

Computer algebra independent integration tests

6-Hyperbolic-functions/6.4-Hyperbolic-cotangent/6.4.2-Hyperbolic-cotangent-functions

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3.146	$\int \frac{\coth^2(x)}{a+b \coth(x)} dx$	722
3.147	$\int \frac{\coth^3(x)}{a+b \coth(x)} dx$	726
3.148	$\int \frac{\coth^4(x)}{a+b \coth(x)} dx$	731
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3.154	$\int \coth(a + 2 \log(x)) dx$	756
3.155	$\int \frac{\coth(a+2 \log(x))}{x} dx$	759
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3.163	$\int \frac{\coth^2(a+2 \log(x))}{x^2} dx$	783
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3.166	$\int (ex)^m \coth^2(a + 2 \log(x)) dx$	792
3.167	$\int (ex)^m \coth^3(a + 2 \log(x)) dx$	795
3.168	$\int \coth^p(a + b \log(x)) dx$	798
3.169	$\int (ex)^m \coth^p(a + b \log(x)) dx$	801
3.170	$\int \coth^p\left(a + \frac{\log(x)}{2}\right) dx$	804
3.171	$\int \coth^p\left(a + \frac{\log(x)}{4}\right) dx$	807
3.172	$\int \coth^p\left(a + \frac{\log(x)}{6}\right) dx$	810
3.173	$\int \coth^p\left(a + \frac{\log(x)}{8}\right) dx$	813
3.174	$\int \coth^p(a + \log(x)) dx$	816
3.175	$\int \coth^p(a + 2 \log(x)) dx$	819
3.176	$\int \coth^p(a + 3 \log(x)) dx$	822
3.177	$\int x^3 \coth(d(a + b \log(cx^n))) dx$	825
3.178	$\int x^2 \coth(d(a + b \log(cx^n))) dx$	828
3.179	$\int x \coth(d(a + b \log(cx^n))) dx$	831
3.180	$\int \coth(d(a + b \log(cx^n))) dx$	834
3.181	$\int \frac{\coth(d(a+b \log(cx^n)))}{x} dx$	837
3.182	$\int \frac{\coth(d(a+b \log(cx^n)))}{x^2} dx$	840
3.183	$\int \frac{\coth(d(a+b \log(cx^n)))}{x^3} dx$	843
3.184	$\int x^3 \coth^2(d(a + b \log(cx^n))) dx$	846
3.185	$\int x^2 \coth^2(d(a + b \log(cx^n))) dx$	849
3.186	$\int x \coth^2(d(a + b \log(cx^n))) dx$	852
3.187	$\int \coth^2(d(a + b \log(cx^n))) dx$	855
3.188	$\int \frac{\coth^2(d(a+b \log(cx^n)))}{x} dx$	858

3.189	$\int \frac{\coth^2(d(a+b \log(cx^n)))}{x^2} dx$	861
3.190	$\int \frac{\coth^2(d(a+b \log(cx^n)))}{x^3} dx$	864
3.191	$\int \frac{\coth^3(a+b \log(cx^n))}{x} dx$	867
3.192	$\int \frac{\coth^4(a+b \log(cx^n))}{x} dx$	871
3.193	$\int \frac{\coth^5(a+b \log(cx^n))}{x} dx$	875
3.194	$\int (ex)^m \coth(d(a+b \log(cx^n))) dx$	880
3.195	$\int (ex)^m \coth^2(d(a+b \log(cx^n))) dx$	883
3.196	$\int (ex)^m \coth^3(d(a+b \log(cx^n))) dx$	886
3.197	$\int \coth^p(d(a+b \log(cx^n))) dx$	890
3.198	$\int (ex)^m \coth^p(d(a+b \log(cx^n))) dx$	893
3.199	$\int \frac{\coth^{\frac{5}{2}}(a+b \log(cx^n))}{x} dx$	896
3.200	$\int \frac{\coth^{\frac{3}{2}}(a+b \log(cx^n))}{x} dx$	901
3.201	$\int \frac{\sqrt{\coth(a+b \log(cx^n))}}{x} dx$	906
3.202	$\int \frac{1}{x\sqrt{\coth(a+b \log(cx^n))}} dx$	910
3.203	$\int \frac{1}{x \coth^{\frac{3}{2}}(a+b \log(cx^n))} dx$	914
3.204	$\int \frac{1}{x \coth^{\frac{5}{2}}(a+b \log(cx^n))} dx$	919
3.205	$\int \frac{\coth^5(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} dx$	925
3.206	$\int \frac{\coth^3(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} dx$	936
3.207	$\int \frac{\coth(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} dx$	945
3.208	$\int \frac{\tanh(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} dx$	950
3.209	$\int \frac{\tanh^3(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} dx$	959
3.210	$\int \coth(x) \sqrt{a+b \coth^2(x)+c \coth^4(x)} dx$	971
3.211	$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2} dx$	981
3.212	$\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2} dx$	988
3.213	$\int e^{c(a+bx)} \sqrt{\coth^2(ac+bcx)} dx$	994
3.214	$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}} dx$	998
3.215	$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}} dx$	1002
3.216	$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}} dx$	1008

3.217	$\int \sin^3(\coth(a + bx)) dx$1014
3.218	$\int \sin^2(\coth(a + bx)) dx$1019
3.219	$\int \sin(\coth(a + bx)) dx$1023
3.220	$\int \csc(\coth(a + bx)) dx$1027
3.221	$\int \cos^3(\coth(a + bx)) dx$1030
3.222	$\int \cos^2(\coth(a + bx)) dx$1035
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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [224]. This is test number [175].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 81.7 (183)	% 18.3 (41)
Mathematica	% 100. (224)	% 0. (0)
Maple	% 73.21 (164)	% 26.79 (60)
Maxima	% 46.88 (105)	% 53.12 (119)
Fricas	% 79.02 (177)	% 20.98 (47)
Sympy	% 13.84 (31)	% 86.16 (193)
Giac	% 56.25 (126)	% 43.75 (98)

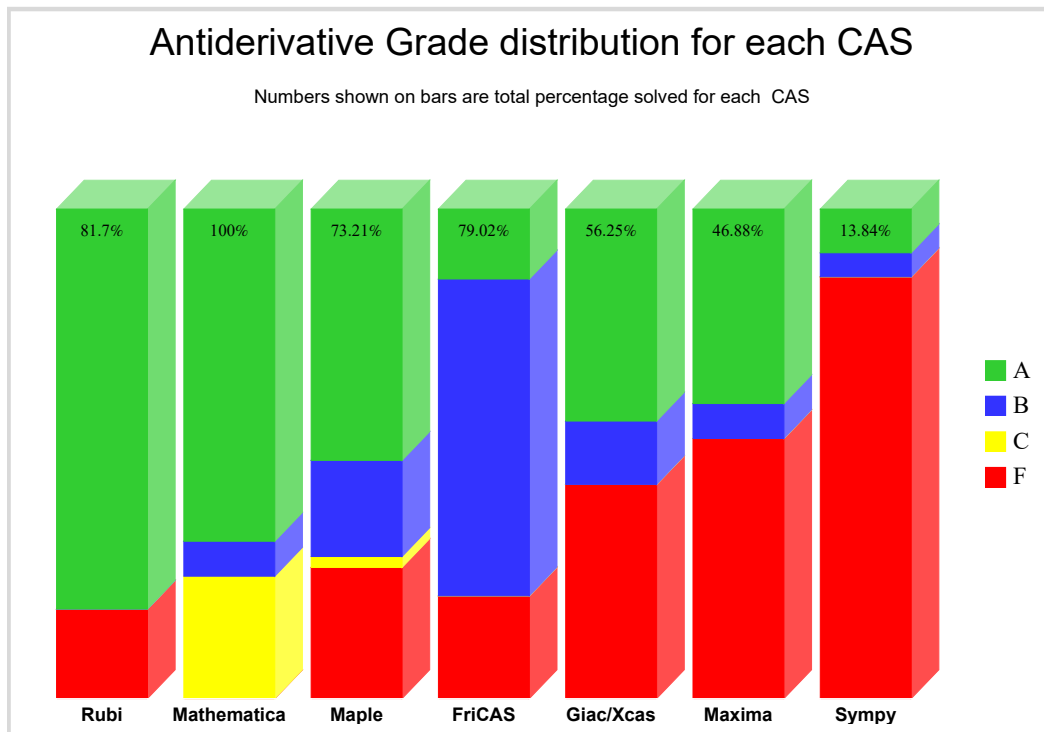
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

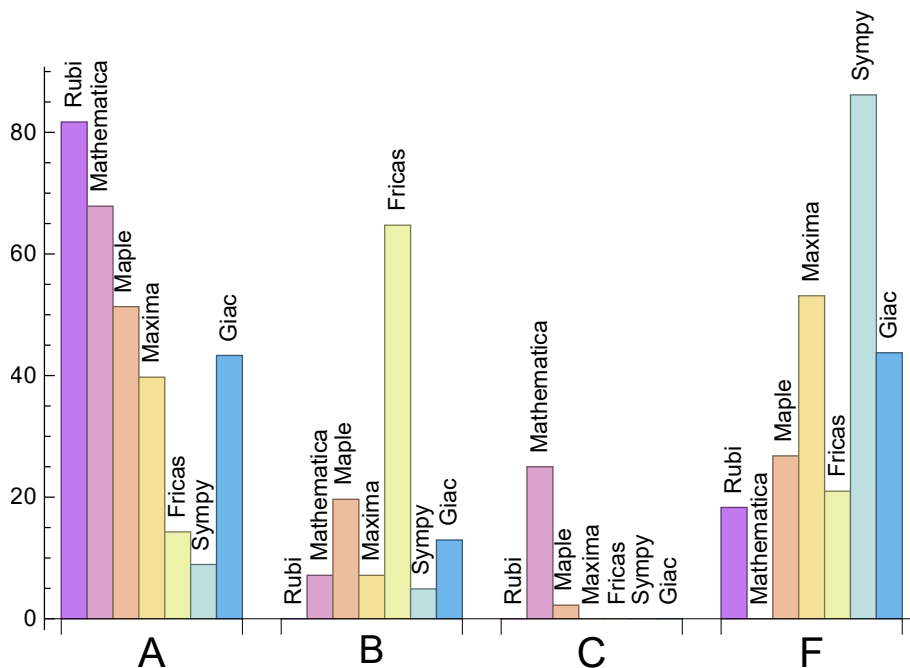
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	81.7	0.	0.	18.3
Mathematica	67.86	7.14	25.	0.
Maple	51.34	19.64	2.23	26.79
Maxima	39.73	7.14	0.	53.12
Fricas	14.29	64.73	0.	20.98
Sympy	8.93	4.91	0.	86.16
Giac	43.3	12.95	0.	43.75

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	84.35	0.99	60.	1.
Mathematica	1.36	96.6	1.31	60.	1.05
Maple	0.04	93.86	1.5	70.5	1.28
Maxima	1.33	111.23	2.05	63.	1.5
Fricas	3.04	2321.98	23.59	828.	14.48
Sympy	3.76	196.35	3.74	104.	2.54
Giac	1.16	107.52	2.01	81.	1.66

1.4 list of integrals that has no closed form antiderivative

{220, 224}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {159, 160, 161, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 196, 197, 198, 209, 212}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

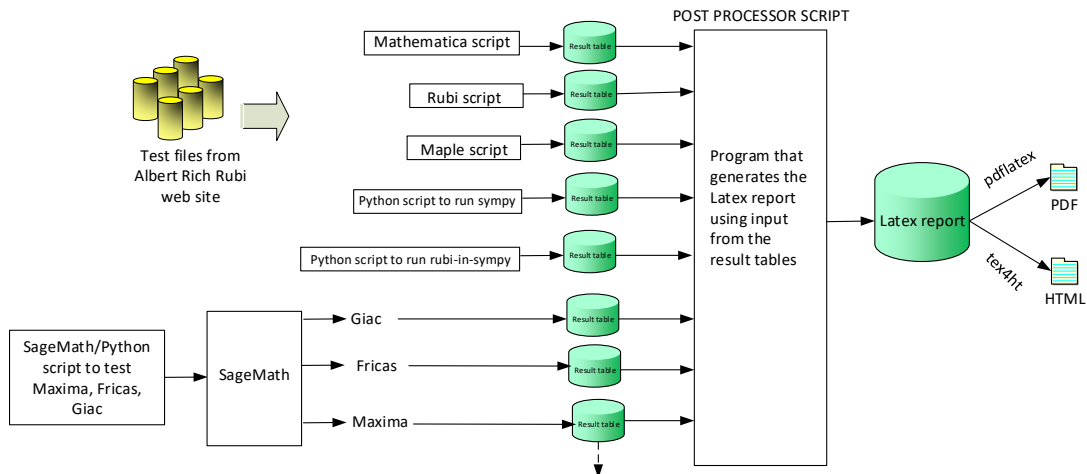
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 162, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { }

C grade: { }

F grade: { 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 10, 12, 15, 16, 17, 18, 19, 20, 21, 22, 24, 28, 29, 30, 35, 36, 37, 38, 39, 42, 43, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 153, 155, 157, 158, 165, 167, 169, 170, 171, 172, 173, 181,

184, 185, 186, 187, 189, 190, 191, 193, 194, 195, 196, 198, 199, 200, 201, 202, 205, 206, 208, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { 151, 166, 168, 174, 175, 176, 177, 178, 179, 180, 182, 183, 197, 207, 209, 210 }

C grade: { 6, 7, 8, 9, 11, 13, 14, 23, 25, 26, 27, 31, 32, 33, 34, 40, 41, 44, 46, 47, 48, 49, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 87, 113, 132, 133, 134, 135, 136, 137, 138, 139, 152, 154, 156, 159, 160, 161, 162, 163, 164, 188, 192, 203, 204, 212 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 29, 30, 31, 32, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 98, 100, 101, 102, 109, 115, 116, 117, 118, 119, 122, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 157, 158, 159, 160, 161, 164, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { 33, 34, 35, 36, 37, 38, 77, 78, 79, 80, 89, 90, 91, 92, 95, 96, 97, 99, 103, 104, 105, 106, 107, 108, 110, 111, 112, 114, 120, 121, 123, 124, 125, 126, 140, 141, 152, 154, 155, 156, 162, 181, 188, 213 }

C grade: { 163, 210, 214, 215, 216 }

F grade: { 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 113, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 208, 209 }

2.1.4 Maxima

A grade: { 18, 19, 20, 21, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 64, 65, 66, 67, 68, 69, 80, 81, 82, 85, 86, 89, 90, 91, 92, 93, 94, 97, 99, 102, 105, 106, 107, 108, 109, 110, 114, 116, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 181, 188, 211, 212, 213, 214, 215, 216, 220, 224 }

B grade: { 61, 62, 63, 77, 78, 79, 83, 84, 95, 96, 104, 111, 112, 191, 192, 193 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 87, 88, 98, 100, 101, 103, 113, 115, 117, 118, 120, 132, 133, 134, 135, 136, 137, 138, 139, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 218, 219, 221, 222, 223 }

2.1.5 FriCAS

A grade: { 9, 10, 46, 81, 85, 86, 91, 92, 93, 101, 107, 108, 118, 122, 143, 144, 145, 146, 151, 152, 153, 155, 156, 158, 163, 164, 213, 214, 218, 220, 222, 224 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 47, 48, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 147, 148, 149, 150, 154, 157, 159, 160, 161, 162, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 216, 217, 219, 221, 223 }

C grade: { }

F grade: { 15, 16, 17, 27, 28, 39, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198 }

2.1.6 Sympy

A grade: { 61, 62, 63, 64, 77, 78, 79, 80, 81, 85, 86, 144, 145, 146, 147, 148, 149, 162, 220, 224 }

B grade: { 65, 66, 67, 68, 69, 127, 128, 129, 130, 131, 155 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223 }

2.1.7 Giac

A grade: { 18, 19, 21, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 188, 192, 211, 212, 213, 214, 215, 216, 220, 224 }

B grade: { 6, 11, 12, 20, 70, 71, 72, 73, 74, 75, 76, 95, 102, 104, 113, 119, 121, 132, 133, 134, 135, 136, 137, 138, 139, 150, 181, 191, 193 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 87, 88, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 218, 219, 221, 222, 223 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	83	80	0	4135	0	0
normalized size	1	1.	0.86	0.82	0.	42.63	0.	0.
time (sec)	N/A	0.07	0.232	0.026	0.	2.507	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	63	0	2654	0	0
normalized size	1	1.	0.87	0.81	0.	34.03	0.	0.
time (sec)	N/A	0.049	0.213	0.01	0.	2.388	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	61	62	0	1754	0	0
normalized size	1	1.	0.81	0.83	0.	23.39	0.	0.
time (sec)	N/A	0.05	0.095	0.011	0.	2.272	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	47	0	1636	0	0
normalized size	1	1.	0.88	0.81	0.	28.21	0.	0.
time (sec)	N/A	0.036	0.042	0.023	0.	2.214	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	0	1646	0	0
normalized size	1	1.	0.86	0.81	0.	28.88	0.	0.
time (sec)	N/A	0.031	0.034	0.033	0.	2.433	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	36	65	0	2576	0	277
normalized size	1	1.	0.46	0.83	0.	33.03	0.	3.55
time (sec)	N/A	0.051	0.074	0.013	0.	2.716	0.	1.335

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	38	64	0	3933	0	0
normalized size	1	1.	0.48	0.81	0.	49.78	0.	0.
time (sec)	N/A	0.05	0.065	0.011	0.	2.854	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	38	83	0	5797	0	0
normalized size	1	1.	0.38	0.83	0.	57.97	0.	0.
time (sec)	N/A	0.071	0.096	0.013	0.	2.599	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	36	209	0	865	0	0
normalized size	1	1.	0.15	0.89	0.	3.67	0.	0.
time (sec)	N/A	0.282	0.026	0.02	0.	1.961	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	149	193	0	871	0	0
normalized size	1	1.	0.68	0.89	0.	4.	0.	0.
time (sec)	N/A	0.294	0.177	0.016	0.	2.047	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	38	115	0	828	0	293
normalized size	1	1.	0.29	0.87	0.	6.27	0.	2.22
time (sec)	N/A	0.109	0.041	0.015	0.	2.001	0.	1.687

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	98	115	0	4398	0	292
normalized size	1	1.	0.74	0.87	0.	33.32	0.	2.21
time (sec)	N/A	0.104	0.121	0.012	0.	2.23	0.	1.642

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	36	193	0	991	0	0
normalized size	1	1.	0.17	0.89	0.	4.55	0.	0.
time (sec)	N/A	0.235	0.029	0.017	0.	2.01	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	36	211	0	9677	0	0
normalized size	1	1.	0.15	0.89	0.	40.66	0.	0.
time (sec)	N/A	0.316	0.062	0.016	0.	2.449	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	45	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.063	0.165	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.041	0.191	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.048	0.247	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	53	131	2068	0	122
normalized size	1	1.	0.92	0.87	2.15	33.9	0.	2.
time (sec)	N/A	0.037	0.118	0.026	1.604	1.981	0.	1.203

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	45	73	304	0	73
normalized size	1	1.	1.26	1.45	2.35	9.81	0.	2.35
time (sec)	N/A	0.02	0.045	0.036	1.59	1.934	0.	1.159

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	46	309	0	81
normalized size	1	1.	1.	1.68	1.48	9.97	0.	2.61
time (sec)	N/A	0.02	0.059	0.04	1.624	1.92	0.	1.2

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	48	79	113	2082	0	149
normalized size	1	1.	0.74	1.22	1.74	32.03	0.	2.29
time (sec)	N/A	0.036	0.143	0.02	1.589	1.963	0.	1.242

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	166	0	0	5673	0	0
normalized size	1	1.	0.56	0.	0.	19.1	0.	0.
time (sec)	N/A	0.284	0.338	0.082	0.	2.207	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	43	0	0	5751	0	0
normalized size	1	1.	0.15	0.	0.	19.9	0.	0.
time (sec)	N/A	0.18	0.039	0.079	0.	2.145	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	151	0	0	4625	0	0
normalized size	1	1.	0.57	0.	0.	17.52	0.	0.
time (sec)	N/A	0.212	0.101	0.106	0.	2.082	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	41	0	0	23547	0	0
normalized size	1	1.	0.16	0.	0.	89.19	0.	0.
time (sec)	N/A	0.169	0.054	0.105	0.	3.482	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	41	0	0	5739	0	0
normalized size	1	1.	0.14	0.	0.	19.86	0.	0.
time (sec)	N/A	0.222	0.063	0.079	0.	2.328	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	43	0	0	0	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.154	0.079	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.041	0.24	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	82	107	0	5846	0	0
normalized size	1	1.	0.61	0.8	0.	43.63	0.	0.
time (sec)	N/A	0.06	0.519	0.04	0.	3.087	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	63	86	0	1743	0	0
normalized size	1	1.	0.61	0.83	0.	16.76	0.	0.
time (sec)	N/A	0.047	0.096	0.045	0.	2.188	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	41	91	0	2554	0	0
normalized size	1	1.	0.39	0.87	0.	24.32	0.	0.
time (sec)	N/A	0.048	0.029	0.041	0.	2.383	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	43	106	0	8204	0	0
normalized size	1	1.	0.3	0.75	0.	58.18	0.	0.
time (sec)	N/A	0.061	0.065	0.024	0.	2.947	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	43	145	117	2689	0	0
normalized size	1	1.	0.58	1.96	1.58	36.34	0.	0.
time (sec)	N/A	0.034	0.061	0.098	1.757	2.272	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	119	46	992	0	0
normalized size	1	1.	0.82	2.38	0.92	19.84	0.	0.
time (sec)	N/A	0.024	0.028	0.097	1.617	2.189	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	192	69	363	0	0
normalized size	1	1.	1.26	6.19	2.23	11.71	0.	0.
time (sec)	N/A	0.02	0.026	0.092	1.741	2.134	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	192	43	460	0	0
normalized size	1	1.	1.	6.19	1.39	14.84	0.	0.
time (sec)	N/A	0.02	0.028	0.093	1.711	2.444	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	40	119	50	724	0	0
normalized size	1	1.	0.8	2.38	1.	14.48	0.	0.
time (sec)	N/A	0.024	0.06	0.093	1.75	2.237	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	51	149	120	4035	0	0
normalized size	1	1.	0.64	1.86	1.5	50.44	0.	0.
time (sec)	N/A	0.035	0.088	0.099	1.77	2.372	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.044	0.264	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	43	77	185	9090	0	108
normalized size	1	1.	0.39	0.7	1.68	82.64	0.	0.98
time (sec)	N/A	0.045	0.063	0.036	1.735	2.65	0.	1.183

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	55	46	1046	0	43
normalized size	1	1.	0.82	1.1	0.92	20.92	0.	0.86
time (sec)	N/A	0.023	0.031	0.041	1.82	2.158	0.	1.145

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	40	59	49	1079	0	46
normalized size	1	1.	0.8	1.18	0.98	21.58	0.	0.92
time (sec)	N/A	0.023	0.073	0.037	1.736	2.193	0.	1.169

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	68	84	209	9179	0	138
normalized size	1	1.	0.58	0.71	1.77	77.79	0.	1.17
time (sec)	N/A	0.046	0.233	0.022	1.801	2.677	0.	1.251

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	68	0	0	7973	0	0
normalized size	1	1.	0.19	0.	0.	22.59	0.	0.
time (sec)	N/A	0.196	0.139	0.099	0.	2.938	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	166	0	0	1783	0	0
normalized size	1	1.	0.57	0.	0.	6.13	0.	0.
time (sec)	N/A	0.215	0.409	0.094	0.	2.264	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	43	0	0	855	0	0
normalized size	1	1.	0.15	0.	0.	2.96	0.	0.
time (sec)	N/A	0.176	0.036	0.099	0.	2.267	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	41	0	0	9634	0	0
normalized size	1	1.	0.14	0.	0.	33.34	0.	0.
time (sec)	N/A	0.218	0.053	0.093	0.	2.911	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	43	0	0	3221	0	0
normalized size	1	1.	0.15	0.	0.	11.07	0.	0.
time (sec)	N/A	0.185	0.051	0.094	0.	2.487	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	43	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	0.042	0.101	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.048	2.588	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.075	0.341	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.04	0.186	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.047	0.195	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.066	0.168	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.07	0.112	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.042	0.113	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.039	0.114	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.039	0.112	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.04	0.142	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.066	0.112	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	94	31	189	1481	48	55
normalized size	1	1.	2.29	0.76	4.61	36.12	1.17	1.34
time (sec)	N/A	0.039	0.238	0.007	1.061	2.049	2.589	1.183

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	84	25	128	915	37	47
normalized size	1	1.	2.71	0.81	4.13	29.52	1.19	1.52
time (sec)	N/A	0.031	0.18	0.004	1.101	2.069	1.884	1.195

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	61	19	74	479	31	39
normalized size	1	1.	2.65	0.83	3.22	20.83	1.35	1.7
time (sec)	N/A	0.021	0.149	0.002	1.04	2.039	1.18	1.17

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	26	189	22	28
normalized size	1	1.	1.	1.	2.	14.54	1.69	2.15
time (sec)	N/A	0.013	0.004	0.003	1.13	1.957	0.746	1.187

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	14	88	27	14
normalized size	1	1.	1.12	1.5	0.88	5.5	1.69	0.88
time (sec)	N/A	0.009	0.031	0.016	1.014	2.012	0.577	1.147

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	32	22	173	88	24
normalized size	1	1.	1.15	1.23	0.85	6.65	3.38	0.92
time (sec)	N/A	0.017	0.058	0.017	1.029	2.395	0.99	1.156

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	44	40	30	278	182	32
normalized size	1	1.	1.22	1.11	0.83	7.72	5.06	0.89
time (sec)	N/A	0.027	0.087	0.017	1.05	2.269	1.472	1.148

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	53	48	38	394	299	41
normalized size	1	1.	1.15	1.04	0.83	8.57	6.5	0.89
time (sec)	N/A	0.036	0.121	0.017	1.179	2.377	2.555	1.152

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	62	56	46	540	444	49
normalized size	1	1.	1.11	1.	0.82	9.64	7.93	0.88
time (sec)	N/A	0.046	0.138	0.024	1.038	2.272	3.234	1.142

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	101	43	0	1469	0	216
normalized size	1	1.	1.77	0.75	0.	25.77	0.	3.79
time (sec)	N/A	0.042	0.259	0.016	0.	2.418	0.	1.17

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	92	35	0	869	0	151
normalized size	1	1.	2.04	0.78	0.	19.31	0.	3.36
time (sec)	N/A	0.032	0.153	0.013	0.	2.39	0.	1.174

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	69	27	0	450	0	85
normalized size	1	1.	2.09	0.82	0.	13.64	0.	2.58
time (sec)	N/A	0.022	0.106	0.013	0.	2.487	0.	1.164

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	45	17	0	181	0	50
normalized size	1	1.	2.14	0.81	0.	8.62	0.	2.38
time (sec)	N/A	0.012	0.074	0.036	0.	2.316	0.	1.177

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	51	27	0	300	0	89
normalized size	1	1.	1.59	0.84	0.	9.38	0.	2.78
time (sec)	N/A	0.023	0.292	0.036	0.	2.361	0.	1.154

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	35	0	578	0	177
normalized size	1	1.	1.76	0.71	0.	11.8	0.	3.61
time (sec)	N/A	0.031	0.308	0.011	0.	2.367	0.	1.191

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	94	43	0	919	0	242
normalized size	1	1.	1.54	0.7	0.	15.07	0.	3.97
time (sec)	N/A	0.042	0.758	0.013	0.	2.456	0.	1.223

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	141	322	470	6413	325	308
normalized size	1	1.	0.99	2.27	3.31	45.16	2.29	2.17
time (sec)	N/A	0.209	0.799	0.006	1.081	2.763	22.938	1.155

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	109	246	296	3318	233	211
normalized size	1	1.	1.08	2.44	2.93	32.85	2.31	2.09
time (sec)	N/A	0.123	0.905	0.004	1.114	2.535	10.181	1.137

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	86	173	184	1563	175	140
normalized size	1	1.	1.25	2.51	2.67	22.65	2.54	2.03
time (sec)	N/A	0.064	0.402	0.004	1.111	2.644	4.431	1.143

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	65	116	66	535	104	84
normalized size	1	1.	1.71	3.05	1.74	14.08	2.74	2.21
time (sec)	N/A	0.024	0.128	0.005	1.013	2.854	1.866	1.124

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	64	76	70	149	236	85
normalized size	1	1.	1.28	1.52	1.4	2.98	4.72	1.7
time (sec)	N/A	0.054	0.079	0.019	1.122	2.886	3.149	1.143

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	100	101	167	980	0	185
normalized size	1	1.	1.18	1.19	1.96	11.53	0.	2.18
time (sec)	N/A	0.095	1.42	0.025	1.165	2.676	0.	1.155

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	134	166	435	3182	0	285
normalized size	1	1.	1.04	1.29	3.37	24.67	0.	2.21
time (sec)	N/A	0.18	3.432	0.031	1.184	3.204	0.	1.18

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	214	230	705	8128	0	424
normalized size	1	1.	1.27	1.36	4.17	48.09	0.	2.51
time (sec)	N/A	0.264	6.208	0.034	1.345	3.468	0.	1.216

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	46	38	130	42	41
normalized size	1	1.	1.71	1.48	1.23	4.19	1.35	1.32
time (sec)	N/A	0.044	0.039	0.017	1.101	2.554	1.362	1.179

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	46	39	126	42	38
normalized size	1	1.	1.71	1.48	1.26	4.06	1.35	1.23
time (sec)	N/A	0.042	0.037	0.016	1.058	2.498	1.183	1.192

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	128	63	0	5682	0	0
normalized size	1	1.	1.73	0.85	0.	76.78	0.	0.
time (sec)	N/A	0.07	3.216	0.053	0.	3.212	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	62	0	5844	0	0
normalized size	1	1.	0.99	0.84	0.	78.97	0.	0.
time (sec)	N/A	0.068	0.142	0.046	0.	3.265	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	110	49	321	0	57
normalized size	1	1.	0.7	1.83	0.82	5.35	0.	0.95
time (sec)	N/A	0.063	0.103	0.033	1.069	2.543	0.	1.199

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	36	80	45	221	0	42
normalized size	1	1.	1.24	2.76	1.55	7.62	0.	1.45
time (sec)	N/A	0.047	0.076	0.03	1.059	2.402	0.	1.154

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	30	70	30	176	0	41
normalized size	1	1.	0.79	1.84	0.79	4.63	0.	1.08
time (sec)	N/A	0.052	0.052	0.03	1.083	2.454	0.	1.128

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	40	23	99	0	26
normalized size	1	1.	1.11	2.11	1.21	5.21	0.	1.37
time (sec)	N/A	0.036	0.048	0.027	1.102	2.478	0.	1.125

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	7	11	8	32	0	8
normalized size	1	1.	0.7	1.1	0.8	3.2	0.	0.8
time (sec)	N/A	0.022	0.003	0.001	1.063	2.515	0.	1.153

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	58	0	16
normalized size	1	1.	1.	1.14	1.29	8.29	0.	2.29
time (sec)	N/A	0.036	0.003	0.017	1.051	2.592	0.	1.158

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	14	23	42	306	0	35
normalized size	1	1.	1.75	2.88	5.25	38.25	0.	4.38
time (sec)	N/A	0.04	0.037	0.019	1.088	2.592	0.	1.144

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	32	55	181	0	14
normalized size	1	1.	1.	2.91	5.	16.45	0.	1.27
time (sec)	N/A	0.034	0.025	0.023	1.102	2.372	0.	1.151

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	354	224	2882	0	309
normalized size	1	1.	1.01	2.28	1.45	18.59	0.	1.99
time (sec)	N/A	0.24	0.266	0.049	1.108	2.847	0.	1.175

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	171	197	0	4251	0	220
normalized size	1	1.	1.28	1.47	0.	31.72	0.	1.64
time (sec)	N/A	0.237	0.788	0.044	0.	2.964	0.	1.135

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	75	175	112	818	0	154
normalized size	1	1.	0.82	1.9	1.22	8.89	0.	1.67
time (sec)	N/A	0.142	0.152	0.039	1.177	2.695	0.	1.173

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	93	0	1103	0	97
normalized size	1	1.	1.1	1.27	0.	15.11	0.	1.33
time (sec)	N/A	0.107	0.436	0.032	0.	2.66	0.	1.154

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	39	0	420	0	47
normalized size	1	1.	1.21	1.03	0.	11.05	0.	1.24
time (sec)	N/A	0.036	0.031	0.017	0.	2.652	0.	1.18

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	16	127	0	62
normalized size	1	1.	1.67	1.08	1.33	10.58	0.	5.17
time (sec)	N/A	0.044	0.05	0.019	1.175	2.702	0.	1.19

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	65	115	0	1230	0	115
normalized size	1	1.	1.14	2.02	0.	21.58	0.	2.02
time (sec)	N/A	0.109	0.105	0.028	0.	2.926	0.	1.149

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	50	116	149	1087	0	143
normalized size	1	1.	1.25	2.9	3.72	27.18	0.	3.58
time (sec)	N/A	0.067	0.137	0.031	1.033	2.723	0.	1.176

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	118	49	319	0	57
normalized size	1	1.	0.7	1.97	0.82	5.32	0.	0.95
time (sec)	N/A	0.067	0.088	0.032	0.998	2.506	0.	1.177

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	34	82	36	200	0	34
normalized size	1	1.	1.36	3.28	1.44	8.	0.	1.36
time (sec)	N/A	0.177	0.058	0.028	1.031	2.536	0.	1.187

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	24	78	30	176	0	41
normalized size	1	1.	0.63	2.05	0.79	4.63	0.	1.08
time (sec)	N/A	0.059	0.047	0.029	1.03	2.536	0.	1.113

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	42	15	90	0	15
normalized size	1	1.	1.12	2.47	0.88	5.29	0.	0.88
time (sec)	N/A	0.113	0.017	0.024	1.015	2.486	0.	1.13

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	16	19	16	101	0	14
normalized size	1	1.	1.6	1.9	1.6	10.1	0.	1.4
time (sec)	N/A	0.114	0.023	0.023	1.534	2.58	0.	1.163

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	9	36	24	269	0	36
normalized size	1	1.	0.6	2.4	1.6	17.93	0.	2.4
time (sec)	N/A	0.041	0.03	0.028	1.564	2.698	0.	1.188

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	45	45	508	0	34
normalized size	1	1.	1.	2.25	2.25	25.4	0.	1.7
time (sec)	N/A	0.167	0.039	0.029	1.523	2.507	0.	1.149

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	38	101	286	0	24
normalized size	1	1.	1.	2.24	5.94	16.82	0.	1.41
time (sec)	N/A	0.045	0.05	0.029	1.035	2.523	0.	1.144

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	160	0	0	813	0	201
normalized size	1	1.	7.62	0.	0.	38.71	0.	9.57
time (sec)	N/A	0.046	5.006	0.177	0.	2.552	0.	1.162

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	144	319	208	2747	0	292
normalized size	1	1.	0.98	2.17	1.41	18.69	0.	1.99
time (sec)	N/A	0.339	0.522	0.047	1.199	2.856	0.	1.16

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	167	200	0	4208	0	221
normalized size	1	1.	1.24	1.48	0.	31.17	0.	1.64
time (sec)	N/A	0.251	1.403	0.041	0.	3.019	0.	1.164

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	73	146	108	826	0	140
normalized size	1	1.	0.86	1.72	1.27	9.72	0.	1.65
time (sec)	N/A	0.164	0.229	0.041	1.156	2.651	0.	1.149

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	1106	0	96
normalized size	1	1.	1.1	1.28	0.	15.36	0.	1.33
time (sec)	N/A	0.112	0.301	0.033	0.	2.871	0.	1.13

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	54	0	564	0	65
normalized size	1	1.	1.2	1.08	0.	11.28	0.	1.3
time (sec)	N/A	0.138	0.113	0.033	0.	2.863	0.	1.145

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	59	62	362	0	103
normalized size	1	1.	0.93	2.03	2.14	12.48	0.	3.55
time (sec)	N/A	0.055	0.081	0.04	1.766	2.617	0.	1.147

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	85	187	0	2423	0	138
normalized size	1	1.	1.02	2.25	0.	29.19	0.	1.66
time (sec)	N/A	0.237	0.182	0.047	0.	3.13	0.	1.154

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	68	257	180	2187	0	271
normalized size	1	1.	0.86	3.25	2.28	27.68	0.	3.43
time (sec)	N/A	0.1	0.258	0.053	1.705	2.768	0.	1.148

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	38	41	57	169	0	35
normalized size	1	1.	1.23	1.32	1.84	5.45	0.	1.13
time (sec)	N/A	0.103	0.051	0.052	1.708	2.796	0.	1.134

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	96	74	1897	0	63
normalized size	1	1.	0.93	2.23	1.72	44.12	0.	1.47
time (sec)	N/A	0.115	0.096	0.043	1.701	2.654	0.	1.11

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	80	58	1168	0	53
normalized size	1	1.	0.89	2.16	1.57	31.57	0.	1.43
time (sec)	N/A	0.099	0.053	0.037	1.54	2.648	0.	1.117

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	65	39	632	0	47
normalized size	1	1.	0.93	2.24	1.34	21.79	0.	1.62
time (sec)	N/A	0.074	0.046	0.035	1.675	2.697	0.	1.145

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	47	23	259	0	23
normalized size	1	1.	1.21	2.47	1.21	13.63	0.	1.21
time (sec)	N/A	0.042	0.028	0.029	1.672	2.959	0.	1.158

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	14	88	27	14
normalized size	1	1.	1.12	1.5	0.88	5.5	1.69	0.88
time (sec)	N/A	0.008	0.025	0.	1.126	2.769	0.614	1.14

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	14	88	27	14
normalized size	1	1.	1.12	1.5	0.88	5.5	1.69	0.88
time (sec)	N/A	0.022	0.015	0.018	1.136	2.805	0.597	1.112

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	24	32	259	92	24
normalized size	1	1.	1.21	1.26	1.68	13.63	4.84	1.26
time (sec)	N/A	0.038	0.029	0.017	1.109	3.092	0.906	1.124

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	28	51	632	162	49
normalized size	1	1.	0.87	0.9	1.65	20.39	5.23	1.58
time (sec)	N/A	0.055	0.041	0.017	1.093	3.02	1.381	1.133

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	73	1168	197	54
normalized size	1	1.	0.89	0.86	1.97	31.57	5.32	1.46
time (sec)	N/A	0.068	0.058	0.02	1.112	3.043	1.859	1.114

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	90	35	0	869	0	182
normalized size	1	1.	2.	0.78	0.	19.31	0.	4.04
time (sec)	N/A	0.052	0.136	0.014	0.	3.052	0.	1.185

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	53	26	0	455	0	96
normalized size	1	1.	1.66	0.81	0.	14.22	0.	3.
time (sec)	N/A	0.038	0.119	0.036	0.	2.891	0.	1.166

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	97	25	0	300	0	119
normalized size	1	1.	3.23	0.83	0.	10.	0.	3.97
time (sec)	N/A	0.039	0.195	0.039	0.	2.99	0.	1.162

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	84	35	0	578	0	144
normalized size	1	1.	1.71	0.71	0.	11.8	0.	2.94
time (sec)	N/A	0.051	0.332	0.013	0.	2.957	0.	1.184

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	70	35	0	1449	0	266
normalized size	1	1.	1.56	0.78	0.	32.2	0.	5.91
time (sec)	N/A	0.063	0.254	0.019	0.	2.953	0.	1.2

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	61	26	0	817	0	180
normalized size	1	1.	1.79	0.76	0.	24.03	0.	5.29
time (sec)	N/A	0.048	0.169	0.039	0.	3.169	0.	1.181

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	81	35	0	637	0	119
normalized size	1	1.	1.93	0.83	0.	15.17	0.	2.83
time (sec)	N/A	0.059	0.359	0.041	0.	3.119	0.	1.206

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	35	0	578	0	182
normalized size	1	1.	1.76	0.71	0.	11.8	0.	3.71
time (sec)	N/A	0.082	0.349	0.017	0.	2.929	0.	1.209

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	105	283	197	3087	0	190
normalized size	1	1.	1.08	2.92	2.03	31.82	0.	1.96
time (sec)	N/A	0.525	0.328	0.056	1.752	3.182	0.	1.131

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	88	167	127	1539	0	131
normalized size	1	1.	1.16	2.2	1.67	20.25	0.	1.72
time (sec)	N/A	0.326	0.303	0.053	1.733	3.245	0.	1.176

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	64	110	90	660	0	100
normalized size	1	1.	1.07	1.83	1.5	11.	0.	1.67
time (sec)	N/A	0.193	0.132	0.046	1.785	2.986	0.	1.134

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	88	68	185	0	77
normalized size	1	1.	0.9	1.73	1.33	3.63	0.	1.51
time (sec)	N/A	0.082	0.076	0.039	1.742	2.817	0.	1.199

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	50	108	148	58
normalized size	1	1.	0.74	1.41	1.28	2.77	3.79	1.49
time (sec)	N/A	0.046	0.053	0.013	1.269	2.539	1.227	1.13

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	49	109	134	58
normalized size	1	1.	0.74	1.41	1.26	2.79	3.44	1.49
time (sec)	N/A	0.058	0.045	0.017	1.238	2.573	1.342	1.127

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	60	85	186	372	80
normalized size	1	1.	0.78	0.95	1.35	2.95	5.9	1.27
time (sec)	N/A	0.092	0.076	0.019	1.276	2.803	2.94	1.148

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	67	111	660	639	103
normalized size	1	1.	1.	1.05	1.73	10.31	9.98	1.61
time (sec)	N/A	0.13	0.12	0.022	1.171	2.83	5.45	1.115

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	88	76	161	1538	882	135
normalized size	1	1.	1.16	1.	2.12	20.24	11.61	1.78
time (sec)	N/A	0.221	0.201	0.021	1.195	2.93	7.993	1.179

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	108	96	228	3089	1013	193
normalized size	1	1.	1.15	1.02	2.43	32.86	10.78	2.05
time (sec)	N/A	0.392	0.287	0.023	1.264	3.096	13.074	1.176

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	73	92	491	0	228
normalized size	1	1.	0.91	1.35	1.7	9.09	0.	4.22
time (sec)	N/A	0.085	0.159	0.112	1.952	2.684	0.	1.174

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	30	0	64	24	49	72	0	32
normalized size	1	0.	2.13	0.8	1.63	2.4	0.	1.07
time (sec)	N/A	0.029	0.024	0.026	1.215	2.535	0.	1.143

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	B	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	64	83	65	171	0	73
normalized size	1	0.	1.42	1.84	1.44	3.8	0.	1.62
time (sec)	N/A	0.021	0.232	0.048	1.757	2.685	0.	1.141

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	23	0	26	37	49	81	0	50
normalized size	1	0.	1.13	1.61	2.13	3.52	0.	2.17
time (sec)	N/A	0.016	0.182	0.024	1.044	2.594	0.	1.117

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	B	A	B	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	40	0	58	71	61	159	0	69
normalized size	1	0.	1.45	1.78	1.52	3.98	0.	1.72
time (sec)	N/A	0.007	0.179	0.045	1.609	2.582	0.	1.121

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	21	26	14	47	27	28
normalized size	1	1.	1.75	2.17	1.17	3.92	2.25	2.33
time (sec)	N/A	0.014	0.027	0.003	1.051	2.535	1.689	1.095

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	B	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	41	0	62	93	63	149	0	70
normalized size	1	0.	1.51	2.27	1.54	3.63	0.	1.71
time (sec)	N/A	0.022	0.162	0.042	1.733	2.656	0.	1.158

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	B	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	21	0	27	35	41	92	0	45
normalized size	1	0.	1.29	1.67	1.95	4.38	0.	2.14
time (sec)	N/A	0.021	0.156	0.026	1.03	2.584	0.	1.108

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	47	0	86	41	72	140	0	54
normalized size	1	0.	1.83	0.87	1.53	2.98	0.	1.15
time (sec)	N/A	0.066	0.103	0.018	1.051	2.468	0.	1.122

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	A	B	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	68	0	154	100	89	266	0	97
normalized size	1	0.	2.26	1.47	1.31	3.91	0.	1.43
time (sec)	N/A	0.048	2.899	0.046	1.624	2.596	0.	1.144

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	A	B	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	41	0	163	54	72	169	0	73
normalized size	1	0.	3.98	1.32	1.76	4.12	0.	1.78
time (sec)	N/A	0.03	2.933	0.02	1.061	2.572	0.	1.111

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	A	B	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	60	0	153	86	81	246	0	89
normalized size	1	0.	2.55	1.43	1.35	4.1	0.	1.48
time (sec)	N/A	0.01	1.951	0.043	1.57	2.641	0.	1.119

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	28	35	26	68	32	28
normalized size	1	1.	2.	2.5	1.86	4.86	2.29	2.
time (sec)	N/A	0.024	0.054	0.003	1.032	2.521	17.227	1.116

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	A	A	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	86	0	153	101	93	236	0	104
normalized size	1	0.	1.78	1.17	1.08	2.74	0.	1.21
time (sec)	N/A	0.045	3.127	0.033	1.564	2.604	0.	1.121

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	A	A	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	60	0	155	55	68	178	0	77
normalized size	1	0.	2.58	0.92	1.13	2.97	0.	1.28
time (sec)	N/A	0.046	2.863	0.023	1.078	2.581	0.	1.152

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	59	0	46	0	0	0	0	0
normalized size	1	0.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.093	0.038	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	79	0	165	0	0	0	0	0
normalized size	1	0.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.369	0.027	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	177	0	215	0	0	0	0	0
normalized size	1	0.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.825	0.037	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	79	0	259	0	0	0	0	0
normalized size	1	0.	3.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	1.889	0.106	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	99	0	126	0	0	0	0	0
normalized size	1	0.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	2.977	0.03	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	52	0	83	0	0	0	0	0
normalized size	1	0.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.391	0.036	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	108	0	125	0	0	0	0	0
normalized size	1	0.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.527	0.027	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	162	0	142	0	0	0	0	0
normalized size	1	0.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.561	0.027	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	194	0	223	0	0	0	0	0
normalized size	1	0.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.907	0.024	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0
normalized size	1	0.	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	1.548	0.05	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0
normalized size	1	0.	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	1.734	0.052	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0
normalized size	1	0.	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	1.814	0.05	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	58	0	198	0	0	0	0	0
normalized size	1	0.	3.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	7.015	1.095	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	62	0	207	0	0	0	0	0
normalized size	1	0.	3.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	4.896	1.046	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	54	0	193	0	0	0	0	0
normalized size	1	0.	3.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	6.939	1.	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	52	0	198	0	0	0	0	0
normalized size	1	0.	3.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	8.329	0.915	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	40	56	32	205	0	99
normalized size	1	1.	1.6	2.24	1.28	8.2	0.	3.96
time (sec)	N/A	0.021	0.057	0.004	1.158	2.914	0.	1.241

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	58	0	197	0	0	0	0	0
normalized size	1	0.	3.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	3.83	1.026	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	55	0	191	0	0	0	0	0
normalized size	1	0.	3.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	3.678	1.026	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	132	0	155	0	0	0	0	0
normalized size	1	0.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	6.606	0.365	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	136	0	165	0	0	0	0	0
normalized size	1	0.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	4.532	0.113	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	130	0	151	0	0	0	0	0
normalized size	1	0.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	6.481	0.11	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	126	0	160	0	0	0	0	0
normalized size	1	0.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	7.686	0.099	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	49	80	50	197	0	50
normalized size	1	1.	1.75	2.86	1.79	7.04	0.	1.79
time (sec)	N/A	0.029	0.104	0.005	1.3	2.512	0.	1.33

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	134	0	158	0	0	0	0	0
normalized size	1	0.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	3.563	0.12	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	135	0	156	0	0	0	0	0
normalized size	1	0.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	3.554	0.127	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	52	67	446	1841	0	170
normalized size	1	1.	1.21	1.56	10.37	42.81	0.	3.95
time (sec)	N/A	0.04	0.207	0.009	1.372	2.673	0.	1.429

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	86	674	541	0	90
normalized size	1	1.	0.98	1.91	14.98	12.02	0.	2.
time (sec)	N/A	0.037	0.114	0.003	1.511	2.508	0.	1.434

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	88	1154	5060	0	216
normalized size	1	1.	1.02	1.33	17.48	76.67	0.	3.27
time (sec)	N/A	0.057	0.28	0.003	1.639	2.687	0.	1.344

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	87	0	158	0	0	0	0	0
normalized size	1	0.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	13.231	1.436	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	168	0	312	0	0	0	0	0
normalized size	1	0.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	14.437	0.221	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	306	0	600	0	0	0	0	0
normalized size	1	0.	1.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	16.586	0.24	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	115	0	387	0	0	0	0	0
normalized size	1	0.	3.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	3.629	0.081	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	135	0	174	0	0	0	0	0
normalized size	1	0.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	5.072	0.069	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	64	93	0	2090	0	0
normalized size	1	1.	0.88	1.27	0.	28.63	0.	0.
time (sec)	N/A	0.051	0.286	0.028	0.	2.436	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	57	92	0	1100	0	0
normalized size	1	1.	0.81	1.31	0.	15.71	0.	0.
time (sec)	N/A	0.05	0.146	0.015	0.	2.512	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	72	0	999	0	0
normalized size	1	1.	1.	1.5	0.	20.81	0.	0.
time (sec)	N/A	0.039	0.075	0.013	0.	2.441	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	0	1000	0	0
normalized size	1	1.	1.	0.94	0.	21.28	0.	0.
time (sec)	N/A	0.04	0.111	0.015	0.	2.348	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	44	93	0	2087	0	0
normalized size	1	1.	0.62	1.31	0.	29.39	0.	0.
time (sec)	N/A	0.052	0.144	0.015	0.	2.842	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	92	0	3644	0	0
normalized size	1	1.	0.64	1.28	0.	50.61	0.	0.
time (sec)	N/A	0.051	0.2	0.013	0.	2.889	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	266	149	0	24521	0	0
normalized size	1	1.	1.97	1.1	0.	181.64	0.	0.
time (sec)	N/A	0.353	9.015	0.104	0.	18.619	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	199	90	0	18436	0	0
normalized size	1	1.	1.9	0.86	0.	175.58	0.	0.
time (sec)	N/A	0.217	25.823	0.063	0.	14.573	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	141	52	0	4811	0	0
normalized size	1	1.	2.43	0.9	0.	82.95	0.	0.
time (sec)	N/A	0.117	18.196	0.065	0.	9.668	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	203	0	0	18421	0	0
normalized size	1	1.	1.92	0.	0.	173.78	0.	0.
time (sec)	N/A	0.24	7.83	0.304	0.	13.848	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	2574	0	0	24602	0	0
normalized size	1	1.	14.07	0.	0.	134.44	0.	0.
time (sec)	N/A	0.332	20.687	0.274	0.	19.634	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	304	559	0	22316	0	0
normalized size	1	1.	2.3	4.23	0.	169.06	0.	0.
time (sec)	N/A	0.231	7.991	0.061	0.	24.07	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	164	320	225	4176	0	244
normalized size	1	1.	0.51	1.	0.71	13.09	0.	0.76
time (sec)	N/A	0.912	10.266	0.255	1.609	2.677	0.	1.432

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	334	298	151	1574	0	209
normalized size	1	1.	1.7	1.51	0.77	7.99	0.	1.06
time (sec)	N/A	0.284	3.77	0.199	1.826	2.422	0.	1.27

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	213	76	196	0	127
normalized size	1	1.	0.61	2.57	0.92	2.36	0.	1.53
time (sec)	N/A	0.144	0.058	0.238	1.705	2.373	0.	1.192

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	218	47	132	0	81
normalized size	1	1.	0.61	2.63	0.57	1.59	0.	0.98
time (sec)	N/A	0.194	0.118	0.231	1.72	2.26	0.	1.183

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	104	301	122	1175	0	176
normalized size	1	1.	0.54	1.56	0.63	6.09	0.	0.91
time (sec)	N/A	0.864	0.297	0.209	1.666	2.542	0.	1.218

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	133	324	196	3186	0	250
normalized size	1	1.	0.43	1.04	0.63	10.24	0.	0.8
time (sec)	N/A	1.743	0.461	0.22	1.762	2.616	0.	1.253

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	929	0	0
normalized size	1	1.	0.79	0.75	0.	5.92	0.	0.
time (sec)	N/A	0.375	0.242	0.025	0.	2.75	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	102	0	474	0	0
normalized size	1	1.	0.77	0.89	0.	4.12	0.	0.
time (sec)	N/A	0.262	0.167	0.02	0.	2.526	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	58	0	464	0	0
normalized size	1	1.	0.77	0.75	0.	6.03	0.	0.
time (sec)	N/A	0.144	0.128	0.016	0.	2.438	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	2.68	0.165	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	929	0	0
normalized size	1	1.	0.79	0.75	0.	5.92	0.	0.
time (sec)	N/A	0.373	0.244	0.023	0.	2.968	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	102	0	474	0	0
normalized size	1	1.	0.77	0.89	0.	4.12	0.	0.
time (sec)	N/A	0.251	0.166	0.021	0.	2.78	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	463	0	0
normalized size	1	1.	0.81	0.75	0.	6.01	0.	0.
time (sec)	N/A	0.14	0.106	0.022	0.	2.698	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	5.484	0.067	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [109] had the largest ratio of [0.7778]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.	12	0.5
2	A	6	6	1.	12	0.5
3	A	6	6	1.	12	0.5
4	A	5	5	1.	12	0.417
5	A	5	5	1.	12	0.417
6	A	6	6	1.	12	0.5
7	A	6	6	1.	12	0.5
8	A	7	6	1.	12	0.5
9	A	13	9	1.	12	0.75
10	A	12	8	1.	12	0.667
11	A	9	9	1.	12	0.75
12	A	9	9	1.	12	0.75
13	A	12	8	1.	12	0.667
14	A	13	9	1.	12	0.75
15	A	2	2	1.	8	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	2	2	1.	10	0.2
17	A	3	3	1.	12	0.25
18	A	3	3	1.	14	0.214
19	A	2	2	1.	14	0.143
20	A	2	2	1.	14	0.143
21	A	3	3	1.	14	0.214
22	A	14	10	1.	14	0.714
23	A	14	10	1.	14	0.714
24	A	13	9	1.	14	0.643
25	A	13	9	1.	14	0.643
26	A	14	10	1.	14	0.714
27	A	14	10	1.	14	0.714
28	A	3	3	1.	12	0.25
29	A	8	7	1.	14	0.5
30	A	7	7	1.	14	0.5
31	A	7	7	1.	14	0.5
32	A	8	7	1.	14	0.5
33	A	4	3	1.	14	0.214
34	A	3	3	1.	14	0.214
35	A	2	2	1.	14	0.143
36	A	2	2	1.	14	0.143
37	A	3	3	1.	14	0.214
38	A	4	3	1.	14	0.214
39	A	3	3	1.	12	0.25
40	A	5	3	1.	14	0.214
41	A	3	3	1.	14	0.214
42	A	3	3	1.	14	0.214
43	A	5	3	1.	14	0.214
44	A	16	10	1.	14	0.714
45	A	14	10	1.	14	0.714
46	A	14	10	1.	14	0.714
47	A	14	10	1.	14	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	14	10	1.	14	0.714
49	A	16	10	1.	14	0.714
50	A	3	3	1.	12	0.25
51	A	3	3	1.	14	0.214
52	A	3	3	1.	14	0.214
53	A	3	3	1.	14	0.214
54	A	3	3	1.	14	0.214
55	A	3	3	1.	14	0.214
56	A	3	3	1.	14	0.214
57	A	3	3	1.	14	0.214
58	A	3	3	1.	14	0.214
59	A	3	3	1.	14	0.214
60	A	3	3	1.	14	0.214
61	A	5	3	1.	6	0.5
62	A	4	3	1.	6	0.5
63	A	3	3	1.	6	0.5
64	A	2	2	1.	6	0.333
65	A	2	2	1.	6	0.333
66	A	3	2	1.	6	0.333
67	A	4	2	1.	6	0.333
68	A	5	2	1.	6	0.333
69	A	6	2	1.	6	0.333
70	A	5	3	1.	8	0.375
71	A	4	3	1.	8	0.375
72	A	3	3	1.	8	0.375
73	A	2	2	1.	8	0.25
74	A	3	3	1.	8	0.375
75	A	4	3	1.	8	0.375
76	A	5	3	1.	8	0.375
77	A	5	4	1.	12	0.333
78	A	4	4	1.	12	0.333
79	A	3	3	1.	12	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	2	2	1.	12	0.167
81	A	2	2	1.	12	0.167
82	A	3	3	1.	12	0.25
83	A	4	4	1.	12	0.333
84	A	5	4	1.	12	0.333
85	A	2	2	1.	12	0.167
86	A	2	2	1.	12	0.167
87	A	5	4	1.	14	0.286
88	A	5	4	1.	14	0.286
89	A	4	3	1.	11	0.273
90	A	3	2	1.	11	0.182
91	A	4	3	1.	11	0.273
92	A	2	2	1.	9	0.222
93	A	1	1	1.	9	0.111
94	A	2	2	1.	11	0.182
95	A	2	2	1.	11	0.182
96	A	2	1	1.	11	0.091
97	A	5	4	1.	13	0.308
98	A	9	6	1.	13	0.462
99	A	4	3	1.	13	0.231
100	A	5	5	1.	11	0.454
101	A	2	2	1.	11	0.182
102	A	2	2	1.	13	0.154
103	A	5	5	1.	13	0.385
104	A	3	2	1.	13	0.154
105	A	5	4	1.	11	0.364
106	A	9	7	1.	11	0.636
107	A	5	4	1.	11	0.364
108	A	8	6	1.	9	0.667
109	A	8	7	1.	9	0.778
110	A	3	2	1.	11	0.182
111	A	8	7	1.	11	0.636

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	4	3	1.	11	0.273
113	A	4	4	1.	13	0.308
114	A	5	3	1.	13	0.231
115	A	10	9	1.	13	0.692
116	A	4	3	1.	13	0.231
117	A	6	6	1.	11	0.546
118	A	6	5	1.	11	0.454
119	A	3	2	1.	13	0.154
120	A	9	7	1.	13	0.538
121	A	3	2	1.	13	0.154
122	A	6	5	1.	13	0.385
123	A	6	4	1.	11	0.364
124	A	5	4	1.	11	0.364
125	A	4	4	1.	11	0.364
126	A	4	4	1.	9	0.444
127	A	2	2	1.	6	0.333
128	A	2	2	1.	9	0.222
129	A	3	2	1.	11	0.182
130	A	3	3	1.	11	0.273
131	A	4	4	1.	11	0.364
132	A	4	4	1.	11	0.364
133	A	3	3	1.	11	0.273
134	A	3	3	1.	11	0.273
135	A	4	4	1.	11	0.364
136	A	4	4	1.	13	0.308
137	A	3	3	1.	13	0.231
138	A	4	4	1.	13	0.308
139	A	4	4	1.	13	0.308
140	A	6	6	1.	13	0.462
141	A	5	5	1.	13	0.385
142	A	4	4	1.	13	0.308
143	A	3	3	1.	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	2	2	1.	8	0.25
145	A	2	2	1.	11	0.182
146	A	4	4	1.	13	0.308
147	A	5	5	1.	13	0.385
148	A	6	6	1.	13	0.462
149	A	7	7	1.	13	0.538
150	A	3	3	1.	14	0.214
151	F	0	0	N/A	0	N/A
152	F	0	0	N/A	0	N/A
153	F	0	0	N/A	0	N/A
154	F	0	0	N/A	0	N/A
155	A	2	1	1.	11	0.091
156	F	0	0	N/A	0	N/A
157	F	0	0	N/A	0	N/A
158	F	0	0	N/A	0	N/A
159	F	0	0	N/A	0	N/A
160	F	0	0	N/A	0	N/A
161	F	0	0	N/A	0	N/A
162	A	3	2	1.	13	0.154
163	F	0	0	N/A	0	N/A
164	F	0	0	N/A	0	N/A
165	F	0	0	N/A	0	N/A
166	F	0	0	N/A	0	N/A
167	F	0	0	N/A	0	N/A
168	F	0	0	N/A	0	N/A
169	F	0	0	N/A	0	N/A
170	F	0	0	N/A	0	N/A
171	F	0	0	N/A	0	N/A
172	F	0	0	N/A	0	N/A
173	F	0	0	N/A	0	N/A
174	F	0	0	N/A	0	N/A
175	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	F	0	0	N/A	0	N/A
177	F	0	0	N/A	0	N/A
178	F	0	0	N/A	0	N/A
179	F	0	0	N/A	0	N/A
180	F	0	0	N/A	0	N/A
181	A	2	1	1.	17	0.059
182	F	0	0	N/A	0	N/A
183	F	0	0	N/A	0	N/A
184	F	0	0	N/A	0	N/A
185	F	0	0	N/A	0	N/A
186	F	0	0	N/A	0	N/A
187	F	0	0	N/A	0	N/A
188	A	3	2	1.	19	0.105
189	F	0	0	N/A	0	N/A
190	F	0	0	N/A	0	N/A
191	A	3	2	1.	17	0.118
192	A	4	2	1.	17	0.118
193	A	4	2	1.	17	0.118
194	F	0	0	N/A	0	N/A
195	F	0	0	N/A	0	N/A
196	F	0	0	N/A	0	N/A
197	F	0	0	N/A	0	N/A
198	F	0	0	N/A	0	N/A
199	A	7	6	1.	19	0.316
200	A	7	6	1.	19	0.316
201	A	6	5	1.	19	0.263
202	A	6	5	1.	19	0.263
203	A	7	6	1.	19	0.316
204	A	7	6	1.	19	0.316
205	A	8	7	1.	23	0.304
206	A	7	6	1.	23	0.261
207	A	4	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	8	5	1.	21	0.238
209	A	11	6	1.	23	0.261
210	A	8	7	1.	21	0.333
211	A	9	7	1.	25	0.28
212	A	8	7	1.	25	0.28
213	A	4	4	1.	25	0.16
214	A	4	4	1.	25	0.16
215	A	8	7	1.	25	0.28
216	A	9	7	1.	25	0.28
217	A	19	5	1.	9	0.556
218	A	13	5	1.	9	0.556
219	A	9	4	1.	7	0.571
220	A	0	0	0.	0	0.
221	A	19	5	1.	9	0.556
222	A	13	5	1.	9	0.556
223	A	9	4	1.	7	0.571
224	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int (b \coth(c + dx))^{7/2} dx$

Optimal. Leaf size=97

$$-\frac{2b^3\sqrt{b \coth(c + dx)}}{d} + \frac{b^{7/2} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{7/2} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b \coth(c + dx))^{5/2}}{5d}$$

[Out] $(b^{(7/2)}*\text{ArcTan}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]])/d + (b^{(7/2)}*\text{ArcTanh}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]])/d - (2*b^3*\text{Sqrt}[b*\text{Coth}[c + d*x]])/d - (2*b*(b*\text{Cotanh}[c + d*x])^{(5/2)})/(5*d)$

Rubi [A] time = 0.07041, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3473, 3476, 329, 212, 206, 203}

$$-\frac{2b^3\sqrt{b \coth(c + dx)}}{d} + \frac{b^{7/2} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{7/2} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b \coth(c + dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x])^{(7/2)}, x]$

[Out] $(b^{(7/2)}*\text{ArcTan}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]])/d + (b^{(7/2)}*\text{ArcTanh}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]])/d - (2*b^3*\text{Sqrt}[b*\text{Coth}[c + d*x]])/d - (2*b*(b*\text{Cotanh}[c + d*x])^{(5/2)})/(5*d)$

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int (b \coth(c + dx))^{7/2} dx &= -\frac{2b(b \coth(c + dx))^{5/2}}{5d} + b^2 \int (b \coth(c + dx))^{3/2} dx \\
&= -\frac{2b^3 \sqrt{b \coth(c + dx)}}{d} - \frac{2b(b \coth(c + dx))^{5/2}}{5d} + b^4 \int \frac{1}{\sqrt{b \coth(c + dx)}} dx \\
&= -\frac{2b^3 \sqrt{b \coth(c + dx)}}{d} - \frac{2b(b \coth(c + dx))^{5/2}}{5d} - \frac{b^5 \operatorname{Subst}\left(\int \frac{1}{\sqrt{x(-b^2+x^2)}} dx, x, b \coth(c + dx)\right)}{d} \\
&= -\frac{2b^3 \sqrt{b \coth(c + dx)}}{d} - \frac{2b(b \coth(c + dx))^{5/2}}{5d} - \frac{(2b^5) \operatorname{Subst}\left(\int \frac{1}{-b^2+x^4} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} \\
&= -\frac{2b^3 \sqrt{b \coth(c + dx)}}{d} - \frac{2b(b \coth(c + dx))^{5/2}}{5d} + \frac{b^4 \operatorname{Subst}\left(\int \frac{1}{b-x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} + \\
&= \frac{b^{7/2} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{7/2} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b^3 \sqrt{b \coth(c + dx)}}{d} - \frac{2b(b \coth(c + dx))^{5/2}}{5d}
\end{aligned}$$

Mathematica [A] time = 0.232407, size = 83, normalized size = 0.86

$$\frac{b^3 \sqrt{b \coth(c + dx)} \left(-2 \coth^{\frac{5}{2}}(c + dx) - 10 \sqrt{\coth(c + dx)} + 5 \tan^{-1}(\sqrt{\coth(c + dx)}) + 5 \tanh^{-1}(\sqrt{\coth(c + dx)}) \right)}{5d \sqrt{\coth(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(7/2), x]

[Out] (b^3*Sqrt[b*Coth[c + d*x]]*(5*ArcTan[Sqrt[Coth[c + d*x]]] + 5*ArcTanh[Sqrt[Coth[c + d*x]]] - 10*Sqrt[Coth[c + d*x]] - 2*Coth[c + d*x]^(5/2)))/(5*d*Sqrt[Coth[c + d*x]])

Maple [A] time = 0.026, size = 80, normalized size = 0.8

$$\frac{1}{d} b^{\frac{7}{2}} \arctan\left(\sqrt{b \coth(dx + c)} \frac{1}{\sqrt{b}}\right) + \frac{1}{d} b^{\frac{7}{2}} \operatorname{Artanh}\left(\sqrt{b \coth(dx + c)} \frac{1}{\sqrt{b}}\right) - \frac{2b}{5d} (b \coth(dx + c))^{\frac{5}{2}} - 2 \frac{b^3 \sqrt{b \coth(dx + c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c))^(7/2), x)

[Out] $b^{7/2} \arctan((b \coth(dx+c))^{1/2}/b^{1/2})/d + b^{7/2} \operatorname{arctanh}((b \coth(dx+c))^{1/2}/b^{1/2})/d - 2/5 b (b \coth(dx+c))^{5/2}/d - 2 b^3 (b \coth(dx+c))^{1/2}/d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx+c))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c))^(7/2),x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c))^(7/2), x)`

Fricas [B] time = 2.50709, size = 4135, normalized size = 42.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c))^(7/2),x, algorithm="fricas")`

[Out] $[-1/20*(10*(b^3 \cosh(dx+c))^4 + 4*b^3 \cosh(dx+c)*\sinh(dx+c)^3 + b^3 \sinh(dx+c)^4 - 2*b^3 \cosh(dx+c)^2 + b^3 + 2*(3*b^3 \cosh(dx+c)^2 - b^3)*\sinh(dx+c)^2 + 4*(b^3 \cosh(dx+c)^3 - b^3 \cosh(dx+c))*\sinh(dx+c))*\sqrt{-b} \arctan(\frac{\cosh(dx+c)^2 + 2*\cosh(dx+c)*\sinh(dx+c) + \sinh(dx+c)^2}{\sqrt{-b} \sqrt{b \cosh(dx+c)/\sinh(dx+c)}}) / (b \cosh(dx+c)^2 + 2*b \cosh(dx+c)*\sinh(dx+c) + b*\sinh(dx+c)^2 + b) - 5*(b^3 \cosh(dx+c)^4 + 4*b^3 \cosh(dx+c)*\sinh(dx+c)^3 + b^3 \sinh(dx+c)^4 - 2*b^3 \cosh(dx+c)^2 + b^3 + 2*(3*b^3 \cosh(dx+c)^2 - b^3)*\sinh(dx+c)^2 + 4*(b^3 \cosh(dx+c)^3 - b^3 \cosh(dx+c))*\sinh(dx+c))*\sqrt{-b} \log(\frac{-(b \cosh(dx+c))^4 + 4*b \cosh(dx+c)^3 \sinh(dx+c) + 6*b \cosh(dx+c)^2 \sinh(dx+c)^2 + 4*b \cosh(dx+c)*\sinh(dx+c)^3 + b*\sinh(dx+c)^4 - 2*(\cosh(dx+c)^2 + 2*\cosh(dx+c)*\sinh(dx+c) + \sinh(dx+c)^2 - 1)*\sqrt{-b} \sqrt{b \cosh(dx+c)/\sinh(dx+c)} - 2*b}{(\cosh(dx+c))^4 + 4*\cosh(dx+c)^3 \sinh(dx+c) + 6*\cosh(dx+c)^2 \sinh(dx+c)^2 + 4*\cosh(dx+c)*\sinh(dx+c)^3 + \sinh(dx+c)^4}) + 16*(3*b^3 \cosh(dx+c)^4 + 12*b^3 \cosh(dx+c)*\sinh(dx+c)^3 + 3*b^3 \sinh(dx+c)^4 - 4*b^3 \cosh(dx+c)^2 + 3*b^3 + 2*(9*b^3 \cosh(dx+c)^2 - 2*b^3)*\sinh(dx+c)^2 +$

$$4*(3*b^3*cosh(d*x + c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d), 1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 5*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 16*(3*b^3*cosh(d*x + c)^4 + 12*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 - 4*b^3*cosh(d*x + c)^2 + 3*b^3 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*sinh(d*x + c)^2 + 4*(3*b^3*cosh(d*x + c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c))^(7/2), x)
```

3.2 $\int (b \coth(c + dx))^{5/2} dx$

Optimal. Leaf size=78

$$-\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b \coth(c + dx))^{3/2}}{3d}$$

[Out] $-\left(\frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b \coth(c + dx)}}{\sqrt{b}}\right]}{d}\right) + \frac{b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \coth(c + dx)}}{\sqrt{b}}\right]}{d} - \frac{2b(b \coth(c + dx))^{3/2}}{3d}$

Rubi [A] time = 0.0490394, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3473, 3476, 329, 298, 203, 206}

$$-\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b \coth(c + dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(b \coth(c + dx))^{5/2}, x]$

[Out] $-\left(\frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b \coth(c + dx)}}{\sqrt{b}}\right]}{d}\right) + \frac{b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \coth(c + dx)}}{\sqrt{b}}\right]}{d} - \frac{2b(b \coth(c + dx))^{3/2}}{3d}$

Rule 3473

$\operatorname{Int}[(b \cdot \tan(c + dx))^{n}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(b \cdot (b \cdot \tan(c + dx))^{n-1}) / (d \cdot (n-1)), x] - \operatorname{Dist}[b^2, \operatorname{Int}[(b \cdot \tan(c + dx))^{n-2}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d, x\} \ \&\& \ \operatorname{GtQ}[n, 1]$

Rule 3476

$\operatorname{Int}[(b \cdot \tan(c + dx))^{n}, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[b/d, \operatorname{Subst}[\operatorname{Int}[x^n / (b^2 + x^2), x], x, b \cdot \tan(c + dx)], x] /;$ $\operatorname{FreeQ}\{b, c, d, n, x\} \ \&\& \ ! \operatorname{IntegerQ}[n]$

Rule 329

$\operatorname{Int}[(c \cdot x)^m \cdot (a + (b \cdot x)^n)^p, x_{\text{Symbol}}] \rightarrow \operatorname{With}\{k = \operatorname{Denominator}[m]\}, \operatorname{Dist}[k/c, \operatorname{Subst}[\operatorname{Int}[x^{k \cdot (m+1) - 1} \cdot (a + (b \cdot x^{k \cdot n})) / c^{k \cdot n}]^p, x], x, (c \cdot x)^{1/k}], x] /;$ $\operatorname{FreeQ}\{a, b, c, p, x\} \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{F}$

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 298

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int (b \coth(c + dx))^{5/2} dx &= -\frac{2b(b \coth(c + dx))^{3/2}}{3d} + b^2 \int \sqrt{b \coth(c + dx)} dx \\
 &= -\frac{2b(b \coth(c + dx))^{3/2}}{3d} - \frac{b^3 \operatorname{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2} dx, x, b \coth(c + dx)\right)}{d} \\
 &= -\frac{2b(b \coth(c + dx))^{3/2}}{3d} - \frac{(2b^3) \operatorname{Subst}\left(\int \frac{x^2}{-b^2+x^4} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} \\
 &= -\frac{2b(b \coth(c + dx))^{3/2}}{3d} + \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{b-x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} - \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{b+x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} \\
 &= -\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b \coth(c + dx))^{3/2}}{3d}
 \end{aligned}$$

Mathematica [A] time = 0.212902, size = 68, normalized size = 0.87

$$\frac{(b \coth(c + dx))^{5/2} \left(2 \coth^{\frac{3}{2}}(c + dx) + 3 \tan^{-1}\left(\sqrt{\coth(c + dx)}\right) - 3 \tanh^{-1}\left(\sqrt{\coth(c + dx)}\right) \right)}{3d \coth^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(5/2),x]

[Out] -((b*Coth[c + d*x])^(5/2)*(3*ArcTan[Sqrt[Coth[c + d*x]]] - 3*ArcTanh[Sqrt[Coth[c + d*x]]] + 2*Coth[c + d*x]^(3/2)))/(3*d*Coth[c + d*x]^(5/2))

Maple [A] time = 0.01, size = 63, normalized size = 0.8

$$-\frac{1}{d}b^{\frac{5}{2}}\arctan\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)+\frac{1}{d}b^{\frac{5}{2}}\operatorname{Artanh}\left(\sqrt{b\coth(dx+c)}\frac{1}{\sqrt{b}}\right)-\frac{2b}{3d}(b\coth(dx+c))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c))^(5/2),x)

[Out] -b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*coth(d*x+c))^(3/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(5/2), x)

Fricas [B] time = 2.38775, size = 2654, normalized size = 34.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="fricas")

```
[Out] [-1/12*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4) + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d), -1/12*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(5/2), x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c))^(5/2), x)
```

3.3 $\int (b \coth(c + dx))^{3/2} dx$

Optimal. Leaf size=75

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b \coth(c+dx)}}{d}$$

[Out] (b^(3/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Coth[c + d*x]])/d

Rubi [A] time = 0.0500011, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3473, 3476, 329, 212, 206, 203}

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b \coth(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x])^(3/2), x]

[Out] (b^(3/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Coth[c + d*x]])/d

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int (b \coth(c + dx))^{3/2} dx &= -\frac{2b\sqrt{b \coth(c + dx)}}{d} + b^2 \int \frac{1}{\sqrt{b \coth(c + dx)}} dx \\
 &= -\frac{2b\sqrt{b \coth(c + dx)}}{d} - \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{\sqrt{x(-b^2+x^2)}} dx, x, b \coth(c + dx)\right)}{d} \\
 &= -\frac{2b\sqrt{b \coth(c + dx)}}{d} - \frac{(2b^3) \operatorname{Subst}\left(\int \frac{1}{-b^2+x^4} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} \\
 &= -\frac{2b\sqrt{b \coth(c + dx)}}{d} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{b-x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{b+x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} \\
 &= \frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b \coth(c + dx)}}{d}
 \end{aligned}$$

Mathematica [A] time = 0.0947051, size = 61, normalized size = 0.81

$$\frac{(b \coth(c + dx))^{3/2} \left(-2\sqrt{\coth(c + dx)} + \tan^{-1}\left(\sqrt{\coth(c + dx)}\right) + \tanh^{-1}\left(\sqrt{\coth(c + dx)}\right)\right)}{d \coth^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(3/2),x]

[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth[c + d*x]])*(b*Coth[c + d*x])^(3/2))/(d*Coth[c + d*x]^(3/2))

Maple [A] time = 0.011, size = 62, normalized size = 0.8

$$\frac{1}{d} b^{\frac{3}{2}} \arctan\left(\sqrt{b \coth(dx+c)} \frac{1}{\sqrt{b}}\right) + \frac{1}{d} b^{\frac{3}{2}} \operatorname{Artanh}\left(\sqrt{b \coth(dx+c)} \frac{1}{\sqrt{b}}\right) - 2 \frac{b \sqrt{b \coth(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c))^(3/2),x)

[Out] b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))/d-2*b*(b*coth(d*x+c))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx+c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(3/2), x)

Fricas [B] time = 2.27195, size = 1754, normalized size = 23.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="fricas")

```
[Out] [-1/4*(2*sqrt(-b)*b*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt
(-b)*b*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x +
c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
+ 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*b*sqrt(b*cosh(d*
x + c)/sinh(d*x + c)))/d, 1/4*(2*b^(3/2)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c
)/sinh(d*x + c)))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + b)) + b^(3/2)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^
3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*
sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))
*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*b*sqrt(b*cosh(d*x + c
)/sinh(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c))^(3/2), x)
```

3.4 $\int \sqrt{b \coth(c + dx)} dx$

Optimal. Leaf size=58

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

[Out] $-\left(\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right]}{d}\right) + \left(\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right]}{d}\right)$

Rubi [A] time = 0.0356433, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3476, 329, 298, 203, 206}

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[b*Coth[c + d*x]], x]`

[Out] $-\left(\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right]}{d}\right) + \left(\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right]}{d}\right)$

Rule 3476

`Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]`

Rule 329

`Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]`

Rule 298

`Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x`

], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{b \coth(c + dx)} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2} dx, x, b \coth(c + dx)\right)}{d} \\ &= -\frac{(2b) \operatorname{Subst}\left(\int \frac{x^2}{-b^2+x^4} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \frac{1}{b-x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} - \frac{b \operatorname{Subst}\left(\int \frac{1}{b+x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{d} \\ &= -\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{d} \end{aligned}$$

Mathematica [A] time = 0.0419602, size = 51, normalized size = 0.88

$$\frac{\sqrt{b \coth(c + dx)} \left(\tanh^{-1} \left(\sqrt{\coth(c + dx)} \right) - \tan^{-1} \left(\sqrt{\coth(c + dx)} \right) \right)}{d \sqrt{\coth(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Coth[c + d*x]],x]

[Out] ((-ArcTan[Sqrt[Coth[c + d*x]])] + ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[b*Coth[c + d*x]]/(d*Sqrt[Coth[c + d*x]])

Maple [A] time = 0.023, size = 47, normalized size = 0.8

$$-\frac{1}{d} \arctan\left(\sqrt{b \coth(dx+c)} \frac{1}{\sqrt{b}}\right) \sqrt{b} + \frac{1}{d} \operatorname{Artanh}\left(\sqrt{b \coth(dx+c)} \frac{1}{\sqrt{b}}\right) \sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*coth(d*x+c))^(1/2),x)`

[Out] `-arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+artanh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*coth(d*x + c)), x)`

Fricas [B] time = 2.21435, size = 1636, normalized size = 28.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `[-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c))^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^`

$$4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4)/d, -1/4*(2*\sqrt{b}*\arctan(\sqrt{b}*\sqrt{b*\cosh(d*x + c)/\sinh(d*x + c)})/(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + b)) - \sqrt{b}*\log(2*b*\cosh(d*x + c)^4 + 8*b*\cosh(d*x + c)^3*\sinh(d*x + c) + 12*b*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 8*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + 2*b*\sinh(d*x + c)^4 + 2*(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + (6*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c)^2 - \cosh(d*x + c)^2 + 2*(2*\cosh(d*x + c)^3 - \cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b}*\sqrt{b*\cosh(d*x + c)/\sinh(d*x + c)} - b))/d]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))**(1/2),x)

[Out] Integral(sqrt(b*coth(c + d*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*coth(d*x + c)), x)

3.5 $\int \frac{1}{\sqrt{b \coth(c+dx)}} dx$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}}$$

[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d)

Rubi [A] time = 0.0308044, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3476, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*Coth[c + d*x]],x]

[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d)

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212


```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
  2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
  x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
  [a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
  , 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{b \coth(c+dx)}} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{x(-b^2+x^2)}} dx, x, b \coth(c+dx)\right)}{d} \\ &= -\frac{(2b) \operatorname{Subst}\left(\int \frac{1}{-b^2+x^4} dx, x, \sqrt{b \coth(c+dx)}\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{b-x^2} dx, x, \sqrt{b \coth(c+dx)}\right)}{d} + \frac{\operatorname{Subst}\left(\int \frac{1}{b+x^2} dx, x, \sqrt{b \coth(c+dx)}\right)}{d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{bd}} \end{aligned}$$

Mathematica [A] time = 0.0340077, size = 49, normalized size = 0.86

$$\frac{\sqrt{\coth(c+dx)} \left(\tan^{-1}\left(\sqrt{\coth(c+dx)}\right) + \tanh^{-1}\left(\sqrt{\coth(c+dx)}\right) \right)}{d\sqrt{b \coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]], x]
```

[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]])]*Sqrt[Coth[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]])

Maple [A] time = 0.033, size = 46, normalized size = 0.8

$$\frac{1}{d} \arctan\left(\sqrt{b \coth(dx+c)} \frac{1}{\sqrt{b}}\right) \frac{1}{\sqrt{b}} + \frac{1}{d} \operatorname{Artanh}\left(\sqrt{b \coth(dx+c)} \frac{1}{\sqrt{b}}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c))^(1/2), x)

[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(1/sqrt(b*coth(d*x + c)), x)

Fricas [B] time = 2.43283, size = 1646, normalized size = 28.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(1/2), x, algorithm="fricas")

[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c))^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(-

```

b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/(b*d), 1/4*(2*sqrt(b)*a
rctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(b)*log(2*b*cos
h(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*
(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*c
osh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^
3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c
)) - b))/(b*d)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)), x)
```

$$3.6 \quad \int \frac{1}{(b \coth(c+dx))^{3/2}} dx$$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b \coth(c+dx)}}$$

[Out] $-(\text{ArcTan}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]]/(b^{(3/2)*d})) + \text{ArcTanh}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]]/(b^{(3/2)*d}) - 2/(b*d*\text{Sqrt}[b*\text{Coth}[c + d*x]])$

Rubi [A] time = 0.0512037, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3474, 3476, 329, 298, 203, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b \coth(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x])^{(-3/2)}, x]$

[Out] $-(\text{ArcTan}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]]/(b^{(3/2)*d})) + \text{ArcTanh}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]]/(b^{(3/2)*d}) - 2/(b*d*\text{Sqrt}[b*\text{Coth}[c + d*x]])$

Rule 3474

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)*(x_*)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Tan}[c + d*x])^{(n+1)}/(b*d*(n+1)), x] - \text{Dist}[1/b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n+2)}, x], x] /;$ $\text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{LtQ}[n, -1]$

Rule 3476

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)*(x_*)]^{(n_)}, x_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\text{Tan}[c + d*x]], x] /;$ $\text{FreeQ}\{b, c, d, n\}, x \ \&\& \ ! \ \text{IntegerQ}[n]$

Rule 329

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1) - 1)}*(a + (b*x^{(k*n)}))/c^$

$n)^p, x], x, (c*x)^{(1/k)], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 298

$\text{Int}[(x_)^2/((a_)+(b_)*(x_)^4), x_Symbol] \text{ :> With}[\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[s/(2*b), \text{Int}[1/(r + s*x^2), x], x] - \text{Dist}[s/(2*b), \text{Int}[1/(r - s*x^2), x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[a/b, 0]$

Rule 203

$\text{Int}[((a_)+(b_)*(x_)^2)^{-1}, x_Symbol] \text{ :> Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 206

$\text{Int}[((a_)+(b_)*(x_)^2)^{-1}, x_Symbol] \text{ :> Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{(b \coth(c+dx))^{3/2}} dx &= -\frac{2}{bd\sqrt{b \coth(c+dx)}} + \frac{\int \sqrt{b \coth(c+dx)} dx}{b^2} \\ &= -\frac{2}{bd\sqrt{b \coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2} dx, x, b \coth(c+dx)\right)}{bd} \\ &= -\frac{2}{bd\sqrt{b \coth(c+dx)}} - \frac{2 \text{Subst}\left(\int \frac{x^2}{-b^2+x^4} dx, x, \sqrt{b \coth(c+dx)}\right)}{bd} \\ &= -\frac{2}{bd\sqrt{b \coth(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2} dx, x, \sqrt{b \coth(c+dx)}\right)}{bd} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2} dx, x, \sqrt{b \coth(c+dx)}\right)}{bd} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b \coth(c+dx)}} \end{aligned}$$

Mathematica [C] time = 0.0737085, size = 36, normalized size = 0.46

$$\frac{{}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; \coth^2(c + dx)\right)}{bd\sqrt{b \coth(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(-3/2), x]

[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Coth[c + d*x]^2])/(b*d*Sqrt[b*Coth[c + d*x]])

Maple [A] time = 0.013, size = 65, normalized size = 0.8

$$-\frac{1}{d} \arctan\left(\sqrt{b \coth(dx + c)} \frac{1}{\sqrt{b}}\right) b^{-\frac{3}{2}} + \frac{1}{d} \operatorname{Arctanh}\left(\sqrt{b \coth(dx + c)} \frac{1}{\sqrt{b}}\right) b^{-\frac{3}{2}} - 2 \frac{1}{bd\sqrt{b \coth(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c))^(3/2), x)

[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*coth(d*x+c))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(3/2), x)

Fricas [B] time = 2.71562, size = 2576, normalized size = 33.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cosh(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))**(3/2),x)

[Out] Integral((b*coth(c + d*x))**(-3/2), x)

Giac [B] time = 1.33478, size = 277, normalized size = 3.55

$$\frac{(\pi + \log(|b|) + 8) \operatorname{sgn}(e^{2dx+2c} - 1)}{\sqrt{bd}} + \frac{4 \arctan\left(-\frac{\sqrt{b}e^{2dx+2c} - \sqrt{be^{4dx+4c} - b}}{\sqrt{b}}\right)}{\sqrt{bd} \operatorname{sgn}(e^{2dx+2c} - 1)} - \frac{2 \log\left(\left|-\sqrt{b}e^{2dx+2c} + \sqrt{be^{4dx+4c} - b}\right|\right)}{\sqrt{bd} \operatorname{sgn}(e^{2dx+2c} - 1)} - \frac{16}{\left(\sqrt{b}e^{2dx+2c} - \sqrt{be^{4dx+4c} - b} + \sqrt{b}\right)d}$$

$4b$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/4*((pi + log(abs(b)) + 8)*sgn(e^(2*d*x + 2*c) - 1)/(sqrt(b)*d) + 4*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt(b))/(sqrt(b)*d*sgn(e^(2*d*x + 2*c) - 1)) - 2*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))/(sqrt(b)*d*sgn(e^(2*d*x + 2*c) - 1)) - 16/((sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))*d*sgn(e^(2*d*x + 2*c) - 1))/b

$$3.7 \quad \int \frac{1}{(b \coth(c+dx))^{5/2}} dx$$

Optimal. Leaf size=79

$$\frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b \coth(c+dx))^{3/2}}$$

[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Coth[c + d*x])^(3/2))

Rubi [A] time = 0.0495235, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3474, 3476, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b \coth(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x])^(-5/2), x]

[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Coth[c + d*x])^(3/2))

Rule 3474

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^

$n)^p, x], x, (c*x)^{(1/k)}, x]] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 212

$\text{Int}[(a_ + (b_.)*(x_)^4)^{-1}, x_Symbol] \text{:>} \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^2), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}[a/b, 0]$

Rule 206

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] \text{:>} \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 203

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] \text{:>} \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{(b \coth(c + dx))^{5/2}} dx &= -\frac{2}{3bd(b \coth(c + dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{b \coth(c + dx)}} dx}{b^2} \\ &= -\frac{2}{3bd(b \coth(c + dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(-b^2 + x^2)}} dx, x, b \coth(c + dx)\right)}{bd} \\ &= -\frac{2}{3bd(b \coth(c + dx))^{3/2}} - \frac{2 \text{Subst}\left(\int \frac{1}{-b^2 + x^4} dx, x, \sqrt{b \coth(c + dx)}\right)}{bd} \\ &= -\frac{2}{3bd(b \coth(c + dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{b - x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{b^2 d} + \frac{\text{Subst}\left(\int \frac{1}{b + x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{b^2 d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c + dx)}}{\sqrt{b}}\right)}{b^{5/2} d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c + dx)}}{\sqrt{b}}\right)}{b^{5/2} d} - \frac{2}{3bd(b \coth(c + dx))^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.0654699, size = 38, normalized size = 0.48

$$\frac{{}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; \coth^2(c + dx)\right)}{3bd(b \coth(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(-5/2), x]

[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Coth[c + d*x]^2])/(3*b*d*(b*Coth[c + d*x])^(3/2))

Maple [A] time = 0.011, size = 64, normalized size = 0.8

$$\frac{1}{d} \arctan\left(\sqrt{b \coth(dx + c)} \frac{1}{\sqrt{b}}\right) b^{-\frac{5}{2}} + \frac{1}{d} \operatorname{Artanh}\left(\sqrt{b \coth(dx + c)} \frac{1}{\sqrt{b}}\right) b^{-\frac{5}{2}} - \frac{2}{3bd} (b \coth(dx + c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c))^(5/2), x)

[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*coth(d*x+c))^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(5/2), x)

Fricas [B] time = 2.85404, size = 3933, normalized size = 49.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/12*(6*(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 + 1)*\sinh(dx + c)^2 + 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 + \cosh(dx + c))*\sinh(dx + c) + 1)*\sqrt{-b}*\arctan((\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2)*\sqrt{-b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)})/(b*\cosh(dx + c)^2 + 2*b*\cosh(dx + c)*\sinh(dx + c) + b*\sinh(dx + c)^2 + b)) + 3*(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 + 1)*\sinh(dx + c)^2 + 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 + \cosh(dx + c))*\sinh(dx + c) + 1)*\sqrt{-b}*\log(-(b*\cosh(dx + c)^4 + 4*b*\cosh(dx + c)^3*\sinh(dx + c) + 6*b*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*b*\cosh(dx + c)*\sinh(dx + c)^3 + b*\sinh(dx + c)^4 + 2*(\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 - 1)*\sqrt{-b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)}) - 2*b)/(\cosh(dx + c)^4 + 4*\cosh(dx + c)^3*\sinh(dx + c) + 6*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4)) + 8*(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 - 1)*\sinh(dx + c)^2 - 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 - \cosh(dx + c))*\sinh(dx + c) + 1)*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)})/(b^3*d*\cosh(dx + c)^4 + 4*b^3*d*\cosh(dx + c)*\sinh(dx + c)^3 + b^3*d*\sinh(dx + c)^4 + 2*b^3*d*\cosh(dx + c)^2 + b^3*d + 2*(3*b^3*d*\cosh(dx + c)^2 + b^3*d)*\sinh(dx + c)^2 + 4*(b^3*d*\cosh(dx + c)^3 + b^3*d*\cosh(dx + c))*\sinh(dx + c)), 1/12*(6*(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 + 1)*\sinh(dx + c)^2 + 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 + \cosh(dx + c))*\sinh(dx + c) + 1)*\sqrt{b}*\arctan(\sqrt{b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)})/(b*\cosh(dx + c)^2 + 2*b*\cosh(dx + c)*\sinh(dx + c) + b*\sinh(dx + c)^2 + b)) + 3*(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 + 1)*\sinh(dx + c)^2 + 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 + \cosh(dx + c))*\sinh(dx + c) + 1)*\sqrt{b}*\log(2*b*\cosh(dx + c)^4 + 8*b*\cosh(dx + c)^3*\sinh(dx + c) + 12*b*\cosh(dx + c)^2*\sinh(dx + c)^2 + 8*b*\cosh(dx + c)*\sinh(dx + c)^3 + 2*b*\sinh(dx + c)^4 + 2*(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + (6*\cosh(dx + c)^2 - 1)*\sinh(dx + c)^2 - \cosh(dx + c)^2 + 2*(2*\cosh(dx + c)^3 - \cosh(dx + c))*\sinh(dx + c))*\sqrt{b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)}) - b) - 8*(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 - 1)*\sinh(dx + c)^2 - 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 - \cosh(dx + c))*\sinh(dx + c) + 1)*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)})/(b^3*d*\cosh(dx + c)^4 + 4*b^3* \end{aligned}$$

$$d*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^3*d*\sinh(d*x + c)^4 + 2*b^3*d*\cosh(d*x + c)^2 + b^3*d + 2*(3*b^3*d*\cosh(d*x + c)^2 + b^3*d)*\sinh(d*x + c)^2 + 4*(b^3*d*\cosh(d*x + c)^3 + b^3*d*\cosh(d*x + c))*\sinh(d*x + c)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError

3.8 $\int \frac{1}{(b \coth(c+dx))^{7/2}} dx$

Optimal. Leaf size=100

$$-\frac{2}{b^3 d \sqrt{b \coth(c+dx)}} - \frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2} d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2} d} - \frac{2}{5bd(b \coth(c+dx))^{5/2}}$$

[Out] $-(\text{ArcTan}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]]/(b^{(7/2)*d})) + \text{ArcTanh}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]]/(b^{(7/2)*d}) - 2/(5*b*d*(b*\text{Coth}[c + d*x])^{(5/2)}) - 2/(b^3*d*\text{Sqrt}[b*\text{Coth}[c + d*x]])$

Rubi [A] time = 0.0705722, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3474, 3476, 329, 298, 203, 206}

$$-\frac{2}{b^3 d \sqrt{b \coth(c+dx)}} - \frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2} d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2} d} - \frac{2}{5bd(b \coth(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x])^{(-7/2)}, x]$

[Out] $-(\text{ArcTan}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]]/(b^{(7/2)*d})) + \text{ArcTanh}[\text{Sqrt}[b*\text{Coth}[c + d*x]]/\text{Sqrt}[b]]/(b^{(7/2)*d}) - 2/(5*b*d*(b*\text{Coth}[c + d*x])^{(5/2)}) - 2/(b^3*d*\text{Sqrt}[b*\text{Coth}[c + d*x]])$

Rule 3474

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Tan}[c + d*x])^{(n + 1)}/(b*d*(n + 1)), x] - \text{Dist}[1/b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n + 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && LtQ[n, -1]

Rule 3476

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\text{Tan}[c + d*x]], x] /;$ FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

```
Int[((c_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b)
], 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b \coth(c + dx))^{7/2}} dx &= -\frac{2}{5bd(b \coth(c + dx))^{5/2}} + \frac{\int \frac{1}{(b \coth(c+dx))^{3/2}} dx}{b^2} \\
&= -\frac{2}{5bd(b \coth(c + dx))^{5/2}} - \frac{2}{b^3 d \sqrt{b \coth(c + dx)}} + \frac{\int \sqrt{b \coth(c + dx)} dx}{b^4} \\
&= -\frac{2}{5bd(b \coth(c + dx))^{5/2}} - \frac{2}{b^3 d \sqrt{b \coth(c + dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2} dx, x, b \coth(c + dx)\right)}{b^3 d} \\
&= -\frac{2}{5bd(b \coth(c + dx))^{5/2}} - \frac{2}{b^3 d \sqrt{b \coth(c + dx)}} - \frac{2 \text{Subst}\left(\int \frac{x^2}{-b^2+x^4} dx, x, \sqrt{b \coth(c + dx)}\right)}{b^3 d} \\
&= -\frac{2}{5bd(b \coth(c + dx))^{5/2}} - \frac{2}{b^3 d \sqrt{b \coth(c + dx)}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2} dx, x, \sqrt{b \coth(c + dx)}\right)}{b^3 d} - \dots \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} - \frac{2}{5bd(b \coth(c + dx))^{5/2}} - \frac{2}{b^3 d \sqrt{b \coth(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 0.0964058, size = 38, normalized size = 0.38

$$-\frac{{}_2F_1\left(-\frac{5}{4}, 1; -\frac{1}{4}; \coth^2(c + dx)\right)}{5bd(b \coth(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(-7/2), x]

[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, Coth[c + d*x]^2])/(5*b*d*(b*Coth[c + d*x])^(5/2))

Maple [A] time = 0.013, size = 83, normalized size = 0.8

$$-\frac{1}{d} \arctan\left(\sqrt{b \coth(dx + c)} \frac{1}{\sqrt{b}}\right) b^{-\frac{7}{2}} + \frac{1}{d} \text{Artanh}\left(\sqrt{b \coth(dx + c)} \frac{1}{\sqrt{b}}\right) b^{-\frac{7}{2}} - \frac{2}{5bd} (b \coth(dx + c))^{-\frac{5}{2}} - 2 \frac{1}{b^3 d \sqrt{b \coth(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c))^(7/2), x)


```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d-2/5/b/d/(b*coth(d*x+c))^(5/2)-2/b^3/d/(b*coth(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-7/2), x)
```

Fricas [B] time = 2.59864, size = 5797, normalized size = 57.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(cosh(d*x + c))^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 5*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
```

$$\begin{aligned} & \text{sh}(d*x + c)^2 * \sinh(d*x + c)^2 + 4 * \cosh(d*x + c) * \sinh(d*x + c)^3 + \sinh(d*x \\ & + c)^4) + 16 * (3 * \cosh(d*x + c)^6 + 18 * \cosh(d*x + c) * \sinh(d*x + c)^5 + 3 * \sinh \\ & (d*x + c)^6 + (45 * \cosh(d*x + c)^2 + 1) * \sinh(d*x + c)^4 + \cosh(d*x + c)^4 + \\ & 4 * (15 * \cosh(d*x + c)^3 + \cosh(d*x + c)) * \sinh(d*x + c)^3 + (45 * \cosh(d*x + c) \\ & ^4 + 6 * \cosh(d*x + c)^2 - 1) * \sinh(d*x + c)^2 - \cosh(d*x + c)^2 + 2 * (9 * \cosh(d \\ & *x + c)^5 + 2 * \cosh(d*x + c)^3 - \cosh(d*x + c)) * \sinh(d*x + c) - 3) * \sqrt{b * \cosh(d*x + c) / \sinh(d*x + c)}) / (b^4 * d * \cosh(d*x + c)^6 + 6 * b^4 * d * \cosh(d*x + c) * \\ & \sinh(d*x + c)^5 + b^4 * d * \sinh(d*x + c)^6 + 3 * b^4 * d * \cosh(d*x + c)^4 + 3 * b^4 * d \\ & * \cosh(d*x + c)^2 + b^4 * d + 3 * (5 * b^4 * d * \cosh(d*x + c)^2 + b^4 * d) * \sinh(d*x + c \\ &)^4 + 4 * (5 * b^4 * d * \cosh(d*x + c)^3 + 3 * b^4 * d * \cosh(d*x + c)) * \sinh(d*x + c)^3 + \\ & 3 * (5 * b^4 * d * \cosh(d*x + c)^4 + 6 * b^4 * d * \cosh(d*x + c)^2 + b^4 * d) * \sinh(d*x + c \\ &)^2 + 6 * (b^4 * d * \cosh(d*x + c)^5 + 2 * b^4 * d * \cosh(d*x + c)^3 + b^4 * d * \cosh(d*x + c) \\ &) * \sinh(d*x + c)), -1/20 * (10 * (\cosh(d*x + c)^6 + 6 * \cosh(d*x + c) * \sinh(d*x \\ & + c)^5 + \sinh(d*x + c)^6 + 3 * (5 * \cosh(d*x + c)^2 + 1) * \sinh(d*x + c)^4 + 3 * \cosh \\ & (d*x + c)^4 + 4 * (5 * \cosh(d*x + c)^3 + 3 * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 3 \\ & * (5 * \cosh(d*x + c)^4 + 6 * \cosh(d*x + c)^2 + 1) * \sinh(d*x + c)^2 + 3 * \cosh(d*x + c) \\ & ^2 + 6 * (\cosh(d*x + c)^5 + 2 * \cosh(d*x + c)^3 + \cosh(d*x + c)) * \sinh(d*x + c) \\ & + 1) * \sqrt{b} * \arctan(\sqrt{b} * \sqrt{b * \cosh(d*x + c) / \sinh(d*x + c)}) / (b * \cosh(d*x + c)^2 + 2 * b * \cosh(d*x + c) * \sinh(d*x + c) + b * \sinh(d*x + c)^2 + b)) - 5 * \\ & (\cosh(d*x + c)^6 + 6 * \cosh(d*x + c) * \sinh(d*x + c)^5 + \sinh(d*x + c)^6 + 3 * (5 \\ & * \cosh(d*x + c)^2 + 1) * \sinh(d*x + c)^4 + 3 * \cosh(d*x + c)^4 + 4 * (5 * \cosh(d*x + c) \\ & ^3 + 3 * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 3 * (5 * \cosh(d*x + c)^4 + 6 * \cosh(d*x \\ & + c)^2 + 1) * \sinh(d*x + c)^2 + 3 * \cosh(d*x + c)^2 + 6 * (\cosh(d*x + c)^5 + 2 * \\ & \cosh(d*x + c)^3 + \cosh(d*x + c)) * \sinh(d*x + c) + 1) * \sqrt{b} * \log(2 * b * \cosh(d*x \\ & + c)^4 + 8 * b * \cosh(d*x + c)^3 * \sinh(d*x + c) + 12 * b * \cosh(d*x + c)^2 * \sinh(d*x \\ & + c)^2 + 8 * b * \cosh(d*x + c) * \sinh(d*x + c)^3 + 2 * b * \sinh(d*x + c)^4 + 2 * (\cosh \\ & (d*x + c)^4 + 4 * \cosh(d*x + c) * \sinh(d*x + c)^3 + \sinh(d*x + c)^4 + (6 * \cosh(d*x + c)^2 - 1) * \sinh(d*x + c)^2 - \cosh(d*x + c)^2 + 2 * (2 * \cosh(d*x + c)^3 - \\ & \cosh(d*x + c)) * \sinh(d*x + c)) * \sqrt{b} * \sqrt{b * \cosh(d*x + c) / \sinh(d*x + c)}) - \\ & b) + 16 * (3 * \cosh(d*x + c)^6 + 18 * \cosh(d*x + c) * \sinh(d*x + c)^5 + 3 * \sinh(d*x \\ & + c)^6 + (45 * \cosh(d*x + c)^2 + 1) * \sinh(d*x + c)^4 + \cosh(d*x + c)^4 + 4 * (15 \\ & * \cosh(d*x + c)^3 + \cosh(d*x + c)) * \sinh(d*x + c)^3 + (45 * \cosh(d*x + c)^4 + \\ & 6 * \cosh(d*x + c)^2 - 1) * \sinh(d*x + c)^2 - \cosh(d*x + c)^2 + 2 * (9 * \cosh(d*x + c) \\ & ^5 + 2 * \cosh(d*x + c)^3 - \cosh(d*x + c)) * \sinh(d*x + c) - 3) * \sqrt{b * \cosh(d*x \\ & + c) / \sinh(d*x + c)}) / (b^4 * d * \cosh(d*x + c)^6 + 6 * b^4 * d * \cosh(d*x + c) * \sinh(d*x \\ & + c)^5 + b^4 * d * \sinh(d*x + c)^6 + 3 * b^4 * d * \cosh(d*x + c)^4 + 3 * b^4 * d * \cosh \\ & (d*x + c)^2 + b^4 * d + 3 * (5 * b^4 * d * \cosh(d*x + c)^2 + b^4 * d) * \sinh(d*x + c)^4 + \\ & 4 * (5 * b^4 * d * \cosh(d*x + c)^3 + 3 * b^4 * d * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 3 * (5 \\ & * b^4 * d * \cosh(d*x + c)^4 + 6 * b^4 * d * \cosh(d*x + c)^2 + b^4 * d) * \sinh(d*x + c)^2 + \\ & 6 * (b^4 * d * \cosh(d*x + c)^5 + 2 * b^4 * d * \cosh(d*x + c)^3 + b^4 * d * \cosh(d*x + c)) * \\ & \sinh(d*x + c))] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.9 $\int (b \coth(c + dx))^{4/3} dx$

Optimal. Leaf size=236

$$\frac{b^{4/3} \log\left(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3}\right)}{4d} + \frac{b^{4/3} \log\left(b^{2/3} + \sqrt[3]{b} \sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3}\right)}{4d}$$

[Out] $-(\text{Sqrt}[3]*b^{(4/3)}*\text{ArcTan}[(1 - (2*(b*\text{Coth}[c + d*x])^{(1/3)})/b^{(1/3)})/\text{Sqrt}[3]])/(2*d) + (\text{Sqrt}[3]*b^{(4/3)}*\text{ArcTan}[(1 + (2*(b*\text{Coth}[c + d*x])^{(1/3)})/b^{(1/3)})/\text{Sqrt}[3]])/(2*d) + (b^{(4/3)}*\text{ArcTanh}[(b*\text{Coth}[c + d*x])^{(1/3)}/b^{(1/3)}])/d - (3*b*(b*\text{Coth}[c + d*x])^{(1/3)})/d - (b^{(4/3)}*\text{Log}[b^{(2/3)} - b^{(1/3)}*(b*\text{Coth}[c + d*x])^{(1/3)} + (b*\text{Coth}[c + d*x])^{(2/3)}])/(4*d) + (b^{(4/3)}*\text{Log}[b^{(2/3)} + b^{(1/3)}*(b*\text{Coth}[c + d*x])^{(1/3)} + (b*\text{Coth}[c + d*x])^{(2/3)}])/(4*d)$

Rubi [A] time = 0.281901, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$\frac{b^{4/3} \log\left(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3}\right)}{4d} + \frac{b^{4/3} \log\left(b^{2/3} + \sqrt[3]{b} \sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3}\right)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x])^{(4/3)}, x]$

[Out] $-(\text{Sqrt}[3]*b^{(4/3)}*\text{ArcTan}[(1 - (2*(b*\text{Coth}[c + d*x])^{(1/3)})/b^{(1/3)})/\text{Sqrt}[3]])/(2*d) + (\text{Sqrt}[3]*b^{(4/3)}*\text{ArcTan}[(1 + (2*(b*\text{Coth}[c + d*x])^{(1/3)})/b^{(1/3)})/\text{Sqrt}[3]])/(2*d) + (b^{(4/3)}*\text{ArcTanh}[(b*\text{Coth}[c + d*x])^{(1/3)}/b^{(1/3)}])/d - (3*b*(b*\text{Coth}[c + d*x])^{(1/3)})/d - (b^{(4/3)}*\text{Log}[b^{(2/3)} - b^{(1/3)}*(b*\text{Coth}[c + d*x])^{(1/3)} + (b*\text{Coth}[c + d*x])^{(2/3)}])/(4*d) + (b^{(4/3)}*\text{Log}[b^{(2/3)} + b^{(1/3)}*(b*\text{Coth}[c + d*x])^{(1/3)} + (b*\text{Coth}[c + d*x])^{(2/3)}])/(4*d)$

Rule 3473

$\text{Int}[(b*\text{tan}[(c + d*x)])^{(n)}, x_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n-2)}, x], x] /;$ $\text{FreeQ}\{b, c, d\}, x \&\& \text{GtQ}[n, 1]$

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 210

Int[((a_) + (b_.)*(x_)^(n_))^(n_ - 1), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_ - 1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_ - 1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int (b \coth(c + dx))^{4/3} dx &= -\frac{3b\sqrt[3]{b \coth(c + dx)}}{d} + b^2 \int \frac{1}{(b \coth(c + dx))^{2/3}} dx \\
 &= -\frac{3b\sqrt[3]{b \coth(c + dx)}}{d} - \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{x^{2/3}(-b^2+x^2)} dx, x, b \coth(c + dx)\right)}{d} \\
 &= -\frac{3b\sqrt[3]{b \coth(c + dx)}}{d} - \frac{(3b^3) \operatorname{Subst}\left(\int \frac{1}{-b^2+x^6} dx, x, \sqrt[3]{b \coth(c + dx)}\right)}{d} \\
 &= -\frac{3b\sqrt[3]{b \coth(c + dx)}}{d} + \frac{b^{4/3} \operatorname{Subst}\left(\int \frac{\sqrt[3]{b-x}}{b^{2/3}-\sqrt[3]{bx+x^2}} dx, x, \sqrt[3]{b \coth(c + dx)}\right)}{d} + \frac{b^{4/3} \operatorname{Subst}\left(\int \frac{\sqrt[3]{b+x}}{b^{2/3}+\sqrt[3]{bx+x^2}} dx, x, \sqrt[3]{b \coth(c + dx)}\right)}{d} \\
 &= \frac{b^{4/3} \tanh^{-1}\left(\frac{\sqrt[3]{b \coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b \coth(c + dx)}}{d} - \frac{b^{4/3} \operatorname{Subst}\left(\int \frac{-\sqrt[3]{b+2x}}{b^{2/3}-\sqrt[3]{bx+x^2}} dx, x, \sqrt[3]{b \coth(c + dx)}\right)}{4d} \\
 &= \frac{b^{4/3} \tanh^{-1}\left(\frac{\sqrt[3]{b \coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b \coth(c + dx)}}{d} - \frac{b^{4/3} \log\left(b^{2/3} - \sqrt[3]{b}\sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3}\right)}{4d} \\
 &= -\frac{\sqrt{3}b^{4/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b \coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{\sqrt{3}b^{4/3} \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{b \coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{4/3} \tanh^{-1}\left(\frac{\sqrt[3]{b \coth(c+dx)}}{\sqrt[3]{b}}\right)}{d}
 \end{aligned}$$

Mathematica [C] time = 0.0259547, size = 36, normalized size = 0.15

$$\frac{3b\sqrt[3]{b \coth(c + dx)} \left({}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; \coth^2(c + dx)\right) - 1 \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(4/3), x]

[Out] $(3*b*(b*\text{Coth}[c + d*x])^{1/3}*(-1 + \text{Hypergeometric2F1}[1/6, 1, 7/6, \text{Coth}[c + d*x]^2]))/d$

Maple [A] time = 0.02, size = 209, normalized size = 0.9

$$-3 \frac{b\sqrt[3]{b\text{coth}(dx+c)}}{d} + \frac{1}{2d} b^{\frac{4}{3}} \ln\left(\sqrt[3]{b\text{coth}(dx+c)} + \sqrt[3]{b}\right) - \frac{1}{4d} b^{\frac{4}{3}} \ln\left(b^{\frac{2}{3}} - \sqrt[3]{b}\sqrt[3]{b\text{coth}(dx+c)} + (b\text{coth}(dx+c))^{\frac{2}{3}}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*\text{coth}(d*x+c))^{4/3}, x)$

[Out] $-3*b*(b*\text{coth}(d*x+c))^{1/3}/d+1/2*b^{4/3}/d*\ln((b*\text{coth}(d*x+c))^{1/3}+b^{1/3})-1/4*b^{4/3}*\ln(b^{2/3}-b^{1/3}*(b*\text{coth}(d*x+c))^{1/3}+(b*\text{coth}(d*x+c))^{2/3}))/d+1/2*b^{4/3}/d*3^{1/2}*\arctan(1/3*3^{1/2}*(2*(b*\text{coth}(d*x+c))^{1/3}/b^{1/3}-1))-1/2*b^{4/3}/d*\ln((b*\text{coth}(d*x+c))^{1/3}-b^{1/3})+1/4*b^{4/3}*\ln(b^{2/3}+b^{1/3}*(b*\text{coth}(d*x+c))^{1/3}+(b*\text{coth}(d*x+c))^{2/3}))/d+1/2*b^{4/3}*\arctan(1/3*(1+2*(b*\text{coth}(d*x+c))^{1/3}/b^{1/3}))*3^{1/2})*3^{1/2}/d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx+c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b*\text{coth}(d*x+c))^{4/3}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}((b*\text{coth}(d*x + c))^{4/3}, x)$

Fricas [A] time = 1.96127, size = 865, normalized size = 3.67

$$2\sqrt{3}(-b)^{\frac{1}{3}} b \arctan\left(\frac{\sqrt{3}b+2\sqrt{3}(-b)^{\frac{2}{3}}\left(\frac{b \cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) - 2\sqrt{3}b^{\frac{4}{3}} \arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}b^{\frac{2}{3}}\left(\frac{b \cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) + (-b)^{\frac{1}{3}} b \log\left((-b)^{\frac{2}{3}} - (-b)^{\frac{1}{3}}\left(\frac{b \cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right) - (-b)^{\frac{1}{3}} b \log\left((-b)^{\frac{2}{3}} + (-b)^{\frac{1}{3}}\left(\frac{b \cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*b*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(4/3)*arctan(-1/3*(s
qrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-
b)^(1/3)*b*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(4/3)*log(b^(2/3) - b^(1/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/
3)) - 2*(-b)^(1/3)*b*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3
) - 2*b^(4/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*b*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


3.10 $\int (b \coth(c + dx))^{2/3} dx$

Optimal. Leaf size=218

$$\frac{b^{2/3} \log(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3})}{4d} + \frac{b^{2/3} \log(b^{2/3} + \sqrt[3]{b} \sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3})}{4d}$$

[Out] (Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*d) - (Sqrt[3]*b^(2/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*d) + (b^(2/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (b^(2/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/ (4*d) + (b^(2/3)*Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/ (4*d)

Rubi [A] time = 0.293867, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3476, 329, 296, 634, 618, 204, 628, 206}

$$\frac{b^{2/3} \log(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3})}{4d} + \frac{b^{2/3} \log(b^{2/3} + \sqrt[3]{b} \sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3})}{4d}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x])^(2/3), x]

[Out] (Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*d) - (Sqrt[3]*b^(2/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*d) + (b^(2/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (b^(2/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/ (4*d) + (b^(2/3)*Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/ (4*d)

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int (b \coth(c + dx))^{2/3} dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^{2/3}}{-b^2+x^2} dx, x, b \coth(c + dx)\right)}{d} \\
 &= \frac{(3b) \operatorname{Subst}\left(\int \frac{x^4}{-b^2+x^6} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{d} \\
 &= \frac{b^{2/3} \operatorname{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2} - \frac{x}{2}}{b^{2/3} - \sqrt[3]{b}x+x^2} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{d} + \frac{b^{2/3} \operatorname{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2} + \frac{x}{2}}{b^{2/3} + \sqrt[3]{b}x+x^2} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{d} \\
 &= \frac{b^{2/3} \tanh^{-1}\left(\frac{\sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3} \operatorname{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3} - \sqrt[3]{b}x+x^2} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{4d} + \frac{b^{2/3} \operatorname{Subst}\left(\int \frac{\sqrt[3]{b}+2x}{b^{2/3} + \sqrt[3]{b}x+x^2} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{4d} \\
 &= \frac{b^{2/3} \tanh^{-1}\left(\frac{\sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3} \log\left(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b} \coth(c + dx) + (b \coth(c + dx))^{2/3}\right)}{4d} + \frac{b^{2/3} \log\left(b^{2/3} + \sqrt[3]{b} \sqrt[3]{b} \coth(c + dx) + (b \coth(c + dx))^{2/3}\right)}{4d} \\
 &= \frac{\sqrt{3} b^{2/3} \tan^{-1}\left(\frac{1 - \frac{2 \sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt{3} b^{2/3} \tan^{-1}\left(\frac{1 + \frac{2 \sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{2/3} \tanh^{-1}\left(\frac{\sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}\right)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.177229, size = 149, normalized size = 0.68

$$\frac{(b \coth(c + dx))^{2/3} \left(-\log\left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1\right) + \log\left(\coth^{\frac{2}{3}}(c + dx) + \sqrt[3]{\coth(c + dx)} + 1\right) + 2\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{b} \coth(c + dx)}{\sqrt[3]{b}}\right) \right)}{4d \coth^{\frac{2}{3}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(2/3),x]

[Out] ((b*Coth[c + d*x])^(2/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[Coth[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2/3))

))

Maple [A] time = 0.016, size = 193, normalized size = 0.9

$$\frac{1}{2d}b^{\frac{2}{3}}\ln\left(\sqrt[3]{b\coth(dx+c)}+\sqrt[3]{b}\right)-\frac{1}{4d}b^{\frac{2}{3}}\ln\left(b^{\frac{2}{3}}-\sqrt[3]{b}\sqrt[3]{b\coth(dx+c)}+(b\coth(dx+c))^{\frac{2}{3}}\right)-\frac{\sqrt{3}}{2d}b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{b\coth(dx+c)}}{\sqrt[3]{b}}-\sqrt[3]{b\coth(dx+c)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c))^(2/3),x)

[Out] $\frac{1}{2}b^{2/3}/d*\ln((b*\coth(d*x+c))^{1/3}+b^{1/3})-1/4*b^{2/3}*ln(b^{2/3}-b^{1/3}*(b*\coth(d*x+c))^{1/3}+(b*\coth(d*x+c))^{2/3})/d-1/2*b^{2/3}/d*3^{1/2}*arctan(1/3*3^{1/2}*(2*(b*\coth(d*x+c))^{1/3}/b^{1/3}-1))-1/2*b^{2/3}/d*ln((b*\coth(d*x+c))^{1/3}-b^{1/3})+1/4*b^{2/3}*ln(b^{2/3}+b^{1/3}*(b*\coth(d*x+c))^{1/3}+(b*\coth(d*x+c))^{2/3})/d-1/2*b^{2/3}*arctan(1/3*(1+2*(b*\coth(d*x+c))^{1/3}/b^{1/3})*3^{1/2})*3^{1/2}/d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(2/3), x)

Fricas [A] time = 2.0473, size = 871, normalized size = 4.

$$2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(-b^2)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+2\sqrt{3}(b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(b^2)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+(-b^2)^{\frac{1}{3}}\log\left(b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b^2)^(1/3)
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 2*sqrt(3)*(b^2)^(1/3)*arctan(
-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
)/b) + (-b^2)^(1/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^(
1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + (b^2)^(1/3)
*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(1/3)*log(b*(b*cosh(d*x +
c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) - 2*(b^2)^(1/3)*log(b*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(c + dx))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(2/3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.11 $\int \sqrt[3]{b \coth(c + dx)} dx$

Optimal. Leaf size=132

$$-\frac{\sqrt[3]{b} \log(b^{2/3} - (b \coth(c + dx))^{2/3})}{2d} + \frac{\sqrt[3]{b} \log(b^{2/3}(b \coth(c + dx))^{2/3} + b^{4/3} + (b \coth(c + dx))^{4/3})}{4d} - \frac{\sqrt{3} \sqrt[3]{b} \tan^{-1}\left(\frac{b^{2/3} + 2(b \coth(c + dx))^{2/3}}{b^{2/3} + (b \coth(c + dx))^{2/3}}\right)}{2d}$$

[Out] $-(\text{Sqrt}[3]*b^{(1/3)}*\text{ArcTan}[(b^{(2/3)} + 2*(b*\text{Coth}[c + d*x])^{(2/3)})/(\text{Sqrt}[3]*b^{(2/3)})])/(2*d) - (b^{(1/3)}*\text{Log}[b^{(2/3)} - (b*\text{Coth}[c + d*x])^{(2/3)}])/(2*d) + (b^{(1/3)}*\text{Log}[b^{(4/3)} + b^{(2/3)}*(b*\text{Coth}[c + d*x])^{(2/3)} + (b*\text{Coth}[c + d*x])^{(4/3)}])/(4*d)$

Rubi [A] time = 0.109114, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {3476, 329, 275, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b} \log(b^{2/3} - (b \coth(c + dx))^{2/3})}{2d} + \frac{\sqrt[3]{b} \log(b^{2/3}(b \coth(c + dx))^{2/3} + b^{4/3} + (b \coth(c + dx))^{4/3})}{4d} - \frac{\sqrt{3} \sqrt[3]{b} \tan^{-1}\left(\frac{b^{2/3} + 2(b \coth(c + dx))^{2/3}}{b^{2/3} + (b \coth(c + dx))^{2/3}}\right)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x])^{(1/3)}, x]$

[Out] $-(\text{Sqrt}[3]*b^{(1/3)}*\text{ArcTan}[(b^{(2/3)} + 2*(b*\text{Coth}[c + d*x])^{(2/3)})/(\text{Sqrt}[3]*b^{(2/3)})])/(2*d) - (b^{(1/3)}*\text{Log}[b^{(2/3)} - (b*\text{Coth}[c + d*x])^{(2/3)}])/(2*d) + (b^{(1/3)}*\text{Log}[b^{(4/3)} + b^{(2/3)}*(b*\text{Coth}[c + d*x])^{(2/3)} + (b*\text{Coth}[c + d*x])^{(4/3)}])/(4*d)$

Rule 3476

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)*(x_*)]^{(n_)}, x_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\text{Tan}[c + d*x]], x] /;$ $\text{FreeQ}\{b, c, d, n\}, x\} \&\& ! \text{IntegerQ}[n]$

Rule 329

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k*(m+1)-1}*(a + (b*x^{k*n}))/c^n]^p, x], x, (c*x)^{(1/k)}, x]] /;$ $\text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 275

$\text{Int}[(x_)^m * ((a_) + (b_) * (x_)^n)^p, x_Symbol] := \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m+1)/k - 1} * (a + b * x^{n/k})^p, x], x, x^k], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 292

$\text{Int}[(x_)/((a_) + (b_) * (x_)^3), x_Symbol] := -\text{Dist}[(3 * \text{Rt}[a, 3] * \text{Rt}[b, 3])^{(-1)}, \text{Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3] * x), x], x] + \text{Dist}[1/(3 * \text{Rt}[a, 3] * \text{Rt}[b, 3]), \text{Int}[(\text{Rt}[a, 3] + \text{Rt}[b, 3] * x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3] * \text{Rt}[b, 3] * x + \text{Rt}[b, 3]^2 * x^2), x], x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 31

$\text{Int}[(a_) + (b_) * (x_)^{(-1)}, x_Symbol] := \text{Simp}[\text{Log}[\text{RemoveContent}[a + b * x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 634

$\text{Int}(((d_) + (e_) * (x_))/((a_) + (b_) * (x_) + (c_) * (x_)^2), x_Symbol] := \text{Dist}[(2 * c * d - b * e)/(2 * c), \text{Int}[1/(a + b * x + c * x^2), x], x] + \text{Dist}[e/(2 * c), \text{Int}[(b + 2 * c * x)/(a + b * x + c * x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2 * c * d - b * e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 * a * c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4 * a * c]$

Rule 617

$\text{Int}(((a_) + (b_) * (x_) + (c_) * (x_)^2)^{(-1)}, x_Symbol] := \text{With}[\{q = 1 - 4 * \text{Simplify}[(a * c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 * c * x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 * a * c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 * a * c, 0]$

Rule 204

$\text{Int}(((a_) + (b_) * (x_)^2)^{(-1)}, x_Symbol] := -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2] * x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$

Rule 628

$\text{Int}(((d_) + (e_) * (x_))/((a_) + (b_) * (x_) + (c_) * (x_)^2), x_Symbol] := \text{Simp}[(d * \text{Log}[\text{RemoveContent}[a + b * x + c * x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 * c * d - b * e, 0]$

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{b \coth(c+dx)} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{\sqrt[3]{x}}{-b^2+x^2} dx, x, b \coth(c+dx)\right)}{d} \\
&= -\frac{(3b) \operatorname{Subst}\left(\int \frac{x^3}{-b^2+x^6} dx, x, \sqrt[3]{b \coth(c+dx)}\right)}{d} \\
&= -\frac{(3b) \operatorname{Subst}\left(\int \frac{x}{-b^2+x^3} dx, x, (b \coth(c+dx))^{2/3}\right)}{2d} \\
&= -\frac{\sqrt[3]{b} \operatorname{Subst}\left(\int \frac{1}{-b^{2/3}+x} dx, x, (b \coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b} \operatorname{Subst}\left(\int \frac{-b^{2/3}+x}{b^{4/3}+b^{2/3}x+x^2} dx, x, (b \coth(c+dx))^{2/3}\right)}{2d} \\
&= -\frac{\sqrt[3]{b} \log\left(b^{2/3} - (b \coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b} \operatorname{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2} dx, x, (b \coth(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt[3]{b} \log\left(b^{4/3} + b^{2/3}(b \coth(c+dx))^{2/3} + (b \coth(c+dx))^2\right)}{4d} \\
&= -\frac{\sqrt[3]{b} \log\left(b^{2/3} - (b \coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b} \log\left(b^{4/3} + b^{2/3}(b \coth(c+dx))^{2/3} + (b \coth(c+dx))^2\right)}{4d} \\
&= -\frac{\sqrt{3} \sqrt[3]{b} \tan^{-1}\left(\frac{1 + \frac{2(b \coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt[3]{b} \log\left(b^{2/3} - (b \coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b} \log\left(b^{4/3} + b^{2/3}(b \coth(c+dx))^{2/3} + (b \coth(c+dx))^2\right)}{4d}
\end{aligned}$$

Mathematica [C] time = 0.040844, size = 38, normalized size = 0.29

$$\frac{3(b \coth(c+dx))^{4/3} {}_2F_1\left(\frac{2}{3}, 1; \frac{5}{3}; \coth^2(c+dx)\right)}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(1/3), x]

[Out] (3*(b*Coth[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 1, 5/3, Coth[c + d*x]^2])/(4*b*d)

Maple [A] time = 0.015, size = 115, normalized size = 0.9

$$-\frac{b}{2d} \ln\left((b \coth(dx+c))^{2/3} - \sqrt[3]{b^2}\right) \frac{1}{\sqrt[3]{b^2}} + \frac{b}{4d} \ln\left((b \coth(dx+c))^{4/3} + \sqrt[3]{b^2} (b \coth(dx+c))^{2/3} + (b^2)^{2/3}\right) \frac{1}{\sqrt[3]{b^2}} - \frac{b\sqrt{3}}{2d} \arctan\left(\frac{1 + \frac{2(b \coth(dx+c))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*coth(d*x+c))^(1/3),x)`

[Out]
$$-1/2*b/d/(b^2)^{(1/3)}*\ln((b*\coth(d*x+c))^{(2/3)}-(b^2)^{(1/3)})+1/4*b/d/(b^2)^{(1/3)}*\ln((b*\coth(d*x+c))^{(4/3)}+(b^2)^{(1/3)}*(b*\coth(d*x+c))^{(2/3)}+(b^2)^{(2/3)})-1/2*b/d*3^{(1/2)}/(b^2)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(b^2)^{(1/3)}*(b*\coth(d*x+c))^{(2/3)+1}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c))^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c))^(1/3),x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c))^(1/3), x)`

Fricas [B] time = 2.00133, size = 828, normalized size = 6.27

$$2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(-b)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}}{3b}\right)-2(-b)^{\frac{1}{3}}\log\left(-(-b)^{\frac{2}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)+(-b)^{\frac{1}{3}}\log\left(\frac{(\cosh(dx+c)^2+2\coth(dx+c))}{\sinh(dx+c)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c))^(1/3),x, algorithm="fricas")`

[Out]
$$-1/4*(2*\sqrt{3})*(-b)^{(1/3)}*\arctan(-1/3*(\sqrt{3})*b - 2*\sqrt{3})*(-b)^{(1/3)}*(b*\cosh(d*x + c)/\sinh(d*x + c))^{(2/3)}/b - 2*(-b)^{(1/3)}*\log(-(-b)^{(2/3)} + (b*\cosh(d*x + c)/\sinh(d*x + c))^{(2/3)}) + (-b)^{(1/3)}*\log(((\cosh(d*x + c))^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 - 1)*(-b)^{(2/3)}*(b*\cosh(d*x + c)/\sinh(d*x + c))^{(2/3)} - (b*\cosh(d*x + c))^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 - b)*(-b)^{(1/3)} + (b*\cosh(d*x + c))^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + b)*(b*\cosh(d*x + c)/\sinh(d*x + c))^{(1/3)}/(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2)$$

$\sqrt{2 - 1})/d$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \coth(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))**(1/3),x)

[Out] Integral((b*coth(c + d*x))**(1/3), x)

Giac [B] time = 1.68703, size = 293, normalized size = 2.22

$$b \frac{\left(2\sqrt{3}|b|^{\frac{4}{3}} \arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2dx+2c)+b}}{e^{(2dx+2c)-1}}\right)^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{3|b|^{\frac{2}{3}}}\right) - |b|^{\frac{4}{3}} \log\left(\frac{\left(\frac{be^{(2dx+2c)+b}}{e^{(2dx+2c)-1}}\right)^{\frac{2}{3}} |b|^{\frac{2}{3}} + |b|^{\frac{4}{3}} + \frac{(be^{(2dx+2c)+b})\left(\frac{be^{(2dx+2c)+b}}{e^{(2dx+2c)-1}}\right)^{\frac{1}{3}}}{e^{(2dx+2c)-1}}\right)}{2|b|^{\frac{4}{3}} \log\left(\frac{\left(\frac{be^{(2dx+2c)+b}}{e^{(2dx+2c)-1}}\right)^{\frac{2}{3}} - |b|^{\frac{2}{3}}}{-|b|^{\frac{2}{3}}}\right)} \right)}{b^2} + \frac{\dots}{b^2}$$

$4d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="giac")

[Out] $-1/4*b*(2*\sqrt{3}*abs(b)^{(4/3)}*arctan(1/3*\sqrt{3}*(2*((b*e^{(2*d*x + 2*c)} + b)/(e^{(2*d*x + 2*c)} - 1))^{(2/3)} + abs(b)^{(2/3)})/abs(b)^{(2/3)})/b^2 - abs(b)^{(4/3)}*log(((b*e^{(2*d*x + 2*c)} + b)/(e^{(2*d*x + 2*c)} - 1))^{(2/3)}*abs(b)^{(2/3)} + abs(b)^{(4/3)} + (b*e^{(2*d*x + 2*c)} + b)*((b*e^{(2*d*x + 2*c)} + b)/(e^{(2*d*x + 2*c)} - 1))^{(1/3)})/(e^{(2*d*x + 2*c)} - 1)/b^2 + 2*abs(b)^{(4/3)}*log(abs((b*e^{(2*d*x + 2*c)} + b)/(e^{(2*d*x + 2*c)} - 1))^{(2/3)} - abs(b)^{(2/3)})/b^2)/d$

$$3.12 \quad \int \frac{1}{\sqrt[3]{b \coth(c+dx)}} dx$$

Optimal. Leaf size=132

$$\frac{\log\left(b^{2/3} - (b \coth(c+dx))^{2/3}\right)}{2\sqrt[3]{bd}} + \frac{\log\left(b^{2/3}(b \coth(c+dx))^{2/3} + b^{4/3} + (b \coth(c+dx))^{4/3}\right)}{4\sqrt[3]{bd}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{b^{2/3} + 2(b \coth(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{bd}}$$

[Out] (Sqrt[3]*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))])/(2*b^(1/3)*d) - Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)]/(2*b^(1/3)*d) + Log[b^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4/3)]/(4*b^(1/3)*d)

Rubi [A] time = 0.10391, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {3476, 329, 275, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(b^{2/3} - (b \coth(c+dx))^{2/3}\right)}{2\sqrt[3]{bd}} + \frac{\log\left(b^{2/3}(b \coth(c+dx))^{2/3} + b^{4/3} + (b \coth(c+dx))^{4/3}\right)}{4\sqrt[3]{bd}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{b^{2/3} + 2(b \coth(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{bd}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x])^(-1/3), x]

[Out] (Sqrt[3]*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))])/(2*b^(1/3)*d) - Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)]/(2*b^(1/3)*d) + Log[b^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4/3)]/(4*b^(1/3)*d)

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && IntegerQ[n]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt[3]{b \coth(c+dx)}} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{x(-b^2+x^2)}} dx, x, b \coth(c+dx)\right)}{d} \\
 &= -\frac{(3b) \operatorname{Subst}\left(\int \frac{x}{-b^2+x^6} dx, x, \sqrt[3]{b \coth(c+dx)}\right)}{d} \\
 &= -\frac{(3b) \operatorname{Subst}\left(\int \frac{1}{-b^2+x^3} dx, x, (b \coth(c+dx))^{2/3}\right)}{2d} \\
 &= -\frac{\operatorname{Subst}\left(\int \frac{1}{-b^{2/3}+x} dx, x, (b \coth(c+dx))^{2/3}\right)}{2\sqrt[3]{bd}} - \frac{\operatorname{Subst}\left(\int \frac{-2b^{2/3}-x}{b^{4/3}+b^{2/3}x+x^2} dx, x, (b \coth(c+dx))^{2/3}\right)}{2\sqrt[3]{bd}} \\
 &= -\frac{\log\left(b^{2/3} - (b \coth(c+dx))^{2/3}\right)}{2\sqrt[3]{bd}} + \frac{\operatorname{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2} dx, x, (b \coth(c+dx))^{2/3}\right)}{4\sqrt[3]{bd}} + \frac{(3\sqrt[3]{b}) \operatorname{Subst}\left(\int \frac{1}{b^{4/3}+b^{2/3}x+x^2} dx, x, (b \coth(c+dx))^{2/3}\right)}{4\sqrt[3]{bd}} \\
 &= -\frac{\log\left(b^{2/3} - (b \coth(c+dx))^{2/3}\right)}{2\sqrt[3]{bd}} + \frac{\log\left(b^{4/3} + b^{2/3}(b \coth(c+dx))^{2/3} + (b \coth(c+dx))^{4/3}\right)}{4\sqrt[3]{bd}} \\
 &= \frac{\sqrt{3} \tan^{-1}\left(\frac{1 + \frac{2(b \coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2\sqrt[3]{bd}} - \frac{\log\left(b^{2/3} - (b \coth(c+dx))^{2/3}\right)}{2\sqrt[3]{bd}} + \frac{\log\left(b^{4/3} + b^{2/3}(b \coth(c+dx))^{2/3} + (b \coth(c+dx))^{4/3}\right)}{4\sqrt[3]{bd}}
 \end{aligned}$$

Mathematica [A] time = 0.120959, size = 98, normalized size = 0.74

$$\frac{\sqrt[3]{\coth(c+dx)} \left(-2 \log\left(1 - \coth^{\frac{2}{3}}(c+dx)\right) + \log\left(\coth^{\frac{4}{3}}(c+dx) + \coth^{\frac{2}{3}}(c+dx) + 1\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2 \coth^{\frac{2}{3}}(c+dx) + 1}{\sqrt{3}}\right) \right)}{4d \sqrt[3]{b \coth(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(-1/3), x]

[Out] (Coth[c + d*x]^(1/3)*(2*sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(2/3))/sqrt[3]] - 2*Log[1 - Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(2/3) + Coth[c + d*x]^(4/3)]))/(4*d*(b*Coth[c + d*x])^(1/3))

Maple [A] time = 0.012, size = 115, normalized size = 0.9

$$-\frac{b}{2d} \ln\left(\left(b \coth(dx+c)\right)^{\frac{2}{3}} - \sqrt[3]{b^2}\right) (b^2)^{-\frac{2}{3}} + \frac{b}{4d} \ln\left(\left(b \coth(dx+c)\right)^{\frac{4}{3}} + \sqrt[3]{b^2} \left(b \coth(dx+c)\right)^{\frac{2}{3}} + (b^2)^{\frac{2}{3}}\right) (b^2)^{-\frac{2}{3}} + \frac{b\sqrt{3}}{2d} \arctan\left(\frac{\sqrt{3} \left(b \coth(dx+c)\right)^{\frac{2}{3}}}{(b^2)^{\frac{2}{3}} + \sqrt[3]{b^2} \left(b \coth(dx+c)\right)^{\frac{2}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c))^(1/3), x)

[Out] $-\frac{1}{2} \frac{b}{d} (b^2)^{\frac{2}{3}} \ln\left(\left(b \coth(dx+c)\right)^{\frac{2}{3}} - (b^2)^{\frac{1}{3}}\right) + \frac{1}{4} \frac{b}{d} (b^2)^{\frac{2}{3}} \ln\left(\left(b \coth(dx+c)\right)^{\frac{4}{3}} + (b^2)^{\frac{1}{3}} \left(b \coth(dx+c)\right)^{\frac{2}{3}} + (b^2)^{\frac{2}{3}}\right) + \frac{1}{2} \frac{b}{d} (b^2)^{\frac{2}{3}} \sqrt[3]{3}^{\frac{1}{2}} \arctan\left(\frac{1}{\sqrt{3}} \sqrt[3]{3}^{\frac{1}{2}} \frac{2}{(b^2)^{\frac{1}{3}}} \left(b \coth(dx+c)\right)^{\frac{2}{3}} + 1\right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(1/3), x)

Fricas [B] time = 2.23023, size = 4398, normalized size = 33.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(1/3), x, algorithm="fricas")

[Out] $\frac{1}{4} \sqrt{3} b \sqrt{(-b)^{\frac{1}{3}}/b} \log\left(\left(3b \cosh(dx+c)\right)^4 + 12b \cosh(dx+c) \sinh(dx+c)^3 + 3b \sinh(dx+c)^4 + 2b \cosh(dx+c)^2 + 2(9b \cosh(dx+c)^2 + b) \sinh(dx+c)^2 + 3(\cosh(dx+c)^4 + 4 \cosh(dx+c) \sinh(dx+c)^3 + \sinh(dx+c)^4 + 2(3 \cosh(dx+c)^2 - 1) \sinh(dx+c) \right)$

$$\begin{aligned}
&)^2 - 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 - \cosh(d*x + c))*\sinh(d*x + c) \\
&+ 1)*(-b)^{(1/3)}*(b*\cosh(d*x + c)/\sinh(d*x + c))^{(2/3)} - \sqrt{3}*((\cosh(d*x \\
&+ c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(d*x \\
&+ c)^2 - 1)*\sinh(d*x + c)^2 - 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 - \cosh \\
&h(d*x + c))*\sinh(d*x + c) + 1)*(-b)^{(2/3)}*(b*\cosh(d*x + c)/\sinh(d*x + c))^{(\\
&2/3)} - (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x \\
&+ c)^4 - 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c)^2 \\
&+ 4*(b*\cosh(d*x + c)^3 - b*\cosh(d*x + c))*\sinh(d*x + c) + b)*(-b)^{(1/3)} - 2 \\
&*(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*b*\cosh(d*x + c) \\
&^2*\sinh(d*x + c)^2 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 \\
&- b)*(b*\cosh(d*x + c)/\sinh(d*x + c))^{(1/3)}*\sqrt{((-b)^{(1/3)}/b)} + 4*(3*b*\cosh \\
&h(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c) + 3*b)/(\cosh(d*x + c)^2 + 2*\c \\
&osh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2)) - 2*(-b)^{(2/3)}*\log(-(-b)^{(2/ \\
&3)} + (b*\cosh(d*x + c)/\sinh(d*x + c))^{(2/3)}) + (-b)^{(2/3)}*\log(((\cosh(d*x + c) \\
&)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 - 1)*(-b)^{(2/3)}*(b*\cosh \\
&h(d*x + c)/\sinh(d*x + c))^{(2/3)} - (b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh \\
&h(d*x + c) + b*\sinh(d*x + c)^2 - b)*(-b)^{(1/3)} + (b*\cosh(d*x + c)^2 + 2*b \\
&*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + b)*(b*\cosh(d*x + c)/\sinh \\
&(d*x + c))^{(1/3)}/(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d \\
&*x + c)^2 - 1))/ (b*d), 1/4*(2*\sqrt{3}*b*\sqrt{(-(-b)^{(1/3)}/b)}*\arctan((2*\sqrt{ \\
&3})*(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + \\
&2*(3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c)^2 - 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x \\
&+ c)^3 - \cosh(d*x + c))*\sinh(d*x + c) + 1)*(-b)^{(2/3)}*(b*\cosh(d*x + c)/\sinh \\
&h(d*x + c))^{(2/3)}*\sqrt{(-(-b)^{(1/3)}/b)} + \sqrt{3}*(b*\cosh(d*x + c)^4 + 4*b*\cosh \\
&h(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*b*\cosh(d*x + c)^2 + 2*(\\
&3*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - b*\cosh(d*x \\
&+ c))*\sinh(d*x + c) + b)*(-b)^{(1/3)}*\sqrt{(-(-b)^{(1/3)}/b)} - 4*\sqrt{3}*(b*\cosh \\
&h(d*x + c)^4 + 4*b*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*b*\cosh(d*x + c)^2*\sinh \\
&h(d*x + c)^2 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - b)*(\\
&b*\cosh(d*x + c)/\sinh(d*x + c))^{(1/3)}*\sqrt{(-(-b)^{(1/3)}/b)}/(9*b*\cosh(d*x + c) \\
&)^4 + 36*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + 9*b*\sinh(d*x + c)^4 + 14*b*\cosh(\\
&d*x + c)^2 + 2*(27*b*\cosh(d*x + c)^2 + 7*b)*\sinh(d*x + c)^2 + 4*(9*b*\cosh(d \\
&*x + c)^3 + 7*b*\cosh(d*x + c))*\sinh(d*x + c) + 9*b)) - 2*(-b)^{(2/3)}*\log(-(- \\
&b)^{(2/3)} + (b*\cosh(d*x + c)/\sinh(d*x + c))^{(2/3)}) + (-b)^{(2/3)}*\log(((\cosh(d \\
&*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 - 1)*(-b)^{(2/3)} \\
&*(b*\cosh(d*x + c)/\sinh(d*x + c))^{(2/3)} - (b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x \\
&+ c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 - b)*(-b)^{(1/3)} + (b*\cosh(d*x + c)^2 \\
&+ 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + b)*(b*\cosh(d*x + c) \\
&)/\sinh(d*x + c))^{(1/3)}/(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \\
&\sinh(d*x + c)^2 - 1))/ (b*d)]
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b \coth(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))**(1/3),x)

[Out] Integral((b*coth(c + d*x))**(-1/3), x)

Giac [B] time = 1.64217, size = 292, normalized size = 2.21

$$b \frac{\left(2\sqrt{3}|b|^{\frac{2}{3}} \arctan \left(\frac{\sqrt{3} \left(2 \left(\frac{be^{(2dx+2c)+b}}{e^{(2dx+2c)-1}} \right)^{\frac{2}{3}} + |b|^{\frac{2}{3}} \right)}{3|b|^{\frac{2}{3}}} \right) \right)}{b^2} + \frac{|b|^{\frac{2}{3}} \log \left(\frac{\left(\frac{be^{(2dx+2c)+b}}{e^{(2dx+2c)-1}} \right)^{\frac{2}{3}} |b|^{\frac{2}{3}} + |b|^{\frac{4}{3}} + \frac{\left(be^{(2dx+2c)+b} \right) \left(\frac{be^{(2dx+2c)+b}}{e^{(2dx+2c)-1}} \right)^{\frac{1}{3}}}{e^{(2dx+2c)-1}} \right)}{b^2} - \frac{2|b|^{\frac{2}{3}} \log \left(\left(\frac{be^{(2dx+2c)+b}}{e^{(2dx+2c)-1}} \right)^{\frac{2}{3}} - |b|^{\frac{2}{3}} \right)}{b^2}$$

$$4d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="giac")

[Out] 1/4*b*(2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 + abs(b)^(2/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 - 2*abs(b)^(2/3)*log(abs(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/b^2/d

$$3.13 \quad \int \frac{1}{(b \coth(c+dx))^{2/3}} dx$$

Optimal. Leaf size=218

$$\frac{\log\left(b^{2/3} - \sqrt[3]{b}\sqrt[3]{b \coth(c+dx)} + (b \coth(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\log\left(b^{2/3} + \sqrt[3]{b}\sqrt[3]{b \coth(c+dx)} + (b \coth(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

[Out] $-(\text{Sqrt}[3]*\text{ArcTan}[(1 - (2*(b*\text{Coth}[c + d*x])^{1/3})/b^{1/3})/\text{Sqrt}[3]])/(2*b^{2/3}*d) + (\text{Sqrt}[3]*\text{ArcTan}[(1 + (2*(b*\text{Coth}[c + d*x])^{1/3})/b^{1/3})/\text{Sqrt}[3]])/(2*b^{2/3}*d) + \text{ArcTanh}[(b*\text{Coth}[c + d*x])^{1/3}/b^{1/3}]/(b^{2/3}*d) - \text{Log}[b^{2/3} - b^{1/3}*(b*\text{Coth}[c + d*x])^{1/3} + (b*\text{Coth}[c + d*x])^{2/3}]/(4*b^{2/3}*d) + \text{Log}[b^{2/3} + b^{1/3}*(b*\text{Coth}[c + d*x])^{1/3} + (b*\text{Coth}[c + d*x])^{2/3}]/(4*b^{2/3}*d)$

Rubi [A] time = 0.23531, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3476, 329, 210, 634, 618, 204, 628, 206}

$$\frac{\log\left(b^{2/3} - \sqrt[3]{b}\sqrt[3]{b \coth(c+dx)} + (b \coth(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\log\left(b^{2/3} + \sqrt[3]{b}\sqrt[3]{b \coth(c+dx)} + (b \coth(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x])^{-2/3}, x]$

[Out] $-(\text{Sqrt}[3]*\text{ArcTan}[(1 - (2*(b*\text{Coth}[c + d*x])^{1/3})/b^{1/3})/\text{Sqrt}[3]])/(2*b^{2/3}*d) + (\text{Sqrt}[3]*\text{ArcTan}[(1 + (2*(b*\text{Coth}[c + d*x])^{1/3})/b^{1/3})/\text{Sqrt}[3]])/(2*b^{2/3}*d) + \text{ArcTanh}[(b*\text{Coth}[c + d*x])^{1/3}/b^{1/3}]/(b^{2/3}*d) - \text{Log}[b^{2/3} - b^{1/3}*(b*\text{Coth}[c + d*x])^{1/3} + (b*\text{Coth}[c + d*x])^{2/3}]/(4*b^{2/3}*d) + \text{Log}[b^{2/3} + b^{1/3}*(b*\text{Coth}[c + d*x])^{1/3} + (b*\text{Coth}[c + d*x])^{2/3}]/(4*b^{2/3}*d)$

Rule 3476

$\text{Int}[(b*\text{tan}[c + d*x])^n, x] := \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\text{Tan}[c + d*x]], x] /;$ $\text{FreeQ}\{b, c, d, n\}, x \&\amp; \text{IntegerQ}[n]$

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(b \coth(c + dx))^{2/3}} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{1}{x^{2/3}(-b^2+x^2)} dx, x, b \coth(c + dx)\right)}{d} \\
 &= -\frac{(3b) \operatorname{Subst}\left(\int \frac{1}{-b^2+x^6} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \frac{\sqrt[3]{b-\frac{x}{2}}}{b^{2/3}-\sqrt[3]{bx+x^2}} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{b^{2/3}d} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt[3]{b+\frac{x}{2}}}{b^{2/3}+\sqrt[3]{bx+x^2}} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{b^{2/3}d} \\
 &= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\operatorname{Subst}\left(\int \frac{-\sqrt[3]{b+2x}}{b^{2/3}-\sqrt[3]{bx+x^2}} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{4b^{2/3}d} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt[3]{b+2x}}{b^{2/3}+\sqrt[3]{bx+x^2}} dx, x, \sqrt[3]{b} \coth(c + dx)\right)}{4b^{2/3}d} \\
 &= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\log\left(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b} \coth(c + dx) + (b \coth(c + dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\log\left(b^{2/3} + \sqrt[3]{b} \sqrt[3]{b} \coth(c + dx) + (b \coth(c + dx))^{2/3}\right)}{4b^{2/3}d} \\
 &= -\frac{\sqrt{3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b} \coth(c+dx)}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\log\left(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b} \coth(c + dx) + (b \coth(c + dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\log\left(b^{2/3} + \sqrt[3]{b} \sqrt[3]{b} \coth(c + dx) + (b \coth(c + dx))^{2/3}\right)}{4b^{2/3}d}
 \end{aligned}$$

Mathematica [C] time = 0.0286256, size = 36, normalized size = 0.17

$$\frac{3 \sqrt[3]{b} \coth(c + dx) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; \coth^2(c + dx)\right)}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(-2/3), x]

[Out] (3*(b*Coth[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])/(b*d)

Maple [A] time = 0.017, size = 193, normalized size = 0.9

$$\frac{1}{2d} \ln\left(\sqrt[3]{b} \coth(dx + c) + \sqrt[3]{b}\right) b^{-\frac{2}{3}} - \frac{1}{4d} \ln\left(b^{\frac{2}{3}} - \sqrt[3]{b} \sqrt[3]{b} \coth(dx + c) + (b \coth(dx + c))^{\frac{2}{3}}\right) b^{-\frac{2}{3}} + \frac{\sqrt{3}}{2d} \arctan\left(\frac{\sqrt{3}}{3} \left(2 \sqrt[3]{b} \coth(dx + c) + \sqrt[3]{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*coth(d*x+c))^(2/3),x)`

[Out] $\frac{1}{2}b^{-2/3}/d \ln((b \operatorname{coth}(dx+c))^{1/3} + b^{1/3}) - \frac{1}{4} \ln(b^{2/3} - b^{1/3}) * (b \operatorname{coth}(dx+c))^{1/3} + (b \operatorname{coth}(dx+c))^{2/3} / b^{2/3} / d + \frac{1}{2} b^{-2/3} / d * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2 * (b \operatorname{coth}(dx+c))^{1/3} / b^{1/3} - 1)) - \frac{1}{2} b^{-2/3} / d \ln((b \operatorname{coth}(dx+c))^{1/3} - b^{1/3}) + \frac{1}{4} \ln(b^{2/3} + b^{1/3}) * (b \operatorname{coth}(dx+c))^{1/3} + (b \operatorname{coth}(dx+c))^{2/3} / b^{2/3} / d + \frac{1}{2} * \arctan(1/3 * (1 + 2 * (b \operatorname{coth}(dx+c))^{1/3} / b^{1/3})) * 3^{1/2} / b^{2/3} / d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{coth}(dx+c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c))^(2/3), x)`

Fricas [B] time = 2.01017, size = 991, normalized size = 4.55

$$2\sqrt{3}b\sqrt{-(-b^2)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{3}(-b^2)^{\frac{1}{3}}b\sqrt{-(-b^2)^{\frac{1}{3}}}-2\sqrt{3}(-b^2)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\sqrt{-(-b^2)^{\frac{1}{3}}}}{3b^2}\right)+2\sqrt{3}(b^2)^{\frac{1}{6}}b\arctan\left(-\frac{\sqrt{3}(b^2)^{\frac{1}{6}}\left((b^2)^{\frac{1}{3}}b-2(b^2)^{\frac{1}{3}}\right)}{3b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] $\frac{1}{4} * (2 * \sqrt{3} * b * \sqrt{-(-b^2)^{1/3}}) * \arctan(-1/3 * (\sqrt{3} * (-b^2)^{1/3} * b * \sqrt{-(-b^2)^{1/3}}) - 2 * \sqrt{3} * (-b^2)^{2/3} * (b * \cosh(dx+c) / \sinh(dx+c))^{1/3} * \sqrt{-(-b^2)^{1/3}}) / b^2) + 2 * \sqrt{3} * (b^2)^{1/6} * b * \arctan(-1/3 * \sqrt{3} * (b^2)^{1/6} * ((b^2)^{1/3} * b - 2 * (-b^2)^{2/3} * (b * \cosh(dx+c) / \sinh(dx+c))^{1/3}) / b^2) + (-b^2)^{2/3} * \log(b * (b * \cosh(dx+c) / \sinh(dx+c))^{2/3} -$

$$\begin{aligned} & (-b^2)^{1/3}b + (-b^2)^{2/3}(b\cosh(dx + c)/\sinh(dx + c))^{1/3} - (b^2)^{2/3}\log(b(b\cosh(dx + c)/\sinh(dx + c))^{2/3} + (b^2)^{1/3}b - (b^2)^{2/3}(b\cosh(dx + c)/\sinh(dx + c))^{1/3}) - 2(-b^2)^{2/3}\log(b(b\cosh(dx + c)/\sinh(dx + c))^{1/3} - (-b^2)^{2/3}) + 2(b^2)^{2/3}\log(b(b\cosh(dx + c)/\sinh(dx + c))^{1/3} + (b^2)^{2/3})/(b^2d) \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))**(2/3),x)

[Out] Integral((b*coth(c + d*x))**(-2/3), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.14 \quad \int \frac{1}{(b \coth(c+dx))^{4/3}} dx$$

Optimal. Leaf size=238

$$\frac{\log\left(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b \coth(c+dx)} + (b \coth(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\log\left(b^{2/3} + \sqrt[3]{b} \sqrt[3]{b \coth(c+dx)} + (b \coth(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\sqrt{3} \tan^{-1}\left(\frac{b \coth(c+dx) - b^{1/3}}{b^{2/3} - \sqrt[3]{b} \sqrt[3]{b \coth(c+dx)}}\right)}{4b^{4/3}d}$$

[Out] (Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(4/3)*d) - (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(4/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d) - 3/(b*d*(b*Coth[c + d*x])^(1/3)) - Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d)

Rubi [A] time = 0.31572, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$\frac{\log\left(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b \coth(c+dx)} + (b \coth(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\log\left(b^{2/3} + \sqrt[3]{b} \sqrt[3]{b \coth(c+dx)} + (b \coth(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\sqrt{3} \tan^{-1}\left(\frac{b \coth(c+dx) - b^{1/3}}{b^{2/3} - \sqrt[3]{b} \sqrt[3]{b \coth(c+dx)}}\right)}{4b^{4/3}d}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x])^(-4/3), x]

[Out] (Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(4/3)*d) - (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(4/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d) - 3/(b*d*(b*Coth[c + d*x])^(1/3)) - Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d)

Rule 3474

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],

x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 296

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b \coth(c + dx))^{4/3}} dx &= -\frac{3}{bd \sqrt[3]{b \coth(c + dx)}} + \frac{\int (b \coth(c + dx))^{2/3} dx}{b^2} \\
&= -\frac{3}{bd \sqrt[3]{b \coth(c + dx)}} - \frac{\text{Subst}\left(\int \frac{x^{2/3}}{-b^2 + x^2} dx, x, b \coth(c + dx)\right)}{bd} \\
&= -\frac{3}{bd \sqrt[3]{b \coth(c + dx)}} - \frac{3 \text{Subst}\left(\int \frac{x^4}{-b^2 + x^6} dx, x, \sqrt[3]{b \coth(c + dx)}\right)}{bd} \\
&= -\frac{3}{bd \sqrt[3]{b \coth(c + dx)}} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2} - \frac{x}{2}}{b^{2/3} - \sqrt[3]{bx} + x^2} dx, x, \sqrt[3]{b \coth(c + dx)}\right)}{b^{4/3}d} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2} + \frac{x}{2}}{b^{2/3} + \sqrt[3]{bx} + x^2} dx, x, \sqrt[3]{b \coth(c + dx)}\right)}{b^{4/3}d} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b \coth(c + dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd \sqrt[3]{b \coth(c + dx)}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{b} + 2x}{b^{2/3} - \sqrt[3]{bx} + x^2} dx, x, \sqrt[3]{b \coth(c + dx)}\right)}{4b^{4/3}d} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b \coth(c + dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd \sqrt[3]{b \coth(c + dx)}} - \frac{\log\left(b^{2/3} - \sqrt[3]{b} \sqrt[3]{b \coth(c + dx)} + (b \coth(c + dx))^{2/3}\right)}{4b^{4/3}d} \\
&= \frac{\sqrt{3} \tan^{-1}\left(\frac{1 - 2 \frac{\sqrt[3]{b \coth(c + dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} - \frac{\sqrt{3} \tan^{-1}\left(\frac{1 + 2 \frac{\sqrt[3]{b \coth(c + dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b \coth(c + dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd \sqrt[3]{b \coth(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 0.0619372, size = 36, normalized size = 0.15

$$-\frac{{}_3F_1\left(-\frac{1}{6}, 1; \frac{5}{6}; \coth^2(c + dx)\right)}{bd \sqrt[3]{b \coth(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(-4/3),x]

[Out] (-3*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(b*d*(b*Coth[c + d*x])^(1/3))

Maple [A] time = 0.016, size = 211, normalized size = 0.9

$$-3 \frac{1}{bd \sqrt[3]{b \coth(dx+c)}} + \frac{1}{2d} \ln \left(\sqrt[3]{b \coth(dx+c)} + \sqrt[3]{b} \right) b^{-\frac{4}{3}} - \frac{1}{4d} \ln \left(b^{\frac{2}{3}} - \sqrt[3]{b} \sqrt[3]{b \coth(dx+c)} + (b \coth(dx+c))^{\frac{2}{3}} \right) b^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c))^(4/3),x)

[Out] -3/b/d/(b*coth(d*x+c))^(1/3)+1/2/b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/4*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/3)/d-1/2/b^(4/3)/d*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1))-1/2/b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)-b^(1/3))+1/4*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/3)/d-1/2*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx+c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(4/3), x)

Fricas [B] time = 2.44882, size = 9677, normalized size = 40.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(\sqrt{3}*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh \\ & (d*x + c)^2 + b)*\sqrt{(-b)^{1/3}/b}*\log(3*b*\cosh(d*x + c)^2 + 6*b*\cosh(d*x \\ & + c)*\sinh(d*x + c) + 3*b*\sinh(d*x + c)^2 - 3*(\cosh(d*x + c)^2 + 2*\cosh(d*x \\ & + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 - 1)*(-b)^{2/3}*(b*\cosh(d*x + c)/\sinh(\\ & d*x + c))^{1/3} - \sqrt{3}*(2*(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + \\ & c) + \sinh(d*x + c)^2 - 1)*(-b)^{2/3}*(b*\cosh(d*x + c)/\sinh(d*x + c))^{2/3} \\ & + (b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 \\ & - b)*(-b)^{1/3} - (b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b* \\ & \sinh(d*x + c)^2 - b)*(b*\cosh(d*x + c)/\sinh(d*x + c))^{1/3})*\sqrt{(-b)^{1/3} \\ & /b} + b) + \sqrt{3}*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b \\ & *\sinh(d*x + c)^2 + b)*\sqrt{-1/b^{2/3}}*\log(-(2*\sqrt{3}*(\cosh(d*x + c)^2 + 2 \\ & *\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 - 1)*b^{2/3}*(b*\cosh(d*x + c \\ &)/\sinh(d*x + c))^{2/3})*\sqrt{-1/b^{2/3}}) - b*\cosh(d*x + c)^2 - 2*b*\cosh(d*x \\ & + c)*\sinh(d*x + c) - b*\sinh(d*x + c)^2 - \sqrt{3}*(b*\cosh(d*x + c)^2 + 2*b*c \\ & osh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 - b)*b^{1/3}*\sqrt{-1/b^{2/3}} \\ &) + (\sqrt{3}*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(\\ & d*x + c)^2 - b)*\sqrt{-1/b^{2/3}}) + 3*(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sin \\ & h(d*x + c) + \sinh(d*x + c)^2 - 1)*b^{2/3}*(b*\cosh(d*x + c)/\sinh(d*x + c))^{ \\ & (1/3) - 3*b)/(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + \\ & c)^2)) + (\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 \\ & + 1)*(-b)^{2/3}*\log((-b)^{2/3} - (-b)^{1/3}*(b*\cosh(d*x + c)/\sinh(d*x + c) \\ &)^{1/3} + (b*\cosh(d*x + c)/\sinh(d*x + c))^{2/3}) - (\cosh(d*x + c)^2 + 2*cos \\ & h(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1)*b^{2/3}*\log(b^{2/3} - b^{1/ \\ & 3}*(b*\cosh(d*x + c)/\sinh(d*x + c))^{1/3} + (b*\cosh(d*x + c)/\sinh(d*x + c))^{ \\ & (2/3)) - 2*(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c) \\ & ^2 + 1)*(-b)^{2/3}*\log((-b)^{1/3} + (b*\cosh(d*x + c)/\sinh(d*x + c))^{1/3}) \\ & + 2*(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1) \\ & *b^{2/3}*\log(b^{1/3} + (b*\cosh(d*x + c)/\sinh(d*x + c))^{1/3}) - 12*(\cosh(d* \\ & x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 - 1)*(b*\cosh(d*x \\ & + c)/\sinh(d*x + c))^{2/3})/(b^2*d*\cosh(d*x + c)^2 + 2*b^2*d*\cosh(d*x + c)* \\ & \sinh(d*x + c) + b^2*d*\sinh(d*x + c)^2 + b^2*d), -1/4*(2*\sqrt{3}*(b*\cosh(d*x \\ & + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + b)*\sqrt{-(- \\ & b)^{1/3}/b}*\arctan(-1/3*\sqrt{3}*(-b)^{1/3}*\sqrt{-(-b)^{1/3}/b} + 2/3*\sqrt{3} \\ &)*(b*\cosh(d*x + c)/\sinh(d*x + c))^{1/3}*\sqrt{-(-b)^{1/3}/b}) - \sqrt{3}*(b*c \\ & osh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + b)*s \\ & \sqrt{-1/b^{2/3}}*\log(-(2*\sqrt{3}*(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x \\ & + c) + \sinh(d*x + c)^2 - 1)*b^{2/3}*(b*\cosh(d*x + c)/\sinh(d*x + c))^{2/3})* \\ & \sqrt{-1/b^{2/3}}) - b*\cosh(d*x + c)^2 - 2*b*\cosh(d*x + c)*\sinh(d*x + c) - b* \\ & \sinh(d*x + c)^2 - \sqrt{3}*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + \\ & c) + b*\sinh(d*x + c)^2 - b)*b^{1/3}*\sqrt{-1/b^{2/3}}) + (\sqrt{3}*(b*\cosh(d* \end{aligned}$$

$$\begin{aligned}
& x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1 \\
& /b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + \\
& c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x \\
& + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - (cosh(d*x + c) \\
& ^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((- \\
& b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + \\
& c)/sinh(d*x + c))^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c \\
&) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sin \\
& h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x + \\
& c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log(\\
& (-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 + \\
& 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + \\
& (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + \\
& c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2 \\
& /3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*s \\
& inh(d*x + c)^2 + b^2*d), 1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c \\
&)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d* \\
& x + c)^2 + 6*b*cosh(d*x + c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(\\
& d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3 \\
&)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*c \\
& osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + \\
& c)/sinh(d*x + c))^(2/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + \\
& c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x \\
& + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c \\
&))^(1/3))*sqrt((-b)^(1/3)/b) + b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(\\
& d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*c \\
& osh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) \\
& - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b \\
& ^2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cos \\
& h(d*x + c)/sinh(d*x + c))^(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin \\
& h(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + \\
& c)/sinh(d*x + c))^(1/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + \\
& c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x \\
& + c))^(1/3)) - 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + \\
& c) + b*sinh(d*x + c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x + \\
& c)/sinh(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 12*(cosh(d*x + c)^2 + 2*cosh(d* \\
& x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c)) \\
& ^2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d \\
& *sinh(d*x + c)^2 + b^2*d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d \\
& *x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)/b)*arctan(- \\
& 1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cosh(d*x + c)/s \\
& inh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + \\
& c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1 \\
& /3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c)) \\
& ^2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
\end{aligned}$$

```

2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
+ (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cos
sh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/s
inh(d*x + c))^(1/3)) + 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cos
h(d*x + c)/sinh(d*x + c))^(1/3))/b^(1/3))/b^(1/3) + 12*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d
*x + c))^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c
) + b^2*d*sinh(d*x + c)^2 + b^2*d)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))**(4/3),x)

[Out] Integral((b*coth(c + d*x))**(-4/3), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="giac")

[Out] Exception raised: TypeError

3.15 $\int \coth^n(a + bx) dx$

Optimal. Leaf size=43

$$\frac{\coth^{n+1}(a + bx) {}_2F_1\left(1, \frac{n+1}{2}; \frac{n+3}{2}; \coth^2(a + bx)\right)}{b(n+1)}$$

[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a + b*x]^2])/(b*(1 + n))

Rubi [A] time = 0.0223006, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3476, 364}

$$\frac{\coth^{n+1}(a + bx) {}_2F_1\left(1, \frac{n+1}{2}; \frac{n+3}{2}; \coth^2(a + bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[Coth[a + b*x]^n, x]

[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a + b*x]^2])/(b*(1 + n))

Rule 3476

Int[((c_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\int \coth^n(a + bx) dx = -\frac{\text{Subst}\left(\int \frac{x^n}{-1+x^2} dx, x, \coth(a + bx)\right)}{b}$$

$$= \frac{\coth^{1+n}(a + bx) {}_2F_1\left(1, \frac{1+n}{2}; \frac{3+n}{2}; \coth^2(a + bx)\right)}{b(1+n)}$$

Mathematica [A] time = 0.0625768, size = 45, normalized size = 1.05

$$\frac{\coth^{n+1}(a + bx) {}_2F_1\left(1, \frac{n+1}{2}; \frac{n+1}{2} + 1; \coth^2(a + bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + b*x]^n, x]

[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, Coth[a + b*x]^2])/(b*(1 + n))

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int (\coth(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(b*x+a)^n, x)

[Out] int(coth(b*x+a)^n, x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(bx + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^n,x, algorithm="maxima")

[Out] integrate(coth(b*x + a)^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{coth}(bx + a)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^n,x, algorithm="fricas")

[Out] integral(coth(b*x + a)^n, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{coth}^n(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)**n,x)

[Out] Integral(coth(a + b*x)**n, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{coth}(bx + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^n,x, algorithm="giac")

[Out] integrate(coth(b*x + a)^n, x)

3.16 $\int (b \coth(c + dx))^n dx$

Optimal. Leaf size=48

$$\frac{(b \coth(c + dx))^{n+1} {}_2F_1\left(1, \frac{n+1}{2}; \frac{n+3}{2}; \coth^2(c + dx)\right)}{bd(n+1)}$$

[Out] ((b*Coth[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[c + d*x]^2])/(b*d*(1 + n))

Rubi [A] time = 0.0270079, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3476, 364}

$$\frac{(b \coth(c + dx))^{n+1} {}_2F_1\left(1, \frac{n+1}{2}; \frac{n+3}{2}; \coth^2(c + dx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x])^n,x]

[Out] ((b*Coth[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[c + d*x]^2])/(b*d*(1 + n))

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\int (b \coth(c + dx))^n dx = -\frac{b \operatorname{Subst}\left(\int \frac{x^n}{-b^2+x^2} dx, x, b \coth(c + dx)\right)}{d}$$

$$= \frac{(b \coth(c + dx))^{1+n} {}_2F_1\left(1, \frac{1+n}{2}; \frac{3+n}{2}; \coth^2(c + dx)\right)}{bd(1+n)}$$

Mathematica [A] time = 0.041099, size = 51, normalized size = 1.06

$$\frac{\coth(c + dx)(b \coth(c + dx))^n {}_2F_1\left(1, \frac{n+1}{2}; \frac{n+1}{2} + 1; \coth^2(c + dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^n,x]

[Out] (Coth[c + d*x]*(b*Coth[c + d*x])^n*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, Coth[c + d*x]^2])/(d*(1 + n))

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int (b \coth(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c))^n,x)

[Out] int((b*coth(d*x+c))^n,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \coth(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**n,x)
```

```
[Out] Integral((b*coth(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c))^n, x)
```

3.17 $\int (b \coth^2(c + dx))^n dx$

Optimal. Leaf size=57

$$\frac{\coth(c + dx) (b \coth^2(c + dx))^n {}_2F_1\left(1, \frac{1}{2}(2n + 1); \frac{1}{2}(2n + 3); \coth^2(c + dx)\right)}{d(2n + 1)}$$

[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, (1 + 2*n)/2, (3 + 2*n)/2, Coth[c + d*x]^2])/(d*(1 + 2*n))

Rubi [A] time = 0.040331, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3658, 3476, 364}

$$\frac{\coth(c + dx) (b \coth^2(c + dx))^n {}_2F_1\left(1, \frac{1}{2}(2n + 1); \frac{1}{2}(2n + 3); \coth^2(c + dx)\right)}{d(2n + 1)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^2)^n,x]

[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, (1 + 2*n)/2, (3 + 2*n)/2, Coth[c + d*x]^2])/(d*(1 + 2*n))

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (b \coth^2(c + dx))^n dx &= \left(\coth^{-2n}(c + dx) (b \coth^2(c + dx))^n \right) \int \coth^{2n}(c + dx) dx \\ &= - \frac{\left(\coth^{-2n}(c + dx) (b \coth^2(c + dx))^n \right) \text{Subst} \left(\int \frac{x^{2n}}{-1+x^2} dx, x, \coth(c + dx) \right)}{d} \\ &= \frac{\coth(c + dx) (b \coth^2(c + dx))^n {}_2F_1 \left(1, \frac{1}{2}(1 + 2n); \frac{1}{2}(3 + 2n); \coth^2(c + dx) \right)}{d(1 + 2n)} \end{aligned}$$

Mathematica [A] time = 0.0478515, size = 47, normalized size = 0.82

$$\frac{\coth(c + dx) (b \coth^2(c + dx))^n {}_2F_1 \left(1, n + \frac{1}{2}; n + \frac{3}{2}; \coth^2(c + dx) \right)}{2dn + d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, 1/2 + n, 3/2 + n,
Coth[c + d*x]^2])/(d + 2*d*n)
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int (b (\coth(dx + c))^2)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^n,x)
```

```
[Out] int((b*coth(d*x+c)^2)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^2)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^2)^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \coth(dx + c)^2\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="fricas")

[Out] integral((b*coth(d*x + c)^2)^n, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth^2(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**2)**n,x)

[Out] Integral((b*coth(c + d*x)**2)**n, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^2)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^n, x)
```

3.18 $\int (b \coth^2(c + dx))^{3/2} dx$

Optimal. Leaf size=61

$$\frac{b \tanh(c + dx) \sqrt{b \coth^2(c + dx) \log(\sinh(c + dx))}}{d} - \frac{b \coth(c + dx) \sqrt{b \coth^2(c + dx)}}{2d}$$

[Out] $-(b \coth[c + d*x] \sqrt{b \coth[c + d*x]^2}) / (2*d) + (b \sqrt{b \coth[c + d*x]^2} \log[\sinh[c + d*x]] \tanh[c + d*x]) / d$

Rubi [A] time = 0.0368725, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 3475}

$$\frac{b \tanh(c + dx) \sqrt{b \coth^2(c + dx) \log(\sinh(c + dx))}}{d} - \frac{b \coth(c + dx) \sqrt{b \coth^2(c + dx)}}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b \coth[c + d*x]^2)^{(3/2)}, x]$

[Out] $-(b \coth[c + d*x] \sqrt{b \coth[c + d*x]^2}) / (2*d) + (b \sqrt{b \coth[c + d*x]^2} \log[\sinh[c + d*x]] \tanh[c + d*x]) / d$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \coth^2(c + dx))^{3/2} dx &= \left(b \sqrt{b \coth^2(c + dx) \tanh(c + dx)} \right) \int \coth^3(c + dx) dx \\ &= -\frac{b \coth(c + dx) \sqrt{b \coth^2(c + dx)}}{2d} + \left(b \sqrt{b \coth^2(c + dx) \tanh(c + dx)} \right) \int \coth(c + dx) dx \\ &= -\frac{b \coth(c + dx) \sqrt{b \coth^2(c + dx)}}{2d} + \frac{b \sqrt{b \coth^2(c + dx)} \log(\sinh(c + dx)) \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.118465, size = 56, normalized size = 0.92

$$\frac{\tanh^3(c + dx) (b \coth^2(c + dx))^{3/2} (\coth^2(c + dx) - 2 \log(\tanh(c + dx)) - 2 \log(\cosh(c + dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(3/2), x]
```

```
[Out] -((b*Coth[c + d*x]^2)^(3/2)*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] - 2*Log
[Tanh[c + d*x]])*Tanh[c + d*x]^3)/(2*d)
```

Maple [A] time = 0.026, size = 53, normalized size = 0.9

$$-\frac{(\coth(dx + c))^2 + \ln(\coth(dx + c) - 1) + \ln(\coth(dx + c) + 1)}{2d(\coth(dx + c))^3} (b(\coth(dx + c))^2)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(3/2), x)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^2)^(3/2)*(coth(d*x+c)^2+ln(coth(d*x+c)-1)+ln(coth(d*x
+c)+1))/coth(d*x+c)^3
```

Maxima [A] time = 1.60448, size = 131, normalized size = 2.15

$$-\frac{(dx+c)b^{\frac{3}{2}}}{d} - \frac{b^{\frac{3}{2}} \log(e^{-dx-c} + 1)}{d} - \frac{b^{\frac{3}{2}} \log(e^{-dx-c} - 1)}{d} - \frac{2b^{\frac{3}{2}} e^{-2dx-2c}}{d(2e^{-2dx-2c} - e^{-4dx-4c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")

[Out] $-(d*x + c)*b^{(3/2)}/d - b^{(3/2)}*\log(e^{-d*x - c} + 1)/d - b^{(3/2)}*\log(e^{-d*x - c} - 1)/d - 2*b^{(3/2)}*e^{-2*d*x - 2*c}/(d*(2*e^{-2*d*x - 2*c} - e^{-4*d*x - 4*c} - 1))$

Fricas [B] time = 1.98088, size = 2068, normalized size = 33.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")

[Out] $(b*d*x*\cosh(d*x + c)^4 - (b*d*x*e^{(2*d*x + 2*c)} - b*d*x)*\sinh(d*x + c)^4 - 4*(b*d*x*\cosh(d*x + c)*e^{(2*d*x + 2*c)} - b*d*x*\cosh(d*x + c))*\sinh(d*x + c)^3 + b*d*x - 2*(b*d*x - b)*\cosh(d*x + c)^2 + 2*(3*b*d*x*\cosh(d*x + c)^2 - b*d*x - (3*b*d*x*\cosh(d*x + c)^2 - b*d*x + b)*e^{(2*d*x + 2*c)} + b)*\sinh(d*x + c)^2 - (b*d*x*\cosh(d*x + c)^4 + b*d*x - 2*(b*d*x - b)*\cosh(d*x + c)^2)*e^{(2*d*x + 2*c)} - (b*\cosh(d*x + c)^4 - (b*e^{(2*d*x + 2*c)} - b)*\sinh(d*x + c)^4 - 4*(b*\cosh(d*x + c)*e^{(2*d*x + 2*c)} - b*\cosh(d*x + c))*\sinh(d*x + c)^3 - 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - (3*b*\cosh(d*x + c)^2 - b)*e^{(2*d*x + 2*c)} - b)*\sinh(d*x + c)^2 - (b*\cosh(d*x + c)^4 - 2*b*\cosh(d*x + c)^2 + b)*e^{(2*d*x + 2*c)} + 4*(b*\cosh(d*x + c)^3 - b*\cosh(d*x + c) - (b*\cosh(d*x + c)^3 - b*\cosh(d*x + c))*e^{(2*d*x + 2*c)})*\sinh(d*x + c) + b)*\log(2*\sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b*d*x*\cosh(d*x + c)^3 - (b*d*x - b)*\cosh(d*x + c) - (b*d*x*\cosh(d*x + c)^3 - (b*d*x - b)*\cosh(d*x + c))*e^{(2*d*x + 2*c)})*\sinh(d*x + c))*\sqrt{(b*e^{(4*d*x + 4*c)} + 2*b*e^{(2*d*x + 2*c)} + b)/(e^{(4*d*x + 4*c)} - 2*e^{(2*d*x + 2*c)} + 1)}/(d*\cosh(d*x + c)^4 + (d*e^{(2*d*x + 2*c)} + d)*\sinh(d*x + c)^4 + 4*(d*\cosh(d*x + c)*e^{(2*d*x + 2*c)} + d*\cosh(d*x + c))*\sinh(d*x + c)^3 - 2*d*\cosh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^2 + (3*d*\cosh(d*x + c)^2 - d)*e^{(2*d*x + 2*c)} - d)*\sinh(d*x + c)^2$

+ (d*cosh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c) + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth^2(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**2)**(3/2),x)

[Out] Integral((b*coth(c + d*x)**2)**(3/2), x)

Giac [A] time = 1.20253, size = 122, normalized size = 2.

$$\frac{\left((dx + c) \operatorname{sgn}(e^{4dx+4c} - 1) - \log(|e^{2dx+2c} - 1|) \operatorname{sgn}(e^{4dx+4c} - 1) + \frac{2e^{(2dx+2c)\operatorname{sgn}(e^{4dx+4c}-1)}}{(e^{2dx+2c}-1)^2} \right) b^{\frac{3}{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")

