/sgn(e^(2*x) - 1) - 1/12*sqrt(2)*(6*(sqrt(e^(4*x) - e^
(2*x)) - e^(2*x))^2 - 3*sqrt(e^(4*x) - e^(2*x)) + 3*e^(2*x) - 1)/((sqrt(e^(
4*x) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1))
```

3.140 $\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=97

$$\frac{ax}{a^2-b^2}-\frac{(a^2+b^2)\tanh(x)}{a^3}-\frac{b(a^2+b^2)\log(\cosh(x))}{a^4}-\frac{b^5\log(a\sinh(x)+b\cosh(x))}{a^4(a^2-b^2)}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*(a^2 + b^2)*Log[Cosh[x]])/a^4 - (b^5*Log[b*Cosh[x] +
 a*Sinh[x]])/(a^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/
(2*a^2) - Tanh[x]^3/(3*a)
```

Rubi [A] time = 0.525196, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3569, 3649, 3650, 3651, 3530, 3475}

$$\frac{ax}{a^2-b^2}-\frac{(a^2+b^2)\tanh(x)}{a^3}-\frac{b(a^2+b^2)\log(\cosh(x))}{a^4}-\frac{b^5\log(a\sinh(x)+b\cosh(x))}{a^4(a^2-b^2)}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*(a^2 + b^2)*Log[Cosh[x]])/a^4 - (b^5*Log[b*Cosh[x] +
 a*Sinh[x]])/(a^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/
(2*a^2) - Tanh[x]^3/(3*a)
```

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n
+ 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = -\frac{\tanh^3(x)}{3a} - \frac{i\int \frac{\left(-3ib+3ia\coth(x)+3ib\coth^2(x)\right)\tanh^3(x)}{a+b\coth(x)}\,dx}{3a}$$

$$= \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} - \frac{\int \frac{\left(-6\left(a^2+b^2\right)+6b^2\coth^2(x)\right)\tanh^2(x)}{a+b\coth(x)}\,dx}{6a^2}$$

$$= -\frac{\left(a^2+b^2\right)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} + \frac{i\int \frac{\left(6ib\left(a^2+b^2\right)-6ia^3\coth(x)-6ib\left(a^2+b^2\right)\coth^2(x)\right)\tanh(x}{a+b\coth(x)}}{6a^3}$$

$$= \frac{ax}{a^2-b^2} - \frac{\left(a^2+b^2\right)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} - \frac{\left(ib^5\right)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^4\left(a^2-b^2\right)} - \frac{\left(b\left(a^2+b^2\right)\right.}{}$$

$$= \frac{ax}{a^2-b^2} - \frac{b\left(a^2+b^2\right)\log(\cosh(x))}{a^4} - \frac{b^5\log(b\cosh(x)+a\sinh(x))}{a^4\left(a^2-b^2\right)} - \frac{\left(a^2+b^2\right)\tanh(x)}{a^3} + \frac{b\,\text{ta}}{2}$$

Mathematica [A] time = 0.327879, size = 105, normalized size = 1.08

$$\frac{\left(2a^3b^2-8a^5+6ab^4\right)\tanh(x)+6\left(b^5-a^4b\right)\log(\cosh(x))+a^2\left(a^2-b^2\right)\operatorname{sech}^2(x)(2a\tanh(x)-3b)+6a^5x-6b^5\log(a}{6a^4(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (6*a^5*x + 6*(-(a^4*b) + b^5)*Log[Cosh[x]] - 6*b^5*Log[b*Cosh[x] + a*Sinh[x
]] + (-8*a^5 + 2*a^3*b^2 + 6*a*b^4)*Tanh[x] + a^2*(a^2 - b^2)*Sech[x]^2*(-3
*b + 2*a*Tanh[x]))/(6*a^4*(a - b)*(a + b))
```

Maple [B] time = 0.056, size = 283, normalized size = 2.9

$$64\,\frac{\ln(\tanh(x/2)+1)}{64\,a-64\,b}-64\,\frac{\ln(\tanh(x/2)-1)}{64\,a+64\,b}-2\,\frac{(\tanh(x/2))^5}{a\left((\tanh(x/2))^2+1\right)^3}-2\,\frac{(\tanh(x/2))^5\,b^2}{a^3\left((\tanh(x/2))^2+1\right)^3}+2\,\frac{b\,(\tanh(x/2}{a^2\left((\tanh(x/2))\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*coth(x)),x)
```

```
[Out] 64/(64*a-64*b)*ln(tanh(1/2*x)+1)-64/(64*a+64*b)*ln(tanh(1/2*x)-1)-2/a/(tanh
(1/2*x)^2+1)^3*tanh(1/2*x)^5-2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*b^2+2/
a^2/(tanh(1/2*x)^2+1)^3*b*tanh(1/2*x)^4-20/3/a/(tanh(1/2*x)^2+1)^3*tanh(1/2
*x)^3-4/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^2+2/a^2/(tanh(1/2*x)^2+1)^3
*b*tanh(1/2*x)^2-2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)-2/a^3/(tanh(1/2*x)^2+1
)^3*tanh(1/2*x)*b^2-1/a^2*b*ln(tanh(1/2*x)^2+1)-1/a^4*ln(tanh(1/2*x)^2+1)*b
^3-b^5/(a+b)/(a-b)/a^4*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Maxima [A] time = 1.75174, size = 197, normalized size = 2.03

$$-\frac{b^5\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^6-a^4b^2}-\frac{2\left(4\,a^2+3\,b^2+3\left(2\,a^2+ab+2\,b^2\right)e^{(-2\,x)}+3\left(2\,a^2+ab+b^2\right)e^{(-4\,x)}\right)}{3\left(3\,a^3e^{(-2\,x)}+3\,a^3e^{(-4\,x)}+a^3e^{(-6\,x)}+a^3\right)}+\frac{x}{a+b}-\frac{\left(a^2b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - a^4*b^2) - 2/3*(4*a^2 + 3*b^2 +
3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(2*a^2 + a*b + b^2)*e^(-4*x))/(3*a^3*e
^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) + x/(a + b) - (a^2*b + b^3)*
log(e^(-2*x) + 1)/a^4
```

Fricas [B] time = 3.18172, size = 3087, normalized size = 31.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 + a^4*b)*x*cosh(x)^6 + 18*(a^5 + a^4*b)*x*cosh(x)*sinh(x)^5 + 3
*(a^5 + a^4*b)*x*sinh(x)^6 + 8*a^5 - 2*a^3*b^2 - 6*a*b^4 + 3*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^4 + 3*(4*
a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 15*(a^5 + a^4*b)*x*cosh(x
)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^4 + 12*(5*(a^5 + a^4*b)*x*cosh(x)^3 + (4*a
```

```
^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)
)*sinh(x)^3 + 3*(4*a^5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)
*cosh(x)^2 + 3*(15*(a^5 + a^4*b)*x*cosh(x)^4 + 4*a^5 - 2*a^4*b + 2*a^2*b^3
- 4*a*b^4 + 6*(4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 +
 a^4*b)*x)*cosh(x)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^2 + 3*(a^5 + a^4*b)*x - 3
*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4
 + 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cosh(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*c
osh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2
+ b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x
))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^4*b - b^5)*co
sh(x)^6 + 6*(a^4*b - b^5)*cosh(x)*sinh(x)^5 + (a^4*b - b^5)*sinh(x)^6 + a^4
*b - b^5 + 3*(a^4*b - b^5)*cosh(x)^4 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - b^5)*cosh(x)^3 + 3*(a^4*b - b^5)*cosh(x))
*sinh(x)^3 + 3*(a^4*b - b^5)*cosh(x)^2 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*c
osh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b - b^5)*cosh(x)^
5 + 2*(a^4*b - b^5)*cosh(x)^3 + (a^4*b - b^5)*cosh(x))*sinh(x))*log(2*cosh(
x)/(cosh(x) - sinh(x))) + 6*(3*(a^5 + a^4*b)*x*cosh(x)^5 + 2*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^3 + (4*a^
5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x))*sinh(x))/((
a^6 - a^4*b^2)*cosh(x)^6 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4
*b^2)*sinh(x)^6 + a^6 - a^4*b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^
4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(
x)^3 + 3*(a^6 - a^4*b^2)*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 +
 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)
^2)*sinh(x)^2 + 6*((a^6 - a^4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3
+ (a^6 - a^4*b^2)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*coth(x)), x)
```

Giac [A] time = 1.13076, size = 190, normalized size = 1.96

$$-\frac{b^5\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^6-a^4b^2}+\frac{x}{a-b}-\frac{\left(a^2b+b^3\right)\log\left(e^{(2x)}+1\right)}{a^4}+\frac{2\left(4a^3+3ab^2+3\left(2a^3-a^2b+ab^2\right)e^{(4x)}+3\right.}{3a^4\left(e^{(2x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^6 - a^4*b^2) + x/(a - b) -
(a^2*b + b^3)*log(e^(2*x) + 1)/a^4 + 2/3*(4*a^3 + 3*a*b^2 + 3*(2*a^3 - a^2*
b + a*b^2)*e^(4*x) + 3*(2*a^3 - a^2*b + 2*a*b^2)*e^(2*x))/(a^4*(e^(2*x) + 1
)^3)
```

3.141 $\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=76

$$-\frac{bx}{a^2-b^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3}+\frac{b^4\log(a\sinh(x)+b\cosh(x))}{a^3(a^2-b^2)}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}$$

```
[Out] -((b*x)/(a^2 - b^2)) + ((a^2 + b^2)*Log[Cosh[x]])/a^3 + (b^4*Log[b*Cosh[x]
+ a*Sinh[x]])/(a^3*(a^2 - b^2)) + (b*Tanh[x])/a^2 - Tanh[x]^2/(2*a)
```

Rubi [A] time = 0.325755, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3569, 3649, 3652, 3530, 3475}

$$-\frac{bx}{a^2-b^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3}+\frac{b^4\log(a\sinh(x)+b\cosh(x))}{a^3(a^2-b^2)}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Coth[x]),x]
```


```
[Out] -((b*x)/(a^2 - b^2)) + ((a^2 + b^2)*Log[Cosh[x]])/a^3 + (b^4*Log[b*Cosh[x]
+ a*Sinh[x]])/(a^3*(a^2 - b^2)) + (b*Tanh[x])/a^2 - Tanh[x]^2/(2*a)
```


Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3652

```
Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[((a*
(A*c - c*C) - b*(A*d - C*d))*x)/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^
```

```
2 + a^2*C)/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
 + f*x]), x], x] - Dist[(c^2*C + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d -
 c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx &= -\frac{\tanh^2(x)}{2a}-\frac{i\int \frac{\left(-2ib+2ia\coth(x)+2ib\coth^2(x)\right)\tanh^2(x)}{a+b\coth(x)}\,dx}{2a}\\
&= \frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}-\frac{\int \frac{\left(-2\left(a^2+b^2\right)+2b^2\coth^2(x)\right)\tanh(x)}{a+b\coth(x)}\,dx}{2a^2}\\
&= -\frac{bx}{a^2-b^2}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}+\frac{\left(ib^4\right)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^3\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\int \tanh(x)\,dx}{a^3}\\
&= -\frac{bx}{a^2-b^2}+\frac{\left(a^2+b^2\right)\log(\cosh(x))}{a^3}+\frac{b^4\log(b\cosh(x)+a\sinh(x))}{a^3\left(a^2-b^2\right)}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.303121, size = 88, normalized size = 1.16

$$\frac{a^2\left(a^2-b^2\right)\operatorname{sech}^2(x)+2\left(ab\left(a^2-b^2\right)\tanh(x)+\left(a^4-b^4\right)\log(\cosh(x))-a^3bx+b^4\log(a\sinh(x)+b\cosh(x))\right)}{2a^3(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a^2*(a^2 - b^2)*Sech[x]^2 + 2*(-(a^3*b*x) + (a^4 - b^4)*Log[Cosh[x]] + b^4
*Log[b*Cosh[x] + a*Sinh[x]] + a*b*(a^2 - b^2)*Tanh[x]))/(2*a^3*(a - b)*(a +
 b))
```

Maple [B] time = 0.053, size = 167, normalized size = 2.2

$$-32\,\frac{\ln(\tanh(x/2)+1)}{32\,a-32\,b}-32\,\frac{\ln(\tanh(x/2)-1)}{32\,a+32\,b}+2\,\frac{(\tanh(x/2))^3\,b}{a^2\left((\tanh(x/2))^2+1\right)^2}-2\,\frac{(\tanh(x/2))^2}{a\left((\tanh(x/2))^2+1\right)^2}+2\,\frac{\tanh}{a^2\left((\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*coth(x)),x)
```

```
[Out] -32/(32*a-32*b)*ln(tanh(1/2*x)+1)-32/(32*a+32*b)*ln(tanh(1/2*x)-1)+2/a^2/(t
anh(1/2*x)^2+1)^2*tanh(1/2*x)^3*b-2/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2+2/a
^2/(tanh(1/2*x)^2+1)^2*b*tanh(1/2*x)+1/a*ln(tanh(1/2*x)^2+1)+1/a^3*ln(tanh(
1/2*x)^2+1)*b^2+b^4/(a+b)/(a-b)/a^3*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Maxima [A] time = 1.73305, size = 127, normalized size = 1.67

$$\frac{b^4 \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^5 - a^3b^2} + \frac{2\left((a+b)e^{(-2x)} + b\right)}{2\,a^2e^{(-2x)} + a^2e^{(-4x)} + a^2} + \frac{x}{a+b} + \frac{\left(a^2+b^2\right)\log\left(e^{(-2x)} + 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-(a - b)*e^(-2*x) + a + b)/(a^5 - a^3*b^2) + 2*((a + b)*e^(-2*x) +
b)/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a^2) + x/(a + b) + (a^2 + b^2)*log(e^(-
2*x) + 1)/a^3
```

Fricas [B] time = 3.24519, size = 1539, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + a^3*b)*x*cosh(x)^4 + 4*(a^4 + a^3*b)*x*cosh(x)*sinh(x)^3 + (a^4 +
a^3*b)*x*sinh(x)^4 + 2*a^3*b - 2*a*b^3 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 -
 (a^4 + a^3*b)*x)*cosh(x)^2 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 - 3*(a^4 + a
^3*b)*x*cosh(x)^2 - (a^4 + a^3*b)*x)*sinh(x)^2 + (a^4 + a^3*b)*x - (b^4*cos
h(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b^4 +
2*(3*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - ((a^4 - b^4)*cosh(
x)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4
+ 2*(a^4 - b^4)*cosh(x)^2 + 2*(a^4 - b^4 + 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)
^2 + 4*((a^4 - b^4)*cosh(x)^3 + (a^4 - b^4)*cosh(x))*sinh(x))*log(2*cosh(x)
/(cosh(x) - sinh(x))) + 4*((a^4 + a^3*b)*x*cosh(x)^3 - (a^4 - a^3*b - a^2*b
^2 + a*b^3 - (a^4 + a^3*b)*x)*cosh(x))*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3
*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sin
h(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh
(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*coth(x)), x)
```

Giac [A] time = 1.17562, size = 131, normalized size = 1.72

$$\frac{b^4 \log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^5 - a^3b^2} - \frac{x}{a - b} + \frac{\left(a^2 + b^2\right)\log\left(e^{(2x)} + 1\right)}{a^3} - \frac{2\left(ab - \left(a^2 - ab\right)e^{(2x)}\right)}{a^3\left(e^{(2x)} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^5 - a^3*b^2) - x/(a - b) + (
a^2 + b^2)*log(e^(2*x) + 1)/a^3 - 2*(a*b - (a^2 - a*b)*e^(2*x))/(a^3*(e^(2*
x) + 1)^2)
```

3.142 $\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=60

$$\frac{ax}{a^2-b^2}-\frac{b^3\log(a\sinh(x)+b\cosh(x))}{a^2\left(a^2-b^2\right)}-\frac{b\log(\cosh(x))}{a^2}-\frac{\tanh(x)}{a}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/a^2 - (b^3*Log[b*Cosh[x] + a*Sinh[x]])
/(a^2*(a^2 - b^2)) - Tanh[x]/a
```

Rubi [A] time = 0.193089, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3569, 3651, 3530, 3475}

$$\frac{ax}{a^2-b^2}-\frac{b^3\log(a\sinh(x)+b\cosh(x))}{a^2\left(a^2-b^2\right)}-\frac{b\log(\cosh(x))}{a^2}-\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/a^2 - (b^3*Log[b*Cosh[x] + a*Sinh[x]])
/(a^2*(a^2 - b^2)) - Tanh[x]/a
```

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx &= -\frac{\tanh(x)}{a}-\frac{i\int \frac{\left(-ib+ia\coth(x)+ib\coth^2(x)\right)\tanh(x)}{a+b\coth(x)}\,dx}{a}\\ &= \frac{ax}{a^2-b^2}-\frac{\tanh(x)}{a}-\frac{b\int \tanh(x)\,dx}{a^2}-\frac{\left(ib^3\right)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2\left(a^2-b^2\right)}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(\cosh(x))}{a^2}-\frac{b^3\log(b\cosh(x)+a\sinh(x))}{a^2\left(a^2-b^2\right)}-\frac{\tanh(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.131647, size = 64, normalized size = 1.07

$$\frac{\left(ab^2-a^3\right)\tanh(x)+\left(b^3-a^2b\right)\log(\cosh(x))+a^3x-b^3\log(a\sinh(x)+b\cosh(x))}{a^4-a^2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a^3*x + (-(a^2*b) + b^3)*Log[Cosh[x]] - b^3*Log[b*Cosh[x] + a*Sinh[x]] + (
-a^3 + a*b^2)*Tanh[x])/(a^4 - a^2*b^2)
```

Maple [A] time = 0.046, size = 110, normalized size = 1.8

$$16\,\frac{\ln(\tanh(x/2)+1)}{16\,a-16\,b}-16\,\frac{\ln(\tanh(x/2)-1)}{16\,a+16\,b}-2\,\frac{\tanh(x/2)}{a\left((\tanh(x/2))^2+1\right)}-\frac{b}{a^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)-\frac{b^3}{(a+b)\,(a-b)\,a^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*coth(x)),x)
```

```
[Out] 16/(16*a-16*b)*ln(tanh(1/2*x)+1)-16/(16*a+16*b)*ln(tanh(1/2*x)-1)-2/a*tanh(
1/2*x)/(tanh(1/2*x)^2+1)-1/a^2*b*ln(tanh(1/2*x)^2+1)-b^3/(a+b)/(a-b)/a^2*ln
(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Maxima [A] time = 1.78454, size = 90, normalized size = 1.5

$$-\frac{b^3\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^4-a^2b^2}+\frac{x}{a+b}-\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2}-\frac{2}{ae^{(-2\,x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - a^2*b^2) + x/(a + b) - b*log(e^(
-2*x) + 1)/a^2 - 2/(a*e^(-2*x) + a)
```

Fricas [B] time = 2.98597, size = 660, normalized size = 11.

$$\frac{\left(a^3+a^2b\right)x\cosh\left(x\right)^2+2\left(a^3+a^2b\right)x\cosh\left(x\right)\sinh\left(x\right)+\left(a^3+a^2b\right)x\sinh\left(x\right)^2+2a^3-2ab^2+\left(a^3+a^2b\right)x-\left(b^3\cosh\left(x\right.\right.}{a^4-a^2b^2+\left(a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a^2*b)*x*cosh(x)^2 + 2*(a^3 + a^2*b)*x*cosh(x)*sinh(x) + (a^3 + a^2
*b)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2 + (a^3 + a^2*b)*x - (b^3*cosh(x)^2 + 2*b^
3*cosh(x)*sinh(x) + b^3*sinh(x)^2 + b^3)*log(2*(b*cosh(x) + a*sinh(x))/(cos
h(x) - sinh(x))) - (a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b - b^3)
*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x
))))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*cosh(x)
*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh^2\left(x\right)}{a+b\coth\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*coth(x)), x)
```

Giac [A] time = 1.13413, size = 100, normalized size = 1.67

$$-\frac{b^3\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^4-a^2b^2}+\frac{x}{a-b}-\frac{b\log\left(e^{(2x)}+1\right)}{a^2}+\frac{2}{a\left(e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^4 - a^2*b^2) + x/(a - b) -
b*log(e^(2*x) + 1)/a^2 + 2/(a*(e^(2*x) + 1))
```

3.143 $\int \frac{\tanh(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{bx}{a^2-b^2}+\frac{b^2\log(a\sinh(x)+b\cosh(x))}{a\left(a^2-b^2\right)}+\frac{\log(\cosh(x))}{a}$$

```
[Out] -((b*x)/(a^2 - b^2)) + Log[Cosh[x]]/a + (b^2*Log[b*Cosh[x] + a*Sinh[x]])/(a
*(a^2 - b^2))
```

Rubi [A] time = 0.0824597, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3571, 3530, 3475}

$$-\frac{bx}{a^2-b^2}+\frac{b^2\log(a\sinh(x)+b\cosh(x))}{a\left(a^2-b^2\right)}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + Log[Cosh[x]]/a + (b^2*Log[b*Cosh[x] + a*Sinh[x]])/(a
*(a^2 - b^2))
```

Rule 3571

```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[((a*c - b*d)*x)/((a^2 + b^2)*(c^2 + d^2)), x
] + (Dist[b^2/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Ta
n[e + f*x]), x], x] - Dist[d^2/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e
+ f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{a+b\coth(x)}\,dx &= -\frac{bx}{a^2-b^2}+\frac{\int \tanh(x)\,dx}{a}+\frac{\left(ib^2\right)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a\left(a^2-b^2\right)}\\ &= -\frac{bx}{a^2-b^2}+\frac{\log(\cosh(x))}{a}+\frac{b^2\log(b\cosh(x)+a\sinh(x))}{a\left(a^2-b^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0755132, size = 46, normalized size = 0.9

$$\frac{\left(a^2-b^2\right)\log(\cosh(x))+b(b\log(a\sinh(x)+b\cosh(x))-ax)}{a^3-ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Coth[x]),x]
```

```
[Out] ((a^2 - b^2)*Log[Cosh[x]] + b*(-(a*x) + b*Log[b*Cosh[x] + a*Sinh[x]]))/(a^3
 - a*b^2)
```

Maple [A] time = 0.039, size = 88, normalized size = 1.7

$$-8\,\frac{\ln\left(\tanh\left(x/2\right)+1\right)}{8\,a-8\,b}-8\,\frac{\ln\left(\tanh\left(x/2\right)-1\right)}{8\,a+8\,b}+\frac{1}{a}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)+\frac{b^2}{(a+b)\,(a-b)\,a}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2b+2\,a\tanh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*coth(x)),x)
```

```
[Out] -8/(8*a-8*b)*ln(tanh(1/2*x)+1)-8/(8*a+8*b)*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/
2*x)^2+1)+b^2/(a+b)/(a-b)/a*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Maxima [A] time = 1.74219, size = 68, normalized size = 1.33

$$\frac{b^2\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^3-ab^2}+\frac{x}{a+b}+\frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] b^2*log(-(a - b)*e^(-2*x) + a + b)/(a^3 - a*b^2) + x/(a + b) + log(e^(-2*x)
 + 1)/a
```

Fricas [A] time = 2.81707, size = 185, normalized size = 3.63

$$\frac{b^2\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\left(a^2+ab\right)x+\left(a^2-b^2\right)\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^3-ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] (b^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a^2 + a*b)*x + (
a^2 - b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*coth(x)), x)
```

Giac [A] time = 1.19928, size = 77, normalized size = 1.51

$$\frac{b^2\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^3-ab^2}-\frac{x}{a-b}+\frac{\log\left(e^{(2x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 - a*b^2) - x/(a - b) + log
(e^(2*x) + 1)/a
```

3.144 $\int \frac{1}{a+b\coth(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)

Rubi [A] time = 0.0458942, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3484, 3530}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Coth[x])^(-1),x]

[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\coth(x)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.0526497, size = 29, normalized size = 0.74

$$\frac{ax-b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Coth[x])^(-1),x]

[Out] (a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)

Maple [A] time = 0.013, size = 55, normalized size = 1.4

$$\frac{\ln(1+\coth(x))}{2a-2b}-\frac{\ln(\coth(x)-1)}{2b+2a}-\frac{b\ln(a+b\coth(x))}{(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(x)),x)
```

```
[Out] 1/(2*a-2*b)*ln(1+coth(x))-1/(2*b+2*a)*ln(coth(x)-1)-b/(a-b)/(a+b)*ln(a+b*co
th(x))
```

Maxima [A] time = 1.26918, size = 50, normalized size = 1.28

$$-\frac{b\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A] time = 2.53858, size = 108, normalized size = 2.77

$$\frac{(a+b)x-b\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [A] time = 1.22743, size = 148, normalized size = 3.79

$$\begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)) & \text{for } a=0 \wedge b=0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b}+\frac{x}{2b\tanh(x)-2b}-\frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}+\frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{x-\log(\tanh(x)+1)}{b} & \text{for } a=0 \\ \frac{ax}{a^2-b^2}-\frac{bx}{a^2-b^2}+\frac{b\log(\tanh(x)+1)}{a^2-b^2}-\frac{b\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1)), Eq(a, 0) & Eq(b, 0)), (-x*tanh(x)/(2
*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b
)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x)
 + 2*b), Eq(a, b)), ((x - log(tanh(x) + 1))/b, Eq(a, 0)), (a*x/(a**2 - b**2
) - b*x/(a**2 - b**2) + b*log(tanh(x) + 1)/(a**2 - b**2) - b*log(tanh(x) +
b/a)/(a**2 - b**2), True))
```

Giac [A] time = 1.12986, size = 58, normalized size = 1.49

$$-\frac{b \log\left(\left|ae^{(2\,x)} + be^{(2\,x)} - a + b\right|\right)}{a^2 - b^2} + \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) + x/(a - b)
```

3.145 $\int \frac{\coth(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=39

$$\frac{a\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0584835, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3531, 3530}

$$\frac{a\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+b\coth(x)}\,dx &= -\frac{bx}{a^2-b^2}+\frac{(ia)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\ &= -\frac{bx}{a^2-b^2}+\frac{a\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.0449134, size = 29, normalized size = 0.74

$$\frac{a\log(a\sinh(x)+b\cosh(x))-bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Coth[x]),x]
```

```
[Out] (-(b*x) + a*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Maple [A] time = 0.017, size = 55, normalized size = 1.4

$$-\frac{\ln(1+\coth(x))}{2a-2b}-\frac{\ln(\coth(x)-1)}{2b+2a}+\frac{a\ln(a+b\coth(x))}{(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*coth(x)),x)
```

```
[Out] -1/(2*a-2*b)*ln(1+coth(x))-1/(2*b+2*a)*ln(coth(x)-1)+a/(a+b)/(a-b)*ln(a+b*c
oth(x))
```

Maxima [A] time = 1.23757, size = 49, normalized size = 1.26

$$\frac{a\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] a*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A] time = 2.57295, size = 109, normalized size = 2.79

$$-\frac{(a+b)x-a\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((a + b)*x - a*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [A] time = 1.34243, size = 134, normalized size = 3.44

$$\begin{cases} \tilde{\infty}x & \text{for } a=0\wedge b=0 \\ \frac{x\tanh(x)}{2b\tanh(x)-2b}-\frac{x}{2b\tanh(x)-2b}-\frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}-\frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{x}{b} & \text{for } a=0 \\ \frac{ax}{a^2-b^2}-\frac{a\log(\tanh(x)+1)}{a^2-b^2}+\frac{a\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2-b^2}-\frac{bx}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*tanh(x)/(2*b*tanh(x) - 2*b) - x/
(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*ta
nh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (x
/b, Eq(a, 0)), (a*x/(a**2 - b**2) - a*log(tanh(x) + 1)/(a**2 - b**2) + a*lo
g(tanh(x) + b/a)/(a**2 - b**2) - b*x/(a**2 - b**2), True))
```

Giac [A] time = 1.12691, size = 58, normalized size = 1.49

$$\frac{a \log \left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2 - b^2} - \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) - x/(a - b)
```

3.146 $\int \frac{\coth^2(x)}{a+b\coth(x)} dx$

Optimal. Leaf size=63

$$\frac{a^3x}{b^2(a^2-b^2)}-\frac{a^2\log(a\sinh(x)+b\cosh(x))}{b(a^2-b^2)}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

```
[Out] -((a*x)/b^2) + (a^3*x)/(b^2*(a^2 - b^2)) + Log[Sinh[x]]/b - (a^2*Log[b*Cosh
[x] + a*Sinh[x]])/(b*(a^2 - b^2))
```

Rubi [A] time = 0.0920795, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3541, 3475, 3484, 3530}

$$\frac{a^3x}{b^2(a^2-b^2)}-\frac{a^2\log(a\sinh(x)+b\cosh(x))}{b(a^2-b^2)}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -((a*x)/b^2) + (a^3*x)/(b^2*(a^2 - b^2)) + Log[Sinh[x]]/b - (a^2*Log[b*Cosh
[x] + a*Sinh[x]])/(b*(a^2 - b^2))
```

Rule 3541

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(d*(2*b*c - a*d)*x)/b^2, x] + (Dist[d^2/b, In
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = -\frac{ax}{b^2} + \frac{a^2\int \frac{1}{a+b\coth(x)}\,dx}{b^2} + \frac{\int \coth(x)\,dx}{b}$$
$$= -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\sinh(x))}{b} - \frac{(ia^2)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{b(a^2-b^2)}$$
$$= -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\sinh(x))}{b} - \frac{a^2\log(b\cosh(x)+a\sinh(x))}{b(a^2-b^2)}$$

Mathematica [A] time = 0.0760584, size = 49, normalized size = 0.78

$$\frac{-a^2\log(a\sinh(x)+b\cosh(x)) + a^2\log(\sinh(x)) + abx - b^2\log(\sinh(x))}{a^2b-b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a*b*x + a^2*Log[Sinh[x]] - b^2*Log[Sinh[x]] - a^2*Log[b*Cosh[x] + a*Sinh[x
]])/(a^2*b - b^3)
```

Maple [A] time = 0.019, size = 60, normalized size = 1.

$${\frac{\ln(1+\coth(x))}{2\,a-2\,b}}-{\frac{\ln(\coth(x)-1)}{2\,b+2\,a}}-{\frac{a^2\ln(a+b\coth(x))}{(a+b)(a-b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*coth(x)),x)
```

```
[Out] 1/(2*a-2*b)*ln(1+coth(x))-1/(2*b+2*a)*ln(coth(x)-1)-a^2/(a+b)/(a-b)/b*ln(a+
b*coth(x))
```

Maxima [A] time = 1.27629, size = 85, normalized size = 1.35

$$-\frac{a^2\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b-b^3} + \frac{x}{a+b} + \frac{\log\left(e^{(-x)}+1\right)}{b} + \frac{\log\left(e^{(-x)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^2*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b - b^3) + x/(a + b) + log(e^(-x)
+ 1)/b + log(e^(-x) - 1)/b
```

Fricas [A] time = 2.80317, size = 186, normalized size = 2.95

$$-\frac{a^2\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right) - (ab+b^2)x - (a^2-b^2)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2b-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -(a^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a*b + b^2)*x -
(a^2 - b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
```

Sympy [A] time = 2.94032, size = 372, normalized size = 5.9

$$\begin{cases} \tilde{\infty}\left(x-\log\left(\tanh\left(x\right)+1\right)+\log\left(\tanh\left(x\right)\right)\right) \\ \frac{3x\tanh\left(x\right)}{2b\tanh\left(x\right)-2b}-\frac{3x}{2b\tanh\left(x\right)-2b}-\frac{2\log\left(\tanh\left(x\right)+1\right)\tanh\left(x\right)}{2b\tanh\left(x\right)-2b}+\frac{2\log\left(\tanh\left(x\right)+1\right)}{2b\tanh\left(x\right)-2b}+\frac{2\log\left(\tanh\left(x\right)\right)\tanh\left(x\right)}{2b\tanh\left(x\right)-2b}-\frac{2\log\left(\tanh\left(x\right)\right)}{2b\tanh\left(x\right)-2b}-\frac{1}{2b\tanh\left(x\right)-2b} \\ \frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)+2b}+\frac{x}{2b\tanh\left(x\right)+2b}-\frac{2\log\left(\tanh\left(x\right)+1\right)\tanh\left(x\right)}{2b\tanh\left(x\right)+2b}-\frac{2\log\left(\tanh\left(x\right)+1\right)}{2b\tanh\left(x\right)+2b}+\frac{2\log\left(\tanh\left(x\right)\right)\tanh\left(x\right)}{2b\tanh\left(x\right)+2b}+\frac{2\log\left(\tanh\left(x\right)\right)}{2b\tanh\left(x\right)+2b}+\frac{1}{2b\tanh\left(x\right)+2b} \\ \frac{x-\frac{1}{\tanh\left(x\right)}}{a} \\ \frac{x-\log\left(\tanh\left(x\right)+1\right)+\log\left(\tanh\left(x\right)\right)}{b} \\ -\frac{a^{2}\log\left(\tanh\left(x\right)+\frac{b}{a}\right)}{a^{2}b-b^{3}}+\frac{a^{2}\log\left(\tanh\left(x\right)\right)}{a^{2}b-b^{3}}+\frac{abx}{a^{2}b-b^{3}}-\frac{b^{2}x}{a^{2}b-b^{3}}+\frac{b^{2}\log\left(\tanh\left(x\right)+1\right)}{a^{2}b-b^{3}}-\frac{b^{2}\log\left(\tanh\left(x\right)\right)}{a^{2}b-b^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 (3*x*tanh(x)/(2*b*tanh(x) - 2*b) - 3*x/(2*b*tanh(x) - 2*b) - 2*log(tanh(x)
 + 1)*tanh(x)/(2*b*tanh(x) - 2*b) + 2*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b)
+ 2*log(tanh(x))*tanh(x)/(2*b*tanh(x) - 2*b) - 2*log(tanh(x))/(2*b*tanh(x)
- 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b)
+ x/(2*b*tanh(x) + 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) + 2*b) -
2*log(tanh(x) + 1)/(2*b*tanh(x) + 2*b) + 2*log(tanh(x))*tanh(x)/(2*b*tanh(x
) + 2*b) + 2*log(tanh(x))/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a
, b)), ((x - 1/tanh(x))/a, Eq(b, 0)), ((x - log(tanh(x) + 1) + log(tanh(x))
)/b, Eq(a, 0)), (-a**2*log(tanh(x) + b/a)/(a**2*b - b**3) + a**2*log(tanh(x
))/(a**2*b - b**3) + a*b*x/(a**2*b - b**3) - b**2*x/(a**2*b - b**3) + b**2*
log(tanh(x) + 1)/(a**2*b - b**3) - b**2*log(tanh(x))/(a**2*b - b**3), True)
)
```

Giac [A] time = 1.14789, size = 80, normalized size = 1.27

$$-\frac{a^{2}\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^{2}b-b^{3}}+\frac{x}{a-b}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b - b^3) + x/(a - b) + lo
g(abs(e^(2*x) - 1))/b
```

3.147 $\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=64

$$-\frac{bx}{a^2-b^2}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^3\log(a+b\coth(x))}{b^2\left(a^2-b^2\right)}-\frac{\coth(x)}{b}$$

```
[Out] -((b*x)/(a^2 - b^2)) - Coth[x]/b + (a^3*Log[a + b*Coth[x]])/(b^2*(a^2 - b^2
)) + (a*Log[Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.129963, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3566, 3626, 3617, 31, 3475}

$$-\frac{bx}{a^2-b^2}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^3\log(a+b\coth(x))}{b^2\left(a^2-b^2\right)}-\frac{\coth(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) - Coth[x]/b + (a^3*Log[a + b*Coth[x]])/(b^2*(a^2 - b^2
)) + (a*Log[Sinh[x]])/(a^2 - b^2)
```

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx &= -\frac{\coth(x)}{b}-\frac{\int \frac{-a-b\coth(x)+a\coth^2(x)}{a+b\coth(x)}\,dx}{b}\\
&= -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a\int \coth(x)\,dx}{a^2-b^2}+\frac{a^3\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b\left(a^2-b^2\right)}\\
&= -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\coth(x)\right)}{b^2\left(a^2-b^2\right)}\\
&= -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a^3\log(a+b\coth(x))}{b^2\left(a^2-b^2\right)}+\frac{a\log(\sinh(x))}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.119717, size = 64, normalized size = 1.

$$\frac{b\left(a^2-b^2\right)\coth(x)+a\left(a^2-b^2\right)\log(\sinh(x))+a^3(-\log(a\sinh(x)+b\cosh(x)))+b^3x}{b^2(b-a)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (b^3*x + b*(a^2 - b^2)*Coth[x] + a*(a^2 - b^2)*Log[Sinh[x]] - a^3*Log[b*Cos
h[x] + a*Sinh[x]])/(b^2*(-a + b)*(a + b))
```

Maple [A] time = 0.022, size = 67, normalized size = 1.1

$$-\frac{\coth(x)}{b}-\frac{\ln(1+\coth(x))}{2a-2b}-\frac{\ln(\coth(x)-1)}{2b+2a}+\frac{a^3\ln(a+b\coth(x))}{b^2(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*coth(x)),x)
```

```
[Out] -coth(x)/b-1/(2*a-2*b)*ln(1+coth(x))-1/(2*b+2*a)*ln(coth(x)-1)+1/b^2*a^3/(a
+b)/(a-b)*ln(a+b*coth(x))
```

