Computer algebra independent integration tests

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/6.3.2-Hyperbolic-tangent-functions

Nasser M. Abbasi

May 24, 2020 Compiled on May 24, 2020 at 1:49pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [247]. This is test number [172].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 83.4 (206)	% 16.6 (41)
Mathematica	% 100. (247)	% 0. (0)
Maple	% 83.81 (207)	% 16.19 (40)
Maxima	% 61.13 (151)	% 38.87 (96)
Fricas	% 84.62 (209)	% 15.38 (38)
Sympy	% 25.91 (64)	% 74.09 (183)
Giac	% 71.66 (177)	% 28.34 (70)

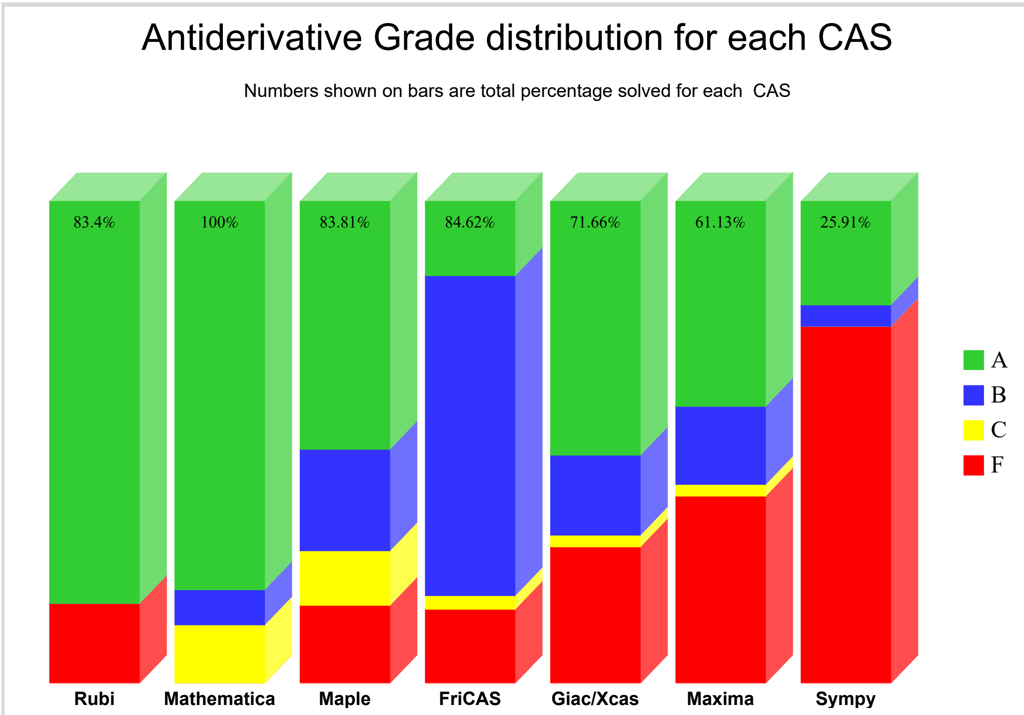
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

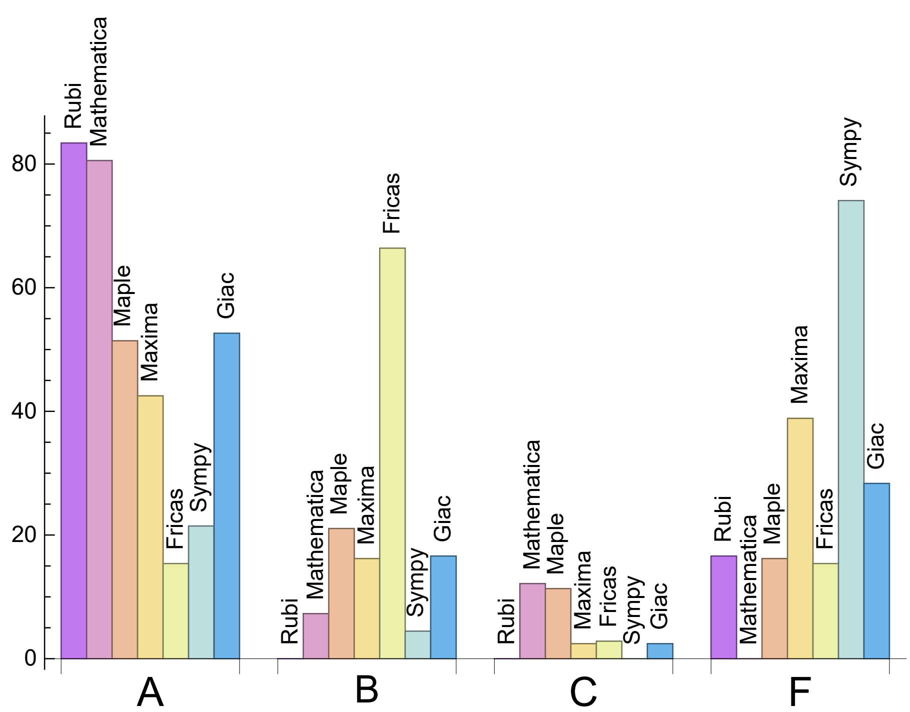
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	83.4	0.	0.	16.6
Mathematica	80.57	7.29	12.15	0.
Maple	51.42	21.05	11.34	16.19
Maxima	42.51	16.19	2.43	38.87
Fricas	15.38	66.4	2.83	15.38
Sympy	21.46	4.45	0.	74.09
Giac	52.63	16.6	2.43	28.34