[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^(4*d*x + 4*c) - 1) + 2*e^(2*d*x + 2*c)*sgn(e^(4*d*x + 4*c) - 1)/(e^(2*d*x + 2*c) - 1)^2)*b^(3/2)/d

3.19 $\int \sqrt{b \coth^2(c + dx)} dx$

Optimal. Leaf size=31

$$\frac{\tanh(c + dx)\sqrt{b \coth^2(c + dx)} \log(\sinh(c + dx))}{d}$$

[Out] (Sqrt[b*Coth[c + d*x]^2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d

Rubi [A] time = 0.0196248, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3658, 3475}

$$\frac{\tanh(c + dx)\sqrt{b \coth^2(c + dx)} \log(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Coth[c + d*x]^2], x]

[Out] (Sqrt[b*Coth[c + d*x]^2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b \coth^2(c + dx)} dx = \left(\sqrt{b \coth^2(c + dx) \tanh(c + dx)} \right) \int \coth(c + dx) dx$$

$$= \frac{\sqrt{b \coth^2(c + dx) \log(\sinh(c + dx)) \tanh(c + dx)}}{d}$$

Mathematica [A] time = 0.0452803, size = 39, normalized size = 1.26

$$\frac{\tanh(c + dx) \sqrt{b \coth^2(c + dx) (\log(\tanh(c + dx)) + \log(\cosh(c + dx)))}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Coth[c + d*x]^2], x]

[Out] (Sqrt[b*Coth[c + d*x]^2]*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c + d*x])/d

Maple [A] time = 0.036, size = 45, normalized size = 1.5

$$\frac{\ln(\coth(dx + c) - 1) + \ln(\coth(dx + c) + 1)}{2d \coth(dx + c)} \sqrt{b(\coth(dx + c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^2)^(1/2), x)

[Out] -1/2/d*(b*coth(d*x+c)^2)^(1/2)*(ln(coth(d*x+c)-1)+ln(coth(d*x+c)+1))/coth(d*x+c)

Maxima [A] time = 1.59022, size = 73, normalized size = 2.35

$$\frac{(dx + c)\sqrt{b}}{d} - \frac{\sqrt{b} \log(e^{-dx-c} + 1)}{d} - \frac{\sqrt{b} \log(e^{-dx-c} - 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")

[Out] $-(d*x + c)*\sqrt{b}/d - \sqrt{b}*\log(e^{-d*x - c} + 1)/d - \sqrt{b}*\log(e^{-d*x - c} - 1)/d$

Fricas [B] time = 1.93362, size = 304, normalized size = 9.81

$$\frac{\left(dx e^{(2dx+2c)} - dx - \left(e^{(2dx+2c)} - 1\right) \log\left(\frac{2 \sinh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)\right) \sqrt{\frac{be^{(4dx+4c)} + 2be^{(2dx+2c)} + b}{e^{(4dx+4c)} - 2e^{(2dx+2c)} + 1}}}{de^{(2dx+2c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")

[Out] $-(d*x*e^{(2*d*x + 2*c)} - d*x - (e^{(2*d*x + 2*c)} - 1)*\log(2*\sinh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))))*\sqrt{((b*e^{(4*d*x + 4*c)} + 2*b*e^{(2*d*x + 2*c)} + b)/(e^{(4*d*x + 4*c)} - 2*e^{(2*d*x + 2*c)} + 1))}/(d*e^{(2*d*x + 2*c)} + d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**2)**(1/2),x)

[Out] Integral(sqrt(b*coth(c + d*x)**2), x)

Giac [A] time = 1.15947, size = 73, normalized size = 2.35

$$\frac{\left((dx + c)\operatorname{sgn}\left(e^{(4dx+4c)} - 1\right) - \log\left(\left|e^{(2dx+2c)} - 1\right|\right)\operatorname{sgn}\left(e^{(4dx+4c)} - 1\right)\right)\sqrt{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^(4*d*x + 4*c) - 1))*sqrt(b)/d
```

$$3.20 \quad \int \frac{1}{\sqrt{b \coth^2(c+dx)}} dx$$

Optimal. Leaf size=31

$$\frac{\coth(c+dx) \log(\cosh(c+dx))}{d\sqrt{b \coth^2(c+dx)}}$$

[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])

Rubi [A] time = 0.0204262, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3658, 3475}

$$\frac{\coth(c+dx) \log(\cosh(c+dx))}{d\sqrt{b \coth^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*Coth[c + d*x]^2],x]

[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b \coth^2(c + dx)}} dx = \frac{\coth(c + dx) \int \tanh(c + dx) dx}{\sqrt{b \coth^2(c + dx)}} \\ = \frac{\coth(c + dx) \log(\cosh(c + dx))}{d\sqrt{b \coth^2(c + dx)}}$$

Mathematica [A] time = 0.0594719, size = 31, normalized size = 1.

$$\frac{\coth(c + dx) \log(\cosh(c + dx))}{d\sqrt{b \coth^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*Coth[c + d*x]^2],x]

[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])

Maple [A] time = 0.04, size = 52, normalized size = 1.7

$$\frac{\coth(dx + c) (\ln(\coth(dx + c) + 1) - 2 \ln(\coth(dx + c)) + \ln(\coth(dx + c) - 1))}{2d} \frac{1}{\sqrt{b(\coth(dx + c))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c)^2)^(1/2),x)

[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)+1)-2*ln(coth(d*x+c))+ln(coth(d*x+c)-1))/ (b*coth(d*x+c)^2)^(1/2)

Maxima [A] time = 1.62442, size = 46, normalized size = 1.48

$$\frac{dx + c}{\sqrt{bd}} - \frac{\log(e^{-2dx-2c} + 1)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")

[Out] -(d*x + c)/(sqrt(b)*d) - log(e^(-2*d*x - 2*c) + 1)/(sqrt(b)*d)

Fricas [B] time = 1.92024, size = 309, normalized size = 9.97

$$\frac{\left(dx e^{(2dx+2c)} - dx - \left(e^{(2dx+2c)} - 1\right) \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)\right) \sqrt{\frac{b e^{(4dx+4c)} + 2 b e^{(2dx+2c)} + b}{e^{(4dx+4c)} - 2 e^{(2dx+2c)} + 1}}}{b d e^{(2dx+2c)} + b d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")

[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b*d*e^(2*d*x + 2*c) + b*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth^2(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**2)**(1/2),x)

[Out] Integral(1/sqrt(b*coth(c + d*x)**2), x)

Giac [B] time = 1.19993, size = 81, normalized size = 2.61

$$\frac{\frac{dx+c}{\sqrt{b \operatorname{sgn}(e^{(4dx+4c)}-1)}} - \frac{\log(e^{(2dx+2c)}+1)}{\sqrt{b \operatorname{sgn}(e^{(4dx+4c)}-1)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)))/d
```

$$3.21 \quad \int \frac{1}{\left(b \coth^2(c+dx)\right)^{3/2}} dx$$

Optimal. Leaf size=65

$$\frac{\coth(c+dx) \log(\cosh(c+dx))}{bd\sqrt{b \coth^2(c+dx)}} - \frac{\tanh(c+dx)}{2bd\sqrt{b \coth^2(c+dx)}}$$

[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(b*d*Sqrt[b*Coth[c + d*x]^2]) - Tanh[c + d*x]/(2*b*d*Sqrt[b*Coth[c + d*x]^2])

Rubi [A] time = 0.0356045, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 3475}

$$\frac{\coth(c+dx) \log(\cosh(c+dx))}{bd\sqrt{b \coth^2(c+dx)}} - \frac{\tanh(c+dx)}{2bd\sqrt{b \coth^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^2)^(-3/2), x]

[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(b*d*Sqrt[b*Coth[c + d*x]^2]) - Tanh[c + d*x]/(2*b*d*Sqrt[b*Coth[c + d*x]^2])

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(b \coth^2(c + dx))^{3/2}} dx &= \frac{\coth(c + dx) \int \tanh^3(c + dx) dx}{b \sqrt{b \coth^2(c + dx)}} \\ &= -\frac{\tanh(c + dx)}{2bd \sqrt{b \coth^2(c + dx)}} + \frac{\coth(c + dx) \int \tanh(c + dx) dx}{b \sqrt{b \coth^2(c + dx)}} \\ &= \frac{\coth(c + dx) \log(\cosh(c + dx))}{bd \sqrt{b \coth^2(c + dx)}} - \frac{\tanh(c + dx)}{2bd \sqrt{b \coth^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.143311, size = 48, normalized size = 0.74

$$\frac{2 \coth(c + dx) \log(\cosh(c + dx)) - \tanh(c + dx)}{2bd \sqrt{b \coth^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-3/2), x]
```

```
[Out] (2*Coth[c + d*x]*Log[Cosh[c + d*x]] - Tanh[c + d*x])/(2*b*d*Sqrt[b*Coth[c +
d*x]^2])
```

Maple [A] time = 0.02, size = 79, normalized size = 1.2

$$\frac{\coth(dx + c) \left(\ln(\coth(dx + c) + 1) (\coth(dx + c))^2 - 2 \ln(\coth(dx + c)) (\coth(dx + c))^2 + \ln(\coth(dx + c) - 1) (\coth(dx + c))^2 \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(3/2), x)
```

[Out] $-1/2/d*\coth(d*x+c)*(ln(\coth(d*x+c)+1)*\coth(d*x+c)^2-2*ln(\coth(d*x+c))*\coth(d*x+c)^2+ln(\coth(d*x+c)-1)*\coth(d*x+c)^2+1)/(b*\coth(d*x+c)^2)^{(3/2)}$

Maxima [A] time = 1.58899, size = 113, normalized size = 1.74

$$\frac{2\sqrt{b}e^{(-2dx-2c)}}{(2b^2e^{(-2dx-2c)} + b^2e^{(-4dx-4c)} + b^2)d} - \frac{dx+c}{b^{\frac{3}{2}}d} - \frac{\log(e^{(-2dx-2c)} + 1)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")`

[Out] $-2*\sqrt{b}*e^{(-2*d*x - 2*c)/((2*b^2*e^{(-2*d*x - 2*c)} + b^2*e^{(-4*d*x - 4*c)} + b^2)*d)} - (d*x + c)/(b^{(3/2)*d)} - \log(e^{(-2*d*x - 2*c)} + 1)/(b^{(3/2)*d)}$

Fricas [B] time = 1.96297, size = 2082, normalized size = 32.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")`

[Out] $(d*x*\cosh(d*x + c)^4 - (d*x*e^{(2*d*x + 2*c)} - d*x)*\sinh(d*x + c)^4 - 4*(d*x*\cosh(d*x + c)*e^{(2*d*x + 2*c)} - d*x*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(d*x - 1)*\cosh(d*x + c)^2 + 2*(3*d*x*\cosh(d*x + c)^2 + d*x - (3*d*x*\cosh(d*x + c)^2 + d*x - 1)*e^{(2*d*x + 2*c)} - 1)*\sinh(d*x + c)^2 + d*x - (d*x*\cosh(d*x + c)^4 + 2*(d*x - 1)*\cosh(d*x + c)^2 + d*x)*e^{(2*d*x + 2*c)} + ((e^{(2*d*x + 2*c)} - 1)*\sinh(d*x + c)^4 - \cosh(d*x + c)^4 + 4*(\cosh(d*x + c)*e^{(2*d*x + 2*c)} - \cosh(d*x + c))*\sinh(d*x + c)^3 - 2*(3*\cosh(d*x + c)^2 - (3*\cosh(d*x + c)^2 + 1)*e^{(2*d*x + 2*c)} + 1)*\sinh(d*x + c)^2 - 2*\cosh(d*x + c)^2 + (\cosh(d*x + c)^4 + 2*\cosh(d*x + c)^2 + 1)*e^{(2*d*x + 2*c)} - 4*(\cosh(d*x + c)^3 - (\cosh(d*x + c)^3 + \cosh(d*x + c))*e^{(2*d*x + 2*c)} + \cosh(d*x + c))*\sinh(d*x + c) - 1)*\log(2*\cosh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 4*(d*x*\cosh(d*x + c)^3 + (d*x - 1)*\cosh(d*x + c) - (d*x*\cosh(d*x + c)^3 + (d*x - 1)*\cosh(d*x + c))*e^{(2*d*x + 2*c)})*\sinh(d*x + c))*\sqrt{(b*e^{(4*d*x + 4*c)} + 2*b*e^{(2*d*x + 2*c)} + b)/(e^{(4*d*x + 4*c)} - 2*e^{(2*d*x + 2*c)} + 1))/(b^2*d*\cosh(d*x + c)^4 + 2*b^2*d*\cosh(d*x + c)^2 + (b^2*d*e^{(2*d*x + 2*c)} + b^2*d)*\sinh(d*x + c)^4 + 4*(b^2*d*\cosh(d*x + c)*e^{(2*d*x + 2*c)} + b^2*d*\cosh(d*x$

+ c))*sinh(d*x + c)^3 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d + (3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c) + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^2(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**2)**(3/2),x)

[Out] Integral((b*coth(c + d*x)**2)**(-3/2), x)

Giac [A] time = 1.2425, size = 149, normalized size = 2.29

$$\frac{\frac{dx+c}{\sqrt{bd}\operatorname{sgn}(e^{4dx+4c}-1)} - \frac{\log(e^{2dx+2c}+1)}{\sqrt{bd}\operatorname{sgn}(e^{4dx+4c}-1)} - \frac{2e^{2dx+2c}}{\sqrt{bd}(e^{2dx+2c}+1)^2\operatorname{sgn}(e^{4dx+4c}-1)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")

[Out] -((d*x + c)/(sqrt(b)*d*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/(sqrt(b)*d*sgn(e^(4*d*x + 4*c) - 1)) - 2*e^(2*d*x + 2*c)/(sqrt(b)*d*(e^(2*d*x + 2*c) + 1)^2*sgn(e^(4*d*x + 4*c) - 1)))/b

3.22 $\int (b \coth^2(c + dx))^{4/3} dx$

Optimal. Leaf size=297

$$\frac{3b \coth(c + dx) \sqrt[3]{b \coth^2(c + dx)}}{5d} - \frac{b \sqrt[3]{b \coth^2(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{\coth(c + dx) + 1}\right)}{4d \coth^{\frac{2}{3}}(c + dx)} + \frac{b \sqrt[3]{b \coth^2(c + dx)}}{5d}$$

[Out] (Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (b*ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^2)^(1/3))/(5*d) - (b*(b*Coth[c + d*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3)) + (b*(b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))

Rubi [A] time = 0.284479, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3473, 3476, 329, 296, 634, 618, 204, 628, 206}

$$\frac{3b \coth(c + dx) \sqrt[3]{b \coth^2(c + dx)}}{5d} - \frac{b \sqrt[3]{b \coth^2(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{\coth(c + dx) + 1}\right)}{4d \coth^{\frac{2}{3}}(c + dx)} + \frac{b \sqrt[3]{b \coth^2(c + dx)}}{5d}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^2)^(4/3), x]

[Out] (Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (b*ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^2)^(1/3))/(5*d) - (b*(b*Coth[c + d*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3)) + (b*(b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))

Rule 3658

Int[(u_)*((b_)*tan[(e_.) + (f_)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p])*(b*Tan[e + f*x]^

```
n)^FracPart[p]]/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618


```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (b \coth^2(c + dx))^{4/3} dx &= \frac{\left(b \sqrt[3]{b \coth^2(c + dx)}\right) \int \coth^{8/3}(c + dx) dx}{\coth^{2/3}(c + dx)} \\
&= -\frac{3b \coth(c + dx) \sqrt[3]{b \coth^2(c + dx)}}{5d} + \frac{\left(b \sqrt[3]{b \coth^2(c + dx)}\right) \int \coth^{2/3}(c + dx) dx}{\coth^{2/3}(c + dx)} \\
&= -\frac{3b \coth(c + dx) \sqrt[3]{b \coth^2(c + dx)}}{5d} - \frac{\left(b \sqrt[3]{b \coth^2(c + dx)}\right) \text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2} dx, x, \coth(c + dx)\right)}{d \coth^{2/3}(c + dx)} \\
&= -\frac{3b \coth(c + dx) \sqrt[3]{b \coth^2(c + dx)}}{5d} - \frac{\left(3b \sqrt[3]{b \coth^2(c + dx)}\right) \text{Subst}\left(\int \frac{x^4}{-1+x^6} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d \coth^{2/3}(c + dx)} \\
&= -\frac{3b \coth(c + dx) \sqrt[3]{b \coth^2(c + dx)}}{5d} + \frac{\left(b \sqrt[3]{b \coth^2(c + dx)}\right) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d \coth^{2/3}(c + dx)} \\
&= \frac{b \tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) \sqrt[3]{b \coth^2(c + dx)}}{d \coth^{2/3}(c + dx)} - \frac{3b \coth(c + dx) \sqrt[3]{b \coth^2(c + dx)}}{5d} - \frac{\left(b \sqrt[3]{b \coth^2(c + dx)}\right) \tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right)}{d \coth^{2/3}(c + dx)} \\
&= \frac{b \tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) \sqrt[3]{b \coth^2(c + dx)}}{d \coth^{2/3}(c + dx)} - \frac{3b \coth(c + dx) \sqrt[3]{b \coth^2(c + dx)}}{5d} - \frac{b \sqrt[3]{b \coth^2(c + dx)} \tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right)}{d \coth^{2/3}(c + dx)} \\
&= \frac{\sqrt{3}b \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^2(c + dx)}}{2d \coth^{2/3}(c + dx)} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^2(c + dx)}}{2d \coth^{2/3}(c + dx)}
\end{aligned}$$

Mathematica [A] time = 0.338393, size = 166, normalized size = 0.56

$$\frac{(b \coth^2(c + dx))^{4/3} \left(12 \coth^{5/3}(c + dx) - 20 \tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) - 5 \left(-\log\left(\coth^{2/3}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1\right) + \log\left(\coth^{2/3}(c + dx) + \sqrt[3]{\coth(c + dx)} + 1\right)\right)\right)}{20d \coth^{8/3}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^2)^(4/3), x]

[Out] -((b*Coth[c + d*x]^2)^(4/3)*(-20*ArcTanh[Coth[c + d*x]^(1/3)] + 12*Coth[c + d*x]^(5/3) - 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*

$\text{Sqrt}[3] \cdot \text{ArcTan}[(1 + 2 \cdot \text{Coth}[c + d \cdot x]^{1/3}) / \text{Sqrt}[3]] - \text{Log}[1 - \text{Coth}[c + d \cdot x]^{1/3} + \text{Coth}[c + d \cdot x]^{2/3}] + \text{Log}[1 + \text{Coth}[c + d \cdot x]^{1/3} + \text{Coth}[c + d \cdot x]^{2/3}] / (20 \cdot d \cdot \text{Coth}[c + d \cdot x]^{8/3})$

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int (b (\coth(dx + c))^2)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^2)^(4/3),x)

[Out] int((b*coth(d*x+c)^2)^(4/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^2)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)

Fricas [B] time = 2.20668, size = 5673, normalized size = 19.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")

[Out] $-1/20 \cdot (10 \cdot (\sqrt{3}) \cdot b \cdot \cosh(dx + c)^2 + 2 \cdot \sqrt{3}) \cdot b \cdot \cosh(dx + c) \cdot \sinh(dx + c) + \sqrt{3} \cdot b \cdot \sinh(dx + c)^2 - \sqrt{3} \cdot b \cdot (-b)^{1/3} \cdot \arctan(1/3 \cdot (\sqrt{3}) \cdot b \cdot \cosh(dx + c)^2 + 2 \cdot \sqrt{3}) \cdot b \cdot \cosh(dx + c) \cdot \sinh(dx + c) + \sqrt{3} \cdot b \cdot \sinh(dx + c)^2$

$$\begin{aligned}
& \text{nh}(d*x + c)^2 + 2*(\text{sqrt}(3)*\text{cosh}(d*x + c)^2 + 2*\text{sqrt}(3)*\text{cosh}(d*x + c)*\text{sinh}(d \\
& *x + c) + \text{sqrt}(3)*\text{sinh}(d*x + c)^2 - \text{sqrt}(3))*(-b)^{(2/3)}*((b*\text{cosh}(d*x + c)^2 \\
& + b*\text{sinh}(d*x + c)^2 + b)/(\text{cosh}(d*x + c)^2 + \text{sinh}(d*x + c)^2 - 1))^{(1/3)} + \\
& \text{sqrt}(3)*b)/ (b*\text{cosh}(d*x + c)^2 + 2*b*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + b*\text{sinh}(d* \\
& x + c)^2 + b)) - 10*(\text{sqrt}(3)*b*\text{cosh}(d*x + c)^2 + 2*\text{sqrt}(3)*b*\text{cosh}(d*x + c)* \\
& \text{sinh}(d*x + c) + \text{sqrt}(3)*b*\text{sinh}(d*x + c)^2 - \text{sqrt}(3)*b)^{(1/3)}*\text{arctan}(-1/3* \\
& (\text{sqrt}(3)*b*\text{cosh}(d*x + c)^2 + 2*\text{sqrt}(3)*b*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + \text{sqrt} \\
& (3)*b*\text{sinh}(d*x + c)^2 - 2*(\text{sqrt}(3)*\text{cosh}(d*x + c)^2 + 2*\text{sqrt}(3)*\text{cosh}(d*x + c \\
&)*\text{sinh}(d*x + c) + \text{sqrt}(3)*\text{sinh}(d*x + c)^2 - \text{sqrt}(3))*b^{(2/3)}*((b*\text{cosh}(d*x + \\
& c)^2 + b*\text{sinh}(d*x + c)^2 + b)/(\text{cosh}(d*x + c)^2 + \text{sinh}(d*x + c)^2 - 1))^{(1/ \\
& 3)} + \text{sqrt}(3)*b)/ (b*\text{cosh}(d*x + c)^2 + 2*b*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + b*\text{si} \\
& \text{nh}(d*x + c)^2 + b)) + 5*(b*\text{cosh}(d*x + c)^2 + 2*b*\text{cosh}(d*x + c)*\text{sinh}(d*x + c \\
&) + b*\text{sinh}(d*x + c)^2 - b)*(-b)^{(1/3)}*\text{log}(((\text{cosh}(d*x + c)^4 + 4*\text{cosh}(d*x + \\
& c)*\text{sinh}(d*x + c)^3 + \text{sinh}(d*x + c)^4 + 2*(3*\text{cosh}(d*x + c)^2 + 1)*\text{sinh}(d*x + \\
& c)^2 + 2*\text{cosh}(d*x + c)^2 + 4*(\text{cosh}(d*x + c)^3 + \text{cosh}(d*x + c))*\text{sinh}(d*x + \\
& c) + 1)*(-b)^{(2/3)} - (\text{cosh}(d*x + c)^4 + 4*\text{cosh}(d*x + c)^3*\text{sinh}(d*x + c) + 6 \\
& *\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c)^2 + 4*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^3 + \text{sinh}(d \\
& *x + c)^4 - 1)*(-b)^{(1/3)}*((b*\text{cosh}(d*x + c)^2 + b*\text{sinh}(d*x + c)^2 + b)/(\text{cos} \\
& h(d*x + c)^2 + \text{sinh}(d*x + c)^2 - 1))^{(1/3)} + (\text{cosh}(d*x + c)^4 + 4*\text{cosh}(d*x \\
& + c)*\text{sinh}(d*x + c)^3 + \text{sinh}(d*x + c)^4 + 2*(3*\text{cosh}(d*x + c)^2 - 1)*\text{sinh}(d*x \\
& + c)^2 - 2*\text{cosh}(d*x + c)^2 + 4*(\text{cosh}(d*x + c)^3 - \text{cosh}(d*x + c))*\text{sinh}(d*x \\
& + c) + 1))*((b*\text{cosh}(d*x + c)^2 + b*\text{sinh}(d*x + c)^2 + b)/(\text{cosh}(d*x + c)^2 + s \\
& \text{inh}(d*x + c)^2 - 1))^{(2/3)})/(\text{cosh}(d*x + c)^4 + 4*\text{cosh}(d*x + c)*\text{sinh}(d*x + c \\
&)^3 + \text{sinh}(d*x + c)^4 + 2*(3*\text{cosh}(d*x + c)^2 + 1)*\text{sinh}(d*x + c)^2 + 2*\text{cosh}(\\
& d*x + c)^2 + 4*(\text{cosh}(d*x + c)^3 + \text{cosh}(d*x + c))*\text{sinh}(d*x + c) + 1)) + 5*(b \\
& *\text{cosh}(d*x + c)^2 + 2*b*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + b*\text{sinh}(d*x + c)^2 - b) \\
& *b^{(1/3)}*\text{log}(((\text{cosh}(d*x + c)^4 + 4*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^3 + \text{sinh}(d*x \\
& + c)^4 + 2*(3*\text{cosh}(d*x + c)^2 + 1)*\text{sinh}(d*x + c)^2 + 2*\text{cosh}(d*x + c)^2 + 4 \\
& *(\text{cosh}(d*x + c)^3 + \text{cosh}(d*x + c))*\text{sinh}(d*x + c) + 1))*b^{(2/3)} - (\text{cosh}(d*x + \\
& c)^4 + 4*\text{cosh}(d*x + c)^3*\text{sinh}(d*x + c) + 6*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c)^2 \\
& + 4*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^3 + \text{sinh}(d*x + c)^4 - 1)*b^{(1/3)}*((b*\text{cosh}(\\
& d*x + c)^2 + b*\text{sinh}(d*x + c)^2 + b)/(\text{cosh}(d*x + c)^2 + \text{sinh}(d*x + c)^2 - 1) \\
&)^{(1/3)} + (\text{cosh}(d*x + c)^4 + 4*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^3 + \text{sinh}(d*x + c \\
&)^4 + 2*(3*\text{cosh}(d*x + c)^2 - 1)*\text{sinh}(d*x + c)^2 - 2*\text{cosh}(d*x + c)^2 + 4*(\text{co} \\
& sh(d*x + c)^3 - \text{cosh}(d*x + c))*\text{sinh}(d*x + c) + 1))*((b*\text{cosh}(d*x + c)^2 + b*s \\
& \text{inh}(d*x + c)^2 + b)/(\text{cosh}(d*x + c)^2 + \text{sinh}(d*x + c)^2 - 1))^{(2/3)})/(\text{cosh}(d \\
& *x + c)^4 + 4*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^3 + \text{sinh}(d*x + c)^4 + 2*(3*\text{cosh}(d \\
& *x + c)^2 + 1)*\text{sinh}(d*x + c)^2 + 2*\text{cosh}(d*x + c)^2 + 4*(\text{cosh}(d*x + c)^3 + c \\
& \text{osh}(d*x + c))*\text{sinh}(d*x + c) + 1)) - 10*(b*\text{cosh}(d*x + c)^2 + 2*b*\text{cosh}(d*x + \\
& c)*\text{sinh}(d*x + c) + b*\text{sinh}(d*x + c)^2 - b)*(-b)^{(1/3)}*\text{log}(((\text{cosh}(d*x + c)^2 \\
& + 2*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + \text{sinh}(d*x + c)^2 + 1)*(-b)^{(1/3)} + (\text{cosh}(d \\
& *x + c)^2 + 2*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + \text{sinh}(d*x + c)^2 - 1))*((b*\text{cosh}(d \\
& *x + c)^2 + b*\text{sinh}(d*x + c)^2 + b)/(\text{cosh}(d*x + c)^2 + \text{sinh}(d*x + c)^2 - 1)) \\
& ^{(1/3)})/(\text{cosh}(d*x + c)^2 + 2*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + \text{sinh}(d*x + c)^2 \\
& + 1)) - 10*(b*\text{cosh}(d*x + c)^2 + 2*b*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + b*\text{sinh}(d*
\end{aligned}$$

$$x + c)^2 - b) * b^{(1/3)} * \log\left(\frac{(\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + c)^2 + 1) * b^{(1/3)} + (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + c)^2 - 1) * ((b * \cosh(dx + c)^2 + b * \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{(1/3)}}{(\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + c)^2 + 1)} + 12 * (b * \cosh(dx + c)^2 + 2 * b * \cosh(dx + c) * \sinh(dx + c) + b * \sinh(dx + c)^2 + b) * ((b * \cosh(dx + c)^2 + b * \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{(1/3)}}{(d * \cosh(dx + c)^2 + 2 * d * \cosh(dx + c) * \sinh(dx + c) + d * \sinh(dx + c)^2 - d)}\right)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**2)**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^2)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)

3.23 $\int (b \coth^2(c + dx))^{2/3} dx$

Optimal. Leaf size=289

$$-\frac{(b \coth^2(c + dx))^{2/3} \log\left(\coth^{2/3}(c + dx) - \sqrt[3]{\coth(c + dx) + 1}\right)}{4d \coth^{4/3}(c + dx)} + \frac{(b \coth^2(c + dx))^{2/3} \log\left(\coth^{2/3}(c + dx) + \sqrt[3]{\coth(c + dx) + 1}\right)}{4d \coth^{4/3}(c + dx)}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(2/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(2/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(2/3))/(d*Coth[c + d*x]^(4/3)) - ((b*Coth[c + d*x]^2)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^2)^(2/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Coth[c + d*x]^2)^(2/3)*Tanh[c + d*x])/d
```

Rubi [A] time = 0.180361, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{(b \coth^2(c + dx))^{2/3} \log\left(\coth^{2/3}(c + dx) - \sqrt[3]{\coth(c + dx) + 1}\right)}{4d \coth^{4/3}(c + dx)} + \frac{(b \coth^2(c + dx))^{2/3} \log\left(\coth^{2/3}(c + dx) + \sqrt[3]{\coth(c + dx) + 1}\right)}{4d \coth^{4/3}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(2/3), x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(2/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(2/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(2/3))/(d*Coth[c + d*x]^(4/3)) - ((b*Coth[c + d*x]^2)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^2)^(2/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Coth[c + d*x]^2)^(2/3)*Tanh[c + d*x])/d
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
```

```
n)^FracPart[p]]/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(n_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x)]/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] \text{ :> } -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \text{ /; } \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \text{ || } \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[(d_ + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \text{ :> } \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \text{ /; } \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 206

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] \text{ :> } \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; } \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \text{ || } \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int (b \coth^2(c + dx))^{2/3} dx &= \frac{(b \coth^2(c + dx))^{2/3} \int \coth^{4/3}(c + dx) dx}{\coth^{4/3}(c + dx)} \\
&= -\frac{3 (b \coth^2(c + dx))^{2/3} \tanh(c + dx)}{d} + \frac{(b \coth^2(c + dx))^{2/3} \int \frac{1}{\coth^{5/3}(c + dx)} dx}{\coth^{4/3}(c + dx)} \\
&= -\frac{3 (b \coth^2(c + dx))^{2/3} \tanh(c + dx)}{d} - \frac{(b \coth^2(c + dx))^{2/3} \text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)} dx, x, \coth(c + dx)\right)}{d \coth^{4/3}(c + dx)} \\
&= -\frac{3 (b \coth^2(c + dx))^{2/3} \tanh(c + dx)}{d} - \frac{\left(3 (b \coth^2(c + dx))^{2/3}\right) \text{Subst}\left(\int \frac{1}{-1+x^6} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d \coth^{4/3}(c + dx)} \\
&= -\frac{3 (b \coth^2(c + dx))^{2/3} \tanh(c + dx)}{d} + \frac{(b \coth^2(c + dx))^{2/3} \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d \coth^{4/3}(c + dx)} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) (b \coth^2(c + dx))^{2/3}}{d \coth^{4/3}(c + dx)} - \frac{3 (b \coth^2(c + dx))^{2/3} \tanh(c + dx)}{d} - \frac{(b \coth^2(c + dx))^{2/3}}{d} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) (b \coth^2(c + dx))^{2/3}}{d \coth^{4/3}(c + dx)} - \frac{(b \coth^2(c + dx))^{2/3} \log\left(1 - \sqrt[3]{\coth(c + dx)}\right)}{4d \coth^{4/3}(c + dx)} \\
&= -\frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) (b \coth^2(c + dx))^{2/3}}{2d \coth^{4/3}(c + dx)} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) (b \coth^2(c + dx))^{2/3}}{2d \coth^{4/3}(c + dx)}
\end{aligned}$$

Mathematica [C] time = 0.0389619, size = 43, normalized size = 0.15

$$\frac{3 \tanh(c + dx) (b \coth^2(c + dx))^{2/3} \left({}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; \coth^2(c + dx)\right) - 1 \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^2)^(2/3), x]

[Out] (3*(b*Coth[c + d*x]^2)^(2/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2]))*Tanh[c + d*x])/d

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (b(\coth(dx+c))^2)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^2)^(2/3),x)

[Out] int((b*coth(d*x+c)^2)^(2/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx+c))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)

Fricas [B] time = 2.14525, size = 5751, normalized size = 19.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(2*(\sqrt{3}*\cosh(d*x + c))^2 + 2*\sqrt{3}*\cosh(d*x + c)*\sinh(d*x + c) + \\ & \sqrt{3}*\sinh(d*x + c)^2 + \sqrt{3})*(-b^2)^{1/3}*\arctan(-1/3*(\sqrt{3}*b*\cosh \\ & (d*x + c)^2 + 2*\sqrt{3}*b*\cosh(d*x + c)*\sinh(d*x + c) + \sqrt{3}*b*\sinh(d*x \\ & + c)^2 - 2*(\sqrt{3}*\cosh(d*x + c))^2 + 2*\sqrt{3}*\cosh(d*x + c)*\sinh(d*x + c) \\ & + \sqrt{3}*\sinh(d*x + c)^2 - \sqrt{3})*(-b^2)^{1/3}*((b*\cosh(d*x + c))^2 + b* \\ & \sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^{1/3} + \sqrt{3} \\ & (3*b)/(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + \\ & c)^2 + b)) + 2*(\sqrt{3}*\cosh(d*x + c))^2 + 2*\sqrt{3}*\cosh(d*x + c)*\sinh(d*x + \end{aligned}$$

$c) + \sqrt{3} \sinh(dx + c)^2 + \sqrt{3} \arctan(-1/3(\sqrt{3}b \cosh(dx + c)^2 + 2\sqrt{3}b \cosh(dx + c) \sinh(dx + c) + \sqrt{3}b \sinh(dx + c)^2 - 2(\sqrt{3} \cosh(dx + c)^2 + 2\sqrt{3} \cosh(dx + c) \sinh(dx + c) + \sqrt{3} \sinh(dx + c)^2 - \sqrt{3})) \cdot (b^2)^{1/3} / ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3} + \sqrt{3}b) / (b \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + b)) + (-b^2)^{1/3} (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) \log(((\cosh(dx + c)^4 + 4 \cosh(dx + c)^3 \sinh(dx + c) + 6 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 - 1) \cdot (-b^2)^{2/3} \cdot ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3} + (b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 - 2b \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 - b) \sinh(dx + c)^2 + 4(b \cosh(dx + c)^3 - b \cosh(dx + c)) \sinh(dx + c) + b) \cdot ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{2/3} - (b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 + 2b \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 + b) \sinh(dx + c)^2 + 4(b \cosh(dx + c)^3 + b \cosh(dx + c)) \sinh(dx + c) + b) \cdot (-b^2)^{1/3}) / (\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2(3 \cosh(dx + c)^2 + 1) \sinh(dx + c)^2 + 2 \cosh(dx + c)^2 + 4(\cosh(dx + c)^3 + \cosh(dx + c)) \sinh(dx + c) + 1)) + (b^2)^{1/3} (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) \log(-((\cosh(dx + c)^4 + 4 \cosh(dx + c)^3 \sinh(dx + c) + 6 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 - 1) \cdot (b^2)^{2/3} \cdot ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3} - (b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 - 2b \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 - b) \sinh(dx + c)^2 + 4(b \cosh(dx + c)^3 - b \cosh(dx + c)) \sinh(dx + c) + b) \cdot ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{2/3} - (b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 + 2b \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 + b) \sinh(dx + c)^2 + 4(b \cosh(dx + c)^3 + b \cosh(dx + c)) \sinh(dx + c) + b) \cdot (b^2)^{1/3}) / (\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2(3 \cosh(dx + c)^2 + 1) \sinh(dx + c)^2 + 2 \cosh(dx + c)^2 + 4(\cosh(dx + c)^3 + \cosh(dx + c)) \sinh(dx + c) + 1)) - 2 \cdot (-b^2)^{1/3} (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) \log(-((-b^2)^{2/3} (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) - (b \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 - b) \cdot ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3}) / (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1)) - 2 \cdot (b^2)^{1/3} (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) \log(((b^2)^{2/3} (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) + (b \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 - b) \cdot ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3}) / (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1))$

```
nh(d*x + c) + sinh(d*x + c)^2 + 1)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(d*cosh(d*x + c)^2
+ 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth^2(c + dx))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^2)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)
```

3.24 $\int \sqrt[3]{b \coth^2(c + dx)} dx$

Optimal. Leaf size=264

$$\frac{\sqrt[3]{b \coth^2(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1\right)}{4d \coth^{\frac{2}{3}}(c + dx)} + \frac{\sqrt[3]{b \coth^2(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) + \sqrt[3]{\coth(c + dx)}\right)}{4d \coth^{\frac{2}{3}}(c + dx)}$$

[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3)) - ((b*Coth[c + d*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3)) + ((b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))

Rubi [A] time = 0.211663, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3658, 3476, 329, 296, 634, 618, 204, 628, 206}

$$\frac{\sqrt[3]{b \coth^2(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1\right)}{4d \coth^{\frac{2}{3}}(c + dx)} + \frac{\sqrt[3]{b \coth^2(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) + \sqrt[3]{\coth(c + dx)}\right)}{4d \coth^{\frac{2}{3}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^2)^(1/3), x]

[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3)) - ((b*Coth[c + d*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3)) + ((b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))

Rule 3658

Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p])*(b*Tan[e + f*x]^n)^FracPart[p]]/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)^m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \sqrt[3]{b \coth^2(c+dx)} dx &= \frac{\sqrt[3]{b \coth^2(c+dx)} \int \coth^{\frac{2}{3}}(c+dx) dx}{\coth^{\frac{2}{3}}(c+dx)} \\
 &= -\frac{\sqrt[3]{b \coth^2(c+dx)} \operatorname{Subst}\left(\int \frac{x^{2/3}}{-1+x^2} dx, x, \coth(c+dx)\right)}{d \coth^{\frac{2}{3}}(c+dx)} \\
 &= -\frac{\left(3\sqrt[3]{b \coth^2(c+dx)}\right) \operatorname{Subst}\left(\int \frac{x^4}{-1+x^6} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \coth^{\frac{2}{3}}(c+dx)} \\
 &= \frac{\sqrt[3]{b \coth^2(c+dx)} \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \coth^{\frac{2}{3}}(c+dx)} + \frac{\sqrt[3]{b \coth^2(c+dx)} \operatorname{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \coth^{\frac{2}{3}}(c+dx)} \\
 &= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \sqrt[3]{b \coth^2(c+dx)}}{d \coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt[3]{b \coth^2(c+dx)} \operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d \coth^{\frac{2}{3}}(c+dx)} \\
 &= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \sqrt[3]{b \coth^2(c+dx)}}{d \coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt[3]{b \coth^2(c+dx)} \log\left(1 - \sqrt[3]{\coth(c+dx)} + \coth(c+dx)\right)}{4d \coth^{\frac{2}{3}}(c+dx)} \\
 &= \frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^2(c+dx)}}{2d \coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^2(c+dx)}}{2d \coth^{\frac{2}{3}}(c+dx)}
 \end{aligned}$$

Mathematica [A] time = 0.101159, size = 151, normalized size = 0.57

$$\frac{\sqrt[3]{b \coth^2(c + dx)} \left(-\log \left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1 \right) + \log \left(\coth^{\frac{2}{3}}(c + dx) + \sqrt[3]{\coth(c + dx)} + 1 \right) + 2\sqrt{3} \tan^{-1} \right)}{4d \coth^{\frac{2}{3}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^2)^(1/3),x]

[Out] ((b*Coth[c + d*x]^2)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[Coth[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2/3))

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \sqrt[3]{b (\coth(dx + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^2)^(1/3),x)

[Out] int((b*coth(d*x+c)^2)^(1/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^2)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^2)^(1/3), x)

Fricas [B] time = 2.08247, size = 4625, normalized size = 17.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cosh(d*x+c)^2)^(1/3),x, algorithm="fricas")

[Out]
$$-1/4*(2*\sqrt{3})*(-b)^{1/3}*\arctan(1/3*(\sqrt{3}*b*\cosh(dx+c)^2 + 2*\sqrt{3})*b*\cosh(dx+c)*\sinh(dx+c) + \sqrt{3}*b*\sinh(dx+c)^2 + 2*(\sqrt{3}*\cosh(dx+c)^2 + 2*\sqrt{3}*\cosh(dx+c)*\sinh(dx+c) + \sqrt{3}*\sinh(dx+c)^2 - \sqrt{3}))*(-b)^{2/3}*((b*\cosh(dx+c)^2 + b*\sinh(dx+c)^2 + b)/(\cosh(dx+c)^2 + \sinh(dx+c)^2 - 1))^{1/3} + \sqrt{3}*b/(b*\cosh(dx+c)^2 + 2*b*\cosh(dx+c)*\sinh(dx+c) + b*\sinh(dx+c)^2 + b)) - 2*\sqrt{3}*b^{1/3}*\arctan(-1/3*(\sqrt{3}*b*\cosh(dx+c)^2 + 2*\sqrt{3})*b*\cosh(dx+c)*\sinh(dx+c) + \sqrt{3}*b*\sinh(dx+c)^2 - 2*(\sqrt{3}*\cosh(dx+c)^2 + 2*\sqrt{3}*\cosh(dx+c)*\sinh(dx+c) + \sqrt{3}*\sinh(dx+c)^2 - \sqrt{3}))*b^{2/3}*((b*\cosh(dx+c)^2 + b*\sinh(dx+c)^2 + b)/(\cosh(dx+c)^2 + \sinh(dx+c)^2 - 1))^{1/3} + \sqrt{3}*b/(b*\cosh(dx+c)^2 + 2*b*\cosh(dx+c)*\sinh(dx+c) + b*\sinh(dx+c)^2 + b)) + (-b)^{1/3}*\log(((\cosh(dx+c)^4 + 4*\cosh(dx+c)*\sinh(dx+c)^3 + \sinh(dx+c)^4 + 2*(3*\cosh(dx+c)^2 + 1)*\sinh(dx+c)^2 + 2*\cosh(dx+c)^2 + 4*(\cosh(dx+c)^3 + \cosh(dx+c))*\sinh(dx+c) + 1))*(-b)^{2/3} - (\cosh(dx+c)^4 + 4*\cosh(dx+c)^3*\sinh(dx+c) + 6*\cosh(dx+c)^2*\sinh(dx+c)^2 + 4*\cosh(dx+c)*\sinh(dx+c)^3 + \sinh(dx+c)^4 - 1))*(-b)^{1/3}*((b*\cosh(dx+c)^2 + b*\sinh(dx+c)^2 + b)/(\cosh(dx+c)^2 + \sinh(dx+c)^2 - 1))^{1/3} + (\cosh(dx+c)^4 + 4*\cosh(dx+c)*\sinh(dx+c)^3 + \sinh(dx+c)^4 + 2*(3*\cosh(dx+c)^2 - 1)*\sinh(dx+c)^2 - 2*\cosh(dx+c)^2 + 4*(\cosh(dx+c)^3 - \cosh(dx+c))*\sinh(dx+c) + 1))*((b*\cosh(dx+c)^2 + b*\sinh(dx+c)^2 + b)/(\cosh(dx+c)^2 + \sinh(dx+c)^2 - 1))^{2/3})/(\cosh(dx+c)^4 + 4*\cosh(dx+c)*\sinh(dx+c)^3 + \sinh(dx+c)^4 + 2*(3*\cosh(dx+c)^2 + 1)*\sinh(dx+c)^2 + 2*\cosh(dx+c)^2 + 4*(\cosh(dx+c)^3 + \cosh(dx+c))*\sinh(dx+c) + 1)) + b^{1/3}*\log(((\cosh(dx+c)^4 + 4*\cosh(dx+c)*\sinh(dx+c)^3 + \sinh(dx+c)^4 + 2*(3*\cosh(dx+c)^2 + 1)*\sinh(dx+c)^2 + 2*\cosh(dx+c)^2 + 4*(\cosh(dx+c)^3 + \cosh(dx+c))*\sinh(dx+c) + 1))*b^{2/3} - (\cosh(dx+c)^4 + 4*\cosh(dx+c)^3*\sinh(dx+c) + 6*\cosh(dx+c)^2*\sinh(dx+c)^2 + 4*\cosh(dx+c)*\sinh(dx+c)^3 + \sinh(dx+c)^4 - 1))*b^{1/3}*((b*\cosh(dx+c)^2 + b*\sinh(dx+c)^2 + b)/(\cosh(dx+c)^2 + \sinh(dx+c)^2 - 1))^{1/3} + (\cosh(dx+c)^4 + 4*\cosh(dx+c)*\sinh(dx+c)^3 + \sinh(dx+c)^4 + 2*(3*\cosh(dx+c)^2 - 1)*\sinh(dx+c)^2 - 2*\cosh(dx+c)^2 + 4*(\cosh(dx+c)^3 - \cosh(dx+c))*\sinh(dx+c) + 1))*((b*\cosh(dx+c)^2 + b*\sinh(dx+c)^2 + b)/(\cosh(dx+c)^2 + \sinh(dx+c)^2 - 1))^{2/3})/(\cosh(dx+c)^4 + 4*\cosh(dx+c)*\sinh(dx+c)^3 + \sinh(dx+c)^4 + 2*(3*\cosh(dx+c)^2 + 1)*\sinh(dx+c)^2 + 2*\cosh(dx+c)^2 + 4*(\cosh(dx+c)$$

$$\begin{aligned} & c)^3 + \cosh(dx + c) \sinh(dx + c) + 1)) - 2(-b)^{1/3} \log(((\cosh(dx + c) \\ &)^2 + 2\cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1)(-b)^{1/3} + (\co \\ & sh(dx + c)^2 + 2\cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1)((b \co \\ & sh(dx + c)^2 + b \sinh(dx + c)^2 + b)/(\cosh(dx + c)^2 + \sinh(dx + c)^2 - \\ & 1))^{1/3})/(\cosh(dx + c)^2 + 2\cosh(dx + c) \sinh(dx + c) + \sinh(dx + c \\ &)^2 + 1)) - 2b^{1/3} \log(((\cosh(dx + c)^2 + 2\cosh(dx + c) \sinh(dx + c) \\ & + \sinh(dx + c)^2 + 1)b^{1/3} + (\cosh(dx + c)^2 + 2\cosh(dx + c) \sinh(d \\ & *x + c) + \sinh(dx + c)^2 - 1)((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) \\ & /(\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3})/(\cosh(dx + c)^2 + 2\cosh(\\ & dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1))) / d \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \coth^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(dx+c)**2)**(1/3),x)

[Out] Integral((b*coth(c + dx)**2)**(1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^2)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(dx+c)^2)^(1/3),x, algorithm="giac")

[Out] integrate((b*coth(dx + c)^2)^(1/3), x)

$$3.25 \quad \int \frac{1}{\sqrt[3]{b \coth^2(c+dx)}} dx$$

Optimal. Leaf size=264

$$\frac{\coth^{\frac{2}{3}}(c+dx) \log\left(\coth^{\frac{2}{3}}(c+dx) - \sqrt[3]{\coth(c+dx)+1}\right)}{4d\sqrt[3]{b \coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx) \log\left(\coth^{\frac{2}{3}}(c+dx) + \sqrt[3]{\coth(c+dx)+1}\right)}{4d\sqrt[3]{b \coth^2(c+dx)}}$$

[Out] $-(\text{Sqrt}[3] * \text{ArcTan}[(1 - 2 * \text{Coth}[c + d * x]^{(1/3)}) / \text{Sqrt}[3]] * \text{Coth}[c + d * x]^{(2/3)}) / (2 * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{Sqrt}[3] * \text{ArcTan}[(1 + 2 * \text{Coth}[c + d * x]^{(1/3)}) / \text{Sqrt}[3]] * \text{Coth}[c + d * x]^{(2/3)}) / (2 * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{ArcTanh}[\text{Coth}[c + d * x]^{(1/3)}] * \text{Coth}[c + d * x]^{(2/3)}) / (d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) - (\text{Coth}[c + d * x]^{(2/3)} * \text{Log}[1 - \text{Coth}[c + d * x]^{(1/3)} + \text{Coth}[c + d * x]^{(2/3)}]) / (4 * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{Coth}[c + d * x]^{(2/3)} * \text{Log}[1 + \text{Coth}[c + d * x]^{(1/3)} + \text{Coth}[c + d * x]^{(2/3)}]) / (4 * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)})$

Rubi [A] time = 0.169139, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3658, 3476, 329, 210, 634, 618, 204, 628, 206}

$$\frac{\coth^{\frac{2}{3}}(c+dx) \log\left(\coth^{\frac{2}{3}}(c+dx) - \sqrt[3]{\coth(c+dx)+1}\right)}{4d\sqrt[3]{b \coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx) \log\left(\coth^{\frac{2}{3}}(c+dx) + \sqrt[3]{\coth(c+dx)+1}\right)}{4d\sqrt[3]{b \coth^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b * \text{Coth}[c + d * x]^2)^{-1/3}, x]$

[Out] $-(\text{Sqrt}[3] * \text{ArcTan}[(1 - 2 * \text{Coth}[c + d * x]^{(1/3)}) / \text{Sqrt}[3]] * \text{Coth}[c + d * x]^{(2/3)}) / (2 * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{Sqrt}[3] * \text{ArcTan}[(1 + 2 * \text{Coth}[c + d * x]^{(1/3)}) / \text{Sqrt}[3]] * \text{Coth}[c + d * x]^{(2/3)}) / (2 * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{ArcTanh}[\text{Coth}[c + d * x]^{(1/3)}] * \text{Coth}[c + d * x]^{(2/3)}) / (d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) - (\text{Coth}[c + d * x]^{(2/3)} * \text{Log}[1 - \text{Coth}[c + d * x]^{(1/3)} + \text{Coth}[c + d * x]^{(2/3)}]) / (4 * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{Coth}[c + d * x]^{(2/3)} * \text{Log}[1 + \text{Coth}[c + d * x]^{(1/3)} + \text{Coth}[c + d * x]^{(2/3)}]) / (4 * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)})$

Rule 3658

$\text{Int}[(u_.) * ((b_.) * \tan[(e_.) + (f_.) * (x_)]^{(n_)})^{(p_)}, x_Symbol] := \text{With}[\{\{ff = \text{FreeFactors}[\text{Tan}[e + f * x], x]\}, \text{Dist}[(b * ff^n)^{\text{IntPart}[p]} * (b * \text{Tan}[e + f * x]^{\text{IntPart}[p]})^{\text{IntPart}[p]}]$

```
n)^FracPart[p]]/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(n_ - 1), x_Symbol] := Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_ - 1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(n_ - 1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt[3]{b \coth^2(c+dx)}} dx &= \frac{\coth^{\frac{2}{3}}(c+dx) \int \frac{1}{\coth^{\frac{2}{3}}(c+dx)} dx}{\sqrt[3]{b \coth^2(c+dx)}} \\
 &= \frac{\coth^{\frac{2}{3}}(c+dx) \operatorname{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)} dx, x, \coth(c+dx)\right)}{d \sqrt[3]{b \coth^2(c+dx)}} \\
 &= \frac{\left(3 \coth^{\frac{2}{3}}(c+dx)\right) \operatorname{Subst}\left(\int \frac{1}{-1+x^6} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \sqrt[3]{b \coth^2(c+dx)}} \\
 &= \frac{\coth^{\frac{2}{3}}(c+dx) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \sqrt[3]{b \coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx) \operatorname{Subst}\left(\int \frac{1-x}{1-x+x^2} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \sqrt[3]{b \coth^2(c+dx)}} \\
 &= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \coth^{\frac{2}{3}}(c+dx)}{d \sqrt[3]{b \coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx) \operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d \sqrt[3]{b \coth^2(c+dx)}} \\
 &= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \coth^{\frac{2}{3}}(c+dx)}{d \sqrt[3]{b \coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx) \log\left(1 - \sqrt[3]{\coth(c+dx)} + \coth^{\frac{2}{3}}(c+dx)\right)}{4d \sqrt[3]{b \coth^2(c+dx)}} \\
 &= \frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{\frac{2}{3}}(c+dx)}{2d \sqrt[3]{b \coth^2(c+dx)}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{\frac{2}{3}}(c+dx)}{2d \sqrt[3]{b \coth^2(c+dx)}} + \dots
 \end{aligned}$$

Mathematica [C] time = 0.0540409, size = 41, normalized size = 0.16

$$\frac{3 \coth(c + dx) {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; \coth^2(c + dx)\right)}{d \sqrt[3]{b \coth^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^2)^(-1/3), x]

[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])/(d*(b*Coth[c + d*x]^2)^(1/3))

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b (\coth(dx + c))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c)^2)^(1/3), x)

[Out] int(1/(b*coth(d*x+c)^2)^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^2)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(1/3), x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)

Fricas [B] time = 3.48159, size = 23547, normalized size = 89.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(\sqrt{3}*b*\sqrt{-1/b^{(2/3)}})*\log((b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c) \\ & * \sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*\sqrt{3}*(\cosh(d*x + c)^4 + 4*\cosh(d*x + c) \\ & * \sinh(d*x + c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c)^2 - 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 - \cosh(d*x + c))*\sinh(d*x + c) + 1)*b^{(2/3)}*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^{(2/3)}*\sqrt{-1/b^{(2/3)}} - 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c)^2 + \sqrt{3}*(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c) + b)*b^{(1/3)}*\sqrt{-1/b^{(2/3)}} + 4*(b*\cosh(d*x + c)^3 - b*\cosh(d*x + c))*\sinh(d*x + c) - (\sqrt{3}*(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*b*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - b)*\sqrt{-1/b^{(2/3)}} + 3*(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 - 1)*b^{(2/3)}*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^{(1/3)} - 3*b)/(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + (6*\cosh(d*x + c)^2 + 1)*\sinh(d*x + c)^2 + \cosh(d*x + c)^2 + 2*(2*\cosh(d*x + c)^3 + \cosh(d*x + c))*\sinh(d*x + c))] + \sqrt{3})*b*\sqrt{((-b)^{(1/3)}/b)*\log(-(3*b*\cosh(d*x + c)^4 + 12*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + 3*b*\sinh(d*x + c)^4 + 2*b*\cosh(d*x + c)^2 + 2*(9*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^2 - 3*(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 - 1)*(-b)^{(2/3)}*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^{(1/3)} - \sqrt{3}*(2*(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c)^2 - 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 - \cosh(d*x + c))*\sinh(d*x + c) + 1)*(-b)^{(2/3)}*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^{(2/3)} + (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c) + b)*(-b)^{(1/3)} - (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*b*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - b)*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^{(1/3)})*\sqrt{((-b)^{(1/3)}/b) + 4*(3*b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c)} \end{aligned}$$

$$\begin{aligned}
& - b)/(\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 + \\
& 1)) + (-b)^{(2/3)}*\log(((\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \\
& \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 + 1)*\sinh(dx + c)^2 + 2*\cosh(dx + \\
& c)^2 + 4*(\cosh(dx + c)^3 + \cosh(dx + c))*\sinh(dx + c) + 1)*(-b)^{(2/3)} - \\
& (\cosh(dx + c)^4 + 4*\cosh(dx + c)^3*\sinh(dx + c) + 6*\cosh(dx + c)^2*\sinh \\
& (dx + c)^2 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 - 1)*(-b)^{(\\
& 1/3)}*((b*\cosh(dx + c)^2 + b*\sinh(dx + c)^2 + b)/(\cosh(dx + c)^2 + \sinh(dx \\
& *x + c)^2 - 1))^{(1/3)} + (\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 \\
& + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 - 1)*\sinh(dx + c)^2 - 2*\cosh(dx \\
& + c)^2 + 4*(\cosh(dx + c)^3 - \cosh(dx + c))*\sinh(dx + c) + 1)*((b*\cosh(dx \\
& x + c)^2 + b*\sinh(dx + c)^2 + b)/(\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{(\\
& 2/3)})/(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 \\
& + 2*(3*\cosh(dx + c)^2 + 1)*\sinh(dx + c)^2 + 2*\cosh(dx + c)^2 + 4*(\cosh \\
& dx + c)^3 + \cosh(dx + c))*\sinh(dx + c) + 1)) - b^{(2/3)}*\log(((\cosh(dx + \\
& c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + \\
& c)^2 + 1)*\sinh(dx + c)^2 + 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 + \cosh(dx \\
& *x + c))*\sinh(dx + c) + 1)*b^{(2/3)} - (\cosh(dx + c)^4 + 4*\cosh(dx + c)^3* \\
& \sinh(dx + c) + 6*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*\cosh(dx + c)*\sinh(dx \\
& x + c)^3 + \sinh(dx + c)^4 - 1)*b^{(1/3)}*((b*\cosh(dx + c)^2 + b*\sinh(dx + \\
& c)^2 + b)/(\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{(1/3)} + (\cosh(dx + c)^4 \\
& + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 \\
& - 1)*\sinh(dx + c)^2 - 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 - \cosh(dx + \\
& c))*\sinh(dx + c) + 1)*((b*\cosh(dx + c)^2 + b*\sinh(dx + c)^2 + b)/(\cosh \\
& dx + c)^2 + \sinh(dx + c)^2 - 1))^{(2/3)})/(\cosh(dx + c)^4 + 4*\cosh(dx + c \\
&)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 + 1)*\sinh(dx + \\
& c)^2 + 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 + \cosh(dx + c))*\sinh(dx + c \\
&) + 1)) - 2*(-b)^{(2/3)}*\log(((\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c \\
&) + \sinh(dx + c)^2 + 1)*(-b)^{(1/3)} + (\cosh(dx + c)^2 + 2*\cosh(dx + c)*\si \\
& nh(dx + c) + \sinh(dx + c)^2 - 1)*((b*\cosh(dx + c)^2 + b*\sinh(dx + c)^2 \\
& + b)/(\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{(1/3)})/(\cosh(dx + c)^2 + 2*c \\
& osh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 + 1)) + 2*b^{(2/3)}*\log(((\cosh(dx \\
& *x + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 + 1)*b^{(1/3)} + \\
& (\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 - 1)*((b \\
& *cosh(dx + c)^2 + b*\sinh(dx + c)^2 + b)/(\cosh(dx + c)^2 + \sinh(dx + c)^ \\
& 2 - 1))^{(1/3)})/(\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx \\
& + c)^2 + 1)))/(b*d), -1/4*(2*sqrt(3)*b*sqrt(-(-b)^{(1/3)}/b)*arctan(-1/3*(sqr \\
& t(3)*(cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 + 1 \\
&)*(-b)^{(1/3)}*sqrt(-(-b)^{(1/3)}/b) - 2*sqrt(3)*(cosh(dx + c)^2 + 2*\cosh(dx \\
& + c)*\sinh(dx + c) + \sinh(dx + c)^2 - 1)*((b*\cosh(dx + c)^2 + b*\sinh(dx \\
& + c)^2 + b)/(\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{(1/3)}*sqrt(-(-b)^{(1/3) \\
& /b)})/(\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 + 1 \\
&)) - sqrt(3)*b*sqrt(-1/b^{(2/3)})*\log((b*\cosh(dx + c)^4 + 4*b*\cosh(dx + c)* \\
& \sinh(dx + c)^3 + b*\sinh(dx + c)^4 - 2*sqrt(3)*(cosh(dx + c)^4 + 4*\cosh(dx \\
& *x + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 - 1)*\sinh \\
& dx + c)^2 - 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 - \cosh(dx + c))*\sinh(dx
\end{aligned}$$

$$\begin{aligned}
& *x + c) + 1)*b^{(2/3)}*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x \\
& + c)^2 + \sinh(d*x + c)^2 - 1))^{(2/3)}*\sqrt{-1/b^{(2/3)}} - 2*b*\cosh(d*x + c)^2 \\
& + 2*(3*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c)^2 + \sqrt{3}*(b*\cosh(d*x + c)^4 \\
& + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*b*\cosh(d*x + \\
& c)^2 + 2*(3*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + \\
& b*\cosh(d*x + c))*\sinh(d*x + c) + b)*b^{(1/3)}*\sqrt{-1/b^{(2/3)}} + 4*(b*\cosh(d \\
& *x + c)^3 - b*\cosh(d*x + c))*\sinh(d*x + c) - (\sqrt{3}*(b*\cosh(d*x + c)^4 + \\
& 4*b*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*b*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4 \\
& *b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - b)*\sqrt{-1/b^{(2/3)}}) \\
& + 3*(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*\cosh(d*x + c)^2* \\
& \sinh(d*x + c)^2 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 - 1)*b^{(\\
& 2/3)}*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh \\
& (d*x + c)^2 - 1))^{(1/3)} - 3*b)/(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x \\
& + c)^3 + \sinh(d*x + c)^4 + (6*\cosh(d*x + c)^2 + 1)*\sinh(d*x + c)^2 + \cosh(d \\
& *x + c)^2 + 2*(2*\cosh(d*x + c)^3 + \cosh(d*x + c))*\sinh(d*x + c))) - (-b)^{(2 \\
& /3)}*\log(((\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c) \\
& ^4 + 2*(3*\cosh(d*x + c)^2 + 1)*\sinh(d*x + c)^2 + 2*\cosh(d*x + c)^2 + 4*(\cos \\
& h(d*x + c)^3 + \cosh(d*x + c))*\sinh(d*x + c) + 1)*(-b)^{(2/3)} - (\cosh(d*x + c) \\
&)^4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + \\
& 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 - 1)*(-b)^{(1/3)}*((b*\cosh \\
& (d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1 \\
&))^{(1/3)} + (\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + \\
& c)^4 + 2*(3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c)^2 - 2*\cosh(d*x + c)^2 + 4*(\c \\
& osh(d*x + c)^3 - \cosh(d*x + c))*\sinh(d*x + c) + 1)*((b*\cosh(d*x + c)^2 + b* \\
& \sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^{(2/3)})/(\cosh(\\
& d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(\\
& d*x + c)^2 + 1)*\sinh(d*x + c)^2 + 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 + \\
& \cosh(d*x + c))*\sinh(d*x + c) + 1)) + b^{(2/3)}*\log(((\cosh(d*x + c)^4 + 4*\cosh \\
& (d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(d*x + c)^2 + 1)*\sin \\
& h(d*x + c)^2 + 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 + \cosh(d*x + c))*\sinh \\
& (d*x + c) + 1)*b^{(2/3)} - (\cosh(d*x + c)^4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) \\
& + 6*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \si \\
& nh(d*x + c)^4 - 1)*b^{(1/3)}*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\co \\
& sh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^{(1/3)} + (\cosh(d*x + c)^4 + 4*\cosh(d*x \\
& + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(d*x + c)^2 - 1)*\sinh(d* \\
& x + c)^2 - 2*\cosh(d*x + c)^2 + 4*(\cosh(d*x + c)^3 - \cosh(d*x + c))*\sinh(d*x \\
& + c) + 1)*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \\
& \sinh(d*x + c)^2 - 1))^{(2/3)})/(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + \\
& c)^3 + \sinh(d*x + c)^4 + 2*(3*\cosh(d*x + c)^2 + 1)*\sinh(d*x + c)^2 + 2*\cosh \\
& (d*x + c)^2 + 4*(\cosh(d*x + c)^3 + \cosh(d*x + c))*\sinh(d*x + c) + 1)) + 2*(\\
& -b)^{(2/3)}*\log(((\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x \\
& + c)^2 + 1)*(-b)^{(1/3)} + (\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \\
& \sinh(d*x + c)^2 - 1)*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d* \\
& x + c)^2 + \sinh(d*x + c)^2 - 1))^{(1/3)})/(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)* \\
& \sinh(d*x + c) + \sinh(d*x + c)^2 + 1)) - 2*b^{(2/3)}*\log(((\cosh(d*x + c)^2 + 2
\end{aligned}$$

$$\begin{aligned}
& * \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) b^{1/3} + (\cosh(dx + c) \\
&)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) ((b \cosh(dx + c) \\
&)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3} \\
&) / (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1)) \\
& / (b d), 1/4 (\sqrt{3} b \sqrt{(-b)^{1/3}/b}) \log(-3 b \cosh(dx + c)^4 + 12 b \cosh \\
& (dx + c) \sinh(dx + c)^3 + 3 b \sinh(dx + c)^4 + 2 b \cosh(dx + c)^2 + \\
& 2 (9 b \cosh(dx + c)^2 + b) \sinh(dx + c)^2 - 3 (\cosh(dx + c)^4 + 4 \cosh \\
& (dx + c)^3 \sinh(dx + c) + 6 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4 \cosh(dx + \\
& c) \sinh(dx + c)^3 + \sinh(dx + c)^4 - 1) (-b)^{2/3} ((b \cosh(dx + c)^2 + \\
& b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3} - \sqrt{3} \\
& (2 (\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2 (3 \cosh(dx + c)^2 - 1) \sinh(dx + c)^2 - 2 \cosh(dx + c)^2 + 4 (\cosh(dx + c)^3 - \cosh(dx + c)) \sinh(dx + c) + 1) (-b)^{2/3} ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{2/3}) + (b \cosh(dx + c)^4 + 4 b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 + 2 b \cosh(dx + c)^2 + 2 (3 b \cosh(dx + c)^2 + b) \sinh(dx + c)^2 + 4 (b \cosh(dx + c)^3 + b \cosh(dx + c)) \sinh(dx + c) + b) (-b)^{1/3} - (b \cosh(dx + c)^4 + 4 b \cosh(dx + c)^3 \sinh(dx + c) + 6 b \cosh(dx + c)^2 \sinh(dx + c)^2 + 4 b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 - b) ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3}) \sqrt{(-b)^{1/3}/b} + 4 (3 b \cosh(dx + c)^3 + b \cosh(dx + c) \sinh(dx + c) - b) / (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) - 2 \sqrt{3} b^{2/3} \arctan(-1/3 \sqrt{3} ((\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) b^{1/3} - 2 (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3}) / ((\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) b^{1/3})) + (-b)^{2/3} \log(((\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2 (3 \cosh(dx + c)^2 + 1) \sinh(dx + c)^2 + 2 \cosh(dx + c)^2 + 4 (\cosh(dx + c)^3 + \cosh(dx + c)) \sinh(dx + c) + 1) (-b)^{2/3} - (\cosh(dx + c)^4 + 4 \cosh(dx + c)^3 \sinh(dx + c) + 6 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 - 1) (-b)^{1/3} ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3} + (\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2 (3 \cosh(dx + c)^2 - 1) \sinh(dx + c)^2 - 2 \cosh(dx + c)^2 + 4 (\cosh(dx + c)^3 - \cosh(dx + c)) \sinh(dx + c) + 1) ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{2/3}) / (\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2 (3 \cosh(dx + c)^2 + 1) \sinh(dx + c)^2 + 2 \cosh(dx + c)^2 + 4 (\cosh(dx + c)^3 + \cosh(dx + c)) \sinh(dx + c) + 1) - b^{2/3} \log(((\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2 (3 \cosh(dx + c)^2 + 1) \sinh(dx + c)^2 + 2 \cosh(dx + c)^2 + 4 (\cosh(dx + c)^3 + \cosh(dx + c)) \sinh(dx + c) + 1) b^{2/3} - (\cosh(dx + c)^4 + 4 \cosh(dx + c)^3 \sinh(dx + c) + 6 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 - 1) b^{1/3} ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3})
\end{aligned}$$

$$\begin{aligned}
& 2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3} + \\
& (\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2 * (\\
& 3 \cosh(dx + c)^2 - 1) \sinh(dx + c)^2 - 2 \cosh(dx + c)^2 + 4 * (\cosh(dx + \\
& c)^3 - \cosh(dx + c)) \sinh(dx + c) + 1) * ((b \cosh(dx + c)^2 + b \sinh(dx + \\
& c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{2/3}) / (\cosh(dx + c)^4 \\
& + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2 * (3 \cosh(dx + c)^2 \\
& + 1) \sinh(dx + c)^2 + 2 \cosh(dx + c)^2 + 4 * (\cosh(dx + c)^3 + \cosh(dx + \\
& c)) \sinh(dx + c) + 1)) - 2 * (-b)^{2/3} * \log(((\cosh(dx + c)^2 + 2 \cosh(dx \\
& + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) * (-b)^{1/3} + (\cosh(dx + c)^2 + 2 \\
& * \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) * ((b \cosh(dx + c)^2 + b \\
& * \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3}) / (\cosh \\
& (dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1)) + 2 * b^{(\\
& 2/3) * \log(((\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^ \\
& 2 + 1) * b^{1/3} + (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx \\
& + c)^2 - 1) * ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 \\
& + \sinh(dx + c)^2 - 1))^{1/3}) / (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx \\
& + c) + \sinh(dx + c)^2 + 1))) / (b * d), -1/4 * (2 * \sqrt{3} * b * \sqrt{-(-b)^{1/3} / b} \\
& * \arctan(-1/3 * (\sqrt{3} * (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \si \\
& nh(dx + c)^2 + 1) * (-b)^{1/3} * \sqrt{-(-b)^{1/3} / b} - 2 * \sqrt{3} * (\cosh(dx + c \\
&)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) * ((b \cosh(dx + c \\
&)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1))^{1/3} \\
& * \sqrt{-(-b)^{1/3} / b}) / (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \si \\
& nh(dx + c)^2 + 1)) + 2 * \sqrt{3} * b^{2/3} * \arctan(-1/3 * \sqrt{3} * ((\cosh(dx + c) \\
& ^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) * b^{1/3} - 2 * (\cosh \\
& (dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) * ((b \cosh \\
& (dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx + c)^2 - 1 \\
&))^{1/3}) / ((\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c) \\
& ^2 + 1) * b^{1/3})) - (-b)^{2/3} * \log(((\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh \\
& (dx + c)^3 + \sinh(dx + c)^4 + 2 * (3 \cosh(dx + c)^2 + 1) \sinh(dx + c)^2 + \\
& 2 \cosh(dx + c)^2 + 4 * (\cosh(dx + c)^3 + \cosh(dx + c)) \sinh(dx + c) + 1) \\
& * (-b)^{2/3} - (\cosh(dx + c)^4 + 4 \cosh(dx + c)^3 \sinh(dx + c) + 6 \cosh(dx \\
& + c)^2 \sinh(dx + c)^2 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c) \\
& ^4 - 1) * (-b)^{1/3} * ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + \\
& c)^2 + \sinh(dx + c)^2 - 1))^{1/3} + (\cosh(dx + c)^4 + 4 \cosh(dx + c) \si \\
& nh(dx + c)^3 + \sinh(dx + c)^4 + 2 * (3 \cosh(dx + c)^2 - 1) \sinh(dx + c)^2 \\
& - 2 \cosh(dx + c)^2 + 4 * (\cosh(dx + c)^3 - \cosh(dx + c)) \sinh(dx + c) + \\
& 1) * ((b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + b) / (\cosh(dx + c)^2 + \sinh(dx \\
& + c)^2 - 1))^{2/3}) / (\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + s \\
& inh(dx + c)^4 + 2 * (3 \cosh(dx + c)^2 + 1) \sinh(dx + c)^2 + 2 \cosh(dx + c \\
&)^2 + 4 * (\cosh(dx + c)^3 + \cosh(dx + c)) \sinh(dx + c) + 1)) + b^{2/3} * \log \\
& (((\cosh(dx + c)^4 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 + 2 * \\
& (3 \cosh(dx + c)^2 + 1) \sinh(dx + c)^2 + 2 \cosh(dx + c)^2 + 4 * (\cosh(dx + \\
& c)^3 + \cosh(dx + c)) \sinh(dx + c) + 1) * b^{2/3} - (\cosh(dx + c)^4 + 4 \co \\
& sh(dx + c)^3 \sinh(dx + c) + 6 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4 \cosh(dx \\
& + c) \sinh(dx + c)^3 + \sinh(dx + c)^4 - 1) * b^{1/3} * ((b \cosh(dx + c)^2 +
\end{aligned}$$

```

b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (c
osh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*c
osh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 +
4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 +
1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c)
)*sinh(d*x + c) + 1)) + 2*(-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)
)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2*co
sh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*si
nh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*b^(2/3)
)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 +
sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)))/(b*d)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b \coth^2(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**2)**(1/3),x)

[Out] Integral((b*coth(c + d*x)**2)**(-1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^2)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)

$$3.26 \quad \int \frac{1}{\left(b \coth^2(c+dx)\right)^{2/3}} dx$$

Optimal. Leaf size=289

$$\frac{3 \coth(c+dx)}{d \left(b \coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{4/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)+1}\right)}{4d \left(b \coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{4/3}(c+dx) \log\left(\coth^{2/3}(c+dx) + \sqrt[3]{\coth(c+dx)+1}\right)}{4d \left(b \coth^2(c+dx)\right)^{2/3}}$$

[Out] $(-3*\text{Coth}[c + d*x])/(d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) - (\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) + (\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}]*\text{Coth}[c + d*x]^{(4/3)})/(d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) - (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) + (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^2)^{(2/3)})$

Rubi [A] time = 0.221971, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$\frac{3 \coth(c+dx)}{d \left(b \coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{4/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)+1}\right)}{4d \left(b \coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{4/3}(c+dx) \log\left(\coth^{2/3}(c+dx) + \sqrt[3]{\coth(c+dx)+1}\right)}{4d \left(b \coth^2(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^2)^(-2/3), x]

[Out] $(-3*\text{Coth}[c + d*x])/(d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) - (\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) + (\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}]*\text{Coth}[c + d*x]^{(4/3)})/(d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) - (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^2)^{(2/3)}) + (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^2)^{(2/3)})$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)^(n_)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_)^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b \coth^2(c + dx))^{2/3}} dx &= \frac{\coth^{4/3}(c + dx) \int \frac{1}{\coth^{4/3}(c + dx)} dx}{(b \coth^2(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{d (b \coth^2(c + dx))^{2/3}} + \frac{\coth^{4/3}(c + dx) \int \coth^{2/3}(c + dx) dx}{(b \coth^2(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{d (b \coth^2(c + dx))^{2/3}} - \frac{\coth^{4/3}(c + dx) \operatorname{Subst}\left(\int \frac{x^{2/3}}{-1+x^2} dx, x, \coth(c + dx)\right)}{d (b \coth^2(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{d (b \coth^2(c + dx))^{2/3}} - \frac{\left(3 \coth^{4/3}(c + dx)\right) \operatorname{Subst}\left(\int \frac{x^4}{-1+x^6} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d (b \coth^2(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{d (b \coth^2(c + dx))^{2/3}} + \frac{\coth^{4/3}(c + dx) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d (b \coth^2(c + dx))^{2/3}} + \frac{\coth^{4/3}(c + dx)}{d (b \coth^2(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{d (b \coth^2(c + dx))^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) \coth^{4/3}(c + dx)}{d (b \coth^2(c + dx))^{2/3}} - \frac{\coth^{4/3}(c + dx) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{4d (b \coth^2(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{d (b \coth^2(c + dx))^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) \coth^{4/3}(c + dx)}{d (b \coth^2(c + dx))^{2/3}} - \frac{\coth^{4/3}(c + dx) \log\left(1 - \sqrt[3]{\coth(c + dx)}\right)}{4d (b \coth^2(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{d (b \coth^2(c + dx))^{2/3}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{4/3}(c + dx)}{2d (b \coth^2(c + dx))^{2/3}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{4/3}(c + dx)}{2d (b \coth^2(c + dx))^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.0634714, size = 41, normalized size = 0.14

$$-\frac{3 \coth(c + dx) {}_2F_1\left(-\frac{1}{6}, 1; \frac{5}{6}; \coth^2(c + dx)\right)}{d (b \coth^2(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^2)^(-2/3), x]

[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(d*(b*Coth[c + d*x]^2)^(2/3))

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (b(\coth(dx+c))^2)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c)^2)^(2/3),x)

[Out] int(1/(b*coth(d*x+c)^2)^(2/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx+c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)

Fricas [B] time = 2.32815, size = 5739, normalized size = 19.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")

[Out] $\frac{1}{4} \cdot (2 \cdot \sqrt{3}) \cdot (b \cdot \cosh(dx+c)^2 + 2 \cdot b \cdot \cosh(dx+c) \cdot \sinh(dx+c) + b \cdot \sinh(dx+c)^2 + b) \cdot \sqrt{-(-b^2)^{1/3}} \cdot \arctan\left(\frac{1}{3} \cdot (2 \cdot \sqrt{3}) \cdot (-b^2)^{2/3} \cdot (\cosh(dx+c)^2 + 2 \cdot \cosh(dx+c) \cdot \sinh(dx+c) + \sinh(dx+c)^2 - 1) \cdot \sqrt{-(-b^2)^{1/3}} \cdot ((b \cdot \cosh(dx+c)^2 + b \cdot \sinh(dx+c)^2 + b) / (\cosh(dx+c)^2 + \sinh(dx+c)^2 - 1))^{1/3} - \sqrt{3} \cdot (b \cdot \cosh(dx+c)^2 + 2 \cdot b \cdot \cosh(dx+c) \cdot \sinh(dx+c) + b \cdot \sinh(dx+c)^2 + b) \cdot (-b^2)^{1/3} \cdot \sqrt{-(-b^2)^{1/3}}\right) / (b^2 \cdot \cosh(dx+c)^2 + 2 \cdot b^2 \cdot \cosh(dx+c) \cdot \sinh(dx+c) + b^2 \cdot \sinh(dx+c)^2)$

$$\begin{aligned}
& + c)^2 + b^2)) + 2\sqrt{3}*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x \\
& + c) + b*\sinh(d*x + c)^2 + b)*(b^2)^{(1/6)}*\arctan(1/3*\sqrt{3}*(b^2)^{(1/6)}*(\\
& 2*(b^2)^{(2/3)}*(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + \\
& c)^2 - 1)*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \\
& \sinh(d*x + c)^2 - 1))^{(1/3)} - (b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d \\
& *x + c) + b*\sinh(d*x + c)^2 + b)*(b^2)^{(1/3)})/(b^2*\cosh(d*x + c)^2 + 2*b^2* \\
& \cosh(d*x + c)*\sinh(d*x + c) + b^2*\sinh(d*x + c)^2 + b^2)) + (-b^2)^{(2/3)}*(c \\
& osh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1)*\log((\\
& (\cosh(d*x + c)^4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*\cosh(d*x + c)^2*\sinh \\
& (d*x + c)^2 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 - 1)*(-b^2) \\
& ^{(2/3)}*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh \\
& (d*x + c)^2 - 1))^{(1/3)} + (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + \\
& c)^3 + b*\sinh(d*x + c)^4 - 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - \\
& b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - b*\cosh(d*x + c))*\sinh(d*x + c) \\
& + b)*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d \\
& *x + c)^2 - 1))^{(2/3)} - (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c \\
&)^3 + b*\sinh(d*x + c)^4 + 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + b) \\
& *\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c) + \\
& b)*(-b^2)^{(1/3)})/(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(\\
& d*x + c)^4 + 2*(3*\cosh(d*x + c)^2 + 1)*\sinh(d*x + c)^2 + 2*\cosh(d*x + c)^2 \\
& + 4*(\cosh(d*x + c)^3 + \cosh(d*x + c))*\sinh(d*x + c) + 1)) - (b^2)^{(2/3)}*(co \\
& sh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1)*\log(- \\
& (\cosh(d*x + c)^4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*\cosh(d*x + c)^2*\sinh \\
& (d*x + c)^2 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4 - 1)*(b^2)^{(\\
& 2/3)}*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(\\
& d*x + c)^2 - 1))^{(1/3)} - (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + \\
& c)^3 + b*\sinh(d*x + c)^4 - 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - b \\
&)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - b*\cosh(d*x + c))*\sinh(d*x + c) + \\
& b)*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d \\
& x + c)^2 - 1))^{(2/3)} - (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c) \\
& ^3 + b*\sinh(d*x + c)^4 + 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + b)* \\
& \sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c) + b \\
&)*(b^2)^{(1/3)})/(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d \\
& x + c)^4 + 2*(3*\cosh(d*x + c)^2 + 1)*\sinh(d*x + c)^2 + 2*\cosh(d*x + c)^2 + \\
& 4*(\cosh(d*x + c)^3 + \cosh(d*x + c))*\sinh(d*x + c) + 1)) - 2*(-b^2)^{(2/3)}*(c \\
& osh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1)*\log(- \\
& ((-b^2)^{(2/3)}*(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + \\
& c)^2 + 1) - (b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(\\
& d*x + c)^2 - b)*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c) \\
& ^2 + \sinh(d*x + c)^2 - 1))^{(1/3)})/(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d \\
& *x + c) + \sinh(d*x + c)^2 + 1)) + 2*(b^2)^{(2/3)}*(\cosh(d*x + c)^2 + 2*\cosh(d \\
& *x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1)*\log(((b^2)^{(2/3)}*(\cosh(d*x + c \\
&)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1) + (b*\cosh(d*x + \\
& c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 - b)*((b*\cosh(d \\
& x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^{(1/3)})
\end{aligned}$$

$(1/3))/(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 + 1)) - 12*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 - b)*((b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + b)/(\cosh(d*x + c)^2 + \sinh(d*x + c)^2 - 1))^(1/3))/ (b^2*d*\cosh(d*x + c)^2 + 2*b^2*d*\cosh(d*x + c)*\sinh(d*x + c) + b^2*d*\sinh(d*x + c)^2 + b^2*d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^2(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**2)**(2/3),x)

[Out] Integral((b*coth(c + d*x)**2)**(-2/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^2)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)

$$3.27 \quad \int \frac{1}{\left(b \coth^2(c+dx)\right)^{4/3}} dx$$

Optimal. Leaf size=309

$$\frac{\coth^{2/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)} + 1\right)}{4bd\sqrt[3]{b \coth^2(c+dx)}} + \frac{\coth^{2/3}(c+dx) \log\left(\coth^{2/3}(c+dx) + \sqrt[3]{\coth(c+dx)} + 1\right)}{4bd\sqrt[3]{b \coth^2(c+dx)}}$$

[Out] $-(\text{Sqrt}[3] * \text{ArcTan}[(1 - 2 * \text{Coth}[c + d * x]^{(1/3)}) / \text{Sqrt}[3]] * \text{Coth}[c + d * x]^{(2/3)}) / (2 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{Sqrt}[3] * \text{ArcTan}[(1 + 2 * \text{Coth}[c + d * x]^{(1/3)}) / \text{Sqrt}[3]] * \text{Coth}[c + d * x]^{(2/3)}) / (2 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{ArcTanh}[\text{Coth}[c + d * x]^{(1/3)}] * \text{Coth}[c + d * x]^{(2/3)}) / (b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) - (\text{Coth}[c + d * x]^{(2/3)} * \text{Log}[1 - \text{Coth}[c + d * x]^{(1/3)} + \text{Coth}[c + d * x]^{(2/3)}]) / (4 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{Coth}[c + d * x]^{(2/3)} * \text{Log}[1 + \text{Coth}[c + d * x]^{(1/3)} + \text{Coth}[c + d * x]^{(2/3)}]) / (4 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) - (3 * \text{Tanh}[c + d * x]) / (5 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)})$

Rubi [A] time = 0.182234, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3474, 3476, 329, 210, 634, 618, 204, 628, 206}

$$\frac{\coth^{2/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)} + 1\right)}{4bd\sqrt[3]{b \coth^2(c+dx)}} + \frac{\coth^{2/3}(c+dx) \log\left(\coth^{2/3}(c+dx) + \sqrt[3]{\coth(c+dx)} + 1\right)}{4bd\sqrt[3]{b \coth^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^2)^(-4/3), x]

[Out] $-(\text{Sqrt}[3] * \text{ArcTan}[(1 - 2 * \text{Coth}[c + d * x]^{(1/3)}) / \text{Sqrt}[3]] * \text{Coth}[c + d * x]^{(2/3)}) / (2 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{Sqrt}[3] * \text{ArcTan}[(1 + 2 * \text{Coth}[c + d * x]^{(1/3)}) / \text{Sqrt}[3]] * \text{Coth}[c + d * x]^{(2/3)}) / (2 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{ArcTanh}[\text{Coth}[c + d * x]^{(1/3)}] * \text{Coth}[c + d * x]^{(2/3)}) / (b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) - (\text{Coth}[c + d * x]^{(2/3)} * \text{Log}[1 - \text{Coth}[c + d * x]^{(1/3)} + \text{Coth}[c + d * x]^{(2/3)}]) / (4 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) + (\text{Coth}[c + d * x]^{(2/3)} * \text{Log}[1 + \text{Coth}[c + d * x]^{(1/3)} + \text{Coth}[c + d * x]^{(2/3)}]) / (4 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)}) - (3 * \text{Tanh}[c + d * x]) / (5 * b * d * (b * \text{Coth}[c + d * x]^2)^{(1/3)})$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.)] /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_))^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_))^(n_))^(p_), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b \coth^2(c + dx))^{4/3}} dx &= \frac{\coth^{2/3}(c + dx) \int \frac{1}{\coth^{8/3}(c+dx)} dx}{b \sqrt[3]{b \coth^2(c + dx)}} \\
&= -\frac{3 \tanh(c + dx)}{5bd \sqrt[3]{b \coth^2(c + dx)}} + \frac{\coth^{2/3}(c + dx) \int \frac{1}{\coth^{2/3}(c+dx)} dx}{b \sqrt[3]{b \coth^2(c + dx)}} \\
&= -\frac{3 \tanh(c + dx)}{5bd \sqrt[3]{b \coth^2(c + dx)}} - \frac{\coth^{2/3}(c + dx) \operatorname{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)} dx, x, \coth(c + dx)\right)}{bd \sqrt[3]{b \coth^2(c + dx)}} \\
&= -\frac{3 \tanh(c + dx)}{5bd \sqrt[3]{b \coth^2(c + dx)}} - \frac{\left(3 \coth^{2/3}(c + dx)\right) \operatorname{Subst}\left(\int \frac{1}{-1+x^6} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{bd \sqrt[3]{b \coth^2(c + dx)}} \\
&= -\frac{3 \tanh(c + dx)}{5bd \sqrt[3]{b \coth^2(c + dx)}} + \frac{\coth^{2/3}(c + dx) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{bd \sqrt[3]{b \coth^2(c + dx)}} + \frac{\coth^{2/3}(c + dx)}{4bd \sqrt[3]{b \coth^2(c + dx)}} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) \coth^{2/3}(c + dx)}{bd \sqrt[3]{b \coth^2(c + dx)}} - \frac{3 \tanh(c + dx)}{5bd \sqrt[3]{b \coth^2(c + dx)}} - \frac{\coth^{2/3}(c + dx) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{4bd \sqrt[3]{b \coth^2(c + dx)}} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) \coth^{2/3}(c + dx)}{bd \sqrt[3]{b \coth^2(c + dx)}} - \frac{\coth^{2/3}(c + dx) \log\left(1 - \sqrt[3]{\coth(c + dx)} + \coth^{2/3}(c + dx)\right)}{4bd \sqrt[3]{b \coth^2(c + dx)}} \\
&= -\frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{2/3}(c + dx)}{2bd \sqrt[3]{b \coth^2(c + dx)}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{2/3}(c + dx)}{2bd \sqrt[3]{b \coth^2(c + dx)}} + \frac{\coth^{2/3}(c + dx)}{4bd \sqrt[3]{b \coth^2(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 0.153863, size = 43, normalized size = 0.14

$$-\frac{3 \coth(c + dx) {}_2F_1\left(-\frac{5}{6}, 1; \frac{1}{6}; \coth^2(c + dx)\right)}{5d (b \coth^2(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^2)^(-4/3), x]

[Out] $(-3*\text{Coth}[c + d*x]*\text{Hypergeometric2F1}[-5/6, 1, 1/6, \text{Coth}[c + d*x]^2])/(5*d*(b*\text{Coth}[c + d*x]^2)^{(4/3)})$

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (b(\coth(dx + c))^2)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*coth(d*x+c)^2)^(4/3), x)`

[Out] `int(1/(b*coth(d*x+c)^2)^(4/3), x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^2)^(4/3), x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c)^2)^(-4/3), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^2)^(4/3), x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^2(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**2)**(4/3),x)

[Out] Integral((b*coth(c + d*x)**2)**(-4/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^2)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^2)^(-4/3), x)

3.28 $\int (b \coth^3(c + dx))^n dx$

Optimal. Leaf size=55

$$\frac{\coth(c + dx) (b \coth^3(c + dx))^n {}_2F_1\left(1, \frac{1}{2}(3n + 1); \frac{3(n+1)}{2}; \coth^2(c + dx)\right)}{d(3n + 1)}$$

[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))

Rubi [A] time = 0.0394953, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3658, 3476, 364}

$$\frac{\coth(c + dx) (b \coth^3(c + dx))^n {}_2F_1\left(1, \frac{1}{2}(3n + 1); \frac{3(n+1)}{2}; \coth^2(c + dx)\right)}{d(3n + 1)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^3)^n,x]

[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (b \coth^3(c + dx))^n dx &= \left(\coth^{-3n}(c + dx) (b \coth^3(c + dx))^n \right) \int \coth^{3n}(c + dx) dx \\ &= \frac{\left(\coth^{-3n}(c + dx) (b \coth^3(c + dx))^n \right) \text{Subst} \left(\int \frac{x^{3n}}{-1+x^2} dx, x, \coth(c + dx) \right)}{d} \\ &= \frac{\coth(c + dx) (b \coth^3(c + dx))^n {}_2F_1 \left(1, \frac{1}{2}(1 + 3n); \frac{3(1+n)}{2}; \coth^2(c + dx) \right)}{d(1 + 3n)} \end{aligned}$$

Mathematica [A] time = 0.0411529, size = 53, normalized size = 0.96

$$\frac{\coth(c + dx) (b \coth^3(c + dx))^n {}_2F_1 \left(1, \frac{1}{2}(3n + 1); \frac{3(n+1)}{2}; \coth^2(c + dx) \right)}{3dn + d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/ (d + 3*d*n)
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int (b (\coth(dx + c))^3)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^n,x)
```

```
[Out] int((b*coth(d*x+c)^3)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^3)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^3)^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \coth(dx + c)^3\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="fricas")

[Out] integral((b*coth(d*x + c)^3)^n, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth^3(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**3)**n,x)

[Out] Integral((b*coth(c + d*x)**3)**n, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^3)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^n, x)
```

3.29 $\int (b \coth^3(c + dx))^{3/2} dx$

Optimal. Leaf size=134

$$\frac{2b \coth^2(c + dx) \sqrt{b \coth^3(c + dx)}}{7d} - \frac{2b \sqrt{b \coth^3(c + dx)}}{3d} - \frac{b \sqrt{b \coth^3(c + dx)} \tan^{-1}(\sqrt{\coth(c + dx)})}{d \coth^{\frac{3}{2}}(c + dx)} + \frac{b \sqrt{b \coth^3(c + dx)}}{d \coth^{\frac{3}{2}}(c + dx)}$$

[Out] $(-2*b*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])/(3*d) - (b*\text{ArcTan}[\text{Sqrt}[\text{Coth}[c + d*x]]]*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])/(d*\text{Coth}[c + d*x]^{(3/2)}) + (b*\text{ArcTanh}[\text{Sqrt}[\text{Coth}[c + d*x]]]*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])/(d*\text{Coth}[c + d*x]^{(3/2)}) - (2*b*\text{Coth}[c + d*x]^2*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])/(7*d)$

Rubi [A] time = 0.0602026, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3658, 3473, 3476, 329, 298, 203, 206}

$$\frac{2b \coth^2(c + dx) \sqrt{b \coth^3(c + dx)}}{7d} - \frac{2b \sqrt{b \coth^3(c + dx)}}{3d} - \frac{b \sqrt{b \coth^3(c + dx)} \tan^{-1}(\sqrt{\coth(c + dx)})}{d \coth^{\frac{3}{2}}(c + dx)} + \frac{b \sqrt{b \coth^3(c + dx)}}{d \coth^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x]^3)^{(3/2)}, x]$

[Out] $(-2*b*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])/(3*d) - (b*\text{ArcTan}[\text{Sqrt}[\text{Coth}[c + d*x]]]*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])/(d*\text{Coth}[c + d*x]^{(3/2)}) + (b*\text{ArcTanh}[\text{Sqrt}[\text{Coth}[c + d*x]]]*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])/(d*\text{Coth}[c + d*x]^{(3/2)}) - (2*b*\text{Coth}[c + d*x]^2*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])/(7*d)$

Rule 3658

$\text{Int}[(u_*)*((b_*)*\tan[(e_*) + (f_*)*(x_)]^{(n_)]^{(p_)}, x_Symbol] :> \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[(b*ff^n)^{\text{IntPart}[p]}*(b*\text{Tan}[e + f*x]^n)^{\text{FracPart}[p]}]/(\text{Tan}[e + f*x]/ff)^{(n*\text{FracPart}[p])}, \text{Int}[\text{ActivateTrig}[u]*(\text{Tan}[e + f*x]/ff)^{(n*p)}, x], x]] /;$ $\text{FreeQ}[\{b, e, f, n, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{EqQ}[u, 1] \ || \ \text{MatchQ}[u, ((d_*)*(\text{trig}_)[e + f*x])^{(m_)}]) /;$ $\text{FreeQ}[\{d, m\}, x] \ \&\& \ \text{MemberQ}[\{\sin, \cos, \tan, \cot, \sec, \csc\}, \text{trig}]])$

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && ! GtQ[a/b, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int (b \coth^3(c + dx))^{3/2} dx &= \frac{\left(b\sqrt{b \coth^3(c + dx)}\right) \int \coth^{\frac{9}{2}}(c + dx) dx}{\coth^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2b \coth^2(c + dx)\sqrt{b \coth^3(c + dx)}}{7d} + \frac{\left(b\sqrt{b \coth^3(c + dx)}\right) \int \coth^{\frac{5}{2}}(c + dx) dx}{\coth^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2b\sqrt{b \coth^3(c + dx)}}{3d} - \frac{2b \coth^2(c + dx)\sqrt{b \coth^3(c + dx)}}{7d} + \frac{\left(b\sqrt{b \coth^3(c + dx)}\right) \int \sqrt{\coth(c + dx)} dx}{\coth^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2b\sqrt{b \coth^3(c + dx)}}{3d} - \frac{2b \coth^2(c + dx)\sqrt{b \coth^3(c + dx)}}{7d} - \frac{\left(b\sqrt{b \coth^3(c + dx)}\right) \text{Subst}\left(\int \sqrt{\coth(u)} du\right)}{d \coth^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2b\sqrt{b \coth^3(c + dx)}}{3d} - \frac{2b \coth^2(c + dx)\sqrt{b \coth^3(c + dx)}}{7d} - \frac{\left(2b\sqrt{b \coth^3(c + dx)}\right) \text{Subst}\left(\int \sqrt{\coth(u)} du\right)}{d \coth^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2b\sqrt{b \coth^3(c + dx)}}{3d} - \frac{2b \coth^2(c + dx)\sqrt{b \coth^3(c + dx)}}{7d} + \frac{\left(b\sqrt{b \coth^3(c + dx)}\right) \text{Subst}\left(\int \sqrt{\coth(u)} du\right)}{d \coth^{\frac{3}{2}}(c + dx)} \\
&= -\frac{2b\sqrt{b \coth^3(c + dx)}}{3d} - \frac{b \tan^{-1}\left(\sqrt{\coth(c + dx)}\right)\sqrt{b \coth^3(c + dx)}}{d \coth^{\frac{3}{2}}(c + dx)} + \frac{b \tanh^{-1}\left(\sqrt{\coth(c + dx)}\right)\sqrt{b \coth^3(c + dx)}}{d \coth^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

Mathematica [A] time = 0.518629, size = 82, normalized size = 0.61

$$\frac{(b \coth^3(c + dx))^{3/2} \left(6 \coth^{\frac{7}{2}}(c + dx) + 14 \coth^{\frac{3}{2}}(c + dx) + 21 \tan^{-1}\left(\sqrt{\coth(c + dx)}\right) - 21 \tanh^{-1}\left(\sqrt{\coth(c + dx)}\right)\right)}{21d \coth^{\frac{9}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^3)^(3/2),x]

[Out] -((b*Coth[c + d*x]^3)^(3/2)*(21*ArcTan[Sqrt[Coth[c + d*x]]] - 21*ArcTanh[Sqrt[Coth[c + d*x]]] + 14*Coth[c + d*x]^(3/2) + 6*Coth[c + d*x]^(7/2)))/(21*d*Coth[c + d*x]^(9/2))

Maple [A] time = 0.04, size = 107, normalized size = 0.8

$$\frac{1}{21 d (\coth(dx + c))^3 b^2} \left(b (\coth(dx + c))^3 \right)^{\frac{3}{2}} \left(21 b^{7/2} \operatorname{Arctanh} \left(\frac{\sqrt{b \coth(dx + c)}}{\sqrt{b}} \right) - 21 b^{7/2} \arctan \left(\frac{\sqrt{b \coth(dx + c)}}{\sqrt{b}} \right) \right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*coth(d*x+c)^3)^(3/2), x)`

[Out] $\frac{1}{21} \frac{1}{d} (b \coth(dx + c)^3)^{3/2} \left(21 b^{7/2} \operatorname{arctanh}(\coth(dx + c))^{1/2} / b^{1/2} - 21 b^{7/2} \arctan(\coth(dx + c))^{1/2} / b^{1/2} - 6 (b \coth(dx + c))^{7/2} - 14 b^2 (b \coth(dx + c))^{3/2} / \coth(dx + c)^3 / (b \coth(dx + c))^{3/2} / b^2 \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^3)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c)^3)^(3/2), x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c)^3)^(3/2), x)`

Fricas [B] time = 3.08659, size = 5846, normalized size = 43.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c)^3)^(3/2), x, algorithm="fricas")`

[Out] $[-1/84 * (42 * (b \cosh(dx + c))^6 + 6 * b * \cosh(dx + c) * \sinh(dx + c)^5 + b * \sinh(dx + c)^6 - 3 * b * \cosh(dx + c)^4 + 3 * (5 * b * \cosh(dx + c)^2 - b) * \sinh(dx + c)^4 + 4 * (5 * b * \cosh(dx + c)^3 - 3 * b * \cosh(dx + c)) * \sinh(dx + c)^3 + 3 * b * \cosh(dx + c)^2 + 3 * (5 * b * \cosh(dx + c)^4 - 6 * b * \cosh(dx + c)^2 + b) * \sinh(dx + c)^2 + 6 * (b * \cosh(dx + c)^5 - 2 * b * \cosh(dx + c)^3 + b * \cosh(dx + c)) * \sinh(dx + c) - b) * \sqrt{-b} * \arctan(\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + c)^2) * \sqrt{-b} * \sqrt{b * \cosh(dx + c) / \sinh(dx + c)}) / (b * \cosh(dx + c)^3)^{3/2}$

$$\begin{aligned}
& (d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2 \\
& 1*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 \\
& - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(\\
& 5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c \\
&)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6 \\
& *(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) \\
& - b)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + \\
& 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + \\
& b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si \\
& nh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cos \\
& h(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x \\
& + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(5*b*cos \\
& h(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d*x + c)^6 + b \\
& *cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 4*(25*b*cos \\
& h(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (75*b \\
& *cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(15*b*cosh(\\
& d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 5*b)*sq \\
& rt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*s \\
& inh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x \\
& + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*s \\
& inh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d* \\
& x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 \\
& + d*cosh(d*x + c))*sinh(d*x + c) - d), -1/84*(42*(b*cosh(d*x + c)^6 + 6*b*c \\
& osh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3* \\
& (5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*co \\
& sh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6 \\
& *b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*co \\
& sh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*arctan(sqrt(b)* \\
& sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)* \\
& sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 21*(b*cosh(d*x + c)^6 + 6*b*cosh(\\
& d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b \\
& *cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d \\
& *x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6 \\
& *b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d \\
& *x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*log(2*b*cosh(d*x + \\
& c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + \\
& c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d* \\
& x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x \\
& + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh \\
& (d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) \\
& + 16*(5*b*cosh(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d \\
& *x + c)^6 + b*cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 \\
& + 4*(25*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + \\
& c)^2 + (75*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + \\
& 2*(15*b*cosh(d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x +
\end{aligned}$$

$c) + 5*b)*\text{sqrt}(b*\cosh(d*x + c)/\sinh(d*x + c)))/(d*\cosh(d*x + c)^6 + 6*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + d*\sinh(d*x + c)^6 - 3*d*\cosh(d*x + c)^4 + 3*(5*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^4 + 4*(5*d*\cosh(d*x + c)^3 - 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*d*\cosh(d*x + c)^2 + 3*(5*d*\cosh(d*x + c)^4 - 6*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 6*(d*\cosh(d*x + c)^5 - 2*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) - d]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth^3(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**3)**(3/2),x)

[Out] Integral((b*coth(c + d*x)**3)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^3)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^3)^(3/2), x)

3.30 $\int \sqrt{b \coth^3(c + dx)} dx$

Optimal. Leaf size=104

$$\frac{\sqrt{b \coth^3(c + dx)} \tan^{-1}(\sqrt{\coth(c + dx)})}{d \coth^{\frac{3}{2}}(c + dx)} + \frac{\sqrt{b \coth^3(c + dx)} \tanh^{-1}(\sqrt{\coth(c + dx)})}{d \coth^{\frac{3}{2}}(c + dx)} - \frac{2 \tanh(c + dx) \sqrt{b \coth^3(c + dx)}}{d}$$

[Out] (ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) + (ArcTanh[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) - (2*Sqrt[b*Coth[c + d*x]^3]*Tanh[c + d*x])/d

Rubi [A] time = 0.0474185, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3658, 3473, 3476, 329, 212, 206, 203}

$$\frac{\sqrt{b \coth^3(c + dx)} \tan^{-1}(\sqrt{\coth(c + dx)})}{d \coth^{\frac{3}{2}}(c + dx)} + \frac{\sqrt{b \coth^3(c + dx)} \tanh^{-1}(\sqrt{\coth(c + dx)})}{d \coth^{\frac{3}{2}}(c + dx)} - \frac{2 \tanh(c + dx) \sqrt{b \coth^3(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Coth[c + d*x]^3], x]

[Out] (ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) + (ArcTanh[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) - (2*Sqrt[b*Coth[c + d*x]^3]*Tanh[c + d*x])/d

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

$x]$ /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \sqrt{b \coth^3(c+dx)} dx &= \frac{\sqrt{b \coth^3(c+dx)} \int \coth^{\frac{3}{2}}(c+dx) dx}{\coth^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2\sqrt{b \coth^3(c+dx)} \tanh(c+dx)}{d} + \frac{\sqrt{b \coth^3(c+dx)} \int \frac{1}{\sqrt{\coth(c+dx)}} dx}{\coth^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2\sqrt{b \coth^3(c+dx)} \tanh(c+dx)}{d} - \frac{\sqrt{b \coth^3(c+dx)} \operatorname{Subst}\left(\int \frac{1}{\sqrt{x(-1+x^2)}} dx, x, \coth(c+dx)\right)}{d \coth^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2\sqrt{b \coth^3(c+dx)} \tanh(c+dx)}{d} - \frac{\left(2\sqrt{b \coth^3(c+dx)}\right) \operatorname{Subst}\left(\int \frac{1}{-1+x^4} dx, x, \sqrt{\coth(c+dx)}\right)}{d \coth^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2\sqrt{b \coth^3(c+dx)} \tanh(c+dx)}{d} + \frac{\sqrt{b \coth^3(c+dx)} \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(c+dx)}\right)}{d \coth^{\frac{3}{2}}(c+dx)} + \dots \\
&= \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right) \sqrt{b \coth^3(c+dx)}}{d \coth^{\frac{3}{2}}(c+dx)} + \frac{\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right) \sqrt{b \coth^3(c+dx)}}{d \coth^{\frac{3}{2}}(c+dx)} - \frac{2\sqrt{b \coth^3(c+dx)}}{d \coth^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.0962737, size = 63, normalized size = 0.61

$$\frac{\sqrt{b \coth^3(c+dx)} \left(-2\sqrt{\coth(c+dx)} + \tan^{-1}\left(\sqrt{\coth(c+dx)}\right) + \tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{d \coth^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Coth[c + d*x]^3], x]

[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth[c + d*x]])*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2))

Maple [A] time = 0.045, size = 86, normalized size = 0.8

$$\frac{1}{d \coth(dx+c)} \sqrt{b (\coth(dx+c))^3} \left(-2\sqrt{b \coth(dx+c)} + \sqrt{b} \operatorname{Artanh}\left(\sqrt{b \coth(dx+c)} \frac{1}{\sqrt{b}}\right) + \sqrt{b} \operatorname{arctan}\left(\sqrt{b \coth(dx+c)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*coth(d*x+c)^3)^(1/2),x)`

[Out] $1/d*(b*\coth(d*x+c)^3)^{(1/2)}/\coth(d*x+c)/(b*\coth(d*x+c))^{(1/2)}*(-2*(b*\coth(d*x+c))^{(1/2)}+b^{(1/2)}*\operatorname{arctanh}((b*\coth(d*x+c))^{(1/2)}/b^{(1/2)}))+b^{(1/2)}*\operatorname{arctan}((b*\coth(d*x+c))^{(1/2)}/b^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*coth(d*x + c)^3), x)`

Fricas [B] time = 2.18796, size = 1743, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")`

[Out] $[-1/4*(2*\sqrt{-b}*\operatorname{arctan}((\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2)*\sqrt{-b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)})/(b*\cosh(dx + c)^2 + 2*b*\cosh(dx + c)*\sinh(dx + c) + b*\sinh(dx + c)^2 + b)) - \sqrt{-b}*\log(-(b*\cosh(dx + c)^4 + 4*b*\cosh(dx + c)^3*\sinh(dx + c) + 6*b*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*b*\cosh(dx + c)*\sinh(dx + c)^3 + b*\sinh(dx + c)^4 - 2*(\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 - 1)*\sqrt{-b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)} - 2*b)/(\cosh(dx + c)^4 + 4*\cosh(dx + c)^3*\sinh(dx + c) + 6*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4)) + 8*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)})/d, 1/4*(2*\sqrt{b}*\operatorname{arctan}(\sqrt{b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)})/(b*\cosh(dx + c)^2 + 2*b*\cosh(dx + c)*\sinh(dx + c) + b*\sinh(dx + c)^2 + b)) + \sqrt{b}*\log(2*b*\cosh(dx + c)^4 + 8*b*\cosh(dx + c)^3*\sinh(dx + c) + 12*b*\cosh(dx + c)^2*\sinh(dx + c)^2 + 8*b*\cosh(dx + c)*\sinh(dx + c)^3 + b*\sinh(dx + c)^4) - \sqrt{b}*\log(2*b*\cosh(dx + c)^4 + 4*b*\cosh(dx + c)^3*\sinh(dx + c) + 6*b*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*b*\cosh(dx + c)*\sinh(dx + c)^3 + b*\sinh(dx + c)^4 - 2*(\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 - 1)*\sqrt{b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)} - 2*b)/(\cosh(dx + c)^4 + 4*\cosh(dx + c)^3*\sinh(dx + c) + 6*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4))]/d$

```
*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh
(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - c
osh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(
b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth^3(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^3), x)
```


$$3.31 \quad \int \frac{1}{\sqrt{b \coth^3(c+dx)}} dx$$

Optimal. Leaf size=105

$$\frac{2 \coth(c+dx)}{d\sqrt{b \coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx) \tan^{-1}(\sqrt{\coth(c+dx)})}{d\sqrt{b \coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx) \tanh^{-1}(\sqrt{\coth(c+dx)})}{d\sqrt{b \coth^3(c+dx)}}$$

[Out] $(-2*\text{Coth}[c + d*x])/(d*\text{Sqrt}[b*\text{Coth}[c + d*x]^3]) - (\text{ArcTan}[\text{Sqrt}[\text{Coth}[c + d*x]]]*\text{Coth}[c + d*x]^{(3/2)})/(d*\text{Sqrt}[b*\text{Coth}[c + d*x]^3]) + (\text{ArcTanh}[\text{Sqrt}[\text{Coth}[c + d*x]]]*\text{Coth}[c + d*x]^{(3/2)})/(d*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])$

Rubi [A] time = 0.0480527, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3658, 3474, 3476, 329, 298, 203, 206}

$$\frac{2 \coth(c+dx)}{d\sqrt{b \coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx) \tan^{-1}(\sqrt{\coth(c+dx)})}{d\sqrt{b \coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx) \tanh^{-1}(\sqrt{\coth(c+dx)})}{d\sqrt{b \coth^3(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/\text{Sqrt}[b*\text{Coth}[c + d*x]^3], x]$

[Out] $(-2*\text{Coth}[c + d*x])/(d*\text{Sqrt}[b*\text{Coth}[c + d*x]^3]) - (\text{ArcTan}[\text{Sqrt}[\text{Coth}[c + d*x]]]*\text{Coth}[c + d*x]^{(3/2)})/(d*\text{Sqrt}[b*\text{Coth}[c + d*x]^3]) + (\text{ArcTanh}[\text{Sqrt}[\text{Coth}[c + d*x]]]*\text{Coth}[c + d*x]^{(3/2)})/(d*\text{Sqrt}[b*\text{Coth}[c + d*x]^3])$

Rule 3658

$\text{Int}[(u_.)*((b_.)*\tan[(e_.) + (f_.)*(x_)]^{(n_)})^{(p_)}, x_Symbol] := \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[(b*ff^n)^{\text{IntPart}[p]}*(b*\text{Tan}[e + f*x]^n)^{\text{FracPart}[p]}]/(\text{Tan}[e + f*x]/ff)^{(n*\text{FracPart}[p])}, \text{Int}[\text{ActivateTrig}[u]*(\text{Tan}[e + f*x]/ff)^{(n*p)}, x], x]] /; \text{FreeQ}\{b, e, f, n, p\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{EqQ}[u, 1] \ || \ \text{MatchQ}[u, ((d_.)*(\text{trig}_)[e + f*x])^{(m_)}]) /; \text{FreeQ}\{d, m\}, x \ \&\& \ \text{MemberQ}\{\{\sin, \cos, \tan, \cot, \sec, \csc\}, \text{trig}\}$

Rule 3474

$\text{Int}[(b_.)*\tan[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] := \text{Simp}[(b*\text{Tan}[c + d*x])^{(n+1)}/(b*d*(n+1)), x] - \text{Dist}[1/b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n+2)}, x],$

x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && ! GtQ[a/b, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{b \coth^3(c+dx)}} dx &= \frac{\coth^{\frac{3}{2}}(c+dx) \int \frac{1}{\coth^{\frac{3}{2}}(c+dx)} dx}{\sqrt{b \coth^3(c+dx)}} \\
&= -\frac{2 \coth(c+dx)}{d\sqrt{b \coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx) \int \sqrt{\coth(c+dx)} dx}{\sqrt{b \coth^3(c+dx)}} \\
&= -\frac{2 \coth(c+dx)}{d\sqrt{b \coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx) \operatorname{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2} dx, x, \coth(c+dx)\right)}{d\sqrt{b \coth^3(c+dx)}} \\
&= -\frac{2 \coth(c+dx)}{d\sqrt{b \coth^3(c+dx)}} - \frac{\left(2 \coth^{\frac{3}{2}}(c+dx)\right) \operatorname{Subst}\left(\int \frac{x^2}{-1+x^4} dx, x, \sqrt{\coth(c+dx)}\right)}{d\sqrt{b \coth^3(c+dx)}} \\
&= -\frac{2 \coth(c+dx)}{d\sqrt{b \coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(c+dx)}\right)}{d\sqrt{b \coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b \coth^3(c+dx)}} \\
&= -\frac{2 \coth(c+dx)}{d\sqrt{b \coth^3(c+dx)}} - \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right) \coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b \coth^3(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right) \coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b \coth^3(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.0294052, size = 41, normalized size = 0.39

$$-\frac{2 \coth(c+dx) {}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; \coth^2(c+dx)\right)}{d\sqrt{b \coth^3(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*Coth[c + d*x]^3], x]

[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, Coth[c + d*x]^2])/(d*Sqrt[b*Coth[c + d*x]^3])

Maple [A] time = 0.041, size = 91, normalized size = 0.9

$$\frac{\coth(dx+c)}{d} \left(-2b^{5/2} + \operatorname{Artanh}\left(\sqrt{b \coth(dx+c)} \frac{1}{\sqrt{b}}\right) b^2 \sqrt{b \coth(dx+c)} - \arctan\left(\sqrt{b \coth(dx+c)} \frac{1}{\sqrt{b}}\right) b^2 \sqrt{b \coth(dx+c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*coth(d*x+c)^3)^(1/2),x)`

[Out] $1/d*\coth(d*x+c)*(-2*b^(5/2)+\operatorname{arctanh}((b*\coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*\coth(d*x+c))^(1/2)-\operatorname{arctan}((b*\coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*\coth(d*x+c))^(1/2))/(b*\coth(d*x+c)^3)^(1/2)/b^(5/2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth(dx + c)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(b*coth(d*x + c)^3), x)`

Fricas [B] time = 2.38283, size = 2554, normalized size = 24.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")`

[Out] $[-1/4*(2*(\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 + 1)*\sqrt{-b}*\arctan((\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2)*\sqrt{-b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)})/(b*\cosh(dx + c)^2 + 2*b*\cosh(dx + c)*\sinh(dx + c) + b*\sinh(dx + c)^2 + b)) + (\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 + 1)*\sqrt{-b}*\log(-b*\cosh(dx + c)^4 + 4*b*\cosh(dx + c)^3*\sinh(dx + c) + 6*b*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*b*\cosh(dx + c)*\sinh(dx + c)^3 + b*\sinh(dx + c)^4 - 2*(\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 - 1)*\sqrt{-b}*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)} - 2*b)/(\cosh(dx + c)^4 + 4*\cosh(dx + c)^3*\sinh(dx + c) + 6*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4) + 8*(\cosh(dx + c)^2 + 2*\cosh(dx + c)*\sinh(dx + c) + \sinh(dx + c)^2 - 1)*\sqrt{b*\cosh(dx + c)/\sinh(dx + c)}$

```

+ c)))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh
(d*x + c)^2 + b*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x
+ c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x +
c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d
*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sq
rt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))
)/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x +
c)^2 + b*d)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth^3(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(1/2), x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2), x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.32 \quad \int \frac{1}{\left(b \coth^3(c+dx)\right)^{3/2}} dx$$

Optimal. Leaf size=141

$$-\frac{2}{3bd\sqrt{b \coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx) \tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b \coth^3(c+dx)}} - \frac{2 \tanh^2(c+dx)}{7bd\sqrt{b \coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx) \tanh^{-1}\left(\sqrt{\cot}\right)}{bd\sqrt{b \coth^3(c+dx)}}$$

[Out] $-2/(3*b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTan[Sqrt[Coth[c + d*x]]]*Coth[c + d*x]^{(3/2)})/(b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c + d*x]]]*Coth[c + d*x]^{(3/2)})/(b*d*Sqrt[b*Coth[c + d*x]^3]) - (2*Tanh[c + d*x]^2)/(7*b*d*Sqrt[b*Coth[c + d*x]^3])$

Rubi [A] time = 0.0608829, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3658, 3474, 3476, 329, 212, 206, 203}

$$-\frac{2}{3bd\sqrt{b \coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx) \tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b \coth^3(c+dx)}} - \frac{2 \tanh^2(c+dx)}{7bd\sqrt{b \coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx) \tanh^{-1}\left(\sqrt{\cot}\right)}{bd\sqrt{b \coth^3(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^3)^(-3/2), x]

[Out] $-2/(3*b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTan[Sqrt[Coth[c + d*x]]]*Coth[c + d*x]^{(3/2)})/(b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c + d*x]]]*Coth[c + d*x]^{(3/2)})/(b*d*Sqrt[b*Coth[c + d*x]^3]) - (2*Tanh[c + d*x]^2)/(7*b*d*Sqrt[b*Coth[c + d*x]^3])$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3474

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b \coth^3(c + dx))^{3/2}} dx &= \frac{\coth^{\frac{3}{2}}(c + dx) \int \frac{1}{\coth^{\frac{9}{2}}(c + dx)} dx}{b\sqrt{b \coth^3(c + dx)}} \\
&= -\frac{2 \tanh^2(c + dx)}{7bd\sqrt{b \coth^3(c + dx)}} + \frac{\coth^{\frac{3}{2}}(c + dx) \int \frac{1}{\coth^{\frac{5}{2}}(c + dx)} dx}{b\sqrt{b \coth^3(c + dx)}} \\
&= -\frac{2}{3bd\sqrt{b \coth^3(c + dx)}} - \frac{2 \tanh^2(c + dx)}{7bd\sqrt{b \coth^3(c + dx)}} + \frac{\coth^{\frac{3}{2}}(c + dx) \int \frac{1}{\sqrt{\coth(c + dx)}} dx}{b\sqrt{b \coth^3(c + dx)}} \\
&= -\frac{2}{3bd\sqrt{b \coth^3(c + dx)}} - \frac{2 \tanh^2(c + dx)}{7bd\sqrt{b \coth^3(c + dx)}} - \frac{\coth^{\frac{3}{2}}(c + dx) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x(-1+x^2)}} dx, x, \coth(c + dx)\right)}{bd\sqrt{b \coth^3(c + dx)}} \\
&= -\frac{2}{3bd\sqrt{b \coth^3(c + dx)}} - \frac{2 \tanh^2(c + dx)}{7bd\sqrt{b \coth^3(c + dx)}} - \frac{(2 \coth^{\frac{3}{2}}(c + dx)) \operatorname{Subst}\left(\int \frac{1}{-1+x^4} dx, x, \coth(c + dx)\right)}{bd\sqrt{b \coth^3(c + dx)}} \\
&= -\frac{2}{3bd\sqrt{b \coth^3(c + dx)}} - \frac{2 \tanh^2(c + dx)}{7bd\sqrt{b \coth^3(c + dx)}} + \frac{\coth^{\frac{3}{2}}(c + dx) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(c + dx)}\right)}{bd\sqrt{b \coth^3(c + dx)}} \\
&= -\frac{2}{3bd\sqrt{b \coth^3(c + dx)}} + \frac{\tan^{-1}\left(\sqrt{\coth(c + dx)}\right) \coth^{\frac{3}{2}}(c + dx)}{bd\sqrt{b \coth^3(c + dx)}} + \frac{\tanh^{-1}\left(\sqrt{\coth(c + dx)}\right)}{bd\sqrt{b \coth^3(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 0.0653703, size = 43, normalized size = 0.3

$$-\frac{2 \coth(c + dx) {}_2F_1\left(-\frac{7}{4}, 1; -\frac{3}{4}; \coth^2(c + dx)\right)}{7d (b \coth^3(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^3)^(-3/2), x]

[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[-7/4, 1, -3/4, Coth[c + d*x]^2])/(7*d*(b*Coth[c + d*x]^3)^(3/2))

Maple [A] time = 0.024, size = 106, normalized size = 0.8

$$\frac{\coth(dx+c)}{21d} \left(-14b^{15/2} (\coth(dx+c))^2 - 6b^{15/2} + 21 \operatorname{Artanh} \left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}} \right) \right) b^4 (b\coth(dx+c))^{7/2} + 21 \arctan \left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c)^3)^(3/2), x)

[Out] 1/21/d*coth(d*x+c)/b^(15/2)*(-14*b^(15/2)*coth(d*x+c)^2-6*b^(15/2)+21*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2)+21*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2))/(b*coth(d*x+c)^3)^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^3)^(3/2), x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^3)^(-3/2), x)

Fricas [B] time = 2.94701, size = 8204, normalized size = 58.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^3)^(3/2), x, algorithm="fricas")

[Out] [-1/84*(42*(cosh(d*x + c))^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*c

$$\begin{aligned}
& \text{osh}(d*x + c)^6 + 15*\text{cosh}(d*x + c)^4 + 9*\text{cosh}(d*x + c)^2 + 1)*\text{sinh}(d*x + c)^2 \\
& + 4*\text{cosh}(d*x + c)^2 + 8*(\text{cosh}(d*x + c)^7 + 3*\text{cosh}(d*x + c)^5 + 3*\text{cosh}(d*x \\
& + c)^3 + \text{cosh}(d*x + c))*\text{sinh}(d*x + c) + 1)*\text{sqrt}(-b)*\text{arctan}((\text{cosh}(d*x + c)^2 \\
& + 2*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + \text{sinh}(d*x + c)^2)*\text{sqrt}(-b)*\text{sqrt}(b*\text{cosh}(d \\
& *x + c)/\text{sinh}(d*x + c)))/(b*\text{cosh}(d*x + c)^2 + 2*b*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) \\
& + b*\text{sinh}(d*x + c)^2 + b)) + 21*(\text{cosh}(d*x + c)^8 + 8*\text{cosh}(d*x + c)*\text{sinh}(d*x \\
& + c)^7 + \text{sinh}(d*x + c)^8 + 4*(7*\text{cosh}(d*x + c)^2 + 1)*\text{sinh}(d*x + c)^6 + 4*c \\
& \text{osh}(d*x + c)^6 + 8*(7*\text{cosh}(d*x + c)^3 + 3*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^5 + \\
& 2*(35*\text{cosh}(d*x + c)^4 + 30*\text{cosh}(d*x + c)^2 + 3)*\text{sinh}(d*x + c)^4 + 6*\text{cosh}(d* \\
& x + c)^4 + 8*(7*\text{cosh}(d*x + c)^5 + 10*\text{cosh}(d*x + c)^3 + 3*\text{cosh}(d*x + c))*\text{sin} \\
& \text{h}(d*x + c)^3 + 4*(7*\text{cosh}(d*x + c)^6 + 15*\text{cosh}(d*x + c)^4 + 9*\text{cosh}(d*x + c)^ \\
& 2 + 1)*\text{sinh}(d*x + c)^2 + 4*\text{cosh}(d*x + c)^2 + 8*(\text{cosh}(d*x + c)^7 + 3*\text{cosh}(d* \\
& x + c)^5 + 3*\text{cosh}(d*x + c)^3 + \text{cosh}(d*x + c))*\text{sinh}(d*x + c) + 1)*\text{sqrt}(-b)*\text{l} \\
& \text{og}(-(b*\text{cosh}(d*x + c)^4 + 4*b*\text{cosh}(d*x + c)^3*\text{sinh}(d*x + c) + 6*b*\text{cosh}(d*x + \\
& c)^2*\text{sinh}(d*x + c)^2 + 4*b*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^3 + b*\text{sinh}(d*x + c) \\
& ^4 + 2*(\text{cosh}(d*x + c)^2 + 2*\text{cosh}(d*x + c)*\text{sinh}(d*x + c) + \text{sinh}(d*x + c)^2 - \\
& 1)*\text{sqrt}(-b)*\text{sqrt}(b*\text{cosh}(d*x + c)/\text{sinh}(d*x + c)) - 2*b)/(\text{cosh}(d*x + c)^4 + \\
& 4*\text{cosh}(d*x + c)^3*\text{sinh}(d*x + c) + 6*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c)^2 + 4*\text{cos} \\
& \text{h}(d*x + c)*\text{sinh}(d*x + c)^3 + \text{sinh}(d*x + c)^4)) + 16*(5*\text{cosh}(d*x + c)^8 + 40 \\
& *\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^7 + 5*\text{sinh}(d*x + c)^8 + 2*(70*\text{cosh}(d*x + c)^2 \\
& - 3)*\text{sinh}(d*x + c)^6 - 6*\text{cosh}(d*x + c)^6 + 4*(70*\text{cosh}(d*x + c)^3 - 9*\text{cosh}(d \\
& *x + c))*\text{sinh}(d*x + c)^5 + 2*(175*\text{cosh}(d*x + c)^4 - 45*\text{cosh}(d*x + c)^2 + 1) \\
& *\text{sinh}(d*x + c)^4 + 2*\text{cosh}(d*x + c)^4 + 8*(35*\text{cosh}(d*x + c)^5 - 15*\text{cosh}(d*x \\
& + c)^3 + \text{cosh}(d*x + c))*\text{sinh}(d*x + c)^3 + 2*(70*\text{cosh}(d*x + c)^6 - 45*\text{cosh}(d \\
& *x + c)^4 + 6*\text{cosh}(d*x + c)^2 - 3)*\text{sinh}(d*x + c)^2 - 6*\text{cosh}(d*x + c)^2 + 4* \\
& (10*\text{cosh}(d*x + c)^7 - 9*\text{cosh}(d*x + c)^5 + 2*\text{cosh}(d*x + c)^3 - 3*\text{cosh}(d*x + \\
& c))*\text{sinh}(d*x + c) + 5)*\text{sqrt}(b*\text{cosh}(d*x + c)/\text{sinh}(d*x + c)))/(b^2*d*\text{cosh}(d*x \\
& + c)^8 + 8*b^2*d*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^7 + b^2*d*\text{sinh}(d*x + c)^8 + 4 \\
& *b^2*d*\text{cosh}(d*x + c)^6 + 6*b^2*d*\text{cosh}(d*x + c)^4 + 4*(7*b^2*d*\text{cosh}(d*x + c) \\
& ^2 + b^2*d)*\text{sinh}(d*x + c)^6 + 8*(7*b^2*d*\text{cosh}(d*x + c)^3 + 3*b^2*d*\text{cosh}(d*x \\
& + c))*\text{sinh}(d*x + c)^5 + 4*b^2*d*\text{cosh}(d*x + c)^2 + 2*(35*b^2*d*\text{cosh}(d*x + c) \\
&)^4 + 30*b^2*d*\text{cosh}(d*x + c)^2 + 3*b^2*d)*\text{sinh}(d*x + c)^4 + 8*(7*b^2*d*\text{cosh} \\
& (d*x + c)^5 + 10*b^2*d*\text{cosh}(d*x + c)^3 + 3*b^2*d*\text{cosh}(d*x + c))*\text{sinh}(d*x + \\
& c)^3 + b^2*d + 4*(7*b^2*d*\text{cosh}(d*x + c)^6 + 15*b^2*d*\text{cosh}(d*x + c)^4 + 9*b^ \\
& 2*d*\text{cosh}(d*x + c)^2 + b^2*d)*\text{sinh}(d*x + c)^2 + 8*(b^2*d*\text{cosh}(d*x + c)^7 + 3 \\
& *b^2*d*\text{cosh}(d*x + c)^5 + 3*b^2*d*\text{cosh}(d*x + c)^3 + b^2*d*\text{cosh}(d*x + c))*\text{sin} \\
& \text{h}(d*x + c)), 1/84*(42*(\text{cosh}(d*x + c)^8 + 8*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^7 + \\
& \text{sinh}(d*x + c)^8 + 4*(7*\text{cosh}(d*x + c)^2 + 1)*\text{sinh}(d*x + c)^6 + 4*\text{cosh}(d*x + \\
& c)^6 + 8*(7*\text{cosh}(d*x + c)^3 + 3*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^5 + 2*(35*\text{cosh} \\
& (d*x + c)^4 + 30*\text{cosh}(d*x + c)^2 + 3)*\text{sinh}(d*x + c)^4 + 6*\text{cosh}(d*x + c)^4 + \\
& 8*(7*\text{cosh}(d*x + c)^5 + 10*\text{cosh}(d*x + c)^3 + 3*\text{cosh}(d*x + c))*\text{sinh}(d*x + c) \\
& ^3 + 4*(7*\text{cosh}(d*x + c)^6 + 15*\text{cosh}(d*x + c)^4 + 9*\text{cosh}(d*x + c)^2 + 1)*\text{sin} \\
& \text{h}(d*x + c)^2 + 4*\text{cosh}(d*x + c)^2 + 8*(\text{cosh}(d*x + c)^7 + 3*\text{cosh}(d*x + c)^5 + \\
& 3*\text{cosh}(d*x + c)^3 + \text{cosh}(d*x + c))*\text{sinh}(d*x + c) + 1)*\text{sqrt}(b)*\text{arctan}(\text{sqrt}(\\
& b)*\text{sqrt}(b*\text{cosh}(d*x + c)/\text{sinh}(d*x + c)))/(b*\text{cosh}(d*x + c)^2 + 2*b*\text{cosh}(d*x +
\end{aligned}$$

```

c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 21*(cosh(d*x + c)^8 + 8*cosh(d
*x + c)*sinh(d*x + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(
d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x +
c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 +
9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x +
c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
+ 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) +
12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 +
2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c
)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*c
osh(d*x + c)/sinh(d*x + c)) - b) - 16*(5*cosh(d*x + c)^8 + 40*cosh(d*x + c)
*sinh(d*x + c)^7 + 5*sinh(d*x + c)^8 + 2*(70*cosh(d*x + c)^2 - 3)*sinh(d*x
+ c)^6 - 6*cosh(d*x + c)^6 + 4*(70*cosh(d*x + c)^3 - 9*cosh(d*x + c))*sinh(
d*x + c)^5 + 2*(175*cosh(d*x + c)^4 - 45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)
^4 + 2*cosh(d*x + c)^4 + 8*(35*cosh(d*x + c)^5 - 15*cosh(d*x + c)^3 + cosh(
d*x + c))*sinh(d*x + c)^3 + 2*(70*cosh(d*x + c)^6 - 45*cosh(d*x + c)^4 + 6*
cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*cosh(d*x + c)^2 + 4*(10*cosh(d*x +
c)^7 - 9*cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x +
c) + 5)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x + c)^8 + 8*b^
2*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*d*sinh(d*x + c)^8 + 4*b^2*d*cosh(d*
x + c)^6 + 6*b^2*d*cosh(d*x + c)^4 + 4*(7*b^2*d*cosh(d*x + c)^2 + b^2*d)*si
nh(d*x + c)^6 + 8*(7*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*
x + c)^5 + 4*b^2*d*cosh(d*x + c)^2 + 2*(35*b^2*d*cosh(d*x + c)^4 + 30*b^2*d
*cosh(d*x + c)^2 + 3*b^2*d)*sinh(d*x + c)^4 + 8*(7*b^2*d*cosh(d*x + c)^5 +
10*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*d +
4*(7*b^2*d*cosh(d*x + c)^6 + 15*b^2*d*cosh(d*x + c)^4 + 9*b^2*d*cosh(d*x +
c)^2 + b^2*d)*sinh(d*x + c)^2 + 8*(b^2*d*cosh(d*x + c)^7 + 3*b^2*d*cosh(d*
x + c)^5 + 3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^3(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**3)**(3/2),x)

```
[Out] Integral((b*coth(c + d*x)**3)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.33 $\int (b \coth^3(c + dx))^{4/3} dx$

Optimal. Leaf size=74

$$-\frac{b \coth^2(c + dx) \sqrt[3]{b \coth^3(c + dx)}}{3d} - \frac{b \sqrt[3]{b \coth^3(c + dx)}}{d} + bx \tanh(c + dx) \sqrt[3]{b \coth^3(c + dx)}$$

[Out] $-\left(\frac{b \coth^2(c + dx) \sqrt[3]{b \coth^3(c + dx)}}{3d}\right) - \frac{b \sqrt[3]{b \coth^3(c + dx)}}{d} + bx \tanh(c + dx) \sqrt[3]{b \coth^3(c + dx)}$

Rubi [A] time = 0.0335459, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 8}

$$-\frac{b \coth^2(c + dx) \sqrt[3]{b \coth^3(c + dx)}}{3d} - \frac{b \sqrt[3]{b \coth^3(c + dx)}}{d} + bx \tanh(c + dx) \sqrt[3]{b \coth^3(c + dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b \coth[c + d*x]^3)^{4/3}, x]$

[Out] $-\left(\frac{b \coth^2(c + dx) \sqrt[3]{b \coth^3(c + dx)}}{3d}\right) - \frac{b \sqrt[3]{b \coth^3(c + dx)}}{d} + bx \tanh(c + dx) \sqrt[3]{b \coth^3(c + dx)}$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int (b \coth^3(c + dx))^{4/3} dx &= \left(b \sqrt[3]{b \coth^3(c + dx) \tanh(c + dx)} \right) \int \coth^4(c + dx) dx \\
 &= -\frac{b \coth^2(c + dx) \sqrt[3]{b \coth^3(c + dx)}}{3d} + \left(b \sqrt[3]{b \coth^3(c + dx) \tanh(c + dx)} \right) \int \coth^2(c + dx) dx \\
 &= -\frac{b \sqrt[3]{b \coth^3(c + dx)}}{d} - \frac{b \coth^2(c + dx) \sqrt[3]{b \coth^3(c + dx)}}{3d} + \left(b \sqrt[3]{b \coth^3(c + dx) \tanh(c + dx)} \right) \int \coth^2(c + dx) dx \\
 &= -\frac{b \sqrt[3]{b \coth^3(c + dx)}}{d} - \frac{b \coth^2(c + dx) \sqrt[3]{b \coth^3(c + dx)}}{3d} + bx \sqrt[3]{b \coth^3(c + dx) \tanh(c + dx)}
 \end{aligned}$$

Mathematica [C] time = 0.0607881, size = 43, normalized size = 0.58

$$\frac{\tanh(c + dx) (b \coth^3(c + dx))^{4/3} {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \tanh^2(c + dx)\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^3)^(4/3), x]

[Out] -((b*Coth[c + d*x]^3)^(4/3)*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^2]*Tanh[c + d*x])/(3*d)

Maple [B] time = 0.098, size = 145, normalized size = 2.

$$\frac{b(e^{2dx+2c}-1)x \sqrt[3]{b(1+e^{2dx+2c})^3}}{1+e^{2dx+2c}} - \frac{4b(3e^{4dx+4c}-3e^{2dx+2c}+2) \sqrt[3]{b(1+e^{2dx+2c})^3}}{(3+3e^{2dx+2c})(e^{2dx+2c}-1)^2 d \sqrt[3]{(e^{2dx+2c}-1)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^3)^(4/3), x)

[Out] $b/(1+\exp(2*d*x+2*c))*(\exp(2*d*x+2*c)-1)*(b*(1+\exp(2*d*x+2*c)))^3/(\exp(2*d*x+2*c)-1)^3)^{(1/3)*x-4/3*b/(1+\exp(2*d*x+2*c))}/(\exp(2*d*x+2*c)-1)^2*(b*(1+\exp(2*d*x+2*c)))^3/(\exp(2*d*x+2*c)-1)^3)^{(1/3)*(3*\exp(4*d*x+4*c)-3*\exp(2*d*x+2*c)+2)/d}$

Maxima [A] time = 1.75654, size = 117, normalized size = 1.58

$$\frac{(dx+c)b^{\frac{4}{3}}}{d} - \frac{4\left(3b^{\frac{4}{3}}e^{(-2dx-2c)} - 3b^{\frac{4}{3}}e^{(-4dx-4c)} - 2b^{\frac{4}{3}}\right)}{3d\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")

[Out] $(d*x + c)*b^{(4/3)}/d - 4/3*(3*b^{(4/3)}*e^{(-2*d*x - 2*c)} - 3*b^{(4/3)}*e^{(-4*d*x - 4*c)} - 2*b^{(4/3)})/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1))$

Fricas [B] time = 2.27201, size = 2689, normalized size = 36.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")

[Out] $-1/3*(3*b*d*x*cosh(d*x + c)^6 - 3*(b*d*x*e^{(2*d*x + 2*c)} - b*d*x)*sinh(d*x + c)^6 - 18*(b*d*x*cosh(d*x + c)*e^{(2*d*x + 2*c)} - b*d*x*cosh(d*x + c))*sinh(d*x + c)^5 - 3*(3*b*d*x + 4*b)*cosh(d*x + c)^4 + 3*(15*b*d*x*cosh(d*x + c))^2 - 3*b*d*x - (15*b*d*x*cosh(d*x + c)^2 - 3*b*d*x - 4*b)*e^{(2*d*x + 2*c)} - 4*b)*sinh(d*x + c)^4 + 12*(5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh(d*x + c) - (5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh(d*x + c))*e^{(2*d*x + 2*c)})*sinh(d*x + c)^3 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c)^2 + 3*(15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2 - (15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2 + 4*b)*e^{(2*d*x + 2*c)} + 4*b)*sinh(d*x + c)^2 - (3*b*d*x*cosh(d*x + c)^6 - 3*(3*b*d*x + 4*b)*cosh(d*x + c)^4 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c))^2 - 8*b)*e^{(2*d*x + 2*c)} + 6*(3*b*d*x*cosh(d*x + c)^5 - 2*(3*b*d*x + 4*b)*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c) - (3*b*d*x*cosh(d*x + c))^5$

$$\begin{aligned}
& - 2*(3*b*d*x + 4*b)*\cosh(d*x + c)^3 + (3*b*d*x + 4*b)*\cosh(d*x + c))*e^{(2*d*x + 2*c))*\sinh(d*x + c) - 8*b)*((b*e^{(6*d*x + 6*c)} + 3*b*e^{(4*d*x + 4*c)} \\
& + 3*b*e^{(2*d*x + 2*c)} + b)/(e^{(6*d*x + 6*c)} - 3*e^{(4*d*x + 4*c)} + 3*e^{(2*d*x + 2*c)} - 1))^{(1/3)}/(d*\cosh(d*x + c)^6 + (d*e^{(2*d*x + 2*c)} + d)*\sinh(d*x \\
& + c)^6 + 6*(d*\cosh(d*x + c)*e^{(2*d*x + 2*c)} + d*\cosh(d*x + c))*\sinh(d*x + c)^5 - 3*d*\cosh(d*x + c)^4 + 3*(5*d*\cosh(d*x + c)^2 + (5*d*\cosh(d*x + c)^2 - \\
& d)*e^{(2*d*x + 2*c)} - d)*\sinh(d*x + c)^4 + 4*(5*d*\cosh(d*x + c)^3 - 3*d*\cosh(d*x + c) + (5*d*\cosh(d*x + c)^3 - 3*d*\cosh(d*x + c))*e^{(2*d*x + 2*c))*\sinh(d*x + c)^3 + 3*d*\cosh(d*x + c)^2 + 3*(5*d*\cosh(d*x + c)^4 - 6*d*\cosh(d*x + c)^2 + (5*d*\cosh(d*x + c)^4 - 6*d*\cosh(d*x + c)^2 + d)*e^{(2*d*x + 2*c)} + d)*\sinh(d*x + c)^2 + (d*\cosh(d*x + c)^6 - 3*d*\cosh(d*x + c)^4 + 3*d*\cosh(d*x + c)^2 - d)*e^{(2*d*x + 2*c)} + 6*(d*\cosh(d*x + c)^5 - 2*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c) + (d*\cosh(d*x + c)^5 - 2*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*e^{(2*d*x + 2*c))*\sinh(d*x + c) - d)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**3)**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^3)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^3)^(4/3), x)

3.34 $\int (b \coth^3(c + dx))^{2/3} dx$

Optimal. Leaf size=50

$$x \tanh^2(c + dx) (b \coth^3(c + dx))^{2/3} - \frac{\tanh(c + dx) (b \coth^3(c + dx))^{2/3}}{d}$$

[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Tanh[c + d*x])/d) + x*(b*Coth[c + d*x]^3)^(2/3)*Tanh[c + d*x]^2

Rubi [A] time = 0.0244887, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 8}

$$x \tanh^2(c + dx) (b \coth^3(c + dx))^{2/3} - \frac{\tanh(c + dx) (b \coth^3(c + dx))^{2/3}}{d}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^3)^(2/3),x]

[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Tanh[c + d*x])/d) + x*(b*Coth[c + d*x]^3)^(2/3)*Tanh[c + d*x]^2

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int (b \coth^3(c + dx))^{2/3} dx &= \left((b \coth^3(c + dx))^{2/3} \tanh^2(c + dx) \right) \int \coth^2(c + dx) dx \\ &= -\frac{(b \coth^3(c + dx))^{2/3} \tanh(c + dx)}{d} + \left((b \coth^3(c + dx))^{2/3} \tanh^2(c + dx) \right) \int 1 dx \\ &= -\frac{(b \coth^3(c + dx))^{2/3} \tanh(c + dx)}{d} + x (b \coth^3(c + dx))^{2/3} \tanh^2(c + dx) \end{aligned}$$

Mathematica [C] time = 0.0275691, size = 41, normalized size = 0.82

$$\frac{\tanh(c + dx) (b \coth^3(c + dx))^{2/3} {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(c + dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^3)^(2/3), x]

[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2]*Tanh[c + d*x])/d)

Maple [B] time = 0.097, size = 119, normalized size = 2.4

$$\frac{(e^{2dx+2c}-1)^2}{(1+e^{2dx+2c})^2} x \left(\frac{b(1+e^{2dx+2c})^3}{(e^{2dx+2c}-1)^3} \right)^{\frac{2}{3}} - 2 \frac{e^{2dx+2c}-1}{(1+e^{2dx+2c})^2} \frac{1}{d} \left(\frac{b(1+e^{2dx+2c})^3}{(e^{2dx+2c}-1)^3} \right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^3)^(2/3), x)

[Out] (b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(1+exp(2*d*x+2*c))^2*(exp(2*d*x+2*c)-1)^2*x-2*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(1+exp(2*d*x+2*c))^2*(exp(2*d*x+2*c)-1)/d

Maxima [A] time = 1.61666, size = 46, normalized size = 0.92

$$\frac{(dx + c)b^{\frac{2}{3}}}{d} + \frac{2b^{\frac{2}{3}}}{d(e^{-2dx-2c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")

[Out] (d*x + c)*b^(2/3)/d + 2*b^(2/3)/(d*(e^(-2*d*x - 2*c) - 1))

Fricas [B] time = 2.18871, size = 992, normalized size = 19.84

$$\frac{(dx \cosh(dx + c)^2 + (dx e^{4dx+4c} - 2 dx e^{2dx+2c} + dx) \sinh(dx + c)^2 - dx + (dx \cosh(dx + c)^2 - dx - 2)e^{4dx+4c} - 2)(d \cosh(dx + c)^2 + (d e^{4dx+4c} + 2 d e^{2dx+2c} + d) \sinh(dx + c)^2 + (d \cosh(dx + c)^2 - d))}{d \cosh(dx + c)^2 + (d e^{4dx+4c} + 2 d e^{2dx+2c} + d) \sinh(dx + c)^2 + (d \cosh(dx + c)^2 - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")

[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c) - 2)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(d*cosh(d*x + c)^2 + (d*e^(4*d*x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c) - d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^3)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(2/3), x)
```

$$3.35 \quad \int \sqrt[3]{b \coth^3(c + dx)} dx$$

Optimal. Leaf size=31

$$\frac{\tanh(c + dx) \sqrt[3]{b \coth^3(c + dx)} \log(\sinh(c + dx))}{d}$$

[Out] ((b*Coth[c + d*x]^3)^(1/3)*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d

Rubi [A] time = 0.0202028, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3658, 3475}

$$\frac{\tanh(c + dx) \sqrt[3]{b \coth^3(c + dx)} \log(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^3)^(1/3), x]

[Out] ((b*Coth[c + d*x]^3)^(1/3)*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt[3]{b \coth^3(c + dx)} dx = \left(\sqrt[3]{b \coth^3(c + dx) \tanh(c + dx)} \right) \int \coth(c + dx) dx$$

$$= \frac{\sqrt[3]{b \coth^3(c + dx) \log(\sinh(c + dx)) \tanh(c + dx)}}{d}$$

Mathematica [A] time = 0.0258951, size = 39, normalized size = 1.26

$$\frac{\tanh(c + dx) \sqrt[3]{b \coth^3(c + dx) (\log(\tanh(c + dx)) + \log(\cosh(c + dx)))}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^3)^(1/3), x]

[Out] ((b*Coth[c + d*x]^3)^(1/3)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c + d*x])/d

Maple [B] time = 0.092, size = 192, normalized size = 6.2

$$\frac{(e^{2dx+2c} - 1)x \sqrt[3]{b(1 + e^{2dx+2c})^3}}{1 + e^{2dx+2c}} - 2 \frac{(e^{2dx+2c} - 1)(dx + c) \sqrt[3]{b(1 + e^{2dx+2c})^3}}{(1 + e^{2dx+2c})d} + \frac{(e^{2dx+2c} - 1) \ln(e^{2dx+2c} - 1) \sqrt[3]{b(1 + e^{2dx+2c})^3}}{(1 + e^{2dx+2c})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^3)^(1/3), x)

[Out] (b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(1+exp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)*x-2*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(1+exp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)/d*(d*x+c)+(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(1+exp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)/d*ln(exp(2*d*x+2*c)-1)

Maxima [A] time = 1.74058, size = 69, normalized size = 2.23

$$\frac{(dx + c)b^{\frac{1}{3}}}{d} + \frac{b^{\frac{1}{3}} \log(e^{-dx-c} + 1)}{d} + \frac{b^{\frac{1}{3}} \log(e^{-dx-c} - 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")

[Out] (d*x + c)*b^(1/3)/d + b^(1/3)*log(e^(-d*x - c) + 1)/d + b^(1/3)*log(e^(-d*x - c) - 1)/d

Fricas [B] time = 2.13365, size = 363, normalized size = 11.71

$$\frac{\left(dx e^{(2dx+2c)} - dx - \left(e^{(2dx+2c)} - 1\right) \log\left(\frac{2 \sinh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)\right) \left(\frac{be^{(6dx+6c)} + 3be^{(4dx+4c)} + 3be^{(2dx+2c)} + b}{e^{(6dx+6c)} - 3e^{(4dx+4c)} + 3e^{(2dx+2c)} - 1}\right)^{\frac{1}{3}}}{de^{(2dx+2c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")

[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(1/3)/(d*e^(2*d*x + 2*c) + d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \coth^3(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**3)**(1/3),x)

[Out] Integral((b*coth(c + d*x)**3)**(1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^3)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(1/3), x)
```


$$3.36 \quad \int \frac{1}{\sqrt[3]{b \coth^3(c+dx)}} dx$$

Optimal. Leaf size=31

$$\frac{\coth(c+dx) \log(\cosh(c+dx))}{d \sqrt[3]{b \coth^3(c+dx)}}$$

[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))

Rubi [A] time = 0.0202842, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3658, 3475}

$$\frac{\coth(c+dx) \log(\cosh(c+dx))}{d \sqrt[3]{b \coth^3(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^3)^(-1/3), x]

[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b \coth^3(c + dx)}} dx = \frac{\coth(c + dx) \int \tanh(c + dx) dx}{\sqrt[3]{b \coth^3(c + dx)}} \\ = \frac{\coth(c + dx) \log(\cosh(c + dx))}{d \sqrt[3]{b \coth^3(c + dx)}}$$

Mathematica [A] time = 0.0284736, size = 31, normalized size = 1.

$$\frac{\coth(c + dx) \log(\cosh(c + dx))}{d \sqrt[3]{b \coth^3(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^3)^(-1/3),x]

[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))

Maple [B] time = 0.093, size = 192, normalized size = 6.2

$$\frac{(1 + e^{2dx+2c})x}{e^{2dx+2c} - 1} \frac{1}{\sqrt[3]{\frac{b(1+e^{2dx+2c})^3}{(e^{2dx+2c}-1)^3}}} - 2 \frac{(1 + e^{2dx+2c})(dx + c)}{(e^{2dx+2c} - 1)d} \frac{1}{\sqrt[3]{\frac{b(1+e^{2dx+2c})^3}{(e^{2dx+2c}-1)^3}}} + \frac{(1 + e^{2dx+2c}) \ln(1 + e^{2dx+2c})}{(e^{2dx+2c} - 1)d} \frac{1}{\sqrt[3]{\frac{b(1+e^{2dx+2c})^3}{(e^{2dx+2c}-1)^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c)^3)^(1/3),x)

[Out] 1/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(1+exp(2*d*x+2*c))*x-2/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(1+exp(2*d*x+2*c))/d*(d*x+c)+1/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(1+exp(2*d*x+2*c))/d*ln(1+exp(2*d*x+2*c))

Maxima [A] time = 1.71109, size = 43, normalized size = 1.39

$$\frac{dx + c}{b^{\frac{1}{3}}d} + \frac{\log(e^{(-2dx-2c)} + 1)}{b^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")

[Out] (d*x + c)/(b^(1/3)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^(1/3)*d)

Fricas [B] time = 2.44438, size = 460, normalized size = 14.84

$$\frac{\left(dx e^{(4dx+4c)} - 2 dx e^{(2dx+2c)} + dx - \left(e^{(4dx+4c)} - 2e^{(2dx+2c)} + 1\right) \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)\right) \left(\frac{be^{(6dx+6c)} + 3be^{(4dx+4c)} + 3be^{(2dx+2c)}}{e^{(6dx+6c)} - 3e^{(4dx+4c)} + 3e^{(2dx+2c)}}\right)}{bde^{(4dx+4c)} + 2bde^{(2dx+2c)} + bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")

[Out] -(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x - (e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))*(b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)^(2/3)/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b \coth^3(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**3)**(1/3),x)

[Out] Integral((b*coth(c + d*x)**3)**(-1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^3)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^3)^(-1/3), x)

$$3.37 \quad \int \frac{1}{\left(b \coth^3(c+dx)\right)^{2/3}} dx$$

Optimal. Leaf size=50

$$\frac{x \coth^2(c+dx)}{\left(b \coth^3(c+dx)\right)^{2/3}} - \frac{\coth(c+dx)}{d \left(b \coth^3(c+dx)\right)^{2/3}}$$

[Out] $-(\text{Coth}[c + d*x]/(d*(b*\text{Coth}[c + d*x]^3)^{(2/3)})) + (x*\text{Coth}[c + d*x]^2)/(b*\text{Coth}[c + d*x]^3)^{(2/3)}$

Rubi [A] time = 0.0242453, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 8}

$$\frac{x \coth^2(c+dx)}{\left(b \coth^3(c+dx)\right)^{2/3}} - \frac{\coth(c+dx)}{d \left(b \coth^3(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^3)^(-2/3), x]

[Out] $-(\text{Coth}[c + d*x]/(d*(b*\text{Coth}[c + d*x]^3)^{(2/3)})) + (x*\text{Coth}[c + d*x]^2)/(b*\text{Coth}[c + d*x]^3)^{(2/3)}$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rubi steps

$$\begin{aligned} \int \frac{1}{(b \coth^3(c + dx))^{2/3}} dx &= \frac{\coth^2(c + dx) \int \tanh^2(c + dx) dx}{(b \coth^3(c + dx))^{2/3}} \\ &= -\frac{\coth(c + dx)}{d (b \coth^3(c + dx))^{2/3}} + \frac{\coth^2(c + dx) \int 1 dx}{(b \coth^3(c + dx))^{2/3}} \\ &= -\frac{\coth(c + dx)}{d (b \coth^3(c + dx))^{2/3}} + \frac{x \coth^2(c + dx)}{(b \coth^3(c + dx))^{2/3}} \end{aligned}$$

Mathematica [A] time = 0.0599557, size = 40, normalized size = 0.8

$$\frac{\coth(c + dx) (\tanh^{-1}(\tanh(c + dx)) \coth(c + dx) - 1)}{d (b \coth^3(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^3)^(-2/3), x]

[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*(b*Coth[c + d*x]^3)^(2/3))

Maple [B] time = 0.093, size = 119, normalized size = 2.4

$$\frac{(1 + e^{2dx+2c})^2}{(e^{2dx+2c} - 1)^2} x \left(\frac{b(1 + e^{2dx+2c})^3}{(e^{2dx+2c} - 1)^3} \right)^{-2/3} + 2 \frac{1 + e^{2dx+2c}}{(e^{2dx+2c} - 1)^2} \frac{1}{d} \left(\frac{b(1 + e^{2dx+2c})^3}{(e^{2dx+2c} - 1)^3} \right)^{-2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c)^3)^(2/3), x)

[Out] $1/(b*(1+\exp(2*d*x+2*c))^{3/2}/(\exp(2*d*x+2*c)-1)^{3/2})/(\exp(2*d*x+2*c)-1)^{2/3}/(1+\exp(2*d*x+2*c))^{2*x+2}/(b*(1+\exp(2*d*x+2*c))^{3/2}/(\exp(2*d*x+2*c)-1)^{3/2})^{2/3}/(\exp(2*d*x+2*c)-1)^{2/3}/(1+\exp(2*d*x+2*c))/d$

Maxima [A] time = 1.74955, size = 50, normalized size = 1.

$$\frac{dx + c}{b^{2/3}d} - \frac{2}{\left(b^{2/3}e^{-2dx-2c} + b^{2/3}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")`

[Out] $(d*x + c)/(b^{2/3}*d) - 2/((b^{2/3}*e^{-2*d*x - 2*c}) + b^{2/3})*d$

Fricas [B] time = 2.23736, size = 724, normalized size = 14.48

$$\frac{(dx \cosh(dx + c))^2 - (dx e^{2dx+2c} - dx) \sinh(dx + c)^2 + dx - (dx \cosh(dx + c)^2 + dx + 2)e^{2dx+2c} - 2(dx \cosh(dx + c) + dx + 2)e^{2dx+2c}}{bd \cosh(dx + c)^2 + (bd e^{2dx+2c} + bd) \sinh(dx + c)^2 + bd + (bd \cosh(dx + c)^2 + bd)e^{2dx+2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")`

[Out] $-(d*x*\cosh(d*x + c)^2 - (d*x*e^{(2*d*x + 2*c)} - d*x)*\sinh(d*x + c)^2 + d*x - (d*x*\cosh(d*x + c)^2 + d*x + 2)*e^{(2*d*x + 2*c)} - 2*(d*x*\cosh(d*x + c)*e^{(2*d*x + 2*c)} - d*x*\cosh(d*x + c))*\sinh(d*x + c) + 2)/((b*e^{(6*d*x + 6*c)} + 3*b*e^{(4*d*x + 4*c)} + 3*b*e^{(2*d*x + 2*c)} + b)/(e^{(6*d*x + 6*c)} - 3*e^{(4*d*x + 4*c)} + 3*e^{(2*d*x + 2*c)} - 1))^{1/3}/(b*d*\cosh(d*x + c)^2 + (b*d*e^{(2*d*x + 2*c)} + b*d)*\sinh(d*x + c)^2 + b*d + (b*d*\cosh(d*x + c)^2 + b*d)*e^{(2*d*x + 2*c)} + 2*(b*d*\cosh(d*x + c)*e^{(2*d*x + 2*c)} + b*d*\cosh(d*x + c))*\sinh(d*x + c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^3(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**3)**(2/3),x)

[Out] Integral((b*coth(c + d*x)**3)**(-2/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^3)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^3)^(-2/3), x)

$$3.38 \quad \int \frac{1}{\left(b \coth^3(c+dx)\right)^{4/3}} dx$$

Optimal. Leaf size=80

$$\frac{x \coth(c+dx)}{b\sqrt[3]{b \coth^3(c+dx)}} - \frac{1}{bd\sqrt[3]{b \coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b \coth^3(c+dx)}}$$

[Out] $-(1/(b*d*(b*\text{Coth}[c + d*x]^3)^{(1/3)})) + (x*\text{Coth}[c + d*x])/(b*(b*\text{Coth}[c + d*x]^3)^{(1/3)}) - \text{Tanh}[c + d*x]^2/(3*b*d*(b*\text{Coth}[c + d*x]^3)^{(1/3)})$

Rubi [A] time = 0.0353556, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 8}

$$\frac{x \coth(c+dx)}{b\sqrt[3]{b \coth^3(c+dx)}} - \frac{1}{bd\sqrt[3]{b \coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b \coth^3(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x]^3)^{-4/3}, x]$

[Out] $-(1/(b*d*(b*\text{Coth}[c + d*x]^3)^{(1/3)})) + (x*\text{Coth}[c + d*x])/(b*(b*\text{Coth}[c + d*x]^3)^{(1/3)}) - \text{Tanh}[c + d*x]^2/(3*b*d*(b*\text{Coth}[c + d*x]^3)^{(1/3)})$

Rule 3658

$\text{Int}[(u_*)*((b_*)*\tan[(e_*) + (f_*)(x_)]^{(n_)})^{(p_)}, x_Symbol] := \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[(b*ff^n)^{\text{IntPart}[p]}*(b*\text{Tan}[e + f*x]^{n^{\text{FracPart}[p]}})/(\text{Tan}[e + f*x]/ff)^{(n*\text{FracPart}[p])}, \text{Int}[\text{ActivateTrig}[u]*(\text{Tan}[e + f*x]/ff)^{(n*p)}, x], x]] /; \text{FreeQ}\{b, e, f, n, p\}, x \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{EqQ}[u, 1] \ || \ \text{MatchQ}[u, ((d_*)*(\text{trig}_)[e + f*x])^{(m_)}]) /; \text{FreeQ}\{d, m\}, x \ \&\& \ \text{MemberQ}\{\sin, \cos, \tan, \cot, \sec, \csc\}, \text{trig}]]$

Rule 3473

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)(x_)]^{(n_)}, x_Symbol] := \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1]$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(b \coth^3(c + dx))^{4/3}} dx &= \frac{\coth(c + dx) \int \tanh^4(c + dx) dx}{b \sqrt[3]{b \coth^3(c + dx)}} \\
 &= -\frac{\tanh^2(c + dx)}{3bd \sqrt[3]{b \coth^3(c + dx)}} + \frac{\coth(c + dx) \int \tanh^2(c + dx) dx}{b \sqrt[3]{b \coth^3(c + dx)}} \\
 &= -\frac{1}{bd \sqrt[3]{b \coth^3(c + dx)}} - \frac{\tanh^2(c + dx)}{3bd \sqrt[3]{b \coth^3(c + dx)}} + \frac{\coth(c + dx) \int 1 dx}{b \sqrt[3]{b \coth^3(c + dx)}} \\
 &= -\frac{1}{bd \sqrt[3]{b \coth^3(c + dx)}} + \frac{x \coth(c + dx)}{b \sqrt[3]{b \coth^3(c + dx)}} - \frac{\tanh^2(c + dx)}{3bd \sqrt[3]{b \coth^3(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.0878472, size = 51, normalized size = 0.64

$$\frac{-\tanh^2(c + dx) + 3 \tanh^{-1}(\tanh(c + dx)) \coth(c + dx) - 3}{3bd \sqrt[3]{b \coth^3(c + dx)}}$$

Antiderivative was successfully verified.

[In] `Integrate[(b*Coth[c + d*x]^3)^(-4/3), x]`

[Out] `(-3 + 3*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x] - Tanh[c + d*x]^2)/(3*b*d*(b*Coth[c + d*x]^3)^(1/3))`

Maple [B] time = 0.099, size = 149, normalized size = 1.9

$$\frac{(1 + e^{2dx+2c})x}{b(e^{2dx+2c} - 1)} \frac{1}{\sqrt[3]{\frac{b(1+e^{2dx+2c})}{(e^{2dx+2c}-1)^3}}} + \frac{12e^{4dx+4c} + 12e^{2dx+2c} + 8}{3b(1 + e^{2dx+2c})^2 (e^{2dx+2c} - 1)} d \frac{1}{\sqrt[3]{\frac{b(1+e^{2dx+2c})}{(e^{2dx+2c}-1)^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& c)^4 + 6*(3*d*x + 4)*\cosh(d*x + c)^2 + 3*d*x + 4)*e^{(4*d*x + 4*c)} - 2*(15*d*x*\cosh(d*x + c)^4 + 6*(3*d*x + 4)*\cosh(d*x + c)^2 + 3*d*x + 4)*e^{(2*d*x + 2*c)} + 4)*\sinh(d*x + c)^2 + 3*d*x + (3*d*x*\cosh(d*x + c)^6 + 3*(3*d*x + 4)*\cosh(d*x + c)^4 + 3*(3*d*x + 4)*\cosh(d*x + c)^2 + 3*d*x + 8)*e^{(4*d*x + 4*c)} - 2*(3*d*x*\cosh(d*x + c)^6 + 3*(3*d*x + 4)*\cosh(d*x + c)^4 + 3*(3*d*x + 4)*\cosh(d*x + c)^2 + 3*d*x + 8)*e^{(2*d*x + 2*c)} + 6*(3*d*x*\cosh(d*x + c)^5 + 2*(3*d*x + 4)*\cosh(d*x + c)^3 + (3*d*x + 4)*\cosh(d*x + c) + (3*d*x*\cosh(d*x + c)^5 + 2*(3*d*x + 4)*\cosh(d*x + c)^3 + (3*d*x + 4)*\cosh(d*x + c))*e^{(4*d*x + 4*c)} - 2*(3*d*x*\cosh(d*x + c)^5 + 2*(3*d*x + 4)*\cosh(d*x + c)^3 + (3*d*x + 4)*\cosh(d*x + c))*e^{(2*d*x + 2*c)})*\sinh(d*x + c) + 8)*((b*e^{(6*d*x + 6*c)} + 3*b*e^{(4*d*x + 4*c)} + 3*b*e^{(2*d*x + 2*c)} + b)/(e^{(6*d*x + 6*c)} - 3*e^{(4*d*x + 4*c)} + 3*e^{(2*d*x + 2*c)} - 1))^{(2/3)}/(b^2*d*\cosh(d*x + c)^6 + 3*b^2*d*\cosh(d*x + c)^4 + (b^2*d*e^{(4*d*x + 4*c)} + 2*b^2*d*e^{(2*d*x + 2*c)} + b^2*d)*\sinh(d*x + c)^6 + 6*(b^2*d*\cosh(d*x + c)*e^{(4*d*x + 4*c)} + 2*b^2*d*\cosh(d*x + c)*e^{(2*d*x + 2*c)} + b^2*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 3*b^2*d*\cosh(d*x + c)^2 + 3*(5*b^2*d*\cosh(d*x + c)^2 + b^2*d + (5*b^2*d*\cosh(d*x + c)^2 + b^2*d)*e^{(4*d*x + 4*c)} + 2*(5*b^2*d*\cosh(d*x + c)^2 + b^2*d)*e^{(2*d*x + 2*c)})*\sinh(d*x + c)^4 + 4*(5*b^2*d*\cosh(d*x + c)^3 + 3*b^2*d*\cosh(d*x + c) + (5*b^2*d*\cosh(d*x + c)^3 + 3*b^2*d*\cosh(d*x + c))*e^{(4*d*x + 4*c)} + 2*(5*b^2*d*\cosh(d*x + c)^3 + 3*b^2*d*\cosh(d*x + c))*e^{(2*d*x + 2*c)})*\sinh(d*x + c)^3 + b^2*d + 3*(5*b^2*d*\cosh(d*x + c)^4 + 6*b^2*d*\cosh(d*x + c)^2 + b^2*d + (5*b^2*d*\cosh(d*x + c)^4 + 6*b^2*d*\cosh(d*x + c)^2 + b^2*d)*e^{(4*d*x + 4*c)} + 2*(5*b^2*d*\cosh(d*x + c)^4 + 6*b^2*d*\cosh(d*x + c)^2 + b^2*d)*e^{(2*d*x + 2*c)})*\sinh(d*x + c)^2 + (b^2*d*\cosh(d*x + c)^6 + 3*b^2*d*\cosh(d*x + c)^4 + 3*b^2*d*\cosh(d*x + c)^2 + b^2*d)*e^{(2*d*x + 2*c)} + 6*(b^2*d*\cosh(d*x + c)^5 + 2*b^2*d*\cosh(d*x + c)^3 + b^2*d*\cosh(d*x + c) + (b^2*d*\cosh(d*x + c)^5 + 2*b^2*d*\cosh(d*x + c)^3 + b^2*d*\cosh(d*x + c))*e^{(4*d*x + 4*c)} + 2*(b^2*d*\cosh(d*x + c)^5 + 2*b^2*d*\cosh(d*x + c)^3 + b^2*d*\cosh(d*x + c))*e^{(2*d*x + 2*c)})*\sinh(d*x + c))
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^3(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**3)**(4/3),x)

[Out] Integral((b*coth(c + d*x)**3)**(-4/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^3)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")`

[Out] `integrate((b*coth(d*x + c)^3)^(-4/3), x)`

3.39 $\int (b \coth^4(c + dx))^n dx$

Optimal. Leaf size=57

$$\frac{\coth(c + dx) (b \coth^4(c + dx))^n {}_2F_1\left(1, \frac{1}{2}(4n + 1); \frac{1}{2}(4n + 3); \coth^2(c + dx)\right)}{d(4n + 1)}$$

[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, (1 + 4*n)/2, (3 + 4*n)/2, Coth[c + d*x]^2])/(d*(1 + 4*n))

Rubi [A] time = 0.0388827, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3658, 3476, 364}

$$\frac{\coth(c + dx) (b \coth^4(c + dx))^n {}_2F_1\left(1, \frac{1}{2}(4n + 1); \frac{1}{2}(4n + 3); \coth^2(c + dx)\right)}{d(4n + 1)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^4)^n, x]

[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, (1 + 4*n)/2, (3 + 4*n)/2, Coth[c + d*x]^2])/(d*(1 + 4*n))

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (b \coth^4(c + dx))^n dx &= \left(\coth^{-4n}(c + dx) (b \coth^4(c + dx))^n \right) \int \coth^{4n}(c + dx) dx \\ &= \frac{\left(\coth^{-4n}(c + dx) (b \coth^4(c + dx))^n \right) \text{Subst} \left(\int \frac{x^{4n}}{-1+x^2} dx, x, \coth(c + dx) \right)}{d} \\ &= \frac{\coth(c + dx) (b \coth^4(c + dx))^n {}_2F_1 \left(1, \frac{1}{2}(1 + 4n); \frac{1}{2}(3 + 4n); \coth^2(c + dx) \right)}{d(1 + 4n)} \end{aligned}$$

Mathematica [A] time = 0.0437062, size = 51, normalized size = 0.89

$$\frac{\coth(c + dx) (b \coth^4(c + dx))^n {}_2F_1 \left(1, 2n + \frac{1}{2}; 2n + \frac{3}{2}; \coth^2(c + dx) \right)}{4dn + d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, 1/2 + 2*n, 3/2 +
2*n, Coth[c + d*x]^2])/ (d + 4*d*n)
```

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int (b(\coth(dx + c))^4)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^n,x)
```

```
[Out] int((b*coth(d*x+c)^4)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^4)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^4)^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \coth(dx + c)^4\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="fricas")

[Out] integral((b*coth(d*x + c)^4)^n, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth^4(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**4)**n,x)

[Out] Integral((b*coth(c + d*x)**4)**n, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^4)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^n, x)
```

3.40 $\int (b \coth^4(c + dx))^{3/2} dx$

Optimal. Leaf size=110

$$\frac{b \coth^3(c + dx) \sqrt{b \coth^4(c + dx)}}{5d} - \frac{b \coth(c + dx) \sqrt{b \coth^4(c + dx)}}{3d} + b x \tanh^2(c + dx) \sqrt{b \coth^4(c + dx)} - \frac{b \tanh(c + dx)}{d}$$

[Out] $-(b \cdot \text{Coth}[c + d \cdot x] \cdot \text{Sqrt}[b \cdot \text{Coth}[c + d \cdot x]^4]) / (3 \cdot d) - (b \cdot \text{Coth}[c + d \cdot x]^3 \cdot \text{Sqrt}[b \cdot \text{Coth}[c + d \cdot x]^4]) / (5 \cdot d) - (b \cdot \text{Sqrt}[b \cdot \text{Coth}[c + d \cdot x]^4] \cdot \text{Tanh}[c + d \cdot x]) / d + b \cdot x \cdot \text{Sqrt}[b \cdot \text{Coth}[c + d \cdot x]^4] \cdot \text{Tanh}[c + d \cdot x]^2$

Rubi [A] time = 0.0452879, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 8}

$$\frac{b \coth^3(c + dx) \sqrt{b \coth^4(c + dx)}}{5d} - \frac{b \coth(c + dx) \sqrt{b \coth^4(c + dx)}}{3d} + b x \tanh^2(c + dx) \sqrt{b \coth^4(c + dx)} - \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b \cdot \text{Coth}[c + d \cdot x]^4)^{(3/2)}, x]$

[Out] $-(b \cdot \text{Coth}[c + d \cdot x] \cdot \text{Sqrt}[b \cdot \text{Coth}[c + d \cdot x]^4]) / (3 \cdot d) - (b \cdot \text{Coth}[c + d \cdot x]^3 \cdot \text{Sqrt}[b \cdot \text{Coth}[c + d \cdot x]^4]) / (5 \cdot d) - (b \cdot \text{Sqrt}[b \cdot \text{Coth}[c + d \cdot x]^4] \cdot \text{Tanh}[c + d \cdot x]) / d + b \cdot x \cdot \text{Sqrt}[b \cdot \text{Coth}[c + d \cdot x]^4] \cdot \text{Tanh}[c + d \cdot x]^2$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n-1))/(d*(n-1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int (b \coth^4(c + dx))^{3/2} dx &= \left(b \sqrt{b \coth^4(c + dx)} \tanh^2(c + dx) \right) \int \coth^6(c + dx) dx \\
 &= -\frac{b \coth^3(c + dx) \sqrt{b \coth^4(c + dx)}}{5d} + \left(b \sqrt{b \coth^4(c + dx)} \tanh^2(c + dx) \right) \int \coth^4(c + dx) dx \\
 &= -\frac{b \coth(c + dx) \sqrt{b \coth^4(c + dx)}}{3d} - \frac{b \coth^3(c + dx) \sqrt{b \coth^4(c + dx)}}{5d} + \left(b \sqrt{b \coth^4(c + dx)} \tanh^2(c + dx) \right) \int \coth^2(c + dx) dx \\
 &= -\frac{b \coth(c + dx) \sqrt{b \coth^4(c + dx)}}{3d} - \frac{b \coth^3(c + dx) \sqrt{b \coth^4(c + dx)}}{5d} - \frac{b \sqrt{b \coth^4(c + dx)} \tanh^2(c + dx)}{d} + \left(b \sqrt{b \coth^4(c + dx)} \tanh^2(c + dx) \right) \int \coth(c + dx) dx \\
 &= -\frac{b \coth(c + dx) \sqrt{b \coth^4(c + dx)}}{3d} - \frac{b \coth^3(c + dx) \sqrt{b \coth^4(c + dx)}}{5d} - \frac{b \sqrt{b \coth^4(c + dx)} \tanh^2(c + dx)}{d} - \frac{b \sqrt{b \coth^4(c + dx)} \tanh(c + dx)}{d}
 \end{aligned}$$

Mathematica [C] time = 0.0633555, size = 43, normalized size = 0.39

$$\frac{\tanh(c + dx) (b \coth^4(c + dx))^{3/2} {}_2F_1\left(-\frac{5}{2}, 1, -\frac{3}{2}; \tanh^2(c + dx)\right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^4)^(3/2), x]

[Out] -((b*Coth[c + d*x]^4)^(3/2)*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d*x]^2]*Tanh[c + d*x])/(5*d)

Maple [A] time = 0.036, size = 77, normalized size = 0.7

$$\frac{6 (\coth(dx + c))^5 + 10 (\coth(dx + c))^3 + 15 \ln(\coth(dx + c) - 1) - 15 \ln(\coth(dx + c) + 1) + 30 \coth(dx + c)}{30 d (\coth(dx + c))^6} (b \coth^4(dx + c))^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*coth(d*x+c)^4)^(3/2),x)`

[Out] $-1/30/d*(b*\coth(d*x+c)^4)^{(3/2)}*(6*\coth(d*x+c)^5+10*\coth(d*x+c)^3+15*\ln(\coth(d*x+c)-1)-15*\ln(\coth(d*x+c)+1)+30*\coth(d*x+c))/\coth(d*x+c)^6$

Maxima [A] time = 1.73484, size = 185, normalized size = 1.68

$$\frac{(dx+c)b^{\frac{3}{2}}}{d} - \frac{2\left(70b^{\frac{3}{2}}e^{(-2dx-2c)} - 140b^{\frac{3}{2}}e^{(-4dx-4c)} + 90b^{\frac{3}{2}}e^{(-6dx-6c)} - 45b^{\frac{3}{2}}e^{(-8dx-8c)} - 23b^{\frac{3}{2}}\right)}{15d\left(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")`

[Out] $(d*x + c)*b^{(3/2)}/d - 2/15*(70*b^{(3/2)}*e^{(-2*d*x - 2*c)} - 140*b^{(3/2)}*e^{(-4*d*x - 4*c)} + 90*b^{(3/2)}*e^{(-6*d*x - 6*c)} - 45*b^{(3/2)}*e^{(-8*d*x - 8*c)} - 23*b^{(3/2)})/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1))$

Fricas [B] time = 2.65019, size = 9090, normalized size = 82.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")`

[Out] $1/15*(15*b*d*x*\cosh(d*x + c)^{10} + 15*(b*d*x*e^{(4*d*x + 4*c)} - 2*b*d*x*e^{(2*d*x + 2*c)} + b*d*x)*\sinh(d*x + c)^{10} + 150*(b*d*x*\cosh(d*x + c)*e^{(4*d*x + 4*c)} - 2*b*d*x*\cosh(d*x + c)*e^{(2*d*x + 2*c)} + b*d*x*\cosh(d*x + c))*\sinh(d*x + c)^9 - 15*(5*b*d*x + 6*b)*\cosh(d*x + c)^8 + 15*(45*b*d*x*\cosh(d*x + c)^2 - 5*b*d*x + (45*b*d*x*\cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^{(4*d*x + 4*c)} - 2*(45*b*d*x*\cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^{(2*d*x + 2*c)} - 6*b)*\sinh(d*x + c)^8 + 120*(15*b*d*x*\cosh(d*x + c)^3 - (5*b*d*x + 6*b)*\cosh(d*x + c) + (15*b*d*x*\cosh(d*x + c)^3 - (5*b*d*x + 6*b)*\cosh(d*x + c))*e^{(4*d*x + 4*c)} - 2*(15*b*d*x*\cosh(d*x + c)^3 - (5*b*d*x + 6*b)*\cosh(d*x + c))*e^{(2*d*x + 2*c)}))*\sinh(d*x + c)^7 + 30*(5*b*d*x + 6*b)*\cosh(d*x + c)^6 + 30*(105*b*d*x*\cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*\cosh(d*x + c)^2 + (105*b*d*x*$

$$\begin{aligned}
& \cosh(dx + c)^4 + 5*b*dx - 14*(5*b*dx + 6*b)*\cosh(dx + c)^2 + 6*b)*e^{(4* \\
& dx + 4*c) - 2*(105*b*dx*\cosh(dx + c)^4 + 5*b*dx - 14*(5*b*dx + 6*b)*\co \\
& sh(dx + c)^2 + 6*b)*e^{(2*dx + 2*c) + 6*b)*\sinh(dx + c)^6 + 60*(63*b*dx* \\
& \cosh(dx + c)^5 - 14*(5*b*dx + 6*b)*\cosh(dx + c)^3 + 3*(5*b*dx + 6*b)*\co \\
& sh(dx + c) + (63*b*dx*\cosh(dx + c)^5 - 14*(5*b*dx + 6*b)*\cosh(dx + c)^ \\
& 3 + 3*(5*b*dx + 6*b)*\cosh(dx + c))*e^{(4*dx + 4*c) - 2*(63*b*dx*\cosh(dx \\
& + c)^5 - 14*(5*b*dx + 6*b)*\cosh(dx + c)^3 + 3*(5*b*dx + 6*b)*\cosh(dx + \\
& c))*e^{(2*dx + 2*c))*\sinh(dx + c)^5 - 10*(15*b*dx + 28*b)*\cosh(dx + c)^ \\
& 4 + 10*(315*b*dx*\cosh(dx + c)^6 - 105*(5*b*dx + 6*b)*\cosh(dx + c)^4 - 1 \\
& 5*b*dx + 45*(5*b*dx + 6*b)*\cosh(dx + c)^2 + (315*b*dx*\cosh(dx + c)^6 - \\
& 105*(5*b*dx + 6*b)*\cosh(dx + c)^4 - 15*b*dx + 45*(5*b*dx + 6*b)*\cosh(d \\
& *x + c)^2 - 28*b)*e^{(4*dx + 4*c) - 2*(315*b*dx*\cosh(dx + c)^6 - 105*(5*b \\
& *dx + 6*b)*\cosh(dx + c)^4 - 15*b*dx + 45*(5*b*dx + 6*b)*\cosh(dx + c)^2 \\
& - 28*b)*e^{(2*dx + 2*c) - 28*b)*\sinh(dx + c)^4 + 40*(45*b*dx*\cosh(dx + \\
& c)^7 - 21*(5*b*dx + 6*b)*\cosh(dx + c)^5 + 15*(5*b*dx + 6*b)*\cosh(dx + c \\
&)^3 - (15*b*dx + 28*b)*\cosh(dx + c) + (45*b*dx*\cosh(dx + c)^7 - 21*(5*b \\
& *dx + 6*b)*\cosh(dx + c)^5 + 15*(5*b*dx + 6*b)*\cosh(dx + c)^3 - (15*b*d \\
& x + 28*b)*\cosh(dx + c))*e^{(4*dx + 4*c) - 2*(45*b*dx*\cosh(dx + c)^7 - 21 \\
& *(5*b*dx + 6*b)*\cosh(dx + c)^5 + 15*(5*b*dx + 6*b)*\cosh(dx + c)^3 - (15 \\
& *b*dx + 28*b)*\cosh(dx + c))*e^{(2*dx + 2*c))*\sinh(dx + c)^3 - 15*b*dx + \\
& 5*(15*b*dx + 28*b)*\cosh(dx + c)^2 + 5*(135*b*dx*\cosh(dx + c)^8 - 84*(5 \\
& *b*dx + 6*b)*\cosh(dx + c)^6 + 90*(5*b*dx + 6*b)*\cosh(dx + c)^4 + 15*b*d \\
& *x - 12*(15*b*dx + 28*b)*\cosh(dx + c)^2 + (135*b*dx*\cosh(dx + c)^8 - 84 \\
& *(5*b*dx + 6*b)*\cosh(dx + c)^6 + 90*(5*b*dx + 6*b)*\cosh(dx + c)^4 + 15* \\
& b*dx - 12*(15*b*dx + 28*b)*\cosh(dx + c)^2 + 28*b)*e^{(4*dx + 4*c) - 2*(1 \\
& 35*b*dx*\cosh(dx + c)^8 - 84*(5*b*dx + 6*b)*\cosh(dx + c)^6 + 90*(5*b*dx \\
& + 6*b)*\cosh(dx + c)^4 + 15*b*dx - 12*(15*b*dx + 28*b)*\cosh(dx + c)^2 + \\
& 28*b)*e^{(2*dx + 2*c) + 28*b)*\sinh(dx + c)^2 + (15*b*dx*\cosh(dx + c)^10 \\
& - 15*(5*b*dx + 6*b)*\cosh(dx + c)^8 + 30*(5*b*dx + 6*b)*\cosh(dx + c)^6 \\
& - 10*(15*b*dx + 28*b)*\cosh(dx + c)^4 - 15*b*dx + 5*(15*b*dx + 28*b)*\cos \\
& h(dx + c)^2 - 46*b)*e^{(4*dx + 4*c) - 2*(15*b*dx*\cosh(dx + c)^10 - 15*(5 \\
& *b*dx + 6*b)*\cosh(dx + c)^8 + 30*(5*b*dx + 6*b)*\cosh(dx + c)^6 - 10*(15 \\
& *b*dx + 28*b)*\cosh(dx + c)^4 - 15*b*dx + 5*(15*b*dx + 28*b)*\cosh(dx + \\
& c)^2 - 46*b)*e^{(2*dx + 2*c) + 10*(15*b*dx*\cosh(dx + c)^9 - 12*(5*b*dx + \\
& 6*b)*\cosh(dx + c)^7 + 18*(5*b*dx + 6*b)*\cosh(dx + c)^5 - 4*(15*b*dx + \\
& 28*b)*\cosh(dx + c)^3 + (15*b*dx + 28*b)*\cosh(dx + c) + (15*b*dx*\cosh(dx \\
& + c)^9 - 12*(5*b*dx + 6*b)*\cosh(dx + c)^7 + 18*(5*b*dx + 6*b)*\cosh(dx \\
& + c)^5 - 4*(15*b*dx + 28*b)*\cosh(dx + c)^3 + (15*b*dx + 28*b)*\cosh(dx \\
& + c))*e^{(4*dx + 4*c) - 2*(15*b*dx*\cosh(dx + c)^9 - 12*(5*b*dx + 6*b)*\co \\
& sh(dx + c)^7 + 18*(5*b*dx + 6*b)*\cosh(dx + c)^5 - 4*(15*b*dx + 28*b)*\co \\
& sh(dx + c)^3 + (15*b*dx + 28*b)*\cosh(dx + c))*e^{(2*dx + 2*c))*\sinh(dx \\
& + c) - 46*b)*\sqrt{(b*e^{(8*dx + 8*c) + 4*b*e^{(6*dx + 6*c) + 6*b*e^{(4*dx + \\
& 4*c) + 4*b*e^{(2*dx + 2*c) + b)/(e^{(8*dx + 8*c) - 4*e^{(6*dx + 6*c) + 6*e \\
& ^{(4*dx + 4*c) - 4*e^{(2*dx + 2*c) + 1)))/(d*\cosh(dx + c)^10 + (d*e^{(4*dx \\
& + 4*c) + 2*d*e^{(2*dx + 2*c) + d)*\sinh(dx + c)^10 + 10*(d*\cosh(dx + c))*e^{
\end{aligned}$$