Maxima [A] time = 1.17138, size = 111, normalized size = 1.73

$$\frac{a^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^2-b^4}+\frac{x}{a+b}-\frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{2}{be^{(-2x)}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] a^3*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^2 - b^4) + x/(a + b) - a*log(e^(-
x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2/(b*e^(-2*x) - b)
```

Fricas [B] time = 2.83034, size = 660, normalized size = 10.31

$$\frac{\left(ab^2+b^3\right)x\cosh\left(x\right)^2+2\left(ab^2+b^3\right)x\cosh\left(x\right)\sinh\left(x\right)+\left(ab^2+b^3\right)x\sinh\left(x\right)^2+2\,a^2b-2\,b^3-\left(ab^2+b^3\right)x-\left(a^3\cosh\right.}{a^2b^2-b^4-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a*b^2 + b^3)*x*cosh(x)^2 + 2*(a*b^2 + b^3)*x*cosh(x)*sinh(x) + (a*b^2 + b
^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3 - (a*b^2 + b^3)*x - (a^3*cosh(x)^2 + 2*a^
3*cosh(x)*sinh(x) + a^3*sinh(x)^2 - a^3)*log(2*(b*cosh(x) + a*sinh(x))/(cos
h(x) - sinh(x))) - (a^3 - a*b^2 - (a^3 - a*b^2)*cosh(x)^2 - 2*(a^3 - a*b^2)
*cosh(x)*sinh(x) - (a^3 - a*b^2)*sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x
))))/(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cosh(x)^2 - 2*(a^2*b^2 - b^4)*cosh(x)
*sinh(x) - (a^2*b^2 - b^4)*sinh(x)^2)
```

Sympy [A] time = 5.44979, size = 639, normalized size = 9.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - 1/tanh(x)), Eq(a, 0) & Eq(b, 0)), (5*x*tanh(x)**2/(2*b*
tanh(x)**2 - 2*b*tanh(x)) - 5*x*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 2*
log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) + 2*log(tanh(x)
+ 1)*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) + 2*log(tanh(x))*tanh(x)**2/(2*
b*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*b*tanh(x)**2 - 2*b*
tanh(x)) - 3*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) + 2/(2*b*tanh(x)**2
- 2*b*tanh(x)), Eq(a, -b)), (x*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) +
x*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)**2/(2
*b*tanh(x)**2 + 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x)**2 +
 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) -
2*log(tanh(x))*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 3*tanh(x)**2/(2*b*t
anh(x)**2 + 2*b*tanh(x)) - 2/(2*b*tanh(x)**2 + 2*b*tanh(x)), Eq(a, b)), ((x
 - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2))/a, Eq(b, 0)), ((x -
1/tanh(x))/b, Eq(a, 0)), (a**3*log(tanh(x) + b/a)*tanh(x)/(a**2*b**2*tanh(x
) - b**4*tanh(x)) - a**3*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b**4*tan
h(x)) - a**2*b/(a**2*b**2*tanh(x) - b**4*tanh(x)) + a*b**2*x*tanh(x)/(a**2*
b**2*tanh(x) - b**4*tanh(x)) - a*b**2*log(tanh(x) + 1)*tanh(x)/(a**2*b**2*t
anh(x) - b**4*tanh(x)) + a*b**2*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b
**4*tanh(x)) - b**3*x*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) + b**3/(a*
*2*b**2*tanh(x) - b**4*tanh(x)), True))
```

Giac [A] time = 1.11489, size = 103, normalized size = 1.61

$$\frac{a^3 \log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2b^2 - b^4} - \frac{x}{a - b} - \frac{a \log\left(\left|e^{(2x)} - 1\right|\right)}{b^2} - \frac{2}{b\left(e^{(2x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^2 - b^4) - x/(a - b) - a
*log(abs(e^(2*x) - 1))/b^2 - 2/(b*(e^(2*x) - 1))
```

3.148 $\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=76

$$\frac{ax}{a^2-b^2}-\frac{b\log(\sinh(x))}{a^2-b^2}-\frac{a^4\log(a+b\coth(x))}{b^3\left(a^2-b^2\right)}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

```
[Out] (a*x)/(a^2 - b^2) + (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - (a^4*Log[a + b*Coth
[x]])/(b^3*(a^2 - b^2)) - (b*Log[Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.221255, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3566, 3647, 3627, 3617, 31, 3475}

$$\frac{ax}{a^2-b^2}-\frac{b\log(\sinh(x))}{a^2-b^2}-\frac{a^4\log(a+b\coth(x))}{b^3\left(a^2-b^2\right)}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) + (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - (a^4*Log[a + b*Coth
[x]])/(b^3*(a^2 - b^2)) - (b*Log[Sinh[x]])/(a^2 - b^2)
```

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3627

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*(A - C)*x)/(a^2 + b^2), x] + (Dist[(a^2*C +
A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] -
 Dist[(b*(A - C))/(a^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e,
```

```
f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx &= -\frac{\coth^2(x)}{2b}-\frac{\int \frac{\coth(x)\left(-2a-2b\coth(x)+2a\coth^2(x)\right)}{a+b\coth(x)}\,dx}{2b}\\
&= \frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{\int \frac{2a^2-2\left(a^2+b^2\right)\coth^2(x)}{a+b\coth(x)}\,dx}{2b^2}\\
&= \frac{ax}{a^2-b^2}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{a^4\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b^2\left(a^2-b^2\right)}-\frac{b\int \coth(x)\,dx}{a^2-b^2}\\
&= \frac{ax}{a^2-b^2}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{b\log(\sinh(x))}{a^2-b^2}-\frac{a^4\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\coth(x)\right)}{b^3\left(a^2-b^2\right)}\\
&= \frac{ax}{a^2-b^2}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{a^4\log(a+b\coth(x))}{b^3\left(a^2-b^2\right)}-\frac{b\log(\sinh(x))}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.2014, size = 88, normalized size = 1.16

$$\frac{2ab\left(a^2-b^2\right)\coth(x)+\left(b^4-a^2b^2\right)\operatorname{csch}^2(x)+2\left(a^4-b^4\right)\log(\sinh(x))-2a^4\log(a\sinh(x)+b\cosh(x))+2ab^3x}{2b^3(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b^3*x + 2*a*b*(a^2 - b^2)*Coth[x] + (-(a^2*b^2) + b^4)*Csch[x]^2 + 2*(
a^4 - b^4)*Log[Sinh[x]] - 2*a^4*Log[b*Cosh[x] + a*Sinh[x]])/(2*(a - b)*b^3*
(a + b))
```

Maple [A] time = 0.021, size = 76, normalized size = 1.

$$-\frac{(\coth(x))^2}{2b}+\frac{a\coth(x)}{b^2}+\frac{\ln(1+\coth(x))}{2a-2b}-\frac{\ln(\coth(x)-1)}{2b+2a}-\frac{a^4\ln(a+b\coth(x))}{b^3(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*coth(x)),x)
```

```
[Out] -1/2*coth(x)^2/b+a*coth(x)/b^2+1/(2*a-2*b)*ln(1+coth(x))-1/(2*b+2*a)*ln(cot
h(x)-1)-1/b^3*a^4/(a+b)/(a-b)*ln(a+b*coth(x))
```

Maxima [A] time = 1.19526, size = 161, normalized size = 2.12

$$-\frac{a^4 \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^2b^3 - b^5} + \frac{2\left((a+b)e^{(-2x)} - a\right)}{2\,b^2e^{(-2x)} - b^2e^{(-4x)} - b^2} + \frac{x}{a+b} + \frac{\left(a^2+b^2\right)\log\left(e^{(-x)}+1\right)}{b^3} + \frac{\left(a^2+b^2\right)\log\left(e^{(-x)} -\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^4*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^3 - b^5) + 2*((a + b)*e^(-2*x) -
 a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) + x/(a + b) + (a^2 + b^2)*log(e^(
-x) + 1)/b^3 + (a^2 + b^2)*log(e^(-x) - 1)/b^3
```

Fricas [B] time = 2.93003, size = 1538, normalized size = 20.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a*b^3 + b^4)*x*cosh(x)^4 + 4*(a*b^3 + b^4)*x*cosh(x)*sinh(x)^3 + (a*b^3 +
 b^4)*x*sinh(x)^4 - 2*a^3*b + 2*a*b^3 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 -
(a*b^3 + b^4)*x)*cosh(x)^2 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 + 3*(a*b^3 +
b^4)*x*cosh(x)^2 - (a*b^3 + b^4)*x)*sinh(x)^2 + (a*b^3 + b^4)*x - (a^4*cosh
(x)^4 + 4*a^4*cosh(x)*sinh(x)^3 + a^4*sinh(x)^4 - 2*a^4*cosh(x)^2 + a^4 + 2
*(3*a^4*cosh(x)^2 - a^4)*sinh(x)^2 + 4*(a^4*cosh(x)^3 - a^4*cosh(x))*sinh(x
))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + ((a^4 - b^4)*cosh(x
)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4 -
 2*(a^4 - b^4)*cosh(x)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)^
2 + 4*((a^4 - b^4)*cosh(x)^3 - (a^4 - b^4)*cosh(x))*sinh(x))*log(2*sinh(x)/
(cosh(x) - sinh(x))) + 4*((a*b^3 + b^4)*x*cosh(x)^3 + (a^3*b - a^2*b^2 - a*
b^3 + b^4 - (a*b^3 + b^4)*x)*cosh(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 -
b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh
(x)^4 - 2*(a^2*b^3 - b^5)*cosh(x)^2 - 2*(a^2*b^3 - b^5 - 3*(a^2*b^3 - b^5)*
cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3 - (a^2*b^3 - b^5)*cosh(
x))*sinh(x))
```

Sympy [A] time = 7.99349, size = 882, normalized size = 11.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2)), Eq
(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2
))/b, Eq(a, 0)), (7*x*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 7*x*ta
nh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/
(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh
(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*
tanh(x)**2) - 4*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) -
 3*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 -
 2*b*tanh(x)**2) + 1/(2*b*tanh(x)**3 - 2*b*tanh(x)**2), Eq(a, -b)), (x*tanh
(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + x*tanh(x)**2/(2*b*tanh(x)**3 + 2
*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x
)**2) - 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4
*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4*log(tanh(x))
*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 3*tanh(x)**2/(2*b*tanh(x)**
3 + 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) - 1/(2*b*ta
nh(x)**3 + 2*b*tanh(x)**2), Eq(a, b)), ((x - 1/tanh(x) - 1/(3*tanh(x)**3))/
a, Eq(b, 0)), (-2*a**4*log(tanh(x) + b/a)*tanh(x)**2/(2*a**2*b**3*tanh(x)**
2 - 2*b**5*tanh(x)**2) + 2*a**4*log(tanh(x))*tanh(x)**2/(2*a**2*b**3*tanh(x
)**2 - 2*b**5*tanh(x)**2) + 2*a**3*b*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b*
*5*tanh(x)**2) - a**2*b**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2
*a*b**3*x*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*a*b**
3*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*x*tanh(x)**
2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*b**4*log(tanh(x) + 1)*ta
nh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*log(tanh(x))
*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + b**4/(2*a**2*b**
3*tanh(x)**2 - 2*b**5*tanh(x)**2), True))
```

Giac [A] time = 1.17949, size = 135, normalized size = 1.78

$$-\frac{a^4 \log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2b^3 - b^5} + \frac{x}{a-b} + \frac{\left(a^2+b^2\right)\log\left(\left|e^{(2x)} - 1\right|\right)}{b^3} - \frac{2\left(ab - \left(ab - b^2\right)e^{(2x)}\right)}{b^3\left(e^{(2x)} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^3 - b^5) + x/(a - b) +
(a^2 + b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2*x))/(b^3*
(e^(2*x) - 1)^2)
```

3.149 $\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{bx}{a^2-b^2}-\frac{\left(a^2+b^2\right)\coth(x)}{b^3}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^5\log(a+b\coth(x))}{b^4\left(a^2-b^2\right)}+\frac{a\coth^2(x)}{2b^2}-\frac{\coth^3(x)}{3b}$$

```
[Out] -((b*x)/(a^2 - b^2)) - ((a^2 + b^2)*Coth[x])/b^3 + (a*Coth[x]^2)/(2*b^2) -
Coth[x]^3/(3*b) + (a^5*Log[a + b*Coth[x]])/(b^4*(a^2 - b^2)) + (a*Log[Sinh[
x]])/(a^2 - b^2)
```

Rubi [A] time = 0.391848, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3566, 3647, 3648, 3626, 3617, 31, 3475}

$$-\frac{bx}{a^2-b^2}-\frac{\left(a^2+b^2\right)\coth(x)}{b^3}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^5\log(a+b\coth(x))}{b^4\left(a^2-b^2\right)}+\frac{a\coth^2(x)}{2b^2}-\frac{\coth^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) - ((a^2 + b^2)*Coth[x])/b^3 + (a*Coth[x]^2)/(2*b^2) -
Coth[x]^3/(3*b) + (a^5*Log[a + b*Coth[x]])/(b^4*(a^2 - b^2)) + (a*Log[Sinh[
x]])/(a^2 - b^2)
```

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m + n +
 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx &= -\frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth^2(x)\left(-3a-3b\coth(x)+3a\coth^2(x)\right)}{a+b\coth(x)}\,dx}{3b} \\
&= \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth(x)\left(6a^2-6\left(a^2+b^2\right)\coth^2(x)\right)}{a+b\coth(x)}\,dx}{6b^2} \\
&= -\frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{-6a\left(a^2+b^2\right)-6b^3\coth(x)+6a\left(a^2+b^2\right)\coth^2(x)}{a+b\coth(x)}\,dx}{6b^3} \\
&= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a\int \coth(x)\,dx}{a^2-b^2} + \frac{a^5\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b^3\left(a^2-b^2\right)} \\
&= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a\log(\sinh(x))}{a^2-b^2} + \frac{a^5\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x,\right.}{b^4\left(a^2-b^2\right)} \\
&= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a^5\log(a+b\coth(x))}{b^4\left(a^2-b^2\right)} + \frac{a\log(\sinh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] time = 0.286693, size = 108, normalized size = 1.15

$$\frac{-3ab^2\left(a^2-b^2\right)\operatorname{csch}^2(x)+6a\left(a^4-b^4\right)\log(\sinh(x))+2b\left(a^2-b^2\right)\coth(x)\left(3a^2+b^2\operatorname{csch}^2(x)+4b^2\right)-6a^5\log(a\sinh}{6b^4(b-a)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(a + b*Coth[x]),x]
```

```
[Out] (6*b^5*x - 3*a*b^2*(a^2 - b^2)*Csch[x]^2 + 2*b*(a^2 - b^2)*Coth[x]*(3*a^2 +
 4*b^2 + b^2*Csch[x]^2) + 6*a*(a^4 - b^4)*Log[Sinh[x]] - 6*a^5*Log[b*Cosh[x
] + a*Sinh[x]])/(6*b^4*(-a + b)*(a + b))
```

Maple [A] time = 0.023, size = 96, normalized size = 1.

$$-\frac{(\coth(x))^3}{3b}+\frac{a(\coth(x))^2}{2b^2}-\frac{a^2\coth(x)}{b^3}-\frac{\coth(x)}{b}-\frac{\ln(1+\coth(x))}{2a-2b}-\frac{\ln(\coth(x)-1)}{2b+2a}+\frac{a^5\ln(a+b\coth(x))}{b^4(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*coth(x)),x)
```

```
[Out] -1/3*coth(x)^3/b+1/2*a*coth(x)^2/b^2-1/b^3*a^2*coth(x)-coth(x)/b-1/(2*a-2*b
)*ln(1+coth(x))-1/(2*b+2*a)*ln(coth(x)-1)+1/b^4*a^5/(a+b)/(a-b)*ln(a+b*coth
(x))
```

Maxima [A] time = 1.26407, size = 228, normalized size = 2.43

$$\frac{a^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^4-b^6}+\frac{2\left(3a^2+4b^2-3\left(2a^2+ab+2b^2\right)e^{(-2x)}+3\left(a^2+ab+2b^2\right)e^{(-4x)}\right)}{3\left(3b^3e^{(-2x)}-3b^3e^{(-4x)}+b^3e^{(-6x)}-b^3\right)}+\frac{x}{a+b}-\frac{(a^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] a^5*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^4 - b^6) + 2/3*(3*a^2 + 4*b^2 - 3
*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(a^2 + a*b + 2*b^2)*e^(-4*x))/(3*b^3*e^
(-2*x) - 3*b^3*e^(-4*x) + b^3*e^(-6*x) - b^3) + x/(a + b) - (a^3 + a*b^2)*l
og(e^(-x) + 1)/b^4 - (a^3 + a*b^2)*log(e^(-x) - 1)/b^4
```

Fricas [B] time = 3.09601, size = 3089, normalized size = 32.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a*b^4 + b^5)*x*cosh(x)^6 + 18*(a*b^4 + b^5)*x*cosh(x)*sinh(x)^5 +
3*(a*b^4 + b^5)*x*sinh(x)^6 + 6*a^4*b + 2*a^2*b^3 - 8*b^5 + 3*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^4 + 3*(2
```

```
*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 + 15*(a*b^4 + b^5)*x*cosh(
x)^2 - 3*(a*b^4 + b^5)*x)*sinh(x)^4 + 12*(5*(a*b^4 + b^5)*x*cosh(x)^3 + (2*
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x
))*sinh(x)^3 - 3*(4*a^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x
)*cosh(x)^2 + 3*(15*(a*b^4 + b^5)*x*cosh(x)^4 - 4*a^4*b + 2*a^3*b^2 - 2*a*b
^4 + 4*b^5 + 6*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^
4 + b^5)*x)*cosh(x)^2 + 3*(a*b^4 + b^5)*x)*sinh(x)^2 - 3*(a*b^4 + b^5)*x -
3*(a^5*cosh(x)^6 + 6*a^5*cosh(x)*sinh(x)^5 + a^5*sinh(x)^6 - 3*a^5*cosh(x)^
4 + 3*a^5*cosh(x)^2 - a^5 + 3*(5*a^5*cosh(x)^2 - a^5)*sinh(x)^4 + 4*(5*a^5*
cosh(x)^3 - 3*a^5*cosh(x))*sinh(x)^3 + 3*(5*a^5*cosh(x)^4 - 6*a^5*cosh(x)^2
 + a^5)*sinh(x)^2 + 6*(a^5*cosh(x)^5 - 2*a^5*cosh(x)^3 + a^5*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + 3*((a^5 - a*b^4)*c
osh(x)^6 + 6*(a^5 - a*b^4)*cosh(x)*sinh(x)^5 + (a^5 - a*b^4)*sinh(x)^6 - a^
5 + a*b^4 - 3*(a^5 - a*b^4)*cosh(x)^4 - 3*(a^5 - a*b^4 - 5*(a^5 - a*b^4)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a*b^4)*cosh(x)^3 - 3*(a^5 - a*b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)^2 + 3*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*
cosh(x)^4 - 6*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*((a^5 - a*b^4)*cosh(x)
^5 - 2*(a^5 - a*b^4)*cosh(x)^3 + (a^5 - a*b^4)*cosh(x))*sinh(x))*log(2*sinh
(x)/(cosh(x) - sinh(x))) + 6*(3*(a*b^4 + b^5)*x*cosh(x)^5 + 2*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^3 - (4*a
^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x))*sinh(x))/(
(a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^2*b^4
- b^6)*sinh(x)^6 - a^2*b^4 + b^6 - 3*(a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^2*b^4
 - b^6 - 5*(a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b^4 - b^6)*cosh
(x)^3 - 3*(a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^2*b^4 - b^6)*cosh(x)^2
+ 3*(a^2*b^4 - b^6 + 5*(a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^2*b^4 - b^6)*cosh(x
)^2)*sinh(x)^2 + 6*((a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^2*b^4 - b^6)*cosh(x)^3
 + (a^2*b^4 - b^6)*cosh(x))*sinh(x))
```

Sympy [A] time = 13.0737, size = 1013, normalized size = 10.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - 1/tanh(x) - 1/(3*tanh(x)**3)), Eq(a, 0) & Eq(b, 0)), ((
x - 1/tanh(x) - 1/(3*tanh(x)**3))/b, Eq(a, 0)), (27*x*tanh(x)**4/(6*b*tanh(
x)**4 - 6*b*tanh(x)**3) - 27*x*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3)
 - 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*lo
g(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*log(tanh(x
))*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**
3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 15*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*t
anh(x)**3) + 9*tanh(x)**2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + tanh(x)/(6*b*
tanh(x)**4 - 6*b*tanh(x)**3) + 2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3), Eq(a, -
b)), (3*x*tanh(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 3*x*tanh(x)**3/(6*
b*tanh(x)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x
)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 + 6
*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**
3) - 12*log(tanh(x))*tanh(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 15*tanh
(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 9*tanh(x)**2/(6*b*tanh(x)**4 + 6
*b*tanh(x)**3) + tanh(x)/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 2/(6*b*tanh(x)
**4 + 6*b*tanh(x)**3), Eq(a, b)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1
/(2*tanh(x)**2) - 1/(4*tanh(x)**4))/a, Eq(b, 0)), (6*a**5*log(tanh(x) + b/a
)*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**5*log(tanh
(x))*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**4*b*tan
```

```
h(x)**2/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) + 3*a**3*b**2*tanh(x)/
(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 2*a**2*b**3/(6*a**2*b**4*tan
h(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*x*tanh(x)**3/(6*a**2*b**4*tanh(x)**
3 - 6*b**6*tanh(x)**3) - 6*a*b**4*log(tanh(x) + 1)*tanh(x)**3/(6*a**2*b**4*
tanh(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*log(tanh(x))*tanh(x)**3/(6*a**2*
b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 3*a*b**4*tanh(x)/(6*a**2*b**4*tanh(x
)**3 - 6*b**6*tanh(x)**3) - 6*b**5*x*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6
*b**6*tanh(x)**3) + 6*b**5*tanh(x)**2/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh
(x)**3) + 2*b**5/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3), True))
```

Giac [A] time = 1.17641, size = 193, normalized size = 2.05

$$\frac{a^5 \log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2b^4 - b^6} - \frac{x}{a-b} - \frac{\left(a^3 + ab^2\right)\log\left(\left|e^{(2x)} - 1\right|\right)}{b^4} - \frac{2\left(3a^2b + 4b^3 + 3\left(a^2b - ab^2 + 2b^3\right)e^{(4x)} - 3\left(\right.}{3b^4\left(e^{(2x)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^4 - b^6) - x/(a - b) - (
a^3 + a*b^2)*log(abs(e^(2*x) - 1))/b^4 - 2/3*(3*a^2*b + 4*b^3 + 3*(a^2*b -
a*b^2 + 2*b^3)*e^(4*x) - 3*(2*a^2*b - a*b^2 + 2*b^3)*e^(2*x))/(b^4*(e^(2*x)
 - 1)^3)
```

3.150 $\int \frac{x\operatorname{csch}^2(x)}{(a+b\coth(x))^2}\,dx$

Optimal. Leaf size=54

$$-\frac{ax}{b\left(a^2-b^2\right)}+\frac{\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}+\frac{x}{b(a+b\coth(x))}$$

```
[Out] -((a*x)/(b*(a^2 - b^2))) + x/(b*(a + b*Coth[x])) + Log[b*Cosh[x] + a*Sinh[x
]]/(a^2 - b^2)
```

Rubi [A] time = 0.0850082, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5467, 3484, 3530}

$$-\frac{ax}{b\left(a^2-b^2\right)}+\frac{\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}+\frac{x}{b(a+b\coth(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

```
[Out] -((a*x)/(b*(a^2 - b^2))) + x/(b*(a + b*Coth[x])) + Log[b*Cosh[x] + a*Sinh[x
]]/(a^2 - b^2)
```

Rule 5467

```
Int[Csch[(c_.) + (d_.)*(x_)]^2*(Coth[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.
)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Coth[c
 + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*
x)^(m - 1)*(a + b*Coth[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{x}{b(a+b\coth(x))} - \frac{\int \frac{1}{a+b\coth(x)}\,dx}{b}$$
$$= -\frac{ax}{b\left(a^2-b^2\right)} + \frac{x}{b(a+b\coth(x))} + \frac{i\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}$$
$$= -\frac{ax}{b\left(a^2-b^2\right)} + \frac{x}{b(a+b\coth(x))} + \frac{\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

Mathematica [A] time = 0.158962, size = 49, normalized size = 0.91

$$\frac{ax-b\log(a\sinh(x)+b\cosh(x))}{b^3-a^2b} + \frac{x\sinh(x)}{ab\sinh(x)+b^2\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

```
[Out] (a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(-(a^2*b) + b^3) + (x*Sinh[x])/(b^2*Co
sh[x] + a*b*Sinh[x])
```

Maple [A] time = 0.112, size = 73, normalized size = 1.4

$$-2\,\frac{x}{a^2-b^2} - 2\,\frac{x}{\left(a\mathrm{e}^{2\,x}+b\mathrm{e}^{2\,x}-a+b\right)(a+b)} + \frac{1}{a^2-b^2}\ln\left(\mathrm{e}^{2\,x}-\frac{a-b}{a+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)^2/(a+b*coth(x))^2,x)
```

```
[Out] -2/(a^2-b^2)*x-2*x/(a*exp(2*x)+b*exp(2*x)-a+b)/(a+b)+1/(a^2-b^2)*ln(exp(2*x
)-(a-b)/(a+b))
```

Maxima [A] time = 1.952, size = 92, normalized size = 1.7

$$\frac{2\,xe^{(2\,x)}}{a^2-2\,ab+b^2-\left(a^2-b^2\right)e^{(2\,x)}} + \frac{\log\left(\frac{(a+b)e^{(2\,x)}-a+b}{a+b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x*e^(2*x)/(a^2 - 2*a*b + b^2 - (a^2 - b^2)*e^(2*x)) + log(((a + b)*e^(2*x
) - a + b)/(a + b))/(a^2 - b^2)
```

Fricas [B] time = 2.68442, size = 491, normalized size = 9.09

$$\frac{2\,(a+b)x\cosh{(x)}^2+4\,(a+b)x\cosh{(x)}\sinh{(x)}+2\,(a+b)x\sinh{(x)}^2-\left((a+b)\cosh{(x)}^2+2\,(a+b)\cosh{(x)}\sinh{(}\right.}{a^3-a^2b-ab^2+b^3-\left(a^3+a^2b-ab^2-b^3\right)\cosh{(x)}^2-2\left(a^3+a^2b-ab^2-b^3\right)\cosh{(x)}\sinh{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="fricas")
```

```
[Out] (2*(a + b)*x*cosh(x)^2 + 4*(a + b)*x*cosh(x)*sinh(x) + 2*(a + b)*x*sinh(x)^
2 - ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a
+ b)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^3 - a^2*b - a*b
^2 + b^3 - (a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 - 2*(a^3 + a^2*b - a*b^2 -
 b^3)*cosh(x)*sinh(x) - (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{csch}^2(x)}{(a + b \coth(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)**2/(a+b*coth(x))**2,x)
```

```
[Out] Integral(x*csch(x)**2/(a + b*coth(x))**2, x)
```

Giac [B] time = 1.17361, size = 228, normalized size = 4.22

$$-\frac{2\,axe^{(2\,x)} + 2\,bxe^{(2\,x)} - ae^{(2\,x)}\log\left(ae^{(2\,x)} + be^{(2\,x)} - a + b\right) - be^{(2\,x)}\log\left(ae^{(2\,x)} + be^{(2\,x)} - a + b\right) + a\log\left(ae^{(2\,x)} + be^{(2\,x)} - a\right.}{a^3 e^{(2\,x)} + a^2 b e^{(2\,x)} - ab^2 e^{(2\,x)} - b^3 e^{(2\,x)} - a^3 + a^2 b + ab^2 - b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="giac")
```

```
[Out] -(2*a*x*e^(2*x) + 2*b*x*e^(2*x) - a*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a +
 b) - b*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a + b) + a*log(a*e^(2*x) + b*e^
(2*x) - a + b) - b*log(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3*e^(2*x) + a^2*b
*e^(2*x) - a*b^2*e^(2*x) - b^3*e^(2*x) - a^3 + a^2*b + a*b^2 - b^3)
```

3.151 $\int x^3 \coth(a + 2\log(x))\,dx$

Optimal. Leaf size=30

$$\frac{1}{2}e^{-2a}\log\left(1 - e^{2a}x^4\right) + \frac{x^4}{4}$$

```
[Out] x^4/4 + Log[1 - E^(2*a)*x^4]/(2*E^(2*a))
```

Rubi [F] time = 0.0285749, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3 \coth(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Coth[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][x^3*Coth[a + 2*Log[x]], x]
```

Rubi steps

$$\int x^3 \coth(a + 2\log(x))\,dx = \int x^3 \coth(a + 2\log(x))\,dx$$

Mathematica [B] time = 0.0241018, size = 64, normalized size = 2.13

$$\frac{1}{2}\cosh(2a)\log\left(x^4\sinh(a) + x^4\cosh(a) + \sinh(a) - \cosh(a)\right) - \frac{1}{2}\sinh(2a)\log\left(x^4\sinh(a) + x^4\cosh(a) + \sinh(a) - \text{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + 2*Log[x]],x]
```

```
[Out] x^4/4 + (Cosh[2*a]*Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]])/2 -
 (Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]]*Sinh[2*a])/2
```

Maple [A] time = 0.026, size = 24, normalized size = 0.8

$$\frac{x^4}{4} + \frac{e^{-2a}\ln\left(e^{2a}x^4 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a+2*ln(x)),x)
```

```
[Out] 1/4*x^4+1/2*exp(-2*a)*ln(exp(2*a)*x^4-1)
```

Maxima [A] time = 1.21452, size = 49, normalized size = 1.63

$$\frac{1}{4}x^4 + \frac{1}{2}e^{(-2a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}e^{(-2a)}\log\left(x^2e^a - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/2*e^(-2*a)*log(x^2*e^a + 1) + 1/2*e^(-2*a)*log(x^2*e^a - 1)
```

Fricas [A] time = 2.53483, size = 72, normalized size = 2.4

$$\frac{1}{4}\left(x^4e^{(2a)} + 2\log\left(x^4e^{(2a)} - 1\right)\right)e^{(-2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^4*e^(2*a) + 2*log(x^4*e^(2*a) - 1))*e^(-2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\coth\left(a + 2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(a+2*ln(x)),x)
```

```
[Out] Integral(x**3*coth(a + 2*log(x)), x)
```

Giac [A] time = 1.14286, size = 32, normalized size = 1.07

$$\frac{1}{4}x^4 + \frac{1}{2}e^{(-2a)}\log\left(\left|x^4e^{(2a)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/2*e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

3.152 $\int x^2 \coth(a + 2\log(x))\,dx$

Optimal. Leaf size=45

$$e^{-3a/2}\tan^{-1}\left(e^{a/2}x\right) - e^{-3a/2}\tanh^{-1}\left(e^{a/2}x\right) + \frac{x^3}{3}$$

```
[Out] x^3/3 + ArcTan[E^(a/2)*x]/E^((3*a)/2) - ArcTanh[E^(a/2)*x]/E^((3*a)/2)
```

Rubi [F] time = 0.0213207, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \coth(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Coth[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][x^2*Coth[a + 2*Log[x]], x]
```

Rubi steps

$$\int x^2 \coth(a + 2\log(x))\,dx = \int x^2 \coth(a + 2\log(x))\,dx$$

Mathematica [C] time = 0.23153, size = 64, normalized size = 1.42

$$\frac{1}{6}\left(3(\sinh(2a) - \cosh(2a))\text{RootSum}\left[\#1^4\sinh(a) + \#1^4\cosh(a) + \sinh(a) - \cosh(a)\&, \frac{\log(x) - \log(x - \#1)}{\#1}\&\right] + 2x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + 2*Log[x]],x]
```

```
[Out] (2*x^3 + 3*RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Lo
g[x] - Log[x - #1])/#1 & ]*(-Cosh[2*a] + Sinh[2*a]))/6
```

Maple [B] time = 0.048, size = 83, normalized size = 1.8

$$\frac{x^3}{3} + \frac{1}{2}\ln\left(-x\sqrt{e^a} + 1\right)\left(e^a\right)^{-\frac{3}{2}} - \frac{1}{2}\ln\left(x\sqrt{e^a} + 1\right)\left(e^a\right)^{-\frac{3}{2}} + \frac{1}{2}\ln\left(-e^{2a}x + \left(-e^a\right)^{\frac{3}{2}}\right)\left(-e^a\right)^{-\frac{3}{2}} - \frac{1}{2}\ln\left(e^{2a}x + \left(-e^a\right)^{\frac{3}{2}}\right)\left(-e^a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a+2*ln(x)),x)
```

```
[Out] 1/3*x^3+1/2/exp(a)^(3/2)*ln(-x*exp(a)^(1/2)+1)-1/2/exp(a)^(3/2)*ln(x*exp(a)
^(1/2)+1)+1/2/(-exp(a))^(3/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))-1/2/(-exp(a))
^(3/2)*ln(exp(2*a)*x+(-exp(a))^(3/2))
```

Maxima [A] time = 1.75688, size = 65, normalized size = 1.44

$$\frac{1}{3}x^3 + \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{1}{2}e^{\left(-\frac{3}{2}a\right)}\log\left(\frac{xe^a - e^{\left(\frac{1}{2}a\right)}}{xe^a + e^{\left(\frac{1}{2}a\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log((x*e^a - e^(1
/2*a))/(x*e^a + e^(1/2*a)))
```

Fricas [A] time = 2.68469, size = 171, normalized size = 3.8

$$\frac{1}{6}\left(2x^3e^{(2a)} + 6\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + 3e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a - 2xe^{\left(\frac{1}{2}a\right)} + 1}{x^2e^a - 1}\right)\right)e^{(-2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3*e^(2*a) + 6*arctan(x*e^(1/2*a))*e^(1/2*a) + 3*e^(1/2*a)*log((x^2
*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\coth(a + 2\log(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(a+2*ln(x)),x)
```

```
[Out] Integral(x**2*coth(a + 2*log(x)), x)
```

Giac [A] time = 1.14067, size = 73, normalized size = 1.62

$$\frac{1}{3}x^3 + \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{1}{2}e^{\left(-\frac{3}{2}a\right)}\log\left(\frac{\left|2xe^a - 2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^a + 2e^{\left(\frac{1}{2}a\right)}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log(abs(2*x*e^a -
 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

3.153 $\int x \coth(a + 2\log(x))\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2} - e^{-a}\tanh^{-1}\left(e^a x^2\right)$$

```
[Out] x^2/2 - ArcTanh[E^a*x^2]/E^a
```

Rubi [F] time = 0.0156127, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \coth(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Coth[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][x*Coth[a + 2*Log[x]], x]
```

Rubi steps

$$\int x \coth(a + 2\log(x))\,dx = \int x \coth(a + 2\log(x))\,dx$$

Mathematica [A] time = 0.181986, size = 26, normalized size = 1.13

$$(\sinh(a) - \cosh(a))\tanh^{-1}\left(x^2(\sinh(a) + \cosh(a))\right) + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + 2*Log[x]],x]
```

```
[Out] x^2/2 + ArcTanh[x^2*(Cosh[a] + Sinh[a])]*(-Cosh[a] + Sinh[a])
```

Maple [A] time = 0.024, size = 37, normalized size = 1.6

$$\frac{x^2}{2} - \frac{e^{-a}\ln\left(e^a x^2 + 1\right)}{2} + \frac{e^{-a}\ln\left(e^a x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a+2*ln(x)),x)
```

```
[Out] 1/2*x^2-1/2*exp(-a)*ln(exp(a)*x^2+1)+1/2*exp(-a)*ln(exp(a)*x^2-1)
```

Maxima [A] time = 1.04383, size = 49, normalized size = 2.13

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}e^{(-a)}\log\left(x^2e^a - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1)
```

Fricas [A] time = 2.59376, size = 81, normalized size = 3.52

$$\frac{1}{2}\left(x^2e^a - \log\left(x^2e^a + 1\right) + \log\left(x^2e^a - 1\right)\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*e^a - log(x^2*e^a + 1) + log(x^2*e^a - 1))*e^(-a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\coth\left(a + 2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*ln(x)),x)
```

```
[Out] Integral(x*coth(a + 2*log(x)), x)
```

Giac [A] time = 1.1168, size = 50, normalized size = 2.17

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}e^{(-a)}\log\left(\left|x^2e^a - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1))
```

3.154 $\int \coth(a+2\log(x))\,dx$

Optimal. Leaf size=40

$$-e^{-a/2}\tan^{-1}\left(e^{a/2}x\right)-e^{-a/2}\tanh^{-1}\left(e^{a/2}x\right)+x$$

```
[Out] x - ArcTan[E^(a/2)*x]/E^(a/2) - ArcTanh[E^(a/2)*x]/E^(a/2)
```

Rubi [F] time = 0.0071696, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]],x]