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	72.9	0.99	57.	1.
Mathematica	1.07	83.34	1.14	56.	1.
Maple	0.04	103.29	1.49	60.	1.31
Maxima	1.4	119.17	2.12	88.	1.55
Fricas	2.66	1966.44	23.1	666.	13.09
Sympy	5.96	170.28	2.99	70.5	1.77
Giac	1.25	120.86	1.92	81.	1.65

1.4 list of integrals that has no closed form antiderivative

{243, 247}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {163, 164, 165, 167, 168, 169, 170, 171, 191, 192, 193, 211, 212, 217, 221, 225, 238}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

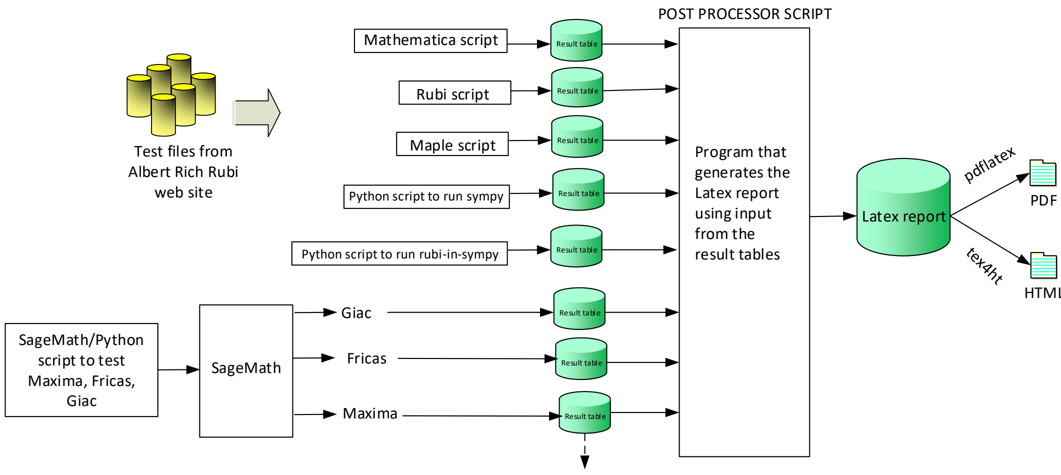
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 150, 157, 176, 183, 186, 187, 188, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247 }

B grade: { }

C grade: { }

F grade: { 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 11, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 148, 150, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 164, 165, 166, 176, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 193, 194, 195, 196, 197, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 213, 215, 216, 226, 228, 229, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247 }

B grade: { 41, 42, 146, 161, 163, 169, 170, 171, 172, 173, 174, 175, 177, 178, 190, 192, 230, 231 }

C grade: { 8, 10, 12, 18, 19, 20, 21, 36, 145, 147, 149, 151, 167, 168, 198, 199, 211, 212, 214, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 238 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 4, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 73, 81, 82, 83, 84, 85, 86, 88, 90, 95, 96, 105, 106, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 146, 153, 186, 187, 188, 194, 195, 196, 197, 198, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247 }

B grade: { 3, 5, 6, 7, 8, 10, 45, 57, 58, 59, 60, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 87, 89, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 121, 122, 123, 124, 144, 145, 150, 157, 176, 183, 237 }

C grade: { 147, 148, 149, 151, 152, 154, 155, 156, 158, 159, 205, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236 }

F grade: { 22, 23, 40, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193, 203, 204, 228, 229, 230, 231, 232, 233 }

2.1.4 Maxima

A grade: { 5, 6, 7, 8, 24, 25, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 56, 59, 60, 61, 62, 65, 66, 69, 70, 71, 72, 73, 74, 80, 82, 90, 91, 92, 93, 94, 95, 96, 105, 106, 107, 108, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 176, 183, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 234, 235, 236, 237, 238, 239, 243, 247

B grade: { 1, 2, 3, 4, 9, 10, 11, 12, 26, 41, 42, 43, 51, 52, 53, 54, 55, 57, 58, 63, 64, 75, 76, 77, 78, 79, 85, 87, 89, 97, 98, 99, 100, 101, 102, 103, 104, 186, 187, 188 }

C grade: { 27, 28, 29, 30, 31, 32 }

F grade: { 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 33, 34, 35, 36, 40, 67, 68, 81, 83, 84, 86, 88, 109, 110, 111, 112, 113, 114, 125, 126, 127, 128, 129, 130, 131, 132, 144, 145, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 222, 223, 226, 227, 228, 229, 230, 231, 232, 233, 240, 241, 242, 244, 245, 246 }