```
(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d
*x + c)^9 - 5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + (9*d*cosh(d*x +
c)^2 - d)*e^(4*d*x + 4*c) + 2*(9*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d
)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d
*x + c)^3 - d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*d*cosh(d*x + c)^3 - d*c
osh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*
(21*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 + (21*d*cosh(d*x + c)^4 - 14*d*
cosh(d*x + c)^2 + d)*e^(4*d*x + 4*c) + 2*(21*d*cosh(d*x + c)^4 - 14*d*cosh
(d*x + c)^2 + d)*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x +
c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c) + (63*d*cosh(d*x + c)^5 -
70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(63*d*cosh(d
*x + c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^5 - 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 - 35*d*cosh(
d*x + c)^4 + 15*d*cosh(d*x + c)^2 + (21*d*cosh(d*x + c)^6 - 35*d*cosh(d*x +
c)^4 + 15*d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(21*d*cosh(d*x + c)^6
- 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d)*si
nh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*
x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5
*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*d*cosh(d*x + c
)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(2*d*x
+ 2*c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 - 2
8*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + (9*d*co
sh(d*x + c)^8 - 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x
+ c)^2 + d)*e^(4*d*x + 4*c) + 2*(9*d*cosh(d*x + c)^8 - 28*d*cosh(d*x + c)^
6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) + d)*s
inh(d*x + c)^2 + (d*cosh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 10*d*cosh(d*x
+ c)^6 - 10*d*cosh(d*x + c)^4 + 5*d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) +
2*(d*cosh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 10*d*cosh(d*x + c)^6 - 10*d*c
osh(d*x + c)^4 + 5*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 10*(d*cosh(d*x
+ c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 +
d*cosh(d*x + c) + (d*cosh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x +
c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(d*cosh(
d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth^4(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**4)**(3/2),x)

[Out] Integral((b*coth(c + d*x)**4)**(3/2), x)

Giac [A] time = 1.18266, size = 108, normalized size = 0.98

$$\frac{1}{15} b^{\frac{3}{2}} \left(\frac{15(dx+c)}{d} - \frac{2(45e^{(8dx+8c)} - 90e^{(6dx+6c)} + 140e^{(4dx+4c)} - 70e^{(2dx+2c)} + 23)}{d(e^{(2dx+2c)} - 1)^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")

[Out] 1/15*b^(3/2)*(15*(d*x + c)/d - 2*(45*e^(8*d*x + 8*c) - 90*e^(6*d*x + 6*c) + 140*e^(4*d*x + 4*c) - 70*e^(2*d*x + 2*c) + 23)/(d*(e^(2*d*x + 2*c) - 1)^5)
)

3.41 $\int \sqrt{b \coth^4(c + dx)} dx$

Optimal. Leaf size=50

$$x \tanh^2(c + dx) \sqrt{b \coth^4(c + dx)} - \frac{\tanh(c + dx) \sqrt{b \coth^4(c + dx)}}{d}$$

[Out] -((Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d) + x*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x]^2

Rubi [A] time = 0.0228098, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 8}

$$x \tanh^2(c + dx) \sqrt{b \coth^4(c + dx)} - \frac{\tanh(c + dx) \sqrt{b \coth^4(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Coth[c + d*x]^4], x]

[Out] -((Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d) + x*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x]^2

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \sqrt{b \coth^4(c + dx)} dx &= \left(\sqrt{b \coth^4(c + dx) \tanh^2(c + dx)} \right) \int \coth^2(c + dx) dx \\ &= -\frac{\sqrt{b \coth^4(c + dx) \tanh(c + dx)}}{d} + \left(\sqrt{b \coth^4(c + dx) \tanh^2(c + dx)} \right) \int 1 dx \\ &= -\frac{\sqrt{b \coth^4(c + dx) \tanh(c + dx)}}{d} + x \sqrt{b \coth^4(c + dx) \tanh^2(c + dx)} \end{aligned}$$

Mathematica [C] time = 0.0306745, size = 41, normalized size = 0.82

$$-\frac{\tanh(c + dx) \sqrt{b \coth^4(c + dx)} {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(c + dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Coth[c + d*x]^4], x]

[Out] -((Sqrt[b*Coth[c + d*x]^4]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2]*Tanh[c + d*x])/d)

Maple [A] time = 0.041, size = 55, normalized size = 1.1

$$-\frac{2 \coth(dx + c) + \ln(\coth(dx + c) - 1) - \ln(\coth(dx + c) + 1)}{2d(\coth(dx + c))^2} \sqrt{b(\coth(dx + c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^4)^(1/2), x)

[Out] -1/2/d*(b*coth(d*x+c)^4)^(1/2)*(2*coth(d*x+c)+ln(coth(d*x+c)-1)-ln(coth(d*x+c)+1))/coth(d*x+c)^2

Maxima [A] time = 1.81973, size = 46, normalized size = 0.92

$$\frac{(dx+c)\sqrt{b}}{d} + \frac{2\sqrt{b}}{d(e^{(-2dx-2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")

[Out] (d*x + c)*sqrt(b)/d + 2*sqrt(b)/(d*(e^(-2*d*x - 2*c) - 1))

Fricas [B] time = 2.15774, size = 1046, normalized size = 20.92

$$\frac{(dx \cosh(dx+c))^2 + (dx e^{(4dx+4c)} - 2 dx e^{(2dx+2c)} + dx) \sinh(dx+c)^2 - dx + (dx \cosh(dx+c)^2 - dx - 2)e^{(4dx+4c)} - 2(dx \cosh(dx+c)^2 + (d e^{(4dx+4c)} + 2 d e^{(2dx+2c)} + d) \sinh(dx+c)^2 + (d \cosh(dx+c))^2 - d)}{d \cosh(dx+c)^2 + (d e^{(4dx+4c)} + 2 d e^{(2dx+2c)} + d) \sinh(dx+c)^2 + (d \cosh(dx+c))^2 - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")

[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c) - 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) + 6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^2 + (d*e^(4*d*x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^2 + (d*cosh(d*x + c))^2 - d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c) - d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth^4(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**4), x)
```

Giac [A] time = 1.14477, size = 43, normalized size = 0.86

$$\sqrt{b} \left(\frac{dx + c}{d} - \frac{2}{d(e^{2dx+2c} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b)*((d*x + c)/d - 2/(d*(e^(2*d*x + 2*c) - 1)))
```

$$3.42 \quad \int \frac{1}{\sqrt{b \coth^4(c+dx)}} dx$$

Optimal. Leaf size=50

$$\frac{x \coth^2(c+dx)}{\sqrt{b \coth^4(c+dx)}} - \frac{\coth(c+dx)}{d\sqrt{b \coth^4(c+dx)}}$$

[Out] -(Coth[c + d*x]/(d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/Sqrt[b*Coth[c + d*x]^4]

Rubi [A] time = 0.0226638, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 8}

$$\frac{x \coth^2(c+dx)}{\sqrt{b \coth^4(c+dx)}} - \frac{\coth(c+dx)}{d\sqrt{b \coth^4(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*Coth[c + d*x]^4], x]

[Out] -(Coth[c + d*x]/(d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/Sqrt[b*Coth[c + d*x]^4]

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{b \coth^4(c+dx)}} dx &= \frac{\coth^2(c+dx) \int \tanh^2(c+dx) dx}{\sqrt{b \coth^4(c+dx)}} \\ &= -\frac{\coth(c+dx)}{d\sqrt{b \coth^4(c+dx)}} + \frac{\coth^2(c+dx) \int 1 dx}{\sqrt{b \coth^4(c+dx)}} \\ &= -\frac{\coth(c+dx)}{d\sqrt{b \coth^4(c+dx)}} + \frac{x \coth^2(c+dx)}{\sqrt{b \coth^4(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.072736, size = 40, normalized size = 0.8

$$\frac{\coth(c+dx) \left(\tanh^{-1}(\tanh(c+dx)) \coth(c+dx) - 1 \right)}{d\sqrt{b \coth^4(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*Coth[c + d*x]^4], x]

[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*Sqrt[b*Coth[c + d*x]^4])

Maple [A] time = 0.037, size = 59, normalized size = 1.2

$$\frac{\coth(dx+c) (\ln(\coth(dx+c)+1) \coth(dx+c) - \ln(\coth(dx+c)-1) \coth(dx+c) - 2)}{2d} \frac{1}{\sqrt{b(\coth(dx+c))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c)^4)^(1/2), x)

[Out] $1/2/d*\coth(d*x+c)*(ln(\coth(d*x+c)+1)*\coth(d*x+c)-ln(\coth(d*x+c)-1)*\coth(d*x+c)-2)/(b*\coth(d*x+c)^4)^{(1/2)}$

Maxima [A] time = 1.73557, size = 49, normalized size = 0.98

$$\frac{dx + c}{\sqrt{bd}} - \frac{2\sqrt{b}}{(be^{(-2dx-2c)} + b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")`

[Out] $(d*x + c)/(\text{sqrt}(b)*d) - 2*\text{sqrt}(b)/((b*e^{(-2*d*x - 2*c)} + b)*d)$

Fricas [B] time = 2.19262, size = 1079, normalized size = 21.58

$$\frac{(dx \cosh(dx + c))^2 + (dx e^{4dx+4c} - 2 dx e^{2dx+2c} + dx) \sinh(dx + c)^2 + dx + (dx \cosh(dx + c)^2 + dx + 2) e^{4dx+4c} - 2(dx \cosh(dx + c)^2 + dx + 2) e^{2dx+2c}}{bd \cosh(dx + c)^2 + (bde^{4dx+4c} + 2 bde^{2dx+2c} + bd) \sinh(dx + c)^2 + bd + (bd \cosh(dx + c)^2 + (bd \cosh(dx + c)^2 + dx + 2) e^{4dx+4c} - 2(dx \cosh(dx + c)^2 + dx + 2) e^{2dx+2c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")`

[Out] $(d*x*\cosh(d*x + c)^2 + (d*x*e^{(4*d*x + 4*c)} - 2*d*x*e^{(2*d*x + 2*c)} + d*x)*\sinh(d*x + c)^2 + d*x + (d*x*\cosh(d*x + c)^2 + d*x + 2)*e^{(4*d*x + 4*c)} - 2*(d*x*\cosh(d*x + c)^2 + d*x + 2)*e^{(2*d*x + 2*c)} + 2*(d*x*\cosh(d*x + c)*e^{(4*d*x + 4*c)} - 2*d*x*\cosh(d*x + c)*e^{(2*d*x + 2*c)} + d*x*\cosh(d*x + c))*\sinh(d*x + c) + 2)*\text{sqrt}((b*e^{(8*d*x + 8*c)} + 4*b*e^{(6*d*x + 6*c)} + 6*b*e^{(4*d*x + 4*c)} + 4*b*e^{(2*d*x + 2*c)} + b)/(e^{(8*d*x + 8*c)} - 4*e^{(6*d*x + 6*c)} + 6*e^{(4*d*x + 4*c)} - 4*e^{(2*d*x + 2*c)} + 1))/(b*d*\cosh(d*x + c)^2 + (b*d*e^{(4*d*x + 4*c)} + 2*b*d*e^{(2*d*x + 2*c)} + b*d)*\sinh(d*x + c)^2 + b*d + (b*d*\cosh(d*x + c)^2 + b*d)*e^{(4*d*x + 4*c)} + 2*(b*d*\cosh(d*x + c)^2 + b*d)*e^{(2*d*x + 2*c)} + 2*(b*d*\cosh(d*x + c)*e^{(4*d*x + 4*c)} + 2*b*d*\cosh(d*x + c)*e^{(2*d*x + 2*c)} + b*d*\cosh(d*x + c))*\sinh(d*x + c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth^4(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**4)**(1/2), x)

[Out] Integral(1/sqrt(b*coth(c + d*x)**4), x)

Giac [A] time = 1.16942, size = 46, normalized size = 0.92

$$\frac{dx + c}{\sqrt{bd}} + \frac{2}{\sqrt{bd}(e^{2dx+2c} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^4)^(1/2), x, algorithm="giac")

[Out] (d*x + c)/(sqrt(b)*d) + 2/(sqrt(b)*d*(e^(2*d*x + 2*c) + 1))

$$3.43 \quad \int \frac{1}{\left(b \coth^4(c+dx)\right)^{3/2}} dx$$

Optimal. Leaf size=118

$$\frac{x \coth^2(c+dx)}{b\sqrt{b \coth^4(c+dx)}} - \frac{\coth(c+dx)}{bd\sqrt{b \coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b \coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b \coth^4(c+dx)}}$$

[Out] $-(\text{Coth}[c + d*x]/(b*d*\text{Sqrt}[b*\text{Coth}[c + d*x]^4])) + (x*\text{Coth}[c + d*x]^2)/(b*\text{Sqrt}[b*\text{Coth}[c + d*x]^4]) - \text{Tanh}[c + d*x]/(3*b*d*\text{Sqrt}[b*\text{Coth}[c + d*x]^4]) - \text{Tanh}[c + d*x]^3/(5*b*d*\text{Sqrt}[b*\text{Coth}[c + d*x]^4])$

Rubi [A] time = 0.0459973, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 8}

$$\frac{x \coth^2(c+dx)}{b\sqrt{b \coth^4(c+dx)}} - \frac{\coth(c+dx)}{bd\sqrt{b \coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b \coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b \coth^4(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x]^4)^{-3/2}, x]$

[Out] $-(\text{Coth}[c + d*x]/(b*d*\text{Sqrt}[b*\text{Coth}[c + d*x]^4])) + (x*\text{Coth}[c + d*x]^2)/(b*\text{Sqrt}[b*\text{Coth}[c + d*x]^4]) - \text{Tanh}[c + d*x]/(3*b*d*\text{Sqrt}[b*\text{Coth}[c + d*x]^4]) - \text{Tanh}[c + d*x]^3/(5*b*d*\text{Sqrt}[b*\text{Coth}[c + d*x]^4])$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n-1))/(d*(n-1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n-2), x],
```


$x]$ /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(b \coth^4(c + dx))^{3/2}} dx &= \frac{\coth^2(c + dx) \int \tanh^6(c + dx) dx}{b\sqrt{b \coth^4(c + dx)}} \\
 &= -\frac{\tanh^3(c + dx)}{5bd\sqrt{b \coth^4(c + dx)}} + \frac{\coth^2(c + dx) \int \tanh^4(c + dx) dx}{b\sqrt{b \coth^4(c + dx)}} \\
 &= -\frac{\tanh(c + dx)}{3bd\sqrt{b \coth^4(c + dx)}} - \frac{\tanh^3(c + dx)}{5bd\sqrt{b \coth^4(c + dx)}} + \frac{\coth^2(c + dx) \int \tanh^2(c + dx) dx}{b\sqrt{b \coth^4(c + dx)}} \\
 &= -\frac{\coth(c + dx)}{bd\sqrt{b \coth^4(c + dx)}} - \frac{\tanh(c + dx)}{3bd\sqrt{b \coth^4(c + dx)}} - \frac{\tanh^3(c + dx)}{5bd\sqrt{b \coth^4(c + dx)}} + \frac{\coth^2(c + dx) \int 1 dx}{b\sqrt{b \coth^4(c + dx)}} \\
 &= -\frac{\coth(c + dx)}{bd\sqrt{b \coth^4(c + dx)}} + \frac{x \coth^2(c + dx)}{b\sqrt{b \coth^4(c + dx)}} - \frac{\tanh(c + dx)}{3bd\sqrt{b \coth^4(c + dx)}} - \frac{\tanh^3(c + dx)}{5bd\sqrt{b \coth^4(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.232707, size = 68, normalized size = 0.58

$$\frac{-3 \tanh^3(c + dx) - 5 \tanh(c + dx) - 15 \coth(c + dx) + 15 \tanh^{-1}(\tanh(c + dx)) \coth^2(c + dx)}{15bd\sqrt{b \coth^4(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^4)^(-3/2), x]

[Out] (-15*Coth[c + d*x] + 15*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]^2 - 5*Tanh[c + d*x] - 3*Tanh[c + d*x]^3)/(15*b*d*Sqrt[b*Coth[c + d*x]^4])

Maple [A] time = 0.022, size = 84, normalized size = 0.7

$$\frac{\coth(dx + c) \left(15 \ln(\coth(dx + c) + 1) (\coth(dx + c))^5 - 15 \ln(\coth(dx + c) - 1) (\coth(dx + c))^5 - 30 (\coth(dx + c))^5 \right)}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*coth(d*x+c)^4)^(3/2),x)`

[Out] $\frac{1}{30} \frac{1}{d} \frac{\coth(d*x+c) * (15 * \ln(\coth(d*x+c)+1) * \coth(d*x+c)^5 - 15 * \ln(\coth(d*x+c)-1) * \coth(d*x+c)^5 - 30 * \coth(d*x+c)^4 - 10 * \coth(d*x+c)^2 - 6)}{\coth(d*x+c)^4}^{3/2}$

Maxima [A] time = 1.80108, size = 209, normalized size = 1.77

$$\frac{2 \left(70 \sqrt{b} e^{(-2dx-2c)} + 140 \sqrt{b} e^{(-4dx-4c)} + 90 \sqrt{b} e^{(-6dx-6c)} + 45 \sqrt{b} e^{(-8dx-8c)} + 23 \sqrt{b} \right)}{15 \left(5 b^2 e^{(-2dx-2c)} + 10 b^2 e^{(-4dx-4c)} + 10 b^2 e^{(-6dx-6c)} + 5 b^2 e^{(-8dx-8c)} + b^2 e^{(-10dx-10c)} + b^2 \right) d} + \frac{dx+c}{b^{\frac{3}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")`

[Out] $\frac{-2}{15} \frac{(70 \sqrt{b} e^{(-2dx-2c)} + 140 \sqrt{b} e^{(-4dx-4c)} + 90 \sqrt{b} e^{(-6dx-6c)} + 45 \sqrt{b} e^{(-8dx-8c)} + 23 \sqrt{b})}{(5 b^2 e^{(-2dx-2c)} + 10 b^2 e^{(-4dx-4c)} + 10 b^2 e^{(-6dx-6c)} + 5 b^2 e^{(-8dx-8c)} + b^2 e^{(-10dx-10c)} + b^2) d} + \frac{dx+c}{b^{3/2} d}$

Fricas [B] time = 2.67664, size = 9179, normalized size = 77.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")`

[Out] $\frac{1}{15} (15 d x \cosh(d x + c)^{10} + 15 (d x e^{(4 d x + 4 c)} - 2 d x e^{(2 d x + 2 c)} + d x) \sinh(d x + c)^{10} + 150 (d x \cosh(d x + c) e^{(4 d x + 4 c)} - 2 d x \cosh(d x + c) e^{(2 d x + 2 c)} + d x \cosh(d x + c)) \sinh(d x + c)^9 + 15 (5 d x + 6) \cosh(d x + c)^8 + 15 (45 d x \cosh(d x + c)^2 + 5 d x + (45 d x \cosh(d x + c)^2 + 5 d x + 6) e^{(4 d x + 4 c)} - 2 (45 d x \cosh(d x + c)^2 + 5 d x + 6) e^{(2 d x + 2 c)} + 6) \sinh(d x + c)^8 + 120 (15 d x \cosh(d x + c)^3 + (5 d x + 6) \cosh(d x + c) + (15 d x \cosh(d x + c)^3 + (5 d x + 6) \cosh(d x + c)) e^{(4 d x + 4 c)} - 2 (15 d x \cosh(d x + c)^3 + (5 d x + 6) \cosh(d x + c)) e^{(2 d x + 2 c)}) \sinh(d x + c)^7 + 30 (5 d x + 6) \cosh(d x + c)^6 + 30 (105 d x \cosh(d x + c)^4 + 14 (5 d x + 6) \cosh(d x + c)^2 + 5 d x + (1$

$$\begin{aligned}
& 05*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + 6)*e^(4*d*x + 4*c) - 2*(105*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + 6)*e^(2*d*x + 2*c) + 6)*sinh(d*x + c)^6 + 60*(63*d*x*cosh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x + c) + (63*d*x*cosh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(63*d*x*cosh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 10*(15*d*x + 28)*cosh(d*x + c)^4 + 10*(315*d*x*cosh(d*x + c)^6 + 105*(5*d*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + (315*d*x*cosh(d*x + c)^6 + 105*(5*d*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(315*d*x*cosh(d*x + c)^6 + 105*(5*d*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(2*d*x + 2*c) + 28)*sinh(d*x + c)^4 + 40*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c) + (45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^3 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 5*(135*d*x*cosh(d*x + c)^8 + 84*(5*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + (135*d*x*cosh(d*x + c)^8 + 84*(5*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(135*d*x*cosh(d*x + c)^8 + 84*(5*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(2*d*x + 2*c) + 28)*sinh(d*x + c)^2 + 15*d*x + (15*d*x*cosh(d*x + c)^10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d*x + 6)*cosh(d*x + c)^6 + 10*(15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 46)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d*x + 6)*cosh(d*x + c)^6 + 10*(15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 46)*e^(2*d*x + 2*c) + 10*(15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c) + (15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + 46)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) + 6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b^2*d*cosh(d*x + c)^10 + 5*b^2*d*cosh(d*x + c)^8 + (b^2*d*e^(4*d*x + 4*c) + 2*b^2*d*e^(2*d*x + 2*c) + b^2*d)*sinh(d*x + c)^10 + 10*(b^2*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b^2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^9 + 10*b^2*d*cosh(d*x + c)^6 + 5*(9*b^2*d*cosh(d*x + c)^2 + b^2*d + (9*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(9*b^2*d*cosh(d*x + c)^2 + b^2*d
\end{aligned}$$

```

)*e^(2*d*x + 2*c))*sinh(d*x + c)^8 + 40*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*co
sh(d*x + c) + (3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(4*d*x + 4*
c) + 2*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^7 + 10*b^2*d*cosh(d*x + c)^4 + 10*(21*b^2*d*cosh(d*x + c)^4 + 14
*b^2*d*cosh(d*x + c)^2 + b^2*d + (21*b^2*d*cosh(d*x + c)^4 + 14*b^2*d*cosh(
d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(21*b^2*d*cosh(d*x + c)^4 + 14*b^2*
d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^6 + 4*(63*b^2*d*c
osh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c) + (63*b^
2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c))*e^
(4*d*x + 4*c) + 2*(63*b^2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15
*b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 5*b^2*d*cosh(d*x +
c)^2 + 10*(21*b^2*d*cosh(d*x + c)^6 + 35*b^2*d*cosh(d*x + c)^4 + 15*b^2*d*
cosh(d*x + c)^2 + b^2*d + (21*b^2*d*cosh(d*x + c)^6 + 35*b^2*d*cosh(d*x + c
)^4 + 15*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(21*b^2*d*cosh(
d*x + c)^6 + 35*b^2*d*cosh(d*x + c)^4 + 15*b^2*d*cosh(d*x + c)^2 + b^2*d)*e
^(2*d*x + 2*c))*sinh(d*x + c)^4 + 40*(3*b^2*d*cosh(d*x + c)^7 + 7*b^2*d*cos
h(d*x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (3*b^2*d*cos
h(d*x + c)^7 + 7*b^2*d*cosh(d*x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*co
sh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*b^2*d*cosh(d*x + c)^7 + 7*b^2*d*cosh(d*
x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*
sinh(d*x + c)^3 + b^2*d + 5*(9*b^2*d*cosh(d*x + c)^8 + 28*b^2*d*cosh(d*x +
c)^6 + 30*b^2*d*cosh(d*x + c)^4 + 12*b^2*d*cosh(d*x + c)^2 + b^2*d + (9*b^2
*d*cosh(d*x + c)^8 + 28*b^2*d*cosh(d*x + c)^6 + 30*b^2*d*cosh(d*x + c)^4 +
12*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(9*b^2*d*cosh(d*x + c
)^8 + 28*b^2*d*cosh(d*x + c)^6 + 30*b^2*d*cosh(d*x + c)^4 + 12*b^2*d*cosh(d
*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^
10 + 5*b^2*d*cosh(d*x + c)^8 + 10*b^2*d*cosh(d*x + c)^6 + 10*b^2*d*cosh(d*x
+ c)^4 + 5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(
d*x + c)^10 + 5*b^2*d*cosh(d*x + c)^8 + 10*b^2*d*cosh(d*x + c)^6 + 10*b^2*d
*cosh(d*x + c)^4 + 5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c) + 10*(b
^2*d*cosh(d*x + c)^9 + 4*b^2*d*cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 +
4*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^9 + 4*
b^2*d*cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^3 +
b^2*d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(d*x + c)^9 + 4*b^2*d*
cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^3 + b^2*d
*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^4(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**4)**(3/2),x)

[Out] Integral((b*coth(c + d*x)**4)**(-3/2), x)

Giac [A] time = 1.25118, size = 138, normalized size = 1.17

$$\frac{\frac{15(dx+c)}{\sqrt{bd}} + \frac{2(45\sqrt{be^{8dx+8c}}+90\sqrt{be^{6dx+6c}}+140\sqrt{be^{4dx+4c}}+70\sqrt{be^{2dx+2c}}+23\sqrt{b})}{bd(e^{2dx+2c}+1)^5}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")

[Out] 1/15*(15*(d*x + c)/(sqrt(b)*d) + 2*(45*sqrt(b)*e^(8*d*x + 8*c) + 90*sqrt(b)*e^(6*d*x + 6*c) + 140*sqrt(b)*e^(4*d*x + 4*c) + 70*sqrt(b)*e^(2*d*x + 2*c) + 23*sqrt(b))/(b*d*(e^(2*d*x + 2*c) + 1)^5)/b

3.44 $\int (b \coth^4(c + dx))^{4/3} dx$

Optimal. Leaf size=353

$$\frac{3b \coth^3(c + dx) \sqrt[3]{b \coth^4(c + dx)}}{13d} - \frac{3b \coth(c + dx) \sqrt[3]{b \coth^4(c + dx)}}{7d} - \frac{b \sqrt[3]{b \coth^4(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{c}\right)}{4d \coth^{\frac{4}{3}}(c + dx)}$$

[Out] $-(\text{Sqrt}[3]*b*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(2*d*\text{Coth}[c + d*x]^{(4/3)}) + (\text{Sqrt}[3]*b*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(2*d*\text{Coth}[c + d*x]^{(4/3)}) + (b*\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}]*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(d*\text{Coth}[c + d*x]^{(4/3)}) - (3*b*\text{Coth}[c + d*x]*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(7*d) - (3*b*\text{Coth}[c + d*x]^3*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(13*d) - (b*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/ (4*d*\text{Coth}[c + d*x]^{(4/3)}) + (b*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/ (4*d*\text{Coth}[c + d*x]^{(4/3)}) - (3*b*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)}*\text{Tanh}[c + d*x])/d$

Rubi [A] time = 0.196008, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$\frac{3b \coth^3(c + dx) \sqrt[3]{b \coth^4(c + dx)}}{13d} - \frac{3b \coth(c + dx) \sqrt[3]{b \coth^4(c + dx)}}{7d} - \frac{b \sqrt[3]{b \coth^4(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{c}\right)}{4d \coth^{\frac{4}{3}}(c + dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x]^{(4/3)})^{(4/3)}, x]$

[Out] $-(\text{Sqrt}[3]*b*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(2*d*\text{Coth}[c + d*x]^{(4/3)}) + (\text{Sqrt}[3]*b*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(2*d*\text{Coth}[c + d*x]^{(4/3)}) + (b*\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}]*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(d*\text{Coth}[c + d*x]^{(4/3)}) - (3*b*\text{Coth}[c + d*x]*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(7*d) - (3*b*\text{Coth}[c + d*x]^3*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)})/(13*d) - (b*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/ (4*d*\text{Coth}[c + d*x]^{(4/3)}) + (b*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/ (4*d*\text{Coth}[c + d*x]^{(4/3)}) - (3*b*(b*\text{Coth}[c + d*x]^{(4/3)})^{(1/3)}*\text{Tanh}[c + d*x])/d$

d*x])/d

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_))^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_))^(n_))^(p_), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

$[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[(d_) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int (b \coth^4(c+dx))^{4/3} dx &= \frac{\left(b \sqrt[3]{b \coth^4(c+dx)}\right) \int \coth^{16/3}(c+dx) dx}{\coth^{4/3}(c+dx)} \\
&= -\frac{3b \coth^3(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{13d} + \frac{\left(b \sqrt[3]{b \coth^4(c+dx)}\right) \int \coth^{10/3}(c+dx) dx}{\coth^{4/3}(c+dx)} \\
&= -\frac{3b \coth(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{7d} - \frac{3b \coth^3(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{13d} + \frac{\left(b \sqrt[3]{b \coth^4(c+dx)}\right) \int \coth^{4/3}(c+dx) dx}{\coth^{4/3}(c+dx)} \\
&= -\frac{3b \coth(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{7d} - \frac{3b \coth^3(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{13d} - \frac{3b \sqrt[3]{b \coth^4(c+dx)}}{d} \\
&= -\frac{3b \coth(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{7d} - \frac{3b \coth^3(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{13d} - \frac{3b \sqrt[3]{b \coth^4(c+dx)}}{d} \\
&= -\frac{3b \coth(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{7d} - \frac{3b \coth^3(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{13d} - \frac{3b \sqrt[3]{b \coth^4(c+dx)}}{d} \\
&= -\frac{3b \coth(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{7d} - \frac{3b \coth^3(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{13d} - \frac{3b \sqrt[3]{b \coth^4(c+dx)}}{d} \\
&= \frac{b \tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \sqrt[3]{b \coth^4(c+dx)}}{d \coth^{4/3}(c+dx)} - \frac{3b \coth(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{7d} - \frac{3b \coth^3(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{13d} - \frac{3b \sqrt[3]{b \coth^4(c+dx)}}{d} \\
&= \frac{b \tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \sqrt[3]{b \coth^4(c+dx)}}{d \coth^{4/3}(c+dx)} - \frac{3b \coth(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{7d} - \frac{3b \coth^3(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{13d} - \frac{3b \sqrt[3]{b \coth^4(c+dx)}}{d} \\
&= -\frac{\sqrt{3}b \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^4(c+dx)}}{2d \coth^{4/3}(c+dx)} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^4(c+dx)}}{2d \coth^{4/3}(c+dx)}
\end{aligned}$$

Mathematica [C] time = 0.139246, size = 68, normalized size = 0.19

$$\frac{3b \tanh(c+dx) \sqrt[3]{b \coth^4(c+dx)} \left(-91 {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; \coth^2(c+dx)\right) + 7 \coth^4(c+dx) + 13 \coth^2(c+dx) + 91\right)}{91d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^4)^(4/3),x]

[Out] $(-3*b*(b*\text{Coth}[c + d*x]^4)^{(1/3)}*(91 + 13*\text{Coth}[c + d*x]^2 + 7*\text{Coth}[c + d*x]^4 - 91*\text{Hypergeometric2F1}[1/6, 1, 7/6, \text{Coth}[c + d*x]^2])*\text{Tanh}[c + d*x])/(91*d)$

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (b(\coth(dx + c))^4)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^4)^(4/3),x)

[Out] int((b*coth(d*x+c)^4)^(4/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^4)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)

Fricas [B] time = 2.93785, size = 7973, normalized size = 22.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")

```
[Out] -1/364*(182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*sinh(d*x
+ c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4*(7*sq
rt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b*cosh(
d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b*cosh(
d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x + c)^2
+ 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10*sqrt(
3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*sqrt(
3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b*cosh(d
*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2 + 8*(s
qrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b*cosh(d
*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b*(-b)^(1/3)*
arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b) - 182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*si
nh(d*x + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4
*(7*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b
*cosh(d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b
*cosh(d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x
+ c)^2 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10
*sqrt(3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4
*sqrt(3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b
*cosh(d*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2
+ 8*(sqrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b
*cosh(d*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b*b^(1/
3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b) + 91*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 +
b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(
d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 +
6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)
*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh
(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 -
15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cos
h(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))
*sinh(d*x + c) + b*(-b)^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)
/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 91*(b*cosh
(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*c
osh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(
d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(
35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b
*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^
3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9
*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d
*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/
3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(2/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
```

```

+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(1/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)
/sinh(d*x + c))^(1/3)) + 12*(111*b*cosh(d*x + c)^8 + 888*b*cosh(d*x + c)*si
nh(d*x + c)^7 + 111*b*sinh(d*x + c)^8 - 336*b*cosh(d*x + c)^6 + 84*(37*b*co
sh(d*x + c)^2 - 4*b)*sinh(d*x + c)^6 + 168*(37*b*cosh(d*x + c)^3 - 12*b*cos
h(d*x + c))*sinh(d*x + c)^5 + 562*b*cosh(d*x + c)^4 + 2*(3885*b*cosh(d*x +
c)^4 - 2520*b*cosh(d*x + c)^2 + 281*b)*sinh(d*x + c)^4 + 8*(777*b*cosh(d*x
+ c)^5 - 840*b*cosh(d*x + c)^3 + 281*b*cosh(d*x + c))*sinh(d*x + c)^3 - 336
*b*cosh(d*x + c)^2 + 12*(259*b*cosh(d*x + c)^6 - 420*b*cosh(d*x + c)^4 + 28
1*b*cosh(d*x + c)^2 - 28*b)*sinh(d*x + c)^2 + 8*(111*b*cosh(d*x + c)^7 - 25
2*b*cosh(d*x + c)^5 + 281*b*cosh(d*x + c)^3 - 84*b*cosh(d*x + c))*sinh(d*x
+ c) + 111*b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/(d*cosh(d*x + c)^8 + 8
*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6
+ 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x +
c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^
5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*
x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c
)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d
*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**4)**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^4)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)

3.45 $\int (b \coth^4(c + dx))^{2/3} dx$

Optimal. Leaf size=291

$$-\frac{(b \coth^4(c + dx))^{2/3} \log\left(\coth^{2/3}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1\right)}{4d \coth^{8/3}(c + dx)} + \frac{(b \coth^4(c + dx))^{2/3} \log\left(\coth^{2/3}(c + dx) + \sqrt[3]{\coth(c + dx)}\right)}{4d \coth^{8/3}(c + dx)}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2/3))/(2*d*Coth[c + d*x]^(8/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2/3))/(2*d*Coth[c + d*x]^(8/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(2/3))/(d*Coth[c + d*x]^(8/3)) - ((b*Coth[c + d*x]^4)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(8/3)) + ((b*Coth[c + d*x]^4)^(2/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(8/3)) - (3*(b*Coth[c + d*x]^4)^(2/3)*Tanh[c + d*x])/(5*d)
```

Rubi [A] time = 0.215087, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3473, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{(b \coth^4(c + dx))^{2/3} \log\left(\coth^{2/3}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1\right)}{4d \coth^{8/3}(c + dx)} + \frac{(b \coth^4(c + dx))^{2/3} \log\left(\coth^{2/3}(c + dx) + \sqrt[3]{\coth(c + dx)}\right)}{4d \coth^{8/3}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(2/3), x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2/3))/(2*d*Coth[c + d*x]^(8/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2/3))/(2*d*Coth[c + d*x]^(8/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(2/3))/(d*Coth[c + d*x]^(8/3)) - ((b*Coth[c + d*x]^4)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(8/3)) + ((b*Coth[c + d*x]^4)^(2/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(8/3)) - (3*(b*Coth[c + d*x]^4)^(2/3)*Tanh[c + d*x])/(5*d)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
```

$n^{\text{FracPart}[p]} / (\text{Tan}[e + f*x] / \text{ff})^{(n*\text{FracPart}[p])}$, $\text{Int}[\text{ActivateTrig}[u]*(\text{Tan}[e + f*x] / \text{ff})^{(n*p)}, x, x] /;$ $\text{FreeQ}[\{b, e, f, n, p\}, x] \ \&\& \ \text{!IntegerQ}[p]$
 $\&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{EqQ}[u, 1] \ || \ \text{MatchQ}[u, ((d_.)*(trig_)[e + f*x])^{(m_.)}]) /;$
 $\text{FreeQ}[\{d, m\}, x] \ \&\& \ \text{MemberQ}[\{\sin, \cos, \tan, \cot, \sec, \csc\}, \text{trig}]])$

Rule 3473

$\text{Int}[(b_.)*\text{tan}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> } \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{(n - 1)}) / (d*(n - 1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n - 2)}, x], x] /;$ $\text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1]$

Rule 3476

$\text{Int}[(b_.)*\text{tan}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> } \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n / (b^2 + x^2), x], x, b*\text{Tan}[c + d*x]], x] /;$ $\text{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ \text{!IntegerQ}[n]$

Rule 329

$\text{Int}[(c_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \text{ :> } \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /;$ $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{Fractio}[\text{ractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]]$

Rule 296

$\text{Int}[(x_.)^{(m_.)} / ((a_.) + (b_.)*(x_.)^{(n_.)}), x_Symbol] \text{ :> } \text{Module}[\{r = \text{Numerator}[\text{Rt}[-(a/b), n]], s = \text{Denominator}[\text{Rt}[-(a/b), n]], k, u\}, \text{Simp}[u = \text{Int}[(r*\text{Cos}[(2*k*m*Pi)/n] - s*\text{Cos}[(2*k*(m + 1)*Pi)/n]*x) / (r^2 - 2*r*s*\text{Cos}[(2*k*Pi)/n]*x + s^2*x^2), x] + \text{Int}[(r*\text{Cos}[(2*k*m*Pi)/n] + s*\text{Cos}[(2*k*(m + 1)*Pi)/n]*x) / (r^2 + 2*r*s*\text{Cos}[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^{(m + 2)}*\text{Int}[1/(r^2 - s^2*x^2), x]) / (a*n*s^m) + \text{Dist}[(2*r^{(m + 1)}) / (a*n*s^m), \text{Sum}[u, \{k, 1, (n - 2)/4\}], x], x]] /;$ $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[(n - 2)/4, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{LtQ}[m, n - 1] \ \&\& \ \text{NegQ}[a/b]$

Rule 634

$\text{Int}[(d_.) + (e_.)*(x_.)] / ((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \text{ :> } \text{Dist}[(2*c*d - b*e) / (2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e / (2*c), \text{Int}[(b + 2*c*x) / (a + b*x + c*x^2), x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (b \coth^4(c + dx))^{2/3} dx &= \frac{(b \coth^4(c + dx))^{2/3} \int \coth^{8/3}(c + dx) dx}{\coth^{8/3}(c + dx)} \\
&= -\frac{3(b \coth^4(c + dx))^{2/3} \tanh(c + dx)}{5d} + \frac{(b \coth^4(c + dx))^{2/3} \int \coth^{2/3}(c + dx) dx}{\coth^{8/3}(c + dx)} \\
&= -\frac{3(b \coth^4(c + dx))^{2/3} \tanh(c + dx)}{5d} - \frac{(b \coth^4(c + dx))^{2/3} \operatorname{Subst}\left(\int \frac{x^{2/3}}{-1+x^2} dx, x, \coth(c + dx)\right)}{d \coth^{8/3}(c + dx)} \\
&= -\frac{3(b \coth^4(c + dx))^{2/3} \tanh(c + dx)}{5d} - \frac{\left(3(b \coth^4(c + dx))^{2/3}\right) \operatorname{Subst}\left(\int \frac{x^4}{-1+x^6} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d \coth^{8/3}(c + dx)} \\
&= -\frac{3(b \coth^4(c + dx))^{2/3} \tanh(c + dx)}{5d} + \frac{(b \coth^4(c + dx))^{2/3} \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d \coth^{8/3}(c + dx)} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) (b \coth^4(c + dx))^{2/3}}{d \coth^{8/3}(c + dx)} - \frac{3(b \coth^4(c + dx))^{2/3} \tanh(c + dx)}{5d} - \frac{(b \coth^4(c + dx))^{2/3} \log\left(1 - \sqrt[3]{\coth(c + dx)}\right)}{d \coth^{8/3}(c + dx)} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) (b \coth^4(c + dx))^{2/3}}{d \coth^{8/3}(c + dx)} - \frac{(b \coth^4(c + dx))^{2/3} \log\left(1 - \sqrt[3]{\coth(c + dx)}\right)}{d \coth^{8/3}(c + dx)} \\
&= \frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) (b \coth^4(c + dx))^{2/3}}{2d \coth^{8/3}(c + dx)} - \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) (b \coth^4(c + dx))^{2/3}}{2d \coth^{8/3}(c + dx)}
\end{aligned}$$

Mathematica [A] time = 0.408708, size = 166, normalized size = 0.57

$$\frac{(b \coth^4(c + dx))^{2/3} \left(-12 \coth^{5/3}(c + dx) + 20 \tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) + 5 \left(-\log\left(\coth^{2/3}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1\right)\right)\right)}{20d \coth^{8/3}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^4)^(2/3), x]

[Out] ((b*Coth[c + d*x]^4)^(2/3)*(20*ArcTanh[Coth[c + d*x]^(1/3)] - 12*Coth[c + d*x]^(5/3) + 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]

2/3]])))/(20*d*Coth[c + d*x]^(8/3))

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int (b(\coth(dx + c))^4)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^4)^(2/3),x)

[Out] int((b*coth(d*x+c)^4)^(2/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^4)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)

Fricas [B] time = 2.26442, size = 1783, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")

[Out] -1/20*(10*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 10*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b^2)^(1/3)*

```
(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 5*(-b^2)^(1/3)*(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh
(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3)) + 5*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c
))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
)) - 10*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3
)) - 10*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3))
+ 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x
+ c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth^4(c + dx))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**4)**(2/3),x)

[Out] Integral((b*coth(c + d*x)**4)**(2/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^4)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)

3.46 $\int \sqrt[3]{b \coth^4(c + dx)} dx$

Optimal. Leaf size=289

$$\frac{\sqrt[3]{b \coth^4(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1\right)}{4d \coth^{\frac{4}{3}}(c + dx)} + \frac{\sqrt[3]{b \coth^4(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) + \sqrt[3]{\coth(c + dx)}\right)}{4d \coth^{\frac{4}{3}}(c + dx)}$$

[Out] $-(\text{Sqrt}[3] \cdot \text{ArcTan}[(1 - 2 \cdot \text{Coth}[c + d \cdot x]^{(1/3)})/\text{Sqrt}[3]] \cdot (b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)})/(2 \cdot d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) + (\text{Sqrt}[3] \cdot \text{ArcTan}[(1 + 2 \cdot \text{Coth}[c + d \cdot x]^{(1/3)})/\text{Sqrt}[3]] \cdot (b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)})/(2 \cdot d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) + (\text{ArcTanh}[\text{Coth}[c + d \cdot x]^{(1/3)}] \cdot (b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)})/(d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) - ((b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)} \cdot \text{Log}[1 - \text{Coth}[c + d \cdot x]^{(1/3)} + \text{Coth}[c + d \cdot x]^{(2/3)}])/ (4 \cdot d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) + ((b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)} \cdot \text{Log}[1 + \text{Coth}[c + d \cdot x]^{(1/3)} + \text{Coth}[c + d \cdot x]^{(2/3)}])/ (4 \cdot d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) - (3 \cdot (b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)} \cdot \text{Tanh}[c + d \cdot x])/d$

Rubi [A] time = 0.175738, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$\frac{\sqrt[3]{b \coth^4(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) - \sqrt[3]{\coth(c + dx)} + 1\right)}{4d \coth^{\frac{4}{3}}(c + dx)} + \frac{\sqrt[3]{b \coth^4(c + dx)} \log\left(\coth^{\frac{2}{3}}(c + dx) + \sqrt[3]{\coth(c + dx)}\right)}{4d \coth^{\frac{4}{3}}(c + dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)}, x]$

[Out] $-(\text{Sqrt}[3] \cdot \text{ArcTan}[(1 - 2 \cdot \text{Coth}[c + d \cdot x]^{(1/3)})/\text{Sqrt}[3]] \cdot (b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)})/(2 \cdot d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) + (\text{Sqrt}[3] \cdot \text{ArcTan}[(1 + 2 \cdot \text{Coth}[c + d \cdot x]^{(1/3)})/\text{Sqrt}[3]] \cdot (b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)})/(2 \cdot d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) + (\text{ArcTanh}[\text{Coth}[c + d \cdot x]^{(1/3)}] \cdot (b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)})/(d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) - ((b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)} \cdot \text{Log}[1 - \text{Coth}[c + d \cdot x]^{(1/3)} + \text{Coth}[c + d \cdot x]^{(2/3)}])/ (4 \cdot d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) + ((b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)} \cdot \text{Log}[1 + \text{Coth}[c + d \cdot x]^{(1/3)} + \text{Coth}[c + d \cdot x]^{(2/3)}])/ (4 \cdot d \cdot \text{Coth}[c + d \cdot x]^{(4/3)}) - (3 \cdot (b \cdot \text{Coth}[c + d \cdot x]^4)^{(1/3)} \cdot \text{Tanh}[c + d \cdot x])/d$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.)] /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_))^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_))^(n_))^(p_), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{b \coth^4(c+dx)} dx &= \frac{\sqrt[3]{b \coth^4(c+dx)} \int \coth^{\frac{4}{3}}(c+dx) dx}{\coth^{\frac{4}{3}}(c+dx)} \\
&= -\frac{3\sqrt[3]{b \coth^4(c+dx)} \tanh(c+dx)}{d} + \frac{\sqrt[3]{b \coth^4(c+dx)} \int \frac{1}{\coth^{\frac{2}{3}}(c+dx)} dx}{\coth^{\frac{4}{3}}(c+dx)} \\
&= -\frac{3\sqrt[3]{b \coth^4(c+dx)} \tanh(c+dx)}{d} - \frac{\sqrt[3]{b \coth^4(c+dx)} \operatorname{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)} dx, x, \coth(c+dx)\right)}{d \coth^{\frac{4}{3}}(c+dx)} \\
&= -\frac{3\sqrt[3]{b \coth^4(c+dx)} \tanh(c+dx)}{d} - \frac{\left(3\sqrt[3]{b \coth^4(c+dx)}\right) \operatorname{Subst}\left(\int \frac{1}{-1+x^6} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \coth^{\frac{4}{3}}(c+dx)} \\
&= -\frac{3\sqrt[3]{b \coth^4(c+dx)} \tanh(c+dx)}{d} + \frac{\sqrt[3]{b \coth^4(c+dx)} \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \coth^{\frac{4}{3}}(c+dx)} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \sqrt[3]{b \coth^4(c+dx)}}{d \coth^{\frac{4}{3}}(c+dx)} - \frac{3\sqrt[3]{b \coth^4(c+dx)} \tanh(c+dx)}{d} - \frac{\sqrt[3]{b \coth^4(c+dx)}}{d} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \sqrt[3]{b \coth^4(c+dx)}}{d \coth^{\frac{4}{3}}(c+dx)} - \frac{\sqrt[3]{b \coth^4(c+dx)} \log\left(1 - \sqrt[3]{\coth(c+dx)} + \coth(c+dx)\right)}{4d \coth^{\frac{4}{3}}(c+dx)} \\
&= -\frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^4(c+dx)}}{2d \coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^4(c+dx)}}{2d \coth^{\frac{4}{3}}(c+dx)}
\end{aligned}$$

Mathematica [C] time = 0.0356893, size = 43, normalized size = 0.15

$$\frac{3 \tanh(c+dx) \sqrt[3]{b \coth^4(c+dx)} \left({}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; \coth^2(c+dx)\right) - 1 \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^4)^(1/3), x]

[Out] (3*(b*Coth[c + d*x]^4)^(1/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])*Tanh[c + d*x])/d

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \sqrt[3]{b (\coth(dx + c))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*coth(d*x+c)^4)^(1/3), x)

[Out] int((b*coth(d*x+c)^4)^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^4)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(1/3), x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)

Fricas [A] time = 2.26736, size = 855, normalized size = 2.96

$$2\sqrt{3}(-b)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}b+2\sqrt{3}(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) - 2\sqrt{3}b^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) + (-b)^{\frac{1}{3}} \log\left((-b)^{\frac{2}{3}} - (-b)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(1/3), x, algorithm="fricas")

[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-b)^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(1/3)*log(b^(2/3) - b^(1/3)*(b*c


```

osh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3))
- 2*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*
b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(b*cosh(d
*x + c)/sinh(d*x + c))^(1/3))/d

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \coth^4(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(1/3), x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^4)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3), x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)
```

$$3.47 \quad \int \frac{1}{\sqrt[3]{b \coth^4(c+dx)}} dx$$

Optimal. Leaf size=289

$$\frac{3 \coth(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx) \log\left(\coth^{\frac{2}{3}}(c+dx) - \sqrt[3]{\coth(c+dx)+1}\right)}{4d \sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx) \log\left(\coth^{\frac{2}{3}}(c+dx) + \sqrt[3]{\coth(c+dx)+1}\right)}{4d \sqrt[3]{b \coth^4(c+dx)}}$$

[Out] $(-3*\text{Coth}[c + d*x])/(d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}]*\text{Coth}[c + d*x]^{(4/3)})/(d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)})$

Rubi [A] time = 0.217884, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$\frac{3 \coth(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx) \log\left(\coth^{\frac{2}{3}}(c+dx) - \sqrt[3]{\coth(c+dx)+1}\right)}{4d \sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx) \log\left(\coth^{\frac{2}{3}}(c+dx) + \sqrt[3]{\coth(c+dx)+1}\right)}{4d \sqrt[3]{b \coth^4(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Coth}[c + d*x]^4)^{(-1/3)}, x]$

[Out] $(-3*\text{Coth}[c + d*x])/(d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}]*\text{Coth}[c + d*x]^{(4/3)})/(d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)})$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.)] /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_))^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c)^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_))^(n_), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x, x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{b \coth^4(c+dx)}} dx &= \frac{\coth^{\frac{4}{3}}(c+dx) \int \frac{1}{\coth^{\frac{4}{3}}(c+dx)} dx}{\sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx) \int \coth^{\frac{2}{3}}(c+dx) dx}{\sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx) \operatorname{Subst}\left(\int \frac{x^{2/3}}{-1+x^2} dx, x, \coth(c+dx)\right)}{d \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} - \frac{\left(3 \coth^{\frac{4}{3}}(c+dx)\right) \operatorname{Subst}\left(\int \frac{x^4}{-1+x^6} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d \sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \coth^{\frac{4}{3}}(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) \coth^{\frac{4}{3}}(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx) \log\left(1 - \sqrt[3]{\coth(c+dx)}\right)}{4d \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{d \sqrt[3]{b \coth^4(c+dx)}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{\frac{4}{3}}(c+dx)}{2d \sqrt[3]{b \coth^4(c+dx)}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{\frac{4}{3}}(c+dx)}{2d \sqrt[3]{b \coth^4(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.0532765, size = 41, normalized size = 0.14

$$-\frac{3 \coth(c+dx) {}_2F_1\left(-\frac{1}{6}, 1; \frac{5}{6}; \coth^2(c+dx)\right)}{d \sqrt[3]{b \coth^4(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^4)^(-1/3), x]

[Out] $(-3*\text{Coth}[c + d*x]*\text{Hypergeometric2F1}[-1/6, 1, 5/6, \text{Coth}[c + d*x]^2])/(d*(b*\text{Coth}[c + d*x]^4)^{(1/3)})$

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b(\text{coth}(dx+c))^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*coth(d*x+c)^4)^(1/3),x)`

[Out] `int(1/(b*coth(d*x+c)^4)^(1/3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \text{coth}(dx+c)^4)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c)^4)^(-1/3), x)`

Fricas [B] time = 2.91105, size = 9634, normalized size = 33.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")`

[Out] $[1/4*(\sqrt{3}*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + b)*\sqrt{(-b)^{(1/3)}/b}*\log(3*b*\cosh(d*x + c)^2 + 6*b*\cosh(d*x + c)*\sinh(d*x + c) + 3*b*\sinh(d*x + c)^2 - 3*(\cosh(d*x + c)^2 + 2*\cosh(d*x$

$$\begin{aligned}
& + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) (-b)^{2/3} (b \cosh(dx + c) / \sinh(dx + c))^{1/3} - \sqrt{3} (2 (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) (-b)^{2/3} (b \cosh(dx + c) / \sinh(dx + c))^{2/3} \\
& + (b \cosh(dx + c)^2 + 2 b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 - b) (-b)^{1/3} - (b \cosh(dx + c)^2 + 2 b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 - b) (b \cosh(dx + c) / \sinh(dx + c))^{1/3}) \sqrt{(-b)^{1/3} / b} + b) + \sqrt{3} (b \cosh(dx + c)^2 + 2 b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + b) \sqrt{-1/b^{2/3}} \log(-2 \sqrt{3} (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) b^{2/3} (b \cosh(dx + c) / \sinh(dx + c))^{2/3} \sqrt{-1/b^{2/3}}) - b \cosh(dx + c)^2 - 2 b \cosh(dx + c) \sinh(dx + c) - b \sinh(dx + c)^2 - \sqrt{3} (b \cosh(dx + c)^2 + 2 b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 - b) b^{1/3} \sqrt{-1/b^{2/3}}) + (\sqrt{3} (b \cosh(dx + c)^2 + 2 b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 - b) \sqrt{-1/b^{2/3}}) + 3 (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) b^{2/3} (b \cosh(dx + c) / \sinh(dx + c))^{1/3} - 3 b) / (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2) + (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) (-b)^{2/3} \log((-b)^{2/3} - (-b)^{1/3} (b \cosh(dx + c) / \sinh(dx + c))^{1/3} + (b \cosh(dx + c) / \sinh(dx + c))^{2/3}) - (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) b^{2/3} \log(b^{2/3} - b^{1/3} (b \cosh(dx + c) / \sinh(dx + c))^{1/3} + (b \cosh(dx + c) / \sinh(dx + c))^{2/3}) - 2 (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) (-b)^{2/3} \log((-b)^{1/3} + (b \cosh(dx + c) / \sinh(dx + c))^{1/3}) + 2 (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) b^{2/3} \log(b^{1/3} + (b \cosh(dx + c) / \sinh(dx + c))^{1/3}) - 12 (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) (b \cosh(dx + c) / \sinh(dx + c))^{2/3} / (b d \cosh(dx + c)^2 + 2 b d \cosh(dx + c) \sinh(dx + c) + b d \sinh(dx + c)^2 + b d), -1/4 (2 \sqrt{3} (b \cosh(dx + c)^2 + 2 b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + b) \sqrt{-(-b)^{1/3} / b} \arctan(-1/3 \sqrt{3} (-b)^{1/3} \sqrt{-(-b)^{1/3} / b} + 2/3 \sqrt{3} (b \cosh(dx + c) / \sinh(dx + c))^{1/3} \sqrt{-(-b)^{1/3} / b}) - \sqrt{3} (b \cosh(dx + c)^2 + 2 b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + b) \sqrt{-1/b^{2/3}} \log(-2 \sqrt{3} (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) b^{2/3} (b \cosh(dx + c) / \sinh(dx + c))^{2/3} \sqrt{-1/b^{2/3}}) - b \cosh(dx + c)^2 - 2 b \cosh(dx + c) \sinh(dx + c) - b \sinh(dx + c)^2 - \sqrt{3} (b \cosh(dx + c)^2 + 2 b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 - b) b^{1/3} \sqrt{-1/b^{2/3}}) + (\sqrt{3} (b \cosh(dx + c)^2 + 2 b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 - b) \sqrt{-1/b^{2/3}}) + 3 (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1) b^{2/3} (b \cosh(dx + c) / \sinh(dx + c))^{1/3} - 3 b) / (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2) - (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) (-b)^{2/3} \log((-b)^{2/3} - (-b)^{1/3} (b \cosh(dx + c) / \sinh(dx + c))^{1/3} + (b \cosh(dx + c) / \sinh(dx + c))^{2/3}) + (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) b^{2/3} \log(b^{2/3} - b^{1/3} (b \cosh(dx + c) / \sinh(dx + c))^{1/3} + (b \cosh(dx + c) / \sinh(dx + c))^{2/3}) + (\cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 + 1) b^{2/3} \log(b^{2/3} - b^{1/3} (b \cosh(dx + c) / \sinh(dx + c))^{1/3} + (b \cosh(dx + c) / \sinh(dx + c))^{2/3})
\end{aligned}$$

$$\begin{aligned}
& c))^{\frac{1}{3}} + (b \cosh(dx + c) / \sinh(dx + c))^{\frac{2}{3}}) + 2 * (\cosh(dx + c)^2 + 2 \\
& * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + c)^2 + 1) * (-b)^{\frac{2}{3}} * \log((-b)^{\frac{1}{3}} / \\
& 3) + (b \cosh(dx + c) / \sinh(dx + c))^{\frac{1}{3}}) - 2 * (\cosh(dx + c)^2 + 2 * \cosh(dx \\
& * x + c) * \sinh(dx + c) + \sinh(dx + c)^2 + 1) * b^{\frac{2}{3}} * \log(b^{\frac{1}{3}} + (b \cosh(dx \\
& + c) / \sinh(dx + c))^{\frac{1}{3}}) + 12 * (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh \\
& (dx + c) + \sinh(dx + c)^2 - 1) * (b \cosh(dx + c) / \sinh(dx + c))^{\frac{2}{3}}) / (b * \\
& d * \cosh(dx + c)^2 + 2 * b * d * \cosh(dx + c) * \sinh(dx + c) + b * d * \sinh(dx + c)^2 \\
& + b * d), 1/4 * (\sqrt{3} * (b \cosh(dx + c)^2 + 2 * b * \cosh(dx + c) * \sinh(dx + c) \\
& + b * \sinh(dx + c)^2 + b) * \sqrt{((-b)^{\frac{1}{3}} / b) * \log(3 * b * \cosh(dx + c)^2 + 6 * b * c \\
& osh(dx + c) * \sinh(dx + c) + 3 * b * \sinh(dx + c)^2 - 3 * (\cosh(dx + c)^2 + 2 * c \\
& osh(dx + c) * \sinh(dx + c) + \sinh(dx + c)^2 - 1) * (-b)^{\frac{2}{3}} * (b \cosh(dx + \\
& c) / \sinh(dx + c))^{\frac{1}{3}} - \sqrt{3} * (2 * (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sin \\
& h(dx + c) + \sinh(dx + c)^2 - 1) * (-b)^{\frac{2}{3}} * (b \cosh(dx + c) / \sinh(dx + c) \\
&)^{\frac{2}{3}} + (b \cosh(dx + c)^2 + 2 * b * \cosh(dx + c) * \sinh(dx + c) + b * \sinh(dx * \\
& + c)^2 - b) * (-b)^{\frac{1}{3}} - (b \cosh(dx + c)^2 + 2 * b * \cosh(dx + c) * \sinh(dx + \\
& c) + b * \sinh(dx + c)^2 - b) * (b \cosh(dx + c) / \sinh(dx + c))^{\frac{1}{3}}) * \sqrt{((- \\
& b)^{\frac{1}{3}} / b) + b) + (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh \\
& (dx + c)^2 + 1) * (-b)^{\frac{2}{3}} * \log((-b)^{\frac{2}{3}} - (-b)^{\frac{1}{3}} * (b \cosh(dx + c) / \sin \\
& h(dx + c))^{\frac{1}{3}} + (b \cosh(dx + c) / \sinh(dx + c))^{\frac{2}{3}}) - (\cosh(dx + c) \\
& ^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + c)^2 + 1) * b^{\frac{2}{3}} * \log(b^{\frac{2}{3}} / \\
& 3) - b^{\frac{1}{3}} * (b \cosh(dx + c) / \sinh(dx + c))^{\frac{1}{3}} + (b \cosh(dx + c) / \sinh \\
& (dx + c))^{\frac{2}{3}}) - 2 * (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sin \\
& h(dx + c)^2 + 1) * (-b)^{\frac{2}{3}} * \log((-b)^{\frac{1}{3}} + (b \cosh(dx + c) / \sinh(dx + c \\
&))^{\frac{1}{3}}) + 2 * (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + \\
& c)^2 + 1) * b^{\frac{2}{3}} * \log(b^{\frac{1}{3}} + (b \cosh(dx + c) / \sinh(dx + c))^{\frac{1}{3}}) - 2 \\
& * \sqrt{3} * (b \cosh(dx + c)^2 + 2 * b * \cosh(dx + c) * \sinh(dx + c) + b * \sinh(dx \\
& + c)^2 + b) * \arctan(-1/3 * \sqrt{3} * (b^{\frac{1}{3}} - 2 * (b \cosh(dx + c) / \sinh(dx + c) \\
&)^{\frac{1}{3}}) / b^{\frac{1}{3}}) / b^{\frac{1}{3}} - 12 * (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx \\
& + c) + \sinh(dx + c)^2 - 1) * (b \cosh(dx + c) / \sinh(dx + c))^{\frac{2}{3}}) / (b * d * \cos \\
& h(dx + c)^2 + 2 * b * d * \cosh(dx + c) * \sinh(dx + c) + b * d * \sinh(dx + c)^2 + b * \\
& d), -1/4 * (2 * \sqrt{3} * (b \cosh(dx + c)^2 + 2 * b * \cosh(dx + c) * \sinh(dx + c) + \\
& b * \sinh(dx + c)^2 + b) * \sqrt{-(-b)^{\frac{1}{3}} / b} * \arctan(-1/3 * \sqrt{3} * (-b)^{\frac{1}{3}} * s \\
& \sqrt{-(-b)^{\frac{1}{3}} / b} + 2/3 * \sqrt{3} * (b \cosh(dx + c) / \sinh(dx + c))^{\frac{1}{3}} * \sqrt{ \\
& -(-b)^{\frac{1}{3}} / b}) - (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh \\
& (dx + c)^2 + 1) * (-b)^{\frac{2}{3}} * \log((-b)^{\frac{2}{3}} - (-b)^{\frac{1}{3}} * (b \cosh(dx + c) / \sin \\
& h(dx + c))^{\frac{1}{3}} + (b \cosh(dx + c) / \sinh(dx + c))^{\frac{2}{3}}) + (\cosh(dx + c) \\
& ^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + c)^2 + 1) * b^{\frac{2}{3}} * \log(b^{\frac{2}{3}} / \\
& 3) - b^{\frac{1}{3}} * (b \cosh(dx + c) / \sinh(dx + c))^{\frac{1}{3}} + (b \cosh(dx + c) / \sinh \\
& (dx + c))^{\frac{2}{3}}) + 2 * (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sin \\
& h(dx + c)^2 + 1) * (-b)^{\frac{2}{3}} * \log((-b)^{\frac{1}{3}} + (b \cosh(dx + c) / \sinh(dx + c \\
&))^{\frac{1}{3}}) - 2 * (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + \\
& c)^2 + 1) * b^{\frac{2}{3}} * \log(b^{\frac{1}{3}} + (b \cosh(dx + c) / \sinh(dx + c))^{\frac{1}{3}}) + 2 \\
& * \sqrt{3} * (b \cosh(dx + c)^2 + 2 * b * \cosh(dx + c) * \sinh(dx + c) + b * \sinh(dx \\
& + c)^2 + b) * \arctan(-1/3 * \sqrt{3} * (b^{\frac{1}{3}} - 2 * (b \cosh(dx + c) / \sinh(dx + c) \\
&)^{\frac{1}{3}}) / b^{\frac{1}{3}}) / b^{\frac{1}{3}} + 12 * (\cosh(dx + c)^2 + 2 * \cosh(dx + c) * \sinh(dx
\end{aligned}$$

+ c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2 + b*d)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b \coth^4(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**4)**(1/3),x)

[Out] Integral((b*coth(c + d*x)**4)**(-1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^4)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^4)^(-1/3), x)

$$3.48 \quad \int \frac{1}{\left(b \coth^4(c+dx)\right)^{2/3}} dx$$

Optimal. Leaf size=291

$$\frac{3 \coth(c+dx)}{5d \left(b \coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{8/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)} + 1\right)}{4d \left(b \coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{8/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)} + 1\right)}{4d \left(b \coth^4(c+dx)\right)^{2/3}}$$

[Out] $(-3*\text{Coth}[c + d*x])/(5*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) - (\text{Sqrt}[3]*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(8/3)})/(2*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(8/3)})/(2*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) + (\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}]*\text{Coth}[c + d*x]^{(8/3)})/(d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) - (\text{Coth}[c + d*x]^{(8/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) + (\text{Coth}[c + d*x]^{(8/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)})$

Rubi [A] time = 0.184945, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3474, 3476, 329, 210, 634, 618, 204, 628, 206}

$$\frac{3 \coth(c+dx)}{5d \left(b \coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{8/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)} + 1\right)}{4d \left(b \coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{8/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)} + 1\right)}{4d \left(b \coth^4(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^4)^(-2/3), x]

[Out] $(-3*\text{Coth}[c + d*x])/(5*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) - (\text{Sqrt}[3]*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(8/3)})/(2*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(8/3)})/(2*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) + (\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}]*\text{Coth}[c + d*x]^{(8/3)})/(d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) - (\text{Coth}[c + d*x]^{(8/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)}) + (\text{Coth}[c + d*x]^{(8/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)})$

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_))^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_))^(n_))^(p_), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b \coth^4(c + dx))^{2/3}} dx &= \frac{\coth^{\frac{8}{3}}(c + dx) \int \frac{1}{\coth^{\frac{8}{3}}(c+dx)} dx}{(b \coth^4(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{5d (b \coth^4(c + dx))^{2/3}} + \frac{\coth^{\frac{8}{3}}(c + dx) \int \frac{1}{\coth^{\frac{2}{3}}(c+dx)} dx}{(b \coth^4(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{5d (b \coth^4(c + dx))^{2/3}} - \frac{\coth^{\frac{8}{3}}(c + dx) \text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)} dx, x, \coth(c + dx)\right)}{d (b \coth^4(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{5d (b \coth^4(c + dx))^{2/3}} - \frac{\left(3 \coth^{\frac{8}{3}}(c + dx)\right) \text{Subst}\left(\int \frac{1}{-1+x^6} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d (b \coth^4(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{5d (b \coth^4(c + dx))^{2/3}} + \frac{\coth^{\frac{8}{3}}(c + dx) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{d (b \coth^4(c + dx))^{2/3}} + \frac{\coth^{\frac{8}{3}}(c + dx)}{4d (b \coth^4(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{5d (b \coth^4(c + dx))^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) \coth^{\frac{8}{3}}(c + dx)}{d (b \coth^4(c + dx))^{2/3}} - \frac{\coth^{\frac{8}{3}}(c + dx) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt[3]{\coth(c + dx)}\right)}{4d (b \coth^4(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{5d (b \coth^4(c + dx))^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c + dx)}\right) \coth^{\frac{8}{3}}(c + dx)}{d (b \coth^4(c + dx))^{2/3}} - \frac{\coth^{\frac{8}{3}}(c + dx) \log\left(\frac{1-\sqrt[3]{\coth(c + dx)}}{1+\sqrt[3]{\coth(c + dx)}}\right)}{4d (b \coth^4(c + dx))^{2/3}} \\
&= -\frac{3 \coth(c + dx)}{5d (b \coth^4(c + dx))^{2/3}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{\frac{8}{3}}(c + dx)}{2d (b \coth^4(c + dx))^{2/3}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{\frac{8}{3}}(c + dx)}{2d (b \coth^4(c + dx))^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.0508721, size = 43, normalized size = 0.15

$$-\frac{3 \coth(c + dx) {}_2F_1\left(-\frac{5}{6}, 1; \frac{1}{6}; \coth^2(c + dx)\right)}{5d (b \coth^4(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^4)^(-2/3), x]

[Out] $(-3*\text{Coth}[c + d*x]*\text{Hypergeometric2F1}[-5/6, 1, 1/6, \text{Coth}[c + d*x]^2])/(5*d*(b*\text{Coth}[c + d*x]^4)^{(2/3)})$

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int (b(\coth(dx + c))^4)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*coth(d*x+c)^4)^(2/3),x)`

[Out] `int(1/(b*coth(d*x+c)^4)^(2/3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^4)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c)^4)^(-2/3), x)`

Fricas [B] time = 2.48705, size = 3221, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")`

[Out] $1/20*(10*\sqrt{3}*(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*b*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c) + b)*\sqrt{3}$

```
(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sqrt(-(-b^2)^(1/3)) - 2
*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b^2)^(1
/3)))/b^2) + 10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) +
b)*(b^2)^(1/6)*arctan(-1/3*sqrt(3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b^2) + 5*(cosh(d*x + c)^4 + 4*cos
h(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*si
nh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sin
h(d*x + c) + 1)*(-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) -
(-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 5*(c
osh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*c
osh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/si
nh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x
+ c))^(1/3)) - 10*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh
(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2
+ 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b^2)^(2/3)*log(
b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 10*(cosh(d*x + c)
^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)
^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x
+ c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))
^(1/3) + (b^2)^(2/3)) - 12*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)
+ b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/(b^2*d*cosh(d*x + c)^4 + 4*b^2
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x
+ c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(
b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^4(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)**4)**(2/3), x)

[Out] Integral((b*coth(c + d*x)**4)**(-2/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^4)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^4)^(-2/3), x)

$$3.49 \quad \int \frac{1}{\left(b \coth^4(c+dx)\right)^{4/3}} dx$$

Optimal. Leaf size=369

$$\frac{3 \coth(c+dx)}{bd\sqrt[3]{b \coth^4(c+dx)}} - \frac{\coth^{4/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)+1}\right)}{4bd\sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{4/3}(c+dx) \log\left(\coth^{2/3}(c+dx) + \sqrt[3]{\coth(c+dx)+1}\right)}{4bd\sqrt[3]{b \coth^4(c+dx)}}$$

[Out] $(-3*\text{Coth}[c + d*x])/(b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}])*\text{Coth}[c + d*x]^{(4/3)}/(b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (3*\text{Tanh}[c + d*x])/(7*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (3*\text{Tanh}[c + d*x]^3)/(13*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)})$

Rubi [A] time = 0.244683, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3658, 3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$\frac{3 \coth(c+dx)}{bd\sqrt[3]{b \coth^4(c+dx)}} - \frac{\coth^{4/3}(c+dx) \log\left(\coth^{2/3}(c+dx) - \sqrt[3]{\coth(c+dx)+1}\right)}{4bd\sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{4/3}(c+dx) \log\left(\coth^{2/3}(c+dx) + \sqrt[3]{\coth(c+dx)+1}\right)}{4bd\sqrt[3]{b \coth^4(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^4)^(-4/3),x]

[Out] $(-3*\text{Coth}[c + d*x])/(b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(1 - 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*\text{Coth}[c + d*x]^{(1/3)})/\text{Sqrt}[3]]*\text{Coth}[c + d*x]^{(4/3)})/(2*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{ArcTanh}[\text{Coth}[c + d*x]^{(1/3)}])*\text{Coth}[c + d*x]^{(4/3)}/(b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 - \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) + (\text{Coth}[c + d*x]^{(4/3)}*\text{Log}[1 + \text{Coth}[c + d*x]^{(1/3)} + \text{Coth}[c + d*x]^{(2/3)}])/(4*b*d*(b*\text{Coth}[c + d*x]^4)^{(1/3)}) - (3*\text{Tanh}[c + d*x])/(7*b*d$

$(b \operatorname{Coth}[c + d x]^4)^{1/3} - (3 \operatorname{Tanh}[c + d x]^3) / (13 b d (b \operatorname{Coth}[c + d x]^4)^{1/3})$

Rule 3658

Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 3474

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 329

Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 296

Int[(x_)^(m_.)/((a_.) + (b_.)*(x_)^(n_.)), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b \coth^4(c+dx))^{4/3}} dx &= \frac{\coth^{4/3}(c+dx) \int \frac{1}{\coth^{16/3}(c+dx)} dx}{b^3 \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \tanh^3(c+dx)}{13bd^3 \sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{4/3}(c+dx) \int \frac{1}{\coth^{10/3}(c+dx)} dx}{b^3 \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \tanh(c+dx)}{7bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh^3(c+dx)}{13bd^3 \sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{4/3}(c+dx) \int \frac{1}{\coth^{4/3}(c+dx)} dx}{b^3 \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh(c+dx)}{7bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh^3(c+dx)}{13bd^3 \sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{4/3}(c+dx) \int \frac{1}{\coth^{4/3}(c+dx)} dx}{b^3 \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh(c+dx)}{7bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh^3(c+dx)}{13bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{\coth^{4/3}(c+dx) \int \frac{1}{\coth^{4/3}(c+dx)} dx}{b^3 \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh(c+dx)}{7bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh^3(c+dx)}{13bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \left(3 \coth^{4/3}(c+dx) \int \frac{1}{\coth^{4/3}(c+dx)} dx \right) \\
&= -\frac{3 \coth(c+dx)}{bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh(c+dx)}{7bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh^3(c+dx)}{13bd^3 \sqrt[3]{b \coth^4(c+dx)}} + \frac{\coth^{4/3}(c+dx) \int \frac{1}{\coth^{4/3}(c+dx)} dx}{b^3 \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{bd^3 \sqrt[3]{b \coth^4(c+dx)}} + \frac{\tanh^{-1}(\sqrt[3]{\coth(c+dx)}) \coth^{4/3}(c+dx)}{bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{3 \tanh(c+dx)}{7bd^3 \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{bd^3 \sqrt[3]{b \coth^4(c+dx)}} + \frac{\tanh^{-1}(\sqrt[3]{\coth(c+dx)}) \coth^{4/3}(c+dx)}{bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{\coth^{4/3}(c+dx) \log(1 - \coth^2(c+dx))}{4bd^3 \sqrt[3]{b \coth^4(c+dx)}} \\
&= -\frac{3 \coth(c+dx)}{bd^3 \sqrt[3]{b \coth^4(c+dx)}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{4/3}(c+dx)}{2bd^3 \sqrt[3]{b \coth^4(c+dx)}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \coth^{4/3}(c+dx)}{2bd^3 \sqrt[3]{b \coth^4(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.0421127, size = 43, normalized size = 0.12

$$-\frac{3 \operatorname{coth}(c + dx) {}_2F_1\left(-\frac{13}{6}, 1; -\frac{7}{6}; \operatorname{coth}^2(c + dx)\right)}{13d \left(b \operatorname{coth}^4(c + dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^4)^(-4/3),x]

[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-13/6, 1, -7/6, Coth[c + d*x]^2])/(13*d*(b*Coth[c + d*x]^4)^(4/3))

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int \left(b (\operatorname{coth}(dx + c))^4\right)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c)^4)^(4/3),x)

[Out] int(1/(b*coth(d*x+c)^4)^(4/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \operatorname{coth}(dx + c)^4\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^4(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^4)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

3.50 $\int (b \coth^m(c + dx))^n dx$

Optimal. Leaf size=57

$$\frac{\coth(c + dx) (b \coth^m(c + dx))^n {}_2F_1\left(1, \frac{1}{2}(mn + 1); \frac{1}{2}(mn + 3); \coth^2(c + dx)\right)}{d(mn + 1)}$$

[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 + m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))

Rubi [A] time = 0.0423468, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3659, 3476, 364}

$$\frac{\coth(c + dx) (b \coth^m(c + dx))^n {}_2F_1\left(1, \frac{1}{2}(mn + 1); \frac{1}{2}(mn + 3); \coth^2(c + dx)\right)}{d(mn + 1)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^m)^n,x]

[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 + m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
)]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (b \coth^m(c + dx))^n dx &= \left(\coth^{-mn}(c + dx) (b \coth^m(c + dx))^n \right) \int \coth^{mn}(c + dx) dx \\ &= \frac{\left(\coth^{-mn}(c + dx) (b \coth^m(c + dx))^n \right) \text{Subst} \left(\int \frac{x^{mn}}{-1+x^2} dx, x, \coth(c + dx) \right)}{d} \\ &= \frac{\coth(c + dx) (b \coth^m(c + dx))^n {}_2F_1 \left(1, \frac{1}{2}(1 + mn); \frac{1}{2}(3 + mn); \coth^2(c + dx) \right)}{d(1 + mn)} \end{aligned}$$

Mathematica [A] time = 0.0481855, size = 55, normalized size = 0.96

$$\frac{\coth(c + dx) (b \coth^m(c + dx))^n {}_2F_1 \left(1, \frac{1}{2}(mn + 1); \frac{1}{2}(mn + 3); \coth^2(c + dx) \right)}{dmn + d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
m*n)/2, Coth[c + d*x]^2])/(d + d*m*n)
```

Maple [F] time = 2.588, size = 0, normalized size = 0.

$$\int (b (\coth(dx + c))^m)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^n,x)
```

```
[Out] int((b*coth(d*x+c)^m)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^m)^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \coth(dx + c)^m\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="fricas")

[Out] integral((b*coth(d*x + c)^m)^n, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth^m(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**m)**n,x)

[Out] Integral((b*coth(c + d*x)**m)**n, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^n, x)
```

3.51 $\int (b \coth^m(c + dx))^{3/2} dx$

Optimal. Leaf size=63

$$\frac{2b \coth^{m+1}(c + dx) \sqrt{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{1}{4}(3m + 2); \frac{3(m+2)}{4}; \coth^2(c + dx)\right)}{d(3m + 2)}$$

[Out] (2*b*Coth[c + d*x]^(1 + m)*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + 3*m)/4, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))

Rubi [A] time = 0.0454474, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{2b \coth^{m+1}(c + dx) \sqrt{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{1}{4}(3m + 2); \frac{3(m+2)}{4}; \coth^2(c + dx)\right)}{d(3m + 2)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^m)^(3/2), x]

[Out] (2*b*Coth[c + d*x]^(1 + m)*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + 3*m)/4, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
)]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (b \coth^m(c + dx))^{3/2} dx &= \left(b \coth^{-\frac{m}{2}}(c + dx) \sqrt{b \coth^m(c + dx)} \right) \int \coth^{\frac{3m}{2}}(c + dx) dx \\ &= -\frac{\left(b \coth^{-\frac{m}{2}}(c + dx) \sqrt{b \coth^m(c + dx)} \right) \text{Subst}\left(\int \frac{x^{3m/2}}{-1+x^2} dx, x, \coth(c + dx)\right)}{d} \\ &= \frac{2b \coth^{1+m}(c + dx) \sqrt{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{1}{4}(2 + 3m); \frac{3(2+m)}{4}; \coth^2(c + dx)\right)}{d(2 + 3m)} \end{aligned}$$

Mathematica [A] time = 0.0747188, size = 58, normalized size = 0.92

$$\frac{2 \coth(c + dx) (b \coth^m(c + dx))^{3/2} {}_2F_1\left(1, \frac{1}{4}(3m + 2); \frac{3(m+2)}{4}; \coth^2(c + dx)\right)}{d(3m + 2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(3/2), x]
```

```
[Out] (2*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(3/2)*Hypergeometric2F1[1, (2 + 3*m)/4, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int (b(\coth(dx + c))^m)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(3/2), x)
```

```
[Out] int((b*coth(d*x+c)^m)^(3/2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**m)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)
```

3.52 $\int \sqrt{b \coth^m(c + dx)} dx$

Optimal. Leaf size=54

$$\frac{2 \coth(c + dx) \sqrt{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{m+2}{4}; \frac{m+6}{4}; \coth^2(c + dx)\right)}{d(m+2)}$$

[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))

Rubi [A] time = 0.0407944, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{2 \coth(c + dx) \sqrt{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{m+2}{4}; \frac{m+6}{4}; \coth^2(c + dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Coth[c + d*x]^m], x]

[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p]]/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \sqrt{b \coth^m(c + dx)} dx &= \left(\coth^{-\frac{m}{2}}(c + dx) \sqrt{b \coth^m(c + dx)} \right) \int \coth^{\frac{m}{2}}(c + dx) dx \\ &= -\frac{\left(\coth^{-\frac{m}{2}}(c + dx) \sqrt{b \coth^m(c + dx)} \right) \text{Subst} \left(\int \frac{x^{m/2}}{-1+x^2} dx, x, \coth(c + dx) \right)}{d} \\ &= \frac{2 \coth(c + dx) \sqrt{b \coth^m(c + dx)} {}_2F_1 \left(1, \frac{2+m}{4}; \frac{6+m}{4}; \coth^2(c + dx) \right)}{d(2 + m)} \end{aligned}$$

Mathematica [A] time = 0.0397064, size = 54, normalized size = 1.

$$\frac{2 \coth(c + dx) \sqrt{b \coth^m(c + dx)} {}_2F_1 \left(1, \frac{m+2}{4}; \frac{m+6}{4}; \coth^2(c + dx) \right)}{d(m + 2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^m], x]
```

```
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
+ m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Maple [F] time = 0.186, size = 0, normalized size = 0.

$$\int \sqrt{b (\coth(dx + c))^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(1/2), x)
```

```
[Out] int((b*coth(d*x+c)^m)^(1/2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx + c)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*coth(d*x + c)^m), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth^m(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**m)**(1/2),x)

[Out] Integral(sqrt(b*coth(c + d*x)**m), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx + c)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^m), x)
```

$$3.53 \quad \int \frac{1}{\sqrt{b \coth^m(c+dx)}} dx$$

Optimal. Leaf size=60

$$\frac{2 \coth(c+dx) {}_2F_1\left(1, \frac{2-m}{4}; \frac{6-m}{4}; \coth^2(c+dx)\right)}{d(2-m)\sqrt{b \coth^m(c+dx)}}$$

[Out] (2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^2])/ (d*(2 - m)*Sqrt[b*Coth[c + d*x]^m])

Rubi [A] time = 0.047278, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{2 \coth(c+dx) {}_2F_1\left(1, \frac{2-m}{4}; \frac{6-m}{4}; \coth^2(c+dx)\right)}{d(2-m)\sqrt{b \coth^m(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*Coth[c + d*x]^m], x]

[Out] (2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^2])/ (d*(2 - m)*Sqrt[b*Coth[c + d*x]^m])

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{b \coth^m(c + dx)}} dx &= \frac{\coth^{\frac{m}{2}}(c + dx) \int \coth^{-\frac{m}{2}}(c + dx) dx}{\sqrt{b \coth^m(c + dx)}} \\ &= \frac{\coth^{\frac{m}{2}}(c + dx) \operatorname{Subst}\left(\int \frac{x^{-m/2}}{-1+x^2} dx, x, \coth(c + dx)\right)}{d\sqrt{b \coth^m(c + dx)}} \\ &= \frac{2 \coth(c + dx) {}_2F_1\left(1, \frac{2-m}{4}; \frac{6-m}{4}; \coth^2(c + dx)\right)}{d(2-m)\sqrt{b \coth^m(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.0473323, size = 58, normalized size = 0.97

$$\frac{2 \coth(c + dx) {}_2F_1\left(1, \frac{2-m}{4}; \frac{6-m}{4}; \coth^2(c + dx)\right)}{d(m-2)\sqrt{b \coth^m(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^m], x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^
2])/(d*(-2 + m)*Sqrt[b*Coth[c + d*x]^m])
```

Maple [F] time = 0.195, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b (\coth(dx + c))^m}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(1/2), x)
```

[Out] `int(1/(b*coth(d*x+c)^m)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth(dx + c)^m}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(b*coth(d*x + c)^m), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth^m(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)**m)**(1/2),x)`

[Out] `Integral(1/sqrt(b*coth(c + d*x)**m), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth(dx + c)^m}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

$$3.54 \quad \int \frac{1}{(b \coth^m(c+dx))^{3/2}} dx$$

Optimal. Leaf size=69

$$\frac{2 \coth^{1-m}(c+dx) {}_2F_1\left(1, \frac{1}{4}(2-3m); \frac{3(2-m)}{4}; \coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b \coth^m(c+dx)}}$$

[Out] (2*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (2 - 3*m)/4, (3*(2 - m))/4, Coth[c + d*x]^2])/(b*d*(2 - 3*m)*Sqrt[b*Coth[c + d*x]^m])

Rubi [A] time = 0.0481878, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{2 \coth^{1-m}(c+dx) {}_2F_1\left(1, \frac{1}{4}(2-3m); \frac{3(2-m)}{4}; \coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b \coth^m(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^m)^(-3/2), x]

[Out] (2*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (2 - 3*m)/4, (3*(2 - m))/4, Coth[c + d*x]^2])/(b*d*(2 - 3*m)*Sqrt[b*Coth[c + d*x]^m])

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x]^(n*FracPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(b \coth^m(c + dx))^{3/2}} dx &= \frac{\coth^{\frac{m}{2}}(c + dx) \int \coth^{-\frac{3m}{2}}(c + dx) dx}{b\sqrt{b \coth^m(c + dx)}} \\ &= -\frac{\coth^{\frac{m}{2}}(c + dx) \operatorname{Subst}\left(\int \frac{x^{-3m/2}}{-1+x^2} dx, x, \coth(c + dx)\right)}{bd\sqrt{b \coth^m(c + dx)}} \\ &= \frac{2 \coth^{1-m}(c + dx) {}_2F_1\left(1, \frac{1}{4}(2 - 3m); \frac{3(2-m)}{4}; \coth^2(c + dx)\right)}{bd(2 - 3m)\sqrt{b \coth^m(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.0661005, size = 58, normalized size = 0.84

$$\frac{2 \coth(c + dx) {}_2F_1\left(1, \frac{1}{4}(2 - 3m); -\frac{3}{4}(m - 2); \coth^2(c + dx)\right)}{d(3m - 2)(b \coth^m(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^m)^(-3/2),x]

[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - 3*m)/4, (-3*(-2 + m))/4, Coth[c + d*x]^2])/(d*(-2 + 3*m)*(b*Coth[c + d*x]^m)^(3/2))

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int (b(\coth(dx + c))^m)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*coth(d*x+c)^m)^(3/2),x)`

[Out] `int(1/(b*coth(d*x+c)^m)^(3/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^m)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c)^m)^(-3/2), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)**m)**(3/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^m)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^m)^(-3/2), x)

3.55 $\int (b \coth^m(c + dx))^{4/3} dx$

Optimal. Leaf size=65

$$\frac{3b \coth^{m+1}(c + dx) \sqrt[3]{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{1}{6}(4m + 3); \frac{1}{6}(4m + 9); \coth^2(c + dx)\right)}{d(4m + 3)}$$

[Out] (3*b*Coth[c + d*x]^(1 + m)*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + 4*m)/6, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))

Rubi [A] time = 0.0453596, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{3b \coth^{m+1}(c + dx) \sqrt[3]{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{1}{6}(4m + 3); \frac{1}{6}(4m + 9); \coth^2(c + dx)\right)}{d(4m + 3)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^m)^(4/3), x]

[Out] (3*b*Coth[c + d*x]^(1 + m)*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + 4*m)/6, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))

Rule 3659

Int[(u_)*((b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := Dist[b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_)*(trig_)[e + f*x])^(m_) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 3476

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 364

```
Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (b \coth^m(c + dx))^{4/3} dx &= \left(b \coth^{-\frac{m}{3}}(c + dx) \sqrt[3]{b \coth^m(c + dx)} \right) \int \coth^{\frac{4m}{3}}(c + dx) dx \\ &= -\frac{\left(b \coth^{-\frac{m}{3}}(c + dx) \sqrt[3]{b \coth^m(c + dx)} \right) \text{Subst}\left(\int \frac{x^{4m/3}}{-1+x^2} dx, x, \coth(c + dx)\right)}{d} \\ &= \frac{3b \coth^{1+m}(c + dx) \sqrt[3]{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{1}{6}(3 + 4m); \frac{1}{6}(9 + 4m); \coth^2(c + dx)\right)}{d(3 + 4m)} \end{aligned}$$

Mathematica [A] time = 0.0703297, size = 60, normalized size = 0.92

$$\frac{3 \coth(c + dx) (b \coth^m(c + dx))^{4/3} {}_2F_1\left(1, \frac{1}{6}(4m + 3); \frac{1}{6}(4m + 9); \coth^2(c + dx)\right)}{d(4m + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(4/3), x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(4/3)*Hypergeometric2F1[1, (3 + 4*m)/6, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int (b(\coth(dx + c))^m)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(4/3), x)
```

```
[Out] int((b*coth(d*x+c)^m)^(4/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**m)**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)
```

3.56 $\int (b \coth^m(c + dx))^{2/3} dx$

Optimal. Leaf size=60

$$\frac{3 \coth(c + dx) (b \coth^m(c + dx))^{2/3} {}_2F_1\left(1, \frac{1}{6}(2m + 3); \frac{1}{6}(2m + 9); \coth^2(c + dx)\right)}{d(2m + 3)}$$

[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))

Rubi [A] time = 0.0430382, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{3 \coth(c + dx) (b \coth^m(c + dx))^{2/3} {}_2F_1\left(1, \frac{1}{6}(2m + 3); \frac{1}{6}(2m + 9); \coth^2(c + dx)\right)}{d(2m + 3)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^m)^(2/3), x]

[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p]]/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
)]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (b \coth^m(c + dx))^{2/3} dx &= \left(\coth^{-\frac{2m}{3}}(c + dx) (b \coth^m(c + dx))^{2/3} \right) \int \coth^{\frac{2m}{3}}(c + dx) dx \\ &= \frac{\left(\coth^{-\frac{2m}{3}}(c + dx) (b \coth^m(c + dx))^{2/3} \right) \text{Subst} \left(\int \frac{x^{2m/3}}{-1+x^2} dx, x, \coth(c + dx) \right)}{d} \\ &= \frac{3 \coth(c + dx) (b \coth^m(c + dx))^{2/3} {}_2F_1 \left(1, \frac{1}{6}(3 + 2m); \frac{1}{6}(9 + 2m); \coth^2(c + dx) \right)}{d(3 + 2m)} \end{aligned}$$

Mathematica [A] time = 0.0417521, size = 60, normalized size = 1.

$$\frac{3 \coth(c + dx) (b \coth^m(c + dx))^{2/3} {}_2F_1 \left(1, \frac{1}{6}(2m + 3); \frac{1}{6}(2m + 9); \coth^2(c + dx) \right)}{d(2m + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(2/3), x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int (b(\coth(dx + c))^m)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(2/3), x)
```

```
[Out] int((b*coth(d*x+c)^m)^(2/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**m)**(2/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)
```

3.57 $\int \sqrt[3]{b \coth^m(c + dx)} dx$

Optimal. Leaf size=54

$$\frac{3 \coth(c + dx) \sqrt[3]{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{m+3}{6}; \frac{m+9}{6}; \coth^2(c + dx)\right)}{d(m+3)}$$

[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6, (9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))

Rubi [A] time = 0.0437366, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{3 \coth(c + dx) \sqrt[3]{b \coth^m(c + dx)} {}_2F_1\left(1, \frac{m+3}{6}; \frac{m+9}{6}; \coth^2(c + dx)\right)}{d(m+3)}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^m)^(1/3), x]

[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6, (9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))

Rule 3659

```
Int[(u_)*((b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := Dist[b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p]]/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_)*(trig_)[e + f*x])^(m_) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \sqrt[3]{b \coth^m(c + dx)} dx &= \left(\coth^{-\frac{m}{3}}(c + dx) \sqrt[3]{b \coth^m(c + dx)} \right) \int \coth^{\frac{m}{3}}(c + dx) dx \\ &= -\frac{\left(\coth^{-\frac{m}{3}}(c + dx) \sqrt[3]{b \coth^m(c + dx)} \right) \text{Subst} \left(\int \frac{x^{m/3}}{-1+x^2} dx, x, \coth(c + dx) \right)}{d} \\ &= \frac{3 \coth(c + dx) \sqrt[3]{b \coth^m(c + dx)} {}_2F_1 \left(1, \frac{3+m}{6}; \frac{9+m}{6}; \coth^2(c + dx) \right)}{d(3 + m)} \end{aligned}$$

Mathematica [A] time = 0.0386344, size = 54, normalized size = 1.

$$\frac{3 \coth(c + dx) \sqrt[3]{b \coth^m(c + dx)} {}_2F_1 \left(1, \frac{m+3}{6}; \frac{m+9}{6}; \coth^2(c + dx) \right)}{d(m + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(1/3), x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int \sqrt[3]{b (\coth(dx + c))^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(1/3), x)
```

```
[Out] int((b*coth(d*x+c)^m)^(1/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \coth^m(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)**m)**(1/3),x)

[Out] Integral((b*coth(c + d*x)**m)**(1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \coth(dx + c)^m)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)
```

$$3.58 \quad \int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}} dx$$

Optimal. Leaf size=60

$$\frac{3 \coth(c+dx) {}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b \coth^m(c+dx)}}$$

[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^2])/ (d*(3 - m)*(b*Coth[c + d*x]^m)^(1/3))

Rubi [A] time = 0.0426262, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{3 \coth(c+dx) {}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b \coth^m(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^m)^(-1/3), x]

[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^2])/ (d*(3 - m)*(b*Coth[c + d*x]^m)^(1/3))

Rule 3659

Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{b \coth^m(c + dx)}} dx &= \frac{\coth^{\frac{m}{3}}(c + dx) \int \coth^{-\frac{m}{3}}(c + dx) dx}{\sqrt[3]{b \coth^m(c + dx)}} \\ &= \frac{\coth^{\frac{m}{3}}(c + dx) \operatorname{Subst}\left(\int \frac{x^{-m/3}}{-1+x^2} dx, x, \coth(c + dx)\right)}{d \sqrt[3]{b \coth^m(c + dx)}} \\ &= \frac{3 \coth(c + dx) {}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c + dx)\right)}{d(3-m) \sqrt[3]{b \coth^m(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.0390112, size = 58, normalized size = 0.97

$$\frac{3 \coth(c + dx) {}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c + dx)\right)}{d(m-3) \sqrt[3]{b \coth^m(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^m)^(-1/3), x]

[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^2])/(d*(-3 + m)*(b*Coth[c + d*x]^m)^(1/3))

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b (\coth(dx + c))^m}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c)^m)^(1/3), x)

[Out] `int(1/(b*coth(d*x+c)^m)^(1/3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^m)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c)^m)^(-1/3), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b \coth^m(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)**m)**(1/3),x)`

[Out] `Integral((b*coth(c + d*x)**m)**(-1/3), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

$$3.59 \quad \int \frac{1}{(b \coth^m(c+dx))^{2/3}} dx$$

Optimal. Leaf size=60

$$\frac{3 \coth(c+dx) {}_2F_1\left(1, \frac{1}{6}(3-2m); \frac{1}{6}(9-2m); \coth^2(c+dx)\right)}{d(3-2m) (b \coth^m(c+dx))^{2/3}}$$

[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*x]^2])/(d*(3 - 2*m)*(b*Coth[c + d*x]^m)^(2/3))

Rubi [A] time = 0.0445252, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{3 \coth(c+dx) {}_2F_1\left(1, \frac{1}{6}(3-2m); \frac{1}{6}(9-2m); \coth^2(c+dx)\right)}{d(3-2m) (b \coth^m(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^m)^(-2/3), x]

[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*x]^2])/(d*(3 - 2*m)*(b*Coth[c + d*x]^m)^(2/3))

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(b \coth^m(c + dx))^{2/3}} dx &= \frac{\coth^{\frac{2m}{3}}(c + dx) \int \coth^{-\frac{2m}{3}}(c + dx) dx}{(b \coth^m(c + dx))^{2/3}} \\ &= \frac{\coth^{\frac{2m}{3}}(c + dx) \operatorname{Subst}\left(\int \frac{x^{-2m/3}}{-1+x^2} dx, x, \coth(c + dx)\right)}{d (b \coth^m(c + dx))^{2/3}} \\ &= \frac{3 \coth(c + dx) {}_2F_1\left(1, \frac{1}{6}(3 - 2m); \frac{1}{6}(9 - 2m); \coth^2(c + dx)\right)}{d(3 - 2m) (b \coth^m(c + dx))^{2/3}} \end{aligned}$$

Mathematica [A] time = 0.0398727, size = 60, normalized size = 1.

$$\frac{3 \coth(c + dx) {}_2F_1\left(1, \frac{1}{6}(3 - 2m); \frac{1}{6}(9 - 2m); \coth^2(c + dx)\right)}{d(2m - 3) (b \coth^m(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^m)^(-2/3), x]

[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*x]^2])/(d*(-3 + 2*m)*(b*Coth[c + d*x]^m)^(2/3))

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int (b (\coth(dx + c))^m)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*coth(d*x+c)^m)^(2/3),x)`

[Out] `int(1/(b*coth(d*x+c)^m)^(2/3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^m)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c)^m)^(-2/3), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth^m(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)**m)**(2/3),x)`

[Out] `Integral((b*coth(c + d*x)**m)**(-2/3), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^m)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^m)^(-2/3), x)

$$3.60 \quad \int \frac{1}{(b \coth^m(c+dx))^{4/3}} dx$$

Optimal. Leaf size=69

$$\frac{3 \coth^{1-m}(c+dx) {}_2F_1\left(1, \frac{1}{6}(3-4m); \frac{1}{6}(9-4m); \coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b \coth^m(c+dx)}}$$

[Out] (3*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Coth[c + d*x]^2])/(b*d*(3 - 4*m)*(b*Coth[c + d*x]^m)^(1/3))

Rubi [A] time = 0.0472706, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{3 \coth^{1-m}(c+dx) {}_2F_1\left(1, \frac{1}{6}(3-4m); \frac{1}{6}(9-4m); \coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b \coth^m(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^m)^(-4/3), x]

[Out] (3*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Coth[c + d*x]^2])/(b*d*(3 - 4*m)*(b*Coth[c + d*x]^m)^(1/3))

Rule 3659

Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(b \coth^m(c + dx))^{4/3}} dx &= \frac{\coth^{\frac{m}{3}}(c + dx) \int \coth^{-\frac{4m}{3}}(c + dx) dx}{b \sqrt[3]{b \coth^m(c + dx)}} \\ &= -\frac{\coth^{\frac{m}{3}}(c + dx) \operatorname{Subst}\left(\int \frac{x^{-4m/3}}{-1+x^2} dx, x, \coth(c + dx)\right)}{bd \sqrt[3]{b \coth^m(c + dx)}} \\ &= \frac{3 \coth^{1-m}(c + dx) {}_2F_1\left(1, \frac{1}{6}(3 - 4m); \frac{1}{6}(9 - 4m); \coth^2(c + dx)\right)}{bd(3 - 4m) \sqrt[3]{b \coth^m(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.0659447, size = 60, normalized size = 0.87

$$\frac{3 \coth(c + dx) {}_2F_1\left(1, \frac{1}{6}(3 - 4m); \frac{1}{6}(9 - 4m); \coth^2(c + dx)\right)}{d(4m - 3) (b \coth^m(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x]^m)^(-4/3), x]

[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Coth[c + d*x]^2])/(d*(-3 + 4*m)*(b*Coth[c + d*x]^m)^(4/3))

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int (b (\coth(dx + c))^m)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*coth(d*x+c)^m)^(4/3),x)`

[Out] `int(1/(b*coth(d*x+c)^m)^(4/3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^m)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")`

[Out] `integrate((b*coth(d*x + c)^m)^(-4/3), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*coth(d*x+c)**m)**(4/3),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \coth(dx + c)^m)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")

[Out] integrate((b*coth(d*x + c)^m)^(-4/3), x)

3.61 $\int (1 + \coth(x))^5 dx$

Optimal. Leaf size=41

$$16x - \frac{1}{4}(\coth(x) + 1)^4 - \frac{2}{3}(\coth(x) + 1)^3 - 2(\coth(x) + 1)^2 - 8 \coth(x) + 16 \log(\sinh(x))$$

[Out] 16*x - 8*Coth[x] - 2*(1 + Coth[x])^2 - (2*(1 + Coth[x])^3)/3 - (1 + Coth[x])^4/4 + 16*Log[Sinh[x]]

Rubi [A] time = 0.0394656, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3478, 3477, 3475}

$$16x - \frac{1}{4}(\coth(x) + 1)^4 - \frac{2}{3}(\coth(x) + 1)^3 - 2(\coth(x) + 1)^2 - 8 \coth(x) + 16 \log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^5,x]

[Out] 16*x - 8*Coth[x] - 2*(1 + Coth[x])^2 - (2*(1 + Coth[x])^3)/3 - (1 + Coth[x])^4/4 + 16*Log[Sinh[x]]

Rule 3478

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]

Rule 3477

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^2, x_Symbol] := Simp[(a^2 - b^2)*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d, x]) /; FreeQ[{a, b, c, d}, x]

Rule 3475

Int[tan[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (1 + \coth(x))^5 dx &= -\frac{1}{4}(1 + \coth(x))^4 + 2 \int (1 + \coth(x))^4 dx \\
&= -\frac{2}{3}(1 + \coth(x))^3 - \frac{1}{4}(1 + \coth(x))^4 + 4 \int (1 + \coth(x))^3 dx \\
&= -2(1 + \coth(x))^2 - \frac{2}{3}(1 + \coth(x))^3 - \frac{1}{4}(1 + \coth(x))^4 + 8 \int (1 + \coth(x))^2 dx \\
&= 16x - 8 \coth(x) - 2(1 + \coth(x))^2 - \frac{2}{3}(1 + \coth(x))^3 - \frac{1}{4}(1 + \coth(x))^4 + 16 \int \coth(x) dx \\
&= 16x - 8 \coth(x) - 2(1 + \coth(x))^2 - \frac{2}{3}(1 + \coth(x))^3 - \frac{1}{4}(1 + \coth(x))^4 + 16 \log(\sinh(x))
\end{aligned}$$

Mathematica [C] time = 0.238116, size = 94, normalized size = 2.29

$$\frac{\sinh(x)(\coth(x) + 1)^5 \left(-20 \sinh(x) \cosh^3(x) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \tanh^2(x)\right) - 120 \sinh^3(x) \cosh(x) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(x)\right) \right) - 12(\sinh(x) + \cosh(x))^5}{12(\sinh(x) + \cosh(x))^5}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^5, x]

[Out] ((1 + Coth[x])^5*Sinh[x]*(-3*Cosh[x]^4 - 20*Cosh[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[x]^2]*Sinh[x] - 66*Cosh[x]^2*Sinh[x]^2 - 120*Cosh[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2]*Sinh[x]^3 + 12*(x + 16*Log[Cosh[x]] + 16*Log[Tanh[x]])*Sinh[x]^4))/(12*(Cosh[x] + Sinh[x])^5)

Maple [A] time = 0.007, size = 31, normalized size = 0.8

$$-\frac{(\coth(x))^4}{4} - \frac{5(\coth(x))^3}{3} - \frac{11(\coth(x))^2}{2} - 15 \coth(x) - 16 \ln(\coth(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+coth(x))^5, x)

[Out] -1/4*coth(x)^4-5/3*coth(x)^3-11/2*coth(x)^2-15*coth(x)-16*ln(coth(x)-1)

Maxima [B] time = 1.0613, size = 189, normalized size = 4.61

$$27x - \frac{20(3e^{-2x} - 3e^{-4x} - 2)}{3(3e^{-2x} - 3e^{-4x} + e^{-6x} - 1)} + \frac{4(e^{-2x} - e^{-4x} + e^{-6x})}{4e^{-2x} - 6e^{-4x} + 4e^{-6x} - e^{-8x} - 1} + \frac{20e^{-2x}}{2e^{-2x} - e^{-4x} - 1} + \frac{20}{e^{-2x} - 1} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^5,x, algorithm="maxima")

[Out] 27*x - 20/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 20*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 20/(e^(-2*x) - 1) + 11*log(e^(-x) + 1) + 11*log(e^(-x) - 1) + 5*log(sinh(x))

Fricas [B] time = 2.04947, size = 1481, normalized size = 36.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^5,x, algorithm="fricas")

[Out] -4/3*(48*cosh(x)^6 + 288*cosh(x)*sinh(x)^5 + 48*sinh(x)^6 + 36*(20*cosh(x)^2 - 3)*sinh(x)^4 - 108*cosh(x)^4 + 48*(20*cosh(x)^3 - 9*cosh(x))*sinh(x)^3 + 8*(90*cosh(x)^4 - 81*cosh(x)^2 + 11)*sinh(x)^2 + 88*cosh(x)^2 - 12*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 16*(18*cosh(x)^5 - 27*cosh(x)^3 + 11*cosh(x))*sinh(x) - 25)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)

Sympy [A] time = 2.58923, size = 48, normalized size = 1.17

$$32x - 16 \log(\tanh(x) + 1) + 16 \log(\tanh(x)) - \frac{15}{\tanh(x)} - \frac{11}{2 \tanh^2(x)} - \frac{5}{3 \tanh^3(x)} - \frac{1}{4 \tanh^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))**5,x)

[Out] 32*x - 16*log(tanh(x) + 1) + 16*log(tanh(x)) - 15/tanh(x) - 11/(2*tanh(x)**2) - 5/(3*tanh(x)**3) - 1/(4*tanh(x)**4)

Giac [A] time = 1.18271, size = 55, normalized size = 1.34

$$-\frac{4(48e^{6x} - 108e^{4x} + 88e^{2x} - 25)}{3(e^{2x} - 1)^4} + 16 \log(|e^{2x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^5,x, algorithm="giac")

[Out] -4/3*(48*e^(6*x) - 108*e^(4*x) + 88*e^(2*x) - 25)/(e^(2*x) - 1)^4 + 16*log(abs(e^(2*x) - 1))

3.62 $\int (1 + \coth(x))^4 dx$

Optimal. Leaf size=31

$$8x - \frac{1}{3}(\coth(x) + 1)^3 - (\coth(x) + 1)^2 - 4\coth(x) + 8\log(\sinh(x))$$

[Out] $8*x - 4*\text{Coth}[x] - (1 + \text{Coth}[x])^2 - (1 + \text{Coth}[x])^3/3 + 8*\text{Log}[\text{Sinh}[x]]$

Rubi [A] time = 0.0309066, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3478, 3477, 3475}

$$8x - \frac{1}{3}(\coth(x) + 1)^3 - (\coth(x) + 1)^2 - 4\coth(x) + 8\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + \text{Coth}[x])^4, x]$

[Out] $8*x - 4*\text{Coth}[x] - (1 + \text{Coth}[x])^2 - (1 + \text{Coth}[x])^3/3 + 8*\text{Log}[\text{Sinh}[x]]$

Rule 3478

$\text{Int}[(a + (b \cdot \tan[c + d \cdot x])^n), x_Symbol] \rightarrow \text{Simp}[(b \cdot (a + b \cdot \tan[c + d \cdot x])^{n-1}) / (d \cdot (n-1)), x] + \text{Dist}[2 \cdot a, \text{Int}[(a + b \cdot \tan[c + d \cdot x])^{n-1}], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]

Rule 3477

$\text{Int}[(a + (b \cdot \tan[c + d \cdot x])^2), x_Symbol] \rightarrow \text{Simp}[(a^2 - b^2) \cdot x, x] + (\text{Dist}[2 \cdot a \cdot b, \text{Int}[\tan[c + d \cdot x], x], x] + \text{Simp}[(b^2 \cdot \tan[c + d \cdot x]) / d, x]) /;$ FreeQ[{a, b, c, d}, x]

Rule 3475

$\text{Int}[\tan[c + d \cdot x], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d \cdot x], x]] / d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (1 + \coth(x))^4 dx &= -\frac{1}{3}(1 + \coth(x))^3 + 2 \int (1 + \coth(x))^3 dx \\
&= -(1 + \coth(x))^2 - \frac{1}{3}(1 + \coth(x))^3 + 4 \int (1 + \coth(x))^2 dx \\
&= 8x - 4 \coth(x) - (1 + \coth(x))^2 - \frac{1}{3}(1 + \coth(x))^3 + 8 \int \coth(x) dx \\
&= 8x - 4 \coth(x) - (1 + \coth(x))^2 - \frac{1}{3}(1 + \coth(x))^3 + 8 \log(\sinh(x))
\end{aligned}$$

Mathematica [C] time = 0.179641, size = 84, normalized size = 2.71

$$\frac{\sinh(x)(\coth(x) + 1)^4 \left(3 \sinh(x) \left(-6 \sinh(x) \cosh(x) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(x) \right) - 2 \cosh^2(x) + \sinh^2(x)(x + 8 \log(\tanh(x))) \right) \right)}{3(\sinh(x) + \cosh(x))^4}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^4, x]

[Out] ((1 + Coth[x])^4*Sinh[x]*(-(Cosh[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[x]^2]) + 3*Sinh[x]*(-2*Cosh[x]^2 - 6*Cosh[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2]*Sinh[x] + (x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]])*Sinh[x]^2)))/(3*(Cosh[x] + Sinh[x])^4)

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$-\frac{(\coth(x))^3}{3} - 2(\coth(x))^2 - 7\coth(x) - 8\ln(\coth(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+coth(x))^4,x)

[Out] -1/3*coth(x)^3-2*coth(x)^2-7*coth(x)-8*ln(coth(x)-1)

Maxima [B] time = 1.10124, size = 128, normalized size = 4.13

$$12x - \frac{4(3e^{(-2x)} - 3e^{(-4x)} - 2)}{3(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1)} + \frac{8e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{12}{e^{(-2x)} - 1} + 4 \log(e^{(-x)} + 1) + 4 \log(e^{(-x)} - 1) + 4 \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^4,x, algorithm="maxima")

[Out] $12x - \frac{4}{3}(3e^{-2x} - 3e^{-4x} - 2)/(3e^{-2x} - 3e^{-4x} + e^{-6x} - 1) + \frac{8e^{-2x}}{2e^{-2x} - e^{-4x} - 1} + \frac{12}{e^{-2x} - 1} + 4\log(e^{-x} + 1) + 4\log(e^{-x} - 1) + 4\log(\sinh(x))$

Fricas [B] time = 2.06893, size = 915, normalized size = 29.52

$$\frac{4 \left(18 \cosh(x)^4 + 72 \cosh(x) \sinh(x)^3 + 18 \sinh(x)^4 + 27 (4 \cosh(x)^2 - 1) \sinh(x)^2 - 27 \cosh(x)^2 - 6 (\cosh(x)^6 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) \right)}{3 (\cosh(x)^6 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^4,x, algorithm="fricas")

[Out] $-4/3(18*\cosh(x)^4 + 72*\cosh(x)*\sinh(x)^3 + 18*\sinh(x)^4 + 27*(4*\cosh(x)^2 - 1)*\sinh(x)^2 - 27*\cosh(x)^2 - 6*(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6) + 3*(5*\cosh(x)^2 - 1)*\sinh(x)^4 - 3*\cosh(x)^4 + 4*(5*\cosh(x)^3 - 3*\cosh(x))*\sinh(x)^3 + 3*(5*\cosh(x)^4 - 6*\cosh(x)^2 + 1)*\sinh(x)^2 + 3*\cosh(x)^2 + 6*(\cosh(x)^5 - 2*\cosh(x)^3 + \cosh(x))*\sinh(x) - 1)*\log(2*\sinh(x)/(\cosh(x) - \sinh(x))) + 18*(4*\cosh(x)^3 - 3*\cosh(x))*\sinh(x) + 11)/(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + 3*(5*\cosh(x)^2 - 1)*\sinh(x)^4 - 3*\cosh(x)^4 + 4*(5*\cosh(x)^3 - 3*\cosh(x))*\sinh(x)^3 + 3*(5*\cosh(x)^4 - 6*\cosh(x)^2 + 1)*\sinh(x)^2 + 3*\cosh(x)^2 + 6*(\cosh(x)^5 - 2*\cosh(x)^3 + \cosh(x))*\sinh(x) - 1)$

Sympy [A] time = 1.88352, size = 37, normalized size = 1.19

$$16x - 8 \log(\tanh(x) + 1) + 8 \log(\tanh(x)) - \frac{7}{\tanh(x)} - \frac{2}{\tanh^2(x)} - \frac{1}{3 \tanh^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))**4,x)

[Out] $16x - 8\log(\tanh(x) + 1) + 8\log(\tanh(x)) - 7/\tanh(x) - 2/\tanh(x)**2 - 1/(3*\tanh(x)**3)$

Giac [A] time = 1.1945, size = 47, normalized size = 1.52

$$-\frac{4(18e^{4x} - 27e^{2x} + 11)}{3(e^{2x} - 1)^3} + 8 \log(|e^{2x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+coth(x))^4,x, algorithm="giac")`

[Out] $-4/3*(18*e^{(4*x)} - 27*e^{(2*x)} + 11)/(e^{(2*x)} - 1)^3 + 8*\log(\text{abs}(e^{(2*x)} - 1))$

3.63 $\int (1 + \coth(x))^3 dx$

Optimal. Leaf size=23

$$4x - \frac{1}{2}(\coth(x) + 1)^2 - 2 \coth(x) + 4 \log(\sinh(x))$$

[Out] 4*x - 2*Coth[x] - (1 + Coth[x])^2/2 + 4*Log[Sinh[x]]

Rubi [A] time = 0.0214076, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3478, 3477, 3475}

$$4x - \frac{1}{2}(\coth(x) + 1)^2 - 2 \coth(x) + 4 \log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^3, x]

[Out] 4*x - 2*Coth[x] - (1 + Coth[x])^2/2 + 4*Log[Sinh[x]]

Rule 3478

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]

Rule 3477

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^2, x_Symbol] := Simp[(a^2 - b^2)*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d, x]) /; FreeQ[{a, b, c, d}, x]

Rule 3475

Int[tan[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (1 + \coth(x))^3 dx &= -\frac{1}{2}(1 + \coth(x))^2 + 2 \int (1 + \coth(x))^2 dx \\
&= 4x - 2 \coth(x) - \frac{1}{2}(1 + \coth(x))^2 + 4 \int \coth(x) dx \\
&= 4x - 2 \coth(x) - \frac{1}{2}(1 + \coth(x))^2 + 4 \log(\sinh(x))
\end{aligned}$$

Mathematica [C] time = 0.149008, size = 61, normalized size = 2.65

$$\frac{1}{4} \operatorname{csch}^2(x) \left(-6 \sinh(2x) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(x) \right) - 2x - 8 \log(\tanh(x)) - 8 \log(\cosh(x)) + \cosh(2x)(2x + 8 \log(\tanh(x))) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^3, x]

[Out] (Csch[x]^2*(-1 - 2*x - 8*Log[Cosh[x]] - 8*Log[Tanh[x]] + Cosh[2*x]*(-1 + 2*x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]])) - 6*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2]*Sinh[2*x])/4

Maple [A] time = 0.002, size = 19, normalized size = 0.8

$$-\frac{(\coth(x))^2}{2} - 3 \coth(x) - 4 \ln(\coth(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+coth(x))^3, x)

[Out] -1/2*coth(x)^2-3*coth(x)-4*ln(coth(x)-1)

Maxima [B] time = 1.03956, size = 74, normalized size = 3.22

$$5x + \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{6}{e^{(-2x)} - 1} + \log(e^{(-x)} + 1) + \log(e^{(-x)} - 1) + 3 \log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^3,x, algorithm="maxima")

[Out] $5x + 2e^{-2x}/(2e^{-2x} - e^{-4x} - 1) + 6/(e^{-2x} - 1) + \log(e^{-x} + 1) + \log(e^{-x} - 1) + 3\log(\sinh(x))$

Fricas [B] time = 2.03912, size = 479, normalized size = 20.83

$$\frac{2\left(4 \cosh(x)^2 - 2\left(\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2\left(3 \cosh(x)^2 - 1\right) \sinh(x)^2 - 2 \cosh(x)^2 + 4\left(\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2\left(3 \cosh(x)^2 - 1\right) \sinh(x)^2 - \dots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^3,x, algorithm="fricas")

[Out] $-2*(4*\cosh(x)^2 - 2*(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)*\log(2*\sinh(x)/(\cosh(x) - \sinh(x))) + 8*\cosh(x)*\sinh(x) + 4*\sinh(x)^2 - 3)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)$

Sympy [A] time = 1.17971, size = 31, normalized size = 1.35

$$8x - 4 \log(\tanh(x) + 1) + 4 \log(\tanh(x)) - \frac{3}{\tanh(x)} - \frac{1}{2 \tanh^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))**3,x)

[Out] $8x - 4*\log(\tanh(x) + 1) + 4*\log(\tanh(x)) - 3/\tanh(x) - 1/(2*\tanh(x)**2)$

Giac [A] time = 1.1699, size = 39, normalized size = 1.7

$$-\frac{2\left(4e^{2x} - 3\right)}{\left(e^{2x} - 1\right)^2} + 4 \log\left(\left|e^{2x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="giac")
```

```
[Out] -2*(4*e^(2*x) - 3)/(e^(2*x) - 1)^2 + 4*log(abs(e^(2*x) - 1))
```

3.64 $\int (1 + \coth(x))^2 dx$

Optimal. Leaf size=13

$$2x - \coth(x) + 2 \log(\sinh(x))$$

[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]

Rubi [A] time = 0.0133208, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3477, 3475}

$$2x - \coth(x) + 2 \log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^2, x]

[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]

Rule 3477

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^2, x_Symbol] := Simp[(a^2 - b^2)*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d, x]) /; FreeQ[{a, b, c, d}, x]

Rule 3475

Int[tan[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (1 + \coth(x))^2 dx &= 2x - \coth(x) + 2 \int \coth(x) dx \\ &= 2x - \coth(x) + 2 \log(\sinh(x)) \end{aligned}$$

Mathematica [A] time = 0.0043895, size = 13, normalized size = 1.

$$2x - \coth(x) + 2 \log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^2,x]

[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]

Maple [A] time = 0.003, size = 13, normalized size = 1.

$$-\coth(x) - 2 \ln(\coth(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+coth(x))^2,x)

[Out] -coth(x)-2*ln(coth(x)-1)

Maxima [A] time = 1.13003, size = 26, normalized size = 2.

$$2x + \frac{2}{e^{(-2x)} - 1} + 2 \log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^2,x, algorithm="maxima")

[Out] 2*x + 2/(e^(-2*x) - 1) + 2*log(sinh(x))

Fricas [B] time = 1.95735, size = 189, normalized size = 14.54

$$\frac{2 \left((\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \log\left(\frac{2 \sinh(x)}{\cosh(x) - \sinh(x)}\right) - 1 \right)}{\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^2,x, algorithm="fricas")

[Out] $2*((\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\log(2*\sinh(x)/(\cosh(x) - \sinh(x))) - 1)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)$

Sympy [A] time = 0.745568, size = 22, normalized size = 1.69

$$4x - 2 \log(\tanh(x) + 1) + 2 \log(\tanh(x)) - \frac{1}{\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))**2,x)

[Out] $4*x - 2*\log(\tanh(x) + 1) + 2*\log(\tanh(x)) - 1/\tanh(x)$

Giac [A] time = 1.18667, size = 28, normalized size = 2.15

$$-\frac{2}{e^{(2*x)} - 1} + 2 \log(|e^{(2*x)} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^2,x, algorithm="giac")

[Out] $-2/(e^{(2*x)} - 1) + 2*\log(\text{abs}(e^{(2*x)} - 1))$

$$3.65 \quad \int \frac{1}{1+\coth(x)} dx$$

Optimal. Leaf size=16

$$\frac{x}{2} - \frac{1}{2(\coth(x)+1)}$$

[Out] x/2 - 1/(2*(1 + Coth[x]))

Rubi [A] time = 0.0086758, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3479, 8}

$$\frac{x}{2} - \frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(-1), x]

[Out] x/2 - 1/(2*(1 + Coth[x]))

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\coth(x)} dx &= -\frac{1}{2(1+\coth(x))} + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} - \frac{1}{2(1+\coth(x))} \end{aligned}$$

Mathematica [A] time = 0.0309119, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x - \sinh(2x) + \cosh(2x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(-1), x]

[Out] (2*x + Cosh[2*x] - Sinh[2*x])/4

Maple [A] time = 0.016, size = 24, normalized size = 1.5

$$-\frac{1}{2 + 2 \coth(x)} + \frac{\ln(1 + \coth(x))}{4} - \frac{\ln(\coth(x) - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+coth(x)), x)

[Out] -1/2/(1+coth(x))+1/4*ln(1+coth(x))-1/4*ln(coth(x)-1)

Maxima [A] time = 1.01401, size = 14, normalized size = 0.88

$$\frac{1}{2}x + \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x)), x, algorithm="maxima")

[Out] 1/2*x + 1/4*e^(-2*x)

Fricas [B] time = 2.01175, size = 88, normalized size = 5.5

$$\frac{(2x + 1) \cosh(x) + (2x - 1) \sinh(x)}{4(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x)),x, algorithm="fricas")

[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))

Sympy [B] time = 0.57725, size = 27, normalized size = 1.69

$$\frac{x \tanh(x)}{2 \tanh(x) + 2} + \frac{x}{2 \tanh(x) + 2} + \frac{1}{2 \tanh(x) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x)),x)

[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)

Giac [A] time = 1.14728, size = 14, normalized size = 0.88

$$\frac{1}{2}x + \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x)),x, algorithm="giac")

[Out] 1/2*x + 1/4*e^(-2*x)

$$3.66 \quad \int \frac{1}{(1+\coth(x))^2} dx$$

Optimal. Leaf size=26

$$\frac{x}{4} - \frac{1}{4(\coth(x)+1)} - \frac{1}{4(\coth(x)+1)^2}$$

[Out] x/4 - 1/(4*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))

Rubi [A] time = 0.0169901, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3479, 8}

$$\frac{x}{4} - \frac{1}{4(\coth(x)+1)} - \frac{1}{4(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(-2), x]

[Out] x/4 - 1/(4*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+\coth(x))^2} dx &= -\frac{1}{4(1+\coth(x))^2} + \frac{1}{2} \int \frac{1}{1+\coth(x)} dx \\ &= -\frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))} + \frac{\int 1 dx}{4} \\ &= \frac{x}{4} - \frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))} \end{aligned}$$

Mathematica [A] time = 0.0576583, size = 30, normalized size = 1.15

$$\frac{1}{16}(4x - 4 \sinh(2x) + \sinh(4x) + 4 \cosh(2x) - \cosh(4x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(-2), x]

[Out] (4*x + 4*Cosh[2*x] - Cosh[4*x] - 4*Sinh[2*x] + Sinh[4*x])/16

Maple [A] time = 0.017, size = 32, normalized size = 1.2

$$-\frac{1}{4(1 + \coth(x))^2} - \frac{1}{4 + 4 \coth(x)} + \frac{\ln(1 + \coth(x))}{8} - \frac{\ln(\coth(x) - 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+coth(x))^2,x)

[Out] -1/4/(1+coth(x))^2-1/4/(1+coth(x))+1/8*ln(1+coth(x))-1/8*ln(coth(x)-1)

Maxima [A] time = 1.02873, size = 22, normalized size = 0.85

$$\frac{1}{4}x + \frac{1}{4}e^{(-2x)} - \frac{1}{16}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^2,x, algorithm="maxima")

[Out] 1/4*x + 1/4*e^(-2*x) - 1/16*e^(-4*x)

Fricas [B] time = 2.39471, size = 173, normalized size = 6.65

$$\frac{(4x - 1) \cosh(x)^2 + 2(4x + 1) \cosh(x) \sinh(x) + (4x - 1) \sinh(x)^2 + 4}{16(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^2,x, algorithm="fricas")

[Out] 1/16*((4*x - 1)*cosh(x)^2 + 2*(4*x + 1)*cosh(x)*sinh(x) + (4*x - 1)*sinh(x)^2 + 4)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)

Sympy [B] time = 0.990182, size = 88, normalized size = 3.38

$$\frac{x \tanh^2(x)}{4 \tanh^2(x) + 8 \tanh(x) + 4} + \frac{2x \tanh(x)}{4 \tanh^2(x) + 8 \tanh(x) + 4} + \frac{x}{4 \tanh^2(x) + 8 \tanh(x) + 4} + \frac{3 \tanh(x)}{4 \tanh^2(x) + 8 \tanh(x) + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))**2,x)

[Out] x*tanh(x)**2/(4*tanh(x)**2 + 8*tanh(x) + 4) + 2*x*tanh(x)/(4*tanh(x)**2 + 8*tanh(x) + 4) + x/(4*tanh(x)**2 + 8*tanh(x) + 4) + 3*tanh(x)/(4*tanh(x)**2 + 8*tanh(x) + 4) + 2/(4*tanh(x)**2 + 8*tanh(x) + 4)

Giac [A] time = 1.15641, size = 24, normalized size = 0.92

$$\frac{1}{16} (4e^{(2x)} - 1)e^{(-4x)} + \frac{1}{4} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^2,x, algorithm="giac")

[Out] 1/16*(4*e^(2*x) - 1)*e^(-4*x) + 1/4*x

$$3.67 \quad \int \frac{1}{(1+\coth(x))^3} dx$$

Optimal. Leaf size=36

$$\frac{x}{8} - \frac{1}{8(\coth(x)+1)} - \frac{1}{8(\coth(x)+1)^2} - \frac{1}{6(\coth(x)+1)^3}$$

[Out] x/8 - 1/(6*(1 + Coth[x])^3) - 1/(8*(1 + Coth[x])^2) - 1/(8*(1 + Coth[x]))

Rubi [A] time = 0.0274045, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3479, 8}

$$\frac{x}{8} - \frac{1}{8(\coth(x)+1)} - \frac{1}{8(\coth(x)+1)^2} - \frac{1}{6(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(-3), x]

[Out] x/8 - 1/(6*(1 + Coth[x])^3) - 1/(8*(1 + Coth[x])^2) - 1/(8*(1 + Coth[x]))

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1 + \coth(x))^3} dx &= -\frac{1}{6(1 + \coth(x))^3} + \frac{1}{2} \int \frac{1}{(1 + \coth(x))^2} dx \\
&= -\frac{1}{6(1 + \coth(x))^3} - \frac{1}{8(1 + \coth(x))^2} + \frac{1}{4} \int \frac{1}{1 + \coth(x)} dx \\
&= -\frac{1}{6(1 + \coth(x))^3} - \frac{1}{8(1 + \coth(x))^2} - \frac{1}{8(1 + \coth(x))} + \frac{\int 1 dx}{8} \\
&= \frac{x}{8} - \frac{1}{6(1 + \coth(x))^3} - \frac{1}{8(1 + \coth(x))^2} - \frac{1}{8(1 + \coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.0870341, size = 44, normalized size = 1.22

$$\frac{1}{96}(12x - 18 \sinh(2x) + 9 \sinh(4x) - 2 \sinh(6x) + 18 \cosh(2x) - 9 \cosh(4x) + 2 \cosh(6x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(-3), x]

[Out] (12*x + 18*Cosh[2*x] - 9*Cosh[4*x] + 2*Cosh[6*x] - 18*Sinh[2*x] + 9*Sinh[4*x] - 2*Sinh[6*x])/96

Maple [A] time = 0.017, size = 40, normalized size = 1.1

$$-\frac{1}{6(1 + \coth(x))^3} - \frac{1}{8(1 + \coth(x))^2} - \frac{1}{8 + 8 \coth(x)} + \frac{\ln(1 + \coth(x))}{16} - \frac{\ln(\coth(x) - 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+coth(x))^3,x)

[Out] -1/6/(1+coth(x))^3-1/8/(1+coth(x))^2-1/8/(1+coth(x))+1/16*ln(1+coth(x))-1/16*ln(coth(x)-1)

Maxima [A] time = 1.05014, size = 30, normalized size = 0.83

$$\frac{1}{8}x + \frac{3}{16}e^{(-2x)} - \frac{3}{32}e^{(-4x)} + \frac{1}{48}e^{(-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^3,x, algorithm="maxima")

[Out] 1/8*x + 3/16*e^(-2*x) - 3/32*e^(-4*x) + 1/48*e^(-6*x)

Fricas [B] time = 2.26864, size = 278, normalized size = 7.72

$$\frac{2(6x+1)\cosh(x)^3 + 6(6x+1)\cosh(x)\sinh(x)^2 + 2(6x-1)\sinh(x)^3 + 3(2(6x-1)\cosh(x)^2 + 9)\sinh(x) + 9\cosh(x)}{96(\cosh(x)^3 + 3\cosh(x)^2\sinh(x) + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^3,x, algorithm="fricas")

[Out] 1/96*(2*(6*x + 1)*cosh(x)^3 + 6*(6*x + 1)*cosh(x)*sinh(x)^2 + 2*(6*x - 1)*sinh(x)^3 + 3*(2*(6*x - 1)*cosh(x)^2 + 9)*sinh(x) + 9*cosh(x))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)

Sympy [B] time = 1.47227, size = 182, normalized size = 5.06

$$\frac{3x \tanh^3(x)}{24 \tanh^3(x) + 72 \tanh^2(x) + 72 \tanh(x) + 24} + \frac{9x \tanh^2(x)}{24 \tanh^3(x) + 72 \tanh^2(x) + 72 \tanh(x) + 24} + \frac{9}{24 \tanh^3(x) + 72 \tanh^2(x) + 72 \tanh(x) + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))**3,x)

[Out] 3*x*tanh(x)**3/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh(x)**2/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh(x)/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 3*x/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) - 7*tanh(x)**3/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 6*tanh(x)/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 3/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24)

Giac [A] time = 1.14846, size = 32, normalized size = 0.89

$$\frac{1}{96} (18e^{4x} - 9e^{2x} + 2)e^{-6x} + \frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^3,x, algorithm="giac")

[Out] 1/96*(18*e^(4*x) - 9*e^(2*x) + 2)*e^(-6*x) + 1/8*x

$$3.68 \quad \int \frac{1}{(1+\coth(x))^4} dx$$

Optimal. Leaf size=46

$$\frac{x}{16} - \frac{1}{16(\coth(x)+1)} - \frac{1}{16(\coth(x)+1)^2} - \frac{1}{12(\coth(x)+1)^3} - \frac{1}{8(\coth(x)+1)^4}$$

[Out] x/16 - 1/(8*(1 + Coth[x])^4) - 1/(12*(1 + Coth[x])^3) - 1/(16*(1 + Coth[x])^2) - 1/(16*(1 + Coth[x]))

Rubi [A] time = 0.0359077, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3479, 8}

$$\frac{x}{16} - \frac{1}{16(\coth(x)+1)} - \frac{1}{16(\coth(x)+1)^2} - \frac{1}{12(\coth(x)+1)^3} - \frac{1}{8(\coth(x)+1)^4}$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(-4),x]

[Out] x/16 - 1/(8*(1 + Coth[x])^4) - 1/(12*(1 + Coth[x])^3) - 1/(16*(1 + Coth[x])^2) - 1/(16*(1 + Coth[x]))

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1 + \coth(x))^4} dx &= -\frac{1}{8(1 + \coth(x))^4} + \frac{1}{2} \int \frac{1}{(1 + \coth(x))^3} dx \\
&= -\frac{1}{8(1 + \coth(x))^4} - \frac{1}{12(1 + \coth(x))^3} + \frac{1}{4} \int \frac{1}{(1 + \coth(x))^2} dx \\
&= -\frac{1}{8(1 + \coth(x))^4} - \frac{1}{12(1 + \coth(x))^3} - \frac{1}{16(1 + \coth(x))^2} + \frac{1}{8} \int \frac{1}{1 + \coth(x)} dx \\
&= -\frac{1}{8(1 + \coth(x))^4} - \frac{1}{12(1 + \coth(x))^3} - \frac{1}{16(1 + \coth(x))^2} - \frac{1}{16(1 + \coth(x))} + \frac{\int 1 dx}{16} \\
&= \frac{x}{16} - \frac{1}{8(1 + \coth(x))^4} - \frac{1}{12(1 + \coth(x))^3} - \frac{1}{16(1 + \coth(x))^2} - \frac{1}{16(1 + \coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.121232, size = 53, normalized size = 1.15

$$\frac{1}{384}(\cosh(4x) - \sinh(4x))(32 \sinh(2x) + 24x \sinh(4x) + 3 \sinh(4x) + 64 \cosh(2x) + 3(8x - 1) \cosh(4x) - 36)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(-4), x]

[Out] ((Cosh[4*x] - Sinh[4*x])*(-36 + 64*Cosh[2*x] + 3*(-1 + 8*x)*Cosh[4*x] + 32*Sinh[2*x] + 3*Sinh[4*x] + 24*x*Sinh[4*x]))/384

Maple [A] time = 0.017, size = 48, normalized size = 1.

$$-\frac{1}{8(1 + \coth(x))^4} - \frac{1}{12(1 + \coth(x))^3} - \frac{1}{16(1 + \coth(x))^2} - \frac{1}{16 + 16 \coth(x)} + \frac{\ln(1 + \coth(x))}{32} - \frac{\ln(\coth(x) - 1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+coth(x))^4, x)

[Out] -1/8/(1+coth(x))^4-1/12/(1+coth(x))^3-1/16/(1+coth(x))^2-1/16/(1+coth(x))+1/32*ln(1+coth(x))-1/32*ln(coth(x)-1)

Maxima [A] time = 1.17884, size = 38, normalized size = 0.83

$$\frac{1}{16}x + \frac{1}{8}e^{(-2x)} - \frac{3}{32}e^{(-4x)} + \frac{1}{24}e^{(-6x)} - \frac{1}{128}e^{(-8x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^4,x, algorithm="maxima")

[Out] 1/16*x + 1/8*e^(-2*x) - 3/32*e^(-4*x) + 1/24*e^(-6*x) - 1/128*e^(-8*x)

Fricas [B] time = 2.37733, size = 394, normalized size = 8.57

$$\frac{3(8x-1)\cosh(x)^4 + 12(8x+1)\cosh(x)\sinh(x)^3 + 3(8x-1)\sinh(x)^4 + 2(9(8x-1)\cosh(x)^2 + 32)\sinh(x)^2 + 64}{384(\cosh(x)^4 + 4\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^4,x, algorithm="fricas")

[Out] 1/384*(3*(8*x - 1)*cosh(x)^4 + 12*(8*x + 1)*cosh(x)*sinh(x)^3 + 3*(8*x - 1)*sinh(x)^4 + 2*(9*(8*x - 1)*cosh(x)^2 + 32)*sinh(x)^2 + 64*cosh(x)^2 + 4*(3*(8*x + 1)*cosh(x)^3 + 16*cosh(x))*sinh(x) - 36)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)

Sympy [B] time = 2.55462, size = 299, normalized size = 6.5

$$\frac{3x \tanh^4(x)}{48 \tanh^4(x) + 192 \tanh^3(x) + 288 \tanh^2(x) + 192 \tanh(x) + 48} + \frac{12x \tanh^3(x)}{48 \tanh^4(x) + 192 \tanh^3(x) + 288 \tanh^2(x) + 192 \tanh(x) + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))**4,x)

[Out] 3*x*tanh(x)**4/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 12*x*tanh(x)**3/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 18*x*tanh(x)**2/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 12*x*tanh(x)/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 3*x/(48*tanh(x)**4 + 192*tanh(x)

)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 45*tanh(x)**3/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 84*tanh(x)**2/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 61*tanh(x)/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 16/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48)

Giac [A] time = 1.15168, size = 41, normalized size = 0.89

$$\frac{1}{384} (48 e^{(6x)} - 36 e^{(4x)} + 16 e^{(2x)} - 3) e^{(-8x)} + \frac{1}{16} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^4,x, algorithm="giac")

[Out] 1/384*(48*e^(6*x) - 36*e^(4*x) + 16*e^(2*x) - 3)*e^(-8*x) + 1/16*x

$$3.69 \quad \int \frac{1}{(1+\coth(x))^5} dx$$

Optimal. Leaf size=56

$$\frac{x}{32} - \frac{1}{32(\coth(x)+1)} - \frac{1}{32(\coth(x)+1)^2} - \frac{1}{24(\coth(x)+1)^3} - \frac{1}{16(\coth(x)+1)^4} - \frac{1}{10(\coth(x)+1)^5}$$

[Out] x/32 - 1/(10*(1 + Coth[x])^5) - 1/(16*(1 + Coth[x])^4) - 1/(24*(1 + Coth[x])^3) - 1/(32*(1 + Coth[x])^2) - 1/(32*(1 + Coth[x]))

Rubi [A] time = 0.0462366, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3479, 8}

$$\frac{x}{32} - \frac{1}{32(\coth(x)+1)} - \frac{1}{32(\coth(x)+1)^2} - \frac{1}{24(\coth(x)+1)^3} - \frac{1}{16(\coth(x)+1)^4} - \frac{1}{10(\coth(x)+1)^5}$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(-5),x]

[Out] x/32 - 1/(10*(1 + Coth[x])^5) - 1/(16*(1 + Coth[x])^4) - 1/(24*(1 + Coth[x])^3) - 1/(32*(1 + Coth[x])^2) - 1/(32*(1 + Coth[x]))

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1 + \coth(x))^5} dx &= -\frac{1}{10(1 + \coth(x))^5} + \frac{1}{2} \int \frac{1}{(1 + \coth(x))^4} dx \\
&= -\frac{1}{10(1 + \coth(x))^5} - \frac{1}{16(1 + \coth(x))^4} + \frac{1}{4} \int \frac{1}{(1 + \coth(x))^3} dx \\
&= -\frac{1}{10(1 + \coth(x))^5} - \frac{1}{16(1 + \coth(x))^4} - \frac{1}{24(1 + \coth(x))^3} + \frac{1}{8} \int \frac{1}{(1 + \coth(x))^2} dx \\
&= -\frac{1}{10(1 + \coth(x))^5} - \frac{1}{16(1 + \coth(x))^4} - \frac{1}{24(1 + \coth(x))^3} - \frac{1}{32(1 + \coth(x))^2} + \frac{1}{16} \int \frac{1}{1 + \coth(x)} dx \\
&= -\frac{1}{10(1 + \coth(x))^5} - \frac{1}{16(1 + \coth(x))^4} - \frac{1}{24(1 + \coth(x))^3} - \frac{1}{32(1 + \coth(x))^2} - \frac{1}{32(1 + \coth(x))} \\
&= \frac{x}{32} - \frac{1}{10(1 + \coth(x))^5} - \frac{1}{16(1 + \coth(x))^4} - \frac{1}{24(1 + \coth(x))^3} - \frac{1}{32(1 + \coth(x))^2} - \frac{1}{32(1 + \coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.138169, size = 62, normalized size = 1.11

$$\frac{(\cosh(5x) - \sinh(5x))(-500 \sinh(x) + 375 \sinh(3x) + 120x \sinh(5x) - 12 \sinh(5x) - 100 \cosh(x) + 225 \cosh(3x) + 120x \cosh(5x) - 500 \sinh(x) + 375 \sinh(3x) - 12 \sinh(5x) + 120x \sinh(5x))}{3840}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(-5), x]

[Out] ((Cosh[5*x] - Sinh[5*x])*(-100*Cosh[x] + 225*Cosh[3*x] + 12*Cosh[5*x] + 120*x*Cosh[5*x] - 500*Sinh[x] + 375*Sinh[3*x] - 12*Sinh[5*x] + 120*x*Sinh[5*x]))/3840

Maple [A] time = 0.024, size = 56, normalized size = 1.

$$-\frac{1}{10(1 + \coth(x))^5} - \frac{1}{16(1 + \coth(x))^4} - \frac{1}{24(1 + \coth(x))^3} - \frac{1}{32(1 + \coth(x))^2} - \frac{1}{32 + 32 \coth(x)} + \frac{\ln(1 + \coth(x))}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+coth(x))^5,x)

[Out] -1/10/(1+coth(x))^5-1/16/(1+coth(x))^4-1/24/(1+coth(x))^3-1/32/(1+coth(x))^2-1/32/(1+coth(x))+1/64*ln(1+coth(x))-1/64*ln(coth(x)-1)

Maxima [A] time = 1.03789, size = 46, normalized size = 0.82

$$\frac{1}{32}x + \frac{5}{64}e^{(-2x)} - \frac{5}{64}e^{(-4x)} + \frac{5}{96}e^{(-6x)} - \frac{5}{256}e^{(-8x)} + \frac{1}{320}e^{(-10x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^5,x, algorithm="maxima")

[Out] 1/32*x + 5/64*e^(-2*x) - 5/64*e^(-4*x) + 5/96*e^(-6*x) - 5/256*e^(-8*x) + 1/320*e^(-10*x)

Fricas [B] time = 2.2716, size = 540, normalized size = 9.64

$$\frac{12(10x + 1)\cosh(x)^5 + 60(10x + 1)\cosh(x)\sinh(x)^4 + 12(10x - 1)\sinh(x)^5 + 15(8(10x - 1)\cosh(x)^2 + 25)\sinh(x)}{3840(\cosh(x)^5 + 5\cosh(x)^4\sinh(x) + 10\cosh(x)^3\sinh(x)^2 + 10\cosh(x)^2\sinh(x)^3 + 5\cosh(x)\sinh(x)^4 + \sinh(x)^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^5,x, algorithm="fricas")

[Out] 1/3840*(12*(10*x + 1)*cosh(x)^5 + 60*(10*x + 1)*cosh(x)*sinh(x)^4 + 12*(10*x - 1)*sinh(x)^5 + 15*(8*(10*x - 1)*cosh(x)^2 + 25)*sinh(x)^3 + 225*cosh(x)^3 + 15*(8*(10*x + 1)*cosh(x)^3 + 45*cosh(x))*sinh(x)^2 + 5*(12*(10*x - 1)*cosh(x)^4 + 225*cosh(x)^2 - 100)*sinh(x) - 100*cosh(x))/(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5)

Sympy [B] time = 3.23362, size = 444, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))**5,x)

[Out] 15*x*tanh(x)**5/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tanh(x)**4/(480*tanh(x)**5 + 2400*t

$$\begin{aligned} & \operatorname{anh}(x)^{**4} + 4800*\operatorname{tanh}(x)^{**3} + 4800*\operatorname{tanh}(x)^{**2} + 2400*\operatorname{tanh}(x) + 480) + 150*x \\ & * \operatorname{tanh}(x)^{**3} / (480*\operatorname{tanh}(x)^{**5} + 2400*\operatorname{tanh}(x)^{**4} + 4800*\operatorname{tanh}(x)^{**3} + 4800*\operatorname{tanh} \\ & (x)^{**2} + 2400*\operatorname{tanh}(x) + 480) + 150*x*\operatorname{tanh}(x)^{**2} / (480*\operatorname{tanh}(x)^{**5} + 2400*\operatorname{tanh} \\ & (x)^{**4} + 4800*\operatorname{tanh}(x)^{**3} + 4800*\operatorname{tanh}(x)^{**2} + 2400*\operatorname{tanh}(x) + 480) + 75*x*\operatorname{tanh} \\ & h(x) / (480*\operatorname{tanh}(x)^{**5} + 2400*\operatorname{tanh}(x)^{**4} + 4800*\operatorname{tanh}(x)^{**3} + 4800*\operatorname{tanh}(x)^{**2} \\ & + 2400*\operatorname{tanh}(x) + 480) + 15*x / (480*\operatorname{tanh}(x)^{**5} + 2400*\operatorname{tanh}(x)^{**4} + 4800*\operatorname{tanh}(\\ & x)^{**3} + 4800*\operatorname{tanh}(x)^{**2} + 2400*\operatorname{tanh}(x) + 480) - 93*\operatorname{tanh}(x)^{**5} / (480*\operatorname{tanh}(x)^{**5} \\ & + 2400*\operatorname{tanh}(x)^{**4} + 4800*\operatorname{tanh}(x)^{**3} + 4800*\operatorname{tanh}(x)^{**2} + 2400*\operatorname{tanh}(x) + 4 \\ & 80) + 195*\operatorname{tanh}(x)^{**3} / (480*\operatorname{tanh}(x)^{**5} + 2400*\operatorname{tanh}(x)^{**4} + 4800*\operatorname{tanh}(x)^{**3} + \\ & 4800*\operatorname{tanh}(x)^{**2} + 2400*\operatorname{tanh}(x) + 480) + 275*\operatorname{tanh}(x)^{**2} / (480*\operatorname{tanh}(x)^{**5} + 24 \\ & 00*\operatorname{tanh}(x)^{**4} + 4800*\operatorname{tanh}(x)^{**3} + 4800*\operatorname{tanh}(x)^{**2} + 2400*\operatorname{tanh}(x) + 480) + 1 \\ & 60*\operatorname{tanh}(x) / (480*\operatorname{tanh}(x)^{**5} + 2400*\operatorname{tanh}(x)^{**4} + 4800*\operatorname{tanh}(x)^{**3} + 4800*\operatorname{tanh}(\\ & x)^{**2} + 2400*\operatorname{tanh}(x) + 480) + 35 / (480*\operatorname{tanh}(x)^{**5} + 2400*\operatorname{tanh}(x)^{**4} + 4800*t \\ & \operatorname{anh}(x)^{**3} + 4800*\operatorname{tanh}(x)^{**2} + 2400*\operatorname{tanh}(x) + 480) \end{aligned}$$

Giac [A] time = 1.14209, size = 49, normalized size = 0.88

$$\frac{1}{3840} \left(300 e^{(8x)} - 300 e^{(6x)} + 200 e^{(4x)} - 75 e^{(2x)} + 12 \right) e^{(-10x)} + \frac{1}{32} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^5,x, algorithm="giac")

[Out] 1/3840*(300*e^(8*x) - 300*e^(6*x) + 200*e^(4*x) - 75*e^(2*x) + 12)*e^(-10*x) + 1/32*x

3.70 $\int (1 + \coth(x))^{7/2} dx$

Optimal. Leaf size=57

$$-\frac{2}{5}(\coth(x) + 1)^{5/2} - \frac{4}{3}(\coth(x) + 1)^{3/2} - 8\sqrt{\coth(x) + 1} + 8\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}}\right)$$

[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 8*Sqrt[1 + Coth[x]] - (4*(1 + Coth[x])^(3/2))/3 - (2*(1 + Coth[x])^(5/2))/5

Rubi [A] time = 0.0419785, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3478, 3480, 206}

$$-\frac{2}{5}(\coth(x) + 1)^{5/2} - \frac{4}{3}(\coth(x) + 1)^{3/2} - 8\sqrt{\coth(x) + 1} + 8\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(7/2), x]

[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 8*Sqrt[1 + Coth[x]] - (4*(1 + Coth[x])^(3/2))/3 - (2*(1 + Coth[x])^(5/2))/5

Rule 3478

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]

Rule 3480

Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt

$Q[a, 0] \parallel \text{Lt}Q[b, 0]$

Rubi steps

$$\begin{aligned}
 \int (1 + \coth(x))^{7/2} dx &= -\frac{2}{5}(1 + \coth(x))^{5/2} + 2 \int (1 + \coth(x))^{5/2} dx \\
 &= -\frac{4}{3}(1 + \coth(x))^{3/2} - \frac{2}{5}(1 + \coth(x))^{5/2} + 4 \int (1 + \coth(x))^{3/2} dx \\
 &= -8\sqrt{1 + \coth(x)} - \frac{4}{3}(1 + \coth(x))^{3/2} - \frac{2}{5}(1 + \coth(x))^{5/2} + 8 \int \sqrt{1 + \coth(x)} dx \\
 &= -8\sqrt{1 + \coth(x)} - \frac{4}{3}(1 + \coth(x))^{3/2} - \frac{2}{5}(1 + \coth(x))^{5/2} + 16 \text{Subst} \left(\int \frac{1}{2-x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\
 &= 8\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right) - 8\sqrt{1 + \coth(x)} - \frac{4}{3}(1 + \coth(x))^{3/2} - \frac{2}{5}(1 + \coth(x))^{5/2}
 \end{aligned}$$

Mathematica [C] time = 0.258514, size = 101, normalized size = 1.77

$$\frac{2(\coth(x) + 1)^{7/2} \left((8 \sinh(2x) + 3) \sinh(x) \sqrt{i(\coth(x) + 1)} + 4 \sinh^3(x) \left(19\sqrt{i(\coth(x) + 1)} - (15 - 15i) \tan^{-1} \left(\frac{1}{2} + \frac{i}{2} \sqrt{1 + \coth(x)} \right) \right) \right)}{15\sqrt{i(\coth(x) + 1)}(\sinh(x) + \cosh(x))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(7/2), x]

[Out] $(-2*(1 + \text{Coth}[x])^{7/2}*(4*((-15 + 15*I)*\text{ArcTan}[(1/2 + I/2)*\text{Sqrt}[I*(1 + \text{Coth}[x])]]) + 19*\text{Sqrt}[I*(1 + \text{Coth}[x])])*\text{Sinh}[x]^3 + \text{Sqrt}[I*(1 + \text{Coth}[x])]*\text{Sinh}[x]*(3 + 8*\text{Sinh}[2*x])))/(15*\text{Sqrt}[I*(1 + \text{Coth}[x])]*(\text{Cosh}[x] + \text{Sinh}[x])^3)$

Maple [A] time = 0.016, size = 43, normalized size = 0.8

$$-\frac{4}{3}(1 + \coth(x))^{\frac{3}{2}} - \frac{2}{5}(1 + \coth(x))^{\frac{5}{2}} + 8 \text{Artanh} \left(\frac{1}{2} \sqrt{1 + \coth(x)} \sqrt{2} \right) \sqrt{2} - 8 \sqrt{1 + \coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+coth(x))^(7/2), x)

[Out] $-4/3*(1+\coth(x))^{3/2}-2/5*(1+\coth(x))^{5/2}+8*\operatorname{arctanh}(1/2*(1+\coth(x))^{1/2})^2*(1/2)*2^{1/2}-8*(1+\coth(x))^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + 1)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+coth(x))^(7/2),x, algorithm="maxima")`

[Out] `integrate((coth(x) + 1)^(7/2), x)`

Fricas [B] time = 2.41786, size = 1469, normalized size = 25.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+coth(x))^(7/2),x, algorithm="fricas")`

[Out] $-4/15*(2*\sqrt{2}*(23*\sqrt{2}*\cosh(x)^5 + 115*\sqrt{2}*\cosh(x)*\sinh(x)^4 + 23*\sqrt{2}*\sinh(x)^5 + 5*(46*\sqrt{2}*\cosh(x)^2 - 7*\sqrt{2})*\sinh(x)^3 - 35*\sqrt{2}*\cosh(x)^3 + 5*(46*\sqrt{2}*\cosh(x)^3 - 21*\sqrt{2}*\cosh(x))*\sinh(x)^2 + 5*(23*\sqrt{2}*\cosh(x)^4 - 21*\sqrt{2}*\cosh(x)^2 + 3*\sqrt{2})*\sinh(x) + 15*\sqrt{2}*\cosh(x))*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))} - 15*(\sqrt{2}*\cosh(x)^6 + 6*\sqrt{2}*\cosh(x)*\sinh(x)^5 + \sqrt{2}*\sinh(x)^6 + 3*(5*\sqrt{2}*\cosh(x)^2 - \sqrt{2})*\sinh(x)^4 - 3*\sqrt{2}*\cosh(x)^4 + 4*(5*\sqrt{2}*\cosh(x)^3 - 3*\sqrt{2}*\cosh(x))*\sinh(x)^3 + 3*(5*\sqrt{2}*\cosh(x)^4 - 6*\sqrt{2}*\cosh(x)^2 + \sqrt{2})*\sinh(x)^2 + 3*\sqrt{2}*\cosh(x)^2 + 6*(\sqrt{2}*\cosh(x)^5 - 2*\sqrt{2}*\cosh(x)^3 + \sqrt{2}*\cosh(x))*\sinh(x) - \sqrt{2})*\log(2*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}*(\cosh(x) + \sinh(x)) + 2*\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + 2*\sinh(x)^2 - 1))/(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + 3*(5*\cosh(x)^2 - 1)*\sinh(x)^4 - 3*\cosh(x)^4 + 4*(5*\cosh(x)^3 - 3*\cosh(x))*\sinh(x)^3 + 3*(5*\cosh(x)^4 - 6*\cosh(x)^2 + 1)*\sinh(x)^2 + 3*\cosh(x)^2 + 6*(\cosh(x)^5 - 2*\cosh(x)^3 + \cosh(x))*\sinh(x) - 1)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))**(7/2),x)

[Out] Timed out

Giac [B] time = 1.16966, size = 216, normalized size = 3.79

$$-\frac{4}{15} \sqrt{2} \frac{2 \left(45 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^4 + 135 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^3 + 170 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^2 + 100 \sqrt{e^{4x} - e^{2x}} \right)}{\left(\sqrt{e^{4x} - e^{2x}} - e^{2x} + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^(7/2),x, algorithm="giac")

[Out] -4/15*sqrt(2)*(2*(45*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^4 + 135*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^3 + 170*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^2 + 100*sqrt(e^(4*x) - e^(2*x)) - 100*e^(2*x) + 23)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^5 + 15*log(abs(2*sqrt(e^(4*x)) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x) - 1)

3.71 $\int (1 + \coth(x))^{5/2} dx$

Optimal. Leaf size=45

$$-\frac{2}{3}(\coth(x) + 1)^{3/2} - 4\sqrt{\coth(x) + 1} + 4\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}}\right)$$

[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 4*Sqrt[1 + Coth[x]] - (2*(1 + Coth[x])^(3/2))/3

Rubi [A] time = 0.031965, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3478, 3480, 206}

$$-\frac{2}{3}(\coth(x) + 1)^{3/2} - 4\sqrt{\coth(x) + 1} + 4\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(5/2), x]

[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 4*Sqrt[1 + Coth[x]] - (2*(1 + Coth[x])^(3/2))/3

Rule 3478

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]

Rule 3480

Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

Rubi steps

$$\begin{aligned}
 \int (1 + \coth(x))^{5/2} dx &= -\frac{2}{3}(1 + \coth(x))^{3/2} + 2 \int (1 + \coth(x))^{3/2} dx \\
 &= -4\sqrt{1 + \coth(x)} - \frac{2}{3}(1 + \coth(x))^{3/2} + 4 \int \sqrt{1 + \coth(x)} dx \\
 &= -4\sqrt{1 + \coth(x)} - \frac{2}{3}(1 + \coth(x))^{3/2} + 8 \operatorname{Subst} \left(\int \frac{1}{2-x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\
 &= 4\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right) - 4\sqrt{1 + \coth(x)} - \frac{2}{3}(1 + \coth(x))^{3/2}
 \end{aligned}$$

Mathematica [C] time = 0.153248, size = 92, normalized size = 2.04

$$\frac{2 \sinh(x)(\coth(x) + 1)^{5/2} \left(\cosh(x)\sqrt{i(\coth(x) + 1)} + \sinh(x) \left(7\sqrt{i(\coth(x) + 1)} - (6 - 6i) \tan^{-1} \left(\frac{1}{2} + \frac{i}{2} \right) \sqrt{i(\coth(x) + 1)} \right) \right)}{3\sqrt{i(\coth(x) + 1)}(\sinh(x) + \cosh(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(5/2), x]

[Out] (-2*(1 + Coth[x])^(5/2)*Sinh[x]*(Cosh[x]*Sqrt[I*(1 + Coth[x])]) + ((-6 + 6*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]) + 7*Sqrt[I*(1 + Coth[x])])*Sinh[x])/(3*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x])^2)

Maple [A] time = 0.013, size = 35, normalized size = 0.8

$$-\frac{2}{3}(1 + \coth(x))^{3/2} + 4 \operatorname{Arctanh} \left(\frac{1}{2} \sqrt{1 + \coth(x)} \sqrt{2} \right) \sqrt{2} - 4\sqrt{1 + \coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+coth(x))^(5/2), x)

[Out] -2/3*(1+coth(x))^(3/2)+4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-4*(1+coth(x))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + 1)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^(5/2),x, algorithm="maxima")

[Out] integrate((coth(x) + 1)^(5/2), x)

Fricas [B] time = 2.39008, size = 869, normalized size = 19.31

$$2 \left(2\sqrt{2}(4\sqrt{2}\cosh(x)^3 + 12\sqrt{2}\cosh(x)\sinh(x)^2 + 4\sqrt{2}\sinh(x)^3 + 3(4\sqrt{2}\cosh(x)^2 - \sqrt{2})\sinh(x) - 3\sqrt{2}\cosh(x)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^(5/2),x, algorithm="fricas")

[Out]
$$-2/3*(2*\sqrt{2}*(4*\sqrt{2}*\cosh(x)^3 + 12*\sqrt{2}*\cosh(x)*\sinh(x)^2 + 4*\sqrt{2}*\sinh(x)^3 + 3*(4*\sqrt{2}*\cosh(x)^2 - \sqrt{2})*\sinh(x) - 3*\sqrt{2}*\cosh(x))*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))} - 3*(\sqrt{2}*\cosh(x)^4 + 4*\sqrt{2}*\cosh(x)*\sinh(x)^3 + \sqrt{2}*\sinh(x)^4 + 2*(3*\sqrt{2}*\cosh(x)^2 - \sqrt{2})*\sinh(x)^2 - 2*\sqrt{2}*\cosh(x)^2 + 4*(\sqrt{2}*\cosh(x)^3 - \sqrt{2}*\cosh(x))*\sinh(x) + \sqrt{2})*\log(2*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}*(\cosh(x) + \sinh(x)) + 2*\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + 2*\sinh(x)^2 - 1)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))**(5/2),x)

[Out] Timed out

Giac [B] time = 1.17387, size = 151, normalized size = 3.36

$$-\frac{2}{3}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{4x}}-e^{2x}\right)-e^{2x}\right)^2+9\sqrt{e^{4x}}-e^{2x}-9e^{2x}+4}{\left(\sqrt{e^{4x}}-e^{2x}\right)-e^{2x}+1}\right)^3+3\log\left(\left|2\sqrt{e^{4x}}-e^{2x}\right|-2e^{2x}+1\right)\right)\operatorname{sgn}\left(e^{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^(5/2),x, algorithm="giac")

[Out] -2/3*sqrt(2)*(2*(6*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^2 + 9*sqrt(e^(4*x)) - e^(2*x)) - 9*e^(2*x) + 4)/(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x) + 1)^3 + 3*log(abs(2*sqrt(e^(4*x)) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x) - 1)

3.72 $\int (1 + \coth(x))^{3/2} dx$

Optimal. Leaf size=33

$$2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - 2\sqrt{\coth(x)+1}$$

[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]

Rubi [A] time = 0.0219855, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3478, 3480, 206}

$$2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - 2\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(3/2), x]

[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[(-2*b)/d,
Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (1 + \coth(x))^{3/2} dx &= -2\sqrt{1 + \coth(x)} + 2 \int \sqrt{1 + \coth(x)} dx \\
&= -2\sqrt{1 + \coth(x)} + 4 \operatorname{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\
&= 2\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right) - 2\sqrt{1 + \coth(x)}
\end{aligned}$$

Mathematica [C] time = 0.106497, size = 69, normalized size = 2.09

$$\frac{2 \sinh(x)(\coth(x) + 1)^{3/2} \left(\sqrt{i(\coth(x) + 1)} - (1 - i) \tan^{-1} \left(\left(\frac{1}{2} + \frac{i}{2} \right) \sqrt{i(\coth(x) + 1)} \right) \right)}{\sqrt{i(\coth(x) + 1)}(\sinh(x) + \cosh(x))}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(3/2), x]

[Out] (-2*(1 + Coth[x])^(3/2)*((-1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]] + Sqrt[I*(1 + Coth[x])]*Sinh[x])/(Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x]))

Maple [A] time = 0.013, size = 27, normalized size = 0.8

$$2 \operatorname{Artanh} \left(\frac{1}{2} \sqrt{1 + \coth(x)} \sqrt{2} \right) \sqrt{2} - 2 \sqrt{1 + \coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+coth(x))^(3/2), x)

[Out] 2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + 1)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^(3/2),x, algorithm="maxima")

[Out] integrate((coth(x) + 1)^(3/2), x)

Fricas [B] time = 2.48679, size = 450, normalized size = 13.64

$$2\sqrt{2}\left(\sqrt{2}\cosh(x) + \sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} - \left(\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 - \sqrt{2}\right)\log\left(\frac{2\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^(3/2),x, algorithm="fricas")

[Out] $-(2*\sqrt{2}*(\sqrt{2}*\cosh(x) + \sqrt{2}*\sinh(x))*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}) - (\sqrt{2}*\cosh(x)^2 + 2*\sqrt{2}*\cosh(x)*\sinh(x) + \sqrt{2}*\sinh(x)^2 - \sqrt{2})*\log(2*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}*(\cosh(x) + \sinh(x)) + 2*\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + 2*\sinh(x)^2 - 1)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))**(3/2),x)

[Out] Integral((coth(x) + 1)**(3/2), x)

Giac [B] time = 1.16432, size = 85, normalized size = 2.58

$$-\sqrt{2}\left(\frac{2}{\sqrt{e^{4x}} - e^{2x} - e^{2x} + 1} + \log\left(\left|2\sqrt{e^{4x}} - e^{2x} - 2e^{2x} + 1\right|\right)\right)\operatorname{sgn}(e^{2x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1) + log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

3.73 $\int \sqrt{1 + \coth(x)} dx$

Optimal. Leaf size=21

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}} \right)$$

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]

Rubi [A] time = 0.0120375, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3480, 206}

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Coth[x]],x]

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]

Rule 3480

Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{1 + \coth(x)} dx &= 2 \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\ &= \sqrt{2} \tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right) \end{aligned}$$

Mathematica [C] time = 0.0741134, size = 45, normalized size = 2.14

$$\frac{(1+i)(\coth(x)+1)^{3/2} \tan^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{i(\coth(x)+1)}\right)}{(i(\coth(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Coth[x]], x]

[Out] ((1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*(1 + Coth[x])^(3/2))/(I*(1 + Coth[x]))^(3/2)

Maple [A] time = 0.036, size = 17, normalized size = 0.8

$$\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+coth(x))^(1/2), x)

[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(coth(x) + 1), x)

Fricas [B] time = 2.31598, size = 181, normalized size = 8.62

$$\frac{1}{2} \sqrt{2} \log\left(2 \sqrt{2} \sqrt{\frac{\sinh(x)}{\cosh(x) - \sinh(x)}} (\cosh(x) + \sinh(x)) + 2 \cosh(x)^2 + 4 \cosh(x) \sinh(x) + 2 \sinh(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))**(1/2),x)

[Out] Integral(sqrt(coth(x) + 1), x)

Giac [B] time = 1.17684, size = 50, normalized size = 2.38

$$-\frac{1}{2} \sqrt{2} \log \left(\left| 2 \sqrt{e^{4x} - e^{2x}} - 2e^{2x} + 1 \right| \right) \operatorname{sgn}(e^{2x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+coth(x))^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x) - 1)

$$3.74 \quad \int \frac{1}{\sqrt{1+\coth(x)}} dx$$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{\coth(x)+1}}$$

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]]

Rubi [A] time = 0.0228691, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3479, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + Coth[x]],x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]]

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3480

Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1 + \coth(x)}} dx &= -\frac{1}{\sqrt{1 + \coth(x)}} + \frac{1}{2} \int \sqrt{1 + \coth(x)} dx \\
&= -\frac{1}{\sqrt{1 + \coth(x)}} + \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right)}{\sqrt{2}} - \frac{1}{\sqrt{1 + \coth(x)}}
\end{aligned}$$

Mathematica [C] time = 0.291743, size = 51, normalized size = 1.59

$$\frac{-2 + (-1 - i)\sqrt{i(\coth(x) + 1)} \tan^{-1} \left(\left(\frac{1}{2} + \frac{i}{2} \right) \sqrt{i(\coth(x) + 1)} \right)}{2\sqrt{\coth(x) + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 + Coth[x]], x]

[Out] (-2 - (1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*Sqrt[I*(1 + Coth[x])])/(2*Sqrt[1 + Coth[x]])

Maple [A] time = 0.036, size = 27, normalized size = 0.8

$$\frac{\sqrt{2}}{2} \text{Artanh} \left(\frac{\sqrt{2}}{2} \sqrt{1 + \coth(x)} \right) - \frac{1}{\sqrt{1 + \coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+coth(x))^(1/2), x)

[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\coth(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(coth(x) + 1), x)

Fricas [B] time = 2.36078, size = 300, normalized size = 9.38

$$\frac{(\sqrt{2} \cosh(x) + \sqrt{2} \sinh(x)) \log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x) + \sinh(x)) + 2 \cosh(x)^2 + 4 \cosh(x) \sinh(x) + 2 \sinh(x)^2 - 1\right)}{4(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="fricas")

[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1) - 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\coth(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))**(1/2),x)

[Out] Integral(1/sqrt(coth(x) + 1), x)

Giac [B] time = 1.15375, size = 89, normalized size = 2.78

$$\frac{\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}-e^{(2x)}}} - \log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}} - 2e^{(2x)} + 1\right|\right)\right)}{4 \operatorname{sgn}(e^{(2x)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x)) - log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

$$3.75 \quad \int \frac{1}{(1+\coth(x))^{3/2}} dx$$

Optimal. Leaf size=49

$$-\frac{1}{2\sqrt{\coth(x)+1}} - \frac{1}{3(\coth(x)+1)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2)) - 1/(2*Sqrt[1 + Coth[x]])

Rubi [A] time = 0.0307411, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3479, 3480, 206}

$$-\frac{1}{2\sqrt{\coth(x)+1}} - \frac{1}{3(\coth(x)+1)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(-3/2),x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2)) - 1/(2*Sqrt[1 + Coth[x]])

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3480

Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(1 + \coth(x))^{3/2}} dx &= -\frac{1}{3(1 + \coth(x))^{3/2}} + \frac{1}{2} \int \frac{1}{\sqrt{1 + \coth(x)}} dx \\
 &= -\frac{1}{3(1 + \coth(x))^{3/2}} - \frac{1}{2\sqrt{1 + \coth(x)}} + \frac{1}{4} \int \sqrt{1 + \coth(x)} dx \\
 &= -\frac{1}{3(1 + \coth(x))^{3/2}} - \frac{1}{2\sqrt{1 + \coth(x)}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\
 &= \frac{\tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right)}{2\sqrt{2}} - \frac{1}{3(1 + \coth(x))^{3/2}} - \frac{1}{2\sqrt{1 + \coth(x)}}
 \end{aligned}$$

Mathematica [C] time = 0.307893, size = 86, normalized size = 1.76

$$\left(\frac{1}{4} + \frac{i}{4} \right) \sqrt{\coth(x) + 1} \left(\left(\frac{1}{6} - \frac{i}{6} \right) (-5 \sinh(2x) + \sinh(4x) + 5 \cosh(2x) - \cosh(4x) - 4) - \frac{i \tan^{-1} \left(\left(\frac{1}{2} + \frac{i}{2} \right) \sqrt{i(\coth(x) + 1)} \right)}{\sqrt{i(\coth(x) + 1)}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(-3/2), x]

[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]])/Sqrt[I*(1 + Coth[x])] + (1/6 - I/6)*(-4 + 5*Cosh[2*x] - Cosh[4*x] - 5*Sinh[2*x] + Sinh[4*x])

Maple [A] time = 0.011, size = 35, normalized size = 0.7

$$-\frac{1}{3} (1 + \coth(x))^{-\frac{3}{2}} + \frac{\sqrt{2}}{4} \text{Artanh} \left(\frac{\sqrt{2}}{2} \sqrt{1 + \coth(x)} \right) - \frac{1}{2} \frac{1}{\sqrt{1 + \coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+coth(x))^(3/2), x)

[Out] $-1/3/(1+\coth(x))^{3/2}+1/4*\operatorname{arctanh}(1/2*(1+\coth(x))^{1/2}*2^{1/2})*2^{1/2}-1/2/(1+\coth(x))^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x) + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+coth(x))^(3/2),x, algorithm="maxima")`

[Out] `integrate((coth(x) + 1)^(-3/2), x)`

Fricas [B] time = 2.36685, size = 578, normalized size = 11.8

$$\frac{2\sqrt{2}(4\sqrt{2}\cosh(x)^2 + 8\sqrt{2}\cosh(x)\sinh(x) + 4\sqrt{2}\sinh(x)^2 - \sqrt{2})\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} - 3(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)\sinh(x)^2 + \sinh(x)^3)}{24(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+coth(x))^(3/2),x, algorithm="fricas")`

[Out] $-1/24*(2*\sqrt{2}*(4*\sqrt{2}*\cosh(x)^2 + 8*\sqrt{2}*\cosh(x)*\sinh(x) + 4*\sqrt{2}*\sinh(x)^2 - \sqrt{2})*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))} - 3*(\sqrt{2}*\cosh(x)^3 + 3*\sqrt{2}*\cosh(x)*\sinh(x)^2 + \sinh(x)^3)*\log(2*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}*(\cosh(x) + \sinh(x)) + 2*\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + 2*\sinh(x)^2 - 1))/(\cosh(x)^3 + 3*\cosh(x)*\sinh(x)^2 + \sinh(x)^3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x) + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))**(3/2),x)

[Out] Integral((coth(x) + 1)**(-3/2), x)

Giac [B] time = 1.19078, size = 177, normalized size = 3.61

$$-\frac{1}{24} \sqrt{2} \left(\frac{3 \log \left(\left| 2 \sqrt{e^{4x} - e^{2x}} - 2e^{2x} + 1 \right| \right)}{\operatorname{sgn}(e^{2x} - 1)} - \frac{2 \left(6 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^2 + 3 \sqrt{e^{4x} - e^{2x}} - 3e^{2x} + 1 \right)}{\left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^3 \operatorname{sgn}(e^{2x} - 1)} - 8 \operatorname{sgn}(e^{2x} - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="giac")

[Out] -1/24*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x)) - e^(2*x)) - 2*e^(2*x) + 1))/sgn(e^(2*x) - 1) - 2*(6*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^2 + 3*sqrt(e^(4*x) - e^(2*x)) - 3*e^(2*x) + 1)/((sqrt(e^(4*x)) - e^(2*x))^3*sgn(e^(2*x) - 1)) - 8*sgn(e^(2*x) - 1))

$$3.76 \quad \int \frac{1}{(1+\coth(x))^{5/2}} dx$$

Optimal. Leaf size=61

$$-\frac{1}{4\sqrt{\coth(x)+1}} - \frac{1}{6(\coth(x)+1)^{3/2}} - \frac{1}{5(\coth(x)+1)^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Coth[x])^(5/2)) - 1/(6*(1 + Coth[x])^(3/2)) - 1/(4*Sqrt[1 + Coth[x]])

Rubi [A] time = 0.0419238, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3479, 3480, 206}

$$-\frac{1}{4\sqrt{\coth(x)+1}} - \frac{1}{6(\coth(x)+1)^{3/2}} - \frac{1}{5(\coth(x)+1)^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(-5/2), x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Coth[x])^(5/2)) - 1/(6*(1 + Coth[x])^(3/2)) - 1/(4*Sqrt[1 + Coth[x]])

Rule 3479

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3480

Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(1 + \coth(x))^{5/2}} dx &= -\frac{1}{5(1 + \coth(x))^{5/2}} + \frac{1}{2} \int \frac{1}{(1 + \coth(x))^{3/2}} dx \\
 &= -\frac{1}{5(1 + \coth(x))^{5/2}} - \frac{1}{6(1 + \coth(x))^{3/2}} + \frac{1}{4} \int \frac{1}{\sqrt{1 + \coth(x)}} dx \\
 &= -\frac{1}{5(1 + \coth(x))^{5/2}} - \frac{1}{6(1 + \coth(x))^{3/2}} - \frac{1}{4\sqrt{1 + \coth(x)}} + \frac{1}{8} \int \sqrt{1 + \coth(x)} dx \\
 &= -\frac{1}{5(1 + \coth(x))^{5/2}} - \frac{1}{6(1 + \coth(x))^{3/2}} - \frac{1}{4\sqrt{1 + \coth(x)}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\
 &= \frac{\tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right)}{4\sqrt{2}} - \frac{1}{5(1 + \coth(x))^{5/2}} - \frac{1}{6(1 + \coth(x))^{3/2}} - \frac{1}{4\sqrt{1 + \coth(x)}}
 \end{aligned}$$

Mathematica [C] time = 0.757937, size = 94, normalized size = 1.54

$$-\frac{1}{60} \sqrt{\coth(x) + 1} (\cosh(3x) - \sinh(3x)) (-24 \sinh(x) + 13 \sinh(3x) - 10 \cosh(x) + 10 \cosh(3x)) + \frac{\left(\frac{1}{8} + \frac{i}{8}\right) (\coth(x) + 1)}{(i)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(-5/2), x]

[Out] ((1/8 + I/8)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*(1 + Coth[x])^(3/2)) / (I*(1 + Coth[x])^(3/2) - (Sqrt[1 + Coth[x]]*(Cosh[3*x] - Sinh[3*x])*(-10*Cosh[x] + 10*Cosh[3*x] - 24*Sinh[x] + 13*Sinh[3*x]))/60

Maple [A] time = 0.013, size = 43, normalized size = 0.7

$$-\frac{1}{5} (1 + \coth(x))^{-5/2} - \frac{1}{6} (1 + \coth(x))^{-3/2} + \frac{\sqrt{2}}{8} \text{Artanh} \left(\frac{\sqrt{2}}{2} \sqrt{1 + \coth(x)} \right) - \frac{1}{4} \frac{1}{\sqrt{1 + \coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+coth(x))^(5/2), x)

[Out] $-1/5/(1+\coth(x))^{5/2}-1/6/(1+\coth(x))^{3/2}+1/8*\operatorname{arctanh}(1/2*(1+\coth(x))^{1/2})*2^{1/2}-1/4/(1+\coth(x))^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x) + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+coth(x))^(5/2),x, algorithm="maxima")`

[Out] `integrate((coth(x) + 1)^(-5/2), x)`

Fricas [B] time = 2.45624, size = 919, normalized size = 15.07

$$2\sqrt{2}(23\sqrt{2}\cosh(x)^4 + 92\sqrt{2}\cosh(x)\sinh(x)^3 + 23\sqrt{2}\sinh(x)^4 + (138\sqrt{2}\cosh(x)^2 - 11\sqrt{2})\sinh(x)^2 - 11\sqrt{2}\cosh(x)\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+coth(x))^(5/2),x, algorithm="fricas")`

[Out] $-1/240*(2*\sqrt{2}*(23*\sqrt{2}*\cosh(x)^4 + 92*\sqrt{2}*\cosh(x)*\sinh(x)^3 + 23*\sqrt{2}*\sinh(x)^4 + (138*\sqrt{2}*\cosh(x)^2 - 11*\sqrt{2})*\sinh(x)^2 - 11*\sqrt{2}*\cosh(x)*\sinh(x)) + 3*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))} - 15*(\sqrt{2}*\cosh(x)^5 + 5*\sqrt{2}*\cosh(x)^4*\sinh(x) + 10*\sqrt{2}*\cosh(x)^3*\sinh(x)^2 + 10*\sqrt{2}*\cosh(x)^2*\sinh(x)^3 + 5*\sqrt{2}*\cosh(x)*\sinh(x)^4 + \sqrt{2}*\sinh(x)^5)*\log(2*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}*(\cosh(x) + \sinh(x)) + 2*\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + 2*\sinh(x)^2 - 1))/(\cosh(x)^5 + 5*\cosh(x)^4*\sinh(x) + 10*\cosh(x)^3*\sinh(x)^2 + 10*\cosh(x)^2*\sinh(x)^3 + 5*\cosh(x)*\sinh(x)^4 + \sinh(x)^5)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\coth(x) + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))**(5/2), x)

[Out] Integral((coth(x) + 1)**(-5/2), x)

Giac [B] time = 1.22279, size = 242, normalized size = 3.97

$$-\frac{1}{240} \sqrt{2} \left(\frac{15 \log \left(\left| 2 \sqrt{e^{4x}} - e^{2x} - 2e^{2x} + 1 \right| \right)}{\operatorname{sgn}(e^{2x} - 1)} - \frac{2 \left(45 \left(\sqrt{e^{4x}} - e^{2x} - e^{2x} \right)^4 + 45 \left(\sqrt{e^{4x}} - e^{2x} - e^{2x} \right)^3 + 35 \left(\sqrt{e^{4x}} - e^{2x} - e^{2x} \right)^2 + 15 \left(\sqrt{e^{4x}} - e^{2x} - e^{2x} \right) - 15e^{2x} + 3 \right)}{\left(\sqrt{e^{4x}} - e^{2x} - e^{2x} \right)^5 \operatorname{sgn}(e^{2x} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x))^(5/2), x, algorithm="giac")

[Out] -1/240*sqrt(2)*(15*log(abs(2*sqrt(e^(4*x)) - e^(2*x)) - 2*e^(2*x) + 1))/sgn(e^(2*x) - 1) - 2*(45*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^4 + 45*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^3 + 35*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^2 + 15*sqrt(e^(4*x)) - 15*e^(2*x) + 3)/((sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^5*sgn(e^(2*x) - 1) - 46*sgn(e^(2*x) - 1)

3.77 $\int (a + b \coth(c + dx))^5 dx$

Optimal. Leaf size=142

$$\frac{b(3a^2 + b^2)(a + b \coth(c + dx))^2}{2d} - \frac{4ab^2(a^2 + b^2) \coth(c + dx)}{d} + \frac{b(10a^2b^2 + 5a^4 + b^4) \log(\sinh(c + dx))}{d} + ax(10a^2b^2 + 5a^4 + b^4)$$