[Out] Defer[Int][Coth[a + 2*Log[x]], x]
```

Rubi steps

$$\int \coth(a+2\log(x))\,dx=\int \coth(a+2\log(x))\,dx$$

Mathematica [C] time = 0.178869, size = 58, normalized size = 1.45

$$\frac{1}{2}(\sinh(2a)-\cosh(2a))\text{RootSum}\left[\#1^4\sinh(a)+\#1^4\cosh(a)+\sinh(a)-\cosh(a)\&,\frac{\log(x)-\log(x-\#1)}{\#1^3}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]],x]

[Out] x + (RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Log[x] -
 Log[x - #1])/#1^3 & ]*(-Cosh[2*a] + Sinh[2*a]))/2
```

Maple [B] time = 0.045, size = 71, normalized size = 1.8

$$x-\frac{1}{2}\ln\left(x\sqrt{-e^a}+1\right)\frac{1}{\sqrt{-e^a}}+\frac{1}{2}\ln\left(x\sqrt{-e^a}-1\right)\frac{1}{\sqrt{-e^a}}+\frac{1}{2}\ln\left(x\sqrt{e^a}-1\right)\frac{1}{\sqrt{e^a}}-\frac{1}{2}\ln\left(x\sqrt{e^a}+1\right)\frac{1}{\sqrt{e^a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x)),x)

[Out] x-1/2/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/2/(-exp(a))^(1/2)*ln(x*(-ex
p(a))^(1/2)-1)+1/2/exp(a)^(1/2)*ln(x*exp(a)^(1/2)-1)-1/2/exp(a)^(1/2)*ln(x*
exp(a)^(1/2)+1)
```

Maxima [A] time = 1.60909, size = 61, normalized size = 1.52

$$-\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{xe^{a} - e^{\left(\frac{1}{2}a\right)}}{xe^{a} + e^{\left(\frac{1}{2}a\right)}}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] -arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log((x*e^a - e^(1/2*a))/(x
*e^a + e^(1/2*a))) + x
```

Fricas [B] time = 2.58194, size = 159, normalized size = 3.98

$$-\frac{1}{2}\left(2\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} - 2xe^{a} - e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^{2}e^{a} - 2xe^{\left(\frac{1}{2}a\right)} + 1}{x^{2}e^{a} - 1}\right)\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(x*e^(1/2*a))*e^(1/2*a) - 2*x*e^a - e^(1/2*a)*log((x^2*e^a -
2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a + 2\log\left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x)),x)
```

```
[Out] Integral(coth(a + 2*log(x)), x)
```

Giac [A] time = 1.12146, size = 69, normalized size = 1.72

$$-\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{\left|2xe^{a} - 2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^{a} + 2e^{\left(\frac{1}{2}a\right)}\right|}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] -arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^(1/2
*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x
```

3.155 $\int \frac{\coth(a+2\log(x))}{x}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log(\sinh(a+2\log(x)))$$

```
[Out] Log[Sinh[a + 2*Log[x]]]/2
```

Rubi [A] time = 0.0137687, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3475}

$$\frac{1}{2}\log(\sinh(a+2\log(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]/x,x]
```

```
[Out] Log[Sinh[a + 2*Log[x]]]/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(a+2\log(x))}{x}\,dx &= \text{Subst}\left(\int \coth(a+2x)\,dx, x, \log(x)\right)\\ &= \frac{1}{2}\log(\sinh(a+2\log(x)))\end{aligned}$$

Mathematica [A] time = 0.0266171, size = 21, normalized size = 1.75

$$\frac{1}{2}(\log(\tanh(a+2\log(x)))+\log(\cosh(a+2\log(x))))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]/x,x]
```

```
[Out] (Log[Cosh[a + 2*Log[x]]] + Log[Tanh[a + 2*Log[x]]])/2
```

Maple [B] time = 0.003, size = 26, normalized size = 2.2

$$-\frac{\ln(\coth(a+2\ln(x))-1)}{4}-\frac{\ln(\coth(a+2\ln(x))+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))/x,x)

[Out] -1/4*ln(coth(a+2*ln(x))-1)-1/4*ln(coth(a+2*ln(x))+1)
```

Maxima [A] time = 1.05091, size = 14, normalized size = 1.17

$$\frac{1}{2}\,\log\left(\sinh\left(a+2\,\log\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="maxima")

[Out] 1/2*log(sinh(a + 2*log(x)))
```

Fricas [A] time = 2.53463, size = 47, normalized size = 3.92

$$\frac{1}{2}\,\log\left(x^{4}e^{(2\,a)}-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="fricas")

[Out] 1/2*log(x^4*e^(2*a) - 1) - log(x)
```

Sympy [B] time = 1.68852, size = 27, normalized size = 2.25

$$\log\left(x\right)-\frac{\log\left(\tanh\left(a+2\log\left(x\right)\right)+1\right)}{2}+\frac{\log\left(\tanh\left(a+2\log\left(x\right)\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))/x,x)

[Out] log(x) - log(tanh(a + 2*log(x)) + 1)/2 + log(tanh(a + 2*log(x)))/2
```

Giac [A] time = 1.09541, size = 28, normalized size = 2.33

$$-\frac{1}{4}\,\log\left(x^{4}\right)+\frac{1}{2}\,\log\left(\left|x^{4}e^{(2\,a)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="giac")

[Out] -1/4*log(x^4) + 1/2*log(abs(x^4*e^(2*a) - 1))
```

3.156 $\int \frac{\coth(a+2\log(x))}{x^2}\,dx$

Optimal. Leaf size=41

$$e^{a/2}\tan^{-1}\left(e^{a/2}x\right)-e^{a/2}\tanh^{-1}\left(e^{a/2}x\right)+\frac{1}{x}$$

```
[Out] x^(-1) + E^(a/2)*ArcTan[E^(a/2)*x] - E^(a/2)*ArcTanh[E^(a/2)*x]
```

Rubi [F] time = 0.0222044, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]/x^2,x]

[Out] Defer[Int][Coth[a + 2*Log[x]]/x^2, x]
```

Rubi steps

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx=\int \frac{\coth(a+2\log(x))}{x^2}\,dx$$

Mathematica [C] time = 0.162025, size = 62, normalized size = 1.51

$$\frac{x(\sinh(a)+\cosh(a))^2\text{RootSum}\left[-\#1^4\sinh(a)+\#1^4\cosh(a)-\sinh(a)-\cosh(a)\&,\frac{\log\left(\frac{1}{x}-\#1\right)+\log(x)}{\#1^3}\&\right]+2}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]/x^2,x]

[Out] (2 + x*RootSum[-Cosh[a] - Sinh[a] + Cosh[a]*#1^4 - Sinh[a]*#1^4 & , (Log[x]
 + Log[x^(-1) - #1])/#1^3 & ]*(Cosh[a] + Sinh[a])^2)/(2*x)
```

Maple [B] time = 0.042, size = 93, normalized size = 2.3

$$x^{-1}+\frac{1}{2}\sqrt{e^{a}}\ln\left(-e^{2a}x+(e^{a})^{\frac{3}{2}}\right)-\frac{1}{2}\sqrt{e^{a}}\ln\left(-e^{2a}x-(e^{a})^{\frac{3}{2}}\right)+\frac{1}{2}\sqrt{-e^{a}}\ln\left(-e^{2a}x+(-e^{a})^{\frac{3}{2}}\right)-\frac{1}{2}\sqrt{-e^{a}}\ln\left(-e^{2a}x-(-e^{a})\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))/x^2,x)

[Out] 1/x+1/2*exp(a)^(1/2)*ln(-exp(2*a)*x+exp(a)^(3/2))-1/2*exp(a)^(1/2)*ln(-exp(
2*a)*x-exp(a)^(3/2))+1/2*(-exp(a))^(1/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))-1/
```

```
2*(-exp(a))^(1/2)*ln(-exp(2*a)*x-(-exp(a))^(3/2))
```

Maxima [A] time = 1.73337, size = 63, normalized size = 1.54

$$-\arctan\left(\frac{e^{\left(-\frac{1}{2}a\right)}}{x}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\frac{1}{x} - e^{\left(\frac{1}{2}a\right)}}{\frac{1}{x} + e^{\left(\frac{1}{2}a\right)}}\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-1/2*a)/x)*e^(1/2*a) + 1/2*e^(1/2*a)*log((1/x - e^(1/2*a))/(1/x
+ e^(1/2*a))) + 1/x
```

Fricas [A] time = 2.6559, size = 149, normalized size = 3.63

$$\frac{2\,x\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + xe^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a - 2\,xe^{\left(\frac{1}{2}a\right)} + 1}{x^2e^a - 1}\right) + 2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*arctan(x*e^(1/2*a))*e^(1/2*a) + x*e^(1/2*a)*log((x^2*e^a - 2*x*e^(
1/2*a) + 1)/(x^2*e^a - 1)) + 2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(a + 2\log\left(x\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))/x**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))/x**2, x)
```

Giac [A] time = 1.15757, size = 70, normalized size = 1.71

$$\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^a - 2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^a + 2\,e^{\left(\frac{1}{2}a\right)}\right|}\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="giac")
```

```
[Out] arctan(x*e^(1/2*a))*e^(1/2*a) + 1/2*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a)
)/abs(2*x*e^a + 2*e^(1/2*a))) + 1/x
```

3.157 $\int \frac{\coth(a+2\log(x))}{x^3} dx$

Optimal. Leaf size=21

$$\frac{1}{2x^2} - e^a \tanh^{-1}\left(e^a x^2\right)$$

```
[Out] 1/(2*x^2) - E^a*ArcTanh[E^a*x^2]
```

Rubi [F] time = 0.0213417, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(a+2\log(x))}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]/x^3,x]
```

```
[Out] Defer[Int][Coth[a + 2*Log[x]]/x^3, x]
```

Rubi steps

$$\int \frac{\coth(a+2\log(x))}{x^3} dx = \int \frac{\coth(a+2\log(x))}{x^3} dx$$

Mathematica [A] time = 0.156472, size = 27, normalized size = 1.29

$$\frac{1}{2x^2} - (\sinh(a)+\cosh(a))\tanh^{-1}\left(\frac{\cosh(a)-\sinh(a)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]/x^3,x]
```

```
[Out] 1/(2*x^2) - ArcTanh[(Cosh[a] - Sinh[a])/x^2]*(Cosh[a] + Sinh[a])
```

Maple [A] time = 0.026, size = 35, normalized size = 1.7

$$\frac{1}{2x^2} - \frac{e^a \ln\left(-e^a x^2 - 1\right)}{2} + \frac{e^a \ln\left(-e^a x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))/x^3,x)
```

```
[Out] 1/2/x^2-1/2*exp(a)*ln(-exp(a)*x^2-1)+1/2*exp(a)*ln(-exp(a)*x^2+1)
```

Maxima [A] time = 1.02956, size = 41, normalized size = 1.95

$$-\frac{1}{2}e^{a}\log\left(\frac{1}{x^{2}}+e^{a}\right)+\frac{1}{2}e^{a}\log\left(\frac{1}{x^{2}}-e^{a}\right)+\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*e^a*log(1/x^2 + e^a) + 1/2*e^a*log(1/x^2 - e^a) + 1/2/x^2
```

Fricas [B] time = 2.5844, size = 92, normalized size = 4.38

$$-\frac{x^{2}e^{a}\log\left(x^{2}e^{a}+1\right)-x^{2}e^{a}\log\left(x^{2}e^{a}-1\right)-1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(x^2*e^a*log(x^2*e^a + 1) - x^2*e^a*log(x^2*e^a - 1) - 1)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(a+2\log\left(x\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))/x**3,x)
```

```
[Out] Integral(coth(a + 2*log(x))/x**3, x)
```

Giac [A] time = 1.10794, size = 45, normalized size = 2.14

$$-\frac{1}{2}e^{a}\log\left(x^{2}e^{a}+1\right)+\frac{1}{2}e^{a}\log\left(\left|x^{2}e^{a}-1\right|\right)+\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*e^a*log(x^2*e^a + 1) + 1/2*e^a*log(abs(x^2*e^a - 1)) + 1/2/x^2
```

3.158 $\int x^3 \coth^2(a + 2\log(x))\,dx$

Optimal. Leaf size=47

$$\frac{e^{-2a}}{1-e^{2a}x^4}+e^{-2a}\log\left(1-e^{2a}x^4\right)+\frac{x^4}{4}$$

```
[Out] x^4/4 + 1/(E^(2*a)*(1 - E^(2*a)*x^4)) + Log[1 - E^(2*a)*x^4]/E^(2*a)
```

Rubi [F] time = 0.0662787, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3 \coth^2(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][x^3*Coth[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int x^3 \coth^2(a+2\log(x))\,dx=\int x^3 \coth^2(a+2\log(x))\,dx$$

Mathematica [A] time = 0.102571, size = 86, normalized size = 1.83

$$\frac{\sinh(3a)-\cosh(3a)}{\left(x^4+1\right)\sinh(a)+\left(x^4-1\right)\cosh(a)}+\cosh(2a)\log\left(\left(x^4+1\right)\sinh(a)+\left(x^4-1\right)\cosh(a)\right)-\sinh(2a)\log\left(\left(x^4+1\right)\sinh(a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^4/4 + Cosh[2*a]*Log[(-1 + x^4)*Cosh[a] + (1 + x^4)*Sinh[a]] - Log[(-1 + x
^4)*Cosh[a] + (1 + x^4)*Sinh[a]]*Sinh[2*a] + (-Cosh[3*a] + Sinh[3*a])/((-1
+ x^4)*Cosh[a] + (1 + x^4)*Sinh[a])
```

Maple [A] time = 0.018, size = 41, normalized size = 0.9

$$\frac{x^4}{4}-\frac{e^{-2a}}{e^{2a}x^4-1}+e^{-2a}\ln\left(e^{2a}x^4-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a+2*ln(x))^2,x)
```

```
[Out] 1/4*x^4-exp(-2*a)/(exp(2*a)*x^4-1)+exp(-2*a)*ln(exp(2*a)*x^4-1)
```

Maxima [A] time = 1.05107, size = 72, normalized size = 1.53

$$\frac{1}{4}\,x^4+e^{(-2\,a)}\log\left(x^2e^a+1\right)+e^{(-2\,a)}\log\left(x^2e^a-1\right)-\frac{1}{x^4e^{(4\,a)}-e^{(2\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + e^(-2*a)*log(x^2*e^a + 1) + e^(-2*a)*log(x^2*e^a - 1) - 1/(x^4*e^
(4*a) - e^(2*a))
```

Fricas [A] time = 2.46811, size = 140, normalized size = 2.98

$$\frac{x^8e^{(4\,a)}-x^4e^{(2\,a)}+4\left(x^4e^{(2\,a)}-1\right)\log\left(x^4e^{(2\,a)}-1\right)-4}{4\left(x^4e^{(4\,a)}-e^{(2\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^8*e^(4*a) - x^4*e^(2*a) + 4*(x^4*e^(2*a) - 1)*log(x^4*e^(2*a) - 1) -
 4)/(x^4*e^(4*a) - e^(2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\coth^2\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(x**3*coth(a + 2*log(x))**2, x)
```

Giac [A] time = 1.12165, size = 54, normalized size = 1.15

$$\frac{1}{4}\,x^4-\frac{x^4}{x^4e^{(2\,a)}-1}+e^{(-2\,a)}\log\left(\left|x^4e^{(2\,a)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - x^4/(x^4*e^(2*a) - 1) + e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

3.159 $\int x^2 \coth^2(a + 2\log(x))\,dx$

Optimal. Leaf size=68

$$\frac{x^3}{1-e^{2a}x^4}+\frac{3}{2}e^{-3a/2}\tan^{-1}\left(e^{a/2}x\right)-\frac{3}{2}e^{-3a/2}\tanh^{-1}\left(e^{a/2}x\right)+\frac{x^3}{3}$$

```
[Out] x^3/3 + x^3/(1 - E^(2*a)*x^4) + (3*ArcTan[E^(a/2)*x])/(2*E^((3*a)/2)) - (3*
ArcTanh[E^(a/2)*x])/(2*E^((3*a)/2))
```

Rubi [F] time = 0.0477565, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \coth^2(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][x^2*Coth[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int x^2 \coth^2(a+2\log(x))\,dx=\int x^2 \coth^2(a+2\log(x))\,dx$$

Mathematica [C] time = 2.8989, size = 154, normalized size = 2.26

$$\frac{16e^{2a}x^7\left(e^{2a}x^4+1\right)^2 \text{HypergeometricPFQ}\left(\left\{\frac{7}{4},2,2,2\right\},\left\{1,1,\frac{19}{4}\right\},e^{2a}x^4\right)}{1155}+\frac{e^{-4a}\left(7\left(27e^{8a}x^{16}-632e^{6a}x^{12}-398e^{4a}x^8+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-9317 - 17825*E^(2*a)*x^4 - 4787*E^(4*a)*x^8 + 1481*E^(6*a)*x^12 + 7*(1331
 + 1976*E^(2*a)*x^4 - 398*E^(4*a)*x^8 - 632*E^(6*a)*x^12 + 27*E^(8*a)*x^16)
*Hypergeometric2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(2688*E^(4*a)*x^5) + (16*E^(2
*a)*x^7*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4},
 E^(2*a)*x^4])/1155
```

Maple [A] time = 0.046, size = 100, normalized size = 1.5

$$\frac{x^3}{3}-\frac{x^3}{e^{2a}x^4-1}+\frac{3}{4}\ln\left(-e^{2a}x+(-e^a)^{\frac{3}{2}}\right)(-e^a)^{-\frac{3}{2}}-\frac{3}{4}\ln\left(e^{2a}x+(-e^a)^{\frac{3}{2}}\right)(-e^a)^{-\frac{3}{2}}+\frac{3}{4}\ln\left(-x\sqrt{e^a}+1\right)(e^a)^{-\frac{3}{2}}-\frac{3}{4}\ln\left(x\sqrt{e^a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a+2*ln(x))^2,x)
```

```
[Out] 1/3*x^3-x^3/(exp(2*a)*x^4-1)+3/4/(-exp(a))^(3/2)*ln(-exp(2*a)*x+(-exp(a))^(
3/2))-3/4/(-exp(a))^(3/2)*ln(exp(2*a)*x+(-exp(a))^(3/2))+3/4/exp(a)^(3/2)*l
n(-x*exp(a)^(1/2)+1)-3/4/exp(a)^(3/2)*ln(x*exp(a)^(1/2)+1)
```

Maxima [A] time = 1.62442, size = 89, normalized size = 1.31

$$\frac{1}{3}x^3 - \frac{x^3}{x^4e^{(2a)} - 1} + \frac{3}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{3}{4}e^{\left(-\frac{3}{2}a\right)}\log\left(\frac{xe^a - e^{\left(\frac{1}{2}a\right)}}{xe^a + e^{\left(\frac{1}{2}a\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4*
e^(-3/2*a)*log((x*e^a - e^(1/2*a))/(x*e^a + e^(1/2*a)))
```

Fricas [B] time = 2.59568, size = 266, normalized size = 3.91

$$\frac{4x^7e^{(4a)} - 16x^3e^{(2a)} + 18\left(x^4e^{(2a)} - 1\right)\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + 9\left(x^4e^{(2a)} - 1\right)e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a - 2xe^{\left(\frac{1}{2}a\right)} + 1}{x^2e^a - 1}\right)}{12\left(x^4e^{(4a)} - e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^7*e^(4*a) - 16*x^3*e^(2*a) + 18*(x^4*e^(2*a) - 1)*arctan(x*e^(1/2
*a))*e^(1/2*a) + 9*(x^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a)
 + 1)/(x^2*e^a - 1)))/(x^4*e^(4*a) - e^(2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\coth^2(a + 2\log(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(x**2*coth(a + 2*log(x))**2, x)
```

Giac [A] time = 1.14405, size = 97, normalized size = 1.43

$$\frac{1}{3}x^3 - \frac{x^3}{x^4e^{(2a)} - 1} + \frac{3}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{3}{4}e^{\left(-\frac{3}{2}a\right)}\log\left(\frac{\left|2xe^a - 2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^a + 2e^{\left(\frac{1}{2}a\right)}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4*
e^(-3/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

3.160 $\int x \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=41

$$\frac{x^2}{1-e^{2a}x^4}-e^{-a}\tanh^{-1}\left(e^a x^2\right)+\frac{x^2}{2}$$

```
[Out] x^2/2 + x^2/(1 - E^(2*a)*x^4) - ArcTanh[E^a*x^2]/E^a
```

Rubi [F] time = 0.0298197, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \coth^2(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][x*Coth[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int x \coth^2(a+2\log(x))\,dx=\int x \coth^2(a+2\log(x))\,dx$$

Mathematica [C] time = 2.93267, size = 163, normalized size = 3.98

$$\frac{2}{105}e^{2a}x^6\left(e^{2a}x^4+1\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2\right\},\left\{1,1,\frac{9}{2}\right\},e^{2a}x^4\right)+\frac{e^{-4a}\left(61e^{6a}x^{12}-181e^{4a}x^8-713e^{2a}x^4+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-375 - 713*E^(2*a)*x^4 - 181*E^(4*a)*x^8 + 61*E^(6*a)*x^12 + (3*(125 + 196
*E^(2*a)*x^4 - 14*E^(4*a)*x^8 - 52*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[Sqr
t[E^(2*a)*x^4]])/Sqrt[E^(2*a)*x^4])/(96*E^(4*a)*x^6) + (2*E^(2*a)*x^6*(1 +
E^(2*a)*x^4)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*a)*x^4])
/105
```

Maple [A] time = 0.02, size = 54, normalized size = 1.3

$$\frac{x^2}{2}-\frac{x^2}{e^{2a}x^4-1}+\frac{e^{-a}\ln\left(e^a x^2-1\right)}{2}-\frac{e^{-a}\ln\left(e^a x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a+2*ln(x))^2,x)
```

```
[Out] 1/2*x^2-x^2/(exp(2*a)*x^4-1)+1/2*exp(-a)*ln(exp(a)*x^2-1)-1/2*exp(-a)*ln(ex
p(a)*x^2+1)
```

Maxima [A] time = 1.06055, size = 72, normalized size = 1.76

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2e^a+1\right) + \frac{1}{2}e^{(-a)}\log\left(x^2e^a-1\right) - \frac{x^2}{x^4e^{(2a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1) - x^2/(
x^4*e^(2*a) - 1)
```

Fricas [B] time = 2.57195, size = 169, normalized size = 4.12

$$\frac{x^6e^{(3a)} - 3x^2e^a - \left(x^4e^{(2a)}-1\right)\log\left(x^2e^a+1\right) + \left(x^4e^{(2a)}-1\right)\log\left(x^2e^a-1\right)}{2\left(x^4e^{(3a)}-e^a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^6*e^(3*a) - 3*x^2*e^a - (x^4*e^(2*a) - 1)*log(x^2*e^a + 1) + (x^4*e^
(2*a) - 1)*log(x^2*e^a - 1))/(x^4*e^(3*a) - e^a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\coth^2\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(x*coth(a + 2*log(x))**2, x)
```

Giac [A] time = 1.11119, size = 73, normalized size = 1.78

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2e^a+1\right) + \frac{1}{2}e^{(-a)}\log\left(\left|x^2e^a-1\right|\right) - \frac{x^2}{x^4e^{(2a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1)) -
x^2/(x^4*e^(2*a) - 1)
```

3.161 $\int \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=60

$$\frac{x}{1-e^{2a}x^4}-\frac{1}{2}e^{-a/2}\tan^{-1}\left(e^{a/2}x\right)-\frac{1}{2}e^{-a/2}\tanh^{-1}\left(e^{a/2}x\right)+x$$

```
[Out] x + x/(1 - E^(2*a)*x^4) - ArcTan[E^(a/2)*x]/(2*E^(a/2)) - ArcTanh[E^(a/2)*x
]/(2*E^(a/2))
```

Rubi [F] time = 0.0098684, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^2(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][Coth[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int \coth^2(a+2\log(x))\,dx=\int \coth^2(a+2\log(x))\,dx$$

Mathematica [C] time = 1.95104, size = 153, normalized size = 2.55

$$\frac{16}{585}e^{2a}x^5\left(e^{2a}x^4+1\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{5}{4},2,2,2\right\},\left\{1,1,\frac{17}{4}\right\},e^{2a}x^4\right)+\frac{e^{-4a}\left(5\left(e^{8a}x^{16}-248e^{6a}x^{12}+102e^{4a}x\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-3645 - 6769*E^(2*a)*x^4 - 1483*E^(4*a)*x^8 + 681*E^(6*a)*x^12 + 5*(729 +
1208*E^(2*a)*x^4 + 102*E^(4*a)*x^8 - 248*E^(6*a)*x^12 + E^(8*a)*x^16)*Hyper
geometric2F1[1/4, 1, 5/4, E^(2*a)*x^4])/(640*E^(4*a)*x^7) + (16*E^(2*a)*x^5
*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{5/4, 2, 2, 2}, {1, 1, 17/4}, E^(2*a
)*x^4])/585
```

Maple [A] time = 0.043, size = 86, normalized size = 1.4

$$x-\frac{x}{e^{2a}x^4-1}+\frac{1}{4}\ln\left(x\sqrt{e^a}-1\right)\frac{1}{\sqrt{e^a}}-\frac{1}{4}\ln\left(x\sqrt{e^a}+1\right)\frac{1}{\sqrt{e^a}}-\frac{1}{4}\ln\left(x\sqrt{-e^a}+1\right)\frac{1}{\sqrt{-e^a}}+\frac{1}{4}\ln\left(x\sqrt{-e^a}-1\right)\frac{1}{\sqrt{-e^a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2,x)
```

```
[Out] x-x/(exp(2*a)*x^4-1)+1/4/exp(a)^(1/2)*ln(x*exp(a)^(1/2)-1)-1/4/exp(a)^(1/2)
*ln(x*exp(a)^(1/2)+1)-1/4/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/4/(-exp
(a))^(1/2)*ln(x*(-exp(a))^(1/2)-1)
```

Maxima [A] time = 1.57043, size = 81, normalized size = 1.35

$$-\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}+\frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{xe^{a}-e^{\left(\frac{1}{2}a\right)}}{xe^{a}+e^{\left(\frac{1}{2}a\right)}}\right)+x-\frac{x}{x^{4}e^{(2a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log((x*e^a - e^(1/2*a)
)/(x*e^a + e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

Fricas [B] time = 2.64083, size = 246, normalized size = 4.1

$$\frac{4x^{5}e^{(3a)}-2\left(x^{4}e^{(2a)}-1\right)\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}+\left(x^{4}e^{(2a)}-1\right)e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^{2}e^{a}-2xe^{\left(\frac{1}{2}a\right)}+1}{x^{2}e^{a}-1}\right)-8xe^{a}}{4\left(x^{4}e^{(3a)}-e^{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^5*e^(3*a) - 2*(x^4*e^(2*a) - 1)*arctan(x*e^(1/2*a))*e^(1/2*a) + (x
^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1))
- 8*x*e^a)/(x^4*e^(3*a) - e^a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\coth^{2}\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2, x)
```

Giac [A] time = 1.119, size = 89, normalized size = 1.48

$$-\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}+\frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{\left|2xe^{a}-2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^{a}+2e^{\left(\frac{1}{2}a\right)}\right|}\right)+x-\frac{x}{x^{4}e^{(2a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^
(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

3.162 $\int \frac{\coth^2(a+2\log(x))}{x}\,dx$

Optimal. Leaf size=14

$$\log(x)-\frac{1}{2}\coth(a+2\log(x))$$

```
[Out] -Coth[a + 2*Log[x]]/2 + Log[x]
```

Rubi [A] time = 0.0239965, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3473, 8}

$$\log(x)-\frac{1}{2}\coth(a+2\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]^2/x,x]
```

```
[Out] -Coth[a + 2*Log[x]]/2 + Log[x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(a+2\log(x))}{x}\,dx &= \text{Subst}\left(\int \coth^2(a+2x)\,dx,x,\log(x)\right)\\ &= -\frac{1}{2}\coth(a+2\log(x))+\text{Subst}\left(\int 1\,dx,x,\log(x)\right)\\ &= -\frac{1}{2}\coth(a+2\log(x))+\log(x)\end{aligned}$$

Mathematica [C] time = 0.0539936, size = 28, normalized size = 2.

$$-\frac{1}{2}\coth(a+2\log(x))\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(a+2\log(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x,x]
```

```
[Out] -(Coth[a + 2*Log[x]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + 2*Log[x]]^2])
/2
```

Maple [B] time = 0.003, size = 35, normalized size = 2.5

$$-\frac{\coth\left(a+2\ln\left(x\right)\right)}{2}-\frac{\ln\left(\coth\left(a+2\ln\left(x\right)\right)-1\right)}{4}+\frac{\ln\left(\coth\left(a+2\ln\left(x\right)\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2/x,x)
```

```
[Out] -1/2*coth(a+2*ln(x))-1/4*ln(coth(a+2*ln(x))-1)+1/4*ln(coth(a+2*ln(x))+1)
```

Maxima [A] time = 1.03175, size = 26, normalized size = 1.86

$$\frac{1}{2}a+\frac{1}{e^{(-2a-4\log(x))}-1}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*a + 1/(e^(-2*a - 4*log(x)) - 1) + log(x)
```

Fricas [B] time = 2.52145, size = 68, normalized size = 4.86

$$\frac{\left(x^4e^{(2a)}-1\right)\log\left(x\right)-1}{x^4e^{(2a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="fricas")
```

```
[Out] ((x^4*e^(2*a) - 1)*log(x) - 1)/(x^4*e^(2*a) - 1)
```

Sympy [A] time = 17.2266, size = 32, normalized size = 2.29

$$\begin{cases} \tilde{\infty}\log\left(x\right) & \text{for}\ a=\log\left(-\frac{1}{x^2}\right)\vee a=\log\left(\frac{1}{x^2}\right) \\ \log\left(x\right)-\frac{1}{2\tanh\left(a+2\log\left(x\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2/x,x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, log(x**(-2))) | Eq(a, log(-1/x**2))), (log(x)
- 1/(2*tanh(a + 2*log(x))), True))
```

Giac [A] time = 1.11646, size = 28, normalized size = 2.

$$-\frac{1}{x^4e^{(2a)}-1}+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="giac")
```

```
[Out] -1/(x^4*e^(2*a) - 1) + 1/4*log(x^4)
```

3.163 $\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx$

Optimal. Leaf size=86

$$\frac{2e^{2a}x^3}{1-e^{2a}x^4}-\frac{1}{x\left(1-e^{2a}x^4\right)}-\frac{1}{2}e^{a/2}\tan^{-1}\left(e^{a/2}x\right)+\frac{1}{2}e^{a/2}\tanh^{-1}\left(e^{a/2}x\right)$$

```
[Out] -(1/(x*(1 - E^(2*a)*x^4))) + (2*E^(2*a)*x^3)/(1 - E^(2*a)*x^4) - (E^(a/2)*A
rcTan[E^(a/2)*x])/2 + (E^(a/2)*ArcTanh[E^(a/2)*x])/2
```

Rubi [F] time = 0.0448182, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]^2/x^2,x]

[Out] Defer[Int][Coth[a + 2*Log[x]]^2/x^2, x]
```

Rubi steps

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx=\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx$$

Mathematica [C] time = 3.12673, size = 153, normalized size = 1.78

$$\frac{16}{231}e^{2a}x^3\left(e^{2a}x^4+1\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{3}{4},2,2,2\right\},\left\{1,1,\frac{15}{4}\right\},e^{2a}x^4\right)+\frac{e^{-2a}\left(\left(-e^{8a}x^{16}-56e^{6a}x^{12}+362e^{4a}x^8\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x^2,x]

[Out] (-343 - 1163*E^(2*a)*x^4 - 241*E^(4*a)*x^8 + 3*E^(6*a)*x^12 + (343 + 632*E^
(2*a)*x^4 + 362*E^(4*a)*x^8 - 56*E^(6*a)*x^12 - E^(8*a)*x^16)*Hypergeometri
c2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(384*E^(2*a)*x^5) + (16*E^(2*a)*x^3*(1 + E^
(2*a)*x^4)^2*HypergeometricPFQ[{3/4, 2, 2, 2}, {1, 1, 15/4}, E^(2*a)*x^4])/
231
```

Maple [C] time = 0.033, size = 101, normalized size = 1.2

$$\frac{-2\,e^{2\,a}x^4+1}{x\left(e^{2\,a}x^4-1\right)}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^2+e^{a}\right)}\_R\,\ln\left(\left(-5\_R^4+4\,e^{2\,a}\right)x-\_R^3\right)}{4}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^2-e^{a}\right)}\_R\,\ln\left(\left(-5\_R^4+4\,e^{2\,a}\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2/x^2,x)
```

```
[Out] (-2*exp(2*a)*x^4+1)/x/(exp(2*a)*x^4-1)+1/4*sum(_R*ln((-5*_R^4+4*exp(2*a))*x
-_R^3),_R=RootOf(_Z^2+exp(a)))+1/4*sum(_R*ln((-5*_R^4+4*exp(2*a))*x-_R^3),_
R=RootOf(_Z^2-exp(a)))
```

Maxima [A] time = 1.56413, size = 93, normalized size = 1.08

$$\frac{1}{2}\arctan\left(\frac{e^{\left(-\frac{1}{2}a\right)}}{x}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{4}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\frac{1}{x}-e^{\left(\frac{1}{2}a\right)}}{\frac{1}{x}+e^{\left(\frac{1}{2}a\right)}}\right)-\frac{1}{x}+\frac{e^{(2a)}}{x\left(\frac{1}{x^4}-e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*arctan(e^(-1/2*a)/x)*e^(1/2*a) - 1/4*e^(1/2*a)*log((1/x - e^(1/2*a))/(1
/x + e^(1/2*a))) - 1/x + e^(2*a)/(x*(1/x^4 - e^(2*a)))
```

Fricas [A] time = 2.60401, size = 236, normalized size = 2.74

$$-\frac{8\,x^4e^{(2a)}+2\left(x^5e^{(2a)}-x\right)\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\left(x^5e^{(2a)}-x\right)e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a+2\,xe^{\left(\frac{1}{2}a\right)}+1}{x^2e^a-1}\right)-4}{4\left(x^5e^{(2a)}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*x^4*e^(2*a) + 2*(x^5*e^(2*a) - x)*arctan(x*e^(1/2*a))*e^(1/2*a) - (
x^5*e^(2*a) - x)*e^(1/2*a)*log((x^2*e^a + 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1))
 - 4)/(x^5*e^(2*a) - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth^2\left(a+2\log\left(x\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2/x**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2/x**2, x)
```

Giac [A] time = 1.12148, size = 104, normalized size = 1.21

$$-\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{4}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^a-2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^a+2\,e^{\left(\frac{1}{2}a\right)}\right|}\right)-\frac{2\,x^4e^{(2a)}-1}{x^5e^{(2a)}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(1/2*a) - 1/4*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(1
/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) - (2*x^4*e^(2*a) - 1)/(x^5*e^(2*a) - x)
```

3.164 $\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$

Optimal. Leaf size=60

$$\frac{3e^{2a}x^2}{2\left(1-e^{2a}x^4\right)}-\frac{1}{2x^2\left(1-e^{2a}x^4\right)}+e^a\tanh^{-1}\left(e^ax^2\right)$$

```
[Out] -1/(2*x^2*(1 - E^(2*a)*x^4)) + (3*E^(2*a)*x^2)/(2*(1 - E^(2*a)*x^4)) + E^a*
ArcTanh[E^a*x^2]
```

Rubi [F] time = 0.0460271, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] Defer[Int][Coth[a + 2*Log[x]]^2/x^3, x]
```

Rubi steps

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx=\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$

Mathematica [C] time = 2.86299, size = 155, normalized size = 2.58

$$\frac{64\left(e^{3a}x^6+e^ax^2\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},2,2,2\right\},\left\{1,1,\frac{7}{2}\right\},e^{2a}x^4\right)+15\left(e^{4a}x^8-17e^{2a}x^4-\frac{27e^{-2a}}{x^4}-77\right)-\frac{15\left(e^{8a}x^{16}+4e^{\cdot}\right.}{}}{480x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] (15*(-77 - 27/(E^(2*a)*x^4) - 17*E^(2*a)*x^4 + E^(4*a)*x^8) - (15*(-27 - 52
*E^(2*a)*x^4 - 54*E^(4*a)*x^8 + 4*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[Sqrt
[E^(2*a)*x^4]])/(E^(2*a)*x^4)^(3/2) + 64*(E^a*x^2 + E^(3*a)*x^6)^2*Hypergeo
metricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, E^(2*a)*x^4])/(480*x^2)
```

Maple [A] time = 0.023, size = 55, normalized size = 0.9

$$\frac{1}{x^2\left(e^{2a}x^4-1\right)}\left(-\frac{3e^{2a}x^4}{2}+\frac{1}{2}\right)+\frac{e^a\ln\left(e^ax^2+1\right)}{2}-\frac{e^a\ln\left(e^ax^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2/x^3,x)
```