2.1.5 FriCAS

A grade: { 61, 65, 66, 71, 72, 84, 90, 93, 94, 95, 106, 112, 136, 137, 138, 139, 146, 147, 148, 149, 150, 152, 153, 154, 155, 159, 209, 215, 218, 219, 220, 224, 236, 237, 241, 243, 245, 247 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 91, 92, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 151, 156, 157, 158, 176, 183, 186, 187, 188, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 221, 222, 223, 225, 226, 227, 234, 235, 238, 239, 240, 242, 244, 246 }

C grade: { 27, 28, 29, 30, 31, 32, 145 }

F grade: { 22, 23, 40, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193, 228, 229, 230, 231, 232, 233 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 21, 41, 42, 43, 44, 45, 46, 47, 48, 49, 57, 58, 59, 60, 61, 62, 65, 66, 95, 106, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 150, 157, 176, 186, 187, 196, 197, 247 }

B grade: { 70, 72, 92, 94, 115, 116, 117, 118, 119, 120, 183 }

C grade: { }

F grade: { 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 50, 51, 52, 53, 54, 55, 56, 63, 64, 67, 68, 69, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 121, 122, 123, 124, 133, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

2.1.7 Giac

A grade: { 1, 2, 3, 10, 12, 24, 33, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 88, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 183, 187, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 234, 235, 236, 237, 238, 239, 243, 247 }

B grade: { 4, 5, 6, 7, 8, 9, 11, 18, 21, 25, 26, 34, 35, 50, 51, 55, 56, 75, 85, 87, 89, 96, 97, 102, 103, 104, 105, 109, 125, 126, 127, 128, 129, 130, 132, 143, 176, 186, 188, 226, 227 }

C grade: { 27, 28, 29, 30, 31, 32 }

F grade: { 13, 14, 15, 16, 17, 19, 20, 22, 23, 36, 40, 67, 68, 144, 145, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 218, 219, 228, 229, 230, 231, 232, 233, 240, 241, 242, 244, 245, 246 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	53	67	155	717	39	100
normalized size	1	1.	1.23	1.56	3.6	16.67	0.91	2.33
time (sec)	N/A	0.024	0.019	0.005	1.043	2.366	0.653	1.199

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	56	138	2642	42	99
normalized size	1	1.	0.88	1.33	3.29	62.9	1.	2.36
time (sec)	N/A	0.033	0.086	0.004	1.547	2.38	0.471	1.182

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	38	54	96	327	27	70
normalized size	1	1.	1.36	1.93	3.43	11.68	0.96	2.5
time (sec)	N/A	0.016	0.009	0.003	1.055	2.083	0.342	1.192

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	43	82	930	31	73
normalized size	1	1.	1.	1.59	3.04	34.44	1.15	2.7
time (sec)	N/A	0.017	0.012	0.003	1.525	2.262	0.243	1.191

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	23	41	34	82	15	38
normalized size	1	1.	1.77	3.15	2.62	6.31	1.15	2.92
time (sec)	N/A	0.009	0.007	0.003	1.036	2.328	0.18	1.179

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	30	15	88	17	36
normalized size	1	1.	1.	2.73	1.36	8.	1.55	3.27
time (sec)	N/A	0.006	0.005	0.003	1.018	2.19	0.151	1.222

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	19	30	15	88	27	38
normalized size	1	1.	1.73	2.73	1.36	8.	2.45	3.45
time (sec)	N/A	0.006	0.009	0.002	1.026	2.447	0.441	1.183

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	27	41	34	82	36	38
normalized size	1	1.	2.08	3.15	2.62	6.31	2.77	2.92
time (sec)	N/A	0.008	0.009	0.	1.044	2.277	1.599	1.238

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	43	107	930	61	74
normalized size	1	1.	1.26	1.59	3.96	34.44	2.26	2.74
time (sec)	N/A	0.018	0.069	0.002	1.029	2.385	4.262	1.221

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	31	54	96	286	49	70
normalized size	1	1.	1.11	1.93	3.43	10.21	1.75	2.5
time (sec)	N/A	0.016	0.01	0.002	1.043	2.111	9.256	1.218

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	44	56	165	2642	75	103
normalized size	1	1.	1.05	1.33	3.93	62.9	1.79	2.45
time (sec)	N/A	0.033	0.159	0.	1.041	2.391	19.782	1.208

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	67	155	666	63	100
normalized size	1	1.	0.72	1.56	3.6	15.49	1.47	2.33
time (sec)	N/A	0.024	0.009	0.002	1.038	2.288	44.837	1.168

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	83	80	0	4136	0	0
normalized size	1	1.	0.86	0.82	0.	42.64	0.	0.
time (sec)	N/A	0.07	0.232	0.032	0.	2.673	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	63	0	2653	0	0
normalized size	1	1.	0.87	0.81	0.	34.01	0.	0.
time (sec)	N/A	0.049	0.205	0.018	0.	2.553	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	61	62	0	1755	0	0
normalized size	1	1.	0.81	0.83	0.	23.4	0.	0.
time (sec)	N/A	0.051	0.085	0.017	0.	2.582	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	47	0	1635	0	0
normalized size	1	1.	0.88	0.81	0.	28.19	0.	0.
time (sec)	N/A	0.032	0.038	0.025	0.	2.822	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	0	1647	0	0
normalized size	1	1.	0.86	0.81	0.	28.89	0.	0.
time (sec)	N/A	0.032	0.034	0.033	0.	2.685	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	36	65	0	2574	0	207
normalized size	1	1.	0.46	0.83	0.	33.	0.	2.65
time (sec)	N/A	0.048	0.074	0.019	0.	2.709	0.	1.288