```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x - (4*a*b^2*(a^2 + b^2)*Coth[c + d*x])/d - (b*(3*a^2 + b^2)*(a + b*Coth[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Coth[c + d*x])^3)/(3*d) - (b*(a + b*Coth[c + d*x])^4)/(4*d) + (b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Sinh[c + d*x]])/d
```

Rubi [A] time = 0.208768, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3482, 3528, 3525, 3475}

$$\frac{b(3a^2 + b^2)(a + b \coth(c + dx))^2}{2d} - \frac{4ab^2(a^2 + b^2) \coth(c + dx)}{d} + \frac{b(10a^2b^2 + 5a^4 + b^4) \log(\sinh(c + dx))}{d} + ax(10a^2b^2 + 5a^4 + b^4)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^5, x]
```

```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x - (4*a*b^2*(a^2 + b^2)*Coth[c + d*x])/d - (b*(3*a^2 + b^2)*(a + b*Coth[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Coth[c + d*x])^3)/(3*d) - (b*(a + b*Coth[c + d*x])^4)/(4*d) + (b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Sinh[c + d*x]])/d
```

Rule 3482

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

0] && GtQ[m, 0]

Rule 3525

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e + f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]

Rule 3475

Int[tan[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int (a + b \coth(c + dx))^5 dx &= -\frac{b(a + b \coth(c + dx))^4}{4d} + \int (a + b \coth(c + dx))^3 (a^2 + b^2 + 2ab \coth(c + dx)) dx \\
 &= -\frac{2ab(a + b \coth(c + dx))^3}{3d} - \frac{b(a + b \coth(c + dx))^4}{4d} + \int (a + b \coth(c + dx))^2 (a(a^2 + 3b^2) \\
 &= -\frac{b(3a^2 + b^2)(a + b \coth(c + dx))^2}{2d} - \frac{2ab(a + b \coth(c + dx))^3}{3d} - \frac{b(a + b \coth(c + dx))^4}{4d} + \\
 &= a(a^4 + 10a^2b^2 + 5b^4)x - \frac{4ab^2(a^2 + b^2) \coth(c + dx)}{d} - \frac{b(3a^2 + b^2)(a + b \coth(c + dx))^2}{2d} \\
 &= a(a^4 + 10a^2b^2 + 5b^4)x - \frac{4ab^2(a^2 + b^2) \coth(c + dx)}{d} - \frac{b(3a^2 + b^2)(a + b \coth(c + dx))^2}{2d}
 \end{aligned}$$

Mathematica [A] time = 0.798875, size = 141, normalized size = 0.99

$$\frac{6b^3(10a^2 + b^2) \coth^2(c + dx) + 60ab^2(2a^2 + b^2) \coth(c + dx) - 12b(10a^2b^2 + 5a^4 + b^4) \log(\tanh(c + dx)) + 20ab^4 \coth(c + dx)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Coth[c + d*x])^5, x]

[Out] -(60*a*b^2*(2*a^2 + b^2)*Coth[c + d*x] + 6*b^3*(10*a^2 + b^2)*Coth[c + d*x]^2 + 20*a*b^4*Coth[c + d*x]^3 + 3*b^5*Coth[c + d*x]^4 + 6*(a + b)^5*Log[1 - Tanh[c + d*x]] - 12*b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Tanh[c + d*x]] - 6*(a - b)^5*Log[1 + Tanh[c + d*x]])/(12*d)

Maple [B] time = 0.006, size = 322, normalized size = 2.3

$$-5 \frac{(\coth(dx+c))^2 a^2 b^3}{d} + \frac{\ln(\coth(dx+c)+1) a^5}{2d} - \frac{5 \ln(\coth(dx+c)+1) a^4 b}{2d} + 5 \frac{\ln(\coth(dx+c)+1) a^3 b^2}{d} - 5 \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*coth(d*x+c))^5,x)

[Out] $-5/d*\coth(d*x+c)^2*a^2*b^3+1/2/d*\ln(\coth(d*x+c)+1)*a^5-5/2/d*\ln(\coth(d*x+c)+1)*a^4*b+5/d*\ln(\coth(d*x+c)+1)*a^3*b^2-5/d*\ln(\coth(d*x+c)+1)*a^2*b^3+5/2/d*\ln(\coth(d*x+c)+1)*a*b^4-1/2/d*\ln(\coth(d*x+c)+1)*b^5-1/2/d*\ln(\coth(d*x+c)-1)*a^5-5/2/d*\ln(\coth(d*x+c)-1)*a^4*b-5/d*\ln(\coth(d*x+c)-1)*a^3*b^2-5/d*\ln(\coth(d*x+c)-1)*a^2*b^3-5/2/d*\ln(\coth(d*x+c)-1)*a*b^4-1/2/d*\ln(\coth(d*x+c)-1)*b^5-1/2/d*\coth(d*x+c)^2*b^5-1/4/d*b^5*\coth(d*x+c)^4-5/3/d*\coth(d*x+c)^3*a*b^4-5/d*a*b^4*\coth(d*x+c)-10/d*a^3*b^2*\coth(d*x+c)$

Maxima [B] time = 1.08076, size = 470, normalized size = 3.31

$$\frac{5}{3} ab^4 \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right) + b^5 \left(x + \frac{c}{d} + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} \right) + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="maxima")

[Out] $5/3*a*b^4*(3*x + 3*c/d - 4*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} - 2)/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1))) + b^5*(x + c/d + \log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d + 4*(e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})/(d*(4*e^{(-2*d*x - 2*c)} - 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} - e^{(-8*d*x - 8*c)} - 1))) + 10*a^2*b^3*(x + c/d + \log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d + 2*e^{(-2*d*x - 2*c)})/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1))) + 10*a^3*b^2*(x + c/d + 2/(d*(e^{(-2*d*x - 2*c)} - 1))) + a^5*x + 5*a^4*b*\log(\sinh(d*x + c))/d$

Fricas [B] time = 2.76279, size = 6413, normalized size = 45.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="fricas")

[Out] $\frac{1}{3} \cdot (3 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx \cdot \cosh(dx + c)^8 + 24 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx \cdot \cosh(dx + c) \cdot \sinh(dx + c)^7 + 3 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx \cdot \sinh(dx + c)^8 - 12 \cdot (5a^3b^2 + 5a^2b^3 + 5ab^4 + b^5 + (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)^6 - 12 \cdot (5a^3b^2 + 5a^2b^3 + 5ab^4 + b^5 - 7 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)^2 + (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \sinh(dx + c)^6 + 24 \cdot (7 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx \cdot \cosh(dx + c)^3 - 3 \cdot (5a^3b^2 + 5a^2b^3 + 5ab^4 + b^5 + (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)) \cdot \sinh(dx + c)^5 + 60 \cdot a^3b^2 + 40 \cdot ab^4 + 6 \cdot (30a^3b^2 + 20a^2b^3 + 20ab^4 + 2b^5 + 3 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)^4 + 6 \cdot (35 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx \cdot \cosh(dx + c)^4 + 30a^3b^2 + 20a^2b^3 + 20ab^4 + 2b^5 + 3 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx - 30 \cdot (5a^3b^2 + 5a^2b^3 + 5ab^4 + b^5 + (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^4 + 24 \cdot (7 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx \cdot \cosh(dx + c)^5 - 10 \cdot (5a^3b^2 + 5a^2b^3 + 5ab^4 + b^5 + (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)^3 + (30a^3b^2 + 20a^2b^3 + 20ab^4 + 2b^5 + 3 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)) \cdot \sinh(dx + c)^3 + 3 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx - 4 \cdot (45a^3b^2 + 15a^2b^3 + 25ab^4 + 3b^5 + 3 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)^2 + 4 \cdot (21 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx \cdot \cosh(dx + c)^6 - 45a^3b^2 - 15a^2b^3 - 25ab^4 - 3b^5 - 45 \cdot (5a^3b^2 + 5a^2b^3 + 5ab^4 + b^5 + (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)^4 - 3 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx + 9 \cdot (30a^3b^2 + 20a^2b^3 + 20ab^4 + 2b^5 + 3 \cdot (a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) \cdot dx) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^2 + 3 \cdot ((5a^4b + 10a^2b^3 + b^5) \cdot \cosh(dx + c)^8 + 8 \cdot (5a^4b + 10a^2b^3 + b^5) \cdot \sinh(dx + c)^8 - 4 \cdot (5a^4b + 10a^2b^3 + b^5) \cdot \cosh(dx + c)^6 - 4 \cdot (5a^4b + 10a^2b^3 + b^5 - 7 \cdot (5a^4b + 10a^2b^3 + b^5) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^6 + 8 \cdot (7 \cdot (5a^4b + 10a^2b^3 + b^5) \cdot \cosh(dx + c)^3 - 3 \cdot (5a^4b + 10a^2b^3 + b^5) \cdot \cosh(dx + c)) \cdot \sinh(dx + c)^5 + 5a^4b + 10a^2b^3 + b^5 + 6 \cdot (5a^4b + 10a^2b^3 + b^5) \cdot \cosh(dx + c)^4 + 2 \cdot (15a^4b + 30a^2b^3 + 3b^5 + 35 \cdot (5a^4b + 10a^2b^3 + b^5) \cdot \cosh(dx + c)^4 - 30 \cdot (5a^4b + 10a^2b^3 + b^5) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^4 + 8 \cdot (7 \cdot (5$

$$\begin{aligned}
& a^4b + 10a^2b^3 + b^5) \cosh(dx + c)^5 - 10(5a^4b + 10a^2b^3 + b^5) \\
& \cosh(dx + c)^3 + 3(5a^4b + 10a^2b^3 + b^5) \cosh(dx + c) \sinh(dx + \\
& c)^3 - 4(5a^4b + 10a^2b^3 + b^5) \cosh(dx + c)^2 + 4(7(5a^4b + 10 \\
& a^2b^3 + b^5) \cosh(dx + c)^6 - 5a^4b - 10a^2b^3 - b^5 - 15(5a^4b \\
& + 10a^2b^3 + b^5) \cosh(dx + c)^4 + 9(5a^4b + 10a^2b^3 + b^5) \cosh(d \\
& x + c)^2) \sinh(dx + c)^2 + 8((5a^4b + 10a^2b^3 + b^5) \cosh(dx + c)^ \\
& 7 - 3(5a^4b + 10a^2b^3 + b^5) \cosh(dx + c)^5 + 3(5a^4b + 10a^2b^ \\
& 3 + b^5) \cosh(dx + c)^3 - (5a^4b + 10a^2b^3 + b^5) \cosh(dx + c) \sinh \\
& (dx + c)) \log(2 \sinh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 8(3(a^5 \\
& - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) dx \cosh(dx + c)^7 - \\
& 9(5a^3b^2 + 5a^2b^3 + 5ab^4 + b^5 + (a^5 - 5a^4b + 10a^3b^2 - 1 \\
& 0a^2b^3 + 5ab^4 - b^5) dx) \cosh(dx + c)^5 + 3(30a^3b^2 + 20a^2b^ \\
& 3 + 20ab^4 + 2b^5 + 3(a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 \\
& - b^5) dx) \cosh(dx + c)^3 - (45a^3b^2 + 15a^2b^3 + 25ab^4 + 3b^5 \\
& + 3(a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5) dx) \cosh(dx \\
& + c) \sinh(dx + c)) / (d \cosh(dx + c)^8 + 8d \cosh(dx + c) \sinh(dx + c)^ \\
& 7 + d \sinh(dx + c)^8 - 4d \cosh(dx + c)^6 + 4(7d \cosh(dx + c)^2 - d) s \\
& inh(dx + c)^6 + 8(7d \cosh(dx + c)^3 - 3d \cosh(dx + c)) \sinh(dx + c)^ \\
& 5 + 6d \cosh(dx + c)^4 + 2(35d \cosh(dx + c)^4 - 30d \cosh(dx + c)^2 + \\
& 3d) \sinh(dx + c)^4 + 8(7d \cosh(dx + c)^5 - 10d \cosh(dx + c)^3 + 3d \\
& \cosh(dx + c)) \sinh(dx + c)^3 - 4d \cosh(dx + c)^2 + 4(7d \cosh(dx + c) \\
& ^6 - 15d \cosh(dx + c)^4 + 9d \cosh(dx + c)^2 - d) \sinh(dx + c)^2 + 8(d \\
& \cosh(dx + c)^7 - 3d \cosh(dx + c)^5 + 3d \cosh(dx + c)^3 - d \cosh(dx + \\
& c)) \sinh(dx + c) + d)
\end{aligned}$$

Sympy [A] time = 22.9375, size = 325, normalized size = 2.29

$$\begin{cases} x(a + b \coth(c))^5 \\ a^5x + \tilde{\omega}a^4bx + \tilde{\omega}a^3b^2x + \tilde{\omega}a^2b^3x + \tilde{\omega}ab^4x + \tilde{\omega}b^5x \\ a^5x + 5a^4bx - \frac{5a^4b \log(\tanh(c+dx)+1)}{d} + \frac{5a^4b \log(\tanh(c+dx))}{d} + 10a^3b^2x - \frac{10a^3b^2}{d \tanh(c+dx)} + 10a^2b^3x - \frac{10a^2b^3 \log(\tanh(c+dx)+1)}{d} + 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))**5,x)

[Out] Piecewise((x*(a + b*coth(c))**5, Eq(d, 0)), (a**5*x + zoo*a**4*b*x + zoo*a**3*b**2*x + zoo*a**2*b**3*x + zoo*a*b**4*x + zoo*b**5*x, Eq(c, log(exp(-d*x))) | Eq(c, log(-exp(-d*x)))), (a**5*x + 5*a**4*b*x - 5*a**4*b*log(tanh(c + d*x) + 1)/d + 5*a**4*b*log(tanh(c + d*x))/d + 10*a**3*b**2*x - 10*a**3*b**2/(d*tanh(c + d*x)) + 10*a**2*b**3*x - 10*a**2*b**3*log(tanh(c + d*x) + 1)/d + 10*a**2*b**3*log(tanh(c + d*x))/d - 5*a**2*b**3/(d*tanh(c + d*x)**2) +

```
5*a*b**4*x - 5*a*b**4/(d*tanh(c + d*x)) - 5*a*b**4/(3*d*tanh(c + d*x)**3) +
b**5*x - b**5*log(tanh(c + d*x) + 1)/d + b**5*log(tanh(c + d*x))/d - b**5/
(2*d*tanh(c + d*x)**2) - b**5/(4*d*tanh(c + d*x)**4), True))
```

Giac [A] time = 1.15487, size = 308, normalized size = 2.17

$$\frac{(a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5)(dx + c)}{d} + \frac{(5a^4b + 10a^2b^3 + b^5)\log(|e^{(2dx+2c)} - 1|)}{d} + \frac{4(15a^3b^2 + 10ab^4 - b^5)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="giac")
```

```
[Out] (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(d*x + c)/d + (5*
a^4*b + 10*a^2*b^3 + b^5)*log(abs(e^(2*d*x + 2*c) - 1))/d + 4/3*(15*a^3*b^2
+ 10*a*b^4 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5)*e^(6*d*x + 6*c) + 3
*(15*a^3*b^2 + 10*a^2*b^3 + 10*a*b^4 + b^5)*e^(4*d*x + 4*c) - (45*a^3*b^2 +
15*a^2*b^3 + 25*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/(d*(e^(2*d*x + 2*c) - 1)^4
)
```

3.78 $\int (a + b \coth(c + dx))^4 dx$

Optimal. Leaf size=101

$$-\frac{b^2(3a^2 + b^2) \coth(c + dx)}{d} + \frac{4ab(a^2 + b^2) \log(\sinh(c + dx))}{d} + x(6a^2b^2 + a^4 + b^4) - \frac{b(a + b \coth(c + dx))^3}{3d} - \frac{ab(a + b \coth(c + dx))^2}{2d}$$

[Out] $(a^4 + 6a^2b^2 + b^4)x - (b^2(3a^2 + b^2)\text{Coth}[c + d*x])/d - (a*b*(a + b*\text{Coth}[c + d*x])^2)/d - (b*(a + b*\text{Coth}[c + d*x])^3)/(3*d) + (4*a*b*(a^2 + b^2)*\text{Log}[\text{Sinh}[c + d*x]])/d$

Rubi [A] time = 0.122539, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3482, 3528, 3525, 3475}

$$-\frac{b^2(3a^2 + b^2) \coth(c + dx)}{d} + \frac{4ab(a^2 + b^2) \log(\sinh(c + dx))}{d} + x(6a^2b^2 + a^4 + b^4) - \frac{b(a + b \coth(c + dx))^3}{3d} - \frac{ab(a + b \coth(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Coth[c + d*x])^4, x]

[Out] $(a^4 + 6a^2b^2 + b^4)x - (b^2(3a^2 + b^2)\text{Coth}[c + d*x])/d - (a*b*(a + b*\text{Coth}[c + d*x])^2)/d - (b*(a + b*\text{Coth}[c + d*x])^3)/(3*d) + (4*a*b*(a^2 + b^2)*\text{Log}[\text{Sinh}[c + d*x]])/d$

Rule 3482

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[n, 1]

Rule 3528

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3525

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*
(x_)]), x_Symbol] := Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (a + b \coth(c + dx))^4 dx &= -\frac{b(a + b \coth(c + dx))^3}{3d} + \int (a + b \coth(c + dx))^2 (a^2 + b^2 + 2ab \coth(c + dx)) dx \\ &= -\frac{ab(a + b \coth(c + dx))^2}{d} - \frac{b(a + b \coth(c + dx))^3}{3d} + \int (a + b \coth(c + dx)) (a(a^2 + 3b^2) + \\ &= (a^4 + 6a^2b^2 + b^4)x - \frac{b^2(3a^2 + b^2) \coth(c + dx)}{d} - \frac{ab(a + b \coth(c + dx))^2}{d} - \frac{b(a + b \coth(c + dx))^3}{3d} \\ &= (a^4 + 6a^2b^2 + b^4)x - \frac{b^2(3a^2 + b^2) \coth(c + dx)}{d} - \frac{ab(a + b \coth(c + dx))^2}{d} - \frac{b(a + b \coth(c + dx))^3}{3d} \end{aligned}$$

Mathematica [A] time = 0.905219, size = 109, normalized size = 1.08

$$\frac{6b^2(6a^2 + b^2) \coth(c + dx) - 24ab(a^2 + b^2) \log(\tanh(c + dx)) + 12ab^3 \coth^2(c + dx) - 3(a - b)^4 \log(\tanh(c + dx)) + 1}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^4, x]
```

```
[Out] -(6*b^2*(6*a^2 + b^2)*Coth[c + d*x] + 12*a*b^3*Coth[c + d*x]^2 + 2*b^4*Coth
[c + d*x]^3 + 3*(a + b)^4*Log[1 - Tanh[c + d*x]] - 24*a*b*(a^2 + b^2)*Log[T
anh[c + d*x]] - 3*(a - b)^4*Log[1 + Tanh[c + d*x]])/(6*d)
```

Maple [B] time = 0.004, size = 246, normalized size = 2.4

$$-\frac{b^4(\coth(dx + c))^3}{3d} - 2\frac{(\coth(dx + c))^2 ab^3}{d} - 6\frac{a^2 \coth(dx + c) b^2}{d} - \frac{\coth(dx + c) b^4}{d} - \frac{\ln(\coth(dx + c) - 1) a^4}{2d} - 2\frac{\ln(\coth(dx + c) + 1) a^4}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*coth(d*x+c))^4,x)`

[Out]
$$-1/3/d*b^4*coth(d*x+c)^3-2/d*coth(d*x+c)^2*a*b^3-6/d*coth(d*x+c)*a^2*b^2-1/d*coth(d*x+c)*b^4-1/2/d*\ln(coth(d*x+c)-1)*a^4-2/d*\ln(coth(d*x+c)-1)*a^3*b-3/d*\ln(coth(d*x+c)-1)*a^2*b^2-2/d*\ln(coth(d*x+c)-1)*a*b^3-1/2/d*\ln(coth(d*x+c)-1)*b^4+1/2/d*\ln(coth(d*x+c)+1)*a^4-2/d*\ln(coth(d*x+c)+1)*a^3*b+3/d*\ln(coth(d*x+c)+1)*a^2*b^2-2/d*\ln(coth(d*x+c)+1)*a*b^3+1/2/d*\ln(coth(d*x+c)+1)*b^4$$

Maxima [B] time = 1.11372, size = 296, normalized size = 2.93

$$\frac{1}{3} b^4 \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right) + 4ab^3 \left(x + \frac{c}{d} + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*coth(d*x+c))^4,x, algorithm="maxima")`

[Out]
$$\frac{1}{3} b^4 (3x + 3c/d - 4(3e^{(-2d*x - 2*c)} - 3e^{(-4d*x - 4*c)} - 2)/(d*(3e^{(-2d*x - 2*c)} - 3e^{(-4d*x - 4*c)} + e^{(-6d*x - 6*c)} - 1))) + 4*a*b^3 * (x + c/d + \log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d + 2*e^{(-2d*x - 2*c)}/(d*(2*e^{(-2d*x - 2*c)} - e^{(-4d*x - 4*c)} - 1))) + 6*a^2*b^2*(x + c/d + 2/(d*(e^{(-2d*x - 2*c)} - 1))) + a^4*x + 4*a^3*b*\log(\sinh(d*x + c))/d$$

Fricas [B] time = 2.53483, size = 3318, normalized size = 32.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*coth(d*x+c))^4,x, algorithm="fricas")`

[Out]
$$1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*\cosh(d*x + c)^6 + 18*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^5 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*\sinh(d*x + c)^6 - 3*(12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*\cosh(d*x + c)^4 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4$$

```

)*d*x*cosh(d*x + c)^2 - 12*a^2*b^2 - 8*a*b^3 - 4*b^4 - 3*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*sinh(d*x + c)^4 - 36*a^2*b^2 - 8*b^4 + 12*(5
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 - (12*a^2*
b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x + 3*(24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x*cosh(d*x + c)^4 + 24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x - 6*(12*a^2*b^2 + 8*a*b^3 + 4*b^4
+ 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 12*((a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(a^3*b + a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^3*b + a*b^3)*sinh(d*x + c)^6 - 3*(a^3*b + a*b^
3)*cosh(d*x + c)^4 - 3*(a^3*b + a*b^3 - 5*(a^3*b + a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 - a^3*b - a*b^3 + 4*(5*(a^3*b + a*b^3)*cosh(d*x + c)^3 - 3*
(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3*b + a*b^3)*cosh(d*x
+ c)^2 + 3*(5*(a^3*b + a*b^3)*cosh(d*x + c)^4 + a^3*b + a*b^3 - 6*(a^3*b +
a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((a^3*b + a*b^3)*cosh(d*x + c)
^5 - 2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*cosh(d*x + c))*sin
h(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^5 - 2*(12*a^2*b^
2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*co
sh(d*x + c)^3 + (24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^
2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4
+ 3*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x +
c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2
*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)

```

Sympy [A] time = 10.181, size = 233, normalized size = 2.31

$$\left\{ \begin{array}{l} x(a + b \coth(c))^4 \\ a^4x + \infty a^3bx + \infty a^2b^2x + \infty ab^3x + \infty b^4x \\ a^4x + 4a^3bx - \frac{4a^3b \log(\tanh(c+dx)+1)}{d} + \frac{4a^3b \log(\tanh(c-dx))}{d} + 6a^2b^2x - \frac{6a^2b^2}{d \tanh(c+dx)} + 4ab^3x - \frac{4ab^3 \log(\tanh(c+dx)+1)}{d} + \frac{4ab^3 \log(\tanh(c-dx))}{d} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))**4,x)

[Out] Piecewise((x*(a + b*coth(c))**4, Eq(d, 0)), (a**4*x + zoo*a**3*b*x + zoo*a*
*2*b**2*x + zoo*a*b**3*x + zoo*b**4*x, Eq(c, log(exp(-d*x))) | Eq(c, log(-e


```
xp(-d*x))), (a**4*x + 4*a**3*b*x - 4*a**3*b*log(tanh(c + d*x) + 1)/d + 4*a
**3*b*log(tanh(c + d*x))/d + 6*a**2*b**2*x - 6*a**2*b**2/(d*tanh(c + d*x))
+ 4*a*b**3*x - 4*a*b**3*log(tanh(c + d*x) + 1)/d + 4*a*b**3*log(tanh(c + d*
x))/d - 2*a*b**3/(d*tanh(c + d*x)**2) + b**4*x - b**4/(d*tanh(c + d*x)) - b
**4/(3*d*tanh(c + d*x)**3), True))
```

Giac [A] time = 1.13745, size = 211, normalized size = 2.09

$$\frac{(a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4)(dx + c)}{d} + \frac{4(a^3b + ab^3) \log(|e^{(2dx+2c)} - 1|)}{d} - \frac{4(9a^2b^2 + 2b^4 + 3(3a^2b^2 + 2ab^3 + b^4))}{3d(e^{2dx+2c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(d*x + c)/d + 4*(a^3*b + a*b^3)
*log(abs(e^(2*d*x + 2*c) - 1))/d - 4/3*(9*a^2*b^2 + 2*b^4 + 3*(3*a^2*b^2 +
2*a*b^3 + b^4)*e^(4*d*x + 4*c) - 3*(6*a^2*b^2 + 2*a*b^3 + b^4)*e^(2*d*x + 2
*c))/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.79 $\int (a + b \coth(c + dx))^3 dx$

Optimal. Leaf size=69

$$\frac{b(3a^2 + b^2) \log(\sinh(c + dx))}{d} + ax(a^2 + 3b^2) - \frac{2ab^2 \coth(c + dx)}{d} - \frac{b(a + b \coth(c + dx))^2}{2d}$$

[Out] $a*(a^2 + 3*b^2)*x - (2*a*b^2*Coth[c + d*x])/d - (b*(a + b*Coth[c + d*x])^2)/(2*d) + (b*(3*a^2 + b^2)*Log[Sinh[c + d*x]])/d$

Rubi [A] time = 0.0638579, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3482, 3525, 3475}

$$\frac{b(3a^2 + b^2) \log(\sinh(c + dx))}{d} + ax(a^2 + 3b^2) - \frac{2ab^2 \coth(c + dx)}{d} - \frac{b(a + b \coth(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Coth}[c + d*x])^3, x]$

[Out] $a*(a^2 + 3*b^2)*x - (2*a*b^2*Coth[c + d*x])/d - (b*(a + b*Coth[c + d*x])^2)/(2*d) + (b*(3*a^2 + b^2)*Log[Sinh[c + d*x]])/d$

Rule 3482

$\text{Int}[(a + b*\tan[(c + d*x)])^n, x] \rightarrow \text{Simp}[(b*(a + b*\tan[c + d*x])^{n-1})/(d*(n-1)), x] + \text{Int}[(a^2 - b^2 + 2*a*b*\tan[c + d*x])*(a + b*\tan[c + d*x])^{n-2}, x] /;$ $\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{GtQ}[n, 1]$

Rule 3525

$\text{Int}[(a + b*\tan[(e + f*x)]*(c + d*\tan[(e + f*x)]))^n, x] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\tan[e + f*x], x], x] + \text{Simp}[(b*d*\tan[e + f*x])/f, x]) /;$ $\text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[b*c + a*d, 0]$

Rule 3475

$\text{Int}[\tan[(c + d*x)], x] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]], x] /;$ $\text{FreeQ}\{c, d, x\}$

Rubi steps

$$\begin{aligned}
\int (a + b \coth(c + dx))^3 dx &= -\frac{b(a + b \coth(c + dx))^2}{2d} + \int (a + b \coth(c + dx)) (a^2 + b^2 + 2ab \coth(c + dx)) dx \\
&= a(a^2 + 3b^2)x - \frac{2ab^2 \coth(c + dx)}{d} - \frac{b(a + b \coth(c + dx))^2}{2d} + (b(3a^2 + b^2)) \int \coth(c + dx) dx \\
&= a(a^2 + 3b^2)x - \frac{2ab^2 \coth(c + dx)}{d} - \frac{b(a + b \coth(c + dx))^2}{2d} + \frac{b(3a^2 + b^2) \log(\sinh(c + dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.402468, size = 86, normalized size = 1.25

$$\frac{-2b(3a^2 + b^2) \log(\tanh(c + dx)) + 6ab^2 \coth(c + dx) + (a - b)^3(-\log(\tanh(c + dx) + 1)) + (a + b)^3 \log(1 - \tanh(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Coth[c + d*x])^3,x]

[Out] -(6*a*b^2*Coth[c + d*x] + b^3*Coth[c + d*x]^2 + (a + b)^3*Log[1 - Tanh[c + d*x]] - 2*b*(3*a^2 + b^2)*Log[Tanh[c + d*x]] - (a - b)^3*Log[1 + Tanh[c + d*x]])/(2*d)

Maple [B] time = 0.004, size = 173, normalized size = 2.5

$$-\frac{b^3 (\coth(dx + c))^2}{2d} - 3 \frac{ab^2 \coth(dx + c)}{d} - \frac{\ln(\coth(dx + c) - 1) a^3}{2d} - \frac{3 \ln(\coth(dx + c) - 1) a^2 b}{2d} - \frac{3 \ln(\coth(dx + c) - 1) a b^2}{2d} - \frac{3 \ln(\coth(dx + c) - 1) b^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*coth(d*x+c))^3,x)

[Out] -1/2/d*b^3*coth(d*x+c)^2-3*a*b^2*coth(d*x+c)/d-1/2/d*ln(coth(d*x+c)-1)*a^3-3/2/d*ln(coth(d*x+c)-1)*a^2*b-3/2/d*ln(coth(d*x+c)-1)*a*b^2-1/2/d*ln(coth(d*x+c)-1)*b^3+1/2/d*ln(coth(d*x+c)+1)*a^3-3/2/d*ln(coth(d*x+c)+1)*a^2*b+3/2/d*ln(coth(d*x+c)+1)*a*b^2-1/2/d*ln(coth(d*x+c)+1)*b^3

Maxima [B] time = 1.11061, size = 184, normalized size = 2.67

$$b^3 \left(x + \frac{c}{d} + \frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} + \frac{2e^{-2dx-2c}}{d(2e^{-2dx-2c} - e^{-4dx-4c} - 1)} \right) + 3ab^2 \left(x + \frac{c}{d} + \frac{2}{d(e^{-2dx-2c} - 1)} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="maxima")

[Out] b^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 3*a*b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^3*x + 3*a^2*b*log(sinh(d*x + c))/d

Fricas [B] time = 2.64428, size = 1563, normalized size = 22.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="fricas")

[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*sinh(d*x + c)^4 + 6*a*b^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x - 2*(3*a*b^2 + b^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 + 2*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^2 - 3*a*b^2 - b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((3*a^2*b + b^3)*cosh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3 - 2*(3*a^2*b + b^3)*cosh(d*x + c)^2 - 2*(3*a^2*b + b^3 - 3*(3*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^2*b + b^3)*cosh(d*x + c)^3 - (3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^3 - (3*a*b^2 + b^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)

Sympy [A] time = 4.43112, size = 175, normalized size = 2.54

$$\left\{ \begin{array}{l} x(a + b \coth(c))^3 \\ a^3x + \tilde{\infty}a^2bx + \tilde{\infty}ab^2x + \tilde{\infty}b^3x \\ a^3x + 3a^2bx - \frac{3a^2b \log(\tanh(c+dx)+1)}{d} + \frac{3a^2b \log(\tanh(c-dx))}{d} + 3ab^2x - \frac{3ab^2}{d \tanh(c+dx)} + b^3x - \frac{b^3 \log(\tanh(c+dx)+1)}{d} + \frac{b^3 \log(\tanh(c-dx))}{d} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))**3,x)

[Out] Piecewise((x*(a + b*coth(c))**3, Eq(d, 0)), (a**3*x + zoo*a**2*b*x + zoo*a*b**2*x + zoo*b**3*x, Eq(c, log(exp(-d*x))) | Eq(c, log(-exp(-d*x)))), (a**3*x + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d + 3*a**2*b*log(tanh(c + d*x))/d + 3*a*b**2*x - 3*a*b**2/(d*tanh(c + d*x)) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d + b**3*log(tanh(c + d*x))/d - b**3/(2*d*tanh(c + d*x)**2), True))

Giac [A] time = 1.14267, size = 140, normalized size = 2.03

$$\frac{(a^3 - 3a^2b + 3ab^2 - b^3)(dx + c)}{d} + \frac{(3a^2b + b^3) \log(|e^{(2dx+2c)} - 1|)}{d} + \frac{2(3ab^2 - (3ab^2 + b^3)e^{(2dx+2c)})}{d(e^{(2dx+2c)} - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="giac")

[Out] (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(d*x + c)/d + (3*a^2*b + b^3)*log(abs(e^(2*d*x + 2*c) - 1))/d + 2*(3*a*b^2 - (3*a*b^2 + b^3)*e^(2*d*x + 2*c))/(d*(e^(2*d*x + 2*c) - 1)^2)

3.80 $\int (a + b \coth(c + dx))^2 dx$

Optimal. Leaf size=38

$$x(a^2 + b^2) + \frac{2ab \log(\sinh(c + dx))}{d} - \frac{b^2 \coth(c + dx)}{d}$$

[Out] $(a^2 + b^2)*x - (b^2*\text{Coth}[c + d*x])/d + (2*a*b*\text{Log}[\text{Sinh}[c + d*x]])/d$

Rubi [A] time = 0.024292, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3477, 3475}

$$x(a^2 + b^2) + \frac{2ab \log(\sinh(c + dx))}{d} - \frac{b^2 \coth(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Coth}[c + d*x])^2, x]$

[Out] $(a^2 + b^2)*x - (b^2*\text{Coth}[c + d*x])/d + (2*a*b*\text{Log}[\text{Sinh}[c + d*x]])/d$

Rule 3477

$\text{Int}[(a + b*\text{tan}[(c + d*x)])^2, x_Symbol] \rightarrow \text{Simp}[(a^2 - b^2)*x, x] + (\text{Dist}[2*a*b, \text{Int}[\text{Tan}[c + d*x], x], x] + \text{Simp}[(b^2*\text{Tan}[c + d*x])/d, x]) /;$ $\text{FreeQ}\{a, b, c, d, x\}$

Rule 3475

$\text{Int}[\text{tan}[(c + d*x)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /;$ $\text{FreeQ}\{c, d, x\}$

Rubi steps

$$\begin{aligned} \int (a + b \coth(c + dx))^2 dx &= (a^2 + b^2)x - \frac{b^2 \coth(c + dx)}{d} + (2ab) \int \coth(c + dx) dx \\ &= (a^2 + b^2)x - \frac{b^2 \coth(c + dx)}{d} + \frac{2ab \log(\sinh(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.127926, size = 65, normalized size = 1.71

$$\frac{(a-b)^2 \log(\tanh(c+dx)+1) - (a+b)^2 \log(1-\tanh(c+dx)) + 4ab \log(\tanh(c+dx)) - 2b^2 \coth(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Coth[c + d*x])^2,x]

[Out] (-2*b^2*Coth[c + d*x] - (a + b)^2*Log[1 - Tanh[c + d*x]] + 4*a*b*Log[Tanh[c + d*x]] + (a - b)^2*Log[1 + Tanh[c + d*x]])/(2*d)

Maple [B] time = 0.005, size = 116, normalized size = 3.1

$$-\frac{b^2 \coth(dx+c)}{d} - \frac{\ln(\coth(dx+c)-1)a^2}{2d} - \frac{\ln(\coth(dx+c)-1)ab}{d} - \frac{\ln(\coth(dx+c)-1)b^2}{2d} + \frac{\ln(\coth(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*coth(d*x+c))^2,x)

[Out] -b^2*coth(d*x+c)/d-1/2/d*ln(coth(d*x+c)-1)*a^2-1/d*ln(coth(d*x+c)-1)*a*b-1/2/d*ln(coth(d*x+c)-1)*b^2+1/2/d*ln(coth(d*x+c)+1)*a^2-1/d*ln(coth(d*x+c)+1)*a*b+1/2/d*ln(coth(d*x+c)+1)*b^2

Maxima [A] time = 1.01268, size = 66, normalized size = 1.74

$$b^2 \left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)} \right) + a^2 x + \frac{2ab \log(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="maxima")

[Out] b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^2*x + 2*a*b*log(sinh(d*x + c))/d

Fricas [B] time = 2.85372, size = 535, normalized size = 14.08

$$\frac{(a^2 - 2ab + b^2)dx \cosh(dx + c)^2 + 2(a^2 - 2ab + b^2)dx \cosh(dx + c) \sinh(dx + c) + (a^2 - 2ab + b^2)dx \sinh(dx + c)^2 - d \cosh(dx + c)^2 + 2d \cosh(dx + c) \sinh(dx + c) - d \sinh(dx + c)^2}{d \cosh(dx + c)^2 + 2d \cosh(dx + c) \sinh(dx + c) + d \sinh(dx + c)^2 - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="fricas")

[Out] ((a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^2 - 2*a*b + b^2)*d*x*sinh(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d*x - 2*b^2 + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)

Sympy [A] time = 1.86648, size = 104, normalized size = 2.74

$$\begin{cases} x(a + b \coth(c))^2 & \text{for } d = 0 \\ a^2x + \infty abx + \infty b^2x & \text{for } c = \log(-e^{-dx}) \vee c = \log(e^{-dx}) \\ a^2x + 2abx - \frac{2ab \log(\tanh(c+dx)+1)}{d} + \frac{2ab \log(\tanh(c+dx))}{d} + b^2x - \frac{b^2}{d \tanh(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))**2,x)

[Out] Piecewise((x*(a + b*coth(c))**2, Eq(d, 0)), (a**2*x + zoo*a*b*x + zoo*b**2*x, Eq(c, log(exp(-d*x))) | Eq(c, log(-exp(-d*x)))), (a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d + 2*a*b*log(tanh(c + d*x))/d + b**2*x - b**2/(d*tanh(c + d*x)), True))

Giac [A] time = 1.12428, size = 84, normalized size = 2.21

$$\frac{2ab \log(|e^{(2dx+2c)} - 1|)}{d} + \frac{(a^2 - 2ab + b^2)(dx + c)}{d} - \frac{2b^2}{d(e^{(2dx+2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*a*b*log(abs(e^(2*d*x + 2*c) - 1))/d + (a^2 - 2*a*b + b^2)*(d*x + c)/d - 2  
*b^2/(d*(e^(2*d*x + 2*c) - 1))
```

$$3.81 \quad \int \frac{1}{a+b \coth(c+dx)} dx$$

Optimal. Leaf size=50

$$\frac{ax}{a^2 - b^2} - \frac{b \log(a \sinh(c + dx) + b \cosh(c + dx))}{d(a^2 - b^2)}$$

[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)*d)

Rubi [A] time = 0.0539133, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3484, 3530}

$$\frac{ax}{a^2 - b^2} - \frac{b \log(a \sinh(c + dx) + b \cosh(c + dx))}{d(a^2 - b^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Coth[c + d*x])^(-1), x]

[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)*d)

Rule 3484

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3530

Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rubi steps

$$\int \frac{1}{a + b \coth(c + dx)} dx = \frac{ax}{a^2 - b^2} - \frac{(ib) \int \frac{-ib - ia \coth(c + dx)}{a + b \coth(c + dx)} dx}{a^2 - b^2}$$

$$= \frac{ax}{a^2 - b^2} - \frac{b \log(b \cosh(c + dx) + a \sinh(c + dx))}{(a^2 - b^2)d}$$

Mathematica [A] time = 0.0792178, size = 64, normalized size = 1.28

$$\frac{(b - a) \log(1 - \coth(c + dx)) + (a + b) \log(\coth(c + dx) + 1) - 2b \log(a + b \coth(c + dx))}{2d(a - b)(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Coth[c + d*x])^(-1), x]

[Out] ((-a + b)*Log[1 - Coth[c + d*x]] + (a + b)*Log[1 + Coth[c + d*x]] - 2*b*Log[a + b*Coth[c + d*x]])/(2*(a - b)*(a + b)*d)

Maple [A] time = 0.019, size = 76, normalized size = 1.5

$$\frac{\ln(\coth(dx + c) + 1)}{d(2a - 2b)} - \frac{\ln(\coth(dx + c) - 1)}{d(2b + 2a)} - \frac{b \ln(a + b \coth(dx + c))}{d(a - b)(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*coth(d*x+c)), x)

[Out] 1/d/(2*a-2*b)*ln(coth(d*x+c)+1)-1/d/(2*b+2*a)*ln(coth(d*x+c)-1)-1/d*b/(a-b)/(a+b)*ln(a+b*coth(d*x+c))

Maxima [A] time = 1.12232, size = 70, normalized size = 1.4

$$-\frac{b \log(-(a - b)e^{(-2dx - 2c)} + a + b)}{(a^2 - b^2)d} + \frac{dx + c}{(a + b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="maxima")

[Out] $-b \log(-(a - b) * e^{(-2*d*x - 2*c)} + a + b) / ((a^2 - b^2) * d) + (d*x + c) / ((a + b) * d)$

Fricas [A] time = 2.88622, size = 149, normalized size = 2.98

$$\frac{(a + b)dx - b \log\left(\frac{2(b \cosh(dx+c) + a \sinh(dx+c))}{\cosh(dx+c) - \sinh(dx+c)}\right)}{(a^2 - b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="fricas")

[Out] $((a + b) * d * x - b * \log(2 * (b * \cosh(d * x + c) + a * \sinh(d * x + c)) / (\cosh(d * x + c) - \sinh(d * x + c)))) / ((a^2 - b^2) * d)$

Sympy [A] time = 3.14948, size = 236, normalized size = 4.72

$$\left\{ \begin{array}{ll} \frac{\infty x}{\coth(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ -\frac{\frac{\coth(c)}{dx \tanh(c+dx)}}{2bd \tanh(c+dx) - 2bd} + \frac{dx}{2bd \tanh(c+dx) - 2bd} - \frac{1}{2bd \tanh(c+dx) - 2bd} & \text{for } a = -b \\ \frac{\frac{\coth(c)}{dx \tanh(c+dx)}}{2bd \tanh(c+dx) + 2bd} + \frac{dx}{2bd \tanh(c+dx) + 2bd} + \frac{1}{2bd \tanh(c+dx) + 2bd} & \text{for } a = b \\ \frac{x}{2bd \tanh(c+dx) + 2bd} & \text{for } d = 0 \\ \frac{\frac{a+b \coth(c)}{x - \frac{\log(\tanh(c+dx)+1)}{d}}}{b} & \text{for } a = 0 \\ \frac{\frac{adx}{a^2d-b^2d} - \frac{bdx}{a^2d-b^2d} + \frac{b \log(\tanh(c+dx)+1)}{a^2d-b^2d} - \frac{b \log\left(\tanh(c+dx) + \frac{b}{a}\right)}{a^2d-b^2d}}{b} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c)),x)

[Out] Piecewise((zoo*x/coth(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*x*tanh(c + d*x)/(2*b*d*tanh(c + d*x) - 2*b*d) + d*x/(2*b*d*tanh(c + d*x) - 2*b*d) - 1/(2*b*d*tanh(c + d*x) - 2*b*d), Eq(a, -b)), (d*x*tanh(c + d*x)/(2*b*d*tanh(c + d*x) + 2*b*d) + d*x/(2*b*d*tanh(c + d*x) + 2*b*d) + 1/(2*b*d*tanh(c + d*x) + 2*b*d), Eq(a, b)), (x/(a + b*coth(c)), Eq(d, 0)), ((x - log(tanh(c + d*x) + 1/d)) / b, True))

```
x) + 1)/d)/b, Eq(a, 0)), (a*d*x/(a**2*d - b**2*d) - b*d*x/(a**2*d - b**2*d)
+ b*log(tanh(c + d*x) + 1)/(a**2*d - b**2*d) - b*log(tanh(c + d*x) + b/a)/
(a**2*d - b**2*d), True))
```

Giac [A] time = 1.14268, size = 85, normalized size = 1.7

$$-\frac{b \log(|ae^{(2dx+2c)} + be^{(2dx+2c)} - a + b|)}{a^2d - b^2d} + \frac{dx + c}{ad - bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^2*d - b^2*d)
+ (d*x + c)/(a*d - b*d)
```

$$3.82 \quad \int \frac{1}{(a+b \coth(c+dx))^2} dx$$

Optimal. Leaf size=85

$$\frac{b}{d(a^2 - b^2)(a + b \coth(c + dx))} - \frac{2ab \log(a \sinh(c + dx) + b \cosh(c + dx))}{d(a^2 - b^2)^2} + \frac{x(a^2 + b^2)}{(a^2 - b^2)^2}$$

[Out] $((a^2 + b^2)*x)/(a^2 - b^2)^2 + b/((a^2 - b^2)*d*(a + b*\text{Coth}[c + d*x])) - (2*a*b*\text{Log}[b*\text{Cosh}[c + d*x] + a*\text{Sinh}[c + d*x]])/((a^2 - b^2)^2*d)$

Rubi [A] time = 0.0946267, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3483, 3531, 3530}

$$\frac{b}{d(a^2 - b^2)(a + b \coth(c + dx))} - \frac{2ab \log(a \sinh(c + dx) + b \cosh(c + dx))}{d(a^2 - b^2)^2} + \frac{x(a^2 + b^2)}{(a^2 - b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Coth[c + d*x])^(-2), x]

[Out] $((a^2 + b^2)*x)/(a^2 - b^2)^2 + b/((a^2 - b^2)*d*(a + b*\text{Coth}[c + d*x])) - (2*a*b*\text{Log}[b*\text{Cosh}[c + d*x] + a*\text{Sinh}[c + d*x]])/((a^2 - b^2)^2*d)$

Rule 3483

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a + b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3531

Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a*c + b*d, 0]

Rule 3530

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \coth(c + dx))^2} dx &= \frac{b}{(a^2 - b^2) d(a + b \coth(c + dx))} + \frac{\int \frac{a-b \coth(c+dx)}{a+b \coth(c+dx)} dx}{a^2 - b^2} \\ &= \frac{(a^2 + b^2)x}{(a^2 - b^2)^2} + \frac{b}{(a^2 - b^2) d(a + b \coth(c + dx))} - \frac{(2iab) \int \frac{-ib-ia \coth(c+dx)}{a+b \coth(c+dx)} dx}{(a^2 - b^2)^2} \\ &= \frac{(a^2 + b^2)x}{(a^2 - b^2)^2} + \frac{b}{(a^2 - b^2) d(a + b \coth(c + dx))} - \frac{2ab \log(b \cosh(c + dx) + a \sinh(c + dx))}{(a^2 - b^2)^2 d} \end{aligned}$$

Mathematica [A] time = 1.4201, size = 100, normalized size = 1.18

$$\frac{2b \left(\frac{b^3 - a^2 b}{a \tanh(c+dx) + b} - 2a^2 \log(a \tanh(c+dx) + b) \right)}{a(a^2 - b^2)^2} - \frac{\log(1 - \tanh(c+dx))}{(a+b)^2} + \frac{\log(\tanh(c+dx) + 1)}{(a-b)^2}$$

$$2d$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-2), x]
```

```
[Out] (-(Log[1 - Tanh[c + d*x]]/(a + b)^2) + Log[1 + Tanh[c + d*x]]/(a - b)^2 + (
2*b*(-2*a^2*Log[b + a*Tanh[c + d*x]] + (-a^2*b) + b^3)/(b + a*Tanh[c + d*x
])))/(a*(a^2 - b^2)^2)/(2*d)
```

Maple [A] time = 0.025, size = 101, normalized size = 1.2

$$\frac{\ln(\coth(dx + c) + 1)}{2d(a - b)^2} - \frac{\ln(\coth(dx + c) - 1)}{2d(a + b)^2} + \frac{b}{d(a - b)(a + b)(a + b \coth(dx + c))} - 2 \frac{ab \ln(a + b \coth(dx + c))}{d(a + b)^2(a - b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*coth(d*x+c))^2,x)

[Out] 1/2/d/(a-b)^2*ln(coth(d*x+c)+1)-1/2/d/(a+b)^2*ln(coth(d*x+c)-1)+1/d*b/(a-b)/(a+b)/(a+b*coth(d*x+c))-2/d*a*b/(a+b)^2/(a-b)^2*ln(a+b*coth(d*x+c))

Maxima [A] time = 1.16493, size = 167, normalized size = 1.96

$$\frac{2ab \log\left(-\frac{(a-b)e^{-2dx-2c} + a + b}{(a^4 - 2a^2b^2 + b^4)d}\right) - \frac{2b^2}{(a^4 - 2a^2b^2 + b^4 - (a^4 - 2a^3b + 2ab^3 - b^4)e^{-2dx-2c})d} + \frac{dx + c}{(a^2 + 2ab + b^2)d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="maxima")

[Out] -2*a*b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^4 - 2*a^2*b^2 + b^4)*d) - 2*b^2/((a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*d*x - 2*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)

Fricas [B] time = 2.67566, size = 980, normalized size = 11.53

$$\frac{(a^3 + 3a^2b + 3ab^2 + b^3)dx \cosh(dx + c)^2 + 2(a^3 + 3a^2b + 3ab^2 + b^3)dx \cosh(dx + c) \sinh(dx + c) + (a^3 + 3a^2b + 3ab^2 + b^3)dx \sinh(dx + c)^2}{(a^5 + a^4b - 2a^3b^2 - 2a^2b^3 + ab^4 + b^5)d \cosh(dx + c)^2 + 2(a^5 + a^4b - 2a^3b^2 - 2a^2b^3 + ab^4 + b^5)d \cosh(dx + c) \sinh(dx + c) + (a^5 + a^4b - 2a^3b^2 - 2a^2b^3 + ab^4 + b^5)d \sinh(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="fricas")

[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^2 - 2*a*b^2 + 2*b^3 - (a^3 + a^2*b - a*b^2 - b^3)*d*x + 2*(a^2*b - a*b^2 - (a^2*b + a*b^2)*cosh(d*x + c)^2 - 2*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) - (a^2*b + a*b^2)*sinh(d*x + c)^2)*log(2*(b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))))/((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*sinh(d*x + c)^2 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*d)

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))**2,x)

[Out] Exception raised: TypeError

Giac [A] time = 1.1549, size = 185, normalized size = 2.18

$$-\frac{2ab \log\left(\left|ae^{(2dx+2c)} + be^{(2dx+2c)} - a + b\right|\right)}{a^4d - 2a^2b^2d + b^4d} + \frac{dx + c}{a^2d - 2abd + b^2d} - \frac{2(ab^2 - b^3)}{\left(ae^{(2dx+2c)} + be^{(2dx+2c)} - a + b\right)(a+b)^2(a-b)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="giac")

[Out] $-2*a*b*\log(\text{abs}(a*e^{(2*d*x + 2*c)} + b*e^{(2*d*x + 2*c)} - a + b))/(a^{4*d} - 2*a^{2*d}*b^{2*d} + b^{4*d}) + (d*x + c)/(a^{2*d} - 2*a*b*d + b^{2*d}) - 2*(a*b^2 - b^3)/((a*e^{(2*d*x + 2*c)} + b*e^{(2*d*x + 2*c)} - a + b)*(a + b)^2*(a - b)^{2*d})$

$$3.83 \quad \int \frac{1}{(a+b \coth(c+dx))^3} dx$$

Optimal. Leaf size=129

$$\frac{2ab}{d(a^2 - b^2)^2 (a + b \coth(c + dx))} + \frac{b}{2d(a^2 - b^2)(a + b \coth(c + dx))^2} - \frac{b(3a^2 + b^2) \log(a \sinh(c + dx) + b \cosh(c + dx))}{d(a^2 - b^2)^3}$$

[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*d*(a + b*Coth[c + d*x])^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])) - (b*(3*a^2 + b^2)*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^3*d)

Rubi [A] time = 0.179718, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3483, 3529, 3531, 3530}

$$\frac{2ab}{d(a^2 - b^2)^2 (a + b \coth(c + dx))} + \frac{b}{2d(a^2 - b^2)(a + b \coth(c + dx))^2} - \frac{b(3a^2 + b^2) \log(a \sinh(c + dx) + b \cosh(c + dx))}{d(a^2 - b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Coth[c + d*x])^(-3), x]

[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*d*(a + b*Coth[c + d*x])^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])) - (b*(3*a^2 + b^2)*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^3*d)

Rule 3483

Int[((a_) + (b_.)*tan[(c_) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(a + b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3529

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a,

b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3531

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a*c + b*d, 0]

Rule 3530

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \coth(c + dx))^3} dx &= \frac{b}{2(a^2 - b^2)d(a + b \coth(c + dx))^2} + \frac{\int \frac{a - b \coth(c + dx)}{(a + b \coth(c + dx))^2} dx}{a^2 - b^2} \\ &= \frac{b}{2(a^2 - b^2)d(a + b \coth(c + dx))^2} + \frac{2ab}{(a^2 - b^2)^2 d(a + b \coth(c + dx))} + \frac{\int \frac{a^2 + b^2 - 2ab \coth(c + dx)}{a + b \coth(c + dx)} dx}{(a^2 - b^2)^2} \\ &= \frac{a(a^2 + 3b^2)x}{(a^2 - b^2)^3} + \frac{b}{2(a^2 - b^2)d(a + b \coth(c + dx))^2} + \frac{2ab}{(a^2 - b^2)^2 d(a + b \coth(c + dx))} - \\ &= \frac{a(a^2 + 3b^2)x}{(a^2 - b^2)^3} + \frac{b}{2(a^2 - b^2)d(a + b \coth(c + dx))^2} + \frac{2ab}{(a^2 - b^2)^2 d(a + b \coth(c + dx))} - \end{aligned}$$

Mathematica [A] time = 3.43228, size = 134, normalized size = 1.04

$$\frac{b \left(\frac{b(b^2 - a^2)((2ab^2 - 6a^3) \tanh(c + dx) - 5a^2 b + b^3)}{a^2(a \tanh(c + dx) + b)^2} + 2(3a^2 + b^2) \log(a \tanh(c + dx) + b) \right)}{(a^2 - b^2)^3} + \frac{\log(1 - \tanh(c + dx))}{(a + b)^3} - \frac{\log(\tanh(c + dx) + 1)}{(a - b)^3}$$

$2d$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Coth[c + d*x])^(-3), x]

[Out] $-(\text{Log}[1 - \text{Tanh}[c + d*x]]/(a + b)^3 - \text{Log}[1 + \text{Tanh}[c + d*x]]/(a - b)^3 + (b*(2*(3*a^2 + b^2)*\text{Log}[b + a*\text{Tanh}[c + d*x]] + (b*(-a^2 + b^2)*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*\text{Tanh}[c + d*x]))/(a^2*(b + a*\text{Tanh}[c + d*x])^2)))/(a^2 - b^2)^3)/(2*d)$

Maple [A] time = 0.031, size = 166, normalized size = 1.3

$$\frac{\ln(\coth(dx + c) + 1)}{2d(a - b)^3} - \frac{\ln(\coth(dx + c) - 1)}{2d(a + b)^3} + \frac{b}{2d(a - b)(a + b)(a + b\coth(dx + c))^2} + 2\frac{ab}{d(a + b)^2(a - b)^2(a + b\coth(dx + c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*coth(d*x+c))^3,x)`

[Out] $1/2/d/(a-b)^3*\ln(\coth(d*x+c)+1)-1/2/d/(a+b)^3*\ln(\coth(d*x+c)-1)+1/2/d*b/(a-b)/(a+b)/(a+b*\coth(d*x+c))^2+2/d*a*b/(a+b)^2/(a-b)^2/(a+b*\coth(d*x+c))-3/d*b/(a+b)^3/(a-b)^3*\ln(a+b*\coth(d*x+c))*a^2-1/d*b^3/(a+b)^3/(a-b)^3*\ln(a+b*\coth(d*x+c))$

Maxima [B] time = 1.18432, size = 435, normalized size = 3.37

$$\frac{(3a^2b + b^3) \log(-(a - b)e^{-2dx - 2c} + a + b)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)d} - \frac{(a^7 + a^6b - 3a^5b^2 - 3a^4b^3 + 3a^3b^4 + 3a^2b^5 - ab^6 - b^7 - 2(a^7 - a^6b - 3a^5b^2 - 3a^4b^3 + 3a^3b^4 + 3a^2b^5 - ab^6 - b^7))e^{-2dx - 2c}}{(a^7 + a^6b - 3a^5b^2 - 3a^4b^3 + 3a^3b^4 + 3a^2b^5 - ab^6 - b^7 - 2(a^7 - a^6b - 3a^5b^2 - 3a^4b^3 + 3a^3b^4 + 3a^2b^5 - ab^6 - b^7))d} + (d*x + c)/((a^3 + 3a^2b + 3ab^2 + b^3)*d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="maxima")`

[Out] $-(3*a^2*b + b^3)*\log(-(a - b)*e^{-2*d*x - 2*c} + a + b)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d) - 2*(3*a^2*b^2 + 3*a*b^3 - (3*a^2*b^2 - 2*a*b^3 - b^4)*e^{-2*d*x - 2*c})/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 - 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7))*e^{-2*d*x - 2*c} + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*e^{-4*d*x - 4*c})*d) + (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)$

Fricas [B] time = 3.20442, size = 3182, normalized size = 24.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & ((a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx \cosh(dx + c) \\ &)^4 + 4(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx \cosh(\\ & dx + c) \sinh(dx + c)^3 + (a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab \\ & ^4 + b^5)dx \sinh(dx + c)^4 + 6a^3b^2 - 12a^2b^3 + 6ab^4 + (a^5 + a \\ & ^4b - 2a^3b^2 - 2a^2b^3 + ab^4 + b^5)dx - 2(3a^3b^2 - a^2b^3 - \\ & 3ab^4 + b^5 + (a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5)dx \\ &) \cosh(dx + c)^2 - 2(3a^3b^2 - a^2b^3 - 3ab^4 + b^5 - 3(a^5 + 5a^4 \\ & *b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx \cosh(dx + c)^2 + (a^5 + \\ & 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5)dx) \sinh(dx + c)^2 - (3 \\ & a^4b - 6a^3b^2 + 4a^2b^3 - 2ab^4 + b^5 + (3a^4b + 6a^3b^2 + 4a^ \\ & 2b^3 + 2ab^4 + b^5) \cosh(dx + c)^4 + 4(3a^4b + 6a^3b^2 + 4a^2b^3 \\ & + 2ab^4 + b^5) \cosh(dx + c) \sinh(dx + c)^3 + (3a^4b + 6a^3b^2 + 4 \\ & a^2b^3 + 2ab^4 + b^5) \sinh(dx + c)^4 - 2(3a^4b - 2a^2b^3 - b^5) \co \\ & sh(dx + c)^2 - 2(3a^4b - 2a^2b^3 - b^5 - 3(3a^4b + 6a^3b^2 + 4a \\ & ^2b^3 + 2ab^4 + b^5) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((3a^4b + 6 \\ & a^3b^2 + 4a^2b^3 + 2ab^4 + b^5) \cosh(dx + c)^3 - (3a^4b - 2a^2b^3 \\ & - b^5) \cosh(dx + c)) \sinh(dx + c) \log(2(b \cosh(dx + c) + a \sinh(dx + \\ & c)) / (\cosh(dx + c) - \sinh(dx + c))) + 4((a^5 + 5a^4b + 10a^3b^2 + 10 \\ & a^2b^3 + 5ab^4 + b^5)dx \cosh(dx + c)^3 - (3a^3b^2 - a^2b^3 - 3a \\ & b^4 + b^5 + (a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5)dx) \co \\ & sh(dx + c) \sinh(dx + c)) / ((a^8 + 2a^7b - 2a^6b^2 - 6a^5b^3 + 6a^3 \\ & *b^5 + 2a^2b^6 - 2ab^7 - b^8)dx \cosh(dx + c)^4 + 4(a^8 + 2a^7b - 2 \\ & a^6b^2 - 6a^5b^3 + 6a^3b^5 + 2a^2b^6 - 2ab^7 - b^8)dx \cosh(dx + c \\ &) \sinh(dx + c)^3 + (a^8 + 2a^7b - 2a^6b^2 - 6a^5b^3 + 6a^3b^5 + 2 \\ & a^2b^6 - 2ab^7 - b^8)dx \sinh(dx + c)^4 - 2(a^8 - 4a^6b^2 + 6a^4b^4 \\ & - 4a^2b^6 + b^8)dx \cosh(dx + c)^2 + 2(3(a^8 + 2a^7b - 2a^6b^2 - 6 \\ & a^5b^3 + 6a^3b^5 + 2a^2b^6 - 2ab^7 - b^8)dx \cosh(dx + c)^2 - (a^8 \\ & - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8)dx) \sinh(dx + c)^2 + (a^8 - 2a^ \\ & 7b - 2a^6b^2 + 6a^5b^3 - 6a^3b^5 + 2a^2b^6 + 2ab^7 - b^8)dx + 4 \\ & ((a^8 + 2a^7b - 2a^6b^2 - 6a^5b^3 + 6a^3b^5 + 2a^2b^6 - 2ab^7 - \\ & b^8)dx \cosh(dx + c)^3 - (a^8 - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8)dx \\ & * \cosh(dx + c)) \sinh(dx + c) \end{aligned}$$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))**3,x)

[Out] Exception raised: TypeError

Giac [A] time = 1.18017, size = 285, normalized size = 2.21

$$-\frac{(3a^2b + b^3) \log(|ae^{(2dx+2c)} + be^{(2dx+2c)} - a + b|)}{a^6d - 3a^4b^2d + 3a^2b^4d - b^6d} + \frac{dx + c}{a^3d - 3a^2bd + 3ab^2d - b^3d} - \frac{2 \left((3a^2b^2 - 4ab^3 + b^4)e^{(2dx+2c)} - 3 \right)}{(ae^{(2dx+2c)} + be^{(2dx+2c)} - a + b)^2 (a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="giac")

[Out] $-(3a^2b + b^3) \log(\text{abs}(a e^{(2d*x + 2*c)} + b e^{(2d*x + 2*c)} - a + b)) / (a^6*d - 3a^4*b^2*d + 3a^2*b^4*d - b^6*d) + (d*x + c) / (a^3*d - 3a^2*b*d + 3a*b^2*d - b^3*d) - 2 * ((3a^2*b^2 - 4a*b^3 + b^4) * e^{(2*d*x + 2*c)} - 3 * (a^3*b^2 - 2a^2*b^3 + a*b^4) / (a + b)) / ((a * e^{(2*d*x + 2*c)} + b * e^{(2*d*x + 2*c)} - a + b)^2 * (a + b)^2 * (a - b)^3 * d)$

$$3.84 \quad \int \frac{1}{(a+b \coth(c+dx))^4} dx$$

Optimal. Leaf size=169

$$\frac{b(3a^2 + b^2)}{d(a^2 - b^2)^3 (a + b \coth(c + dx))} + \frac{ab}{d(a^2 - b^2)^2 (a + b \coth(c + dx))^2} + \frac{b}{3d(a^2 - b^2)(a + b \coth(c + dx))^3} - \frac{4ab(a^2 + b^2)}{d(a^2 - b^2)^4 (a + b \coth(c + dx))^4}$$

[Out] $((a^4 + 6a^2b^2 + b^4)x)/(a^2 - b^2)^4 + b/(3(a^2 - b^2)d(a + b \coth[c + dx])^3) + (ab)/((a^2 - b^2)^2 d(a + b \coth[c + dx])^2) + (b(3a^2 + b^2))/((a^2 - b^2)^3 d(a + b \coth[c + dx])) - (4ab(a^2 + b^2) \log[b \cosh[c + dx] + a \sinh[c + dx]])/(a^2 - b^2)^4 d$

Rubi [A] time = 0.264259, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3483, 3529, 3531, 3530}

$$\frac{b(3a^2 + b^2)}{d(a^2 - b^2)^3 (a + b \coth(c + dx))} + \frac{ab}{d(a^2 - b^2)^2 (a + b \coth(c + dx))^2} + \frac{b}{3d(a^2 - b^2)(a + b \coth(c + dx))^3} - \frac{4ab(a^2 + b^2)}{d(a^2 - b^2)^4 (a + b \coth(c + dx))^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Coth[c + d*x])^(-4), x]

[Out] $((a^4 + 6a^2b^2 + b^4)x)/(a^2 - b^2)^4 + b/(3(a^2 - b^2)d(a + b \coth[c + dx])^3) + (ab)/((a^2 - b^2)^2 d(a + b \coth[c + dx])^2) + (b(3a^2 + b^2))/((a^2 - b^2)^3 d(a + b \coth[c + dx])) - (4ab(a^2 + b^2) \log[b \cosh[c + dx] + a \sinh[c + dx]])/(a^2 - b^2)^4 d$

Rule 3483

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a + b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]

Rule 3529

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])

$^{(m+1)}\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3531

$\text{Int}[\frac{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}, x_Symbol] :> \text{Simp}[\frac{(a*c + b*d)*x}{a^2 + b^2}, x] + \text{Dist}[\frac{(b*c - a*d)}{a^2 + b^2}, \text{Int}[\frac{(b - a*\text{Tan}[e + f*x])}{(a + b*\text{Tan}[e + f*x])}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

Rule 3530

$\text{Int}[\frac{(c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}, x_Symbol] :> \text{Simp}[\frac{(c*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]])}{(b*f)}, x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[a*c + b*d, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \coth(c + dx))^4} dx &= \frac{b}{3(a^2 - b^2) d(a + b \coth(c + dx))^3} + \frac{\int \frac{a - b \coth(c + dx)}{(a + b \coth(c + dx))^3} dx}{a^2 - b^2} \\ &= \frac{b}{3(a^2 - b^2) d(a + b \coth(c + dx))^3} + \frac{ab}{(a^2 - b^2)^2 d(a + b \coth(c + dx))^2} + \frac{\int \frac{a^2 + b^2 - 2ab \coth(c + dx)}{(a + b \coth(c + dx))^2} dx}{(a^2 - b^2)^2} \\ &= \frac{b}{3(a^2 - b^2) d(a + b \coth(c + dx))^3} + \frac{ab}{(a^2 - b^2)^2 d(a + b \coth(c + dx))^2} + \frac{b(3a^2 + b^2)}{(a^2 - b^2)^3 d(a + b \coth(c + dx))} \\ &= \frac{(a^4 + 6a^2b^2 + b^4)x}{(a^2 - b^2)^4} + \frac{b}{3(a^2 - b^2) d(a + b \coth(c + dx))^3} + \frac{ab}{(a^2 - b^2)^2 d(a + b \coth(c + dx))} \\ &= \frac{(a^4 + 6a^2b^2 + b^4)x}{(a^2 - b^2)^4} + \frac{b}{3(a^2 - b^2) d(a + b \coth(c + dx))^3} + \frac{ab}{(a^2 - b^2)^2 d(a + b \coth(c + dx))} \end{aligned}$$

Mathematica [A] time = 6.20822, size = 214, normalized size = 1.27

$$\frac{b^4}{3a^3d(a^2 - b^2)(a \tanh(c + dx) + b)^3} + \frac{b^3(2a^2 - b^2)}{a^3d(a^2 - b^2)^2(a \tanh(c + dx) + b)^2} - \frac{b^2(-3a^2b^2 + 6a^4 + b^4)}{a^3d(a^2 - b^2)^3(a \tanh(c + dx) + b)} - \frac{4ab}{a^3d(a^2 - b^2)^3(a \tanh(c + dx) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Coth[c + d*x])^(-4),x]

[Out] $-\frac{\log[1 - \tanh[c + dx]]}{2(a+b)^{4d}} + \frac{\log[1 + \tanh[c + dx]]}{2(a-b)^{4d}} - \frac{4ab(a^2 + b^2)\log[b + a\tanh[c + dx]]}{(a^2 - b^2)^{4d}} - \frac{b^4}{3a^3(a^2 - b^2)d(b + a\tanh[c + dx])^3} + \frac{b^3(2a^2 - b^2)}{(a^3(a^2 - b^2)^2d(b + a\tanh[c + dx])^2} - \frac{b^2(6a^4 - 3a^2b^2 + b^4)}{(a^3(a^2 - b^2)^3d(b + a\tanh[c + dx])}$

Maple [A] time = 0.034, size = 230, normalized size = 1.4

$$\frac{\ln(\coth(dx+c)+1)}{2d(a-b)^4} - \frac{\ln(\coth(dx+c)-1)}{2d(a+b)^4} + \frac{b}{3d(a-b)(a+b)(a+b\coth(dx+c))^3} + \frac{ab}{d(a+b)^2(a-b)^2(a+b\coth(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*coth(d*x+c))^4,x)

[Out] $\frac{1}{2} \frac{d}{(a-b)^4} \ln(\coth(dx+c)+1) - \frac{1}{2} \frac{d}{(a+b)^4} \ln(\coth(dx+c)-1) + \frac{1}{3} \frac{d*b}{(a-b)(a+b)(a+b\coth(dx+c))^3} + \frac{1}{d} \frac{a*b}{(a+b)^2} \frac{1}{(a-b)^2} \frac{1}{(a+b\coth(dx+c))^2} + \frac{d*b}{(a+b)^3} \frac{1}{(a-b)^3} \frac{1}{(a+b\coth(dx+c))} * \frac{a^2+1}{d*b^3} \frac{1}{(a+b)^3} \frac{1}{(a-b)^3} \frac{1}{(a+b\coth(dx+c))} - \frac{4}{d*b} \frac{a^3}{(a+b)^4} \frac{1}{(a-b)^4} \ln(a+b\coth(dx+c)) - \frac{4}{d*b^3} \frac{a}{(a+b)^4} \frac{1}{(a-b)^4} \ln(a+b\coth(dx+c))$

Maxima [B] time = 1.3454, size = 705, normalized size = 4.17

$$\frac{4(a^3b + ab^3) \log(-(a-b)e^{-2dx-2c} + a+b)}{(a^8 - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8)d} - \frac{3(a^{10} + 2a^9b - 3a^8b^2 - 8a^7b^3 + 2a^6b^4 + 12a^5b^5 + 2a^4b^6 - 8a^3b^7 - \dots)}{3(a^{10} + 2a^9b - 3a^8b^2 - 8a^7b^3 + 2a^6b^4 + 12a^5b^5 + 2a^4b^6 - 8a^3b^7 - \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="maxima")

[Out] $-4(a^3b + a^2b^2) \log(-(a-b)e^{-2dx-2c} + a+b) / ((a^8 - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8)d) - \frac{4}{3} (9a^4b^2 + 18a^3b^3 + 11a^2b^4 + 4a^2b^5 + 2b^6 - 3(6a^4b^2 + 2a^3b^3 - 5a^2b^4 - 2ab^5 - b^6)) e^{-2dx-2c} + 3(3a^4b^2 - 4a^3b^3 + b^6) e^{-4dx-4c} / ((a^8 - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8)d)$

$$10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^{10} - 3*(a^{10} - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^{10})*e^{(-2*d*x - 2*c)} + 3*(a^{10} - 2*a^9*b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 2*a*b^9 + b^{10})*e^{(-4*d*x - 4*c)} - (a^{10} - 4*a^9*b + 3*a^8*b^2 + 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^{10})*e^{(-6*d*x - 6*c)})*d) + (d*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)$$

Fricas [B] time = 3.4678, size = 8128, normalized size = 48.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="fricas")

[Out] $\frac{1}{3}*(3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*\cosh(d*x + c)^6 + 18*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^5 + 3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*\sinh(d*x + c)^6 - 36*a^5*b^2 + 108*a^4*b^3 - 116*a^3*b^4 + 60*a^2*b^5 - 24*a*b^6 + 8*b^7 - 3*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*\cosh(d*x + c)^4 - 3*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 - 15*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*\cosh(d*x + c)^2 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*\sinh(d*x + c)^4 + 12*(5*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*\cosh(d*x + c)^3 - (12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*d*x + 3*(24*a^5*b^2 - 32*a^4*b^3 - 12*a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*\cosh(d*x + c)^2 + 3*(24*a^5*b^2 - 32*a^4*b^3 - 12*a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 15*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*\cosh(d*x + c)^4 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x - 6*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 12*(a^6*b - 3*a^5*b^2 + 4$

$$\begin{aligned}
& *a^4*b^3 - 4*a^3*b^4 + 3*a^2*b^5 - a*b^6 - (a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + \\
& 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*\cosh(d*x + c)^6 - 6*(a^6*b + 3*a^5*b^2 + 4* \\
& a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*\cosh(d*x + c)*\sinh(d*x + c)^5 - (a \\
& ^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*\sinh(d*x + c) \\
& ^6 + 3*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*\cosh(d*x + c)^4 + 3*(a^6*b + a^5 \\
& *b^2 - a^2*b^5 - a*b^6 - 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a \\
& ^2*b^5 + a*b^6)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - 4*(5*(a^6*b + 3*a^5*b^2 \\
& + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*\cosh(d*x + c)^3 - 3*(a^6*b + a \\
& ^5*b^2 - a^2*b^5 - a*b^6)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 3*(a^6*b - a^5*b \\
& ^2 - a^2*b^5 + a*b^6)*\cosh(d*x + c)^2 - 3*(a^6*b - a^5*b^2 - a^2*b^5 + a*b^ \\
& 6 + 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*\cosh(\\
& d*x + c)^4 - 6*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*\cosh(d*x + c)^2)*\sinh(d* \\
& x + c)^2 - 6*((a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^ \\
& 6)*\cosh(d*x + c)^5 - 2*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*\cosh(d*x + c)^3 \\
& + (a^6*b - a^5*b^2 - a^2*b^5 + a*b^6)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*(\\
& b*\cosh(d*x + c) + a*\sinh(d*x + c))/(\cosh(d*x + c) - \sinh(d*x + c))) + 6*(3* \\
& (a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^ \\
& 6 + b^7)*d*x*\cosh(d*x + c)^5 - 2*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a \\
& *b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2 \\
& *b^5 - 5*a*b^6 - b^7)*d*x)*\cosh(d*x + c)^3 + (24*a^5*b^2 - 32*a^4*b^3 - 12* \\
& a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^ \\
& 4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*\cosh(d*x + c))*\sinh(d*x + \\
& c))/((a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14 \\
& *a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*\cosh(d*x + \\
& c)^6 + 6*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 \\
& + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*\cosh(d \\
& *x + c)*\sinh(d*x + c)^5 + (a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b \\
& ^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 \\
& + b^11)*d*\sinh(d*x + c)^6 - 3*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a \\
& ^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b \\
& ^10 - b^11)*d*\cosh(d*x + c)^4 + 3*(5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^ \\
& 3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 \\
& + 3*a*b^10 + b^11)*d*\cosh(d*x + c)^2 - (a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b \\
& ^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2* \\
& b^9 - a*b^10 - b^11)*d)*\sinh(d*x + c)^4 + 3*(a^11 - a^10*b - 5*a^9*b^2 + 5* \\
& a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5 \\
& *a^2*b^9 - a*b^10 + b^11)*d*\cosh(d*x + c)^2 + 4*(5*(a^11 + 3*a^10*b - a^9*b \\
& ^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3* \\
& b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*\cosh(d*x + c)^3 - 3*(a^11 + a^10*b - 5*a \\
& ^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5* \\
& a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(\\
& 5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5 \\
& *b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*\cosh(d*x + c)^ \\
& 4 - 6*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10 \\
& *a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*\cosh(d*x +
\end{aligned}$$

$$\begin{aligned} & c)^2 + (a^{11} - a^{10}b - 5a^9b^2 + 5a^8b^3 + 10a^7b^4 - 10a^6b^5 - \\ & 10a^5b^6 + 10a^4b^7 + 5a^3b^8 - 5a^2b^9 - ab^{10} + b^{11})d) \sinh(dx + c)^2 - (a^{11} - 3a^{10}b - a^9b^2 + 11a^8b^3 - 6a^7b^4 - 14a^6b^5 \\ & + 14a^5b^6 + 6a^4b^7 - 11a^3b^8 + a^2b^9 + 3ab^{10} - b^{11})d + 6(\\ & (a^{11} + 3a^{10}b - a^9b^2 - 11a^8b^3 - 6a^7b^4 + 14a^6b^5 + 14a^5b^6 \\ & - 6a^4b^7 - 11a^3b^8 - a^2b^9 + 3ab^{10} + b^{11})d \cosh(dx + c)^5 \\ & - 2(a^{11} + a^{10}b - 5a^9b^2 - 5a^8b^3 + 10a^7b^4 + 10a^6b^5 - 10a^5b^6 \\ & - 10a^4b^7 + 5a^3b^8 + 5a^2b^9 - ab^{10} - b^{11})d \cosh(dx + c) \\ &)^3 + (a^{11} - a^{10}b - 5a^9b^2 + 5a^8b^3 + 10a^7b^4 - 10a^6b^5 - 10 \\ & a^5b^6 + 10a^4b^7 + 5a^3b^8 - 5a^2b^9 - ab^{10} + b^{11})d \cosh(dx + \\ & c)) \sinh(dx + c) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 1.21574, size = 424, normalized size = 2.51

$$-\frac{4(a^3b + ab^3) \log(|ae^{(2dx+2c)} + be^{(2dx+2c)} - a + b|)}{a^8d - 4a^6b^2d + 6a^4b^4d - 4a^2b^6d + b^8d} + \frac{dx + c}{a^4d - 4a^3bd + 6a^2b^2d - 4ab^3d + b^4d} - \frac{4(3(3a^4b^2 - 2a^3b^3 - 2a^2b^4 + 2a^3b^3 - 2a^2b^4 + 2ab^5 - b^6)e^{(4dx+4c)} - 3(6a^4b^2 - 14a^3b^3 + 11a^2b^4 - 4ab^5 + b^6)e^{(2dx+2c)} + (9a^5b^2 - 27a^4b^3 + 29a^3b^4 - 15a^2b^5 + 6ab^6 - 2b^7)/(a+b))/(ae^{(2dx+2c)} + be^{(2dx+2c)} - a + b)^3(a+b)^3(a-b)^4d}{a^4d - 4a^3bd + 6a^2b^2d - 4ab^3d + b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="giac")

[Out]
$$-4(a^3b + ab^3) \log(\text{abs}(ae^{(2dx+2c)} + be^{(2dx+2c)} - a + b)) / (a^8d - 4a^6b^2d + 6a^4b^4d - 4a^2b^6d + b^8d) + (dx + c) / (a^4d - 4a^3bd + 6a^2b^2d - 4ab^3d + b^4d) - 4/3(3(3a^4b^2 - 2a^3b^3 - 2a^2b^4 + 2ab^5 - b^6)e^{(4dx+4c)} - 3(6a^4b^2 - 14a^3b^3 + 11a^2b^4 - 4ab^5 + b^6)e^{(2dx+2c)} + (9a^5b^2 - 27a^4b^3 + 29a^3b^4 - 15a^2b^5 + 6ab^6 - 2b^7)/(a+b)) / ((ae^{(2dx+2c)} + be^{(2dx+2c)} - a + b)^3(a+b)^3(a-b)^4d)$$

$$3.85 \quad \int \frac{1}{4+6 \coth(c+dx)} dx$$

Optimal. Leaf size=31

$$\frac{3 \log(2 \sinh(c + dx) + 3 \cosh(c + dx))}{10d} - \frac{x}{5}$$

[Out] $-x/5 + (3*\text{Log}[3*\text{Cosh}[c + d*x] + 2*\text{Sinh}[c + d*x]])/(10*d)$

Rubi [A] time = 0.0438151, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3484, 3530}

$$\frac{3 \log(2 \sinh(c + dx) + 3 \cosh(c + dx))}{10d} - \frac{x}{5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(4 + 6*\text{Coth}[c + d*x])^{-1}, x]$

[Out] $-x/5 + (3*\text{Log}[3*\text{Cosh}[c + d*x] + 2*\text{Sinh}[c + d*x]])/(10*d)$

Rule 3484

$\text{Int}[(a + (b \cdot \tan[c + d \cdot x])^{-1}), x_Symbol] \rightarrow \text{Simp}[(a \cdot x)/(a^2 + b^2), x] + \text{Dist}[b/(a^2 + b^2), \text{Int}[(b - a \cdot \tan[c + d \cdot x])/(a + b \cdot \tan[c + d \cdot x]), x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3530

$\text{Int}[(c + (d \cdot \tan[e + f \cdot x])/(a + (b \cdot \tan[e + f \cdot x]) \cdot x)), x_Symbol] \rightarrow \text{Simp}[(c \cdot \text{Log}[\text{RemoveContent}[a \cdot \text{Cos}[e + f \cdot x] + b \cdot \text{Sin}[e + f \cdot x], x]])/(b \cdot f), x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b \cdot c - a \cdot d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a \cdot c + b \cdot d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{4+6 \coth(c+dx)} dx &= -\frac{x}{5} + \frac{3}{10} i \int \frac{-6i - 4i \coth(c+dx)}{4+6 \coth(c+dx)} dx \\ &= -\frac{x}{5} + \frac{3 \log(3 \cosh(c+dx) + 2 \sinh(c+dx))}{10d} \end{aligned}$$

Mathematica [A] time = 0.0387123, size = 53, normalized size = 1.71

$$-\frac{\log(1 - \tanh(c + dx))}{20d} - \frac{\log(\tanh(c + dx) + 1)}{4d} + \frac{3 \log(2 \tanh(c + dx) + 3)}{10d}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 6*Coth[c + d*x])^(-1),x]

[Out] -Log[1 - Tanh[c + d*x]]/(20*d) - Log[1 + Tanh[c + d*x]]/(4*d) + (3*Log[3 + 2*Tanh[c + d*x]])/(10*d)

Maple [A] time = 0.017, size = 46, normalized size = 1.5

$$-\frac{\ln(\coth(dx + c) + 1)}{4d} - \frac{\ln(\coth(dx + c) - 1)}{20d} + \frac{3 \ln(2 + 3 \coth(dx + c))}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4+6*coth(d*x+c)),x)

[Out] -1/4/d*ln(coth(d*x+c)+1)-1/20/d*ln(coth(d*x+c)-1)+3/10/d*ln(2+3*coth(d*x+c))

Maxima [A] time = 1.10128, size = 38, normalized size = 1.23

$$\frac{dx + c}{10d} + \frac{3 \log(e^{(-2dx-2c)} + 5)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="maxima")

[Out] 1/10*(d*x + c)/d + 3/10*log(e^(-2*d*x - 2*c) + 5)/d

Fricas [A] time = 2.55375, size = 130, normalized size = 4.19

$$-\frac{5 dx - 3 \log\left(\frac{2(3 \cosh(dx+c)+2 \sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="fricas")

[Out] $-1/10*(5*d*x - 3*\log(2*(3*\cosh(d*x + c) + 2*\sinh(d*x + c))/(\cosh(d*x + c) - \sinh(d*x + c))))/d$

Sympy [A] time = 1.36237, size = 42, normalized size = 1.35

$$\begin{cases} \frac{x}{10} - \frac{3 \log(\tanh(c+dx)+1)}{10d} + \frac{3 \log\left(\tanh(c+dx)+\frac{3}{2}\right)}{10d} & \text{for } d \neq 0 \\ \frac{x}{6 \coth(c)+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+6*coth(d*x+c)),x)

[Out] Piecewise((x/10 - 3*log(tanh(c + d*x) + 1)/(10*d) + 3*log(tanh(c + d*x) + 3/2)/(10*d), Ne(d, 0)), (x/(6*coth(c) + 4), True))

Giac [A] time = 1.17926, size = 41, normalized size = 1.32

$$-\frac{dx+c}{2d} + \frac{3 \log(5e^{(2dx+2c)}+1)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="giac")

[Out] $-1/2*(d*x + c)/d + 3/10*\log(5*e^{(2*d*x + 2*c)} + 1)/d$

$$3.86 \quad \int \frac{1}{4-6 \coth(c+dx)} dx$$

Optimal. Leaf size=31

$$-\frac{3 \log(3 \cosh(c+dx) - 2 \sinh(c+dx))}{10d} - \frac{x}{5}$$

[Out] $-x/5 - (3*\text{Log}[3*\text{Cosh}[c + d*x] - 2*\text{Sinh}[c + d*x]])/(10*d)$

Rubi [A] time = 0.0424359, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3484, 3530}

$$-\frac{3 \log(3 \cosh(c+dx) - 2 \sinh(c+dx))}{10d} - \frac{x}{5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(4 - 6*\text{Coth}[c + d*x])^{-1}, x]$

[Out] $-x/5 - (3*\text{Log}[3*\text{Cosh}[c + d*x] - 2*\text{Sinh}[c + d*x]])/(10*d)$

Rule 3484

$\text{Int}[(a + (b_*)\tan[(c_*) + (d_*)(x_*)])^{-1}, x_Symbol] \rightarrow \text{Simp}[(a*x)/(a^2 + b^2), x] + \text{Dist}[b/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[c + d*x])/(a + b*\text{Tan}[c + d*x]), x], x] /;$ $\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

Rule 3530

$\text{Int}[(c + (d_*)\tan[(e_*) + (f_*)(x_*)])]/((a + (b_*)\tan[(e_*) + (f_*)(x_*)]), x_Symbol] \rightarrow \text{Simp}[(c*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]])/(b*f), x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{EqQ}[a*c + b*d, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{4-6 \coth(c+dx)} dx &= -\frac{x}{5} - \frac{3}{10} i \int \frac{6i - 4i \coth(c+dx)}{4-6 \coth(c+dx)} dx \\ &= -\frac{x}{5} - \frac{3 \log(3 \cosh(c+dx) - 2 \sinh(c+dx))}{10d} \end{aligned}$$

Mathematica [A] time = 0.0374029, size = 53, normalized size = 1.71

$$-\frac{3 \log(3 - 2 \tanh(c + dx))}{10d} + \frac{\log(1 - \tanh(c + dx))}{4d} + \frac{\log(\tanh(c + dx) + 1)}{20d}$$

Antiderivative was successfully verified.

[In] Integrate[(4 - 6*Coth[c + d*x])^(-1),x]

[Out] (-3*Log[3 - 2*Tanh[c + d*x]])/(10*d) + Log[1 - Tanh[c + d*x]]/(4*d) + Log[1 + Tanh[c + d*x]]/(20*d)

Maple [A] time = 0.016, size = 46, normalized size = 1.5

$$\frac{\ln(\coth(dx + c) + 1)}{20d} - \frac{3 \ln(-2 + 3 \coth(dx + c))}{10d} + \frac{\ln(\coth(dx + c) - 1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4-6*coth(d*x+c)),x)

[Out] 1/20/d*ln(coth(d*x+c)+1)-3/10/d*ln(-2+3*coth(d*x+c))+1/4/d*ln(coth(d*x+c)-1)

Maxima [A] time = 1.0579, size = 39, normalized size = 1.26

$$-\frac{1}{2}x - \frac{c}{2d} - \frac{3 \log(5e^{(-2dx-2c)} + 1)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="maxima")

[Out] -1/2*x - 1/2*c/d - 3/10*log(5*e^(-2*d*x - 2*c) + 1)/d

Fricas [A] time = 2.49815, size = 126, normalized size = 4.06

$$\frac{dx - 3 \log\left(\frac{2(3 \cosh(dx+c) - 2 \sinh(dx+c))}{\cosh(dx+c) - \sinh(dx+c)}\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="fricas")

[Out] 1/10*(d*x - 3*log(2*(3*cosh(d*x + c) - 2*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))))/d

Sympy [A] time = 1.18326, size = 42, normalized size = 1.35

$$\begin{cases} -\frac{x}{2} - \frac{3 \log\left(\tanh\left(c+dx\right) - \frac{3}{2}\right)}{10d} + \frac{3 \log(\tanh(c+dx)+1)}{10d} & \text{for } d \neq 0 \\ \frac{x}{4-6 \coth(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-6*coth(d*x+c)),x)

[Out] Piecewise((-x/2 - 3*log(tanh(c + d*x) - 3/2)/(10*d) + 3*log(tanh(c + d*x) + 1)/(10*d), Ne(d, 0)), (x/(4 - 6*coth(c)), True))

Giac [A] time = 1.19247, size = 38, normalized size = 1.23

$$\frac{dx + c}{10d} - \frac{3 \log\left(e^{(2dx+2c)} + 5\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="giac")

[Out] 1/10*(d*x + c)/d - 3/10*log(e^(2*d*x + 2*c) + 5)/d

3.87 $\int \sqrt{a + b \coth(c + dx)} dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b} \coth(c+dx)}{\sqrt{a+b}}\right)}{d} - \frac{\sqrt{a-b} \tanh^{-1}\left(\frac{\sqrt{a+b} \coth(c+dx)}{\sqrt{a-b}}\right)}{d}$$

[Out] $-\left(\frac{\sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \coth(c+dx)}{\sqrt{a-b}}\right]}{d}\right) + \left(\frac{\sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \coth(c+dx)}{\sqrt{a+b}}\right]}{d}\right)$

Rubi [A] time = 0.0700508, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3485, 700, 1130, 207}

$$\frac{\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b} \coth(c+dx)}{\sqrt{a+b}}\right)}{d} - \frac{\sqrt{a-b} \tanh^{-1}\left(\frac{\sqrt{a+b} \coth(c+dx)}{\sqrt{a-b}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Coth[c + d*x]], x]

[Out] $-\left(\frac{\sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \coth(c+dx)}{\sqrt{a-b}}\right]}{d}\right) + \left(\frac{\sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \coth(c+dx)}{\sqrt{a+b}}\right]}{d}\right)$

Rule 3485

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]

Rule 700

Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[2*e, Subst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 1130

Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -

$q/2 + c*x^2), x], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{GeQ}[m, 2]$

Rule 207

$\text{Int}[(a_ + (b_ .)*(x_)^2)^{-1}, x_Symbol] \ :> \ -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \sqrt{a + b \coth(c + dx)} dx &= -\frac{b \text{Subst}\left(\int \frac{\sqrt{a+x}}{-b^2+x^2} dx, x, b \coth(c + dx)\right)}{d} \\ &= -\frac{(2b) \text{Subst}\left(\int \frac{x^2}{a^2-b^2-2ax^2+x^4} dx, x, \sqrt{a + b \coth(c + dx)}\right)}{d} \\ &= \frac{(a-b) \text{Subst}\left(\int \frac{1}{-a+b+x^2} dx, x, \sqrt{a + b \coth(c + dx)}\right)}{d} - \frac{(a+b) \text{Subst}\left(\int \frac{1}{-a-b+x^2} dx, x, \sqrt{a + b \coth(c + dx)}\right)}{d} \\ &= -\frac{\sqrt{a-b} \tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a+b}}\right)}{d} \end{aligned}$$

Mathematica [C] time = 3.21564, size = 128, normalized size = 1.73

$$\frac{\sqrt{a + b \coth(c + dx)} \left(\sqrt{i(a + b)} \tanh^{-1} \left(\frac{\sqrt{i(a + b \coth(c + dx))}}{\sqrt{i(a + b)}} \right) - \sqrt{i(a - b)} \tanh^{-1} \left(\frac{\sqrt{i(a + b \coth(c + dx))}}{\sqrt{i(a - b)}} \right) \right)}{d \sqrt{i(a + b \coth(c + dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Coth[c + d*x]],x]

[Out] ((-(Sqrt[I*(a - b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x]])]/Sqrt[I*(a - b)]]) + Sqrt[I*(a + b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x]])]/Sqrt[I*(a + b)])*Sqrt[a + b*Coth[c + d*x]]/(d*Sqrt[I*(a + b*Coth[c + d*x])])

Maple [A] time = 0.053, size = 63, normalized size = 0.9

$$\frac{1}{d} \text{Artanh} \left(\sqrt{a + b \coth(dx + c)} \frac{1}{\sqrt{a + b}} \right) \sqrt{a + b} - \frac{1}{d} \sqrt{-a + b} \arctan \left(\sqrt{a + b \coth(dx + c)} \frac{1}{\sqrt{-a + b}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*coth(d*x+c))^(1/2),x)`

[Out] `arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d-1/d*(-a+b)^(1/2)*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*coth(d*x + c) + a), x)`

Fricas [B] time = 3.21213, size = 5682, normalized size = 76.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `[1/4*(sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c) + sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*x + c)^2 + (6*`

$$\begin{aligned}
& a \cosh(dx + c)^2 - 2a + b) \sinh(dx + c)^2 + 2(2a \cosh(dx + c)^3 - (2a - b) \cosh(dx + c)) \sinh(dx + c) + a - b) \sqrt{a - b} \sqrt{(b \cosh(dx + c) + a \sinh(dx + c)) / \sinh(dx + c)} \\
& + 4((2a^2 - b^2) \cosh(dx + c)^3 - 2(a^2 - ab) \cosh(dx + c)) \sinh(dx + c) / (\cosh(dx + c)^4 + 4 \cosh(dx + c)^3 \sinh(dx + c) + 6 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4) \\
&) / d, -1/4(2 \sqrt{-a - b}) \arctan(((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 - a) \sqrt{-a - b} \sqrt{(b \cosh(dx + c) + a \sinh(dx + c)) / \sinh(dx + c)}) / ((a^2 + 2ab + b^2) \cosh(dx + c)^2 + 2(a^2 + 2ab + b^2) \cosh(dx + c) \sinh(dx + c) + (a^2 + 2ab + b^2) \sinh(dx + c)^2 - a^2 + b^2)) - \\
& \sqrt{a - b} \log(((2a^2 - b^2) \cosh(dx + c)^4 + 4(2a^2 - b^2) \cosh(dx + c) \sinh(dx + c)^3 + (2a^2 - b^2) \sinh(dx + c)^4 - 4(a^2 - ab) \cosh(dx + c)^2 + 2(3(2a^2 - b^2) \cosh(dx + c)^2 - 2a^2 + 2ab) \sinh(dx + c)^2 + 2a^2 - 4ab + 2b^2 - 2(a \cosh(dx + c)^4 + 4a \cosh(dx + c) \sinh(dx + c)^3 + a \sinh(dx + c)^4 - (2a - b) \cosh(dx + c)^2 + (6a \cosh(dx + c)^2 - 2a + b) \sinh(dx + c)^2 + 2(2a \cosh(dx + c)^3 - (2a - b) \cosh(dx + c)) \sinh(dx + c) + a - b) \sqrt{a - b} \sqrt{(b \cosh(dx + c) + a \sinh(dx + c)) / \sinh(dx + c)} \\
& + 4((2a^2 - b^2) \cosh(dx + c)^3 - 2(a^2 - ab) \cosh(dx + c)) \sinh(dx + c) / (\cosh(dx + c)^4 + 4 \cosh(dx + c)^3 \sinh(dx + c) + 6 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4 \cosh(dx + c) \sinh(dx + c)^3 + \sinh(dx + c)^4) \\
&) / d, -1/4(2 \sqrt{-a + b}) \arctan(-(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 - a + b) \sqrt{-a + b} \sqrt{(b \cosh(dx + c) + a \sinh(dx + c)) / \sinh(dx + c)}) / ((a^2 - b^2) \cosh(dx + c)^2 + 2(a^2 - b^2) \cosh(dx + c) \sinh(dx + c) + (a^2 - b^2) \sinh(dx + c)^2 - a^2 + 2ab - b^2)) - \\
& \sqrt{a + b} \log(2(a^2 + 2ab + b^2) \cosh(dx + c)^4 + 8(a^2 + 2ab + b^2) \cosh(dx + c) \sinh(dx + c)^3 + 2(a^2 + 2ab + b^2) \sinh(dx + c)^4 - 4(a^2 + ab) \cosh(dx + c)^2 + 4(3(a^2 + 2ab + b^2) \cosh(dx + c)^2 - a^2 - ab) \sinh(dx + c)^2 + 2a^2 - b^2 + 2((a + b) \cosh(dx + c)^4 + 4(a + b) \cosh(dx + c) \sinh(dx + c)^3 + (a + b) \sinh(dx + c)^4 - (2a + b) \cosh(dx + c)^2 + (6(a + b) \cosh(dx + c)^2 - 2a - b) \sinh(dx + c)^2 + 2(2(a + b) \cosh(dx + c)^3 - (2a + b) \cosh(dx + c)) \sinh(dx + c) + a) \sqrt{a + b} \sqrt{(b \cosh(dx + c) + a \sinh(dx + c)) / \sinh(dx + c)} \\
& + 8((a^2 + 2ab + b^2) \cosh(dx + c)^3 - (a^2 + ab) \cosh(dx + c)) \sinh(dx + c) / d, -1/2(\sqrt{-a + b}) \arctan(-(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 - a + b) \sqrt{-a + b} \sqrt{(b \cosh(dx + c) + a \sinh(dx + c)) / \sinh(dx + c)}) / ((a^2 - b^2) \cosh(dx + c)^2 + 2(a^2 - b^2) \cosh(dx + c) \sinh(dx + c) + (a^2 - b^2) \sinh(dx + c)^2 - a^2 + 2ab - b^2)) + \\
& \sqrt{-a - b} \arctan(((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 - a) \sqrt{-a - b} \sqrt{(b \cosh(dx + c) + a \sinh(dx + c)) / \sinh(dx + c)}) / ((a^2 + 2ab + b^2) \cosh(dx + c)^2 + 2(a^2 + 2ab + b^2) \cosh(dx + c) \sinh(dx + c) + (a^2 + 2ab + b^2) \sinh(dx + c)^2 - a^2 + b^2)) / d \\
&]
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \coth(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a + b*coth(c + d*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \coth(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*coth(d*x + c) + a), x)

$$3.88 \quad \int \frac{1}{\sqrt{a+b \coth(c+dx)}} dx$$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Coth}[c + d*x]]/\text{Sqrt}[a - b]]/(\text{Sqrt}[a - b]*d)) + \text{ArcTanh}[\text{Sqrt}[a + b*\text{Coth}[c + d*x]]/\text{Sqrt}[a + b]]/(\text{Sqrt}[a + b]*d)$

Rubi [A] time = 0.0678776, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3485, 708, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] `Int[1/Sqrt[a + b*Coth[c + d*x]],x]`

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Coth}[c + d*x]]/\text{Sqrt}[a - b]]/(\text{Sqrt}[a - b]*d)) + \text{ArcTanh}[\text{Sqrt}[a + b*\text{Coth}[c + d*x]]/\text{Sqrt}[a + b]]/(\text{Sqrt}[a + b]*d)$

Rule 3485

`Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 + b^2, 0]`

Rule 708

`Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2*e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]`

Rule 1093

`Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int`

$[1/(b/2 + q/2 + c*x^2), x], x]] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

Rule 207

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \text{ :> } -\text{Simp}[\text{ArcTanh}[\text{Rt}[b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a + b \coth(c + dx)}} dx &= -\frac{b \text{Subst}\left(\int \frac{1}{\sqrt{a+x(-b^2+x^2)}} dx, x, b \coth(c + dx)\right)}{d} \\ &= -\frac{(2b) \text{Subst}\left(\int \frac{1}{a^2-b^2-2ax^2+x^4} dx, x, \sqrt{a + b \coth(c + dx)}\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \frac{1}{-a-b+x^2} dx, x, \sqrt{a + b \coth(c + dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2} dx, x, \sqrt{a + b \coth(c + dx)}\right)}{d} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-bd}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+bd}} \end{aligned}$$

Mathematica [A] time = 0.141617, size = 73, normalized size = 0.99

$$-\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*Coth[c + d*x]], x]

[Out] -((ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b] - ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]]/Sqrt[a + b])/d)

Maple [A] time = 0.046, size = 62, normalized size = 0.8

$$\frac{1}{d} \text{Artanh}\left(\sqrt{a + b \coth(dx + c)} \frac{1}{\sqrt{a + b}}\right) \frac{1}{\sqrt{a + b}} + \frac{1}{d} \arctan\left(\sqrt{a + b \coth(dx + c)} \frac{1}{\sqrt{-a + b}}\right) \frac{1}{\sqrt{-a + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*coth(d*x+c))^(1/2),x)`

[Out] `arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/d/(-a+b)^(1/2)*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(b*coth(d*x + c) + a), x)`

Fricas [B] time = 3.26532, size = 5844, normalized size = 78.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `[1/4*(sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c) + (a + b)*sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^`

$$\begin{aligned}
& 4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 - (2*a - b)*\cosh(d*x + c)^2 + (6*a*\cosh(d*x + c)^2 - 2*a + b)*\sinh(d*x + c)^2 + 2*(2*a*\cosh(d*x + c)^3 - (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a - b*\sqrt{a - b}*\sqrt{(b*\cosh(d*x + c) + a*\sinh(d*x + c))/\sinh(d*x + c)} + 4*((2*a^2 - b^2)*\cosh(d*x + c)^3 - 2*(a^2 - a*b)*\cosh(d*x + c))*\sinh(d*x + c))/(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4))/((a^2 - b^2)*d), -1/4*(2*(a - b)*\sqrt{-a - b}*\arctan(((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 - a)*\sqrt{-a - b}*\sqrt{(b*\cosh(d*x + c) + a*\sinh(d*x + c))/\sinh(d*x + c)}))/((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + 2*a*b + b^2)*\sinh(d*x + c)^2 - a^2 + b^2)) - (a + b)*\sqrt{a - b}*\log(((2*a^2 - b^2)*\cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a^2 - b^2)*\sinh(d*x + c)^4 - 4*(a^2 - a*b)*\cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*\cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*\sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*\cosh(d*x + c)^4 + 4*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*\sinh(d*x + c)^4 - (2*a - b)*\cosh(d*x + c)^2 + (6*a*\cosh(d*x + c)^2 - 2*a + b)*\sinh(d*x + c)^2 + 2*(2*a*\cosh(d*x + c)^3 - (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a - b)*\sqrt{a - b}*\sqrt{(b*\cosh(d*x + c) + a*\sinh(d*x + c))/\sinh(d*x + c)} + 4*((2*a^2 - b^2)*\cosh(d*x + c)^3 - 2*(a^2 - a*b)*\cosh(d*x + c))*\sinh(d*x + c))/(\cosh(d*x + c)^4 + 4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*\cosh(d*x + c)*\sinh(d*x + c)^3 + \sinh(d*x + c)^4)))/((a^2 - b^2)*d), -1/4*(2*(a + b)*\sqrt{-a + b}*\arctan(-(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 - a + b)*\sqrt{-a + b}*\sqrt{(b*\cosh(d*x + c) + a*\sinh(d*x + c))/\sinh(d*x + c)}))/((a^2 - b^2)*\cosh(d*x + c)^2 + 2*(a^2 - b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 - b^2)*\sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) - \sqrt{a + b}*(a - b)*\log(2*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*\sinh(d*x + c)^4 - 4*(a^2 + a*b)*\cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 - a^2 - a*b)*\sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 - (2*a + b)*\cosh(d*x + c)^2 + (6*(a + b)*\cosh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 + 2*(2*(a + b)*\cosh(d*x + c)^3 - (2*a + b)*\cosh(d*x + c))*\sinh(d*x + c) + a)*\sqrt{a + b}*\sqrt{(b*\cosh(d*x + c) + a*\sinh(d*x + c))/\sinh(d*x + c)} + 8*((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^3 - (a^2 + a*b)*\cosh(d*x + c))*\sinh(d*x + c))/((a^2 - b^2)*d), -1/2*((a + b)*\sqrt{-a + b}*\arctan(-(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 - a + b)*\sqrt{-a + b}*\sqrt{(b*\cosh(d*x + c) + a*\sinh(d*x + c))/\sinh(d*x + c)}))/((a^2 - b^2)*\cosh(d*x + c)^2 + 2*(a^2 - b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 - b^2)*\sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) + (a - b)*\sqrt{-a - b}*\arctan(((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 - a)*\sqrt{-a - b}*\sqrt{(b*\cosh(d*x + c) + a*\sinh(d*x + c))/\sinh(d*x + c)}))/((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + 2*a*b + b^2)*\sinh(d*x + c)^2 - a^2 + b^2))/((a^2 - b^2)*d)]
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \coth(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))**(1/2),x)

[Out] Integral(1/sqrt(a + b*coth(c + d*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \coth(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*coth(d*x + c) + a), x)

$$3.89 \quad \int \frac{\sinh^4(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=60

$$\frac{5x}{16} + \frac{1}{8(1-\coth(x))} - \frac{3}{16(\coth(x)+1)} + \frac{1}{32(1-\coth(x))^2} - \frac{3}{32(\coth(x)+1)^2} - \frac{1}{24(\coth(x)+1)^3}$$

[Out] (5*x)/16 + 1/(32*(1 - Coth[x])^2) + 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x])^3) - 3/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))

Rubi [A] time = 0.062674, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3487, 44, 207}

$$\frac{5x}{16} + \frac{1}{8(1-\coth(x))} - \frac{3}{16(\coth(x)+1)} + \frac{1}{32(1-\coth(x))^2} - \frac{3}{32(\coth(x)+1)^2} - \frac{1}{24(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^4/(1 + Coth[x]),x]

[Out] (5*x)/16 + 1/(32*(1 - Coth[x])^2) + 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x])^3) - 3/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))

Rule 3487

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^4(x)}{1 + \coth(x)} dx &= \text{Subst} \left(\int \frac{1}{(1-x)^3(1+x)^4} dx, x, \coth(x) \right) \\
 &= \text{Subst} \left(\int \left(-\frac{1}{16(-1+x)^3} + \frac{1}{8(-1+x)^2} + \frac{1}{8(1+x)^4} + \frac{3}{16(1+x)^3} + \frac{3}{16(1+x)^2} - \frac{5}{16(-1+x^2)} \right) dx, \right. \\
 &= \frac{1}{32(1 - \coth(x))^2} + \frac{1}{8(1 - \coth(x))} - \frac{1}{24(1 + \coth(x))^3} - \frac{3}{32(1 + \coth(x))^2} - \frac{3}{16(1 + \coth(x))} - \frac{5}{16} \\
 &= \frac{5x}{16} + \frac{1}{32(1 - \coth(x))^2} + \frac{1}{8(1 - \coth(x))} - \frac{1}{24(1 + \coth(x))^3} - \frac{3}{32(1 + \coth(x))^2} - \frac{3}{16(1 + \coth(x))}
 \end{aligned}$$

Mathematica [A] time = 0.103101, size = 42, normalized size = 0.7

$$\frac{1}{192}(60x - 45 \sinh(2x) + 9 \sinh(4x) - \sinh(6x) + 15 \cosh(2x) - 6 \cosh(4x) + \cosh(6x))$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^4/(1 + Coth[x]), x]

[Out] (60*x + 15*Cosh[2*x] - 6*Cosh[4*x] + Cosh[6*x] - 45*Sinh[2*x] + 9*Sinh[4*x] - Sinh[6*x])/192

Maple [B] time = 0.033, size = 110, normalized size = 1.8

$$\frac{1}{3} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-6} - \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-5} + \frac{5}{8} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-4} + \frac{5}{12} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-3} - \frac{3}{8} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1} + \frac{5}{16} \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^4/(1+coth(x)), x)

[Out] 1/3/(tanh(1/2*x)+1)^6-1/(tanh(1/2*x)+1)^5+5/8/(tanh(1/2*x)+1)^4+5/12/(tanh(1/2*x)+1)^3-3/8/(tanh(1/2*x)+1)+5/16*ln(tanh(1/2*x)+1)+1/8/(tanh(1/2*x)-1)^4+1/4/(tanh(1/2*x)-1)^3-1/8/(tanh(1/2*x)-1)^2-1/4/(tanh(1/2*x)-1)-5/16*ln(t

$\operatorname{anh}(1/2*x)-1)$

Maxima [A] time = 1.06854, size = 49, normalized size = 0.82

$$-\frac{1}{128} (10e^{(-2x)} - 1)e^{(4x)} + \frac{5}{16}x + \frac{5}{32}e^{(-2x)} - \frac{5}{128}e^{(-4x)} + \frac{1}{192}e^{(-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="maxima")

[Out] $-1/128*(10*e^{(-2*x)} - 1)*e^{(4*x)} + 5/16*x + 5/32*e^{(-2*x)} - 5/128*e^{(-4*x)} + 1/192*e^{(-6*x)}$

Fricas [B] time = 2.54278, size = 321, normalized size = 5.35

$$\frac{5 \cosh(x)^5 + 25 \cosh(x) \sinh(x)^4 + \sinh(x)^5 + 5(2 \cosh(x)^2 - 3) \sinh(x)^3 - 45 \cosh(x)^3 + 5(10 \cosh(x)^3 - 27 \cosh(x)) \sinh(x)^2 + 60(2x + 1) \cosh(x) + 5(\cosh(x)^4 - 9 \cosh(x)^2 + 24x - 12) \sinh(x)}{384(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="fricas")

[Out] $1/384*(5*\cosh(x)^5 + 25*\cosh(x)*\sinh(x)^4 + \sinh(x)^5 + 5*(2*\cosh(x)^2 - 3)*\sinh(x)^3 - 45*\cosh(x)^3 + 5*(10*\cosh(x)^3 - 27*\cosh(x))*\sinh(x)^2 + 60*(2*x + 1)*\cosh(x) + 5*(\cosh(x)^4 - 9*\cosh(x)^2 + 24*x - 12)*\sinh(x))/(\cosh(x) + \sinh(x))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^4(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)**4/(1+coth(x)),x)

[Out] Integral(sinh(x)**4/(coth(x) + 1), x)

Giac [A] time = 1.19885, size = 57, normalized size = 0.95

$$-\frac{1}{384} (110 e^{6x} - 60 e^{4x} + 15 e^{2x} - 2) e^{-6x} + \frac{5}{16} x + \frac{1}{128} e^{4x} - \frac{5}{64} e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="giac")

[Out] -1/384*(110*e^(6*x) - 60*e^(4*x) + 15*e^(2*x) - 2)*e^(-6*x) + 5/16*x + 1/128*e^(4*x) - 5/64*e^(2*x)

$$3.90 \quad \int \frac{\sinh^3(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=29

$$\frac{4 \cosh^3(x)}{15} - \frac{4 \cosh(x)}{5} - \frac{\sinh^3(x)}{5(\coth(x) + 1)}$$

[Out] $(-4*\text{Cosh}[x])/5 + (4*\text{Cosh}[x]^3)/15 - \text{Sinh}[x]^3/(5*(1 + \text{Coth}[x]))$

Rubi [A] time = 0.0474609, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3502, 2633}

$$\frac{4 \cosh^3(x)}{15} - \frac{4 \cosh(x)}{5} - \frac{\sinh^3(x)}{5(\coth(x) + 1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[x]^3/(1 + \text{Coth}[x]), x]$

[Out] $(-4*\text{Cosh}[x])/5 + (4*\text{Cosh}[x]^3)/15 - \text{Sinh}[x]^3/(5*(1 + \text{Coth}[x]))$

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{1 + \coth(x)} dx &= -\frac{\sinh^3(x)}{5(1 + \coth(x))} + \frac{4}{5} \int \sinh^3(x) dx \\
&= -\frac{\sinh^3(x)}{5(1 + \coth(x))} - \frac{4}{5} \text{Subst} \left(\int (1 - x^2) dx, x, \cosh(x) \right) \\
&= -\frac{4 \cosh(x)}{5} + \frac{4 \cosh^3(x)}{15} - \frac{\sinh^3(x)}{5(1 + \coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.0763492, size = 36, normalized size = 1.24

$$\frac{\text{csch}(x)(-40 \sinh(2x) + 4 \sinh(4x) - 20 \cosh(2x) + \cosh(4x) - 45)}{120(\coth(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^3/(1 + Coth[x]), x]

[Out] (Csch[x]*(-45 - 20*Cosh[2*x] + Cosh[4*x] - 40*Sinh[2*x] + 4*Sinh[4*x]))/(120*(1 + Coth[x]))

Maple [B] time = 0.03, size = 80, normalized size = 2.8

$$-\frac{2}{5} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-5} + \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-4} - \frac{1}{3} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-3} - \frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-2} - \frac{3}{8} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1} - \frac{1}{6} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^0$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^3/(1+coth(x)), x)

[Out] -2/5/(tanh(1/2*x)+1)^5+1/(tanh(1/2*x)+1)^4-1/3/(tanh(1/2*x)+1)^3-1/2/(tanh(1/2*x)+1)^2-3/8/(tanh(1/2*x)+1)-1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2+3/8/(tanh(1/2*x)-1)

Maxima [A] time = 1.05927, size = 45, normalized size = 1.55

$$-\frac{1}{48} (12e^{(-2x)} - 1)e^{(3x)} - \frac{3}{8} e^{(-x)} + \frac{1}{12} e^{(-3x)} - \frac{1}{80} e^{(-5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="maxima")

[Out] -1/48*(12*e^(-2*x) - 1)*e^(3*x) - 3/8*e^(-x) + 1/12*e^(-3*x) - 1/80*e^(-5*x)

Fricas [B] time = 2.40185, size = 221, normalized size = 7.62

$$\frac{\cosh(x)^4 + 16 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 - 10) \sinh(x)^2 - 20 \cosh(x)^2 + 16(\cosh(x)^3 - 5 \cosh(x))}{120(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="fricas")

[Out] 1/120*(cosh(x)^4 + 16*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 10)*sinh(x)^2 - 20*cosh(x)^2 + 16*(cosh(x)^3 - 5*cosh(x))*sinh(x) - 45)/(cosh(x) + sinh(x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)**3/(1+coth(x)),x)

[Out] Integral(sinh(x)**3/(coth(x) + 1), x)

Giac [A] time = 1.15439, size = 42, normalized size = 1.45

$$-\frac{1}{240} (90 e^{4x} - 20 e^{2x} + 3) e^{-5x} + \frac{1}{48} e^{3x} - \frac{1}{4} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/240*(90*e^(4*x) - 20*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) - 1/4*e^x
```

$$3.91 \quad \int \frac{\sinh^2(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=38

$$-\frac{3x}{8} - \frac{1}{8(1-\coth(x))} + \frac{1}{4(\coth(x)+1)} + \frac{1}{8(\coth(x)+1)^2}$$

[Out] (-3*x)/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) + 1/(4*(1 + Coth[x]))
)

Rubi [A] time = 0.0522239, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3487, 44, 207}

$$-\frac{3x}{8} - \frac{1}{8(1-\coth(x))} + \frac{1}{4(\coth(x)+1)} + \frac{1}{8(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^2/(1 + Coth[x]),x]

[Out] (-3*x)/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) + 1/(4*(1 + Coth[x]))
)

Rule 3487

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m-2)*b*f), Subst[Int[(a-x)^(m/2-1)*(a+x)^(n+m/2-1), x], x, b*Tan[e+f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^2(x)}{1 + \coth(x)} dx &= -\text{Subst} \left(\int \frac{1}{(1-x)^2(1+x)^3} dx, x, \coth(x) \right) \\
 &= -\text{Subst} \left(\int \left(\frac{1}{8(-1+x)^2} + \frac{1}{4(1+x)^3} + \frac{1}{4(1+x)^2} - \frac{3}{8(-1+x^2)} \right) dx, x, \coth(x) \right) \\
 &= -\frac{1}{8(1-\coth(x))} + \frac{1}{8(1+\coth(x))^2} + \frac{1}{4(1+\coth(x))} + \frac{3}{8} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \coth(x) \right) \\
 &= -\frac{3x}{8} - \frac{1}{8(1-\coth(x))} + \frac{1}{8(1+\coth(x))^2} + \frac{1}{4(1+\coth(x))}
 \end{aligned}$$

Mathematica [A] time = 0.0523216, size = 30, normalized size = 0.79

$$\frac{1}{32}(-12x + 8 \sinh(2x) - \sinh(4x) - 4 \cosh(2x) + \cosh(4x))$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^2/(1 + Coth[x]), x]

[Out] (-12*x - 4*Cosh[2*x] + Cosh[4*x] + 8*Sinh[2*x] - Sinh[4*x])/32

Maple [B] time = 0.03, size = 70, normalized size = 1.8

$$\frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-4} - \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-3} + \frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1} - \frac{3}{8} \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + \frac{1}{4} \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^{-2} + \frac{1}{4} \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^2/(1+coth(x)), x)

[Out] 1/2/(tanh(1/2*x)+1)^4-1/(tanh(1/2*x)+1)^3+1/2/(tanh(1/2*x)+1)-3/8*ln(tanh(1/2*x)+1)+1/4/(tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)+3/8*ln(tanh(1/2*x)-1)

Maxima [A] time = 1.08251, size = 30, normalized size = 0.79

$$-\frac{3}{8}x + \frac{1}{16}e^{(2x)} - \frac{3}{16}e^{(-2x)} + \frac{1}{32}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="maxima")

[Out] -3/8*x + 1/16*e^(2*x) - 3/16*e^(-2*x) + 1/32*e^(-4*x)

Fricas [A] time = 2.45374, size = 176, normalized size = 4.63

$$\frac{3 \cosh(x)^3 + 9 \cosh(x) \sinh(x)^2 + \sinh(x)^3 - 6(2x + 1) \cosh(x) + 3(\cosh(x)^2 - 4x + 2) \sinh(x)}{32(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="fricas")

[Out] 1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 - 6*(2*x + 1)*cosh(x) + 3*(cosh(x)^2 - 4*x + 2)*sinh(x))/(cosh(x) + sinh(x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)**2/(1+coth(x)),x)

[Out] Integral(sinh(x)**2/(coth(x) + 1), x)

Giac [A] time = 1.12842, size = 41, normalized size = 1.08

$$\frac{1}{32}(9e^{(4x)} - 6e^{(2x)} + 1)e^{(-4x)} - \frac{3}{8}x + \frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/32*(9*e^(4*x) - 6*e^(2*x) + 1)*e^(-4*x) - 3/8*x + 1/16*e^(2*x)
```


$$3.92 \quad \int \frac{\sinh(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=19

$$\frac{2 \cosh(x)}{3} - \frac{\sinh(x)}{3(\coth(x) + 1)}$$

[Out] (2*Cosh[x])/3 - Sinh[x]/(3*(1 + Coth[x]))

Rubi [A] time = 0.0355857, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3502, 2638}

$$\frac{2 \cosh(x)}{3} - \frac{\sinh(x)}{3(\coth(x) + 1)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(1 + Coth[x]),x]

[Out] (2*Cosh[x])/3 - Sinh[x]/(3*(1 + Coth[x]))

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{1 + \coth(x)} dx &= -\frac{\sinh(x)}{3(1 + \coth(x))} + \frac{2}{3} \int \sinh(x) dx \\ &= \frac{2 \cosh(x)}{3} - \frac{\sinh(x)}{3(1 + \coth(x))}\end{aligned}$$

Mathematica [A] time = 0.0476313, size = 21, normalized size = 1.11

$$\frac{1}{12} (4 \sinh^3(x) + 9 \cosh(x) - \cosh(3x))$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]/(1 + Coth[x]), x]

[Out] (9*Cosh[x] - Cosh[3*x] + 4*Sinh[x]^3)/12

Maple [B] time = 0.027, size = 40, normalized size = 2.1

$$-\frac{2}{3} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-3} + \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-2} + \frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1} - \frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)/(1+coth(x)), x)

[Out] -2/3/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2+1/2/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)-1)

Maxima [A] time = 1.10248, size = 23, normalized size = 1.21

$$\frac{1}{2} e^{(-x)} - \frac{1}{12} e^{(-3x)} + \frac{1}{4} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)/(1+coth(x)), x, algorithm="maxima")

[Out] $1/2*e^{-x} - 1/12*e^{-3*x} + 1/4*e^x$

Fricas [A] time = 2.47767, size = 99, normalized size = 5.21

$$\frac{\cosh(x)^2 + 4 \cosh(x) \sinh(x) + \sinh(x)^2 + 3}{6(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)/(1+coth(x)),x, algorithm="fricas")`

[Out] $1/6*(\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + \sinh(x)^2 + 3)/(\cosh(x) + \sinh(x))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)/(1+coth(x)),x)`

[Out] `Integral(sinh(x)/(coth(x) + 1), x)`

Giac [A] time = 1.12542, size = 26, normalized size = 1.37

$$\frac{1}{12} (6e^{2x} - 1)e^{-3x} + \frac{1}{4} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)/(1+coth(x)),x, algorithm="giac")`

[Out] $1/12*(6*e^{2*x} - 1)*e^{-3*x} + 1/4*e^x$

$$3.93 \quad \int \frac{\operatorname{csch}(x)}{1+\operatorname{coth}(x)} dx$$

Optimal. Leaf size=10

$$-\frac{\operatorname{csch}(x)}{\operatorname{coth}(x)+1}$$

[Out] -(Csch[x]/(1 + Coth[x]))

Rubi [A] time = 0.0221637, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3488}

$$-\frac{\operatorname{csch}(x)}{\operatorname{coth}(x)+1}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]/(1 + Coth[x]),x]

[Out] -(Csch[x]/(1 + Coth[x]))

Rule 3488

```
Int[((d_)*sec[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{1+\operatorname{coth}(x)} dx = -\frac{\operatorname{csch}(x)}{1+\operatorname{coth}(x)}$$

Mathematica [A] time = 0.0031722, size = 7, normalized size = 0.7

$$\sinh(x) - \cosh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]/(1 + Coth[x]),x]

[Out] -Cosh[x] + Sinh[x]

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$-\frac{\operatorname{csch}(x)}{1 + \operatorname{coth}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)/(1+coth(x)),x)

[Out] -csch(x)/(1+coth(x))

Maxima [A] time = 1.06341, size = 8, normalized size = 0.8

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)/(1+coth(x)),x, algorithm="maxima")

[Out] -e^(-x)

Fricas [A] time = 2.51462, size = 32, normalized size = 3.2

$$-\frac{1}{\cosh(x) + \sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)/(1+coth(x)),x, algorithm="fricas")

[Out] -1/(cosh(x) + sinh(x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{\operatorname{coth}(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)/(coth(x) + 1), x)
```

Giac [A] time = 1.15261, size = 8, normalized size = 0.8

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -e^(-x)
```

$$3.94 \quad \int \frac{\operatorname{csch}^2(x)}{1+\operatorname{coth}(x)} dx$$

Optimal. Leaf size=7

$$-\log(\operatorname{coth}(x) + 1)$$

[Out] -Log[1 + Coth[x]]

Rubi [A] time = 0.0357077, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3487, 31}

$$-\log(\operatorname{coth}(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^2/(1 + Coth[x]),x]

[Out] -Log[1 + Coth[x]]

Rule 3487

Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rule 31

Int[((a_.) + (b_.)*(x_.))^(n_.), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^2(x)}{1+\operatorname{coth}(x)} dx &= -\operatorname{Subst}\left(\int \frac{1}{1+x} dx, x, \operatorname{coth}(x)\right) \\ &= -\log(1 + \operatorname{coth}(x)) \end{aligned}$$

Mathematica [A] time = 0.0029622, size = 7, normalized size = 1.

$$\log(\sinh(x)) - x$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]^2/(1 + Coth[x]),x]

[Out] -x + Log[Sinh[x]]

Maple [A] time = 0.017, size = 8, normalized size = 1.1

$$-\ln(1 + \coth(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)^2/(1+coth(x)),x)

[Out] -ln(1+coth(x))

Maxima [A] time = 1.05112, size = 9, normalized size = 1.29

$$-\log(\coth(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="maxima")

[Out] -log(coth(x) + 1)

Fricas [B] time = 2.59185, size = 58, normalized size = 8.29

$$-2x + \log\left(\frac{2 \sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -2*x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(x)}{\operatorname{coth}(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)**2/(coth(x) + 1), x)
```

Giac [A] time = 1.15829, size = 16, normalized size = 2.29

$$-2x + \log(|e^{2x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2*x + log(abs(e^(2*x) - 1))
```

$$3.95 \quad \int \frac{\operatorname{csch}^3(x)}{1+\operatorname{coth}(x)} dx$$

Optimal. Leaf size=8

$$\tanh^{-1}(\cosh(x)) - \operatorname{csch}(x)$$

[Out] ArcTanh[Cosh[x]] - Csch[x]

Rubi [A] time = 0.0400128, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3501, 3770}

$$\tanh^{-1}(\cosh(x)) - \operatorname{csch}(x)$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^3/(1 + Coth[x]), x]

[Out] ArcTanh[Cosh[x]] - Csch[x]

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)), Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] && !ILtQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^3(x)}{1+\operatorname{coth}(x)} dx &= -\operatorname{csch}(x) - \int \operatorname{csch}(x) dx \\ &= \tanh^{-1}(\cosh(x)) - \operatorname{csch}(x) \end{aligned}$$

Mathematica [A] time = 0.0373046, size = 14, normalized size = 1.75

$$-\operatorname{csch}(x) - \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]^3/(1 + Coth[x]), x]

[Out] -Csch[x] - Log[Tanh[x/2]]

Maple [B] time = 0.019, size = 23, normalized size = 2.9

$$\frac{1}{2} \tanh\left(\frac{x}{2}\right) - \frac{1}{2} \left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)^3/(1+coth(x)), x)

[Out] 1/2*tanh(1/2*x)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))

Maxima [B] time = 1.0878, size = 42, normalized size = 5.25

$$\frac{2e^{-x}}{e^{-2x}-1} + \log(e^{-x}+1) - \log(e^{-x}-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3/(1+coth(x)), x, algorithm="maxima")

[Out] 2*e^(-x)/(e^(-2*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)

Fricas [B] time = 2.59187, size = 306, normalized size = 38.25

$$\frac{(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \log(\cosh(x) + \sinh(x) + 1) - (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1)}{\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="fricas")

[Out] ((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) - 1) - 2*cosh(x) - 2*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(x)}{\operatorname{coth}(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)**3/(1+coth(x)),x)

[Out] Integral(csch(x)**3/(coth(x) + 1), x)

Giac [B] time = 1.14436, size = 35, normalized size = 4.38

$$-\frac{2e^x}{e^{(2x)} - 1} + \log(e^x + 1) - \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="giac")

[Out] -2*e^x/(e^(2*x) - 1) + log(e^x + 1) - log(abs(e^x - 1))

$$3.96 \quad \int \frac{\operatorname{csch}^4(x)}{1+\operatorname{coth}(x)} dx$$

Optimal. Leaf size=11

$$\operatorname{coth}(x) - \frac{\operatorname{coth}^2(x)}{2}$$

[Out] Coth[x] - Coth[x]^2/2

Rubi [A] time = 0.0343565, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3487}

$$\operatorname{coth}(x) - \frac{\operatorname{coth}^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^4/(1 + Coth[x]), x]

[Out] Coth[x] - Coth[x]^2/2

Rule 3487

Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^4(x)}{1+\operatorname{coth}(x)} dx &= \operatorname{Subst}\left(\int (1-x) dx, x, \operatorname{coth}(x)\right) \\ &= \operatorname{coth}(x) - \frac{\operatorname{coth}^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.0245922, size = 11, normalized size = 1.

$$\operatorname{coth}(x) - \frac{\operatorname{csch}^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]^4/(1 + Coth[x]),x]

[Out] Coth[x] - Csch[x]^2/2

Maple [B] time = 0.023, size = 32, normalized size = 2.9

$$-\frac{1}{8} \left(\tanh\left(\frac{x}{2}\right) \right)^2 + \frac{1}{2} \tanh\left(\frac{x}{2}\right) - \frac{1}{8} \left(\tanh\left(\frac{x}{2}\right) \right)^{-2} + \frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)^4/(1+coth(x)),x)

[Out] -1/8*tanh(1/2*x)^2+1/2*tanh(1/2*x)-1/8/tanh(1/2*x)^2+1/2/tanh(1/2*x)

Maxima [B] time = 1.10184, size = 55, normalized size = 5.

$$\frac{4e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} - \frac{2}{2e^{(-2x)} - e^{(-4x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="maxima")

[Out] 4*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2/(2*e^(-2*x) - e^(-4*x) - 1)

Fricas [B] time = 2.37182, size = 181, normalized size = 16.45

$$\frac{2}{\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 - 1) \sinh(x)^2 - 2 \cosh(x)^2 + 4(\cosh(x)^3 - \cosh(x)) \sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="fricas")

[Out] $-2/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(x)}{\operatorname{coth}(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(x)**4/(1+coth(x)),x)`

[Out] `Integral(csch(x)**4/(coth(x) + 1), x)`

Giac [A] time = 1.15129, size = 14, normalized size = 1.27

$$-\frac{2}{(e^{2x} - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(x)^4/(1+coth(x)),x, algorithm="giac")`

[Out] `-2/(e^(2*x) - 1)^2`

$$3.97 \quad \int \frac{\sinh^4(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=155

$$-\frac{b^5 \log(a+b \coth(x))}{(a^2-b^2)^3} - \frac{(3a^2+9ab+8b^2) \log(1-\coth(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2) \log(\coth(x)+1)}{16(a-b)^3} - \frac{\sinh^4(x)(b-a \coth(x))}{4(a^2-b^2)}$$

[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Coth[x]])/(16*(a + b)^3) + ((3*a^2 - 9*a*b + 8*b^2)*Log[1 + Coth[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Coth[x]])/(a^2 - b^2)^3 - ((4*b^3 - a*(7 - (3*a^2)/b^2)*b^2*Coth[x])*Sinh[x]^2)/(8*(a^2 - b^2)^2) - ((b - a*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))

Rubi [A] time = 0.239974, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3506, 741, 823, 801}

$$-\frac{b^5 \log(a+b \coth(x))}{(a^2-b^2)^3} - \frac{(3a^2+9ab+8b^2) \log(1-\coth(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2) \log(\coth(x)+1)}{16(a-b)^3} - \frac{\sinh^4(x)(b-a \coth(x))}{4(a^2-b^2)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^4/(a + b*Coth[x]),x]

[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Coth[x]])/(16*(a + b)^3) + ((3*a^2 - 9*a*b + 8*b^2)*Log[1 + Coth[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Coth[x]])/(a^2 - b^2)^3 - ((4*b^3 - a*(7 - (3*a^2)/b^2)*b^2*Coth[x])*Sinh[x]^2)/(8*(a^2 - b^2)^2) - ((b - a*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))

Rule 3506

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rule 741

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2


```
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(x)}{a + b \coth(x)} dx &= \frac{\text{Subst} \left(\int \frac{1}{(a+x) \left(1 - \frac{x^2}{b^2}\right)^3} dx, x, b \coth(x) \right)}{b} \\
&= -\frac{(b - a \coth(x)) \sinh^4(x)}{4(a^2 - b^2)} + \frac{b \text{Subst} \left(\int \frac{-4 + \frac{3a^2}{b^2} + \frac{3ax}{b^2}}{(a+x) \left(1 - \frac{x^2}{b^2}\right)^2} dx, x, b \coth(x) \right)}{4(a^2 - b^2)} \\
&= -\frac{\left(4b^3 - a \left(7 - \frac{3a^2}{b^2}\right) b^2 \coth(x)\right) \sinh^2(x)}{8(a^2 - b^2)^2} - \frac{(b - a \coth(x)) \sinh^4(x)}{4(a^2 - b^2)} - \frac{b^5 \text{Subst} \left(\int \frac{-\frac{3a^4 - 7a^2 b^2 + 8b^4}{b^6} + \frac{a}{(a+x) \left(1 - \frac{x^2}{b^2}\right)} dx, x, b \coth(x) \right)}{8(a^2 - b^2)} \\
&= -\frac{\left(4b^3 - a \left(7 - \frac{3a^2}{b^2}\right) b^2 \coth(x)\right) \sinh^2(x)}{8(a^2 - b^2)^2} - \frac{(b - a \coth(x)) \sinh^4(x)}{4(a^2 - b^2)} - \frac{b^5 \text{Subst} \left(\int \left(-\frac{(a-b)^2(3a^2 + 9ab)}{2b^5(a+b)(b-x)} \right) dx, x, b \coth(x) \right)}{8(a^2 - b^2)} \\
&= -\frac{(3a^2 + 9ab + 8b^2) \log(1 - \coth(x))}{16(a+b)^3} + \frac{(3a^2 - 9ab + 8b^2) \log(1 + \coth(x))}{16(a-b)^3} - \frac{b^5 \log(a + b \coth(x))}{(a^2 - b^2)^3}
\end{aligned}$$

Mathematica [A] time = 0.265987, size = 156, normalized size = 1.01

$$\frac{-40a^3b^2x + 24a^3b^2 \sinh(2x) - 2a^3b^2 \sinh(4x) + 4b(-4a^2b^2 + a^4 + 3b^4) \cosh(2x) - b(a^2 - b^2)^2 \cosh(4x) + 12a^5x - 8a^5 \sinh(2x)}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^4/(a + b*Coth[x]),x]

[Out] (12*a^5*x - 40*a^3*b^2*x + 60*a*b^4*x + 4*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Cosh[2*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*b^5*Log[b*Cosh[x] + a*Sinh[x]] - 8*a^5*Sinh[2*x] + 24*a^3*b^2*Sinh[2*x] - 16*a*b^4*Sinh[2*x] + a^5*Sinh[4*x] - 2*a^3*b^2*Sinh[4*x] + a*b^4*Sinh[4*x])/(32*(a - b)^3*(a + b)^3)

Maple [B] time = 0.049, size = 354, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(x)^4/(a+b*coth(x)),x)`

[Out]
$$-16/(64*a-64*b)/(\tanh(1/2*x)+1)^4+64/(128*a-128*b)/(\tanh(1/2*x)+1)^3+1/8/(a-b)^2/(\tanh(1/2*x)+1)^{2*a-3/8}/(a-b)^2/(\tanh(1/2*x)+1)^{2*b-3/8}/(a-b)^2/(\tanh(1/2*x)+1)*a+5/8/(a-b)^2/(\tanh(1/2*x)+1)*b+3/8/(a-b)^3*\ln(\tanh(1/2*x)+1)*a^{2-9/8}/(a-b)^3*\ln(\tanh(1/2*x)+1)*a*b+1/(a-b)^3*\ln(\tanh(1/2*x)+1)*b^2+16/(64*a+64*b)/(\tanh(1/2*x)-1)^4+64/(128*a+128*b)/(\tanh(1/2*x)-1)^3-1/8/(a+b)^2/(\tanh(1/2*x)-1)^{2*a-3/8}/(a+b)^2/(\tanh(1/2*x)-1)^{2*b-3/8}/(a+b)^2/(\tanh(1/2*x)-1)*a-5/8/(a+b)^2/(\tanh(1/2*x)-1)*b-3/8/(a+b)^3*\ln(\tanh(1/2*x)-1)*a^{2-9/8}/(a+b)^3*\ln(\tanh(1/2*x)-1)*a*b-1/(a+b)^3*\ln(\tanh(1/2*x)-1)*b^2-b^5/(a-b)^3/(a+b)^3*\ln(\tanh(1/2*x)^{2*b+2*a*\tanh(1/2*x)+b})$$

Maxima [A] time = 1.10772, size = 224, normalized size = 1.45

$$-\frac{b^5 \log(-(a-b)e^{-2x} + a + b)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{(3a^2 + 9ab + 8b^2)x}{8(a^3 + 3a^2b + 3ab^2 + b^3)} - \frac{(4(2a + 3b)e^{-2x} - a - b)e^{4x}}{64(a^2 + 2ab + b^2)} + \frac{4(2a - 3b)e^{-2x} - (a - b)e^{4x}}{64(a^2 - 2ab + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="maxima")`

[Out]
$$-b^5*\log(-(a-b)*e^{-2*x} + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(3*a^2 + 9*a*b + 8*b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*(2*a + 3*b)*e^{-2*x} - a - b)*e^{4*x}/(a^2 + 2*a*b + b^2) + 1/64*(4*(2*a - 3*b)*e^{-2*x} - (a - b)*e^{4*x})/(a^2 - 2*a*b + b^2)$$

Fricas [B] time = 2.84713, size = 2882, normalized size = 18.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="fricas")`

[Out]
$$1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^8 + 8*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)*\sinh(x)^7 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\sinh(x)^8 - 4*(2*a^5 - a^4*b$$

$$\begin{aligned}
& - 6a^3b^2 + 4a^2b^3 + 4ab^4 - 3b^5) \cosh(x)^6 - 4(2a^5 - a^4b - 6a^3b^2 + 4a^2b^3 + 4ab^4 - 3b^5) \cosh(x)^2 \sinh(x)^6 + 8(3a^5 - 10a^3b^2 + 15ab^4 + 8b^5) x \cosh(x)^4 + 8(7(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5) \cosh(x)^3 - 3(2a^5 - a^4b - 6a^3b^2 + 4a^2b^3 + 4ab^4 - 3b^5) \cosh(x)) \sinh(x)^5 - a^5 - a^4b + 2a^3b^2 + 2a^2b^3 - ab^4 - b^5 + 2(35(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5) \cosh(x)^4 - 30(2a^5 - a^4b - 6a^3b^2 + 4a^2b^3 + 4ab^4 - 3b^5) \cosh(x)^2 + 4(3a^5 - 10a^3b^2 + 15ab^4 + 8b^5) x) \sinh(x)^4 + 8(7(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5) \cosh(x)^5 - 10(2a^5 - a^4b - 6a^3b^2 + 4a^2b^3 + 4ab^4 - 3b^5) \cosh(x)^3 + 4(3a^5 - 10a^3b^2 + 15ab^4 + 8b^5) x \cosh(x)) \sinh(x)^3 + 4(2a^5 + a^4b - 6a^3b^2 - 4a^2b^3 + 4ab^4 + 3b^5) \cosh(x)^2 + 4(7(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5) \cosh(x)^6 + 2a^5 + a^4b - 6a^3b^2 - 4a^2b^3 + 4ab^4 + 3b^5 - 15(2a^5 - a^4b - 6a^3b^2 + 4a^2b^3 + 4ab^4 - 3b^5) \cosh(x)^4 + 12(3a^5 - 10a^3b^2 + 15ab^4 + 8b^5) x \cosh(x)^2) \sinh(x)^2 - 64(b^5 \cosh(x)^4 + 4b^5 \cosh(x)^3 \sinh(x) + 6b^5 \cosh(x)^2 \sinh(x)^2 + 4b^5 \cosh(x) \sinh(x)^3 + b^5 \sinh(x)^4) \log(2(b \cosh(x) + a \sinh(x)) / (\cosh(x) - \sinh(x))) + 8((a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5) \cosh(x)^7 - 3(2a^5 - a^4b - 6a^3b^2 + 4a^2b^3 + 4ab^4 - 3b^5) \cosh(x)^5 + 4(3a^5 - 10a^3b^2 + 15ab^4 + 8b^5) x \cosh(x)^3 + (2a^5 + a^4b - 6a^3b^2 - 4a^2b^3 + 4ab^4 + 3b^5) \cosh(x)) \sinh(x)) / ((a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \cosh(x)^4 + 4(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \cosh(x)^3 \sinh(x) + 6(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \cosh(x)^2 \sinh(x)^2 + 4(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \cosh(x) \sinh(x)^3 + (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \sinh(x)^4)
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^4(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)**4/(a+b*coth(x)),x)

[Out] Integral(sinh(x)**4/(a + b*coth(x)), x)

Giac [A] time = 1.17534, size = 309, normalized size = 1.99

$$\frac{b^5 \log(|-ae^{(2x)} - be^{(2x)} + a - b|)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{(3a^2 - 9ab + 8b^2)x}{8(a^3 - 3a^2b + 3ab^2 - b^3)} - \frac{(18a^2e^{(4x)} - 54abe^{(4x)} + 48b^2e^{(4x)} - 8a^2e^{(2x)} + 20ab^2e^{(2x)} - 12b^2e^{(2x)} + a^2 - 2ab + b^2)e^{(-4x)}}{64(a^3 - 3a^2b + 3ab^2 - b^3)} + \frac{(a^2e^{(4x)} + be^{(4x)} - 8ae^{(2x)} - 12be^{(2x)})}{(a^2 + 2ab + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="giac")

[Out] $-b^5 \log(\text{abs}(-a e^{(2x)} - b e^{(2x)} + a - b)) / (a^6 - 3a^4b^2 + 3a^2b^4 - b^6) + 1/8 * (3a^2 - 9ab + 8b^2) * x / (a^3 - 3a^2b + 3ab^2 - b^3) - 1/64 * (18a^2e^{(4x)} - 54ab^2e^{(4x)} + 48b^2e^{(4x)} - 8a^2e^{(2x)} + 20ab^2e^{(2x)} - 12b^2e^{(2x)} + a^2 - 2ab + b^2) * e^{(-4x)} / (a^3 - 3a^2b + 3ab^2 - b^3) + 1/64 * (a e^{(4x)} + b e^{(4x)} - 8a e^{(2x)} - 12b e^{(2x)}) / (a^2 + 2ab + b^2)$

3.98 $\int \frac{\sinh^3(x)}{a+b \coth(x)} dx$

Optimal. Leaf size=134

$$-\frac{b \sinh^3(x)}{3(a^2 - b^2)} - \frac{b^3 \sinh(x)}{(a^2 - b^2)^2} + \frac{a \cosh^3(x)}{3(a^2 - b^2)} + \frac{ab^2 \cosh(x)}{(a^2 - b^2)^2} - \frac{a \cosh(x)}{a^2 - b^2} - \frac{b^4 \tanh^{-1}\left(\frac{\sinh(x)(a \coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2 - b^2)^{5/2}}$$

[Out] -((b^4*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)) + (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3)/(3*(a^2 - b^2)) - (b^3*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 - b^2))

Rubi [A] time = 0.237369, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3511, 3486, 2633, 2638, 3509, 206}

$$-\frac{b \sinh^3(x)}{3(a^2 - b^2)} - \frac{b^3 \sinh(x)}{(a^2 - b^2)^2} + \frac{a \cosh^3(x)}{3(a^2 - b^2)} + \frac{ab^2 \cosh(x)}{(a^2 - b^2)^2} - \frac{a \cosh(x)}{a^2 - b^2} - \frac{b^4 \tanh^{-1}\left(\frac{\sinh(x)(a \coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2 - b^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^3/(a + b*Coth[x]),x]

[Out] -((b^4*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)) + (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3)/(3*(a^2 - b^2)) - (b^3*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 - b^2))

Rule 3511

Int[((d_)*sec[(e_.) + (f_.)*(x_)]))^m/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e + f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[m, 0]

Rule 3486

Int[((d_.)*sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] | NeQ[a^2 + b^2, 0])

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3509

Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^3(x)}{a + b \coth(x)} dx &= \frac{\int (a - b \coth(x)) \sinh^3(x) dx}{a^2 - b^2} + \frac{b^2 \int \frac{\sinh(x)}{a + b \coth(x)} dx}{a^2 - b^2} \\
 &= -\frac{b \sinh^3(x)}{3(a^2 - b^2)} + \frac{b^2 \int (a - b \coth(x)) \sinh(x) dx}{(a^2 - b^2)^2} + \frac{b^4 \int \frac{\operatorname{csch}(x)}{a + b \coth(x)} dx}{(a^2 - b^2)^2} + \frac{a \int \sinh^3(x) dx}{a^2 - b^2} \\
 &= -\frac{b^3 \sinh(x)}{(a^2 - b^2)^2} - \frac{b \sinh^3(x)}{3(a^2 - b^2)} + \frac{(ab^2) \int \sinh(x) dx}{(a^2 - b^2)^2} - \frac{b^4 \operatorname{Subst}\left(\int \frac{1}{a^2 - b^2 - x^2} dx, x, i(-ib - ia \coth(x))\right)}{(a^2 - b^2)^2} \\
 &= -\frac{b^4 \tanh^{-1}\left(\frac{(b + a \coth(x)) \sinh(x)}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{5/2}} + \frac{ab^2 \cosh(x)}{(a^2 - b^2)^2} - \frac{a \cosh(x)}{a^2 - b^2} + \frac{a \cosh^3(x)}{3(a^2 - b^2)} - \frac{b^3 \sinh(x)}{(a^2 - b^2)^2} - \frac{b \sinh^3(x)}{3(a^2 - b^2)}
 \end{aligned}$$

Mathematica [A] time = 0.787788, size = 171, normalized size = 1.28

$$\frac{3b\sqrt{b-a}(a^2b + a^3 - 5ab^2 - 5b^3)\sinh(x) - 3a\sqrt{b-a}(3a^2b + 3a^3 - 7ab^2 - 7b^3)\cosh(x) + 24b^4\sqrt{a+b}\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{12(b-a)^{5/2}(a+b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^3/(a + b*Coth[x]),x]

[Out] (24*b^4*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] - 3*a*Sqrt[-a + b]*(3*a^3 + 3*a^2*b - 7*a*b^2 - 7*b^3)*Cosh[x] - a*(-a + b)^(3/2)*(a + b)^2*Cosh[3*x] + 3*b*Sqrt[-a + b]*(a^3 + a^2*b - 5*a*b^2 - 5*b^3)*Sinh[x] + b*(-a + b)^(3/2)*(a + b)^2*Sinh[3*x])/(12*(-a + b)^(5/2)*(a + b)^3)

Maple [A] time = 0.044, size = 197, normalized size = 1.5

$$-16 \frac{1}{(32a - 32b)(\tanh(x/2) + 1)^2} + \frac{32}{96a - 96b} \left(\tanh\left(\frac{x}{2}\right) + 1\right)^{-3} - \frac{a}{2(a-b)^2} \left(\tanh\left(\frac{x}{2}\right) + 1\right)^{-1} + \frac{b}{(a-b)^2} \left(\tanh\left(\frac{x}{2}\right) + 1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^3/(a+b*coth(x)),x)

[Out] -16/(32*a-32*b)/(tanh(1/2*x)+1)^2+32/3/(tanh(1/2*x)+1)^3/(32*a-32*b)-1/2/(a-b)^2/(tanh(1/2*x)+1)*a+1/(a-b)^2/(tanh(1/2*x)+1)*b-32/3/(tanh(1/2*x)-1)^3/(32*a+32*b)-16/(32*a+32*b)/(tanh(1/2*x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)*a+1/(a+b)^2/(tanh(1/2*x)-1)*b+2*b^4/(a-b)^2/(a+b)^2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.96357, size = 4251, normalized size = 31.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)*\sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*\cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^2)*\sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*\cosh(x))*\sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*\cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^4 + 6*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*\cosh(x)^2)*\sinh(x)^2 + 24*(b^4*\cosh(x)^3 + 3*b^4*\cosh(x)^2*\sinh(x) + 3*b^4*\cosh(x)*\sinh(x))^2 + b^4*\sinh(x)^3)*\sqrt{a^2 - b^2}*\log(((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 - 2*\sqrt{a^2 - b^2}*(\cosh(x) + \sinh(x)) + a - b)/((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 - a + b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^5 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*\cosh(x)^3 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*\cosh(x))*\sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*\cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*\cosh(x)^2*\sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*\cosh(x)*\sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*\sinh(x)^3), 1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)*\sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*\cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^2)*\sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*\cosh(x))*\sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*\cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5 - 5 \end{aligned}$$

$$\begin{aligned}
 &*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^4 + 6*(3*a^5 - \\
 &a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*\cosh(x)^2*\sinh(x)^2 + 4 \\
 &8*(b^4*\cosh(x)^3 + 3*b^4*\cosh(x)^2*\sinh(x) + 3*b^4*\cosh(x)*\sinh(x)^2 + b^4* \\
 &\sinh(x)^3)*\sqrt{-a^2 + b^2}*\arctan(\sqrt{-a^2 + b^2}/((a + b)*\cosh(x) + (a + \\
 &b)*\sinh(x))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh \\
 &(x)^5 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*\cosh(x) \\
 &)^3 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*\cosh(x))*s \\
 &\sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*\cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 \\
 &+ 3*a^2*b^4 - b^6)*\cosh(x)^2*\sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^ \\
 &6)*\cosh(x)*\sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*\sinh(x)^3)]
 \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)**3/(a+b*coth(x)),x)

[Out] Integral(sinh(x)**3/(a + b*coth(x)), x)

Giac [A] time = 1.13538, size = 220, normalized size = 1.64

$$\frac{2b^4 \arctan\left(-\frac{ae^x + be^x}{\sqrt{-a^2 + b^2}}\right)}{(a^4 - 2a^2b^2 + b^4)\sqrt{-a^2 + b^2}} - \frac{(9ae^{2x} - 15be^{2x} - a + b)e^{-3x}}{24(a^2 - 2ab + b^2)} + \frac{a^2e^{3x} + 2abe^{3x} + b^2e^{3x} - 9a^2e^x - 24abe^x - 15b^2e^x}{24(a^3 + 3a^2b + 3ab^2 + b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="giac")

[Out]
$$\begin{aligned}
 &-2*b^4*\arctan(-(a*e^x + b*e^x)/\sqrt{-a^2 + b^2})/((a^4 - 2*a^2*b^2 + b^4)*s \\
 &qrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 15*b*e^(2*x) - a + b)*e^(-3*x)/(a^2 \\
 &- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^ \\
 &x - 24*a*b*e^x - 15*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
 \end{aligned}$$

$$3.99 \quad \int \frac{\sinh^2(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=92

$$\frac{b^3 \log(a + b \coth(x))}{(a^2 - b^2)^2} - \frac{\sinh^2(x)(b - a \coth(x))}{2(a^2 - b^2)} + \frac{(a + 2b) \log(1 - \coth(x))}{4(a + b)^2} - \frac{(a - 2b) \log(\coth(x) + 1)}{4(a - b)^2}$$

[Out] ((a + 2*b)*Log[1 - Coth[x]])/(4*(a + b)^2) - ((a - 2*b)*Log[1 + Coth[x]])/(4*(a - b)^2) - (b^3*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Sinh[x]^2)/(2*(a^2 - b^2))

Rubi [A] time = 0.141564, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3506, 741, 801}

$$\frac{b^3 \log(a + b \coth(x))}{(a^2 - b^2)^2} - \frac{\sinh^2(x)(b - a \coth(x))}{2(a^2 - b^2)} + \frac{(a + 2b) \log(1 - \coth(x))}{4(a + b)^2} - \frac{(a - 2b) \log(\coth(x) + 1)}{4(a - b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^2/(a + b*Coth[x]),x]

[Out] ((a + 2*b)*Log[1 - Coth[x]])/(4*(a + b)^2) - ((a - 2*b)*Log[1 + Coth[x]])/(4*(a - b)^2) - (b^3*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Sinh[x]^2)/(2*(a^2 - b^2))

Rule 3506

Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rule 741

Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&

LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (c_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^2(x)}{a + b \coth(x)} dx &= -\frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2} dx, x, b \coth(x)\right)}{b} \\ &= -\frac{(b - a \coth(x)) \sinh^2(x)}{2(a^2 - b^2)} - \frac{b \text{Subst}\left(\int \frac{-2 + \frac{a^2}{b^2} + \frac{ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)} dx, x, b \coth(x)\right)}{2(a^2 - b^2)} \\ &= -\frac{(b - a \coth(x)) \sinh^2(x)}{2(a^2 - b^2)} - \frac{b \text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right) dx, x, b \coth(x)\right)}{2(a^2 - b^2)} \\ &= \frac{(a + 2b) \log(1 - \coth(x))}{4(a + b)^2} - \frac{(a - 2b) \log(1 + \coth(x))}{4(a - b)^2} - \frac{b^3 \log(a + b \coth(x))}{(a^2 - b^2)^2} - \frac{(b - a \coth(x)) \sinh^2(x)}{2(a^2 - b^2)} \end{aligned}$$

Mathematica [A] time = 0.151654, size = 75, normalized size = 0.82

$$\frac{a(a^2 - b^2) \sinh(2x) + (b^3 - a^2b) \cosh(2x) - 2a^3x + 6ab^2x - 4b^3 \log(a \sinh(x) + b \cosh(x))}{4(a - b)^2(a + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^2/(a + b*Coth[x]), x]

[Out] (-2*a^3*x + 6*a*b^2*x + (-a^2*b) + b^3)*Cosh[2*x] - 4*b^3*Log[b*Cosh[x] + a*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x]/(4*(a - b)^2*(a + b)^2)

Maple [B] time = 0.039, size = 175, normalized size = 1.9

$$-8 \frac{1}{(16a - 16b)(\tanh(x/2) + 1)^2} + 16 \frac{1}{(32a - 32b)(\tanh(x/2) + 1)} - \frac{a}{2(a-b)^2} \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + \frac{b}{(a-b)^2} \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^2/(a+b*coth(x)),x)

[Out] $-8/(16*a-16*b)/(\tanh(1/2*x)+1)^2+16/(32*a-32*b)/(\tanh(1/2*x)+1)-1/2*a/(a-b)^2*\ln(\tanh(1/2*x)+1)+1/(a-b)^2*\ln(\tanh(1/2*x)+1)*b+8/(16*a+16*b)/(\tanh(1/2*x)-1)^2+16/(32*a+32*b)/(\tanh(1/2*x)-1)+1/2*a/(a+b)^2*\ln(\tanh(1/2*x)-1)+1/(a+b)^2*\ln(\tanh(1/2*x)-1)*b-b^3/(a-b)^2/(a+b)^2*\ln(\tanh(1/2*x)^2*b+2*a*\tanh(1/2*x)+b)$

Maxima [A] time = 1.17738, size = 112, normalized size = 1.22

$$-\frac{b^3 \log(-(a-b)e^{(-2x)} + a + b)}{a^4 - 2a^2b^2 + b^4} - \frac{(a + 2b)x}{2(a^2 + 2ab + b^2)} + \frac{e^{(2x)}}{8(a + b)} - \frac{e^{(-2x)}}{8(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="maxima")

[Out] $-b^3*\log(-(a - b)*e^{(-2*x)} + a + b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(a + 2*b)*x/(a^2 + 2*a*b + b^2) + 1/8*e^{(2*x)}/(a + b) - 1/8*e^{(-2*x)}/(a - b)$

Fricas [B] time = 2.69496, size = 818, normalized size = 8.89

$$(a^3 - a^2b - ab^2 + b^3) \cosh(x)^4 + 4(a^3 - a^2b - ab^2 + b^3) \cosh(x) \sinh(x)^3 + (a^3 - a^2b - ab^2 + b^3) \sinh(x)^4 - 4(a^3 - 3a^2b - 3ab^2 + b^3) \cosh(x) \sinh(x)^3 + 4(a^3 - a^2b - ab^2 + b^3) \sinh(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="fricas")

[Out] $1/8*((a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)*\sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*\sinh(x)^4 - 4*(a^3 - 3*a*b^2 + 3*a^2*b - b^3)*\cosh(x)*\sinh(x)^3 + 4*(a^3 - a^2*b - a*b^2 + b^3)*\sinh(x)^4)$

$$2 - 2b^3)x \cosh(x)^2 - a^3 - a^2b + ab^2 + b^3 + 2(3(a^3 - a^2b - ab^2 + b^3) \cosh(x)^2 - 2(a^3 - 3ab^2 - 2b^3)x) \sinh(x)^2 - 8(b^3 \cosh(x)^2 + 2b^3 \cosh(x) \sinh(x) + b^3 \sinh(x)^2) \log(2(b \cosh(x) + a \sinh(x)) / (\cosh(x) - \sinh(x))) + 4((a^3 - a^2b - ab^2 + b^3) \cosh(x)^3 - 2(a^3 - 3ab^2 - 2b^3)x \cosh(x)) \sinh(x) / ((a^4 - 2a^2b^2 + b^4) \cosh(x)^2 + 2(a^4 - 2a^2b^2 + b^4) \cosh(x) \sinh(x) + (a^4 - 2a^2b^2 + b^4) \sinh(x)^2)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)**2/(a+b*coth(x)),x)

[Out] Integral(sinh(x)**2/(a + b*coth(x)), x)

Giac [A] time = 1.17309, size = 154, normalized size = 1.67

$$-\frac{b^3 \log(|-ae^{(2x)} - be^{(2x)} + a - b|)}{a^4 - 2a^2b^2 + b^4} - \frac{(a - 2b)x}{2(a^2 - 2ab + b^2)} + \frac{(2ae^{(2x)} - 4be^{(2x)} - a + b)e^{(-2x)}}{8(a^2 - 2ab + b^2)} + \frac{e^{(2x)}}{8(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="giac")

[Out] $-b^3 \log(\text{abs}(-a e^{(2x)} - b e^{(2x)} + a - b)) / (a^4 - 2a^2b^2 + b^4) - 1/2 * (a - 2b)x / (a^2 - 2ab + b^2) + 1/8 * (2a e^{(2x)} - 4b e^{(2x)} - a + b) e^{(-2x)} / (a^2 - 2ab + b^2) + 1/8 * e^{(2x)} / (a + b)$

$$3.100 \quad \int \frac{\sinh(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=73

$$-\frac{b \sinh(x)}{a^2 - b^2} + \frac{a \cosh(x)}{a^2 - b^2} - \frac{b^2 \tanh^{-1}\left(\frac{\sinh(x)(a \coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2 - b^2)^{3/2}}$$

[Out] -((b^2*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)) + (a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)

Rubi [A] time = 0.106747, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {3511, 3486, 2638, 3509, 206}

$$-\frac{b \sinh(x)}{a^2 - b^2} + \frac{a \cosh(x)}{a^2 - b^2} - \frac{b^2 \tanh^{-1}\left(\frac{\sinh(x)(a \coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2 - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(a + b*Coth[x]),x]

[Out] -((b^2*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)) + (a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)

Rule 3511

Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e + f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[m, 0]

Rule 3486

Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] | NeQ[a^2 + b^2, 0])

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3509

Int[sec[(e_.) + (f_.)*(x_)]/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sinh(x)}{a + b \coth(x)} dx &= \frac{\int (a - b \coth(x)) \sinh(x) dx}{a^2 - b^2} + \frac{b^2 \int \frac{\operatorname{csch}(x)}{a + b \coth(x)} dx}{a^2 - b^2} \\ &= -\frac{b \sinh(x)}{a^2 - b^2} + \frac{a \int \sinh(x) dx}{a^2 - b^2} - \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{a^2 - b^2 - x^2} dx, x, i(-ib - ia \coth(x)) \sinh(x)\right)}{a^2 - b^2} \\ &= -\frac{b^2 \tanh^{-1}\left(\frac{(b + a \coth(x)) \sinh(x)}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{3/2}} + \frac{a \cosh(x)}{a^2 - b^2} - \frac{b \sinh(x)}{a^2 - b^2} \end{aligned}$$

Mathematica [A] time = 0.435651, size = 80, normalized size = 1.1

$$\frac{a \cosh(x)}{a^2 - b^2} + b \left(\frac{\sinh(x)}{b^2 - a^2} - \frac{2b \tan^{-1}\left(\frac{a + b \tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{(b-a)^{3/2}(a+b)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]/(a + b*Coth[x]), x]

[Out] (a*Cosh[x])/(a^2 - b^2) + b*((-2*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)^(3/2)*(a + b)^(3/2)) + Sinh[x]/(-a^2 + b^2))

Maple [A] time = 0.032, size = 93, normalized size = 1.3

$$-8 \frac{1}{(8a+8b)(\tanh(x/2)-1)} + 8 \frac{1}{(8a-8b)(\tanh(x/2)+1)} + 2 \frac{b^2}{(a+b)(a-b)\sqrt{-a^2+b^2}} \arctan\left(\frac{1}{2} \frac{2 \tanh(x/2) b}{\sqrt{-a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(x)/(a+b*coth(x)),x)`

[Out] `-8/(8*a+8*b)/(tanh(1/2*x)-1)+8/(8*a-8*b)/(tanh(1/2*x)+1)+2*b^2/(a+b)/(a-b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)/(a+b*coth(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 2.66047, size = 1103, normalized size = 15.11

$$\frac{a^3 + a^2b - ab^2 - b^3 + (a^3 - a^2b - ab^2 + b^3) \cosh(x)^2 + 2(a^3 - a^2b - ab^2 + b^3) \cosh(x) \sinh(x) + (a^3 - a^2b - ab^2 + b^3) \sinh(x)^2 - 2(b^2 \cosh(x) + b^2 \sinh(x)) \sqrt{a^2 - b^2} \log((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 + 2\sqrt{a^2 - b^2} \cosh(x) \sinh(x))}{2((a^4 - 2a^2b^2 + b^4) \cosh(x)^2 + 2(a^3 - a^2b - ab^2 + b^3) \cosh(x) \sinh(x) + (a^3 - a^2b - ab^2 + b^3) \sinh(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(x)/(a+b*coth(x)),x, algorithm="fricas")`

[Out] `[1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(`

```
cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*
a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3
- a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2
+ b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
- 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)/(a+b*coth(x)),x)

[Out] Integral(sinh(x)/(a + b*coth(x)), x)

Giac [A] time = 1.15365, size = 97, normalized size = 1.33

$$\frac{2b^2 \arctan\left(\frac{ae^x + be^x}{\sqrt{-a^2 + b^2}}\right)}{(a^2 - b^2)\sqrt{-a^2 + b^2}} + \frac{e^{-x}}{2(a - b)} + \frac{e^x}{2(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="giac")

[Out] 2*b^2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) + 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)

$$3.101 \quad \int \frac{\operatorname{csch}(x)}{a+b \operatorname{coth}(x)} dx$$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{\sinh(x)(a \operatorname{coth}(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

[Out] -(ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])

Rubi [A] time = 0.035892, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3509, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sinh(x)(a \operatorname{coth}(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]/(a + b*Coth[x]),x]

[Out] -(ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])

Rule 3509

Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{a + b \operatorname{coth}(x)} dx = -\operatorname{Subst}\left(\int \frac{1}{a^2 - b^2 - x^2} dx, x, i(-ib - ia \operatorname{coth}(x)) \sinh(x)\right)$$

$$= -\frac{i \tan^{-1}\left(\frac{(-ib - ia \operatorname{coth}(x)) \sinh(x)}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}}$$

Mathematica [A] time = 0.0311623, size = 46, normalized size = 1.21

$$\frac{2 \tan^{-1}\left(\frac{a+b \tanh\left(\frac{x}{2}\right)}{\sqrt{b-a} \sqrt{a+b}}\right)}{\sqrt{b-a} \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]/(a + b*Coth[x]), x]

[Out] (2*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*Sqrt[a + b])

Maple [A] time = 0.017, size = 39, normalized size = 1.

$$2 \frac{1}{\sqrt{-a^2 + b^2}} \arctan\left(1/2 \frac{2 \tanh(x/2) b + 2 a}{\sqrt{-a^2 + b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)/(a+b*coth(x)), x)

[Out] 2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.65223, size = 420, normalized size = 11.05

$$\left[\frac{\log\left(\frac{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)}{\sqrt{a^2-b^2}}, \frac{2\sqrt{-a^2+b^2}\arctan\left(\frac{\sqrt{-a^2+b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)}{a^2-b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="fricas")

[Out] [log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b))/sqrt(a^2 - b^2), 2*sqrt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/(a^2 - b^2)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{a + b \operatorname{coth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)/(a+b*coth(x)),x)

[Out] Integral(csch(x)/(a + b*coth(x)), x)

Giac [A] time = 1.17974, size = 47, normalized size = 1.24

$$\frac{2 \arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

$$3.102 \quad \int \frac{\operatorname{csch}^2(x)}{a+b \operatorname{coth}(x)} dx$$

Optimal. Leaf size=12

$$-\frac{\log(a+b \operatorname{coth}(x))}{b}$$

[Out] -(Log[a + b*Coth[x]]/b)

Rubi [A] time = 0.0438356, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3506, 31}

$$-\frac{\log(a+b \operatorname{coth}(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^2/(a + b*Coth[x]), x]

[Out] -(Log[a + b*Coth[x]]/b)

Rule 3506

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^2(x)}{a+b \operatorname{coth}(x)} dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{a+x} dx, x, b \operatorname{coth}(x)\right)}{b} \\ &= -\frac{\log(a+b \operatorname{coth}(x))}{b} \end{aligned}$$

Mathematica [A] time = 0.0496965, size = 20, normalized size = 1.67

$$\frac{\log(\sinh(x)) - \log(a \sinh(x) + b \cosh(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]^2/(a + b*Coth[x]),x]

[Out] (Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b

Maple [A] time = 0.019, size = 13, normalized size = 1.1

$$-\frac{\ln(a + b \coth(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)^2/(a+b*coth(x)),x)

[Out] -ln(a+b*coth(x))/b

Maxima [A] time = 1.17506, size = 16, normalized size = 1.33

$$-\frac{\log(b \coth(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")

[Out] -log(b*coth(x) + a)/b

Fricas [B] time = 2.70178, size = 127, normalized size = 10.58

$$-\frac{\log\left(\frac{2(b \cosh(x) + a \sinh(x))}{\cosh(x) - \sinh(x)}\right) - \log\left(\frac{2 \sinh(x)}{\cosh(x) - \sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(x) - sinh(x))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(x)}{a + b \operatorname{coth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*coth(x)), x)
```

Giac [B] time = 1.19027, size = 62, normalized size = 5.17

$$-\frac{(a+b)\log(|ae^{2x} + be^{2x} - a + b|)}{ab + b^2} + \frac{\log(|e^{2x} - 1|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(2*x) - 1))/b
```

3.103 $\int \frac{\operatorname{csch}^3(x)}{a+b \operatorname{coth}(x)} dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{a^2-b^2} \tanh^{-1}\left(\frac{\sinh(x)(a \operatorname{coth}(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2} + \frac{a \tanh^{-1}(\cosh(x))}{b^2} - \frac{\operatorname{csch}(x)}{b}$$

[Out] (a*ArcTanh[Cosh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/b^2 - Csch[x]/b

Rubi [A] time = 0.109173, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3510, 3486, 3770, 3509, 206}

$$-\frac{\sqrt{a^2-b^2} \tanh^{-1}\left(\frac{\sinh(x)(a \operatorname{coth}(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2} + \frac{a \tanh^{-1}(\cosh(x))}{b^2} - \frac{\operatorname{csch}(x)}{b}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^3/(a + b*Coth[x]),x]

[Out] (a*ArcTanh[Cosh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/b^2 - Csch[x]/b

Rule 3510

Int[((d_)*sec[(e_) + (f_)*(x_)])^(m_)/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> -Dist[d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e + f*x]), x], x] + Dist[(d^2*(a^2 + b^2))/b^2, Int[(d*Sec[e + f*x])^(m - 2)/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2, 0] && IGtQ[m, 1]

Rule 3486

Int[((d_)*sec[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] | NeQ[a^2 + b^2, 0])

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_.)]/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^3(x)}{a + b \operatorname{coth}(x)} dx &= -\frac{\int (a - b \operatorname{coth}(x)) \operatorname{csch}(x) dx}{b^2} + \frac{(a^2 - b^2) \int \frac{\operatorname{csch}(x)}{a + b \operatorname{coth}(x)} dx}{b^2} \\ &= -\frac{\operatorname{csch}(x)}{b} - \frac{a \int \operatorname{csch}(x) dx}{b^2} - \frac{(a^2 - b^2) \operatorname{Subst}\left(\int \frac{1}{a^2 - b^2 - x^2} dx, x, i(-ib - ia \operatorname{coth}(x)) \sinh(x)\right)}{b^2} \\ &= \frac{a \tanh^{-1}(\cosh(x))}{b^2} - \frac{\sqrt{a^2 - b^2} \tanh^{-1}\left(\frac{(b + a \operatorname{coth}(x)) \sinh(x)}{\sqrt{a^2 - b^2}}\right)}{b^2} - \frac{\operatorname{csch}(x)}{b} \end{aligned}$$

Mathematica [A] time = 0.104844, size = 65, normalized size = 1.14

$$\frac{2\sqrt{b-a}\sqrt{a+b} \tan^{-1}\left(\frac{a+b \tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right) + a \log\left(\tanh\left(\frac{x}{2}\right)\right) + b \operatorname{csch}(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Coth[x]), x]
```

```
[Out] -((2*sqrt[-a + b]*sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])]/(sqrt[-a + b]*sqrt[a + b])) + b*Csch[x] + a*Log[Tanh[x/2]])/b^2)
```

Maple [B] time = 0.028, size = 115, normalized size = 2.

$$\frac{1}{2b} \tanh\left(\frac{x}{2}\right) - \frac{1}{2b} \left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - \frac{a}{b^2} \ln\left(\tanh\left(\frac{x}{2}\right)\right) + 2 \frac{a^2}{b^2 \sqrt{-a^2 + b^2}} \arctan\left(\frac{1}{2} \frac{2 \tanh(x/2) b + 2a}{\sqrt{-a^2 + b^2}}\right) - 2 \frac{1}{\sqrt{-a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)^3/(a+b*coth(x)),x)

[Out] 1/2/b*tanh(1/2*x)-1/2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))+2/b^2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))*a^2-2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.92631, size = 1230, normalized size = 21.58

$$\left[\frac{\sqrt{a^2 - b^2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \log\left(\frac{(a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - 2\sqrt{a^2 - b^2} \cosh(x)}{(a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - a + b}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="fricas")

[Out] [(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) - 2*b*cosh(x) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (a*cosh(x)^2

```
+ 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) - 1) - 2*b*s
inh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2), (2*s
qrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(sqrt
(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 2*b*cosh(x) + (a*cosh(x)
)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x)
- 1) - 2*b*sinh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
- b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(x)}{a + b \operatorname{coth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*coth(x)), x)
```

Giac [A] time = 1.14891, size = 115, normalized size = 2.02

$$\frac{a \log(e^x + 1)}{b^2} - \frac{a \log(|e^x - 1|)}{b^2} + \frac{2(a^2 - b^2) \arctan\left(\frac{ae^x + be^x}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}b^2} - \frac{2e^x}{b(e^{2x} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1))/b^2 + 2*(a^2 - b^2)*arctan((a*e^x
+ b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - 2*e^x/(b*(e^(2*x) - 1))
```

$$3.104 \quad \int \frac{\operatorname{csch}^4(x)}{a+b \operatorname{coth}(x)} dx$$

Optimal. Leaf size=40

$$-\frac{(a^2 - b^2) \log(a + b \operatorname{coth}(x))}{b^3} + \frac{a \operatorname{coth}(x)}{b^2} - \frac{\operatorname{coth}^2(x)}{2b}$$

[Out] (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - ((a^2 - b^2)*Log[a + b*Coth[x]])/b^3

Rubi [A] time = 0.0674236, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3506, 697}

$$-\frac{(a^2 - b^2) \log(a + b \operatorname{coth}(x))}{b^3} + \frac{a \operatorname{coth}(x)}{b^2} - \frac{\operatorname{coth}^2(x)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^4/(a + b*Coth[x]),x]

[Out] (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - ((a^2 - b^2)*Log[a + b*Coth[x]])/b^3

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a + b \operatorname{coth}(x)} dx = \frac{\operatorname{Subst}\left(\int \frac{1 - \frac{x^2}{b^2}}{a+x} dx, x, b \operatorname{coth}(x)\right)}{b}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{b^2} - \frac{x}{b^2} + \frac{-a^2+b^2}{b^2(a+x)}\right) dx, x, b \operatorname{coth}(x)\right)}{b}$$

$$= \frac{a \operatorname{coth}(x)}{b^2} - \frac{\operatorname{coth}^2(x)}{2b} - \frac{(a^2 - b^2) \log(a + b \operatorname{coth}(x))}{b^3}$$