```
[Out] (-3/2*exp(2*a)*x^4+1/2)/x^2/(exp(2*a)*x^4-1)+1/2*exp(a)*ln(exp(a)*x^2+1)-1/
2*exp(a)*ln(exp(a)*x^2-1)
```

Maxima [A] time = 1.07843, size = 68, normalized size = 1.13

$$\frac{1}{2}e^{a}\log\left(\frac{1}{x^{2}}+e^{a}\right)-\frac{1}{2}e^{a}\log\left(\frac{1}{x^{2}}-e^{a}\right)-\frac{1}{2x^{2}}+\frac{e^{(2a)}}{x^{2}\left(\frac{1}{x^{4}}-e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*e^a*log(1/x^2 + e^a) - 1/2*e^a*log(1/x^2 - e^a) - 1/2/x^2 + e^(2*a)/(x^
2*(1/x^4 - e^(2*a)))
```

Fricas [A] time = 2.58052, size = 178, normalized size = 2.97

$$-\frac{3x^{4}e^{(2a)}-\left(x^{6}e^{(3a)}-x^{2}e^{a}\right)\log\left(x^{2}e^{a}+1\right)+\left(x^{6}e^{(3a)}-x^{2}e^{a}\right)\log\left(x^{2}e^{a}-1\right)-1}{2\left(x^{6}e^{(2a)}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4*e^(2*a) - (x^6*e^(3*a) - x^2*e^a)*log(x^2*e^a + 1) + (x^6*e^(3*
a) - x^2*e^a)*log(x^2*e^a - 1) - 1)/(x^6*e^(2*a) - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth^{2}(a+2\log(x))}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2/x**3,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2/x**3, x)
```

Giac [A] time = 1.15239, size = 77, normalized size = 1.28

$$\frac{1}{2}e^{a}\log\left(x^{2}e^{a}+1\right)-\frac{1}{2}e^{a}\log\left(\left|x^{2}e^{a}-1\right|\right)-\frac{3x^{4}e^{(2a)}-1}{2\left(x^{6}e^{(2a)}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*e^a*log(x^2*e^a + 1) - 1/2*e^a*log(abs(x^2*e^a - 1)) - 1/2*(3*x^4*e^(2*
a) - 1)/(x^6*e^(2*a) - x^2)
```

3.165 $\int (ex)^m \coth(a+2\log(x))\,dx$

Optimal. Leaf size=59

$$\frac{(ex)^{m+1}}{e(m+1)}-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};e^{2a}x^4\right)}{e(m+1)}$$

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/4
, (5 + m)/4, E^(2*a)*x^4])/(e*(1 + m))
```

Rubi [F] time = 0.0387854, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \coth(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[a + 2*Log[x]], x]
```

Rubi steps

$$\int (ex)^m \coth(a+2\log(x))\,dx=\int (ex)^m \coth(a+2\log(x))\,dx$$

Mathematica [A] time = 0.0933579, size = 46, normalized size = 0.78

$$-\frac{x(ex)^m\left(2\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};x^4(\cosh(2a)+\sinh(2a))\right)-1\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]],x]
```

```
[Out] -((x*(e*x)^m*(-1 + 2*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])]))/(1 + m))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (ex)^m \coth(a+2\ln(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x)),x)
```

```
[Out] int((e*x)^m*coth(a+2*ln(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth (a + 2 \log (x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \coth (a + 2 \log (x)), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(a + 2*log(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth (a + 2 \log (x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x)),x)
```

```
[Out] Integral((e*x)**m*coth(a + 2*log(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth (a + 2 \log (x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x)), x)
```

3.166 $\int (ex)^m \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=79

$$-\frac{(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};e^{2a}x^4\right)}{e}+\frac{(ex)^{m+1}}{e\left(1-e^{2a}x^4\right)}+\frac{(ex)^{m+1}}{e(m+1)}$$

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) + (e*x)^(1 + m)/(e*(1 - E^(2*a)*x^4)) - ((e*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, E^(2*a)*x^4])/e
```

Rubi [F] time = 0.0679443, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \coth^2(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^2,x]

[Out] Defer[Int][(e*x)^m*Coth[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int (ex)^m \coth^2(a+2\log(x))\,dx=\int (ex)^m \coth^2(a+2\log(x))\,dx$$

Mathematica [B] time = 0.369473, size = 165, normalized size = 2.09

$$\frac{x(ex)^m\left(\frac{x^4(\sinh(a)+\cosh(a))\left((m+5)x^4(\sinh(a)+\cosh(a))\,{}_2F_1\left(2,\frac{m+9}{4};\frac{m+13}{4};x^4(\cosh(2a)+\sinh(2a))\right)+2(m+9)(\cosh(a)-\sinh(a))\,{}_2F_1\left(2,\frac{m+5}{4};\frac{m+9}{4};x^4(\cosh\right.\right.}{(m+5)(m+9)}\right.}{(\cosh(a)-\sinh(a))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^2,x]

[Out] (x*(e*x)^m*((x^4*(Cosh[a] + Sinh[a])*(2*(9 + m)*Hypergeometric2F1[2, (5 + m
)/4, (9 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])]*(Cosh[a] - Sinh[a]) + (5 + m)*
x^4*Hypergeometric2F1[2, (9 + m)/4, (13 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])
]*(Cosh[a] + Sinh[a])))/((5 + m)*(9 + m)) + (Hypergeometric2F1[2, (1 + m)/4
, (5 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])]*(Cosh[2*a] - Sinh[2*a]))/(1 + m))
)/(Cosh[a] - Sinh[a])^2
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int (ex)^m\left(\coth\left(a+2\ln(x)\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x))^2,x)
```

```
[Out] int((e*x)^m*coth(a+2*ln(x))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \log(x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \coth(a + 2 \log(x))^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(a + 2*log(x))^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \log(x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

3.167 $\int (ex)^m \coth^3(a+2\log(x))\,dx$

Optimal. Leaf size=177

$$-\frac{(m^2+2m+9)(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};e^{2a}x^4\right)}{4e(m+1)}-\frac{\left(e^{2a}x^4+1\right)^2(ex)^{m+1}}{4e\left(1-e^{2a}x^4\right)^2}-\frac{e^{-2a}\left(e^{2a}(3-m)-e^{4a}(m+5)x^4\right)(ex)^{m+1}}{8e\left(1-e^{2a}x^4\right)}+$$

```
[Out] ((3 + m)*(5 + m)*(e*x)^(1 + m))/(8*e*(1 + m)) - ((e*x)^(1 + m)*(1 + E^(2*a)
*x^4)^2)/(4*e*(1 - E^(2*a)*x^4)^2) - ((e*x)^(1 + m)*(E^(2*a)*(3 - m) - E^(4
*a)*(5 + m)*x^4))/(8*e*E^(2*a)*(1 - E^(2*a)*x^4)) - ((9 + 2*m + m^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, E^(2*a)*x^4])/(4*e*(1 +
m))
```

Rubi [F] time = 0.0728497, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \coth^3(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[a + 2*Log[x]]^3, x]
```

Rubi steps

$$\int (ex)^m \coth^3(a+2\log(x))\,dx=\int (ex)^m \coth^3(a+2\log(x))\,dx$$

Mathematica [A] time = 0.825247, size = 215, normalized size = 1.21

$$\frac{x(ex)^m\left(-\frac{x^8(\sinh(2a)+\cosh(2a))\left((m+9)x^4(\sinh(a)+\cosh(a))\,{}_2F_1\left(3,\frac{m+13}{4};\frac{m+17}{4};x^4(\cosh(2a)+\sinh(2a))\right)+3(m+13)(\cosh(a)-\sinh(a))\,{}_2F_1\left(3,\frac{m+9}{4};\frac{m+13}{4};x\right.\right.}{(m+9)(m+13)}\right.}{(\cosh(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
```

```
[Out] (x*(e*x)^m*((-3*x^4*Hypergeometric2F1[3, (5 + m)/4, (9 + m)/4, x^4*(Cosh[2*
a] + Sinh[2*a])]*(Cosh[a] - Sinh[a]))/(5 + m) - (Hypergeometric2F1[3, (1 +
m)/4, (5 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])]*(Cosh[a] - Sinh[a])^3)/(1 + m
) - (x^8*(3*(13 + m)*Hypergeometric2F1[3, (9 + m)/4, (13 + m)/4, x^4*(Cosh[
2*a] + Sinh[2*a])]*(Cosh[a] - Sinh[a]) + (9 + m)*x^4*Hypergeometric2F1[3, (
13 + m)/4, (17 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])]*(Cosh[a] + Sinh[a]))*(C
osh[2*a] + Sinh[2*a]))/((9 + m)*(13 + m))))/(Cosh[a] - Sinh[a])^3
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (ex)^m (\coth (a+2\ln (x)))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x))^3,x)
```

```
[Out] int((e*x)^m*coth(a+2*ln(x))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth (a+2\log (x))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex)^m \coth (a+2\log (x))^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(a + 2*log(x))^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth (a+2\log (x))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

3.168 $\int \coth^p(a + b\log(x))\,dx$

Optimal. Leaf size=79

$$x\left(-e^{2a}x^{2b}-1\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}F_1\left(\frac{1}{2b};p,-p;\frac{1}{2}\left(2+\frac{1}{b}\right);e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

```
[Out] (x*(-1 - E^(2*a)*x^(2*b))^p*AppellF1[1/(2*b), p, -p, (2 + b^(-1))/2, E^(2*a
)*x^(2*b), -(E^(2*a)*x^(2*b))])/(1 + E^(2*a)*x^(2*b))^p
```

Rubi [F] time = 0.0221232, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^p(a + b\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*Log[x]]^p,x]
```

```
[Out] Defer[Int][Coth[a + b*Log[x]]^p, x]
```

Rubi steps

$$\int \coth^p(a + b\log(x))\,dx = \int \coth^p(a + b\log(x))\,dx$$

Mathematica [B] time = 1.88852, size = 259, normalized size = 3.28

$$\frac{(2b+1)x\left(\frac{e^{2a}x^{2b}+1}{e^{2a}x^{2b}-1}\right)^p F_1\left(\frac{1}{2b};p,-p;1+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{2e^{2a}bpx^{2b}F_1\left(1+\frac{1}{2b};p,1-p;2+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)+2e^{2a}bpx^{2b}F_1\left(1+\frac{1}{2b};p+1,-p;2+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)+(2b+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + b*Log[x]]^p,x]
```

```
[Out] ((1 + 2*b)*x*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(2*b)))^p*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(2*b*E^(2*a)
*p*x^(2*b)*AppellF1[1 + 1/(2*b), p, 1 - p, 2 + 1/(2*b), E^(2*a)*x^(2*b), -(
E^(2*a)*x^(2*b))] + 2*b*E^(2*a)*p*x^(2*b)*AppellF1[1 + 1/(2*b), 1 + p, -p,
2 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))] + (1 + 2*b)*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \left(\coth\left(a + b\ln(x)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(x))^p,x)

[Out] int(coth(a+b*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(b\log\left(x\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="maxima")

[Out] integrate(coth(b*log(x) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\coth\left(b\log\left(x\right)+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="fricas")

[Out] integral(coth(b*log(x) + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^{p}\left(a+b\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(x))**p,x)

[Out] Integral(coth(a + b*log(x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(b\log\left(x\right)+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="giac")

[Out] integrate(coth(b*log(x) + a)^p, x)
```

3.169 $\int (ex)^m \coth^p(a + b\log(x))\, dx$

Optimal. Leaf size=99

$$\frac{(ex)^{m+1}\left(-e^{2a}x^{2b}-1\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}F_1\left(\frac{m+1}{2b};p,-p;\frac{m+1}{2b}+1;e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(m+1)}$$

```
[Out] ((e*x)^(1 + m)*(-1 - E^(2*a)*x^(2*b))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 +
(1 + m)/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(e*(1 + m)*(1 + E^(2*a
)*x^(2*b))^p)
```

Rubi [F] time = 0.115831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \coth^p(a + b\log(x))\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[a + b*Log[x]]^p,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[a + b*Log[x]]^p, x]
```

Rubi steps

$$\int (ex)^m \coth^p(a + b\log(x))\, dx = \int (ex)^m \coth^p(a + b\log(x))\, dx$$

Mathematica [A] time = 2.97686, size = 126, normalized size = 1.27

$$\frac{x(ex)^m\left(1-e^{2a}x^{2b}\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}\left(\frac{e^{2a}x^{2b}+1}{e^{2a}x^{2b}-1}\right)^p F_1\left(\frac{m+1}{2b};p,-p;\frac{m+1}{2b}+1;e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Coth[a + b*Log[x]]^p,x]
```

```
[Out] (x*(e*x)^m*(1 - E^(2*a)*x^(2*b))^p*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(
2*b)))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 + (1 + m)/(2*b), E^(2*a)*x^(2*b),
 -(E^(2*a)*x^(2*b))])/((1 + m)*(1 + E^(2*a)*x^(2*b))^p)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (ex)^m (\coth(a + b\ln(x)))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+b*ln(x))^p,x)
```

```
[Out] int((e*x)^m*coth(a+b*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth (b \log (x) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth(b*log(x) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \coth (b \log (x) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*log(x) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+b*ln(x))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth (b \log (x) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(b*log(x) + a)^p, x)
```

3.170 $\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx$

Optimal. Leaf size=52

$$-\frac{e^{-2a}2^{-p}\left(-e^{2a}x-1\right)^{p+1}\,{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}x+1\right)\right)}{p+1}$$

```
[Out] -(((-1 - E^(2*a)*x)^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 + E^(2*a)
*x)/2])/(2^p*E^(2*a)*(1 + p)))
```

Rubi [F] time = 0.046479, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]/2]^p,x]
```

```
[Out] Defer[Int][Coth[(2*a + Log[x])/2]^p, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \coth^p\left(\frac{1}{2}(2a+\log(x))\right)\,dx$$

Mathematica [A] time = 0.391243, size = 83, normalized size = 1.6

$$-\frac{e^{-2a}2^p\left(e^{2a}x+1\right)^{1-p}\left(\frac{e^{2a}x+1}{e^{2a}x-1}\right)^{p-1}\,{}_2F_1\left(1-p,-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}x\right)}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/2]^p,x]
```

```
[Out] -((2^p*(1 + E^(2*a)*x)^(1 - p)*((1 + E^(2*a)*x)/(-1 + E^(2*a)*x))^(-1 + p)*
Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x)/2])/(E^(2*a)*(-1 + p)
))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \left(\coth\left(a+\frac{\ln(x)}{2}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/2*ln(x))^p,x)
```

```
[Out] int(coth(a+1/2*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a + \frac{1}{2}\log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 1/2*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth\left(a + \frac{1}{2}\log(x)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/2*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p\left(a + \frac{\log(x)}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a + \frac{1}{2}\log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/2*log(x))^p, x)
```

3.171 $\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx$

Optimal. Leaf size=108

$$e^{-4a}\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt{x}\right)^{1-p}-\frac{e^{-4a}2^{1-p}p\left(-e^{2a}\sqrt{x}-1\right)^{p+1}{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}\sqrt{x}+1\right)\right)}{p+1}$$

```
[Out] ((-1 - E^(2*a)*Sqrt[x])^(1 + p)*(1 - E^(2*a)*Sqrt[x])^(1 - p))/E^(4*a) - (2
^(1 - p)*p*(-1 - E^(2*a)*Sqrt[x])^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p
, (1 + E^(2*a)*Sqrt[x])/2])/(E^(4*a)*(1 + p))
```

Rubi [F] time = 0.0508788, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]/4]^p,x]
```

```
[Out] Defer[Int][Coth[(4*a + Log[x])/4]^p, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth^p\left(\frac{1}{4}(4a+\log(x))\right)\,dx$$

Mathematica [A] time = 0.526514, size = 125, normalized size = 1.16

$$\frac{e^{-4a}\left(e^{2a}\sqrt{x}+1\right)^{1-p}\left(\frac{e^{2a}\sqrt{x}+1}{e^{2a}\sqrt{x}-1}\right)^{p-1}\left((p-1)\left(e^{2a}\sqrt{x}+1\right)^{p+1}-2^{p+1}p\,{}_2F_1\left(1-p,-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt{x}\right)\right)}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/4]^p,x]
```

```
[Out] ((1 + E^(2*a)*Sqrt[x])^(1 - p)*((1 + E^(2*a)*Sqrt[x])/(-1 + E^(2*a)*Sqrt[x]
))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*Sqrt[x])^(1 + p) - 2^(1 + p)*p*Hypergeom
etric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*Sqrt[x])/2]))/(E^(4*a)*(-1 + p))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \left(\coth\left(a+\frac{\ln(x)}{4}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/4*ln(x))^p,x)
```

```
[Out] int(coth(a+1/4*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a + \frac{1}{4} \log (x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 1/4*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth \left(a + \frac{1}{4} \log (x)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/4*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p \left(a + \frac{\log (x)}{4}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/4)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a + \frac{1}{4} \log (x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/4*log(x))^p, x)
```

3.172 $\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx$

Optimal. Leaf size=162

$$-\frac{e^{-6a}2^{-p}\left(2p^2+1\right)\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}\sqrt[3]{x}+1\right)\right)}{p+1}+e^{-6a}p\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}$$

```
[Out] (p*(-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1 - p))/E^(6*a) +
((-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1 - p)*x^(1/3))/E^(4
*a) - ((1 + 2*p^2)*(-1 - E^(2*a)*x^(1/3))^(1 + p)*Hypergeometric2F1[p, 1 +
p, 2 + p, (1 + E^(2*a)*x^(1/3))/2])/(2^p*E^(6*a)*(1 + p))
```

Rubi [F] time = 0.0499177, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]/6]^p,x]
```

```
[Out] Defer[Int][Coth[(6*a + Log[x])/6]^p, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)dx=\int \coth^p\left(\frac{1}{6}(6a+\log(x))\right)dx$$

Mathematica [A] time = 0.561147, size = 142, normalized size = 0.88

$$\frac{e^{-6a}\left(e^{2a}\sqrt[3]{x}+1\right)^{1-p}\left(\frac{e^{2a}\sqrt[3]{x}+1}{e^{2a}\sqrt[3]{x}-1}\right)^{p-1}\left((p-1)\left(e^{2a}\sqrt[3]{x}+1\right)^{p+1}\left(e^{2a}\sqrt[3]{x}+p\right)-2^p\left(2p^2+1\right){}_2F_1\left(1-p,-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt[3]{x}\right)\right.}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/6]^p,x]
```

```
[Out] ((1 + E^(2*a)*x^(1/3))^(1 - p)*((1 + E^(2*a)*x^(1/3))/(-1 + E^(2*a)*x^(1/3)
))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*x^(1/3))^(1 + p)*(p + E^(2*a)*x^(1/3)) -
 2^p*(1 + 2*p^2)*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x^(1/3)
)/2]))/(E^(6*a)*(-1 + p))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \left(\coth\left(a+\frac{\ln(x)}{6}\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/6*ln(x))^p,x)
```

```
[Out] int(coth(a+1/6*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a + \frac{1}{6}\log\left(x\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 1/6*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\coth\left(a + \frac{1}{6}\log\left(x\right)\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/6*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^{p}\left(a + \frac{\log\left(x\right)}{6}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/6)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a + \frac{1}{6}\log\left(x\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/6*log(x))^p, x)
```

3.173 $\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$

Optimal. Leaf size=194

$$-\frac{e^{-8a}2^{2-p}p\left(p^2+2\right)\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}\sqrt[4]{x}+1\right)\right)}{3(p+1)}+\frac{1}{3}e^{-12a}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(e^{4a}\left(2p^2+3\right)+2e\right.$$

```
[Out] ((-1 - E^(2*a)*x^(1/4))^(1 + p)*(1 - E^(2*a)*x^(1/4))^(1 - p)*(E^(4*a)*(3 +
 2*p^2) + 2*E^(6*a)*p*x^(1/4)))/(3*E^(12*a)) + ((-1 - E^(2*a)*x^(1/4))^(1 +
 p)*(1 - E^(2*a)*x^(1/4))^(1 - p)*Sqrt[x])/E^(4*a) - (2^(2 - p)*p*(2 + p^2)
*(-1 - E^(2*a)*x^(1/4))^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 + E^(
2*a)*x^(1/4))/2])/(3*E^(8*a)*(1 + p))
```

Rubi [F] time = 0.0507109, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]/8]^p,x]
```

```
[Out] Defer[Int][Coth[(8*a + Log[x])/8]^p, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \coth^p\left(\frac{1}{8}(8a+\log(x))\right)\,dx$$

Mathematica [A] time = 0.906553, size = 223, normalized size = 1.15

$$\frac{e^{-8a}\left(e^{2a}\sqrt[4]{x}+1\right)^{1-p}\left(\frac{e^{2a}\sqrt[4]{x}+1}{e^{2a}\sqrt[4]{x}-1}\right)^{p-1}\left(-2^{p+3}p\,{}_2F_1\left(-p-2,1-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt[4]{x}\right)+2^{p+2}(2p-1)\,{}_2F_1\left(-p-1,1-p;2-p\right.\right.}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/8]^p,x]
```

```
[Out] ((1 + E^(2*a)*x^(1/4))^(1 - p)*((1 + E^(2*a)*x^(1/4))/(-1 + E^(2*a)*x^(1/4)
))^(-1 + p)*(-(2^(3 + p)*p*Hypergeometric2F1[-2 - p, 1 - p, 2 - p, 1/2 - (E
^(2*a)*x^(1/4))/2]) + 2^(2 + p)*(-1 + 2*p)*Hypergeometric2F1[-1 - p, 1 - p,
 2 - p, 1/2 - (E^(2*a)*x^(1/4))/2] + (-1 + p)*(E^(4*a)*(1 + E^(2*a)*x^(1/4)
)^(1 + p)*Sqrt[x] - 2^(1 + p)*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^
(2*a)*x^(1/4))/2])))/(E^(8*a)*(-1 + p))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(\coth\left(a+\frac{\ln(x)}{8}\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/8*ln(x))^p,x)
```

```
[Out] int(coth(a+1/8*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a+\frac{1}{8}\log(x)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 1/8*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth\left(a+\frac{1}{8}\log(x)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/8*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^{p}\left(a+\frac{\log(x)}{8}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/8)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a+\frac{1}{8}\log(x)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/8*log(x))^p, x)
```

3.174 $\int \coth^p(a+\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(-e^{2a}x^2-1\right)^p\left(e^{2a}x^2+1\right)^{-p}F_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)$$

```
[Out] (x*(-1 - E^(2*a)*x^2)^p*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2*a)*x^
2)])/(1 + E^(2*a)*x^2)^p
```

Rubi [F] time = 0.0153368, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^p(a+\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]]^p,x]
```

```
[Out] Defer[Int][Coth[a + Log[x]]^p, x]
```

Rubi steps

$$\int \coth^p(a+\log(x))\,dx=\int \coth^p(a+\log(x))\,dx$$

Mathematica [B] time = 1.54812, size = 171, normalized size = 2.8

$$\frac{3x\left(\frac{e^{2a}x^2+1}{e^{2a}x^2-1}\right)^p F_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)}{2e^{2a}px^2\left(F_1\left(\frac{3}{2};p,1-p;\frac{5}{2};e^{2a}x^2,-e^{2a}x^2\right)+F_1\left(\frac{3}{2};p+1,-p;\frac{5}{2};e^{2a}x^2,-e^{2a}x^2\right)\right)+3F_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]]^p,x]
```

```
[Out] (3*x*((1 + E^(2*a)*x^2)/(-1 + E^(2*a)*x^2))^p*AppellF1[1/2, p, -p, 3/2, E^(
2*a)*x^2, -(E^(2*a)*x^2)])/(3*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2
*a)*x^2)] + 2*E^(2*a)*p*x^2*(AppellF1[3/2, p, 1 - p, 5/2, E^(2*a)*x^2, -(E^
(2*a)*x^2)] + AppellF1[3/2, 1 + p, -p, 5/2, E^(2*a)*x^2, -(E^(2*a)*x^2)]))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int (\coth(a+\ln(x)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+ln(x))^p,x)
```

```
[Out] int(coth(a+ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a + \log(x))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth(a + \log(x))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p(a + \log(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a + \log(x))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + log(x))^p, x)
```

3.175 $\int \coth^p(a+2\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(-e^{2a}x^4-1\right)^p\left(e^{2a}x^4+1\right)^{-p}F_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)$$

```
[Out] (x*(-1 - E^(2*a)*x^4)^p*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2*a)*x^
4)])/(1 + E^(2*a)*x^4)^p
```

Rubi [F] time = 0.0105149, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^p(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]^p,x]
```

```
[Out] Defer[Int][Coth[a + 2*Log[x]]^p, x]
```

Rubi steps

$$\int \coth^p(a+2\log(x))\,dx=\int \coth^p(a+2\log(x))\,dx$$

Mathematica [B] time = 1.7343, size = 171, normalized size = 2.8

$$\frac{5x\left(\frac{e^{2a}x^4+1}{e^{2a}x^4-1}\right)^pF_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)}{4e^{2a}px^4\left(F_1\left(\frac{5}{4};p,1-p;\frac{9}{4};e^{2a}x^4,-e^{2a}x^4\right)+F_1\left(\frac{5}{4};p+1,-p;\frac{9}{4};e^{2a}x^4,-e^{2a}x^4\right)\right)+5F_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 2*Log[x]]^p,x]
```

```
[Out] (5*x*((1 + E^(2*a)*x^4)/(-1 + E^(2*a)*x^4))^p*AppellF1[1/4, p, -p, 5/4, E^(
2*a)*x^4, -(E^(2*a)*x^4)])/(5*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2
*a)*x^4)] + 4*E^(2*a)*p*x^4*(AppellF1[5/4, p, 1 - p, 9/4, E^(2*a)*x^4, -(E^
(2*a)*x^4)] + AppellF1[5/4, 1 + p, -p, 9/4, E^(2*a)*x^4, -(E^(2*a)*x^4)]))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int (\coth(a+2\ln(x)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^p,x)
```

```
[Out] int(coth(a+2*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a+2 \log \left(x\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 2*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth \left(a+2 \log \left(x\right)\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 2*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^{p}\left(a+2\log \left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**p,x)
```

```
[Out] Integral(coth(a + 2*log(x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a+2 \log \left(x\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 2*log(x))^p, x)
```

3.176 $\int \coth^p(a+3\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(-e^{2a}x^6-1\right)^p\left(e^{2a}x^6+1\right)^{-p}F_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)$$

```
[Out] (x*(-1 - E^(2*a)*x^6)^p*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2*a)*x^
6)])/(1 + E^(2*a)*x^6)^p
```

Rubi [F] time = 0.0179362, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^p(a+3\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 3*Log[x]]^p,x]
```

```
[Out] Defer[Int][Coth[a + 3*Log[x]]^p, x]
```

Rubi steps

$$\int \coth^p(a+3\log(x))\,dx=\int \coth^p(a+3\log(x))\,dx$$

Mathematica [B] time = 1.81414, size = 171, normalized size = 2.8

$$\frac{7x\left(\frac{e^{2a}x^6+1}{e^{2a}x^6-1}\right)^pF_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)}{6e^{2a}px^6\left(F_1\left(\frac{7}{6};p,1-p;\frac{13}{6};e^{2a}x^6,-e^{2a}x^6\right)+F_1\left(\frac{7}{6};p+1,-p;\frac{13}{6};e^{2a}x^6,-e^{2a}x^6\right)\right)+7F_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 3*Log[x]]^p,x]
```

```
[Out] (7*x*((1 + E^(2*a)*x^6)/(-1 + E^(2*a)*x^6))^p*AppellF1[1/6, p, -p, 7/6, E^(
2*a)*x^6, -(E^(2*a)*x^6)])/(7*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2
*a)*x^6)] + 6*E^(2*a)*p*x^6*(AppellF1[7/6, p, 1 - p, 13/6, E^(2*a)*x^6, -(E
^(2*a)*x^6)] + AppellF1[7/6, 1 + p, -p, 13/6, E^(2*a)*x^6, -(E^(2*a)*x^6)])
)
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int (\coth(a+3\ln(x)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+3*ln(x))^p,x)
```

```
[Out] int(coth(a+3*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a + 3\log\left(x\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 3*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth\left(a + 3\log\left(x\right)\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 3*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^{p}\left(a + 3\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*ln(x))**p,x)
```

```
[Out] Integral(coth(a + 3*log(x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a + 3\log\left(x\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 3*log(x))^p, x)
```

3.177 $\int x^3 \coth(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=58

$$\frac{x^4}{4}-\frac{1}{2}x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] x^4/4 - (x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*a*d)*(c*x^
n)^(2*b*d)])/2
```

Rubi [F] time = 0.0390568, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3\coth(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][x^3*Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int x^3\coth(d(a+b\log(cx^n)))\,dx=\int x^3\coth(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] time = 7.01525, size = 198, normalized size = 3.41

$$-\frac{x^4\left(2e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{2}{bdn};2+\frac{2}{bdn};e^{2d(a+b\log(cx^n))}\right)+(bdn+2)\left({}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2d(a+b\log(cx^n))}\right)+\coth(d(a\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^4*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2
+ 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(Coth[d*(a + b*Log[c
*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1, 2/
(b*d*n), 1 + 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x
^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(8 + 4*
b*d*n))
```

Maple [F] time = 1.095, size = 0, normalized size = 0.

$$\int x^3\coth(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^4 - \int \frac{x^3}{c^{bd}e^{(bd\log(x^n)+ad)} + 1}\,dx + \int \frac{x^3}{c^{bd}e^{(bd\log(x^n)+ad)} - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - integrate(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrat
e(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\coth\left(bd\log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^3*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\coth\left(\left(b\log\left(cx^n\right) + a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^3*coth((b*log(c*x^n) + a)*d), x)
```

3.178 $\int x^2 \coth(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=62

$$\frac{x^3}{3}-\frac{2}{3}x^3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] x^3/3 - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*a*d)
*(c*x^n)^(2*b*d)])/3
```

Rubi [F] time = 0.0299628, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \coth(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][x^2*Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int x^2 \coth(d(a+b\log(cx^n)))\,dx=\int x^2 \coth(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] time = 4.89553, size = 207, normalized size = 3.34

$$-\frac{x^3\left(3e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{3}{2bdn};2+\frac{3}{2bdn};e^{2d(a+b\log(cx^n))}\right)+(2bdn+3)\left({}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2d(a+b\log(cx^n))}\right)+\coth\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^3*(3*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n),
2 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(Coth[d*(a + b
*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1
[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a +
 b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])
))/(9 + 6*b*d*n))
```

Maple [F] time = 1.046, size = 0, normalized size = 0.

$$\int x^2\coth(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3 - \int \frac{x^2}{c^{bd}e^{(bd\log(x^n)+ad)}+1}\,dx + \int \frac{x^2}{c^{bd}e^{(bd\log(x^n)+ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - integrate(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrat
e(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\coth\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\coth\left(ad+bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*coth(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\coth\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*coth((b*log(c*x^n) + a)*d), x)
```

3.179 $\int x \coth(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=54

$$\frac{x^2}{2}-x^2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] x^2/2 - x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*a*d)*(c*x^n
)^(2*b*d)]
```

Rubi [F] time = 0.0239149, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\coth(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][x*Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int x\coth(d(a+b\log(cx^n)))\,dx=\int x\coth(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] time = 6.93912, size = 193, normalized size = 3.57

$$-\frac{x^2\left(e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{1}{bdn};2+\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)+(bdn+1)\left({}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)+\coth(d(a+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^2*(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(Coth[d*(a + b*Log[c*x
^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1, 1/(b
*d*n), 1 + 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n
])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(2 + 2*b*
d*n))
```

Maple [F] time = 1., size = 0, normalized size = 0.

$$\int x\coth(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2 - \int \frac{x}{c^{bd}e^{(bd\log(x^n)+ad)} + 1}\,dx + \int \frac{x}{c^{bd}e^{(bd\log(x^n)+ad)} - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - integrate(x/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(
x/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\coth\left(bd\log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\coth\left(ad + bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*coth(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\coth\left((b\log\left(cx^n\right) + a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x*coth((b*log(c*x^n) + a)*d), x)
```

3.180 $\int \coth (d (a+b \log (c x^n))) \, dx$

Optimal. Leaf size=52

$$x-2 x \, _2F_1\left(1,\frac{1}{2 b d n};1+\frac{1}{2 b d n};e^{2 a d} \left(c x^n\right)^{2 b d}\right)$$

```
[Out] x - 2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n
)^(2*b*d)]
```

Rubi [F] time = 0.0114539, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth (d (a+b \log (c x^n))) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int \coth (d (a+b \log (c x^n))) \, dx=\int \coth (d (a+b \log (c x^n))) \, dx$$

Mathematica [B] time = 8.32863, size = 198, normalized size = 3.81

$$-\frac{x e^{2 d (a+b \log (c x^n))} \, _2F_1\left(1,1+\frac{1}{2 b d n};2+\frac{1}{2 b d n};e^{2 d (a+b \log (c x^n))}\right)}{2 b d n+1}-x \left(\, _2F_1\left(1,\frac{1}{2 b d n};1+\frac{1}{2 b d n};e^{2 d (a+b \log (c x^n))}\right)+\coth (d (a+b \log$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((E^(2*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 + 1
/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(1 + 2*b*d*n)) - x*(Coth[d*(a + b*
Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[
1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a +
b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])
```

Maple [F] time = 0.915, size = 0, normalized size = 0.

$$\int \coth (d (a+b \ln (c x^n))) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x - \int \frac{1}{c^{bd}e^{(bd\log(x^n)+ad)} + 1}\,dx + \int \frac{1}{c^{bd}e^{(bd\log(x^n)+ad)} - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] x - integrate(1/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(1/(c^(
b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth\left(bd\log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(d\left(a + b\log\left(cx^n\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(coth(d*(a + b*log(c*x**n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left((b\log\left(cx^n\right) + a)d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d), x)
```

3.181 $\int \frac{\coth(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=25

$$\frac{\log(\sinh(ad+bd\log(cx^n)))}{bdn}$$

```
[Out] Log[Sinh[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Rubi [A] time = 0.0206705, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3475}

$$\frac{\log(\sinh(ad+bd\log(cx^n)))}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Log[Sinh[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int \coth(d(a+bx))\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\log(\sinh(ad+bd\log(cx^n)))}{bdn}\end{aligned}$$

Mathematica [A] time = 0.0569006, size = 40, normalized size = 1.6

$$\frac{\log(\tanh(ad+bd\log(cx^n)))+\log(\cosh(d(a+b\log(cx^n))))}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (Log[Cosh[d*(a + b*Log[c*x^n])]] + Log[Tanh[a*d + b*d*Log[c*x^n]]])/(b*d*n)
```

Maple [B] time = 0.004, size = 56, normalized size = 2.2

$$-\frac{\ln(\coth(d(a+b\ln(cx^n)))-1)}{2dbn}-\frac{\ln(\coth(d(a+b\ln(cx^n)))+1)}{2dbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] -1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))-1)-1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)
))+1)
```

Maxima [A] time = 1.15765, size = 32, normalized size = 1.28

$$\frac{\log\left(\sinh\left((b\log\left(cx^{n}\right)+a)d\right)\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] log(sinh((b*log(c*x^n) + a)*d))/(b*d*n)
```

Fricas [B] time = 2.91355, size = 205, normalized size = 8.2

$$-\frac{bdn\log\left(x\right)-\log\left(\frac{2\,\sinh(bdn\log(x)+bd\log(c)+ad)}{\cosh(bdn\log(x)+bd\log(c)+ad)-\sinh(bdn\log(x)+bd\log(c)+ad)}\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] -(b*d*n*log(x) - log(2*sinh(b*d*n*log(x) + b*d*log(c) + a*d)/(cosh(b*d*n*lo
g(x) + b*d*log(c) + a*d) - sinh(b*d*n*log(x) + b*d*log(c) + a*d))))/(b*d*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(ad+bd\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x, x)
```

Giac [B] time = 1.24077, size = 99, normalized size = 3.96

$$\frac{\log\left(-2\,x^{2\,bdn}|c|^{2\,bd}\cos\left(\pi bd\mathrm{sgn}\left(c\right)-\pi bd\right)e^{(2\,ad)}+x^{4\,bdn}|c|^{4\,bd}e^{(4\,ad)}+1\right)}{2\,bdn}-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] 1/2*log(-2*x^(2*b*d*n)*abs(c)^(2*b*d)*cos(pi*b*d*sgn(c) - pi*b*d)*e^(2*a*d)
 + x^(4*b*d*n)*abs(c)^(4*b*d)*e^(4*a*d) + 1)/(b*d*n) - log(x)
```

3.182 $\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=58

$$\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{x}-\frac{1}{x}$$

[Out] -x^(-1) + (2*Hypergeometric2F1[1, -1/(2*b*d*n), 1 - 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/x

---

Rubi [F] time = 0.0313433, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^2,x]

[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]/x^2, x]

Rubi steps

$$\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx=\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Mathematica [B] time = 3.82998, size = 197, normalized size = 3.4

$$\frac{-\frac{e^{2d\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(1,1-\frac{1}{2bdn};2-\frac{1}{2bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)}{2bdn-1}+{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)+\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\coth(}{x}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^2,x]

[Out] (Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(-1 + 2*b*d*n) + Hypergeometric2F1[1, -1/(2*b*d*n), 1 - 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/x

---

Maple [F] time = 1.026, size = 0, normalized size = 0.