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	38	64	0	3934	0	0
normalized size	1	1.	0.48	0.81	0.	49.8	0.	0.
time (sec)	N/A	0.049	0.077	0.02	0.	2.555	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	38	83	0	5796	0	0
normalized size	1	1.	0.38	0.83	0.	57.96	0.	0.
time (sec)	N/A	0.067	0.11	0.02	0.	2.693	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	26	102	0	563	63	258
normalized size	1	1.	0.38	1.48	0.	8.16	0.91	3.74
time (sec)	N/A	0.06	0.026	0.023	0.	2.217	2.941	1.243

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	45	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.045	0.174	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.042	0.176	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	30	57	1368	0	70
normalized size	1	1.	0.8	0.86	1.63	39.09	0.	2.
time (sec)	N/A	0.021	0.023	0.025	1.574	2.426	0.	1.209

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	26	26	196	0	42
normalized size	1	1.	1.	1.62	1.62	12.25	0.	2.62
time (sec)	N/A	0.015	0.006	0.033	1.574	2.25	0.	1.196

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	42	198	0	50
normalized size	1	1.	1.	1.81	2.62	12.38	0.	3.12
time (sec)	N/A	0.015	0.008	0.039	1.599	2.27	0.	1.192

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	67	153	467	0	192
normalized size	1	1.	0.64	0.76	1.74	5.31	0.	2.18
time (sec)	N/A	0.049	0.295	0.026	1.694	2.338	0.	1.36

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	46	53	89	252	0	124
normalized size	1	1.	0.77	0.88	1.48	4.2	0.	2.07
time (sec)	N/A	0.032	0.094	0.015	1.601	2.27	0.	1.298

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	45	38	55	0	73
normalized size	1	1.	1.	1.45	1.23	1.77	0.	2.35
time (sec)	N/A	0.018	0.033	0.029	1.622	2.344	0.	1.184

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	52	61	54	0	85
normalized size	1	1.	1.26	1.68	1.97	1.74	0.	2.74
time (sec)	N/A	0.017	0.079	0.027	1.573	2.321	0.	1.229

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	79	115	252	0	151
normalized size	1	1.	0.85	1.32	1.92	4.2	0.	2.52
time (sec)	N/A	0.031	0.126	0.019	1.63	2.276	0.	1.266

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	63	91	177	466	0	182
normalized size	1	1.	0.72	1.03	2.01	5.3	0.	2.07
time (sec)	N/A	0.048	0.236	0.017	1.607	2.328	0.	1.385

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	38	43	0	374	0	74
normalized size	1	1.	0.67	0.75	0.	6.56	0.	1.3
time (sec)	N/A	0.038	0.038	0.039	0.	2.233	0.	1.211

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	55	76	0	4099	0	462
normalized size	1	1.	0.64	0.88	0.	47.66	0.	5.37
time (sec)	N/A	0.037	0.059	0.03	0.	2.724	0.	1.31

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	40	62	0	1237	0	155
normalized size	1	1.	0.63	0.98	0.	19.63	0.	2.46
time (sec)	N/A	0.03	0.033	0.036	0.	2.318	0.	1.242

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	26	65	0	1763	0	0
normalized size	1	1.	0.41	1.02	0.	27.55	0.	0.
time (sec)	N/A	0.03	0.017	0.04	0.	2.451	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	56	46	111	6368	0	61
normalized size	1	1.	0.81	0.67	1.61	92.29	0.	0.88
time (sec)	N/A	0.026	0.148	0.026	1.61	2.527	0.	1.185

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	19	32	26	647	0	22
normalized size	1	1.	0.61	1.03	0.84	20.87	0.	0.71
time (sec)	N/A	0.014	0.017	0.032	1.603	2.321	0.	1.194

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	19	32	31	668	0	26
normalized size	1	1.	0.61	1.03	1.	21.55	0.	0.84
time (sec)	N/A	0.014	0.033	0.039	1.648	2.133	0.	1.261

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.052	3.984	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	202	81	408	2376	95	101
normalized size	1	1.	2.02	0.81	4.08	23.76	0.95	1.01
time (sec)	N/A	0.072	1.608	0.006	1.591	2.358	0.616	1.203

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	178	65	265	1470	76	86
normalized size	1	1.	2.31	0.84	3.44	19.09	0.99	1.12
time (sec)	N/A	0.053	1.088	0.003	1.658	2.341	0.422	1.175

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	103	49	157	780	61	72
normalized size	1	1.	1.84	0.88	2.8	13.93	1.09	1.29
time (sec)	N/A	0.036	0.898	0.001	1.612	2.156	0.28	1.171