Mathematica [A] time = 0.137139, size = 50, normalized size = 1.25

$$\frac{2(a^2 - b^2)(\log(\sinh(x)) - \log(a \sinh(x) + b \cosh(x))) + 2ab \operatorname{coth}(x) - b^2 \operatorname{csch}^2(x)}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]^4/(a + b*Coth[x]), x]

[Out] (2*a*b*Coth[x] - b^2*Csch[x]^2 + 2*(a^2 - b^2)*(Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]]))/(2*b^3)

Maple [B] time = 0.031, size = 116, normalized size = 2.9

$$-\frac{1}{8b} \left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{a}{2b^2} \tanh\left(\frac{x}{2}\right) - \frac{1}{8b} \left(\tanh\left(\frac{x}{2}\right)\right)^{-2} + \frac{a^2}{b^3} \ln\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{1}{b} \ln\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{a}{2b^2} \left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - \frac{a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)^4/(a+b*coth(x)), x)

[Out] -1/8/b*tanh(1/2*x)^2+1/2/b^2*a*tanh(1/2*x)-1/8/b/tanh(1/2*x)^2+1/b^3*ln(tanh(1/2*x))*a^2-1/b*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x)-1/b^3*ln(tanh(1/2*x))^2*b+2*a*tanh(1/2*x)+b)*a^2+1/b*ln(tanh(1/2*x))^2*b+2*a*tanh(1/2*x)+b)

Maxima [B] time = 1.0331, size = 149, normalized size = 3.72

$$\frac{2((a+b)e^{(-2x)}-a)}{2b^2e^{(-2x)}-b^2e^{(-4x)}-b^2} - \frac{(a^2-b^2)\log(-(a-b)e^{(-2x)}+a+b)}{b^3} + \frac{(a^2-b^2)\log(e^{(-x)}+1)}{b^3} + \frac{(a^2-b^2)\log(e^{(-x)}-1)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] 2*((a + b)*e^(-2*x) - a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) - (a^2 - b^2)
*log(-(a - b)*e^(-2*x) + a + b)/b^3 + (a^2 - b^2)*log(e^(-x) + 1)/b^3 + (a
^2 - b^2)*log(e^(-x) - 1)/b^3
```

Fricas [B] time = 2.72258, size = 1087, normalized size = 27.18

$$2(ab - b^2) \cosh(x)^2 + 4(ab - b^2) \cosh(x) \sinh(x) + 2(ab - b^2) \sinh(x)^2 - 2ab - ((a^2 - b^2) \cosh(x)^4 + 4(a^2 - b^2) \cosh(x)^2 \sinh(x)^2 + 4(a^2 - b^2) \sinh(x)^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] (2*(a*b - b^2)*cosh(x)^2 + 4*(a*b - b^2)*cosh(x)*sinh(x) + 2*(a*b - b^2)*si
nh(x)^2 - 2*a*b - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3
+ (a^2 - b^2)*sinh(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)
)^2 - a^2 + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 -
b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) +
((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*sin
h(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 - a^2 + b^2)*
sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 - b^2)*cosh(x))*sin
h(x))*log(2*sinh(x)/(cosh(x) - sinh(x)))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)*si
nh(x)^3 + b^3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)
*sinh(x)^2 + 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(x)}{a + b \operatorname{coth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*coth(x)),x)
```


[Out] Integral(csch(x)**4/(a + b*coth(x)), x)

Giac [B] time = 1.17598, size = 143, normalized size = 3.58

$$-\frac{(a^3 + a^2b - ab^2 - b^3) \log(|ae^{(2x)} + be^{(2x)} - a + b|)}{ab^3 + b^4} + \frac{(a^2 - b^2) \log(|e^{(2x)} - 1|)}{b^3} - \frac{2(ab - (ab - b^2)e^{(2x)})}{b^3(e^{(2x)} - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="giac")

[Out] $-(a^3 + a^2b - a*b^2 - b^3)*\log(\text{abs}(a*e^{(2*x)} + b*e^{(2*x)} - a + b))/(a*b^3 + b^4) + (a^2 - b^2)*\log(\text{abs}(e^{(2*x)} - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^{(2*x)})/(b^3*(e^{(2*x)} - 1)^2)$

$$3.105 \quad \int \frac{\cosh^4(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=60

$$\frac{x}{16} - \frac{1}{8(1 - \coth(x))} - \frac{3}{16(\coth(x) + 1)} + \frac{1}{32(1 - \coth(x))^2} + \frac{5}{32(\coth(x) + 1)^2} - \frac{1}{24(\coth(x) + 1)^3}$$

[Out] x/16 + 1/(32*(1 - Coth[x])^2) - 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x])^3) + 5/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))

Rubi [A] time = 0.0669269, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3516, 848, 88, 207}

$$\frac{x}{16} - \frac{1}{8(1 - \coth(x))} - \frac{3}{16(\coth(x) + 1)} + \frac{1}{32(1 - \coth(x))^2} + \frac{5}{32(\coth(x) + 1)^2} - \frac{1}{24(\coth(x) + 1)^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^4/(1 + Coth[x]),x]

[Out] x/16 + 1/(32*(1 - Coth[x])^2) - 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x])^3) + 5/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 848

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x]]

$x)^p, x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \text{IntegersQ}\{m, n\} \&\& (\text{IntegerQ}\{p\} \mid\mid (\text{GtQ}\{m, 0\} \&\& \text{GeQ}\{n, -1\}))$

Rule 207

$\text{Int}[(a_ + (b_ \cdot x)^{-2})^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2] \cdot x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2] \cdot \text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b] \&\& (\text{LtQ}[a, 0] \mid\mid \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\cosh^4(x)}{1 + \coth(x)} dx &= -\text{Subst} \left(\int \frac{x^4}{(1+x)(-1+x^2)^3} dx, x, \coth(x) \right) \\ &= -\text{Subst} \left(\int \frac{x^4}{(-1+x)^3(1+x)^4} dx, x, \coth(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{1}{16(-1+x)^3} + \frac{1}{8(-1+x)^2} - \frac{1}{8(1+x)^4} + \frac{5}{16(1+x)^3} - \frac{3}{16(1+x)^2} + \frac{1}{16(-1+x^2)} \right) dx, x, \coth(x) \right) \\ &= \frac{1}{32(1 - \coth(x))^2} - \frac{1}{8(1 - \coth(x))} - \frac{1}{24(1 + \coth(x))^3} + \frac{5}{32(1 + \coth(x))^2} - \frac{3}{16(1 + \coth(x))} - \frac{1}{16} \\ &= \frac{x}{16} + \frac{1}{32(1 - \coth(x))^2} - \frac{1}{8(1 - \coth(x))} - \frac{1}{24(1 + \coth(x))^3} + \frac{5}{32(1 + \coth(x))^2} - \frac{3}{16(1 + \coth(x))} \end{aligned}$$

Mathematica [A] time = 0.088029, size = 42, normalized size = 0.7

$$\frac{1}{192}(12x + 3 \sinh(2x) - 3 \sinh(4x) - \sinh(6x) + 15 \cosh(2x) + 6 \cosh(4x) + \cosh(6x))$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^4/(1 + Coth[x]), x]

[Out] (12*x + 15*Cosh[2*x] + 6*Cosh[4*x] + Cosh[6*x] + 3*Sinh[2*x] - 3*Sinh[4*x] - Sinh[6*x])/192

Maple [B] time = 0.032, size = 118, normalized size = 2.

$$\frac{1}{3} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-6} - \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-5} + \frac{13}{8} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-4} - \frac{19}{12} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-3} + \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-2} - \frac{3}{8} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(x)^4/(1+coth(x)),x)`

[Out] $\frac{1}{3}/(\tanh(1/2*x)+1)^6 - 1/(\tanh(1/2*x)+1)^5 + 13/8/(\tanh(1/2*x)+1)^4 - 19/12/(\tanh(1/2*x)+1)^3 + 1/(\tanh(1/2*x)+1)^2 - 3/8/(\tanh(1/2*x)+1) + 1/16*\ln(\tanh(1/2*x)+1) + 1/8/(\tanh(1/2*x)-1)^4 + 1/4/(\tanh(1/2*x)-1)^3 + 3/8/(\tanh(1/2*x)-1)^2 + 1/4/(\tanh(1/2*x)-1) - 1/16*\ln(\tanh(1/2*x)-1)$

Maxima [A] time = 0.997534, size = 49, normalized size = 0.82

$$\frac{1}{128} (6e^{(-2x)} + 1)e^{(4x)} + \frac{1}{16}x + \frac{1}{32}e^{(-2x)} + \frac{3}{128}e^{(-4x)} + \frac{1}{192}e^{(-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^4/(1+coth(x)),x, algorithm="maxima")`

[Out] $\frac{1}{128}*(6*e^{(-2*x)} + 1)*e^{(4*x)} + \frac{1}{16}*x + \frac{1}{32}*e^{(-2*x)} + \frac{3}{128}*e^{(-4*x)} + \frac{1}{192}*e^{(-6*x)}$

Fricas [B] time = 2.50562, size = 319, normalized size = 5.32

$$\frac{5 \cosh(x)^5 + 25 \cosh(x) \sinh(x)^4 + \sinh(x)^5 + (10 \cosh(x)^2 + 9) \sinh(x)^3 + 27 \cosh(x)^3 + (50 \cosh(x)^3 + 81 \cosh(x)) \sinh(x)}{384 (\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^4/(1+coth(x)),x, algorithm="fricas")`

[Out] $\frac{1}{384}*(5*\cosh(x)^5 + 25*\cosh(x)*\sinh(x)^4 + \sinh(x)^5 + (10*\cosh(x)^2 + 9)*\sinh(x)^3 + 27*\cosh(x)^3 + (50*\cosh(x)^3 + 81*\cosh(x))*\sinh(x)^2 + 12*(2*x + 1)*\cosh(x) + (5*\cosh(x)^4 + 27*\cosh(x)^2 + 24*x - 12)*\sinh(x))/(\cosh(x) + \sinh(x))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^4(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**4/(1+coth(x)),x)

[Out] Integral(cosh(x)**4/(coth(x) + 1), x)

Giac [A] time = 1.17667, size = 57, normalized size = 0.95

$$-\frac{1}{384} (22e^{(6x)} - 12e^{(4x)} - 9e^{(2x)} - 2)e^{(-6x)} + \frac{1}{16}x + \frac{1}{128}e^{(4x)} + \frac{3}{64}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="giac")

[Out] -1/384*(22*e^(6*x) - 12*e^(4*x) - 9*e^(2*x) - 2)*e^(-6*x) + 1/16*x + 1/128*e^(4*x) + 3/64*e^(2*x)

$$3.106 \quad \int \frac{\cosh^3(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=25

$$-\frac{\sinh^5(x)}{5} - \frac{\sinh^3(x)}{3} + \frac{\cosh^5(x)}{5}$$

[Out] Cosh[x]^5/5 - Sinh[x]^3/3 - Sinh[x]^5/5

Rubi [A] time = 0.17683, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3518, 3108, 3107, 2565, 30, 2564, 14}

$$-\frac{\sinh^5(x)}{5} - \frac{\sinh^3(x)}{3} + \frac{\cosh^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^3/(1 + Coth[x]),x]

[Out] Cosh[x]^5/5 - Sinh[x]^3/3 - Sinh[x]^5/5

Rule 3518

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3108

Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> Dist[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILtQ[p, 0]

Rule 3107

Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In

```
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{1 + \coth(x)} dx &= - \left(i \int \frac{\cosh^3(x) \sinh(x)}{-i \cosh(x) - i \sinh(x)} dx \right) \\
&= - \int \cosh^3(x) \sinh(x) (-\cosh(x) + \sinh(x)) dx \\
&= i \int (-i \cosh^4(x) \sinh(x) + i \cosh^3(x) \sinh^2(x)) dx \\
&= \int \cosh^4(x) \sinh(x) dx - \int \cosh^3(x) \sinh^2(x) dx \\
&= - \left(i \text{Subst} \left(\int x^2 (1 - x^2) dx, x, i \sinh(x) \right) \right) + \text{Subst} \left(\int x^4 dx, x, \cosh(x) \right) \\
&= \frac{\cosh^5(x)}{5} - i \text{Subst} \left(\int (x^2 - x^4) dx, x, i \sinh(x) \right) \\
&= \frac{\cosh^5(x)}{5} - \frac{\sinh^3(x)}{3} - \frac{\sinh^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.0579108, size = 34, normalized size = 1.36

$$\frac{1}{120} (\cosh(x) - \sinh(x))(10 \sinh(2x) + \sinh(4x) + 20 \cosh(2x) + 4 \cosh(4x))$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^3/(1 + Coth[x]), x]

[Out] ((Cosh[x] - Sinh[x])*(20*Cosh[2*x] + 4*Cosh[4*x] + 10*Sinh[2*x] + Sinh[4*x]))/120

Maple [B] time = 0.028, size = 82, normalized size = 3.3

$$-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4} + \frac{2}{5}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5} + \frac{4}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} - \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \frac{3}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{1}{6}\left(\tanh\left(\frac{x}{2}\right)+1\right)^0$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^3/(1+coth(x)), x)

[Out] -1/(tanh(1/2*x)+1)^4+2/5/(tanh(1/2*x)+1)^5+4/3/(tanh(1/2*x)+1)^3-1/(tanh(1/2*x)+1)^2+3/8/(tanh(1/2*x)+1)-1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2-3/8/(tanh(1/2*x)-1)

Maxima [A] time = 1.03129, size = 36, normalized size = 1.44

$$\frac{1}{48} (6e^{-2x} + 1)e^{3x} + \frac{1}{24} e^{-3x} + \frac{1}{80} e^{-5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="maxima")

[Out] 1/48*(6*e^(-2*x) + 1)*e^(3*x) + 1/24*e^(-3*x) + 1/80*e^(-5*x)

Fricas [B] time = 2.53558, size = 200, normalized size = 8.

$$\frac{\cosh(x)^4 + \cosh(x)\sinh(x)^3 + \sinh(x)^4 + (6\cosh(x)^2 + 5)\sinh(x)^2 + 5\cosh(x)^2 + (\cosh(x)^3 + 5\cosh(x))\sinh(x)}{30(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="fricas")

[Out] 1/30*(cosh(x)^4 + cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 5)*sinh(x)^2 + 5*cosh(x)^2 + (cosh(x)^3 + 5*cosh(x))*sinh(x))/(cosh(x) + sinh(x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**3/(1+coth(x)),x)

[Out] Integral(cosh(x)**3/(coth(x) + 1), x)

Giac [A] time = 1.18732, size = 34, normalized size = 1.36

$$\frac{1}{240} (10 e^{2x} + 3) e^{-5x} + \frac{1}{48} e^{3x} + \frac{1}{8} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="giac")

[Out] 1/240*(10*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) + 1/8*e^x

$$3.107 \quad \int \frac{\cosh^2(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=38

$$\frac{x}{8} - \frac{1}{8(1 - \coth(x))} - \frac{1}{4(\coth(x) + 1)} + \frac{1}{8(\coth(x) + 1)^2}$$

[Out] x/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))

Rubi [A] time = 0.0592763, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3516, 848, 88, 207}

$$\frac{x}{8} - \frac{1}{8(1 - \coth(x))} - \frac{1}{4(\coth(x) + 1)} + \frac{1}{8(\coth(x) + 1)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^2/(1 + Coth[x]), x]

[Out] x/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 848

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{1 + \coth(x)} dx &= -\text{Subst} \left(\int \frac{x^2}{(1+x)(-1+x^2)^2} dx, x, \coth(x) \right) \\
&= -\text{Subst} \left(\int \frac{x^2}{(-1+x)^2(1+x)^3} dx, x, \coth(x) \right) \\
&= -\text{Subst} \left(\int \left(\frac{1}{8(-1+x)^2} + \frac{1}{4(1+x)^3} - \frac{1}{4(1+x)^2} + \frac{1}{8(-1+x^2)} \right) dx, x, \coth(x) \right) \\
&= -\frac{1}{8(1 - \coth(x))} + \frac{1}{8(1 + \coth(x))^2} - \frac{1}{4(1 + \coth(x))} - \frac{1}{8} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \coth(x) \right) \\
&= \frac{x}{8} - \frac{1}{8(1 - \coth(x))} + \frac{1}{8(1 + \coth(x))^2} - \frac{1}{4(1 + \coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.047073, size = 24, normalized size = 0.63

$$\frac{1}{32}(4x - \sinh(4x) + 4 \cosh(2x) + \cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 + Coth[x]), x]
```

```
[Out] (4*x + 4*Cosh[2*x] + Cosh[4*x] - Sinh[4*x])/32
```

Maple [B] time = 0.029, size = 78, normalized size = 2.1

$$\frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-4} - \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-3} + \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-2} - \frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1} + \frac{1}{8} \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + \frac{1}{4} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1+coth(x)), x)
```

[Out] $\frac{1}{2}/(\tanh(1/2*x)+1)^4 - 1/(\tanh(1/2*x)+1)^3 + 1/(\tanh(1/2*x)+1)^2 - 1/2/(\tanh(1/2*x)+1) + 1/8*\ln(\tanh(1/2*x)+1) + 1/4/(\tanh(1/2*x)-1)^2 + 1/4/(\tanh(1/2*x)-1) - 1/8*\ln(\tanh(1/2*x)-1)$

Maxima [A] time = 1.02968, size = 30, normalized size = 0.79

$$\frac{1}{8}x + \frac{1}{16}e^{(2x)} + \frac{1}{16}e^{(-2x)} + \frac{1}{32}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^2/(1+coth(x)),x, algorithm="maxima")`

[Out] $1/8*x + 1/16*e^{(2*x)} + 1/16*e^{(-2*x)} + 1/32*e^{(-4*x)}$

Fricas [A] time = 2.53566, size = 176, normalized size = 4.63

$$\frac{3 \cosh(x)^3 + 9 \cosh(x) \sinh(x)^2 + \sinh(x)^3 + 2(2x + 1) \cosh(x) + (3 \cosh(x)^2 + 4x - 2) \sinh(x)}{32(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^2/(1+coth(x)),x, algorithm="fricas")`

[Out] $1/32*(3*\cosh(x)^3 + 9*\cosh(x)*\sinh(x)^2 + \sinh(x)^3 + 2*(2*x + 1)*\cosh(x) + (3*\cosh(x)^2 + 4*x - 2)*\sinh(x))/(\cosh(x) + \sinh(x))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)**2/(1+coth(x)),x)`

[Out] `Integral(cosh(x)**2/(coth(x) + 1), x)`

Giac [A] time = 1.11309, size = 41, normalized size = 1.08

$$-\frac{1}{32} (3e^{4x} - 2e^{2x} - 1)e^{-4x} + \frac{1}{8}x + \frac{1}{16}e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="giac")

[Out] -1/32*(3*e^(4*x) - 2*e^(2*x) - 1)*e^(-4*x) + 1/8*x + 1/16*e^(2*x)

$$3.108 \quad \int \frac{\cosh(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=17

$$\frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

[Out] Cosh[x]^3/3 - Sinh[x]^3/3

Rubi [A] time = 0.113401, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3518, 3108, 3107, 2565, 30, 2564}

$$\frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(1 + Coth[x]),x]

[Out] Cosh[x]^3/3 - Sinh[x]^3/3

Rule 3518

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3108

Int[cos[(c_.) + (d_.)*(x_.)]^(m_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.)*(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(p_.), x_Symbol] := Dist[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILtQ[p, 0]

Rule 3107

Int[cos[(c_.) + (d_.)*(x_.)]^(m_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.)*(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(p_.), x_Symbol] := In

```
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_.)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(x)}{1 + \coth(x)} dx &= - \left(i \int \frac{\cosh(x) \sinh(x)}{-i \cosh(x) - i \sinh(x)} dx \right) \\
 &= - \int \cosh(x) \sinh(x) (-\cosh(x) + \sinh(x)) dx \\
 &= i \int (-i \cosh^2(x) \sinh(x) + i \cosh(x) \sinh^2(x)) dx \\
 &= \int \cosh^2(x) \sinh(x) dx - \int \cosh(x) \sinh^2(x) dx \\
 &= - \left(i \operatorname{Subst} \left(\int x^2 dx, x, i \sinh(x) \right) \right) + \operatorname{Subst} \left(\int x^2 dx, x, \cosh(x) \right) \\
 &= \frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}
 \end{aligned}$$

Mathematica [A] time = 0.0166151, size = 19, normalized size = 1.12

$$\frac{1}{12} (-4 \sinh^3(x) + 3 \cosh(x) + \cosh(3x))$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]/(1 + Coth[x]),x]

[Out] (3*Cosh[x] + Cosh[3*x] - 4*Sinh[x]^3)/12

Maple [B] time = 0.024, size = 42, normalized size = 2.5

$$\frac{2}{3} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-3} - \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-2} + \frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1} - \frac{1}{2} \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)/(1+coth(x)),x)

[Out] 2/3/(tanh(1/2*x)+1)^3-1/(tanh(1/2*x)+1)^2+1/2/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)-1)

Maxima [A] time = 1.01463, size = 15, normalized size = 0.88

$$\frac{1}{12} e^{(-3x)} + \frac{1}{4} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="maxima")

[Out] 1/12*e^(-3*x) + 1/4*e^x

Fricas [A] time = 2.48556, size = 90, normalized size = 5.29

$$\frac{\cosh(x)^2 + \cosh(x)\sinh(x) + \sinh(x)^2}{3(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="fricas")

[Out] 1/3*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) + sinh(x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(1+coth(x)),x)

[Out] Integral(cosh(x)/(coth(x) + 1), x)

Giac [A] time = 1.13034, size = 15, normalized size = 0.88

$$\frac{1}{12} e^{(-3x)} + \frac{1}{4} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="giac")

[Out] 1/12*e^(-3*x) + 1/4*e^x

$$3.109 \quad \int \frac{\operatorname{sech}(x)}{1+\operatorname{coth}(x)} dx$$

Optimal. Leaf size=10

$$-\sinh(x) + \cosh(x) + \tan^{-1}(\sinh(x))$$

[Out] ArcTan[Sinh[x]] + Cosh[x] - Sinh[x]

Rubi [A] time = 0.114155, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3518, 3108, 3107, 2638, 2592, 321, 203}

$$-\sinh(x) + \cosh(x) + \tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[Sech[x]/(1 + Coth[x]),x]

[Out] ArcTan[Sinh[x]] + Cosh[x] - Sinh[x]

Rule 3518

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3108

Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> Dist[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILtQ[p, 0]

Rule 3107

Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]

)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a² - ff²*x²)^{((n + 1)/2)}, x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 321

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*xⁿ)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*cⁿ*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*xⁿ)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 203

Int[((a_) + (b_.)*(x_)²)⁽⁻¹⁾, x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{1 + \operatorname{coth}(x)} dx &= - \left(i \int \frac{\tanh(x)}{-i \cosh(x) - i \sinh(x)} dx \right) \\
&= - \int (-\cosh(x) + \sinh(x)) \tanh(x) dx \\
&= i \int (-i \sinh(x) + i \sinh(x) \tanh(x)) dx \\
&= \int \sinh(x) dx - \int \sinh(x) \tanh(x) dx \\
&= \cosh(x) - \operatorname{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, \sinh(x) \right) \\
&= \cosh(x) - \sinh(x) + \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sinh(x) \right) \\
&= \tan^{-1}(\sinh(x)) + \cosh(x) - \sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.0233032, size = 16, normalized size = 1.6

$$-\sinh(x) + \cosh(x) + 2 \tan^{-1} \left(\tanh \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]/(1 + Coth[x]), x]

[Out] 2*ArcTan[Tanh[x/2]] + Cosh[x] - Sinh[x]

Maple [A] time = 0.023, size = 19, normalized size = 1.9

$$2 \arctan(\tanh(x/2)) + 2 (\tanh(x/2) + 1)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)/(1+coth(x)), x)

[Out] 2*arctan(tanh(1/2*x))+2/(tanh(1/2*x)+1)

Maxima [A] time = 1.53425, size = 16, normalized size = 1.6

$$-2 \arctan(e^{-x}) + e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)/(1+coth(x)),x, algorithm="maxima")`

[Out] `-2*arctan(e^(-x)) + e^(-x)`

Fricas [B] time = 2.57969, size = 101, normalized size = 10.1

$$\frac{2(\cosh(x) + \sinh(x)) \arctan(\cosh(x) + \sinh(x)) + 1}{\cosh(x) + \sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)/(1+coth(x)),x, algorithm="fricas")`

[Out] `(2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) + 1)/(cosh(x) + sinh(x))`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{\operatorname{coth}(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)/(1+coth(x)),x)`

[Out] `Integral(sech(x)/(coth(x) + 1), x)`

Giac [A] time = 1.16349, size = 14, normalized size = 1.4

$$2 \arctan(e^x) + e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)/(1+coth(x)),x, algorithm="giac")`

[Out] `2*arctan(e^x) + e^(-x)`

$$3.110 \quad \int \frac{\operatorname{sech}^2(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=15

$$\tanh(x) - \log(\tanh(x)) - \log(\coth(x) + 1)$$

[Out] -Log[1 + Coth[x]] - Log[Tanh[x]] + Tanh[x]

Rubi [A] time = 0.0411416, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3516, 44}

$$\tanh(x) - \log(\tanh(x)) - \log(\coth(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/(1 + Coth[x]),x]

[Out] -Log[1 + Coth[x]] - Log[Tanh[x]] + Tanh[x]

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{1 + \operatorname{coth}(x)} dx &= -\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)} dx, x, \operatorname{coth}(x)\right) \\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{1}{1+x}\right) dx, x, \operatorname{coth}(x)\right) \\
&= -\log(1 + \operatorname{coth}(x)) - \log(\tanh(x)) + \tanh(x)
\end{aligned}$$

Mathematica [A] time = 0.0296206, size = 9, normalized size = 0.6

$$-x + \tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^2/(1 + Coth[x]), x]

[Out] -x + Log[Cosh[x]] + Tanh[x]

Maple [B] time = 0.028, size = 36, normalized size = 2.4

$$-2 \ln(\tanh(x/2) + 1) + 2 \frac{\tanh(x/2)}{(\tanh(x/2))^2 + 1} + \ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^2/(1+coth(x)), x)

[Out] -2*ln(tanh(1/2*x)+1)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)+ln(tanh(1/2*x)^2+1)

Maxima [A] time = 1.56431, size = 24, normalized size = 1.6

$$\frac{2}{e^{(-2x)} + 1} + \log(e^{(-2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^2/(1+coth(x)), x, algorithm="maxima")

[Out] $2/(e^{-2x} + 1) + \log(e^{-2x} + 1)$

Fricas [B] time = 2.6983, size = 269, normalized size = 17.93

$$\frac{2x \cosh(x)^2 + 4x \cosh(x) \sinh(x) + 2x \sinh(x)^2 - (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right)}{\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)^2/(1+coth(x)),x, algorithm="fricas")`

[Out] $-(2x \cosh(x)^2 + 4x \cosh(x) \sinh(x) + 2x \sinh(x)^2 - (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \log(2 \cosh(x) / (\cosh(x) - \sinh(x))) + 2x + 2) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{\operatorname{coth}(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)**2/(1+coth(x)),x)`

[Out] `Integral(sech(x)**2/(coth(x) + 1), x)`

Giac [A] time = 1.18812, size = 36, normalized size = 2.4

$$-2x - \frac{e^{(2x)} + 3}{e^{(2x)} + 1} + \log(e^{(2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)^2/(1+coth(x)),x, algorithm="giac")`

[Out] $-2x - (e^{(2x)} + 3)/(e^{(2x)} + 1) + \log(e^{(2x)} + 1)$

$$3.111 \quad \int \frac{\operatorname{sech}^3(x)}{1+\operatorname{coth}(x)} dx$$

Optimal. Leaf size=20

$$-\operatorname{sech}(x) - \frac{1}{2} \tan^{-1}(\sinh(x)) + \frac{1}{2} \tanh(x)\operatorname{sech}(x)$$

[Out] `-ArcTan[Sinh[x]]/2 - Sech[x] + (Sech[x]*Tanh[x])/2`

Rubi [A] time = 0.167309, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3518, 3108, 3107, 2606, 8, 2611, 3770}

$$-\operatorname{sech}(x) - \frac{1}{2} \tan^{-1}(\sinh(x)) + \frac{1}{2} \tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

[In] `Int[Sech[x]^3/(1 + Coth[x]), x]`

[Out] `-ArcTan[Sinh[x]]/2 - Sech[x] + (Sech[x]*Tanh[x])/2`

Rule 3518

`Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*cos[e + f*x] + b*sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))`

Rule 3108

`Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> Dist[a^p*b^p, Int[(Cos[c + d*x]^m*sin[c + d*x]^n)/(b*cos[c + d*x] + a*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILtQ[p, 0]`

Rule 3107

`Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x])^p, x], x]`

)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :=> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 8

Int[a_, x_Symbol] :=> Simp[a*x, x] /; FreeQ[a, x]

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :=> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :=> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}^3(x)}{1 + \operatorname{coth}(x)} dx &= - \left(i \int \frac{\operatorname{sech}^2(x) \tanh(x)}{-i \cosh(x) - i \sinh(x)} dx \right) \\
 &= - \int \operatorname{sech}^2(x) (-\cosh(x) + \sinh(x)) \tanh(x) dx \\
 &= i \int (-i \operatorname{sech}(x) \tanh(x) + i \operatorname{sech}(x) \tanh^2(x)) dx \\
 &= \int \operatorname{sech}(x) \tanh(x) dx - \int \operatorname{sech}(x) \tanh^2(x) dx \\
 &= \frac{1}{2} \operatorname{sech}(x) \tanh(x) - \frac{1}{2} \int \operatorname{sech}(x) dx - \operatorname{Subst} \left(\int 1 dx, x, \operatorname{sech}(x) \right) \\
 &= -\frac{1}{2} \tan^{-1}(\sinh(x)) - \operatorname{sech}(x) + \frac{1}{2} \operatorname{sech}(x) \tanh(x)
 \end{aligned}$$

Mathematica [A] time = 0.0391703, size = 20, normalized size = 1.

$$\frac{1}{2}(\tanh(x) - 2)\operatorname{sech}(x) - \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^3/(1 + Coth[x]), x]

[Out] -ArcTan[Tanh[x/2]] + (Sech[x]*(-2 + Tanh[x]))/2

Maple [B] time = 0.029, size = 45, normalized size = 2.3

$$4 \frac{-1/4 (\tanh(x/2))^3 - 1/2 (\tanh(x/2))^2 + 1/4 \tanh(x/2) - 1/2}{((\tanh(x/2))^2 + 1)^2} - \arctan\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^3/(1+coth(x)), x)

[Out] 4*(-1/4*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2+1/4*tanh(1/2*x)-1/2)/(tanh(1/2*x)^2+1)^2-arctan(tanh(1/2*x))

Maxima [B] time = 1.52306, size = 45, normalized size = 2.25

$$-\frac{e^{-x} + 3e^{-3x}}{2e^{-2x} + e^{-4x} + 1} + \arctan(e^{-x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^3/(1+coth(x)), x, algorithm="maxima")

[Out] -(e^(-x) + 3*e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + arctan(e^(-x))

Fricas [B] time = 2.5068, size = 508, normalized size = 25.4

$$\frac{\cosh(x)^3 + 3 \cosh(x) \sinh(x)^2 + \sinh(x)^3 + (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x))}{\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)^3/(1+coth(x)),x, algorithm="fricas")`

[Out] $-(\cosh(x)^3 + 3*\cosh(x)*\sinh(x)^2 + \sinh(x)^3 + (\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)*\arctan(\cosh(x) + \sinh(x)) + 3*(\cosh(x)^2 + 1)*\sinh(x) + 3*\cosh(x))/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{\operatorname{coth}(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)**3/(1+coth(x)),x)`

[Out] `Integral(sech(x)**3/(coth(x) + 1), x)`

Giac [A] time = 1.14922, size = 34, normalized size = 1.7

$$-\frac{e^{(3x)} + 3e^x}{(e^{(2x)} + 1)^2} - \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)^3/(1+coth(x)),x, algorithm="giac")`

[Out] $-(e^{(3*x)} + 3*e^x)/(e^{(2*x)} + 1)^2 - \arctan(e^x)$

$$3.112 \quad \int \frac{\operatorname{sech}^4(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=17

$$\frac{\tanh^2(x)}{2} - \frac{\tanh^3(x)}{3}$$

[Out] Tanh[x]^2/2 - Tanh[x]^3/3

Rubi [A] time = 0.0454175, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3516, 848, 43}

$$\frac{\tanh^2(x)}{2} - \frac{\tanh^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^4/(1 + Coth[x]),x]

[Out] Tanh[x]^2/2 - Tanh[x]^3/3

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 848

Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le

Q[7*m + 4*n + 4, 0] || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}^4(x)}{1 + \operatorname{coth}(x)} dx &= -\operatorname{Subst} \left(\int \frac{-1 + x^2}{x^4(1 + x)} dx, x, \operatorname{coth}(x) \right) \\
 &= -\operatorname{Subst} \left(\int \frac{-1 + x}{x^4} dx, x, \operatorname{coth}(x) \right) \\
 &= -\operatorname{Subst} \left(\int \left(-\frac{1}{x^4} + \frac{1}{x^3} \right) dx, x, \operatorname{coth}(x) \right) \\
 &= \frac{\tanh^2(x)}{2} - \frac{\tanh^3(x)}{3}
 \end{aligned}$$

Mathematica [A] time = 0.0498663, size = 17, normalized size = 1.

$$\frac{1}{6} (-2 \tanh^3(x) - 3 \operatorname{sech}^2(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^4/(1 + Coth[x]), x]

[Out] (-3*Sech[x]^2 - 2*Tanh[x]^3)/6

Maple [B] time = 0.029, size = 38, normalized size = 2.2

$$-4 \frac{-1/2 (\tanh(x/2))^4 + 2/3 (\tanh(x/2))^3 - 1/2 (\tanh(x/2))^2}{((\tanh(x/2))^2 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^4/(1+coth(x)), x)

[Out] -4*(-1/2*tanh(1/2*x)^4+2/3*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2)/(tanh(1/2*x)^2+1)^3

Maxima [B] time = 1.03484, size = 101, normalized size = 5.94

$$\frac{2e^{-2x}}{3e^{-2x} + 3e^{-4x} + e^{-6x} + 1} - \frac{4e^{-4x}}{3e^{-2x} + 3e^{-4x} + e^{-6x} + 1} - \frac{2}{3(3e^{-2x} + 3e^{-4x} + e^{-6x} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="maxima")

[Out] $-2e^{-2x}/(3e^{-2x} + 3e^{-4x} + e^{-6x} + 1) - 4e^{-4x}/(3e^{-2x} + 3e^{-4x} + e^{-6x} + 1) - 2/3/(3e^{-2x} + 3e^{-4x} + e^{-6x} + 1)$

Fricas [B] time = 2.52306, size = 286, normalized size = 16.82

$$\frac{4(\cosh(x) + 2\sinh(x))}{3(\cosh(x)^5 + 5\cosh(x)\sinh(x)^4 + \sinh(x)^5 + (10\cosh(x)^2 + 3)\sinh(x)^3 + 3\cosh(x)^3 + (10\cosh(x)^3 + 9\cosh(x))\sinh(x)^2 + (5\cosh(x)^4 + 9\cosh(x)^2 + 2)\sinh(x) + 4\cosh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="fricas")

[Out] $-4/3*(\cosh(x) + 2*\sinh(x))/(\cosh(x)^5 + 5*\cosh(x)*\sinh(x)^4 + \sinh(x)^5 + (10*\cosh(x)^2 + 3)*\sinh(x)^3 + 3*\cosh(x)^3 + (10*\cosh(x)^3 + 9*\cosh(x))*\sinh(x)^2 + (5*\cosh(x)^4 + 9*\cosh(x)^2 + 2)*\sinh(x) + 4*\cosh(x))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{\operatorname{coth}(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)**4/(1+coth(x)),x)

[Out] Integral(sech(x)**4/(coth(x) + 1), x)

Giac [A] time = 1.14445, size = 24, normalized size = 1.41

$$-\frac{2(3e^{2x}-1)}{3(e^{2x}+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="giac")

[Out] -2/3*(3*e^(2*x) - 1)/(e^(2*x) + 1)^3

3.113 $\int \sqrt{1 + \coth(x)} \operatorname{sech}^2(x) dx$

Optimal. Leaf size=21

$$\tanh^{-1}(\sqrt{\coth(x)+1}) + \tanh(x)\sqrt{\coth(x)+1}$$

[Out] ArcTanh[Sqrt[1 + Coth[x]]] + Sqrt[1 + Coth[x]]*Tanh[x]

Rubi [A] time = 0.045848, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3516, 47, 63, 207}

$$\tanh^{-1}(\sqrt{\coth(x)+1}) + \tanh(x)\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Coth[x]]*Sech[x]^2,x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]] + Sqrt[1 + Coth[x]]*Tanh[x]

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{1 + \coth(x)} \operatorname{sech}^2(x) dx &= -\operatorname{Subst} \left(\int \frac{\sqrt{1+x}}{x^2} dx, x, \coth(x) \right) \\ &= \sqrt{1 + \coth(x)} \tanh(x) - \frac{1}{2} \operatorname{Subst} \left(\int \frac{1}{x\sqrt{1+x}} dx, x, \coth(x) \right) \\ &= \sqrt{1 + \coth(x)} \tanh(x) - \operatorname{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\ &= \tanh^{-1}(\sqrt{1 + \coth(x)}) + \sqrt{1 + \coth(x)} \tanh(x) \end{aligned}$$

Mathematica [C] time = 5.00565, size = 160, normalized size = 7.62

$$\frac{1}{2} \sqrt{\coth(x) + 1} \left(2 \tanh(x) + \frac{(1-i) \tan^{-1} \left(\left(\frac{1}{2} + \frac{i}{2} \right) \sqrt{i(\coth(x) + 1)} \right)}{\sqrt{i(\coth(x) + 1)}} \right) + \frac{\sinh\left(\frac{x}{2}\right) \left(4 \tanh^{-1} \left(\sqrt{\tanh\left(\frac{x}{2}\right)} \right) + \sqrt{2} \left(\log \left(\tanh\left(\frac{x}{2}\right) \right) \right) \right)}{\sqrt{i(\coth(x) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Coth[x]]*Sech[x]^2,x]

[Out] (Sqrt[1 + Coth[x]]*(((1 - I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]))/Sqrt[I*(1 + Coth[x])] + ((4*ArcTanh[Sqrt[Tanh[x/2]]] + Sqrt[2]*(Log[1 - Sqrt[2]*Sqrt[Tanh[x/2]] + Tanh[x/2]] - Log[1 + Sqrt[2]*Sqrt[Tanh[x/2]] + Tanh[x/2]]))*Sinh[x/2]*(-Cosh[x/2] + Sinh[x/2]))/Sqrt[Tanh[x/2]] + 2*Tanh[x])/2

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(x))^2 \sqrt{1 + \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(x)^2*(1+coth(x))^(1/2),x)`

[Out] `int(sech(x)^2*(1+coth(x))^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x) + 1} \operatorname{sech}(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(coth(x) + 1)*sech(x)^2, x)`

Fricas [B] time = 2.55175, size = 813, normalized size = 38.71

$$4\sqrt{2}(\sqrt{2}\cosh(x) + \sqrt{2}\sinh(x))\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} + (\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1)\log\left(\frac{2\sqrt{2}(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x))\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} + (\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1)}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")`

[Out] `1/4*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((2*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(-(2*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x)))) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)**2*(1+coth(x))**(1/2),x)

[Out] Timed out

Giac [B] time = 1.16151, size = 201, normalized size = 9.57

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2\sqrt{2}-\log\left(-\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)\right)+\sqrt{2}\log\left(\frac{\left(\sqrt{e^{(2x)}-1}-e^x\right)^2-2\sqrt{2}+3}{\left(\sqrt{e^{(2x)}-1}-e^x\right)^2+2\sqrt{2}+3}\right)-\frac{8\left(3\left(\sqrt{e^{(2x)}-1}-e^x\right)^2+1\right)}{\left(\sqrt{e^{(2x)}-1}-e^x\right)^4+6\left(\sqrt{e^{(2x)}-1}-e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")

[Out] -1/4*sqrt(2)*(sqrt(2)*(2*sqrt(2) - log(-(sqrt(2) - 1)/(sqrt(2) + 1))) + sqrt(2)*log(((sqrt(e^(2*x) - 1) - e^x)^2 - 2*sqrt(2) + 3)/((sqrt(e^(2*x) - 1) - e^x)^2 + 2*sqrt(2) + 3)) - 8*(3*(sqrt(e^(2*x) - 1) - e^x)^2 + 1)/((sqrt(e^(2*x) - 1) - e^x)^4 + 6*(sqrt(e^(2*x) - 1) - e^x)^2 + 1))*sgn(e^(2*x) - 1)

$$3.114 \quad \int \frac{\cosh^4(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=147

$$\frac{a^4 b \log(a + b \coth(x))}{(a^2 - b^2)^3} - \frac{\sinh^4(x)(b - a \coth(x))}{4(a^2 - b^2)} - \frac{\sinh^2(x)(4b(2a^2 - b^2) - a(5a^2 - b^2) \coth(x))}{8(a^2 - b^2)^2} - \frac{a(3a + b) \log(1 - \coth(x))}{16(a + b)}$$

[Out] $-(a*(3*a + b)*\text{Log}[1 - \text{Coth}[x]])/(16*(a + b)^3) + (a*(3*a - b)*\text{Log}[1 + \text{Coth}[x]])/(16*(a - b)^3) - (a^4*b*\text{Log}[a + b*\text{Coth}[x]])/(a^2 - b^2)^3 - ((4*b*(2*a^2 - b^2) - a*(5*a^2 - b^2)*\text{Coth}[x])*\text{Sinh}[x]^2)/(8*(a^2 - b^2)^2) - ((b - a)*\text{Coth}[x])*\text{Sinh}[x]^4/(4*(a^2 - b^2))$

Rubi [A] time = 0.339408, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3516, 1647, 801}

$$\frac{a^4 b \log(a + b \coth(x))}{(a^2 - b^2)^3} - \frac{\sinh^4(x)(b - a \coth(x))}{4(a^2 - b^2)} - \frac{\sinh^2(x)(4b(2a^2 - b^2) - a(5a^2 - b^2) \coth(x))}{8(a^2 - b^2)^2} - \frac{a(3a + b) \log(1 - \coth(x))}{16(a + b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cosh}[x]^4/(a + b*\text{Coth}[x]), x]$

[Out] $-(a*(3*a + b)*\text{Log}[1 - \text{Coth}[x]])/(16*(a + b)^3) + (a*(3*a - b)*\text{Log}[1 + \text{Coth}[x]])/(16*(a - b)^3) - (a^4*b*\text{Log}[a + b*\text{Coth}[x]])/(a^2 - b^2)^3 - ((4*b*(2*a^2 - b^2) - a*(5*a^2 - b^2)*\text{Coth}[x])*\text{Sinh}[x]^2)/(8*(a^2 - b^2)^2) - ((b - a)*\text{Coth}[x])*\text{Sinh}[x]^4/(4*(a^2 - b^2))$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}], x], x, b*\text{Tan}[e + f*x], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1647

$\text{Int}[(Pq_)*((d_) + (e_.)*(x_.))^{(m_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{Q = \text{PolynomialQuotient}[(d + e*x)^m*Pq, a + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = \text{Coeff}[\text{Polynomial}$

Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1], Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 801

Int[(((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.)))/((a_.) + (c_.)*(x_.)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^4(x)}{a + b \coth(x)} dx &= - \left(b \operatorname{Subst} \left(\int \frac{x^4}{(a+x)(-b^2+x^2)^3} dx, x, b \coth(x) \right) \right) \\
 &= - \frac{(b - a \coth(x)) \sinh^4(x)}{4(a^2 - b^2)} - \frac{\operatorname{Subst} \left(\int \frac{\frac{a^2 b^4}{a^2 - b^2} - \frac{3ab^4 x}{a^2 - b^2} + 4b^2 x^2}{(a+x)(-b^2+x^2)^2} dx, x, b \coth(x) \right)}{4b} \\
 &= - \frac{(4b(2a^2 - b^2) - a(5a^2 - b^2) \coth(x)) \sinh^2(x)}{8(a^2 - b^2)^2} - \frac{(b - a \coth(x)) \sinh^4(x)}{4(a^2 - b^2)} - \operatorname{Subst} \left(\int \frac{\frac{a^2 b^4 (3a^2 + b^2)}{(a^2 - b^2)^2}}{(a+x)} dx, x, b \coth(x) \right) \\
 &= - \frac{(4b(2a^2 - b^2) - a(5a^2 - b^2) \coth(x)) \sinh^2(x)}{8(a^2 - b^2)^2} - \frac{(b - a \coth(x)) \sinh^4(x)}{4(a^2 - b^2)} - \operatorname{Subst} \left(\int \left(-\frac{ab^3(3a^2 + b^2)}{2(a+b)} \right) dx, x, b \coth(x) \right) \\
 &= - \frac{a(3a + b) \log(1 - \coth(x))}{16(a + b)^3} + \frac{a(3a - b) \log(1 + \coth(x))}{16(a - b)^3} - \frac{a^4 b \log(a + b \coth(x))}{(a^2 - b^2)^3} - \frac{(4b(2a^2 - b^2) \coth(x) - a(5a^2 - b^2)) \sinh^2(x)}{8(a^2 - b^2)^2} - \frac{(b - a \coth(x)) \sinh^4(x)}{4(a^2 - b^2)}
 \end{aligned}$$

Mathematica [A] time = 0.521728, size = 144, normalized size = 0.98

$$\frac{24a^3 b^2 x + 8a^3 (a^2 - b^2) \sinh(2x) - 2a^3 b^2 \sinh(4x) - 4b (-4a^2 b^2 + 3a^4 + b^4) \cosh(2x) - b (a^2 - b^2)^2 \cosh(4x) - 32a^4 b \log(a + b \coth(x))}{32(a - b)^3 (a + b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^4/(a + b*Coth[x]),x]

[Out] $(12a^5x + 24a^3b^2x - 4ab^4x - 4b(3a^4 - 4a^2b^2 + b^4)\text{Cosh}[2x] - b(a^2 - b^2)^2\text{Cosh}[4x] - 32a^4b\text{Log}[b\text{Cosh}[x] + a\text{Sinh}[x]] + 8a^3(a^2 - b^2)\text{Sinh}[2x] + a^5\text{Sinh}[4x] - 2a^3b^2\text{Sinh}[4x] + ab^4\text{Sinh}[4x]) / (32(a - b)^3(a + b)^3)$

Maple [B] time = 0.047, size = 319, normalized size = 2.2

$$-\frac{1}{4a-4b} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-4} + 4 \frac{1}{(8a-8b)(\tanh(x/2)+1)^3} + \frac{5a}{8(a-b)^2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1} - \frac{3b}{8(a-b)^2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^4/(a+b*coth(x)),x)

[Out] $-1/(4a-4b)/(\tanh(1/2*x)+1)^4 + 4/(8a-8b)/(\tanh(1/2*x)+1)^3 + 5/8/(a-b)^2/(\tanh(1/2*x)+1)*a - 3/8/(a-b)^2/(\tanh(1/2*x)+1)*b - 7/8/(a-b)^2/(\tanh(1/2*x)+1)^2*a + 5/8/(a-b)^2/(\tanh(1/2*x)+1)^2*b + 3/8/(a-b)^3*\ln(\tanh(1/2*x)+1)*a^2 - 1/8/(a-b)^3*\ln(\tanh(1/2*x)+1)*a*b + 1/(4a+4b)/(\tanh(1/2*x)-1)^4 + 4/(8a+8b)/(\tanh(1/2*x)-1)^3 + 7/8/(a+b)^2/(\tanh(1/2*x)-1)^2*a + 5/8/(a+b)^2/(\tanh(1/2*x)-1)^2*b + 5/8/(a+b)^2/(\tanh(1/2*x)-1)*a + 3/8/(a+b)^2/(\tanh(1/2*x)-1)*b - 3/8/(a+b)^3*\ln(\tanh(1/2*x)-1)*a^2 - 1/8/(a+b)^3*\ln(\tanh(1/2*x)-1)*a*b - a^4*b/(a-b)^3/(a+b)^3*\ln(\tanh(1/2*x)^2*b + 2*a*tanh(1/2*x) + b)$

Maxima [A] time = 1.19885, size = 208, normalized size = 1.41

$$-\frac{a^4b \log(-(a-b)e^{(-2x)} + a + b)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{(3a^2 + ab)x}{8(a^3 + 3a^2b + 3ab^2 + b^3)} + \frac{(4(2a + b)e^{(-2x)} + a + b)e^{(4x)}}{64(a^2 + 2ab + b^2)} - \frac{4(2a - b)e^{(-2x)} + (a - b)}{64(a^2 - 2ab + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="maxima")

[Out] $-a^4*b*\log(-(a - b)*e^{(-2*x)} + a + b)/(a^6 - 3a^4*b^2 + 3a^2*b^4 - b^6) + 1/8*(3a^2 + a*b)*x/(a^3 + 3a^2*b + 3a*b^2 + b^3) + 1/64*(4*(2*a + b)*e^{(-2*x)} + a + b)*e^{(4*x)}/(a^2 + 2*a*b + b^2) - 1/64*(4*(2*a - b)*e^{(-2*x)} + (a - b)*e^{(-4*x)})/(a^2 - 2*a*b + b^2)$

Fricas [B] time = 2.85631, size = 2747, normalized size = 18.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="fricas")

[Out] $\frac{1}{64}((a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5)\cosh(x)^8 + 8(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5)\cosh(x)\sinh(x)^7 + (a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5)\sinh(x)^8 + 4(2a^5 - 3a^4b - 2a^3b^2 + 4a^2b^3 - b^5)\cosh(x)^6 + 4(2a^5 - 3a^4b - 2a^3b^2 + 4a^2b^3 - b^5 + 7(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5)\cosh(x)^2)\sinh(x)^6 + 8(3a^5 + 8a^4b + 6a^3b^2 - ab^4)xx\cosh(x)^4 + 8(7(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5)\cosh(x)^3 + 3(2a^5 - 3a^4b - 2a^3b^2 + 4a^2b^3 - b^5)\cosh(x))\sinh(x)^5 - a^5 - a^4b + 2a^3b^2 + 2a^2b^3 - ab^4 - b^5 + 2(35(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5)\cosh(x)^4 + 30(2a^5 - 3a^4b - 2a^3b^2 + 4a^2b^3 - b^5)\cosh(x)^2 + 4(3a^5 + 8a^4b + 6a^3b^2 - ab^4)xx)\sinh(x)^4 + 8(7(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5)\cosh(x)^5 + 10(2a^5 - 3a^4b - 2a^3b^2 + 4a^2b^3 - b^5)\cosh(x)^3 + 4(3a^5 + 8a^4b + 6a^3b^2 - ab^4)xx\cosh(x))\sinh(x)^3 - 4(2a^5 + 3a^4b - 2a^3b^2 - 4a^2b^3 + b^5)\cosh(x)^2 + 4(7(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5)\cosh(x)^6 - 2a^5 - 3a^4b + 2a^3b^2 + 4a^2b^3 - b^5 + 15(2a^5 - 3a^4b - 2a^3b^2 + 4a^2b^3 - b^5)\cosh(x)^4 + 12(3a^5 + 8a^4b + 6a^3b^2 - ab^4)xx\cosh(x)^2)\sinh(x)^2 - 64(a^4b\cosh(x)^4 + 4a^4b\cosh(x)^3\sinh(x) + 6a^4b\cosh(x)^2\sinh(x)^2 + 4a^4b\cosh(x)\sinh(x)^3 + a^4b\sinh(x)^4)\log(2(b\cosh(x) + a\sinh(x)))/(\cosh(x) - \sinh(x)) + 8((a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5)\cosh(x)^7 + 3(2a^5 - 3a^4b - 2a^3b^2 + 4a^2b^3 - b^5)\cosh(x)^5 + 4(3a^5 + 8a^4b + 6a^3b^2 - ab^4)xx\cosh(x)^3 - (2a^5 + 3a^4b - 2a^3b^2 - 4a^2b^3 + b^5)\cosh(x))\sinh(x))/((a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\cosh(x)^4 + 4(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\cosh(x)^3\sinh(x) + 6(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\cosh(x)^2\sinh(x)^2 + 4(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\cosh(x)\sinh(x)^3 + (a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\sinh(x)^4)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^4(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**4/(a+b*coth(x)),x)

[Out] Integral(cosh(x)**4/(a + b*coth(x)), x)

Giac [A] time = 1.15977, size = 292, normalized size = 1.99

$$-\frac{a^4 b \log\left(|-ae^{(2x)} - be^{(2x)} + a - b|\right)}{a^6 - 3a^4 b^2 + 3a^2 b^4 - b^6} + \frac{(3a^2 - ab)x}{8(a^3 - 3a^2 b + 3ab^2 - b^3)} - \frac{(18a^2 e^{(4x)} - 6abe^{(4x)} + 8a^2 e^{(2x)} - 12abe^{(2x)} + 4b^2 e^{(2x)})}{64(a^3 - 3a^2 b + 3ab^2 - b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="giac")

[Out] $-a^4 b \log(\text{abs}(-a e^{(2x)} - b e^{(2x)} + a - b)) / (a^6 - 3a^4 b^2 + 3a^2 b^4 - b^6) + 1/8 * (3a^2 - a*b) * x / (a^3 - 3a^2 b + 3a*b^2 - b^3) - 1/64 * (18a^2 e^{(4x)} - 6a*b e^{(4x)} + 8a^2 e^{(2x)} - 12a*b e^{(2x)} + 4b^2 e^{(2x)} + a^2 - 2a*b + b^2) * e^{(-4x)} / (a^3 - 3a^2 b + 3a*b^2 - b^3) + 1/64 * (a e^{(4x)} + b e^{(4x)} + 8a e^{(2x)} + 4b e^{(2x)}) / (a^2 + 2a*b + b^2)$

$$3.115 \quad \int \frac{\cosh^3(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=135

$$\frac{a \sinh^3(x)}{3(a^2 - b^2)} + \frac{a \sinh(x)}{a^2 - b^2} + \frac{ab^2 \sinh(x)}{(a^2 - b^2)^2} - \frac{b \cosh^3(x)}{3(a^2 - b^2)} - \frac{a^2 b \cosh(x)}{(a^2 - b^2)^2} + \frac{a^3 b \tanh^{-1}\left(\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{5/2}}$$

[Out] (a^3*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*b^2*Sinh[x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^2))

Rubi [A] time = 0.250881, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3518, 3109, 2633, 2565, 30, 3100, 2637, 3074, 206}

$$\frac{a \sinh^3(x)}{3(a^2 - b^2)} + \frac{a \sinh(x)}{a^2 - b^2} + \frac{ab^2 \sinh(x)}{(a^2 - b^2)^2} - \frac{b \cosh^3(x)}{3(a^2 - b^2)} - \frac{a^2 b \cosh(x)}{(a^2 - b^2)^2} + \frac{a^3 b \tanh^{-1}\left(\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^3/(a + b*Coth[x]),x]

[Out] (a^3*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*b^2*Sinh[x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^2))

Rule 3518

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*cos[e + f*x] + b*sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3109

Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b

```
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] := Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 + b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x] + Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^3(x)}{a + b \coth(x)} dx &= - \left(i \int \frac{\cosh^3(x) \sinh(x)}{-ib \cosh(x) - ia \sinh(x)} dx \right) \\
 &= \frac{a \int \cosh^3(x) dx}{a^2 - b^2} - \frac{b \int \cosh^2(x) \sinh(x) dx}{a^2 - b^2} + \frac{(iab) \int \frac{\cosh^2(x)}{-ib \cosh(x) - ia \sinh(x)} dx}{a^2 - b^2} \\
 &= -\frac{a^2 b \cosh(x)}{(a^2 - b^2)^2} + \frac{(ia^3 b) \int \frac{1}{-ib \cosh(x) - ia \sinh(x)} dx}{(a^2 - b^2)^2} + \frac{(ab^2) \int \cosh(x) dx}{(a^2 - b^2)^2} + \frac{(ia) \text{Subst} \left(\int (1 - x^2) dx \right)}{a^2 - b^2} \\
 &= -\frac{a^2 b \cosh(x)}{(a^2 - b^2)^2} - \frac{b \cosh^3(x)}{3(a^2 - b^2)} + \frac{ab^2 \sinh(x)}{(a^2 - b^2)^2} + \frac{a \sinh(x)}{a^2 - b^2} + \frac{a \sinh^3(x)}{3(a^2 - b^2)} - \frac{(a^3 b) \text{Subst} \left(\int \frac{1}{a^2 - b^2 - x^2} dx \right)}{(a^2 - b^2)^{5/2}} \\
 &= \frac{a^3 b \tanh^{-1} \left(\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}} \right)}{(a^2 - b^2)^{5/2}} - \frac{a^2 b \cosh(x)}{(a^2 - b^2)^2} - \frac{b \cosh^3(x)}{3(a^2 - b^2)} + \frac{ab^2 \sinh(x)}{(a^2 - b^2)^2} + \frac{a \sinh(x)}{a^2 - b^2} + \frac{a \sinh^3(x)}{3(a^2 - b^2)}
 \end{aligned}$$

Mathematica [A] time = 1.40314, size = 167, normalized size = 1.24

$$\frac{1}{12} \left(\frac{3a(3a^2 + b^2) \sinh(x)}{(a-b)^2(a+b)^2} + \frac{3b(b^2 - 5a^2) \cosh(x)}{(a-b)^2(a+b)^2} + \frac{a^3 \sinh(3x)}{(a-b)^2(a+b)^2} - \frac{24a^3 b \tan^{-1} \left(\frac{a+b \tanh(\frac{x}{2})}{\sqrt{b-a} \sqrt{a+b}} \right)}{(b-a)^{5/2}(a+b)^{5/2}} - \frac{ab^2 \sinh(3x)}{(a-b)^2(a+b)^2} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^3/(a + b*Coth[x]),x]

[Out] ((-24*a^3*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)^(5/2)*(a + b)^(5/2)) + (3*b*(-5*a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2) + (b*Cosh[3*x])/((-a + b)*(a + b)) + (3*a*(3*a^2 + b^2)*Sinh[x])/((a - b)^2*(a + b)^2) + (a^3*Sinh[3*x])/((a - b)^2*(a + b)^2) - (a*b^2*Sinh[3*x])/((a - b)^2*(a + b)^2))/12

Maple [A] time = 0.041, size = 200, normalized size = 1.5

$$-\frac{4}{12a-12b} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-3} + 2 \frac{1}{(4a-4b)(\tanh(x/2)+1)^2} - \frac{a}{(a-b)^2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1} + \frac{b}{2(a-b)^2} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^3/(a+b*coth(x)),x)

[Out]
$$-4/3/(\tanh(1/2*x)+1)^3/(4*a-4*b)+2/(4*a-4*b)/(\tanh(1/2*x)+1)^2-1/(a-b)^2/(\tanh(1/2*x)+1)*a+1/2/(a-b)^2/(\tanh(1/2*x)+1)*b-4/3/(\tanh(1/2*x)-1)^3/(4*a+4*b)-2/(4*a+4*b)/(\tanh(1/2*x)-1)^2-1/(a+b)^2/(\tanh(1/2*x)-1)*a-1/2/(a+b)^2/(\tanh(1/2*x)-1)*b-2*a^3*b/(a-b)^2/(a+b)^2/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2*\tanh(1/2*x)*b+2*a)/(-a^2+b^2)^{(1/2)})$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 3.01861, size = 4208, normalized size = 31.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="fricas")

[Out]
$$[1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\cosh(x)*\sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*\sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3$$

```

b^3 - a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a
+ b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) +
6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^
5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2
+ (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^
4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)
^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 + 5*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 +
5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a^4
*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^
3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cosh(
x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(-a^2 + b^2
)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^5 - 5*a^4*b
- 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**3/(a+b*coth(x)),x)

[Out] Integral(cosh(x)**3/(a + b*coth(x)), x)

Giac [A] time = 1.1639, size = 221, normalized size = 1.64

$$\frac{2a^3b \arctan\left(-\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{(9ae^{2x}-3be^{2x}+a-b)e^{-3x}}{24(a^2-2ab+b^2)} + \frac{a^2e^{3x}+2abe^{3x}+b^2e^{3x}+9a^2e^x+12abe^x+3b^2e^x}{24(a^3+3a^2b+3ab^2+b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="giac")

[Out] $2a^3b \arctan(-\frac{a e^x + b e^x}{\sqrt{-a^2 + b^2}}) / ((a^4 - 2a^2b^2 + b^4) \sqrt{-a^2 + b^2}) - 1/24 * (9a e^{2x} - 3b e^{2x} + a - b) e^{-3x} / (a^2 - 2a*b + b^2) + 1/24 * (a^2 e^{3x} + 2a*b e^{3x} + b^2 e^{3x} + 9a^2 e^x + 12a*b e^x + 3b^2 e^x) / (a^3 + 3a^2*b + 3a*b^2 + b^3)$

$$3.116 \quad \int \frac{\cosh^2(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=85

$$-\frac{a^2 b \log(a + b \coth(x))}{(a^2 - b^2)^2} - \frac{\sinh^2(x)(b - a \coth(x))}{2(a^2 - b^2)} - \frac{a \log(1 - \coth(x))}{4(a + b)^2} + \frac{a \log(\coth(x) + 1)}{4(a - b)^2}$$

[Out] $-(a*\text{Log}[1 - \text{Coth}[x]])/(4*(a + b)^2) + (a*\text{Log}[1 + \text{Coth}[x]])/(4*(a - b)^2) - (a^2*b*\text{Log}[a + b*\text{Coth}[x]])/(a^2 - b^2)^2 - ((b - a*\text{Coth}[x])*\text{Sinh}[x]^2)/(2*(a^2 - b^2))$

Rubi [A] time = 0.163791, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3516, 1647, 801}

$$-\frac{a^2 b \log(a + b \coth(x))}{(a^2 - b^2)^2} - \frac{\sinh^2(x)(b - a \coth(x))}{2(a^2 - b^2)} - \frac{a \log(1 - \coth(x))}{4(a + b)^2} + \frac{a \log(\coth(x) + 1)}{4(a - b)^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cosh}[x]^2/(a + b*\text{Coth}[x]), x]$

[Out] $-(a*\text{Log}[1 - \text{Coth}[x]])/(4*(a + b)^2) + (a*\text{Log}[1 + \text{Coth}[x]])/(4*(a - b)^2) - (a^2*b*\text{Log}[a + b*\text{Coth}[x]])/(a^2 - b^2)^2 - ((b - a*\text{Coth}[x])*\text{Sinh}[x]^2)/(2*(a^2 - b^2))$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] :> \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}, x], x, b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, n\}, x \ \&\& \ \text{IntegerQ}[m/2]$

Rule 1647

$\text{Int}[(Pq_)*((d_) + (e_.)*(x_.))^{(m_.)*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] :> \text{With}\{Q = \text{PolynomialQuotient}[(d + e*x)^m*Pq, a + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]\}, \text{Simp}[(a*g - c*f*x)*(a + c*x^2)^{(p + 1)}/(2*a*c*(p + 1)), x] + \text{Dist}[1/(2*a*c*(p + 1)), \text{Int}[(d + e*x)^m*(a + c*x^2)^{(p + 1)}*\text{ExpandToSum}[(2*a*c*(p + 1)*Q]/(d + e*x)^m + (c*f*(2*p$

+ 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 801

Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_.) + (c_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(x)}{a + b \coth(x)} dx &= - \left(b \operatorname{Subst} \left(\int \frac{x^2}{(a+x)(-b^2+x^2)^2} dx, x, b \coth(x) \right) \right) \\ &= - \frac{(b - a \coth(x)) \sinh^2(x)}{2(a^2 - b^2)} - \frac{\operatorname{Subst} \left(\int \frac{\frac{a^2 b^2}{a^2 - b^2} - \frac{a b^2 x}{a^2 - b^2}}{(a+x)(-b^2+x^2)} dx, x, b \coth(x) \right)}{2b} \\ &= - \frac{(b - a \coth(x)) \sinh^2(x)}{2(a^2 - b^2)} - \frac{\operatorname{Subst} \left(\int \left(-\frac{ab}{2(a+b)^2(b-x)} + \frac{2a^2 b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{ab}{2(a-b)^2(b+x)} \right) dx, x, b \coth(x) \right)}{2b} \\ &= - \frac{a \log(1 - \coth(x))}{4(a+b)^2} + \frac{a \log(1 + \coth(x))}{4(a-b)^2} - \frac{a^2 b \log(a + b \coth(x))}{(a^2 - b^2)^2} - \frac{(b - a \coth(x)) \sinh^2(x)}{2(a^2 - b^2)} \end{aligned}$$

Mathematica [A] time = 0.228807, size = 73, normalized size = 0.86

$$\frac{(b^3 - a^2 b) \cosh(2x) + a(2x(a^2 + b^2) + (a^2 - b^2) \sinh(2x) - 4ab \log(a \sinh(x) + b \cosh(x)))}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^2/(a + b*Coth[x]), x]

[Out] (((-a^2*b) + b^3)*Cosh[2*x] + a*(2*(a^2 + b^2)*x - 4*a*b*Log[b*Cosh[x] + a*Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)

Maple [A] time = 0.041, size = 146, normalized size = 1.7

$$-2 \frac{1}{(4a - 4b)(\tanh(x/2) + 1)^2} + 4 \frac{1}{(8a - 8b)(\tanh(x/2) + 1)} + \frac{a}{2(a-b)^2} \ln \left(\tanh \left(\frac{x}{2} \right) + 1 \right) + 2 \frac{1}{(4a + 4b)(\tanh(x/2) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(x)^2/(a+b*coth(x)),x)`

[Out]
$$-2/(4*a-4*b)/(\tanh(1/2*x)+1)^2+4/(8*a-8*b)/(\tanh(1/2*x)+1)+1/2*a/(a-b)^2*\ln(\tanh(1/2*x)+1)+2/(4*a+4*b)/(\tanh(1/2*x)-1)^2+4/(8*a+8*b)/(\tanh(1/2*x)-1)-1/2*a/(a+b)^2*\ln(\tanh(1/2*x)-1)-a^2*b/(a-b)^2/(a+b)^2*\ln(\tanh(1/2*x)^2*b+2*a*\tanh(1/2*x)+b)$$

Maxima [A] time = 1.15573, size = 108, normalized size = 1.27

$$-\frac{a^2 b \log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4-2a^2 b^2+b^4} + \frac{ax}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="maxima")`

[Out]
$$-a^2*b*\log(-(a-b)*e^{(-2*x)}+a+b)/(a^4-2*a^2*b^2+b^4)+1/2*a*x/(a^2+2*a*b+b^2)+1/8*e^{(2*x)}/(a+b)-1/8*e^{(-2*x)}/(a-b)$$

Fricas [B] time = 2.65051, size = 826, normalized size = 9.72

$$(a^3 - a^2 b - ab^2 + b^3) \cosh(x)^4 + 4(a^3 - a^2 b - ab^2 + b^3) \cosh(x) \sinh(x)^3 + (a^3 - a^2 b - ab^2 + b^3) \sinh(x)^4 + 4(a^3 + 2a^2 b + a*b^2 + b^3) \cosh(x)^2 \sinh(x)^2 + 4(a^3 + 2a^2 b + a*b^2 + b^3) \sinh(x)^2 \cosh(x)^2 + 4(a^3 + 2a^2 b + a*b^2 + b^3) \cosh(x) \sinh(x)^3 + 4(a^3 + 2a^2 b + a*b^2 + b^3) \sinh(x) \cosh(x)^3 + 4(a^3 + 2a^2 b + a*b^2 + b^3) \cosh(x)^4 + 4(a^3 + 2a^2 b + a*b^2 + b^3) \sinh(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="fricas")`

[Out]
$$1/8*((a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)*\sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*\sinh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2 + b^3)*x*\cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)^2 + 2*(a^3 + 2*a^2*b + a*b^2)*x)*\sinh(x)^2 - 8*(a^2*b*\cosh(x)^2 + 2*a^2*b*\cosh(x)*\sinh(x) + a^2*b*\sinh(x)^2)*\log(2*(b*\cosh(x) + a*\sinh(x))/(\cosh(x) - \sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)^3 + 2*(a^3 + 2*a^2*b + a*b^2)*x*\cosh(x))*\sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*\cosh(x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*\cosh(x)*\sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*\sinh(x)^2)$$

$\sinh(x)^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**2/(a+b*coth(x)),x)

[Out] Integral(cosh(x)**2/(a + b*coth(x)), x)

Giac [A] time = 1.14898, size = 140, normalized size = 1.65

$$-\frac{a^2 b \log\left(|-ae^{2x} - be^{2x} + a - b|\right)}{a^4 - 2a^2 b^2 + b^4} + \frac{ax}{2(a^2 - 2ab + b^2)} - \frac{(2ae^{2x} + a - b)e^{-2x}}{8(a^2 - 2ab + b^2)} + \frac{e^{2x}}{8(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="giac")

[Out] $-a^2 b \log(\text{abs}(-a e^{2x} - b e^{2x} + a - b)) / (a^4 - 2 a^2 b^2 + b^4) + 1 / 2 a x / (a^2 - 2 a b + b^2) - 1 / 8 (2 a e^{2x} + a - b) e^{-2x} / (a^2 - 2 a b + b^2) + 1 / 8 e^{2x} / (a + b)$

$$3.117 \quad \int \frac{\cosh(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=72

$$\frac{a \sinh(x)}{a^2 - b^2} - \frac{b \cosh(x)}{a^2 - b^2} + \frac{ab \tanh^{-1} \left(\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}} \right)}{(a^2 - b^2)^{3/2}}$$

[Out] (a*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)

Rubi [A] time = 0.112425, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {3518, 3109, 2637, 2638, 3074, 206}

$$\frac{a \sinh(x)}{a^2 - b^2} - \frac{b \cosh(x)}{a^2 - b^2} + \frac{ab \tanh^{-1} \left(\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}} \right)}{(a^2 - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(a + b*Coth[x]),x]

[Out] (a*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)

Rule 3518

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*cos[e + f*x] + b*sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3109

Int[(cos[(c_.) + (d_.)*(x_.)]^(m_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.))/(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)]), x_Symbol] :> Dist[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*cos[c + d*x] + b*sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&

IGtQ[m, 0] && IGtQ[n, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cosh(x)}{a + b \coth(x)} dx &= - \left(i \int \frac{\cosh(x) \sinh(x)}{-ib \cosh(x) - ia \sinh(x)} dx \right) \\ &= \frac{a \int \cosh(x) dx}{a^2 - b^2} - \frac{b \int \sinh(x) dx}{a^2 - b^2} + \frac{(iab) \int \frac{1}{-ib \cosh(x) - ia \sinh(x)} dx}{a^2 - b^2} \\ &= -\frac{b \cosh(x)}{a^2 - b^2} + \frac{a \sinh(x)}{a^2 - b^2} - \frac{(ab) \text{Subst} \left(\int \frac{1}{a^2 - b^2 - x^2} dx, x, -a \cosh(x) - b \sinh(x) \right)}{a^2 - b^2} \\ &= \frac{ab \tanh^{-1} \left(\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}} \right)}{(a^2 - b^2)^{3/2}} - \frac{b \cosh(x)}{a^2 - b^2} + \frac{a \sinh(x)}{a^2 - b^2} \end{aligned}$$

Mathematica [A] time = 0.301377, size = 79, normalized size = 1.1

$$\frac{a \sinh(x)}{a^2 - b^2} + \frac{b \cosh(x)}{b^2 - a^2} + \frac{2ab \tan^{-1} \left(\frac{a + b \tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}} \right)}{(b-a)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]/(a + b*Coth[x]),x]

[Out] $(2*a*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)^{(3/2)}*(a + b)^{(3/2)}) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)$

Maple [A] time = 0.033, size = 92, normalized size = 1.3

$$-4 \frac{1}{(4a + 4b)(\tanh(x/2) - 1)} - 4 \frac{1}{(4a - 4b)(\tanh(x/2) + 1)} - 2 \frac{ab}{(a + b)(a - b)\sqrt{-a^2 + b^2}} \arctan\left(\frac{1}{2} \frac{2 \tanh(x/2) b - \sqrt{-a^2 + b^2}}{\sqrt{-a^2 + b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)/(a+b*coth(x)),x)

[Out] $-4/(4*a+4*b)/(\tanh(1/2*x)-1) - 4/(4*a-4*b)/(\tanh(1/2*x)+1) - 2*a*b/(a+b)/(a-b)/(-a^2+b^2)^{(1/2)} * \arctan(1/2*(2*\tanh(1/2*x)*b+2*a)/(-a^2+b^2)^{(1/2)})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.87074, size = 1106, normalized size = 15.36

$$\left[\frac{a^3 + a^2b - ab^2 - b^3 - (a^3 - a^2b - ab^2 + b^3) \cosh(x)^2 - 2(a^3 - a^2b - ab^2 + b^3) \cosh(x) \sinh(x) - (a^3 - a^2b - ab^2 + b^3) \cosh(x)^2}{2((a^4 - 2a^2b^2 + b^4) \cosh(x)^2 - 2(a^3 - a^2b - ab^2 + b^3) \cosh(x) \sinh(x) - (a^3 - a^2b - ab^2 + b^3) \cosh(x)^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)^2 - \\ & 2*(a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)*\sinh(x) - (a^3 - a^2*b - a*b^2 + b^3) \\ & *\sinh(x)^2 + 2*(a*b*\cosh(x) + a*b*\sinh(x))*\sqrt{a^2 - b^2}*\log(((a + b)*\cos \\ & h(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 - 2*\sqrt{a^2 - b^2}* \\ & (\cosh(x) + \sinh(x)) + a - b)/((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) \\ & + (a + b)*\sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*\cosh(x) + (a^4 - 2 \\ & *a^2*b^2 + b^4)*\sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - \\ & a*b^2 + b^3)*\cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)*\sinh(x) - (a \\ & ^3 - a^2*b - a*b^2 + b^3)*\sinh(x)^2 + 4*(a*b*\cosh(x) + a*b*\sinh(x))*\sqrt{-a \\ & ^2 + b^2}*\arctan(\sqrt{-a^2 + b^2}/((a + b)*\cosh(x) + (a + b)*\sinh(x))))/(a \\ & ^4 - 2*a^2*b^2 + b^4)*\cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*\sinh(x)] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(a+b*coth(x)),x)

[Out] Integral(cosh(x)/(a + b*coth(x)), x)

Giac [A] time = 1.13028, size = 96, normalized size = 1.33

$$-\frac{2ab \arctan\left(\frac{ae^x + be^x}{\sqrt{-a^2 + b^2}}\right)}{(a^2 - b^2)\sqrt{-a^2 + b^2}} - \frac{e^{(-x)}}{2(a - b)} + \frac{e^x}{2(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="giac")

[Out]
$$-2*a*b*\arctan((a*e^x + b*e^x)/\sqrt{-a^2 + b^2})/((a^2 - b^2)*\sqrt{-a^2 + b^2}) - 1/2*e^{(-x)}/(a - b) + 1/2*e^x/(a + b)$$

$$3.118 \quad \int \frac{\operatorname{sech}(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=50

$$\frac{b \tanh^{-1}\left(\frac{a \cosh(x)+b \sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}} + \frac{\tan^{-1}(\sinh(x))}{a}$$

[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2])

Rubi [A] time = 0.138412, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {3518, 3110, 3770, 3074, 206}

$$\frac{b \tanh^{-1}\left(\frac{a \cosh(x)+b \sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}} + \frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]/(a + b*Coth[x]), x]

[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2])

Rule 3518

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3110

Int[(cos[(c_.) + (d_.)*(x_.)]^(m_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.))/(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)]), x_Symbol] :> Int[ExpandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}(x)}{a + b \coth(x)} dx &= - \left(i \int \frac{\tanh(x)}{-ib \cosh(x) - ia \sinh(x)} dx \right) \\
 &= - \int \left(-\frac{\operatorname{sech}(x)}{a} + \frac{ib}{a(ib \cosh(x) + ia \sinh(x))} \right) dx \\
 &= \frac{\int \operatorname{sech}(x) dx}{a} - \frac{(ib) \int \frac{1}{ib \cosh(x) + ia \sinh(x)} dx}{a} \\
 &= \frac{\tan^{-1}(\sinh(x))}{a} + \frac{b \operatorname{Subst} \left(\int \frac{1}{a^2 - b^2 - x^2} dx, x, a \cosh(x) + b \sinh(x) \right)}{a} \\
 &= \frac{\tan^{-1}(\sinh(x))}{a} + \frac{b \tanh^{-1} \left(\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}} \right)}{a \sqrt{a^2 - b^2}}
 \end{aligned}$$

Mathematica [A] time = 0.112744, size = 60, normalized size = 1.2

$$\frac{2 \left(\tan^{-1} \left(\tanh \left(\frac{x}{2} \right) \right) - \frac{b \tan^{-1} \left(\frac{a + b \tanh \left(\frac{x}{2} \right)}{\sqrt{b-a} \sqrt{a+b}} \right)}{\sqrt{b-a} \sqrt{a+b}} \right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Coth[x]), x]
```

[Out] $(2*(\text{ArcTan}[\text{Tanh}[x/2]] - (b*\text{ArcTan}[(a + b*\text{Tanh}[x/2])]/(\text{Sqrt}[-a + b]*\text{Sqrt}[a + b])))/(\text{Sqrt}[-a + b]*\text{Sqrt}[a + b]))/a$

Maple [A] time = 0.033, size = 54, normalized size = 1.1

$$2 \frac{\arctan(\tanh(x/2))}{a} - 2 \frac{b}{a\sqrt{-a^2 + b^2}} \arctan\left(\frac{1}{2} \frac{2 \tanh(x/2) b + 2 a}{\sqrt{-a^2 + b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(x)/(a+b*coth(x)),x)`

[Out] $2/a*\arctan(\tanh(1/2*x))-2/a*b/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2*\tanh(1/2*x)*b+2*a)/(-a^2+b^2)^{(1/2)})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)/(a+b*coth(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 2.863, size = 564, normalized size = 11.28

$$\left[\frac{\sqrt{a^2 - b^2} b \log\left(\frac{(a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 + 2\sqrt{a^2 - b^2}(\cosh(x) + \sinh(x)) + a - b}{(a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - a + b}\right) + 2(a^2 - b^2) \arctan(\cosh(x) + \sinh(x))}{a^3 - ab^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(x)/(a+b*coth(x)),x, algorithm="fricas")`

[Out] $[(\text{sqrt}(a^2 - b^2)*b*\log(((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + 2*\text{sqrt}(a^2 - b^2)*(\cosh(x) + \sinh(x)) + a - b)/((a + b)*c$

```

osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(a^2
- b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*b*ar
ctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^2)*ar
ctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{a + b \operatorname{coth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*coth(x)), x)
```

Giac [A] time = 1.14499, size = 65, normalized size = 1.3

$$-\frac{2b \arctan\left(\frac{ae^x + be^x}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}a} + \frac{2 \arctan(e^x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arct
an(e^x)/a
```

$$3.119 \quad \int \frac{\operatorname{sech}^2(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=29

$$-\frac{b \log(\tanh(x))}{a^2} - \frac{b \log(a + b \coth(x))}{a^2} + \frac{\tanh(x)}{a}$$

[Out] -((b*Log[a + b*Coth[x]])/a^2) - (b*Log[Tanh[x]])/a^2 + Tanh[x]/a

Rubi [A] time = 0.0554407, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3516, 44}

$$-\frac{b \log(\tanh(x))}{a^2} - \frac{b \log(a + b \coth(x))}{a^2} + \frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/(a + b*Coth[x]),x]

[Out] -((b*Log[a + b*Coth[x]])/a^2) - (b*Log[Tanh[x]])/a^2 + Tanh[x]/a

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^2(x)}{a + b \coth(x)} dx &= - \left(b \operatorname{Subst} \left(\int \frac{1}{x^2(a+x)} dx, x, b \coth(x) \right) \right) \\ &= - \left(b \operatorname{Subst} \left(\int \left(\frac{1}{ax^2} - \frac{1}{a^2x} + \frac{1}{a^2(a+x)} \right) dx, x, b \coth(x) \right) \right) \\ &= - \frac{b \log(a + b \coth(x))}{a^2} - \frac{b \log(\tanh(x))}{a^2} + \frac{\tanh(x)}{a} \end{aligned}$$

Mathematica [A] time = 0.0806487, size = 27, normalized size = 0.93

$$\frac{-b \log(a \sinh(x) + b \cosh(x)) + a \tanh(x) + b \log(\cosh(x))}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^2/(a + b*Coth[x]),x]

[Out] (b*Log[Cosh[x]] - b*Log[b*Cosh[x] + a*Sinh[x]] + a*Tanh[x])/a^2

Maple [A] time = 0.04, size = 59, normalized size = 2.

$$2 \frac{\tanh(x/2)}{a((\tanh(x/2))^2 + 1)} + \frac{b}{a^2} \ln \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 + 1 \right) - \frac{b}{a^2} \ln \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 b + 2 a \tanh(x/2) + b \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^2/(a+b*coth(x)),x)

[Out] 2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/a^2*b*ln(tanh(1/2*x)^2+1)-b/a^2*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)

Maxima [A] time = 1.76628, size = 62, normalized size = 2.14

$$-\frac{b \log(-(a-b)e^{-2x} + a + b)}{a^2} + \frac{b \log(e^{-2x} + 1)}{a^2} + \frac{2}{ae^{-2x} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="maxima")

[Out] $-b \log(-(a - b)e^{-2x} + a + b)/a^2 + b \log(e^{-2x} + 1)/a^2 + 2/(a e^{-2x} + a)$

Fricas [B] time = 2.61689, size = 362, normalized size = 12.48

$$\frac{(b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + b) \log\left(\frac{2(b \cosh(x) + a \sinh(x))}{\cosh(x) - \sinh(x)}\right) - (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + b)}{a^2 \cosh(x)^2 + 2a^2 \cosh(x) \sinh(x) + a^2 \sinh(x)^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="fricas")

[Out] $-((b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + b) \log(2(b \cosh(x) + a \sinh(x))/(\cosh(x) - \sinh(x))) - (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 + b) \log(2 \cosh(x)/(\cosh(x) - \sinh(x))) + 2a)/(a^2 \cosh(x)^2 + 2a^2 \cosh(x) \sinh(x) + a^2 \sinh(x)^2 + a^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{a + b \operatorname{coth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)**2/(a+b*coth(x)),x)

[Out] Integral(sech(x)**2/(a + b*coth(x)), x)

Giac [B] time = 1.1468, size = 103, normalized size = 3.55

$$-\frac{(ab + b^2) \log(|ae^{(2x)} + be^{(2x)} - a + b|)}{a^3 + a^2b} + \frac{b \log(e^{(2x)} + 1)}{a^2} - \frac{be^{(2x)} + 2a + b}{a^2(e^{(2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a*b + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 + a^2*b) + b*log(
e^(2*x) + 1)/a^2 - (b*e^(2*x) + 2*a + b)/(a^2*(e^(2*x) + 1))
```


$$3.120 \quad \int \frac{\operatorname{sech}^3(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=83

$$-\frac{b^2 \tan^{-1}(\sinh(x))}{a^3} + \frac{b\sqrt{a^2-b^2} \tanh^{-1}\left(\frac{a \cosh(x)+b \sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3} - \frac{b \operatorname{sech}(x)}{a^2} + \frac{\tan^{-1}(\sinh(x))}{2a} + \frac{\tanh(x) \operatorname{sech}(x)}{2a}$$

[Out] ArcTan[Sinh[x]]/(2*a) - (b^2*ArcTan[Sinh[x]])/a^3 + (b*Sqrt[a^2 - b^2]*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 - (b*Sech[x])/a^2 + (Sech[x]*Tanh[x])/(2*a)

Rubi [A] time = 0.237053, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3518, 3110, 3768, 3770, 3104, 3074, 206}

$$-\frac{b^2 \tan^{-1}(\sinh(x))}{a^3} + \frac{b\sqrt{a^2-b^2} \tanh^{-1}\left(\frac{a \cosh(x)+b \sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3} - \frac{b \operatorname{sech}(x)}{a^2} + \frac{\tan^{-1}(\sinh(x))}{2a} + \frac{\tanh(x) \operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^3/(a + b*Coth[x]), x]

[Out] ArcTan[Sinh[x]]/(2*a) - (b^2*ArcTan[Sinh[x]])/a^3 + (b*Sqrt[a^2 - b^2]*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 - (b*Sech[x])/a^2 + (Sech[x]*Tanh[x])/(2*a)

Rule 3518

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a + b*tan[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3110

Int[(cos[(c_.) + (d_.)*(x_.)]^(m_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.))/(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)]), x_Symbol] :> Int[ExpandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a + b \operatorname{coth}(x)} dx &= - \left(i \int \frac{\operatorname{sech}^2(x) \tanh(x)}{-ib \cosh(x) - ia \sinh(x)} dx \right) \\
&= - \int \left(-\frac{\operatorname{sech}^3(x)}{a} + \frac{ib \operatorname{sech}^2(x)}{a(ib \cosh(x) + ia \sinh(x))} \right) dx \\
&= \frac{\int \operatorname{sech}^3(x) dx}{a} - \frac{(ib) \int \frac{\operatorname{sech}^2(x)}{ib \cosh(x) + ia \sinh(x)} dx}{a} \\
&= -\frac{b \operatorname{sech}(x)}{a^2} + \frac{\operatorname{sech}(x) \tanh(x)}{2a} + \frac{\int \operatorname{sech}(x) dx}{2a} - \frac{b^2 \int \operatorname{sech}(x) dx}{a^3} - \frac{(ib(a^2 - b^2)) \int \frac{1}{ib \cosh(x) + ia \sinh(x)} dx}{a^3} \\
&= \frac{\tan^{-1}(\sinh(x))}{2a} - \frac{b^2 \tan^{-1}(\sinh(x))}{a^3} - \frac{b \operatorname{sech}(x)}{a^2} + \frac{\operatorname{sech}(x) \tanh(x)}{2a} + \frac{(b(a^2 - b^2)) \operatorname{Subst}\left(\int \frac{1}{a^2} dx\right)}{a^3} \\
&= \frac{\tan^{-1}(\sinh(x))}{2a} - \frac{b^2 \tan^{-1}(\sinh(x))}{a^3} + \frac{b\sqrt{a^2 - b^2} \tanh^{-1}\left(\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}\right)}{a^3} - \frac{b \operatorname{sech}(x)}{a^2} + \frac{\operatorname{sech}(x) \tanh(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.182099, size = 85, normalized size = 1.02

$$\frac{2(a^2 - 2b^2) \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + 4b\sqrt{b-a}\sqrt{a+b} \tan^{-1}\left(\frac{a+b \tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right) + a \operatorname{sech}(x)(a \tanh(x) - 2b)}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^3/(a + b*Coth[x]), x]

[Out] (2*(a^2 - 2*b^2)*ArcTan[Tanh[x/2]] + 4*b*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] + a*Sech[x]*(-2*b + a*Tanh[x]))/(2*a^3)

Maple [B] time = 0.047, size = 187, normalized size = 2.3

$$-\frac{1}{a} \left(\tanh\left(\frac{x}{2}\right) \right)^3 \left(\left(\tanh\left(\frac{x}{2}\right) \right)^2 + 1 \right)^{-2} - 2 \frac{(\tanh(x/2))^2 b}{a^2 \left((\tanh(x/2))^2 + 1 \right)^2} + \frac{1}{a} \tanh\left(\frac{x}{2}\right) \left(\left(\tanh\left(\frac{x}{2}\right) \right)^2 + 1 \right)^{-2} - 2 \frac{b}{a^2 \left((\tanh(x/2))^2 + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^3/(a+b*coth(x)), x)

```
[Out] -1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-2/a^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2*b+1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)-2/a^2/(tanh(1/2*x)^2+1)^2*b+1/a*arctan(tanh(1/2*x))-2/a^3*arctan(tanh(1/2*x))*b^2-2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))+2*b^3/a^3/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.13031, size = 2423, normalized size = 29.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [((a^2 - 2*a*b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*b)*sinh(x)^3 + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*b^2)*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 + a^2 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 - 2*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x) + (3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x)), ((a^2 - 2*a*b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*b)*sinh(x)^3 - 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sq
```

```

rt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))
+ ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*
b^2)*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 +
a^2 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 -
2*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x)
+ (3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^
3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh
(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{a + b \operatorname{coth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)**3/(a+b*coth(x)),x)

[Out] Integral(sech(x)**3/(a + b*coth(x)), x)

Giac [A] time = 1.15398, size = 138, normalized size = 1.66

$$\frac{(a^2 - 2b^2) \arctan(e^x)}{a^3} - \frac{2(a^2b - b^3) \arctan\left(\frac{ae^x + be^x}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}a^3} + \frac{ae^{(3x)} - 2be^{(3x)} - ae^x - 2be^x}{a^2(e^{(2x)} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="giac")

[Out] (a^2 - 2*b^2)*arctan(e^x)/a^3 - 2*(a^2*b - b^3)*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a*e^(3*x) - 2*b*e^(3*x) - a*e^x - 2*b*e^x)/(a^2*(e^(2*x) + 1)^2)

$$3.121 \quad \int \frac{\operatorname{sech}^4(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=79

$$\frac{(a^2 - b^2) \tanh(x)}{a^3} - \frac{b(a^2 - b^2) \log(\tanh(x))}{a^4} - \frac{b(a^2 - b^2) \log(a + b \coth(x))}{a^4} + \frac{b \tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a}$$

[Out] $-\left(\frac{b(a^2 - b^2) \operatorname{Log}[a + b \operatorname{Coth}[x]]}{a^4} - \frac{b(a^2 - b^2) \operatorname{Log}[\operatorname{Tanh}[x]]}{a^4}\right) + \frac{(a^2 - b^2) \operatorname{Tanh}[x]}{a^3} + \frac{b \operatorname{Tanh}[x]^2}{2a^2} - \frac{\operatorname{Tanh}[x]^3}{3a}$

Rubi [A] time = 0.099762, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3516, 894}

$$\frac{(a^2 - b^2) \tanh(x)}{a^3} - \frac{b(a^2 - b^2) \log(\tanh(x))}{a^4} - \frac{b(a^2 - b^2) \log(a + b \coth(x))}{a^4} + \frac{b \tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a}$$

Antiderivative was successfully verified.

[In] `Int[Sech[x]^4/(a + b*Coth[x]),x]`

[Out] $-\left(\frac{b(a^2 - b^2) \operatorname{Log}[a + b \operatorname{Coth}[x]]}{a^4} - \frac{b(a^2 - b^2) \operatorname{Log}[\operatorname{Tanh}[x]]}{a^4}\right) + \frac{(a^2 - b^2) \operatorname{Tanh}[x]}{a^3} + \frac{b \operatorname{Tanh}[x]^2}{2a^2} - \frac{\operatorname{Tanh}[x]^3}{3a}$

Rule 3516

`Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]`

Rule 894

`Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))`

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^4(x)}{a + b \coth(x)} dx &= - \left(b \operatorname{Subst} \left(\int \frac{-b^2 + x^2}{x^4(a+x)} dx, x, b \coth(x) \right) \right) \\ &= - \left(b \operatorname{Subst} \left(\int \left(-\frac{b^2}{ax^4} + \frac{b^2}{a^2x^3} + \frac{a^2 - b^2}{a^3x^2} + \frac{-a^2 + b^2}{a^4x} + \frac{a^2 - b^2}{a^4(a+x)} \right) dx, x, b \coth(x) \right) \right) \\ &= -\frac{b(a^2 - b^2) \log(a + b \coth(x))}{a^4} - \frac{b(a^2 - b^2) \log(\tanh(x))}{a^4} + \frac{(a^2 - b^2) \tanh(x)}{a^3} + \frac{b \tanh^2(x)}{2a^2} - \end{aligned}$$

Mathematica [A] time = 0.258318, size = 68, normalized size = 0.86

$$\frac{(4a^3 - 6ab^2) \tanh(x) - 6b(b^2 - a^2) (\log(\cosh(x)) - \log(a \sinh(x) + b \cosh(x))) + a^2 \operatorname{sech}^2(x) (2a \tanh(x) - 3b)}{6a^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^4/(a + b*Coth[x]), x]

[Out] (-6*b*(-a^2 + b^2)*(Log[Cosh[x]] - Log[b*Cosh[x] + a*Sinh[x]]) + (4*a^3 - 6*a*b^2)*Tanh[x] + a^2*Sech[x]^2*(-3*b + 2*a*Tanh[x]))/(6*a^4)

Maple [B] time = 0.053, size = 257, normalized size = 3.3

$$2 \frac{(\tanh(x/2))^5}{a((\tanh(x/2))^2 + 1)^3} - 2 \frac{(\tanh(x/2))^5 b^2}{a^3((\tanh(x/2))^2 + 1)^3} + 2 \frac{b(\tanh(x/2))^4}{a^2((\tanh(x/2))^2 + 1)^3} + \frac{4}{3a} \left(\tanh\left(\frac{x}{2}\right) \right)^3 \left(\left(\tanh\left(\frac{x}{2}\right) \right)^2 + 1 \right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^4/(a+b*coth(x)), x)

[Out] 2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5-2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*b^2+2/a^2/(tanh(1/2*x)^2+1)^3*b*tanh(1/2*x)^4+4/3/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3-4/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^2+2/a^2/(tanh(1/2*x)^2+1)^3*b*tanh(1/2*x)^2+2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)-2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*b^2+1/a^2*b*ln(tanh(1/2*x)^2+1)-1/a^4*ln(tanh(1/2*x)^2+1)*b^3-b/a^2*ln(tanh(1/2*x)^2+b)+b^3/a^4*ln(tanh(1/2*x)^2+b+2*a*tanh(1/2*x)+b)

Maxima [A] time = 1.7053, size = 180, normalized size = 2.28

$$\frac{2(2a^2 - 3b^2 + 3(2a^2 - ab - 2b^2)e^{-2x}) - 3(ab + b^2)e^{-4x}}{3(3a^3e^{-2x} + 3a^3e^{-4x} + a^3e^{-6x} + a^3)} - \frac{(a^2b - b^3)\log(-(a-b)e^{-2x} + a + b)}{a^4} + \frac{(a^2b - b^3)\log(e^{-2x} + 1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="maxima")

[Out] 2/3*(2*a^2 - 3*b^2 + 3*(2*a^2 - a*b - 2*b^2)*e^(-2*x) - 3*(a*b + b^2)*e^(-4*x))/(3*a^3*e^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) - (a^2*b - b^3)*log(-(a - b)*e^(-2*x) + a + b)/a^4 + (a^2*b - b^3)*log(e^(-2*x) + 1)/a^4

Fricas [B] time = 2.76842, size = 2187, normalized size = 27.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="fricas")

[Out] -1/3*(6*(a^2*b - a*b^2)*cosh(x)^4 + 24*(a^2*b - a*b^2)*cosh(x)*sinh(x)^3 + 6*(a^2*b - a*b^2)*sinh(x)^4 + 4*a^3 - 6*a*b^2 + 6*(2*a^3 + a^2*b - 2*a*b^2)*cosh(x)^2 + 6*(2*a^3 + a^2*b - 2*a*b^2 + 6*(a^2*b - a*b^2)*cosh(x)^2)*sinh(x)^2 + 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^2*b - b^3)*sinh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^2*b - b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^2*b - b^3)*sinh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^2*b - b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 12*(2*(a^2*b - a*b^2)*cosh(x)^3 + (2*a^3 + a^2*b - 2*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 + 3*a^4*cosh(x))*sinh(x)^

$$3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{a + b \operatorname{coth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)**4/(a+b*coth(x)),x)

[Out] Integral(sech(x)**4/(a + b*coth(x)), x)

Giac [B] time = 1.14789, size = 271, normalized size = 3.43

$$-\frac{(a^3b + a^2b^2 - ab^3 - b^4) \log(|ae^{(2x)} + be^{(2x)} - a + b|)}{a^5 + a^4b} + \frac{(a^2b - b^3) \log(e^{(2x)} + 1)}{a^4} - \frac{11a^2be^{(6x)} - 11b^3e^{(6x)} + 45a^2be^{(4x)} - 11a^2b - 12ab^2 - 11b^3}{a^4(e^{(2x)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="giac")

[Out] $-(a^3*b + a^2*b^2 - a*b^3 - b^4)*\log(\operatorname{abs}(a*e^{(2*x)} + b*e^{(2*x)} - a + b))/(a^5 + a^4*b) + (a^2*b - b^3)*\log(e^{(2*x)} + 1)/a^4 - 1/6*(11*a^2*b*e^{(6*x)} - 11*b^3*e^{(6*x)} + 45*a^2*b*e^{(4*x)} - 12*a*b^2*e^{(4*x)} - 33*b^3*e^{(4*x)} + 24*a^3*e^{(2*x)} + 45*a^2*b*e^{(2*x)} - 24*a*b^2*e^{(2*x)} - 33*b^3*e^{(2*x)} + 8*a^3 + 11*a^2*b - 12*a*b^2 - 11*b^3)/(a^4*(e^{(2*x)} + 1)^3)$

$$3.122 \quad \int \frac{\operatorname{sech}(x)}{i+2 \coth(x)} dx$$

Optimal. Leaf size=31

$$-i \tan^{-1}(\sinh(x)) - \frac{2 \tanh^{-1}\left(\frac{\cosh(x)-2i \sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

[Out] (-I)*ArcTan[Sinh[x]] - (2*ArcTanh[(Cosh[x] - (2*I)*Sinh[x])/Sqrt[5]])/Sqrt[5]

Rubi [A] time = 0.103316, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3518, 3110, 3770, 3074, 206}

$$-i \tan^{-1}(\sinh(x)) - \frac{2 \tanh^{-1}\left(\frac{\cosh(x)-2i \sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]/(I + 2*Coth[x]), x]

[Out] (-I)*ArcTan[Sinh[x]] - (2*ArcTanh[(Cosh[x] - (2*I)*Sinh[x])/Sqrt[5]])/Sqrt[5]

Rule 3518

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*cos[e + f*x] + b*sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3110

Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}(x)}{i + 2 \coth(x)} dx &= - \left(i \int \frac{\tanh(x)}{-2i \cosh(x) + \sinh(x)} dx \right) \\
 &= - \int \left(i \operatorname{sech}(x) - \frac{2i}{2 \cosh(x) + i \sinh(x)} \right) dx \\
 &= - (i \int \operatorname{sech}(x) dx) + 2i \int \frac{1}{2 \cosh(x) + i \sinh(x)} dx \\
 &= -i \tan^{-1}(\sinh(x)) - 2 \operatorname{Subst} \left(\int \frac{1}{5 - x^2} dx, x, \cosh(x) - 2i \sinh(x) \right) \\
 &= -i \tan^{-1}(\sinh(x)) - \frac{2 \tanh^{-1} \left(\frac{\cosh(x) - 2i \sinh(x)}{\sqrt{5}} \right)}{\sqrt{5}}
 \end{aligned}$$

Mathematica [A] time = 0.0514349, size = 38, normalized size = 1.23

$$-\frac{4 \tanh^{-1} \left(\frac{1 - 2i \tanh \left(\frac{x}{2} \right)}{\sqrt{5}} \right)}{\sqrt{5}} - 2i \tan^{-1} \left(\tanh \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + 2*Coth[x]), x]
```

```
[Out] (-2*I)*ArcTan[Tanh[x/2]] - (4*ArcTanh[(1 - (2*I)*Tanh[x/2])/Sqrt[5]])/Sqrt[
5]
```

Maple [A] time = 0.052, size = 41, normalized size = 1.3

$$-\ln\left(\tanh\left(\frac{x}{2}\right) - i\right) + \frac{4i}{5}\sqrt{5}\arctan\left(\frac{\sqrt{5}}{5}(2\tanh(x/2) + i)\right) + \ln\left(\tanh\left(\frac{x}{2}\right) + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)/(I+2*coth(x)),x)

[Out] -ln(tanh(1/2*x)-I)+4/5*I*5^(1/2)*arctan(1/5*(2*tanh(1/2*x)+I)*5^(1/2))+ln(tanh(1/2*x)+I)

Maxima [A] time = 1.70802, size = 57, normalized size = 1.84

$$\frac{2}{5}\sqrt{5}\log\left(-\frac{2\sqrt{5} - (4i + 2)e^{-x}}{2\sqrt{5} + (4i + 2)e^{-x}}\right) + 2i\arctan(e^{-x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="maxima")

[Out] 2/5*sqrt(5)*log(-(2*sqrt(5) - (4*I + 2)*e^(-x))/(2*sqrt(5) + (4*I + 2)*e^(-x))) + 2*I*arctan(e^(-x))

Fricas [A] time = 2.79643, size = 169, normalized size = 5.45

$$-\frac{2}{5}\sqrt{5}\log\left(\left(\frac{2}{5}i + \frac{1}{5}\right)\sqrt{5} + e^x\right) + \frac{2}{5}\sqrt{5}\log\left(-\left(\frac{2}{5}i + \frac{1}{5}\right)\sqrt{5} + e^x\right) + \log(e^x + i) - \log(e^x - i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="fricas")

[Out] -2/5*sqrt(5)*log((2/5*I + 1/5)*sqrt(5) + e^x) + 2/5*sqrt(5)*log(-(2/5*I + 1/5)*sqrt(5) + e^x) + log(e^x + I) - log(e^x - I)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{2 \coth(x) + i} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)/(I+2*coth(x)),x)

[Out] Integral(sech(x)/(2*coth(x) + I), x)

Giac [A] time = 1.13408, size = 35, normalized size = 1.13

$$\frac{4}{5}i\sqrt{5}\arctan\left(\left(\frac{1}{5}i + \frac{2}{5}\right)\sqrt{5}e^x\right) + \log(e^x + i) - \log(e^x - i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="giac")

[Out] 4/5*I*sqrt(5)*arctan((1/5*I + 2/5)*sqrt(5)*e^x) + log(e^x + I) - log(e^x - I)

3.123 $\int \frac{\tanh^4(x)}{1+\coth(x)} dx$

Optimal. Leaf size=43

$$\frac{5x}{2} - \frac{5 \tanh^3(x)}{6} + \tanh^2(x) - \frac{5 \tanh(x)}{2} - 2 \log(\cosh(x)) + \frac{\tanh^3(x)}{2(\coth(x)+1)}$$

[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Tanh[x]^3/(2*(1 + Coth[x]))

Rubi [A] time = 0.114614, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3552, 3529, 3531, 3475}

$$\frac{5x}{2} - \frac{5 \tanh^3(x)}{6} + \tanh^2(x) - \frac{5 \tanh(x)}{2} - 2 \log(\cosh(x)) + \frac{\tanh^3(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4/(1 + Coth[x]),x]

[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Tanh[x]^3/(2*(1 + Coth[x]))

Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a*c + b*d, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^4(x)}{1 + \coth(x)} dx &= \frac{\tanh^3(x)}{2(1 + \coth(x))} - \frac{1}{2} \int (-5 + 4 \coth(x)) \tanh^4(x) dx \\
 &= -\frac{5}{6} \tanh^3(x) + \frac{\tanh^3(x)}{2(1 + \coth(x))} - \frac{1}{2} i \int (-4i + 5i \coth(x)) \tanh^3(x) dx \\
 &= \tanh^2(x) - \frac{5 \tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1 + \coth(x))} + \frac{1}{2} \int (5 - 4 \coth(x)) \tanh^2(x) dx \\
 &= -\frac{5 \tanh(x)}{2} + \tanh^2(x) - \frac{5 \tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1 + \coth(x))} + \frac{1}{2} i \int (4i - 5i \coth(x)) \tanh(x) dx \\
 &= \frac{5x}{2} - \frac{5 \tanh(x)}{2} + \tanh^2(x) - \frac{5 \tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1 + \coth(x))} - 2 \int \tanh(x) dx \\
 &= \frac{5x}{2} - 2 \log(\cosh(x)) - \frac{5 \tanh(x)}{2} + \tanh^2(x) - \frac{5 \tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1 + \coth(x))}
 \end{aligned}$$

Mathematica [A] time = 0.095612, size = 40, normalized size = 0.93

$$\frac{1}{12} (30x - 3 \sinh(2x) + 3 \cosh(2x) - 28 \tanh(x) - 24 \log(\cosh(x)) + (4 \tanh(x) - 6) \operatorname{sech}^2(x))$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^4/(1 + Coth[x]), x]

[Out] (30*x + 3*Cosh[2*x] - 24*Log[Cosh[x]] - 3*Sinh[2*x] - 28*Tanh[x] + Sech[x]^2*(-6 + 4*Tanh[x]))/12

Maple [B] time = 0.043, size = 96, normalized size = 2.2

$$\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{9}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-4\frac{(\tanh(x/2))^5-1/2(\tanh(x/2))}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^4/(1+coth(x)),x)`

[Out] `1/(tanh(1/2*x)+1)^2-1/(tanh(1/2*x)+1)+9/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/2*x)-1)-4*(tanh(1/2*x)^5-1/2*tanh(1/2*x)^4+8/3*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2+tanh(1/2*x))/(tanh(1/2*x)^2+1)^3-2*ln(tanh(1/2*x)^2+1)`

Maxima [A] time = 1.70061, size = 74, normalized size = 1.72

$$\frac{1}{2}x - \frac{2(15e^{-2x} + 12e^{-4x} + 7)}{3(3e^{-2x} + 3e^{-4x} + e^{-6x} + 1)} + \frac{1}{4}e^{-2x} - 2\log(e^{-2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^4/(1+coth(x)),x, algorithm="maxima")`

[Out] `1/2*x - 2/3*(15*e^(-2*x) + 12*e^(-4*x) + 7)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 1/4*e^(-2*x) - 2*log(e^(-2*x) + 1)`

Fricas [B] time = 2.6535, size = 1897, normalized size = 44.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^4/(1+coth(x)),x, algorithm="fricas")`

[Out] `1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 + 3*(54*x + 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 + 54*x + 17)*sinh(x)^6 + 18*(168*x*cosh(x)^3 + (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420*x*`


```

cosh(x)^4 + 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*cosh(
x)^5 + 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 + (54*x +
65)*cosh(x)^2 + (1512*x*cosh(x)^6 + 45*(54*x + 17)*cosh(x)^4 + 486*(2*x + 1
)*cosh(x)^2 + 54*x + 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^3 +
9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cosh(x)
^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh(x)^
7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) -
sinh(x))) + 2*(216*x*cosh(x)^7 + 9*(54*x + 17)*cosh(x)^5 + 162*(2*x + 1)*co
sh(x)^3 + (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^
7 + sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^
3 + 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cos
h(x)^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh
(x)^7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**4/(1+coth(x)),x)

[Out] Integral(tanh(x)**4/(coth(x) + 1), x)

Giac [A] time = 1.11034, size = 63, normalized size = 1.47

$$\frac{9}{2}x + \frac{(51e^{6x} + 81e^{4x} + 65e^{2x} + 3)e^{-2x}}{12(e^{2x} + 1)^3} - 2 \log(e^{2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="giac")

[Out] 9/2*x + 1/12*(51*e^(6*x) + 81*e^(4*x) + 65*e^(2*x) + 3)*e^(-2*x)/(e^(2*x) + 1)^3 - 2*log(e^(2*x) + 1)

$$3.124 \quad \int \frac{\tanh^3(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=37

$$-\frac{3x}{2} - \tanh^2(x) + \frac{3 \tanh(x)}{2} + 2 \log(\cosh(x)) + \frac{\tanh^2(x)}{2(\coth(x) + 1)}$$

[Out] $(-3*x)/2 + 2*\text{Log}[\text{Cosh}[x]] + (3*\text{Tanh}[x])/2 - \text{Tanh}[x]^2 + \text{Tanh}[x]^2/(2*(1 + \text{Coth}[x]))$

Rubi [A] time = 0.0991369, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3552, 3529, 3531, 3475}

$$-\frac{3x}{2} - \tanh^2(x) + \frac{3 \tanh(x)}{2} + 2 \log(\cosh(x)) + \frac{\tanh^2(x)}{2(\coth(x) + 1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tanh}[x]^3/(1 + \text{Coth}[x]), x]$

[Out] $(-3*x)/2 + 2*\text{Log}[\text{Cosh}[x]] + (3*\text{Tanh}[x])/2 - \text{Tanh}[x]^2 + \text{Tanh}[x]^2/(2*(1 + \text{Coth}[x]))$

Rule 3552

$\text{Int}[\frac{(c_. + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}}{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]}, x_Symbol] :> -\text{Simp}[(a*(c + d*\text{Tan}[e + f*x])^{(n + 1)})/(2*f*(b*c - a*d)*(a + b*\text{Tan}[e + f*x])), x] + \text{Dist}[1/(2*a*(b*c - a*d)), \text{Int}[(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[b*c + a*d*(n - 1) - b*d*n*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{NeQ}[c^2 + d^2, 0] \&\& !\text{GtQ}[n, 0]$

Rule 3529

$\text{Int}[\frac{(a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((c_.) + (d_.)*\text{tan}[(e_.) + (f_.)*(x_.)])}{(f*(m + 1)*(a^2 + b^2)}, x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)
)*(x_)]), x_Symbol] := Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{1 + \coth(x)} dx &= \frac{\tanh^2(x)}{2(1 + \coth(x))} - \frac{1}{2} \int (-4 + 3 \coth(x)) \tanh^3(x) dx \\
&= -\tanh^2(x) + \frac{\tanh^2(x)}{2(1 + \coth(x))} - \frac{1}{2} \int (-3i + 4i \coth(x)) \tanh^2(x) dx \\
&= \frac{3 \tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1 + \coth(x))} + \frac{1}{2} \int (4 - 3 \coth(x)) \tanh(x) dx \\
&= -\frac{3x}{2} + \frac{3 \tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1 + \coth(x))} + 2 \int \tanh(x) dx \\
&= -\frac{3x}{2} + 2 \log(\cosh(x)) + \frac{3 \tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1 + \coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.0532405, size = 33, normalized size = 0.89

$$\frac{1}{4} (-6x + \sinh(2x) - \cosh(2x) + 4 \tanh(x) + 2 \operatorname{sech}^2(x) + 8 \log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(1 + Coth[x]), x]
```

```
[Out] (-6*x - Cosh[2*x] + 8*Log[Cosh[x]] + 2*Sech[x]^2 + Sinh[2*x] + 4*Tanh[x])/4
```

Maple [B] time = 0.037, size = 80, normalized size = 2.2

$$-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{7}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + 2\frac{(\tanh(x/2))^3 - (\tanh(x/2))^2}{((\tanh(x/2))^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(1+coth(x)),x)

[Out] -1/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)-7/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/2*x)-1)+2*(tanh(1/2*x)^3-tanh(1/2*x)^2+tanh(1/2*x))/(tanh(1/2*x)^2+1)+2*ln(tanh(1/2*x)^2+1)

Maxima [A] time = 1.54025, size = 58, normalized size = 1.57

$$\frac{1}{2}x + \frac{2(2e^{(-2x)} + 1)}{2e^{(-2x)} + e^{(-4x)} + 1} - \frac{1}{4}e^{(-2x)} + 2\log(e^{(-2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="maxima")

[Out] 1/2*x + 2*(2*e^(-2*x) + 1)/(2*e^(-2*x) + e^(-4*x) + 1) - 1/4*e^(-2*x) + 2*log(e^(-2*x) + 1)

Fricas [B] time = 2.64764, size = 1168, normalized size = 31.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="fricas")

[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 + (28*x + 1)*cosh(x)^4 + (210*x*cosh(x)^2 + 28*x + 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 + (28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 + 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh

$$(x)^3 + 2*\cosh(x))*\sinh(x)^3 + (15*\cosh(x)^4 + 12*\cosh(x)^2 + 1)*\sinh(x)^2 + \cosh(x)^2 + 2*(3*\cosh(x)^5 + 4*\cosh(x)^3 + \cosh(x))*\sinh(x))*\log(2*\cosh(x))/(\cosh(x) - \sinh(x)) + 4*(21*x*\cosh(x)^5 + (28*x + 1)*\cosh(x)^3 + (7*x + 5)*\cosh(x))*\sinh(x) + 1)/(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + (15*\cosh(x)^2 + 2)*\sinh(x)^4 + 2*\cosh(x)^4 + 4*(5*\cosh(x)^3 + 2*\cosh(x))*\sinh(x)^3 + (15*\cosh(x)^4 + 12*\cosh(x)^2 + 1)*\sinh(x)^2 + \cosh(x)^2 + 2*(3*\cosh(x)^5 + 4*\cosh(x)^3 + \cosh(x))*\sinh(x))$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3/(1+coth(x)),x)

[Out] Integral(tanh(x)**3/(coth(x) + 1), x)

Giac [A] time = 1.11696, size = 53, normalized size = 1.43

$$-\frac{7}{2}x - \frac{(e^{4x} + 10e^{2x} + 1)e^{-2x}}{4(e^{2x} + 1)^2} + 2 \log(e^{2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="giac")

[Out] -7/2*x - 1/4*(e^(4*x) + 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1)^2 + 2*log(e^(2*x) + 1)

$$3.125 \quad \int \frac{\tanh^2(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=29

$$\frac{3x}{2} - \frac{3 \tanh(x)}{2} - \log(\cosh(x)) + \frac{\tanh(x)}{2(\coth(x) + 1)}$$

[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]/(2*(1 + Coth[x]))

Rubi [A] time = 0.0742931, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3552, 3529, 3531, 3475}

$$\frac{3x}{2} - \frac{3 \tanh(x)}{2} - \log(\cosh(x)) + \frac{\tanh(x)}{2(\coth(x) + 1)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2/(1 + Coth[x]),x]

[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]/(2*(1 + Coth[x]))

Rule 3552

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !GtQ[n, 0]

Rule 3529

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^2(x)}{1 + \coth(x)} dx &= \frac{\tanh(x)}{2(1 + \coth(x))} - \frac{1}{2} \int (-3 + 2 \coth(x)) \tanh^2(x) dx \\ &= -\frac{3 \tanh(x)}{2} + \frac{\tanh(x)}{2(1 + \coth(x))} - \frac{1}{2} i \int (-2i + 3i \coth(x)) \tanh(x) dx \\ &= \frac{3x}{2} - \frac{3 \tanh(x)}{2} + \frac{\tanh(x)}{2(1 + \coth(x))} - \int \tanh(x) dx \\ &= \frac{3x}{2} - \log(\cosh(x)) - \frac{3 \tanh(x)}{2} + \frac{\tanh(x)}{2(1 + \coth(x))} \end{aligned}$$

Mathematica [A] time = 0.0456504, size = 27, normalized size = 0.93

$$\frac{1}{4}(6x - \sinh(2x) + \cosh(2x) - 4 \tanh(x) - 4 \log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (6*x + Cosh[2*x] - 4*Log[Cosh[x]] - Sinh[2*x] - 4*Tanh[x])/4
```

Maple [B] time = 0.035, size = 65, normalized size = 2.2

$$\left(\tanh\left(\frac{x}{2}\right) + 1\right)^{-2} - \left(\tanh\left(\frac{x}{2}\right) + 1\right)^{-1} + \frac{5}{2} \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{1}{2} \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - 2 \frac{\tanh(x/2)}{(\tanh(x/2))^2 + 1} - \ln\left(\left(\tanh\left(\frac{x}{2}\right) + 1\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2/(1+coth(x)),x)`

[Out] $\frac{1}{(\tanh(1/2*x)+1)^2} - \frac{1}{\tanh(1/2*x)+1} + \frac{5}{2} \ln(\tanh(1/2*x)+1) - \frac{1}{2} \ln(\tanh(1/2*x)-1) - 2 \tanh(1/2*x) / (\tanh(1/2*x)^2+1) - \ln(\tanh(1/2*x)^2+1)$

Maxima [A] time = 1.67494, size = 39, normalized size = 1.34

$$\frac{1}{2}x - \frac{2}{e^{(-2x)} + 1} + \frac{1}{4}e^{(-2x)} - \log(e^{(-2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^2/(1+coth(x)),x, algorithm="maxima")`

[Out] $\frac{1}{2}x - \frac{2}{(e^{(-2*x)} + 1)} + \frac{1}{4}e^{(-2*x)} - \log(e^{(-2*x)} + 1)$

Fricas [B] time = 2.69727, size = 632, normalized size = 21.79

$$\frac{10x \cosh(x)^4 + 40x \cosh(x) \sinh(x)^3 + 10x \sinh(x)^4 + (10x + 9) \cosh(x)^2 + (60x \cosh(x)^2 + 10x + 9) \sinh(x)^2 - 4}{4(\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^2/(1+coth(x)),x, algorithm="fricas")`

[Out] $\frac{1}{4} * (10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 + (10*x + 9)*cosh(x)^2 + (60*x*cosh(x)^2 + 10*x + 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(20*x*cosh(x)^3 + (10*x + 9)*cosh(x))*sinh(x) + 1) / (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{\coth(x)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**2/(1+coth(x)),x)`

[Out] `Integral(tanh(x)**2/(coth(x) + 1), x)`

Giac [A] time = 1.14485, size = 47, normalized size = 1.62

$$\frac{5}{2}x + \frac{(9e^{2x} + 1)e^{-2x}}{4(e^{2x} + 1)} - \log(e^{2x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^2/(1+coth(x)),x, algorithm="giac")`

[Out] `5/2*x + 1/4*(9*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1) - log(e^(2*x) + 1)`

$$3.126 \quad \int \frac{\tanh(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=19

$$-\frac{x}{2} + \frac{1}{2(\coth(x)+1)} + \log(\cosh(x))$$

[Out] -x/2 + 1/(2*(1 + Coth[x])) + Log[Cosh[x]]

Rubi [A] time = 0.0417203, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3551, 3479, 8, 3475}

$$-\frac{x}{2} + \frac{1}{2(\coth(x)+1)} + \log(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/(1 + Coth[x]),x]

[Out] -x/2 + 1/(2*(1 + Coth[x])) + Log[Cosh[x]]

Rule 3551

```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*Tan[e + f*x]), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{1 + \coth(x)} dx &= -\int \frac{1}{1 + \coth(x)} dx + \int \tanh(x) dx \\ &= \frac{1}{2(1 + \coth(x))} + \log(\cosh(x)) - \frac{\int 1 dx}{2} \\ &= -\frac{x}{2} + \frac{1}{2(1 + \coth(x))} + \log(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0281422, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x + \sinh(2x) - \cosh(2x) + 4 \log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(1 + Coth[x]),x]
```

```
[Out] (-2*x - Cosh[2*x] + 4*Log[Cosh[x]] + Sinh[2*x])/4
```

Maple [B] time = 0.029, size = 47, normalized size = 2.5

$$-\left(\tanh\left(\frac{x}{2}\right) + 1\right)^{-2} + \left(\tanh\left(\frac{x}{2}\right) + 1\right)^{-1} - \frac{3}{2} \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{1}{2} \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + \ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1+coth(x)),x)
```

```
[Out] -1/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)-3/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)^2+1)
```

Maxima [A] time = 1.67204, size = 23, normalized size = 1.21

$$\frac{1}{2}x - \frac{1}{4}e^{(-2x)} + \log(e^{(-2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="maxima")

[Out] 1/2*x - 1/4*e^(-2*x) + log(e^(-2*x) + 1)

Fricas [B] time = 2.95861, size = 259, normalized size = 13.63

$$\frac{6x \cosh(x)^2 + 12x \cosh(x) \sinh(x) + 6x \sinh(x)^2 - 4(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2) \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right)}{4(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="fricas")

[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\coth(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(1+coth(x)),x)

[Out] Integral(tanh(x)/(coth(x) + 1), x)

Giac [A] time = 1.15774, size = 23, normalized size = 1.21

$$-\frac{3}{2}x - \frac{1}{4}e^{(-2x)} + \log(e^{(2x)} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="giac")

[Out] -3/2*x - 1/4*e^(-2*x) + log(e^(2*x) + 1)

$$3.127 \quad \int \frac{1}{1+\coth(x)} dx$$

Optimal. Leaf size=16

$$\frac{x}{2} - \frac{1}{2(\coth(x) + 1)}$$

[Out] x/2 - 1/(2*(1 + Coth[x]))

Rubi [A] time = 0.0083708, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3479, 8}

$$\frac{x}{2} - \frac{1}{2(\coth(x) + 1)}$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^(-1), x]

[Out] x/2 - 1/(2*(1 + Coth[x]))

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\coth(x)} dx &= -\frac{1}{2(1+\coth(x))} + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} - \frac{1}{2(1+\coth(x))} \end{aligned}$$

Mathematica [A] time = 0.0252688, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x - \sinh(2x) + \cosh(2x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^(-1), x]

[Out] (2*x + Cosh[2*x] - Sinh[2*x])/4

Maple [A] time = 0., size = 24, normalized size = 1.5

$$-\frac{1}{2 + 2 \coth(x)} + \frac{\ln(1 + \coth(x))}{4} - \frac{\ln(\coth(x) - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+coth(x)), x)

[Out] -1/2/(1+coth(x))+1/4*ln(1+coth(x))-1/4*ln(coth(x)-1)

Maxima [A] time = 1.1258, size = 14, normalized size = 0.88

$$\frac{1}{2}x + \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x)), x, algorithm="maxima")

[Out] 1/2*x + 1/4*e^(-2*x)

Fricas [B] time = 2.76851, size = 88, normalized size = 5.5

$$\frac{(2x + 1) \cosh(x) + (2x - 1) \sinh(x)}{4(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x)),x, algorithm="fricas")

[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))

Sympy [B] time = 0.613514, size = 27, normalized size = 1.69

$$\frac{x \tanh(x)}{2 \tanh(x) + 2} + \frac{x}{2 \tanh(x) + 2} + \frac{1}{2 \tanh(x) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x)),x)

[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)

Giac [A] time = 1.1395, size = 14, normalized size = 0.88

$$\frac{1}{2}x + \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+coth(x)),x, algorithm="giac")

[Out] 1/2*x + 1/4*e^(-2*x)

$$3.128 \quad \int \frac{\coth(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=16

$$\frac{x}{2} + \frac{1}{2(\coth(x)+1)}$$

[Out] x/2 + 1/(2*(1 + Coth[x]))

Rubi [A] time = 0.0215589, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3526, 8}

$$\frac{x}{2} + \frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/(1 + Coth[x]),x]

[Out] x/2 + 1/(2*(1 + Coth[x]))

Rule 3526

Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth(x)}{1+\coth(x)} dx &= \frac{1}{2(1+\coth(x))} + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{1}{2(1+\coth(x))} \end{aligned}$$

Mathematica [A] time = 0.0152798, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x + \sinh(2x) - \cosh(2x))$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]/(1 + Coth[x]), x]

[Out] (2*x - Cosh[2*x] + Sinh[2*x])/4

Maple [A] time = 0.018, size = 24, normalized size = 1.5

$$\frac{1}{2 + 2 \coth(x)} + \frac{\ln(1 + \coth(x))}{4} - \frac{\ln(\coth(x) - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(1+coth(x)), x)

[Out] 1/2/(1+coth(x))+1/4*ln(1+coth(x))-1/4*ln(coth(x)-1)

Maxima [A] time = 1.13635, size = 14, normalized size = 0.88

$$\frac{1}{2}x - \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1+coth(x)), x, algorithm="maxima")

[Out] 1/2*x - 1/4*e^(-2*x)

Fricas [B] time = 2.80484, size = 88, normalized size = 5.5

$$\frac{(2x - 1) \cosh(x) + (2x + 1) \sinh(x)}{4(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(1+coth(x)),x, algorithm="fricas")`

[Out] $1/4*((2*x - 1)*\cosh(x) + (2*x + 1)*\sinh(x))/(\cosh(x) + \sinh(x))$

Sympy [B] time = 0.59691, size = 27, normalized size = 1.69

$$\frac{x \tanh(x)}{2 \tanh(x) + 2} + \frac{x}{2 \tanh(x) + 2} - \frac{1}{2 \tanh(x) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(1+coth(x)),x)`

[Out] $x*\tanh(x)/(2*\tanh(x) + 2) + x/(2*\tanh(x) + 2) - 1/(2*\tanh(x) + 2)$

Giac [A] time = 1.11193, size = 14, normalized size = 0.88

$$\frac{1}{2}x - \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(1+coth(x)),x, algorithm="giac")`

[Out] $1/2*x - 1/4*e^{(-2*x)}$

$$3.129 \quad \int \frac{\coth^2(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=19

$$-\frac{x}{2} - \frac{1}{2(\coth(x) + 1)} + \log(\sinh(x))$$

[Out] $-x/2 - 1/(2*(1 + \text{Coth}[x])) + \text{Log}[\text{Sinh}[x]]$

Rubi [A] time = 0.0382739, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3540, 3475}

$$-\frac{x}{2} - \frac{1}{2(\coth(x) + 1)} + \log(\sinh(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[x]^2/(1 + \text{Coth}[x]), x]$

[Out] $-x/2 - 1/(2*(1 + \text{Coth}[x])) + \text{Log}[\text{Sinh}[x]]$

Rule 3540

$\text{Int}[(a + (b \cdot \tan[e + f \cdot x]))^m \cdot (c + (d \cdot \tan[e + f \cdot x]))^2, x_Symbol] \rightarrow -\text{Simp}[(b \cdot (a \cdot c + b \cdot d))^2 \cdot (a + b \cdot \tan[e + f \cdot x])^m / (2 \cdot a^3 \cdot f \cdot m), x] + \text{Dist}[1/(2 \cdot a^2), \text{Int}[(a + b \cdot \tan[e + f \cdot x])^{m+1} \cdot \text{Simp}[a \cdot c^2 - 2 \cdot b \cdot c \cdot d + a \cdot d^2 - 2 \cdot b \cdot d^2 \cdot \tan[e + f \cdot x], x], x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{LeQ}[m, -1] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

Rule 3475

$\text{Int}[\tan[(c + (d \cdot x))], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d \cdot x], x]]/d, x] /;$ $\text{FreeQ}\{c, d\}, x$

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{1 + \coth(x)} dx &= -\frac{1}{2(1 + \coth(x))} - \frac{1}{2} \int (1 - 2 \coth(x)) dx \\
&= -\frac{x}{2} - \frac{1}{2(1 + \coth(x))} + \int \coth(x) dx \\
&= -\frac{x}{2} - \frac{1}{2(1 + \coth(x))} + \log(\sinh(x))
\end{aligned}$$

Mathematica [A] time = 0.0285523, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x - \sinh(2x) + \cosh(2x) + 4 \log(\sinh(x)))$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2/(1 + Coth[x]),x]

[Out] (-2*x + Cosh[2*x] + 4*Log[Sinh[x]] - Sinh[2*x])/4

Maple [A] time = 0.017, size = 24, normalized size = 1.3

$$-\frac{1}{2 + 2 \coth(x)} - \frac{3 \ln(1 + \coth(x))}{4} - \frac{\ln(\coth(x) - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(1+coth(x)),x)

[Out] -1/2/(1+coth(x))-3/4*ln(1+coth(x))-1/4*ln(coth(x)-1)

Maxima [A] time = 1.10925, size = 32, normalized size = 1.68

$$\frac{1}{2}x + \frac{1}{4}e^{(-2x)} + \log(e^{(-x)} + 1) + \log(e^{(-x)} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="maxima")

[Out] $\frac{1}{2}x + \frac{1}{4}e^{-2x} + \log(e^{-x} + 1) + \log(e^{-x} - 1)$

Fricas [B] time = 3.09236, size = 259, normalized size = 13.63

$$\frac{6x \cosh(x)^2 + 12x \cosh(x) \sinh(x) + 6x \sinh(x)^2 - 4(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2) \log\left(\frac{2 \sinh(x)}{\cosh(x) - \sinh(x)}\right)}{4(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^2/(1+coth(x)),x, algorithm="fricas")`

[Out] $-\frac{1}{4}(6x \cosh(x)^2 + 12x \cosh(x) \sinh(x) + 6x \sinh(x)^2 - 4(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2) \log(2 \sinh(x) / (\cosh(x) - \sinh(x))) - 1) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)$

Sympy [B] time = 0.905729, size = 92, normalized size = 4.84

$$\frac{x \tanh(x)}{2 \tanh(x) + 2} + \frac{x}{2 \tanh(x) + 2} - \frac{2 \log(\tanh(x) + 1) \tanh(x)}{2 \tanh(x) + 2} - \frac{2 \log(\tanh(x) + 1)}{2 \tanh(x) + 2} + \frac{2 \log(\tanh(x)) \tanh(x)}{2 \tanh(x) + 2} + \frac{2 \log(\tanh(x))}{2 \tanh(x) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)**2/(1+coth(x)),x)`

[Out] $x \tanh(x) / (2 \tanh(x) + 2) + x / (2 \tanh(x) + 2) - 2 \log(\tanh(x) + 1) \tanh(x) / (2 \tanh(x) + 2) - 2 \log(\tanh(x) + 1) / (2 \tanh(x) + 2) + 2 \log(\tanh(x)) \tanh(x) / (2 \tanh(x) + 2) + 2 \log(\tanh(x)) / (2 \tanh(x) + 2) + 1 / (2 \tanh(x) + 2)$

Giac [A] time = 1.12442, size = 24, normalized size = 1.26

$$-\frac{3}{2}x + \frac{1}{4}e^{-2x} + \log(|e^{2x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^2/(1+coth(x)),x, algorithm="giac")`

[Out] $-3/2*x + 1/4*e^{-2*x} + \log(\text{abs}(e^{2*x} - 1))$

$$3.130 \quad \int \frac{\coth^3(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=31

$$\frac{3x}{2} + \frac{\coth^2(x)}{2(\coth(x)+1)} - \frac{3\coth(x)}{2} - \log(\sinh(x))$$

[Out] (3*x)/2 - (3*Coth[x])/2 + Coth[x]^2/(2*(1 + Coth[x])) - Log[Sinh[x]]

Rubi [A] time = 0.0553428, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3550, 3525, 3475}

$$\frac{3x}{2} + \frac{\coth^2(x)}{2(\coth(x)+1)} - \frac{3\coth(x)}{2} - \log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^3/(1 + Coth[x]),x]

[Out] (3*x)/2 - (3*Coth[x])/2 + Coth[x]^2/(2*(1 + Coth[x])) - Log[Sinh[x]]

Rule 3550

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(c + d*Tan[e + f*x])^(n - 1))/(2*a*f*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e + f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^3(x)}{1 + \coth(x)} dx &= \frac{\coth^2(x)}{2(1 + \coth(x))} - \frac{1}{2} \int (2 - 3 \coth(x)) \coth(x) dx \\ &= \frac{3x}{2} - \frac{3 \coth(x)}{2} + \frac{\coth^2(x)}{2(1 + \coth(x))} - \int \coth(x) dx \\ &= \frac{3x}{2} - \frac{3 \coth(x)}{2} + \frac{\coth^2(x)}{2(1 + \coth(x))} - \log(\sinh(x)) \end{aligned}$$

Mathematica [A] time = 0.0413891, size = 27, normalized size = 0.87

$$\frac{1}{4}(6x + \sinh(2x) - \cosh(2x) - 4 \coth(x) - 4 \log(\sinh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(1 + Coth[x]),x]
```

```
[Out] (6*x - Cosh[2*x] - 4*Coth[x] - 4*Log[Sinh[x]] + Sinh[2*x])/4
```

Maple [A] time = 0.017, size = 28, normalized size = 0.9

$$-\coth(x) + \frac{1}{2 + 2 \coth(x)} + \frac{5 \ln(1 + \coth(x))}{4} - \frac{\ln(\coth(x) - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(1+coth(x)),x)
```

```
[Out] -coth(x)+1/2/(1+coth(x))+5/4*ln(1+coth(x))-1/4*ln(coth(x)-1)
```

Maxima [A] time = 1.09329, size = 51, normalized size = 1.65

$$\frac{1}{2}x + \frac{2}{e^{(-2x)} - 1} - \frac{1}{4}e^{(-2x)} - \log(e^{(-x)} + 1) - \log(e^{(-x)} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="maxima")

[Out] $1/2*x + 2/(e^{-2*x} - 1) - 1/4*e^{-2*x} - \log(e^{-x} + 1) - \log(e^{-x} - 1)$

Fricas [B] time = 3.01997, size = 632, normalized size = 20.39

$$\frac{10x \cosh(x)^4 + 40x \cosh(x) \sinh(x)^3 + 10x \sinh(x)^4 - (10x + 9) \cosh(x)^2 + (60x \cosh(x)^2 - 10x - 9) \sinh(x)^2 - 4(\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + (6 \cosh(x)^2 - 1) \sinh(x)^2 - \cosh(x)^2 + 2(2 \cosh(x)^3 - \cosh(x)) \sinh(x) \log(2 \sinh(x) / (\cosh(x) - \sinh(x))) + 2(20x \cosh(x)^3 - (10x + 9) \cosh(x)) \sinh(x) + 1) / (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + (6 \cosh(x)^2 - 1) \sinh(x)^2 - \cosh(x)^2 + 2(2 \cosh(x)^3 - \cosh(x)) \sinh(x))}{4(\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + (6 \cosh(x)^2 - 1) \sinh(x)^2 - \cosh(x)^2 + 2(2 \cosh(x)^3 - \cosh(x)) \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="fricas")

[Out] $1/4*(10*x*\cosh(x)^4 + 40*x*\cosh(x)*\sinh(x)^3 + 10*x*\sinh(x)^4 - (10*x + 9)*\cosh(x)^2 + (60*x*\cosh(x)^2 - 10*x - 9)*\sinh(x)^2 - 4*(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + (6*\cosh(x)^2 - 1)*\sinh(x)^2 - \cosh(x)^2 + 2*(2*\cosh(x)^3 - \cosh(x))*\sinh(x))*\log(2*\sinh(x)/(\cosh(x) - \sinh(x))) + 2*(20*x*\cosh(x)^3 - (10*x + 9)*\cosh(x))*\sinh(x) + 1)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + (6*\cosh(x)^2 - 1)*\sinh(x)^2 - \cosh(x)^2 + 2*(2*\cosh(x)^3 - \cosh(x))*\sinh(x))$

Sympy [B] time = 1.3806, size = 162, normalized size = 5.23

$$\frac{x \tanh^2(x)}{2 \tanh^2(x) + 2 \tanh(x)} + \frac{x \tanh(x)}{2 \tanh^2(x) + 2 \tanh(x)} + \frac{2 \log(\tanh(x) + 1) \tanh^2(x)}{2 \tanh^2(x) + 2 \tanh(x)} + \frac{2 \log(\tanh(x) + 1) \tanh(x)}{2 \tanh^2(x) + 2 \tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**3/(1+coth(x)),x)

[Out] $x*\tanh(x)**2/(2*\tanh(x)**2 + 2*\tanh(x)) + x*\tanh(x)/(2*\tanh(x)**2 + 2*\tanh(x)) + 2*\log(\tanh(x) + 1)*\tanh(x)**2/(2*\tanh(x)**2 + 2*\tanh(x)) + 2*\log(\tanh(x) + 1)*\tanh(x)/(2*\tanh(x)**2 + 2*\tanh(x)) - 2*\log(\tanh(x))*\tanh(x)**2/(2*\tanh(x)**2 + 2*\tanh(x)) - 2*\log(\tanh(x))*\tanh(x)/(2*\tanh(x)**2 + 2*\tanh(x)) + 3*\tanh(x)**2/(2*\tanh(x)**2 + 2*\tanh(x)) - 2/(2*\tanh(x)**2 + 2*\tanh(x))$

Giac [A] time = 1.13294, size = 49, normalized size = 1.58

$$\frac{5}{2}x - \frac{(9e^{2x} - 1)e^{-2x}}{4(e^{2x} - 1)} - \log(|e^{2x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="giac")

[Out] 5/2*x - 1/4*(9*e^(2*x) - 1)*e^(-2*x)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))

$$3.131 \quad \int \frac{\coth^4(x)}{1+\coth(x)} dx$$

Optimal. Leaf size=37

$$-\frac{3x}{2} + \frac{\coth^3(x)}{2(\coth(x)+1)} - \coth^2(x) + \frac{3\coth(x)}{2} + 2\log(\sinh(x))$$

[Out] $(-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + Coth[x]^3/(2*(1 + Coth[x])) + 2*Log[Sinh[x]]$

Rubi [A] time = 0.0677348, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3550, 3528, 3525, 3475}

$$-\frac{3x}{2} + \frac{\coth^3(x)}{2(\coth(x)+1)} - \coth^2(x) + \frac{3\coth(x)}{2} + 2\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^4/(1 + Coth[x]),x]

[Out] $(-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + Coth[x]^3/(2*(1 + Coth[x])) + 2*Log[Sinh[x]]$

Rule 3550

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(c + d*Tan[e + f*x])^(n - 1))/(2*a*f*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]

Rule 3528

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3525

```
Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*
(x_)]), x_Symbol] := Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^4(x)}{1 + \coth(x)} dx &= \frac{\coth^3(x)}{2(1 + \coth(x))} - \frac{1}{2} \int (3 - 4 \coth(x)) \coth^2(x) dx \\ &= -\coth^2(x) + \frac{\coth^3(x)}{2(1 + \coth(x))} + \frac{1}{2} \int (-4i + 3i \coth(x)) \coth(x) dx \\ &= -\frac{3x}{2} + \frac{3 \coth(x)}{2} - \coth^2(x) + \frac{\coth^3(x)}{2(1 + \coth(x))} + 2 \int \coth(x) dx \\ &= -\frac{3x}{2} + \frac{3 \coth(x)}{2} - \coth^2(x) + \frac{\coth^3(x)}{2(1 + \coth(x))} + 2 \log(\sinh(x)) \end{aligned}$$

Mathematica [A] time = 0.0575645, size = 33, normalized size = 0.89

$$\frac{1}{4} (-6x - \sinh(2x) + \cosh(2x) + 4 \coth(x) - 2 \operatorname{csch}^2(x) + 8 \log(\sinh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(1 + Coth[x]),x]
```

```
[Out] (-6*x + Cosh[2*x] + 4*Coth[x] - 2*Csch[x]^2 + 8*Log[Sinh[x]] - Sinh[2*x])/4
```

Maple [A] time = 0.02, size = 32, normalized size = 0.9

$$-\frac{(\coth(x))^2}{2} + \coth(x) - \frac{1}{2 + 2 \coth(x)} - \frac{7 \ln(1 + \coth(x))}{4} - \frac{\ln(\coth(x) - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^4/(1+coth(x)),x)`

[Out] $-1/2*\coth(x)^2+\coth(x)-1/2/(1+\coth(x))-7/4*\ln(1+\coth(x))-1/4*\ln(\coth(x)-1)$

Maxima [A] time = 1.11246, size = 73, normalized size = 1.97

$$\frac{1}{2}x + \frac{2(2e^{(-2x)} - 1)}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{1}{4}e^{(-2x)} + 2 \log(e^{(-x)} + 1) + 2 \log(e^{(-x)} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^4/(1+coth(x)),x, algorithm="maxima")`

[Out] $1/2*x + 2*(2*e^{(-2*x)} - 1)/(2*e^{(-2*x)} - e^{(-4*x)} - 1) + 1/4*e^{(-2*x)} + 2*\log(e^{(-x)} + 1) + 2*\log(e^{(-x)} - 1)$

Fricas [B] time = 3.04278, size = 1168, normalized size = 31.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^4/(1+coth(x)),x, algorithm="fricas")`

[Out] $-1/4*(14*x*\cosh(x)^6 + 84*x*\cosh(x)*\sinh(x)^5 + 14*x*\sinh(x)^6 - (28*x + 1)*\cosh(x)^4 + (210*x*\cosh(x)^2 - 28*x - 1)*\sinh(x)^4 + 4*(70*x*\cosh(x)^3 - (28*x + 1)*\cosh(x))*\sinh(x)^3 + 2*(7*x + 5)*\cosh(x)^2 + 2*(105*x*\cosh(x)^4 - 3*(28*x + 1)*\cosh(x)^2 + 7*x + 5)*\sinh(x)^2 - 8*(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + (15*\cosh(x)^2 - 2)*\sinh(x)^4 - 2*\cosh(x)^4 + 4*(5*\cosh(x)^3 - 2*\cosh(x))*\sinh(x)^3 + (15*\cosh(x)^4 - 12*\cosh(x)^2 + 1)*\sinh(x)^2 + \cosh(x)^2 + 2*(3*\cosh(x)^5 - 4*\cosh(x)^3 + \cosh(x))*\sinh(x))*\log(2*\sinh(x)/(\cosh(x) - \sinh(x))) + 4*(21*x*\cosh(x)^5 - (28*x + 1)*\cosh(x)^3 + (7*x + 5)*\cosh(x))*\sinh(x) - 1)/(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + (15*\cosh(x)^2 - 2)*\sinh(x)^4 - 2*\cosh(x)^4 + 4*(5*\cosh(x)^3 - 2*\cosh(x))*\sinh(x)^3 + (15*\cosh(x)^4 - 12*\cosh(x)^2 + 1)*\sinh(x)^2 + \cosh(x)^2 + 2*(3*\cosh(x)^5 - 4*\cosh(x)^3 + \cosh(x))*\sinh(x))$

Sympy [B] time = 1.85889, size = 197, normalized size = 5.32

$$\frac{x \tanh^3(x)}{2 \tanh^3(x) + 2 \tanh^2(x)} + \frac{x \tanh^2(x)}{2 \tanh^3(x) + 2 \tanh^2(x)} - \frac{4 \log(\tanh(x) + 1) \tanh^3(x)}{2 \tanh^3(x) + 2 \tanh^2(x)} - \frac{4 \log(\tanh(x) + 1) \tanh^2(x)}{2 \tanh^3(x) + 2 \tanh^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**4/(1+coth(x)),x)

[Out] x*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + x*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) + 3*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) + tanh(x)/(2*tanh(x)**3 + 2*tanh(x)**2) - 1/(2*tanh(x)**3 + 2*tanh(x)**2)

Giac [A] time = 1.11409, size = 54, normalized size = 1.46

$$-\frac{7}{2}x + \frac{(e^{4x} - 10e^{2x} + 1)e^{-2x}}{4(e^{2x} - 1)^2} + 2 \log(|e^{2x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="giac")

[Out] -7/2*x + 1/4*(e^(4*x) - 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) - 1)^2 + 2*log(abs(e^(2*x) - 1))

3.132 $\int \coth(x)(1 + \coth(x))^{3/2} dx$

Optimal. Leaf size=45

$$-\frac{2}{3}(\coth(x) + 1)^{3/2} - 2\sqrt{\coth(x) + 1} + 2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}}\right)$$

[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1 + Coth[x])^(3/2))/3

Rubi [A] time = 0.051792, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3527, 3478, 3480, 206}

$$-\frac{2}{3}(\coth(x) + 1)^{3/2} - 2\sqrt{\coth(x) + 1} + 2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Coth[x]*(1 + Coth[x])^(3/2), x]

[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1 + Coth[x])^(3/2))/3

Rule 3527

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Dist[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

Rule 3478

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]

Rule 3480

Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,

b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \coth(x)(1 + \coth(x))^{3/2} dx &= -\frac{2}{3}(1 + \coth(x))^{3/2} + \int (1 + \coth(x))^{3/2} dx \\ &= -2\sqrt{1 + \coth(x)} - \frac{2}{3}(1 + \coth(x))^{3/2} + 2 \int \sqrt{1 + \coth(x)} dx \\ &= -2\sqrt{1 + \coth(x)} - \frac{2}{3}(1 + \coth(x))^{3/2} + 4 \operatorname{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{1 + \coth(x)}\right) \\ &= 2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1 + \coth(x)} - \frac{2}{3}(1 + \coth(x))^{3/2} \end{aligned}$$

Mathematica [C] time = 0.135816, size = 90, normalized size = 2.

$$\frac{2(\coth(x) + 1)^{3/2} \left(\cosh(x)\sqrt{i(\coth(x) + 1)} + 4 \sinh(x)\sqrt{i(\coth(x) + 1)} - (3 - 3i) \sinh(x) \tan^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{i(\coth(x) + 1)}\right) \right)}{3\sqrt{i(\coth(x) + 1)}(\sinh(x) + \cosh(x))}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]*(1 + Coth[x])^(3/2), x]

[Out] (-2*(1 + Coth[x])^(3/2)*(Cosh[x]*Sqrt[I*(1 + Coth[x])] - (3 - 3*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*Sinh[x] + 4*Sqrt[I*(1 + Coth[x])]*Sinh[x])/ (3*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x]))

Maple [A] time = 0.014, size = 35, normalized size = 0.8

$$-\frac{2}{3}(1 + \coth(x))^{3/2} + 2 \operatorname{Artanh}\left(\frac{1}{2}\sqrt{1 + \coth(x)}\sqrt{2}\right)\sqrt{2} - 2\sqrt{1 + \coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)*(1+coth(x))^(3/2),x)`

[Out] $-2/3*(1+\coth(x))^{3/2}+2*\operatorname{arctanh}(1/2*(1+\coth(x))^{1/2}*2^{1/2})*2^{1/2}-2*(1+\coth(x))^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + 1)^{\frac{3}{2}} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="maxima")`

[Out] `integrate((coth(x) + 1)^(3/2)*coth(x), x)`

Fricas [B] time = 3.052, size = 869, normalized size = 19.31

$$2\sqrt{2}(5\sqrt{2}\cosh(x)^3 + 15\sqrt{2}\cosh(x)\sinh(x)^2 + 5\sqrt{2}\sinh(x)^3 + 3(5\sqrt{2}\cosh(x)^2 - \sqrt{2})\sinh(x) - 3\sqrt{2}\cosh(x))\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))} - 3(\sqrt{2}\cosh(x)^4 + 4\sqrt{2}\cosh(x)\sinh(x)^3 + \sqrt{2}\sinh(x)^4 + 2(3\sqrt{2}\cosh(x)^2 - \sqrt{2})\sinh(x)^2 - 2\sqrt{2}\cosh(x)^2 + 4(\sqrt{2}\cosh(x)^3 - \sqrt{2}\cosh(x))\sinh(x) + \sqrt{2})\log(2\sqrt{2}\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}(\cosh(x) + \sinh(x)) + 2\cosh(x)^2 + 4\cosh(x)\sinh(x) + 2\sinh(x)^2 - 1)/(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 - 1)\sinh(x)^2 - 2\cosh(x)^2 + 4(\cosh(x)^3 - \cosh(x))\sinh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="fricas")`

[Out] $-1/3*(2*\sqrt{2}*(5*\sqrt{2}*\cosh(x)^3 + 15*\sqrt{2}*\cosh(x)*\sinh(x)^2 + 5*\sqrt{2}*\sinh(x)^3 + 3*(5*\sqrt{2}*\cosh(x)^2 - \sqrt{2})*\sinh(x) - 3*\sqrt{2}*\cosh(x))*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))} - 3*(\sqrt{2}*\cosh(x)^4 + 4*\sqrt{2}*\cosh(x)*\sinh(x)^3 + \sqrt{2}*\sinh(x)^4 + 2*(3*\sqrt{2}*\cosh(x)^2 - \sqrt{2})*\sinh(x)^2 - 2*\sqrt{2}*\cosh(x)^2 + 4*(\sqrt{2}*\cosh(x)^3 - \sqrt{2}*\cosh(x))*\sinh(x) + \sqrt{2})*\log(2*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}*(\cosh(x) + \sinh(x)) + 2*\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + 2*\sinh(x)^2 - 1)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + 1)^{\frac{3}{2}} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(1+coth(x))**(3/2),x)

[Out] Integral((coth(x) + 1)**(3/2)*coth(x), x)

Giac [B] time = 1.18529, size = 182, normalized size = 4.04

$$-\frac{1}{3} \sqrt{2} \left(3 \log \left(\left| 2 \sqrt{e^{4x} - e^{2x}} - 2e^{2x} + 1 \right| \right) \operatorname{sgn}(e^{2x} - 1) + \frac{2 \left(9 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^2 \operatorname{sgn}(e^{2x} - 1) + 12 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right) \right)}{\left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right) - e^{2x} + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="giac")

[Out] -1/3*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x)) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x) - 1) + 2*(9*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 12*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))/(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x) + 1)^3

3.133 $\int \coth(x)\sqrt{1 + \coth(x)} dx$

Optimal. Leaf size=32

$$\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - 2\sqrt{\coth(x)+1}$$

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]

Rubi [A] time = 0.0381682, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3527, 3480, 206}

$$\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - 2\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]*Sqrt[1 + Coth[x]],x]

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]

Rule 3527

Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Dist[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && !LtQ[m, 0]

Rule 3480

Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \coth(x)\sqrt{1+\coth(x)} dx &= -2\sqrt{1+\coth(x)} + \int \sqrt{1+\coth(x)} dx \\
&= -2\sqrt{1+\coth(x)} + 2\text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{1+\coth(x)}\right) \\
&= \sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)}
\end{aligned}$$

Mathematica [C] time = 0.119345, size = 53, normalized size = 1.66

$$(1+i)\sqrt{\coth(x)+1} \left(-\frac{i \tan^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{\sqrt{i(\coth(x)+1)}} - (1-i) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]*Sqrt[1 + Coth[x]], x]

[Out] (1 + I)*Sqrt[1 + Coth[x]]*((-1 + I) - (I*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]])/Sqrt[I*(1 + Coth[x])])

Maple [A] time = 0.036, size = 26, normalized size = 0.8

$$\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)\sqrt{2} - 2\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(1+coth(x))^(1/2), x)

[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x)+1} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(coth(x) + 1)*coth(x), x)

Fricas [B] time = 2.89116, size = 455, normalized size = 14.22

$$\frac{4\sqrt{2}(\sqrt{2}\cosh(x) + \sqrt{2}\sinh(x))\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} - (\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 - \sqrt{2})\log\left(\frac{2(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2)}{\cosh(x)-\sinh(x)}\right)}{2(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="fricas")

[Out]
$$-1/2*(4*\sqrt{2}*(\sqrt{2}*\cosh(x) + \sqrt{2}*\sinh(x))*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))} - (\sqrt{2}*\cosh(x)^2 + 2*\sqrt{2}*\cosh(x)*\sinh(x) + \sqrt{2}*\sinh(x)^2 - \sqrt{2})*\log(2*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}*(\cosh(x) + \sinh(x) + 2*\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + 2*\sinh(x)^2 - 1))/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x) + 1} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(1+coth(x))**(1/2),x)

[Out] Integral(sqrt(coth(x) + 1)*coth(x), x)

Giac [B] time = 1.1659, size = 96, normalized size = 3.

$$-\frac{1}{2}\sqrt{2}\left(\log\left(\left|2\sqrt{e^{4x}-e^{2x}}-2e^{2x}+1\right|\right)\operatorname{sgn}\left(e^{2x}-1\right)+\frac{4\operatorname{sgn}\left(e^{2x}-1\right)}{\sqrt{e^{4x}-e^{2x}}-e^{2x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x) - 1) + 4*sgn(e^(2*x) - 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1))
```

$$3.134 \quad \int \frac{\coth(x)}{\sqrt{1+\coth(x)}} dx$$

Optimal. Leaf size=30

$$\frac{1}{\sqrt{\coth(x)+1}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Coth[x]]

Rubi [A] time = 0.0386308, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3526, 3480, 206}

$$\frac{1}{\sqrt{\coth(x)+1}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/Sqrt[1 + Coth[x]], x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Coth[x]]

Rule 3526

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rule 3480

Int[Sqrt[(a_) + (b_)*tan[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\coth(x)}{\sqrt{1 + \coth(x)}} dx &= \frac{1}{\sqrt{1 + \coth(x)}} + \frac{1}{2} \int \sqrt{1 + \coth(x)} dx \\ &= \frac{1}{\sqrt{1 + \coth(x)}} + \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{1}{\sqrt{1 + \coth(x)}} \end{aligned}$$

Mathematica [C] time = 0.194822, size = 97, normalized size = 3.23

$$\frac{\text{csch}(x) \left(\frac{1}{2} \sinh(2x) - \frac{1}{2} \cosh(2x) + \frac{1}{2} \right) (\sinh(x) + \cosh(x))}{\sqrt{\coth(x) + 1}} + \frac{\left(\frac{1}{2} - \frac{i}{2} \right) \text{csch}(x) (\sinh(x) + \cosh(x)) \tan^{-1} \left(\left(\frac{1}{2} + \frac{i}{2} \right) \sqrt{i \coth(x) + 1} \right)}{\sqrt{i \coth(x) + i \sqrt{\coth(x) + 1}}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]/Sqrt[1 + Coth[x]], x]

[Out] ((1/2 - I/2)*ArcTan[(1/2 + I/2)*Sqrt[I + I*Coth[x]]]*Csch[x]*(Cosh[x] + Sinh[x]))/(Sqrt[I + I*Coth[x]]*Sqrt[1 + Coth[x]]) + (Csch[x]*(Cosh[x] + Sinh[x]))*(1/2 - Cosh[2*x]/2 + Sinh[2*x]/2)/Sqrt[1 + Coth[x]]

Maple [A] time = 0.039, size = 25, normalized size = 0.8

$$\frac{\sqrt{2}}{2} \text{Artanh} \left(\frac{\sqrt{2}}{2} \sqrt{1 + \coth(x)} \right) + \frac{1}{\sqrt{1 + \coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(1+coth(x))^(1/2), x)

[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+coth(x))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{\coth(x)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="maxima")

[Out] integrate(coth(x)/sqrt(coth(x) + 1), x)

Fricas [B] time = 2.99012, size = 300, normalized size = 10.

$$\frac{(\sqrt{2} \cosh(x) + \sqrt{2} \sinh(x)) \log\left(2 \sqrt{2} \sqrt{\frac{\sinh(x)}{\cosh(x) - \sinh(x)}} (\cosh(x) + \sinh(x)) + 2 \cosh(x)^2 + 4 \cosh(x) \sinh(x) + 2 \sinh(x)^2 - 1\right)}{4 (\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="fricas")

[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1) + 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{\coth(x)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1+coth(x))**(1/2),x)

[Out] Integral(coth(x)/sqrt(coth(x) + 1), x)

Giac [B] time = 1.16177, size = 119, normalized size = 3.97

$$-\frac{1}{2}\sqrt{2}\operatorname{sgn}(e^{2x}-1) - \frac{\sqrt{2}\log\left(\left|-2\sqrt{e^{4x}-e^{2x}}+2e^{2x}-1\right|\right)}{4\operatorname{sgn}(e^{2x}-1)} - \frac{\sqrt{2}}{2\left(\sqrt{e^{4x}-e^{2x}}-e^{2x}\right)\operatorname{sgn}(e^{2x}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*sgn(e^(2*x) - 1) - 1/4*sqrt(2)*log(abs(-2*sqrt(e^(4*x) - e^(2*x)) + 2*e^(2*x) - 1))/sgn(e^(2*x) - 1) - 1/2*sqrt(2)/((sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1))

$$3.135 \quad \int \frac{\coth(x)}{(1+\coth(x))^{3/2}} dx$$

Optimal. Leaf size=49

$$-\frac{1}{2\sqrt{\coth(x)+1}} + \frac{1}{3(\coth(x)+1)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Coth[x])^(3/2)) - 1/(2*Sqrt[1 + Coth[x]])

Rubi [A] time = 0.0508973, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3526, 3479, 3480, 206}

$$-\frac{1}{2\sqrt{\coth(x)+1}} + \frac{1}{3(\coth(x)+1)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/(1 + Coth[x])^(3/2), x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Coth[x])^(3/2)) - 1/(2*Sqrt[1 + Coth[x]])

Rule 3526

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rule 3479

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_) + (d_.)*(x_)]], x_Symbol] := Dist[(-2*b)/d,
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\coth(x)}{(1 + \coth(x))^{3/2}} dx &= \frac{1}{3(1 + \coth(x))^{3/2}} + \frac{1}{2} \int \frac{1}{\sqrt{1 + \coth(x)}} dx \\ &= \frac{1}{3(1 + \coth(x))^{3/2}} - \frac{1}{2\sqrt{1 + \coth(x)}} + \frac{1}{4} \int \sqrt{1 + \coth(x)} dx \\ &= \frac{1}{3(1 + \coth(x))^{3/2}} - \frac{1}{2\sqrt{1 + \coth(x)}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right)}{2\sqrt{2}} + \frac{1}{3(1 + \coth(x))^{3/2}} - \frac{1}{2\sqrt{1 + \coth(x)}} \end{aligned}$$

Mathematica [C] time = 0.33222, size = 84, normalized size = 1.71

$$\left(\frac{1}{4} + \frac{i}{4}\right) \sqrt{\coth(x) + 1} \left(\left(\frac{1}{6} - \frac{i}{6}\right) (-\sinh(2x) - \sinh(4x) + \cosh(2x) + \cosh(4x) - 2) - \frac{i \tan^{-1} \left(\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{i(\coth(x) + 1)} \right)}{\sqrt{i(\coth(x) + 1)}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 + Coth[x])^(3/2), x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*((( -I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] + (1/6 - I/6)*(-2 + Cosh[2*x] + Cosh[4*x] - Sinh
[2*x] - Sinh[4*x])
```

Maple [A] time = 0.013, size = 35, normalized size = 0.7

$$\frac{1}{3} (1 + \coth(x))^{-\frac{3}{2}} + \frac{\sqrt{2}}{4} \text{Artanh} \left(\frac{\sqrt{2}}{2} \sqrt{1 + \coth(x)} \right) - \frac{1}{2} \frac{1}{\sqrt{1 + \coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)/(1+coth(x))^(3/2),x)`

[Out] $\frac{1}{3}(1+\coth(x))^{-3/2} + \frac{1}{4} \operatorname{arctanh}\left(\frac{1}{2}(1+\coth(x))^{1/2}\right) 2^{1/2} - \frac{1}{2}(1+\coth(x))^{-1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{(\coth(x) + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="maxima")`

[Out] `integrate(coth(x)/(coth(x) + 1)^(3/2), x)`

Fricas [B] time = 2.95701, size = 578, normalized size = 11.8

$$\frac{2\sqrt{2}(2\sqrt{2}\cosh(x)^2 + 4\sqrt{2}\cosh(x)\sinh(x) + 2\sqrt{2}\sinh(x)^2 + \sqrt{2})\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} - 3(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)\sinh(x)^2 + 3\sqrt{2}\sinh(x)^3) \log\left(\frac{\sinh(x)}{\cosh(x)-\sinh(x)}\right) + 3(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)\sinh(x)^2 + 3\sqrt{2}\sinh(x)^3)}{24(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + 3\sinh(x)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="fricas")`

[Out] $-\frac{1}{24}(2\sqrt{2}(2\sqrt{2}\cosh(x)^2 + 4\sqrt{2}\cosh(x)\sinh(x) + 2\sqrt{2}\sinh(x)^2 + \sqrt{2})\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} - 3(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)\sinh(x)^2 + 3\sqrt{2}\sinh(x)^3) \log(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x) + \sinh(x)) + 2\cosh(x)^2 + 4\cosh(x)\sinh(x) + 2\sinh(x)^2 - 1))/(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + 3\sinh(x)^3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{(\coth(x) + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1+coth(x))**(3/2),x)

[Out] Integral(coth(x)/(coth(x) + 1)**(3/2), x)

Giac [B] time = 1.18417, size = 144, normalized size = 2.94

$$-\frac{1}{24} \sqrt{2} \left(\frac{3 \log \left(\left| 2 \sqrt{e^{4x} - e^{2x}} - 2e^{2x} + 1 \right| \right)}{\operatorname{sgn}(e^{2x} - 1)} + \frac{2 \left(3 \sqrt{e^{4x} - e^{2x}} - 3e^{2x} + 1 \right)}{\left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^3 \operatorname{sgn}(e^{2x} - 1)} - 4 \operatorname{sgn}(e^{2x} - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="giac")

[Out] -1/24*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x)) - e^(2*x)) - 2*e^(2*x) + 1))/sgn(e^(2*x) - 1) + 2*(3*sqrt(e^(4*x)) - 3*e^(2*x) + 1)/((sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1) - 4*sgn(e^(2*x) - 1)

3.136 $\int \coth^2(x)(1 + \coth(x))^{3/2} dx$

Optimal. Leaf size=45

$$-\frac{2}{5}(\coth(x) + 1)^{5/2} - 2\sqrt{\coth(x) + 1} + 2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}}\right)$$

[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1 + Coth[x])^(5/2))/5

Rubi [A] time = 0.0628274, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3543, 3478, 3480, 206}

$$-\frac{2}{5}(\coth(x) + 1)^{5/2} - 2\sqrt{\coth(x) + 1} + 2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2*(1 + Coth[x])^(3/2), x]

[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1 + Coth[x])^(5/2))/5

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3478

```
Int[((a_.) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[(-2*b)/d,
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \coth^2(x)(1 + \coth(x))^{3/2} dx &= -\frac{2}{5}(1 + \coth(x))^{5/2} + \int (1 + \coth(x))^{3/2} dx \\
 &= -2\sqrt{1 + \coth(x)} - \frac{2}{5}(1 + \coth(x))^{5/2} + 2 \int \sqrt{1 + \coth(x)} dx \\
 &= -2\sqrt{1 + \coth(x)} - \frac{2}{5}(1 + \coth(x))^{5/2} + 4 \operatorname{Subst}\left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)}\right) \\
 &= 2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1 + \coth(x)} - \frac{2}{5}(1 + \coth(x))^{5/2}
 \end{aligned}$$

Mathematica [C] time = 0.253652, size = 70, normalized size = 1.56

$$\frac{2\left(2\coth^2(x) + \operatorname{csch}^2(x) + (5 + 5i)\sqrt{i(\coth(x) + 1)} \tan^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{i(\coth(x) + 1)}\right) + \coth(x)(\operatorname{csch}^2(x) + 9) + 7\right)}{5\sqrt{\coth(x) + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*(1 + Coth[x])^(3/2), x]
```

```
[Out] (-2*(7 + 2*Coth[x]^2 + (5 + 5*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*
  Sqrt[I*(1 + Coth[x])] + Csch[x]^2 + Coth[x]*(9 + Csch[x]^2))/(5*Sqrt[1 + C
  oth[x]])
```

Maple [A] time = 0.019, size = 35, normalized size = 0.8

$$-\frac{2}{5}(1 + \coth(x))^{\frac{5}{2}} + 2 \operatorname{Artanh}\left(\frac{1}{2}\sqrt{1 + \coth(x)}\sqrt{2}\right)\sqrt{2} - 2\sqrt{1 + \coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^2*(1+coth(x))^(3/2),x)`

[Out] $-2/5*(1+\coth(x))^{5/2}+2*\operatorname{arctanh}(1/2*(1+\coth(x))^{1/2}*2^{1/2})*2^{1/2}-2*(1+\coth(x))^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + 1)^{\frac{3}{2}} \coth(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="maxima")`

[Out] `integrate((coth(x) + 1)^(3/2)*coth(x)^2, x)`

Fricas [B] time = 2.95329, size = 1449, normalized size = 32.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="fricas")`

[Out] $-1/5*(2*\sqrt{2}*(9*\sqrt{2}*\cosh(x)^5 + 45*\sqrt{2}*\cosh(x)*\sinh(x)^4 + 9*\sqrt{2}*\sinh(x)^5 + 10*(9*\sqrt{2}*\cosh(x)^2 - \sqrt{2})*\sinh(x)^3 - 10*\sqrt{2}*\cosh(x)^3 + 30*(3*\sqrt{2}*\cosh(x)^3 - \sqrt{2}*\cosh(x))*\sinh(x)^2 + 5*(9*\sqrt{2}*\cosh(x)^4 - 6*\sqrt{2}*\cosh(x)^2 + \sqrt{2})*\sinh(x) + 5*\sqrt{2}*\cosh(x))*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))} - 5*(\sqrt{2}*\cosh(x)^6 + 6*\sqrt{2}*\cosh(x)*\sinh(x)^5 + \sqrt{2}*\sinh(x)^6 + 3*(5*\sqrt{2}*\cosh(x)^2 - \sqrt{2})*\sinh(x)^4 - 3*\sqrt{2}*\cosh(x)^4 + 4*(5*\sqrt{2}*\cosh(x)^3 - 3*\sqrt{2}*\cosh(x))*\sinh(x)^3 + 3*(5*\sqrt{2}*\cosh(x)^4 - 6*\sqrt{2}*\cosh(x)^2 + \sqrt{2})*\sinh(x)^2 + 3*\sqrt{2}*\cosh(x)^2 + 6*(\sqrt{2}*\cosh(x)^5 - 2*\sqrt{2}*\cosh(x)^3 + \sqrt{2}*\cosh(x))*\sinh(x) - \sqrt{2})*\log(2*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}*(\cosh(x) + \sinh(x)) + 2*\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + 2*\sinh(x)^2 - 1))/(\cosh(x)^6 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6 + 3*(5*\cosh(x)^2 - 1)*\sinh(x)^4 - 3*\cosh(x)^4 + 4*(5*\cosh(x)^3 - 3*\cosh(x))*\sinh(x)^3 + 3*(5*\cosh(x)^4 - 6*\cosh(x)^2 + 1)*\sinh(x)^2 + 3*\cosh(x)^2 + 6*(\cosh(x)^5 - 2*\cosh(x)^3 +$

$\cosh(x) \cdot \sinh(x) - 1$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\coth(x) + 1)^{\frac{3}{2}} \coth^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2*(1+coth(x))**(3/2),x)

[Out] Integral((coth(x) + 1)**(3/2)*coth(x)**2, x)

Giac [B] time = 1.20042, size = 266, normalized size = 5.91

$$-\frac{1}{5} \sqrt{2} \left(5 \log \left(\left| 2 \sqrt{e^{4x} - e^{2x}} - 2e^{2x} + 1 \right| \right) \operatorname{sgn}(e^{2x} - 1) + \frac{2 \left(25 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^4 \operatorname{sgn}(e^{2x} - 1) + 60 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^3 \operatorname{sgn}(e^{2x} - 1) + 70 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right)^2 \operatorname{sgn}(e^{2x} - 1) + 40 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x} \right) \operatorname{sgn}(e^{2x} - 1) + 9 \operatorname{sgn}(e^{2x} - 1) \right)}{\left(\sqrt{e^{4x} - e^{2x}} - e^{2x} + 1 \right)^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="giac")

[Out] -1/5*sqrt(2)*(5*log(abs(2*sqrt(e^(4*x)) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x) - 1) + 2*(25*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^4*sgn(e^(2*x) - 1) + 60*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1) + 70*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 40*(sqrt(e^(4*x)) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + 9*sgn(e^(2*x) - 1))/(sqrt(e^(4*x)) - e^(2*x) + 1)^5)

3.137 $\int \coth^2(x) \sqrt{1 + \coth(x)} dx$

Optimal. Leaf size=34

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}} \right) - \frac{2}{3} (\coth(x) + 1)^{3/2}$$

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*(1 + Coth[x])^(3/2))/3

Rubi [A] time = 0.0478325, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3543, 3480, 206}

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{\coth(x) + 1}}{\sqrt{2}} \right) - \frac{2}{3} (\coth(x) + 1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2*Sqrt[1 + Coth[x]],x]

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*(1 + Coth[x])^(3/2))/3

Rule 3543

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])

Rule 3480

Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \coth^2(x)\sqrt{1+\coth(x)} dx &= -\frac{2}{3}(1+\coth(x))^{3/2} + \int \sqrt{1+\coth(x)} dx \\
&= -\frac{2}{3}(1+\coth(x))^{3/2} + 2 \operatorname{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{1+\coth(x)}\right) \\
&= \sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - \frac{2}{3}(1+\coth(x))^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.168769, size = 61, normalized size = 1.79

$$\frac{-2 \coth^2(x) - 4 \coth(x) - (3 + 3i)\sqrt{i(\coth(x) + 1)} \tan^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{i(\coth(x) + 1)}\right) - 2}{3\sqrt{\coth(x) + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2*Sqrt[1 + Coth[x]],x]

[Out] (-2 - 4*Coth[x] - 2*Coth[x]^2 - (3 + 3*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*Sqrt[I*(1 + Coth[x])])/(3*Sqrt[1 + Coth[x]])

Maple [A] time = 0.039, size = 26, normalized size = 0.8

$$-\frac{2}{3}(1+\coth(x))^{\frac{3}{2}} + \operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\coth(x)}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2*(1+coth(x))^(1/2),x)

[Out] -2/3*(1+coth(x))^(3/2)+arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x) + 1} \coth(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(coth(x) + 1)*coth(x)^2, x)

Fricas [B] time = 3.16877, size = 817, normalized size = 24.03

$$8\sqrt{2}\left(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)^2\sinh(x) + 3\sqrt{2}\cosh(x)\sinh(x)^2 + \sqrt{2}\sinh(x)^3\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} - 3\left(\sqrt{2}\cosh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/6*(8*\sqrt{2}*(\sqrt{2}*\cosh(x)^3 + 3*\sqrt{2}*\cosh(x)^2*\sinh(x) + 3*\sqrt{2}*\cosh(x)*\sinh(x)^2 + \sqrt{2}*\sinh(x)^3)*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}) \\ & - 3*(\sqrt{2}*\cosh(x)^4 + 4*\sqrt{2}*\cosh(x)*\sinh(x)^3 + \sqrt{2}*\sinh(x)^4 + \\ & 2*(3*\sqrt{2}*\cosh(x)^2 - \sqrt{2})*\sinh(x)^2 - 2*\sqrt{2}*\cosh(x)^2 + 4*(\sqrt{2}*\cosh(x)^3 - \sqrt{2}*\cosh(x))*\sinh(x) + \sqrt{2})*\log(2*\sqrt{2}*\sqrt{\sinh(x)/(\cosh(x) - \sinh(x))}*(\cosh(x) + \sinh(x)) + 2*\cosh(x)^2 + 4*\cosh(x)*\sinh(x) + 2*\sinh(x)^2 - 1))/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1) \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\coth(x) + 1} \coth^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2*(1+coth(x))**(1/2),x)

[Out] Integral(sqrt(coth(x) + 1)*coth(x)**2, x)

Giac [B] time = 1.18143, size = 180, normalized size = 5.29

$$-\frac{1}{6}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{4x}-e^{2x}}-2e^{2x}+1\right|\right)\operatorname{sgn}\left(e^{2x}-1\right)+\frac{8\left(3\left(\sqrt{e^{4x}-e^{2x}}-e^{2x}\right)^2\operatorname{sgn}\left(e^{2x}-1\right)+3\left(\sqrt{e^{4x}-e^{2x}}-e^{2x}\right)\right)}{\left(\sqrt{e^{4x}-e^{2x}}-e^{2x}+1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")

[Out] -1/6*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x)) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x) - 1) + 8*(3*(sqrt(e^(4*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 3*(sqrt(e^(4*x)) - e^(2*x))*sgn(e^(2*x) - 1) + sgn(e^(2*x) - 1))/(sqrt(e^(4*x)) - e^(2*x) + 1)^3)

$$3.138 \quad \int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}} dx$$

Optimal. Leaf size=42

$$-2\sqrt{\coth(x)+1} - \frac{1}{\sqrt{\coth(x)+1}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]] - 2*Sqrt[1 + Coth[x]]

Rubi [A] time = 0.058539, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3543, 3479, 3480, 206}

$$-2\sqrt{\coth(x)+1} - \frac{1}{\sqrt{\coth(x)+1}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2/Sqrt[1 + Coth[x]],x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]] - 2*Sqrt[1 + Coth[x]]

Rule 3543

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])

Rule 3479

Int[((a_.) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_) + (d_.)*(x_)]], x_Symbol] := Dist[(-2*b)/d,
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^2(x)}{\sqrt{1 + \coth(x)}} dx &= -2\sqrt{1 + \coth(x)} + \int \frac{1}{\sqrt{1 + \coth(x)}} dx \\
 &= -\frac{1}{\sqrt{1 + \coth(x)}} - 2\sqrt{1 + \coth(x)} + \frac{1}{2} \int \sqrt{1 + \coth(x)} dx \\
 &= -\frac{1}{\sqrt{1 + \coth(x)}} - 2\sqrt{1 + \coth(x)} + \text{Subst}\left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)}\right) \\
 &= \frac{\tanh^{-1}\left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{1 + \coth(x)}} - 2\sqrt{1 + \coth(x)}
 \end{aligned}$$

Mathematica [C] time = 0.359309, size = 81, normalized size = 1.93

$$\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \text{csch}(x)(\sinh(x) + \cosh(x)) \left(\left(\frac{1}{2} - \frac{i}{2}\right) (-\sinh(2x) + \cosh(2x) - 5) - \frac{i \tan^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{i(\coth(x)+1)}\right)}{\sqrt{i(\coth(x)+1)}} \right)}{\sqrt{\coth(x) + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/Sqrt[1 + Coth[x]], x]
```

```
[Out] ((1/2 + I/2)*Csch[x]*(Cosh[x] + Sinh[x])*((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(
  1 + Coth[x])]])/Sqrt[I*(1 + Coth[x])] + (1/2 - I/2)*(-5 + Cosh[2*x] - Sinh[
  2*x]))/Sqrt[1 + Coth[x]]
```


Maple [A] time = 0.041, size = 35, normalized size = 0.8

$$\frac{\sqrt{2}}{2} \operatorname{Arctanh} \left(\frac{\sqrt{2}}{2} \sqrt{1 + \coth(x)} \right) - \frac{1}{\sqrt{1 + \coth(x)}} - 2 \sqrt{1 + \coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^2/(1+coth(x))^(1/2),x)`

[Out] `1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)-2*(1+coth(x))^(1/2)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{\sqrt{\coth(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="maxima")`

[Out] `integrate(coth(x)^2/sqrt(coth(x) + 1), x)`

Fricas [B] time = 3.11865, size = 637, normalized size = 15.17

$$\frac{2\sqrt{2}(5\sqrt{2}\cosh(x)^2 + 10\sqrt{2}\cosh(x)\sinh(x) + 5\sqrt{2}\sinh(x)^2 - \sqrt{2})\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} - (\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)\sinh(x)^2 + 3\sqrt{2}\sinh(x)^3 - \sqrt{2})}{4(\cosh(x)^3 + \sinh(x)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="fricas")`

[Out] `-1/4*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^2 + 10*sqrt(2)*cosh(x)*sinh(x) + 5*sqrt(2)*sinh(x)^2 - sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - (sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + 3*sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2)`

$(x)^2 - 1)/(\cosh(x)^3 + 3*\cosh(x)*\sinh(x)^2 + \sinh(x)^3 + (3*\cosh(x)^2 - 1)*\sinh(x) - \cosh(x))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{\sqrt{\coth(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2/(1+coth(x))**(1/2), x)

[Out] Integral(coth(x)**2/sqrt(coth(x) + 1), x)

Giac [B] time = 1.20583, size = 119, normalized size = 2.83

$$-\frac{\frac{5\sqrt{2}e^{2x}}{\operatorname{sgn}(e^{2x}-1)} - \frac{\sqrt{2}}{\operatorname{sgn}(e^{2x}-1)}}{2\sqrt{e^{4x}-e^{2x}}} - \frac{\sqrt{2}\log\left(\left|2\sqrt{e^{4x}-e^{2x}}-2e^{2x}+1\right|\right)}{4\operatorname{sgn}(e^{2x}-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(1+coth(x))^(1/2), x, algorithm="giac")

[Out] $-1/2*(5*\sqrt{2}*e^{2*x}/\operatorname{sgn}(e^{2*x} - 1) - \sqrt{2}/\operatorname{sgn}(e^{2*x} - 1))/\sqrt{e^{4*x} - e^{2*x}} - 1/4*\sqrt{2}*\log(\operatorname{abs}(2*\sqrt{e^{4*x} - e^{2*x}} - 2*e^{2*x} + 1))/\operatorname{sgn}(e^{2*x} - 1)$

$$3.139 \quad \int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}} dx$$

Optimal. Leaf size=49

$$\frac{3}{2\sqrt{\coth(x)+1}} - \frac{1}{3(\coth(x)+1)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2)) + 3/(2*Sqrt[1 + Coth[x]])