$$\int \frac{\coth\left(d\left(a+b\ln\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x} - \int \frac{1}{c^{bd}x^2e^{(bd\log(x^n)+ad)} + x^2}\,dx + \int \frac{1}{c^{bd}x^2e^{(bd\log(x^n)+ad)} - x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] -1/x - integrate(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) + x^2), x) + integra
te(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) - x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth\left(bd\log\left(cx^n\right) + ad\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(ad + bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left((b\log\left(cx^n\right) + a)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)/x^2, x)
```

3.183 $\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=55

$$\frac{{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{x^2}-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) + Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*a*d)*(c
*x^n)^(2*b*d)]/x^2
```

Rubi [F] time = 0.0296067, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^3,x]

[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]/x^3, x]
```

Rubi steps

$$\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx=\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Mathematica [B] time = 3.67831, size = 191, normalized size = 3.47

$$\frac{-\frac{e^{2d\left(a+b\log\left(cx^n\right)\right)}{}_2F_1\left(1,1-\frac{1}{bdn};2-\frac{1}{bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)}{bdn-1}+{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)+\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\coth\left(d\left(a\right.\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^3,x]

[Out] (Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^
(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*d*n),
 E^(2*d*(a + b*Log[c*x^n]))])/(-1 + b*d*n) + Hypergeometric2F1[1, -(1/(b*d*
n)), 1 - 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n])
]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/(2*x^2)
```

Maple [F] time = 1.026, size = 0, normalized size = 0.

$$\int \frac{\coth\left(d\left(a+b\ln\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2x^2} - \int \frac{1}{c^{bd}x^3e^{(bd\log(x^n)+ad)} + x^3}\,dx + \int \frac{1}{c^{bd}x^3e^{(bd\log(x^n)+ad)} - x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) + x^3), x) + int
egrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) - x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\coth\left(bd\log\left(cx^n\right) + ad\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(\left(b\log\left(cx^n\right) + a\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)/x^3, x)
```

3.184 $\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=132

$$-\frac{2x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x^4\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}+\frac{1}{4}x^4\left(\frac{4}{bdn}+1\right)$$

```
[Out] ((1 + 4/(b*d*n))*x^4)/4 + (x^4*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2
/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rubi [F] time = 0.0864764, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][x^3*Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx=\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 6.60582, size = 155, normalized size = 1.17

$$\frac{x^4\left((bdn+2)\left(-4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2d(a+b\log(cx^n))}\right)-4\coth(d(a+b\log(cx^n)))+bdn\right)-8e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{2}{bdn}\right.\right.}{4bdn(bdn+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^4*(-8*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 +
 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(b*d*n - 4*Coth[d*(a
+ b*Log[c*x^n])] - 4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*d*
(a + b*Log[c*x^n]))])))/(4*b*d*n*(2 + b*d*n))
```

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int x^3 \left(\coth(d(a+b\ln(cx^n)))\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2\,bd}dnx^4e^{(2\,bd\log(x^n)+2\,ad)}-(bdn+8)x^4}{4\left(bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}-bdn\right)}-4\int\frac{x^3}{bc^{bd}dne^{(bd\log(x^n)+ad)}+bdn}\,dx+4\int\frac{x^3}{bc^{bd}dne^{(bd\log(x^n)+ad)}-bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*c^(2*b*d)*d*n*x^4*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 8)*x^4)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 4*integrate(x^3/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 4*integrate(x^3/(b*c^(b*d)*d*n
*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^3\coth\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\coth\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*coth((b*log(c*x^n) + a)*d)^2, x)
```

3.185 $\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=136

$$-\frac{2x^3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x^3\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}+\frac{1}{3}x^3\left(\frac{3}{bdn}+1\right)$$

```
[Out] ((1 + 3/(b*d*n))*x^3)/3 + (x^3*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 +
 3/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rubi [F] time = 0.0584018, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][x^2*Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx=\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 4.53231, size = 165, normalized size = 1.21

$$\frac{x^3\left((2bdn+3)\left(-3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2d(a+b\log(cx^n))}\right)-3\coth(d(a+b\log(cx^n)))+bdn\right)-9e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\right.\right.}{3bdn(2bdn+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^3*(-9*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n), 2
 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(b*d*n - 3*Coth
[d*(a + b*Log[c*x^n])] - 3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n
), E^(2*d*(a + b*Log[c*x^n]))])))/(3*b*d*n*(3 + 2*b*d*n))
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int x^2(\coth(d(a+b\ln(cx^n))))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2\,bd}dnx^3e^{(2\,bd\log(x^n)+2\,ad)}-(bdn+6)x^3}{3\left(bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}-bdn\right)}-3\int\frac{x^2}{bc^{bd}dne^{(bd\log(x^n)+ad)}+bdn}\,dx+3\int\frac{x^2}{bc^{bd}dne^{(bd\log(x^n)+ad)}-bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c^(2*b*d)*d*n*x^3*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 6)*x^3)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 3*integrate(x^2/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 3*integrate(x^2/(b*c^(b*d)*d*n
*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2\coth\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*coth((b*log(c*x^n) + a)*d)^2, x)
```

3.186 $\int x \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=130

$$-\frac{2x^2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x^2\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}+\frac{1}{2}x^2\left(\frac{2}{bdn}+1\right)$$

```
[Out] ((1 + 2/(b*d*n))*x^2)/2 + (x^2*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1
/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rubi [F] time = 0.0427788, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][x*Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx=\int x \coth^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 6.48103, size = 151, normalized size = 1.16

$$\frac{x^2\left((bdn+1)\left(-2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)-2\coth(d(a+b\log(cx^n)))+bdn\right)-2e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{1}{bdn}\right.\right.}{2bdn(bdn+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^2*(-2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(b*d*n - 2*Coth[d*(a
+ b*Log[c*x^n])] - 2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*d*
(a + b*Log[c*x^n]))])))/(2*b*d*n*(1 + b*d*n))
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int x(\coth(d(a+b\ln(cx^n))))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2\,bd}dnx^2e^{(2\,bd\log(x^n)+2\,ad)}-(bdn+4)x^2}{2\left(bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}-bdn\right)}-2\int\frac{x}{bc^{bd}dne^{(bd\log(x^n)+ad)}+bdn}\,dx+2\int\frac{x}{bc^{bd}dne^{(bd\log(x^n)+ad)}-bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c^(2*b*d)*d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 4)*x^2)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 2*integrate(x/(b*c^(b*d)
*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 2*integrate(x/(b*c^(b*d)*d*n*e^(
b*d*log(x^n) + a*d) - b*d*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\coth\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\coth^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x*coth(a*d + b*d*log(c*x**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x*coth((b*log(c*x^n) + a)*d)^2, x)
```

3.187 $\int \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=126

$$-\frac{2x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdn}+\frac{x\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+x\left(\frac{1}{bdn}+1\right)$$

```
[Out] (1 + 1/(b*d*n))*x + (x*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 - E^(2*a*
d)*(c*x^n)^(2*b*d))) - (2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*
n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rubi [F] time = 0.0139827, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int \coth^2(d(a+b\log(cx^n)))\,dx=\int \coth^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 7.68637, size = 160, normalized size = 1.27

$$\frac{x\left((2bdn+1)\left(-{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)-\coth(d(a+b\log(cx^n)))+bdn\right)-e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{1}{2bdn};\right.}{bdn(2bdn+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x*(-(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 +
1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))]) + (1 + 2*b*d*n)*(b*d*n - Coth[d*(
a + b*Log[c*x^n])] - Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(
2*d*(a + b*Log[c*x^n]))])))/(b*d*n*(1 + 2*b*d*n))
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \left(\coth(d(a+b\ln(cx^n)))\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2\,bd}dnxe^{(2\,bd\log(x^n)+2\,ad)}-(bdn+2)x}{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}-bdn}-\int\frac{1}{bc^{bd}dne^{(bd\log(x^n)+ad)}+bdn}\,dx+\int\frac{1}{bc^{bd}dne^{(bd\log(x^n)+ad)}-bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] (b*c^(2*b*d)*d*n*x*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 2)*x)/(b*c^(2*b*d)
*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - integrate(1/(b*c^(b*d)*d*n*e^(b*
d*log(x^n) + a*d) + b*d*n), x) + integrate(1/(b*c^(b*d)*d*n*e^(b*d*log(x^n)
 + a*d) - b*d*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\coth\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(coth(d*(a + b*log(c*x**n)))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\coth\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2, x)
```

3.188 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=28

$$\log(x)-\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn}$$

```
[Out] -(Coth[a*d + b*d*Log[c*x^n]]/(b*d*n)) + Log[x]
```

Rubi [A] time = 0.0291035, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3473, 8}

$$\log(x)-\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -(Coth[a*d + b*d*Log[c*x^n]]/(b*d*n)) + Log[x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx&=\frac{\text{Subst}\left(\int \coth^2(d(a+bx))\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=-\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn}+\frac{\text{Subst}\left(\int 1\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=-\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn}+\log(x)\end{aligned}$$

Mathematica [C] time = 0.103764, size = 49, normalized size = 1.75

$$-\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2\left(ad+b\log\left(cx^n\right)d\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -((Coth[a*d + b*d*Log[c*x^n]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a*d + b*
d*Log[c*x^n]]^2])/(b*d*n))
```

Maple [B] time = 0.005, size = 80, normalized size = 2.9

$$-\frac{\coth\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)}{dbn}-\frac{\ln\left(\coth\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)-1\right)}{2\,dbn}+\frac{\ln\left(\coth\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)+1\right)}{2\,dbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x,x)
```

```
[Out] -1/b/d/n*coth(d*(a+b*ln(c*x^n)))-1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))-1)+1/
2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))+1)
```

Maxima [A] time = 1.30048, size = 50, normalized size = 1.79

$$-\frac{2}{bc^{2\,bd}dne^{\left(2\,bd\log\left(x^{n}\right)+2\,ad\right)}-bdn}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) + log(x)
```

Fricas [B] time = 2.51242, size = 197, normalized size = 7.04

$$\frac{\left(bdn\log\left(x\right)+1\right)\sinh\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)-\cosh\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)}{bdn\sinh\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

```
[Out] ((b*d*n*log(x) + 1)*sinh(b*d*n*log(x) + b*d*log(c) + a*d) - cosh(b*d*n*log(
x) + b*d*log(c) + a*d))/(b*d*n*sinh(b*d*n*log(x) + b*d*log(c) + a*d))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth^{2}\left(ad+bd\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))**2/x, x)
```

Giac [A] time = 1.33036, size = 50, normalized size = 1.79

$$-\frac{2}{\left(c^{2\,bd}x^{2\,bdn}e^{\left(2\,ad\right)}-1\right)bdn}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

```
[Out] -2/((c^(2*b*d)*x^(2*b*d*n)*e^(2*a*d) - 1)*b*d*n) + log(x)
```

3.189 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=134

$$-\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdnx}+\frac{e^{2ad}\left(cx^n\right)^{2bd}+1}{bdnx\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{1-\frac{1}{bdn}}{x}$$

```
[Out] -((1 - 1/(b*d*n))/x) + (1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x*(1 - E^(2*a
*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -1/(2*b*d*n), 1 - 1/(2*b*d*
n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n*x)
```

Rubi [F] time = 0.0506045, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^2/x^2, x]
```

Rubi steps

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx=\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$$

Mathematica [A] time = 3.56269, size = 158, normalized size = 1.18

$$\frac{e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{2bdn};2-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)-(2bdn-1)\left({}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)+\coth(d(\right.\right.}{bdnx(2bdn-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] (E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*
b*d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + 2*b*d*n)*(b*d*n + Coth[d*(a + b
*Log[c*x^n])] + Hypergeometric2F1[1, -1/(2*b*d*n), 1 - 1/(2*b*d*n), E^(2*d*
(a + b*Log[c*x^n]))]))/(b*d*n*(-1 + 2*b*d*n)*x)
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int \frac{\left(\coth\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)} - bdn + 2}{bc^{2\,bd}dnxe^{(2\,bd\log(x^n)+2\,ad)} - bdnx} + \int \frac{1}{bc^{bd}dnx^2e^{(bd\log(x^n)+ad)} + bdnx^2}\,dx - \int \frac{1}{bc^{bd}dnx^2e^{(bd\log(x^n)+ad)} - bdnx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 2)/(b*c^(2*b*d)*d*n*
x*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x) + integrate(1/(b*c^(b*d)*d*n*x^2*e^
(b*d*log(x^n) + a*d) + b*d*n*x^2), x) - integrate(1/(b*c^(b*d)*d*n*x^2*e^(b
*d*log(x^n) + a*d) - b*d*n*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\coth\left(bd\log\left(cx^n\right) + ad\right)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left((b\log\left(cx^n\right) + a)d\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^2, x)
```

3.190 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=135

$$-\frac{2\,{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdnx^2}+\frac{e^{2ad}\left(cx^n\right)^{2bd}+1}{bdnx^2\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{2-bdn}{2bdnx^2}$$

```
[Out] (2 - b*d*n)/(2*b*d*n*x^2) + (1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x^2*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/
(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n*x^2)
```

Rubi [F] time = 0.0530002, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^2/x^3, x]
```

Rubi steps

$$\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx=\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Mathematica [A] time = 3.55429, size = 156, normalized size = 1.16

$$\frac{2e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{bdn};2-\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)-(bdn-1)\left(2\,{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)+2\coth\left(d\left(\right.\right.\right.}{2bdnx^2(bdn-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*
d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + b*d*n)*(b*d*n + 2*Coth[d*(a + b*L
og[c*x^n])] + 2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*d*(a
 + b*Log[c*x^n]))]))/(2*b*d*n*(-1 + b*d*n)*x^2)
```

Maple [F] time = 0.127, size = 0, normalized size = 0.

$$\int \frac{\left(\coth\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}-bdn+4}{2\left(bc^{2\,bd}dnx^2e^{(2\,bd\log(x^n)+2\,ad)}-bdnx^2\right)}+2\int\frac{1}{bc^{bd}dnx^3e^{(bd\log(x^n)+ad)}+bdnx^3}\,dx-2\int\frac{1}{bc^{bd}dnx^3e^{(bd\log(x^n)+ad)}-bdnx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 4)/(b*c^(2*b*d)*
d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x^2) + 2*integrate(1/(b*c^(b*d)*
d*n*x^3*e^(b*d*log(x^n) + a*d) + b*d*n*x^3), x) - 2*integrate(1/(b*c^(b*d)*
d*n*x^3*e^(b*d*log(x^n) + a*d) - b*d*n*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\coth\left(bd\log\left(cx^n\right)+ad\right)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^3, x)
```

3.191 $\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=43

$$\frac{\log(\sinh(a+b\log(cx^n)))}{bn}-\frac{\coth^2(a+b\log(cx^n))}{2bn}$$

```
[Out] -Coth[a + b*Log[c*x^n]]^2/(2*b*n) + Log[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rubi [A] time = 0.0402723, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3473, 3475}

$$\frac{\log(\sinh(a+b\log(cx^n)))}{bn}-\frac{\coth^2(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -Coth[a + b*Log[c*x^n]]^2/(2*b*n) + Log[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \coth^3(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\coth^2(a+b\log(cx^n))}{2bn}+\frac{\operatorname{Subst}\left(\int \coth(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\coth^2(a+b\log(cx^n))}{2bn}+\frac{\log(\sinh(a+b\log(cx^n)))}{bn}\end{aligned}$$

Mathematica [A] time = 0.206839, size = 52, normalized size = 1.21

$$-\frac{-2\log(\tanh(a+b\log(cx^n)))-2\log(\cosh(a+b\log(cx^n)))+\coth^2(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]^2 - 2*Log[Cosh[a + b*Log[c*x^n]]] - 2*Log[Tanh[a +
 b*Log[c*x^n]]])/(2*b*n)
```

Maple [A] time = 0.009, size = 67, normalized size = 1.6

$$-\frac{(\coth(a+b\ln(cx^n)))^2}{2bn}-\frac{\ln(\coth(a+b\ln(cx^n))-1)}{2bn}-\frac{\ln(\coth(a+b\ln(cx^n))+1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] -1/2*coth(a+b*ln(c*x^n))^2/b/n-1/2/n/b*ln(coth(a+b*ln(c*x^n))-1)-1/2/n/b*ln
(coth(a+b*ln(c*x^n))+1)
```

Maxima [B] time = 1.37249, size = 446, normalized size = 10.37

$$-\frac{4\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}-3}{4\left(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)}-\frac{3\left(2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}-1\right)}{4\left(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)}+\frac{}{4\left(bc^{4\,b}ne\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -1/4*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3)/(b*c^(4*b)*n*e^(4*b*log(x^n) +
4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 3/4*(2*c^(2*b)*e^(2*b*
log(x^n) + 2*a) - 1)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^
(2*b*log(x^n) + 2*a) + b*n) + 1/4*(2*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3)/(b
*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) +
b*n) - 3/4/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x
^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) + lo
g((c^b*e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n) - log(x)
```

Fricas [B] time = 2.67292, size = 1841, normalized size = 42.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) + 4*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^4 - 2*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^2 + b*n*log(x) + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) - b*n
*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c
) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) +
 a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c)
 + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c)
```

```
 + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*sinh(b*n*log(x) + b*log(c
) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)))
 + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3*log(x) - (b*n*log(x) - 1)*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*
log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*l
og(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cos
h(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 - b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) +
b*log(c) + a)^3 - b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*
log(c) + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42911, size = 170, normalized size = 3.95

$$\frac{\log\left(-2\,x^{2\,bn}|c|^{2\,b}\cos\left(\pi b\mathrm{sgn}\left(c\right)-\pi b\right)e^{(2\,a)}+x^{4\,bn}|c|^{4\,b}e^{(4\,a)}+1\right)}{2\,bn}-\frac{3\,c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{2\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^{2}bn}-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/2*log(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b*
n)*abs(c)^(4*b)*e^(4*a) + 1)/(b*n) - 1/2*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c
^(2*b)*x^(2*b*n)*e^(2*a) + 3)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^2*b*n) - log
(x)
```

3.192 $\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=45

$$-\frac{\coth^3(a+b\log(cx^n))}{3bn}-\frac{\coth(a+b\log(cx^n))}{bn}+\log(x)$$

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n)) - Coth[a + b*Log[c*x^n]]^3/(3*b*n) + Log[x]
```

Rubi [A] time = 0.0374456, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3473, 8}

$$-\frac{\coth^3(a+b\log(cx^n))}{3bn}-\frac{\coth(a+b\log(cx^n))}{bn}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n)) - Coth[a + b*Log[c*x^n]]^3/(3*b*n) + Log[x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^4(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\coth^3(a+b\log(cx^n))}{3bn}+\frac{\text{Subst}\left(\int \coth^2(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\coth(a+b\log(cx^n))}{bn}-\frac{\coth^3(a+b\log(cx^n))}{3bn}+\frac{\text{Subst}\left(\int 1\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\coth(a+b\log(cx^n))}{bn}-\frac{\coth^3(a+b\log(cx^n))}{3bn}+\log(x)\end{aligned}$$

Mathematica [C] time = 0.114257, size = 44, normalized size = 0.98

$$-\frac{\coth^3(a+b\log(cx^n))\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(a+b\log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[a + b*Log[
c*x^n]]^2])/(3*b*n)
```

Maple [A] time = 0.003, size = 86, normalized size = 1.9

$$-\frac{(\coth(a+b\ln(cx^n)))^3}{3bn}-\frac{\coth(a+b\ln(cx^n))}{bn}-\frac{\ln(\coth(a+b\ln(cx^n))-1)}{2bn}+\frac{\ln(\coth(a+b\ln(cx^n))+1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] -1/3*coth(a+b*ln(c*x^n))^3/b/n-coth(a+b*ln(c*x^n))/b/n-1/2/n/b*ln(coth(a+b*
ln(c*x^n))-1)+1/2/n/b*ln(coth(a+b*ln(c*x^n))+1)
```

Maxima [B] time = 1.51077, size = 674, normalized size = 14.98

$$-\frac{18\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)}-27\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+11}{12\left(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}-3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}-bn\right)}-\frac{6\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)}-}{12\left(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}-3\,bc^{4\,b}ne^{(4\,b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -1/12*(18*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 27*c^(2*b)*e^(2*b*log(x^n) + 2*a
) + 11)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n)
 + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 1/12*(6*c^(4*b)*e^(
4*b*log(x^n) + 4*a) - 15*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 11)/(b*c^(6*b)*n*
e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)
*n*e^(2*b*log(x^n) + 2*a) - b*n) - 2/3*(3*c^(4*b)*e^(4*b*log(x^n) + 4*a) -
3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) -
 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a
) - b*n) - 1/2*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n*e^(6*b*l
og(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*
b*log(x^n) + 2*a) - b*n) - 2/3/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^
(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n
) + log(x)
```

Fricas [B] time = 2.50828, size = 541, normalized size = 12.02

$$\frac{(3\,bn\log(x)+4)\sinh(bn\log(x)+b\log(c)+a)^3-4\,\cosh(bn\log(x)+b\log(c)+a)^3-12\,\cosh(bn\log(x)+b\log(}{3\left(bn\sinh(bn\log(x)+b\log(c)+a)^3+3\left(bn\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*n*log(x) + 4)*sinh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log(
x) + b*log(c) + a)^3 - 12*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + 3*((3*b*n*log(x) + 4)*cosh(b*n*log(x) + b*log(c) + a)^2
- 3*b*n*log(x) - 4)*sinh(b*n*log(x) + b*log(c) + a))/(b*n*sinh(b*n*log(x) +
```

```
 b*log(c) + a)^3 + 3*(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n
*log(x) + b*log(c) + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43402, size = 90, normalized size = 2.

$$-\frac{4\left(3\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 3\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 2\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)} - 1\right)^{3}bn} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 3*c^(2*b)*x^(2*b*n)*e^(2*a) + 2)/((c^(2
*b)*x^(2*b*n)*e^(2*a) - 1)^3*b*n) + log(x)
```

3.193 $\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=66

$$\frac{\log(\sinh(a+b\log(cx^n)))}{bn}-\frac{\coth^4(a+b\log(cx^n))}{4bn}-\frac{\coth^2(a+b\log(cx^n))}{2bn}$$

```
[Out] -Coth[a + b*Log[c*x^n]]^2/(2*b*n) - Coth[a + b*Log[c*x^n]]^4/(4*b*n) + Log[
Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rubi [A] time = 0.0573933, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3473, 3475}

$$\frac{\log(\sinh(a+b\log(cx^n)))}{bn}-\frac{\coth^4(a+b\log(cx^n))}{4bn}-\frac{\coth^2(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] -Coth[a + b*Log[c*x^n]]^2/(2*b*n) - Coth[a + b*Log[c*x^n]]^4/(4*b*n) + Log[
Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^5(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\coth^4(a+b\log(cx^n))}{4bn}+\frac{\text{Subst}\left(\int \coth^3(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\coth^2(a+b\log(cx^n))}{2bn}-\frac{\coth^4(a+b\log(cx^n))}{4bn}+\frac{\text{Subst}\left(\int \coth(a+bx)\,dx,x,\log\right.}{n}\\ &= -\frac{\coth^2(a+b\log(cx^n))}{2bn}-\frac{\coth^4(a+b\log(cx^n))}{4bn}+\frac{\log(\sinh(a+b\log(cx^n)))}{bn}\end{aligned}$$

Mathematica [A] time = 0.279886, size = 67, normalized size = 1.02

$$-\frac{-4\log(\tanh(a+b\log(cx^n)))-4\log(\cosh(a+b\log(cx^n)))+\coth^4(a+b\log(cx^n))+2\coth^2(a+b\log(cx^n))}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] -(2*Coth[a + b*Log[c*x^n]]^2 + Coth[a + b*Log[c*x^n]]^4 - 4*Log[Cosh[a + b*
Log[c*x^n]]] - 4*Log[Tanh[a + b*Log[c*x^n]]])/(4*b*n)
```

Maple [A] time = 0.003, size = 88, normalized size = 1.3

$$-\frac{(\coth(a+b\ln(cx^n)))^4}{4bn}-\frac{(\coth(a+b\ln(cx^n)))^2}{2bn}-\frac{\ln(\coth(a+b\ln(cx^n))-1)}{2bn}-\frac{\ln(\coth(a+b\ln(cx^n))+1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] -1/4*coth(a+b*ln(c*x^n))^4/b/n-1/2*coth(a+b*ln(c*x^n))^2/b/n-1/2/n/b*ln(cot
h(a+b*ln(c*x^n))-1)-1/2/n/b*ln(coth(a+b*ln(c*x^n))+1)
```

Maxima [B] time = 1.63914, size = 1154, normalized size = 17.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] -1/24*(48*c^(6*b)*e^(6*b*log(x^n) + 6*a) - 108*c^(4*b)*e^(4*b*log(x^n) + 4*
a) + 88*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) +
 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n
) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/24*(12*c^(6*b)*e
^(6*b*log(x^n) + 6*a) - 42*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 52*c^(2*b)*e^(2
*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*
n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*
b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 5/8*(4*c^(6*b)*e^(6*b*log(x^n) + 6*a)
- 6*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/
(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a)
+ 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*
a) + b*n) - 5/12*(6*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 4*c^(2*b)*e^(2*b*log(x
^n) + 2*a) + 1)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*
log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2
*b*log(x^n) + 2*a) + b*n) - 5/12*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*
c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6
*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a)
+ b*n) - 5/8/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log
(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*
log(x^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n)
 + log((c^b*e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n) - log(x)
```

Fricas [B] time = 2.68697, size = 5060, normalized size = 76.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^8*log(x) + 8*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^8 - 4*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) - b*n*log(x) + 1)*
sinh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a
)^3*log(x) - 3*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*l
og(x) + b*log(c) + a)^5 + 2*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) +
 a)^4 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) - 30*(b*n*log(x)
 - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n*log(x) - 2)*sinh(b*n*log(x)
 + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5*log(x) - 10
*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^3 + (3*b*n*log(x) - 2)*co
sh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 - 4*(b*n*l
og(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n*log(x) + 4*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^6*log(x) - 15*(b*n*log(x) - 1)*cosh(b*n*log(x) +
b*log(c) + a)^4 + 3*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) + a)^2 -
b*n*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*lo
g(c) + a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c)
+ a)^7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c
) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log
(c) + a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b
*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b
*log(c) + a)^4 - 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x)
+ b*log(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log
(x) + b*log(c) + a)^5 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*l
og(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*lo
g(x) + b*log(c) + a)^6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*
log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(
b*n*log(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b
*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*sinh(
b*n*log(x) + b*log(c) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(
x) + b*log(c) + a))) + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7*log(x) - 3*
(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^5 + (3*b*n*log(x) - 2)*cos
h(b*n*log(x) + b*log(c) + a)^3 - (b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c
) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) +
a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^7 + b*n*sinh(b*n*log(x) + b*log(c) + a)^8 - 4*b*n*cosh(b*n*log(x) + b*log
(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log
(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*
cosh(b*n*log(x) + b*log(c) + a)^3 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) +
a)^4 - 30*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) +
b*log(c) + a)^4 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^5 - 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*
(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 - 15*b*n*cosh(b*n*log(x) + b*log(c
) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 - 3*b*n*c
osh(b*n*log(x) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3
- b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34413, size = 216, normalized size = 3.27

$$\frac{\log\left(-2\,x^{2\,bn}|c|^{2\,b}\cos\left(\pi b\mathrm{sgn}\left(c\right)-\pi b\right)e^{(2\,a)}+x^{4\,bn}|c|^{4\,b}e^{(4\,a)}+1\right)}{2\,bn}-\frac{25\,c^{8\,b}x^{8\,bn}e^{(8\,a)}-52\,c^{6\,b}x^{6\,bn}e^{(6\,a)}+102\,c^{4\,b}x^{4\,bn}e^{(4\,a)}-5}{12\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^{4}bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/2*log(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b*
n)*abs(c)^(4*b)*e^(4*a) + 1)/(b*n) - 1/12*(25*c^(8*b)*x^(8*b*n)*e^(8*a) - 5
2*c^(6*b)*x^(6*b*n)*e^(6*a) + 102*c^(4*b)*x^(4*b*n)*e^(4*a) - 52*c^(2*b)*x^
(2*b*n)*e^(2*a) + 25)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^4*b*n) - log(x)
```

3.194 $\int (ex)^m \coth(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=87

$$\frac{(ex)^{m+1}}{e(m+1)}-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2ad}\left(cx^n\right)^{2bd}\right)}{e(m+1)}$$

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(
2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(e*(1 + m))
```

Rubi [F] time = 0.0464916, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \coth(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]

[Out] Defer[Int][(e*x)^m*Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int (ex)^m \coth(d(a+b\log(cx^n)))\,dx=\int (ex)^m \coth(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 13.2313, size = 158, normalized size = 1.82

$$\frac{x(ex)^m\left(-\frac{(m+1)e^{2ad}(cx^n)^{2bd}\,{}_2F_1\left(1,\frac{m+2bdn+1}{2bdn};\frac{m+4bdn+1}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{2bdn+m+1}-{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2d(a+b\log(cx^n))}\right)\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]

[Out] (x*(e*x)^m*(-Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n),
 E^(2*d*(a + b*Log[c*x^n]))] - (E^(2*a*d)*(1 + m)*(c*x^n)^(2*b*d)*Hypergeom
etric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*
a*d)*(c*x^n)^(2*b*d)])/(1 + m + 2*b*d*n)))/(1 + m)
```

Maple [F] time = 1.436, size = 0, normalized size = 0.

$$\int (ex)^m \coth(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^m x x^m}{m+1} - e^m \int \frac{x^m}{c^{bd} e^{(bd \log(x^n)+ad)} + 1}\, dx + e^m \int \frac{x^m}{c^{bd} e^{(bd \log(x^n)+ad)} - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] e^m*x*x^m/(m + 1) - e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1),
 x) + e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \coth\left(bd \log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth\left(\left(b \log\left(cx^n\right) + a\right)d\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d), x)
```

3.195 $\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=168

$$-\frac{2(ex)^{m+1}{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2ad}(cx^n)^{2bd}\right)}{bden}+\frac{(ex)^{m+1}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bden\left(1-e^{2ad}(cx^n)^{2bd}\right)}+\frac{(ex)^{m+1}(bdn+m+1)}{bde(m+1)n}$$

```
[Out] ((1 + m + b*d*n)*(e*x)^(1 + m))/(b*d*e*(1 + m)*n) + ((e*x)^(1 + m)*(1 + E^(
2*a*d)*(c*x^n)^(2*b*d)))/(b*d*e*n*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*(e*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E
^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*e*n)
```

Rubi [F] time = 0.0769521, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx=\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 14.4369, size = 312, normalized size = 1.86

$$(ex)^m\left(\frac{x}{m+1}-\frac{x^{-2m}\exp\left(-\frac{(2m+1)(a+b\log(cx^n)-bn\log(x))}{bn}\right)\left((m+1)x^{2bdn+2m+1}\exp\left(\frac{(2bdn+2m+1)(a+b\log(cx^n)-bn\log(x))}{bn}\right){}_2F_1\left(1\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (e*x)^m*(x/(1 + m) - (E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))*(1 + m + 2*b
*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))
*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b
*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^(((1 + 2*m + 2*b*d*n)*(a - b*n*Log[x
] + b*Log[c*x^n]))/(b*n))*(1 + m)*x^(1 + 2*m + 2*b*d*n)*Hypergeometric2F1[1
, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*d*(a + b*L
og[c*x^n]))])/(b*d*E^(((1 + 2*m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*n*
(1 + m + 2*b*d*n)*x^(2*m)))
```

Maple [F] time = 0.221, size = 0, normalized size = 0.

$$\int (ex)^m (\coth (d (a + b \ln (cx^n))))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e^m(m+1)\int \frac{x^m}{bc^{bd}dne^{(bd\log(x^n)+ad)}+bdn}\,dx+e^m(m+1)\int \frac{x^m}{bc^{bd}dne^{(bd\log(x^n)+ad)}-bdn}\,dx+\frac{bc^{2\,bd}de^m nxe^{(2\,bd\log(x^n)+2\,ad+}}{(mn+n)bc^{2\,bd}de^{(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n),
x) + e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) - b*d*
n), x) + (b*c^(2*b*d)*d*e^m*n*x*e^(2*b*d*log(x^n) + 2*a*d + m*log(x)) - (b*
d*e^m*n + 2*e^m*(m + 1))*x*x^m)/((m*n + n)*b*c^(2*b*d)*d*e^(2*b*d*log(x^n)
+ 2*a*d) - (m*n + n)*b*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex)^m \coth (bd \log (cx^n) + ad)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth ((b \log (cx^n) + a)d)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^2, x)
```

3.196 $\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=306

$$-\frac{(ex)^{m+1}\left(2b^2d^2n^2+m^2+2m+1\right){}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2ad}\left(cx^n\right)^{2bd}\right)}{b^2d^2e(m+1)n^2}+\frac{e^{-2ad}(ex)^{m+1}\left(\frac{e^{4ad}(2bdn+m+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(-2bdn+m}{n}\right.}{2b^2d^2en\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}$$

```
[Out] ((1 + m + b*d*n)*(1 + m + 2*b*d*n)*(e*x)^(1 + m))/(2*b^2*d^2*e*(1 + m)*n^2)
 - ((e*x)^(1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^2)/(2*b*d*e*n*(1 - E^(2*a
*d)*(c*x^n)^(2*b*d))^2) + ((e*x)^(1 + m)*((E^(2*a*d)*(1 + m - 2*b*d*n))/n +
 (E^(4*a*d)*(1 + m + 2*b*d*n)*(c*x^n)^(2*b*d))/n))/(2*b^2*d^2*e*E^(2*a*d)*n
*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2
*a*d)*(c*x^n)^(2*b*d)])/(b^2*d^2*e*(1 + m)*n^2)
```

Rubi [F] time = 0.0694341, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3, x]
```

Rubi steps

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx=\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 16.5856, size = 600, normalized size = 1.96

$$-\frac{x^{-m}(ex)^m\left(2b^2d^2n^2+m^2+2m+1\right)\operatorname{csch}(d(a+b(\log(cx^n)-n\log(x))))\left(\frac{\sinh(d(a+b(\log(cx^n)-n\log(x))))\exp\left(-\frac{(2m+1)(a+b(\log(cx^n)-}{bn}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (x*(e*x)^m*Coth[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m) - (x*(e*x)^m
*Csch[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n) + (
(1 + m)*x*(e*x)^m*Csch[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csch[b*d*n*Log
[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x]])/(2*b^2*d^2*
n^2) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Csch[d*(a + b*(-(n*Log[x])
+ Log[c*x^n]))]*((x^(1 + m)*Csch[d*(a + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/
(1 + m) + ((E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) +
 Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^((a +
```

```
2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n)
)*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*
b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^((a*(1 + 2*m + 2*b*d*n))/(b*n) + (1
 + m + 2*b*d*n)*Log[x] + ((1 + 2*m + 2*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n
)*(1 + m)*Hypergeometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*
n)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])*Sinh[d*(a + b*(-(n*Log[x]) + Log
[c*x^n]))])/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 +
m)*(1 + m + 2*b*d*n))))/(2*b^2*d^2*n^2*x^m)
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int (ex)^m (\coth (d (a + b \ln (cx^n))))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(2\,b^2d^2e^mn^2 + \left(m^2 + 2\,m + 1\right)e^m\right)\int \frac{x^m}{2\left(b^2c^{bd}d^2n^2e^{(bd\log(x^n)+ad)} + b^2d^2n^2\right)}\,dx + \left(2\,b^2d^2e^mn^2 + \left(m^2 + 2\,m + 1\right)e^m\right)\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] -(2*b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*integrate(1/2*x^m/(b^2*c^(b*d)*d
^2*n^2*e^(b*d*log(x^n) + a*d) + b^2*d^2*n^2), x) + (2*b^2*d^2*e^m*n^2 + (m^
2 + 2*m + 1)*e^m)*integrate(1/2*x^m/(b^2*c^(b*d)*d^2*n^2*e^(b*d*log(x^n) +
a*d) - b^2*d^2*n^2), x) + (b^2*c^(4*b*d)*d^2*e^m*n^2*x*e^(4*b*d*log(x^n) +
4*a*d + m*log(x)) + (b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*x*x^m - (2*b^2*
c^(2*b*d)*d^2*e^m*n^2*e^(2*a*d) + 2*(m*n + n)*b*c^(2*b*d)*d*e^m*e^(2*a*d) +
 (m^2 + 2*m + 1)*c^(2*b*d)*e^m*e^(2*a*d))*x*e^(2*b*d*log(x^n) + m*log(x)))/
((m*n^2 + n^2)*b^2*c^(4*b*d)*d^2*e^(4*b*d*log(x^n) + 4*a*d) - 2*(m*n^2 + n^
2)*b^2*c^(2*b*d)*d^2*e^(2*b*d*log(x^n) + 2*a*d) + (m*n^2 + n^2)*b^2*d^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \coth (bd\log (cx^n) + ad)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth\left((b\log\left(cx^n\right)+a)d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^3, x)
```

3.197 $\int \coth^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=115

$$x\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}F_1\left(\frac{1}{2bdn};p,-p;1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] (x*(-1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2
*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(1 + E^(
2*a*d)*(c*x^n)^(2*b*d))^p
```

Rubi [F] time = 0.0150993, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \coth^p(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^p, x]
```