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	58	33	68	308	44	53
normalized size	1	1.	1.61	0.92	1.89	8.56	1.22	1.47
time (sec)	N/A	0.021	0.505	0.003	1.08	2.199	0.296	1.162

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	39	54	42	136	73	36
normalized size	1	1.	1.39	1.93	1.5	4.86	2.61	1.29
time (sec)	N/A	0.013	0.101	0.017	1.084	2.19	0.827	1.189

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	72	58	270	223	54
normalized size	1	1.	1.18	1.41	1.14	5.29	4.37	1.06
time (sec)	N/A	0.028	0.171	0.018	1.081	2.224	1.645	1.17

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	83	90	76	428	430	69
normalized size	1	1.	1.14	1.23	1.04	5.86	5.89	0.95
time (sec)	N/A	0.044	0.246	0.019	1.136	2.118	2.017	1.196

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	88	108	90	594	694	84
normalized size	1	1.	0.92	1.12	0.94	6.19	7.23	0.88
time (sec)	N/A	0.062	0.249	0.02	1.078	2.156	2.608	1.186

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	109	126	105	798	1018	99
normalized size	1	1.	0.9	1.04	0.87	6.6	8.41	0.82
time (sec)	N/A	0.082	0.276	0.027	1.096	2.197	4.176	1.204

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	65	43	112	1470	0	189
normalized size	1	1.	1.14	0.75	1.96	25.79	0.	3.32
time (sec)	N/A	0.041	0.201	0.019	1.633	2.344	0.	1.277

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	35	95	871	0	130
normalized size	1	1.	0.87	0.78	2.11	19.36	0.	2.89
time (sec)	N/A	0.031	0.164	0.013	1.589	2.327	0.	1.174

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	77	451	0	70
normalized size	1	1.	1.	0.82	2.33	13.67	0.	2.12
time (sec)	N/A	0.021	0.067	0.013	1.566	2.255	0.	1.173

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	58	182	0	36
normalized size	1	1.	1.	0.81	2.76	8.67	0.	1.71
time (sec)	N/A	0.011	0.045	0.032	1.601	2.22	0.	1.202

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	77	301	0	68
normalized size	1	1.	1.	0.84	2.41	9.41	0.	2.12
time (sec)	N/A	0.023	0.072	0.033	1.586	2.291	0.	1.182

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	35	93	579	0	130
normalized size	1	1.	1.08	0.71	1.9	11.82	0.	2.65
time (sec)	N/A	0.031	0.133	0.014	1.607	2.34	0.	1.204

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	62	43	107	921	0	189
normalized size	1	1.	1.02	0.7	1.75	15.1	0.	3.1
time (sec)	N/A	0.041	0.229	0.017	1.64	2.228	0.	1.246

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	114	322	419	6413	211	305
normalized size	1	1.	0.8	2.27	2.95	45.16	1.49	2.15
time (sec)	N/A	0.211	0.64	0.005	1.655	2.616	0.757	1.217

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	246	271	3318	144	209
normalized size	1	1.	0.9	2.44	2.68	32.85	1.43	2.07
time (sec)	N/A	0.124	0.334	0.006	1.871	2.462	0.527	1.199

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	67	173	159	1563	100	138
normalized size	1	1.	0.97	2.51	2.3	22.65	1.45	2.
time (sec)	N/A	0.065	0.327	0.004	1.783	2.293	0.341	1.237

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	54	116	66	535	54	82
normalized size	1	1.	1.42	3.05	1.74	14.08	1.42	2.16
time (sec)	N/A	0.023	0.095	0.004	1.127	2.215	0.224	1.238

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	64	76	76	149	224	85
normalized size	1	1.	1.28	1.52	1.52	2.98	4.48	1.7
time (sec)	N/A	0.054	0.082	0.019	1.177	2.322	2.365	1.241

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	94	101	171	980	3599	188
normalized size	1	1.	1.11	1.19	2.01	11.53	42.34	2.21
time (sec)	N/A	0.09	1.059	0.027	1.158	2.393	32.424	1.215

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	122	166	439	3182	0	288
normalized size	1	1.	0.95	1.29	3.4	24.67	0.	2.23
time (sec)	N/A	0.178	2.291	0.03	1.429	2.575	0.	1.25

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	160	230	709	8128	0	427
normalized size	1	1.	0.95	1.36	4.2	48.09	0.	2.53
time (sec)	N/A	0.269	3.257	0.035	1.416	2.897	0.	1.272

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	46	38	130	42	42
normalized size	1	1.	1.71	1.48	1.23	4.19	1.35	1.35
time (sec)	N/A	0.044	0.033	0.017	1.113	2.258	0.634	1.181

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	46	39	127	42	39
normalized size	1	1.	1.71	1.48	1.26	4.1	1.35	1.26
time (sec)	N/A	0.042	0.032	0.017	1.112	2.265	0.572	1.188