Rubi [A] time = 0.0821066, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3540, 3526, 3480, 206}

$$\frac{3}{2\sqrt{\coth(x)+1}} - \frac{1}{3(\coth(x)+1)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2/(1 + Coth[x])^(3/2),x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2)) + 3/(2*Sqrt[1 + Coth[x]])

Rule 3540

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)])^2, x_Symbol] :> -Simp[(b*(a*c + b*d)^2*(a + b*Tan[e + f*x])^m)/(2*a^3*f*m), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]

Rule 3526

Int[((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0]

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[(-2*b)/d,
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
  b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(x)}{(1 + \coth(x))^{3/2}} dx &= -\frac{1}{3(1 + \coth(x))^{3/2}} - \frac{1}{2} \int \frac{1 - 2 \coth(x)}{\sqrt{1 + \coth(x)}} dx \\ &= -\frac{1}{3(1 + \coth(x))^{3/2}} + \frac{3}{2\sqrt{1 + \coth(x)}} + \frac{1}{4} \int \sqrt{1 + \coth(x)} dx \\ &= -\frac{1}{3(1 + \coth(x))^{3/2}} + \frac{3}{2\sqrt{1 + \coth(x)}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \sqrt{1 + \coth(x)} \right) \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{1 + \coth(x)}}{\sqrt{2}} \right)}{2\sqrt{2}} - \frac{1}{3(1 + \coth(x))^{3/2}} + \frac{3}{2\sqrt{1 + \coth(x)}} \end{aligned}$$

Mathematica [C] time = 0.349202, size = 86, normalized size = 1.76

$$\left(\frac{1}{4} + \frac{i}{4}\right) \sqrt{\coth(x) + 1} \left(-\left(\frac{1}{6} - \frac{i}{6}\right) (-7 \sinh(2x) - \sinh(4x) + 7 \cosh(2x) + \cosh(4x) - 8) - \frac{i \tan^{-1} \left(\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{i(\coth(x))} \right)}{\sqrt{i(\coth(x) + 1)}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(1 + Coth[x])^(3/2), x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*((( -I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])
  ]])]/Sqrt[I*(1 + Coth[x])] - (1/6 - I/6)*(-8 + 7*Cosh[2*x] + Cosh[4*x] - 7*
  Sinh[2*x] - Sinh[4*x])
```

Maple [A] time = 0.017, size = 35, normalized size = 0.7

$$-\frac{1}{3}(1 + \coth(x))^{-\frac{3}{2}} + \frac{\sqrt{2}}{4} \operatorname{Arctanh}\left(\frac{\sqrt{2}}{2}\sqrt{1 + \coth(x)}\right) + \frac{3}{2}\frac{1}{\sqrt{1 + \coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(1+coth(x))^(3/2), x)

[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+3/2/(1+coth(x))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^2}{(\coth(x) + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(1+coth(x))^(3/2), x, algorithm="maxima")

[Out] integrate(coth(x)^2/(coth(x) + 1)^(3/2), x)

Fricas [B] time = 2.92866, size = 578, normalized size = 11.8

$$\frac{2\sqrt{2}(8\sqrt{2}\cosh(x)^2 + 16\sqrt{2}\cosh(x)\sinh(x) + 8\sqrt{2}\sinh(x)^2 + \sqrt{2})\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} + 3(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)\sinh(x)^2 + 3\sqrt{2}\sinh(x)^3) + 3}{24(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + 3\sinh(x)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(1+coth(x))^(3/2), x, algorithm="fricas")

[Out] 1/24*(2*sqrt(2)*(8*sqrt(2)*cosh(x)^2 + 16*sqrt(2)*cosh(x)*sinh(x) + 8*sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*sinh(x)^3)

$\cosh(x)^2 \sinh(x) + 3 \cosh(x) \sinh(x)^2 + \sinh(x)^3$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{(\coth(x) + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2/(1+coth(x))**(3/2),x)

[Out] Integral(coth(x)**2/(coth(x) + 1)**(3/2), x)

Giac [B] time = 1.20938, size = 182, normalized size = 3.71

$$-\frac{2}{3} \sqrt{2} \operatorname{sgn}(e^{2x} - 1) - \frac{\sqrt{2} \log\left(\left|-2\sqrt{e^{4x} - e^{2x}} + 2e^{2x} - 1\right|\right)}{8 \operatorname{sgn}(e^{2x} - 1)} - \frac{\sqrt{2} \left(6 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x}\right)^2 - 3\sqrt{e^{4x} - e^{2x}} + 3e^{2x}\right)}{12 \left(\sqrt{e^{4x} - e^{2x}} - e^{2x}\right)^3 \operatorname{sgn}(e^{2x} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="giac")

[Out] $-\frac{2}{3} \sqrt{2} \operatorname{sgn}(e^{2x} - 1) - \frac{1}{8} \sqrt{2} \log(\operatorname{abs}(-2 \sqrt{e^{4x} - e^{2x}} - e^{2x} + 2e^{2x} - 1)) / \operatorname{sgn}(e^{2x} - 1) - \frac{1}{12} \sqrt{2} (6 (\sqrt{e^{4x} - e^{2x}} - e^{2x})^2 - 3 \sqrt{e^{4x} - e^{2x}} + 3e^{2x} - 1) / ((\sqrt{e^{4x} - e^{2x}} - e^{2x})^3 \operatorname{sgn}(e^{2x} - 1))$

$$3.140 \quad \int \frac{\tanh^4(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=97

$$\frac{ax}{a^2 - b^2} - \frac{(a^2 + b^2) \tanh(x)}{a^3} - \frac{b(a^2 + b^2) \log(\cosh(x))}{a^4} - \frac{b^5 \log(a \sinh(x) + b \cosh(x))}{a^4(a^2 - b^2)} + \frac{b \tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a}$$

[Out] (a*x)/(a^2 - b^2) - (b*(a^2 + b^2)*Log[Cosh[x]])/a^4 - (b^5*Log[b*Cosh[x] + a*Sinh[x]])/(a^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/(2*a^2) - Tanh[x]^3/(3*a)

Rubi [A] time = 0.525196, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3569, 3649, 3650, 3651, 3530, 3475}

$$\frac{ax}{a^2 - b^2} - \frac{(a^2 + b^2) \tanh(x)}{a^3} - \frac{b(a^2 + b^2) \log(\cosh(x))}{a^4} - \frac{b^5 \log(a \sinh(x) + b \cosh(x))}{a^4(a^2 - b^2)} + \frac{b \tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4/(a + b*Coth[x]), x]

[Out] (a*x)/(a^2 - b^2) - (b*(a^2 + b^2)*Log[Cosh[x]])/a^4 - (b^5*Log[b*Cosh[x] + a*Sinh[x]])/(a^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/(2*a^2) - Tanh[x]^3/(3*a)

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3650

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :=
Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n
+ 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
+ n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
-1] && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3651

```

Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] := Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

Rule 3530

```

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] := Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

```

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^4(x)}{a + b \coth(x)} dx &= -\frac{\tanh^3(x)}{3a} - \frac{i \int \frac{(-3ib+3ia \coth(x)+3ib \coth^2(x)) \tanh^3(x)}{a+b \coth(x)} dx}{3a} \\
 &= \frac{b \tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} - \frac{\int \frac{(-6(a^2+b^2)+6b^2 \coth^2(x)) \tanh^2(x)}{a+b \coth(x)} dx}{6a^2} \\
 &= -\frac{(a^2 + b^2) \tanh(x)}{a^3} + \frac{b \tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} + \frac{i \int \frac{(6ib(a^2+b^2)-6ia^3 \coth(x)-6ib(a^2+b^2) \coth^2(x)) \tanh(x)}{a+b \coth(x)} dx}{6a^3} \\
 &= \frac{ax}{a^2 - b^2} - \frac{(a^2 + b^2) \tanh(x)}{a^3} + \frac{b \tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} - \frac{(ib^5) \int \frac{-ib-ia \coth(x)}{a+b \coth(x)} dx}{a^4 (a^2 - b^2)} - \frac{(b(a^2 + b^2))}{a^3} \\
 &= \frac{ax}{a^2 - b^2} - \frac{b(a^2 + b^2) \log(\cosh(x))}{a^4} - \frac{b^5 \log(b \cosh(x) + a \sinh(x))}{a^4 (a^2 - b^2)} - \frac{(a^2 + b^2) \tanh(x)}{a^3} + \frac{b \tanh^2(x)}{2a^2}
 \end{aligned}$$

Mathematica [A] time = 0.327879, size = 105, normalized size = 1.08

$$\frac{(2a^3b^2 - 8a^5 + 6ab^4) \tanh(x) + 6(b^5 - a^4b) \log(\cosh(x)) + a^2(a^2 - b^2) \operatorname{sech}^2(x)(2a \tanh(x) - 3b) + 6a^5x - 6b^5 \log(a \cosh(x) + b \sinh(x))}{6a^4(a - b)(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^4/(a + b*Coth[x]), x]

[Out] (6*a^5*x + 6*(-(a^4*b) + b^5)*Log[Cosh[x]] - 6*b^5*Log[b*Cosh[x] + a*Sinh[x]] + (-8*a^5 + 2*a^3*b^2 + 6*a*b^4)*Tanh[x] + a^2*(a^2 - b^2)*Sech[x]^2*(-3*b + 2*a*Tanh[x]))/(6*a^4*(a - b)*(a + b))

Maple [B] time = 0.056, size = 283, normalized size = 2.9

$$64 \frac{\ln(\tanh(x/2) + 1)}{64a - 64b} - 64 \frac{\ln(\tanh(x/2) - 1)}{64a + 64b} - 2 \frac{(\tanh(x/2))^5}{a((\tanh(x/2))^2 + 1)^3} - 2 \frac{(\tanh(x/2))^5 b^2}{a^3((\tanh(x/2))^2 + 1)^3} + 2 \frac{b(\tanh(x/2))}{a^2((\tanh(x/2))^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^4/(a+b*coth(x)),x)`

[Out] $64/(64*a-64*b)*\ln(\tanh(1/2*x)+1)-64/(64*a+64*b)*\ln(\tanh(1/2*x)-1)-2/a/(\tanh(1/2*x)^2+1)^3*\tanh(1/2*x)^5-2/a^3/(\tanh(1/2*x)^2+1)^3*\tanh(1/2*x)^5*b^2+2/a^2/(\tanh(1/2*x)^2+1)^3*b*\tanh(1/2*x)^4-20/3/a/(\tanh(1/2*x)^2+1)^3*\tanh(1/2*x)^3-4/a^3/(\tanh(1/2*x)^2+1)^3*\tanh(1/2*x)^3*b^2+2/a^2/(\tanh(1/2*x)^2+1)^3*b*\tanh(1/2*x)^2-2/a/(\tanh(1/2*x)^2+1)^3*\tanh(1/2*x)-2/a^3/(\tanh(1/2*x)^2+1)^3*\tanh(1/2*x)*b^2-1/a^2*b*\ln(\tanh(1/2*x)^2+1)-1/a^4*\ln(\tanh(1/2*x)^2+1)*b^3-b^5/(a+b)/(a-b)/a^4*\ln(\tanh(1/2*x)^2*b+2*a*\tanh(1/2*x)+b)$

Maxima [A] time = 1.75174, size = 197, normalized size = 2.03

$$-\frac{b^5 \log\left(-\left(a-b\right)e^{-2x}+a+b\right)}{a^6-a^4b^2} - \frac{2\left(4a^2+3b^2+3\left(2a^2+ab+2b^2\right)e^{-2x}+3\left(2a^2+ab+b^2\right)e^{-4x}\right)}{3\left(3a^3e^{-2x}+3a^3e^{-4x}+a^3e^{-6x}+a^3\right)} + \frac{x}{a+b} - \frac{\left(a^2b+b^3\right)\log\left(e^{-2x}+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="maxima")`

[Out] $-b^5*\log\left(-\left(a-b\right)*e^{-2*x}+a+b\right)/\left(a^6-a^4*b^2\right)-2/3*\left(4*a^2+3*b^2+3*\left(2*a^2+a*b+2*b^2\right)*e^{-2*x}+3*\left(2*a^2+a*b+b^2\right)*e^{-4*x}\right)/\left(3*a^3*e^{-2*x}+3*a^3*e^{-4*x}+a^3*e^{-6*x}+a^3\right)+x/\left(a+b\right)-\left(a^2*b+b^3\right)*\log\left(e^{-2*x}+1\right)/a^4$

Fricas [B] time = 3.18172, size = 3087, normalized size = 31.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="fricas")`

[Out] $1/3*(3*(a^5+a^4*b)*x*\cosh(x)^6+18*(a^5+a^4*b)*x*\cosh(x)*\sinh(x)^5+3*(a^5+a^4*b)*x*\sinh(x)^6+8*a^5-2*a^3*b^2-6*a*b^4+3*(4*a^5-2*a^4*b-2*a^3*b^2+2*a^2*b^3-2*a*b^4+3*(a^5+a^4*b)*x)*\cosh(x)^4+3*(4*a^5-2*a^4*b-2*a^3*b^2+2*a^2*b^3-2*a*b^4+15*(a^5+a^4*b)*x*\cosh(x)^2+3*(a^5+a^4*b)*x)*\sinh(x)^4+12*(5*(a^5+a^4*b)*x*\cosh(x)^3+(4*a$

```

^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)
)*sinh(x)^3 + 3*(4*a^5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)
*cosh(x)^2 + 3*(15*(a^5 + a^4*b)*x*cosh(x)^4 + 4*a^5 - 2*a^4*b + 2*a^2*b^3
- 4*a*b^4 + 6*(4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 +
a^4*b)*x)*cosh(x)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^2 + 3*(a^5 + a^4*b)*x - 3
*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4
+ 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cosh(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*c
osh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2
+ b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x
))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^4*b - b^5)*co
sh(x)^6 + 6*(a^4*b - b^5)*cosh(x)*sinh(x)^5 + (a^4*b - b^5)*sinh(x)^6 + a^4
*b - b^5 + 3*(a^4*b - b^5)*cosh(x)^4 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - b^5)*cosh(x)^3 + 3*(a^4*b - b^5)*cosh(x))
*sinh(x)^3 + 3*(a^4*b - b^5)*cosh(x)^2 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*c
osh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b - b^5)*cosh(x)^
5 + 2*(a^4*b - b^5)*cosh(x)^3 + (a^4*b - b^5)*cosh(x))*sinh(x))*log(2*cosh(
x)/(cosh(x) - sinh(x))) + 6*(3*(a^5 + a^4*b)*x*cosh(x)^5 + 2*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^3 + (4*a^
5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x))*sinh(x))/((
a^6 - a^4*b^2)*cosh(x)^6 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4
*b^2)*sinh(x)^6 + a^6 - a^4*b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^
4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(
x)^3 + 3*(a^6 - a^4*b^2)*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 +
3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)
^2)*sinh(x)^2 + 6*((a^6 - a^4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3
+ (a^6 - a^4*b^2)*cosh(x))*sinh(x))

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**4/(a+b*coth(x)),x)

[Out] Integral(tanh(x)**4/(a + b*coth(x)), x)

Giac [A] time = 1.13076, size = 190, normalized size = 1.96

$$-\frac{b^5 \log(|ae^{(2x)} + be^{(2x)} - a + b|)}{a^6 - a^4 b^2} + \frac{x}{a - b} - \frac{(a^2 b + b^3) \log(e^{(2x)} + 1)}{a^4} + \frac{2(4a^3 + 3ab^2 + 3(2a^3 - a^2 b + ab^2)e^{(4x)} + 3(2a^3 - a^2 b + ab^2)e^{(2x)} + 3(2a^3 - a^2 b + ab^2))}{3a^4(e^{(2x)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="giac")

[Out] -b^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^6 - a^4*b^2) + x/(a - b) - (a^2*b + b^3)*log(e^(2*x) + 1)/a^4 + 2/3*(4*a^3 + 3*a*b^2 + 3*(2*a^3 - a^2*b + a*b^2)*e^(4*x) + 3*(2*a^3 - a^2*b + 2*a*b^2)*e^(2*x))/(a^4*(e^(2*x) + 1)^3)

$$3.141 \quad \int \frac{\tanh^3(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=76

$$-\frac{bx}{a^2-b^2} + \frac{(a^2+b^2)\log(\cosh(x))}{a^3} + \frac{b^4\log(a\sinh(x)+b\cosh(x))}{a^3(a^2-b^2)} + \frac{b\tanh(x)}{a^2} - \frac{\tanh^2(x)}{2a}$$

[Out] $-\frac{(b*x)}{(a^2 - b^2)} + \frac{((a^2 + b^2)*\text{Log}[\text{Cosh}[x]])}{a^3} + \frac{(b^4*\text{Log}[b*\text{Cosh}[x] + a*\text{Sinh}[x]])}{(a^3*(a^2 - b^2))} + \frac{(b*\text{Tanh}[x])}{a^2} - \frac{\text{Tanh}[x]^2}{(2*a)}$

Rubi [A] time = 0.325755, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3569, 3649, 3652, 3530, 3475}

$$-\frac{bx}{a^2-b^2} + \frac{(a^2+b^2)\log(\cosh(x))}{a^3} + \frac{b^4\log(a\sinh(x)+b\cosh(x))}{a^3(a^2-b^2)} + \frac{b\tanh(x)}{a^2} - \frac{\tanh^2(x)}{2a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tanh}[x]^3/(a + b*\text{Coth}[x]), x]$

[Out] $-\frac{(b*x)}{(a^2 - b^2)} + \frac{((a^2 + b^2)*\text{Log}[\text{Cosh}[x]])}{a^3} + \frac{(b^4*\text{Log}[b*\text{Cosh}[x] + a*\text{Sinh}[x]])}{(a^3*(a^2 - b^2))} + \frac{(b*\text{Tanh}[x])}{a^2} - \frac{\text{Tanh}[x]^2}{(2*a)}$

Rule 3569

$\text{Int}[\frac{(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}}{x_Symbol}] := \text{Simp}[\frac{b^2*(a + b*\text{Tan}[e + f*x])^{(m+1)}*(c + d*\text{Tan}[e + f*x])^{(n+1)}}{(f*(m+1)*(a^2 + b^2)*(b*c - a*d)}, x] + \text{Dist}[\frac{1}{(m+1)*(a^2 + b^2)*(b*c - a*d)}, \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m+1)}*(c + d*\text{Tan}[e + f*x])^n*\text{Simp}[a*(b*c - a*d)*(m+1) - b^2*d*(m+n+2) - b*(b*c - a*d)*(m+1)*\text{Tan}[e + f*x] - b^2*d*(m+n+2)*\text{Tan}[e + f*x]^2, x], x], x] /;$
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m] && !ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3649

$\text{Int}[\frac{(a_. + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}}{x_Symbol}] := \text{Simp}[\frac{b^2*(A_. + (B_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m+1)}*(C_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n+1)}}{(f*(m+1)*(a^2 + b^2)*(b*c - a*d)}, x] + \text{Dist}[\frac{1}{(m+1)*(a^2 + b^2)*(b*c - a*d)}, \text{Int}[(A + B*\text{Tan}[e + f*x])^{(m+1)}*(C + D*\text{Tan}[e + f*x])^n*\text{Simp}[A*(b*c - a*d)*(m+1) - b^2*d*(m+n+2) - b*(b*c - a*d)*(m+1)*\text{Tan}[e + f*x] - b^2*d*(m+n+2)*\text{Tan}[e + f*x]^2, x], x], x] /;$

```

+ (f_.)*(x_)^2), x_Symbol] := Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3652

```

Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)^2])/(((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] := Simp[((a*
(A*c - c*C) - b*(A*d - C*d))*x)/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^
2 + a^2*c)/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
+ f*x]), x], x] - Dist[(c^2*c + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d -
c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

Rule 3530

```

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] := Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

```

Rule 3475

```

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]

```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{a + b \coth(x)} dx &= -\frac{\tanh^2(x)}{2a} - \frac{i \int \frac{(-2ib+2ia \coth(x)+2ib \coth^2(x)) \tanh^2(x)}{a+b \coth(x)} dx}{2a} \\
&= \frac{b \tanh(x)}{a^2} - \frac{\tanh^2(x)}{2a} - \frac{\int \frac{(-2(a^2+b^2)+2b^2 \coth^2(x)) \tanh(x)}{a+b \coth(x)} dx}{2a^2} \\
&= -\frac{bx}{a^2 - b^2} + \frac{b \tanh(x)}{a^2} - \frac{\tanh^2(x)}{2a} + \frac{(ib^4) \int \frac{-ib-ia \coth(x)}{a+b \coth(x)} dx}{a^3(a^2 - b^2)} + \frac{(a^2 + b^2) \int \tanh(x) dx}{a^3} \\
&= -\frac{bx}{a^2 - b^2} + \frac{(a^2 + b^2) \log(\cosh(x))}{a^3} + \frac{b^4 \log(b \cosh(x) + a \sinh(x))}{a^3(a^2 - b^2)} + \frac{b \tanh(x)}{a^2} - \frac{\tanh^2(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.303121, size = 88, normalized size = 1.16

$$\frac{a^2(a^2 - b^2) \operatorname{sech}^2(x) + 2(ab(a^2 - b^2) \tanh(x) + (a^4 - b^4) \log(\cosh(x)) - a^3bx + b^4 \log(a \sinh(x) + b \cosh(x)))}{2a^3(a - b)(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^3/(a + b*Coth[x]), x]

[Out] (a^2*(a^2 - b^2)*Sech[x]^2 + 2*(-(a^3*b*x) + (a^4 - b^4)*Log[Cosh[x]] + b^4*Log[b*Cosh[x] + a*Sinh[x]] + a*b*(a^2 - b^2)*Tanh[x]))/(2*a^3*(a - b)*(a + b))

Maple [B] time = 0.053, size = 167, normalized size = 2.2

$$-32 \frac{\ln(\tanh(x/2) + 1)}{32a - 32b} - 32 \frac{\ln(\tanh(x/2) - 1)}{32a + 32b} + 2 \frac{(\tanh(x/2))^3 b}{a^2((\tanh(x/2))^2 + 1)^2} - 2 \frac{(\tanh(x/2))^2}{a((\tanh(x/2))^2 + 1)^2} + 2 \frac{\tanh(x/2)}{a^2((\tanh(x/2))^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a+b*coth(x)), x)

[Out] -32/(32*a-32*b)*ln(tanh(1/2*x)+1)-32/(32*a+32*b)*ln(tanh(1/2*x)-1)+2/a^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*b-2/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2+2/a^2/(tanh(1/2*x)^2+1)^2*b*tanh(1/2*x)+1/a*ln(tanh(1/2*x)^2+1)+1/a^3*ln(tanh(

$$1/2*x)^{2+1}*b^2+b^4/(a+b)/(a-b)/a^3*\ln(\tanh(1/2*x)^2*b+2*a*\tanh(1/2*x)+b)$$

Maxima [A] time = 1.73305, size = 127, normalized size = 1.67

$$\frac{b^4 \log\left(- (a-b)e^{-2x} + a + b\right)}{a^5 - a^3 b^2} + \frac{2\left((a+b)e^{-2x} + b\right)}{2a^2 e^{-2x} + a^2 e^{-4x} + a^2} + \frac{x}{a+b} + \frac{(a^2 + b^2) \log\left(e^{-2x} + 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="maxima")

[Out] $b^4*\log(-(a - b)*e^{-2*x} + a + b)/(a^5 - a^3*b^2) + 2*((a + b)*e^{-2*x} + b)/(2*a^2*e^{-2*x} + a^2*e^{-4*x} + a^2) + x/(a + b) + (a^2 + b^2)*\log(e^{-2*x} + 1)/a^3$

Fricas [B] time = 3.24519, size = 1539, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="fricas")

[Out] $-\left((a^4 + a^3*b)*x*\cosh(x)^4 + 4*(a^4 + a^3*b)*x*\cosh(x)*\sinh(x)^3 + (a^4 + a^3*b)*x*\sinh(x)^4 + 2*a^3*b - 2*a*b^3 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 - (a^4 + a^3*b)*x)*\cosh(x)^2 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 - 3*(a^4 + a^3*b)*x)*\cosh(x)^2 - (a^4 + a^3*b)*x*\sinh(x)^2 + (a^4 + a^3*b)*x - (b^4*\cosh(x)^4 + 4*b^4*\cosh(x)*\sinh(x)^3 + b^4*\sinh(x)^4 + 2*b^4*\cosh(x)^2 + b^4 + 2*(3*b^4*\cosh(x)^2 + b^4)*\sinh(x)^2 + 4*(b^4*\cosh(x)^3 + b^4*\cosh(x))*\sinh(x))*\log(2*(b*\cosh(x) + a*\sinh(x))/(\cosh(x) - \sinh(x))) - ((a^4 - b^4)*\cosh(x)^4 + 4*(a^4 - b^4)*\cosh(x)*\sinh(x)^3 + (a^4 - b^4)*\sinh(x)^4 + a^4 - b^4 + 2*(a^4 - b^4)*\cosh(x)^2 + 2*(a^4 - b^4 + 3*(a^4 - b^4)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^4 - b^4)*\cosh(x)^3 + (a^4 - b^4)*\cosh(x))*\sinh(x))*\log(2*\cosh(x)/(\cosh(x) - \sinh(x))) + 4*((a^4 + a^3*b)*x*\cosh(x)^3 - (a^4 - a^3*b - a^2*b^2 + a*b^3 - (a^4 + a^3*b)*x)*\cosh(x))*\sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*\cosh(x)^4 + 4*(a^5 - a^3*b^2)*\cosh(x)*\sinh(x)^3 + (a^5 - a^3*b^2)*\sinh(x)^4 + 2*(a^5 - a^3*b^2)*\cosh(x)^2 + 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2))*\cosh(x)^2*\sinh(x)^2 + 4*((a^5 - a^3*b^2)*\cosh(x)^3 + (a^5 - a^3*b^2)*\cosh(x))*\sinh(x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3/(a+b*coth(x)),x)

[Out] Integral(tanh(x)**3/(a + b*coth(x)), x)

Giac [A] time = 1.17562, size = 131, normalized size = 1.72

$$\frac{b^4 \log(|ae^{(2x)} + be^{(2x)} - a + b|)}{a^5 - a^3b^2} - \frac{x}{a - b} + \frac{(a^2 + b^2) \log(e^{(2x)} + 1)}{a^3} - \frac{2(ab - (a^2 - ab)e^{(2x)})}{a^3(e^{(2x)} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="giac")

[Out] b^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^5 - a^3*b^2) - x/(a - b) + (a^2 + b^2)*log(e^(2*x) + 1)/a^3 - 2*(a*b - (a^2 - a*b)*e^(2*x))/(a^3*(e^(2*x) + 1)^2)

$$3.142 \quad \int \frac{\tanh^2(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=60

$$\frac{ax}{a^2 - b^2} - \frac{b^3 \log(a \sinh(x) + b \cosh(x))}{a^2 (a^2 - b^2)} - \frac{b \log(\cosh(x))}{a^2} - \frac{\tanh(x)}{a}$$

[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/a^2 - (b^3*Log[b*Cosh[x] + a*Sinh[x]])/(a^2*(a^2 - b^2)) - Tanh[x]/a

Rubi [A] time = 0.193089, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3569, 3651, 3530, 3475}

$$\frac{ax}{a^2 - b^2} - \frac{b^3 \log(a \sinh(x) + b \cosh(x))}{a^2 (a^2 - b^2)} - \frac{b \log(\cosh(x))}{a^2} - \frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2/(a + b*Coth[x]),x]

[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/a^2 - (b^3*Log[b*Cosh[x] + a*Sinh[x]])/(a^2*(a^2 - b^2)) - Tanh[x]/a

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || IntegerQ[m]) && !(ILtQ[n, -1] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
```

```

*(x_)]), x_Symbol] := Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]

```

Rule 3530

```

Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*
(x_)]), x_Symbol] := Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

```

Rule 3475

```

Int[tan[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]

```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{a + b \coth(x)} dx &= -\frac{\tanh(x)}{a} - \frac{i \int \frac{(-ib+ia \coth(x)+ib \coth^2(x)) \tanh(x)}{a+b \coth(x)} dx}{a} \\
&= \frac{ax}{a^2 - b^2} - \frac{\tanh(x)}{a} - \frac{b \int \tanh(x) dx}{a^2} - \frac{(ib^3) \int \frac{-ib-ia \coth(x)}{a+b \coth(x)} dx}{a^2 (a^2 - b^2)} \\
&= \frac{ax}{a^2 - b^2} - \frac{b \log(\cosh(x))}{a^2} - \frac{b^3 \log(b \cosh(x) + a \sinh(x))}{a^2 (a^2 - b^2)} - \frac{\tanh(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.131647, size = 64, normalized size = 1.07

$$\frac{(ab^2 - a^3) \tanh(x) + (b^3 - a^2b) \log(\cosh(x)) + a^3x - b^3 \log(a \sinh(x) + b \cosh(x))}{a^4 - a^2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Coth[x]), x]
```

```
[Out] (a^3*x + (-a^2*b) + b^3)*Log[Cosh[x]] - b^3*Log[b*Cosh[x] + a*Sinh[x]] + (-a^3 + a*b^2)*Tanh[x])/(a^4 - a^2*b^2)
```

Maple [A] time = 0.046, size = 110, normalized size = 1.8

$$16 \frac{\ln(\tanh(x/2) + 1)}{16a - 16b} - 16 \frac{\ln(\tanh(x/2) - 1)}{16a + 16b} - 2 \frac{\tanh(x/2)}{a((\tanh(x/2))^2 + 1)} - \frac{b}{a^2} \ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right) - \frac{b^3}{(a+b)(a-b)a^2} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2/(a+b*coth(x)),x)`

[Out] `16/(16*a-16*b)*ln(tanh(1/2*x)+1)-16/(16*a+16*b)*ln(tanh(1/2*x)-1)-2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)-1/a^2*b*ln(tanh(1/2*x)^2+1)-b^3/(a+b)/(a-b)/a^2*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)`

Maxima [A] time = 1.78454, size = 90, normalized size = 1.5

$$-\frac{b^3 \log(-(a-b)e^{-2x} + a + b)}{a^4 - a^2b^2} + \frac{x}{a + b} - \frac{b \log(e^{-2x} + 1)}{a^2} - \frac{2}{ae^{-2x} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="maxima")`

[Out] `-b^3*log(-(a-b)*e^(-2*x) + a + b)/(a^4 - a^2*b^2) + x/(a + b) - b*log(e^(-2*x) + 1)/a^2 - 2/(a*e^(-2*x) + a)`

Fricas [B] time = 2.98597, size = 660, normalized size = 11.

$$\frac{(a^3 + a^2b)x \cosh(x)^2 + 2(a^3 + a^2b)x \cosh(x) \sinh(x) + (a^3 + a^2b)x \sinh(x)^2 + 2a^3 - 2ab^2 + (a^3 + a^2b)x - (b^3 \cosh(x)^2 + 2b^3 \sinh(x)^2)}{a^4 - a^2b^2 + (a^4 - a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="fricas")`

[Out] `((a^3 + a^2*b)*x*cosh(x)^2 + 2*(a^3 + a^2*b)*x*cosh(x)*sinh(x) + (a^3 + a^2*b)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2 + (a^3 + a^2*b)*x - (b^3*cosh(x)^2 + 2*b^3*sinh(x)^2))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2))`

$$3*\cosh(x)*\sinh(x) + b^3*\sinh(x)^2 + b^3)*\log(2*(b*\cosh(x) + a*\sinh(x))/(\cosh(x) - \sinh(x))) - (a^2*b - b^3 + (a^2*b - b^3)*\cosh(x)^2 + 2*(a^2*b - b^3)*\cosh(x)*\sinh(x) + (a^2*b - b^3)*\sinh(x)^2)*\log(2*\cosh(x)/(\cosh(x) - \sinh(x))))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*\cosh(x)^2 + 2*(a^4 - a^2*b^2)*\cosh(x)*\sinh(x) + (a^4 - a^2*b^2)*\sinh(x)^2)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**2/(a+b*coth(x)),x)

[Out] Integral(tanh(x)**2/(a + b*coth(x)), x)

Giac [A] time = 1.13413, size = 100, normalized size = 1.67

$$-\frac{b^3 \log(|ae^{(2x)} + be^{(2x)} - a + b|)}{a^4 - a^2b^2} + \frac{x}{a - b} - \frac{b \log(e^{(2x)} + 1)}{a^2} + \frac{2}{a(e^{(2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="giac")

[Out] $-b^3*\log(\text{abs}(a*e^{(2*x)} + b*e^{(2*x)} - a + b))/(a^4 - a^2*b^2) + x/(a - b) - b*\log(e^{(2*x)} + 1)/a^2 + 2/(a*(e^{(2*x)} + 1))$

$$3.143 \quad \int \frac{\tanh(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=51

$$-\frac{bx}{a^2-b^2} + \frac{b^2 \log(a \sinh(x) + b \cosh(x))}{a(a^2-b^2)} + \frac{\log(\cosh(x))}{a}$$

[Out] $-\left(\frac{b*x}{a^2 - b^2}\right) + \text{Log}[\text{Cosh}[x]]/a + (b^2*\text{Log}[b*\text{Cosh}[x] + a*\text{Sinh}[x]])/(a*(a^2 - b^2))$

Rubi [A] time = 0.0824597, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3571, 3530, 3475}

$$-\frac{bx}{a^2-b^2} + \frac{b^2 \log(a \sinh(x) + b \cosh(x))}{a(a^2-b^2)} + \frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/(a + b*Coth[x]),x]

[Out] $-\left(\frac{b*x}{a^2 - b^2}\right) + \text{Log}[\text{Cosh}[x]]/a + (b^2*\text{Log}[b*\text{Cosh}[x] + a*\text{Sinh}[x]])/(a*(a^2 - b^2))$

Rule 3571

```
Int[1/(((a_) + (b_)*tan[(e_) + (f_)*(x_)])*((c_) + (d_)*tan[(e_) + (f_)*(x_)])), x_Symbol] :> Simp[((a*c - b*d)*x)/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[b^2/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist[d^2/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/(a_ + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int \frac{\tanh(x)}{a + b \coth(x)} dx &= -\frac{bx}{a^2 - b^2} + \frac{\int \tanh(x) dx}{a} + \frac{(ib^2) \int \frac{-ib - ia \coth(x)}{a + b \coth(x)} dx}{a(a^2 - b^2)} \\ &= -\frac{bx}{a^2 - b^2} + \frac{\log(\cosh(x))}{a} + \frac{b^2 \log(b \cosh(x) + a \sinh(x))}{a(a^2 - b^2)} \end{aligned}$$

Mathematica [A] time = 0.0755132, size = 46, normalized size = 0.9

$$\frac{(a^2 - b^2) \log(\cosh(x)) + b(b \log(a \sinh(x) + b \cosh(x)) - ax)}{a^3 - ab^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]/(a + b*Coth[x]), x]

[Out] ((a^2 - b^2)*Log[Cosh[x]] + b*(-(a*x) + b*Log[b*Cosh[x] + a*Sinh[x]]))/(a^3 - a*b^2)

Maple [A] time = 0.039, size = 88, normalized size = 1.7

$$-8 \frac{\ln(\tanh(x/2) + 1)}{8a - 8b} - 8 \frac{\ln(\tanh(x/2) - 1)}{8a + 8b} + \frac{1}{a} \ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right) + \frac{b^2}{(a+b)(a-b)a} \ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 b + 2a \tanh\left(\frac{x}{2}\right) + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)/(a+b*coth(x)), x)

[Out] -8/(8*a-8*b)*ln(tanh(1/2*x)+1)-8/(8*a+8*b)*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)^2+1)+b^2/(a+b)/(a-b)/a*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+a)

Maxima [A] time = 1.74219, size = 68, normalized size = 1.33

$$\frac{b^2 \log\left(-\frac{(a-b)e^{-2x} + a + b}{a^3 - ab^2}\right) + \frac{x}{a+b} + \frac{\log(e^{-2x} + 1)}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="maxima")

[Out] b^2*log(-(a - b)*e^(-2*x) + a + b)/(a^3 - a*b^2) + x/(a + b) + log(e^(-2*x) + 1)/a

Fricas [A] time = 2.81707, size = 185, normalized size = 3.63

$$\frac{b^2 \log\left(\frac{2(b \cosh(x) + a \sinh(x))}{\cosh(x) - \sinh(x)}\right) - (a^2 + ab)x + (a^2 - b^2) \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right)}{a^3 - ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="fricas")

[Out] (b^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a^2 + a*b)*x + (a^2 - b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{a + b \coth(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*coth(x)),x)

[Out] Integral(tanh(x)/(a + b*coth(x)), x)

Giac [A] time = 1.19928, size = 77, normalized size = 1.51

$$\frac{b^2 \log(|ae^{(2x)} + be^{(2x)} - a + b|)}{a^3 - ab^2} - \frac{x}{a - b} + \frac{\log(e^{(2x)} + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="giac")

[Out] b^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 - a*b^2) - x/(a - b) + log(e^(2*x) + 1)/a

$$3.144 \quad \int \frac{1}{a+b \coth(x)} dx$$

Optimal. Leaf size=39

$$\frac{ax}{a^2 - b^2} - \frac{b \log(a \sinh(x) + b \cosh(x))}{a^2 - b^2}$$

[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)

Rubi [A] time = 0.0458942, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3484, 3530}

$$\frac{ax}{a^2 - b^2} - \frac{b \log(a \sinh(x) + b \cosh(x))}{a^2 - b^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Coth[x])^(-1), x]

[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)

Rule 3484

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3530

Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rubi steps

$$\int \frac{1}{a + b \coth(x)} dx = \frac{ax}{a^2 - b^2} - \frac{(ib) \int \frac{-ib - ia \coth(x)}{a + b \coth(x)} dx}{a^2 - b^2}$$

$$= \frac{ax}{a^2 - b^2} - \frac{b \log(b \cosh(x) + a \sinh(x))}{a^2 - b^2}$$

Mathematica [A] time = 0.0526497, size = 29, normalized size = 0.74

$$\frac{ax - b \log(a \sinh(x) + b \cosh(x))}{a^2 - b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Coth[x])^(-1), x]

[Out] (a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)

Maple [A] time = 0.013, size = 55, normalized size = 1.4

$$\frac{\ln(1 + \coth(x))}{2a - 2b} - \frac{\ln(\coth(x) - 1)}{2b + 2a} - \frac{b \ln(a + b \coth(x))}{(a + b)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*coth(x)), x)

[Out] 1/(2*a-2*b)*ln(1+coth(x))-1/(2*b+2*a)*ln(coth(x)-1)-b/(a-b)/(a+b)*ln(a+b*coth(x))

Maxima [A] time = 1.26918, size = 50, normalized size = 1.28

$$-\frac{b \log(-(a - b)e^{(-2x)} + a + b)}{a^2 - b^2} + \frac{x}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(x)), x, algorithm="maxima")

[Out] $-b \log(-(a - b)e^{-2x} + a + b)/(a^2 - b^2) + x/(a + b)$

Fricas [A] time = 2.53858, size = 108, normalized size = 2.77

$$\frac{(a + b)x - b \log\left(\frac{2(b \cosh(x) + a \sinh(x))}{\cosh(x) - \sinh(x)}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*coth(x)),x, algorithm="fricas")`

[Out] $((a + b)x - b \log(2*(b \cosh(x) + a \sinh(x))/(\cosh(x) - \sinh(x))))/(a^2 - b^2)$

Sympy [A] time = 1.22743, size = 148, normalized size = 3.79

$$\begin{cases} \infty & \text{for } a = 0 \wedge b = 0 \\ -\frac{x \tanh(x)}{2b \tanh(x) - 2b} + \frac{x}{2b \tanh(x) - 2b} - \frac{1}{2b \tanh(x) - 2b} & \text{for } a = -b \\ \frac{x \tanh(x)}{2b \tanh(x) + 2b} + \frac{x}{2b \tanh(x) + 2b} + \frac{1}{2b \tanh(x) + 2b} & \text{for } a = b \\ \frac{x - \log(\tanh(x) + 1)}{b} & \text{for } a = 0 \\ \frac{ax}{a^2 - b^2} - \frac{bx}{a^2 - b^2} + \frac{b \log(\tanh(x) + 1)}{a^2 - b^2} - \frac{b \log\left(\tanh(x) + \frac{b}{a}\right)}{a^2 - b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*coth(x)),x)`

[Out] `Piecewise((zoo*(x - log(tanh(x) + 1)), Eq(a, 0) & Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), ((x - log(tanh(x) + 1))/b, Eq(a, 0)), (a*x/(a**2 - b**2) - b*x/(a**2 - b**2) + b*log(tanh(x) + 1)/(a**2 - b**2) - b*log(tanh(x) + b/a)/(a**2 - b**2), True))`

Giac [A] time = 1.12986, size = 58, normalized size = 1.49

$$-\frac{b \log(|ae^{2x} + be^{2x} - a + b|)}{a^2 - b^2} + \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) + x/(a - b)
```

$$3.145 \quad \int \frac{\coth(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=39

$$\frac{a \log(a \sinh(x) + b \cosh(x))}{a^2 - b^2} - \frac{bx}{a^2 - b^2}$$

[Out] $-\left(\frac{b*x}{a^2 - b^2}\right) + \frac{a*\text{Log}[b*\text{Cosh}[x] + a*\text{Sinh}[x]]}{a^2 - b^2}$

Rubi [A] time = 0.0584835, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3531, 3530}

$$\frac{a \log(a \sinh(x) + b \cosh(x))}{a^2 - b^2} - \frac{bx}{a^2 - b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[x]/(a + b*\text{Coth}[x]), x]$

[Out] $-\left(\frac{b*x}{a^2 - b^2}\right) + \frac{a*\text{Log}[b*\text{Cosh}[x] + a*\text{Sinh}[x]]}{a^2 - b^2}$

Rule 3531

$\text{Int}[\left(\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)]}{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)]}\right), x_Symbol] \rightarrow \text{Simp}[\left(\frac{(a*c + b*d)*x}{a^2 + b^2}, x\right) + \text{Dist}[\left(\frac{b*c - a*d}{a^2 + b^2}, \text{Int}[\left(\frac{b - a*\tan[e + f*x]}{a + b*\tan[e + f*x]}\right), x], x\right) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

Rule 3530

$\text{Int}[\left(\frac{(c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_)]}{(a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)]}\right), x_Symbol] \rightarrow \text{Simp}[\left(\frac{c*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]]}{b*f}, x\right) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[a*c + b*d, 0]$

Rubi steps

$$\int \frac{\coth(x)}{a + b \coth(x)} dx = -\frac{bx}{a^2 - b^2} + \frac{(ia) \int \frac{-ib - ia \coth(x)}{a + b \coth(x)} dx}{a^2 - b^2}$$

$$= -\frac{bx}{a^2 - b^2} + \frac{a \log(b \cosh(x) + a \sinh(x))}{a^2 - b^2}$$

Mathematica [A] time = 0.0449134, size = 29, normalized size = 0.74

$$\frac{a \log(a \sinh(x) + b \cosh(x)) - bx}{a^2 - b^2}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]/(a + b*Coth[x]),x]

[Out] $(-(b*x) + a*\text{Log}[b*\text{Cosh}[x] + a*\text{Sinh}[x]])/(a^2 - b^2)$

Maple [A] time = 0.017, size = 55, normalized size = 1.4

$$-\frac{\ln(1 + \coth(x))}{2a - 2b} - \frac{\ln(\coth(x) - 1)}{2b + 2a} + \frac{a \ln(a + b \coth(x))}{(a + b)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a+b*coth(x)),x)

[Out] $-1/(2*a-2*b)*\ln(1+\coth(x))-1/(2*b+2*a)*\ln(\coth(x)-1)+a/(a+b)/(a-b)*\ln(a+b*\coth(x))$

Maxima [A] time = 1.23757, size = 49, normalized size = 1.26

$$\frac{a \log(-(a - b)e^{(-2x)} + a + b)}{a^2 - b^2} + \frac{x}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="maxima")

[Out] $a \log(-(a - b)e^{-2x} + a + b)/(a^2 - b^2) + x/(a + b)$

Fricas [A] time = 2.57295, size = 109, normalized size = 2.79

$$\frac{(a + b)x - a \log\left(\frac{2(b \cosh(x) + a \sinh(x))}{\cosh(x) - \sinh(x)}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(a+b*coth(x)),x, algorithm="fricas")`

[Out] $-((a + b)x - a \log(2*(b \cosh(x) + a \sinh(x))/(\cosh(x) - \sinh(x))))/(a^2 - b^2)$

Sympy [A] time = 1.34243, size = 134, normalized size = 3.44

$$\begin{cases} \infty x & \text{for } a = 0 \wedge b = 0 \\ \frac{x \tanh(x)}{2b \tanh(x) - 2b} - \frac{x}{2b \tanh(x) - 2b} - \frac{1}{2b \tanh(x) - 2b} & \text{for } a = -b \\ \frac{x \tanh(x)}{2b \tanh(x) + 2b} + \frac{x}{2b \tanh(x) + 2b} - \frac{1}{2b \tanh(x) + 2b} & \text{for } a = b \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{ax}{a^2 - b^2} - \frac{a \log(\tanh(x) + 1)}{a^2 - b^2} + \frac{a \log\left(\tanh(x) + \frac{b}{a}\right)}{a^2 - b^2} - \frac{bx}{a^2 - b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(a+b*coth(x)),x)`

[Out] `Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*tanh(x)/(2*b*tanh(x) - 2*b) - x/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (x/b, Eq(a, 0)), (a*x/(a**2 - b**2) - a*log(tanh(x) + 1)/(a**2 - b**2) + a*log(tanh(x) + b/a)/(a**2 - b**2) - b*x/(a**2 - b**2), True))`

Giac [A] time = 1.12691, size = 58, normalized size = 1.49

$$\frac{a \log\left(|ae^{2x} + be^{2x} - a + b|\right)}{a^2 - b^2} - \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) - x/(a - b)
```

$$3.146 \quad \int \frac{\coth^2(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=63

$$\frac{a^3x}{b^2(a^2-b^2)} - \frac{a^2 \log(a \sinh(x) + b \cosh(x))}{b(a^2-b^2)} - \frac{ax}{b^2} + \frac{\log(\sinh(x))}{b}$$

[Out] $-\left(\frac{a^3x}{b^2} + \frac{a^2 \log(a \sinh(x) + b \cosh(x))}{b(a^2-b^2)}\right) + \frac{\log(\sinh(x))}{b} - \frac{ax}{b^2}$

Rubi [A] time = 0.0920795, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3541, 3475, 3484, 3530}

$$\frac{a^3x}{b^2(a^2-b^2)} - \frac{a^2 \log(a \sinh(x) + b \cosh(x))}{b(a^2-b^2)} - \frac{ax}{b^2} + \frac{\log(\sinh(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2/(a + b*Coth[x]),x]

[Out] $-\left(\frac{a^3x}{b^2} + \frac{a^2 \log(a \sinh(x) + b \cosh(x))}{b(a^2-b^2)}\right) + \frac{\log(\sinh(x))}{b} - \frac{ax}{b^2}$

Rule 3541

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(d*(2*b*c - a*d)*x)/b^2, x] + (Dist[d^2/b, Int[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3484

Int[((a_.) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +

$d*x]), x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

Rule 3530

$\text{Int}[\frac{(c_.) + (d_.)\tan[(e_.) + (f_.)*(x_)]}{(a_.) + (b_.)\tan[(e_.) + (f_.)*(x_)]}, x_Symbol] :> \text{Simp}[(c*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]])/(b*f), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{EqQ}[a*c + b*d, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(x)}{a + b \coth(x)} dx &= -\frac{ax}{b^2} + \frac{a^2 \int \frac{1}{a+b \coth(x)} dx}{b^2} + \frac{\int \coth(x) dx}{b} \\ &= -\frac{ax}{b^2} + \frac{a^3 x}{b^2(a^2 - b^2)} + \frac{\log(\sinh(x))}{b} - \frac{(ia^2) \int \frac{-ib-ia \coth(x)}{a+b \coth(x)} dx}{b(a^2 - b^2)} \\ &= -\frac{ax}{b^2} + \frac{a^3 x}{b^2(a^2 - b^2)} + \frac{\log(\sinh(x))}{b} - \frac{a^2 \log(b \cosh(x) + a \sinh(x))}{b(a^2 - b^2)} \end{aligned}$$

Mathematica [A] time = 0.0760584, size = 49, normalized size = 0.78

$$\frac{-a^2 \log(a \sinh(x) + b \cosh(x)) + a^2 \log(\sinh(x)) + abx - b^2 \log(\sinh(x))}{a^2 b - b^3}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2/(a + b*Coth[x]),x]

[Out] (a*b*x + a^2*Log[Sinh[x]] - b^2*Log[Sinh[x]] - a^2*Log[b*Cosh[x] + a*Sinh[x]])/(a^2*b - b^3)

Maple [A] time = 0.019, size = 60, normalized size = 1.

$$\frac{\ln(1 + \coth(x))}{2a - 2b} - \frac{\ln(\coth(x) - 1)}{2b + 2a} - \frac{a^2 \ln(a + b \coth(x))}{(a + b)(a - b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^2/(a+b*coth(x)),x)`

[Out] $1/(2*a-2*b)*\ln(1+\coth(x))-1/(2*b+2*a)*\ln(\coth(x)-1)-a^2/(a+b)/(a-b)/b*\ln(a+b*\coth(x))$

Maxima [A] time = 1.27629, size = 85, normalized size = 1.35

$$-\frac{a^2 \log\left(-\left(a-b\right)e^{-2x}+a+b\right)}{a^2b-b^3} + \frac{x}{a+b} + \frac{\log\left(e^{-x}+1\right)}{b} + \frac{\log\left(e^{-x}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="maxima")`

[Out] $-a^2*\log\left(-\left(a-b\right)*e^{-2*x}+a+b\right)/\left(a^2*b-b^3\right)+x/\left(a+b\right)+\log\left(e^{-x}+1\right)/b+\log\left(e^{-x}-1\right)/b$

Fricas [A] time = 2.80317, size = 186, normalized size = 2.95

$$-\frac{a^2 \log\left(\frac{2(b \cosh(x)+a \sinh(x))}{\cosh(x)-\sinh(x)}\right) - (ab + b^2)x - (a^2 - b^2) \log\left(\frac{2 \sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2b - b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="fricas")`

[Out] $-\left(a^2*\log\left(2*\left(b*\cosh(x)+a*\sinh(x)\right)/\left(\cosh(x)-\sinh(x)\right)\right) - \left(a*b+b^2\right)*x - \left(a^2-b^2\right)*\log\left(2*\sinh(x)/\left(\cosh(x)-\sinh(x)\right)\right)\right)/\left(a^2*b-b^3\right)$

Sympy [A] time = 2.94032, size = 372, normalized size = 5.9

$$\left(\begin{array}{l} \infty (x - \log(\tanh(x) + 1) + \log(\tanh(x))) \\ \frac{3x \tanh(x)}{2b \tanh(x) - 2b} - \frac{3x}{2b \tanh(x) - 2b} - \frac{2 \log(\tanh(x) + 1) \tanh(x)}{2b \tanh(x) - 2b} + \frac{2 \log(\tanh(x) + 1)}{2b \tanh(x) - 2b} + \frac{2 \log(\tanh(x)) \tanh(x)}{2b \tanh(x) - 2b} - \frac{2 \log(\tanh(x))}{2b \tanh(x) - 2b} - \frac{1}{2b \tanh(x) - 2b} \\ \frac{x \tanh(x)}{2b \tanh(x) + 2b} + \frac{x}{2b \tanh(x) + 2b} - \frac{2 \log(\tanh(x) + 1) \tanh(x)}{2b \tanh(x) + 2b} - \frac{2 \log(\tanh(x) + 1)}{2b \tanh(x) + 2b} + \frac{2 \log(\tanh(x)) \tanh(x)}{2b \tanh(x) + 2b} + \frac{2 \log(\tanh(x))}{2b \tanh(x) + 2b} + \frac{1}{2b \tanh(x) + 2b} \\ \frac{x - \frac{1}{\tanh(x)}}{x - \log(\tanh(x) + 1) + \log(\tanh(x))} \\ - \frac{a^2 \log\left(\tanh(x) + \frac{b}{a}\right)}{a^2 b - b^3} + \frac{a^2 \log(\tanh(x))}{a^2 b - b^3} + \frac{abx}{a^2 b - b^3} - \frac{b^2 x}{a^2 b - b^3} + \frac{b^2 \log(\tanh(x) + 1)}{a^2 b - b^3} - \frac{b^2 \log(\tanh(x))}{a^2 b - b^3} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2/(a+b*coth(x)),x)

[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)), (3*x*tanh(x)/(2*b*tanh(x) - 2*b) - 3*x/(2*b*tanh(x) - 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) - 2*b) + 2*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b) + 2*log(tanh(x))*tanh(x)/(2*b*tanh(x) - 2*b) - 2*log(tanh(x))/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) + 2*b) - 2*log(tanh(x) + 1)/(2*b*tanh(x) + 2*b) + 2*log(tanh(x))*tanh(x)/(2*b*tanh(x) + 2*b) + 2*log(tanh(x))/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), ((x - 1/tanh(x))/a, Eq(b, 0)), ((x - log(tanh(x) + 1) + log(tanh(x)))/b, Eq(a, 0)), (-a**2*log(tanh(x) + b/a)/(a**2*b - b**3) + a**2*log(tanh(x))/(a**2*b - b**3) + a*b*x/(a**2*b - b**3) - b**2*x/(a**2*b - b**3) + b**2*log(tanh(x) + 1)/(a**2*b - b**3) - b**2*log(tanh(x))/(a**2*b - b**3), True))

Giac [A] time = 1.14789, size = 80, normalized size = 1.27

$$-\frac{a^2 \log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2 b - b^3} + \frac{x}{a - b} + \frac{\log\left(\left|e^{(2x)} - 1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="giac")

[Out] -a^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b - b^3) + x/(a - b) + log(abs(e^(2*x) - 1))/b

$$3.147 \quad \int \frac{\coth^3(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=64

$$-\frac{bx}{a^2-b^2} + \frac{a \log(\sinh(x))}{a^2-b^2} + \frac{a^3 \log(a+b \coth(x))}{b^2(a^2-b^2)} - \frac{\coth(x)}{b}$$

[Out] -((b*x)/(a^2 - b^2)) - Coth[x]/b + (a^3*Log[a + b*Coth[x]])/(b^2*(a^2 - b^2)) + (a*Log[Sinh[x]])/(a^2 - b^2)

Rubi [A] time = 0.129963, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3566, 3626, 3617, 31, 3475}

$$-\frac{bx}{a^2-b^2} + \frac{a \log(\sinh(x))}{a^2-b^2} + \frac{a^3 \log(a+b \coth(x))}{b^2(a^2-b^2)} - \frac{\coth(x)}{b}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^3/(a + b*Coth[x]),x]

[Out] -((b*x)/(a^2 - b^2)) - Coth[x]/b + (a^3*Log[a + b*Coth[x]])/(b^2*(a^2 - b^2)) + (a*Log[Sinh[x]])/(a^2 - b^2)

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n - 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || IntegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3626

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(a*A + b*B -
```

```
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
+ Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.) +
(f_.)*(x_)^2], x_Symbol] := Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^3(x)}{a + b \coth(x)} dx &= -\frac{\coth(x)}{b} - \frac{\int \frac{-a - b \coth(x) + a \coth^2(x)}{a + b \coth(x)} dx}{b} \\
 &= -\frac{bx}{a^2 - b^2} - \frac{\coth(x)}{b} + \frac{a \int \coth(x) dx}{a^2 - b^2} + \frac{a^3 \int \frac{1 - \coth^2(x)}{a + b \coth(x)} dx}{b(a^2 - b^2)} \\
 &= -\frac{bx}{a^2 - b^2} - \frac{\coth(x)}{b} + \frac{a \log(\sinh(x))}{a^2 - b^2} + \frac{a^3 \operatorname{Subst}\left(\int \frac{1}{a+x} dx, x, b \coth(x)\right)}{b^2(a^2 - b^2)} \\
 &= -\frac{bx}{a^2 - b^2} - \frac{\coth(x)}{b} + \frac{a^3 \log(a + b \coth(x))}{b^2(a^2 - b^2)} + \frac{a \log(\sinh(x))}{a^2 - b^2}
 \end{aligned}$$

Mathematica [A] time = 0.119717, size = 64, normalized size = 1.

$$\frac{b(a^2 - b^2) \coth(x) + a(a^2 - b^2) \log(\sinh(x)) + a^3(-\log(a \sinh(x) + b \cosh(x))) + b^3 x}{b^2(b - a)(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^3/(a + b*Coth[x]),x]

[Out] (b^3*x + b*(a^2 - b^2)*Coth[x] + a*(a^2 - b^2)*Log[Sinh[x]] - a^3*Log[b*Cosh[x] + a*Sinh[x]])/(b^2*(-a + b)*(a + b))

Maple [A] time = 0.022, size = 67, normalized size = 1.1

$$-\frac{\coth(x)}{b} - \frac{\ln(1 + \coth(x))}{2a - 2b} - \frac{\ln(\coth(x) - 1)}{2b + 2a} + \frac{a^3 \ln(a + b\coth(x))}{b^2(a + b)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^3/(a+b*coth(x)),x)

[Out] -coth(x)/b-1/(2*a-2*b)*ln(1+coth(x))-1/(2*b+2*a)*ln(coth(x)-1)+1/b^2*a^3/(a+b)/(a-b)*ln(a+b*coth(x))

Maxima [A] time = 1.17138, size = 111, normalized size = 1.73

$$\frac{a^3 \log(-(a - b)e^{(-2x)} + a + b)}{a^2 b^2 - b^4} + \frac{x}{a + b} - \frac{a \log(e^{(-x)} + 1)}{b^2} - \frac{a \log(e^{(-x)} - 1)}{b^2} + \frac{2}{be^{(-2x)} - b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="maxima")

[Out] a^3*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^2 - b^4) + x/(a + b) - a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2/(b*e^(-2*x) - b)

Fricas [B] time = 2.83034, size = 660, normalized size = 10.31

$$\frac{(ab^2 + b^3)x \cosh(x)^2 + 2(ab^2 + b^3)x \cosh(x) \sinh(x) + (ab^2 + b^3)x \sinh(x)^2 + 2a^2b - 2b^3 - (ab^2 + b^3)x - (a^3 \cosh(x) - a^3 \sinh(x))}{a^2b^2 - b^4 - (a^2b^2 - b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a*b^2 + b^3)*x*cosh(x)^2 + 2*(a*b^2 + b^3)*x*cosh(x)*sinh(x) + (a*b^2 + b^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3 - (a*b^2 + b^3)*x - (a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2 - a^3)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a^3 - a*b^2 - (a^3 - a*b^2)*cosh(x)^2 - 2*(a^3 - a*b^2)*cosh(x)*sinh(x) - (a^3 - a*b^2)*sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cosh(x)^2 - 2*(a^2*b^2 - b^4)*cosh(x)*sinh(x) - (a^2*b^2 - b^4)*sinh(x)^2)
```

Sympy [A] time = 5.44979, size = 639, normalized size = 9.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - 1/tanh(x)), Eq(a, 0) & Eq(b, 0)), (5*x*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 5*x*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) + 2*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 3*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) + 2/(2*b*tanh(x)**2 - 2*b*tanh(x)), Eq(a, -b)), (x*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) + x*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 3*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2/(2*b*tanh(x)**2 + 2*b*tanh(x)), Eq(a, b)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2))/a, Eq(b, 0)), ((x - 1/tanh(x))/b, Eq(a, 0)), (a**3*log(tanh(x) + b/a)*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) - a**3*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) - a**2*b/(a**2*b**2*tanh(x) - b**4*tanh(x)) + a*b**2*x*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) - a*b**2*log(tanh(x) + 1)*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) + a*b**2*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) - b**3*x*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) + b**3/(a**2*b**2*tanh(x) - b**4*tanh(x)), True))
```

Giac [A] time = 1.11489, size = 103, normalized size = 1.61

$$\frac{a^3 \log(|ae^{(2x)} + be^{(2x)} - a + b|)}{a^2b^2 - b^4} - \frac{x}{a - b} - \frac{a \log(|e^{(2x)} - 1|)}{b^2} - \frac{2}{b(e^{(2x)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="giac")

[Out] a^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^2 - b^4) - x/(a - b) - a*log(abs(e^(2*x) - 1))/b^2 - 2/(b*(e^(2*x) - 1))

$$3.148 \quad \int \frac{\coth^4(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=76

$$\frac{ax}{a^2 - b^2} - \frac{b \log(\sinh(x))}{a^2 - b^2} - \frac{a^4 \log(a + b \coth(x))}{b^3 (a^2 - b^2)} + \frac{a \coth(x)}{b^2} - \frac{\coth^2(x)}{2b}$$

[Out] (a*x)/(a^2 - b^2) + (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - (a^4*Log[a + b*Coth[x]])/(b^3*(a^2 - b^2)) - (b*Log[Sinh[x]])/(a^2 - b^2)

Rubi [A] time = 0.221255, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3566, 3647, 3627, 3617, 31, 3475}

$$\frac{ax}{a^2 - b^2} - \frac{b \log(\sinh(x))}{a^2 - b^2} - \frac{a^4 \log(a + b \coth(x))}{b^3 (a^2 - b^2)} + \frac{a \coth(x)}{b^2} - \frac{\coth^2(x)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^4/(a + b*Coth[x]),x]

[Out] (a*x)/(a^2 - b^2) + (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - (a^4*Log[a + b*Coth[x]])/(b^3*(a^2 - b^2)) - (b*Log[Sinh[x]])/(a^2 - b^2)

Rule 3566

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n - 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || IntegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3647

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.

```
) + (f_.)*(x_)^2), x_Symbol] := Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3627

```
Int[((A_) + (C_)*tan[(e_) + (f_.)*(x_)])^2)/((a_) + (b_.)*tan[(e_) + (f_.)*(x_)]), x_Symbol] := Simp[(a*(A - C)*x)/(a^2 + b^2), x] + (Dist[(a^2*C + A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(b*(A - C))/(a^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

Rule 3617

```
Int[((a_) + (b_.)*tan[(e_) + (f_.)*(x_)])^(m_.)*((A_) + (C_)*tan[(e_) + (f_.)*(x_)])^2), x_Symbol] := Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(x)}{a + b \coth(x)} dx &= -\frac{\coth^2(x)}{2b} - \frac{\int \frac{\coth(x)(-2a-2b \coth(x)+2a \coth^2(x))}{a+b \coth(x)} dx}{2b} \\
&= \frac{a \coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{\int \frac{2a^2-2(a^2+b^2) \coth^2(x)}{a+b \coth(x)} dx}{2b^2} \\
&= \frac{ax}{a^2 - b^2} + \frac{a \coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{a^4 \int \frac{1-\coth^2(x)}{a+b \coth(x)} dx}{b^2(a^2 - b^2)} - \frac{b \int \coth(x) dx}{a^2 - b^2} \\
&= \frac{ax}{a^2 - b^2} + \frac{a \coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{b \log(\sinh(x))}{a^2 - b^2} - \frac{a^4 \text{Subst}\left(\int \frac{1}{a+x} dx, x, b \coth(x)\right)}{b^3(a^2 - b^2)} \\
&= \frac{ax}{a^2 - b^2} + \frac{a \coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{a^4 \log(a + b \coth(x))}{b^3(a^2 - b^2)} - \frac{b \log(\sinh(x))}{a^2 - b^2}
\end{aligned}$$

Mathematica [A] time = 0.2014, size = 88, normalized size = 1.16

$$\frac{2ab(a^2 - b^2) \coth(x) + (b^4 - a^2b^2) \operatorname{csch}^2(x) + 2(a^4 - b^4) \log(\sinh(x)) - 2a^4 \log(a \sinh(x) + b \cosh(x)) + 2ab^3x}{2b^3(a - b)(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^4/(a + b*Coth[x]), x]

[Out] (2*a*b^3*x + 2*a*b*(a^2 - b^2)*Coth[x] + (-a^2*b^2) + b^4)*Csch[x]^2 + 2*(a^4 - b^4)*Log[Sinh[x]] - 2*a^4*Log[b*Cosh[x] + a*Sinh[x]]/(2*(a - b)*b^3*(a + b))

Maple [A] time = 0.021, size = 76, normalized size = 1.

$$-\frac{(\coth(x))^2}{2b} + \frac{a \coth(x)}{b^2} + \frac{\ln(1 + \coth(x))}{2a - 2b} - \frac{\ln(\coth(x) - 1)}{2b + 2a} - \frac{a^4 \ln(a + b \coth(x))}{b^3(a + b)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^4/(a+b*coth(x)), x)

[Out] $-1/2*\coth(x)^2/b+a*\coth(x)/b^2+1/(2*a-2*b)*\ln(1+\coth(x))-1/(2*b+2*a)*\ln(\coth(x)-1)-1/b^3*a^4/(a+b)/(a-b)*\ln(a+b*\coth(x))$

Maxima [A] time = 1.19526, size = 161, normalized size = 2.12

$$-\frac{a^4 \log\left(-(a-b)e^{-2x} + a + b\right)}{a^2 b^3 - b^5} + \frac{2\left((a+b)e^{-2x} - a\right)}{2b^2 e^{-2x} - b^2 e^{-4x} - b^2} + \frac{x}{a+b} + \frac{(a^2 + b^2) \log\left(e^{-x} + 1\right)}{b^3} + \frac{(a^2 + b^2) \log\left(e^{-x} - 1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="maxima")

[Out] $-a^4*\log(-(a-b)*e^{-2*x} + a + b)/(a^2*b^3 - b^5) + 2*((a+b)*e^{-2*x} - a)/(2*b^2*e^{-2*x} - b^2*e^{-4*x} - b^2) + x/(a+b) + (a^2 + b^2)*\log(e^{-x} + 1)/b^3 + (a^2 + b^2)*\log(e^{-x} - 1)/b^3$

Fricas [B] time = 2.93003, size = 1538, normalized size = 20.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="fricas")

[Out] $((a*b^3 + b^4)*x*\cosh(x)^4 + 4*(a*b^3 + b^4)*x*\cosh(x)*\sinh(x)^3 + (a*b^3 + b^4)*x*\sinh(x)^4 - 2*a^3*b + 2*a*b^3 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 - (a*b^3 + b^4)*x)*\cosh(x)^2 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 + 3*(a*b^3 + b^4)*x*\cosh(x)^2 - (a*b^3 + b^4)*x)*\sinh(x)^2 + (a*b^3 + b^4)*x - (a^4*\cosh(x)^4 + 4*a^4*\cosh(x)*\sinh(x)^3 + a^4*\sinh(x)^4 - 2*a^4*\cosh(x)^2 + a^4 + 2*(3*a^4*\cosh(x)^2 - a^4)*\sinh(x)^2 + 4*(a^4*\cosh(x)^3 - a^4*\cosh(x))*\sinh(x))*\log(2*(b*\cosh(x) + a*\sinh(x))/(\cosh(x) - \sinh(x))) + ((a^4 - b^4)*\cosh(x)^4 + 4*(a^4 - b^4)*\cosh(x)*\sinh(x)^3 + (a^4 - b^4)*\sinh(x)^4 + a^4 - b^4 - 2*(a^4 - b^4)*\cosh(x)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^4 - b^4)*\cosh(x)^3 - (a^4 - b^4)*\cosh(x))*\sinh(x))*\log(2*\sinh(x)/(\cosh(x) - \sinh(x))) + 4*((a*b^3 + b^4)*x*\cosh(x)^3 + (a^3*b - a^2*b^2 - a*b^3 + b^4 - (a*b^3 + b^4)*x)*\cosh(x))*\sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*\cosh(x)^4 + 4*(a^2*b^3 - b^5)*\cosh(x)*\sinh(x)^3 + (a^2*b^3 - b^5)*\sinh(x)^4 - 2*(a^2*b^3 - b^5)*\cosh(x)^2 - 2*(a^2*b^3 - b^5 - 3*(a^2*b^3 - b^5)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^2*b^3 - b^5)*\cosh(x)^3 - (a^2*b^3 - b^5)*\cosh(x)$

x))*sinh(x))

Sympy [A] time = 7.99349, size = 882, normalized size = 11.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**4/(a+b*coth(x)),x)

[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2)), Eq(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2))/b, Eq(a, 0)), (7*x*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 7*x*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 4*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 3*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + 1/(2*b*tanh(x)**3 - 2*b*tanh(x)**2), Eq(a, -b)), (x*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + x*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 3*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) - 1/(2*b*tanh(x)**3 + 2*b*tanh(x)**2), Eq(a, b)), ((x - 1/tanh(x) - 1/(3*tanh(x)**3))/a, Eq(b, 0)), (-2*a**4*log(tanh(x) + b/a)*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*a**4*log(tanh(x))*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*a**3*b*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - a**2*b**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*a*b**3*x*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*a*b**3*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*x*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*b**4*log(tanh(x) + 1)*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*log(tanh(x))*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + b**4/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2), True))

Giac [A] time = 1.17949, size = 135, normalized size = 1.78

$$-\frac{a^4 \log(|ae^{(2x)} + be^{(2x)} - a + b|)}{a^2b^3 - b^5} + \frac{x}{a - b} + \frac{(a^2 + b^2) \log(|e^{(2x)} - 1|)}{b^3} - \frac{2(ab - (ab - b^2)e^{(2x)})}{b^3(e^{(2x)} - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="giac")

[Out] -a^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^3 - b^5) + x/(a - b) + (a^2 + b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2*x))/(b^3*(e^(2*x) - 1)^2)

$$3.149 \quad \int \frac{\coth^5(x)}{a+b \coth(x)} dx$$

Optimal. Leaf size=94

$$-\frac{bx}{a^2-b^2} - \frac{(a^2+b^2)\coth(x)}{b^3} + \frac{a \log(\sinh(x))}{a^2-b^2} + \frac{a^5 \log(a+b \coth(x))}{b^4(a^2-b^2)} + \frac{a \coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b}$$

[Out] $-\left(\frac{b*x}{a^2-b^2}\right) - \left(\frac{(a^2+b^2)*\text{Coth}[x]}{b^3} + \frac{a*\text{Coth}[x]^2}{2*b^2} - \text{Coth}[x]^3/(3*b) + \frac{a^5*\text{Log}[a+b*\text{Coth}[x]]}{b^4*(a^2-b^2)} + \frac{a*\text{Log}[\text{Sinh}[x]]}{a^2-b^2}\right)$

Rubi [A] time = 0.391848, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3566, 3647, 3648, 3626, 3617, 31, 3475}

$$-\frac{bx}{a^2-b^2} - \frac{(a^2+b^2)\coth(x)}{b^3} + \frac{a \log(\sinh(x))}{a^2-b^2} + \frac{a^5 \log(a+b \coth(x))}{b^4(a^2-b^2)} + \frac{a \coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^5/(a + b*Coth[x]), x]

[Out] $-\left(\frac{b*x}{a^2-b^2}\right) - \left(\frac{(a^2+b^2)*\text{Coth}[x]}{b^3} + \frac{a*\text{Coth}[x]^2}{2*b^2} - \text{Coth}[x]^3/(3*b) + \frac{a^5*\text{Log}[a+b*\text{Coth}[x]]}{b^4*(a^2-b^2)} + \frac{a*\text{Log}[\text{Sinh}[x]]}{a^2-b^2}\right)$

Rule 3566

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m-2)*(c + d*Tan[e + f*x])^(n+1))/(d*f*(m+n-1)), x] + Dist[1/(d*(m+n-1)), Int[(a + b*Tan[e + f*x])^(m-3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m+n-1) - b^2*(b*c*(m-2) + a*d*(1+n)) + b*d*(m+n-1)*(3*a^2 - b^2)*Tan[e + f*x] - b^2*(b*c*(m-2) - a*d*(3*m+2*n-4))*Tan[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || IntegerQ[m]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))

Rule 3647

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.)
+ (f_.)*(x_)^2), x_Symbol] := Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))

```

Rule 3648

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)^2), x_Symbol] :=
Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m + n +
1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))

```

Rule 3626

```

Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
+ Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]

```

Rule 3617

```

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
(f_.)*(x_)]^2), x_Symbol] := Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]

```

Rule 31

```

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

```

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^5(x)}{a + b \coth(x)} dx &= -\frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth^2(x)(-3a-3b \coth(x)+3a \coth^2(x))}{a+b \coth(x)} dx}{3b} \\
 &= \frac{a \coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth(x)(6a^2-6(a^2+b^2) \coth^2(x))}{a+b \coth(x)} dx}{6b^2} \\
 &= -\frac{(a^2 + b^2) \coth(x)}{b^3} + \frac{a \coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{-6a(a^2+b^2)-6b^3 \coth(x)+6a(a^2+b^2) \coth^2(x)}{a+b \coth(x)} dx}{6b^3} \\
 &= -\frac{bx}{a^2 - b^2} - \frac{(a^2 + b^2) \coth(x)}{b^3} + \frac{a \coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a \int \coth(x) dx}{a^2 - b^2} + \frac{a^5 \int \frac{1-\coth^2(x)}{a+b \coth(x)} dx}{b^3 (a^2 - b^2)} \\
 &= -\frac{bx}{a^2 - b^2} - \frac{(a^2 + b^2) \coth(x)}{b^3} + \frac{a \coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a \log(\sinh(x))}{a^2 - b^2} + \frac{a^5 \text{Subst}\left(\int \frac{1}{a+x} dx, x, \coth(x)\right)}{b^4 (a^2 - b^2)} \\
 &= -\frac{bx}{a^2 - b^2} - \frac{(a^2 + b^2) \coth(x)}{b^3} + \frac{a \coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a^5 \log(a + b \coth(x))}{b^4 (a^2 - b^2)} + \frac{a \log(\sinh(x))}{a^2 - b^2}
 \end{aligned}$$

Mathematica [A] time = 0.286693, size = 108, normalized size = 1.15

$$\frac{-3ab^2(a^2 - b^2) \operatorname{csch}^2(x) + 6a(a^4 - b^4) \log(\sinh(x)) + 2b(a^2 - b^2) \coth(x)(3a^2 + b^2 \operatorname{csch}^2(x) + 4b^2) - 6a^5 \log(a \sinh(x))}{6b^4(b - a)(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^5/(a + b*Coth[x]), x]

[Out] (6*b^5*x - 3*a*b^2*(a^2 - b^2)*Csch[x]^2 + 2*b*(a^2 - b^2)*Coth[x]*(3*a^2 + 4*b^2 + b^2*Csch[x]^2) + 6*a*(a^4 - b^4)*Log[Sinh[x]] - 6*a^5*Log[b*Cosh[x] + a*Sinh[x]])/(6*b^4*(-a + b)*(a + b))

Maple [A] time = 0.023, size = 96, normalized size = 1.

$$-\frac{(\coth(x))^3}{3b} + \frac{a(\coth(x))^2}{2b^2} - \frac{a^2\coth(x)}{b^3} - \frac{\coth(x)}{b} - \frac{\ln(1+\coth(x))}{2a-2b} - \frac{\ln(\coth(x)-1)}{2b+2a} + \frac{a^5 \ln(a+b\coth(x))}{b^4(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^5/(a+b*coth(x)),x)`

[Out] `-1/3*coth(x)^3/b+1/2*a*coth(x)^2/b^2-1/b^3*a^2*coth(x)-coth(x)/b-1/(2*a-2*b)*ln(1+coth(x))-1/(2*b+2*a)*ln(coth(x)-1)+1/b^4*a^5/(a+b)/(a-b)*ln(a+b*coth(x))`

Maxima [A] time = 1.26407, size = 228, normalized size = 2.43

$$\frac{a^5 \log(-(a-b)e^{(-2x)} + a + b)}{a^2b^4 - b^6} + \frac{2(3a^2 + 4b^2 - 3(2a^2 + ab + 2b^2)e^{(-2x)} + 3(a^2 + ab + 2b^2)e^{(-4x)})}{3(3b^3e^{(-2x)} - 3b^3e^{(-4x)} + b^3e^{(-6x)} - b^3)} + \frac{x}{a+b} - \frac{(a^3 + ab^2)\log(e^{-x} + 1)}{b^4} - \frac{(a^3 + ab^2)\log(e^{-x} - 1)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="maxima")`

[Out] `a^5*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^4 - b^6) + 2/3*(3*a^2 + 4*b^2 - 3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(a^2 + a*b + 2*b^2)*e^(-4*x))/(3*b^3*e^(-2*x) - 3*b^3*e^(-4*x) + b^3*e^(-6*x) - b^3) + x/(a + b) - (a^3 + a*b^2)*log(e^(-x) + 1)/b^4 - (a^3 + a*b^2)*log(e^(-x) - 1)/b^4`

Fricas [B] time = 3.09601, size = 3089, normalized size = 32.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="fricas")`

[Out] `-1/3*(3*(a*b^4 + b^5)*x*cosh(x)^6 + 18*(a*b^4 + b^5)*x*cosh(x)*sinh(x)^5 + 3*(a*b^4 + b^5)*x*sinh(x)^6 + 6*a^4*b + 2*a^2*b^3 - 8*b^5 + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^4 + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*sinh(x)^4 + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^2 + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*sinh(x)^2 + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x) + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*sinh(x) + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x) + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*sinh(x)`

```

*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 + 15*(a*b^4 + b^5)*x*cosh(
x)^2 - 3*(a*b^4 + b^5)*x)*sinh(x)^4 + 12*(5*(a*b^4 + b^5)*x*cosh(x)^3 + (2*
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x
))*sinh(x)^3 - 3*(4*a^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x
)*cosh(x)^2 + 3*(15*(a*b^4 + b^5)*x*cosh(x)^4 - 4*a^4*b + 2*a^3*b^2 - 2*a*b
^4 + 4*b^5 + 6*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^
4 + b^5)*x)*cosh(x)^2 + 3*(a*b^4 + b^5)*x)*sinh(x)^2 - 3*(a*b^4 + b^5)*x -
3*(a^5*cosh(x)^6 + 6*a^5*cosh(x)*sinh(x)^5 + a^5*sinh(x)^6 - 3*a^5*cosh(x)^
4 + 3*a^5*cosh(x)^2 - a^5 + 3*(5*a^5*cosh(x)^2 - a^5)*sinh(x)^4 + 4*(5*a^5*
cosh(x)^3 - 3*a^5*cosh(x))*sinh(x)^3 + 3*(5*a^5*cosh(x)^4 - 6*a^5*cosh(x)^2
+ a^5)*sinh(x)^2 + 6*(a^5*cosh(x)^5 - 2*a^5*cosh(x)^3 + a^5*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + 3*((a^5 - a*b^4)*c
osh(x)^6 + 6*(a^5 - a*b^4)*cosh(x)*sinh(x)^5 + (a^5 - a*b^4)*sinh(x)^6 - a^
5 + a*b^4 - 3*(a^5 - a*b^4)*cosh(x)^4 - 3*(a^5 - a*b^4 - 5*(a^5 - a*b^4)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a*b^4)*cosh(x)^3 - 3*(a^5 - a*b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)^2 + 3*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*
cosh(x)^4 - 6*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*((a^5 - a*b^4)*cosh(x)
^5 - 2*(a^5 - a*b^4)*cosh(x)^3 + (a^5 - a*b^4)*cosh(x))*sinh(x))*log(2*sinh
(x)/(cosh(x) - sinh(x))) + 6*(3*(a*b^4 + b^5)*x*cosh(x)^5 + 2*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^3 - (4*a
^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x))*sinh(x))/((
a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^2*b^4
- b^6)*sinh(x)^6 - a^2*b^4 + b^6 - 3*(a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^2*b^4
- b^6 - 5*(a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b^4 - b^6)*cosh
(x)^3 - 3*(a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^2*b^4 - b^6)*cosh(x)^2
+ 3*(a^2*b^4 - b^6 + 5*(a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^2*b^4 - b^6)*cosh(x)
)^2)*sinh(x)^2 + 6*((a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^2*b^4 - b^6)*cosh(x)^3
+ (a^2*b^4 - b^6)*cosh(x))*sinh(x))

```

Sympy [A] time = 13.0737, size = 1013, normalized size = 10.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**5/(a+b*coth(x)),x)

[Out] Piecewise((zoo*(x - 1/tanh(x) - 1/(3*tanh(x)**3)), Eq(a, 0) & Eq(b, 0)), ((x - 1/tanh(x) - 1/(3*tanh(x)**3))/b, Eq(a, 0)), (27*x*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 27*x*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*log(tanh(x)

```

)) * tanh(x)**4 / (6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 12*log(tanh(x)) * tanh(x)**
3 / (6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 15*tanh(x)**4 / (6*b*tanh(x)**4 - 6*b*t
anh(x)**3) + 9*tanh(x)**2 / (6*b*tanh(x)**4 - 6*b*tanh(x)**3) + tanh(x) / (6*b*
tanh(x)**4 - 6*b*tanh(x)**3) + 2 / (6*b*tanh(x)**4 - 6*b*tanh(x)**3), Eq(a, -
b)), (3*x*tanh(x)**4 / (6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 3*x*tanh(x)**3 / (6*
b*tanh(x)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1) * tanh(x)**4 / (6*b*tanh(x)
)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1) * tanh(x)**3 / (6*b*tanh(x)**4 + 6
*b*tanh(x)**3) - 12*log(tanh(x)) * tanh(x)**4 / (6*b*tanh(x)**4 + 6*b*tanh(x)**
3) - 12*log(tanh(x)) * tanh(x)**3 / (6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 15*tanh
(x)**4 / (6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 9*tanh(x)**2 / (6*b*tanh(x)**4 + 6
*b*tanh(x)**3) + tanh(x) / (6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 2 / (6*b*tanh(x)
**4 + 6*b*tanh(x)**3), Eq(a, b)), ((x - log(tanh(x) + 1) + log(tanh(x))) - 1
/ (2*tanh(x)**2) - 1 / (4*tanh(x)**4)) / a, Eq(b, 0)), (6*a**5*log(tanh(x) + b/a
) * tanh(x)**3 / (6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**5*log(tanh
(x)) * tanh(x)**3 / (6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**4*b*tan
h(x)**2 / (6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) + 3*a**3*b**2*tanh(x) /
(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 2*a**2*b**3 / (6*a**2*b**4*tan
h(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*x*tanh(x)**3 / (6*a**2*b**4*tanh(x)**
3 - 6*b**6*tanh(x)**3) - 6*a*b**4*log(tanh(x) + 1) * tanh(x)**3 / (6*a**2*b**4*
tanh(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*log(tanh(x)) * tanh(x)**3 / (6*a**2*
b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 3*a*b**4*tanh(x) / (6*a**2*b**4*tanh(x)
)**3 - 6*b**6*tanh(x)**3) - 6*b**5*x*tanh(x)**3 / (6*a**2*b**4*tanh(x)**3 - 6
*b**6*tanh(x)**3) + 6*b**5*tanh(x)**2 / (6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh
(x)**3) + 2*b**5 / (6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3), True))

```

Giac [A] time = 1.17641, size = 193, normalized size = 2.05

$$\frac{a^5 \log(|ae^{2x} + be^{2x} - a + b|)}{a^2b^4 - b^6} - \frac{x}{a - b} - \frac{(a^3 + ab^2) \log(|e^{2x} - 1|)}{b^4} - \frac{2(3a^2b + 4b^3 + 3(a^2b - ab^2 + 2b^3)e^{4x}) - 3(2a^3 + a^2b^2 + 2b^3)e^{2x}}{3b^4(e^{2x} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="giac")

[Out] a^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^4 - b^6) - x/(a - b) - (a^3 + a*b^2)*log(abs(e^(2*x) - 1))/b^4 - 2/3*(3*a^2*b + 4*b^3 + 3*(a^2*b - a*b^2 + 2*b^3)*e^(4*x) - 3*(2*a^2*b - a*b^2 + 2*b^3)*e^(2*x))/(b^4*(e^(2*x) - 1)^3)

$$3.150 \quad \int \frac{x \operatorname{csch}^2(x)}{(a+b \operatorname{coth}(x))^2} dx$$

Optimal. Leaf size=54

$$-\frac{ax}{b(a^2-b^2)} + \frac{\log(a \sinh(x) + b \cosh(x))}{a^2-b^2} + \frac{x}{b(a+b \operatorname{coth}(x))}$$

[Out] $-\left(\frac{a*x}{b*(a^2 - b^2)}\right) + x/(b*(a + b*\operatorname{Coth}[x])) + \operatorname{Log}[b*\operatorname{Cosh}[x] + a*\operatorname{Sinh}[x]]/(a^2 - b^2)$

Rubi [A] time = 0.0850082, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5467, 3484, 3530}

$$-\frac{ax}{b(a^2-b^2)} + \frac{\log(a \sinh(x) + b \cosh(x))}{a^2-b^2} + \frac{x}{b(a+b \operatorname{coth}(x))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x*\operatorname{Csch}[x]^2)/(a + b*\operatorname{Coth}[x])^2, x]$

[Out] $-\left(\frac{a*x}{b*(a^2 - b^2)}\right) + x/(b*(a + b*\operatorname{Coth}[x])) + \operatorname{Log}[b*\operatorname{Cosh}[x] + a*\operatorname{Sinh}[x]]/(a^2 - b^2)$

Rule 5467

$\operatorname{Int}[\operatorname{Csch}[(c_.) + (d_.)*(x_)]^2*(\operatorname{Coth}[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^{(n_.)}*((e_.) + (f_.)*(x_))^{(m_.)}, x_Symbol] := -\operatorname{Simp}[(e + f*x)^m*(a + b*\operatorname{Coth}[c + d*x])^{(n + 1)}]/(b*d*(n + 1)), x] + \operatorname{Dist}[(f*m)/(b*d*(n + 1)), \operatorname{Int}[(e + f*x)^{(m - 1)}*(a + b*\operatorname{Coth}[c + d*x])^{(n + 1)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, n\}, x] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ \operatorname{NeQ}[n, -1]$

Rule 3484

$\operatorname{Int}[(a_.) + (b_.)*\tan[(c_.) + (d_.)*(x_)]^{(-1)}, x_Symbol] := \operatorname{Simp}[(a*x)/(a^2 + b^2), x] + \operatorname{Dist}[b/(a^2 + b^2), \operatorname{Int}[(b - a*\tan[c + d*x])/(a + b*\tan[c + d*x]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[a^2 + b^2, 0]$

Rule 3530

```
Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x \operatorname{csch}^2(x)}{(a + b \operatorname{coth}(x))^2} dx &= \frac{x}{b(a + b \operatorname{coth}(x))} - \frac{\int \frac{1}{a + b \operatorname{coth}(x)} dx}{b} \\ &= -\frac{ax}{b(a^2 - b^2)} + \frac{x}{b(a + b \operatorname{coth}(x))} + \frac{i \int \frac{-ib - ia \operatorname{coth}(x)}{a + b \operatorname{coth}(x)} dx}{a^2 - b^2} \\ &= -\frac{ax}{b(a^2 - b^2)} + \frac{x}{b(a + b \operatorname{coth}(x))} + \frac{\log(b \cosh(x) + a \sinh(x))}{a^2 - b^2} \end{aligned}$$

Mathematica [A] time = 0.158962, size = 49, normalized size = 0.91

$$\frac{ax - b \log(a \sinh(x) + b \cosh(x))}{b^3 - a^2 b} + \frac{x \sinh(x)}{ab \sinh(x) + b^2 \cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

```
[Out] (a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(-(a^2*b) + b^3) + (x*Sinh[x])/(b^2*Co
sh[x] + a*b*Sinh[x])
```

Maple [A] time = 0.112, size = 73, normalized size = 1.4

$$-2 \frac{x}{a^2 - b^2} - 2 \frac{x}{(ae^{2x} + be^{2x} - a + b)(a + b)} + \frac{1}{a^2 - b^2} \ln \left(e^{2x} - \frac{a - b}{a + b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)^2/(a+b*coth(x))^2,x)
```

```
[Out] -2/(a^2-b^2)*x-2*x/(a*exp(2*x)+b*exp(2*x)-a+b)/(a+b)+1/(a^2-b^2)*ln(exp(2*x)
)-(a-b)/(a+b))
```

Maxima [A] time = 1.952, size = 92, normalized size = 1.7

$$\frac{2xe^{(2x)}}{a^2 - 2ab + b^2 - (a^2 - b^2)e^{(2x)}} + \frac{\log\left(\frac{(a+b)e^{(2x)} - a + b}{a+b}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cscsch(x)^2/(a+b*coth(x))^2,x, algorithm="maxima")

[Out] $2*x*e^{(2*x)}/(a^2 - 2*a*b + b^2 - (a^2 - b^2)*e^{(2*x)}) + \log(((a + b)*e^{(2*x)} - a + b)/(a + b))/(a^2 - b^2)$

Fricas [B] time = 2.68442, size = 491, normalized size = 9.09

$$\frac{2(a+b)x \cosh(x)^2 + 4(a+b)x \cosh(x) \sinh(x) + 2(a+b)x \sinh(x)^2 - ((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) - a^3 - a^2b - ab^2 + b^3 - (a^3 + a^2b - ab^2 - b^3) \cosh(x)^2 - 2(a^3 + a^2b - ab^2 - b^3) \cosh(x) \sinh(x) - (a^3 + a^2b - ab^2 - b^3) \sinh(x)^2)}{a^3 - a^2b - ab^2 + b^3 - (a^3 + a^2b - ab^2 - b^3) \cosh(x)^2 - 2(a^3 + a^2b - ab^2 - b^3) \cosh(x) \sinh(x) - (a^3 + a^2b - ab^2 - b^3) \sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cscsch(x)^2/(a+b*coth(x))^2,x, algorithm="fricas")

[Out] $(2*(a + b)*x*\cosh(x)^2 + 4*(a + b)*x*\cosh(x)*\sinh(x) + 2*(a + b)*x*\sinh(x)^2 - ((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 - a + b)*\log(2*(b*\cosh(x) + a*\sinh(x))/(\cosh(x) - \sinh(x))))/(a^3 - a^2*b - a*b^2 + b^3 - (a^3 + a^2*b - a*b^2 - b^3)*\cosh(x)^2 - 2*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(x)*\sinh(x) - (a^3 + a^2*b - a*b^2 - b^3)*\sinh(x)^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{csch}^2(x)}{(a + b \operatorname{coth}(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cscsch(x)**2/(a+b*coth(x))**2,x)

[Out] Integral(x*cscsch(x)**2/(a + b*coth(x))**2, x)

Giac [B] time = 1.17361, size = 228, normalized size = 4.22

$$\frac{2axe^{(2x)} + 2bx e^{(2x)} - ae^{(2x)} \log(ae^{(2x)} + be^{(2x)} - a + b) - be^{(2x)} \log(ae^{(2x)} + be^{(2x)} - a + b) + a \log(ae^{(2x)} + be^{(2x)} - a + b) - b \log(ae^{(2x)} + be^{(2x)} - a + b)}{a^3e^{(2x)} + a^2be^{(2x)} - ab^2e^{(2x)} - b^3e^{(2x)} - a^3 + a^2b + ab^2 - b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cscsch(x)^2/(a+b*coth(x))^2,x, algorithm="giac")

[Out] $-(2axxe^{(2x)} + 2bx e^{(2x)} - ae^{(2x)} \log(ae^{(2x)} + be^{(2x)} - a + b) - be^{(2x)} \log(ae^{(2x)} + be^{(2x)} - a + b) + a \log(ae^{(2x)} + be^{(2x)} - a + b) - b \log(ae^{(2x)} + be^{(2x)} - a + b)) / (a^3e^{(2x)} + a^2be^{(2x)} - ab^2e^{(2x)} - b^3e^{(2x)} - a^3 + a^2b + ab^2 - b^3)$

3.151 $\int x^3 \coth(a + 2 \log(x)) dx$

Optimal. Leaf size=30

$$\frac{1}{2}e^{-2a} \log(1 - e^{2a}x^4) + \frac{x^4}{4}$$

[Out] $x^4/4 + \text{Log}[1 - E^{(2*a)*x^4}/(2*E^{(2*a)})]$

Rubi [F] time = 0.0285749, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^3 \coth(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[x^3*\text{Coth}[a + 2*\text{Log}[x]], x]$

[Out] $\text{Defer}[\text{Int}[x^3*\text{Coth}[a + 2*\text{Log}[x]], x]$

Rubi steps

$$\int x^3 \coth(a + 2 \log(x)) dx = \int x^3 \coth(a + 2 \log(x)) dx$$

Mathematica [B] time = 0.0241018, size = 64, normalized size = 2.13

$$\frac{1}{2} \cosh(2a) \log(x^4 \sinh(a) + x^4 \cosh(a) + \sinh(a) - \cosh(a)) - \frac{1}{2} \sinh(2a) \log(x^4 \sinh(a) + x^4 \cosh(a) + \sinh(a) - \cosh(a))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^3*\text{Coth}[a + 2*\text{Log}[x]], x]$

[Out] $x^4/4 + (\text{Cosh}[2*a]*\text{Log}[-\text{Cosh}[a] + x^4*\text{Cosh}[a] + \text{Sinh}[a] + x^4*\text{Sinh}[a]])/2 - (\text{Log}[-\text{Cosh}[a] + x^4*\text{Cosh}[a] + \text{Sinh}[a] + x^4*\text{Sinh}[a]]*\text{Sinh}[2*a])/2$

Maple [A] time = 0.026, size = 24, normalized size = 0.8

$$\frac{x^4}{4} + \frac{e^{-2a} \ln(e^{2a} x^4 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*coth(a+2*ln(x)),x)`

[Out] `1/4*x^4+1/2*exp(-2*a)*ln(exp(2*a)*x^4-1)`

Maxima [A] time = 1.21452, size = 49, normalized size = 1.63

$$\frac{1}{4} x^4 + \frac{1}{2} e^{(-2a)} \log(x^2 e^a + 1) + \frac{1}{2} e^{(-2a)} \log(x^2 e^a - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(a+2*log(x)),x, algorithm="maxima")`

[Out] `1/4*x^4 + 1/2*e^(-2*a)*log(x^2*e^a + 1) + 1/2*e^(-2*a)*log(x^2*e^a - 1)`

Fricas [A] time = 2.53483, size = 72, normalized size = 2.4

$$\frac{1}{4} (x^4 e^{(2a)} + 2 \log(x^4 e^{(2a)} - 1)) e^{(-2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(a+2*log(x)),x, algorithm="fricas")`

[Out] `1/4*(x^4*e^(2*a) + 2*log(x^4*e^(2*a) - 1))*e^(-2*a)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*coth(a+2*ln(x)),x)`

[Out] `Integral(x**3*coth(a + 2*log(x)), x)`

Giac [A] time = 1.14286, size = 32, normalized size = 1.07

$$\frac{1}{4}x^4 + \frac{1}{2}e^{(-2a)}\log(|x^4e^{(2a)} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(a+2*log(x)),x, algorithm="giac")`

[Out] `1/4*x^4 + 1/2*e^(-2*a)*log(abs(x^4*e^(2*a) - 1))`

3.152 $\int x^2 \coth(a + 2 \log(x)) dx$

Optimal. Leaf size=45

$$e^{-3a/2} \tan^{-1}(e^{a/2}x) - e^{-3a/2} \tanh^{-1}(e^{a/2}x) + \frac{x^3}{3}$$

[Out] $x^3/3 + \text{ArcTan}[E^{(a/2)*x}]/E^{((3*a)/2)} - \text{ArcTanh}[E^{(a/2)*x}]/E^{((3*a)/2)}$

Rubi [F] time = 0.0213207, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \coth(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[x^2*\text{Coth}[a + 2*\text{Log}[x]], x]$

[Out] $\text{Defer}[\text{Int}[x^2*\text{Coth}[a + 2*\text{Log}[x]], x]$

Rubi steps

$$\int x^2 \coth(a + 2 \log(x)) dx = \int x^2 \coth(a + 2 \log(x)) dx$$

Mathematica [C] time = 0.23153, size = 64, normalized size = 1.42

$$\frac{1}{6} \left(3(\sinh(2a) - \cosh(2a)) \text{RootSum} \left[\#1^4 \sinh(a) + \#1^4 \cosh(a) + \sinh(a) - \cosh(a) \&, \frac{\log(x) - \log(x - \#1)}{\#1} \& \right] + 2x^3 \right)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^2*\text{Coth}[a + 2*\text{Log}[x]], x]$

[Out] $(2*x^3 + 3*\text{RootSum}[-\text{Cosh}[a] + \text{Sinh}[a] + \text{Cosh}[a]*\#1^4 + \text{Sinh}[a]*\#1^4 \&, (\text{Log}[x] - \text{Log}[x - \#1])/ \#1 \&]*(-\text{Cosh}[2*a] + \text{Sinh}[2*a]))/6$

Maple [B] time = 0.048, size = 83, normalized size = 1.8

$$\frac{x^3}{3} + \frac{1}{2} \ln(-x\sqrt{e^a} + 1)(e^a)^{-\frac{3}{2}} - \frac{1}{2} \ln(x\sqrt{e^a} + 1)(e^a)^{-\frac{3}{2}} + \frac{1}{2} \ln(-e^{2a}x + (-e^a)^{\frac{3}{2}})(-e^a)^{-\frac{3}{2}} - \frac{1}{2} \ln(e^{2a}x + (-e^a)^{\frac{3}{2}})(-e^a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*coth(a+2*ln(x)),x)`

[Out] `1/3*x^3+1/2/exp(a)^(3/2)*ln(-x*exp(a)^(1/2)+1)-1/2/exp(a)^(3/2)*ln(x*exp(a)^(1/2)+1)+1/2/(-exp(a))^(3/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))-1/2/(-exp(a))^(3/2)*ln(exp(2*a)*x+(-exp(a))^(3/2))`

Maxima [A] time = 1.75688, size = 65, normalized size = 1.44

$$\frac{1}{3}x^3 + \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{1}{2}e^{\left(-\frac{3}{2}a\right)} \log\left(\frac{xe^a - e^{\left(\frac{1}{2}a\right)}}{xe^a + e^{\left(\frac{1}{2}a\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*coth(a+2*log(x)),x, algorithm="maxima")`

[Out] `1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log((x*e^a - e^(1/2*a))/(x*e^a + e^(1/2*a)))`

Fricas [A] time = 2.68469, size = 171, normalized size = 3.8

$$\frac{1}{6} \left(2x^3e^{(2a)} + 6 \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + 3e^{\left(\frac{1}{2}a\right)} \log\left(\frac{x^2e^a - 2xe^{\left(\frac{1}{2}a\right)} + 1}{x^2e^a - 1}\right) \right) e^{(-2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*coth(a+2*log(x)),x, algorithm="fricas")`

[Out] `1/6*(2*x^3*e^(2*a) + 6*arctan(x*e^(1/2*a))*e^(1/2*a) + 3*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-2*a)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*coth(a+2*ln(x)),x)

[Out] Integral(x**2*coth(a + 2*log(x)), x)

Giac [A] time = 1.14067, size = 73, normalized size = 1.62

$$\frac{1}{3}x^3 + \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{1}{2}e^{\left(-\frac{3}{2}a\right)} \log\left(\frac{\left|2xe^a - 2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^a + 2e^{\left(\frac{1}{2}a\right)}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="giac")

[Out] 1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))

3.153 $\int x \coth(a + 2 \log(x)) dx$

Optimal. Leaf size=23

$$\frac{x^2}{2} - e^{-a} \tanh^{-1}(e^a x^2)$$

[Out] $x^2/2 - \text{ArcTanh}[E^a x^2]/E^a$

Rubi [F] time = 0.0156127, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x \coth(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[x \cdot \text{Coth}[a + 2 \cdot \text{Log}[x]], x]$

[Out] $\text{Defer}[\text{Int}[x \cdot \text{Coth}[a + 2 \cdot \text{Log}[x]], x]$

Rubi steps

$$\int x \coth(a + 2 \log(x)) dx = \int x \coth(a + 2 \log(x)) dx$$

Mathematica [A] time = 0.181986, size = 26, normalized size = 1.13

$$(\sinh(a) - \cosh(a)) \tanh^{-1}(x^2(\sinh(a) + \cosh(a))) + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x \cdot \text{Coth}[a + 2 \cdot \text{Log}[x]], x]$

[Out] $x^2/2 + \text{ArcTanh}[x^2 \cdot (\text{Cosh}[a] + \text{Sinh}[a])] \cdot (-\text{Cosh}[a] + \text{Sinh}[a])$

Maple [A] time = 0.024, size = 37, normalized size = 1.6

$$\frac{x^2}{2} - \frac{e^{-a} \ln(e^a x^2 + 1)}{2} + \frac{e^{-a} \ln(e^a x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*coth(a+2*ln(x)),x)

[Out] 1/2*x^2-1/2*exp(-a)*ln(exp(a)*x^2+1)+1/2*exp(-a)*ln(exp(a)*x^2-1)

Maxima [A] time = 1.04383, size = 49, normalized size = 2.13

$$\frac{1}{2} x^2 - \frac{1}{2} e^{(-a)} \log(x^2 e^a + 1) + \frac{1}{2} e^{(-a)} \log(x^2 e^a - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(a+2*log(x)),x, algorithm="maxima")

[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1)

Fricas [A] time = 2.59376, size = 81, normalized size = 3.52

$$\frac{1}{2} (x^2 e^a - \log(x^2 e^a + 1) + \log(x^2 e^a - 1)) e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(a+2*log(x)),x, algorithm="fricas")

[Out] 1/2*(x^2*e^a - log(x^2*e^a + 1) + log(x^2*e^a - 1))*e^(-a)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(a+2*ln(x)),x)`

[Out] `Integral(x*coth(a + 2*log(x)), x)`

Giac [A] time = 1.1168, size = 50, normalized size = 2.17

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log(x^2e^a + 1) + \frac{1}{2}e^{(-a)}\log(|x^2e^a - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(a+2*log(x)),x, algorithm="giac")`

[Out] `1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1))`

3.154 $\int \coth(a + 2 \log(x)) dx$

Optimal. Leaf size=40

$$-e^{-a/2} \tan^{-1}(e^{a/2}x) - e^{-a/2} \tanh^{-1}(e^{a/2}x) + x$$

[Out] $x - \text{ArcTan}[E^{(a/2)*x}]/E^{(a/2)} - \text{ArcTanh}[E^{(a/2)*x}]/E^{(a/2)}$

Rubi [F] time = 0.0071696, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[\text{Coth}[a + 2*\text{Log}[x]], x]$

[Out] $\text{Defer}[\text{Int}][\text{Coth}[a + 2*\text{Log}[x]], x]$

Rubi steps

$$\int \coth(a + 2 \log(x)) dx = \int \coth(a + 2 \log(x)) dx$$

Mathematica [C] time = 0.178869, size = 58, normalized size = 1.45

$$\frac{1}{2}(\sinh(2a) - \cosh(2a))\text{RootSum}\left[\#1^4 \sinh(a) + \#1^4 \cosh(a) + \sinh(a) - \cosh(a)\&, \frac{\log(x) - \log(x - \#1)}{\#1^3}\&\right] + x$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Coth}[a + 2*\text{Log}[x]], x]$

[Out] $x + (\text{RootSum}[-\text{Cosh}[a] + \text{Sinh}[a] + \text{Cosh}[a]*\#1^4 + \text{Sinh}[a]*\#1^4 \&, (\text{Log}[x] - \text{Log}[x - \#1])/\#1^3 \&]*(-\text{Cosh}[2*a] + \text{Sinh}[2*a]))/2$

Maple [B] time = 0.045, size = 71, normalized size = 1.8

$$x - \frac{1}{2} \ln(x\sqrt{-e^a} + 1) \frac{1}{\sqrt{-e^a}} + \frac{1}{2} \ln(x\sqrt{-e^a} - 1) \frac{1}{\sqrt{-e^a}} + \frac{1}{2} \ln(x\sqrt{e^a} - 1) \frac{1}{\sqrt{e^a}} - \frac{1}{2} \ln(x\sqrt{e^a} + 1) \frac{1}{\sqrt{e^a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(a+2*ln(x)),x)`

[Out] `x-1/2/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/2/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)-1)+1/2/exp(a)^(1/2)*ln(x*exp(a)^(1/2)-1)-1/2/exp(a)^(1/2)*ln(x*exp(a)^(1/2)+1)`

Maxima [A] time = 1.60909, size = 61, normalized size = 1.52

$$-\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{xe^a - e^{\left(\frac{1}{2}a\right)}}{xe^a + e^{\left(\frac{1}{2}a\right)}}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x)),x, algorithm="maxima")`

[Out] `-arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log((x*e^a - e^(1/2*a))/(x*e^a + e^(1/2*a))) + x`

Fricas [B] time = 2.58194, size = 159, normalized size = 3.98

$$-\frac{1}{2}\left(2\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} - 2xe^a - e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a - 2xe^{\left(\frac{1}{2}a\right)} + 1}{x^2e^a - 1}\right)\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x)),x, algorithm="fricas")`

[Out] `-1/2*(2*arctan(x*e^(1/2*a))*e^(1/2*a) - 2*x*e^a - e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-a)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*ln(x)),x)`

[Out] `Integral(coth(a + 2*log(x)), x)`

Giac [A] time = 1.12146, size = 69, normalized size = 1.72

$$-\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(-\frac{1}{2}a\right)} \log\left(\frac{\left|2xe^a - 2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^a + 2e^{\left(\frac{1}{2}a\right)}\right|}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x)),x, algorithm="giac")`

[Out] `-arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x`

$$3.155 \quad \int \frac{\coth(a+2 \log(x))}{x} dx$$

Optimal. Leaf size=12

$$\frac{1}{2} \log(\sinh(a + 2 \log(x)))$$

[Out] Log[Sinh[a + 2*Log[x]]]/2

Rubi [A] time = 0.0137687, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3475}

$$\frac{1}{2} \log(\sinh(a + 2 \log(x)))$$

Antiderivative was successfully verified.

[In] Int[Coth[a + 2*Log[x]]/x,x]

[Out] Log[Sinh[a + 2*Log[x]]]/2

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth(a + 2 \log(x))}{x} dx &= \text{Subst}\left(\int \coth(a + 2x) dx, x, \log(x)\right) \\ &= \frac{1}{2} \log(\sinh(a + 2 \log(x))) \end{aligned}$$

Mathematica [A] time = 0.0266171, size = 21, normalized size = 1.75

$$\frac{1}{2} (\log(\tanh(a + 2 \log(x))) + \log(\cosh(a + 2 \log(x))))$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + 2*Log[x]]/x,x]

[Out] (Log[Cosh[a + 2*Log[x]]] + Log[Tanh[a + 2*Log[x]]])/2

Maple [B] time = 0.003, size = 26, normalized size = 2.2

$$-\frac{\ln(\coth(a + 2 \ln(x)) - 1)}{4} - \frac{\ln(\coth(a + 2 \ln(x)) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+2*ln(x))/x,x)

[Out] -1/4*ln(coth(a+2*ln(x))-1)-1/4*ln(coth(a+2*ln(x))+1)

Maxima [A] time = 1.05091, size = 14, normalized size = 1.17

$$\frac{1}{2} \log(\sinh(a + 2 \log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))/x,x, algorithm="maxima")

[Out] 1/2*log(sinh(a + 2*log(x)))

Fricas [A] time = 2.53463, size = 47, normalized size = 3.92

$$\frac{1}{2} \log(x^4 e^{(2a)} - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))/x,x, algorithm="fricas")

[Out] 1/2*log(x^4*e^(2*a) - 1) - log(x)

Sympy [B] time = 1.68852, size = 27, normalized size = 2.25

$$\log(x) - \frac{\log(\tanh(a + 2\log(x)) + 1)}{2} + \frac{\log(\tanh(a + 2\log(x)))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*ln(x))/x,x)

[Out] log(x) - log(tanh(a + 2*log(x)) + 1)/2 + log(tanh(a + 2*log(x)))/2

Giac [A] time = 1.09541, size = 28, normalized size = 2.33

$$-\frac{1}{4} \log(x^4) + \frac{1}{2} \log(|x^4 e^{(2a)} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))/x,x, algorithm="giac")

[Out] -1/4*log(x^4) + 1/2*log(abs(x^4*e^(2*a) - 1))

$$3.156 \quad \int \frac{\coth(a+2 \log(x))}{x^2} dx$$

Optimal. Leaf size=41

$$e^{a/2} \tan^{-1}(e^{a/2}x) - e^{a/2} \tanh^{-1}(e^{a/2}x) + \frac{1}{x}$$

[Out] $x^{(-1)} + E^{(a/2)} \cdot \text{ArcTan}[E^{(a/2)} \cdot x] - E^{(a/2)} \cdot \text{ArcTanh}[E^{(a/2)} \cdot x]$

Rubi [F] time = 0.0222044, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth(a + 2 \log(x))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + 2*Log[x]]/x^2, x]

[Out] Defer[Int][Coth[a + 2*Log[x]]/x^2, x]

Rubi steps

$$\int \frac{\coth(a + 2 \log(x))}{x^2} dx = \int \frac{\coth(a + 2 \log(x))}{x^2} dx$$

Mathematica [C] time = 0.162025, size = 62, normalized size = 1.51

$$\frac{x(\sinh(a) + \cosh(a))^2 \text{RootSum} \left[-\#1^4 \sinh(a) + \#1^4 \cosh(a) - \sinh(a) - \cosh(a) \&, \frac{\log\left(\frac{1}{x} - \#1\right) + \log(x)}{\#1^3} \& \right] + 2}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + 2*Log[x]]/x^2, x]

[Out] $(2 + x \cdot \text{RootSum}[-\text{Cosh}[a] - \text{Sinh}[a] + \text{Cosh}[a] \cdot \#1^4 - \text{Sinh}[a] \cdot \#1^4 \&, (\text{Log}[x] + \text{Log}[x^{(-1)} - \#1])/\#1^3 \&]) \cdot (\text{Cosh}[a] + \text{Sinh}[a])^2 / (2 \cdot x)$

Maple [B] time = 0.042, size = 93, normalized size = 2.3

$$x^{-1} + \frac{1}{2}\sqrt{e^a} \ln\left(-e^{2a}x + (e^a)^{\frac{3}{2}}\right) - \frac{1}{2}\sqrt{e^a} \ln\left(-e^{2a}x - (e^a)^{\frac{3}{2}}\right) + \frac{1}{2}\sqrt{-e^a} \ln\left(-e^{2a}x + (-e^a)^{\frac{3}{2}}\right) - \frac{1}{2}\sqrt{-e^a} \ln\left(-e^{2a}x - (-e^a)^{\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(a+2*ln(x))/x^2,x)`

[Out] `1/x+1/2*exp(a)^(1/2)*ln(-exp(2*a)*x+exp(a)^(3/2))-1/2*exp(a)^(1/2)*ln(-exp(2*a)*x-exp(a)^(3/2))+1/2*(-exp(a))^(1/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))-1/2*(-exp(a))^(1/2)*ln(-exp(2*a)*x-(-exp(a))^(3/2))`

Maxima [A] time = 1.73337, size = 63, normalized size = 1.54

$$-\arctan\left(\frac{e^{\left(-\frac{1}{2}a\right)}}{x}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\frac{1}{x} - e^{\left(\frac{1}{2}a\right)}}{\frac{1}{x} + e^{\left(\frac{1}{2}a\right)}}\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x))/x^2,x, algorithm="maxima")`

[Out] `-arctan(e^(-1/2*a)/x)*e^(1/2*a) + 1/2*e^(1/2*a)*log((1/x - e^(1/2*a))/(1/x + e^(1/2*a))) + 1/x`

Fricas [A] time = 2.6559, size = 149, normalized size = 3.63

$$\frac{2x \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + xe^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a - 2xe^{\left(\frac{1}{2}a\right)} + 1}{x^2e^a - 1}\right) + 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x))/x^2,x, algorithm="fricas")`

[Out] $\frac{1}{2}*(2*x*\arctan(x*e^{(1/2*a)})*e^{(1/2*a)} + x*e^{(1/2*a)}*\log((x^2*e^a - 2*x*e^{(1/2*a)} + 1)/(x^2*e^a - 1)) + 2)/x$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(a + 2 \log(x))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*ln(x))/x**2,x)`

[Out] `Integral(coth(a + 2*log(x))/x**2, x)`

Giac [A] time = 1.15757, size = 70, normalized size = 1.71

$$\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\left|2xe^a - 2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^a + 2e^{\left(\frac{1}{2}a\right)}\right|}\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x))/x^2,x, algorithm="giac")`

[Out] $\arctan(x*e^{(1/2*a)})*e^{(1/2*a)} + 1/2*e^{(1/2*a)}*\log(\text{abs}(2*x*e^a - 2*e^{(1/2*a)})/\text{abs}(2*x*e^a + 2*e^{(1/2*a)})) + 1/x$

$$3.157 \quad \int \frac{\coth(a+2 \log(x))}{x^3} dx$$

Optimal. Leaf size=21

$$\frac{1}{2x^2} - e^a \tanh^{-1}(e^a x^2)$$

[Out] 1/(2*x^2) - E^a*ArcTanh[E^a*x^2]

Rubi [F] time = 0.0213417, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth(a + 2 \log(x))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + 2*Log[x]]/x^3,x]

[Out] Defer[Int][Coth[a + 2*Log[x]]/x^3, x]

Rubi steps

$$\int \frac{\coth(a + 2 \log(x))}{x^3} dx = \int \frac{\coth(a + 2 \log(x))}{x^3} dx$$

Mathematica [A] time = 0.156472, size = 27, normalized size = 1.29

$$\frac{1}{2x^2} - (\sinh(a) + \cosh(a)) \tanh^{-1}\left(\frac{\cosh(a) - \sinh(a)}{x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + 2*Log[x]]/x^3,x]

[Out] 1/(2*x^2) - ArcTanh[(Cosh[a] - Sinh[a])/x^2]*(Cosh[a] + Sinh[a])

Maple [A] time = 0.026, size = 35, normalized size = 1.7

$$\frac{1}{2x^2} - \frac{e^a \ln(-e^a x^2 - 1)}{2} + \frac{e^a \ln(-e^a x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(a+2*ln(x))/x^3,x)`

[Out] `1/2/x^2-1/2*exp(a)*ln(-exp(a)*x^2-1)+1/2*exp(a)*ln(-exp(a)*x^2+1)`

Maxima [A] time = 1.02956, size = 41, normalized size = 1.95

$$-\frac{1}{2} e^a \log\left(\frac{1}{x^2} + e^a\right) + \frac{1}{2} e^a \log\left(\frac{1}{x^2} - e^a\right) + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x))/x^3,x, algorithm="maxima")`

[Out] `-1/2*e^a*log(1/x^2 + e^a) + 1/2*e^a*log(1/x^2 - e^a) + 1/2/x^2`

Fricas [B] time = 2.5844, size = 92, normalized size = 4.38

$$-\frac{x^2 e^a \log(x^2 e^a + 1) - x^2 e^a \log(x^2 e^a - 1) - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x))/x^3,x, algorithm="fricas")`

[Out] `-1/2*(x^2*e^a*log(x^2*e^a + 1) - x^2*e^a*log(x^2*e^a - 1) - 1)/x^2`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(a + 2 \log(x))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*ln(x))/x**3,x)`

[Out] `Integral(coth(a + 2*log(x))/x**3, x)`

Giac [A] time = 1.10794, size = 45, normalized size = 2.14

$$-\frac{1}{2} e^a \log(x^2 e^a + 1) + \frac{1}{2} e^a \log(|x^2 e^a - 1|) + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x))/x^3,x, algorithm="giac")`

[Out] `-1/2*e^a*log(x^2*e^a + 1) + 1/2*e^a*log(abs(x^2*e^a - 1)) + 1/2/x^2`

3.158 $\int x^3 \coth^2(a + 2 \log(x)) dx$

Optimal. Leaf size=47

$$\frac{e^{-2a}}{1 - e^{2a}x^4} + e^{-2a} \log(1 - e^{2a}x^4) + \frac{x^4}{4}$$

[Out] $x^4/4 + 1/(E^{(2*a)}*(1 - E^{(2*a)}*x^4)) + \text{Log}[1 - E^{(2*a)}*x^4]/E^{(2*a)}$

Rubi [F] time = 0.0662787, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^3 \coth^2(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[x^3*\text{Coth}[a + 2*\text{Log}[x]]^2, x]$

[Out] $\text{Defer}[\text{Int}][x^3*\text{Coth}[a + 2*\text{Log}[x]]^2, x]$

Rubi steps

$$\int x^3 \coth^2(a + 2 \log(x)) dx = \int x^3 \coth^2(a + 2 \log(x)) dx$$

Mathematica [A] time = 0.102571, size = 86, normalized size = 1.83

$$\frac{\sinh(3a) - \cosh(3a)}{(x^4 + 1) \sinh(a) + (x^4 - 1) \cosh(a)} + \cosh(2a) \log((x^4 + 1) \sinh(a) + (x^4 - 1) \cosh(a)) - \sinh(2a) \log((x^4 + 1) \sinh(a) + (x^4 - 1) \cosh(a))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^3*\text{Coth}[a + 2*\text{Log}[x]]^2, x]$

[Out] $x^4/4 + \text{Cosh}[2*a]*\text{Log}[(-1 + x^4)*\text{Cosh}[a] + (1 + x^4)*\text{Sinh}[a]] - \text{Log}[(-1 + x^4)*\text{Cosh}[a] + (1 + x^4)*\text{Sinh}[a]]*\text{Sinh}[2*a] + (-\text{Cosh}[3*a] + \text{Sinh}[3*a])/((-1 + x^4)*\text{Cosh}[a] + (1 + x^4)*\text{Sinh}[a])$

Maple [A] time = 0.018, size = 41, normalized size = 0.9

$$\frac{x^4}{4} - \frac{e^{-2a}}{e^{2a}x^4 - 1} + e^{-2a} \ln(e^{2a}x^4 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*coth(a+2*ln(x))^2,x)`

[Out] `1/4*x^4-exp(-2*a)/(exp(2*a)*x^4-1)+exp(-2*a)*ln(exp(2*a)*x^4-1)`

Maxima [A] time = 1.05107, size = 72, normalized size = 1.53

$$\frac{1}{4}x^4 + e^{(-2a)} \log(x^2e^a + 1) + e^{(-2a)} \log(x^2e^a - 1) - \frac{1}{x^4e^{(4a)} - e^{(2a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(a+2*log(x))^2,x, algorithm="maxima")`

[Out] `1/4*x^4 + e^(-2*a)*log(x^2*e^a + 1) + e^(-2*a)*log(x^2*e^a - 1) - 1/(x^4*e^(4*a) - e^(2*a))`

Fricas [A] time = 2.46811, size = 140, normalized size = 2.98

$$\frac{x^8e^{(4a)} - x^4e^{(2a)} + 4(x^4e^{(2a)} - 1)\log(x^4e^{(2a)} - 1) - 4}{4(x^4e^{(4a)} - e^{(2a)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(a+2*log(x))^2,x, algorithm="fricas")`

[Out] `1/4*(x^8*e^(4*a) - x^4*e^(2*a) + 4*(x^4*e^(2*a) - 1)*log(x^4*e^(2*a) - 1) - 4)/(x^4*e^(4*a) - e^(2*a))`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth^2(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*coth(a+2*ln(x))**2,x)

[Out] Integral(x**3*coth(a + 2*log(x))**2, x)

Giac [A] time = 1.12165, size = 54, normalized size = 1.15

$$\frac{1}{4} x^4 - \frac{x^4}{x^4 e^{(2a)} - 1} + e^{(-2a)} \log(|x^4 e^{(2a)} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="giac")

[Out] 1/4*x^4 - x^4/(x^4*e^(2*a) - 1) + e^(-2*a)*log(abs(x^4*e^(2*a) - 1))

3.159 $\int x^2 \coth^2(a + 2 \log(x)) dx$

Optimal. Leaf size=68

$$\frac{x^3}{1 - e^{2a}x^4} + \frac{3}{2}e^{-3a/2} \tan^{-1}(e^{a/2}x) - \frac{3}{2}e^{-3a/2} \tanh^{-1}(e^{a/2}x) + \frac{x^3}{3}$$

[Out] $x^3/3 + x^3/(1 - E^{(2*a)*x^4}) + (3*ArcTan[E^{(a/2)*x}]/(2*E^{((3*a)/2)})) - (3*ArcTanh[E^{(a/2)*x}]/(2*E^{((3*a)/2)}))$

Rubi [F] time = 0.0477565, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \coth^2(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2*Coth[a + 2*Log[x]]^2,x]

[Out] Defer[Int][x^2*Coth[a + 2*Log[x]]^2, x]

Rubi steps

$$\int x^2 \coth^2(a + 2 \log(x)) dx = \int x^2 \coth^2(a + 2 \log(x)) dx$$

Mathematica [C] time = 2.8989, size = 154, normalized size = 2.26

$$\frac{16e^{2a}x^7 (e^{2a}x^4 + 1)^2 \text{HypergeometricPFQ}\left(\left\{\frac{7}{4}, 2, 2, 2\right\}, \left\{1, 1, \frac{19}{4}\right\}, e^{2a}x^4\right)}{1155} + \frac{e^{-4a} \left(7(27e^{8a}x^{16} - 632e^{6a}x^{12} - 398e^{4a}x^8 - 1976e^{2a}x^4 - 398e^{4a}x^8 - 632e^{6a}x^{12} + 27e^{8a}x^{16})\right)}{1155}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*Coth[a + 2*Log[x]]^2,x]

[Out] $(-9317 - 17825E^{(2*a)*x^4} - 4787E^{(4*a)*x^8} + 1481E^{(6*a)*x^{12}} + 7*(1331 + 1976E^{(2*a)*x^4} - 398E^{(4*a)*x^8} - 632E^{(6*a)*x^{12}} + 27E^{(8*a)*x^{16}})$

*Hypergeometric2F1[3/4, 1, 7/4, E^(2*a)*x^4]/(2688*E^(4*a)*x^5) + (16*E^(2*a)*x^7*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4}, E^(2*a)*x^4])/1155

Maple [A] time = 0.046, size = 100, normalized size = 1.5

$$\frac{x^3}{3} - \frac{x^3}{e^{2a}x^4 - 1} + \frac{3}{4} \ln\left(-e^{2a}x + (-e^a)^{\frac{3}{2}}\right)(-e^a)^{-\frac{3}{2}} - \frac{3}{4} \ln\left(e^{2a}x + (-e^a)^{\frac{3}{2}}\right)(-e^a)^{-\frac{3}{2}} + \frac{3}{4} \ln(-x\sqrt{e^a} + 1)(e^a)^{-\frac{3}{2}} - \frac{3}{4} \ln(x\sqrt{e^a} + 1)(e^a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*coth(a+2*ln(x))^2,x)

[Out] 1/3*x^3-x^3/(exp(2*a)*x^4-1)+3/4/(-exp(a))^(3/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))-3/4/(-exp(a))^(3/2)*ln(exp(2*a)*x+(-exp(a))^(3/2))+3/4/exp(a)^(3/2)*ln(-x*exp(a)^(1/2)+1)-3/4/exp(a)^(3/2)*ln(x*exp(a)^(1/2)+1)

Maxima [A] time = 1.62442, size = 89, normalized size = 1.31

$$\frac{1}{3}x^3 - \frac{x^3}{x^4e^{(2a)} - 1} + \frac{3}{2} \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{3}{4}e^{\left(-\frac{3}{2}a\right)} \log\left(\frac{xe^a - e^{\left(\frac{1}{2}a\right)}}{xe^a + e^{\left(\frac{1}{2}a\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="maxima")

[Out] 1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4*e^(-3/2*a)*log((x*e^a - e^(1/2*a))/(x*e^a + e^(1/2*a)))

Fricas [B] time = 2.59568, size = 266, normalized size = 3.91

$$\frac{4x^7e^{(4a)} - 16x^3e^{(2a)} + 18(x^4e^{(2a)} - 1) \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + 9(x^4e^{(2a)} - 1)e^{\left(\frac{1}{2}a\right)} \log\left(\frac{x^2e^a - 2xe^{\left(\frac{1}{2}a\right)} + 1}{x^2e^a - 1}\right)}{12(x^4e^{(4a)} - e^{(2a)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="fricas")

[Out] $\frac{1}{12}(4x^7e^{4a} - 16x^3e^{2a} + 18(x^4e^{2a} - 1)\arctan(xe^{1/2a}))e^{1/2a} + 9(x^4e^{2a} - 1)e^{1/2a}\log\left(\frac{x^2e^a - 2xe^{1/2a} + 1}{x^2e^a - 1}\right)/(x^4e^{4a} - e^{2a})$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth^2(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*coth(a+2*ln(x))**2,x)

[Out] Integral(x**2*coth(a + 2*log(x))**2, x)

Giac [A] time = 1.14405, size = 97, normalized size = 1.43

$$\frac{1}{3}x^3 - \frac{x^3}{x^4e^{2a} - 1} + \frac{3}{2} \arctan\left(xe^{\frac{1}{2}a}\right)e^{-\frac{3}{2}a} + \frac{3}{4}e^{-\frac{3}{2}a} \log\left(\frac{\left|2xe^a - 2e^{\frac{1}{2}a}\right|}{\left|2xe^a + 2e^{\frac{1}{2}a}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="giac")

[Out] $\frac{1}{3}x^3 - x^3/(x^4e^{2a} - 1) + 3/2*\arctan(xe^{1/2a})*e^{-3/2a} + 3/4*e^{-3/2a}*\log(\text{abs}(2*x*e^a - 2*e^{1/2a}))/\text{abs}(2*x*e^a + 2*e^{1/2a}))$

3.160 $\int x \coth^2(a + 2 \log(x)) dx$

Optimal. Leaf size=41

$$\frac{x^2}{1 - e^{2a}x^4} - e^{-a} \tanh^{-1}(e^a x^2) + \frac{x^2}{2}$$

[Out] $x^2/2 + x^2/(1 - E^{(2*a)*x^4}) - \text{ArcTanh}[E^a*x^2]/E^a$

Rubi [F] time = 0.0298197, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x \coth^2(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x*Coth[a + 2*Log[x]]^2,x]

[Out] Defer[Int][x*Coth[a + 2*Log[x]]^2, x]

Rubi steps

$$\int x \coth^2(a + 2 \log(x)) dx = \int x \coth^2(a + 2 \log(x)) dx$$

Mathematica [C] time = 2.93267, size = 163, normalized size = 3.98

$$\frac{2}{105} e^{2a} x^6 (e^{2a} x^4 + 1)^2 \text{HypergeometricPFQ}\left(\left\{\frac{3}{2}, 2, 2, 2\right\}, \left\{1, 1, \frac{9}{2}\right\}, e^{2a} x^4\right) + \frac{e^{-4a} \left(61 e^{6a} x^{12} - 181 e^{4a} x^8 - 713 e^{2a} x^4 + \dots\right)}{\dots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*Coth[a + 2*Log[x]]^2,x]

[Out] $(-375 - 713 E^{(2*a)*x^4} - 181 E^{(4*a)*x^8} + 61 E^{(6*a)*x^{12}} + (3*(125 + 196 E^{(2*a)*x^4} - 14 E^{(4*a)*x^8} - 52 E^{(6*a)*x^{12}} + E^{(8*a)*x^{16}}) * \text{ArcTanh}[\text{Sqr}$

$t[E^{(2*a)*x^4}]/\text{Sqrt}[E^{(2*a)*x^4}]/(96*E^{(4*a)*x^6} + (2*E^{(2*a)*x^6}*(1 + E^{(2*a)*x^4})^2*\text{HypergeometricPFQ}[\{3/2, 2, 2, 2\}, \{1, 1, 9/2\}, E^{(2*a)*x^4}])$
/105

Maple [A] time = 0.02, size = 54, normalized size = 1.3

$$\frac{x^2}{2} - \frac{x^2}{e^{2a}x^4 - 1} + \frac{e^{-a} \ln(e^a x^2 - 1)}{2} - \frac{e^{-a} \ln(e^a x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*coth(a+2*ln(x))^2,x)`

[Out] $1/2*x^2-x^2/(\exp(2*a)*x^4-1)+1/2*\exp(-a)*\ln(\exp(a)*x^2-1)-1/2*\exp(-a)*\ln(\exp(a)*x^2+1)$

Maxima [A] time = 1.06055, size = 72, normalized size = 1.76

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)} \log(x^2e^a + 1) + \frac{1}{2}e^{(-a)} \log(x^2e^a - 1) - \frac{x^2}{x^4e^{(2a)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(a+2*log(x))^2,x, algorithm="maxima")`

[Out] $1/2*x^2 - 1/2*e^{(-a)}*\log(x^2*e^a + 1) + 1/2*e^{(-a)}*\log(x^2*e^a - 1) - x^2/(x^4*e^{(2*a)} - 1)$

Fricas [B] time = 2.57195, size = 169, normalized size = 4.12

$$\frac{x^6e^{(3a)} - 3x^2e^a - (x^4e^{(2a)} - 1) \log(x^2e^a + 1) + (x^4e^{(2a)} - 1) \log(x^2e^a - 1)}{2(x^4e^{(3a)} - e^a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(a+2*log(x))^2,x, algorithm="fricas")`

[Out] $\frac{1}{2}(x^6 e^{3a} - 3x^2 e^a - (x^4 e^{2a} - 1) \log(x^2 e^a + 1) + (x^4 e^{2a} - 1) \log(x^2 e^a - 1)) / (x^4 e^{3a} - e^a)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth^2(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(a+2*ln(x))**2,x)

[Out] Integral(x*coth(a + 2*log(x))**2, x)

Giac [A] time = 1.11119, size = 73, normalized size = 1.78

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)} \log(x^2 e^a + 1) + \frac{1}{2}e^{(-a)} \log(|x^2 e^a - 1|) - \frac{x^2}{x^4 e^{(2a)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="giac")

[Out] $\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)} \log(x^2 e^a + 1) + \frac{1}{2}e^{(-a)} \log(\text{abs}(x^2 e^a - 1)) - x^2 / (x^4 e^{(2a)} - 1)$

3.161 $\int \coth^2(a + 2 \log(x)) dx$

Optimal. Leaf size=60

$$\frac{x}{1 - e^{2a}x^4} - \frac{1}{2}e^{-a/2} \tan^{-1}(e^{a/2}x) - \frac{1}{2}e^{-a/2} \tanh^{-1}(e^{a/2}x) + x$$

[Out] $x + x/(1 - E^{(2*a)*x^4}) - \text{ArcTan}[E^{(a/2)*x}]/(2*E^{(a/2)}) - \text{ArcTanh}[E^{(a/2)*x}]/(2*E^{(a/2)})$

Rubi [F] time = 0.0098684, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^2(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[\text{Coth}[a + 2*\text{Log}[x]]^2, x]$

[Out] $\text{Defer}[\text{Int}][\text{Coth}[a + 2*\text{Log}[x]]^2, x]$

Rubi steps

$$\int \coth^2(a + 2 \log(x)) dx = \int \coth^2(a + 2 \log(x)) dx$$

Mathematica [C] time = 1.95104, size = 153, normalized size = 2.55

$$\frac{16}{585}e^{2a}x^5 (e^{2a}x^4 + 1)^2 \text{HypergeometricPFQ}\left(\left\{\frac{5}{4}, 2, 2, 2\right\}, \left\{1, 1, \frac{17}{4}\right\}, e^{2a}x^4\right) + \frac{e^{-4a} \left(5(e^{8a}x^{16} - 248e^{6a}x^{12} + 102e^{4a}x^8)\right)}{\dots}$$

Warning: Unable to verify antiderivative.

[In] $\text{Integrate}[\text{Coth}[a + 2*\text{Log}[x]]^2, x]$

[Out] $(-3645 - 6769E^{(2*a)*x^4} - 1483E^{(4*a)*x^8} + 681E^{(6*a)*x^{12}} + 5*(729 + 1208E^{(2*a)*x^4} + 102E^{(4*a)*x^8} - 248E^{(6*a)*x^{12}} + E^{(8*a)*x^{16}})*\text{Hyper}$

geometric2F1[1/4, 1, 5/4, E^(2*a)*x^4])/(640*E^(4*a)*x^7) + (16*E^(2*a)*x^5*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{5/4, 2, 2, 2}, {1, 1, 17/4}, E^(2*a)*x^4])/585

Maple [A] time = 0.043, size = 86, normalized size = 1.4

$$x - \frac{x}{e^{2a}x^4 - 1} + \frac{1}{4} \ln(x\sqrt{e^a} - 1) \frac{1}{\sqrt{e^a}} - \frac{1}{4} \ln(x\sqrt{e^a} + 1) \frac{1}{\sqrt{e^a}} - \frac{1}{4} \ln(x\sqrt{-e^a} + 1) \frac{1}{\sqrt{-e^a}} + \frac{1}{4} \ln(x\sqrt{-e^a} - 1) \frac{1}{\sqrt{-e^a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+2*ln(x))^2,x)

[Out] x-x/(exp(2*a)*x^4-1)+1/4/exp(a)^(1/2)*ln(x*exp(a)^(1/2)-1)-1/4/exp(a)^(1/2)*ln(x*exp(a)^(1/2)+1)-1/4/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/4/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)-1)

Maxima [A] time = 1.57043, size = 81, normalized size = 1.35

$$-\frac{1}{2} \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} + \frac{1}{4}e^{\left(-\frac{1}{2}a\right)} \log\left(\frac{xe^a - e^{\left(\frac{1}{2}a\right)}}{xe^a + e^{\left(\frac{1}{2}a\right)}}\right) + x - \frac{x}{x^4e^{(2a)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2,x, algorithm="maxima")

[Out] -1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log((x*e^a - e^(1/2*a))/(x*e^a + e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)

Fricas [B] time = 2.64083, size = 246, normalized size = 4.1

$$\frac{4x^5e^{(3a)} - 2(x^4e^{(2a)} - 1)\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + (x^4e^{(2a)} - 1)e^{\left(\frac{1}{2}a\right)} \log\left(\frac{x^2e^a - 2xe^{\left(\frac{1}{2}a\right)} + 1}{x^2e^a - 1}\right) - 8xe^a}{4(x^4e^{(3a)} - e^a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2,x, algorithm="fricas")

[Out] $\frac{1}{4}(4x^5e^{3a} - 2(x^4e^{2a} - 1)\arctan(xe^{1/2a}))e^{1/2a} + (x^4e^{2a} - 1)e^{1/2a}\log((x^2e^a - 2xe^{1/2a} + 1)/(x^2e^a - 1)) - 8xe^a/(x^4e^{3a} - e^a)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^2(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*ln(x))**2,x)

[Out] Integral(coth(a + 2*log(x))**2, x)

Giac [A] time = 1.119, size = 89, normalized size = 1.48

$$-\frac{1}{2} \arctan\left(xe^{\frac{1}{2}a}\right)e^{-\frac{1}{2}a} + \frac{1}{4}e^{-\frac{1}{2}a} \log\left(\frac{\left|2xe^a - 2e^{\frac{1}{2}a}\right|}{\left|2xe^a + 2e^{\frac{1}{2}a}\right|}\right) + x - \frac{x}{x^4e^{2a} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2,x, algorithm="giac")

[Out] $-1/2\arctan(xe^{1/2a})e^{-1/2a} + 1/4e^{-1/2a}\log(\text{abs}(2xe^a - 2e^{1/2a})/\text{abs}(2xe^a + 2e^{1/2a})) + x - x/(x^4e^{2a} - 1)$

$$3.162 \quad \int \frac{\coth^2(a+2\log(x))}{x} dx$$

Optimal. Leaf size=14

$$\log(x) - \frac{1}{2} \coth(a + 2\log(x))$$

[Out] -Coth[a + 2*Log[x]]/2 + Log[x]

Rubi [A] time = 0.0239965, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3473, 8}

$$\log(x) - \frac{1}{2} \coth(a + 2\log(x))$$

Antiderivative was successfully verified.

[In] Int[Coth[a + 2*Log[x]]^2/x,x]

[Out] -Coth[a + 2*Log[x]]/2 + Log[x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(a + 2\log(x))}{x} dx &= \text{Subst} \left(\int \coth^2(a + 2x) dx, x, \log(x) \right) \\ &= -\frac{1}{2} \coth(a + 2\log(x)) + \text{Subst} \left(\int 1 dx, x, \log(x) \right) \\ &= -\frac{1}{2} \coth(a + 2\log(x)) + \log(x) \end{aligned}$$

Mathematica [C] time = 0.0539936, size = 28, normalized size = 2.

$$-\frac{1}{2} \coth(a + 2 \log(x)) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(a + 2 \log(x))\right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + 2*Log[x]]^2/x,x]

[Out] -(Coth[a + 2*Log[x]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + 2*Log[x]]^2])/2

Maple [B] time = 0.003, size = 35, normalized size = 2.5

$$-\frac{\coth(a + 2 \ln(x))}{2} - \frac{\ln(\coth(a + 2 \ln(x)) - 1)}{4} + \frac{\ln(\coth(a + 2 \ln(x)) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+2*ln(x))^2/x,x)

[Out] -1/2*coth(a+2*ln(x))-1/4*ln(coth(a+2*ln(x))-1)+1/4*ln(coth(a+2*ln(x))+1)

Maxima [A] time = 1.03175, size = 26, normalized size = 1.86

$$\frac{1}{2}a + \frac{1}{e^{(-2a-4 \log(x))} - 1} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="maxima")

[Out] 1/2*a + 1/(e^(-2*a - 4*log(x)) - 1) + log(x)

Fricas [B] time = 2.52145, size = 68, normalized size = 4.86

$$\frac{(x^4 e^{(2a)} - 1) \log(x) - 1}{x^4 e^{(2a)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="fricas")

[Out] ((x^4*e^(2*a) - 1)*log(x) - 1)/(x^4*e^(2*a) - 1)

Sympy [A] time = 17.2266, size = 32, normalized size = 2.29

$$\begin{cases} \infty \log(x) & \text{for } a = \log\left(-\frac{1}{x^2}\right) \vee a = \log\left(\frac{1}{x^2}\right) \\ \log(x) - \frac{1}{2 \tanh(a+2 \log(x))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*ln(x))**2/x,x)

[Out] Piecewise((zoo*log(x), Eq(a, log(x**(-2))) | Eq(a, log(-1/x**2))), (log(x) - 1/(2*tanh(a + 2*log(x))), True))

Giac [A] time = 1.11646, size = 28, normalized size = 2.

$$-\frac{1}{x^4 e^{(2a)} - 1} + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="giac")

[Out] -1/(x^4*e^(2*a) - 1) + 1/4*log(x^4)

$$3.163 \quad \int \frac{\coth^2(a+2 \log(x))}{x^2} dx$$

Optimal. Leaf size=86

$$\frac{2e^{2a}x^3}{1 - e^{2a}x^4} - \frac{1}{x(1 - e^{2a}x^4)} - \frac{1}{2}e^{a/2} \tan^{-1}(e^{a/2}x) + \frac{1}{2}e^{a/2} \tanh^{-1}(e^{a/2}x)$$

[Out] $-(1/(x*(1 - E^(2*a)*x^4))) + (2*E^(2*a)*x^3)/(1 - E^(2*a)*x^4) - (E^(a/2)*ArcTan[E^(a/2)*x])/2 + (E^(a/2)*ArcTanh[E^(a/2)*x])/2$

Rubi [F] time = 0.0448182, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth^2(a + 2 \log(x))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + 2*Log[x]]^2/x^2, x]

[Out] Defer[Int][Coth[a + 2*Log[x]]^2/x^2, x]

Rubi steps

$$\int \frac{\coth^2(a + 2 \log(x))}{x^2} dx = \int \frac{\coth^2(a + 2 \log(x))}{x^2} dx$$

Mathematica [C] time = 3.12673, size = 153, normalized size = 1.78

$$\frac{16}{231}e^{2a}x^3(e^{2a}x^4 + 1)^2 \text{HypergeometricPFQ}\left(\left\{\frac{3}{4}, 2, 2, 2\right\}, \left\{1, 1, \frac{15}{4}\right\}, e^{2a}x^4\right) + \frac{e^{-2a}\left((-e^{8a}x^{16} - 56e^{6a}x^{12} + 362e^{4a}x^8 + \dots)\right)}{\dots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + 2*Log[x]]^2/x^2, x]

[Out] $(-343 - 1163E^{(2a)}x^4 - 241E^{(4a)}x^8 + 3E^{(6a)}x^{12} + (343 + 632E^{(2a)}x^4 + 362E^{(4a)}x^8 - 56E^{(6a)}x^{12} - E^{(8a)}x^{16})\text{Hypergeometric2F1}[3/4, 1, 7/4, E^{(2a)}x^4]/(384E^{(2a)}x^5) + (16E^{(2a)}x^3(1 + E^{(2a)}x^4)^2\text{HypergeometricPFQ}[\{3/4, 2, 2, 2\}, \{1, 1, 15/4\}, E^{(2a)}x^4])/231$

Maple [C] time = 0.033, size = 101, normalized size = 1.2

$$\frac{-2e^{2a}x^4 + 1}{x(e^{2a}x^4 - 1)} + \frac{\sum_{R=\text{RootOf}(_Z^2+e^a)} -R \ln((-5_R^4 + 4e^{2a})x - _R^3)}{4} + \frac{\sum_{R=\text{RootOf}(_Z^2-e^a)} -R \ln((-5_R^4 + 4e^{2a})x - _R^3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(a+2*ln(x))^2/x^2,x)`

[Out] $(-2\exp(2a)x^4+1)/x/(\exp(2a)x^4-1)+1/4\sum(_R\ln((-5_R^4+4\exp(2a))x-_R^3), _R=\text{RootOf}(_Z^2+\exp(a)))+1/4\sum(_R\ln((-5_R^4+4\exp(2a))x-_R^3), _R=\text{RootOf}(_Z^2-\exp(a)))$

Maxima [A] time = 1.56413, size = 93, normalized size = 1.08

$$\frac{1}{2} \arctan\left(\frac{e^{(-\frac{1}{2}a)}}{x}\right) e^{(\frac{1}{2}a)} - \frac{1}{4} e^{(\frac{1}{2}a)} \log\left(\frac{\frac{1}{x} - e^{(\frac{1}{2}a)}}{\frac{1}{x} + e^{(\frac{1}{2}a)}}\right) - \frac{1}{x} + \frac{e^{(2a)}}{x\left(\frac{1}{x^4} - e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x))^2/x^2,x, algorithm="maxima")`

[Out] $1/2*\arctan(e^{(-1/2*a)}/x)*e^{(1/2*a)} - 1/4*e^{(1/2*a)}*\log((1/x - e^{(1/2*a)})/(1/x + e^{(1/2*a)})) - 1/x + e^{(2a)}/(x*(1/x^4 - e^{(2a)}))$

Fricas [A] time = 2.60401, size = 236, normalized size = 2.74

$$\frac{8x^4e^{(2a)} + 2(x^5e^{(2a)} - x) \arctan\left(xe^{(\frac{1}{2}a)}\right) e^{(\frac{1}{2}a)} - (x^5e^{(2a)} - x) e^{(\frac{1}{2}a)} \log\left(\frac{x^2e^a + 2xe^{(\frac{1}{2}a)} + 1}{x^2e^a - 1}\right) - 4}{4(x^5e^{(2a)} - x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="fricas")

[Out] $-1/4*(8*x^4*e^{(2*a)} + 2*(x^5*e^{(2*a)} - x)*\arctan(x*e^{(1/2*a)})*e^{(1/2*a)} - (x^5*e^{(2*a)} - x)*e^{(1/2*a)}*\log((x^2*e^a + 2*x*e^{(1/2*a)} + 1)/(x^2*e^a - 1)) - 4)/(x^5*e^{(2*a)} - x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(a + 2 \log(x))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*ln(x))**2/x**2,x)

[Out] Integral(coth(a + 2*log(x))**2/x**2, x)

Giac [A] time = 1.12148, size = 104, normalized size = 1.21

$$-\frac{1}{2} \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} - \frac{1}{4}e^{\left(\frac{1}{2}a\right)} \log\left(\frac{\left|2xe^a - 2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^a + 2e^{\left(\frac{1}{2}a\right)}\right|}\right) - \frac{2x^4e^{(2a)} - 1}{x^5e^{(2a)} - x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="giac")

[Out] $-1/2*\arctan(x*e^{(1/2*a)})*e^{(1/2*a)} - 1/4*e^{(1/2*a)}*\log(\text{abs}(2*x*e^a - 2*e^{(1/2*a)})/\text{abs}(2*x*e^a + 2*e^{(1/2*a)})) - (2*x^4*e^{(2*a)} - 1)/(x^5*e^{(2*a)} - x)$

$$3.164 \quad \int \frac{\coth^2(a+2 \log(x))}{x^3} dx$$

Optimal. Leaf size=60

$$\frac{3e^{2a}x^2}{2(1-e^{2a}x^4)} - \frac{1}{2x^2(1-e^{2a}x^4)} + e^a \tanh^{-1}(e^a x^2)$$

[Out] $-1/(2*x^2*(1 - E^(2*a)*x^4)) + (3*E^(2*a)*x^2)/(2*(1 - E^(2*a)*x^4)) + E^a*$
ArcTanh[E^a*x^2]

Rubi [F] time = 0.0460271, antiderivative size = 0, normalized size of antiderivative = 0.,
number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$,
Rules used = {}

$$\int \frac{\coth^2(a + 2 \log(x))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + 2*Log[x]]^2/x^3, x]

[Out] Defer[Int][Coth[a + 2*Log[x]]^2/x^3, x]

Rubi steps

$$\int \frac{\coth^2(a + 2 \log(x))}{x^3} dx = \int \frac{\coth^2(a + 2 \log(x))}{x^3} dx$$

Mathematica [C] time = 2.86299, size = 155, normalized size = 2.58

$$64(e^{3a}x^6 + e^a x^2)^2 \text{HypergeometricPFQ}\left(\left\{\frac{1}{2}, 2, 2, 2\right\}, \left\{1, 1, \frac{7}{2}\right\}, e^{2a}x^4\right) + 15\left(e^{4a}x^8 - 17e^{2a}x^4 - \frac{27e^{-2a}}{x^4} - 77\right) - \frac{15(e^{8a}x^{16} + 4e^a x^4 + 4e^{-2a})}{480x^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + 2*Log[x]]^2/x^3, x]

[Out] $(15*(-77 - 27/(E^{(2*a)}*x^4) - 17*E^{(2*a)}*x^4 + E^{(4*a)}*x^8) - (15*(-27 - 52*E^{(2*a)}*x^4 - 54*E^{(4*a)}*x^8 + 4*E^{(6*a)}*x^{12} + E^{(8*a)}*x^{16})*\text{ArcTanh}[\text{Sqrt}[E^{(2*a)}*x^4]])/(E^{(2*a)}*x^4)^{(3/2)} + 64*(E^a*x^2 + E^{(3*a)}*x^6)^2*\text{HypergeometricPFQ}[\{1/2, 2, 2, 2\}, \{1, 1, 7/2\}, E^{(2*a)}*x^4])/(480*x^2)$

Maple [A] time = 0.023, size = 55, normalized size = 0.9

$$\frac{1}{x^2(e^{2a}x^4 - 1)} \left(-\frac{3e^{2a}x^4}{2} + \frac{1}{2} \right) + \frac{e^a \ln(e^a x^2 + 1)}{2} - \frac{e^a \ln(e^a x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(a+2*ln(x))^2/x^3,x)`

[Out] $(-3/2*\exp(2*a)*x^4+1/2)/x^2/(\exp(2*a)*x^4-1)+1/2*\exp(a)*\ln(\exp(a)*x^2+1)-1/2*\exp(a)*\ln(\exp(a)*x^2-1)$

Maxima [A] time = 1.07843, size = 68, normalized size = 1.13

$$\frac{1}{2} e^a \log\left(\frac{1}{x^2} + e^a\right) - \frac{1}{2} e^a \log\left(\frac{1}{x^2} - e^a\right) - \frac{1}{2x^2} + \frac{e^{(2a)}}{x^2\left(\frac{1}{x^4} - e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+2*log(x))^2/x^3,x, algorithm="maxima")`

[Out] $1/2*e^a*\log(1/x^2 + e^a) - 1/2*e^a*\log(1/x^2 - e^a) - 1/2/x^2 + e^{(2*a)}/(x^2*(1/x^4 - e^{(2*a)}))$

Fricas [A] time = 2.58052, size = 178, normalized size = 2.97

$$\frac{3x^4e^{(2a)} - (x^6e^{(3a)} - x^2e^a)\log(x^2e^a + 1) + (x^6e^{(3a)} - x^2e^a)\log(x^2e^a - 1) - 1}{2(x^6e^{(2a)} - x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="fricas")

[Out] $-\frac{1}{2}*(3*x^4*e^{(2*a)} - (x^6*e^{(3*a)} - x^2*e^a)*\log(x^2*e^a + 1) + (x^6*e^{(3*a)} - x^2*e^a)*\log(x^2*e^a - 1) - 1)/(x^6*e^{(2*a)} - x^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(a + 2 \log(x))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*ln(x))**2/x**3,x)

[Out] Integral(coth(a + 2*log(x))**2/x**3, x)

Giac [A] time = 1.15239, size = 77, normalized size = 1.28

$$\frac{1}{2} e^a \log(x^2 e^a + 1) - \frac{1}{2} e^a \log(|x^2 e^a - 1|) - \frac{3x^4 e^{(2a)} - 1}{2(x^6 e^{(2a)} - x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="giac")

[Out] $\frac{1}{2}*e^a*\log(x^2*e^a + 1) - \frac{1}{2}*e^a*\log(\text{abs}(x^2*e^a - 1)) - \frac{1}{2}*(3*x^4*e^{(2*a)} - 1)/(x^6*e^{(2*a)} - x^2)$

3.165 $\int (ex)^m \coth(a + 2 \log(x)) dx$

Optimal. Leaf size=59

$$\frac{(ex)^{m+1}}{e(m+1)} - \frac{2(ex)^{m+1} {}_2F_1\left(1, \frac{m+1}{4}; \frac{m+5}{4}; e^{2a}x^4\right)}{e(m+1)}$$

[Out] $(e*x)^{(1+m)}/(e*(1+m)) - (2*(e*x)^{(1+m)}*Hypergeometric2F1[1, (1+m)/4, (5+m)/4, E^{(2*a)*x^4}]/(e*(1+m))$

Rubi [F] time = 0.0387854, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \coth(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e*x)^m*Coth[a + 2*Log[x]],x]

[Out] Defer[Int] [(e*x)^m*Coth[a + 2*Log[x]], x]

Rubi steps

$$\int (ex)^m \coth(a + 2 \log(x)) dx = \int (ex)^m \coth(a + 2 \log(x)) dx$$

Mathematica [A] time = 0.0933579, size = 46, normalized size = 0.78

$$\frac{x(ex)^m \left(2 {}_2F_1\left(1, \frac{m+1}{4}; \frac{m+5}{4}; x^4(\cosh(2a) + \sinh(2a))\right) - 1 \right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]],x]

[Out] $-\left(\frac{x \cdot (e^x)^m \cdot (-1 + 2 \cdot \text{Hypergeometric2F1}[1, (1 + m)/4, (5 + m)/4, x^4 \cdot (\text{Cosh}[2 \cdot a] + \text{Sinh}[2 \cdot a])])}{(1 + m)}\right)$

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \ln(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*coth(a+2*ln(x)),x)`

[Out] `int((e*x)^m*coth(a+2*ln(x)),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="maxima")`

[Out] `integrate((e*x)^m*coth(a + 2*log(x)), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \coth(a + 2 \log(x)), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="fricas")`

[Out] `integral((e*x)^m*coth(a + 2*log(x)), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*coth(a+2*ln(x)),x)

[Out] Integral((e*x)**m*coth(a + 2*log(x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="giac")

[Out] integrate((e*x)^m*coth(a + 2*log(x)), x)

3.166 $\int (ex)^m \coth^2(a + 2 \log(x)) dx$

Optimal. Leaf size=79

$$-\frac{(ex)^{m+1} {}_2F_1\left(1, \frac{m+1}{4}; \frac{m+5}{4}; e^{2a}x^4\right)}{e} + \frac{(ex)^{m+1}}{e(1 - e^{2a}x^4)} + \frac{(ex)^{m+1}}{e(m+1)}$$

[Out] $(e*x)^{(1+m)}/(e*(1+m)) + (e*x)^{(1+m)}/(e*(1 - E^{(2*a)*x^4})) - ((e*x)^{(1+m)}*Hypergeometric2F1[1, (1+m)/4, (5+m)/4, E^{(2*a)*x^4}])/e$

Rubi [F] time = 0.0679443, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \coth^2(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^2,x]

[Out] Defer[Int][(e*x)^m*Coth[a + 2*Log[x]]^2, x]

Rubi steps

$$\int (ex)^m \coth^2(a + 2 \log(x)) dx = \int (ex)^m \coth^2(a + 2 \log(x)) dx$$

Mathematica [B] time = 0.369473, size = 165, normalized size = 2.09

$$x(ex)^m \frac{\left(x^4(\sinh(a)+\cosh(a))\left((m+5)x^4(\sinh(a)+\cosh(a)) {}_2F_1\left(2, \frac{m+9}{4}; \frac{m+13}{4}; x^4(\cosh(2a)+\sinh(2a))\right) + 2(m+9)(\cosh(a)-\sinh(a)) {}_2F_1\left(2, \frac{m+5}{4}; \frac{m+9}{4}; x^4(\cosh(2a)+\sinh(2a))\right) \right)}{(m+5)(m+9)} \right)}{(\cosh(a) - \sinh(a))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^2,x]


```
[Out] (x*(e*x)^m*((x^4*(Cosh[a] + Sinh[a]))*(2*(9 + m)*Hypergeometric2F1[2, (5 + m)
)/4, (9 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a]))*(Cosh[a] - Sinh[a]) + (5 + m)*
x^4*Hypergeometric2F1[2, (9 + m)/4, (13 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])
]*(Cosh[a] + Sinh[a])))/((5 + m)*(9 + m)) + (Hypergeometric2F1[2, (1 + m)/4
, (5 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a]))*(Cosh[2*a] - Sinh[2*a]))/(1 + m)
)/(Cosh[a] - Sinh[a])^2
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int (ex)^m (\coth(a + 2 \ln(x)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x))^2,x)
```

```
[Out] int((e*x)^m*coth(a+2*ln(x))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \log(x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \coth(a + 2 \log(x))^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(a + 2*log(x))^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*coth(a+2*ln(x))**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \log(x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="giac")

[Out] integrate((e*x)^m*coth(a + 2*log(x))^2, x)

3.167 $\int (ex)^m \coth^3(a + 2 \log(x)) dx$

Optimal. Leaf size=177

$$\frac{(m^2 + 2m + 9)(ex)^{m+1} {}_2F_1\left(1, \frac{m+1}{4}; \frac{m+5}{4}; e^{2a}x^4\right)}{4e(m+1)} - \frac{(e^{2a}x^4 + 1)^2 (ex)^{m+1}}{4e(1 - e^{2a}x^4)^2} - \frac{e^{-2a}(e^{2a}(3-m) - e^{4a}(m+5)x^4)(ex)^{m+1}}{8e(1 - e^{2a}x^4)} + \dots$$

[Out] $((3 + m) * (5 + m) * (e * x)^{(1 + m)}) / (8 * e * (1 + m)) - ((e * x)^{(1 + m)} * (1 + E^{(2 * a)} * x^4)^2) / (4 * e * (1 - E^{(2 * a)} * x^4)^2) - ((e * x)^{(1 + m)} * (E^{(2 * a)} * (3 - m) - E^{(4 * a)} * (5 + m) * x^4)) / (8 * e * E^{(2 * a)} * (1 - E^{(2 * a)} * x^4)) - ((9 + 2 * m + m^2) * (e * x)^{(1 + m)} * \text{Hypergeometric2F1}[1, (1 + m) / 4, (5 + m) / 4, E^{(2 * a)} * x^4]) / (4 * e * (1 + m))$

Rubi [F] time = 0.0728497, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \coth^3(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^3,x]

[Out] Defer[Int][(e*x)^m*Coth[a + 2*Log[x]]^3, x]

Rubi steps

$$\int (ex)^m \coth^3(a + 2 \log(x)) dx = \int (ex)^m \coth^3(a + 2 \log(x)) dx$$

Mathematica [A] time = 0.825247, size = 215, normalized size = 1.21

$$x(ex)^m \left(-\frac{x^8(\sinh(2a)+\cosh(2a))((m+9)x^4(\sinh(a)+\cosh(a)){}_2F_1\left(3, \frac{m+13}{4}; \frac{m+17}{4}; x^4(\cosh(2a)+\sinh(2a))\right)+3(m+13)(\cosh(a)-\sinh(a)){}_2F_1\left(3, \frac{m+9}{4}; \frac{m+13}{4}; x^4\right)}{(m+9)(m+13)} \right)$$

(cosh(a) -

Antiderivative was successfully verified.