Rubi steps

$$\int \coth^p(d(a+b\log(cx^n)))\,dx=\int \coth^p(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] time = 3.6288, size = 387, normalized size = 3.37

$$\frac{x(2bdn+1)\left(\frac{e^{2ad}(cx^n)^{2bd}+1}{e^{2ad}(cx^n)^{2bd}-1}\right)^p F_1\left(\frac{1}{2bdn};p,-p;1-\right.}{2bdnpe^{2ad}(cx^n)^{2bd}F_1\left(1+\frac{1}{2bdn};p,1-p;2+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)+2bdnpe^{2ad}(cx^n)^{2bd}F_1\left(1+\frac{1}{2bdn};p+1,\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((1 + 2*b*d*n)*x*((1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(-1 + E^(2*a*d)*(c*x^n)^(
2*b*d)))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^
(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d
)*AppellF1[1 + 1/(2*b*d*n), p, 1 - p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2
*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + 2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d)*
AppellF1[1 + 1/(2*b*d*n), 1 + p, -p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*
b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + (1 + 2*b*d*n)*AppellF1[1/(2*b*d*n), p
, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*
d))])
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int (\coth(d(a+b\ln(cx^n))))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left((b\log\left(cx^n\right) + a)d\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth\left(bd\log\left(cx^n\right) + ad\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left((b\log\left(cx^n\right) + a)d\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

3.198 $\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=135

$$\frac{(ex)^{m+1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}F_1\left(\frac{m+1}{2bdn};p,-p;\frac{m+1}{2bdn}+1;e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)}{e(m+1)}$$

```
[Out] ((e*x)^(1 + m)*(-1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[(1 + m)/(2*b*d*n
), p, -p, 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*
x^n)^(2*b*d))])/(e*(1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rubi [F] time = 0.0973732, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p, x]
```

Rubi steps

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx=\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 5.07176, size = 174, normalized size = 1.29

$$\frac{x(ex)^m\left(1-e^{2ad}(cx^n)^{2bd}\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}\left(\frac{e^{2ad}(cx^n)^{2bd}+1}{e^{2ad}(cx^n)^{2bd}-1}\right)^p F_1\left(\frac{m+1}{2bdn};p,-p;\frac{m+1}{2bdn}+1;e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*((1 + E^(2*a*d)*(c*x^n)^(2*b*d
))/(-1 + E^(2*a*d)*(c*x^n)^(2*b*d)))^p*AppellF1[(1 + m)/(2*b*d*n), p, -p, 1
 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d
))])/((1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (ex)^m(\coth(d(a+b\ln(cx^n))))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth \left((b \log (cx^n) + a)d \right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \coth \left(bd \log (cx^n) + ad \right)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth \left((b \log (cx^n) + a)d \right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

3.199 $\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=73

$$-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}-\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - (2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Rubi [A] time = 0.0514403, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3473, 3476, 329, 298, 203, 206}

$$-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}-\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - (2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn} \\
&= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} \\
&= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{}\right.}{} \\
&= -\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2\coth^{\frac{3}{2}}(a+b\,\mathrm{l}}{3bn}
\end{aligned}$$

Mathematica [A] time = 0.285752, size = 64, normalized size = 0.88

$$-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))+3\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)-3\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -(3*ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] - 3*ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]] + 2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Maple [A] time = 0.028, size = 93, normalized size = 1.3

$$-\frac{2}{3\,bn}(\coth(a+b\ln(cx^n)))^{\frac{3}{2}}-\frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}-1\right)+\frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}+1\right)-\frac{1}{bn}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^(5/2)/x,x)
```

```
[Out] -2/3*coth(a+b*ln(c*x^n))^(3/2)/b/n-1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+
1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2))/b
/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(coth(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [B] time = 2.43603, size = 2090, normalized size = 28.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)
*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(co
sh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x
) + b*log(c) + a)^2 - 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*
n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x)
+ b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c
) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(
c) + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
a) - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(x) + b*log(c) +
a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)
/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
+ b*n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.200 $\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=70

$$-\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - (2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] time = 0.0502198, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3473, 3476, 329, 212, 206, 203}

$$-\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - (2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn} \\
&= -\frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} \\
&= -\frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{}{1}\right.}{} \\
&= \frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2\sqrt{\coth(a+b\log}}{bn}
\end{aligned}$$

Mathematica [A] time = 0.146253, size = 57, normalized size = 0.81

$$\frac{-2\sqrt{\coth(a+b\log(cx^n))}+\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)+\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]
]] - 2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A] time = 0.015, size = 92, normalized size = 1.3

$$-2\,\frac{\sqrt{\coth(a+b\ln(cx^n))}}{bn} - \frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}-1\right) + \frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}+1\right) + \frac{1}{bn}\arctan\left(\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] -2*coth(a+b*ln(c*x^n))^(1/2)/b/n-1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/
2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)+arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(coth(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [B] time = 2.51166, size = 1100, normalized size = 15.71

$$-\frac{4\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)}{\sinh(bn\log(x)+b\log(c)+a)}}+2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a
)) + 2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*
log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.201 $\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n)
```

Rubi [A] time = 0.0390491, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3476, 329, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx = \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$
$$= -\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Mathematica [A] time = 0.075032, size = 48, normalized size = 1.

$$\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n)
```

Maple [A] time = 0.013, size = 72, normalized size = 1.5

$$-\frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}-1\right) + \frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}+1\right) - \frac{1}{bn}\arctan\left(\sqrt{\coth(a+b\ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] -1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/
2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\coth(b\log(cx^n)+a)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(b*log(c*x^n) + a))/x, x)
```

Fricas [B] time = 2.44119, size = 999, normalized size = 20.81

$$\frac{2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*
log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\coth\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(coth(a + b*log(c*x**n)))/x, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.202 $\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=47

$$\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n)
```

Rubi [A] time = 0.0398971, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3476, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth(a+b\log(}\right.}{bn}$$
$$= \frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Mathematica [A] time = 0.110676, size = 47, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n)
```

Maple [A] time = 0.015, size = 44, normalized size = 0.9

$$\frac{1}{bn}\arctan\left(\sqrt{\coth(a+b\ln(cx^n))}\right) + \frac{1}{bn}\text{Artanh}\left(\sqrt{\coth(a+b\ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\coth(b\log(cx^n)+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(coth(b*log(c*x^n) + a))), x)
```

Fricas [B] time = 2.34765, size = 1000, normalized size = 21.28

$$-\frac{2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a
)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*
sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + lo
g(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b
*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n
*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(coth(a + b*log(c*x**n)))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\coth\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(coth(b*log(c*x^n) + a))), x)
```

3.203 $\int \frac{1}{x \coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=71

$$-\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}}-\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.0517306, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3474, 3476, 329, 298, 203, 206}

$$-\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}}-\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}$$
$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$
$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subs}}{}$$
$$= -\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{}{bn\sqrt{\coth(}}$$

Mathematica [C] time = 0.144225, size = 44, normalized size = 0.62

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\coth^2(a+b\log(cx^n))\right)}{bn\sqrt{\coth(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Coth[a + b*Log[c*x^n]]^2])/(b*n*Sqrt[Co
th[a + b*Log[c*x^n]]])
```

Maple [A] time = 0.015, size = 93, normalized size = 1.3

$$\frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}+1\right) - 2\,\frac{1}{bn\sqrt{\coth(a+b\ln(cx^n))}} - \frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}-1\right) - \frac{1}{bn}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-2/b/n/coth(a+b*ln(c*x^n))^(1/2)-1/2
/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\coth\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [B] time = 2.84225, size = 2087, normalized size = 29.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)
*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(co
sh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*
n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x)
+ b*log(c) + a)^2 + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) +
b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) +
b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
+ a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c)
 + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
 - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + si
nh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/s
inh(b*n*log(x) + b*log(c) + a)) - 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
b*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\coth\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(3/2)), x)
```

3.204 $\int \frac{1}{x \coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=72

$$-\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - 2/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] time = 0.0510296, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3474, 3476, 329, 212, 206, 203}

$$-\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - 2/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{5}{2}}(a+bx)}\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx,x,\coth(a+b\log(cx^n))\right)}{bn}\\ &= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}\\ &= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\text{Sub}\\ &= \frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}-\frac{}{3bn\coth^{\frac{3}{2}}(a}\end{aligned}$$

Mathematica [C] time = 0.200434, size = 46, normalized size = 0.64

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\coth^2(a+b\log(cx^n))\right)}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Coth[a + b*Log[c*x^n]]^2])/(3*b*n*Coth[
a + b*Log[c*x^n]]^(3/2))
```

Maple [A] time = 0.013, size = 92, normalized size = 1.3

$$\frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}+1\right)-\frac{2}{3\,bn}(\coth(a+b\ln(cx^n)))^{-\frac{3}{2}}-\frac{1}{2\,bn}\ln\left(\sqrt{\coth(a+b\ln(cx^n))}-1\right)+\frac{1}{bn}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] 1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-2/3/b/n/coth(a+b*ln(c*x^n))^(3/2)-1
/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+arctan(coth(a+b*ln(c*x^n))^(1/2))/b/
n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \coth\left(b \log\left(cx^{n}\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [B] time = 2.88856, size = 3644, normalized size = 50.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*n*log(x) + b*log(c) + a)^4 + 16*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*sinh(b*n*log(x) + b*log(c) + a)^4
 + 8*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 + 6*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4
 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c)
 + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)
+ 1)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2
 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*s
inh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqr
t(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + 8*cos
h(b*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*c
osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sin
h(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(c
osh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a) + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*lo
g(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*
log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b
*log(c) + a))) + 16*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) +
b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 4*(cosh(b*n*log(x) + b*log
(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
 a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c)
 + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*sqrt(cosh(b*n*log(x) + b*log(
c) + a)/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(
```

```
c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) +
b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*
n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3
 + b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \coth\left(b \log\left(cx^{n}\right) + a\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(5/2)), x)
```

3.205 $\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=135

$$\frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}}-\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*
Coth[x]^4])])/(4*c^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt
[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c]) - Sqr
t[a + b*Coth[x]^2 + c*Coth[x]^4]/(2*c)
```

Rubi [A] time = 0.353085, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3701, 1251, 1653, 843, 621, 206, 724}

$$\frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}}-\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*
Coth[x]^4])])/(4*c^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt
[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c]) - Sqr
t[a + b*Coth[x]^2 + c*Coth[x]^4]/(2*c)
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
```

```
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx &= -\operatorname{Subst}\left(\int \frac{x^5}{\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)\\
&= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{\operatorname{Subst}\left(\int \frac{\frac{b}{2}+\frac{1}{2}(b-2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)}{2c}\\
&= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\mathrm{co}\right.\\
&= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{(b-2c)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\coth^2(}{\sqrt{a+b\coth^2(x)+c\,\mathrm{c}}}\right.}{2c}\\
&= \frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}} + \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\,\mathrm{cot}}}\right.}{2\sqrt{a+b+c}}
\end{aligned}$$

Mathematica [A] time = 9.01482, size = 266, normalized size = 1.97

$$\frac{\frac{2\operatorname{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\left(2c^{3/2}\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)+(b+2c)\sinh^2(x)+c}}\right)+(b-2c)\sqrt{a+b+c}\tanh^{-1}\left(\frac{(b+2c)\sinh}{2\sqrt{c}\sqrt{\sinh^4(x)(a+b+c)}}\right.\right.}{\sqrt{a+b+c}}}{8c^{3/2}\sqrt{\operatorname{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((2*(2*c^(3/2)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*
Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])] + (b - 2*c)*Sqrt[a
+ b + c]*ArcTanh[(2*c + (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[c + (b + 2*c)*
Sinh[x]^2 + (a + b + c)*Sinh[x]^4])])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2
*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/Sqrt[a + b + c] - Sqrt[2]*Sqrt[c]*(
3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4)/(8*
c^(3/2)*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*
Csch[x]^4])
```

Maple [A] time = 0.104, size = 149, normalized size = 1.1

$$-\frac{1}{2c}\sqrt{a+b(\coth(x))^2+c(\coth(x))^4}+\frac{b}{4}\ln\left(\left(\frac{b}{2}+c(\coth(x))^2\right)\frac{1}{\sqrt{c}}+\sqrt{a+b(\coth(x))^2+c(\coth(x))^4}\right)c^{-\frac{3}{2}}-\frac{1}{2}\ln\Bigg(\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] -1/2*(a+b*coth(x)^2+c*coth(x)^4)^(1/2)/c+1/4*b/c^(3/2)*ln((1/2*b+c*coth(x)^
2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))-1/2*ln((1/2*b+c*coth(x)^2)/c^
(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1/2)+1/2/(a+b+c)^(1/2)*arctanh(
1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x
)^4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] time = 18.6186, size = 24521, normalized size = 181.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*(((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a +
 b)*c - 2*c^2)*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x
)^4 - 2*(a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*
a + b)*c - 2*c^2)*cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*sinh(x)^2 +
a*b + b^2 - (2*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh
(x)^3 - (a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(c)*log(((b
^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cos
h(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 + 4*a*c -
 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 - b^2 - 4*
a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 - 3*(
b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c
^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*(b^2 + 4
*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(
7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2)*cosh(x
)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^2 + 4*a*
c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 - 15*(b
^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x
)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(
b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 +
2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b
 - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4 + (
a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2
*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2*b)*
c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4*a*c - 8
*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b^2 + 4*a
*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8
+ 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))
*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 +
8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*
cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cos
h(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) - 2*(c^2*cosh(x)^4 + 4*c^2*co
sh(x)*sinh(x)^3 + c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 - c^
2)*sinh(x)^2 + c^2 + 4*(c^2*cosh(x)^3 - c^2*cosh(x))*sinh(x))*sqrt(a + b +
c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^
6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b -
 b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2
 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x
)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*
b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*s
inh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*
a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(
x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a
 + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
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+ 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c
 - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2
+ a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 4*sqrt(2)*((a + b)
*c + c^2)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*c
osh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*
c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sin
h(x)^3 + sinh(x)^4)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3
)*cosh(x)*sinh(x)^3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 - 2
*((a + b)*c^2 + c^3)*cosh(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^
3)*cosh(x)^2)*sinh(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2 +
 c^3)*cosh(x))*sinh(x)), -1/8*(4*(c^2*cosh(x)^4 + 4*c^2*cosh(x)*sinh(x)^3 +
 c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 - c^2)*sinh(x)^2 + c^
2 + 4*(c^2*cosh(x)^3 - c^2*cosh(x))*sinh(x))*sqrt(-a - b - c)*arctan(sqrt(2
)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*si
nh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)
^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(
-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b +
3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 +
 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6
 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c
 + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3
- 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2
*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2
 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
 + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b -
 b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2
)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(
a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 -
3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c
+ 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) + ((a*b + b
^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a + b)*c - 2*c^2)*c
osh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^4 - 2*(a*b + b
^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a + b)*c - 2*c^2
)*cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*sinh(x)^2 + a*b + b^2 - (2*a
 + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^3 - (a*b + b
^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(c)*log(((b^2 + 4*(a + 2*b)
*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)*sinh(x)^7 +
 (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^
6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sin
h(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 - 3*(b^2 + 4*a*c - 8*
c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^4 +
2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*(b^2 + 4*a*c - 8*c^2)*co
sh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(7*(b^2 + 4*(a +
2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2)*cosh(x)^3 + (3*b^2 + 4
*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(
x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 - 15*(b^2 + 4*a*c - 8*c
^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^2 - b^2 - 4*a*
c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)
*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*c
osh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))
*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(
x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x
```

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)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2*b)*c + 8*c^2 + 8*((
b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x)^5
+ (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b^2 + 4*a*c - 8*c^2)*cosh
(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2
 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(
35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 -
 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*co
sh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(
x)^3 - cosh(x))*sinh(x) + 1)) + 4*sqrt(2)*((a + b)*c + c^2)*sqrt(((a + b +
c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(((
a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3)*cosh(x)*sinh(x)^3 + ((a
 + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 - 2*((a + b)*c^2 + c^3)*cosh
(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^3)*cosh(x)^2)*sinh(x)^2 +
 4*(((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2 + c^3)*cosh(x))*sinh(x)),
-1/4*(((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a +
b)*c - 2*c^2)*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)
^4 - 2*(a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a
 + b)*c - 2*c^2)*cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*sinh(x)^2 + a
*b + b^2 - (2*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(
x)^3 - (a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(-c)*arctan(
1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c
)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*s
inh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*s
qrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cos
h(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)
/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(
x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*cosh(
x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 - 4*(a*c - c^2)*cosh(x)^6 + 4*(7
*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^6 + 8*(7*((a + b)*c + c^2
)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 - 30*(a*c - c^2)*cosh(x)^2 + (3
*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 - 10*(a*c -
 c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a*c - c^2)*
cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 - 15*(a*c - c^2)*cosh(x)^4 + 3
*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^2 + (a + b)*c + c^2 +
 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*c +
3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(x))) - (c^2*cosh(x)^4 + 4*c^2*
cosh(x)*sinh(x)^3 + c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 -
c^2)*sinh(x)^2 + c^2 + 4*(c^2*cosh(x)^3 - c^2*cosh(x))*sinh(x))*sqrt(a + b
+ c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a
^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh
(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 +
a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(
3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sin
h(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
```

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^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^
2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b
*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^
2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 2*sqrt(2)*((a +
b)*c + c^2)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b +
3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c
^3)*cosh(x)*sinh(x)^3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 -
 2*((a + b)*c^2 + c^3)*cosh(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 +
c^3)*cosh(x)^2)*sinh(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2
 + c^3)*cosh(x))*sinh(x)), -1/4*(2*(c^2*cosh(x)^4 + 4*c^2*cosh(x)*sinh(x)^3
 + c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 - c^2)*sinh(x)^2 +
c^2 + 4*(c^2*cosh(x)^3 - c^2*cosh(x))*sinh(x))*sqrt(-a - b - c)*arctan(sqrt
(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*
sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(
x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqr
t(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a -
c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b
+ 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)
*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8
 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a
*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)
^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b
*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 +
 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b
^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)
*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b
 - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c
^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15
*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c
 + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7
- 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*
c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) + ((a*b +
 b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a + b)*c - 2*c^2)
*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^4 - 2*(a*b +
 b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a + b)*c - 2*c
^2)*cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*sinh(x)^2 + a*b + b^2 - (2
*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^3 - (a*b +
 b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(-c)*arctan(1/2*sqrt(2)*(
(b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 -
 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*
((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(-c)*sqrt(
((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3
*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 -
 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x
)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7
+ ((a + b)*c + c^2)*sinh(x)^8 - 4*(a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c +
 c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 -
 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(
35*((a + b)*c + c^2)*cosh(x)^4 - 30*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3
*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 - 10*(a*c - c^2)*cosh(x)
^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a*c - c^2)*cosh(x)^2 + 4
```

```
*(7*((a + b)*c + c^2)*cosh(x)^6 - 15*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c
 + 3*c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^2 + (a + b)*c + c^2 + 8*(((a + b)*
c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x
)^3 - (a*c - c^2)*cosh(x))*sinh(x))) + 2*sqrt(2)*((a + b)*c + c^2)*sqrt(((a
 + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a
 + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*
cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4
)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3)*cosh(x)*sinh(x)^
3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 - 2*((a + b)*c^2 + c^
3)*cosh(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^3)*cosh(x)^2)*sinh
(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2 + c^3)*cosh(x))*sin
h(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

3.206 $\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=105

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}$$

```
[Out] -ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]
)]/(2*Sqrt[c]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rubi [A] time = 0.216574, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3701, 1251, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```


```
[Out] -ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]
)]/(2*Sqrt[c]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```


Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx,x,-i\coth(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a-bx+cx^2}}\,dx,x,-\coth^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx,x,-\coth^2(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+
\right.\\
&= \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{-b-2c\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)+\text{Subst}\left(\int \frac{1}{4a+4b
\right.\\
&= \frac{\tanh^{-1}\left(\frac{-b-2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}
\end{aligned}$$

Mathematica [A] time = 25.8231, size = 199, normalized size = 1.9

$$\frac{\operatorname{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\left(\frac{\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)+(b+2c)\sinh^2(x)+c}}\right)}{\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{}\right.}{2\sqrt{\operatorname{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[c + (b +
 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]/Sqrt[a + b + c] - ArcTanh[(2*c +
 (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)
*Sinh[x]^4])]/Sqrt[c])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b +
c)*Cosh[4*x]]*Csch[x]^2)/(2*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a
+ b + c)*Cosh[4*x])*Csch[x]^4])
```

Maple [A] time = 0.063, size = 90, normalized size = 0.9

$$-\frac{1}{2}\ln\left(\left(\frac{b}{2}+c\left(\coth\left(x\right)\right)^{2}\right)\frac{1}{\sqrt{c}}+\sqrt{a+b\left(\coth\left(x\right)\right)^{2}+c\left(\coth\left(x\right)\right)^{4}}\right)\frac{1}{\sqrt{c}}+\frac{1}{2}\mathrm{Artanh}\left(\frac{b\left(\coth\left(x\right)\right)^{2}+2c\left(\coth\left(x\right)\right)^{2}+2a}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] -1/2*ln((1/2*b+c*coth(x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1
/2)+1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)
^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(x\right)^{3}}{\sqrt{c\coth\left(x\right)^{4}+b\coth\left(x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] time = 14.5732, size = 18436, normalized size = 175.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((a + b + c)*sqrt(c)*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*
(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*
c^2)*sinh(x)^8 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b
)*c + 8*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a
+ 2*b)*c + 8*c^2)*cosh(x)^3 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 +
2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c
 + 8*c^2)*cosh(x)^4 - 30*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a -
 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5
- 10*(b^2 + 4*a*c - 8*c^2)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*c
osh(x))*sinh(x)^3 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a +
2*b)*c + 8*c^2)*cosh(x)^6 - 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 +
 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*s
qrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sin
h(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x
)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c
)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2
 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh
(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
```

```
sinh(x)^4)) + b^2 + 4*(a + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^
2)*cosh(x)^7 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c
 + 24*c^2)*cosh(x)^3 - (b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 +
 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x
)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^
2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))
*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4
*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) +
1)) + sqrt(a + b + c)*c*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x
)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 +
2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh
(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b
+ b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(
x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*
(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a +
 b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*
c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c
^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a
*b + b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 +
(a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7
 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c
^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*co
sh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))
)/((a + b)*c + c^2), -1/4*(2*sqrt(-a - b - c)*c*arctan(sqrt(2)*((a + b + c)
*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a
 - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b
 + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sq
rt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2
*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^
4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sin
h(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4
 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)
*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x
)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a
+ b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 +
4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*
c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x
)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b -
 b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(
x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) - (a + b + c)*sqrt(c)*log
(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)
*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 + 4*a
*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 - b^2
```

```
- 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 -
 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c +
24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*(b^2
 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 +
 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2)*co
sh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^2 +
4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 - 1
5*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*co
sh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 +
 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^
2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3
- (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4
 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2
 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) +
 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2
*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4*a*c
 - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b^2 +
 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x
)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh
(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^
4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 -
 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3
*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)))/((a + b)*c + c^2), 1/4*(
2*(a + b + c)*sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c
)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b
 + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)
*cosh(x))*sinh(x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b
+ c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2
*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)
^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)
^8 + 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 -
4*(a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sin
h(x)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^
5 + 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 -
 30*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*
c + c^2)*cosh(x)^5 - 10*(a*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(
x))*sinh(x)^3 - 4*(a*c - c^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6
- 15*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)
*sinh(x)^2 + (a + b)*c + c^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^
2)*cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(
x))) + sqrt(a + b + c)*c*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(
x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cos
h(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b
 + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh
(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2
*(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a
+ b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*co
sh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)
*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c -
c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 -
a*b + b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)
*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a
+ b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*
cosh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 +
 (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
```

```
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*
c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*c
osh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
))/((a + b)*c + c^2), -1/2*(sqrt(-a - b - c)*c*arctan(sqrt(2)*((a + b + c)*
cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a
- c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b
+ c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqr
t(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*
(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4
 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh
(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
 b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)
^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
- b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4
- 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*
c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)
^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a +
 b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c
 - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)
^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c
^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b -
b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x
)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) - (a + b + c)*sqrt(-c)*arc
tan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b +
 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*
c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*
c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b +
3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*c
osh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 - 4*(a*c - c^2)*cosh(x)^6 +
4*(7*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^6 + 8*(7*((a + b)*c +
 c^2)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2
)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 - 30*(a*c - c^2)*cosh(x)^2
+ (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 - 10*(a
*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a*c - c
^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 - 15*(a*c - c^2)*cosh(x)^4
 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^2 + (a + b)*c + c
^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*
c + 3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(x))))/((a + b)*c + c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(coth(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.207 $\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rubi [A] time = 0.117339, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3701, 1247, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\operatorname{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)$$
$$= \operatorname{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Mathematica [B] time = 18.1964, size = 141, normalized size = 2.43

$$\frac{\operatorname{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)+(b+2c)\sinh^2(x)+c}}\right)}{2\sqrt{a+b+c}\sqrt{\operatorname{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[c + (b +
2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*Sqrt[3*a - b + 3*c - 4*(a - c)*Co
sh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/(2*Sqrt[a + b + c]*Sqrt[(3*a -
b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4])
```

Maple [A] time = 0.065, size = 52, normalized size = 0.9

$$\frac{1}{2}\operatorname{Artanh}\left(\frac{b(\coth(x))^2+2c(\coth(x))^2+2a+b}{2}\frac{1}{\sqrt{a+b+c}}\frac{1}{\sqrt{a+b(\coth(x))^2+c(\coth(x))^4}}\right)\frac{1}{\sqrt{a+b+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] 1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/
2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] time = 9.66772, size = 4811, normalized size = 82.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a
^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh
(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 +
a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(
3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sin
h(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^
2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b
*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^
2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sqrt(a + b + c), -
1/2*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 +
 (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4
*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh
(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^
2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(
7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c -
 c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*si
nh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2
*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a
^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2
)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*
a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c
```

```
^2)*cosh(x))*sinh(x)))/(a + b + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.208 $\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=106

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]
)]/(2*Sqrt[a]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rubi [A] time = 0.239581, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3701, 1251, 960, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]
)]/(2*Sqrt[a]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a-bx+cx^2}}+\frac{1}{x\sqrt{a-bx+cx^2}}\right)dx, x, -\coth^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+}}\right.$$

$$= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)-\text{Subst}\left(\int \frac{1}{4a+4b+}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{-2a-b+(-b-2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Mathematica [A] time = 7.82955, size = 203, normalized size = 1.92

$$-\frac{\text{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\left(-\frac{\tanh^{-1}\left(\frac{2a-(2a+b)\cosh^2(x)}{2\sqrt{a}\sqrt{\cosh^4(x)(a+b+c)-(2a+b)\cosh^2(x)+a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{}{2\sqrt{a+b+c}}\right.}{}\right.}{2\sqrt{\text{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -((-(ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*Sqrt[a - (2*a + b)*Cosh
[x]^2 + (a + b + c)*Cosh[x]^4])]/Sqrt[a]) - ArcTanh[(-a + c + (a + b + c)*C
osh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cos
h[x]^4])]/Sqrt[a + b + c])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a +
b + c)*Cosh[4*x]]*Csch[x]^2)/(2*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] +
 (a + b + c)*Cosh[4*x])*Csch[x]^4])
```

Maple [F] time = 0.304, size = 0, normalized size = 0.

$$\int \tanh(x)\frac{1}{\sqrt{a+b(\coth(x))^2+c(\coth(x))^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] time = 13.8482, size = 18421, normalized size = 173.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b + c)*sqrt(a)*log(((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*
(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*
a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b +
b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a
*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 +
2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2
+ 4*a*c)*cosh(x)^4 - 30*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b +
3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 -
10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh
(x))*sinh(x)^3 - 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b +
b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^2 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8
*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*sinh(x)^2 - 4*sqrt(
2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)
^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2
+ 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sq
rt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2
*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^
4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sin
h(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*c
osh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c)*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12
*a*c)*cosh(x)^3 - (8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*co
sh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 +
 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3
)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh
(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh
(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
 sqrt(a + b + c)*a*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 +
 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6
```

```
+ 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c
 + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3
- 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a +
b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3
*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2
+ 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*c
osh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b +
b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(
x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x
))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a +
b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a +
 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*
(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*c
osh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(a^
2 + a*b + a*c), -1/4*(2*a*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh
(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)
*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((
a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(
a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4
*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^
4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*
c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 +
 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*
c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^
2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30
*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 -
 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*
c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c
^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 -
 a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 +
 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c
- c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3
- (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) - (a + b + c)*sqrt(a)*log(((8*
a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh
(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 -
4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 + b
^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(8
*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c
)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 - 30*(8*a^2 - b
^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(
8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 - 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3
 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 - 4*(8*a^2 - b^2 -
4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^2
 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 -
 8*a^2 + b^2 + 4*a*c)*sinh(x)^2 - 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a +
 b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*
(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a -
```

```
b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b
 + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a +
2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4
*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c)
*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 - (8*a^2 - b^2 - 4
*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7
*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(
x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*c
osh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x
)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5
 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)))/(a^2 + a*b + a*c), 1/4*(2*sqrt(-a)
*(a + b + c)*arctan(1/2*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*
sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*co
sh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh(x))*
sinh(x) + 2*a + b)*sqrt(-a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(
x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x
)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + a*b + a*c)*cosh(x)^8 + 8*(a^
2 + a*b + a*c)*cosh(x)*sinh(x)^7 + (a^2 + a*b + a*c)*sinh(x)^8 - 4*(a^2 - a
*c)*cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^6 + 8
*(7*(a^2 + a*b + a*c)*cosh(x)^3 - 3*(a^2 - a*c)*cosh(x))*sinh(x)^5 + 2*(3*a
^2 - a*b + 3*a*c)*cosh(x)^4 + 2*(35*(a^2 + a*b + a*c)*cosh(x)^4 - 30*(a^2 -
 a*c)*cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*sinh(x)^4 + 8*(7*(a^2 + a*b + a*c)*c
osh(x)^5 - 10*(a^2 - a*c)*cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*cosh(x))*sinh(x
)^3 - 4*(a^2 - a*c)*cosh(x)^2 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^6 - 15*(a^2
- a*c)*cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^2
 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*cosh(x)^7 - 3*(a^2 - a*c)*cosh(x)
^5 + (3*a^2 - a*b + 3*a*c)*cosh(x)^3 - (a^2 - a*c)*cosh(x))*sinh(x))) + sqr
t(a + b + c)*a*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c
^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*
(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)
^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3
*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 -
10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2
)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*
a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(
x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c
+ c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*s
inh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*c
osh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*s
inh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b +
c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c
)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2
*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a +
 b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2
 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(
x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*s
inh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(a^2 +
a*b + a*c), 1/2*(sqrt(-a)*(a + b + c)*arctan(1/2*sqrt(2)*((2*a + b)*cosh(x)
^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh
(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x
)^3 - (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a)*sqrt(((a + b + c)*cosh
(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh
(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh
```

```
(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + a*b
 + a*c)*cosh(x)^8 + 8*(a^2 + a*b + a*c)*cosh(x)*sinh(x)^7 + (a^2 + a*b + a*
c)*sinh(x)^8 - 4*(a^2 - a*c)*cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^2 -
 a^2 + a*c)*sinh(x)^6 + 8*(7*(a^2 + a*b + a*c)*cosh(x)^3 - 3*(a^2 - a*c)*co
sh(x))*sinh(x)^5 + 2*(3*a^2 - a*b + 3*a*c)*cosh(x)^4 + 2*(35*(a^2 + a*b + a
*c)*cosh(x)^4 - 30*(a^2 - a*c)*cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*sinh(x)^4 +
 8*(7*(a^2 + a*b + a*c)*cosh(x)^5 - 10*(a^2 - a*c)*cosh(x)^3 + (3*a^2 - a*b
 + 3*a*c)*cosh(x))*sinh(x)^3 - 4*(a^2 - a*c)*cosh(x)^2 + 4*(7*(a^2 + a*b +
a*c)*cosh(x)^6 - 15*(a^2 - a*c)*cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*cosh(x)
^2 - a^2 + a*c)*sinh(x)^2 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*cosh(x)^
7 - 3*(a^2 - a*c)*cosh(x)^5 + (3*a^2 - a*b + 3*a*c)*cosh(x)^3 - (a^2 - a*c)
*cosh(x))*sinh(x))) - a*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x
)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*c
osh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*c
osh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a
+ b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a
+ b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*c
osh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
)/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c
- c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)
*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(
a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*
c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 1
0*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c
+ 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2
)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a
^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8
*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c -
c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 -
(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))))/(a^2 + a*b + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

3.209 $\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=183

$$\frac{b\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}}-\frac{\tanh^2(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2a}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}+\frac{\text{ta}}{}$$

```
[Out] -ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]
)]/(2*Sqrt[a]) + (b*ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])])/(4*a^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2
)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b +
c]) - (Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]*Tanh[x]^2)/(2*a)
```

Rubi [A] time = 0.332368, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3701, 1251, 960, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}}-\frac{\tanh^2(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2a}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}+\frac{\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]
)]/(2*Sqrt[a]) + (b*ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])])/(4*a^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2
)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b +
c]) - (Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]*Tanh[x]^2)/(2*a)
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
```

```
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{x^3\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2\sqrt{a-bx+cx^2}} - \frac{1}{x\sqrt{a-bx+cx^2}} + \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\right)\right.\right.$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+\cdots}}\right.$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\tanh^2(x)}{2a} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+cx^2}}\,dx, x, -\text{coth}\cdots\right.}{4a}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} \cdots$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{b\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}} + \cdots$$

Mathematica [B] time = 20.6866, size = 2574, normalized size = 14.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] Sqrt[(3*a - b + 3*c - 4*a*Cosh[2*x] + 4*c*Cosh[2*x] + a*Cosh[4*x] + b*Cosh[
4*x] + c*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x])]*(-1/(2*a) + Sech[x]^2/(2
*a)) + (((2*a - b)*Sqrt[a + b + c]*ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2*S
qrt[a]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4])] + 2*a^(3/2)*
ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a +
 b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4])])*Sqrt[3*a - b + 3*c - 4*(a - c)*Co
sh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2*((-5*Sqrt[(3*a)/(3 - 4*Cosh[2*x]
 + Cosh[4*x]) - b/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (3*c)/(3 - 4*Cosh[2*x] +
Cosh[4*x]) - (4*a*Cosh[2*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (4*c*Cosh[2*x]
)/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (a*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x
]) + (b*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (c*Cosh[4*x])/(3 - 4*Cos
h[2*x] + Cosh[4*x])]*Sech[x]*Sinh[3*x])/(2*(3*a - b + 3*c - 4*a*Cosh[2*x] +
 4*c*Cosh[2*x] + a*Cosh[4*x] + b*Cosh[4*x] + c*Cosh[4*x])) + (b*Sqrt[(3*a)/
(3 - 4*Cosh[2*x] + Cosh[4*x]) - b/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (3*c)/(3
- 4*Cosh[2*x] + Cosh[4*x]) - (4*a*Cosh[2*x])/(3 - 4*Cosh[2*x] + Cosh[4*x])
+ (4*c*Cosh[2*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (a*Cosh[4*x])/(3 - 4*Cosh
[2*x] + Cosh[4*x]) + (b*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (c*Cosh[
4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x])]*Sech[x]*Sinh[3*x])/(a*(3*a - b + 3*c -
 4*a*Cosh[2*x] + 4*c*Cosh[2*x] + a*Cosh[4*x] + b*Cosh[4*x] + c*Cosh[4*x]))
+ (Sqrt[(3*a)/(3 - 4*Cosh[2*x] + Cosh[4*x]) - b/(3 - 4*Cosh[2*x] + Cosh[4*x
]) + (3*c)/(3 - 4*Cosh[2*x] + Cosh[4*x]) - (4*a*Cosh[2*x])/(3 - 4*Cosh[2*x]
 + Cosh[4*x]) + (4*c*Cosh[2*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (a*Cosh[4*x
])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (b*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*
x]) + (c*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x])]*Sech[x]*Sinh[5*x])/(2*(3
*a - b + 3*c - 4*a*Cosh[2*x] + 4*c*Cosh[2*x] + a*Cosh[4*x] + b*Cosh[4*x] +
c*Cosh[4*x])) + (5*Sqrt[(3*a)/(3 - 4*Cosh[2*x] + Cosh[4*x]) - b/(3 - 4*Cosh
[2*x] + Cosh[4*x]) + (3*c)/(3 - 4*Cosh[2*x] + Cosh[4*x]) - (4*a*Cosh[2*x])/
(3 - 4*Cosh[2*x] + Cosh[4*x]) + (4*c*Cosh[2*x])/(3 - 4*Cosh[2*x] + Cosh[4*x
]) + (a*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (b*Cosh[4*x])/(3 - 4*Cos
h[2*x] + Cosh[4*x]) + (c*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x])]*Tanh[x])
/(3*a - b + 3*c - 4*a*Cosh[2*x] + 4*c*Cosh[2*x] + a*Cosh[4*x] + b*Cosh[4*x]
 + c*Cosh[4*x]) - (3*b*Sqrt[(3*a)/(3 - 4*Cosh[2*x] + Cosh[4*x]) - b/(3 - 4*
Cosh[2*x] + Cosh[4*x]) + (3*c)/(3 - 4*Cosh[2*x] + Cosh[4*x]) - (4*a*Cosh[2*
x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (4*c*Cosh[2*x])/(3 - 4*Cosh[2*x] + Cosh
[4*x]) + (a*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (b*Cosh[4*x])/(3 - 4
*Cosh[2*x] + Cosh[4*x]) + (c*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x])]*Tanh
[x])/(a*(3*a - b + 3*c - 4*a*Cosh[2*x] + 4*c*Cosh[2*x] + a*Cosh[4*x] + b*Co
sh[4*x] + c*Cosh[4*x]))))/(4*a^(3/2)*Sqrt[a + b + c]*Sqrt[(3*a - b + 3*c -
4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4]*(-(((2*a - b)*Sqrt[
a + b + c]*ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*Sqrt[a - (2*a + b
)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4])] + 2*a^(3/2)*ArcTanh[(-a + c + (a + b
 + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b +
 c)*Cosh[x]^4])])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Co
sh[4*x]]*Coth[x]*Csch[x]^2)/(2*a^(3/2)*Sqrt[a + b + c]*Sqrt[(3*a - b + 3*c
- 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4]) + (Sqrt[3*a - b
+ 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2*(((2*a - b)*
Sqrt[a + b + c]*(-(((2*a + b)*Cosh[x]*Sinh[x])/(Sqrt[a]*Sqrt[a - (2*a + b)*
Cosh[x]^2 + (a + b + c)*Cosh[x]^4])) - ((2*a - (2*a + b)*Cosh[x]^2)*(-2*(2*
a + b)*Cosh[x]*Sinh[x] + 4*(a + b + c)*Cosh[x]^3*Sinh[x]))/(4*Sqrt[a]*(a -
(2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4)^(3/2))))/(1 - (2*a - (2*a + b)
*Cosh[x]^2)^2/(4*a*(a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4))) + (2
*a^(3/2)*(-((-a + c + (a + b + c)*Cosh[2*x])*(-2*(2*a + b)*Cosh[x]*Sinh[x]
+ 4*(a + b + c)*Cosh[x]^3*Sinh[x]))/(4*Sqrt[a + b + c]*(a - (2*a + b)*Cosh[
x]^2 + (a + b + c)*Cosh[x]^4)^(3/2)) + (Sqrt[a + b + c]*Sinh[2*x])/Sqrt[a -
 (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4]))/(1 - (-a + c + (a + b + c)*
Cosh[2*x])^2/(4*(a + b + c)*(a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^
4)))))/(4*a^(3/2)*Sqrt[a + b + c]*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x]
 + (a + b + c)*Cosh[4*x])*Csch[x]^4]) + (((2*a - b)*Sqrt[a + b + c]*ArcTanh
[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a +
```