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	63	0	5682	0	0
normalized size	1	1.	1.	0.85	0.	76.78	0.	0.
time (sec)	N/A	0.072	0.088	0.052	0.	2.912	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	62	0	5844	0	0
normalized size	1	1.	1.	0.84	0.	78.97	0.	0.
time (sec)	N/A	0.069	0.064	0.039	0.	2.845	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	98	49	319	0	57
normalized size	1	1.	0.7	1.63	0.82	5.32	0.	0.95
time (sec)	N/A	0.072	0.076	0.031	1.121	2.249	0.	1.22

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	34	72	36	200	134	34
normalized size	1	1.	1.36	2.88	1.44	8.	5.36	1.36
time (sec)	N/A	0.167	0.059	0.027	1.182	2.198	1.597	1.194

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	24	68	30	176	0	41
normalized size	1	1.	0.63	1.79	0.79	4.63	0.	1.08
time (sec)	N/A	0.059	0.037	0.027	1.139	2.314	0.	1.227

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	42	15	90	48	15
normalized size	1	1.	1.12	2.47	0.88	5.29	2.82	0.88
time (sec)	N/A	0.111	0.016	0.023	1.156	2.439	0.436	1.187

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	17	28	167	0	24
normalized size	1	1.	1.17	1.42	2.33	13.92	0.	2.
time (sec)	N/A	0.11	0.021	0.023	1.143	2.631	0.	1.281

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	32	39	267	0	39
normalized size	1	1.	0.73	2.13	2.6	17.8	0.	2.6
time (sec)	N/A	0.039	0.029	0.026	1.121	2.633	0.	1.176

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	20	39	65	733	0	46
normalized size	1	1.	1.11	2.17	3.61	40.72	0.	2.56
time (sec)	N/A	0.161	0.079	0.027	1.115	2.644	0.	1.213

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	20	48	101	286	0	24
normalized size	1	1.	1.18	2.82	5.94	16.82	0.	1.41
time (sec)	N/A	0.044	0.048	0.03	1.165	2.284	0.	1.227

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	49	55	100	2136	0	66
normalized size	1	1.	1.44	1.62	2.94	62.82	0.	1.94
time (sec)	N/A	0.192	0.119	0.033	1.079	2.189	0.	1.32

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	80	201	626	0	32
normalized size	1	1.	0.82	2.42	6.09	18.97	0.	0.97
time (sec)	N/A	0.054	0.051	0.033	1.105	1.985	0.	1.216

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	68	103	132	4358	0	82
normalized size	1	1.	1.55	2.34	3.	99.05	0.	1.86
time (sec)	N/A	0.211	0.264	0.033	1.213	2.318	0.	1.288

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	144	320	220	2747	0	289
normalized size	1	1.	0.98	2.18	1.5	18.69	0.	1.97
time (sec)	N/A	0.346	0.54	0.046	1.161	2.541	0.	1.187

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	180	166	0	4208	0	220
normalized size	1	1.	1.31	1.21	0.	30.72	0.	1.61
time (sec)	N/A	0.279	1.148	0.042	0.	2.452	0.	1.217

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	145	112	826	0	136
normalized size	1	1.	0.87	1.73	1.33	9.83	0.	1.62
time (sec)	N/A	0.162	0.222	0.039	1.168	2.633	0.	1.213

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	1103	0	81
normalized size	1	1.	1.1	1.28	0.	15.32	0.	1.12
time (sec)	N/A	0.116	0.221	0.03	0.	2.519	0.	1.187

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	59	53	0	675	0	81
normalized size	1	1.	1.13	1.02	0.	12.98	0.	1.56
time (sec)	N/A	0.143	0.09	0.033	0.	2.451	0.	1.213

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	56	88	360	0	105
normalized size	1	1.	0.97	1.93	3.03	12.41	0.	3.62
time (sec)	N/A	0.054	0.083	0.039	1.04	2.26	0.	1.199

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	123	110	0	3212	0	169
normalized size	1	1.	1.5	1.34	0.	39.17	0.	2.06
time (sec)	N/A	0.293	0.346	0.042	0.	2.78	0.	1.184

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	166	217	2186	0	273
normalized size	1	1.	0.9	2.13	2.78	28.03	0.	3.5
time (sec)	N/A	0.1	0.239	0.045	1.122	2.464	0.	1.295

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	296	311	0	12964	0	369
normalized size	1	1.	1.16	1.22	0.	50.84	0.	1.45
time (sec)	N/A	0.548	0.847	0.047	0.	3.802	0.	1.252

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	119	333	416	7144	0	556
normalized size	1	1.	0.92	2.56	3.2	54.95	0.	4.28
time (sec)	N/A	0.154	0.582	0.051	1.111	2.747	0.	1.219

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	29	46	180	0	38
normalized size	1	1.	1.39	0.88	1.39	5.45	0.	1.15
time (sec)	N/A	0.102	0.068	0.036	1.563	2.322	0.	1.226

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	116	49	323	0	57
normalized size	1	1.	0.7	1.93	0.82	5.38	0.	0.95
time (sec)	N/A	0.061	0.045	0.033	1.114	2.198	0.	1.251