[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^3,x]

[Out] (x*(e*x)^m*((-3*x^4*Hypergeometric2F1[3, (5 + m)/4, (9 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])]*(Cosh[a] - Sinh[a]))/(5 + m) - (Hypergeometric2F1[3, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])]*(Cosh[a] - Sinh[a])^3)/(1 + m) - (x^8*(3*(13 + m)*Hypergeometric2F1[3, (9 + m)/4, (13 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])]*(Cosh[a] - Sinh[a]) + (9 + m)*x^4*Hypergeometric2F1[3, (13 + m)/4, (17 + m)/4, x^4*(Cosh[2*a] + Sinh[2*a])]*(Cosh[a] + Sinh[a]))*(Cosh[2*a] + Sinh[2*a]))/((9 + m)*(13 + m)))/(Cosh[a] - Sinh[a])^3

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (ex)^m (\coth(a + 2 \ln(x)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m*coth(a+2*ln(x))^3,x)

[Out] int((e*x)^m*coth(a+2*ln(x))^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \log(x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="maxima")

[Out] integrate((e*x)^m*coth(a + 2*log(x))^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \coth(a + 2 \log(x))^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(a + 2*log(x))^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(a + 2 \log(x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

3.168 $\int \coth^p(a + b \log(x)) dx$

Optimal. Leaf size=79

$$x(-e^{2a}x^{2b} - 1)^p (e^{2a}x^{2b} + 1)^{-p} F_1\left(\frac{1}{2b}; p, -p; \frac{1}{2}\left(2 + \frac{1}{b}\right); e^{2a}x^{2b}, -e^{2a}x^{2b}\right)$$

[Out] $(x*(-1 - E^{(2*a)*x^{(2*b)}})^p * AppellF1[1/(2*b), p, -p, (2 + b^{-1})/2, E^{(2*a)*x^{(2*b)}}, -(E^{(2*a)*x^{(2*b)}})]) / (1 + E^{(2*a)*x^{(2*b)}})^p$

Rubi [F] time = 0.0221232, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^p(a + b \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + b*Log[x]]^p, x]

[Out] Defer[Int][Coth[a + b*Log[x]]^p, x]

Rubi steps

$$\int \coth^p(a + b \log(x)) dx = \int \coth^p(a + b \log(x)) dx$$

Mathematica [B] time = 1.88852, size = 259, normalized size = 3.28

$$\frac{(2b + 1)x \left(\frac{e^{2a}x^{2b} + 1}{e^{2a}x^{2b} - 1}\right)^p F_1\left(\frac{1}{2b}; p, -p; 1 + \frac{1}{2b}; e^{2a}x^{2b}, -e^{2a}x^{2b}\right)}{2e^{2a}bp x^{2b} F_1\left(1 + \frac{1}{2b}; p, 1 - p; 2 + \frac{1}{2b}; e^{2a}x^{2b}, -e^{2a}x^{2b}\right) + 2e^{2a}bp x^{2b} F_1\left(1 + \frac{1}{2b}; p + 1, -p; 2 + \frac{1}{2b}; e^{2a}x^{2b}, -e^{2a}x^{2b}\right) + (2b + 1)F_1\left(1 + \frac{1}{2b}; p, -p; 2 + \frac{1}{2b}; e^{2a}x^{2b}, -e^{2a}x^{2b}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + b*Log[x]]^p, x]

```
[Out] ((1 + 2*b)*x*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(2*b)))^p*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))]/(2*b*E^(2*a)
*p*x^(2*b)*AppellF1[1 + 1/(2*b), p, 1 - p, 2 + 1/(2*b), E^(2*a)*x^(2*b), -(
E^(2*a)*x^(2*b))] + 2*b*E^(2*a)*p*x^(2*b)*AppellF1[1 + 1/(2*b), 1 + p, -p,
2 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))] + (1 + 2*b)*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int (\coth(a + b \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(x))^p,x)
```

```
[Out] int(coth(a+b*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(b*log(x) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\coth(b \log(x) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(b*log(x) + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p(a + b \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*ln(x))**p,x)

[Out] Integral(coth(a + b*log(x))**p, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(x))^p,x, algorithm="giac")

[Out] integrate(coth(b*log(x) + a)^p, x)

3.169 $\int (ex)^m \coth^p(a + b \log(x)) dx$

Optimal. Leaf size=99

$$\frac{(ex)^{m+1} (-e^{2a}x^{2b} - 1)^p (e^{2a}x^{2b} + 1)^{-p} F_1\left(\frac{m+1}{2b}; p, -p; \frac{m+1}{2b} + 1; e^{2a}x^{2b}, -e^{2a}x^{2b}\right)}{e(m+1)}$$

[Out] $((e*x)^{(1+m)*(-1 - E^{(2*a)*x^{(2*b)}})^p * AppellF1[(1+m)/(2*b), p, -p, 1 + (1+m)/(2*b), E^{(2*a)*x^{(2*b)}}, -(E^{(2*a)*x^{(2*b)}})] / (e*(1+m)*(1 + E^{(2*a)*x^{(2*b)}})^p)$

Rubi [F] time = 0.115831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \coth^p(a + b \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e*x)^m*Coth[a + b*Log[x]]^p, x]

[Out] Defer[Int][(e*x)^m*Coth[a + b*Log[x]]^p, x]

Rubi steps

$$\int (ex)^m \coth^p(a + b \log(x)) dx = \int (ex)^m \coth^p(a + b \log(x)) dx$$

Mathematica [A] time = 2.97686, size = 126, normalized size = 1.27

$$\frac{x(ex)^m (1 - e^{2a}x^{2b})^p (e^{2a}x^{2b} + 1)^{-p} \left(\frac{e^{2a}x^{2b} + 1}{e^{2a}x^{2b} - 1}\right)^p F_1\left(\frac{m+1}{2b}; p, -p; \frac{m+1}{2b} + 1; e^{2a}x^{2b}, -e^{2a}x^{2b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e*x)^m*Coth[a + b*Log[x]]^p, x]

[Out] $(x*(e*x)^m*(1 - E^{(2*a)*x^{(2*b)}})^p*((1 + E^{(2*a)*x^{(2*b)}})/(-1 + E^{(2*a)*x^{(2*b)}}))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 + (1 + m)/(2*b), E^{(2*a)*x^{(2*b)}}, -(E^{(2*a)*x^{(2*b)}})]/((1 + m)*(1 + E^{(2*a)*x^{(2*b)}})^p)$

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (ex)^m (\coth(a + b \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*coth(a+b*ln(x))^p,x)`

[Out] `int((e*x)^m*coth(a+b*ln(x))^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="maxima")`

[Out] `integrate((e*x)^m*coth(b*log(x) + a)^p, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \coth(b \log(x) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="fricas")`

[Out] `integral((e*x)^m*coth(b*log(x) + a)^p, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*coth(a+b*ln(x))**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="giac")

[Out] integrate((e*x)^m*coth(b*log(x) + a)^p, x)

$$3.170 \quad \int \coth^p \left(a + \frac{\log(x)}{2} \right) dx$$

Optimal. Leaf size=52

$$-\frac{e^{-2a} 2^{-p} (-e^{2a} x - 1)^{p+1} {}_2F_1 \left(p, p+1; p+2; \frac{1}{2} (e^{2a} x + 1) \right)}{p+1}$$

[Out] -(((-1 - E^(2*a)*x)^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 + E^(2*a)*x)/2]))/(2^p*E^(2*a)*(1 + p))

Rubi [F] time = 0.046479, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^p \left(a + \frac{\log(x)}{2} \right) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + Log[x]/2]^p, x]

[Out] Defer[Int][Coth[(2*a + Log[x])/2]^p, x]

Rubi steps

$$\int \coth^p \left(a + \frac{\log(x)}{2} \right) dx = \int \coth^p \left(\frac{1}{2} (2a + \log(x)) \right) dx$$

Mathematica [A] time = 0.391243, size = 83, normalized size = 1.6

$$-\frac{e^{-2a} 2^p (e^{2a} x + 1)^{1-p} \left(\frac{e^{2a} x + 1}{e^{2a} x - 1} \right)^{p-1} {}_2F_1 \left(1-p, -p; 2-p; \frac{1}{2} - \frac{1}{2} e^{2a} x \right)}{p-1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + Log[x]/2]^p, x]

[Out] $-\left(2^p(1 + E^{(2a)x})^{(1-p)}\left(\frac{1 + E^{(2a)x}}{-1 + E^{(2a)x}}\right)^{(-1+p)}\right) \cdot \text{Hypergeometric2F1}\left[1-p, -p, 2-p, \frac{1}{2} - \frac{E^{(2a)x}}{2}\right] / \left(E^{(2a)x}(-1+p)\right)$

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \left(\coth\left(a + \frac{\ln(x)}{2}\right) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(a+1/2*ln(x))^p,x)`

[Out] `int(coth(a+1/2*ln(x))^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a + \frac{1}{2} \log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+1/2*log(x))^p,x, algorithm="maxima")`

[Out] `integrate(coth(a + 1/2*log(x))^p, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth\left(a + \frac{1}{2} \log(x)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+1/2*log(x))^p,x, algorithm="fricas")`

[Out] `integral(coth(a + 1/2*log(x))^p, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p \left(a + \frac{\log(x)}{2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+1/2*ln(x))**p,x)

[Out] Integral(coth(a + log(x)/2)**p, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a + \frac{1}{2} \log(x) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+1/2*log(x))~p,x, algorithm="giac")

[Out] integrate(coth(a + 1/2*log(x))~p, x)

$$3.171 \quad \int \coth^p \left(a + \frac{\log(x)}{4} \right) dx$$

Optimal. Leaf size=108

$$e^{-4a} (-e^{2a}\sqrt{x}-1)^{p+1} (1-e^{2a}\sqrt{x})^{1-p} - \frac{e^{-4a} 2^{1-p} p (-e^{2a}\sqrt{x}-1)^{p+1} {}_2F_1\left(p, p+1; p+2; \frac{1}{2}(e^{2a}\sqrt{x}+1)\right)}{p+1}$$

[Out] $((-1 - E^{(2*a)*Sqrt[x]})^{(1 + p)} * (1 - E^{(2*a)*Sqrt[x]})^{(1 - p)}) / E^{(4*a)} - (2^{(1 - p)} * p * (-1 - E^{(2*a)*Sqrt[x]})^{(1 + p)} * \text{Hypergeometric2F1}[p, 1 + p, 2 + p, (1 + E^{(2*a)*Sqrt[x]})/2]) / (E^{(4*a)} * (1 + p))$

Rubi [F] time = 0.0508788, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^p \left(a + \frac{\log(x)}{4} \right) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + Log[x]/4]^p, x]

[Out] Defer[Int][Coth[(4*a + Log[x])/4]^p, x]

Rubi steps

$$\int \coth^p \left(a + \frac{\log(x)}{4} \right) dx = \int \coth^p \left(\frac{1}{4}(4a + \log(x)) \right) dx$$

Mathematica [A] time = 0.526514, size = 125, normalized size = 1.16

$$\frac{e^{-4a} (e^{2a}\sqrt{x}+1)^{1-p} \left(\frac{e^{2a}\sqrt{x}+1}{e^{2a}\sqrt{x}-1} \right)^{p-1} \left((p-1) (e^{2a}\sqrt{x}+1)^{p+1} - 2^{p+1} p {}_2F_1\left(1-p, -p; 2-p; \frac{1}{2} - \frac{1}{2}e^{2a}\sqrt{x}\right) \right)}{p-1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + Log[x]/4]^p,x]

[Out] $((1 + E^{(2*a)*\text{Sqrt}[x]})^{(1 - p)} * ((1 + E^{(2*a)*\text{Sqrt}[x]}) / (-1 + E^{(2*a)*\text{Sqrt}[x]}))^{(-1 + p)} * ((-1 + p) * (1 + E^{(2*a)*\text{Sqrt}[x]})^{(1 + p)} - 2^{(1 + p)} * p * \text{Hypergeometric2F1}[1 - p, -p, 2 - p, 1/2 - (E^{(2*a)*\text{Sqrt}[x]})/2])) / (E^{(4*a)*(-1 + p)})$

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \left(\coth \left(a + \frac{\ln(x)}{4} \right) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+1/4*ln(x))^p,x)

[Out] int(coth(a+1/4*ln(x))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a + \frac{1}{4} \log(x) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="maxima")

[Out] integrate(coth(a + 1/4*log(x))^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\coth \left(a + \frac{1}{4} \log(x) \right)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="fricas")

[Out] `integral(coth(a + 1/4*log(x))p, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p \left(a + \frac{\log(x)}{4} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+1/4*ln(x))p,x)`

[Out] `Integral(coth(a + log(x)/4)p, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a + \frac{1}{4} \log(x) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+1/4*log(x))p,x, algorithm="giac")`

[Out] `integrate(coth(a + 1/4*log(x))p, x)`

$$3.172 \quad \int \coth^p \left(a + \frac{\log(x)}{6} \right) dx$$

Optimal. Leaf size=162

$$\frac{e^{-6a} 2^{-p} (2p^2 + 1) (-e^{2a} \sqrt[3]{x} - 1)^{p+1} {}_2F_1 \left(p, p+1; p+2; \frac{1}{2} (e^{2a} \sqrt[3]{x} + 1) \right)}{p+1} + e^{-6a} p (-e^{2a} \sqrt[3]{x} - 1)^{p+1} (1 - e^{2a} \sqrt[3]{x})^{1-p} + e^{-4a} \sqrt[3]{x}$$

[Out] (p*(-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1 - p))/E^(6*a) + ((-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1 - p)*x^(1/3))/E^(4*a) - ((1 + 2*p^2)*(-1 - E^(2*a)*x^(1/3))^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 + E^(2*a)*x^(1/3))/2])/(2^p*E^(6*a)*(1 + p))

Rubi [F] time = 0.0499177, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^p \left(a + \frac{\log(x)}{6} \right) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + Log[x]/6]^p, x]

[Out] Defer[Int][Coth[(6*a + Log[x])/6]^p, x]

Rubi steps

$$\int \coth^p \left(a + \frac{\log(x)}{6} \right) dx = \int \coth^p \left(\frac{1}{6} (6a + \log(x)) \right) dx$$

Mathematica [A] time = 0.561147, size = 142, normalized size = 0.88

$$\frac{e^{-6a} (e^{2a} \sqrt[3]{x} + 1)^{1-p} \left(\frac{e^{2a} \sqrt[3]{x} + 1}{e^{2a} \sqrt[3]{x} - 1} \right)^{p-1} \left((p-1) (e^{2a} \sqrt[3]{x} + 1)^{p+1} (e^{2a} \sqrt[3]{x} + p) - 2^p (2p^2 + 1) {}_2F_1 \left(1-p, -p; 2-p; \frac{1}{2} - \frac{1}{2} e^{2a} \sqrt[3]{x} \right) \right)}{p-1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + Log[x]/6]^p,x]

[Out] $((1 + E^{(2*a)*x^{(1/3)}})^{(1 - p)} * ((1 + E^{(2*a)*x^{(1/3)}})/(-1 + E^{(2*a)*x^{(1/3)}}))^{(-1 + p)} * ((-1 + p) * (1 + E^{(2*a)*x^{(1/3)}})^{(1 + p)} * (p + E^{(2*a)*x^{(1/3)}}) - 2^p * (1 + 2*p^2) * \text{Hypergeometric2F1}[1 - p, -p, 2 - p, 1/2 - (E^{(2*a)*x^{(1/3)}})/2])) / (E^{(6*a)} * (-1 + p))$

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \left(\coth \left(a + \frac{\ln(x)}{6} \right) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+1/6*ln(x))^p,x)

[Out] int(coth(a+1/6*ln(x))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a + \frac{1}{6} \log(x) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="maxima")

[Out] integrate(coth(a + 1/6*log(x))^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\coth \left(a + \frac{1}{6} \log(x) \right)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="fricas")

[Out] `integral(coth(a + 1/6*log(x))^p, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p \left(a + \frac{\log(x)}{6} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+1/6*ln(x))**p,x)`

[Out] `Integral(coth(a + log(x)/6)**p, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a + \frac{1}{6} \log(x) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(a+1/6*log(x))^p,x, algorithm="giac")`

[Out] `integrate(coth(a + 1/6*log(x))^p, x)`

$$3.173 \quad \int \coth^p \left(a + \frac{\log(x)}{8} \right) dx$$

Optimal. Leaf size=194

$$\frac{e^{-8a} 2^{2-p} p (p^2 + 2) (-e^{2a} \sqrt[4]{x} - 1)^{p+1} {}_2F_1 \left(p, p+1; p+2; \frac{1}{2} (e^{2a} \sqrt[4]{x} + 1) \right)}{3(p+1)} + \frac{1}{3} e^{-12a} (-e^{2a} \sqrt[4]{x} - 1)^{p+1} (e^{4a} (2p^2 + 3) + 2e^{6a})$$

[Out] $((-1 - E^{(2*a)*x^{(1/4)}})^{(1+p)}*(1 - E^{(2*a)*x^{(1/4)}})^{(1-p)}*(E^{(4*a)}*(3 + 2*p^2) + 2*E^{(6*a)*p*x^{(1/4)}})/(3*E^{(12*a)}) + ((-1 - E^{(2*a)*x^{(1/4)}})^{(1+p)}*(1 - E^{(2*a)*x^{(1/4)}})^{(1-p)}*Sqrt[x])/E^{(4*a)} - (2^{(2-p)}*p*(2+p^2)*(-1 - E^{(2*a)*x^{(1/4)}})^{(1+p)}*Hypergeometric2F1[p, 1+p, 2+p, (1+E^{(2*a)*x^{(1/4)}})/2])/(3*E^{(8*a)}*(1+p))$

Rubi [F] time = 0.0507109, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^p \left(a + \frac{\log(x)}{8} \right) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + Log[x]/8]^p, x]

[Out] Defer[Int][Coth[(8*a + Log[x])/8]^p, x]

Rubi steps

$$\int \coth^p \left(a + \frac{\log(x)}{8} \right) dx = \int \coth^p \left(\frac{1}{8}(8a + \log(x)) \right) dx$$

Mathematica [A] time = 0.906553, size = 223, normalized size = 1.15

$$\frac{e^{-8a} (e^{2a} \sqrt[4]{x} + 1)^{1-p} \left(\frac{e^{2a} \sqrt[4]{x} + 1}{e^{2a} \sqrt[4]{x} - 1} \right)^{p-1} \left(-2^{p+3} p {}_2F_1 \left(-p-2, 1-p; 2-p; \frac{1}{2} - \frac{1}{2} e^{2a} \sqrt[4]{x} \right) + 2^{p+2} (2p-1) {}_2F_1 \left(-p-1, 1-p; 2-p; \right) \right)}{p-1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + Log[x]/8]^p,x]

[Out] $((1 + E^{(2*a)*x^{(1/4)}})^{(1 - p)} * ((1 + E^{(2*a)*x^{(1/4)}})/(-1 + E^{(2*a)*x^{(1/4)}}))^{(-1 + p)} * (-2^{(3 + p)*p} * \text{Hypergeometric2F1}[-2 - p, 1 - p, 2 - p, 1/2 - (E^{(2*a)*x^{(1/4)}})/2]) + 2^{(2 + p)*(-1 + 2*p)} * \text{Hypergeometric2F1}[-1 - p, 1 - p, 2 - p, 1/2 - (E^{(2*a)*x^{(1/4)}})/2] + (-1 + p) * (E^{(4*a)} * (1 + E^{(2*a)*x^{(1/4)}}))^{(1 + p)} * \text{Sqrt}[x] - 2^{(1 + p)} * \text{Hypergeometric2F1}[1 - p, -p, 2 - p, 1/2 - (E^{(2*a)*x^{(1/4)}})/2]) / (E^{(8*a)*(-1 + p)})$

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(\coth \left(a + \frac{\ln(x)}{8} \right) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+1/8*ln(x))^p,x)

[Out] int(coth(a+1/8*ln(x))^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth \left(a + \frac{1}{8} \log(x) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="maxima")

[Out] integrate(coth(a + 1/8*log(x))^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\coth \left(a + \frac{1}{8} \log(x) \right)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/8*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p\left(a + \frac{\log(x)}{8}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/8)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(a + \frac{1}{8} \log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/8*log(x))^p, x)
```

3.174 $\int \coth^p(a + \log(x)) dx$

Optimal. Leaf size=61

$$x(-e^{2ax^2} - 1)^p (e^{2ax^2} + 1)^{-p} F_1\left(\frac{1}{2}; p, -p; \frac{3}{2}; e^{2ax^2}, -e^{2ax^2}\right)$$

[Out] $(x*(-1 - E^{(2*a)*x^2})^p * \text{AppellF1}[1/2, p, -p, 3/2, E^{(2*a)*x^2}, -(E^{(2*a)*x^2})]) / (1 + E^{(2*a)*x^2})^p$

Rubi [F] time = 0.0153368, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^p(a + \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + Log[x]]^p, x]

[Out] Defer[Int][Coth[a + Log[x]]^p, x]

Rubi steps

$$\int \coth^p(a + \log(x)) dx = \int \coth^p(a + \log(x)) dx$$

Mathematica [B] time = 1.54812, size = 171, normalized size = 2.8

$$\frac{3x \left(\frac{e^{2ax^2} + 1}{e^{2ax^2} - 1}\right)^p F_1\left(\frac{1}{2}; p, -p; \frac{3}{2}; e^{2ax^2}, -e^{2ax^2}\right)}{2e^{2a} p x^2 \left(F_1\left(\frac{3}{2}; p, 1 - p; \frac{5}{2}; e^{2ax^2}, -e^{2ax^2}\right) + F_1\left(\frac{3}{2}; p + 1, -p; \frac{5}{2}; e^{2ax^2}, -e^{2ax^2}\right)\right) + 3F_1\left(\frac{1}{2}; p, -p; \frac{3}{2}; e^{2ax^2}, -e^{2ax^2}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + Log[x]]^p, x]


```
[Out] (3*x*((1 + E^(2*a)*x^2)/(-1 + E^(2*a)*x^2))^p*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2*a)*x^2)]/(3*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2*a)*x^2)] + 2*E^(2*a)*p*x^2*(AppellF1[3/2, p, 1 - p, 5/2, E^(2*a)*x^2, -(E^(2*a)*x^2)] + AppellF1[3/2, 1 + p, -p, 5/2, E^(2*a)*x^2, -(E^(2*a)*x^2)]))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int (\coth(a + \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+ln(x))^p,x)
```

```
[Out] int(coth(a+ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a + \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\coth(a + \log(x))^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p(a + \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+ln(x))**p,x)

[Out] Integral(coth(a + log(x))**p, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a + \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+log(x))^p,x, algorithm="giac")

[Out] integrate(coth(a + log(x))^p, x)

3.175 $\int \coth^p(a + 2 \log(x)) dx$

Optimal. Leaf size=61

$$x(-e^{2a}x^4 - 1)^p (e^{2a}x^4 + 1)^{-p} F_1\left(\frac{1}{4}; p, -p; \frac{5}{4}; e^{2a}x^4, -e^{2a}x^4\right)$$

[Out] $(x*(-1 - E^{(2*a)*x^4})^p*AppellF1[1/4, p, -p, 5/4, E^{(2*a)*x^4}, -(E^{(2*a)*x^4})])/(1 + E^{(2*a)*x^4})^p$

Rubi [F] time = 0.0105149, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^p(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + 2*Log[x]]^p, x]

[Out] Defer[Int][Coth[a + 2*Log[x]]^p, x]

Rubi steps

$$\int \coth^p(a + 2 \log(x)) dx = \int \coth^p(a + 2 \log(x)) dx$$

Mathematica [B] time = 1.7343, size = 171, normalized size = 2.8

$$\frac{5x \left(\frac{e^{2a}x^4 + 1}{e^{2a}x^4 - 1} \right)^p F_1\left(\frac{1}{4}; p, -p; \frac{5}{4}; e^{2a}x^4, -e^{2a}x^4\right)}{4e^{2a}px^4 \left(F_1\left(\frac{5}{4}; p, 1 - p; \frac{9}{4}; e^{2a}x^4, -e^{2a}x^4\right) + F_1\left(\frac{5}{4}; p + 1, -p; \frac{9}{4}; e^{2a}x^4, -e^{2a}x^4\right) \right) + 5F_1\left(\frac{1}{4}; p, -p; \frac{5}{4}; e^{2a}x^4, -e^{2a}x^4\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + 2*Log[x]]^p, x]

```
[Out] (5*x*((1 + E^(2*a)*x^4)/(-1 + E^(2*a)*x^4))^p*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2*a)*x^4)]/(5*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2*a)*x^4)] + 4*E^(2*a)*p*x^4*(AppellF1[5/4, p, 1 - p, 9/4, E^(2*a)*x^4, -(E^(2*a)*x^4)] + AppellF1[5/4, 1 + p, -p, 9/4, E^(2*a)*x^4, -(E^(2*a)*x^4)]))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int (\coth(a + 2 \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^p,x)
```

```
[Out] int(coth(a+2*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a + 2 \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 2*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\coth(a + 2 \log(x))^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 2*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*ln(x))**p,x)

[Out] Integral(coth(a + 2*log(x))**p, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a + 2 \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+2*log(x))^p,x, algorithm="giac")

[Out] integrate(coth(a + 2*log(x))^p, x)

3.176 $\int \coth^p(a + 3 \log(x)) dx$

Optimal. Leaf size=61

$$x(-e^{2ax^6} - 1)^p (e^{2ax^6} + 1)^{-p} F_1\left(\frac{1}{6}; p, -p; \frac{7}{6}; e^{2ax^6}, -e^{2ax^6}\right)$$

[Out] $(x*(-1 - E^{(2*a)*x^6})^p * \text{AppellF1}[1/6, p, -p, 7/6, E^{(2*a)*x^6}, -(E^{(2*a)*x^6})]) / (1 + E^{(2*a)*x^6})^p$

Rubi [F] time = 0.0179362, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^p(a + 3 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + 3*Log[x]]^p, x]

[Out] Defer[Int][Coth[a + 3*Log[x]]^p, x]

Rubi steps

$$\int \coth^p(a + 3 \log(x)) dx = \int \coth^p(a + 3 \log(x)) dx$$

Mathematica [B] time = 1.81414, size = 171, normalized size = 2.8

$$\frac{7x \left(\frac{e^{2ax^6} + 1}{e^{2ax^6} - 1}\right)^p F_1\left(\frac{1}{6}; p, -p; \frac{7}{6}; e^{2ax^6}, -e^{2ax^6}\right)}{6e^{2a} p x^6 \left(F_1\left(\frac{7}{6}; p, 1 - p; \frac{13}{6}; e^{2ax^6}, -e^{2ax^6}\right) + F_1\left(\frac{7}{6}; p + 1, -p; \frac{13}{6}; e^{2ax^6}, -e^{2ax^6}\right)\right) + 7F_1\left(\frac{1}{6}; p, -p; \frac{7}{6}; e^{2ax^6}, -e^{2ax^6}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[a + 3*Log[x]]^p, x]

```
[Out] (7*x*((1 + E^(2*a)*x^6)/(-1 + E^(2*a)*x^6))^p*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2*a)*x^6)]/(7*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2*a)*x^6)] + 6*E^(2*a)*p*x^6*(AppellF1[7/6, p, 1 - p, 13/6, E^(2*a)*x^6, -(E^(2*a)*x^6)] + AppellF1[7/6, 1 + p, -p, 13/6, E^(2*a)*x^6, -(E^(2*a)*x^6)])
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int (\coth(a + 3 \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+3*ln(x))^p,x)
```

```
[Out] int(coth(a+3*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a + 3 \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 3*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\coth(a + 3 \log(x))^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 3*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^p(a + 3 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+3*ln(x))**p,x)

[Out] Integral(coth(a + 3*log(x))**p, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(a + 3 \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+3*log(x))^p,x, algorithm="giac")

[Out] integrate(coth(a + 3*log(x))^p, x)

3.177 $\int x^3 \coth(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=58

$$\frac{x^4}{4} - \frac{1}{2}x^4 {}_2F_1\left(1, \frac{2}{bdn}; 1 + \frac{2}{bdn}; e^{2ad}(cx^n)^{2bd}\right)$$

[Out] $x^4/4 - (x^4 \text{Hypergeometric2F1}[1, 2/(b*d*n), 1 + 2/(b*d*n), E^{(2*a*d)*(c*x^n)^{(2*b*d)}}])/2$

Rubi [F] time = 0.0390568, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^3 \coth(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])], x]

[Out] Defer[Int][x^3*Coth[d*(a + b*Log[c*x^n])], x]

Rubi steps

$$\int x^3 \coth(d(a + b \log(cx^n))) dx = \int x^3 \coth(d(a + b \log(cx^n))) dx$$

Mathematica [B] time = 7.01525, size = 198, normalized size = 3.41

$$x^4 \left(2e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 + \frac{2}{bdn}; 2 + \frac{2}{bdn}; e^{2d(a+b \log(cx^n))}\right) + (bdn + 2) \left({}_2F_1\left(1, \frac{2}{bdn}; 1 + \frac{2}{bdn}; e^{2d(a+b \log(cx^n))}\right) + \coth(d(a + b \log(cx^n))) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])], x]

[Out] $-((x^4*(2*E^{(2*d*(a + b*Log[c*x^n]))}*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 + 2/(b*d*n), E^{(2*d*(a + b*Log[c*x^n]))}]) + (2 + b*d*n)*(Coth[d*(a + b*Log[c$

$*x^n]) - \text{Coth}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n])] + \text{Hypergeometric2F1}[1, 2/(b*d*n), 1 + 2/(b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n]))}] + \text{Csch}[d*(a + b*\text{Log}[c*x^n])] * \text{Csch}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n])] * \text{Sinh}[b*d*n*\text{Log}[x]]]) / (8 + 4*b*d*n))$

Maple [F] time = 1.095, size = 0, normalized size = 0.

$$\int x^3 \coth(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*coth(d*(a+b*ln(c*x^n))),x)

[Out] int(x^3*coth(d*(a+b*ln(c*x^n))),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^4 - \int \frac{x^3}{c^{bd}e^{(bd \log(x^n)+ad)} + 1} dx + \int \frac{x^3}{c^{bd}e^{(bd \log(x^n)+ad)} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] 1/4*x^4 - integrate(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \coth(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral(x^3*coth(b*d*log(c*x^n) + a*d), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*coth(d*(a+b*ln(c*x**n))),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] `integrate(x^3*coth((b*log(c*x^n) + a)*d), x)`

3.178 $\int x^2 \coth(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=62

$$\frac{x^3}{3} - \frac{2}{3}x^3 {}_2F_1\left(1, \frac{3}{2bdn}; 1 + \frac{3}{2bdn}; e^{2ad}(cx^n)^{2bd}\right)$$

[Out] $x^3/3 - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/3$

Rubi [F] time = 0.0299628, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \coth(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])],x]

[Out] Defer[Int][x^2*Coth[d*(a + b*Log[c*x^n])], x]

Rubi steps

$$\int x^2 \coth(d(a + b \log(cx^n))) dx = \int x^2 \coth(d(a + b \log(cx^n))) dx$$

Mathematica [B] time = 4.89553, size = 207, normalized size = 3.34

$$x^3 \left(3e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 + \frac{3}{2bdn}; 2 + \frac{3}{2bdn}; e^{2d(a+b \log(cx^n))}\right) + (2bdn + 3) \left({}_2F_1\left(1, \frac{3}{2bdn}; 1 + \frac{3}{2bdn}; e^{2d(a+b \log(cx^n))}\right) + \coth \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])],x]

[Out] $-((x^3*(3*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n), 2 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))]) + (3 + 2*b*d*n)*(Coth[d*(a + b$

$*\text{Log}[c*x^n]] - \text{Coth}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n])] + \text{Hypergeometric2F1}$
 $[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n]))}] + \text{Csch}[d*(a +$
 $b*\text{Log}[c*x^n])* \text{Csch}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n])* \text{Sinh}[b*d*n*\text{Log}[x]]]$
 $)]/(9 + 6*b*d*n))$

Maple [F] time = 1.046, size = 0, normalized size = 0.

$$\int x^2 \coth(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*coth(d*(a+b*ln(c*x^n))),x)`

[Out] `int(x^2*coth(d*(a+b*ln(c*x^n))),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3 - \int \frac{x^2}{c^{bd}e^{(bd \log(x^n)+ad)} + 1} dx + \int \frac{x^2}{c^{bd}e^{(bd \log(x^n)+ad)} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `1/3*x^3 - integrate(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrat`
`e(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \coth(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(x^2*coth(b*d*log(c*x^n) + a*d), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*coth(d*(a+b*ln(c*x**n))),x)

[Out] Integral(x**2*coth(a*d + b*d*log(c*x**n)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")

[Out] integrate(x^2*coth((b*log(c*x^n) + a)*d), x)

3.179 $\int x \coth(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=54

$$\frac{x^2}{2} - x^2 {}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; e^{2ad} (cx^n)^{2bd}\right)$$

[Out] $x^2/2 - x^2 \text{Hypergeometric2F1}[1, 1/(b*d*n), 1 + 1/(b*d*n), E^{(2*a*d)}*(c*x^n)^{(2*b*d)}]$

Rubi [F] time = 0.0239149, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x \coth(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[x*\text{Coth}[d*(a + b*\text{Log}[c*x^n])], x]$

[Out] $\text{Defer}[\text{Int}][x*\text{Coth}[d*(a + b*\text{Log}[c*x^n])], x]$

Rubi steps

$$\int x \coth(d(a + b \log(cx^n))) dx = \int x \coth(d(a + b \log(cx^n))) dx$$

Mathematica [B] time = 6.93912, size = 193, normalized size = 3.57

$$x^2 \left(e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 + \frac{1}{bdn}; 2 + \frac{1}{bdn}; e^{2d(a+b \log(cx^n))}\right) + (bdn + 1) \left({}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; e^{2d(a+b \log(cx^n))}\right) + \coth(d(a + b \log(cx^n))) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] $\text{Integrate}[x*\text{Coth}[d*(a + b*\text{Log}[c*x^n])], x]$

[Out] $-((x^2*(E^{(2*d*(a + b*\text{Log}[c*x^n])}))*\text{Hypergeometric2F1}[1, 1 + 1/(b*d*n), 2 + 1/(b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n])}]) + (1 + b*d*n)*(Coth[d*(a + b*\text{Log}[c*x^n])])$

$\wedge n)) - \text{Coth}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^\wedge n])] + \text{Hypergeometric2F1}[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*d*(a + b*\text{Log}[c*x^\wedge n]))] + \text{Csch}[d*(a + b*\text{Log}[c*x^\wedge n])] * \text{Csch}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^\wedge n])] * \text{Sinh}[b*d*n*\text{Log}[x]])) / (2 + 2*b*d*n))$

Maple [F] time = 1., size = 0, normalized size = 0.

$$\int x \coth(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*coth(d*(a+b*ln(c*x^n))),x)`

[Out] `int(x*coth(d*(a+b*ln(c*x^n))),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2 - \int \frac{x}{c^{bd}e^{(bd \log(x^n)+ad)} + 1} dx + \int \frac{x}{c^{bd}e^{(bd \log(x^n)+ad)} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `1/2*x^2 - integrate(x/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(x/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \coth(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(x*coth(b*d*log(c*x^n) + a*d), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(d*(a+b*ln(c*x**n))),x)`

[Out] `Integral(x*coth(a*d + b*d*log(c*x**n)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] `integrate(x*coth((b*log(c*x^n) + a)*d), x)`

3.180 $\int \coth(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=52

$$x - 2x {}_2F_1\left(1, \frac{1}{2bdn}; 1 + \frac{1}{2bdn}; e^{2ad}(cx^n)^{2bd}\right)$$

[Out] x - 2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)]

Rubi [F] time = 0.0114539, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[d*(a + b*Log[c*x^n])], x]

[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])], x]

Rubi steps

$$\int \coth(d(a + b \log(cx^n))) dx = \int \coth(d(a + b \log(cx^n))) dx$$

Mathematica [B] time = 8.32863, size = 198, normalized size = 3.81

$$\frac{x e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 + \frac{1}{2bdn}; 2 + \frac{1}{2bdn}; e^{2d(a+b \log(cx^n))}\right)}{2bdn + 1} - x \left({}_2F_1\left(1, \frac{1}{2bdn}; 1 + \frac{1}{2bdn}; e^{2d(a+b \log(cx^n))}\right) + \coth(d(a + b \log(cx^n))) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])], x]

[Out] -((E^(2*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 + 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(1 + 2*b*d*n)) - x*(Coth[d*(a + b*

$\text{Log}[c*x^n]] - \text{Coth}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n])] + \text{Hypergeometric2F1}[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n]))}] + \text{Csch}[d*(a + b*\text{Log}[c*x^n])]*\text{Csch}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n])]*\text{Sinh}[b*d*n*\text{Log}[x]]]$

Maple [F] time = 0.915, size = 0, normalized size = 0.

$$\int \coth(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*(a+b*ln(c*x^n))),x)

[Out] int(coth(d*(a+b*ln(c*x^n))),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x - \int \frac{1}{c^{bd}e^{(bd \log(x^n)+ad)} + 1} dx + \int \frac{1}{c^{bd}e^{(bd \log(x^n)+ad)} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] x - integrate(1/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(1/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\coth(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral(coth(b*d*log(c*x^n) + a*d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*ln(c*x**n))),x)

[Out] Integral(coth(d*(a + b*log(c*x**n))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="giac")

[Out] integrate(coth((b*log(c*x^n) + a)*d), x)

$$3.181 \quad \int \frac{\coth(d(a+b \log(cx^n)))}{x} dx$$

Optimal. Leaf size=25

$$\frac{\log(\sinh(ad + bd \log(cx^n)))}{bdn}$$

[Out] Log[Sinh[a*d + b*d*Log[c*x^n]]]/(b*d*n)

Rubi [A] time = 0.0206705, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3475}

$$\frac{\log(\sinh(ad + bd \log(cx^n)))}{bdn}$$

Antiderivative was successfully verified.

[In] Int[Coth[d*(a + b*Log[c*x^n])]/x,x]

[Out] Log[Sinh[a*d + b*d*Log[c*x^n]]]/(b*d*n)

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth(d(a+b \log(cx^n)))}{x} dx &= \frac{\text{Subst}\left(\int \coth(d(a+bx)) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\log(\sinh(ad + bd \log(cx^n)))}{bdn} \end{aligned}$$

Mathematica [A] time = 0.0569006, size = 40, normalized size = 1.6

$$\frac{\log(\tanh(ad + bd \log(cx^n))) + \log(\cosh(d(a + b \log(cx^n))))}{bdn}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x,x]

[Out] (Log[Cosh[d*(a + b*Log[c*x^n])]] + Log[Tanh[a*d + b*d*Log[c*x^n]])]/(b*d*n)

Maple [B] time = 0.004, size = 56, normalized size = 2.2

$$-\frac{\ln(\coth(d(a + b \ln(cx^n))) - 1)}{2dbn} - \frac{\ln(\coth(d(a + b \ln(cx^n))) + 1)}{2dbn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*(a+b*ln(c*x^n)))/x,x)

[Out] -1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))-1)-1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))+1)

Maxima [A] time = 1.15765, size = 32, normalized size = 1.28

$$\frac{\log(\sinh((b \log(cx^n) + a)d))}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")

[Out] log(sinh((b*log(c*x^n) + a)*d))/(b*d*n)

Fricas [B] time = 2.91355, size = 205, normalized size = 8.2

$$-\frac{bdn \log(x) - \log\left(\frac{2 \sinh(bdn \log(x) + bd \log(c) + ad)}{\cosh(bdn \log(x) + bd \log(c) + ad) - \sinh(bdn \log(x) + bd \log(c) + ad)}\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")

[Out] $-(b*d*n*\log(x) - \log(2*\sinh(b*d*n*\log(x) + b*d*\log(c) + a*d)/(\cosh(b*d*n*\log(x) + b*d*\log(c) + a*d) - \sinh(b*d*n*\log(x) + b*d*\log(c) + a*d))))/(b*d*n)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(ad + bd \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*ln(c*x**n)))/x,x)

[Out] Integral(coth(a*d + b*d*log(c*x**n))/x, x)

Giac [B] time = 1.24077, size = 99, normalized size = 3.96

$$\frac{\log\left(-2x^{2bdn}|c|^{2bd} \cos(\pi b d \operatorname{sgn}(c) - \pi b d) e^{(2ad)} + x^{4bdn}|c|^{4bd} e^{(4ad)} + 1\right)}{2bdn} - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")

[Out] $1/2*\log(-2*x^{(2*b*d*n)}*abs(c)^{(2*b*d)}*\cos(\pi*b*d*\operatorname{sgn}(c) - \pi*b*d)*e^{(2*a*d)} + x^{(4*b*d*n)}*abs(c)^{(4*b*d)}*e^{(4*a*d)} + 1)/(b*d*n) - \log(x)$

$$3.182 \quad \int \frac{\coth(d(a+b \log(cx^n)))}{x^2} dx$$

Optimal. Leaf size=58

$$\frac{{}_2F_1\left(1, -\frac{1}{2bdn}; 1 - \frac{1}{2bdn}; e^{2ad}(cx^n)^{2bd}\right)}{x} - \frac{1}{x}$$

[Out] $-x^{(-1)} + (2*\text{Hypergeometric2F1}[1, -1/(2*b*d*n), 1 - 1/(2*b*d*n), E^{(2*a*d)*(c*x^n)^{(2*b*d)}}])/x$

Rubi [F] time = 0.0313433, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth(d(a+b \log(cx^n)))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^2,x]

[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]/x^2, x]

Rubi steps

$$\int \frac{\coth(d(a+b \log(cx^n)))}{x^2} dx = \int \frac{\coth(d(a+b \log(cx^n)))}{x^2} dx$$

Mathematica [B] time = 3.82998, size = 197, normalized size = 3.4

$$\frac{e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 - \frac{1}{2bdn}; 2 - \frac{1}{2bdn}; e^{2d(a+b \log(cx^n))}\right)}{2bdn-1} + {}_2F_1\left(1, -\frac{1}{2bdn}; 1 - \frac{1}{2bdn}; e^{2d(a+b \log(cx^n))}\right) + \coth(d(a+b \log(cx^n))) - \coth(\dots)$$

x

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^2,x]

[Out] $(\text{Coth}[d*(a + b*\text{Log}[c*x^n])] - \text{Coth}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n])] - (E^{(2*d*(a + b*\text{Log}[c*x^n]))}*\text{Hypergeometric2F1}[1, 1 - 1/(2*b*d*n), 2 - 1/(2*b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n]))}]))/(-1 + 2*b*d*n) + \text{Hypergeometric2F1}[1, -1/(2*b*d*n), 1 - 1/(2*b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n]))}] + \text{Csch}[d*(a + b*\text{Log}[c*x^n])] * \text{Csch}[d*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n])] * \text{Sinh}[b*d*n*\text{Log}[x]])/x$

Maple [F] time = 1.026, size = 0, normalized size = 0.

$$\int \frac{\coth(d(a + b \ln(cx^n)))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*(a+b*ln(c*x^n)))/x^2,x)`

[Out] `int(coth(d*(a+b*ln(c*x^n)))/x^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x} - \int \frac{1}{c^{bd} x^2 e^{(bd \log(x^n) + ad)} + x^2} dx + \int \frac{1}{c^{bd} x^2 e^{(bd \log(x^n) + ad)} - x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")`

[Out] `-1/x - integrate(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) + x^2), x) + integrate(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) - x^2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bd \log(cx^n) + ad)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")`

[Out] `integral(coth(b*d*log(c*x^n) + a*d)/x^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(ad + bd \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*ln(c*x**n)))/x**2,x)`

[Out] `Integral(coth(a*d + b*d*log(c*x**n))/x**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth((b \log(cx^n) + a)d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")`

[Out] `integrate(coth((b*log(c*x^n) + a)*d)/x^2, x)`

$$3.183 \quad \int \frac{\coth(d(a+b \log(cx^n)))}{x^3} dx$$

Optimal. Leaf size=55

$$\frac{{}_2F_1\left(1, -\frac{1}{bdn}; 1 - \frac{1}{bdn}; e^{2ad} (cx^n)^{2bd}\right)}{x^2} - \frac{1}{2x^2}$$

[Out] $-1/(2*x^2) + \text{Hypergeometric2F1}[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^{(2*a*d)*(c*x^n)^{(2*b*d)}}]/x^2$

Rubi [F] time = 0.0296067, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth(d(a+b \log(cx^n)))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^3, x]

[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]/x^3, x]

Rubi steps

$$\int \frac{\coth(d(a+b \log(cx^n)))}{x^3} dx = \int \frac{\coth(d(a+b \log(cx^n)))}{x^3} dx$$

Mathematica [B] time = 3.67831, size = 191, normalized size = 3.47

$$\frac{e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 - \frac{1}{bdn}; 2 - \frac{1}{bdn}; e^{2d(a+b \log(cx^n))}\right)}{bdn-1} + {}_2F_1\left(1, -\frac{1}{bdn}; 1 - \frac{1}{bdn}; e^{2d(a+b \log(cx^n))}\right) + \coth(d(a+b \log(cx^n))) - \coth(d(a+b \log(cx^n)))$$

$2x^2$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^3, x]

```
[Out] (Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(-1 + b*d*n) + Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/(2*x^2)
```

Maple [F] time = 1.026, size = 0, normalized size = 0.

$$\int \frac{\coth(d(a + b \ln(cx^n)))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2x^2} - \int \frac{1}{c^{bd}x^3e^{(bd \log(x^n)+ad)} + x^3} dx + \int \frac{1}{c^{bd}x^3e^{(bd \log(x^n)+ad)} - x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) + x^3), x) + integrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) - x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bd \log(cx^n) + ad)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

[Out] `integral(coth(b*d*log(c*x^n) + a*d)/x^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*ln(c*x**n)))/x**3,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth((b \log(cx^n) + a)d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")`

[Out] `integrate(coth((b*log(c*x^n) + a)*d)/x^3, x)`

3.184 $\int x^3 \coth^2(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=132

$$-\frac{2x^4 {}_2F_1\left(1, \frac{2}{bdn}; 1 + \frac{2}{bdn}; e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x^4 (e^{2ad}(cx^n)^{2bd} + 1)}{bdn(1 - e^{2ad}(cx^n)^{2bd})} + \frac{1}{4}x^4 \left(\frac{4}{bdn} + 1\right)$$

[Out] $((1 + 4/(b*d*n))*x^4)/4 + (x^4*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)$

Rubi [F] time = 0.0864764, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^3 \coth^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]

[Out] Defer[Int][x^3*Coth[d*(a + b*Log[c*x^n])]^2, x]

Rubi steps

$$\int x^3 \coth^2(d(a + b \log(cx^n))) dx = \int x^3 \coth^2(d(a + b \log(cx^n))) dx$$

Mathematica [A] time = 6.60582, size = 155, normalized size = 1.17

$$\frac{x^4 \left((bdn + 2) \left(-4 {}_2F_1\left(1, \frac{2}{bdn}; 1 + \frac{2}{bdn}; e^{2d(a+b \log(cx^n))}\right) - 4 \coth(d(a + b \log(cx^n))) + bdn \right) - 8e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 + \frac{2}{bdn}\right) \right)}{4bdn(bdn + 2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]

```
[Out] (x^4*(-8*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 +
2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(b*d*n - 4*Coth[d*(a
+ b*Log[c*x^n])) - 4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*d*
(a + b*Log[c*x^n]))])))/(4*b*d*n*(2 + b*d*n))
```

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int x^3 (\coth(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2bd}dnx^4e^{(2bd\log(x^n)+2ad)} - (bdn + 8)x^4}{4(bc^{2bd}dne^{(2bd\log(x^n)+2ad)} - bdn)} - 4 \int \frac{x^3}{bc^{bd}dne^{(bd\log(x^n)+ad)} + bdn} dx + 4 \int \frac{x^3}{bc^{bd}dne^{(bd\log(x^n)+ad)} - bdn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*c^(2*b*d)*d*n*x^4*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 8)*x^4)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 4*integrate(x^3/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 4*integrate(x^3/(b*c^(b*d)*d*n
*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \coth(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

[Out] `integral(x^3*coth(b*d*log(c*x^n) + a*d)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*coth(d*(a+b*ln(c*x**n))))**2,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth((b \log(cx^n) + a)d)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] `integrate(x^3*coth((b*log(c*x^n) + a)*d)^2, x)`

3.185 $\int x^2 \coth^2(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=136

$$-\frac{2x^3 {}_2F_1\left(1, \frac{3}{2bdn}; 1 + \frac{3}{2bdn}; e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x^3 (e^{2ad}(cx^n)^{2bd} + 1)}{bdn(1 - e^{2ad}(cx^n)^{2bd})} + \frac{1}{3}x^3 \left(\frac{3}{bdn} + 1\right)$$

[Out] $((1 + 3/(b*d*n))*x^3)/3 + (x^3*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)$

Rubi [F] time = 0.0584018, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \coth^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]

[Out] Defer[Int][x^2*Coth[d*(a + b*Log[c*x^n])]^2, x]

Rubi steps

$$\int x^2 \coth^2(d(a + b \log(cx^n))) dx = \int x^2 \coth^2(d(a + b \log(cx^n))) dx$$

Mathematica [A] time = 4.53231, size = 165, normalized size = 1.21

$$\frac{x^3 \left((2bdn + 3) \left(-3 {}_2F_1\left(1, \frac{3}{2bdn}; 1 + \frac{3}{2bdn}; e^{2d(a+b \log(cx^n))}\right) - 3 \coth(d(a + b \log(cx^n))) + bdn \right) - 9e^{2d(a+b \log(cx^n))} {}_2F_1(1, 1, \dots) \right)}{3bdn(2bdn + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]

[Out] $(x^3(-9E^{(2d(a+b\log[cx^n])})\text{Hypergeometric2F1}[1, 1 + 3/(2bdn), 2 + 3/(2bdn), E^{(2d(a+b\log[cx^n])})] + (3 + 2bdn)(bdn - 3\text{Coth}[d(a+b\log[cx^n])]) - 3\text{Hypergeometric2F1}[1, 3/(2bdn), 1 + 3/(2bdn), E^{(2d(a+b\log[cx^n])})])))/(3bdn(3 + 2bdn))$

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int x^2 (\coth(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2bd}dnx^3e^{(2bd\log(x^n)+2ad)} - (bdn + 6)x^3}{3(bc^{2bd}dne^{(2bd\log(x^n)+2ad)} - bdn)} - 3 \int \frac{x^2}{bc^{bd}dne^{(bd\log(x^n)+ad)} + bdn} dx + 3 \int \frac{x^2}{bc^{bd}dne^{(bd\log(x^n)+ad)} - bdn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] $1/3*(b*c^{(2*b*d)*d*n}*x^3*e^{(2*b*d*\log(x^n) + 2*a*d)} - (b*d*n + 6)*x^3)/(b*c^{(2*b*d)*d*n}*e^{(2*b*d*\log(x^n) + 2*a*d)} - b*d*n) - 3*\text{integrate}(x^2/(b*c^{(b*d)*d*n}*e^{(b*d*\log(x^n) + a*d)} + b*d*n), x) + 3*\text{integrate}(x^2/(b*c^{(b*d)*d*n}*e^{(b*d*\log(x^n) + a*d)} - b*d*n), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \coth(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(x^2*coth(b*d*log(c*x^n) + a*d)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*coth(d*(a+b*ln(c*x**n))))**2,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth((b \log(cx^n) + a)d)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] `integrate(x^2*coth((b*log(c*x^n) + a)*d)^2, x)`

3.186 $\int x \coth^2(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=130

$$-\frac{2x^2 {}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x^2 (e^{2ad}(cx^n)^{2bd} + 1)}{bdn(1 - e^{2ad}(cx^n)^{2bd})} + \frac{1}{2}x^2 \left(\frac{2}{bdn} + 1\right)$$

[Out] $((1 + 2/(b*d*n))*x^2)/2 + (x^2*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)$

Rubi [F] time = 0.0427788, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x \coth^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x*Coth[d*(a + b*Log[c*x^n])]^2,x]

[Out] Defer[Int][x*Coth[d*(a + b*Log[c*x^n])]^2, x]

Rubi steps

$$\int x \coth^2(d(a + b \log(cx^n))) dx = \int x \coth^2(d(a + b \log(cx^n))) dx$$

Mathematica [A] time = 6.48103, size = 151, normalized size = 1.16

$$\frac{x^2 \left((bdn + 1) \left(-2 {}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; e^{2d(a+b \log(cx^n))}\right) - 2 \coth(d(a + b \log(cx^n))) + bdn \right) - 2e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 + \frac{1}{bdn}\right) \right)}{2bdn(bdn + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x*Coth[d*(a + b*Log[c*x^n])]^2,x]

[Out] $(x^2 * (-2 * E^{(2 * d * (a + b * \log[c * x^n]))}) * \text{Hypergeometric2F1}[1, 1 + 1/(b * d * n), 2 + 1/(b * d * n), E^{(2 * d * (a + b * \log[c * x^n]))}] + (1 + b * d * n) * (b * d * n - 2 * \text{Coth}[d * (a + b * \log[c * x^n])]) - 2 * \text{Hypergeometric2F1}[1, 1/(b * d * n), 1 + 1/(b * d * n), E^{(2 * d * (a + b * \log[c * x^n])}])) / (2 * b * d * n * (1 + b * d * n))$

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int x (\coth(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*coth(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(x*coth(d*(a+b*ln(c*x^n)))^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2bd} dnx^2 e^{(2bd \log(x^n) + 2ad)} - (bdn + 4)x^2}{2(bc^{2bd} dne^{(2bd \log(x^n) + 2ad)} - bdn)} - 2 \int \frac{x}{bc^{bd} dne^{(bd \log(x^n) + ad)} + bdn} dx + 2 \int \frac{x}{bc^{bd} dne^{(bd \log(x^n) + ad)} - bdn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] $1/2 * (b * c^{(2 * b * d)} * d * n * x^2 * e^{(2 * b * d * \log(x^n) + 2 * a * d)} - (b * d * n + 4) * x^2) / (b * c^{(2 * b * d)} * d * n * e^{(2 * b * d * \log(x^n) + 2 * a * d)} - b * d * n) - 2 * \text{integrate}(x / (b * c^{(b * d)} * d * n * e^{(b * d * \log(x^n) + a * d)} + b * d * n), x) + 2 * \text{integrate}(x / (b * c^{(b * d)} * d * n * e^{(b * d * \log(x^n) + a * d)} - b * d * n), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \coth(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(x*coth(b*d*log(c*x^n) + a*d)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth^2(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(d*(a+b*ln(c*x**n))))**2,x)`

[Out] `Integral(x*coth(a*d + b*d*log(c*x**n))**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth((b \log(cx^n) + a)d)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] `integrate(x*coth((b*log(c*x^n) + a)*d)^2, x)`

3.187 $\int \coth^2(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=126

$$-\frac{2x {}_2F_1\left(1, \frac{1}{2bdn}; 1 + \frac{1}{2bdn}; e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x(e^{2ad}(cx^n)^{2bd} + 1)}{bdn(1 - e^{2ad}(cx^n)^{2bd})} + x\left(\frac{1}{bdn} + 1\right)$$

[Out] $(1 + 1/(b*d*n))*x + (x*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)$

Rubi [F] time = 0.0139827, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[d*(a + b*Log[c*x^n])]^2, x]

[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^2, x]

Rubi steps

$$\int \coth^2(d(a + b \log(cx^n))) dx = \int \coth^2(d(a + b \log(cx^n))) dx$$

Mathematica [A] time = 7.68637, size = 160, normalized size = 1.27

$$\frac{x\left((2bdn + 1)\left(-{}_2F_1\left(1, \frac{1}{2bdn}; 1 + \frac{1}{2bdn}; e^{2d(a+b \log(cx^n))}\right) - \coth(d(a + b \log(cx^n))) + bdn\right) - e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 + \frac{1}{2bdn}\right)\right)}{bdn(2bdn + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2, x]

[Out] $(x * (- (E^{(2*d*(a + b*\text{Log}[c*x^n])}) * \text{Hypergeometric2F1}[1, 1 + 1/(2*b*d*n), 2 + 1/(2*b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n])})]) + (1 + 2*b*d*n) * (b*d*n - \text{Coth}[d*(a + b*\text{Log}[c*x^n])]) - \text{Hypergeometric2F1}[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n])})])) / (b*d*n * (1 + 2*b*d*n))$

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (\coth(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(coth(d*(a+b*ln(c*x^n)))^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2bd} d n x e^{(2bd \log(x^n) + 2ad)} - (bdn + 2)x}{bc^{2bd} d n e^{(2bd \log(x^n) + 2ad)} - bdn} - \int \frac{1}{bc^{bd} d n e^{(bd \log(x^n) + ad)} + bdn} dx + \int \frac{1}{bc^{bd} d n e^{(bd \log(x^n) + ad)} - bdn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] $(b*c^{(2*b*d)*d*n*x}*e^{(2*b*d*\log(x^n) + 2*a*d)} - (b*d*n + 2)*x) / (b*c^{(2*b*d)*d*n}*e^{(2*b*d*\log(x^n) + 2*a*d)} - b*d*n) - \text{integrate}(1/(b*c^{(b*d)*d*n}*e^{(b*d*\log(x^n) + a*d)} + b*d*n), x) + \text{integrate}(1/(b*c^{(b*d)*d*n}*e^{(b*d*\log(x^n) + a*d)} - b*d*n), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\coth(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(coth(b*d*log(c*x^n) + a*d)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \coth^2(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*ln(c*x**n)))**2,x)`

[Out] `Integral(coth(d*(a + b*log(c*x**n)))**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth((b \log(cx^n) + a)d)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] `integrate(coth((b*log(c*x^n) + a)*d)^2, x)`

$$3.188 \quad \int \frac{\coth^2(d(a+b \log(cx^n)))}{x} dx$$

Optimal. Leaf size=28

$$\log(x) - \frac{\coth(ad + bd \log(cx^n))}{bdn}$$

[Out] -(Coth[a*d + b*d*Log[c*x^n]]/(b*d*n)) + Log[x]

Rubi [A] time = 0.0291035, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3473, 8}

$$\log(x) - \frac{\coth(ad + bd \log(cx^n))}{bdn}$$

Antiderivative was successfully verified.

[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x,x]

[Out] -(Coth[a*d + b*d*Log[c*x^n]]/(b*d*n)) + Log[x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(d(a+b \log(cx^n)))}{x} dx &= \frac{\text{Subst}\left(\int \coth^2(d(a+bx)) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\coth(ad + bd \log(cx^n))}{bdn} + \frac{\text{Subst}\left(\int 1 dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\coth(ad + bd \log(cx^n))}{bdn} + \log(x) \end{aligned}$$

Mathematica [C] time = 0.103764, size = 49, normalized size = 1.75

$$-\frac{\coth(ad + bd \log(cx^n)) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(ad + b \log(cx^n)) d\right)}{bdn}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x,x]

[Out] -((Coth[a*d + b*d*Log[c*x^n]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a*d + b*d*Log[c*x^n]]^2])/(b*d*n))

Maple [B] time = 0.005, size = 80, normalized size = 2.9

$$\frac{\coth(d(a + b \ln(cx^n)))}{dbn} - \frac{\ln(\coth(d(a + b \ln(cx^n))) - 1)}{2 dbn} + \frac{\ln(\coth(d(a + b \ln(cx^n))) + 1)}{2 dbn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*(a+b*ln(c*x^n)))^2/x,x)

[Out] -1/b/d/n*coth(d*(a+b*ln(c*x^n)))-1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))-1)+1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))+1)

Maxima [A] time = 1.30048, size = 50, normalized size = 1.79

$$-\frac{2}{bc^{2bd}dne^{(2bd \log(x^n)+2ad)} - bdn} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")

[Out] -2/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) + log(x)

Fricas [B] time = 2.51242, size = 197, normalized size = 7.04

$$\frac{(bdn \log(x) + 1) \sinh(bdn \log(x) + bd \log(c) + ad) - \cosh(bdn \log(x) + bd \log(c) + ad)}{bdn \sinh(bdn \log(x) + bd \log(c) + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")

[Out] ((b*d*n*log(x) + 1)*sinh(b*d*n*log(x) + b*d*log(c) + a*d) - cosh(b*d*n*log(x) + b*d*log(c) + a*d))/(b*d*n*sinh(b*d*n*log(x) + b*d*log(c) + a*d))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(ad + bd \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x,x)

[Out] Integral(coth(a*d + b*d*log(c*x**n))**2/x, x)

Giac [A] time = 1.33036, size = 50, normalized size = 1.79

$$-\frac{2}{(c^{2bd}x^{2bdn}e^{(2ad)} - 1)bdn} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")

[Out] -2/((c^(2*b*d)*x^(2*b*d*n)*e^(2*a*d) - 1)*b*d*n) + log(x)

$$3.189 \quad \int \frac{\coth^2(d(a+b \log(cx^n)))}{x^2} dx$$

Optimal. Leaf size=134

$$-\frac{{}_2F_1\left(1, -\frac{1}{2bdn}; 1 - \frac{1}{2bdn}; e^{2ad}(cx^n)^{2bd}\right)}{bdnx} + \frac{e^{2ad}(cx^n)^{2bd} + 1}{bdnx(1 - e^{2ad}(cx^n)^{2bd})} - \frac{1 - \frac{1}{bdn}}{x}$$

[Out] $-\left(\left(1 - \frac{1}{b*d*n}\right)/x\right) + \left(1 + E^{(2*a*d)*(c*x^n)^{(2*b*d)}}\right)/(b*d*n*x*(1 - E^{(2*a*d)*(c*x^n)^{(2*b*d)}}) - \left(2*Hypergeometric2F1\left[1, -1/(2*b*d*n), 1 - 1/(2*b*d*n), E^{(2*a*d)*(c*x^n)^{(2*b*d)}}\right]\right)/(b*d*n*x)$

Rubi [F] time = 0.0506045, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth^2(d(a+b \log(cx^n)))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^2, x]

[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^2/x^2, x]

Rubi steps

$$\int \frac{\coth^2(d(a+b \log(cx^n)))}{x^2} dx = \int \frac{\coth^2(d(a+b \log(cx^n)))}{x^2} dx$$

Mathematica [A] time = 3.56269, size = 158, normalized size = 1.18

$$\frac{e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 - \frac{1}{2bdn}; 2 - \frac{1}{2bdn}; e^{2d(a+b \log(cx^n))}\right) - (2bdn - 1) \left({}_2F_1\left(1, -\frac{1}{2bdn}; 1 - \frac{1}{2bdn}; e^{2d(a+b \log(cx^n))}\right) + \coth(d(a+b \log(cx^n)))\right)}{bdnx(2bdn - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^2, x]

[Out] $(E^{(2*d*(a + b*\text{Log}[c*x^n]))}*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n]))}] - (-1 + 2*b*d*n)*(b*d*n + \text{Coth}[d*(a + b*\text{Log}[c*x^n])]) + Hypergeometric2F1[1, -1/(2*b*d*n), 1 - 1/(2*b*d*n), E^{(2*d*(a + b*\text{Log}[c*x^n])])]))/(b*d*n*(-1 + 2*b*d*n)*x)$

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int \frac{(\coth(d(a + b \ln(cx^n))))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)`

[Out] `int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bc^{2bd}dne^{(2bd\log(x^n)+2ad)} - bdn + 2}{bc^{2bd}dnxe^{(2bd\log(x^n)+2ad)} - bdnx} + \int \frac{1}{bc^{bd}dnx^2e^{(bd\log(x^n)+ad)} + bdnx^2} dx - \int \frac{1}{bc^{bd}dnx^2e^{(bd\log(x^n)+ad)} - bdnx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")`

[Out] $-(b*c^{(2*b*d)*d*n}*e^{(2*b*d*\log(x^n) + 2*a*d)} - b*d*n + 2)/(b*c^{(2*b*d)*d*n}*x*e^{(2*b*d*\log(x^n) + 2*a*d)} - b*d*n*x) + \text{integrate}(1/(b*c^{(b*d)*d*n}*x^2*e^{(b*d*\log(x^n) + a*d)} + b*d*n*x^2), x) - \text{integrate}(1/(b*c^{(b*d)*d*n}*x^2*e^{(b*d*\log(x^n) + a*d)} - b*d*n*x^2), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bd \log(cx^n) + ad)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth((b \log(cx^n) + a)d)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^2, x)
```

$$3.190 \quad \int \frac{\coth^2(d(a+b \log(cx^n)))}{x^3} dx$$

Optimal. Leaf size=135

$$-\frac{{}_2F_1\left(1, -\frac{1}{bdn}; 1 - \frac{1}{bdn}; e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2} + \frac{e^{2ad}(cx^n)^{2bd} + 1}{bdnx^2(1 - e^{2ad}(cx^n)^{2bd})} + \frac{2 - bdn}{2bdnx^2}$$

[Out] $(2 - b*d*n)/(2*b*d*n*x^2) + (1 + E^{(2*a*d)*(c*x^n)^{(2*b*d)}})/(b*d*n*x^2*(1 - E^{(2*a*d)*(c*x^n)^{(2*b*d)}})) - (2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^{(2*a*d)*(c*x^n)^{(2*b*d)}}]/(b*d*n*x^2))$

Rubi [F] time = 0.0530002, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth^2(d(a+b \log(cx^n)))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^3, x]

[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^2/x^3, x]

Rubi steps

$$\int \frac{\coth^2(d(a+b \log(cx^n)))}{x^3} dx = \int \frac{\coth^2(d(a+b \log(cx^n)))}{x^3} dx$$

Mathematica [A] time = 3.55429, size = 156, normalized size = 1.16

$$\frac{2e^{2d(a+b \log(cx^n))} {}_2F_1\left(1, 1 - \frac{1}{bdn}; 2 - \frac{1}{bdn}; e^{2d(a+b \log(cx^n))}\right) - (bdn - 1) \left({}_2F_1\left(1, -\frac{1}{bdn}; 1 - \frac{1}{bdn}; e^{2d(a+b \log(cx^n))}\right) + 2 \coth(d(a + b \log(cx^n))) \right)}{2bdnx^2(bdn - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^3, x]

[Out] $(2 * E^{(2 * d * (a + b * \text{Log}[c * x^n]))} * \text{Hypergeometric2F1}[1, 1 - 1/(b * d * n), 2 - 1/(b * d * n), E^{(2 * d * (a + b * \text{Log}[c * x^n]))}] - (-1 + b * d * n) * (b * d * n + 2 * \text{Coth}[d * (a + b * \text{Log}[c * x^n])]) + 2 * \text{Hypergeometric2F1}[1, -(1/(b * d * n)), 1 - 1/(b * d * n), E^{(2 * d * (a + b * \text{Log}[c * x^n]))}])) / (2 * b * d * n * (-1 + b * d * n) * x^2)$

Maple [F] time = 0.127, size = 0, normalized size = 0.

$$\int \frac{(\coth(d(a + b \ln(cx^n))))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)`

[Out] `int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bc^{2bd} dne^{(2bd \log(x^n)+2ad)} - bdn + 4}{2(bc^{2bd} dnx^2 e^{(2bd \log(x^n)+2ad)} - bdnx^2)} + 2 \int \frac{1}{bc^{bd} dnx^3 e^{(bd \log(x^n)+ad)} + bdnx^3} dx - 2 \int \frac{1}{bc^{bd} dnx^3 e^{(bd \log(x^n)+ad)} - bdnx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")`

[Out] $-1/2 * (b * c^{(2 * b * d)} * d * n * e^{(2 * b * d * \log(x^n) + 2 * a * d)} - b * d * n + 4) / (b * c^{(2 * b * d)} * d * n * x^2 * e^{(2 * b * d * \log(x^n) + 2 * a * d)} - b * d * n * x^2) + 2 * \text{integrate}(1 / (b * c^{(b * d)} * d * n * x^3 * e^{(b * d * \log(x^n) + a * d)} + b * d * n * x^3), x) - 2 * \text{integrate}(1 / (b * c^{(b * d)} * d * n * x^3 * e^{(b * d * \log(x^n) + a * d)} - b * d * n * x^3), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bd \log(cx^n) + ad)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth((b \log(cx^n) + a)d)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^3, x)
```

$$3.191 \quad \int \frac{\coth^3(a+b \log(cx^n))}{x} dx$$

Optimal. Leaf size=43

$$\frac{\log(\sinh(a+b \log(cx^n)))}{bn} - \frac{\coth^2(a+b \log(cx^n))}{2bn}$$

[Out] $-\text{Coth}[a + b \cdot \text{Log}[c \cdot x^n]]^2 / (2 \cdot b \cdot n) + \text{Log}[\text{Sinh}[a + b \cdot \text{Log}[c \cdot x^n]]] / (b \cdot n)$

Rubi [A] time = 0.0402723, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3473, 3475}

$$\frac{\log(\sinh(a+b \log(cx^n)))}{bn} - \frac{\coth^2(a+b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[a + b \cdot \text{Log}[c \cdot x^n]]^3 / x, x]$

[Out] $-\text{Coth}[a + b \cdot \text{Log}[c \cdot x^n]]^2 / (2 \cdot b \cdot n) + \text{Log}[\text{Sinh}[a + b \cdot \text{Log}[c \cdot x^n]]] / (b \cdot n)$

Rule 3473

$\text{Int}[(b \cdot \tan(c \cdot x) + d \cdot x)^n, x_Symbol] \rightarrow \text{Simp}[(b \cdot (b \cdot \tan(c \cdot x) + d \cdot x))^{n-1} / (d \cdot (n-1)), x] - \text{Dist}[b^2, \text{Int}[(b \cdot \tan(c \cdot x) + d \cdot x)^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3475

$\text{Int}[\tan(c \cdot x) + d \cdot x], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c \cdot x + d \cdot x], x]] / d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth^3(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \coth^3(a + bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\coth^2(a + b \log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int \coth(a + bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\coth^2(a + b \log(cx^n))}{2bn} + \frac{\log(\sinh(a + b \log(cx^n)))}{bn} \end{aligned}$$

Mathematica [A] time = 0.206839, size = 52, normalized size = 1.21

$$-\frac{-2 \log(\tanh(a + b \log(cx^n))) - 2 \log(\cosh(a + b \log(cx^n))) + \coth^2(a + b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + b*Log[c*x^n]]^3/x,x]

[Out] -(Coth[a + b*Log[c*x^n]]^2 - 2*Log[Cosh[a + b*Log[c*x^n]]] - 2*Log[Tanh[a + b*Log[c*x^n]]])/(2*b*n)

Maple [A] time = 0.009, size = 67, normalized size = 1.6

$$-\frac{(\coth(a + b \ln(cx^n)))^2}{2bn} - \frac{\ln(\coth(a + b \ln(cx^n)) - 1)}{2bn} - \frac{\ln(\coth(a + b \ln(cx^n)) + 1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+b*ln(c*x^n))^3/x,x)

[Out] -1/2*coth(a+b*ln(c*x^n))^2/b/n-1/2/n/b*ln(coth(a+b*ln(c*x^n))-1)-1/2/n/b*ln(coth(a+b*ln(c*x^n))+1)

Maxima [B] time = 1.37249, size = 446, normalized size = 10.37

$$-\frac{4c^{2b}e^{(2b \log(x^n)+2a)} - 3}{4(bc^4bne^{(4b \log(x^n)+4a)} - 2bc^2bne^{(2b \log(x^n)+2a)} + bn)} - \frac{3(2c^{2b}e^{(2b \log(x^n)+2a)} - 1)}{4(bc^4bne^{(4b \log(x^n)+4a)} - 2bc^2bne^{(2b \log(x^n)+2a)} + bn)} + \frac{1}{4(bc^4bne^{(4b \log(x^n)+4a)} - 2bc^2bne^{(2b \log(x^n)+2a)} + bn)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="maxima")

[Out]
$$-1/4*(4*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a) - 3}/(b*c^{(4*b)*n}*e^{(4*b*\log(x^n) + 4*a) - 2*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a) + b*n} - 3/4*(2*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a) - 1}/(b*c^{(4*b)*n}*e^{(4*b*\log(x^n) + 4*a) - 2*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a) + b*n} + 1/4*(2*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a) - 3}/(b*c^{(4*b)*n}*e^{(4*b*\log(x^n) + 4*a) - 2*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a) + b*n} - 3/4/(b*c^{(4*b)*n}*e^{(4*b*\log(x^n) + 4*a) - 2*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a) + b*n} + \log((c^b*e^{(b*\log(x^n) + a) + 1)*e^{-a})/c^b)/(b*n) + \log((c^b*e^{(b*\log(x^n) + a) - 1)*e^{-a})/c^b)/(b*n) - \log(x)$$

Fricas [B] time = 2.67292, size = 1841, normalized size = 42.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="fricas")

[Out]
$$-(b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^4*\log(x) + 4*b*n*\cosh(b*n*\log(x) + b*\log(c) + a)*\log(x)*\sinh(b*n*\log(x) + b*\log(c) + a)^3 + b*n*\log(x)*\sinh(b*n*\log(x) + b*\log(c) + a)^4 - 2*(b*n*\log(x) - 1)*\cosh(b*n*\log(x) + b*\log(c) + a)^2 + b*n*\log(x) + 2*(3*b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^2*\log(x) - b*n*\log(x) + 1)*\sinh(b*n*\log(x) + b*\log(c) + a)^2 - (\cosh(b*n*\log(x) + b*\log(c) + a)^4 + 4*\cosh(b*n*\log(x) + b*\log(c) + a)*\sinh(b*n*\log(x) + b*\log(c) + a)^3 + \sinh(b*n*\log(x) + b*\log(c) + a)^4 + 2*(3*\cosh(b*n*\log(x) + b*\log(c) + a)^2 - 1)*\sinh(b*n*\log(x) + b*\log(c) + a)^2 - 2*\cosh(b*n*\log(x) + b*\log(c) + a)^2 + 4*(\cosh(b*n*\log(x) + b*\log(c) + a)^3 - \cosh(b*n*\log(x) + b*\log(c) + a))*\sinh(b*n*\log(x) + b*\log(c) + a) + 1)*\log(2*\sinh(b*n*\log(x) + b*\log(c) + a))/(\cosh(b*n*\log(x) + b*\log(c) + a) - \sinh(b*n*\log(x) + b*\log(c) + a))) + 4*(b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^3*\log(x) - (b*n*\log(x) - 1)*\cosh(b*n*\log(x) + b*\log(c) + a))*\sinh(b*n*\log(x) + b*\log(c) + a))/(b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^4 + 4*b*n*\cosh(b*n*\log(x) + b*\log(c) + a)*\sinh(b*n*\log(x) + b*\log(c) + a)^3 + b*n*\sinh(b*n*\log(x) + b*\log(c) + a)^4 - 2*b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^2 + 2*(3*b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^2 - b*n)*\sinh(b*n*\log(x) + b*\log(c) + a)^2 + b*n + 4*(b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^3 - b*n*\cosh(b*n*\log(x) + b*\log(c) + a))*\sinh(b*n*\log(x) + b*\log(c) + a))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*ln(c*x**n))**3/x,x)

[Out] Timed out

Giac [B] time = 1.42911, size = 170, normalized size = 3.95

$$\frac{\log\left(-2x^{2bn}|c|^{2b}\cos(\pi b\operatorname{sgn}(c)-\pi b)e^{(2a)}+x^{4bn}|c|^{4b}e^{(4a)}+1\right)}{2bn}-\frac{3c^{4b}x^{4bn}e^{(4a)}-2c^{2b}x^{2bn}e^{(2a)}+3}{2\left(c^{2b}x^{2bn}e^{(2a)}-1\right)^2bn}-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="giac")

[Out] 1/2*log(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b*n)*abs(c)^(4*b)*e^(4*a) + 1)/(b*n) - 1/2*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 3)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^2*b*n) - log(x)

$$3.192 \quad \int \frac{\coth^4(a+b \log(cx^n))}{x} dx$$

Optimal. Leaf size=45

$$-\frac{\coth^3(a+b \log(cx^n))}{3bn} - \frac{\coth(a+b \log(cx^n))}{bn} + \log(x)$$

[Out] $-(\text{Coth}[a + b*\text{Log}[c*x^n]]/(b*n)) - \text{Coth}[a + b*\text{Log}[c*x^n]]^3/(3*b*n) + \text{Log}[x]$

Rubi [A] time = 0.0374456, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3473, 8}

$$-\frac{\coth^3(a+b \log(cx^n))}{3bn} - \frac{\coth(a+b \log(cx^n))}{bn} + \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[a + b*\text{Log}[c*x^n]]^4/x, x]$

[Out] $-(\text{Coth}[a + b*\text{Log}[c*x^n]]/(b*n)) - \text{Coth}[a + b*\text{Log}[c*x^n]]^3/(3*b*n) + \text{Log}[x]$

Rule 3473

$\text{Int}[(b_* \tan[(c_*) + (d_*)(x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \coth^4(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\coth^3(a + b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \coth^2(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\coth(a + b \log(cx^n))}{bn} - \frac{\coth^3(a + b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int 1 dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\coth(a + b \log(cx^n))}{bn} - \frac{\coth^3(a + b \log(cx^n))}{3bn} + \log(x)
\end{aligned}$$

Mathematica [C] time = 0.114257, size = 44, normalized size = 0.98

$$-\frac{\coth^3(a + b \log(cx^n)) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \tanh^2(a + b \log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + b*Log[c*x^n]]^4/x,x]

[Out] -(Coth[a + b*Log[c*x^n]]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[a + b*Log[c*x^n]]^2])/(3*b*n)

Maple [A] time = 0.003, size = 86, normalized size = 1.9

$$-\frac{(\coth(a + b \ln(cx^n)))^3}{3bn} - \frac{\coth(a + b \ln(cx^n))}{bn} - \frac{\ln(\coth(a + b \ln(cx^n)) - 1)}{2bn} + \frac{\ln(\coth(a + b \ln(cx^n)) + 1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+b*ln(c*x^n))^4/x,x)

[Out] -1/3*coth(a+b*ln(c*x^n))^3/b/n-coth(a+b*ln(c*x^n))/b/n-1/2/n/b*ln(coth(a+b*ln(c*x^n))-1)+1/2/n/b*ln(coth(a+b*ln(c*x^n))+1)

Maxima [B] time = 1.51077, size = 674, normalized size = 14.98

$$-\frac{18c^{4b}e^{(4b \log(x^n)+4a)} - 27c^{2b}e^{(2b \log(x^n)+2a)} + 11}{12(bc^6bne^{(6b \log(x^n)+6a)} - 3bc^4bne^{(4b \log(x^n)+4a)} + 3bc^2bne^{(2b \log(x^n)+2a)} - bn)} - \frac{6c^{4b}e^{(4b \log(x^n)+4a)} - 15c^{2b}e^{(2b \log(x^n)+2a)} + 11}{12(bc^6bne^{(6b \log(x^n)+6a)} - 3bc^4bne^{(4b \log(x^n)+4a)} + 3bc^2bne^{(2b \log(x^n)+2a)} - bn)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="maxima")

[Out]
$$\frac{-1/12*(18*c^{(4*b)}*e^{(4*b*\log(x^n) + 4*a)} - 27*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a)} + 11)/(b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} - 3*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} + 3*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} - b*n) - 1/12*(6*c^{(4*b)}*e^{(4*b*\log(x^n) + 4*a)} - 15*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a)} + 11)/(b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} - 3*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} + 3*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} - b*n) - 2/3*(3*c^{(4*b)}*e^{(4*b*\log(x^n) + 4*a)} - 3*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a)} + 1)/(b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} - 3*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} + 3*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} - b*n) - 1/2*(3*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a)} - 1)/(b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} - 3*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} + 3*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} - b*n) - 2/3/(b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} - 3*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} + 3*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} - b*n) + \log(x)$$

Fricas [B] time = 2.50828, size = 541, normalized size = 12.02

$$\frac{(3bn \log(x) + 4) \sinh(bn \log(x) + b \log(c) + a)^3 - 4 \cosh(bn \log(x) + b \log(c) + a)^3 - 12 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a)^2 + 3((3bn \log(x) + 4) \cosh(bn \log(x) + b \log(c) + a)^2 - 3bn \log(x) - 4) \sinh(bn \log(x) + b \log(c) + a))}{3(bn \sinh(bn \log(x) + b \log(c) + a)^3 + 3(bn \cosh(bn \log(x) + b \log(c) + a)^2 - bn \sinh(bn \log(x) + b \log(c) + a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="fricas")

[Out]
$$\frac{1/3*((3*b*n*\log(x) + 4)*\sinh(b*n*\log(x) + b*\log(c) + a)^3 - 4*\cosh(b*n*\log(x) + b*\log(c) + a)*\sinh(b*n*\log(x) + b*\log(c) + a)^2 + 3*((3*b*n*\log(x) + 4)*\cosh(b*n*\log(x) + b*\log(c) + a)^2 - 3*b*n*\log(x) - 4)*\sinh(b*n*\log(x) + b*\log(c) + a))/(b*n*\sinh(b*n*\log(x) + b*\log(c) + a)^3 + 3*(b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^2 - b*n)*\sinh(b*n*\log(x) + b*\log(c) + a))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*ln(c*x**n))**4/x,x)

[Out] Timed out

Giac [A] time = 1.43402, size = 90, normalized size = 2.

$$-\frac{4 \left(3 c^{4b} x^{4bn} e^{(4a)} - 3 c^{2b} x^{2bn} e^{(2a)} + 2 \right)}{3 \left(c^{2b} x^{2bn} e^{(2a)} - 1 \right)^3 bn} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="giac")

[Out] $-4/3*(3*c^{(4*b)}*x^{(4*b*n)}*e^{(4*a)} - 3*c^{(2*b)}*x^{(2*b*n)}*e^{(2*a)} + 2)/((c^{(2*b)}*x^{(2*b*n)}*e^{(2*a)} - 1)^{3*b*n}) + \log(x)$

$$3.193 \quad \int \frac{\coth^5(a+b \log(cx^n))}{x} dx$$

Optimal. Leaf size=66

$$\frac{\log(\sinh(a+b \log(cx^n)))}{bn} - \frac{\coth^4(a+b \log(cx^n))}{4bn} - \frac{\coth^2(a+b \log(cx^n))}{2bn}$$

[Out] $-\text{Coth}[a + b \cdot \text{Log}[c \cdot x^n]]^2 / (2 \cdot b \cdot n) - \text{Coth}[a + b \cdot \text{Log}[c \cdot x^n]]^4 / (4 \cdot b \cdot n) + \text{Log}[\text{Sinh}[a + b \cdot \text{Log}[c \cdot x^n]]] / (b \cdot n)$

Rubi [A] time = 0.0573933, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3473, 3475}

$$\frac{\log(\sinh(a+b \log(cx^n)))}{bn} - \frac{\coth^4(a+b \log(cx^n))}{4bn} - \frac{\coth^2(a+b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[a + b \cdot \text{Log}[c \cdot x^n]]^5 / x, x]$

[Out] $-\text{Coth}[a + b \cdot \text{Log}[c \cdot x^n]]^2 / (2 \cdot b \cdot n) - \text{Coth}[a + b \cdot \text{Log}[c \cdot x^n]]^4 / (4 \cdot b \cdot n) + \text{Log}[\text{Sinh}[a + b \cdot \text{Log}[c \cdot x^n]]] / (b \cdot n)$

Rule 3473

$\text{Int}[(b \cdot \tan[(c \cdot x) + (d \cdot x)])^n, x_Symbol] \rightarrow \text{Simp}[(b \cdot (b \cdot \tan[c + d \cdot x])^{n-1}) / (d \cdot (n-1)), x] - \text{Dist}[b^2, \text{Int}[(b \cdot \tan[c + d \cdot x])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3475

$\text{Int}[\tan[(c \cdot x) + (d \cdot x)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d \cdot x], x]] / d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^5(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \coth^5(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\coth^4(a + b \log(cx^n))}{4bn} + \frac{\text{Subst}\left(\int \coth^3(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\coth^2(a + b \log(cx^n))}{2bn} - \frac{\coth^4(a + b \log(cx^n))}{4bn} + \frac{\text{Subst}\left(\int \coth(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\coth^2(a + b \log(cx^n))}{2bn} - \frac{\coth^4(a + b \log(cx^n))}{4bn} + \frac{\log(\sinh(a + b \log(cx^n)))}{bn}
\end{aligned}$$

Mathematica [A] time = 0.279886, size = 67, normalized size = 1.02

$$\frac{-4 \log(\tanh(a + b \log(cx^n))) - 4 \log(\cosh(a + b \log(cx^n))) + \coth^4(a + b \log(cx^n)) + 2 \coth^2(a + b \log(cx^n))}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + b*Log[c*x^n]]^5/x, x]

[Out] -(2*Coth[a + b*Log[c*x^n]]^2 + Coth[a + b*Log[c*x^n]]^4 - 4*Log[Cosh[a + b*Log[c*x^n]]] - 4*Log[Tanh[a + b*Log[c*x^n]]])/(4*b*n)

Maple [A] time = 0.003, size = 88, normalized size = 1.3

$$\frac{(\coth(a + b \ln(cx^n)))^4}{4bn} - \frac{(\coth(a + b \ln(cx^n)))^2}{2bn} - \frac{\ln(\coth(a + b \ln(cx^n)) - 1)}{2bn} - \frac{\ln(\coth(a + b \ln(cx^n)) + 1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+b*ln(c*x^n))^5/x, x)

[Out] -1/4*coth(a+b*ln(c*x^n))^4/b/n-1/2*coth(a+b*ln(c*x^n))^2/b/n-1/2/n/b*ln(coth(a+b*ln(c*x^n))-1)-1/2/n/b*ln(coth(a+b*ln(c*x^n))+1)

Maxima [B] time = 1.63914, size = 1154, normalized size = 17.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/24*(48*c^{(6*b)}*e^{(6*b*\log(x^n) + 6*a)} - 108*c^{(4*b)}*e^{(4*b*\log(x^n) + 4*a)} + 88*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a)} - 25)/(b*c^{(8*b)}*n*e^{(8*b*\log(x^n) + 8*a)} - 4*b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} + 6*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} + 4*a) - 4*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} + b*n) + 1/24*(12*c^{(6*b)}*e^{(6*b*\log(x^n) + 6*a)} - 42*c^{(4*b)}*e^{(4*b*\log(x^n) + 4*a)} + 52*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a)} - 25)/(b*c^{(8*b)}*n*e^{(8*b*\log(x^n) + 8*a)} - 4*b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} + 6*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} - 4*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} + b*n) - 5/8*(4*c^{(6*b)}*e^{(6*b*\log(x^n) + 6*a)} - 6*c^{(4*b)}*e^{(4*b*\log(x^n) + 4*a)} + 4*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a)} - 1)/(b*c^{(8*b)}*n*e^{(8*b*\log(x^n) + 8*a)} - 4*b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} + 6*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} - 4*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} + b*n) - 5/12*(6*c^{(4*b)}*e^{(4*b*\log(x^n) + 4*a)} - 4*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a)} + 1)/(b*c^{(8*b)}*n*e^{(8*b*\log(x^n) + 8*a)} - 4*b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} + 6*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} - 4*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} + b*n) - 5/12*(4*c^{(2*b)}*e^{(2*b*\log(x^n) + 2*a)} - 1)/(b*c^{(8*b)}*n*e^{(8*b*\log(x^n) + 8*a)} - 4*b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} + 6*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} - 4*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} + b*n) - 5/8/(b*c^{(8*b)}*n*e^{(8*b*\log(x^n) + 8*a)} - 4*b*c^{(6*b)}*n*e^{(6*b*\log(x^n) + 6*a)} + 6*b*c^{(4*b)}*n*e^{(4*b*\log(x^n) + 4*a)} - 4*b*c^{(2*b)}*n*e^{(2*b*\log(x^n) + 2*a)} + b*n) + \log((c^b*e^{(b*\log(x^n) + a)} + 1)*e^{-a}/c^b)/(b*n) + \log((c^b*e^{(b*\log(x^n) + a)} - 1)*e^{-a}/c^b)/(b*n) - \log(x) \end{aligned}$$

Fricas [B] time = 2.68697, size = 5060, normalized size = 76.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -(b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^8*\log(x) + 8*b*n*\cosh(b*n*\log(x) + b*\log(c) + a)*\log(x)*\sinh(b*n*\log(x) + b*\log(c) + a)^7 + b*n*\log(x)*\sinh(b*n*\log(x) + b*\log(c) + a)^8 - 4*(b*n*\log(x) - 1)*\cosh(b*n*\log(x) + b*\log(c) + a)^6 + 4*(7*b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^2*\log(x) - b*n*\log(x) + 1)*\sinh(b*n*\log(x) + b*\log(c) + a)^6 + 8*(7*b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^3*\log(x) - 3*(b*n*\log(x) - 1)*\cosh(b*n*\log(x) + b*\log(c) + a))*\sinh(b*n*\log(x) + b*\log(c) + a)^5 + 2*(3*b*n*\log(x) - 2)*\cosh(b*n*\log(x) + b*\log(c) + a)^4 + 2*(35*b*n*\cosh(b*n*\log(x) + b*\log(c) + a)^4*\log(x) - 30*(b*n*\log(x) \end{aligned}$$

$$\begin{aligned}
& - 1) * \cosh(b * n * \log(x) + b * \log(c) + a)^2 + 3 * b * n * \log(x) - 2) * \sinh(b * n * \log(x) \\
& + b * \log(c) + a)^4 + 8 * (7 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^5 * \log(x) - 10 \\
& * (b * n * \log(x) - 1) * \cosh(b * n * \log(x) + b * \log(c) + a)^3 + (3 * b * n * \log(x) - 2) * \cosh(b * n * \log(x) + b * \log(c) + a) * \sinh(b * n * \log(x) + b * \log(c) + a)^3 - 4 * (b * n * \log(x) - 1) * \cosh(b * n * \log(x) + b * \log(c) + a)^2 + b * n * \log(x) + 4 * (7 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^6 * \log(x) - 15 * (b * n * \log(x) - 1) * \cosh(b * n * \log(x) + b * \log(c) + a)^4 + 3 * (3 * b * n * \log(x) - 2) * \cosh(b * n * \log(x) + b * \log(c) + a)^2 - b * n * \log(x) + 1) * \sinh(b * n * \log(x) + b * \log(c) + a)^2 - (\cosh(b * n * \log(x) + b * \log(c) + a)^8 + 8 * \cosh(b * n * \log(x) + b * \log(c) + a) * \sinh(b * n * \log(x) + b * \log(c) + a)^7 + \sinh(b * n * \log(x) + b * \log(c) + a)^8 + 4 * (7 * \cosh(b * n * \log(x) + b * \log(c) + a)^2 - 1) * \sinh(b * n * \log(x) + b * \log(c) + a)^6 - 4 * \cosh(b * n * \log(x) + b * \log(c) + a)^6 + 8 * (7 * \cosh(b * n * \log(x) + b * \log(c) + a)^3 - 3 * \cosh(b * n * \log(x) + b * \log(c) + a)) * \sinh(b * n * \log(x) + b * \log(c) + a)^5 + 2 * (35 * \cosh(b * n * \log(x) + b * \log(c) + a)^4 - 30 * \cosh(b * n * \log(x) + b * \log(c) + a)^2 + 3) * \sinh(b * n * \log(x) + b * \log(c) + a)^4 + 6 * \cosh(b * n * \log(x) + b * \log(c) + a)^4 + 8 * (7 * \cosh(b * n * \log(x) + b * \log(c) + a)^5 - 10 * \cosh(b * n * \log(x) + b * \log(c) + a)^3 + 3 * \cosh(b * n * \log(x) + b * \log(c) + a)) * \sinh(b * n * \log(x) + b * \log(c) + a)^3 + 4 * (7 * \cosh(b * n * \log(x) + b * \log(c) + a)^6 - 15 * \cosh(b * n * \log(x) + b * \log(c) + a)^4 + 9 * \cosh(b * n * \log(x) + b * \log(c) + a)^2 - 1) * \sinh(b * n * \log(x) + b * \log(c) + a)^2 - 4 * \cosh(b * n * \log(x) + b * \log(c) + a)^2 + 8 * (\cosh(b * n * \log(x) + b * \log(c) + a)^7 - 3 * \cosh(b * n * \log(x) + b * \log(c) + a)^5 + 3 * \cosh(b * n * \log(x) + b * \log(c) + a)^3 - \cosh(b * n * \log(x) + b * \log(c) + a)) * \sinh(b * n * \log(x) + b * \log(c) + a) + 1) * \log(2 * \sinh(b * n * \log(x) + b * \log(c) + a) / (\cosh(b * n * \log(x) + b * \log(c) + a) - \sinh(b * n * \log(x) + b * \log(c) + a))) + 8 * (b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^7 * \log(x) - 3 * (b * n * \log(x) - 1) * \cosh(b * n * \log(x) + b * \log(c) + a)^5 + (3 * b * n * \log(x) - 2) * \cosh(b * n * \log(x) + b * \log(c) + a)^3 - (b * n * \log(x) - 1) * \cosh(b * n * \log(x) + b * \log(c) + a)) * \sinh(b * n * \log(x) + b * \log(c) + a)) / (b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^8 + 8 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a) * \sinh(b * n * \log(x) + b * \log(c) + a)^7 + b * n * \sinh(b * n * \log(x) + b * \log(c) + a)^8 - 4 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^6 + 4 * (7 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^2 - b * n) * \sinh(b * n * \log(x) + b * \log(c) + a)^6 + 6 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^4 + 8 * (7 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^3 - 3 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)) * \sinh(b * n * \log(x) + b * \log(c) + a)^5 + 2 * (35 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^4 - 30 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^2 + 3 * b * n) * \sinh(b * n * \log(x) + b * \log(c) + a)^4 - 4 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^2 + 8 * (7 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^5 - 10 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^3 + 3 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)) * \sinh(b * n * \log(x) + b * \log(c) + a)^3 + 4 * (7 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^6 - 15 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^4 + 9 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^2 - b * n) * \sinh(b * n * \log(x) + b * \log(c) + a)^2 + b * n + 8 * (b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^7 - 3 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^5 + 3 * b * n * \cosh(b * n * \log(x) + b * \log(c) + a)^3 - b * n * \cosh(b * n * \log(x) + b * \log(c) + a)) * \sinh(b * n * \log(x) + b * \log(c) + a))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*ln(c*x**n))**5/x,x)

[Out] Timed out

Giac [B] time = 1.34413, size = 216, normalized size = 3.27

$$\frac{\log\left(-2x^{2bn}|c|^{2b}\cos(\pi b\operatorname{sgn}(c)-\pi b)e^{(2a)}+x^{4bn}|c|^{4b}e^{(4a)}+1\right)}{2bn}-\frac{25c^{8b}x^{8bn}e^{(8a)}-52c^{6b}x^{6bn}e^{(6a)}+102c^{4b}x^{4bn}e^{(4a)}-12\left(c^{2b}x^{2bn}e^{(2a)}-1\right)^4bn}{12\left(c^{2b}x^{2bn}e^{(2a)}-1\right)^4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="giac")

[Out] $\frac{1}{2}\log(-2x^{(2bn)}\operatorname{abs}(c)^{(2b)}\cos(\pi b\operatorname{sgn}(c)-\pi b)e^{(2a)}+x^{(4bn)}\operatorname{abs}(c)^{(4b)}e^{(4a)}+1)/(bn)-\frac{1}{12}(25c^{(8b)}x^{(8bn)}e^{(8a)}-52c^{(6b)}x^{(6bn)}e^{(6a)}+102c^{(4b)}x^{(4bn)}e^{(4a)}-52c^{(2b)}x^{(2bn)}e^{(2a)}+25)/((c^{(2b)}x^{(2bn)}e^{(2a)}-1)^{4bn})-\log(x)$

3.194 $\int (ex)^m \coth(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=87

$$\frac{(ex)^{m+1}}{e(m+1)} - \frac{2(ex)^{m+1} {}_2F_1\left(1, \frac{m+1}{2bdn}; \frac{m+1}{2bdn} + 1; e^{2ad}(cx^n)^{2bd}\right)}{e(m+1)}$$

[Out] $(e*x)^{(1+m)}/(e*(1+m)) - (2*(e*x)^{(1+m)}*Hypergeometric2F1[1, (1+m)/(2*b*d*n), 1 + (1+m)/(2*b*d*n), E^{(2*a*d)*(c*x^n)^{(2*b*d)}])/(e*(1+m))$

Rubi [F] time = 0.0464916, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \coth(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])], x]

[Out] Defer[Int] [(e*x)^m*Coth[d*(a + b*Log[c*x^n])], x]

Rubi steps

$$\int (ex)^m \coth(d(a + b \log(cx^n))) dx = \int (ex)^m \coth(d(a + b \log(cx^n))) dx$$

Mathematica [A] time = 13.2313, size = 158, normalized size = 1.82

$$\frac{x(ex)^m \left(-\frac{(m+1)e^{2ad}(cx^n)^{2bd} {}_2F_1\left(1, \frac{m+2bdn+1}{2bdn}; \frac{m+4bdn+1}{2bdn}; e^{2ad}(cx^n)^{2bd}\right)}{2bdn+m+1} - {}_2F_1\left(1, \frac{m+1}{2bdn}; \frac{m+1}{2bdn} + 1; e^{2d(a+b \log(cx^n))}\right) \right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])], x]

[Out] $(x*(e*x)^m*(-\text{Hypergeometric2F1}[1, (1+m)/(2*b*d*n), 1+(1+m)/(2*b*d*n), E^{(2*d*(a+b*\text{Log}[c*x^n])}] - (E^{(2*a*d)}*(1+m)*(c*x^n)^{(2*b*d)}*\text{Hypergeometric2F1}[1, (1+m+2*b*d*n)/(2*b*d*n), (1+m+4*b*d*n)/(2*b*d*n), E^{(2*a*d)}*(c*x^n)^{(2*b*d)}])/(1+m+2*b*d*n)))/(1+m)$

Maple [F] time = 1.436, size = 0, normalized size = 0.

$$\int (ex)^m \coth(d(a+b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)`

[Out] `int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^m x x^m}{m+1} - e^m \int \frac{x^m}{c^{bd} e^{(bd \log(x^n) + ad)} + 1} dx + e^m \int \frac{x^m}{c^{bd} e^{(bd \log(x^n) + ad)} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `e^m*x*x^m/(m+1) - e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \coth(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral((e*x)^m*coth(b*d*log(c*x^n) + a*d), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n))),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")

[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d), x)

3.195 $\int (ex)^m \coth^2(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=168

$$-\frac{2(ex)^{m+1} {}_2F_1\left(1, \frac{m+1}{2bdn}; \frac{m+1}{2bdn} + 1; e^{2ad}(cx^n)^{2bd}\right)}{bden} + \frac{(ex)^{m+1} (e^{2ad}(cx^n)^{2bd} + 1)}{bden(1 - e^{2ad}(cx^n)^{2bd})} + \frac{(ex)^{m+1}(bdn + m + 1)}{bde(m + 1)n}$$

[Out] $((1 + m + b*d*n)*(e*x)^{(1 + m)})/(b*d*e*(1 + m)*n) + ((e*x)^{(1 + m)}*(1 + E^{(2*a*d)*(c*x^n)^{(2*b*d)}})/(b*d*e*n*(1 - E^{(2*a*d)*(c*x^n)^{(2*b*d)}})) - (2*(e*x)^{(1 + m)}*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^{(2*a*d)*(c*x^n)^{(2*b*d)}}]/(b*d*e*n))$

Rubi [F] time = 0.0769521, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \coth^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]

[Out] Defer[Int] [(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2, x]

Rubi steps

$$\int (ex)^m \coth^2(d(a + b \log(cx^n))) dx = \int (ex)^m \coth^2(d(a + b \log(cx^n))) dx$$

Mathematica [A] time = 14.4369, size = 312, normalized size = 1.86

$$(ex)^m \left(\frac{x}{m+1} - \frac{x^{-2m} \exp\left(-\frac{(2m+1)(a+b \log(cx^n)-bn \log(x))}{bn}\right) \left((m+1)x^{2bdn+2m+1} \exp\left(\frac{(2bdn+2m+1)(a+b \log(cx^n)-bn \log(x))}{bn}\right) {}_2F_1\left(1, \frac{m+1}{2bdn}\right) \right)}{m+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]

[Out] (e*x)^m*(x/(1 + m) - (E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n)))*(1 + m + 2*b*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n)))*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^(((1 + 2*m + 2*b*d*n)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*(1 + m)*x^(1 + 2*m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(b*d*E^(((1 + 2*m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*n*(1 + m + 2*b*d*n)*x^(2*m)))

Maple [F] time = 0.221, size = 0, normalized size = 0.

$$\int (ex)^m (\coth(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)

[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e^m(m+1) \int \frac{x^m}{bc^{bd} dne^{(bd \log(x^n)+ad)} + bdn} dx + e^m(m+1) \int \frac{x^m}{bc^{bd} dne^{(bd \log(x^n)+ad)} - bdn} dx + \frac{bc^{2bd} de^m n x e^{(2bd \log(x^n)+2ad+n \log(x))}}{(mn+n)bc^{2bd} de^{2(ad+n \log(x))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")

[Out] -e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) - b*d*n), x) + (b*c^(2*b*d)*d*e^m*n*x*e^(2*b*d*log(x^n) + 2*a*d + m*log(x)) - (b*d*e^m*n + 2*e^m*(m + 1))*x*x^m)/((m*n + n)*b*c^(2*b*d)*d*e^(2*b*d*log(x^n) + 2*a*d) - (m*n + n)*b*d)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((ex)^m \coth (bd \log (cx^n) + ad)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")

[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n))))**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth ((b \log (cx^n) + a)d)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")

[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^2, x)

3.196 $\int (ex)^m \coth^3(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=306

$$\frac{(ex)^{m+1} (2b^2d^2n^2 + m^2 + 2m + 1) {}_2F_1\left(1, \frac{m+1}{2bdn}; \frac{m+1}{2bdn} + 1; e^{2ad} (cx^n)^{2bd}\right)}{b^2d^2e(m+1)n^2} + \frac{e^{-2ad} (ex)^{m+1} \left(\frac{e^{4ad} (2bdn+m+1)(cx^n)^{2bd}}{n} + \frac{e^{2ad} (-2bdn+m)}{n}\right)}{2b^2d^2en(1 - e^{2ad} (cx^n)^{2bd})}$$

[Out] $((1 + m + b*d*n)*(1 + m + 2*b*d*n)*(e*x)^{(1 + m)})/(2*b^2*d^2*e*(1 + m)*n^2) - ((e*x)^{(1 + m)}*(1 + E^{(2*a*d)}*(c*x^n)^{(2*b*d)})^2)/(2*b*d*e*n*(1 - E^{(2*a*d)}*(c*x^n)^{(2*b*d)})^2) + ((e*x)^{(1 + m)}*((E^{(2*a*d)}*(1 + m - 2*b*d*n))/n + (E^{(4*a*d)}*(1 + m + 2*b*d*n)*(c*x^n)^{(2*b*d)})/n))/(2*b^2*d^2*e*E^{(2*a*d)}*n*(1 - E^{(2*a*d)}*(c*x^n)^{(2*b*d)})) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^{(1 + m)}*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^{(2*a*d)}*(c*x^n)^{(2*b*d)}])/(b^2*d^2*e*(1 + m)*n^2)$

Rubi [F] time = 0.0694341, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \coth^3(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]

[Out] Defer[Int] [(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3, x]

Rubi steps

$$\int (ex)^m \coth^3(d(a + b \log(cx^n))) dx = \int (ex)^m \coth^3(d(a + b \log(cx^n))) dx$$

Mathematica [A] time = 16.5856, size = 600, normalized size = 1.96

$$x^{-m} (ex)^m (2b^2d^2n^2 + m^2 + 2m + 1) \operatorname{csch}(d(a + b(\log(cx^n) - n \log(x)))) \left(\frac{\sinh(d(a + b(\log(cx^n) - n \log(x)))) \exp\left(-\frac{(2m+1)(a + b(\log(cx^n) - n \log(x)))}{bn}\right)}{\dots} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]

[Out] (x*(e*x)^m*Coth[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m) - (x*(e*x)^m *Csch[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n) + ((1 + m)*x*(e*x)^m*Csch[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csch[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x]])/(2*b^2*d^2*n^2) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Csch[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*((x^(1 + m)*Csch[d*(a + b*Log[c*x^n]))*Sinh[b*d*n*Log[x]])/(1 + m) + (E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m + 2*b*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^((a*(1 + 2*m + 2*b*d*n))/(b*n) + (1 + m + 2*b*d*n)*Log[x] + ((1 + 2*m + 2*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))]*Sinh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*(1 + m + 2*b*d*n)))/(2*b^2*d^2*n^2*x^m)

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int (ex)^m (\coth(d(a + b \ln(cx^n))))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)

[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-(2b^2d^2e^mn^2 + (m^2 + 2m + 1)e^m) \int \frac{x^m}{2(b^2c^{bd}d^2n^2e^{(bd \log(x^n)+ad)} + b^2d^2n^2)} dx + (2b^2d^2e^mn^2 + (m^2 + 2m + 1)e^m) \int \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")

[Out] $-(2*b^2*d^2*e^{m*n^2} + (m^2 + 2*m + 1)*e^m)*\text{integrate}(1/2*x^m/(b^2*c^{(b*d)*d^2*n^2}*e^{(b*d*\log(x^n) + a*d) + b^2*d^2*n^2}), x) + (2*b^2*d^2*e^{m*n^2} + (m^2 + 2*m + 1)*e^m)*\text{integrate}(1/2*x^m/(b^2*c^{(b*d)*d^2*n^2}*e^{(b*d*\log(x^n) + a*d) - b^2*d^2*n^2}), x) + (b^2*c^{(4*b*d)*d^2}*e^{m*n^2}*x*e^{(4*b*d*\log(x^n) + 4*a*d + m*\log(x))} + (b^2*d^2*e^{m*n^2} + (m^2 + 2*m + 1)*e^m)*x*x^m - (2*b^2*c^{(2*b*d)*d^2}*e^{m*n^2}*e^{(2*a*d) + 2*(m*n + n)*b*c^{(2*b*d)*d}*e^m*e^{(2*a*d) + (m^2 + 2*m + 1)*c^{(2*b*d)*e^m*e^{(2*a*d)}})*x*e^{(2*b*d*\log(x^n) + m*\log(x))})/((m*n^2 + n^2)*b^2*c^{(4*b*d)*d^2}*e^{(4*b*d*\log(x^n) + 4*a*d) - 2*(m*n^2 + n^2)*b^2*c^{(2*b*d)*d^2}*e^{(2*b*d*\log(x^n) + 2*a*d) + (m*n^2 + n^2)*b^2*d^2})$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \coth(bd \log(cx^n) + ad)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")

[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n))))**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth((b \log(cx^n) + a)d)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^3, x)
```

3.197 $\int \coth^p(d(a + b \log(cx^n))) dx$

Optimal. Leaf size=115

$$x(-e^{2ad}(cx^n)^{2bd} - 1)^p (e^{2ad}(cx^n)^{2bd} + 1)^{-p} F_1\left(\frac{1}{2bdn}; p, -p; 1 + \frac{1}{2bdn}; e^{2ad}(cx^n)^{2bd}, -e^{2ad}(cx^n)^{2bd}\right)$$

[Out] $(x*(-1 - E^{(2*a*d)*(c*x^n)^{(2*b*d)})}^p * \text{AppellF1}[1/(2*b*d*n), p, -p, 1 + 1/(2*b*d*n), E^{(2*a*d)*(c*x^n)^{(2*b*d)}}, -(E^{(2*a*d)*(c*x^n)^{(2*b*d)})}]) / (1 + E^{(2*a*d)*(c*x^n)^{(2*b*d)})}^p$

Rubi [F] time = 0.0150993, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \coth^p(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[Coth[d*(a + b*Log[c*x^n])]^p, x]

[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^p, x]

Rubi steps

$$\int \coth^p(d(a + b \log(cx^n))) dx = \int \coth^p(d(a + b \log(cx^n))) dx$$

Mathematica [B] time = 3.6288, size = 387, normalized size = 3.37

$$\frac{x(2bdn + 1) \left(\frac{e^{2ad}(cx^n)^{2bd} + 1}{e^{2ad}(cx^n)^{2bd} - 1} \right)^p F_1\left(\frac{1}{2bdn}; p, -p; 1 + \frac{1}{2bdn}; e^{2ad}(cx^n)^{2bd}, -e^{2ad}(cx^n)^{2bd}\right)}{2bdnpe^{2ad}(cx^n)^{2bd} F_1\left(1 + \frac{1}{2bdn}; p, 1 - p; 2 + \frac{1}{2bdn}; e^{2ad}(cx^n)^{2bd}, -e^{2ad}(cx^n)^{2bd}\right) + 2bdnpe^{2ad}(cx^n)^{2bd} F_1\left(1 + \frac{1}{2bdn}; p + 1, -p; 2 + \frac{1}{2bdn}; e^{2ad}(cx^n)^{2bd}, -e^{2ad}(cx^n)^{2bd}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^p, x]

```
[Out] ((1 + 2*b*d*n)*x*((1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(-1 + E^(2*a*d)*(c*x^n)^(2*b*d)))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))]/(2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d))*AppellF1[1 + 1/(2*b*d*n), p, 1 - p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + 2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d)*AppellF1[1 + 1/(2*b*d*n), 1 + p, -p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + (1 + 2*b*d*n)*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])]
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int (\coth(d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \coth((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\coth(bd \log(cx^n) + ad)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n))))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

3.198 $\int (ex)^m \coth^p (d(a + b \log(cx^n))) dx$

Optimal. Leaf size=135

$$\frac{(ex)^{m+1} \left(-e^{2ad} (cx^n)^{2bd} - 1\right)^p \left(e^{2ad} (cx^n)^{2bd} + 1\right)^{-p} F_1\left(\frac{m+1}{2bdn}; p, -p; \frac{m+1}{2bdn} + 1; e^{2ad} (cx^n)^{2bd}, -e^{2ad} (cx^n)^{2bd}\right)}{e(m+1)}$$

[Out] $((e*x)^{(1+m)*(-1 - E^{(2*a*d)*(c*x^n)^{(2*b*d)})})^p \text{AppellF1}[(1+m)/(2*b*d*n), p, -p, 1 + (1+m)/(2*b*d*n), E^{(2*a*d)*(c*x^n)^{(2*b*d)}}, -(E^{(2*a*d)*(c*x^n)^{(2*b*d)})})]/(e*(1+m)*(1 + E^{(2*a*d)*(c*x^n)^{(2*b*d)})})^p$

Rubi [F] time = 0.0973732, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (ex)^m \coth^p (d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p, x]

[Out] Defer[Int] [(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p, x]

Rubi steps

$$\int (ex)^m \coth^p (d(a + b \log(cx^n))) dx = \int (ex)^m \coth^p (d(a + b \log(cx^n))) dx$$

Mathematica [A] time = 5.07176, size = 174, normalized size = 1.29

$$\frac{x(ex)^m \left(1 - e^{2ad} (cx^n)^{2bd}\right)^p \left(e^{2ad} (cx^n)^{2bd} + 1\right)^{-p} \left(\frac{e^{2ad} (cx^n)^{2bd} + 1}{e^{2ad} (cx^n)^{2bd} - 1}\right)^p F_1\left(\frac{m+1}{2bdn}; p, -p; \frac{m+1}{2bdn} + 1; e^{2ad} (cx^n)^{2bd}, -e^{2ad} (cx^n)^{2bd}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p, x]

```
[Out] (x*(e*x)^m*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*((1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(-1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[(1 + m)/(2*b*d*n), p, -p, 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d)))]/((1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (ex)^m (\coth(d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \coth(bd \log(cx^n) + ad)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n))))**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \coth((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")

[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)

$$3.199 \quad \int \frac{\coth^{\frac{5}{2}}(a+b \log(cx^n))}{x} dx$$

Optimal. Leaf size=73

$$-\frac{2 \coth^{\frac{3}{2}}(a+b \log(cx^n))}{3bn} - \frac{\tan^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn}$$

[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) - (2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b*n)

Rubi [A] time = 0.0514403, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3473, 3476, 329, 298, 203, 206}

$$-\frac{2 \coth^{\frac{3}{2}}(a+b \log(cx^n))}{3bn} - \frac{\tan^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]

[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) - (2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b*n)

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^

$n)^p, x], x, (c*x)^{(1/k)}, x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 298

$\text{Int}[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] \text{ :> With}[\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[s/(2*b), \text{Int}[1/(r + s*x^2), x], x] - \text{Dist}[s/(2*b), \text{Int}[1/(r - s*x^2), x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[a/b, 0]$

Rule 203

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \text{ :> Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \text{ :> Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \coth^{\frac{5}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n} \\
 &= -\frac{2 \coth^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \sqrt{\coth(a + bx)} dx, x, \log(cx^n)\right)}{n} \\
 &= -\frac{2 \coth^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2} dx, x, \coth(a + b \log(cx^n))\right)}{bn} \\
 &= -\frac{2 \coth^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} - \frac{2 \text{Subst}\left(\int \frac{x^2}{-1+x^4} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
 &= -\frac{2 \coth^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
 &= -\frac{\tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} - \frac{2 \coth^{\frac{3}{2}}(a + b \log(cx^n))}{3bn}
 \end{aligned}$$

Mathematica [A] time = 0.285752, size = 64, normalized size = 0.88

$$\frac{2 \coth^3(a + b \log(cx^n)) + 3 \tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right) - 3 \tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]

[Out] -(3*ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] - 3*ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]] + 2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b*n)

Maple [A] time = 0.028, size = 93, normalized size = 1.3

$$-\frac{2}{3bn} (\coth(a + b \ln(cx^n)))^3 - \frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} - 1\right) + \frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} + 1\right) - \frac{1}{bn} \arctan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+b*ln(c*x^n))^(5/2)/x,x)

[Out] -2/3*coth(a+b*ln(c*x^n))^(3/2)/b/n-1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(b \log(cx^n) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")

[Out] integrate(coth(b*log(c*x^n) + a)^(5/2)/x, x)

Fricas [B] time = 2.43603, size = 2090, normalized size = 28.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")

[Out]
$$\frac{1}{6} \cdot (6 \cdot (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 1) \cdot \arctan(-\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) - \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 1) \cdot \sqrt{\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) / \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)}) - 4 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 3 \cdot (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 1) \cdot \log(-\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) - \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 1) \cdot \sqrt{\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) / \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)}) - 8 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) - 4 \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 4 \cdot (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + 1) \cdot \sqrt{\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) / \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)} + 4) / (b \cdot n \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + 2 \cdot b \cdot n \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + b \cdot n \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - b \cdot n)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*ln(c*x**n))**(5/2)/x,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.200 \quad \int \frac{\coth^2(a+b \log(cx^n))}{x} dx$$

Optimal. Leaf size=70

$$-\frac{2\sqrt{\coth(a+b \log(cx^n))}}{bn} + \frac{\tan^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn} + \frac{\tanh^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn}$$

[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) - (2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)

Rubi [A] time = 0.0502198, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3473, 3476, 329, 212, 206, 203}

$$-\frac{2\sqrt{\coth(a+b \log(cx^n))}}{bn} + \frac{\tan^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn} + \frac{\tanh^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn}$$

Antiderivative was successfully verified.

[In] Int[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]

[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) - (2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^

$n)^p, x], x, (c*x)^{(1/k)}, x]] /; \text{FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 212

$\text{Int}[(a_) + (b_.)*(x_)^4)^{-1}, x_Symbol] \ :> \ \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^2), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ !\text{GtQ}[a/b, 0]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 203

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \coth^{\frac{3}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n} \\
 &= -\frac{2\sqrt{\coth(a + b \log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\
 &= -\frac{2\sqrt{\coth(a + b \log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(-1+x^2)}} dx, x, \coth(a + b \log(cx^n))\right)}{bn} \\
 &= -\frac{2\sqrt{\coth(a + b \log(cx^n))}}{bn} - \frac{2 \text{Subst}\left(\int \frac{1}{-1+x^4} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
 &= -\frac{2\sqrt{\coth(a + b \log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
 &= \frac{\tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} - \frac{2\sqrt{\coth(a + b \log(cx^n))}}{bn}
 \end{aligned}$$

Mathematica [A] time = 0.146253, size = 57, normalized size = 0.81

$$\frac{-2\sqrt{\coth(a + b \log(cx^n))} + \tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right) + \tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]

[Out] (ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]] - 2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)

Maple [A] time = 0.015, size = 92, normalized size = 1.3

$$-2 \frac{\sqrt{\coth(a + b \ln(cx^n))}}{bn} - \frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} - 1\right) + \frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} + 1\right) + \frac{1}{bn} \arctan\left(\sqrt{\coth(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+b*ln(c*x^n))^(3/2)/x,x)

[Out] -2*coth(a+b*ln(c*x^n))^(1/2)/b/n-1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)+arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(b \log(cx^n) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")

[Out] integrate(coth(b*log(c*x^n) + a)^(3/2)/x, x)

Fricas [B] time = 2.51166, size = 1100, normalized size = 15.71

$$4 \sqrt{\frac{\cosh(bn \log(x) + b \log(c) + a)}{\sinh(bn \log(x) + b \log(c) + a)}} + 2 \arctan\left(-\cosh(bn \log(x) + b \log(c) + a)^2 - 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) - \sinh(bn \log(x) + b \log(c) + a)^2 + (\cosh(bn \log(x) + b \log(c) + a)^2 + 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) + \sinh(bn \log(x) + b \log(c) + a)^2 - 1) \sqrt{\cosh(bn \log(x) + b \log(c) + a) / \sinh(bn \log(x) + b \log(c) + a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")

[Out] -1/2*(4*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a)) + 2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*ln(c*x**n))**(3/2)/x,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")

[Out] Timed out

$$3.201 \quad \int \frac{\sqrt{\coth(a+b \log(cx^n))}}{x} dx$$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn} - \frac{\tan^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn}$$

[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)

Rubi [A] time = 0.0390491, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3476, 329, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn} - \frac{\tan^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]

[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x

], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{\coth(a + b \log(cx^n))}}{x} dx &= \frac{\text{Subst}\left(\int \sqrt{\coth(a + bx)} dx, x, \log(cx^n)\right)}{n} \\
 &= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2} dx, x, \coth(a + b \log(cx^n))\right)}{bn} \\
 &= -\frac{2 \text{Subst}\left(\int \frac{x^2}{-1+x^4} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
 &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
 &= -\frac{\tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn}
 \end{aligned}$$

Mathematica [A] time = 0.075032, size = 48, normalized size = 1.

$$\frac{\tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} - \frac{\tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Coth[a + b*Log[c*x^n]]]/x, x]

[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)

Maple [A] time = 0.013, size = 72, normalized size = 1.5

$$-\frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} - 1\right) + \frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} + 1\right) - \frac{1}{bn} \arctan\left(\sqrt{\coth(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+b*ln(c*x^n))^(1/2)/x,x)

[Out] -1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\coth(b \log(cx^n) + a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(coth(b*log(c*x^n) + a))/x, x)

Fricas [B] time = 2.44119, size = 999, normalized size = 20.81

$$2 \arctan\left(-\cosh(bn \log(x) + b \log(c) + a)^2 - 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) - \sinh(bn \log(x) + b \log(c) + a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")

[Out] 1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - log

$$\frac{(-\cosh(b*n*\log(x) + b*\log(c) + a)^2 - 2*\cosh(b*n*\log(x) + b*\log(c) + a)*\sinh(b*n*\log(x) + b*\log(c) + a) - \sinh(b*n*\log(x) + b*\log(c) + a)^2 + (\cosh(b*n*\log(x) + b*\log(c) + a)^2 + 2*\cosh(b*n*\log(x) + b*\log(c) + a)*\sinh(b*n*\log(x) + b*\log(c) + a) + \sinh(b*n*\log(x) + b*\log(c) + a)^2 - 1)*\sqrt{\cosh(b*n*\log(x) + b*\log(c) + a)/\sinh(b*n*\log(x) + b*\log(c) + a)}}{(b*n)}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\coth(a + b \log(cx^n))}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*ln(c*x**n))**(1/2)/x,x)

[Out] Integral(sqrt(coth(a + b*log(c*x**n)))/x, x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")

[Out] Timed out

$$3.202 \quad \int \frac{1}{x\sqrt{\coth(a+b \log(cx^n))}} dx$$

Optimal. Leaf size=47

$$\frac{\tan^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn}$$

[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)

Rubi [A] time = 0.0398971, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3476, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b \log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]

[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_.))^(m_)*((a_) + (b_.)*(x_.)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k), x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],

$x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}[a/b, 0]$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{\coth(a + b \log(cx^n))}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(-1+x^2)}} dx, x, \coth(a + b \log(cx^n))\right)}{bn} \\ &= \frac{2 \text{Subst}\left(\int \frac{1}{-1+x^4} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\ &= \frac{\tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \end{aligned}$$

Mathematica [A] time = 0.110676, size = 47, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]), x]

[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)

Maple [A] time = 0.015, size = 44, normalized size = 0.9

$$\frac{1}{bn} \arctan\left(\sqrt{\coth(a + b \ln(cx^n))}\right) + \frac{1}{bn} \operatorname{Arctanh}\left(\sqrt{\coth(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/coth(a+b*ln(c*x^n))^(1/2),x)

[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/n

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\coth(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(x*sqrt(coth(b*log(c*x^n) + a))), x)

Fricas [B] time = 2.34765, size = 1000, normalized size = 21.28

$$2 \arctan\left(-\cosh(bn \log(x) + b \log(c) + a)^2 - 2 \cosh(bn \log(x) + b \log(c) + a) \sinh(bn \log(x) + b \log(c) + a) - \sinh(bn \log(x) + b \log(c) + a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")

[Out] -1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)

$$\begin{aligned} &)^2 + (\cosh(b*n*\log(x) + b*\log(c) + a)^2 + 2*\cosh(b*n*\log(x) + b*\log(c) + a) \\ &)*\sinh(b*n*\log(x) + b*\log(c) + a) + \sinh(b*n*\log(x) + b*\log(c) + a)^2 - 1)* \\ &\sqrt{\cosh(b*n*\log(x) + b*\log(c) + a)/\sinh(b*n*\log(x) + b*\log(c) + a)}) + \log \\ &(-\cosh(b*n*\log(x) + b*\log(c) + a)^2 - 2*\cosh(b*n*\log(x) + b*\log(c) + a)*\sinh \\ &(b*n*\log(x) + b*\log(c) + a) - \sinh(b*n*\log(x) + b*\log(c) + a)^2 + (\cosh(b \\ &*n*\log(x) + b*\log(c) + a)^2 + 2*\cosh(b*n*\log(x) + b*\log(c) + a)*\sinh(b*n*\log \\ &(x) + b*\log(c) + a) + \sinh(b*n*\log(x) + b*\log(c) + a)^2 - 1)*\sqrt{\cosh(b*n \\ &*\log(x) + b*\log(c) + a)/\sinh(b*n*\log(x) + b*\log(c) + a)})))/(b*n) \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\coth(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/coth(a+b*ln(c*x**n))**(1/2),x)

[Out] Integral(1/(x*sqrt(coth(a + b*log(c*x**n))))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\coth(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="giac")

[Out] integrate(1/(x*sqrt(coth(b*log(c*x^n) + a))), x)

$$3.203 \quad \int \frac{1}{x \coth^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

Optimal. Leaf size=71

$$-\frac{2}{bn\sqrt{\coth(a+b \log(cx^n))}} - \frac{\tan^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn} + \frac{\tanh^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn}$$

[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])

Rubi [A] time = 0.0517306, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3474, 3476, 329, 298, 203, 206}

$$-\frac{2}{bn\sqrt{\coth(a+b \log(cx^n))}} - \frac{\tan^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn} + \frac{\tanh^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]

[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])

Rule 3474

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^

$n)^p, x], x, (c*x)^{(1/k)], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 298

$\text{Int}[(x_)^2/((a_)+(b_)*(x_)^4), x_Symbol] \ :> \ \text{With}[\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[s/(2*b), \text{Int}[1/(r + s*x^2), x], x] - \text{Dist}[s/(2*b), \text{Int}[1/(r - s*x^2), x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{!GtQ}[a/b, 0]$

Rule 203

$\text{Int}[(a_)+(b_)*(x_)^2]^{-1}, x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 206

$\text{Int}[(a_)+(b_)*(x_)^2]^{-1}, x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x \coth^{\frac{3}{2}}(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{3}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
 &= -\frac{2}{bn\sqrt{\coth(a + b \log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\coth(a + bx)} dx, x, \log(cx^n)\right)}{n} \\
 &= -\frac{2}{bn\sqrt{\coth(a + b \log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2} dx, x, \coth(a + b \log(cx^n))\right)}{bn} \\
 &= -\frac{2}{bn\sqrt{\coth(a + b \log(cx^n))}} - \frac{2 \text{Subst}\left(\int \frac{x^2}{-1+x^4} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
 &= -\frac{2}{bn\sqrt{\coth(a + b \log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
 &= -\frac{\tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn}
 \end{aligned}$$

Mathematica [C] time = 0.144225, size = 44, normalized size = 0.62

$$\frac{{}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; \coth^2(a + b \log(cx^n))\right)}{bn\sqrt{\coth(a + b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]

[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Coth[a + b*Log[c*x^n]]^2])/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])

Maple [A] time = 0.015, size = 93, normalized size = 1.3

$$\frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} + 1\right) - 2 \frac{1}{bn\sqrt{\coth(a + b \ln(cx^n))}} - \frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} - 1\right) - \frac{1}{bn} \arctan\left(\sqrt{\coth(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/coth(a+b*ln(c*x^n))^(3/2),x)

[Out] 1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-2/b/n/coth(a+b*ln(c*x^n))^(1/2)-1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \coth(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(3/2)), x)

Fricas [B] time = 2.84225, size = 2087, normalized size = 29.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")

[Out] $\frac{1}{2} \cdot (2 \cdot (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + 1) \cdot \arctan(-\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) - \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 1) \cdot \sqrt{\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) / \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)}) - 4 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + 1) \cdot \log(-\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) - \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 1) \cdot \sqrt{\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) / \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)}) - 8 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) - 4 \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 4 \cdot (\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a))^2 + 2 \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 - 1) \cdot \sqrt{\cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) / \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)} - 4) / (b \cdot n \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + 2 \cdot b \cdot n \cdot \cosh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + b \cdot n \cdot \sinh(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)^2 + b \cdot n)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/coth(a+b*ln(c*x**n))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \coth(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(3/2)), x)
```

$$3.204 \quad \int \frac{1}{x \coth^{\frac{5}{2}}(a+b \log(cx^n))} dx$$

Optimal. Leaf size=72

$$-\frac{2}{3bn \coth^{\frac{3}{2}}(a+b \log(cx^n))} + \frac{\tan^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn} + \frac{\tanh^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn}$$

[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) - 2/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))

Rubi [A] time = 0.0510296, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3474, 3476, 329, 212, 206, 203}

$$-\frac{2}{3bn \coth^{\frac{3}{2}}(a+b \log(cx^n))} + \frac{\tan^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn} + \frac{\tanh^{-1}(\sqrt{\coth(a+b \log(cx^n))})}{bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]

[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) - 2/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))

Rule 3474

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

Rule 3476

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^

$n^p, x], x, (c*x)^{(1/k)}, x]] /; \text{FreeQ}\{a, b, c, p\}, x] \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{FractionQ}[m] \ \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 212

$\text{Int}[(a_ + (b_.)*(x_)^4)^{-1}, x_Symbol] \ :> \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^2), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ !\text{GtQ}[a/b, 0]$

Rule 206

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \text{NegQ}[a/b] \ \&\& (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 203

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \text{PosQ}[a/b] \ \&\& (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \coth^{\frac{5}{2}}(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{5}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2}{3bn \coth^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2}{3bn \coth^{\frac{3}{2}}(a + b \log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(-1+x^2)}} dx, x, \coth(a + b \log(cx^n))\right)}{bn} \\
&= -\frac{2}{3bn \coth^{\frac{3}{2}}(a + b \log(cx^n))} - \frac{2 \text{Subst}\left(\int \frac{1}{-1+x^4} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
&= -\frac{2}{3bn \coth^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\coth(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{\tan^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a + b \log(cx^n))}\right)}{bn} - \frac{2}{3bn \coth^{\frac{3}{2}}(a + b \log(cx^n))}
\end{aligned}$$

Mathematica [C] time = 0.200434, size = 46, normalized size = 0.64

$$-\frac{{}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; \coth^2(a + b \log(cx^n))\right)}{3bn \coth^{\frac{3}{2}}(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)), x]

[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Coth[a + b*Log[c*x^n]]^2])/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))

Maple [A] time = 0.013, size = 92, normalized size = 1.3

$$\frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} + 1\right) - \frac{2}{3bn} (\coth(a + b \ln(cx^n)))^{-\frac{3}{2}} - \frac{1}{2bn} \ln\left(\sqrt{\coth(a + b \ln(cx^n))} - 1\right) + \frac{1}{bn} \arctan\left(\frac{\sqrt{\coth(a + b \ln(cx^n))}}{\coth(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/coth(a+b*ln(c*x^n))^(5/2),x)`

[Out] $\frac{1}{2} \frac{1}{b n} \ln(\coth(a+b \ln(c x^n))^{1/2+1}) - \frac{2}{3} \frac{1}{b n} \coth(a+b \ln(c x^n))^{3/2-1} - \frac{1}{2} \frac{1}{b n} \ln(\coth(a+b \ln(c x^n))^{1/2-1}) + \arctan(\coth(a+b \ln(c x^n))^{1/2}) / b n$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \coth(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")`

[Out] `integrate(1/(x*coth(b*log(c*x^n) + a)^(5/2)), x)`

Fricas [B] time = 2.88856, size = 3644, normalized size = 50.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & -\frac{1}{6} (4 \cosh(b n \log(x) + b \log(c) + a)^4 + 16 \cosh(b n \log(x) + b \log(c) + a) \sinh(b n \log(x) + b \log(c) + a)^3 \\ & + 4 \sinh(b n \log(x) + b \log(c) + a)^4 + 8 (3 \cosh(b n \log(x) + b \log(c) + a)^2 + 1) \sinh(b n \log(x) + b \log(c) + a)^2 \\ & + 6 (\cosh(b n \log(x) + b \log(c) + a)^4 + 4 \cosh(b n \log(x) + b \log(c) + a) \sinh(b n \log(x) + b \log(c) + a)^3 \\ & + \sinh(b n \log(x) + b \log(c) + a)^4 + 2 (3 \cosh(b n \log(x) + b \log(c) + a)^2 + 1) \sinh(b n \log(x) + b \log(c) + a)^2 \\ & + 2 \cosh(b n \log(x) + b \log(c) + a)^2 + 4 (\cosh(b n \log(x) + b \log(c) + a)^3 + \cosh(b n \log(x) + b \log(c) + a) \sinh(b n \log(x) + b \log(c) + a) \\ & + 1) \arctan(-\cosh(b n \log(x) + b \log(c) + a)^2 - 2 \cosh(b n \log(x) + b \log(c) + a) \sinh(b n \log(x) + b \log(c) + a) \\ & - \sinh(b n \log(x) + b \log(c) + a)^2 + (\cosh(b n \log(x) + b \log(c) + a)^2 + 2 \cosh(b n \log(x) + b \log(c) + a) \sinh(b n \log(x) + b \log(c) + a) \\ & + \sinh(b n \log(x) + b \log(c) + a)^2 - 1) \sqrt{\cosh(b n \log(x) + b \log(c) + a) / \sinh(b n \log(x) + b \log(c) + a)}) \\ & + 8 \cosh(b n \log(x) + b \log(c) + a)^2 + 3 (\cosh(b n \log(x) + b \log(c) + a)^4 + 4 \cosh(b n \log(x) + b \log(c) + a) \sinh(b n \log(x) + b \log(c) + a)^3 \\ & + \sinh(b n \log(x) + b \log(c) + a)^4 + 2 (3 \cosh(b n \log(x) + b \log(c) + a)^2 + 1) \sinh(b n \log(x) + b \log(c) + a)^2 \\ & + 2 \cosh(b n \log(x) + b \log(c) + a)^2 + 4 (\cosh(b n \log(x) + b \log(c) + a)^3 + \cosh(b n \log(x) + b \log(c) + a) \sinh(b n \log(x) + b \log(c) + a) \\ & + 1) \arctan(-\cosh(b n \log(x) + b \log(c) + a)^2 - 2 \cosh(b n \log(x) + b \log(c) + a) \sinh(b n \log(x) + b \log(c) + a) \\ & - \sinh(b n \log(x) + b \log(c) + a)^2 + (\cosh(b n \log(x) + b \log(c) + a)^2 + 2 \cosh(b n \log(x) + b \log(c) + a) \sinh(b n \log(x) + b \log(c) + a) \\ & + \sinh(b n \log(x) + b \log(c) + a)^2 - 1) \sqrt{\cosh(b n \log(x) + b \log(c) + a) / \sinh(b n \log(x) + b \log(c) + a)}) \end{aligned}$$

```

osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sin
h(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(c
osh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a) + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*lo
g(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*
log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b
*log(c) + a))) + 16*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) +
b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 4*(cosh(b*n*log(x) + b*log
(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c)
+ a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*sqrt(cosh(b*n*log(x) + b*log(
c) + a)/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(
c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) +
b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*
n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3
+ b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/coth(a+b*ln(c*x**n))**(5/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \coth(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(5/2)), x)
```

$$3.205 \quad \int \frac{\coth^5(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} dx$$

Optimal. Leaf size=135

$$\frac{(b-2c) \tanh^{-1}\left(\frac{b+2c \coth^2(x)}{2\sqrt{c}\sqrt{a+b \coth^2(x)+c \coth^4(x)}}\right)}{4c^{3/2}} - \frac{\sqrt{a+b \coth^2(x)+c \coth^4(x)}}{2c} + \frac{\tanh^{-1}\left(\frac{2a+(b+2c) \coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b \coth^2(x)+c \coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]])/(4*c^(3/2))) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]])/(2*Sqrt[a + b + c])] - Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]/(2*c)

Rubi [A] time = 0.353085, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3701, 1251, 1653, 843, 621, 206, 724}

$$\frac{(b-2c) \tanh^{-1}\left(\frac{b+2c \coth^2(x)}{2\sqrt{c}\sqrt{a+b \coth^2(x)+c \coth^4(x)}}\right)}{4c^{3/2}} - \frac{\sqrt{a+b \coth^2(x)+c \coth^4(x)}}{2c} + \frac{\tanh^{-1}\left(\frac{2a+(b+2c) \coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b \coth^2(x)+c \coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]])/(4*c^(3/2))) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]])/(2*Sqrt[a + b + c])] - Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]/(2*c)

Rule 3701

Int[cot[(d_.) + (e_.)*(x_.)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_.)]*(f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_.)]*(f_.))^(n2_.))^(p_), x_Symbol]
 := -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^5(x)}{\sqrt{a + b \coth^2(x) + c \coth^4(x)}} dx &= -\text{Subst} \left(\int \frac{x^5}{(1+x^2)\sqrt{a-bx^2+cx^4}} dx, x, -i \coth(x) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(1+x)\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right) \right) \\
&= -\frac{\sqrt{a + b \coth^2(x) + c \coth^4(x)}}{2c} - \frac{\text{Subst} \left(\int \frac{\frac{b}{2} + \frac{1}{2}(b-2c)x}{(1+x)\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right)}{2c} \\
&= -\frac{\sqrt{a + b \coth^2(x) + c \coth^4(x)}}{2c} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right) \\
&= -\frac{\sqrt{a + b \coth^2(x) + c \coth^4(x)}}{2c} - \frac{(b-2c) \text{Subst} \left(\int \frac{1}{4c-x^2} dx, x, \frac{-b-2c \coth^2(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} \right)}{2c} \\
&= \frac{(b-2c) \tanh^{-1} \left(\frac{b+2c \coth^2(x)}{2\sqrt{c}\sqrt{a+b \coth^2(x)+c \coth^4(x)}} \right)}{4c^{3/2}} + \frac{\tanh^{-1} \left(\frac{2a+b+(b+2c) \coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b \coth^2(x)+c \coth^4(x)}} \right)}{2\sqrt{a+b+c}}
\end{aligned}$$

Mathematica [A] time = 9.01482, size = 266, normalized size = 1.97

$$\frac{2c \operatorname{sch}^2(x) \sqrt{\cosh(4x)(a+b+c) - 4(a-c) \cosh(2x) + 3a - b + 3c} \left(2c^{3/2} \tanh^{-1} \left(\frac{\cosh(2x)(a+b+c) - a + c}{2\sqrt{a+b+c} \sqrt{\sinh^4(x)(a+b+c) + (b+2c) \sinh^2(x) + c}} \right) + (b-2c) \sqrt{a+b+c} \tanh^{-1} \left(\frac{(b+2c) \sinh^2(x)}{2\sqrt{c} \sqrt{\sinh^4(x)(a+b+c) + (b+2c) \sinh^2(x) + c}} \right) \right)}{\sqrt{a+b+c}}$$

$$8c^{3/2} \sqrt{\operatorname{csch}^4(x)(\cosh(4x)(a+b+c) - 4(a-c) \cosh(2x) + 3a - b + 3c)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] ((2*(2*c^(3/2)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])) + (b - 2*c)*Sqrt[a + b + c]*ArcTanh[(2*c + (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])))*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/Sqrt[a + b + c] - Sqrt[2]*Sqrt[c]*(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4)/(8*c^(3/2)*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*

Csch[x]^4))

Maple [A] time = 0.104, size = 149, normalized size = 1.1

$$-\frac{1}{2c}\sqrt{a+b(\coth(x))^2+c(\coth(x))^4}+\frac{b}{4}\ln\left(\frac{b}{2}+c(\coth(x))^2\right)\frac{1}{\sqrt{c}}+\sqrt{a+b(\coth(x))^2+c(\coth(x))^4}c^{-\frac{3}{2}}-\frac{1}{2}\ln\left(\frac{b}{2}+c(\coth(x))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)

[Out] $-\frac{1}{2}(a+b\coth(x)^2+c\coth(x)^4)^{1/2}/c+1/4*b/c^{3/2}*\ln((1/2*b+c*\coth(x)^2)/c^{1/2}+(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2})-1/2*\ln((1/2*b+c*\coth(x)^2)/c^{1/2}+(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2})/c^{1/2}+1/2/(a+b+c)^{1/2}*\operatorname{arctanh}(1/2*(b*\coth(x)^2+2*c*\coth(x)^2+2*a+b)/(a+b+c)^{1/2}/(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)

Fricas [B] time = 18.6186, size = 24521, normalized size = 181.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/8*((a*b + b^2 - (2*a + b)*c - 2*c^2)*\cosh(x)^4 + 4*(a*b + b^2 - (2*a + \\ & b)*c - 2*c^2)*\cosh(x)*\sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*\sinh(x) \\ &)^4 - 2*(a*b + b^2 - (2*a + b)*c - 2*c^2)*\cosh(x)^2 + 2*(3*(a*b + b^2 - (2* \\ & a + b)*c - 2*c^2)*\cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*\sinh(x)^2 + \\ & a*b + b^2 - (2*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*\cosh \\ & (x)^3 - (a*b + b^2 - (2*a + b)*c - 2*c^2)*\cosh(x))*\sinh(x))*\sqrt{c}*\log(((b \\ & ^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)*\cos \\ & h(x)*\sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*\sinh(x)^8 - 4*(b^2 + 4*a*c - \\ & 8*c^2)*\cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^2 - b^2 - 4* \\ & a*c + 8*c^2)*\sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^3 - 3*(\\ & b^2 + 4*a*c - 8*c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c \\ & ^2)*\cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^4 - 30*(b^2 + 4 \\ & *a*c - 8*c^2)*\cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*\sinh(x)^4 + 8*(\\ & 7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2)*\cosh(x) \\ &)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*\cosh(x))*\sinh(x)^3 - 4*(b^2 + 4*a* \\ & c - 8*c^2)*\cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^6 - 15*(b \\ & ^2 + 4*a*c - 8*c^2)*\cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*\cosh(x) \\ &)^2 - b^2 - 4*a*c + 8*c^2)*\sinh(x)^2 - 4*\sqrt{2}*((b + 2*c)*\cosh(x)^4 + 4*(\\ & b + 2*c)*\cosh(x)*\sinh(x)^3 + (b + 2*c)*\sinh(x)^4 - 2*(b - 2*c)*\cosh(x)^2 + \\ & 2*(3*(b + 2*c)*\cosh(x)^2 - b + 2*c)*\sinh(x)^2 + 4*((b + 2*c)*\cosh(x)^3 - (b \\ & - 2*c)*\cosh(x))*\sinh(x) + b + 2*c)*\sqrt{c}*\sqrt{((a + b + c)*\cosh(x)^4 + (\\ & a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2 \\ & *a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*c \\ & \cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)} + b^2 + 4*(a + 2*b)* \\ & c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^7 - 3*(b^2 + 4*a*c - 8 \\ & *c^2)*\cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*\cosh(x)^3 - (b^2 + 4*a \\ & *c - 8*c^2)*\cosh(x))*\sinh(x))/(\cosh(x)^8 + 8*\cosh(x)*\sinh(x)^7 + \sinh(x)^8 \\ & + 4*(7*\cosh(x)^2 - 1)*\sinh(x)^6 - 4*\cosh(x)^6 + 8*(7*\cosh(x)^3 - 3*\cosh(x)) \\ & *\sinh(x)^5 + 2*(35*\cosh(x)^4 - 30*\cosh(x)^2 + 3)*\sinh(x)^4 + 6*\cosh(x)^4 + \\ & 8*(7*\cosh(x)^5 - 10*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^3 + 4*(7*\cosh(x)^6 - 15* \\ & \cosh(x)^4 + 9*\cosh(x)^2 - 1)*\sinh(x)^2 - 4*\cosh(x)^2 + 8*(\cosh(x)^7 - 3*\cos \\ & h(x)^5 + 3*\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)) - 2*(c^2*\cosh(x)^4 + 4*c^2*\co \\ & sh(x)*\sinh(x)^3 + c^2*\sinh(x)^4 - 2*c^2*\cosh(x)^2 + 2*(3*c^2*\cosh(x)^2 - c^ \\ & 2)*\sinh(x)^2 + c^2 + 4*(c^2*\cosh(x)^3 - c^2*\cosh(x))*\sinh(x))*\sqrt{a + b + \\ & c}*\log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^8 + 8*(a^2 + 2*a*b \\ & + b^2 + 2*(a + b)*c + c^2)*\cosh(x)*\sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + \\ & b)*c + c^2)*\sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^6 + 4*(7*(a^2 + 2 \\ & *a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(x)^ \\ & 6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^3 - 3*(a^2 + a*b - \\ & b*c - c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\co \\ & sh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^4 - 30*(a^2 \end{aligned}$$

$$\begin{aligned}
& + a*b - b*c - c^2)*\cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\sinh(x) \\
& ^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^5 - 10*(a^2 + a* \\
& b - b*c - c^2)*\cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x))*s \\
& \sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + \\
& 2*(a + b)*c + c^2)*\cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*\cosh(x)^4 + 3*(3* \\
& a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(\\
& x)^2 + \sqrt{2}*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)*\cosh(x)*\sinh(x)^3 + (\\
& a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - a \\
& + c)*\sinh(x)^2 + 4*((a + b + c)*\cosh(x)^3 - (a - c)*\cosh(x))*\sinh(x) + a + \\
& b + c)*\sqrt{a + b + c}*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 \\
& - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 \\
& + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - \\
& 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 \\
& + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^7 - 3*(a^2 + a*b - b*c \\
& - c^2)*\cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^3 - (a^2 \\
& + a*b - b*c - c^2)*\cosh(x))*\sinh(x))/(\cosh(x)^4 + 4*\cosh(x)^3*\sinh(x) + 6*c \\
& osh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + 4*\sqrt{2}*((a + b) \\
& *c + c^2)*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*c \\
& osh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3* \\
& c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sin \\
& h(x)^3 + \sinh(x)^4)))/(((a + b)*c^2 + c^3)*\cosh(x)^4 + 4*((a + b)*c^2 + c^3 \\
&)*\cosh(x)*\sinh(x)^3 + ((a + b)*c^2 + c^3)*\sinh(x)^4 + (a + b)*c^2 + c^3 - 2 \\
& *((a + b)*c^2 + c^3)*\cosh(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^ \\
& 3)*\cosh(x)^2)*\sinh(x)^2 + 4*((a + b)*c^2 + c^3)*\cosh(x)^3 - ((a + b)*c^2 + \\
& c^3)*\cosh(x))*\sinh(x)), -1/8*(4*(c^2*\cosh(x)^4 + 4*c^2*\cosh(x)*\sinh(x)^3 + \\
& c^2*\sinh(x)^4 - 2*c^2*\cosh(x)^2 + 2*(3*c^2*\cosh(x)^2 - c^2)*\sinh(x)^2 + c^ \\
& 2 + 4*(c^2*\cosh(x)^3 - c^2*\cosh(x))*\sinh(x))*\sqrt{-a - b - c}*\arctan(\sqrt{2} \\
&)*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)*\cosh(x)*\sinh(x)^3 + (a + b + c)*\si \\
& nh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - a + c)*\sinh(x) \\
& ^2 + 4*((a + b + c)*\cosh(x)^3 - (a - c)*\cosh(x))*\sinh(x) + a + b + c)*\sqrt{ \\
& -a - b - c}*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c) \\
& *\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + \\
& 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*s \\
& \sinh(x)^3 + \sinh(x)^4)))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^8 + \\
& 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)*\sinh(x)^7 + (a^2 + 2*a*b \\
& + b^2 + 2*(a + b)*c + c^2)*\sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^6 \\
& + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^2 - a^2 - a*b + b*c \\
& + c^2)*\sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^3 \\
& - 3*(a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2 \\
& *(3*a + b)*c + 3*c^2)*\cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + \\
& c^2)*\cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 3*a^2 + 2*a*b - b^2 \\
& + 2*(3*a + b)*c + 3*c^2)*\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c \\
& + c^2)*\cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*\cosh(x)^3 + (3*a^2 + 2*a*b - \\
& b^2 + 2*(3*a + b)*c + 3*c^2)*\cosh(x))*\sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2 \\
&)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^6 - 15*(
\end{aligned}$$

$$\begin{aligned}
& a^2 + a*b - b*c - c^2) * \cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + \\
& 3*c^2) * \cosh(x)^2 - a^2 - a*b + b*c + c^2) * \sinh(x)^2 + a^2 + 2*a*b + b^2 + \\
& 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2) * \cosh(x)^7 - \\
& 3*(a^2 + a*b - b*c - c^2) * \cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c \\
& + 3*c^2) * \cosh(x)^3 - (a^2 + a*b - b*c - c^2) * \cosh(x)) * \sinh(x)) + ((a*b + b \\
& ^2 - (2*a + b)*c - 2*c^2) * \cosh(x)^4 + 4*(a*b + b^2 - (2*a + b)*c - 2*c^2) * \c \\
& osh(x) * \sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2) * \sinh(x)^4 - 2*(a*b + b \\
& ^2 - (2*a + b)*c - 2*c^2) * \cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a + b)*c - 2*c^2 \\
&) * \cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2) * \sinh(x)^2 + a*b + b^2 - (2*a \\
& + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2) * \cosh(x)^3 - (a*b + b \\
& ^2 - (2*a + b)*c - 2*c^2) * \cosh(x)) * \sinh(x)) * \sqrt{c} * \log((b^2 + 4*(a + 2*b) \\
& *c + 8*c^2) * \cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2) * \cosh(x) * \sinh(x)^7 + \\
& (b^2 + 4*(a + 2*b)*c + 8*c^2) * \sinh(x)^8 - 4*(b^2 + 4*a*c - 8*c^2) * \cosh(x)^ \\
& 6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2) * \cosh(x)^2 - b^2 - 4*a*c + 8*c^2) * \sin \\
& h(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2) * \cosh(x)^3 - 3*(b^2 + 4*a*c - 8* \\
& c^2) * \cosh(x)) * \sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2) * \cosh(x)^4 + \\
& 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2) * \cosh(x)^4 - 30*(b^2 + 4*a*c - 8*c^2) * \co \\
& sh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2) * \sinh(x)^4 + 8*(7*(b^2 + 4*(a + \\
& 2*b)*c + 8*c^2) * \cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2) * \cosh(x)^3 + (3*b^2 + 4 \\
& *(3*a - 2*b)*c + 24*c^2) * \cosh(x)) * \sinh(x)^3 - 4*(b^2 + 4*a*c - 8*c^2) * \cosh(\\
& x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2) * \cosh(x)^6 - 15*(b^2 + 4*a*c - 8*c \\
& ^2) * \cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2) * \cosh(x)^2 - b^2 - 4*a* \\
& c + 8*c^2) * \sinh(x)^2 - 4*\sqrt{2}*((b + 2*c) * \cosh(x)^4 + 4*(b + 2*c) * \cosh(x) \\
& * \sinh(x)^3 + (b + 2*c) * \sinh(x)^4 - 2*(b - 2*c) * \cosh(x)^2 + 2*(3*(b + 2*c) * c \\
& osh(x)^2 - b + 2*c) * \sinh(x)^2 + 4*((b + 2*c) * \cosh(x)^3 - (b - 2*c) * \cosh(x)) \\
& * \sinh(x) + b + 2*c) * \sqrt{c} * \sqrt{((a + b + c) * \cosh(x)^4 + (a + b + c) * \sinh(\\
& x)^4 - 4*(a - c) * \cosh(x)^2 + 2*(3*(a + b + c) * \cosh(x)^2 - 2*a + 2*c) * \sinh(x) \\
&)^2 + 3*a - b + 3*c) / (\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x) \\
& ^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + b^2 + 4*(a + 2*b)*c + 8*c^2 + 8*((\\
& b^2 + 4*(a + 2*b)*c + 8*c^2) * \cosh(x)^7 - 3*(b^2 + 4*a*c - 8*c^2) * \cosh(x)^5 \\
& + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2) * \cosh(x)^3 - (b^2 + 4*a*c - 8*c^2) * \cosh \\
& (x)) * \sinh(x)) / (\cosh(x)^8 + 8*\cosh(x) * \sinh(x)^7 + \sinh(x)^8 + 4*(7*\cosh(x)^2 \\
& - 1) * \sinh(x)^6 - 4*\cosh(x)^6 + 8*(7*\cosh(x)^3 - 3*\cosh(x)) * \sinh(x)^5 + 2*(\\
& 35*\cosh(x)^4 - 30*\cosh(x)^2 + 3) * \sinh(x)^4 + 6*\cosh(x)^4 + 8*(7*\cosh(x)^5 - \\
& 10*\cosh(x)^3 + 3*\cosh(x)) * \sinh(x)^3 + 4*(7*\cosh(x)^6 - 15*\cosh(x)^4 + 9*\co \\
& sh(x)^2 - 1) * \sinh(x)^2 - 4*\cosh(x)^2 + 8*(\cosh(x)^7 - 3*\cosh(x)^5 + 3*\cosh(\\
& x)^3 - \cosh(x)) * \sinh(x) + 1)) + 4*\sqrt{2}*((a + b)*c + c^2) * \sqrt{((a + b + \\
& c) * \cosh(x)^4 + (a + b + c) * \sinh(x)^4 - 4*(a - c) * \cosh(x)^2 + 2*(3*(a + b + \\
& c) * \cosh(x)^2 - 2*a + 2*c) * \sinh(x)^2 + 3*a - b + 3*c) / (\cosh(x)^4 - 4*\cosh(x) \\
& ^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) / (((\\
& a + b)*c^2 + c^3) * \cosh(x)^4 + 4*((a + b)*c^2 + c^3) * \cosh(x) * \sinh(x)^3 + ((a \\
& + b)*c^2 + c^3) * \sinh(x)^4 + (a + b)*c^2 + c^3 - 2*((a + b)*c^2 + c^3) * \cosh \\
& (x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^3) * \cosh(x)^2) * \sinh(x)^2 + \\
& 4*(((a + b)*c^2 + c^3) * \cosh(x)^3 - ((a + b)*c^2 + c^3) * \cosh(x)) * \sinh(x)), \\
& -1/4*((a*b + b^2 - (2*a + b)*c - 2*c^2) * \cosh(x)^4 + 4*(a*b + b^2 - (2*a +
\end{aligned}$$


```

((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3
*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 -
4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)
)^4)/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7
+ ((a + b)*c + c^2)*sinh(x)^8 - 4*(a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c +
c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 -
3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(
35*((a + b)*c + c^2)*cosh(x)^4 - 30*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3
*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 - 10*(a*c - c^2)*cosh(x)
^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a*c - c^2)*cosh(x)^2 + 4
*(7*((a + b)*c + c^2)*cosh(x)^6 - 15*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c
+ 3*c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^2 + (a + b)*c + c^2 + 8*((a + b)*
c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x)
^3 - (a*c - c^2)*cosh(x))*sinh(x))) + 2*sqrt(2)*((a + b)*c + c^2)*sqrt(((a
+ b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a
+ b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*
cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4
)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3)*cosh(x)*sinh(x)^
3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 - 2*((a + b)*c^2 + c^
3)*cosh(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^3)*cosh(x)^2)*sinh
(x)^2 + 4*((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2 + c^3)*cosh(x))*sin
h(x))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**5/(a+b*coth(x)**2+c*coth(x)**4)**(1/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^5}{\sqrt{c \coth(x)^4 + b \coth(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

$$3.206 \quad \int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}} dx$$

Optimal. Leaf size=105

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}$$

[Out] -ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[c]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])

Rubi [A] time = 0.216574, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3701, 1251, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] -ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[c]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])

Rule 3701

Int[cot[(d_.) + (e_.)*(x_.)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_.)]*(f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_.)]*(f_.))^(n2_.))^(p_), x_Symbol] :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n 2, 2*n] && NeQ[b^2 - 4*a*c, 0]

Rule 1251

Int[(x_.)^(m_.)*((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +

$b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, d, e, p, q\}, x\} \&\& \text{IntegerQ}[(m - 1)/2]$

Rule 843

$\text{Int}[(d_.) + (e_.)*(x_.)^{(m_.)}*((f_.) + (g_.)*(x_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}), x_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + e*x)^{(m + 1)}*(a + b*x + c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 621

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2], x_Symbol] \rightarrow \text{Dist}[2, \text{Subst}[\text{Int}[1/(4*c - x^2), x], x, (b + 2*c*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 724

$\text{Int}[1/(((d_.) + (e_.)*(x_.)*\text{Sqrt}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2])), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{\sqrt{a + b \coth^2(x) + c \coth^4(x)}} dx &= \text{Subst} \left(\int \frac{x^3}{(1+x^2)\sqrt{a-bx^2+cx^4}} dx, x, -i \coth(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(1+x)\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{4c-x^2} dx, x, \frac{-b-2c \coth^2(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} \right) + \text{Subst} \left(\int \frac{1}{4a+4b+4c-x^2} dx, x, \frac{-b-2c \coth^2(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} \right) \\
&= \frac{\tanh^{-1} \left(\frac{-b-2c \coth^2(x)}{2\sqrt{c}\sqrt{a+b \coth^2(x)+c \coth^4(x)}} \right)}{2\sqrt{c}} + \frac{\tanh^{-1} \left(\frac{2a+b+(b+2c) \coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b \coth^2(x)+c \coth^4(x)}} \right)}{2\sqrt{a+b+c}}
\end{aligned}$$

Mathematica [A] time = 25.8231, size = 199, normalized size = 1.9

$$\frac{\text{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c} \left(\frac{\tanh^{-1} \left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)+(b+2c)\sinh^2(x)+c}} \right)}{\sqrt{a+b+c}} - \frac{\tanh^{-1} \left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{c}\sqrt{a+b \coth^2(x)+c \coth^4(x)}} \right)}{2\sqrt{c}} \right)}{2\sqrt{\text{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] ((ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])]/(2*Sqrt[a + b + c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4]))/Sqrt[a + b + c] - ArcTanh[(2*c + (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])/Sqrt[c])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/(2*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4])

Maple [A] time = 0.063, size = 90, normalized size = 0.9

$$-\frac{1}{2} \ln \left(\left(\frac{b}{2} + c (\coth(x))^2 \right) \frac{1}{\sqrt{c}} + \sqrt{a + b (\coth(x))^2 + c (\coth(x))^4} \right) \frac{1}{\sqrt{c}} + \frac{1}{2} \text{Artanh} \left(\frac{b (\coth(x))^2 + 2c (\coth(x))^2 + 2a}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] -1/2*ln((1/2*b+c*coth(x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1/2)+1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)^3}{\sqrt{c \coth(x)^4 + b \coth(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] time = 14.5732, size = 18436, normalized size = 175.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b + c)*sqrt(c)*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*c
```

$$\begin{aligned}
& \text{osh}(x) * \sinh(x)^3 - 4*(b^2 + 4*a*c - 8*c^2)*\cosh(x)^2 + 4*(7*(b^2 + 4*(a + \\
& 2*b)*c + 8*c^2)*\cosh(x)^6 - 15*(b^2 + 4*a*c - 8*c^2)*\cosh(x)^4 + 3*(3*b^2 + \\
& 4*(3*a - 2*b)*c + 24*c^2)*\cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*\sinh(x)^2 - 4*s \\
& \text{qrt}(2)*((b + 2*c)*\cosh(x)^4 + 4*(b + 2*c)*\cosh(x)*\sinh(x)^3 + (b + 2*c)*\sin \\
& h(x)^4 - 2*(b - 2*c)*\cosh(x)^2 + 2*(3*(b + 2*c)*\cosh(x)^2 - b + 2*c)*\sinh(x) \\
&)^2 + 4*((b + 2*c)*\cosh(x)^3 - (b - 2*c)*\cosh(x))*\sinh(x) + b + 2*c)*\text{sqrt}(c \\
&)*\text{sqrt}(((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 \\
& + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh \\
& (x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \\
& \sinh(x)^4)) + b^2 + 4*(a + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^ \\
& 2)*\cosh(x)^7 - 3*(b^2 + 4*a*c - 8*c^2)*\cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c \\
& + 24*c^2)*\cosh(x)^3 - (b^2 + 4*a*c - 8*c^2)*\cosh(x))*\sinh(x))/(\cosh(x)^8 + \\
& 8*\cosh(x)*\sinh(x)^7 + \sinh(x)^8 + 4*(7*\cosh(x)^2 - 1)*\sinh(x)^6 - 4*\cosh(x) \\
&)^6 + 8*(7*\cosh(x)^3 - 3*\cosh(x))*\sinh(x)^5 + 2*(35*\cosh(x)^4 - 30*\cosh(x)^ \\
& 2 + 3)*\sinh(x)^4 + 6*\cosh(x)^4 + 8*(7*\cosh(x)^5 - 10*\cosh(x)^3 + 3*\cosh(x)) \\
& *\sinh(x)^3 + 4*(7*\cosh(x)^6 - 15*\cosh(x)^4 + 9*\cosh(x)^2 - 1)*\sinh(x)^2 - 4 \\
& *\cosh(x)^2 + 8*(\cosh(x)^7 - 3*\cosh(x)^5 + 3*\cosh(x)^3 - \cosh(x))*\sinh(x) + \\
& 1)) + \text{sqrt}(a + b + c)*c*\log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x) \\
&)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)*\sinh(x)^7 + (a^2 + \\
& 2*a*b + b^2 + 2*(a + b)*c + c^2)*\sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*\cosh \\
& (x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^2 - a^2 - a*b \\
& + b*c + c^2)*\sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh \\
& (x)^3 - 3*(a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2* \\
& (a + b)*c + 3*c^2)*\cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2) \\
&)*\cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + \\
& b)*c + 3*c^2)*\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cos \\
& h(x)^5 - 10*(a^2 + a*b - b*c - c^2)*\cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)* \\
& c + 3*c^2)*\cosh(x))*\sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 4*(7* \\
& (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^6 - 15*(a^2 + a*b - b*c - c \\
& ^2)*\cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^2 - a^2 - a \\
& *b + b*c + c^2)*\sinh(x)^2 + \text{sqrt}(2)*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)* \\
& \cosh(x)*\sinh(x)^3 + (a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + \\
& b + c)*\cosh(x)^2 - a + c)*\sinh(x)^2 + 4*((a + b + c)*\cosh(x)^3 - (a - c)* \\
& \cosh(x))*\sinh(x) + a + b + c)*\text{sqrt}(a + b + c)*\text{sqrt}(((a + b + c)*\cosh(x)^4 + \\
& (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - \\
& 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6* \\
& \cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + a^2 + 2*a*b + b^2 \\
& + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^7 \\
& - 3*(a^2 + a*b - b*c - c^2)*\cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c \\
& ^2)*\cosh(x)^3 - (a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x))/(\cosh(x)^4 + 4*\cosh(x) \\
&)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) \\
&)/((a + b)*c + c^2), -1/4*(2*\text{sqrt}(-a - b - c)*c*\arctan(\text{sqrt}(2)*((a + b + c) \\
&)*\cosh(x)^4 + 4*(a + b + c)*\cosh(x)*\sinh(x)^3 + (a + b + c)*\sinh(x)^4 - 2*(a \\
& - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - a + c)*\sinh(x)^2 + 4*((a + b \\
& + c)*\cosh(x)^3 - (a - c)*\cosh(x))*\sinh(x) + a + b + c)*\text{sqrt}(-a - b - c)*\text{sq}
\end{aligned}$$

$$\begin{aligned} & \text{rt}(((a + b + c) \cosh(x)^4 + (a + b + c) \sinh(x)^4 - 4(a - c) \cosh(x)^2 + 2 \\ & * (3(a + b + c) \cosh(x)^2 - 2a + 2c) \sinh(x)^2 + 3a - b + 3c) / (\cosh(x)^4 \\ & - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4)) / ((a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^8 + 8(a^2 + 2ab \\ & + b^2 + 2(a + b)c + c^2) \cosh(x) \sinh(x)^7 + (a^2 + 2ab + b^2 + 2(a \\ & + b)c + c^2) \sinh(x)^8 - 4(a^2 + ab - bc - c^2) \cosh(x)^6 + 4(7(a^2 + \\ & 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^2 - a^2 - ab + bc + c^2) \sinh(x) \\ &)^6 + 8(7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^3 - 3(a^2 + ab \\ & - bc - c^2) \cosh(x)) \sinh(x)^5 + 2(3a^2 + 2ab - b^2 + 2(3a + b)c + \\ & 3c^2) \cosh(x)^4 + 2(35(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^4 \\ & - 30(a^2 + ab - bc - c^2) \cosh(x)^2 + 3a^2 + 2ab - b^2 + 2(3a + b) \\ &)c + 3c^2) \sinh(x)^4 + 8(7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x) \\ &)^5 - 10(a^2 + ab - bc - c^2) \cosh(x)^3 + (3a^2 + 2ab - b^2 + 2(3a \\ & + b)c + 3c^2) \cosh(x)) \sinh(x)^3 - 4(a^2 + ab - bc - c^2) \cosh(x)^2 + \\ & 4(7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^6 - 15(a^2 + ab - bc \\ & - c^2) \cosh(x)^4 + 3(3a^2 + 2ab - b^2 + 2(3a + b)c + 3c^2) \cosh(x) \\ &)^2 - a^2 - ab + bc + c^2) \sinh(x)^2 + a^2 + 2ab + b^2 + 2(a + b)c + \\ & c^2 + 8((a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^7 - 3(a^2 + ab - \\ & bc - c^2) \cosh(x)^5 + (3a^2 + 2ab - b^2 + 2(3a + b)c + 3c^2) \cosh(x) \\ &)^3 - (a^2 + ab - bc - c^2) \cosh(x)) \sinh(x)) - (a + b + c) \sqrt{c} \log \\ & (((b^2 + 4(a + 2b)c + 8c^2) \cosh(x)^8 + 8(b^2 + 4(a + 2b)c + 8c^2) \\ &) \cosh(x) \sinh(x)^7 + (b^2 + 4(a + 2b)c + 8c^2) \sinh(x)^8 - 4(b^2 + 4a \\ &)c - 8c^2) \cosh(x)^6 + 4(7(b^2 + 4(a + 2b)c + 8c^2) \cosh(x)^2 - b^2 \\ & - 4ac + 8c^2) \sinh(x)^6 + 8(7(b^2 + 4(a + 2b)c + 8c^2) \cosh(x)^3 - \\ & 3(b^2 + 4ac - 8c^2) \cosh(x)) \sinh(x)^5 + 2(3b^2 + 4(3a - 2b)c + \\ & 24c^2) \cosh(x)^4 + 2(35(b^2 + 4(a + 2b)c + 8c^2) \cosh(x)^4 - 30(b^2 \\ & + 4ac - 8c^2) \cosh(x)^2 + 3b^2 + 4(3a - 2b)c + 24c^2) \sinh(x)^4 + \\ & 8(7(b^2 + 4(a + 2b)c + 8c^2) \cosh(x)^5 - 10(b^2 + 4ac - 8c^2) \cosh(x) \\ &)^3 + (3b^2 + 4(3a - 2b)c + 24c^2) \cosh(x)) \sinh(x)^3 - 4(b^2 + \\ & 4ac - 8c^2) \cosh(x)^2 + 4(7(b^2 + 4(a + 2b)c + 8c^2) \cosh(x)^6 - 1 \\ & 5(b^2 + 4ac - 8c^2) \cosh(x)^4 + 3(3b^2 + 4(3a - 2b)c + 24c^2) \cosh(x) \\ &)^2 - b^2 - 4ac + 8c^2) \sinh(x)^2 - 4\sqrt{2}((b + 2c) \cosh(x)^4 + \\ & 4(b + 2c) \cosh(x) \sinh(x)^3 + (b + 2c) \sinh(x)^4 - 2(b - 2c) \cosh(x)^2 \\ & + 2(3(b + 2c) \cosh(x)^2 - b + 2c) \sinh(x)^2 + 4((b + 2c) \cosh(x)^3 \\ & - (b - 2c) \cosh(x)) \sinh(x) + b + 2c) \sqrt{c} \sqrt{((a + b + c) \cosh(x)^4 \\ & + (a + b + c) \sinh(x)^4 - 4(a - c) \cosh(x)^2 + 2(3(a + b + c) \cosh(x)^2 \\ & - 2a + 2c) \sinh(x)^2 + 3a - b + 3c) / (\cosh(x)^4 - 4 \cosh(x)^3 \sinh(x) + \\ & 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4)) + b^2 + 4(a + 2 \\ &)b)c + 8c^2 + 8((b^2 + 4(a + 2b)c + 8c^2) \cosh(x)^7 - 3(b^2 + 4ac \\ & - 8c^2) \cosh(x)^5 + (3b^2 + 4(3a - 2b)c + 24c^2) \cosh(x)^3 - (b^2 + \\ & 4ac - 8c^2) \cosh(x)) \sinh(x)) / (\cosh(x)^8 + 8 \cosh(x) \sinh(x)^7 + \sinh(x) \\ &)^8 + 4(7 \cosh(x)^2 - 1) \sinh(x)^6 - 4 \cosh(x)^6 + 8(7 \cosh(x)^3 - 3 \cosh(x)) \sinh(x)^5 \\ & + 2(35 \cosh(x)^4 - 30 \cosh(x)^2 + 3) \sinh(x)^4 + 6 \cosh(x)^4 \\ & + 8(7 \cosh(x)^5 - 10 \cosh(x)^3 + 3 \cosh(x)) \sinh(x)^3 + 4(7 \cosh(x)^6 - \\ & 15 \cosh(x)^4 + 9 \cosh(x)^2 - 1) \sinh(x)^2 - 4 \cosh(x)^2 + 8(\cosh(x)^7 - 3 \end{aligned}$$

$$\begin{aligned}
& * \cosh(x)^5 + 3 \cosh(x)^3 - \cosh(x) \sinh(x) + 1)) / ((a + b)c + c^2), 1/4 * \\
& 2 * (a + b + c) * \sqrt{-c} * \arctan(1/2 * \sqrt{2} * ((b + 2c) * \cosh(x)^4 + 4 * (b + 2c) \\
&) * \cosh(x) * \sinh(x)^3 + (b + 2c) * \sinh(x)^4 - 2 * (b - 2c) * \cosh(x)^2 + 2 * (3 * (b \\
& + 2c) * \cosh(x)^2 - b + 2c) * \sinh(x)^2 + 4 * ((b + 2c) * \cosh(x)^3 - (b - 2c) \\
&) * \cosh(x) * \sinh(x) + b + 2c) * \sqrt{-c} * \sqrt{((a + b + c) * \cosh(x)^4 + (a + b \\
& + c) * \sinh(x)^4 - 4 * (a - c) * \cosh(x)^2 + 2 * (3 * (a + b + c) * \cosh(x)^2 - 2 * a + 2 \\
& * c) * \sinh(x)^2 + 3 * a - b + 3 * c) / (\cosh(x)^4 - 4 * \cosh(x)^3 * \sinh(x) + 6 * \cosh(x) \\
& ^2 * \sinh(x)^2 - 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4)) / (((a + b)c + c^2) * \cosh(x) \\
& ^8 + 8 * ((a + b)c + c^2) * \cosh(x) * \sinh(x)^7 + ((a + b)c + c^2) * \sinh(x)^8 - \\
& 4 * (a * c - c^2) * \cosh(x)^6 + 4 * (7 * ((a + b)c + c^2) * \cosh(x)^2 - a * c + c^2) * \sin \\
& h(x)^6 + 8 * (7 * ((a + b)c + c^2) * \cosh(x)^3 - 3 * (a * c - c^2) * \cosh(x)) * \sinh(x)^5 \\
& + 2 * ((3 * a - b) * c + 3 * c^2) * \cosh(x)^4 + 2 * (35 * ((a + b)c + c^2) * \cosh(x)^4 - \\
& 30 * (a * c - c^2) * \cosh(x)^2 + (3 * a - b) * c + 3 * c^2) * \sinh(x)^4 + 8 * (7 * ((a + b) * \\
& c + c^2) * \cosh(x)^5 - 10 * (a * c - c^2) * \cosh(x)^3 + ((3 * a - b) * c + 3 * c^2) * \cosh(\\
& x) * \sinh(x)^3 - 4 * (a * c - c^2) * \cosh(x)^2 + 4 * (7 * ((a + b) * c + c^2) * \cosh(x)^6 \\
& - 15 * (a * c - c^2) * \cosh(x)^4 + 3 * ((3 * a - b) * c + 3 * c^2) * \cosh(x)^2 - a * c + c^2) \\
& * \sinh(x)^2 + (a + b) * c + c^2 + 8 * (((a + b) * c + c^2) * \cosh(x)^7 - 3 * (a * c - c^2) \\
&) * \cosh(x)^5 + ((3 * a - b) * c + 3 * c^2) * \cosh(x)^3 - (a * c - c^2) * \cosh(x) * \sinh(\\
& x))) + \sqrt{a + b + c} * c * \log(((a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(\\
& x)^8 + 8 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x) * \sinh(x)^7 + (a^2 + \\
& 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \sinh(x)^8 - 4 * (a^2 + a * b - b * c - c^2) * \cos \\
& h(x)^6 + 4 * (7 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^2 - a^2 - a * b \\
& + b * c + c^2) * \sinh(x)^6 + 8 * (7 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh \\
& (x)^3 - 3 * (a^2 + a * b - b * c - c^2) * \cosh(x)) * \sinh(x)^5 + 2 * (3 * a^2 + 2 * a * b + 2 \\
& * (a + b) * c + 3 * c^2) * \cosh(x)^4 + 2 * (35 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) \\
&) * \cosh(x)^4 - 30 * (a^2 + a * b - b * c - c^2) * \cosh(x)^2 + 3 * a^2 + 2 * a * b + 2 * (a \\
& + b) * c + 3 * c^2) * \sinh(x)^4 + 8 * (7 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \co \\
& sh(x)^5 - 10 * (a^2 + a * b - b * c - c^2) * \cosh(x)^3 + (3 * a^2 + 2 * a * b + 2 * (a + b) \\
&) * c + 3 * c^2) * \cosh(x) * \sinh(x)^3 - 4 * (a^2 + a * b - b * c - c^2) * \cosh(x)^2 + 4 * (7 \\
& * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^6 - 15 * (a^2 + a * b - b * c - \\
& c^2) * \cosh(x)^4 + 3 * (3 * a^2 + 2 * a * b + 2 * (a + b) * c + 3 * c^2) * \cosh(x)^2 - a^2 - \\
& a * b + b * c + c^2) * \sinh(x)^2 + \sqrt{2} * ((a + b + c) * \cosh(x)^4 + 4 * (a + b + c) \\
&) * \cosh(x) * \sinh(x)^3 + (a + b + c) * \sinh(x)^4 - 2 * (a - c) * \cosh(x)^2 + 2 * (3 * (a \\
& + b + c) * \cosh(x)^2 - a + c) * \sinh(x)^2 + 4 * ((a + b + c) * \cosh(x)^3 - (a - c) * \\
& \cosh(x)) * \sinh(x) + a + b + c) * \sqrt{a + b + c} * \sqrt{((a + b + c) * \cosh(x)^4 + \\
& (a + b + c) * \sinh(x)^4 - 4 * (a - c) * \cosh(x)^2 + 2 * (3 * (a + b + c) * \cosh(x)^2 - \\
& 2 * a + 2 * c) * \sinh(x)^2 + 3 * a - b + 3 * c) / (\cosh(x)^4 - 4 * \cosh(x)^3 * \sinh(x) + 6 \\
& * \cosh(x)^2 * \sinh(x)^2 - 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4)) + a^2 + 2 * a * b + b^2 \\
& + 2 * (a + b) * c + c^2 + 8 * ((a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^7 \\
& - 3 * (a^2 + a * b - b * c - c^2) * \cosh(x)^5 + (3 * a^2 + 2 * a * b + 2 * (a + b) * c + 3 * \\
& c^2) * \cosh(x)^3 - (a^2 + a * b - b * c - c^2) * \cosh(x)) * \sinh(x)) / (\cosh(x)^4 + 4 * \c \\
& osh(x)^3 * \sinh(x) + 6 * \cosh(x)^2 * \sinh(x)^2 + 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4) \\
&)) / ((a + b) * c + c^2), -1/2 * (\sqrt{-a - b - c} * c * \arctan(\sqrt{2} * ((a + b + c) * \\
& \cosh(x)^4 + 4 * (a + b + c) * \cosh(x) * \sinh(x)^3 + (a + b + c) * \sinh(x)^4 - 2 * (a \\
& - c) * \cosh(x)^2 + 2 * (3 * (a + b + c) * \cosh(x)^2 - a + c) * \sinh(x)^2 + 4 * ((a + b
\end{aligned}$$

$$\begin{aligned}
& + c) \cosh(x)^3 - (a - c) \cosh(x) \sinh(x) + a + b + c) \sqrt{-a - b - c} \sqrt{ \\
& t(((a + b + c) \cosh(x)^4 + (a + b + c) \sinh(x)^4 - 4(a - c) \cosh(x)^2 + 2 \\
& (3(a + b + c) \cosh(x)^2 - 2a + 2c) \sinh(x)^2 + 3a - b + 3c) / (\cosh(x)^4 \\
& - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh \\
& (x)^4)) / ((a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^8 + 8(a^2 + 2ab \\
& + b^2 + 2(a + b)c + c^2) \cosh(x) \sinh(x)^7 + (a^2 + 2ab + b^2 + 2(a + \\
& b)c + c^2) \sinh(x)^8 - 4(a^2 + ab - bc - c^2) \cosh(x)^6 + 4(7(a^2 + \\
& 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^2 - a^2 - ab + bc + c^2) \sinh(x) \\
& ^6 + 8(7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^3 - 3(a^2 + ab \\
& - bc - c^2) \cosh(x)) \sinh(x)^5 + 2(3a^2 + 2ab - b^2 + 2(3a + b)c + \\
& 3c^2) \cosh(x)^4 + 2(35(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^4 \\
& - 30(a^2 + ab - bc - c^2) \cosh(x)^2 + 3a^2 + 2ab - b^2 + 2(3a + b)c \\
& + 3c^2) \sinh(x)^4 + 8(7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x) \\
& ^5 - 10(a^2 + ab - bc - c^2) \cosh(x)^3 + (3a^2 + 2ab - b^2 + 2(3a + \\
& b)c + 3c^2) \cosh(x)) \sinh(x)^3 - 4(a^2 + ab - bc - c^2) \cosh(x)^2 + 4 \\
& * (7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^6 - 15(a^2 + ab - bc \\
& - c^2) \cosh(x)^4 + 3(3a^2 + 2ab - b^2 + 2(3a + b)c + 3c^2) \cosh(x) \\
& ^2 - a^2 - ab + bc + c^2) \sinh(x)^2 + a^2 + 2ab + b^2 + 2(a + b)c + c \\
& ^2 + 8((a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^7 - 3(a^2 + ab - \\
& bc - c^2) \cosh(x)^5 + (3a^2 + 2ab - b^2 + 2(3a + b)c + 3c^2) \cosh(x) \\
&)^3 - (a^2 + ab - bc - c^2) \cosh(x)) \sinh(x))) - (a + b + c) \sqrt{-c} \arctan \\
& (1/2 \sqrt{2}) * ((b + 2c) \cosh(x)^4 + 4(b + 2c) \cosh(x) \sinh(x)^3 + (b + \\
& 2c) \sinh(x)^4 - 2(b - 2c) \cosh(x)^2 + 2(3(b + 2c) \cosh(x)^2 - b + 2c) \\
&) \sinh(x)^2 + 4((b + 2c) \cosh(x)^3 - (b - 2c) \cosh(x)) \sinh(x) + b + 2c) \\
&) \sqrt{-c} \sqrt{(((a + b + c) \cosh(x)^4 + (a + b + c) \sinh(x)^4 - 4(a - c) \\
&) \cosh(x)^2 + 2(3(a + b + c) \cosh(x)^2 - 2a + 2c) \sinh(x)^2 + 3a - b + \\
& 3c) / (\cosh(x)^4 - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh \\
& (x)^4)) / (((a + b)c + c^2) \cosh(x)^8 + 8((a + b)c + c^2) \c \\
& osh(x) \sinh(x)^7 + ((a + b)c + c^2) \sinh(x)^8 - 4(a*c - c^2) \cosh(x)^6 + \\
& 4(7((a + b)c + c^2) \cosh(x)^2 - a*c + c^2) \sinh(x)^6 + 8(7((a + b)c + \\
& c^2) \cosh(x)^3 - 3(a*c - c^2) \cosh(x)) \sinh(x)^5 + 2((3a - b)c + 3c^2 \\
&) \cosh(x)^4 + 2(35((a + b)c + c^2) \cosh(x)^4 - 30(a*c - c^2) \cosh(x)^2 \\
& + (3a - b)c + 3c^2) \sinh(x)^4 + 8(7((a + b)c + c^2) \cosh(x)^5 - 10(a \\
& *c - c^2) \cosh(x)^3 + ((3a - b)c + 3c^2) \cosh(x)) \sinh(x)^3 - 4(a*c - c \\
& ^2) \cosh(x)^2 + 4(7((a + b)c + c^2) \cosh(x)^6 - 15(a*c - c^2) \cosh(x)^4 \\
& + 3((3a - b)c + 3c^2) \cosh(x)^2 - a*c + c^2) \sinh(x)^2 + (a + b)c + c \\
& ^2 + 8(((a + b)c + c^2) \cosh(x)^7 - 3(a*c - c^2) \cosh(x)^5 + ((3a - b)c \\
& + 3c^2) \cosh(x)^3 - (a*c - c^2) \cosh(x)) \sinh(x)) / ((a + b)c + c^2)]
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{\sqrt{a + b \coth^2(x) + c \coth^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2), x)

[Out] Integral(coth(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2), x, algorithm="giac")

[Out] Timed out

$$3.207 \quad \int \frac{\coth(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} dx$$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

[Out] ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])

Rubi [A] time = 0.117339, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3701, 1247, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])

Rule 3701

Int[cot[(d_.) + (e_.)*(x_.)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_.)]*(f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_.)]*(f_.))^(n2_.))^(p_.), x_Symbol] :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p]/(f^2 + x^2), x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n 2, 2*n] && NeQ[b^2 - 4*a*c, 0]

Rule 1247

Int[(x_.)*((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\coth(x)}{\sqrt{a + b \coth^2(x) + c \coth^4(x)}} dx &= -\text{Subst} \left(\int \frac{x}{(1+x^2)\sqrt{a-bx^2+cx^4}} dx, x, -i \coth(x) \right) \\ &= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right) \right) \\ &= \text{Subst} \left(\int \frac{1}{4a+4b+4c-x^2} dx, x, \frac{2a+b+(b+2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \right) \\ &= \frac{\tanh^{-1} \left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \right)}{2\sqrt{a+b+c}} \end{aligned}$$

Mathematica [B] time = 18.1964, size = 141, normalized size = 2.43

$$\frac{\text{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)+(b+2c)\sinh^2(x)+c}}\right)}{2\sqrt{a+b+c}\sqrt{\text{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] (ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*Sqrt[3*a - b + 3*c - 4*(a - c)*Co

$\text{sh}[2*x] + (a + b + c)*\text{Cosh}[4*x]]*\text{Csch}[x]^2)/(2*\text{Sqrt}[a + b + c]*\text{Sqrt}[(3*a - b + 3*c - 4*(a - c)*\text{Cosh}[2*x] + (a + b + c)*\text{Cosh}[4*x])*\text{Csch}[x]^4])$

Maple [A] time = 0.065, size = 52, normalized size = 0.9

$$\frac{1}{2} \text{Arctanh} \left(\frac{b(\coth(x))^2 + 2c(\coth(x))^2 + 2a + b}{2} \frac{1}{\sqrt{a + b + c}} \frac{1}{\sqrt{a + b(\coth(x))^2 + c(\coth(x))^4}} \right) \frac{1}{\sqrt{a + b + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2), x)`

[Out] `1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{c \coth(x)^4 + b \coth(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2), x, algorithm="maxima")`

[Out] `integrate(coth(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)`

Fricas [B] time = 9.66772, size = 4811, normalized size = 82.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2), x, algorithm="fricas")`

[Out] `[1/4*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a`

$$\begin{aligned}
& + b)*c + c^2)*\sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^6 + 4*(7*(a^2 + \\
& 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(x) \\
&)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^3 - 3*(a^2 + a*b \\
& - b*c - c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)* \\
& \cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^4 - 30*(a \\
& ^2 + a*b - b*c - c^2)*\cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\sinh \\
& (x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^5 - 10*(a^2 + \\
& a*b - b*c - c^2)*\cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)) \\
& *\sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 \\
& + 2*(a + b)*c + c^2)*\cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*\cosh(x)^4 + 3*(\\
& 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh \\
& h(x)^2 + \sqrt{2}*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)*\cosh(x)*\sinh(x)^3 + \\
& (a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - \\
& a + c)*\sinh(x)^2 + 4*((a + b + c)*\cosh(x)^3 - (a - c)*\cosh(x))*\sinh(x) + a \\
& + b + c)*\sqrt{a + b + c}*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x) \\
& ^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^ \\
& 2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 \\
& - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^ \\
& 2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^7 - 3*(a^2 + a*b - b \\
& *c - c^2)*\cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^3 - (a^ \\
& 2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x))/(\cosh(x)^4 + 4*\cosh(x)^3*\sinh(x) + 6 \\
& *\cosh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4))/\sqrt{a + b + c}, - \\
& 1/2*\sqrt{-a - b - c}*\arctan(\sqrt{2}*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)* \\
& \cosh(x)*\sinh(x)^3 + (a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + \\
& b + c)*\cosh(x)^2 - a + c)*\sinh(x)^2 + 4*((a + b + c)*\cosh(x)^3 - (a - c)*c \\
& osh(x))*\sinh(x) + a + b + c)*\sqrt{-a - b - c}*\sqrt{((a + b + c)*\cosh(x)^4 + \\
& (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - \\
& 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6 \\
& *\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)))/((a^2 + 2*a*b + b^ \\
& 2 + 2*(a + b)*c + c^2)*\cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 \\
&)*\cosh(x)*\sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\sinh(x)^8 - 4 \\
& *(a^2 + a*b - b*c - c^2)*\cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c \\
& + c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b \\
& ^2 + 2*(a + b)*c + c^2)*\cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh \\
& (x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*\cosh(x)^4 + 2*(35*(\\
& a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^4 - 30*(a^2 + a*b - b*c - c^ \\
& 2)*\cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*\sinh(x)^4 + 8*(\\
& 7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^5 - 10*(a^2 + a*b - b*c - \\
& c^2)*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*\cosh(x))*\si \\
& nh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2 \\
& *(a + b)*c + c^2)*\cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*\cosh(x)^4 + 3*(3*a \\
& ^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2 \\
&)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 \\
& + 2*(a + b)*c + c^2)*\cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*\cosh(x)^5 + (3* \\
& a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*\cosh(x)^3 - (a^2 + a*b - b*c - c
\end{aligned}$$

$^2) * \cosh(x) * \sinh(x)) / (a + b + c)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a + b \coth^2(x) + c \coth^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)

[Out] Integral(coth(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")

[Out] Timed out

$$3.208 \quad \int \frac{\tanh(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} dx$$

Optimal. Leaf size=106

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}$$

[Out] -ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])

Rubi [A] time = 0.239581, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3701, 1251, 960, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] -ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])

Rule 3701

Int[cot[(d_.) + (e_.)*(x_.)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_.)]*(f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_.)]*(f_.))^(n2_.))^(p_), x_Symbol] :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n 2, 2*n] && NeQ[b^2 - 4*a*c, 0]

Rule 1251

Int[(x_.)^(m_.)*((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +

$b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, d, e, p, q\}, x] \&\& \text{IntegerQ}[(m - 1)/2]$

Rule 960

$\text{Int}[(d + e*x)^m * (f + g*x)^n * (a + b*x + c*x^2)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m * (f + g*x)^n * (a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& (\text{IntegerQ}[p] \parallel (\text{ILtQ}[m, 0] \&\& \text{ILtQ}[n, 0])) \&\& !(\text{IGtQ}[m, 0] \parallel \text{IGtQ}[n, 0])$

Rule 724

$\text{Int}[1/((d + e*x)*\text{Sqrt}[a + b*x + c*x^2]), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

Rule 206

$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{\sqrt{a + b \coth^2(x) + c \coth^4(x)}} dx &= \text{Subst} \left(\int \frac{1}{x(1+x^2)\sqrt{a-bx^2+cx^4}} dx, x, -i \coth(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(1+x)\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{(-1-x)\sqrt{a-bx+cx^2}} + \frac{1}{x\sqrt{a-bx+cx^2}} \right) dx, x, -\coth^2(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(-1-x)\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{a-bx+cx^2}} dx, x, -\coth^2(x) \right) \\
&= -\text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+b\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \right) - \text{Subst} \left(\int \frac{1}{4a+4b+4c-x^2} dx, x, \frac{2a+b\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \right)}{2\sqrt{a}} - \frac{\tanh^{-1} \left(\frac{-2a-b+(-b-2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \right)}{2\sqrt{a+b+c}}
\end{aligned}$$

Mathematica [A] time = 7.82955, size = 203, normalized size = 1.92

$$\frac{\text{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c} \left(-\frac{\tanh^{-1} \left(\frac{2a-(2a+b)\cosh^2(x)}{2\sqrt{a}\sqrt{\cosh^4(x)(a+b+c)-(2a+b)\cosh^2(x)+a}} \right)}{\sqrt{a}} - \frac{\tanh^{-1} \left(\frac{-2a-b+(-b-2c)\cosh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \right)}{2\sqrt{a+b+c}} \right)}{2\sqrt{\text{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] -((-(ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4]])/Sqrt[a]) - ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4]])/Sqrt[a + b + c])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/(2*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4])

Maple [F] time = 0.304, size = 0, normalized size = 0.

$$\int \tanh(x) \frac{1}{\sqrt{a + b(\coth(x))^2 + c(\coth(x))^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)`

[Out] `int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{c \coth(x)^4 + b \coth(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")`

[Out] `integrate(tanh(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)`

Fricas [B] time = 13.8482, size = 18421, normalized size = 173.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")`

[Out] `[1/4*((a + b + c)*sqrt(a)*log(((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 - 30*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 -`

$$\begin{aligned}
& 10*(8*a^2 - b^2 - 4*a*c)*\cosh(x)^3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*\cosh(x)*\sinh(x)^3 - 4*(8*a^2 - b^2 - 4*a*c)*\cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*\cosh(x)^6 - 15*(8*a^2 - b^2 - 4*a*c)*\cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*\cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*\sinh(x)^2 - 4*\sqrt{2}*((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + b)*\sinh(x)^4 - 2*(2*a - b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 - 2*a + b)*\sinh(x)^2 + 4*((2*a + b)*\cosh(x)^3 - (2*a - b)*\cosh(x))*\sinh(x) + 2*a + b)*\sqrt{a}*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4))} + 8*a^2 + 8*a*b + b^2 + 4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*\cosh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c)*\cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*\cosh(x)^3 - (8*a^2 - b^2 - 4*a*c)*\cosh(x))*\sinh(x))/(\cosh(x)^8 + 8*\cosh(x)*\sinh(x)^7 + \sinh(x)^8 + 4*(7*\cosh(x)^2 + 1)*\sinh(x)^6 + 4*\cosh(x)^6 + 8*(7*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^5 + 2*(35*\cosh(x)^4 + 30*\cosh(x)^2 + 3)*\sinh(x)^4 + 6*\cosh(x)^4 + 8*(7*\cosh(x)^5 + 10*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^3 + 4*(7*\cosh(x)^6 + 15*\cosh(x)^4 + 9*\cosh(x)^2 + 1)*\sinh(x)^2 + 4*\cosh(x)^2 + 8*(\cosh(x)^7 + 3*\cosh(x)^5 + 3*\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)) + \sqrt{a + b + c}*a*\log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)*\sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*\cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x))*\sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*\cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(x)^2 + \sqrt{2}*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)*\cosh(x)*\sinh(x)^3 + (a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - a + c)*\sinh(x)^2 + 4*((a + b + c)*\cosh(x)^3 - (a - c)*\cosh(x))*\sinh(x) + a + b + c)*\sqrt{a + b + c}*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4))} + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*\cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^3 - (a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x))/(\cosh(x)^4 + 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)))/(a^2 + a*b + a*c), -1/4*(2*a*\sqrt{-a - b - c})*\arctan(\sqrt{2}*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)*\cosh(x)*\sinh(x)^3 + (a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - a + c)*\sinh(x)^2 + 4*((a + b + c)
\end{aligned}$$

$$\begin{aligned}
& * \cosh(x)^3 - (a - c) * \cosh(x) * \sinh(x) + a + b + c) * \sqrt{-a - b - c} * \sqrt{((a + b + c) * \cosh(x)^4 + (a + b + c) * \sinh(x)^4 - 4 * (a - c) * \cosh(x)^2 + 2 * (3 * (a + b + c) * \cosh(x)^2 - 2 * a + 2 * c) * \sinh(x)^2 + 3 * a - b + 3 * c) / (\cosh(x)^4 - 4 * \cosh(x)^3 * \sinh(x) + 6 * \cosh(x)^2 * \sinh(x)^2 - 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4)) / ((a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^8 + 8 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x) * \sinh(x)^7 + (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \sinh(x)^8 - 4 * (a^2 + a * b - b * c - c^2) * \cosh(x)^6 + 4 * (7 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^2 - a^2 - a * b + b * c + c^2) * \sinh(x)^6 + 8 * (7 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^3 - 3 * (a^2 + a * b - b * c - c^2) * \cosh(x)) * \sinh(x)^5 + 2 * (3 * a^2 + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \cosh(x)^4 + 2 * (35 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^4 - 30 * (a^2 + a * b - b * c - c^2) * \cosh(x)^2 + 3 * a^2 + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \sinh(x)^4 + 8 * (7 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^5 - 10 * (a^2 + a * b - b * c - c^2) * \cosh(x)^3 + (3 * a^2 + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \cosh(x)) * \sinh(x)^3 - 4 * (a^2 + a * b - b * c - c^2) * \cosh(x)^2 + 4 * (7 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^6 - 15 * (a^2 + a * b - b * c - c^2) * \cosh(x)^4 + 3 * (3 * a^2 + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \cosh(x)^2 - a^2 - a * b + b * c + c^2) * \sinh(x)^2 + a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2 + 8 * ((a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^7 - 3 * (a^2 + a * b - b * c - c^2) * \cosh(x)^5 + (3 * a^2 + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \cosh(x)^3 - (a^2 + a * b - b * c - c^2) * \cosh(x)) * \sinh(x))) - (a + b + c) * \sqrt{a} * \log(((8 * a^2 + 8 * a * b + b^2 + 4 * a * c) * \cosh(x)^8 + 8 * (8 * a^2 + 8 * a * b + b^2 + 4 * a * c) * \cosh(x) * \sinh(x)^7 + (8 * a^2 + 8 * a * b + b^2 + 4 * a * c) * \sinh(x)^8 - 4 * (8 * a^2 - b^2 - 4 * a * c) * \cosh(x)^6 + 4 * (7 * (8 * a^2 + 8 * a * b + b^2 + 4 * a * c) * \cosh(x)^2 - 8 * a^2 + b^2 + 4 * a * c) * \sinh(x)^6 + 8 * (7 * (8 * a^2 + 8 * a * b + b^2 + 4 * a * c) * \cosh(x)^3 - 3 * (8 * a^2 - b^2 - 4 * a * c) * \cosh(x)) * \sinh(x)^5 + 2 * (24 * a^2 - 8 * a * b + 3 * b^2 + 12 * a * c) * \cosh(x)^4 + 2 * (35 * (8 * a^2 + 8 * a * b + b^2 + 4 * a * c) * \cosh(x)^4 - 30 * (8 * a^2 - b^2 - 4 * a * c) * \cosh(x)^2 + 24 * a^2 - 8 * a * b + 3 * b^2 + 12 * a * c) * \sinh(x)^4 + 8 * (7 * (8 * a^2 + 8 * a * b + b^2 + 4 * a * c) * \cosh(x)^5 - 10 * (8 * a^2 - b^2 - 4 * a * c) * \cosh(x)^3 + (24 * a^2 - 8 * a * b + 3 * b^2 + 12 * a * c) * \cosh(x)) * \sinh(x)^3 - 4 * (8 * a^2 - b^2 - 4 * a * c) * \cosh(x)^2 + 4 * (7 * (8 * a^2 + 8 * a * b + b^2 + 4 * a * c) * \cosh(x)^6 - 15 * (8 * a^2 - b^2 - 4 * a * c) * \cosh(x)^4 + 3 * (24 * a^2 - 8 * a * b + 3 * b^2 + 12 * a * c) * \cosh(x)^2 - 8 * a^2 + b^2 + 4 * a * c) * \sinh(x)^2 - 4 * \sqrt{2} * ((2 * a + b) * \cosh(x)^4 + 4 * (2 * a + b) * \cosh(x) * \sinh(x)^3 + (2 * a + b) * \sinh(x)^4 - 2 * (2 * a - b) * \cosh(x)^2 + 2 * (3 * (2 * a + b) * \cosh(x)^2 - 2 * a + b) * \sinh(x)^2 + 4 * ((2 * a + b) * \cosh(x)^3 - (2 * a - b) * \cosh(x)) * \sinh(x) + 2 * a + b) * \sqrt{a} * \sqrt{((a + b + c) * \cosh(x)^4 + (a + b + c) * \sinh(x)^4 - 4 * (a - c) * \cosh(x)^2 + 2 * (3 * (a + b + c) * \cosh(x)^2 - 2 * a + 2 * c) * \sinh(x)^2 + 3 * a - b + 3 * c) / (\cosh(x)^4 - 4 * \cosh(x)^3 * \sinh(x) + 6 * \cosh(x)^2 * \sinh(x)^2 - 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4)) + 8 * a^2 + 8 * a * b + b^2 + 4 * a * c + 8 * ((8 * a^2 + 8 * a * b + b^2 + 4 * a * c) * \cosh(x)^7 - 3 * (8 * a^2 - b^2 - 4 * a * c) * \cosh(x)^5 + (24 * a^2 - 8 * a * b + 3 * b^2 + 12 * a * c) * \cosh(x)^3 - (8 * a^2 - b^2 - 4 * a * c) * \cosh(x)) * \sinh(x)) / (\cosh(x)^8 + 8 * \cosh(x) * \sinh(x)^7 + \sinh(x)^8 + 4 * (7 * \cosh(x)^2 + 1) * \sinh(x)^6 + 4 * \cosh(x)^6 + 8 * (7 * \cosh(x)^3 + 3 * \cosh(x)) * \sinh(x)^5 + 2 * (35 * \cosh(x)^4 + 30 * \cosh(x)^2 + 3) * \sinh(x)^4 + 6 * \cosh(x)^4 + 8 * (7 * \cosh(x)^5 + 10 * \cosh(x)^3 + 3 * \cosh(x)) * \sinh(x)^3 + 4 * (7 * \cosh(x)^6 + 15 * \cosh(x)
\end{aligned}$$

$$\begin{aligned}
&)^4 + 9*\cosh(x)^2 + 1)*\sinh(x)^2 + 4*\cosh(x)^2 + 8*(\cosh(x)^7 + 3*\cosh(x)^5 \\
&+ 3*\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)))/(a^2 + a*b + a*c), 1/4*(2*\sqrt{-a} \\
&*(a + b + c)*\arctan(1/2*\sqrt{2}*((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)* \\
&\sinh(x)^3 + (2*a + b)*\sinh(x)^4 - 2*(2*a - b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 - 2*a + b)*\sinh(x)^2 + 4*((2*a + b)*\cosh(x)^3 - (2*a - b)*\cosh(x))* \\
&\sinh(x) + 2*a + b)*\sqrt{-a}*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)))/((a^2 + a*b + a*c)*\cosh(x)^8 + 8*(a^2 + a*b + a*c)*\cosh(x)*\sinh(x)^7 + (a^2 + a*b + a*c)*\sinh(x)^8 - 4*(a^2 - a*c)*\cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*\cosh(x)^2 - a^2 + a*c)*\sinh(x)^6 + 8*(7*(a^2 + a*b + a*c)*\cosh(x)^3 - 3*(a^2 - a*c)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 - a*b + 3*a*c)*\cosh(x)^4 + 2*(35*(a^2 + a*b + a*c)*\cosh(x)^4 - 30*(a^2 - a*c)*\cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*\sinh(x)^4 + 8*(7*(a^2 + a*b + a*c)*\cosh(x)^5 - 10*(a^2 - a*c)*\cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*\cosh(x))*\sinh(x)^3 - 4*(a^2 - a*c)*\cosh(x)^2 + 4*(7*(a^2 + a*b + a*c)*\cosh(x)^6 - 15*(a^2 - a*c)*\cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*\cosh(x)^2 - a^2 + a*c)*\sinh(x)^2 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*\cosh(x)^7 - 3*(a^2 - a*c)*\cosh(x)^5 + (3*a^2 - a*b + 3*a*c)*\cosh(x)^3 - (a^2 - a*c)*\cosh(x))*\sinh(x))) + \sqrt{a + b + c}*a*\log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)*\sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*\cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x))*\sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*\cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(x)^2 + \sqrt{2}*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)*\cosh(x)*\sinh(x)^3 + (a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - a + c)*\sinh(x)^2 + 4*((a + b + c)*\cosh(x)^3 - (a - c)*\cosh(x))*\sinh(x) + a + b + c)*\sqrt{a + b + c}*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*\cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^3 - (a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x))/(\cosh(x)^4 + 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)))/(a^2 + a*b + a*c), 1/2*(\sqrt{-a}*(a + b + c)*\arctan(1/2*\sqrt{2}*((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + b)*\sinh(x)^4 - 2*(2*a - b)*\cosh
\end{aligned}$$

$$\begin{aligned}
& (x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 - 2*a + b)*\sinh(x)^2 + 4*((2*a + b)*\cosh(x) \\
&)^3 - (2*a - b)*\cosh(x))*\sinh(x) + 2*a + b)*\sqrt{-a}*\sqrt{((a + b + c)*\cosh \\
& (x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh \\
& (x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh \\
& (x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4))/((a^2 + a*b \\
& + a*c)*\cosh(x)^8 + 8*(a^2 + a*b + a*c)*\cosh(x)*\sinh(x)^7 + (a^2 + a*b + a* \\
& c)*\sinh(x)^8 - 4*(a^2 - a*c)*\cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*\cosh(x)^2 - \\
& a^2 + a*c)*\sinh(x)^6 + 8*(7*(a^2 + a*b + a*c)*\cosh(x)^3 - 3*(a^2 - a*c)*\co \\
& sh(x))*\sinh(x)^5 + 2*(3*a^2 - a*b + 3*a*c)*\cosh(x)^4 + 2*(35*(a^2 + a*b + a \\
& *c)*\cosh(x)^4 - 30*(a^2 - a*c)*\cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*\sinh(x)^4 + \\
& 8*(7*(a^2 + a*b + a*c)*\cosh(x)^5 - 10*(a^2 - a*c)*\cosh(x)^3 + (3*a^2 - a*b \\
& + 3*a*c)*\cosh(x))*\sinh(x)^3 - 4*(a^2 - a*c)*\cosh(x)^2 + 4*(7*(a^2 + a*b + \\
& a*c)*\cosh(x)^6 - 15*(a^2 - a*c)*\cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*\cosh(x) \\
& ^2 - a^2 + a*c)*\sinh(x)^2 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*\cosh(x)^ \\
& 7 - 3*(a^2 - a*c)*\cosh(x)^5 + (3*a^2 - a*b + 3*a*c)*\cosh(x)^3 - (a^2 - a*c) \\
& *\cosh(x))*\sinh(x))) - a*\sqrt{-a - b - c}*\arctan(\sqrt{2}*((a + b + c)*\cosh(x) \\
&)^4 + 4*(a + b + c)*\cosh(x)*\sinh(x)^3 + (a + b + c)*\sinh(x)^4 - 2*(a - c)*\c \\
& osh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - a + c)*\sinh(x)^2 + 4*((a + b + c)*\c \\
& osh(x)^3 - (a - c)*\cosh(x))*\sinh(x) + a + b + c)*\sqrt{-a - b - c}*\sqrt{((a \\
& + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a \\
& + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\c \\
& osh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4) \\
&))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 \\
& + 2*(a + b)*c + c^2)*\cosh(x)*\sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c \\
& + c^2)*\sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^6 + 4*(7*(a^2 + 2*a*b \\
& + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(x)^6 + 8 \\
& *(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^3 - 3*(a^2 + a*b - b*c \\
& - c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2) \\
& *\cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^4 - 30*(\\
& a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3* \\
& c^2)*\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^5 - 1 \\
& 0*(a^2 + a*b - b*c - c^2)*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c \\
& + 3*c^2)*\cosh(x))*\sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 4*(7*(a \\
& ^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2) \\
&)*\cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*\cosh(x)^2 - a \\
& ^2 - a*b + b*c + c^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8 \\
& *((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^7 - 3*(a^2 + a*b - b*c - \\
& c^2)*\cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*\cosh(x)^3 - \\
& (a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x)))/((a^2 + a*b + a*c)]
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{a + b \coth^2(x) + c \coth^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)

[Out] Integral(tanh(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{c \coth(x)^4 + b \coth(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")

[Out] integrate(tanh(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)

$$3.209 \quad \int \frac{\tanh^3(x)}{\sqrt{a+b \coth^2(x)+c \coth^4(x)}} dx$$

Optimal. Leaf size=183

$$\frac{b \tanh^{-1}\left(\frac{2a+b \coth^2(x)}{2\sqrt{a}\sqrt{a+b \coth^2(x)+c \coth^4(x)}}\right)}{4a^{3/2}} - \frac{\tanh^2(x)\sqrt{a+b \coth^2(x)+c \coth^4(x)}}{2a} - \frac{\tanh^{-1}\left(\frac{2a+b \coth^2(x)}{2\sqrt{a}\sqrt{a+b \coth^2(x)+c \coth^4(x)}}\right)}{2\sqrt{a}} +$$

[Out] $-\text{ArcTanh}[(2*a + b*\text{Coth}[x]^2)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*\text{Coth}[x]^2 + c*\text{Coth}[x]^4])]/(2*\text{Sqrt}[a]) + (b*\text{ArcTanh}[(2*a + b*\text{Coth}[x]^2)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*\text{Coth}[x]^2 + c*\text{Coth}[x]^4)])/(4*a^{(3/2)}) + \text{ArcTanh}[(2*a + b + (b + 2*c)*\text{Coth}[x]^2)/(2*\text{Sqrt}[a + b + c]*\text{Sqrt}[a + b*\text{Coth}[x]^2 + c*\text{Coth}[x]^4])]/(2*\text{Sqrt}[a + b + c]) - (\text{Sqrt}[a + b*\text{Coth}[x]^2 + c*\text{Coth}[x]^4]*\text{Tanh}[x]^2)/(2*a)$

Rubi [A] time = 0.332368, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3701, 1251, 960, 730, 724, 206}

$$\frac{b \tanh^{-1}\left(\frac{2a+b \coth^2(x)}{2\sqrt{a}\sqrt{a+b \coth^2(x)+c \coth^4(x)}}\right)}{4a^{3/2}} - \frac{\tanh^2(x)\sqrt{a+b \coth^2(x)+c \coth^4(x)}}{2a} - \frac{\tanh^{-1}\left(\frac{2a+b \coth^2(x)}{2\sqrt{a}\sqrt{a+b \coth^2(x)+c \coth^4(x)}}\right)}{2\sqrt{a}} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tanh}[x]^3/\text{Sqrt}[a + b*\text{Coth}[x]^2 + c*\text{Coth}[x]^4], x]$

[Out] $-\text{ArcTanh}[(2*a + b*\text{Coth}[x]^2)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*\text{Coth}[x]^2 + c*\text{Coth}[x]^4])]/(2*\text{Sqrt}[a]) + (b*\text{ArcTanh}[(2*a + b*\text{Coth}[x]^2)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*\text{Coth}[x]^2 + c*\text{Coth}[x]^4)])/(4*a^{(3/2)}) + \text{ArcTanh}[(2*a + b + (b + 2*c)*\text{Coth}[x]^2)/(2*\text{Sqrt}[a + b + c]*\text{Sqrt}[a + b*\text{Coth}[x]^2 + c*\text{Coth}[x]^4])]/(2*\text{Sqrt}[a + b + c]) - (\text{Sqrt}[a + b*\text{Coth}[x]^2 + c*\text{Coth}[x]^4]*\text{Tanh}[x]^2)/(2*a)$

Rule 3701

$\text{Int}[\text{cot}[(d_.) + (e_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\text{cot}[(d_.) + (e_.)*(x_.)]*(f_.))^{(n_.) + (c_.)*\text{cot}[(d_.) + (e_.)*(x_.)]*(f_.)}^{(n2_.)}]^{(p_.)}, x_Symbol]$
 $:= -\text{Dist}[f/e, \text{Subst}[\text{Int}[(x/f)^m*(a + b*x^n + c*x^{(2*n)})^p]/(f^2 + x^2), x], x, f*\text{Cot}[d + e*x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, f, m, n, p\}, x] \ \&\& \ \text{EqQ}[n, 2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 730