```
 b + c)*Cosh[x]^4])] + 2*a^(3/2)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(
2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4])])*
Csch[x]^2*(-8*(a - c)*Sinh[2*x] + 4*(a + b + c)*Sinh[4*x]))/(8*a^(3/2)*Sqrt
[a + b + c]*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x
]]*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[
x]^4]) - (((2*a - b)*Sqrt[a + b + c]*ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2
*Sqrt[a]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4])] + 2*a^(3/2
)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a
 + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4])])*Sqrt[3*a - b + 3*c - 4*(a - c)*
Cosh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2*(-4*(3*a - b + 3*c - 4*(a - c)
*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Coth[x]*Csch[x]^4 + Csch[x]^4*(-8*(a -
c)*Sinh[2*x] + 4*(a + b + c)*Sinh[4*x])))/(8*a^(3/2)*Sqrt[a + b + c]*((3*a
- b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4)^(3/2)))
)
```

Maple [F] time = 0.274, size = 0, normalized size = 0.

$$\int (\tanh(x))^3 \frac{1}{\sqrt{a + b(\coth(x))^2 + c(\coth(x))^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(tanh(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] time = 19.6342, size = 24602, normalized size = 134.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*(((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2
+ (2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x
)^4 + 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b
^2 + (2*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^2 +
```

```
2*a^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh
(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(a)*log(((8
*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cos
h(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 -
 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 +
b^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(
8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*
c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 - 30*(8*a^2 -
b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*
(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 - 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^
3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 - 4*(8*a^2 - b^2 -
 4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^
2 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2
- 8*a^2 + b^2 + 4*a*c)*sinh(x)^2 + 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a
+ b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3
*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a -
 b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a +
b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a +
 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 +
4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c
)*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 - (8*a^2 - b^2 -
4*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(
7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh
(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*
cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(
x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^
5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*(a^2*cosh(x)^4 + 4*a^2*cosh(x)
*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + a^2)*si
nh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(a + b + c)*lo
g(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c
- c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)
^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*
b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x)^4 +
 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b
*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*sinh(x
)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 +
 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2
+ sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b
 + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)
*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b +
c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*
(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a
 - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*co
sh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*(
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^
2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 4*sqrt(2)*(a^2 + a*b +
a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x
)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(c
osh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^
3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cosh(x)^4 + 4*(a^3 + a^2*b + a^2*c)
*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*sinh(x)^4 + a^3 + a^2*b + a^2*c
```

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+ 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 + 2*(a^3 + a^2*b + a^2*c + 3*(a^3 + a^2
*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b + a^2*c)*cosh(x)^3 + (a^
3 + a^2*b + a^2*c)*cosh(x))*sinh(x)), -1/8*(4*(a^2*cosh(x)^4 + 4*a^2*cosh(x
)*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + a^2)*s
inh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(-a - b - c)*
arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a
 + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^
4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2
 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
- 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 +
 (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c
^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^
2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*
a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2
*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2
+ 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*
a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*
b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*
(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)
*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2
*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))
) + ((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2 + (
2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^4
 + 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2
+ (2*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^2 + 2*a
^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)
^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(a)*log(((8*a^
2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x
)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 - 4*
a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 + b^2
 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(8*a
^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*
cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 - 30*(8*a^2 - b^2
 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(8*
a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 - 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3 +
 (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 - 4*(8*a^2 - b^2 - 4*
a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^2 -
 b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 - 8
*a^2 + b^2 + 4*a*c)*sinh(x)^2 + 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b
)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2
*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a - b)
*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b +
 c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*
c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4*a
*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c)*c
osh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 - (8*a^2 - b^2 - 4*a
*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*c
osh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)
^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cos
h(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^
4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 +
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 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 4*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((
a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(
a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4
*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^
4)))/((a^3 + a^2*b + a^2*c)*cosh(x)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sin
h(x)^3 + (a^3 + a^2*b + a^2*c)*sinh(x)^4 + a^3 + a^2*b + a^2*c + 2*(a^3 + a
^2*b + a^2*c)*cosh(x)^2 + 2*(a^3 + a^2*b + a^2*c + 3*(a^3 + a^2*b + a^2*c)*
cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b + a^2*c)*cosh(x)^3 + (a^3 + a^2*b +
a^2*c)*cosh(x))*sinh(x)), 1/4*(((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4
 + 4*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b
^2 + (2*a - b)*c)*sinh(x)^4 + 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2
 + 2*(3*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (
2*a - b)*c)*sinh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b -
 b^2 + (2*a - b)*c)*cosh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*
sinh(x))*sqrt(-a)*arctan(1/2*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cos
h(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a +
b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh
(x))*sinh(x) + 2*a + b)*sqrt(-a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*
sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*s
inh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*si
nh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + a*b + a*c)*cosh(x)^8 +
8*(a^2 + a*b + a*c)*cosh(x)*sinh(x)^7 + (a^2 + a*b + a*c)*sinh(x)^8 - 4*(a^
2 - a*c)*cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^
6 + 8*(7*(a^2 + a*b + a*c)*cosh(x)^3 - 3*(a^2 - a*c)*cosh(x))*sinh(x)^5 + 2
*(3*a^2 - a*b + 3*a*c)*cosh(x)^4 + 2*(35*(a^2 + a*b + a*c)*cosh(x)^4 - 30*(
a^2 - a*c)*cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*sinh(x)^4 + 8*(7*(a^2 + a*b + a
*c)*cosh(x)^5 - 10*(a^2 - a*c)*cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*cosh(x))*s
inh(x)^3 - 4*(a^2 - a*c)*cosh(x)^2 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^6 - 15*
(a^2 - a*c)*cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*cosh(x)^2 - a^2 + a*c)*sinh
(x)^2 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*cosh(x)^7 - 3*(a^2 - a*c)*co
sh(x)^5 + (3*a^2 - a*b + 3*a*c)*cosh(x)^3 - (a^2 - a*c)*cosh(x))*sinh(x)))
+ (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^
2 + 2*(3*a^2*cosh(x)^2 + a^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh
(x))*sinh(x))*sqrt(a + b + c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*
cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2
)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2
- a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)
*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*
b + 2*(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
 + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b +
2*(a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a
 + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 +
 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b
*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a
^2 - a*b + b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b
 + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(
3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a
- c)*cosh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x
)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x
)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x
) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4
+ 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4)) - 2*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b +
 c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*
```

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c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cos
h(x)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*
sinh(x)^4 + a^3 + a^2*b + a^2*c + 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 + 2*(a^
3 + a^2*b + a^2*c + 3*(a^3 + a^2*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
+ a^2*b + a^2*c)*cosh(x)^3 + (a^3 + a^2*b + a^2*c)*cosh(x))*sinh(x)), 1/4*(
((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2 + (2*a
- b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^4 + 2
*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2 + (2
*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^2 + 2*a^2 +
 a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^3 +
 (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(-a)*arctan(1/2*sq
rt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh
(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)
^2 + 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a
)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2
 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh
(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4))/((a^2 + a*b + a*c)*cosh(x)^8 + 8*(a^2 + a*b + a*c)*cosh(x)*sin
h(x)^7 + (a^2 + a*b + a*c)*sinh(x)^8 - 4*(a^2 - a*c)*cosh(x)^6 + 4*(7*(a^2
+ a*b + a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^6 + 8*(7*(a^2 + a*b + a*c)*cosh
(x)^3 - 3*(a^2 - a*c)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - a*b + 3*a*c)*cosh(x)^
4 + 2*(35*(a^2 + a*b + a*c)*cosh(x)^4 - 30*(a^2 - a*c)*cosh(x)^2 + 3*a^2 -
a*b + 3*a*c)*sinh(x)^4 + 8*(7*(a^2 + a*b + a*c)*cosh(x)^5 - 10*(a^2 - a*c)*
cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*cosh(x))*sinh(x)^3 - 4*(a^2 - a*c)*cosh(x
)^2 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^6 - 15*(a^2 - a*c)*cosh(x)^4 + 3*(3*a^
2 - a*b + 3*a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^2 + a^2 + a*b + a*c + 8*((a
^2 + a*b + a*c)*cosh(x)^7 - 3*(a^2 - a*c)*cosh(x)^5 + (3*a^2 - a*b + 3*a*c)
*cosh(x)^3 - (a^2 - a*c)*cosh(x))*sinh(x))) - 2*(a^2*cosh(x)^4 + 4*a^2*cosh
(x)*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + a^2)
*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(-a - b - c
)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x
)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)
^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^
2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c
^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7
 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c -
 c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 -
a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 +
2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 +
 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^
2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (
3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 +
a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*c
osh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 +
2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 +
 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x
))) - 2*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)
*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*
sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*s
inh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cosh(x
)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*sin
```

```
h(x)^4 + a^3 + a^2*b + a^2*c + 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 + 2*(a^3 +
 a^2*b + a^2*c + 3*(a^3 + a^2*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a
^2*b + a^2*c)*cosh(x)^3 + (a^3 + a^2*b + a^2*c)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(tanh(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.210 $\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}-\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\tanh^{-1}\left(\frac{2a+(b}{2\sqrt{a+b+c}\sqrt{}}\right.$$

```
[Out] -((b + 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c
*Coth[x]^4])])/(4*Sqrt[c]) + (Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*
Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])])/2 - Sq
rt[a + b*Coth[x]^2 + c*Coth[x]^4]/2
```

Rubi [A] time = 0.230797, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3701, 1247, 734, 843, 621, 206, 724}

$$-\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}-\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\tanh^{-1}\left(\frac{2a+(b}{2\sqrt{a+b+c}\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -((b + 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c
*Coth[x]^4])])/(4*Sqrt[c]) + (Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*
Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])])/2 - Sq
rt[a + b*Coth[x]^2 + c*Coth[x]^4]/2
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = -\,\text{Subst}\left(\int \frac{x\sqrt{a-bx^2+cx^4}}{1+x^2}\,dx, x, -i\coth(x)\right)$$

$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a-bx+cx^2}}{1+x}\,dx, x, -\coth^2(x)\right)\right)$$

$$= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}+\frac{1}{4}\,\text{Subst}\left(\int \frac{-2a-b+(b+2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, \right.$$

$$= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}+\frac{1}{2}(-a-b-c)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-b}} \right.$$

$$= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}+(a+b+c)\,\text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2} \right.$$

$$= -\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\tanh^{-1}\left(\frac{}{2\sqrt{a}} \right.$$

Mathematica [B] time = 7.99061, size = 304, normalized size = 2.3

$$\frac{\operatorname{csch}^2(x)\left(4\sqrt{c}(a+b+c)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)+(b+2c)\sinh}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (Csch[x]^2*(4*Sqrt[c]*(a + b + c)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/
(2*Sqrt[a + b + c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*
Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]] + Sqrt[a
+ b + c]*(-2*(b + 2*c)*ArcTanh[(2*c + (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[
c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*Sqrt[3*a - b + 3*c - 4*(
a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]] - Sqrt[2]*Sqrt[c]*(3*a - b + 3*c
- 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^2)))/(8*Sqrt[c]*Sqrt
[a + b + c]*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*
x])*Csch[x]^4])
```

Maple [C] time = 0.061, size = 559, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] -1/2*(a+b*coth(x)^2+c*coth(x)^4)^(1/2)-1/8*(-b-c)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)*(4+2*(b+(-
4*a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2)*Elli
pticF(1/2*coth(x)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*ln((b+2*c*coth(x)^2)/c^(1/2)+2*(a+b*cot
h(x)^2+c*coth(x)^4)^(1/2))*c^(1/2)-1/4*ln((b+2*c*coth(x)^2)/c^(1/2)+2*(a+b*
coth(x)^2+c*coth(x)^4)^(1/2))/c^(1/2)*b+1/2*a/(a+b+c)^(1/2)*arctanh(1/2*(b*
coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1
/2))+1/2*b/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b
+c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))+1/2*c/(a+b+c)^(1/2)*arctanh(1/
2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^
4)^(1/2))-1/8*(b+c)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*coth(x)^2)
^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2)*EllipticF(1/2*coth(x)*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \coth(x)^4 + b \coth(x)^2 + a} \coth(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

Fricas [B] time = 24.0699, size = 22316, normalized size = 169.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh
(x)^4 - 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b - 2*c)*sinh(x)
^2 + 4*((b + 2*c)*cosh(x)^3 - (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)
*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*
c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 +
 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 -
b^2 - 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)
^3 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*
c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*
(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)
^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2
)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^
2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6
 - 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2
)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)
^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh
(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x
)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(
x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(
x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(
x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a
 + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4
*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b
^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + si
nh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*
cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh
(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)
^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7
 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*(c*cosh(x)^4 + 4*
c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*cosh(x)^2 - c)*s
inh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c)*sqrt(a + b + c)*log(((a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2
)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^4 +
2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b
*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c -
c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 -
 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*
c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*
b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + sqr
t(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)
*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh
(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sq
rt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a -
c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b
+ 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)
*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2
+ 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*co
sh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*
c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*s
inh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 4*sqrt(2)*c*sqrt(((a + b + c
)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c
)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^
```

```
3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(c*c
osh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*cos
h(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c), -1/8*(4*(
c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*
cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c)*sqrt(-a
 - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sin
h(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cos
h(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sin
h(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c
)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)
*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*
sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*s
inh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b
 - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh
(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a +
 b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(
3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^
2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*
a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(
x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4
*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b
- b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2
 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b
)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b
 - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x)
)*sinh(x))) - ((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2
*c)*sinh(x)^4 - 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b - 2*c)
*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)
*sqrt(c)*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b
)*c + 8*c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 -
4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh
(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)
*cosh(x)^3 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a
 - 2*b)*c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)
^4 - 30*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)
*sinh(x)^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c
 - 8*c^2)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3
 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*c
osh(x)^6 - 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c
+ 24*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)
*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2
*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c
)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b +
c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^
2 + 4*(a + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*
(b^2 + 4*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x
)^3 - (b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x
)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x
)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4
+ 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7
*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(c
osh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 4*sqrt(2)*c
*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2
+ 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(
```

```
x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
sinh(x)^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(
x)^2 + 2*(3*c*cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x
) + c), 1/4*(((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*
c)*sinh(x)^4 - 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b - 2*c)*
sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)*
sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh
(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x
)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh
(x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b
)*c + c^2)*cosh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 - 4*(a*c - c^2)*
cosh(x)^6 + 4*(7*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^6 + 8*(7*
((a + b)*c + c^2)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a -
b)*c + 3*c^2)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 - 30*(a*c - c^2
)*cosh(x)^2 + (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(
x)^5 - 10*(a*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3
 - 4*(a*c - c^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 - 15*(a*c - c^
2)*cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^2 + (
a + b)*c + c^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*cosh(x)^5 +
 ((3*a - b)*c + 3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(x))) + (c*cosh
(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*cosh(x
)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c)*sqrt(a + b +
c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^
6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b -
 b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2
 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x
)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*
b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*s
inh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*
a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(
x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a
 + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
+ 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c
 - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2
+ a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 2*sqrt(2)*c*sqrt((
(a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*
(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 -
4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)
^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 +
2*(3*c*cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c),
 -1/4*(2*(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2
 + 2*(3*c*cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) +
c)*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*c
osh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a +
b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*co
```

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sh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 +
(a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)
*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*
(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
 c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(
x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2
)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c -
c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sin
h(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^
2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)
*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a
^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^
2)*cosh(x))*sinh(x))) - ((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^
3 + (b + 2*c)*sinh(x)^4 - 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2
- b - 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b + 2*c)*cosh(x))*sinh(x)
+ b + 2*c)*sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*c
osh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b +
2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*co
sh(x))*sinh(x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c
)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)
*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*
sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8
+ 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 - 4*(
a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sinh(x
)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^5 +
 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 - 30
*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c +
 c^2)*cosh(x)^5 - 10*(a*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 - 4*(a*c - c^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 - 1
5*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)*si
nh(x)^2 + (a + b)*c + c^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*
cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(x))
) + 2*sqrt(2)*c*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a
- c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a -
b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(
x)*sinh(x)^3 + sinh(x)^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x
)^4 - 2*c*cosh(x)^2 + 2*(3*c*cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*
cosh(x))*sinh(x) + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\coth^2(x) + c\coth^4(x)}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*coth(x)**2 + c*coth(x)**4)*coth(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \coth(x)^4 + b \coth(x)^2 + a} \coth(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

3.211 $\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=319

$$-\frac{15\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{4bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{25e^{c(a+bx)}\tanh}{4b}$$

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (4*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^4) + (26*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c +
b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x))*Sqrt[Coth[a
*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(6*b*c*(1 - E^(2*c*(a + b*x)))^2) + (25*E
^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(4*b*c*(1 - E^(
2*c*(a + b*x)))) - (15*ArcTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*T
anh[a*c + b*c*x])/(4*b*c)
```

Rubi [A] time = 0.912047, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6720, 2282, 390, 1814, 1157, 385, 207}

$$-\frac{15\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{4bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{25e^{c(a+bx)}\tanh}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
```


```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (4*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^4) + (26*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c +
b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x))*Sqrt[Coth[a
*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(6*b*c*(1 - E^(2*c*(a + b*x)))^2) + (25*E
^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(4*b*c*(1 - E^(
2*c*(a + b*x)))) - (15*ArcTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*T
anh[a*c + b*c*x])/(4*b*c)
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx = \left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth^5(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{(1+x^2)^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int \left(1+\frac{2(1+10x^4+5x^8)}{(-1+x^2)^5}\right)dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} + \frac{\left(2\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{S}}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+b}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+b}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+b}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+b}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+b}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

Mathematica [A] time = 10.2662, size = 164, normalized size = 0.51

$$\frac{\left(66e^{c(a+bx)} - 314e^{3c(a+bx)} + 374e^{5c(a+bx)} - 246e^{7c(a+bx)} + 24e^{9c(a+bx)} + 45\left(e^{2c(a+bx)}-1\right)^4\log\left(1-e^{c(a+bx)}\right) - 45\left(e^{2c(a+bx)}\right.}{24bc\left(e^{2c(a+bx)}-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[Coth[c*(a + b*x)]^2]*(66*E^(c*(a + b*x)) - 314*E^(3*c*(a + b*x)) + 37
4*E^(5*c*(a + b*x)) - 246*E^(7*c*(a + b*x)) + 24*E^(9*c*(a + b*x)) + 45*(-1
 + E^(2*c*(a + b*x)))^4*Log[1 - E^(c*(a + b*x))] - 45*(-1 + E^(2*c*(a + b*x
)))^4*Log[1 + E^(c*(a + b*x))])*Tanh[c*(a + b*x)])/(24*b*c*(-1 + E^(2*c*(a
+ b*x)))^4)
```

Maple [A] time = 0.255, size = 320, normalized size = 1.

$$\frac{\left(e^{2c(bx+a)}-1\right)e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}} - \frac{e^{c(bx+a)}\left(75e^{6c(bx+a)} - 115e^{4c(bx+a)} + 109e^{2c(bx+a)} - 21\right)}{\left(12+12e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)^3 cb}\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)/c/b*exp(c*(b*x+a))-1/12/(1+exp(2*c*(b*x+a)))/(exp(2*
c*(b*x+a))-1)^3*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c
*(b*x+a))*(75*exp(6*c*(b*x+a))-115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))-21
)/c/b+15/8/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^
2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-1)-15/8/(1+exp(2*c*(b
*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2
)^(1/2)/c/b*ln(exp(c*(b*x+a))+1)
```

Maxima [A] time = 1.60907, size = 225, normalized size = 0.71

$$-\frac{15 \log\left(e^{(bcx+ac)}+1\right)}{8\,bc}+\frac{15 \log\left(e^{(bcx+ac)}-1\right)}{8\,bc}+\frac{12\,e^{(9\,bcx+9\,ac)}-123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}-157\,e^{(3\,bcx+3\,ac)}+33\,e^{(bc}}{12\,bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(b*c*x + a*c) + 1)/(b*c) + 15/8*log(e^(b*c*x + a*c) - 1)/(b*c)
+ 1/12*(12*e^(9*b*c*x + 9*a*c) - 123*e^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x +
 5*a*c) - 157*e^(3*b*c*x + 3*a*c) + 33*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x +
8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a
*c) + 1))
```

Fricas [B] time = 2.67679, size = 4176, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*cosh(b*c*x + a*c)^9 + 216*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
24*sinh(b*c*x + a*c)^9 + 6*(144*cosh(b*c*x + a*c)^2 - 41)*sinh(b*c*x + a*c)
^7 - 246*cosh(b*c*x + a*c)^7 + 42*(48*cosh(b*c*x + a*c)^3 - 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + 2*(1512*cosh(b*c*x + a*c)^4 - 2583*cosh(b*c*x
+ a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 374*cosh(b*c*x + a*c)^5 + 2*(1512*cos
h(b*c*x + a*c)^5 - 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b
*c*x + a*c)^4 + 2*(1008*cosh(b*c*x + a*c)^6 - 4305*cosh(b*c*x + a*c)^4 + 18
70*cosh(b*c*x + a*c)^2 - 157)*sinh(b*c*x + a*c)^3 - 314*cosh(b*c*x + a*c)^3
 + 2*(432*cosh(b*c*x + a*c)^7 - 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x
+ a*c)^3 - 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*
c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7
*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(
7*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*co
sh(b*c*x + a*c)^4 - 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cos
h(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b
*c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c
*x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x
```

```
 + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c)
 + sinh(b*c*x + a*c) + 1) + 45*(cosh(b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*c*x + a*c)^2 - 1)*si
nh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 - 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x + a*c)^4 - 30*cosh(
b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh
(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x
 + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x
+ a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c
))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) +
2*(108*cosh(b*c*x + a*c)^8 - 861*cosh(b*c*x + a*c)^6 + 935*cosh(b*c*x + a*c
)^4 - 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 66*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 +
 b*c*sinh(b*c*x + a*c)^8 - 4*b*c*cosh(b*c*x + a*c)^6 + 4*(7*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)^4 + 8*(7*b*c*
cosh(b*c*x + a*c)^3 - 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*
b*c*cosh(b*c*x + a*c)^4 - 30*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x +
a*c)^4 - 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x + a*c)^5 - 10*b*c*
cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*b
*c*cosh(b*c*x + a*c)^6 - 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*c*cosh(b*c*x + a*
c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x + a*c)^7 - 3*b*c*
cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43158, size = 244, normalized size = 0.76

$$\frac{\frac{24\,e^{(bcx+ac)}}{\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{45\,\log\left(e^{(bcx+ac)}+1\right)}{\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}+\frac{45\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{2\left(75\,e^{(7\,bcx+7\,ac)}-115\,e^{(5\,bcx+5\,ac)}+109\,e^{(3\,bcx+3\,ac)}-21\,e^{(bcx+ac)}\right)}{\left(e^{(2\,bcx+2\,ac)}-1\right)^{4}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}}{24\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 45*log(e^(b*c*x + a
*c) + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 45*log(abs(e^(b*c*x + a*c) - 1))/sg
n(e^(2*b*c*x + 2*a*c) - 1) - 2*(75*e^(7*b*c*x + 7*a*c) - 115*e^(5*b*c*x + 5
*a*c) + 109*e^(3*b*c*x + 3*a*c) - 21*e^(b*c*x + a*c))/((e^(2*b*c*x + 2*a*c)
 - 1)^4*sgn(e^(2*b*c*x + 2*a*c) - 1)))/(b*c)
```

3.212 $\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\, dx$

Optimal. Leaf size=197

$$-\frac{3\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{3e^{c(a+bx)}\tanh(ac+}{bc\left(1-\right.}$$

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^2) + (3*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b
*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))) - (3*ArcTanh[E^(c*(a + b*x))]*Sqrt[Cot
h[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Rubi [A] time = 0.2836, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6720, 2282, 390, 1158, 12, 288, 207}

$$-\frac{3\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{3e^{c(a+bx)}\tanh(ac+}{bc\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```


```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^2) + (3*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b
*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))) - (3*ArcTanh[E^(c*(a + b*x))]*Sqrt[Cot
h[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\,dx = \left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth^3(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{(1+x^2)^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int \left(1+\frac{2(1+3x^4)}{(-1+x^2)^3}\right) dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} + \frac{\left(2\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{S}}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+b}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+b}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+b}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+b}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

Mathematica [C] time = 3.77009, size = 334, normalized size = 1.7

$$-\frac{e^{-5c(a+bx)}\tanh^3(c(a+bx))\coth^2(c(a+bx))^{3/2}\left(256e^{8c(a+bx)}\left(e^{2c(a+bx)}+1\right)^3\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2,2,2\right\},\left\{1,\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -((Coth[c*(a + b*x)]^2)^(3/2)*(-21*(252105 + 507305*E^(2*c*(a + b*x)) + 173
916*E^(4*c*(a + b*x)) - 154296*E^(6*c*(a + b*x)) - 73885*E^(8*c*(a + b*x))
+ 4887*E^(10*c*(a + b*x))) - (315*(-16807 - 28218*E^(2*c*(a + b*x)) + 1173*
E^(4*c*(a + b*x)) + 17748*E^(6*c*(a + b*x)) + 4299*E^(8*c*(a + b*x)) - 1434
*E^(10*c*(a + b*x)) + 7*E^(12*c*(a + b*x)))*ArcTanh[Sqrt[E^(2*c*(a + b*x))]
])/Sqrt[E^(2*c*(a + b*x))] + 384*E^(8*c*(a + b*x))*(1 + E^(2*c*(a + b*x)))^
2*(7 + 5*E^(2*c*(a + b*x)))*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1,
11/2}, E^(2*c*(a + b*x))] + 256*E^(8*c*(a + b*x))*(1 + E^(2*c*(a + b*x)))^3
*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, E^(2*c*(a + b*
x))])*Tanh[c*(a + b*x)]^3)/(60480*b*c*E^(5*c*(a + b*x)))
```

Maple [A] time = 0.199, size = 298, normalized size = 1.5

$$\frac{\left(e^{2c(bx+a)}-1\right)e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}-\frac{e^{c(bx+a)}\left(3e^{2c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)cb\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}}-\frac{\left(3e^{2c(bx+a)}-3\right)\ln\left(e^{c(bx+\right.}{\left(2+2e^{2c(bx+a)}\right)cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)/c/b*exp(c*(b*x+a))-1/(1+exp(2*c*(b*x+a)))/(exp(2*c*(
b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x
+a))*(3*exp(2*c*(b*x+a))-1)/c/b-3/2/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-
1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+
a))+1)+3/2/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^
2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-1)
```

Maxima [A] time = 1.82596, size = 151, normalized size = 0.77

$$-\frac{3\log\left(e^{(bcx+ac)}+1\right)}{2bc}+\frac{3\log\left(e^{(bcx+ac)}-1\right)}{2bc}+\frac{e^{(5bcx+5ac)}-5e^{(3bcx+3ac)}+2e^{(bcx+ac)}}{bc\left(e^{(4bcx+4ac)}-2e^{(2bcx+2ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(e^(b*c*x + a*c) + 1)/(b*c) + 3/2*log(e^(b*c*x + a*c) - 1)/(b*c) +
(e^(5*b*c*x + 5*a*c) - 5*e^(3*b*c*x + 3*a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(
4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 2.42228, size = 1574, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + 2*s
inh(b*c*x + a*c)^5 + 10*(2*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^3 - 1
0*cosh(b*c*x + a*c)^3 + 10*(2*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x
 + a*c)^3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x
+ a*c)^2 - 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*
c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) +
 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + sinh(b*
c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 2*cosh(b
*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*
c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + 2*(5*cosh(b*c*x +
a*c)^4 - 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 4*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
 b*c*sinh(b*c*x + a*c)^4 - 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3 - b*
c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27028, size = 209, normalized size = 1.06

$$\frac{\frac{2\,e^{(bcx+ac)}}{\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{3\,\log\left(e^{(bcx+ac)}+1\right)}{\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} + \frac{3\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{2\left(3\,e^{(3\,bcx+3\,ac)}-e^{(bcx+ac)}\right)}{\left(e^{(2\,bcx+2\,ac)}-1\right)^{2}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 3*log(e^(b*c*x + a*c)
 + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 3*log(abs(e^(b*c*x + a*c) - 1))/sgn(e^
(2*b*c*x + 2*a*c) - 1) - 2*(3*e^(3*b*c*x + 3*a*c) - e^(b*c*x + a*c))/((e^(2
*b*c*x + 2*a*c) - 1)^2*sgn(e^(2*b*c*x + 2*a*c) - 1)))/(b*c)
```

3.213 $\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}-\frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}$$

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*Ar
cTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Rubi [A] time = 0.143621, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 388, 206}

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}-\frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*Ar
cTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth(ac+bcx)\,dx$$
$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{\left(2\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\operatorname{Su}}{bc}$$
$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\,\mathrm{ta}}{bc}$$

Mathematica [A] time = 0.0577475, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)} - 2\tanh^{-1}\left(e^{c(a+bx)}\right)\right)\tanh(c(a+bx))\sqrt{\coth^2(c(a+bx))}}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTanh[E^(c*(a + b*x))])*Sqrt[Coth[c*(a + b*x)]^2]*T
anh[c*(a + b*x)])/(b*c)
```

Maple [B] time = 0.238, size = 213, normalized size = 2.6

$$\frac{\left(e^{2c(bx+a)}-1\right)e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}} + \frac{\left(e^{2c(bx+a)}-1\right)\ln\left(e^{c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)cb}\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}} - \frac{\left(e^{2c(bx+a)}-1\right)\ln\left(e^{c(bx+}\right.}{\left(1+e^{2c(bx+a)}\right)cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)/c/b*exp(c*(b*x+a))+1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(
b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp
(c*(b*x+a))-1)-1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x
+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+1)
```

Maxima [A] time = 1.70541, size = 76, normalized size = 0.92

$$\frac{e^{(bcx+ac)}}{bc} - \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(b*c*x + a*c)/(b*c) - log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c
) - 1)/(b*c)
```

Fricas [A] time = 2.37335, size = 196, normalized size = 2.36

$$\frac{\cosh\left(bcx+ac\right)-\log\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)+1\right)+\log\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)-1\right)+\sinh\left(bcx+\right.}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c) - log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) + log(c
osh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + sinh(b*c*x + a*c))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1916, size = 127, normalized size = 1.53

$$\frac{\frac{e^{\left(bcx+ac\right)}}{\operatorname{sgn}\left(e^{\left(2\,bcx+2\,ac\right)}-1\right)}-\frac{\log\left(e^{\left(bcx+ac\right)}+1\right)}{\operatorname{sgn}\left(e^{\left(2\,bcx+2\,ac\right)}-1\right)}+\frac{\log\left(\left|e^{\left(bcx+ac\right)}-1\right|\right)}{\operatorname{sgn}\left(e^{\left(2\,bcx+2\,ac\right)}-1\right)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - log(e^(b*c*x + a*c) + 1)/sg
n(e^(2*b*c*x + 2*a*c) - 1) + log(abs(e^(b*c*x + a*c) - 1))/sgn(e^(2*b*c*x +
 2*a*c) - 1))/(b*c)
```

3.214 $\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*Ar
cTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rubi [A] time = 0.194186, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 388, 203}

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*Ar
cTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$
$$= \frac{\coth(ac+bcx)\,\text{Subst}\left(\int \frac{-1+x^2}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$
$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$
$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

Mathematica [A] time = 0.117547, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)} - 2\tan^{-1}\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))}{bc\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*Sqrt
[Coth[c*(a + b*x)]^2])
```

Maple [C] time = 0.231, size = 218, normalized size = 2.6

$$\frac{\left(1+e^{2c(bx+a)}\right)e^{c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)cb}\frac{1}{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}} + \frac{i\left(1+e^{2c(bx+a)}\right)\ln\left(e^{c(bx+a)}-i\right)}{\left(e^{2c(bx+a)}-1\right)cb}\frac{1}{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}} - \frac{i\left(1+e^{2c(bx+a)}\right)\ln\left(e^{c(bx+a)}+i\right)}{\left(e^{2c(bx+a)}-1\right)cb} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1
)*(1+exp(2*c*(b*x+a)))/c/b*exp(c*(b*x+a))+I/((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp
(c*(b*x+a))-I)-I/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp
(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp(c*(b*x+a))+I)
```

Maxima [A] time = 1.71976, size = 47, normalized size = 0.57

$$-\frac{2\arctan\left(e^{(bcx+ac)}\right)}{bc} + \frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(b*c*x + a*c))/(b*c) + e^(b*c*x + a*c)/(b*c)
```

Fricas [A] time = 2.25958, size = 132, normalized size = 1.59

$$-\frac{2\arctan\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\coth^2\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(coth(a*c + b*c*x)**2), x)
```

Giac [A] time = 1.18302, size = 81, normalized size = 0.98

$$-\frac{2\arctan\left(e^{(bcx+ac)}\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-e^{(bcx+ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*x + a*c)*
sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c)
```

3.215 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=193

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}-\frac{3\tan^{-1}\left(e^{c(a+bx)}\right)c}{bc\sqrt{\coth^2(a}}$$

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a
*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a
 + b*x)))*Sqrt[Coth[a*c + b*c*x]^2]) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c
+ b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rubi [A] time = 0.86382, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6720, 2282, 390, 1158, 12, 288, 203}

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}-\frac{3\tan^{-1}\left(e^{c(a+bx)}\right)c}{bc\sqrt{\coth^2(a}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a
*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a
 + b*x)))*Sqrt[Coth[a*c + b*c*x]^2]) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c
+ b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^3(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x^2)^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\operatorname{Subst}\left(\int \left(1-\frac{2(1+3x^4)}{(1+x^2)^3}\right)dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\operatorname{Subst}\left(\int \frac{1+3x^4}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx)\operatorname{Subst}\left(\int \cdots\right.}{2bc\sqrt{\coth^2(ac}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} - \frac{(6\coth(ac+bcx))\operatorname{Subst}\left(\int\right.}{bc\sqrt{\coth^2(ac}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\coth(ac+bcx}{bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\coth(ac+bcx}{bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac}$$

Mathematica [A] time = 0.297194, size = 104, normalized size = 0.54

$$\frac{\left(e^{c(a+bx)}\left(5e^{2c(a+bx)}+e^{4c(a+bx)}+2\right)-3\left(e^{2c(a+bx)}+1\right)^2\tan^{-1}\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(2 + 5*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))) - 3*(1 + E^
(2*c*(a + b*x)))^2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*(1 + E^
(2*c*(a + b*x)))^2*Sqrt[Coth[c*(a + b*x)]^2])
```