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	36	80	45	221	134	42
normalized size	1	1.	1.24	2.76	1.55	7.62	4.62	1.45
time (sec)	N/A	0.042	0.047	0.032	1.063	2.061	1.591	1.178

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	30	76	30	178	0	41
normalized size	1	1.	0.79	2.	0.79	4.68	0.	1.08
time (sec)	N/A	0.049	0.03	0.032	1.018	2.149	0.	1.208

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	40	23	99	48	26
normalized size	1	1.	1.21	2.11	1.21	5.21	2.53	1.37
time (sec)	N/A	0.03	0.027	0.027	1.01	2.218	0.436	1.259

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	7	11	8	32	8	8
normalized size	1	1.	0.7	1.1	0.8	3.2	0.8	0.8
time (sec)	N/A	0.02	0.003	0.003	1.179	2.231	0.382	1.19

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	7	6	7	57	0	18
normalized size	1	1.	1.4	1.2	1.4	11.4	0.	3.6
time (sec)	N/A	0.033	0.004	0.018	1.167	2.316	0.	1.267

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	12	21	30	198	0	24
normalized size	1	1.	2.	3.5	5.	33.	0.	4.
time (sec)	N/A	0.035	0.025	0.022	1.583	2.082	0.	1.222

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	34	50	181	0	14
normalized size	1	1.	1.	3.09	4.55	16.45	0.	1.27
time (sec)	N/A	0.033	0.024	0.025	1.183	2.022	0.	1.255

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	41	66	968	0	42
normalized size	1	1.	1.	1.71	2.75	40.33	0.	1.75
time (sec)	N/A	0.042	0.027	0.026	1.637	2.215	0.	1.229

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	20	56	126	467	0	24
normalized size	1	1.	0.8	2.24	5.04	18.68	0.	0.96
time (sec)	N/A	0.041	0.03	0.024	1.118	2.153	0.	1.204

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	67	99	2298	0	61
normalized size	1	1.	1.	1.97	2.91	67.59	0.	1.79
time (sec)	N/A	0.048	0.033	0.028	1.745	2.156	0.	1.227

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	137	925	521	12186	0	801
normalized size	1	1.	0.98	6.61	3.72	87.04	0.	5.72
time (sec)	N/A	0.158	0.551	0.056	1.73	3.166	0.	1.231

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	92	438	275	4520	0	427
normalized size	1	1.	1.11	5.28	3.31	54.46	0.	5.14
time (sec)	N/A	0.111	0.347	0.046	1.615	2.643	0.	1.257

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	49	143	120	1088	0	140
normalized size	1	1.	1.22	3.58	3.	27.2	0.	3.5
time (sec)	N/A	0.066	0.157	0.039	1.679	2.434	0.	1.207

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	20	12	15	126	0	61
normalized size	1	1.	1.82	1.09	1.36	11.45	0.	5.55
time (sec)	N/A	0.042	0.044	0.023	1.166	2.311	0.	1.21

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	55	108	146	58
normalized size	1	1.	0.74	1.41	1.41	2.77	3.74	1.49
time (sec)	N/A	0.046	0.045	0.011	1.14	2.359	0.743	1.214

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	75	175	116	818	0	150
normalized size	1	1.	0.82	1.92	1.27	8.99	0.	1.65
time (sec)	N/A	0.146	0.113	0.046	1.226	2.304	0.	1.21

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	354	223	2882	0	306
normalized size	1	1.	1.01	2.28	1.44	18.59	0.	1.97
time (sec)	N/A	0.236	0.194	0.049	1.168	2.393	0.	1.24

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	187	166	715	0	16338	0	440
normalized size	1	1.19	1.06	4.55	0.	104.06	0.	2.8
time (sec)	N/A	0.273	0.496	0.053	0.	4.489	0.	1.236

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	109	116	316	0	5131	0	205
normalized size	1	1.07	1.14	3.1	0.	50.3	0.	2.01
time (sec)	N/A	0.166	0.259	0.042	0.	3.163	0.	1.265

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	65	110	0	1000	0	85
normalized size	1	1.	1.16	1.96	0.	17.86	0.	1.52
time (sec)	N/A	0.088	0.084	0.03	0.	2.56	0.	1.248

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	46	39	0	423	0	47
normalized size	1	1.	1.24	1.05	0.	11.43	0.	1.27
time (sec)	N/A	0.032	0.029	0.018	0.	2.165	0.	1.293

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	93	0	1106	0	82
normalized size	1	1.	1.1	1.27	0.	15.15	0.	1.12
time (sec)	N/A	0.09	0.225	0.038	0.	2.496	0.	1.211

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	258	198	0	4251	0	219
normalized size	1	1.	1.95	1.5	0.	32.2	0.	1.66
time (sec)	N/A	0.156	0.459	0.044	0.	2.566	0.	1.269

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	40	74	1897	104	63
normalized size	1	1.	0.93	0.93	1.72	44.12	2.42	1.47
time (sec)	N/A	0.089	0.095	0.023	1.646	2.368	0.637	1.239

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	58	1168	85	53
normalized size	1	1.	0.89	0.86	1.57	31.57	2.3	1.43
time (sec)	N/A	0.066	0.052	0.019	1.762	2.322	0.539	1.271