```
Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{\sqrt{a + b \coth^2(x) + c \coth^4(x)}} dx &= -\text{Subst} \left(\int \frac{1}{x^3 (1+x^2) \sqrt{a - bx^2 + cx^4}} dx, x, -i \coth(x) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(1+x) \sqrt{a - bx + cx^2}} dx, x, -\coth^2(x) \right) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x^2 \sqrt{a - bx + cx^2}} - \frac{1}{x \sqrt{a - bx + cx^2}} + \frac{1}{(1+x) \sqrt{a - bx + cx^2}} \right) dx, x, -\coth^2(x) \right) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 \sqrt{a - bx + cx^2}} dx, x, -\coth^2(x) \right) \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x \sqrt{a - bx + cx^2}} dx, x, -\coth^2(x) \right) \\
&= -\frac{\sqrt{a + b \coth^2(x) + c \coth^4(x)} \tanh^2(x)}{2a} - \frac{b \text{Subst} \left(\int \frac{1}{x \sqrt{a - bx + cx^2}} dx, x, -\coth^2(x) \right)}{4a} \\
&= -\frac{\tanh^{-1} \left(\frac{2a + b \coth^2(x)}{2\sqrt{a} \sqrt{a + b \coth^2(x) + c \coth^4(x)}} \right)}{2\sqrt{a}} + \frac{\tanh^{-1} \left(\frac{2a + b + (b+2c) \coth^2(x)}{2\sqrt{a+b+c} \sqrt{a + b \coth^2(x) + c \coth^4(x)}} \right)}{2\sqrt{a+b+c}} \\
&= -\frac{\tanh^{-1} \left(\frac{2a + b \coth^2(x)}{2\sqrt{a} \sqrt{a + b \coth^2(x) + c \coth^4(x)}} \right)}{2\sqrt{a}} + \frac{b \tanh^{-1} \left(\frac{2a + b \coth^2(x)}{2\sqrt{a} \sqrt{a + b \coth^2(x) + c \coth^4(x)}} \right)}{4a^{3/2}} + \dots
\end{aligned}$$

Mathematica [B] time = 20.6866, size = 2574, normalized size = 14.07

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] Sqrt[(3*a - b + 3*c - 4*a*Cosh[2*x] + 4*c*Cosh[2*x] + a*Cosh[4*x] + b*Cosh[4*x] + c*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x])] * (-1/(2*a) + Sech[x]^2/(2*a)) + (((2*a - b)*Sqrt[a + b + c]*ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4]]) + 2*a^(3/2)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4]])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2*((-5*Sqrt[(3*a)/(3 - 4*Cosh[2*x] + Cosh[4*x]) - b/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (3*c)/(3 - 4*Cosh[2*x] + Cosh[4*x]) - (4*a*Cosh[2*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (4*c*Cosh[2*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (a*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (b*Cosh[4*x])/(3 - 4*Cosh[2*x] + Cosh[4*x]) + (c*Cosh[4*x])/(3 - 4*Cos

$$\begin{aligned}
& h[2x] + \text{Cosh}[4x]) * \text{Sech}[x] * \text{Sinh}[3x]) / (2 * (3a - b + 3c - 4a * \text{Cosh}[2x] + \\
& 4c * \text{Cosh}[2x] + a * \text{Cosh}[4x] + b * \text{Cosh}[4x] + c * \text{Cosh}[4x])) + (b * \text{Sqrt}[(3a) / \\
& (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) - b / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) + (3c) / (3 \\
& - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) - (4a * \text{Cosh}[2x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) \\
& + (4c * \text{Cosh}[2x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) + (a * \text{Cosh}[4x]) / (3 - 4 * \text{Cosh} \\
& [2x] + \text{Cosh}[4x]) + (b * \text{Cosh}[4x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) + (c * \text{Cosh}[\\
& 4x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x])) * \text{Sech}[x] * \text{Sinh}[3x]) / (a * (3a - b + 3c - \\
& 4a * \text{Cosh}[2x] + 4c * \text{Cosh}[2x] + a * \text{Cosh}[4x] + b * \text{Cosh}[4x] + c * \text{Cosh}[4x])) \\
& + (\text{Sqrt}[(3a) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) - b / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x] \\
&) + (3c) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) - (4a * \text{Cosh}[2x]) / (3 - 4 * \text{Cosh}[2x] \\
& + \text{Cosh}[4x]) + (4c * \text{Cosh}[2x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) + (a * \text{Cosh}[4x] \\
&) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) + (b * \text{Cosh}[4x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x] \\
& x]) + (c * \text{Cosh}[4x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x])) * \text{Sech}[x] * \text{Sinh}[5x]) / (2 * (3 \\
& * a - b + 3c - 4a * \text{Cosh}[2x] + 4c * \text{Cosh}[2x] + a * \text{Cosh}[4x] + b * \text{Cosh}[4x] + \\
& c * \text{Cosh}[4x])) + (5 * \text{Sqrt}[(3a) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) - b / (3 - 4 * \text{Cosh} \\
& [2x] + \text{Cosh}[4x]) + (3c) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) - (4a * \text{Cosh}[2x]) / \\
& (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) + (4c * \text{Cosh}[2x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x] \\
&) + (a * \text{Cosh}[4x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) + (b * \text{Cosh}[4x]) / (3 - 4 * \text{Cos} \\
& h[2x] + \text{Cosh}[4x]) + (c * \text{Cosh}[4x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x])) * \text{Tanh}[x] / \\
& (3a - b + 3c - 4a * \text{Cosh}[2x] + 4c * \text{Cosh}[2x] + a * \text{Cosh}[4x] + b * \text{Cosh}[4x] \\
& + c * \text{Cosh}[4x]) - (3 * b * \text{Sqrt}[(3a) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) - b / (3 - 4 * \\
& \text{Cosh}[2x] + \text{Cosh}[4x]) + (3c) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) - (4a * \text{Cosh}[2x] \\
& x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) + (4c * \text{Cosh}[2x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh} \\
& [4x]) + (a * \text{Cosh}[4x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x]) + (b * \text{Cosh}[4x]) / (3 - 4 \\
& * \text{Cosh}[2x] + \text{Cosh}[4x]) + (c * \text{Cosh}[4x]) / (3 - 4 * \text{Cosh}[2x] + \text{Cosh}[4x])) * \text{Tanh} \\
& [x]) / (a * (3a - b + 3c - 4a * \text{Cosh}[2x] + 4c * \text{Cosh}[2x] + a * \text{Cosh}[4x] + b * \text{Co} \\
& sh[4x] + c * \text{Cosh}[4x]))) / (4a^(3/2) * \text{Sqrt}[a + b + c] * \text{Sqrt}[(3a - b + 3c - \\
& 4 * (a - c) * \text{Cosh}[2x] + (a + b + c) * \text{Cosh}[4x]) * \text{Csch}[x]^4 * (-(((2a - b) * \text{Sqrt}[\\
& a + b + c] * \text{ArcTanh}[(2a - (2a + b) * \text{Cosh}[x]^2) / (2 * \text{Sqrt}[a] * \text{Sqrt}[a - (2a + b) \\
&) * \text{Cosh}[x]^2 + (a + b + c) * \text{Cosh}[x]^4)]) + 2a^(3/2) * \text{ArcTanh}[(-a + c + (a + b \\
& + c) * \text{Cosh}[2x]) / (2 * \text{Sqrt}[a + b + c] * \text{Sqrt}[a - (2a + b) * \text{Cosh}[x]^2 + (a + b + \\
& c) * \text{Cosh}[x]^4)]) * \text{Sqrt}[3a - b + 3c - 4 * (a - c) * \text{Cosh}[2x] + (a + b + c) * \text{Co} \\
& sh[4x]] * \text{Coth}[x] * \text{Csch}[x]^2) / (2a^(3/2) * \text{Sqrt}[a + b + c] * \text{Sqrt}[(3a - b + 3c \\
& - 4 * (a - c) * \text{Cosh}[2x] + (a + b + c) * \text{Cosh}[4x]) * \text{Csch}[x]^4]) + (\text{Sqrt}[3a - b \\
& + 3c - 4 * (a - c) * \text{Cosh}[2x] + (a + b + c) * \text{Cosh}[4x]) * \text{Csch}[x]^2 * (((2a - b) * \\
& \text{Sqrt}[a + b + c] * (-(((2a + b) * \text{Cosh}[x] * \text{Sinh}[x]) / (\text{Sqrt}[a] * \text{Sqrt}[a - (2a + b) * \\
& \text{Cosh}[x]^2 + (a + b + c) * \text{Cosh}[x]^4))) - ((2a - (2a + b) * \text{Cosh}[x]^2) * (-2 * (2 \\
& a + b) * \text{Cosh}[x] * \text{Sinh}[x] + 4 * (a + b + c) * \text{Cosh}[x]^3 * \text{Sinh}[x])) / (4 * \text{Sqrt}[a] * (a - \\
& (2a + b) * \text{Cosh}[x]^2 + (a + b + c) * \text{Cosh}[x]^4)^(3/2)))) / (1 - (2a - (2a + b) \\
& * \text{Cosh}[x]^2)^2 / (4a * (a - (2a + b) * \text{Cosh}[x]^2 + (a + b + c) * \text{Cosh}[x]^4)) + (2 \\
& * a^(3/2) * (-((-a + c + (a + b + c) * \text{Cosh}[2x]) * (-2 * (2a + b) * \text{Cosh}[x] * \text{Sinh}[x] \\
& + 4 * (a + b + c) * \text{Cosh}[x]^3 * \text{Sinh}[x])) / (4 * \text{Sqrt}[a + b + c] * (a - (2a + b) * \text{Cosh} \\
& [x]^2 + (a + b + c) * \text{Cosh}[x]^4)^(3/2)) + (\text{Sqrt}[a + b + c] * \text{Sinh}[2x]) / \text{Sqrt}[a - \\
& (2a + b) * \text{Cosh}[x]^2 + (a + b + c) * \text{Cosh}[x]^4])) / (1 - (-a + c + (a + b + c) * \\
& \text{Cosh}[2x])^2 / (4 * (a + b + c) * (a - (2a + b) * \text{Cosh}[x]^2 + (a + b + c) * \text{Cosh}[x]^
\end{aligned}$$

```

4)))))/(4*a^(3/2)*Sqrt[a + b + c]*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x]
+ (a + b + c)*Cosh[4*x])*Csch[x]^4)) + (((2*a - b)*Sqrt[a + b + c]*ArcTanh
[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a +
b + c)*Cosh[x]^4]]) + 2*a^(3/2)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(
2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4]])*
Csch[x]^2*(-8*(a - c)*Sinh[2*x] + 4*(a + b + c)*Sinh[4*x]))/(8*a^(3/2)*Sqrt
[a + b + c]*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x
]]*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[
x]^4)) - (((2*a - b)*Sqrt[a + b + c]*ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2
*Sqrt[a]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4]]) + 2*a^(3/2
)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a
+ b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4]])*Sqrt[3*a - b + 3*c - 4*(a - c)*
Cosh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2*(-4*(3*a - b + 3*c - 4*(a - c)
*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Coth[x]*Csch[x]^4 + Csch[x]^4*(-8*(a -
c)*Sinh[2*x] + 4*(a + b + c)*Sinh[4*x])))/(8*a^(3/2)*Sqrt[a + b + c]*((3*a
- b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4)^(3/2)))
)

```

Maple [F] time = 0.274, size = 0, normalized size = 0.

$$\int (\tanh(x))^3 \frac{1}{\sqrt{a + b(\coth(x))^2 + c(\coth(x))^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)

[Out] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\sqrt{c \coth(x)^4 + b \coth(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")

```
[Out] integrate(tanh(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] time = 19.6342, size = 24602, normalized size = 134.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^4 + 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(a)*log(((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 - 30*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 - 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 - 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^2 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*sinh(x)^2 + 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c)*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 - (8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^
```

$$\begin{aligned}
& 5 + 3\cosh(x)^3 + \cosh(x))\sinh(x) + 1)) - 2*(a^2\cosh(x)^4 + 4*a^2\cosh(x) \\
& * \sinh(x)^3 + a^2\sinh(x)^4 + 2*a^2\cosh(x)^2 + 2*(3*a^2\cosh(x)^2 + a^2)*\sinh(x)^2 + a^2 + 4*(a^2\cosh(x)^3 + a^2\cosh(x))\sinh(x))\sqrt{a + b + c} \log \\
& \left((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)\sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)\cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)^2 - a^2 - a*b + b*c + c^2)\sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)\cosh(x))\sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)\cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)\cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)\cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)\cosh(x))\sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)\cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)\cosh(x)^2 - a^2 - a*b + b*c + c^2)\sinh(x)^2 + \sqrt{2}*((a + b + c)\cosh(x)^4 + 4*(a + b + c)\cosh(x)\sinh(x)^3 + (a + b + c)\sinh(x)^4 - 2*(a - c)\cosh(x)^2 + 2*(3*(a + b + c)\cosh(x)^2 - a + c)\sinh(x)^2 + 4*((a + b + c)\cosh(x)^3 - (a - c)\cosh(x))\sinh(x) + a + b + c)\sqrt{a + b + c}\sqrt{((a + b + c)\cosh(x)^4 + (a + b + c)\sinh(x)^4 - 4*(a - c)\cosh(x)^2 + 2*(3*(a + b + c)\cosh(x)^2 - 2*a + 2*c)\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)} + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)\cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)\cosh(x)^3 - (a^2 + a*b - b*c - c^2)\cosh(x))\sinh(x)/(\cosh(x)^4 + 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4) + 4*\sqrt{2}*(a^2 + a*b + a*c)\sqrt{((a + b + c)\cosh(x)^4 + (a + b + c)\sinh(x)^4 - 4*(a - c)\cosh(x)^2 + 2*(3*(a + b + c)\cosh(x)^2 - 2*a + 2*c)\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4))/((a^3 + a^2*b + a^2*c)\cosh(x)^4 + 4*(a^3 + a^2*b + a^2*c)\cosh(x)\sinh(x)^3 + (a^3 + a^2*b + a^2*c)\sinh(x)^4 + a^3 + a^2*b + a^2*c + 2*(a^3 + a^2*b + a^2*c)\cosh(x)^2 + 2*(a^3 + a^2*b + a^2*c + 3*(a^3 + a^2*b + a^2*c)\cosh(x)^2)\sinh(x)^2 + 4*((a^3 + a^2*b + a^2*c)\cosh(x)^3 + (a^3 + a^2*b + a^2*c)\cosh(x))\sinh(x)), -1/8*(4*(a^2\cosh(x)^4 + 4*a^2\cosh(x))\sinh(x)^3 + a^2\sinh(x)^4 + 2*a^2\cosh(x)^2 + 2*(3*a^2\cosh(x)^2 + a^2)\sinh(x)^2 + a^2 + 4*(a^2\cosh(x)^3 + a^2\cosh(x))\sinh(x))\sqrt{-a - b - c} \arctan(\sqrt{2}*((a + b + c)\cosh(x)^4 + 4*(a + b + c)\cosh(x)\sinh(x)^3 + (a + b + c)\sinh(x)^4 - 2*(a - c)\cosh(x)^2 + 2*(3*(a + b + c)\cosh(x)^2 - a + c)\sinh(x)^2 + 4*((a + b + c)\cosh(x)^3 - (a - c)\cosh(x))\sinh(x) + a + b + c)\sqrt{-a - b - c}\sqrt{((a + b + c)\cosh(x)^4 + (a + b + c)\sinh(x)^4 - 4*(a - c)\cosh(x)^2 + 2*(3*(a + b + c)\cosh(x)^2 - 2*a + 2*c)\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)\cosh(x)\sinh(x)^7 +
\end{aligned}$$

$$\begin{aligned}
& (a^2 + 2ab + b^2 + 2(a + b)c + c^2) \sinh(x)^8 - 4(a^2 + ab - bc - c^2) \cosh(x)^6 + 4(7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^2 - a^2 - ab + bc + c^2) \sinh(x)^6 + 8(7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^3 - 3(a^2 + ab - bc - c^2) \cosh(x)) \sinh(x)^5 + 2(3a^2 + 2ab - b^2 + 2(3a + b)c + 3c^2) \cosh(x)^4 + 2(35(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^4 - 30(a^2 + ab - bc - c^2) \cosh(x)^2 + 3a^2 + 2ab - b^2 + 2(3a + b)c + 3c^2) \sinh(x)^4 + 8(7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^5 - 10(a^2 + ab - bc - c^2) \cosh(x)^3 + (3a^2 + 2ab - b^2 + 2(3a + b)c + 3c^2) \cosh(x)) \sinh(x)^3 - 4(a^2 + ab - bc - c^2) \cosh(x)^2 + 4(7(a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^6 - 15(a^2 + ab - bc - c^2) \cosh(x)^4 + 3(3a^2 + 2ab - b^2 + 2(3a + b)c + 3c^2) \cosh(x)^2 - a^2 - ab + bc + c^2) \sinh(x)^2 + a^2 + 2ab + b^2 + 2(a + b)c + c^2 + 8((a^2 + 2ab + b^2 + 2(a + b)c + c^2) \cosh(x)^7 - 3(a^2 + ab - bc - c^2) \cosh(x)^5 + (3a^2 + 2ab - b^2 + 2(3a + b)c + 3c^2) \cosh(x)^3 - (a^2 + ab - bc - c^2) \cosh(x)) \sinh(x)) + ((2a^2 + ab - b^2 + (2a - b)c) \cosh(x)^4 + 4(2a^2 + ab - b^2 + (2a - b)c) \cosh(x) \sinh(x)^3 + (2a^2 + ab - b^2 + (2a - b)c) \sinh(x)^4 + 2(2a^2 + ab - b^2 + (2a - b)c) \cosh(x)^2 + 2(3(2a^2 + ab - b^2 + (2a - b)c) \cosh(x)^2 + 2a^2 + ab - b^2 + (2a - b)c) \sinh(x)^2 + 2a^2 + ab - b^2 + (2a - b)c + 4((2a^2 + ab - b^2 + (2a - b)c) \cosh(x)^3 + (2a^2 + ab - b^2 + (2a - b)c) \cosh(x)) \sinh(x)) \sqrt{a} \log(((8a^2 + 8ab + b^2 + 4ac) \cosh(x)^8 + 8(8a^2 + 8ab + b^2 + 4ac) \cosh(x) \sinh(x)^7 + (8a^2 + 8ab + b^2 + 4ac) \sinh(x)^8 - 4(8a^2 - b^2 - 4ac) \cosh(x)^6 + 4(7(8a^2 + 8ab + b^2 + 4ac) \cosh(x)^2 - 8a^2 + b^2 + 4ac) \sinh(x)^6 + 8(7(8a^2 + 8ab + b^2 + 4ac) \cosh(x)^3 - 3(8a^2 - b^2 - 4ac) \cosh(x)) \sinh(x)^5 + 2(24a^2 - 8ab + 3b^2 + 12ac) \cosh(x)^4 + 2(35(8a^2 + 8ab + b^2 + 4ac) \cosh(x)^4 - 30(8a^2 - b^2 - 4ac) \cosh(x)^2 + 24a^2 - 8ab + 3b^2 + 12ac) \sinh(x)^4 + 8(7(8a^2 + 8ab + b^2 + 4ac) \cosh(x)^5 - 10(8a^2 - b^2 - 4ac) \cosh(x)^3 + (24a^2 - 8ab + 3b^2 + 12ac) \cosh(x)) \sinh(x)^3 - 4(8a^2 - b^2 - 4ac) \cosh(x)^2 + 4(7(8a^2 + 8ab + b^2 + 4ac) \cosh(x)^6 - 15(8a^2 - b^2 - 4ac) \cosh(x)^4 + 3(24a^2 - 8ab + 3b^2 + 12ac) \cosh(x)^2 - 8a^2 + b^2 + 4ac) \sinh(x)^2 + 4\sqrt{2}((2a + b) \cosh(x)^4 + 4(2a + b) \cosh(x) \sinh(x)^3 + (2a + b) \sinh(x)^4 - 2(2a - b) \cosh(x)^2 + 2(3(2a + b) \cosh(x)^2 - 2a + b) \sinh(x)^2 + 4((2a + b) \cosh(x)^3 - (2a - b) \cosh(x)) \sinh(x) + 2a + b) \sqrt{a} \sqrt{((a + b + c) \cosh(x)^4 + (a + b + c) \sinh(x)^4 - 4(a - c) \cosh(x)^2 + 2(3(a + b + c) \cosh(x)^2 - 2a + 2c) \sinh(x)^2 + 3a - b + 3c) / (\cosh(x)^4 - 4\cosh(x)^3 \sinh(x) + 6\cosh(x)^2 \sinh(x)^2 - 4\cosh(x) \sinh(x)^3 + \sinh(x)^4))} + 8a^2 + 8ab + b^2 + 4ac + 8((8a^2 + 8ab + b^2 + 4ac) \cosh(x)^7 - 3(8a^2 - b^2 - 4ac) \cosh(x)^5 + (24a^2 - 8ab + 3b^2 + 12ac) \cosh(x)^3 - (8a^2 - b^2 - 4ac) \cosh(x)) \sinh(x) / (\cosh(x)^8 + 8\cosh(x) \sinh(x)^7 + \sinh(x)^8 + 4(7\cosh(x)^2 + 1) \sinh(x)^6 + 4\cosh(x)^6 + 8(7\cosh(x)^3 + 3\cosh(x)) \sinh(x)^5 + 2(35\cosh(x)^4 + 30\cosh(x)^2 + 3) \sinh(x)^4 + 6\cosh(x)^4 + 8(7\cosh(x)^5 + 10\cosh(x)^3 + 3\cosh(x)) \sinh(x)^3 + 4(7\cosh(x)^6 + 15\cosh(x)^4
\end{aligned}$$

$$\begin{aligned}
& 4 + 9\cosh(x)^2 + 1)\sinh(x)^2 + 4\cosh(x)^2 + 8(\cosh(x)^7 + 3\cosh(x)^5 + \\
& 3\cosh(x)^3 + \cosh(x))\sinh(x) + 1)) + 4\sqrt{2}(a^2 + a*b + a*c)\sqrt{((a + b + c)\cosh(x)^4 + (a + b + c)\sinh(x)^4 - 4(a - c)\cosh(x)^2 + 2(3(a + b + c)\cosh(x)^2 - 2*a + 2*c)\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4 \\
& * \cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 4\cosh(x)\sinh(x)^3 + \sinh(x)^4)))/((a^3 + a^2*b + a^2*c)\cosh(x)^4 + 4(a^3 + a^2*b + a^2*c)\cosh(x)\sinh(x)^3 + (a^3 + a^2*b + a^2*c)\sinh(x)^4 + a^3 + a^2*b + a^2*c + 2(a^3 + a^2*b + a^2*c)\cosh(x)^2 + 2(a^3 + a^2*b + a^2*c)\cosh(x)^2)\sinh(x)^2 + 4((a^3 + a^2*b + a^2*c)\cosh(x)^3 + (a^3 + a^2*b + a^2*c)\cosh(x))\sinh(x)), 1/4(((2*a^2 + a*b - b^2 + (2*a - b)*c)\cosh(x)^4 + 4(2*a^2 + a*b - b^2 + (2*a - b)*c)\cosh(x)\sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)\sinh(x)^4 + 2(2*a^2 + a*b - b^2 + (2*a - b)*c)\cosh(x)^2 + 2(3(2*a^2 + a*b - b^2 + (2*a - b)*c)\cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)\sinh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c + 4((2*a^2 + a*b - b^2 + (2*a - b)*c)\cosh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)\cosh(x))\sinh(x))\sqrt{-a}\arctan(1/2\sqrt{2}((2*a + b)\cosh(x)^4 + 4(2*a + b)\cosh(x)\sinh(x)^3 + (2*a + b)\sinh(x)^4 - 2(2*a - b)\cosh(x)^2 + 2(3(2*a + b)\cosh(x)^2 - 2*a + b)\sinh(x)^2 + 4((2*a + b)\cosh(x)^3 - (2*a - b)\cosh(x))\sinh(x) + 2*a + b)\sqrt{-a}\sqrt{((a + b + c)\cosh(x)^4 + (a + b + c)\sinh(x)^4 - 4(a - c)\cosh(x)^2 + 2(3(a + b + c)\cosh(x)^2 - 2*a + 2*c)\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 4\cosh(x)\sinh(x)^3 + \sinh(x)^4)))/((a^2 + a*b + a*c)\cosh(x)^8 + 8(a^2 + a*b + a*c)\cosh(x)\sinh(x)^7 + (a^2 + a*b + a*c)\sinh(x)^8 - 4(a^2 - a*c)\cosh(x)^6 + 4(7(a^2 + a*b + a*c)\cosh(x)^2 - a^2 + a*c)\sinh(x)^6 + 8(7(a^2 + a*b + a*c)\cosh(x)^3 - 3(a^2 - a*c)\cosh(x))\sinh(x)^5 + 2(3*a^2 - a*b + 3*a*c)\cosh(x)^4 + 2(35(a^2 + a*b + a*c)\cosh(x)^4 - 30(a^2 - a*c)\cosh(x)^2 + 3*a^2 - a*b + 3*a*c)\sinh(x)^4 + 8(7(a^2 + a*b + a*c)\cosh(x)^5 - 10(a^2 - a*c)\cosh(x)^3 + (3*a^2 - a*b + 3*a*c)\cosh(x))\sinh(x)^3 - 4(a^2 - a*c)\cosh(x)^2 + 4(7(a^2 + a*b + a*c)\cosh(x)^6 - 15(a^2 - a*c)\cosh(x)^4 + 3(3*a^2 - a*b + 3*a*c)\cosh(x)^2 - a^2 + a*c)\sinh(x)^2 + a^2 + a*b + a*c + 8((a^2 + a*b + a*c)\cosh(x)^7 - 3(a^2 - a*c)\cosh(x)^5 + (3*a^2 - a*b + 3*a*c)\cosh(x)^3 - (a^2 - a*c)\cosh(x))\sinh(x))) + (a^2\cosh(x)^4 + 4*a^2\cosh(x)\sinh(x)^3 + a^2\sinh(x)^4 + 2*a^2\cosh(x)^2 + 2(3*a^2\cosh(x)^2 + a^2)\sinh(x)^2 + a^2 + 4(a^2\cosh(x)^3 + a^2\cosh(x))\sinh(x))\sqrt{a + b + c}\log(((a^2 + 2*a*b + b^2 + 2(a + b)*c + c^2)\cosh(x)^8 + 8(a^2 + 2*a*b + b^2 + 2(a + b)*c + c^2)\cosh(x)\sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2(a + b)*c + c^2)\sinh(x)^8 - 4(a^2 + a*b - b*c - c^2)\cosh(x)^6 + 4(7(a^2 + 2*a*b + b^2 + 2(a + b)*c + c^2)\cosh(x)^2 - a^2 - a*b + b*c + c^2)\sinh(x)^6 + 8(7(a^2 + 2*a*b + b^2 + 2(a + b)*c + c^2)\cosh(x)^3 - 3(a^2 + a*b - b*c - c^2)\cosh(x))\sinh(x)^5 + 2(3*a^2 + 2*a*b + 2(a + b)*c + 3*c^2)\cosh(x)^4 + 2(35(a^2 + 2*a*b + b^2 + 2(a + b)*c + c^2)\cosh(x)^4 - 30(a^2 + a*b - b*c - c^2)\cosh(x)^2 + 3*a^2 + 2*a*b + 2(a + b)*c + 3*c^2)\sinh(x)^4 + 8(7(a^2 + 2*a*b + b^2 + 2(a + b)*c + c^2)\cosh(x)^5 - 10(a^2 + a*b - b*c - c^2)\cosh(x)^3 + (3*a^2 + 2*a*b + 2(a + b)*c + 3*c^2)\cosh(x))\sinh(x)^3 - 4(a^2 + a*b - b*c - c^2)\cosh(x)^2 +
\end{aligned}$$

$$\begin{aligned}
& 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^6 - 15*(a^2 + a*b - b \\
& *c - c^2)*\cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^2 - a \\
& ^2 - a*b + b*c + c^2)*\sinh(x)^2 + \sqrt{2}*((a + b + c)*\cosh(x)^4 + 4*(a + b \\
& + c)*\cosh(x)*\sinh(x)^3 + (a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(\\
& 3*(a + b + c)*\cosh(x)^2 - a + c)*\sinh(x)^2 + 4*((a + b + c)*\cosh(x)^3 - (a \\
& - c)*\cosh(x))*\sinh(x) + a + b + c)*\sqrt{a + b + c}*\sqrt{((a + b + c)*\cosh(x) \\
&)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x) \\
&)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) \\
&) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + a^2 + 2*a*b \\
& + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cos \\
& h(x)^7 - 3*(a^2 + a*b - b*c - c^2)*\cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c \\
& + 3*c^2)*\cosh(x)^3 - (a^2 + a*b - b*c - c^2)*\cosh(x))*\sinh(x))/(\cosh(x)^4 \\
& + 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(\\
& x)^4)) - 2*\sqrt{2}*(a^2 + a*b + a*c)*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + \\
& c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2* \\
& c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^ \\
& 2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*\cos \\
& h(x)^4 + 4*(a^3 + a^2*b + a^2*c)*\cosh(x)*\sinh(x)^3 + (a^3 + a^2*b + a^2*c)* \\
& \sinh(x)^4 + a^3 + a^2*b + a^2*c + 2*(a^3 + a^2*b + a^2*c)*\cosh(x)^2 + 2*(a^ \\
& 3 + a^2*b + a^2*c + 3*(a^3 + a^2*b + a^2*c)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 \\
& + a^2*b + a^2*c)*\cosh(x)^3 + (a^3 + a^2*b + a^2*c)*\cosh(x))*\sinh(x)), 1/4*(\\
& ((2*a^2 + a*b - b^2 + (2*a - b)*c)*\cosh(x)^4 + 4*(2*a^2 + a*b - b^2 + (2*a \\
& - b)*c)*\cosh(x)*\sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*\sinh(x)^4 + 2 \\
& *(2*a^2 + a*b - b^2 + (2*a - b)*c)*\cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2 + (2 \\
& *a - b)*c)*\cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)*\sinh(x)^2 + 2*a^2 + \\
& a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*\cosh(x)^3 + \\
& (2*a^2 + a*b - b^2 + (2*a - b)*c)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(1/2*\sqrt{2})* \\
& ((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + b)*\sinh \\
& (x)^4 - 2*(2*a - b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 - 2*a + b)*\sinh(x) \\
& ^2 + 4*((2*a + b)*\cosh(x)^3 - (2*a - b)*\cosh(x))*\sinh(x) + 2*a + b)*\sqrt{-a} \\
&)*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 \\
& + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh \\
& (x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \\
& \sinh(x)^4)))/((a^2 + a*b + a*c)*\cosh(x)^8 + 8*(a^2 + a*b + a*c)*\cosh(x)*\sin \\
& h(x)^7 + (a^2 + a*b + a*c)*\sinh(x)^8 - 4*(a^2 - a*c)*\cosh(x)^6 + 4*(7*(a^2 \\
& + a*b + a*c)*\cosh(x)^2 - a^2 + a*c)*\sinh(x)^6 + 8*(7*(a^2 + a*b + a*c)*\cosh \\
& (x)^3 - 3*(a^2 - a*c)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 - a*b + 3*a*c)*\cosh(x)^ \\
& 4 + 2*(35*(a^2 + a*b + a*c)*\cosh(x)^4 - 30*(a^2 - a*c)*\cosh(x)^2 + 3*a^2 - \\
& a*b + 3*a*c)*\sinh(x)^4 + 8*(7*(a^2 + a*b + a*c)*\cosh(x)^5 - 10*(a^2 - a*c)* \\
& \cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*\cosh(x))*\sinh(x)^3 - 4*(a^2 - a*c)*\cosh(x) \\
&)^2 + 4*(7*(a^2 + a*b + a*c)*\cosh(x)^6 - 15*(a^2 - a*c)*\cosh(x)^4 + 3*(3*a^ \\
& 2 - a*b + 3*a*c)*\cosh(x)^2 - a^2 + a*c)*\sinh(x)^2 + a^2 + a*b + a*c + 8*((a \\
& ^2 + a*b + a*c)*\cosh(x)^7 - 3*(a^2 - a*c)*\cosh(x)^5 + (3*a^2 - a*b + 3*a*c) \\
& *\cosh(x)^3 - (a^2 - a*c)*\cosh(x))*\sinh(x)) - 2*(a^2*\cosh(x)^4 + 4*a^2*\cosh \\
& (x)*\sinh(x)^3 + a^2*\sinh(x)^4 + 2*a^2*\cosh(x)^2 + 2*(3*a^2*\cosh(x)^2 + a^2)
\end{aligned}$$


```

*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(-a - b - c
)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
(a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a
+ b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)
^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c
^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7
+ (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c -
c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 -
a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 +
2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^
2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (
3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 +
a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*c
osh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 +
2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 +
2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 +
2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x
))) - 2*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)
*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*
sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*s
inh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^3 + a^2*b + a^2*c)*cosh(x)
)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*sin
h(x)^4 + a^3 + a^2*b + a^2*c + 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 + 2*(a^3 +
a^2*b + a^2*c + 3*(a^3 + a^2*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a
^2*b + a^2*c)*cosh(x)^3 + (a^3 + a^2*b + a^2*c)*cosh(x))*sinh(x)]]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\sqrt{a + b \coth^2(x) + c \coth^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)

[Out] $\text{Integral}(\tanh(x)**3/\text{sqrt}(a + b*\text{coth}(x)**2 + c*\text{coth}(x)**4), x)$

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\tanh(x)^3/(a+b*\text{coth}(x)^2+c*\text{coth}(x)^4)^{(1/2)}, x, \text{algorithm}="giac")$

[Out] Timed out

$$3.210 \quad \int \coth(x) \sqrt{a + b \coth^2(x) + c \coth^4(x)} dx$$

Optimal. Leaf size=132

$$-\frac{1}{2} \sqrt{a + b \coth^2(x) + c \coth^4(x)} - \frac{(b + 2c) \tanh^{-1} \left(\frac{b + 2c \coth^2(x)}{2\sqrt{c} \sqrt{a + b \coth^2(x) + c \coth^4(x)}} \right)}{4\sqrt{c}} + \frac{1}{2} \sqrt{a + b + c} \tanh^{-1} \left(\frac{2a + (b + 2c) \coth^2(x)}{2\sqrt{a + b + c} \sqrt{a + b \coth^2(x) + c \coth^4(x)}} \right)$$

[Out] $-\left(\left(b + 2c\right) \operatorname{ArcTanh}\left[\left(b + 2c \operatorname{Coth}[x]^2\right) / \left(2 \sqrt{c} \sqrt{a + b \operatorname{Coth}[x]^2 + c \operatorname{Coth}[x]^4}\right)\right]\right) / \left(4 \sqrt{c}\right) + \left(\sqrt{a + b + c} \operatorname{ArcTanh}\left[\left(2a + b + \left(b + 2c\right) \operatorname{Coth}[x]^2\right) / \left(2 \sqrt{a + b + c} \sqrt{a + b \operatorname{Coth}[x]^2 + c \operatorname{Coth}[x]^4}\right)\right]\right) / 2 - \operatorname{Sqrt}\left[a + b \operatorname{Coth}[x]^2 + c \operatorname{Coth}[x]^4\right] / 2$

Rubi [A] time = 0.230797, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3701, 1247, 734, 843, 621, 206, 724}

$$-\frac{1}{2} \sqrt{a + b \coth^2(x) + c \coth^4(x)} - \frac{(b + 2c) \tanh^{-1} \left(\frac{b + 2c \coth^2(x)}{2\sqrt{c} \sqrt{a + b \coth^2(x) + c \coth^4(x)}} \right)}{4\sqrt{c}} + \frac{1}{2} \sqrt{a + b + c} \tanh^{-1} \left(\frac{2a + (b + 2c) \coth^2(x)}{2\sqrt{a + b + c} \sqrt{a + b \coth^2(x) + c \coth^4(x)}} \right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\operatorname{Coth}[x] \operatorname{Sqrt}\left[a + b \operatorname{Coth}[x]^2 + c \operatorname{Coth}[x]^4\right], x\right]$

[Out] $-\left(\left(b + 2c\right) \operatorname{ArcTanh}\left[\left(b + 2c \operatorname{Coth}[x]^2\right) / \left(2 \sqrt{c} \sqrt{a + b \operatorname{Coth}[x]^2 + c \operatorname{Coth}[x]^4}\right)\right]\right) / \left(4 \sqrt{c}\right) + \left(\sqrt{a + b + c} \operatorname{ArcTanh}\left[\left(2a + b + \left(b + 2c\right) \operatorname{Coth}[x]^2\right) / \left(2 \sqrt{a + b + c} \sqrt{a + b \operatorname{Coth}[x]^2 + c \operatorname{Coth}[x]^4}\right)\right]\right) / 2 - \operatorname{Sqrt}\left[a + b \operatorname{Coth}[x]^2 + c \operatorname{Coth}[x]^4\right] / 2$

Rule 3701

$\operatorname{Int}\left[\cot\left[\left(d_{.}\right) + \left(e_{.}\right) \cdot \left(x_{.}\right)\right]^{\left(m_{.}\right)} \cdot \left(\left(a_{.}\right) + \left(b_{.}\right) \cdot \cot\left[\left(d_{.}\right) + \left(e_{.}\right) \cdot \left(x_{.}\right)\right] \cdot \left(f_{.}\right)\right)^{\left(n_{.}\right)} + \left(c_{.}\right) \cdot \cot\left[\left(d_{.}\right) + \left(e_{.}\right) \cdot \left(x_{.}\right)\right] \cdot \left(f_{.}\right)^{\left(n_{2_{.}}\right)}\right]^{\left(p_{.}\right)}, x_{\text{Symbol}}]$
 $\rightarrow -\operatorname{Dist}\left[f/e, \operatorname{Subst}\left[\operatorname{Int}\left[\left(\left(x/f\right)^m \cdot \left(a + b \cdot x^n + c \cdot x^{2n}\right)^p\right) / \left(f^2 + x^2\right), x\right], x, f \cdot \cot[d + e \cdot x], x\right] /; \operatorname{FreeQ}\left[\{a, b, c, d, e, f, m, n, p\}, x\right] \&\& \operatorname{EqQ}\left[n^2, 2n\right] \&\& \operatorname{NeQ}\left[b^2 - 4ac, 0\right]$

Rule 1247

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(
p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 734

```
Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) &
& !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \coth(x) \sqrt{a + b \coth^2(x) + c \coth^4(x)} dx &= -\text{Subst} \left(\int \frac{x \sqrt{a - bx^2 + cx^4}}{1 + x^2} dx, x, -i \coth(x) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a - bx + cx^2}}{1 + x} dx, x, -\coth^2(x) \right) \right) \\
&= -\frac{1}{2} \sqrt{a + b \coth^2(x) + c \coth^4(x)} + \frac{1}{4} \text{Subst} \left(\int \frac{-2a - b + (b + 2c)x}{(1 + x) \sqrt{a - bx + cx^2}} dx \right) \\
&= -\frac{1}{2} \sqrt{a + b \coth^2(x) + c \coth^4(x)} + \frac{1}{2} (-a - b - c) \text{Subst} \left(\int \frac{1}{(1 + x) \sqrt{a - bx + cx^2}} dx \right) \\
&= -\frac{1}{2} \sqrt{a + b \coth^2(x) + c \coth^4(x)} + (a + b + c) \text{Subst} \left(\int \frac{1}{4a + 4b + 4c - (b + 2c)x} dx \right) \\
&= -\frac{(b + 2c) \tanh^{-1} \left(\frac{b + 2c \coth^2(x)}{2\sqrt{c} \sqrt{a + b \coth^2(x) + c \coth^4(x)}} \right)}{4\sqrt{c}} + \frac{1}{2} \sqrt{a + b + c} \tanh^{-1} \left(\frac{\coth(x)}{\sqrt{a + b + c}} \right)
\end{aligned}$$

Mathematica [B] time = 7.99061, size = 304, normalized size = 2.3

$$\text{csch}^2(x) \left(4\sqrt{c}(a + b + c) \sqrt{\cosh(4x)(a + b + c) - 4(a - c) \cosh(2x) + 3a - b + 3c} \tanh^{-1} \left(\frac{\cosh(2x)(a + b + c) - a + c}{2\sqrt{a + b + c} \sqrt{\sinh^4(x)(a + b + c) + (b + 2c) \sinh^2(x)}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4], x]

[Out] (Csch[x]^2*(4*Sqrt[c]*(a + b + c)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])]/(2*Sqrt[a + b + c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4]))*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]] + Sqrt[a + b + c]*(-2*(b + 2*c)*ArcTanh[(2*c + (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4]))*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]] - Sqrt[2]*Sqrt[c]*(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^2)/(8*Sqrt[c]*Sqrt[a + b + c]*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4])

Maple [C] time = 0.061, size = 559, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)`

[Out]
$$-1/2*(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2}-1/8*(-b-c)*2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2}*(4-2*(-b+(-4*a*c+b^2)^{1/2})/a*\coth(x)^2)^{1/2}*(4+2*(b+(-4*a*c+b^2)^{1/2})/a*\coth(x)^2)^{1/2}/(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2}*EllipticF(1/2*\coth(x)*2^{1/2}*((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2},1/2*(-4+2*b*(b+(-4*a*c+b^2)^{1/2})/a/c)^{1/2})-1/2*\ln((b+2*c*\coth(x)^2)/c^{1/2}+2*(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2})*c^{1/2}-1/4*\ln((b+2*c*\coth(x)^2)/c^{1/2}+2*(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2})/c^{1/2}*b+1/2*a/(a+b+c)^{1/2}*\operatorname{arctanh}(1/2*(b*\coth(x)^2+2*c*\coth(x)^2+2*a+b)/(a+b+c)^{1/2})/(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2}))+1/2*b/(a+b+c)^{1/2}*\operatorname{arctanh}(1/2*(b*\coth(x)^2+2*c*\coth(x)^2+2*a+b)/(a+b+c)^{1/2})/(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2}))+1/2*c/(a+b+c)^{1/2}*\operatorname{arctanh}(1/2*(b*\coth(x)^2+2*c*\coth(x)^2+2*a+b)/(a+b+c)^{1/2})/(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2}))-1/8*(b+c)*2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2}*(4-2*(-b+(-4*a*c+b^2)^{1/2})/a*\coth(x)^2)^{1/2}*(4+2*(b+(-4*a*c+b^2)^{1/2})/a*\coth(x)^2)^{1/2}/(a+b*\coth(x)^2+c*\coth(x)^4)^{1/2}*EllipticF(1/2*\coth(x)*2^{1/2}*((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2},1/2*(-4+2*b*(b+(-4*a*c+b^2)^{1/2})/a/c)^{1/2})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \coth(x)^4 + b \coth(x)^2 + a} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)`

Fricas [B] time = 24.0699, size = 22316, normalized size = 169.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")

[Out]
$$\frac{1}{8} \left((b+2c) \cosh(x)^4 + 4(b+2c) \cosh(x) \sinh(x)^3 + (b+2c) \sinh(x)^4 - 2(b+2c) \cosh(x)^2 + 2(3(b+2c) \cosh(x)^2 - b - 2c) \sinh(x)^2 + 4((b+2c) \cosh(x)^3 - (b+2c) \cosh(x)) \sinh(x) + b + 2c \right) \sqrt{c} \log \left((b^2 + 4(a+2b)c + 8c^2) \cosh(x)^8 + 8(b^2 + 4(a+2b)c + 8c^2) \cosh(x) \sinh(x)^7 + (b^2 + 4(a+2b)c + 8c^2) \sinh(x)^8 - 4(b^2 + 4ac - 8c^2) \cosh(x)^6 + 4(7(b^2 + 4(a+2b)c + 8c^2) \cosh(x)^2 - b^2 - 4ac + 8c^2) \sinh(x)^6 + 8(7(b^2 + 4(a+2b)c + 8c^2) \cosh(x)^3 - 3(b^2 + 4ac - 8c^2) \cosh(x)) \sinh(x)^5 + 2(3b^2 + 4(3a - 2b)c + 24c^2) \cosh(x)^4 + 2(35(b^2 + 4(a+2b)c + 8c^2) \cosh(x)^4 - 30(b^2 + 4ac - 8c^2) \cosh(x)^2 + 3b^2 + 4(3a - 2b)c + 24c^2) \sinh(x)^4 + 8(7(b^2 + 4(a+2b)c + 8c^2) \cosh(x)^5 - 10(b^2 + 4ac - 8c^2) \cosh(x)^3 + (3b^2 + 4(3a - 2b)c + 24c^2) \cosh(x)) \sinh(x)^3 - 4(b^2 + 4ac - 8c^2) \cosh(x)^2 + 4(7(b^2 + 4(a+2b)c + 8c^2) \cosh(x)^6 - 15(b^2 + 4ac - 8c^2) \cosh(x)^4 + 3(3b^2 + 4(3a - 2b)c + 24c^2) \cosh(x)^2 - b^2 - 4ac + 8c^2) \sinh(x)^2 - 4\sqrt{2} \left((b+2c) \cosh(x)^4 + 4(b+2c) \cosh(x) \sinh(x)^3 + (b+2c) \sinh(x)^4 - 2(b-2c) \cosh(x)^2 + 2(3(b+2c) \cosh(x)^2 - b + 2c) \sinh(x)^2 + 4((b+2c) \cosh(x))^3 - (b-2c) \cosh(x)) \sinh(x) + b + 2c \right) \sqrt{c} \sqrt{\left((a+b+c) \cosh(x)^4 + (a+b+c) \sinh(x)^4 - 4(a-c) \cosh(x)^2 + 2(3(a+b+c) \cosh(x)^2 - 2a + 2c) \sinh(x)^2 + 3a - b + 3c \right) / \left(\cosh(x)^4 - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 \right) + b^2 + 4(a+2b)c + 8c^2 + 8((b^2 + 4(a+2b)c + 8c^2) \cosh(x)^7 - 3(b^2 + 4ac - 8c^2) \cosh(x)^5 + (3b^2 + 4(3a - 2b)c + 24c^2) \cosh(x)^3 - (b^2 + 4ac - 8c^2) \cosh(x)) \sinh(x) / \left(\cosh(x)^8 + 8 \cosh(x) \sinh(x)^7 + \sinh(x)^8 + 4(7 \cosh(x)^2 - 1) \sinh(x)^6 - 4 \cosh(x)^6 + 8(7 \cosh(x)^3 - 3 \cosh(x)) \sinh(x)^5 + 2(35 \cosh(x)^4 - 30 \cosh(x)^2 + 3) \sinh(x)^4 + 6 \cosh(x)^4 + 8(7 \cosh(x)^5 - 10 \cosh(x)^3 + 3 \cosh(x)) \sinh(x)^3 + 4(7 \cosh(x)^6 - 15 \cosh(x)^4 + 9 \cosh(x)^2 - 1) \sinh(x)^2 - 4 \cosh(x)^2 + 8(\cosh(x)^7 - 3 \cosh(x)^5 + 3 \cosh(x)^3 - \cosh(x)) \sinh(x) + 1 \right) + 2(c \cosh(x)^4 + 4c \cosh(x) \sinh(x)^3 + c \sinh(x)^4 - 2c \cosh(x)^2 + 2(3c \cosh(x)^2 - c) \sinh(x)^2 + 4(c \cosh(x)^3 - c \cosh(x)) \sinh(x) + c) \sqrt{a+b+c} \log \left((a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^8 + 8(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x) \sinh(x)^7 + (a^2 + 2ab + b^2 + 2(a+b)c + c^2) \sinh(x)^8 - 4(a^2 + ab - bc - c^2) \cosh(x)^6 + 4(7(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^2 - a^2 - ab + bc + c^2) \sinh(x)^6 + 8(7(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^3 - 3(a^2 + ab - bc - c^2) \cosh(x)) \sinh(x)^5 + 2(3a^2 + 2ab + 2(a+b)c + 3c^2) \cosh(x)^4 + 2(35(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^4 - 30(a^2 + ab - bc - c^2) \cosh(x)^2 + 3a^2 + 2ab + 2(a+b)c + 3c^2) \sinh(x)^4 + 8(7(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^5 - 10(a^2 + ab - bc -$$

$$\begin{aligned}
& c^2 \cosh(x)^3 + (3a^2 + 2ab + 2(a+b)c + 3c^2) \cosh(x) \sinh(x)^3 - \\
& 4(a^2 + ab - bc - c^2) \cosh(x)^2 + 4(7(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^6 - \\
& 15(a^2 + ab - bc - c^2) \cosh(x)^4 + 3(3a^2 + 2ab + 2(a+b)c + 3c^2) \cosh(x)^2 - \\
& a^2 - ab + bc + c^2) \sinh(x)^2 + \sqrt{2}((a+b+c) \cosh(x)^4 + 4(a+b+c) \cosh(x) \sinh(x)^3 + \\
& (a+b+c) \sinh(x)^4 - 2(a-c) \cosh(x)^2 + 2(3(a+b+c) \cosh(x)^2 - a+c) \sinh(x)^2 + \\
& 4((a+b+c) \cosh(x)^3 - (a-c) \cosh(x)) \sinh(x) + a+b+c) \sqrt{a+b+c} \sqrt{((a+b+c) \cosh(x)^4 + \\
& (a+b+c) \sinh(x)^4 - 4(a-c) \cosh(x)^2 + 2(3(a+b+c) \cosh(x)^2 - 2a+2c) \sinh(x)^2 + \\
& 3a-b+3c) / (\cosh(x)^4 - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \\
& \sinh(x)^4)) + a^2 + 2ab + b^2 + 2(a+b)c + c^2 + 8((a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^7 - \\
& 3(a^2 + ab - bc - c^2) \cosh(x)^5 + (3a^2 + 2ab + 2(a+b)c + 3c^2) \cosh(x)^3 - (a^2 + ab - bc - \\
& c^2) \cosh(x)) \sinh(x) / (\cosh(x)^4 + 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 + \\
& 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4) - 4 \sqrt{2} c \sqrt{((a+b+c) \cosh(x)^4 + (a+b+c) \sinh(x)^4 - \\
& 4(a-c) \cosh(x)^2 + 2(3(a+b+c) \cosh(x)^2 - 2a+2c) \sinh(x)^2 + 3a-b+3c) / (\cosh(x)^4 - \\
& 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4)) / (c \cosh(x)^4 + \\
& 4c \cosh(x) \sinh(x)^3 + c \sinh(x)^4 - 2c \cosh(x)^2 + 2(3c \cosh(x)^2 - c) \sinh(x)^2 + \\
& 4(c \cosh(x)^3 - c \cosh(x)) \sinh(x) + c), -1/8(4(c \cosh(x)^4 + 4c \cosh(x) \sinh(x)^3 + \\
& c \sinh(x)^4 - 2c \cosh(x)^2 + 2(3c \cosh(x)^2 - c) \sinh(x)^2 + 4(c \cosh(x)^3 - c \cosh(x)) \sinh(x) + \\
& c) \sqrt{-a-b-c} \arctan(\sqrt{2}((a+b+c) \cosh(x)^4 + 4(a+b+c) \cosh(x) \sinh(x)^3 + (a+b+c) \sinh(x)^4 - \\
& 2(a-c) \cosh(x)^2 + 2(3(a+b+c) \cosh(x)^2 - a+c) \sinh(x)^2 + 4((a+b+c) \cosh(x)^3 - (a-c) \cosh(x)) \sinh(x) + \\
& a+b+c) \sqrt{-a-b-c} \sqrt{((a+b+c) \cosh(x)^4 + (a+b+c) \sinh(x)^4 - 4(a-c) \cosh(x)^2 + 2(3(a+b+c) \cosh(x)^2 - \\
& 2a+2c) \sinh(x)^2 + 3a-b+3c) / (\cosh(x)^4 - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - \\
& 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4)) / ((a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^8 + 8(a^2 + 2ab + b^2 + \\
& 2(a+b)c + c^2) \cosh(x) \sinh(x)^7 + (a^2 + 2ab + b^2 + 2(a+b)c + c^2) \sinh(x)^8 - 4(a^2 + ab - bc - \\
& c^2) \cosh(x)^6 + 4(7(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^2 - a^2 - ab + bc + c^2) \sinh(x)^6 + \\
& 8(7(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^3 - 3(a^2 + ab - bc - c^2) \cosh(x)) \sinh(x)^5 + 2(3a^2 + \\
& 2ab - b^2 + 2(3a+b)c + 3c^2) \cosh(x)^4 + 2(35(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^4 - \\
& 30(a^2 + ab - bc - c^2) \cosh(x)^2 + 3a^2 + 2ab - b^2 + 2(3a+b)c + 3c^2) \sinh(x)^4 + 8(7(a^2 + 2ab + b^2 + \\
& 2(a+b)c + c^2) \cosh(x)^5 - 10(a^2 + ab - bc - c^2) \cosh(x)^3 + (3a^2 + 2ab - b^2 + 2(3a+b)c + 3c^2) \cosh(x)) \sinh(x)^3 - \\
& 4(a^2 + ab - bc - c^2) \cosh(x)^2 + 4(7(a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^6 - 15(a^2 + ab - bc - \\
& c^2) \cosh(x)^4 + 3(3a^2 + 2ab - b^2 + 2(3a+b)c + 3c^2) \cosh(x)^2 - a^2 - ab + bc + c^2) \sinh(x)^2 + \\
& a^2 + 2ab + b^2 + 2(a+b)c + c^2 + 8((a^2 + 2ab + b^2 + 2(a+b)c + c^2) \cosh(x)^7 - 3(a^2 + ab - bc - \\
& c^2) \cosh(x)^5 + (3a^2 + 2ab - b^2 + 2(3a+b)c + 3c^2) \cosh(x)^3 - (a^2 + ab - bc - c^2) \cosh(x)
\end{aligned}$$

$$\begin{aligned}
&)*\sinh(x)) - ((b + 2*c)*\cosh(x)^4 + 4*(b + 2*c)*\cosh(x)*\sinh(x)^3 + (b + 2 \\
& *c)*\sinh(x)^4 - 2*(b + 2*c)*\cosh(x)^2 + 2*(3*(b + 2*c)*\cosh(x)^2 - b - 2*c) \\
& *\sinh(x)^2 + 4*((b + 2*c)*\cosh(x)^3 - (b + 2*c)*\cosh(x))*\sinh(x) + b + 2*c) \\
& *sqrt(c)*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^8 + 8*(b^2 + 4*(a + 2*b) \\
&)*c + 8*c^2)*\cosh(x)*\sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*\sinh(x)^8 - \\
& 4*(b^2 + 4*a*c - 8*c^2)*\cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh \\
& (x)^2 - b^2 - 4*a*c + 8*c^2)*\sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2) \\
& *\cosh(x)^3 - 3*(b^2 + 4*a*c - 8*c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*b^2 + 4*(3*a \\
& - 2*b)*c + 24*c^2)*\cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x) \\
& ^4 - 30*(b^2 + 4*a*c - 8*c^2)*\cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2) \\
& *\sinh(x)^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^5 - 10*(b^2 + 4*a*c \\
& - 8*c^2)*\cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*\cosh(x))*\sinh(x)^3 \\
& - 4*(b^2 + 4*a*c - 8*c^2)*\cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*c \\
& osh(x)^6 - 15*(b^2 + 4*a*c - 8*c^2)*\cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c \\
& + 24*c^2)*\cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*\sinh(x)^2 - 4*sqrt(2)*((b + 2*c) \\
& *\cosh(x)^4 + 4*(b + 2*c)*\cosh(x)*\sinh(x)^3 + (b + 2*c)*\sinh(x)^4 - 2*(b - 2 \\
& *c)*\cosh(x)^2 + 2*(3*(b + 2*c)*\cosh(x)^2 - b + 2*c)*\sinh(x)^2 + 4*((b + 2*c) \\
&)*\cosh(x)^3 - (b - 2*c)*\cosh(x))*\sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + \\
& c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + \\
& c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x) \\
& ^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*\sinh(x)^3 + sinh(x)^4)) + b^ \\
& 2 + 4*(a + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*\cosh(x)^7 - 3* \\
& (b^2 + 4*a*c - 8*c^2)*\cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*\cosh(x) \\
&)^3 - (b^2 + 4*a*c - 8*c^2)*\cosh(x))*\sinh(x))/(cosh(x)^8 + 8*cosh(x)*\sinh(x) \\
&)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*\sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x) \\
&)^3 - 3*cosh(x))*\sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*\sinh(x)^4 \\
& + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*\sinh(x)^3 + 4*(7 \\
& *cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*\sinh(x)^2 - 4*cosh(x)^2 + 8*(c \\
& osh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*\sinh(x) + 1)) + 4*sqrt(2)*c \\
& *sqrt(((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 \\
& + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(cosh(\\
& x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*\sinh(x)^3 + \\
& sinh(x)^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*\sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(\\
& x)^2 + 2*(3*c*cosh(x)^2 - c)*\sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*\sinh(x) \\
&) + c), 1/4*(((b + 2*c)*\cosh(x)^4 + 4*(b + 2*c)*\cosh(x)*\sinh(x)^3 + (b + 2* \\
& c)*\sinh(x)^4 - 2*(b + 2*c)*\cosh(x)^2 + 2*(3*(b + 2*c)*\cosh(x)^2 - b - 2*c)* \\
& sinh(x)^2 + 4*((b + 2*c)*\cosh(x)^3 - (b + 2*c)*\cosh(x))*\sinh(x) + b + 2*c)* \\
& sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*\cosh(x)^4 + 4*(b + 2*c)*\cosh(x)*\sinh \\
& (x)^3 + (b + 2*c)*\sinh(x)^4 - 2*(b - 2*c)*\cosh(x)^2 + 2*(3*(b + 2*c)*\cosh(x) \\
&)^2 - b + 2*c)*\sinh(x)^2 + 4*((b + 2*c)*\cosh(x)^3 - (b - 2*c)*\cosh(x))*\sinh \\
& (x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 \\
& - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 \\
& + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - \\
& 4*cosh(x)*\sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*\cosh(x)^8 + 8*((a + b) \\
&)*c + c^2)*\cosh(x)*\sinh(x)^7 + ((a + b)*c + c^2)*\sinh(x)^8 - 4*(a*c - c^2)*
\end{aligned}$$

$$\begin{aligned}
& \cosh(x)^6 + 4*(7*((a + b)*c + c^2)*\cosh(x)^2 - a*c + c^2)*\sinh(x)^6 + 8*(7* \\
& ((a + b)*c + c^2)*\cosh(x)^3 - 3*(a*c - c^2)*\cosh(x))*\sinh(x)^5 + 2*((3*a - \\
& b)*c + 3*c^2)*\cosh(x)^4 + 2*(35*((a + b)*c + c^2)*\cosh(x)^4 - 30*(a*c - c^2) \\
&)*\cosh(x)^2 + (3*a - b)*c + 3*c^2)*\sinh(x)^4 + 8*(7*((a + b)*c + c^2)*\cosh(\\
& x)^5 - 10*(a*c - c^2)*\cosh(x)^3 + ((3*a - b)*c + 3*c^2)*\cosh(x))*\sinh(x)^3 \\
& - 4*(a*c - c^2)*\cosh(x)^2 + 4*(7*((a + b)*c + c^2)*\cosh(x)^6 - 15*(a*c - c^ \\
& 2)*\cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*\cosh(x)^2 - a*c + c^2)*\sinh(x)^2 + (\\
& a + b)*c + c^2 + 8*((a + b)*c + c^2)*\cosh(x)^7 - 3*(a*c - c^2)*\cosh(x)^5 + \\
& ((3*a - b)*c + 3*c^2)*\cosh(x)^3 - (a*c - c^2)*\cosh(x))*\sinh(x)) + (c*\cosh \\
& (x)^4 + 4*c*\cosh(x)*\sinh(x)^3 + c*\sinh(x)^4 - 2*c*\cosh(x)^2 + 2*(3*c*\cosh(x) \\
&)^2 - c)*\sinh(x)^2 + 4*(c*\cosh(x)^3 - c*\cosh(x))*\sinh(x) + c)*\sqrt{a + b + \\
& c}*\log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^8 + 8*(a^2 + 2*a*b \\
& + b^2 + 2*(a + b)*c + c^2)*\cosh(x)*\sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + \\
& b)*c + c^2)*\sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^6 + 4*(7*(a^2 + 2 \\
& *a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(x)^ \\
& 6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^3 - 3*(a^2 + a*b - \\
& b*c - c^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\co \\
& sh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^4 - 30*(a^2 \\
& + a*b - b*c - c^2)*\cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\sinh(x) \\
&)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^5 - 10*(a^2 + a* \\
& b - b*c - c^2)*\cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x))*\s \\
& inh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + \\
& 2*(a + b)*c + c^2)*\cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*\cosh(x)^4 + 3*(3* \\
& a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^2 - a^2 - a*b + b*c + c^2)*\sinh(\\
& x)^2 + \sqrt{2}*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)*\cosh(x)*\sinh(x)^3 + (\\
& a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - a \\
& + c)*\sinh(x)^2 + 4*((a + b + c)*\cosh(x)^3 - (a - c)*\cosh(x))*\sinh(x) + a + \\
& b + c)*\sqrt{a + b + c}*\sqrt{((a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 \\
& - 4*(a - c)*\cosh(x)^2 + 2*(3*(a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 \\
& + 3*a - b + 3*c)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - \\
& 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 \\
& + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*\cosh(x)^7 - 3*(a^2 + a*b - b*c \\
& - c^2)*\cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*\cosh(x)^3 - (a^2 \\
& + a*b - b*c - c^2)*\cosh(x))*\sinh(x))/(\cosh(x)^4 + 4*\cosh(x)^3*\sinh(x) + 6*c \\
& osh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) - 2*\sqrt{2}*c*\sqrt{((\\
& (a + b + c)*\cosh(x)^4 + (a + b + c)*\sinh(x)^4 - 4*(a - c)*\cosh(x)^2 + 2*(3* \\
& (a + b + c)*\cosh(x)^2 - 2*a + 2*c)*\sinh(x)^2 + 3*a - b + 3*c)/(\cosh(x)^4 - \\
& 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x) \\
& ^4)))/(c*\cosh(x)^4 + 4*c*\cosh(x)*\sinh(x)^3 + c*\sinh(x)^4 - 2*c*\cosh(x)^2 + \\
& 2*(3*c*\cosh(x)^2 - c)*\sinh(x)^2 + 4*(c*\cosh(x)^3 - c*\cosh(x))*\sinh(x) + \\
& c)*\sqrt{-a - b - c}*\arctan(\sqrt{2}*((a + b + c)*\cosh(x)^4 + 4*(a + b + c)*\co \\
& sh(x)*\sinh(x)^3 + (a + b + c)*\sinh(x)^4 - 2*(a - c)*\cosh(x)^2 + 2*(3*(a + \\
& b + c)*\cosh(x)^2 - a + c)*\sinh(x)^2 + 4*((a + b + c)*\cosh(x)^3 - (a - c)*\co
\end{aligned}$$

$$\begin{aligned}
& \text{sh}(x) * \sinh(x) + a + b + c) * \text{sqrt}(-a - b - c) * \text{sqrt}(((a + b + c) * \cosh(x)^4 + \\
& (a + b + c) * \sinh(x)^4 - 4 * (a - c) * \cosh(x)^2 + 2 * (3 * (a + b + c) * \cosh(x)^2 - \\
& 2 * a + 2 * c) * \sinh(x)^2 + 3 * a - b + 3 * c) / (\cosh(x)^4 - 4 * \cosh(x)^3 * \sinh(x) + 6 * \\
& \cosh(x)^2 * \sinh(x)^2 - 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4)) / ((a^2 + 2 * a * b + b^2 \\
& + 2 * (a + b) * c + c^2) * \cosh(x)^8 + 8 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) \\
& * \cosh(x) * \sinh(x)^7 + (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \sinh(x)^8 - 4 * \\
& (a^2 + a * b - b * c - c^2) * \cosh(x)^6 + 4 * (7 * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + \\
& c^2) * \cosh(x)^2 - a^2 - a * b + b * c + c^2) * \sinh(x)^6 + 8 * (7 * (a^2 + 2 * a * b + b^2 \\
& + 2 * (a + b) * c + c^2) * \cosh(x)^3 - 3 * (a^2 + a * b - b * c - c^2) * \cosh(x)) * \sinh(x) \\
& ^5 + 2 * (3 * a^2 + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \cosh(x)^4 + 2 * (35 * (a \\
& ^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^4 - 30 * (a^2 + a * b - b * c - c^2) \\
&) * \cosh(x)^2 + 3 * a^2 + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \sinh(x)^4 + 8 * (7 \\
& * (a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2) * \cosh(x)^5 - 10 * (a^2 + a * b - b * c - \\
& c^2) * \cosh(x)^3 + (3 * a^2 + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \cosh(x)) * \sinh(x) \\
& ^3 - 4 * (a^2 + a * b - b * c - c^2) * \cosh(x)^2 + 4 * (7 * (a^2 + 2 * a * b + b^2 + 2 * \\
& (a + b) * c + c^2) * \cosh(x)^6 - 15 * (a^2 + a * b - b * c - c^2) * \cosh(x)^4 + 3 * (3 * a^2 \\
& + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \cosh(x)^2 - a^2 - a * b + b * c + c^2) \\
& * \sinh(x)^2 + a^2 + 2 * a * b + b^2 + 2 * (a + b) * c + c^2 + 8 * ((a^2 + 2 * a * b + b^2 \\
& + 2 * (a + b) * c + c^2) * \cosh(x)^7 - 3 * (a^2 + a * b - b * c - c^2) * \cosh(x)^5 + (3 * a \\
& ^2 + 2 * a * b - b^2 + 2 * (3 * a + b) * c + 3 * c^2) * \cosh(x)^3 - (a^2 + a * b - b * c - c^2) \\
& * \cosh(x)) * \sinh(x))) - ((b + 2 * c) * \cosh(x)^4 + 4 * (b + 2 * c) * \cosh(x) * \sinh(x)^3 \\
& + (b + 2 * c) * \sinh(x)^4 - 2 * (b + 2 * c) * \cosh(x)^2 + 2 * (3 * (b + 2 * c) * \cosh(x)^2 \\
& - b - 2 * c) * \sinh(x)^2 + 4 * ((b + 2 * c) * \cosh(x)^3 - (b + 2 * c) * \cosh(x)) * \sinh(x) \\
& + b + 2 * c) * \text{sqrt}(-c) * \arctan(1/2 * \text{sqrt}(2) * ((b + 2 * c) * \cosh(x)^4 + 4 * (b + 2 * c) * c \\
& \cosh(x) * \sinh(x)^3 + (b + 2 * c) * \sinh(x)^4 - 2 * (b - 2 * c) * \cosh(x)^2 + 2 * (3 * (b + \\
& 2 * c) * \cosh(x)^2 - b + 2 * c) * \sinh(x)^2 + 4 * ((b + 2 * c) * \cosh(x)^3 - (b - 2 * c) * c \\
& \cosh(x)) * \sinh(x) + b + 2 * c) * \text{sqrt}(-c) * \text{sqrt}(((a + b + c) * \cosh(x)^4 + (a + b + c) \\
&) * \sinh(x)^4 - 4 * (a - c) * \cosh(x)^2 + 2 * (3 * (a + b + c) * \cosh(x)^2 - 2 * a + 2 * c) \\
& * \sinh(x)^2 + 3 * a - b + 3 * c) / (\cosh(x)^4 - 4 * \cosh(x)^3 * \sinh(x) + 6 * \cosh(x)^2 * \\
& \sinh(x)^2 - 4 * \cosh(x) * \sinh(x)^3 + \sinh(x)^4)) / (((a + b) * c + c^2) * \cosh(x)^8 \\
& + 8 * ((a + b) * c + c^2) * \cosh(x) * \sinh(x)^7 + ((a + b) * c + c^2) * \sinh(x)^8 - 4 * (\\
& a * c - c^2) * \cosh(x)^6 + 4 * (7 * ((a + b) * c + c^2) * \cosh(x)^2 - a * c + c^2) * \sinh(x) \\
&)^6 + 8 * (7 * ((a + b) * c + c^2) * \cosh(x)^3 - 3 * (a * c - c^2) * \cosh(x)) * \sinh(x)^5 + \\
& 2 * ((3 * a - b) * c + 3 * c^2) * \cosh(x)^4 + 2 * (35 * ((a + b) * c + c^2) * \cosh(x)^4 - 30 \\
& * (a * c - c^2) * \cosh(x)^2 + (3 * a - b) * c + 3 * c^2) * \sinh(x)^4 + 8 * (7 * ((a + b) * c + \\
& c^2) * \cosh(x)^5 - 10 * (a * c - c^2) * \cosh(x)^3 + ((3 * a - b) * c + 3 * c^2) * \cosh(x)) \\
& * \sinh(x)^3 - 4 * (a * c - c^2) * \cosh(x)^2 + 4 * (7 * ((a + b) * c + c^2) * \cosh(x)^6 - 1 \\
& 5 * (a * c - c^2) * \cosh(x)^4 + 3 * ((3 * a - b) * c + 3 * c^2) * \cosh(x)^2 - a * c + c^2) * \sinh(x) \\
& ^2 + (a + b) * c + c^2 + 8 * (((a + b) * c + c^2) * \cosh(x)^7 - 3 * (a * c - c^2) * \\
& \cosh(x)^5 + ((3 * a - b) * c + 3 * c^2) * \cosh(x)^3 - (a * c - c^2) * \cosh(x)) * \sinh(x) \\
&) + 2 * \text{sqrt}(2) * c * \text{sqrt}(((a + b + c) * \cosh(x)^4 + (a + b + c) * \sinh(x)^4 - 4 * (a \\
& - c) * \cosh(x)^2 + 2 * (3 * (a + b + c) * \cosh(x)^2 - 2 * a + 2 * c) * \sinh(x)^2 + 3 * a - \\
& b + 3 * c) / (\cosh(x)^4 - 4 * \cosh(x)^3 * \sinh(x) + 6 * \cosh(x)^2 * \sinh(x)^2 - 4 * \cosh(x) \\
&) * \sinh(x)^3 + \sinh(x)^4)) / (c * \cosh(x)^4 + 4 * c * \cosh(x) * \sinh(x)^3 + c * \sinh(x) \\
&)^4 - 2 * c * \cosh(x)^2 + 2 * (3 * c * \cosh(x)^2 - c) * \sinh(x)^2 + 4 * (c * \cosh(x)^3 - c *
\end{aligned}$$

`cosh(x))*sinh(x) + c]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \coth^2(x) + c \coth^4(x)} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)*(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)`

[Out] `Integral(sqrt(a + b*coth(x)**2 + c*coth(x)**4)*coth(x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \coth(x)^4 + b \coth(x)^2 + a} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)`

3.211 $\int e^{c(a+bx)} \coth^2(ac + bcx)^{5/2} dx$

Optimal. Leaf size=319

$$\frac{15 \tanh^{-1}\left(e^{c(a+bx)}\right) \tanh(ac + bcx) \sqrt{\coth^2(ac + bcx)}}{4bc} + \frac{e^{c(a+bx)} \tanh(ac + bcx) \sqrt{\coth^2(ac + bcx)}}{bc} + \frac{25e^{c(a+bx)} \tanh(ac + bcx)}{4bc}$$

[Out] $(E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(b*c) - (4*E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(b*c*(1 - E^{(2*c*(a + b*x))})^4) + (26*E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(3*b*c*(1 - E^{(2*c*(a + b*x))})^3) - (55*E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(6*b*c*(1 - E^{(2*c*(a + b*x))})^2) + (25*E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(4*b*c*(1 - E^{(2*c*(a + b*x))})) - (15*\text{ArcTanh}[E^{(c*(a + b*x))}]*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(4*b*c)$

Rubi [A] time = 0.912047, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {6720, 2282, 390, 1814, 1157, 385, 207}

$$\frac{15 \tanh^{-1}\left(e^{c(a+bx)}\right) \tanh(ac + bcx) \sqrt{\coth^2(ac + bcx)}}{4bc} + \frac{e^{c(a+bx)} \tanh(ac + bcx) \sqrt{\coth^2(ac + bcx)}}{bc} + \frac{25e^{c(a+bx)} \tanh(ac + bcx)}{4bc}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(c*(a + b*x))}*(\text{Coth}[a*c + b*c*x]^2)^{(5/2)}, x]$

[Out] $(E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(b*c) - (4*E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(b*c*(1 - E^{(2*c*(a + b*x))})^4) + (26*E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(3*b*c*(1 - E^{(2*c*(a + b*x))})^3) - (55*E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(6*b*c*(1 - E^{(2*c*(a + b*x))})^2) + (25*E^{(c*(a + b*x))}*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(4*b*c*(1 - E^{(2*c*(a + b*x))})) - (15*\text{ArcTanh}[E^{(c*(a + b*x))}]*\text{Sqrt}[\text{Coth}[a*c + b*c*x]^2]*\text{Tanh}[a*c + b*c*x])/(4*b*c)$

Rule 6720

$\text{Int}[(u_*)*((a_*)(v_)^{(m_*)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a*v^m)^{\text{FracPart}[p]})/v^{(m*\text{FracPart}[p])}, \text{Int}[u*v^{(m*p)}, x], x] /; \text{FreeQ}\{a, m, p\}, x$

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2} dx &= \left(\sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)} \right) \int e^{c(a+bx)} \coth^5(ac+bcx) dx \\
 &= \frac{\left(\sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)} \right) \text{Subst} \left(\int \frac{(1+x^2)^5}{(-1+x^2)^5} dx, x, e^{c(a+bx)} \right)}{bc} \\
 &= \frac{\left(\sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)} \right) \text{Subst} \left(\int \left(1 + \frac{2(1+10x^4+5x^8)}{(-1+x^2)^5} \right) dx, x, e^{c(a+bx)} \right)}{bc} \\
 &= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} + \frac{\left(2 \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)} \right) \text{Subst} \left(\int \frac{1+10x^4+5x^8}{(-1+x^2)^5} dx, x, e^{c(a+bx)} \right)}{bc} \\
 &= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^4} \\
 &= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^4} \\
 &= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^4} \\
 &= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^4} \\
 &= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^4}
 \end{aligned}$$

Mathematica [A] time = 10.2662, size = 164, normalized size = 0.51

$$\frac{(66e^{c(a+bx)} - 314e^{3c(a+bx)} + 374e^{5c(a+bx)} - 246e^{7c(a+bx)} + 24e^{9c(a+bx)} + 45(e^{2c(a+bx)} - 1)^4 \log(1 - e^{c(a+bx)}) - 45(e^{2c(a+bx)} - 1)^4 \log(1 + e^{c(a+bx)}))}{24bc(e^{2c(a+bx)} - 1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2), x]

[Out] (Sqrt[Coth[c*(a + b*x)]^2]*(66*E^(c*(a + b*x)) - 314*E^(3*c*(a + b*x)) + 374*E^(5*c*(a + b*x)) - 246*E^(7*c*(a + b*x)) + 24*E^(9*c*(a + b*x)) + 45*(-1 + E^(2*c*(a + b*x)))^4*Log[1 - E^(c*(a + b*x))] - 45*(-1 + E^(2*c*(a + b*x)))^4*Log[1 + E^(c*(a + b*x))])*Tanh[c*(a + b*x)]/(24*b*c*(-1 + E^(2*c*(a + b*x)))^4)

Maple [A] time = 0.255, size = 320, normalized size = 1.

$$\frac{(e^{2c(bx+a)} - 1)e^{c(bx+a)}}{(1 + e^{2c(bx+a)})cb} \sqrt{\frac{(1 + e^{2c(bx+a)})^2}{(e^{2c(bx+a)} - 1)^2}} - \frac{e^{c(bx+a)}(75e^{6c(bx+a)} - 115e^{4c(bx+a)} + 109e^{2c(bx+a)} - 21)}{(12 + 12e^{2c(bx+a)})(e^{2c(bx+a)} - 1)^3 cb} \sqrt{\frac{(1 + e^{2c(bx+a)})^2}{(e^{2c(bx+a)} - 1)^2}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2), x)

[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*exp(c*(b*x+a))-1/12/(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a))-1)^3*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))*(75*exp(6*c*(b*x+a))-115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))-21)/c/b+15/8/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-1)-15/8/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+1)

Maxima [A] time = 1.60907, size = 225, normalized size = 0.71

$$-\frac{15 \log(e^{(bcx+ac)} + 1)}{8bc} + \frac{15 \log(e^{(bcx+ac)} - 1)}{8bc} + \frac{12e^{9bcx+9ac} - 123e^{7bcx+7ac} + 187e^{5bcx+5ac} - 157e^{3bcx+3ac} + 33e^{bcx+ac}}{12bc(e^{8bcx+8ac} - 4e^{6bcx+6ac} + 6e^{4bcx+4ac} - 4e^{2bcx+2ac} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(b*c*x + a*c) + 1)/(b*c) + 15/8*log(e^(b*c*x + a*c) - 1)/(b*c)
+ 1/12*(12*e^(9*b*c*x + 9*a*c) - 123*e^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x +
  5*a*c) - 157*e^(3*b*c*x + 3*a*c) + 33*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x +
  8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a
  *c) + 1))
```

Fricas [B] time = 2.67679, size = 4176, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*cosh(b*c*x + a*c)^9 + 216*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
24*sinh(b*c*x + a*c)^9 + 6*(144*cosh(b*c*x + a*c)^2 - 41)*sinh(b*c*x + a*c)
^7 - 246*cosh(b*c*x + a*c)^7 + 42*(48*cosh(b*c*x + a*c)^3 - 41*cosh(b*c*x +
  a*c))*sinh(b*c*x + a*c)^6 + 2*(1512*cosh(b*c*x + a*c)^4 - 2583*cosh(b*c*x
  + a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 374*cosh(b*c*x + a*c)^5 + 2*(1512*cos
  h(b*c*x + a*c)^5 - 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b
  *c*x + a*c)^4 + 2*(1008*cosh(b*c*x + a*c)^6 - 4305*cosh(b*c*x + a*c)^4 + 18
  70*cosh(b*c*x + a*c)^2 - 157)*sinh(b*c*x + a*c)^3 - 314*cosh(b*c*x + a*c)^3
  + 2*(432*cosh(b*c*x + a*c)^7 - 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x
  + a*c)^3 - 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*
  c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7
  *cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(
  7*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*co
  sh(b*c*x + a*c)^4 - 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cos
  h(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*co
  sh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b
  *c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c
  *x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x
  + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c)
  + sinh(b*c*x + a*c) + 1) + 45*(cosh(b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*s
  inh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*c*x + a*c)^2 - 1)*si
  nh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 - 3*co
  sh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x + a*c)^4 - 30*cosh(
  b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh
```

$$\begin{aligned} & (b^2c^2x^2 + a^2c^2)^5 - 10\cosh(b^2c^2x^2 + a^2c^2)^3 + 3\cosh(b^2c^2x^2 + a^2c^2)\sinh(b^2c^2x^2 + a^2c^2)^3 + 4(7\cosh(b^2c^2x^2 + a^2c^2)^6 - 15\cosh(b^2c^2x^2 + a^2c^2)^4 + 9\cosh(b^2c^2x^2 + a^2c^2)^2 - 1)\sinh(b^2c^2x^2 + a^2c^2)^2 - 4\cosh(b^2c^2x^2 + a^2c^2)^2 + 8(\cosh(b^2c^2x^2 + a^2c^2)^7 - 3\cosh(b^2c^2x^2 + a^2c^2)^5 + 3\cosh(b^2c^2x^2 + a^2c^2)^3 - \cosh(b^2c^2x^2 + a^2c^2))\sinh(b^2c^2x^2 + a^2c^2) + 1)\log(\cosh(b^2c^2x^2 + a^2c^2) + \sinh(b^2c^2x^2 + a^2c^2) - 1) + 2(108\cosh(b^2c^2x^2 + a^2c^2)^8 - 861\cosh(b^2c^2x^2 + a^2c^2)^6 + 935\cosh(b^2c^2x^2 + a^2c^2)^4 - 471\cosh(b^2c^2x^2 + a^2c^2)^2 + 33)\sinh(b^2c^2x^2 + a^2c^2) + 66\cosh(b^2c^2x^2 + a^2c^2)) / (b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^8 + 8b^2c^2\cosh(b^2c^2x^2 + a^2c^2)\sinh(b^2c^2x^2 + a^2c^2)^7 + b^2c^2\sinh(b^2c^2x^2 + a^2c^2)^8 - 4b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^6 + 4(7b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^6 - b^2c^2)\sinh(b^2c^2x^2 + a^2c^2)^6 + 6b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^4 + 8(7b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^3 - 3b^2c^2\cosh(b^2c^2x^2 + a^2c^2))\sinh(b^2c^2x^2 + a^2c^2)^5 + 2(35b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^4 - 30b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^2 + 3b^2c^2)\sinh(b^2c^2x^2 + a^2c^2)^4 - 4b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^2 + 8(7b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^5 - 10b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^3 + 3b^2c^2\cosh(b^2c^2x^2 + a^2c^2))\sinh(b^2c^2x^2 + a^2c^2)^3 + 4(7b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^6 - 15b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^4 + 9b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^2 - b^2c^2)\sinh(b^2c^2x^2 + a^2c^2)^2 + b^2c^2 + 8(b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^7 - 3b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^5 + 3b^2c^2\cosh(b^2c^2x^2 + a^2c^2)^3 - b^2c^2\cosh(b^2c^2x^2 + a^2c^2))\sinh(b^2c^2x^2 + a^2c^2)) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(5/2), x)

[Out] Timed out

Giac [A] time = 1.43158, size = 244, normalized size = 0.76

$$\frac{24 e^{(bcx+ac)}}{\operatorname{sgn}(e^{(2bcx+2ac)}-1)} - \frac{45 \log(e^{(bcx+ac)}+1)}{\operatorname{sgn}(e^{(2bcx+2ac)}-1)} + \frac{45 \log(|e^{(bcx+ac)}-1|)}{\operatorname{sgn}(e^{(2bcx+2ac)}-1)} - \frac{2(75 e^{(7bcx+7ac)} - 115 e^{(5bcx+5ac)} + 109 e^{(3bcx+3ac)} - 21 e^{(bcx+ac)})}{(e^{(2bcx+2ac)}-1)^4 \operatorname{sgn}(e^{(2bcx+2ac)}-1)}$$

$24bc$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2), x, algorithm="giac")

```
[Out] 1/24*(24*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 45*log(e^(b*c*x + a*c) + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 45*log(abs(e^(b*c*x + a*c) - 1))/sgn(e^(2*b*c*x + 2*a*c) - 1) - 2*(75*e^(7*b*c*x + 7*a*c) - 115*e^(5*b*c*x + 5*a*c) + 109*e^(3*b*c*x + 3*a*c) - 21*e^(b*c*x + a*c))/((e^(2*b*c*x + 2*a*c) - 1)^4*sgn(e^(2*b*c*x + 2*a*c) - 1)))/(b*c)
```

3.212 $\int e^{c(a+bx)} \coth^2(ac + bcx)^{3/2} dx$

Optimal. Leaf size=197

$$\frac{3 \tanh^{-1}\left(e^{c(a+bx)}\right) \tanh(ac + bcx) \sqrt{\coth^2(ac + bcx)}}{bc} + \frac{e^{c(a+bx)} \tanh(ac + bcx) \sqrt{\coth^2(ac + bcx)}}{bc} + \frac{3e^{c(a+bx)} \tanh(ac + bcx)}{bc(1 - e^{2c(a+bx)})}$$

[Out] $(E^{(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*E^{(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^{(2*c*(a + b*x))})^2) + (3*E^{(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^{(2*c*(a + b*x))})) - (3*ArcTanh[E^{(c*(a + b*x))}]*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)$

Rubi [A] time = 0.2836, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {6720, 2282, 390, 1158, 12, 288, 207}

$$\frac{3 \tanh^{-1}\left(e^{c(a+bx)}\right) \tanh(ac + bcx) \sqrt{\coth^2(ac + bcx)}}{bc} + \frac{e^{c(a+bx)} \tanh(ac + bcx) \sqrt{\coth^2(ac + bcx)}}{bc} + \frac{3e^{c(a+bx)} \tanh(ac + bcx)}{bc(1 - e^{2c(a+bx)})}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^{(3/2)}, x]$

[Out] $(E^{(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*E^{(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^{(2*c*(a + b*x))})^2) + (3*E^{(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^{(2*c*(a + b*x))})) - (3*ArcTanh[E^{(c*(a + b*x))}]*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)$

Rule 6720

$\text{Int}[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a*v^m)^{\text{FracPart}[p]})/v^{(m*\text{FracPart}[p])}, \text{Int}[u*v^{(m*p)}, x], x] /;$ FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, c, d, e},
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2} dx &= \left(\sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)} \right) \int e^{c(a+bx)} \coth^3(ac+bcx) dx \\
&= \frac{\left(\sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)} \right) \text{Subst} \left(\int \frac{(1+x^2)^3}{(-1+x^2)^3} dx, x, e^{c(a+bx)} \right)}{bc} \\
&= \frac{\left(\sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)} \right) \text{Subst} \left(\int \left(1 + \frac{2(1+3x^4)}{(-1+x^2)^3} \right) dx, x, e^{c(a+bx)} \right)}{bc} \\
&= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} + \frac{\left(2 \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)} \right) \text{Subst} \left(\int \frac{2x^4}{(-1+x^2)^3} dx, x, e^{c(a+bx)} \right)}{bc} \\
&= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^2} \\
&= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^2} \\
&= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^2} \\
&= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^2} \\
&= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)} \sqrt{\coth^2(ac+bcx) \tanh(ac+bcx)}}{bc(1-e^{2c(a+bx)})^2}
\end{aligned}$$

Mathematica [C] time = 3.77009, size = 334, normalized size = 1.7

$$e^{-5c(a+bx)} \tanh^3(c(a+bx)) \coth^2(c(a+bx))^{3/2} \left(256e^{8c(a+bx)} (e^{2c(a+bx)} + 1) \right)^3 \text{HypergeometricPFQ} \left(\left\{ \frac{3}{2}, 2, 2, 2, 2 \right\}, \left\{ 1, \dots \right\} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2), x]

[Out] -((Coth[c*(a + b*x)]^2)^(3/2)*(-21*(252105 + 507305*E^(2*c*(a + b*x))) + 173916*E^(4*c*(a + b*x)) - 154296*E^(6*c*(a + b*x)) - 73885*E^(8*c*(a + b*x)) + 4887*E^(10*c*(a + b*x))) - (315*(-16807 - 28218*E^(2*c*(a + b*x))) + 1173*

$E^{(4c(a+bx))} + 17748E^{(6c(a+bx))} + 4299E^{(8c(a+bx))} - 1434E^{(10c(a+bx))} + 7E^{(12c(a+bx))} \cdot \text{ArcTanh}[\text{Sqrt}[E^{(2c(a+bx))}]] / \text{Sqrt}[E^{(2c(a+bx))}] + 384E^{(8c(a+bx))} \cdot (1 + E^{(2c(a+bx))})^2 \cdot (7 + 5E^{(2c(a+bx))}) \cdot \text{HypergeometricPFQ}[\{3/2, 2, 2, 2, 2\}, \{1, 1, 1, 11/2\}, E^{(2c(a+bx))}] + 256E^{(8c(a+bx))} \cdot (1 + E^{(2c(a+bx))})^3 \cdot \text{HypergeometricPFQ}[\{3/2, 2, 2, 2, 2, 2\}, \{1, 1, 1, 1, 11/2\}, E^{(2c(a+bx))}] \cdot \text{Tanh}[c(a+bx)]^3 / (60480 \cdot b \cdot c \cdot E^{(5c(a+bx))})$

Maple [A] time = 0.199, size = 298, normalized size = 1.5

$$\frac{(e^{2c(bx+a)} - 1)e^{c(bx+a)}}{(1 + e^{2c(bx+a)})cb} \sqrt{\frac{(1 + e^{2c(bx+a)})^2}{(e^{2c(bx+a)} - 1)^2}} - \frac{e^{c(bx+a)}(3e^{2c(bx+a)} - 1)}{(1 + e^{2c(bx+a)})(e^{2c(bx+a)} - 1)cb} \sqrt{\frac{(1 + e^{2c(bx+a)})^2}{(e^{2c(bx+a)} - 1)^2}} - \frac{(3e^{2c(bx+a)} - 3)\ln(e^{c(bx+a)})}{(2 + 2e^{2c(bx+a)})c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2), x)

[Out] $1/(1+\exp(2c(b*x+a))) \cdot (\exp(2c(b*x+a))-1) \cdot ((1+\exp(2c(b*x+a)))^2/(\exp(2c(b*x+a))-1)^2)^{(1/2)}/c/b \cdot \exp(c(b*x+a))-1/(1+\exp(2c(b*x+a)))/(\exp(2c(b*x+a))-1) \cdot ((1+\exp(2c(b*x+a)))^2/(\exp(2c(b*x+a))-1)^2)^{(1/2)} \cdot \exp(c(b*x+a)) \cdot (3 \cdot \exp(2c(b*x+a))-1)/c/b - 3/2/(1+\exp(2c(b*x+a))) \cdot (\exp(2c(b*x+a))-1) \cdot ((1+\exp(2c(b*x+a)))^2/(\exp(2c(b*x+a))-1)^2)^{(1/2)}/c/b \cdot \ln(\exp(c(b*x+a))+1) + 3/2/(1+\exp(2c(b*x+a))) \cdot (\exp(2c(b*x+a))-1) \cdot ((1+\exp(2c(b*x+a)))^2/(\exp(2c(b*x+a))-1)^2)^{(1/2)}/c/b \cdot \ln(\exp(c(b*x+a))-1)$

Maxima [A] time = 1.82596, size = 151, normalized size = 0.77

$$-\frac{3 \log(e^{(bcx+ac)} + 1)}{2bc} + \frac{3 \log(e^{(bcx+ac)} - 1)}{2bc} + \frac{e^{(5bcx+5ac)} - 5e^{(3bcx+3ac)} + 2e^{(bcx+ac)}}{bc(e^{(4bcx+4ac)} - 2e^{(2bcx+2ac)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2), x, algorithm="maxima")

[Out] $-3/2 \cdot \log(e^{(b*c*x + a*c)} + 1)/(b*c) + 3/2 \cdot \log(e^{(b*c*x + a*c)} - 1)/(b*c) + (e^{(5*b*c*x + 5*a*c)} - 5 \cdot e^{(3*b*c*x + 3*a*c)} + 2 \cdot e^{(b*c*x + a*c)})/(b*c \cdot (e^{(4*b*c*x + 4*a*c)} - 2 \cdot e^{(2*b*c*x + 2*a*c)} + 1))$

Fricas [B] time = 2.42228, size = 1574, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2), x, algorithm="fricas")

[Out]
$$\frac{1}{2} \cdot (2 \cosh(bcx + ac)^5 + 10 \cosh(bcx + ac) \sinh(bcx + ac)^4 + 2 \sinh(bcx + ac)^5 + 10(2 \cosh(bcx + ac)^2 - 1) \sinh(bcx + ac)^3 - 10 \cosh(bcx + ac)^3 + 10(2 \cosh(bcx + ac)^3 - 3 \cosh(bcx + ac)) \sinh(bcx + ac)^2 - 3(\cosh(bcx + ac)^4 + 4 \cosh(bcx + ac) \sinh(bcx + ac)^3 + \sinh(bcx + ac)^4 + 2(3 \cosh(bcx + ac)^2 - 1) \sinh(bcx + ac)^2 - 2 \cosh(bcx + ac)^2 + 4(\cosh(bcx + ac)^3 - \cosh(bcx + ac)) \sinh(bcx + ac) + 1) \log(\cosh(bcx + ac) + \sinh(bcx + ac) + 1) + 3(\cosh(bcx + ac)^4 + 4 \cosh(bcx + ac) \sinh(bcx + ac)^3 + \sinh(bcx + ac)^4 + 2(3 \cosh(bcx + ac)^2 - 1) \sinh(bcx + ac)^2 - 2 \cosh(bcx + ac)^2 + 4(\cosh(bcx + ac)^3 - \cosh(bcx + ac)) \sinh(bcx + ac) + 1) \log(\cosh(bcx + ac) + \sinh(bcx + ac) - 1) + 2(5 \cosh(bcx + ac)^4 - 15 \cosh(bcx + ac)^2 + 2) \sinh(bcx + ac) + 4 \cosh(bcx + ac)) / (bc \cosh(bcx + ac)^4 + 4bc \cosh(bcx + ac) \sinh(bcx + ac)^3 + bc \sinh(bcx + ac)^4 - 2bc \cosh(bcx + ac)^2 + 2(3bc \cosh(bcx + ac)^2 + 2bc \sinh(bcx + ac)^2 - bc) \sinh(bcx + ac)^2 + bc + 4(bc \cosh(bcx + ac)^3 - bc \cosh(bcx + ac)) \sinh(bcx + ac))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(3/2), x)

[Out] Timed out

Giac [A] time = 1.27028, size = 209, normalized size = 1.06

$$\frac{2e^{(bcx+ac)}}{\operatorname{sgn}(e^{(2bcx+2ac)}-1)} - \frac{3 \log(e^{(bcx+ac)}+1)}{\operatorname{sgn}(e^{(2bcx+2ac)}-1)} + \frac{3 \log(|e^{(bcx+ac)}-1|)}{\operatorname{sgn}(e^{(2bcx+2ac)}-1)} - \frac{2(3e^{(3bcx+3ac)}-e^{(bcx+ac)})}{(e^{(2bcx+2ac)}-1)^2 \operatorname{sgn}(e^{(2bcx+2ac)}-1)}$$

$2bc$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 3*log(e^(b*c*x + a*c)
+ 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 3*log(abs(e^(b*c*x + a*c) - 1))/sgn(e^(
2*b*c*x + 2*a*c) - 1) - 2*(3*e^(3*b*c*x + 3*a*c) - e^(b*c*x + a*c))/((e^(2
*b*c*x + 2*a*c) - 1)^2*sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c)
```

$$3.213 \quad \int e^{c(a+bx)} \sqrt{\coth^2(ac+bcx)} dx$$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)} \tanh(ac+bcx) \sqrt{\coth^2(ac+bcx)}}{bc} - \frac{2 \tanh^{-1}(e^{c(a+bx)}) \tanh(ac+bcx) \sqrt{\coth^2(ac+bcx)}}{bc}$$

[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*ArcTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)

Rubi [A] time = 0.143621, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {6720, 2282, 388, 206}

$$\frac{e^{c(a+bx)} \tanh(ac+bcx) \sqrt{\coth^2(ac+bcx)}}{bc} - \frac{2 \tanh^{-1}(e^{c(a+bx)}) \tanh(ac+bcx) \sqrt{\coth^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2], x]

[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*ArcTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int e^{c(a+bx)} \sqrt{\coth^2(ac+bcx)} dx &= \left(\sqrt{\coth^2(ac+bcx)} \tanh(ac+bcx) \right) \int e^{c(a+bx)} \coth(ac+bcx) dx \\ &= \frac{\left(\sqrt{\coth^2(ac+bcx)} \tanh(ac+bcx) \right) \text{Subst} \left(\int \frac{-1-x^2}{1-x^2} dx, x, e^{c(a+bx)} \right)}{bc} \\ &= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx)} \tanh(ac+bcx)}{bc} - \frac{\left(2 \sqrt{\coth^2(ac+bcx)} \tanh(ac+bcx) \right) \text{Subst} \left(\int \frac{-1-x^2}{1-x^2} dx, x, e^{c(a+bx)} \right)}{bc} \\ &= \frac{e^{c(a+bx)} \sqrt{\coth^2(ac+bcx)} \tanh(ac+bcx)}{bc} - \frac{2 \tanh^{-1} \left(e^{c(a+bx)} \right) \sqrt{\coth^2(ac+bcx)} \tanh(ac+bcx)}{bc} \end{aligned}$$

Mathematica [A] time = 0.0577475, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)} - 2 \tanh^{-1} \left(e^{c(a+bx)} \right) \right) \tanh(c(a+bx)) \sqrt{\coth^2(c(a+bx))}}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2], x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTanh[E^(c*(a + b*x))])*Sqrt[Coth[c*(a + b*x)]^2]*T
anh[c*(a + b*x)])/ (b*c)
```

Maple [B] time = 0.238, size = 213, normalized size = 2.6

$$\frac{(e^{2c(bx+a)} - 1)e^{c(bx+a)}}{(1 + e^{2c(bx+a)})cb} \sqrt{\frac{(1 + e^{2c(bx+a)})^2}{(e^{2c(bx+a)} - 1)^2}} + \frac{(e^{2c(bx+a)} - 1)\ln(e^{c(bx+a)} - 1)}{(1 + e^{2c(bx+a)})cb} \sqrt{\frac{(1 + e^{2c(bx+a)})^2}{(e^{2c(bx+a)} - 1)^2}} - \frac{(e^{2c(bx+a)} - 1)\ln(e^{c(bx+a)} + 1)}{(1 + e^{2c(bx+a)})cb} \sqrt{\frac{(1 + e^{2c(bx+a)})^2}{(e^{2c(bx+a)} - 1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x)

[Out] $\frac{1}{(1+\exp(2*c*(b*x+a)))} * (\exp(2*c*(b*x+a))-1) * ((1+\exp(2*c*(b*x+a)))^2 / (\exp(2*c*(b*x+a))-1)^2)^{(1/2)} / c / b * \exp(c*(b*x+a)) + \frac{1}{(1+\exp(2*c*(b*x+a)))} * (\exp(2*c*(b*x+a))-1) * ((1+\exp(2*c*(b*x+a)))^2 / (\exp(2*c*(b*x+a))-1)^2)^{(1/2)} / c / b * \ln(\exp(c*(b*x+a))-1) - \frac{1}{(1+\exp(2*c*(b*x+a)))} * (\exp(2*c*(b*x+a))-1) * ((1+\exp(2*c*(b*x+a)))^2 / (\exp(2*c*(b*x+a))-1)^2)^{(1/2)} / c / b * \ln(\exp(c*(b*x+a))+1)$

Maxima [A] time = 1.70541, size = 76, normalized size = 0.92

$$\frac{e^{(bcx+ac)}}{bc} - \frac{\log(e^{(bcx+ac)} + 1)}{bc} + \frac{\log(e^{(bcx+ac)} - 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")

[Out] $e^{(b*c*x + a*c)} / (b*c) - \log(e^{(b*c*x + a*c)} + 1) / (b*c) + \log(e^{(b*c*x + a*c)} - 1) / (b*c)$

Fricas [A] time = 2.37335, size = 196, normalized size = 2.36

$$\frac{\cosh(bc x + ac) - \log(\cosh(bc x + ac) + \sinh(bc x + ac) + 1) + \log(\cosh(bc x + ac) + \sinh(bc x + ac) - 1) + \sinh(bc x + ac)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")

[Out] $(\cosh(b*c*x + a*c) - \log(\cosh(b*c*x + a*c) + \sinh(b*c*x + a*c) + 1) + \log(\cosh(b*c*x + a*c) + \sinh(b*c*x + a*c) - 1) + \sinh(b*c*x + a*c)) / (b*c)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(1/2), x)

[Out] Timed out

Giac [A] time = 1.1916, size = 127, normalized size = 1.53

$$\frac{\frac{e^{bcx+ac}}{\operatorname{sgn}(e^{2bcx+2ac}-1)} - \frac{\log(e^{bcx+ac}+1)}{\operatorname{sgn}(e^{2bcx+2ac}-1)} + \frac{\log(|e^{bcx+ac}-1|)}{\operatorname{sgn}(e^{2bcx+2ac}-1)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2), x, algorithm="giac")

[Out] (e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - log(e^(b*c*x + a*c) + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + log(abs(e^(b*c*x + a*c) - 1))/sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c)

$$3.214 \quad \int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}} dx$$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2 \tan^{-1}(e^{c(a+bx)}) \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*ArcTan[E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])

Rubi [A] time = 0.194186, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {6720, 2282, 388, 203}

$$\frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2 \tan^{-1}(e^{c(a+bx)}) \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

Antiderivative was successfully verified.

[In] Int[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2], x]

[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*ArcTan[E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x]
&& !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}} dx &= \frac{\coth(ac+bcx) \int e^{c(a+bx)} \tanh(ac+bcx) dx}{\sqrt{\coth^2(ac+bcx)}} \\ &= \frac{\coth(ac+bcx) \text{Subst}\left(\int \frac{-1+x^2}{1+x^2} dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}} \\ &= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2 \coth(ac+bcx)) \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}} \\ &= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2 \tan^{-1}\left(e^{c(a+bx)}\right) \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} \end{aligned}$$

Mathematica [A] time = 0.117547, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)} - 2 \tan^{-1}\left(e^{c(a+bx)}\right)\right) \coth(c(a+bx))}{bc\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2], x]

[Out] ((E^(c*(a + b*x)) - 2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*Sqrt[Coth[c*(a + b*x)]^2])

Maple [C] time = 0.231, size = 218, normalized size = 2.6

$$\frac{(1 + e^{2c(bx+a)}) e^{c(bx+a)}}{(e^{2c(bx+a)} - 1) cb} \frac{1}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}} + \frac{i(1 + e^{2c(bx+a)}) \ln(e^{c(bx+a)} - i)}{(e^{2c(bx+a)} - 1) cb} \frac{1}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}} - \frac{i(1 + e^{2c(bx+a)}) \ln(e^{c(bx+a)} + i)}{(e^{2c(bx+a)} - 1) cb} \frac{1}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2), x)

[Out] $\frac{1}{((1+\exp(2*c*(b*x+a)))^2/(\exp(2*c*(b*x+a))-1)^2)^{1/2}} \frac{1}{(\exp(2*c*(b*x+a))-1)* (1+\exp(2*c*(b*x+a)))/c/b*\exp(c*(b*x+a))+I/((1+\exp(2*c*(b*x+a)))^2/(\exp(2*c*(b*x+a))-1)^2)^{1/2}} + \frac{1}{((1+\exp(2*c*(b*x+a)))^2/(\exp(2*c*(b*x+a))-1)^2)^{1/2}} \frac{1}{(\exp(2*c*(b*x+a))-1)* (1+\exp(2*c*(b*x+a)))/c/b*\ln(\exp(c*(b*x+a))-I)-I/((1+\exp(2*c*(b*x+a)))^2/(\exp(2*c*(b*x+a))-1)^2)^{1/2}} + \frac{1}{((1+\exp(2*c*(b*x+a)))^2/(\exp(2*c*(b*x+a))-1)^2)^{1/2}} \frac{1}{(\exp(2*c*(b*x+a))-1)* (1+\exp(2*c*(b*x+a)))/c/b*\ln(\exp(c*(b*x+a))+I)}$

Maxima [A] time = 1.71976, size = 47, normalized size = 0.57

$$-\frac{2 \arctan(e^{(bcx+ac)})}{bc} + \frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2), x, algorithm="maxima")

[Out] $-2*\arctan(e^{(b*c*x + a*c)})/(b*c) + e^{(b*c*x + a*c)}/(b*c)$

Fricas [A] time = 2.25958, size = 132, normalized size = 1.59

$$\frac{2 \arctan(\cosh(bc x + ac) + \sinh(bc x + ac)) - \cosh(bc x + ac) - \sinh(bc x + ac)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2), x, algorithm="fricas")

[Out] $-(2*\arctan(\cosh(b*c*x + a*c) + \sinh(b*c*x + a*c)) - \cosh(b*c*x + a*c) - \sinh(b*c*x + a*c))/(b*c)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac} \int \frac{e^{bcx}}{\sqrt{\coth^2(ac + bcx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(1/2), x)`

[Out] `exp(a*c)*Integral(exp(b*c*x)/sqrt(coth(a*c + b*c*x)**2), x)`

Giac [A] time = 1.18302, size = 81, normalized size = 0.98

$$\frac{2 \arctan\left(e^{(bcx+ac)}\right) \operatorname{sgn}\left(e^{(2bcx+2ac)} - 1\right) - e^{(bcx+ac)} \operatorname{sgn}\left(e^{(2bcx+2ac)} - 1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2), x, algorithm="giac")`

[Out] $-(2*\arctan(e^{(b*c*x + a*c)})*\operatorname{sgn}(e^{(2*b*c*x + 2*a*c)} - 1) - e^{(b*c*x + a*c)}*\operatorname{sgn}(e^{(2*b*c*x + 2*a*c)} - 1))/(b*c)$

$$3.215 \quad \int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}} dx$$

Optimal. Leaf size=193

$$\frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)} \coth(ac+bcx)}{bc(e^{2c(a+bx)}+1)\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)} \coth(ac+bcx)}{bc(e^{2c(a+bx)}+1)^2\sqrt{\coth^2(ac+bcx)}} - \frac{3 \tan^{-1}(e^{c(a+bx)})}{bc\sqrt{\coth^2(ac+bcx)}}$$

[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x))))*Sqrt[Coth[a*c + b*c*x]^2]) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])

Rubi [A] time = 0.86382, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {6720, 2282, 390, 1158, 12, 288, 203}

$$\frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)} \coth(ac+bcx)}{bc(e^{2c(a+bx)}+1)\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)} \coth(ac+bcx)}{bc(e^{2c(a+bx)}+1)^2\sqrt{\coth^2(ac+bcx)}} - \frac{3 \tan^{-1}(e^{c(a+bx)})}{bc\sqrt{\coth^2(ac+bcx)}}$$

Antiderivative was successfully verified.

[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2), x]

[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x))))*Sqrt[Coth[a*c + b*c*x]^2]) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])

Rule 6720

Int[(u_)*((a_)*(v_)^(m_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, c, d, e},
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}} dx &= \frac{\coth(ac+bcx) \int e^{c(a+bx)} \tanh^3(ac+bcx) dx}{\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{\coth(ac+bcx) \operatorname{Subst}\left(\int \frac{(-1+x^2)^3}{(1+x^2)^3} dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{\coth(ac+bcx) \operatorname{Subst}\left(\int \left(1 - \frac{2(1+3x^4)}{(1+x^2)^3}\right) dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2 \coth(ac+bcx)) \operatorname{Subst}\left(\int \frac{1+3x^4}{(1+x^2)^3} dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)})^2 \sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx) \operatorname{Subst}\left(\int \frac{1+3x^4}{(1+x^2)^3} dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)})^2 \sqrt{\coth^2(ac+bcx)}} - \frac{(6 \coth(ac+bcx)) \operatorname{Subst}\left(\int \frac{1+3x^4}{(1+x^2)^3} dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)})^2 \sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)}) \sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)})^2 \sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)}) \sqrt{\coth^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] time = 0.297194, size = 104, normalized size = 0.54

$$\frac{\left(e^{c(a+bx)} \left(5e^{2c(a+bx)} + e^{4c(a+bx)} + 2\right) - 3 \left(e^{2c(a+bx)} + 1\right)^2 \tan^{-1}\left(e^{c(a+bx)}\right)\right) \coth(c(a+bx))}{bc \left(e^{2c(a+bx)} + 1\right)^2 \sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2), x]

[Out] $((E^{(c*(a + b*x))}*(2 + 5*E^{(2*c*(a + b*x))} + E^{(4*c*(a + b*x))}) - 3*(1 + E^{(2*c*(a + b*x))})^2*ArcTan[E^{(c*(a + b*x))}])*Coth[c*(a + b*x)]/(b*c*(1 + E^{(2*c*(a + b*x))})^2*Sqrt[Coth[c*(a + b*x)]^2])$

Maple [C] time = 0.209, size = 301, normalized size = 1.6

$$\frac{(1 + e^{2c(bx+a)})e^{c(bx+a)}}{(e^{2c(bx+a)} - 1)cb} \frac{1}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}} + \frac{e^{c(bx+a)}(3e^{2c(bx+a)} + 1)}{(1 + e^{2c(bx+a)})(e^{2c(bx+a)} - 1)cb} \frac{1}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}} + \frac{\frac{3i}{2}(1 + e^{2c(bx+a)}) \ln(e^{c(bx+a)} - 1)}{(e^{2c(bx+a)} - 1)cb}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\exp(c*(b*x+a))/(\coth(b*c*x+a*c)^2)^{(3/2)}, x)$

[Out] $1/((1+\exp(2*c*(b*x+a)))^2/(\exp(2*c*(b*x+a))-1)^2)^{(1/2)}/(\exp(2*c*(b*x+a))-1)*(1+\exp(2*c*(b*x+a)))/c/b*\exp(c*(b*x+a))+1/(1+\exp(2*c*(b*x+a)))/(\exp(2*c*(b*x+a))-1)/((1+\exp(2*c*(b*x+a)))^2/(\exp(2*c*(b*x+a))-1)^2)^{(1/2)}*\exp(c*(b*x+a))*(3*\exp(2*c*(b*x+a))+1)/c/b+3/2*I*(1+\exp(2*c*(b*x+a)))/(\exp(2*c*(b*x+a))-1)/((1+\exp(2*c*(b*x+a)))^2/(\exp(2*c*(b*x+a))-1)^2)^{(1/2)}/c/b*\ln(\exp(c*(b*x+a))-1)-3/2*I*(1+\exp(2*c*(b*x+a)))/(\exp(2*c*(b*x+a))-1)/((1+\exp(2*c*(b*x+a)))^2/(\exp(2*c*(b*x+a))-1)^2)^{(1/2)}/c/b*\ln(\exp(c*(b*x+a))+1)$

Maxima [A] time = 1.66612, size = 122, normalized size = 0.63

$$-\frac{3 \arctan(e^{(bcx+ac)})}{bc} + \frac{e^{(5bcx+5ac)} + 5e^{(3bcx+3ac)} + 2e^{(bcx+ac)}}{bc(e^{(4bcx+4ac)} + 2e^{(2bcx+2ac)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\exp(c*(b*x+a))/(\coth(b*c*x+a*c)^2)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] $-3*\arctan(e^{(b*c*x + a*c)})/(b*c) + (e^{(5*b*c*x + 5*a*c)} + 5*e^{(3*b*c*x + 3*a*c)} + 2*e^{(b*c*x + a*c)})/(b*c*(e^{(4*b*c*x + 4*a*c)} + 2*e^{(2*b*c*x + 2*a*c)} + 1))$

Fricas [B] time = 2.54231, size = 1175, normalized size = 6.09

$$\cosh(bc x + ac)^5 + 5 \cosh(bc x + ac) \sinh(bc x + ac)^4 + \sinh(bc x + ac)^5 + 5(2 \cosh(bc x + ac)^2 + 1) \sinh(bc x + ac)^3 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2), x, algorithm="fricas")

[Out] (cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + sinh(b*c*x + a*c)^5 + 5*(2*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^3 + 5*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + (5*cosh(b*c*x + a*c)^4 + 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 2*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + b*c*sinh(b*c*x + a*c)^4 + 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(3/2), x)

[Out] Timed out

Giac [A] time = 1.21802, size = 176, normalized size = 0.91

$$\left(3 \arctan\left(e^{(bcx+ac)}\right) e^{(-ac)} \operatorname{sgn}\left(e^{(2bcx+2ac)} - 1\right) - e^{(bcx)} \operatorname{sgn}\left(e^{(2bcx+2ac)} - 1\right) - \frac{3 e^{(3bcx+2ac)} \operatorname{sgn}\left(e^{(2bcx+2ac)} - 1\right) + e^{(bcx)} \operatorname{sgn}\left(e^{(2bcx+2ac)} - 1\right)}{\left(e^{(2bcx+2ac)} + 1\right)^2} \right) / bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*arctan(e^(b*c*x + a*c))*e^(-a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (3*e^(3*b*c*x + 2*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) + 1)^2*e^(a*c)/(b*c)
```

$$3.216 \quad \int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}} dx$$

Optimal. Leaf size=311

$$\frac{e^{c(a+bx)} \coth(ac+bcx)}{bc \sqrt{\coth^2(ac+bcx)}} + \frac{25e^{c(a+bx)} \coth(ac+bcx)}{4bc (e^{2c(a+bx)} + 1) \sqrt{\coth^2(ac+bcx)}} - \frac{55e^{c(a+bx)} \coth(ac+bcx)}{6bc (e^{2c(a+bx)} + 1)^2 \sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}}{3bc (e^{2c(a+bx)} + 1)}$$

[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (4*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Coth[a*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3*Sqrt[Coth[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a*c + b*c*x]^2]) + (25*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(4*b*c*(1 + E^(2*c*(a + b*x))) * Sqrt[Coth[a*c + b*c*x]^2]) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(4*b*c*Sqrt[Coth[a*c + b*c*x]^2])

Rubi [A] time = 1.74274, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {6720, 2282, 390, 1814, 1157, 385, 203}

$$\frac{e^{c(a+bx)} \coth(ac+bcx)}{bc \sqrt{\coth^2(ac+bcx)}} + \frac{25e^{c(a+bx)} \coth(ac+bcx)}{4bc (e^{2c(a+bx)} + 1) \sqrt{\coth^2(ac+bcx)}} - \frac{55e^{c(a+bx)} \coth(ac+bcx)}{6bc (e^{2c(a+bx)} + 1)^2 \sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}}{3bc (e^{2c(a+bx)} + 1)}$$

Antiderivative was successfully verified.

[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2), x]

[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (4*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Coth[a*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3*Sqrt[Coth[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a*c + b*c*x]^2]) + (25*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(4*b*c*(1 + E^(2*c*(a + b*x))) * Sqrt[Coth[a*c + b*c*x]^2]) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(4*b*c*Sqrt[Coth[a*c + b*c*x]^2])

Rule 6720

Int[(u_)*((a_)*(v_)^(m_))^(p_), x_Symbol] :=> Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x]


```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x)]*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}} dx &= \frac{\coth(ac+bcx) \int e^{c(a+bx)} \tanh^5(ac+bcx) dx}{\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{\coth(ac+bcx) \operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{(1+x^2)^5} dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{\coth(ac+bcx) \operatorname{Subst}\left(\int \left(1 - \frac{2(1+10x^4+5x^8)}{(1+x^2)^5}\right) dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2 \coth(ac+bcx)) \operatorname{Subst}\left(\int \frac{1+10x^4+5x^8}{(1+x^2)^5} dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)})^4 \sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx) \operatorname{Subst}\left(\int \frac{8-1}{4bc\sqrt{\coth^2(ac+bcx)}} dx, x, e^{c(a+bx)}\right)}{4bc\sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)})^4 \sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)} \coth(ac+bcx)}{3bc(1+e^{2c(a+bx)})^3 \sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)})^4 \sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)} \coth(ac+bcx)}{3bc(1+e^{2c(a+bx)})^3 \sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)})^4 \sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)} \coth(ac+bcx)}{3bc(1+e^{2c(a+bx)})^3 \sqrt{\coth^2(ac+bcx)}} \\
&= \frac{e^{c(a+bx)} \coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)} \coth(ac+bcx)}{bc(1+e^{2c(a+bx)})^4 \sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)} \coth(ac+bcx)}{3bc(1+e^{2c(a+bx)})^3 \sqrt{\coth^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] time = 0.461341, size = 133, normalized size = 0.43

$$\frac{\left(e^{c(a+bx)} \left(157e^{2c(a+bx)} + 187e^{4c(a+bx)} + 123e^{6c(a+bx)} + 12e^{8c(a+bx)} + 33\right) - 45 \left(e^{2c(a+bx)} + 1\right)^4 \tan^{-1}\left(e^{c(a+bx)}\right)\right) \coth(c(a+bx))}{12bc \left(e^{2c(a+bx)} + 1\right)^4 \sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2), x]

[Out] ((E^(c*(a + b*x)))*(33 + 157*E^(2*c*(a + b*x)) + 187*E^(4*c*(a + b*x)) + 123*E^(6*c*(a + b*x)) + 12*E^(8*c*(a + b*x))) - 45*(1 + E^(2*c*(a + b*x)))^4*ArcTan[E^(c*(a + b*x))]*Coth[c*(a + b*x)]/(12*b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Coth[c*(a + b*x)]^2])

Maple [C] time = 0.22, size = 324, normalized size = 1.

$$\frac{(1 + e^{2c(bx+a)})e^{c(bx+a)}}{(e^{2c(bx+a)} - 1)cb} \frac{1}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}} + \frac{e^{c(bx+a)}(75e^{6c(bx+a)} + 115e^{4c(bx+a)} + 109e^{2c(bx+a)} + 21)}{12(1 + e^{2c(bx+a)})^3(e^{2c(bx+a)} - 1)cb} \frac{1}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}} + \frac{15i}{8}(1 + e^{2c(bx+a)})e^{c(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2), x)

[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))/c/b*exp(c*(b*x+a))+1/12/(1+exp(2*c*(b*x+a)))^3/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))*(75*exp(6*c*(b*x+a))+115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))+21)/c/b+15/8*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-I)-15/8*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+I)

Maxima [A] time = 1.76172, size = 196, normalized size = 0.63

$$\frac{15 \arctan\left(e^{(bcx+ac)}\right)}{4bc} + \frac{12e^{(9bcx+9ac)} + 123e^{(7bcx+7ac)} + 187e^{(5bcx+5ac)} + 157e^{(3bcx+3ac)} + 33e^{(bcx+ac)}}{12bc\left(e^{(8bcx+8ac)} + 4e^{(6bcx+6ac)} + 6e^{(4bcx+4ac)} + 4e^{(2bcx+2ac)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/4*arctan(e^(b*c*x + a*c))/(b*c) + 1/12*(12*e^(9*b*c*x + 9*a*c) + 123*e^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x + 5*a*c) + 157*e^(3*b*c*x + 3*a*c) + 33*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 2.61625, size = 3186, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*cosh(b*c*x + a*c)^9 + 108*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 + 12*sinh(b*c*x + a*c)^9 + 3*(144*cosh(b*c*x + a*c)^2 + 41)*sinh(b*c*x + a*c)^7 + 123*cosh(b*c*x + a*c)^7 + 21*(48*cosh(b*c*x + a*c)^3 + 41*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^6 + (1512*cosh(b*c*x + a*c)^4 + 2583*cosh(b*c*x + a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 187*cosh(b*c*x + a*c)^5 + (1512*cosh(b*c*x + a*c)^5 + 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^4 + (1008*cosh(b*c*x + a*c)^6 + 4305*cosh(b*c*x + a*c)^4 + 1870*cosh(b*c*x + a*c)^2 + 157)*sinh(b*c*x + a*c)^3 + 157*cosh(b*c*x + a*c)^3 + (432*cosh(b*c*x + a*c)^7 + 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x + a*c)^3 + 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^6 + 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x + a*c)^4 + 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 + 15*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 4*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 + 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + (108*cosh(b*c*x + a*c)^8 + 861*cosh(b*c*x + a*c)^6 + 935*cosh(b*c*x + a*c)^4 + 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 33*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + b*c*sinh(b*c*x + a*c)^8 + 4*b*c*cosh(b*c*x + a*c)^6 + 4*(7*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)^4 + 8*(7*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
```

$$+ a*c)^5 + 2*(35*b*c*cosh(b*c*x + a*c)^4 + 30*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x + a*c)^5 + 10*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*b*c*cosh(b*c*x + a*c)^6 + 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x + a*c)^7 + 3*b*c*cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(5/2), x)

[Out] Timed out

Giac [A] time = 1.25344, size = 250, normalized size = 0.8

$$\frac{\left(45 \arctan\left(e^{(bcx+ac)}\right) e^{-ac} \operatorname{sgn}\left(e^{(2bcx+2ac)} - 1\right) - 12 e^{(bcx)} \operatorname{sgn}\left(e^{(2bcx+2ac)} - 1\right) - \frac{75 e^{(7bcx+6ac)} \operatorname{sgn}\left(e^{(2bcx+2ac)} - 1\right) + 115 e^{(5bcx+4ac)}}{12bc}\right)}{12bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2), x, algorithm="giac")

[Out]
$$-1/12*(45*\arctan(e^{(b*c*x + a*c)})*e^{-a*c}*sgn(e^{(2*b*c*x + 2*a*c)} - 1) - 12*e^{(b*c*x)}*sgn(e^{(2*b*c*x + 2*a*c)} - 1) - (75*e^{(7*b*c*x + 6*a*c)}*sgn(e^{(2*b*c*x + 2*a*c)} - 1) + 115*e^{(5*b*c*x + 4*a*c)}*sgn(e^{(2*b*c*x + 2*a*c)} - 1) + 109*e^{(3*b*c*x + 2*a*c)}*sgn(e^{(2*b*c*x + 2*a*c)} - 1) + 21*e^{(b*c*x)}*sgn(e^{(2*b*c*x + 2*a*c)} - 1))/(e^{(2*b*c*x + 2*a*c)} + 1)^4)*e^{(a*c)}/(b*c)$$

3.217 $\int \sin^3(\coth(a + bx)) dx$

Optimal. Leaf size=157

$$\frac{\sin(3)\text{CosIntegral}(3 - 3 \coth(a + bx))}{8b} + \frac{\sin(3)\text{CosIntegral}(3 \coth(a + bx) + 3)}{8b} - \frac{3 \sin(1)\text{CosIntegral}(1 - \coth(a + bx))}{8b}$$

```
[Out] (-3*CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Coth[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3])/(8*b) +
(CosIntegral[3 + 3*Coth[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
(3*Cos[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rubi [A] time = 0.374672, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\sin(3)\text{CosIntegral}(3 - 3 \coth(a + bx))}{8b} + \frac{\sin(3)\text{CosIntegral}(3 \coth(a + bx) + 3)}{8b} - \frac{3 \sin(1)\text{CosIntegral}(1 - \coth(a + bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Coth[a + b*x]]^3,x]
```

```
[Out] (-3*CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Coth[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3])/(8*b) +
(CosIntegral[3 + 3*Coth[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
(3*Cos[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \sin^3(\coth(a + bx)) dx &= \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1-x^2} dx, x, \coth(a + bx)\right)}{b} \\
 &= \frac{\text{Subst}\left(\int \left(-\frac{\sin^3(x)}{2(-1+x)} + \frac{\sin^3(x)}{2(1+x)}\right) dx, x, \coth(a + bx)\right)}{b} \\
 &= -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{-1+x} dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1+x} dx, x, \coth(a + bx)\right)}{2b} \\
 &= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(-1+x)} - \frac{\sin(3x)}{4(-1+x)}\right) dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(1+x)} - \frac{\sin(3x)}{4(1+x)}\right) dx, x, \coth(a + bx)\right)}{2b} \\
 &= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{-1+x} dx, x, \coth(a + bx)\right)}{8b} - \frac{\text{Subst}\left(\int \frac{\sin(3x)}{1+x} dx, x, \coth(a + bx)\right)}{8b} - \frac{3 \text{Subst}\left(\int \frac{\sin(x)}{-1+x} dx, x, \coth(a + bx)\right)}{8b} + \frac{3 \text{Subst}\left(\int \frac{\sin(x)}{1+x} dx, x, \coth(a + bx)\right)}{8b} \\
 &= \frac{(3 \cos(1)) \text{Subst}\left(\int \frac{\sin(1-x)}{-1+x} dx, x, \coth(a + bx)\right)}{8b} + \frac{(3 \cos(1)) \text{Subst}\left(\int \frac{\sin(1+x)}{1+x} dx, x, \coth(a + bx)\right)}{8b} \\
 &= -\frac{3\text{Ci}(1 - \coth(a + bx)) \sin(1)}{8b} - \frac{3\text{Ci}(1 + \coth(a + bx)) \sin(1)}{8b} + \frac{\text{Ci}(3 - 3 \coth(a + bx)) \sin(1)}{8b}
 \end{aligned}$$

Mathematica [A] time = 0.242156, size = 124, normalized size = 0.79

$$2 \sin(3) \operatorname{CosIntegral}(3 - 3 \operatorname{coth}(a + bx)) + 2 \sin(3) \operatorname{CosIntegral}(3 \operatorname{coth}(a + bx) + 3) - 6 \sin(1) \operatorname{CosIntegral}(1 - \operatorname{coth}(a + bx)) + 6 \sin(1) \operatorname{CosIntegral}(1 + \operatorname{coth}(a + bx))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Coth[a + b*x]]^3,x]

[Out] (-6*CosIntegral[1 - Coth[a + b*x]]*Sin[1] - 6*CosIntegral[1 + Coth[a + b*x]]*Sin[1] + 2*CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3] + 2*CosIntegral[3 + 3*Coth[a + b*x]]*Sin[3] - 2*Cos[3]*SinIntegral[3 - 3*Coth[a + b*x]] + 6*Cos[1]*SinIntegral[1 - Coth[a + b*x]] + 6*Cos[1]*SinIntegral[1 + Coth[a + b*x]] - 2*Cos[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)

Maple [A] time = 0.025, size = 118, normalized size = 0.8

$$\frac{1}{b} \left(-\frac{\operatorname{Si}(3 + 3 \operatorname{coth}(bx + a)) \cos(3)}{8} + \frac{\operatorname{Ci}(3 + 3 \operatorname{coth}(bx + a)) \sin(3)}{8} + \frac{\operatorname{Si}(-3 + 3 \operatorname{coth}(bx + a)) \cos(3)}{8} + \frac{\operatorname{Ci}(-3 + 3 \operatorname{coth}(bx + a)) \sin(3)}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(coth(b*x+a))^3,x)

[Out] 1/b*(-1/8*Si(3+3*coth(b*x+a))*cos(3)+1/8*Ci(3+3*coth(b*x+a))*sin(3)+1/8*Si(-3+3*coth(b*x+a))*cos(3)+1/8*Ci(-3+3*coth(b*x+a))*sin(3)+3/8*Si(coth(b*x+a)+1)*cos(1)-3/8*Ci(coth(b*x+a)+1)*sin(1)-3/8*Si(coth(b*x+a)-1)*cos(1)-3/8*Ci(coth(b*x+a)-1)*sin(1))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\operatorname{coth}(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(coth(b*x+a))^3,x, algorithm="maxima")

[Out] integrate(sin(coth(b*x + a))^3, x)

Fricas [B] time = 2.7501, size = 929, normalized size = 5.92

$$\operatorname{Ci}\left(\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(3) + \operatorname{Ci}\left(-\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(3) + \operatorname{Ci}\left(\frac{6}{e^{(2bx+2a)}-1}\right)\sin(3) + \operatorname{Ci}\left(-\frac{6}{e^{(2bx+2a)}-1}\right)\sin(3) - 3 \operatorname{Ci}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(coth(b*x+a))^3,x, algorithm="fricas")

[Out] $\frac{1}{16}(\cos_integral(6e^{(2bx+2a)} / (e^{(2bx+2a)} - 1))\sin(3) + \cos_integral(-6e^{(2bx+2a)} / (e^{(2bx+2a)} - 1))\sin(3) + \cos_integral(6 / (e^{(2bx+2a)} - 1))\sin(3) + \cos_integral(-6 / (e^{(2bx+2a)} - 1))\sin(3) - 3\cos_integral(2e^{(2bx+2a)} / (e^{(2bx+2a)} - 1))\sin(1) - 3\cos_integral(-2e^{(2bx+2a)} / (e^{(2bx+2a)} - 1))\sin(1) - 3\cos_integral(2 / (e^{(2bx+2a)} - 1))\sin(1) - 3\cos_integral(-2 / (e^{(2bx+2a)} - 1))\sin(1) - 2\cos(3)\sin_integral(6e^{(2bx+2a)} / (e^{(2bx+2a)} - 1)) + 6\cos(1)\sin_integral(2e^{(2bx+2a)} / (e^{(2bx+2a)} - 1)) + 2\cos(3)\sin_integral(6 / (e^{(2bx+2a)} - 1)) - 6\cos(1)\sin_integral(2 / (e^{(2bx+2a)} - 1))) / b$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^3(\coth(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(coth(b*x+a))**3,x)

[Out] Integral(sin(coth(a + b*x))**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\coth(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a))^3, x)
```

3.218 $\int \sin^2(\coth(a + bx)) dx$

Optimal. Leaf size=115

$$\frac{\cos(2)\text{CosIntegral}(2 - 2\coth(a + bx))}{4b} - \frac{\cos(2)\text{CosIntegral}(2\coth(a + bx) + 2)}{4b} + \frac{\sin(2)\text{Si}(2 - 2\coth(a + bx))}{4b} - \frac{\sin(2)\text{Si}(2\coth(a + bx) + 2)}{4b}$$

[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b*x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Sin[2]*SinIntegral[2 + 2*Coth[a + b*x]])/(4*b)

Rubi [A] time = 0.262045, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\cos(2)\text{CosIntegral}(2 - 2\coth(a + bx))}{4b} - \frac{\cos(2)\text{CosIntegral}(2\coth(a + bx) + 2)}{4b} + \frac{\sin(2)\text{Si}(2 - 2\coth(a + bx))}{4b} - \frac{\sin(2)\text{Si}(2\coth(a + bx) + 2)}{4b}$$

Antiderivative was successfully verified.

[In] Int[Sin[Coth[a + b*x]]^2,x]

[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b*x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Sin[2]*SinIntegral[2 + 2*Coth[a + b*x]])/(4*b)

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rule 3312

Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)^(n_)], x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(\coth(a + bx)) dx &= \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1-x^2} dx, x, \coth(a + bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sin^2(x)}{2(-1+x)} + \frac{\sin^2(x)}{2(1+x)}\right) dx, x, \coth(a + bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{-1+x} dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1+x} dx, x, \coth(a + bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)} - \frac{\cos(2x)}{2(-1+x)}\right) dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)} - \frac{\cos(2x)}{2(1+x)}\right) dx, x, \coth(a + bx)\right)}{2b} \\
&= -\frac{\log(1 - \coth(a + bx))}{4b} + \frac{\log(1 + \coth(a + bx))}{4b} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x} dx, x, \coth(a + bx)\right)}{4b} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{1+x} dx, x, \coth(a + bx)\right)}{4b} \\
&= -\frac{\log(1 - \coth(a + bx))}{4b} + \frac{\log(1 + \coth(a + bx))}{4b} + \frac{\cos(2) \text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x} dx, x, \coth(a + bx)\right)}{4b} - \frac{\cos(2) \text{Subst}\left(\int \frac{\cos(2-2x)}{1+x} dx, x, \coth(a + bx)\right)}{4b} \\
&= \frac{\cos(2)\text{Ci}(2 - 2 \coth(a + bx))}{4b} - \frac{\cos(2)\text{Ci}(2 + 2 \coth(a + bx))}{4b} - \frac{\log(1 - \coth(a + bx))}{4b} + \frac{\log(1 + \coth(a + bx))}{4b}
\end{aligned}$$

Mathematica [A] time = 0.166621, size = 88, normalized size = 0.77

$$\frac{\cos(2)\text{CosIntegral}(2 - 2 \coth(a + bx)) - \cos(2)\text{CosIntegral}(2(\coth(a + bx) + 1)) + \sin(2)\text{Si}(2 - 2 \coth(a + bx)) - \sin(2)\text{Si}(2(\coth(a + bx) + 1))}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Coth[a + b*x]]^2,x]

[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]] - Cos[2]*CosIntegral[2*(1 + Coth[a + b*x])]) - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] + Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]] - Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/(4*b)

Maple [A] time = 0.02, size = 102, normalized size = 0.9

$$-\frac{\ln(\coth(bx+a)-1)}{4b} + \frac{\ln(\coth(bx+a)+1)}{4b} - \frac{\text{Si}(2+2\coth(bx+a))\sin(2)}{4b} - \frac{\text{Ci}(2+2\coth(bx+a))\cos(2)}{4b} - \frac{\text{Si}(-2+2\coth(bx+a))\sin(2)}{4b} + \frac{\text{Ci}(-2+2\coth(bx+a))\cos(2)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(coth(b*x+a))^2,x)

[Out] -1/4/b*ln(coth(b*x+a)-1)+1/4/b*ln(coth(b*x+a)+1)-1/4*Si(2+2*coth(b*x+a))*sin(2)/b-1/4*Ci(2+2*coth(b*x+a))*cos(2)/b-1/4*Si(-2+2*coth(b*x+a))*sin(2)/b+1/4/b*Ci(-2+2*coth(b*x+a))*cos(2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x - \frac{1}{2} \int \cos\left(\frac{2(e^{2bx+2a} + 1)}{e^{2bx+2a} - 1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(coth(b*x+a))^2,x, algorithm="maxima")

[Out] 1/2*x - 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)), x)

Fricas [A] time = 2.52604, size = 474, normalized size = 4.12

$$4bx - \cos(2) \text{Ci}\left(\frac{4e^{2bx+2a}}{e^{2bx+2a}-1}\right) - \cos(2) \text{Ci}\left(-\frac{4e^{2bx+2a}}{e^{2bx+2a}-1}\right) + \cos(2) \text{Ci}\left(\frac{4}{e^{2bx+2a}-1}\right) + \cos(2) \text{Ci}\left(-\frac{4}{e^{2bx+2a}-1}\right) - 2 \sin(2) \text{Si}\left(\frac{4}{e^{2bx+2a}-1}\right) - 2 \sin(2) \text{Si}\left(-\frac{4}{e^{2bx+2a}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x - cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) -
cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) - 1)) + cos(2)*cos_integral(-4/(e^(2*b*x + 2*a)
) - 1)) - 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) -
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^2(\operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))**2,x)
```

```
[Out] Integral(sin(coth(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\operatorname{coth}(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a))^2, x)
```

3.219 $\int \sin(\coth(a + bx)) dx$

Optimal. Leaf size=77

$$\frac{\sin(1)\text{CosIntegral}(1 - \coth(a + bx))}{2b} - \frac{\sin(1)\text{CosIntegral}(\coth(a + bx) + 1)}{2b} + \frac{\cos(1)\text{Si}(1 - \coth(a + bx))}{2b} + \frac{\cos(1)\text{Si}(\coth(a + bx) + 1)}{2b}$$

[Out] $-(\text{CosIntegral}[1 - \text{Coth}[a + b*x]]*\text{Sin}[1])/(2*b) - (\text{CosIntegral}[1 + \text{Coth}[a + b*x]]*\text{Sin}[1])/(2*b) + (\text{Cos}[1]*\text{SinIntegral}[1 - \text{Coth}[a + b*x]])/(2*b) + (\text{Cos}[1]*\text{SinIntegral}[1 + \text{Coth}[a + b*x]])/(2*b)$

Rubi [A] time = 0.144187, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3333, 3303, 3299, 3302}

$$\frac{\sin(1)\text{CosIntegral}(1 - \coth(a + bx))}{2b} - \frac{\sin(1)\text{CosIntegral}(\coth(a + bx) + 1)}{2b} + \frac{\cos(1)\text{Si}(1 - \coth(a + bx))}{2b} + \frac{\cos(1)\text{Si}(\coth(a + bx) + 1)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sin[Coth[a + b*x]],x]

[Out] $-(\text{CosIntegral}[1 - \text{Coth}[a + b*x]]*\text{Sin}[1])/(2*b) - (\text{CosIntegral}[1 + \text{Coth}[a + b*x]]*\text{Sin}[1])/(2*b) + (\text{Cos}[1]*\text{SinIntegral}[1 - \text{Coth}[a + b*x]])/(2*b) + (\text{Cos}[1]*\text{SinIntegral}[1 + \text{Coth}[a + b*x]])/(2*b)$

Rule 3333

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] := Int[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \sin(\coth(a + bx)) dx &= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x^2} dx, x, \coth(a + bx)\right)}{b} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{2(1-x)} + \frac{\sin(x)}{2(1+x)}\right) dx, x, \coth(a + bx)\right)}{b} \\
 &= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x} dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x} dx, x, \coth(a + bx)\right)}{2b} \\
 &= -\frac{\cos(1) \text{Subst}\left(\int \frac{\sin(1-x)}{1-x} dx, x, \coth(a + bx)\right)}{2b} + \frac{\cos(1) \text{Subst}\left(\int \frac{\sin(1+x)}{1+x} dx, x, \coth(a + bx)\right)}{2b} \\
 &= -\frac{\text{Ci}(1 - \coth(a + bx)) \sin(1)}{2b} - \frac{\text{Ci}(1 + \coth(a + bx)) \sin(1)}{2b} + \frac{\cos(1) \text{Si}(1 - \coth(a + bx))}{2b} + \frac{\cos(1) \text{Si}(1 + \coth(a + bx))}{2b}
 \end{aligned}$$

Mathematica [A] time = 0.128473, size = 59, normalized size = 0.77

$$\frac{\sin(1) \text{CosIntegral}(1 - \coth(a + bx)) + \sin(1) \text{CosIntegral}(\coth(a + bx) + 1) - \cos(1) (\text{Si}(1 - \coth(a + bx)) + \text{Si}(\coth(a + bx) + 1))}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Coth[a + b*x]], x]

[Out] -(CosIntegral[1 - Coth[a + b*x]]*Sin[1] + CosIntegral[1 + Coth[a + b*x]]*Sin[1] - Cos[1]*(SinIntegral[1 - Coth[a + b*x]] + SinIntegral[1 + Coth[a + b*x]]))/(2*b)

Maple [A] time = 0.016, size = 58, normalized size = 0.8

$$\frac{1}{b} \left(\frac{\text{Si}(\coth(bx + a) + 1) \cos(1)}{2} - \frac{\text{Ci}(\coth(bx + a) + 1) \sin(1)}{2} - \frac{\text{Si}(\coth(bx + a) - 1) \cos(1)}{2} - \frac{\text{Ci}(\coth(bx + a) - 1) \sin(1)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(coth(b*x+a)),x)`

[Out] `1/b*(1/2*Si(coth(b*x+a)+1)*cos(1)-1/2*Ci(coth(b*x+a)+1)*sin(1)-1/2*Si(coth(b*x+a)-1)*cos(1)-1/2*Ci(coth(b*x+a)-1)*sin(1))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\operatorname{coth}(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(coth(b*x+a)),x, algorithm="maxima")`

[Out] `integrate(sin(coth(b*x + a)), x)`

Fricas [B] time = 2.43796, size = 464, normalized size = 6.03

$$\frac{\operatorname{Ci}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(1) + \operatorname{Ci}\left(-\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(1) + \operatorname{Ci}\left(\frac{2}{e^{(2bx+2a)}-1}\right)\sin(1) + \operatorname{Ci}\left(-\frac{2}{e^{(2bx+2a)}-1}\right)\sin(1) - 2\cos(1)\operatorname{Si}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(coth(b*x+a)),x, algorithm="fricas")`

[Out] `-1/4*(cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) + cos_integral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) + cos_integral(2/(e^(2*b*x + 2*a) - 1))*sin(1) + cos_integral(-2/(e^(2*b*x + 2*a) - 1))*sin(1) - 2*cos(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*cos(1)*sin_integral(2/(e^(2*b*x + 2*a) - 1)))/b`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x)
```

```
[Out] Integral(sin(coth(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\operatorname{coth}(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a)), x)
```

3.220 $\int \csc(\coth(a + bx)) dx$

Optimal. Leaf size=65

$$\frac{1}{2} \text{Unintegrable} \left(\frac{\text{csch}^2(a + bx) \csc(\coth(a + bx))}{\coth(a + bx) - 1}, x \right) - \frac{1}{2} \text{Unintegrable} \left(\frac{\text{csch}^2(a + bx) \csc(\coth(a + bx))}{\coth(a + bx) + 1}, x \right)$$

[Out] Unintegrable[(Csc[Coth[a + b*x]]*Csch[a + b*x]^2)/(-1 + Coth[a + b*x]), x]/2 - Unintegrable[(Csc[Coth[a + b*x]]*Csch[a + b*x]^2)/(1 + Coth[a + b*x]), x]/2

Rubi [A] time = 0.0772917, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \csc(\coth(a + bx)) dx$$

Verification is Not applicable to the result.

[In] Int[Csc[Coth[a + b*x]], x]

[Out] -Defer[Subst][Defer[Int][Csc[x]/(-1 + x), x], x, Coth[a + b*x]]/(2*b) + Defer[Subst][Defer[Int][Csc[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)

Rubi steps

$$\begin{aligned} \int \csc(\coth(a + bx)) dx &= \frac{\text{Subst} \left(\int \frac{\csc(x)}{1-x^2} dx, x, \coth(a + bx) \right)}{b} \\ &= \frac{\text{Subst} \left(\int \left(-\frac{\csc(x)}{2(-1+x)} + \frac{\csc(x)}{2(1+x)} \right) dx, x, \coth(a + bx) \right)}{b} \\ &= -\frac{\text{Subst} \left(\int \frac{\csc(x)}{-1+x} dx, x, \coth(a + bx) \right)}{2b} + \frac{\text{Subst} \left(\int \frac{\csc(x)}{1+x} dx, x, \coth(a + bx) \right)}{2b} \end{aligned}$$

Mathematica [A] time = 2.68009, size = 0, normalized size = 0.

$$\int \csc(\coth(a + bx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[Coth[a + b*x]],x]

[Out] Integrate[Csc[Coth[a + b*x]], x]

Maple [A] time = 0.165, size = 0, normalized size = 0.

$$\int \csc(\coth(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(coth(b*x+a)),x)

[Out] int(csc(coth(b*x+a)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \csc(\coth(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(coth(b*x+a)),x, algorithm="maxima")

[Out] integrate(csc(coth(b*x + a)), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\csc(\coth(bx + a)), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(coth(b*x+a)),x, algorithm="fricas")

[Out] integral(csc(coth(b*x + a)), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \csc(\operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(coth(b*x+a)),x)`

[Out] `Integral(csc(coth(a + b*x)), x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \csc(\operatorname{coth}(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(coth(b*x+a)),x, algorithm="giac")`

[Out] `integrate(csc(coth(b*x + a)), x)`

3.221 $\int \cos^3(\coth(a + bx)) dx$

Optimal. Leaf size=157

$$\frac{\cos(3)\text{CosIntegral}(3 - 3\coth(a + bx))}{8b} - \frac{3\cos(1)\text{CosIntegral}(1 - \coth(a + bx))}{8b} + \frac{3\cos(1)\text{CosIntegral}(\coth(a + bx))}{8b}$$

```
[Out] -(Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]])/(8*b) - (3*Cos[1]*CosIntegral[1 - Coth[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Coth[a + b*x]])/(8*b) + (Cos[3]*CosIntegral[3 + 3*Coth[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 - 3*Coth[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) + (3*Sin[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(8*b)
```

Rubi [A] time = 0.373451, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\cos(3)\text{CosIntegral}(3 - 3\coth(a + bx))}{8b} - \frac{3\cos(1)\text{CosIntegral}(1 - \coth(a + bx))}{8b} + \frac{3\cos(1)\text{CosIntegral}(\coth(a + bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]]^3,x]
```

```
[Out] -(Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]])/(8*b) - (3*Cos[1]*CosIntegral[1 - Coth[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Coth[a + b*x]])/(8*b) + (Cos[3]*CosIntegral[3 + 3*Coth[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 - 3*Coth[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) + (3*Sin[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(8*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)^(n_)], x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \cos^3(\coth(a + bx)) dx &= \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1-x^2} dx, x, \coth(a + bx)\right)}{b} \\
 &= \frac{\text{Subst}\left(\int \left(-\frac{\cos^3(x)}{2(-1+x)} + \frac{\cos^3(x)}{2(1+x)}\right) dx, x, \coth(a + bx)\right)}{b} \\
 &= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{-1+x} dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1+x} dx, x, \coth(a + bx)\right)}{2b} \\
 &= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(-1+x)} + \frac{\cos(3x)}{4(-1+x)}\right) dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(1+x)} + \frac{\cos(3x)}{4(1+x)}\right) dx, x, \coth(a + bx)\right)}{2b} \\
 &= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{-1+x} dx, x, \coth(a + bx)\right)}{8b} + \frac{\text{Subst}\left(\int \frac{\cos(3x)}{1+x} dx, x, \coth(a + bx)\right)}{8b} - \frac{3\text{Subst}\left(\int \frac{\cos(x)}{-1+x} dx, x, \coth(a + bx)\right)}{8b} \\
 &= -\frac{(3\cos(1))\text{Subst}\left(\int \frac{\cos(1-x)}{-1+x} dx, x, \coth(a + bx)\right)}{8b} + \frac{(3\cos(1))\text{Subst}\left(\int \frac{\cos(1+x)}{1+x} dx, x, \coth(a + bx)\right)}{8b} \\
 &= -\frac{\cos(3)\text{Ci}(3 - 3\coth(a + bx))}{8b} - \frac{3\cos(1)\text{Ci}(1 - \coth(a + bx))}{8b} + \frac{3\cos(1)\text{Ci}(1 + \coth(a + bx))}{8b}
 \end{aligned}$$

Mathematica [A] time = 0.24361, size = 124, normalized size = 0.79

$$-2 \cos(3) \operatorname{CosIntegral}(3 - 3 \operatorname{coth}(a + bx)) - 6 \cos(1) \operatorname{CosIntegral}(1 - \operatorname{coth}(a + bx)) + 6 \cos(1) \operatorname{CosIntegral}(\operatorname{coth}(a + bx))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Coth[a + b*x]]^3, x]

[Out] $(-2 \operatorname{Cos}[3] \operatorname{CosIntegral}[3 - 3 \operatorname{Coth}[a + b*x]] - 6 \operatorname{Cos}[1] \operatorname{CosIntegral}[1 - \operatorname{Coth}[a + b*x]] + 6 \operatorname{Cos}[1] \operatorname{CosIntegral}[1 + \operatorname{Coth}[a + b*x]] + 2 \operatorname{Cos}[3] \operatorname{CosIntegral}[3 + 3 \operatorname{Coth}[a + b*x]] - 2 \operatorname{Sin}[3] \operatorname{SinIntegral}[3 - 3 \operatorname{Coth}[a + b*x]] - 6 \operatorname{Sin}[1] \operatorname{SinIntegral}[1 - \operatorname{Coth}[a + b*x]] + 6 \operatorname{Sin}[1] \operatorname{SinIntegral}[1 + \operatorname{Coth}[a + b*x]] + 2 \operatorname{Sin}[3] \operatorname{SinIntegral}[3 + 3 \operatorname{Coth}[a + b*x]]) / (16*b)$

Maple [A] time = 0.023, size = 118, normalized size = 0.8

$$\frac{1}{b} \left(\frac{\operatorname{Si}(3 + 3 \operatorname{coth}(bx + a)) \sin(3)}{8} + \frac{\operatorname{Ci}(3 + 3 \operatorname{coth}(bx + a)) \cos(3)}{8} + \frac{\operatorname{Si}(-3 + 3 \operatorname{coth}(bx + a)) \sin(3)}{8} - \frac{\operatorname{Ci}(-3 + 3 \operatorname{coth}(bx + a)) \cos(3)}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(coth(b*x+a))^3, x)

[Out] $1/b * (1/8 * \operatorname{Si}(3 + 3 \operatorname{coth}(b*x+a)) * \sin(3) + 1/8 * \operatorname{Ci}(3 + 3 \operatorname{coth}(b*x+a)) * \cos(3) + 1/8 * \operatorname{Si}(-3 + 3 \operatorname{coth}(b*x+a)) * \sin(3) - 1/8 * \operatorname{Ci}(-3 + 3 \operatorname{coth}(b*x+a)) * \cos(3) + 3/8 * \operatorname{Si}(\operatorname{coth}(b*x+a) + 1) * \sin(1) + 3/8 * \operatorname{Ci}(\operatorname{coth}(b*x+a) + 1) * \cos(1) + 3/8 * \operatorname{Si}(\operatorname{coth}(b*x+a) - 1) * \sin(1) - 3/8 * \operatorname{Ci}(\operatorname{coth}(b*x+a) - 1) * \cos(1))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\operatorname{coth}(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(coth(b*x+a))^3, x, algorithm="maxima")

[Out] integrate(cos(coth(b*x + a))^3, x)

Fricas [B] time = 2.96805, size = 929, normalized size = 5.92

$$\cos(3) \operatorname{Ci}\left(\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right) + 3 \cos(1) \operatorname{Ci}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right) + 3 \cos(1) \operatorname{Ci}\left(-\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right) + \cos(3) \operatorname{Ci}\left(-\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right) - \cos(3) \operatorname{Ci}\left(\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(coth(b*x+a))^3,x, algorithm="fricas")

[Out] 1/16*(cos(3)*cos_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 3*cos(1)*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 3*cos(1)*cos_integral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(3)*cos_integral(-6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(3)*cos_integral(6/(e^(2*b*x + 2*a) - 1)) - 3*cos(1)*cos_integral(2/(e^(2*b*x + 2*a) - 1)) - 3*cos(1)*cos_integral(-2/(e^(2*b*x + 2*a) - 1)) - cos(3)*cos_integral(-6/(e^(2*b*x + 2*a) - 1)) + 2*sin(3)*sin_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 6*sin(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*sin(3)*sin_integral(6/(e^(2*b*x + 2*a) - 1)) + 6*sin(1)*sin_integral(2/(e^(2*b*x + 2*a) - 1))))/b

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^3(\operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(coth(b*x+a))**3,x)

[Out] Integral(cos(coth(a + b*x))**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\operatorname{coth}(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a))^3, x)
```

3.222 $\int \cos^2(\coth(a + bx)) dx$

Optimal. Leaf size=115

$$\frac{\cos(2)\text{CosIntegral}(2 - 2\coth(a + bx))}{4b} + \frac{\cos(2)\text{CosIntegral}(2\coth(a + bx) + 2)}{4b} - \frac{\sin(2)\text{Si}(2 - 2\coth(a + bx))}{4b} +$$

```
[Out] -(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Cos[2]*CosIntegral[2 +
2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rubi [A] time = 0.251206, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\cos(2)\text{CosIntegral}(2 - 2\coth(a + bx))}{4b} + \frac{\cos(2)\text{CosIntegral}(2\coth(a + bx) + 2)}{4b} - \frac{\sin(2)\text{Si}(2 - 2\coth(a + bx))}{4b} +$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]]^2,x]
```

```
[Out] -(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Cos[2]*CosIntegral[2 +
2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)^(n_)], x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(\coth(a + bx)) dx &= \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1-x^2} dx, x, \coth(a + bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cos^2(x)}{2(-1+x)} + \frac{\cos^2(x)}{2(1+x)}\right) dx, x, \coth(a + bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{-1+x} dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1+x} dx, x, \coth(a + bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)} + \frac{\cos(2x)}{2(-1+x)}\right) dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)} + \frac{\cos(2x)}{2(1+x)}\right) dx, x, \coth(a + bx)\right)}{2b} \\
&= -\frac{\log(1 - \coth(a + bx))}{4b} + \frac{\log(1 + \coth(a + bx))}{4b} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x} dx, x, \coth(a + bx)\right)}{4b} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{1+x} dx, x, \coth(a + bx)\right)}{4b} \\
&= -\frac{\log(1 - \coth(a + bx))}{4b} + \frac{\log(1 + \coth(a + bx))}{4b} - \frac{\cos(2) \text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x} dx, x, \coth(a + bx)\right)}{4b} + \frac{\cos(2) \text{Subst}\left(\int \frac{\cos(2-2x)}{1+x} dx, x, \coth(a + bx)\right)}{4b} \\
&= -\frac{\cos(2)\text{Ci}(2 - 2 \coth(a + bx))}{4b} + \frac{\cos(2)\text{Ci}(2 + 2 \coth(a + bx))}{4b} - \frac{\log(1 - \coth(a + bx))}{4b} + \frac{\log(1 + \coth(a + bx))}{4b}
\end{aligned}$$

Mathematica [A] time = 0.165737, size = 88, normalized size = 0.77

$$\frac{-\cos(2)\text{CosIntegral}(2 - 2 \coth(a + bx)) + \cos(2)\text{CosIntegral}(2(\coth(a + bx) + 1)) - \sin(2)\text{Si}(2 - 2 \coth(a + bx)) + \sin(2)\text{Si}(2(\coth(a + bx) + 1))}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Coth[a + b*x]]^2,x]

[Out] $(-\text{Cos}[2] \cdot \text{CosIntegral}[2 - 2 \cdot \text{Coth}[a + b \cdot x]]) + \text{Cos}[2] \cdot \text{CosIntegral}[2 \cdot (1 + \text{Coth}[a + b \cdot x])] - \text{Log}[1 - \text{Coth}[a + b \cdot x]] + \text{Log}[1 + \text{Coth}[a + b \cdot x]] - \text{Sin}[2] \cdot \text{SinIntegral}[2 - 2 \cdot \text{Coth}[a + b \cdot x]] + \text{Sin}[2] \cdot \text{SinIntegral}[2 \cdot (1 + \text{Coth}[a + b \cdot x])]) / (4 \cdot b)$

Maple [A] time = 0.021, size = 102, normalized size = 0.9

$$\frac{\text{Si}(2 + 2 \coth(bx + a)) \sin(2)}{4b} + \frac{\text{Ci}(2 + 2 \coth(bx + a)) \cos(2)}{4b} + \frac{\text{Si}(-2 + 2 \coth(bx + a)) \sin(2)}{4b} - \frac{\text{Ci}(-2 + 2 \coth(bx + a)) \cos(2)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(coth(b*x+a))^2,x)

[Out] $\frac{1}{4} \text{Si}(2 + 2 \coth(b \cdot x + a)) \cdot \sin(2) / b + \frac{1}{4} \text{Ci}(2 + 2 \coth(b \cdot x + a)) \cdot \cos(2) / b + \frac{1}{4} \text{Si}(-2 + 2 \coth(b \cdot x + a)) \cdot \sin(2) / b - \frac{1}{4} \text{Ci}(-2 + 2 \coth(b \cdot x + a)) \cdot \cos(2) / b - \frac{1}{4} \ln(\coth(b \cdot x + a) - 1) + \frac{1}{4} \ln(\coth(b \cdot x + a) + 1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x + \frac{1}{2} \int \cos\left(\frac{2(e^{2bx+2a} + 1)}{e^{2bx+2a} - 1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(coth(b*x+a))^2,x, algorithm="maxima")

[Out] $\frac{1}{2}x + \frac{1}{2} \int \cos(2 \cdot (e^{2 \cdot b \cdot x + 2 \cdot a} + 1) / (e^{2 \cdot b \cdot x + 2 \cdot a} - 1)), x$

Fricas [A] time = 2.77952, size = 474, normalized size = 4.12

$$\frac{4bx + \cos(2) \text{Ci}\left(\frac{4e^{2bx+2a}}{e^{2bx+2a}-1}\right) + \cos(2) \text{Ci}\left(-\frac{4e^{2bx+2a}}{e^{2bx+2a}-1}\right) - \cos(2) \text{Ci}\left(\frac{4}{e^{2bx+2a}-1}\right) - \cos(2) \text{Ci}\left(-\frac{4}{e^{2bx+2a}-1}\right) + 2 \sin(2) \text{Si}\left(\frac{4}{e^{2bx+2a}-1}\right) + 2 \sin(2) \text{Si}\left(-\frac{4}{e^{2bx+2a}-1}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x + cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) +
cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) - 1)) - cos(2)*cos_integral(-4/(e^(2*b*x + 2*a)
) - 1)) + 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) +
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^2(\operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))**2,x)
```

```
[Out] Integral(cos(coth(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\operatorname{coth}(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a))^2, x)
```

3.223 $\int \cos(\coth(a + bx)) dx$

Optimal. Leaf size=77

$$-\frac{\cos(1)\text{CosIntegral}(1 - \coth(a + bx))}{2b} + \frac{\cos(1)\text{CosIntegral}(\coth(a + bx) + 1)}{2b} - \frac{\sin(1)\text{Si}(1 - \coth(a + bx))}{2b} + \frac{\sin(1)\text{Si}(\coth(a + bx) + 1)}{2b}$$

[Out] $-(\text{Cos}[1]*\text{CosIntegral}[1 - \text{Coth}[a + b*x]])/(2*b) + (\text{Cos}[1]*\text{CosIntegral}[1 + \text{Coth}[a + b*x]])/(2*b) - (\text{Sin}[1]*\text{SinIntegral}[1 - \text{Coth}[a + b*x]])/(2*b) + (\text{Sin}[1]*\text{SinIntegral}[1 + \text{Coth}[a + b*x]])/(2*b)$

Rubi [A] time = 0.140022, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3334, 3303, 3299, 3302}

$$-\frac{\cos(1)\text{CosIntegral}(1 - \coth(a + bx))}{2b} + \frac{\cos(1)\text{CosIntegral}(\coth(a + bx) + 1)}{2b} - \frac{\sin(1)\text{Si}(1 - \coth(a + bx))}{2b} + \frac{\sin(1)\text{Si}(\coth(a + bx) + 1)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[Coth[a + b*x]],x]

[Out] $-(\text{Cos}[1]*\text{CosIntegral}[1 - \text{Coth}[a + b*x]])/(2*b) + (\text{Cos}[1]*\text{CosIntegral}[1 + \text{Coth}[a + b*x]])/(2*b) - (\text{Sin}[1]*\text{SinIntegral}[1 - \text{Coth}[a + b*x]])/(2*b) + (\text{Sin}[1]*\text{SinIntegral}[1 + \text{Coth}[a + b*x]])/(2*b)$

Rule 3334

Int[Cos[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rubi steps

$$\begin{aligned}
 \int \cos(\coth(a + bx)) dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x^2} dx, x, \coth(a + bx)\right)}{b} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{2(1-x)} + \frac{\cos(x)}{2(1+x)}\right) dx, x, \coth(a + bx)\right)}{b} \\
 &= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x} dx, x, \coth(a + bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cos(x)}{1+x} dx, x, \coth(a + bx)\right)}{2b} \\
 &= \frac{\cos(1) \text{Subst}\left(\int \frac{\cos(1-x)}{1-x} dx, x, \coth(a + bx)\right)}{2b} + \frac{\cos(1) \text{Subst}\left(\int \frac{\cos(1+x)}{1+x} dx, x, \coth(a + bx)\right)}{2b} \\
 &= -\frac{\cos(1)\text{Ci}(1 - \coth(a + bx))}{2b} + \frac{\cos(1)\text{Ci}(1 + \coth(a + bx))}{2b} - \frac{\sin(1)\text{Si}(1 - \coth(a + bx))}{2b} + \frac{\sin(1)\text{Si}(1 + \coth(a + bx))}{2b}
 \end{aligned}$$

Mathematica [A] time = 0.105566, size = 62, normalized size = 0.81

$$\frac{\cos(1)\text{CosIntegral}(1 - \coth(a + bx)) - \cos(1)\text{CosIntegral}(\coth(a + bx) + 1) + \sin(1)\text{Si}(1 - \coth(a + bx)) - \sin(1)\text{Si}(\coth(a + bx) + 1)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Coth[a + b*x]], x]

[Out] -(Cos[1]*CosIntegral[1 - Coth[a + b*x]] - Cos[1]*CosIntegral[1 + Coth[a + b*x]]) + Sin[1]*SinIntegral[1 - Coth[a + b*x]] - Sin[1]*SinIntegral[1 + Coth[a + b*x]]/(2*b)

Maple [A] time = 0.022, size = 58, normalized size = 0.8

$$\frac{1}{b} \left(\frac{\text{Si}(\coth(bx + a) + 1) \sin(1)}{2} + \frac{\text{Ci}(\coth(bx + a) + 1) \cos(1)}{2} + \frac{\text{Si}(\coth(bx + a) - 1) \sin(1)}{2} - \frac{\text{Ci}(\coth(bx + a) - 1) \cos(1)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(coth(b*x+a)),x)`

[Out] $1/b*(1/2*Si(\coth(b*x+a)+1)*\sin(1)+1/2*Ci(\coth(b*x+a)+1)*\cos(1)+1/2*Si(\coth(b*x+a)-1)*\sin(1)-1/2*Ci(\coth(b*x+a)-1)*\cos(1))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\coth(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(coth(b*x+a)),x, algorithm="maxima")`

[Out] `integrate(cos(coth(b*x + a)), x)`

Fricas [B] time = 2.69787, size = 463, normalized size = 6.01

$$\frac{\cos(1) Ci\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right) + \cos(1) Ci\left(-\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right) - \cos(1) Ci\left(\frac{2}{e^{(2bx+2a)}-1}\right) - \cos(1) Ci\left(-\frac{2}{e^{(2bx+2a)}-1}\right) + 2 \sin(1) Si\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right) + 2 \sin(1) Si\left(-\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right) - 2 \sin(1) Si\left(\frac{2}{e^{(2bx+2a)}-1}\right) - 2 \sin(1) Si\left(-\frac{2}{e^{(2bx+2a)}-1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(coth(b*x+a)),x, algorithm="fricas")`

[Out] $1/4*(\cos(1)*\cos_integral(2*e^{(2*b*x + 2*a)}/(e^{(2*b*x + 2*a)} - 1)) + \cos(1)*\cos_integral(-2*e^{(2*b*x + 2*a)}/(e^{(2*b*x + 2*a)} - 1)) - \cos(1)*\cos_integral(2/(e^{(2*b*x + 2*a)} - 1)) - \cos(1)*\cos_integral(-2/(e^{(2*b*x + 2*a)} - 1)) + 2*\sin(1)*\sin_integral(2*e^{(2*b*x + 2*a)}/(e^{(2*b*x + 2*a)} - 1)) + 2*\sin(1)*\sin_integral(-2*e^{(2*b*x + 2*a)}/(e^{(2*b*x + 2*a)} - 1)) + 2*\sin(1)*\sin_integral(2/(e^{(2*b*x + 2*a)} - 1))) / b$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\coth(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x)
```

```
[Out] Integral(cos(coth(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\operatorname{coth}(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a)), x)
```

3.224 $\int \sec(\coth(a + bx)) dx$

Optimal. Leaf size=65

$$\frac{1}{2} \text{Unintegrable} \left(\frac{\text{csch}^2(a + bx) \sec(\coth(a + bx))}{\coth(a + bx) - 1}, x \right) - \frac{1}{2} \text{Unintegrable} \left(\frac{\text{csch}^2(a + bx) \sec(\coth(a + bx))}{\coth(a + bx) + 1}, x \right)$$

[Out] Unintegrable[(Csch[a + b*x]^2*Sec[Coth[a + b*x]])/(-1 + Coth[a + b*x]), x]/2 - Unintegrable[(Csch[a + b*x]^2*Sec[Coth[a + b*x]])/(1 + Coth[a + b*x]), x]/2

Rubi [A] time = 0.0770718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sec(\coth(a + bx)) dx$$

Verification is Not applicable to the result.

[In] Int[Sec[Coth[a + b*x]], x]

[Out] -Defer[Subst][Defer[Int][Sec[x]/(-1 + x), x], x, Coth[a + b*x]]/(2*b) + Defer[Subst][Defer[Int][Sec[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)

Rubi steps

$$\begin{aligned} \int \sec(\coth(a + bx)) dx &= \frac{\text{Subst} \left(\int \frac{\sec(x)}{1-x^2} dx, x, \coth(a + bx) \right)}{b} \\ &= \frac{\text{Subst} \left(\int \left(-\frac{\sec(x)}{2(-1+x)} + \frac{\sec(x)}{2(1+x)} \right) dx, x, \coth(a + bx) \right)}{b} \\ &= -\frac{\text{Subst} \left(\int \frac{\sec(x)}{-1+x} dx, x, \coth(a + bx) \right)}{2b} + \frac{\text{Subst} \left(\int \frac{\sec(x)}{1+x} dx, x, \coth(a + bx) \right)}{2b} \end{aligned}$$

Mathematica [A] time = 5.48446, size = 0, normalized size = 0.

$$\int \sec(\coth(a + bx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sec[Coth[a + b*x]],x]

[Out] Integrate[Sec[Coth[a + b*x]], x]

Maple [A] time = 0.067, size = 0, normalized size = 0.

$$\int \sec(\coth(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(coth(b*x+a)),x)

[Out] int(sec(coth(b*x+a)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sec(\coth(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(coth(b*x+a)),x, algorithm="maxima")

[Out] integrate(sec(coth(b*x + a)), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sec(\coth(bx + a)), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(coth(b*x+a)),x, algorithm="fricas")

[Out] integral(sec(coth(b*x + a)), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sec(\operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(coth(b*x+a)),x)`

[Out] `Integral(sec(coth(a + b*x)), x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sec(\operatorname{coth}(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(coth(b*x+a)),x, algorithm="giac")`

[Out] `integrate(sec(coth(b*x + a)), x)`

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```



```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```
56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65 else #result contains complex but optimal is not
66     if debug then
67         print("result contains complex but optimal is not");
68     fi;
69     return "C";
70 end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do not
as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+') or type(expn,'*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

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```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by


```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
          sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```