Maple [C] time = 0.209, size = 301, normalized size = 1.6

$$\frac{\left(1+e^{2c(bx+a)}\right)e^{c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)cb}\frac{1}{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}}+\frac{e^{c(bx+a)}\left(3e^{2c(bx+a)}+1\right)}{\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)cb}\frac{1}{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}}+\frac{\frac{3i}{2}\left(1+e^{2c(bx+a)}\right)\ln\left(e^{c(bx+a)}-i\right)}{\left(e^{2c(bx+a)}-1\right)cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1
)*(1+exp(2*c*(b*x+a)))/c/b*exp(c*(b*x+a))+1/(1+exp(2*c*(b*x+a)))/(exp(2*c*(
b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x
+a))*(3*exp(2*c*(b*x+a))+1)/c/b+3/2*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a)
)-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*
x+a))-I)-3/2*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a
)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+I)
```

Maxima [A] time = 1.66612, size = 122, normalized size = 0.63

$$-\frac{3\arctan\left(e^{(bcx+ac)}\right)}{bc}+\frac{e^{(5\,bcx+5\,ac)}+5\,e^{(3\,bcx+3\,ac)}+2\,e^{(bcx+ac)}}{bc\left(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3*arctan(e^(b*c*x + a*c))/(b*c) + (e^(5*b*c*x + 5*a*c) + 5*e^(3*b*c*x + 3*
a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c)
 + 1))
```

Fricas [B] time = 2.54231, size = 1175, normalized size = 6.09

$$\frac{\cosh\left(bcx+ac\right)^{5}+5\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{4}+\sinh\left(bcx+ac\right)^{5}+5\left(2\,\cosh\left(bcx+ac\right)^{2}+1\right)\sinh\left(bcx+ac\right)^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + sinh(b*c*x
 + a*c)^5 + 5*(2*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^3 + 5*cosh(b*c*
x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a
*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 2
*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*sinh(b*c
*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + (5*cosh(b*c*
x + a*c)^4 + 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 2*cosh(b*c*x +
 a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
^3 + b*c*sinh(b*c*x + a*c)^4 + 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*
c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3
+ b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21802, size = 176, normalized size = 0.91

$$-\frac{\left(3 \arctan\left(e^{(bcx+ac)}\right) e^{(-ac)} \mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - e^{(bcx)} \mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - \frac{3\,e^{(3\,bcx+2\,ac)} \mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) + e^{(bcx)} \mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{\left(e^{(2\,bcx+2\,ac)}+1\right)^{2}}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*arctan(e^(b*c*x + a*c))*e^(-a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*
x)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (3*e^(3*b*c*x + 2*a*c)*sgn(e^(2*b*c*x + 2
*a*c) - 1) + e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) +
 1)^2)*e^(a*c)/(b*c)
```

3.216 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=311

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}+\frac{26e^{c(a+bx)}}{3bc\left(e^{2c(a+bx)}\right.}$$

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (4*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Coth[a
*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3*Sqrt[Coth[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Coth[a*c +
b*c*x])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a*c + b*c*x]^2]) + (25*E
^(c*(a + b*x))*Coth[a*c + b*c*x])/(4*b*c*(1 + E^(2*c*(a + b*x)))*Sqrt[Coth[
a*c + b*c*x]^2]) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(4*b*c*Sq
rt[Coth[a*c + b*c*x]^2])
```

Rubi [A] time = 1.74274, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6720, 2282, 390, 1814, 1157, 385, 203}

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}+\frac{26e^{c(a+bx)}}{3bc\left(e^{2c(a+bx)}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (4*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Coth[a
*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3*Sqrt[Coth[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Coth[a*c +
b*c*x])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a*c + b*c*x]^2]) + (25*E
^(c*(a + b*x))*Coth[a*c + b*c*x])/(4*b*c*(1 + E^(2*c*(a + b*x)))*Sqrt[Coth[
a*c + b*c*x]^2]) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(4*b*c*Sq
rt[Coth[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^5(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\,\text{Subst}\left(\int \frac{(-1+x^2)^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\,\text{Subst}\left(\int \left(1-\frac{2(1+10x^4+5x^8)}{(1+x^2)^5}\right)dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\,\text{Subst}\left(\int \frac{1+10x^4+5x^8}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx)\,\text{Subst}\left(\int\right.}{4bc\sqrt{\text{co}}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\text{cot}}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\text{cot}}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\text{cot}}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\text{cot}}}$$

Mathematica [A] time = 0.461341, size = 133, normalized size = 0.43

$$\frac{\left(e^{c(a+bx)}\left(157e^{2c(a+bx)}+187e^{4c(a+bx)}+123e^{6c(a+bx)}+12e^{8c(a+bx)}+33\right)-45\left(e^{2c(a+bx)}+1\right)^4\tan^{-1}\left(e^{c(a+bx)}\right)\right)\coth(c(a+b}{12bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(33 + 157*E^(2*c*(a + b*x)) + 187*E^(4*c*(a + b*x)) + 123
*E^(6*c*(a + b*x)) + 12*E^(8*c*(a + b*x))) - 45*(1 + E^(2*c*(a + b*x)))^4*A
rcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(12*b*c*(1 + E^(2*c*(a + b*x)))^
4*Sqrt[Coth[c*(a + b*x)]^2])
```

Maple [C] time = 0.22, size = 324, normalized size = 1.

$$\frac{\left(1+e^{2c(bx+a)}\right)e^{c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)cb}\frac{1}{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}}+\frac{e^{c(bx+a)}\left(75\,e^{6c(bx+a)}+115\,e^{4c(bx+a)}+109\,e^{2c(bx+a)}+21\right)}{12\left(1+e^{2c(bx+a)}\right)^3\left(e^{2c(bx+a)}-1\right)cb}\frac{1}{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}}+\frac{\frac{15i}{8}(1+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1
)*(1+exp(2*c*(b*x+a)))/c/b*exp(c*(b*x+a))+1/12/(1+exp(2*c*(b*x+a)))^3/(exp(
2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c
*(b*x+a))*(75*exp(6*c*(b*x+a))+115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))+21
)/c/b+15/8*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a))
)^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-I)-15/8*I*(1+exp(2*
c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-
1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+I)
```

Maxima [A] time = 1.76172, size = 196, normalized size = 0.63

$$-\frac{15\arctan\left(e^{(bcx+ac)}\right)}{4\,bc}+\frac{12\,e^{(9\,bcx+9\,ac)}+123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}+157\,e^{(3\,bcx+3\,ac)}+33\,e^{(bcx+ac)}}{12\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/4*arctan(e^(b*c*x + a*c))/(b*c) + 1/12*(12*e^(9*b*c*x + 9*a*c) + 123*e^
(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x + 5*a*c) + 157*e^(3*b*c*x + 3*a*c) + 33*
e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4
*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 2.61625, size = 3186, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*cosh(b*c*x + a*c)^9 + 108*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
12*sinh(b*c*x + a*c)^9 + 3*(144*cosh(b*c*x + a*c)^2 + 41)*sinh(b*c*x + a*c)
^7 + 123*cosh(b*c*x + a*c)^7 + 21*(48*cosh(b*c*x + a*c)^3 + 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + (1512*cosh(b*c*x + a*c)^4 + 2583*cosh(b*c*x +
a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 187*cosh(b*c*x + a*c)^5 + (1512*cosh(b*
c*x + a*c)^5 + 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b*c*x
 + a*c)^4 + (1008*cosh(b*c*x + a*c)^6 + 4305*cosh(b*c*x + a*c)^4 + 1870*cos
h(b*c*x + a*c)^2 + 157)*sinh(b*c*x + a*c)^3 + 157*cosh(b*c*x + a*c)^3 + (43
2*cosh(b*c*x + a*c)^7 + 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x + a*c)^3
 + 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*c)^8 + 8
*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*
```

```
c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^6 + 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b
*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x
 + a*c)^4 + 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x
+ a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x
 + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 + 15*cosh(b*c*x + a
*c)^4 + 9*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 4*cosh(b*c*x + a*c
)^2 + 8*(cosh(b*c*x + a*c)^7 + 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^
3 + cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + si
nh(b*c*x + a*c)) + (108*cosh(b*c*x + a*c)^8 + 861*cosh(b*c*x + a*c)^6 + 935
*cosh(b*c*x + a*c)^4 + 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 33
*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh
(b*c*x + a*c)^7 + b*c*sinh(b*c*x + a*c)^8 + 4*b*c*cosh(b*c*x + a*c)^6 + 4*(
7*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a
*c)^4 + 8*(7*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^5 + 2*(35*b*c*cosh(b*c*x + a*c)^4 + 30*b*c*cosh(b*c*x + a*c)^2 + 3*b
*c)*sinh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x +
 a*c)^5 + 10*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*b*c*cosh(b*c*x + a*c)^6 + 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*
c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x
+ a*c)^7 + 3*b*c*cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh
(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25344, size = 250, normalized size = 0.8

$$-\frac{\left(45 \arctan\left(e^{(bcx+ac)}\right) e^{(-ac)} \mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right) - 12 e^{(bcx)} \mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right) - \frac{75 e^{(7bcx+6ac)} \mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right) + 115 e^{(5bcx+4a}}{}}{12bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(45*arctan(e^(b*c*x + a*c))*e^(-a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 1
2*e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (75*e^(7*b*c*x + 6*a*c)*sgn(e^(2
*b*c*x + 2*a*c) - 1) + 115*e^(5*b*c*x + 4*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1)
 + 109*e^(3*b*c*x + 2*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 21*e^(b*c*x)*sgn(
e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) + 1)^4)*e^(a*c)/(b*c)
```

3.217 $\int \sin^3(\coth(a+bx))\,dx$

Optimal. Leaf size=157

$$\frac{\sin(3)\text{CosIntegral}(3-3\coth(a+bx))}{8b}+\frac{\sin(3)\text{CosIntegral}(3\coth(a+bx)+3)}{8b}-\frac{3\sin(1)\text{CosIntegral}(1-\coth(a+\dots}{8b}$$

```
[Out] (-3*CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Coth[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3])/(8*b) +
 (CosIntegral[3 + 3*Coth[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Cos[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rubi [A] time = 0.374672, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\sin(3)\text{CosIntegral}(3-3\coth(a+bx))}{8b}+\frac{\sin(3)\text{CosIntegral}(3\coth(a+bx)+3)}{8b}-\frac{3\sin(1)\text{CosIntegral}(1-\coth(a+\dots}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Coth[a + b*x]]^3,x]
```


```
[Out] (-3*CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Coth[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3])/(8*b) +
 (CosIntegral[3 + 3*Coth[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Cos[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```


Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin^3(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sin^3(x)}{2(-1+x)}+\frac{\sin^3(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(-1+x)}-\frac{\sin(3x)}{4(-1+x)}\right)dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(1+x)}-\frac{\sin(3x)}{4(1+x)}\right)dx, x, \text{co}\right.}{2b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\frac{\text{Subst}\left(\int \frac{\sin(3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\frac{3\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\sin(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \text{coth}\right.}{8b}$$
$$= -\frac{3\text{Ci}(1-\coth(a+bx))\sin(1)}{8b}-\frac{3\text{Ci}(1+\coth(a+bx))\sin(1)}{8b}+\frac{\text{Ci}(3-3\coth(a+bx))\,\text{s}}{8b}$$

Mathematica [A] time = 0.242156, size = 124, normalized size = 0.79

$$\frac{2\sin(3)\text{CosIntegral}(3-3\coth(a+bx))+2\sin(3)\text{CosIntegral}(3\coth(a+bx)+3)-6\sin(1)\text{CosIntegral}(1-\text{cot}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Coth[a + b*x]]^3,x]
```

```
[Out] (-6*CosIntegral[1 - Coth[a + b*x]]*Sin[1] - 6*CosIntegral[1 + Coth[a + b*x]
]*Sin[1] + 2*CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3] + 2*CosIntegral[3 + 3*
Coth[a + b*x]]*Sin[3] - 2*Cos[3]*SinIntegral[3 - 3*Coth[a + b*x]] + 6*Cos[1
]*SinIntegral[1 - Coth[a + b*x]] + 6*Cos[1]*SinIntegral[1 + Coth[a + b*x]]
- 2*Cos[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

Maple [A] time = 0.025, size = 118, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{\text{Si}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\text{Ci}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\text{Si}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\text{Ci}(-3+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(b*x+a))^3,x)
```

```
[Out] 1/b*(-1/8*Si(3+3*coth(b*x+a))*cos(3)+1/8*Ci(3+3*coth(b*x+a))*sin(3)+1/8*Si(
-3+3*coth(b*x+a))*cos(3)+1/8*Ci(-3+3*coth(b*x+a))*sin(3)+3/8*Si(coth(b*x+a)
+1)*cos(1)-3/8*Ci(coth(b*x+a)+1)*sin(1)-3/8*Si(coth(b*x+a)-1)*cos(1)-3/8*Ci
(coth(b*x+a)-1)*sin(1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\coth(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(coth(b*x + a))^3, x)
```

Fricas [B] time = 2.7501, size = 929, normalized size = 5.92

$$\frac{\mathrm{Ci}\left(\frac{6\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)\sin(3)+\mathrm{Ci}\left(-\frac{6\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)\sin(3)+\mathrm{Ci}\left(\frac{6}{e^{(2\,bx+2\,a)}-1}\right)\sin(3)+\mathrm{Ci}\left(-\frac{6}{e^{(2\,bx+2\,a)}-1}\right)\sin(3)-3\,\mathrm{Ci}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(cos_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(3) + cos_in
tegral(-6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(3) + cos_integral(6/(e
^(2*b*x + 2*a) - 1))*sin(3) + cos_integral(-6/(e^(2*b*x + 2*a) - 1))*sin(3)
 - 3*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) - 3*cos_i
ntegral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) - 3*cos_integral(2
/(e^(2*b*x + 2*a) - 1))*sin(1) - 3*cos_integral(-2/(e^(2*b*x + 2*a) - 1))*s
in(1) - 2*cos(3)*sin_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 6*
cos(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*cos(3)*sin
_integral(6/(e^(2*b*x + 2*a) - 1)) - 6*cos(1)*sin_integral(2/(e^(2*b*x + 2*
a) - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^3(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))**3,x)
```

```
[Out] Integral(sin(coth(a + b*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\coth(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a))^3, x)
```

3.218 $\int \sin^2(\coth(a+bx))\,dx$

Optimal. Leaf size=115

$$\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b} - \frac{\cos(2)\text{CosIntegral}(2\coth(a+bx)+2)}{4b} + \frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b} -$$

```
[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2
*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b*
x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Sin[2]*SinIn
tegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rubi [A] time = 0.262045, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b} - \frac{\cos(2)\text{CosIntegral}(2\coth(a+bx)+2)}{4b} + \frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b} -$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Coth[a + b*x]]^2,x]
```


```
[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2
*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b*
x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Sin[2]*SinIn
tegral[2 + 2*Coth[a + b*x]])/(4*b)
```


Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin^2(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1-x^2}\,dx,x,\coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sin^2(x)}{2(-1+x)}+\frac{\sin^2(x)}{2(1+x)}\right)dx,x,\coth(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1+x}\,dx,x,\coth(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}-\frac{\cos(2x)}{2(-1+x)}\right)dx,x,\coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}-\frac{\cos(2x)}{2(1+x)}\right)dx,x,\coth(\right.}{2b}$$
$$= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{4b}-\frac{\text{S}}{}$$
$$= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}+\frac{\cos(2)\,\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx,x,\coth(a+\right.}{4b}$$
$$= \frac{\cos(2)\text{Ci}(2-2\coth(a+bx))}{4b}-\frac{\cos(2)\text{Ci}(2+2\coth(a+bx))}{4b}-\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log}{}$$

Mathematica [A] time = 0.166621, size = 88, normalized size = 0.77

$$\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))-\cos(2)\text{CosIntegral}(2(\coth(a+bx)+1))+\sin(2)\text{Si}(2-2\coth(a+bx))-\sin}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Coth[a + b*x]]^2,x]
```

```
[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]] - Cos[2]*CosIntegral[2*(1 + Coth[a
 + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] + Sin[2]*SinInt
egral[2 - 2*Coth[a + b*x]] - Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/(4*
b)
```

Maple [A] time = 0.02, size = 102, normalized size = 0.9

$$-\frac{\ln(\coth(bx+a)-1)}{4b}+\frac{\ln(\coth(bx+a)+1)}{4b}-\frac{\text{Si}(2+2\coth(bx+a))\sin(2)}{4b}-\frac{\text{Ci}(2+2\coth(bx+a))\cos(2)}{4b}-\frac{\text{Si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(b*x+a))^2,x)
```

```
[Out] -1/4/b*ln(coth(b*x+a)-1)+1/4/b*ln(coth(b*x+a)+1)-1/4*Si(2+2*coth(b*x+a))*si
n(2)/b-1/4*Ci(2+2*coth(b*x+a))*cos(2)/b-1/4*Si(-2+2*coth(b*x+a))*sin(2)/b+1
/4/b*Ci(-2+2*coth(b*x+a))*cos(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x-\frac{1}{2}\int \cos\left(\frac{2\left(e^{(2bx+2a)}+1\right)}{e^{(2bx+2a)}-1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)), x
)
```

Fricas [A] time = 2.52604, size = 474, normalized size = 4.12

$$\frac{4\,bx-\cos\left(2\right)\operatorname{Ci}\left(\frac{4\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right)-\cos\left(2\right)\operatorname{Ci}\left(-\frac{4\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right)+\cos\left(2\right)\operatorname{Ci}\left(\frac{4}{e^{\left(2\,bx+2\,a\right)}-1}\right)+\cos\left(2\right)\operatorname{Ci}\left(-\frac{4}{e^{\left(2\,bx+2\,a\right)}-1}\right)-2\,\sin\left(2\right)\,\ldots}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x - cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) -
 cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) - 1)) + cos(2)*cos_integral(-4/(e^(2*b*x + 2*a
) - 1)) - 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) -
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{2}\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))**2,x)
```

```
[Out] Integral(sin(coth(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\coth\left(bx+a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a))^2, x)
```

3.219 $\int \sin(\coth(a+bx))\,dx$

Optimal. Leaf size=77

$$-\frac{\sin(1)\text{CosIntegral}(1-\coth(a+bx))}{2b}-\frac{\sin(1)\text{CosIntegral}(\coth(a+bx)+1)}{2b}+\frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{S}}{}$$

```
[Out] -(CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(2*b) - (CosIntegral[1 + Coth[a +
b*x]]*Sin[1])/(2*b) + (Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Cos[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rubi [A] time = 0.144187, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3333, 3303, 3299, 3302}

$$-\frac{\sin(1)\text{CosIntegral}(1-\coth(a+bx))}{2b}-\frac{\sin(1)\text{CosIntegral}(\coth(a+bx)+1)}{2b}+\frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{S}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Coth[a + b*x]],x]
```

```
[Out] -(CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(2*b) - (CosIntegral[1 + Coth[a +
b*x]]*Sin[1])/(2*b) + (Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Cos[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{2(1-x)} + \frac{\sin(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{\cos(1)\,\text{Subst}\left(\int \frac{\sin(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\cos(1)\,\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \coth(a+b$$
$$= -\frac{\text{Ci}(1-\coth(a+bx))\sin(1)}{2b} - \frac{\text{Ci}(1+\coth(a+bx))\sin(1)}{2b} + \frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b}$$

Mathematica [A] time = 0.128473, size = 59, normalized size = 0.77

$$-\frac{\sin(1)\text{CosIntegral}(1-\coth(a+bx)) + \sin(1)\text{CosIntegral}(\coth(a+bx)+1) - \cos(1)(\text{Si}(1-\coth(a+bx)) + \text{Si}(c}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Coth[a + b*x]],x]
```

```
[Out] -(CosIntegral[1 - Coth[a + b*x]]*Sin[1] + CosIntegral[1 + Coth[a + b*x]]*Si
n[1] - Cos[1]*(SinIntegral[1 - Coth[a + b*x]] + SinIntegral[1 + Coth[a + b*
x]]))/(2*b)
```

Maple [A] time = 0.016, size = 58, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\text{Si}(\coth(bx+a)+1)\cos(1)}{2} - \frac{\text{Ci}(\coth(bx+a)+1)\sin(1)}{2} - \frac{\text{Si}(\coth(bx+a)-1)\cos(1)}{2} - \frac{\text{Ci}(\coth(bx+a)-}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(b*x+a)),x)
```

```
[Out] 1/b*(1/2*Si(coth(b*x+a)+1)*cos(1)-1/2*Ci(coth(b*x+a)+1)*sin(1)-1/2*Si(coth(
b*x+a)-1)*cos(1)-1/2*Ci(coth(b*x+a)-1)*sin(1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(sin(coth(b*x + a)), x)
```

Fricas [B] time = 2.43796, size = 464, normalized size = 6.03

$$-\frac{\operatorname{Ci}\left(\frac{2 e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(1)+\operatorname{Ci}\left(-\frac{2 e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(1)+\operatorname{Ci}\left(\frac{2}{e^{(2bx+2a)}-1}\right)\sin(1)+\operatorname{Ci}\left(-\frac{2}{e^{(2bx+2a)}-1}\right)\sin(1)-2\cos(1)\operatorname{Si}\left(\frac{2 e^{(2bx+}}{e^{(2bx+2a}}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/4*(cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) + cos_in
tegral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) + cos_integral(2/(e
^(2*b*x + 2*a) - 1))*sin(1) + cos_integral(-2/(e^(2*b*x + 2*a) - 1))*sin(1)
 - 2*cos(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*cos(1
)*sin_integral(2/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x)
```

```
[Out] Integral(sin(coth(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a)), x)
```

3.220 $\int \csc(\coth(a+bx))\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}\text{Unintegrable}\left(\frac{\text{csch}^2(a+bx)\csc(\coth(a+bx))}{\coth(a+bx)-1},x\right)-\frac{1}{2}\text{Unintegrable}\left(\frac{\text{csch}^2(a+bx)\csc(\coth(a+bx))}{\coth(a+bx)+1},x\right)$$

```
[Out] Unintegrable[(Csc[Coth[a + b*x]]*Csch[a + b*x]^2)/(-1 + Coth[a + b*x]), x]/
2 - Unintegrable[(Csc[Coth[a + b*x]]*Csch[a + b*x]^2)/(1 + Coth[a + b*x]),
x]/2
```

Rubi [A] time = 0.0772917, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \csc(\coth(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[Coth[a + b*x]],x]
```

```
[Out] -Defer[Subst][Defer[Int][Csc[x]/(-1 + x), x], x, Coth[a + b*x]]/(2*b) + Def
er[Subst][Defer[Int][Csc[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}\int \csc(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\csc(x)}{1-x^2}\,dx,x,\coth(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{\csc(x)}{2(-1+x)}+\frac{\csc(x)}{2(1+x)}\right)dx,x,\coth(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \frac{\csc(x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\csc(x)}{1+x}\,dx,x,\coth(a+bx)\right)}{2b}\end{aligned}$$

Mathematica [A] time = 2.68009, size = 0, normalized size = 0.

$$\int \csc(\coth(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[Coth[a + b*x]],x]
```

```
[Out] Integrate[Csc[Coth[a + b*x]], x]
```

Maple [A] time = 0.165, size = 0, normalized size = 0.

$$\int \csc(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(coth(b*x+a)),x)
```

```
[Out] int(csc(coth(b*x+a)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \csc(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(csc(coth(b*x + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\csc(\coth(bx+a)),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(csc(coth(b*x + a)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \csc(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x)
```

```
[Out] Integral(csc(coth(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \csc(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(csc(coth(b*x + a)), x)
```

3.221 $\int \cos^3(\coth(a+bx))\,dx$

Optimal. Leaf size=157

$$-\frac{\cos(3)\text{CosIntegral}(3-3\coth(a+bx))}{8b}-\frac{3\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{CosIntegral}(\coth(a}{8b}$$

```
[Out] -(Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]])/(8*b) - (3*Cos[1]*CosIntegral[1
- Coth[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Coth[a + b*x]])/(8*b) +
 (Cos[3]*CosIntegral[3 + 3*Coth[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Sin[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rubi [A] time = 0.373451, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$-\frac{\cos(3)\text{CosIntegral}(3-3\coth(a+bx))}{8b}-\frac{3\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{CosIntegral}(\coth(a}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]]^3,x]
```

```
[Out] -(Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]])/(8*b) - (3*Cos[1]*CosIntegral[1
- Coth[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Coth[a + b*x]])/(8*b) +
 (Cos[3]*CosIntegral[3 + 3*Coth[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Sin[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \cos^3(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\cos^3(x)}{2(-1+x)}+\frac{\cos^3(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(-1+x)}+\frac{\cos(3x)}{4(-1+x)}\right)dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(1+x)}+\frac{\cos(3x)}{4(1+x)}\right)dx, x, \text{cotl}\right.}{2b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{\text{Subst}\left(\int \frac{\cos(3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\frac{3\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\cos(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \coth(\right.}{8b}$$
$$= -\frac{\cos(3)\text{Ci}(3-3\coth(a+bx))}{8b}-\frac{3\cos(1)\text{Ci}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{Ci}(1+\coth(a+b}{8b}$$

Mathematica [A] time = 0.24361, size = 124, normalized size = 0.79

$$-2\cos(3)\text{CosIntegral}(3-3\coth(a+bx))-6\cos(1)\text{CosIntegral}(1-\coth(a+bx))+6\cos(1)\text{CosIntegral}(\coth(a+b$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Coth[a + b*x]]^3,x]
```

```
[Out] (-2*Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]] - 6*Cos[1]*CosIntegral[1 - Coth
[a + b*x]] + 6*Cos[1]*CosIntegral[1 + Coth[a + b*x]] + 2*Cos[3]*CosIntegral
[3 + 3*Coth[a + b*x]] - 2*Sin[3]*SinIntegral[3 - 3*Coth[a + b*x]] - 6*Sin[1
]*SinIntegral[1 - Coth[a + b*x]] + 6*Sin[1]*SinIntegral[1 + Coth[a + b*x]]
+ 2*Sin[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

Maple [A] time = 0.023, size = 118, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\text{Si}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\text{Ci}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\text{Si}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\text{Ci}(-3+3\text{cot}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(b*x+a))^3,x)
```

```
[Out] 1/b*(1/8*Si(3+3*coth(b*x+a))*sin(3)+1/8*Ci(3+3*coth(b*x+a))*cos(3)+1/8*Si(-
3+3*coth(b*x+a))*sin(3)-1/8*Ci(-3+3*coth(b*x+a))*cos(3)+3/8*Si(coth(b*x+a)+
1)*sin(1)+3/8*Ci(coth(b*x+a)+1)*cos(1)+3/8*Si(coth(b*x+a)-1)*sin(1)-3/8*Ci(
coth(b*x+a)-1)*cos(1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\coth\left(bx+a\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(coth(b*x + a))^3, x)
```

Fricas [B] time = 2.96805, size = 929, normalized size = 5.92

$$\frac{\cos\left(3\right)\operatorname{Ci}\left(\frac{6\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right)+3\,\cos\left(1\right)\operatorname{Ci}\left(\frac{2\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right)+3\,\cos\left(1\right)\operatorname{Ci}\left(-\frac{2\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right)+\cos\left(3\right)\operatorname{Ci}\left(-\frac{6\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right)-\cos\left(3\right)\operatorname{Ci}\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(cos(3)*cos_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 3*cos(
1)*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 3*cos(1)*cos_int
egral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(3)*cos_integral(-6*e^
(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(3)*cos_integral(6/(e^(2*b*x + 2*
a) - 1)) - 3*cos(1)*cos_integral(2/(e^(2*b*x + 2*a) - 1)) - 3*cos(1)*cos_in
tegral(-2/(e^(2*b*x + 2*a) - 1)) - cos(3)*cos_integral(-6/(e^(2*b*x + 2*a)
- 1)) + 2*sin(3)*sin_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 6*
sin(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*sin(3)*sin
_integral(6/(e^(2*b*x + 2*a) - 1)) + 6*sin(1)*sin_integral(2/(e^(2*b*x + 2*
a) - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^{3}\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))**3,x)
```

```
[Out] Integral(cos(coth(a + b*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\coth\left(bx+a\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a))^3, x)
```

3.222 $\int \cos^2(\coth(a+bx))\,dx$

Optimal. Leaf size=115

$$-\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b}+\frac{\cos(2)\text{CosIntegral}(2\coth(a+bx)+2)}{4b}-\frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b}+$$

```
[Out] -(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Cos[2]*CosIntegral[2 +
2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rubi [A] time = 0.251206, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$-\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b}+\frac{\cos(2)\text{CosIntegral}(2\coth(a+bx)+2)}{4b}-\frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]]^2,x]
```

```
[Out] -(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Cos[2]*CosIntegral[2 +
2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \cos^2(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\cos^2(x)}{2(-1+x)}+\frac{\cos^2(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}+\frac{\cos(2x)}{2(-1+x)}\right)dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}+\frac{\cos(2x)}{2(1+x)}\right)dx, x, \text{co}\right.}{2b}$$
$$= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{4b}$$
$$= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}-\frac{\cos(2)\,\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx, x, \coth(a\right.}{4b}$$
$$= -\frac{\cos(2)\text{Ci}(2-2\coth(a+bx))}{4b}+\frac{\cos(2)\text{Ci}(2+2\coth(a+bx))}{4b}-\frac{\log(1-\coth(a+bx))}{4b}$$

Mathematica [A] time = 0.165737, size = 88, normalized size = 0.77

$$\frac{-\cos(2)\text{CosIntegral}(2-2\coth(a+bx))+\cos(2)\text{CosIntegral}(2(\coth(a+bx)+1))-\sin(2)\text{Si}(2-2\coth(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Coth[a + b*x]]^2,x]
```

```
[Out] (-(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]]) + Cos[2]*CosIntegral[2*(1 + Cot
h[a + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] - Sin[2]*Sin
Integral[2 - 2*Coth[a + b*x]] + Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/
(4*b)
```

Maple [A] time = 0.021, size = 102, normalized size = 0.9

$$\frac{\text{Si}(2+2\coth(bx+a))\sin(2)}{4b}+\frac{\text{Ci}(2+2\coth(bx+a))\cos(2)}{4b}+\frac{\text{Si}(-2+2\coth(bx+a))\sin(2)}{4b}-\frac{\text{Ci}(-2+2\text{cot}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(b*x+a))^2,x)
```

```
[Out] 1/4*Si(2+2*coth(b*x+a))*sin(2)/b+1/4*Ci(2+2*coth(b*x+a))*cos(2)/b+1/4*Si(-2
+2*coth(b*x+a))*sin(2)/b-1/4/b*Ci(-2+2*coth(b*x+a))*cos(2)-1/4/b*ln(coth(b*
x+a)-1)+1/4/b*ln(coth(b*x+a)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x+\frac{1}{2}\int \cos\left(\frac{2\left(e^{(2bx+2a)}+1\right)}{e^{(2bx+2a)}-1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)), x
)
```

Fricas [A] time = 2.77952, size = 474, normalized size = 4.12

$$\frac{4\,bx + \cos\left(2\right)\operatorname{Ci}\left(\frac{4\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right) + \cos\left(2\right)\operatorname{Ci}\left(-\frac{4\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right) - \cos\left(2\right)\operatorname{Ci}\left(\frac{4}{e^{\left(2\,bx+2\,a\right)}-1}\right) - \cos\left(2\right)\operatorname{Ci}\left(-\frac{4}{e^{\left(2\,bx+2\,a\right)}-1}\right) + 2\,\sin\left(2\right)\operatorname{Si}\left(\right.}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x + cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) +
 cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) - 1)) - cos(2)*cos_integral(-4/(e^(2*b*x + 2*a
) - 1)) + 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) +
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^2\left(\coth\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))**2,x)
```

```
[Out] Integral(cos(coth(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\coth\left(bx + a\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a))^2, x)
```

3.223 $\int \cos(\coth(a+bx))\,dx$

Optimal. Leaf size=77

$$-\frac{\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{CosIntegral}(\coth(a+bx)+1)}{2b}-\frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b}+\frac{\sin\text{(}}{}$$

```
[Out] -(Cos[1]*CosIntegral[1 - Coth[a + b*x]])/(2*b) + (Cos[1]*CosIntegral[1 + Co
th[a + b*x]])/(2*b) - (Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Sin[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rubi [A] time = 0.140022, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3334, 3303, 3299, 3302}

$$-\frac{\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{CosIntegral}(\coth(a+bx)+1)}{2b}-\frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b}+\frac{\sin\text{(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]],x]
```

```
[Out] -(Cos[1]*CosIntegral[1 - Coth[a + b*x]])/(2*b) + (Cos[1]*CosIntegral[1 + Co
th[a + b*x]])/(2*b) - (Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Sin[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \cos(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{2(1-x)} + \frac{\cos(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cos(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= \frac{\cos(1)\,\text{Subst}\left(\int \frac{\cos(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\cos(1)\,\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{\cos(1)\text{Ci}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\text{Ci}(1+\coth(a+bx))}{2b} - \frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b} + $$

Mathematica [A] time = 0.105566, size = 62, normalized size = 0.81

$$-\frac{\cos(1)\text{CosIntegral}(1-\coth(a+bx)) - \cos(1)\text{CosIntegral}(\coth(a+bx)+1) + \sin(1)\text{Si}(1-\coth(a+bx)) - \sin(1)\text{Si}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Coth[a + b*x]],x]
```

```
[Out] -(Cos[1]*CosIntegral[1 - Coth[a + b*x]] - Cos[1]*CosIntegral[1 + Coth[a + b
*x]] + Sin[1]*SinIntegral[1 - Coth[a + b*x]] - Sin[1]*SinIntegral[1 + Coth[
a + b*x]])/(2*b)
```

Maple [A] time = 0.022, size = 58, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\text{Si}(\coth(bx+a)+1)\sin(1)}{2} + \frac{\text{Ci}(\coth(bx+a)+1)\cos(1)}{2} + \frac{\text{Si}(\coth(bx+a)-1)\sin(1)}{2} - \frac{\text{Ci}(\coth(bx+a)-1)}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(b*x+a)),x)
```

```
[Out] 1/b*(1/2*Si(coth(b*x+a)+1)*sin(1)+1/2*Ci(coth(b*x+a)+1)*cos(1)+1/2*Si(coth(
b*x+a)-1)*sin(1)-1/2*Ci(coth(b*x+a)-1)*cos(1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(cos(coth(b*x + a)), x)
```

Fricas [B] time = 2.69787, size = 463, normalized size = 6.01

$$\frac{\cos(1)\,\mathrm{Ci}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)+\cos(1)\,\mathrm{Ci}\left(-\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)-\cos(1)\,\mathrm{Ci}\left(\frac{2}{e^{(2\,bx+2\,a)}-1}\right)-\cos(1)\,\mathrm{Ci}\left(-\frac{2}{e^{(2\,bx+2\,a)}-1}\right)+2\,\sin(1)\,\mathrm{Si}\left(\frac{2\,e^{(2}}{e^{(2\,bx}}\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(cos(1)*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(1)*
cos_integral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(1)*cos_integra
l(2/(e^(2*b*x + 2*a) - 1)) - cos(1)*cos_integral(-2/(e^(2*b*x + 2*a) - 1))
+ 2*sin(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*sin(1)
*sin_integral(2/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x)
```

```
[Out] Integral(cos(coth(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a)), x)
```

3.224 $\int \sec(\coth(a+bx))\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}\text{Unintegrable}\left(\frac{\text{csch}^2(a+bx)\sec(\coth(a+bx))}{\coth(a+bx)-1},x\right)-\frac{1}{2}\text{Unintegrable}\left(\frac{\text{csch}^2(a+bx)\sec(\coth(a+bx))}{\coth(a+bx)+1},x\right)$$

```
[Out] Unintegrable[(Csch[a + b*x]^2*Sec[Coth[a + b*x]])/(-1 + Coth[a + b*x]), x]/
2 - Unintegrable[(Csch[a + b*x]^2*Sec[Coth[a + b*x]])/(1 + Coth[a + b*x]),
x]/2
```

Rubi [A] time = 0.0770718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec(\coth(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[Coth[a + b*x]],x]
```

```
[Out] -Defer[Subst][Defer[Int][Sec[x]/(-1 + x), x], x, Coth[a + b*x]]/(2*b) + Def
er[Subst][Defer[Int][Sec[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}
\int \sec(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\sec(x)}{1-x^2}\,dx,x,\coth(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sec(x)}{2(-1+x)}+\frac{\sec(x)}{2(1+x)}\right)dx,x,\coth(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\sec(x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sec(x)}{1+x}\,dx,x,\coth(a+bx)\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 5.48446, size = 0, normalized size = 0.

$$\int \sec(\coth(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[Coth[a + b*x]],x]
```

```
[Out] Integrate[Sec[Coth[a + b*x]], x]
```

Maple [A] time = 0.067, size = 0, normalized size = 0.

$$\int \sec(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(coth(b*x+a)),x)
```

```
[Out] int(sec(coth(b*x+a)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sec(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(sec(coth(b*x + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sec(\coth(bx+a)),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(sec(coth(b*x + a)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sec(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x)
```

```
[Out] Integral(sec(coth(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sec(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(sec(coth(b*x + a)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
    not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```