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	28	39	632	75	47
normalized size	1	1.	0.87	0.9	1.26	20.39	2.42	1.52
time (sec)	N/A	0.051	0.045	0.017	1.921	2.282	0.47	1.2

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	24	23	259	61	23
normalized size	1	1.	1.21	1.26	1.21	13.63	3.21	1.21
time (sec)	N/A	0.038	0.028	0.016	1.85	2.322	0.428	1.209

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	14	88	27	14
normalized size	1	1.	1.12	1.5	0.88	5.5	1.69	0.88
time (sec)	N/A	0.02	0.026	0.016	1.267	2.211	0.388	1.206

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	14	88	27	14
normalized size	1	1.	1.12	1.5	0.88	5.5	1.69	0.88
time (sec)	N/A	0.008	0.015	0.014	1.181	2.104	0.396	1.197

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	43	32	259	0	24
normalized size	1	1.	1.21	2.26	1.68	13.63	0.	1.26
time (sec)	N/A	0.04	0.028	0.027	1.27	2.375	0.	1.205

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	59	51	632	0	49
normalized size	1	1.	0.93	2.03	1.76	21.79	0.	1.69
time (sec)	N/A	0.068	0.043	0.029	1.218	2.254	0.	1.208

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	75	73	1168	0	54
normalized size	1	1.	0.89	2.03	1.97	31.57	0.	1.46
time (sec)	N/A	0.089	0.059	0.032	1.398	2.218	0.	1.183

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	91	86	1897	0	65
normalized size	1	1.	0.98	2.12	2.	44.12	0.	1.51
time (sec)	N/A	0.101	0.128	0.036	1.142	2.211	0.	1.271

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	35	0	871	82	130
normalized size	1	1.	0.87	0.78	0.	19.36	1.82	2.89
time (sec)	N/A	0.048	0.18	0.011	0.	2.34	11.353	1.355

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	0	456	70	72
normalized size	1	1.	1.	0.81	0.	14.25	2.19	2.25
time (sec)	N/A	0.035	0.044	0.026	0.	2.298	2.139	1.264

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	301	66	78
normalized size	1	1.	1.	0.83	0.	10.03	2.2	2.6
time (sec)	N/A	0.037	0.054	0.043	0.	2.157	2.408	1.267

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	51	35	0	579	82	100
normalized size	1	1.	1.04	0.71	0.	11.82	1.67	2.04
time (sec)	N/A	0.049	0.119	0.016	0.	2.247	10.032	1.269

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	53	35	0	1450	82	189
normalized size	1	1.	1.18	0.78	0.	32.22	1.82	4.2
time (sec)	N/A	0.058	0.198	0.019	0.	2.348	17.353	1.291

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	0	818	71	130
normalized size	1	1.	1.	0.76	0.	24.06	2.09	3.82
time (sec)	N/A	0.044	0.068	0.034	0.	2.206	2.999	1.223

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	35	0	639	78	73
normalized size	1	1.	0.88	0.83	0.	15.21	1.86	1.74
time (sec)	N/A	0.056	0.088	0.039	0.	2.35	3.403	1.26

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	35	0	579	82	136
normalized size	1	1.	1.08	0.71	0.	11.82	1.67	2.78
time (sec)	N/A	0.08	0.096	0.021	0.	2.264	11.425	1.268

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	105	96	203	3089	0	192
normalized size	1	1.	1.12	1.02	2.16	32.86	0.	2.04
time (sec)	N/A	0.367	0.453	0.023	1.765	2.626	0.	1.21

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	88	76	135	1538	442	132
normalized size	1	1.	1.16	1.	1.78	20.24	5.82	1.74
time (sec)	N/A	0.213	0.274	0.022	1.83	2.514	105.921	1.311

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	67	96	662	330	101
normalized size	1	1.	1.02	1.05	1.5	10.34	5.16	1.58
time (sec)	N/A	0.129	0.12	0.02	1.846	2.495	1.36	1.204

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	60	76	186	243	78
normalized size	1	1.	0.78	0.95	1.21	2.95	3.86	1.24
time (sec)	N/A	0.092	0.075	0.017	1.815	2.287	0.871	1.181

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	54	109	141	58
normalized size	1	1.	0.74	1.41	1.38	2.79	3.62	1.49
time (sec)	N/A	0.059	0.054	0.019	1.277	2.164	0.702	1.19

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	55	108	146	58
normalized size	1	1.	0.74	1.41	1.41	2.77	3.74	1.49
time (sec)	N/A	0.046	0.041	0.	1.142	2.172	0.687	1.167

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	78	88	185	0	78
normalized size	1	1.	0.9	1.53	1.73	3.63	0.	1.53
time (sec)	N/A	0.079	0.077	0.039	1.261	2.357	0.	1.18

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	64	100	116	662	0	101
normalized size	1	1.	1.07	1.67	1.93	11.03	0.	1.68
time (sec)	N/A	0.182	0.123	0.042	1.259	2.481	0.	1.152

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	91	134	163	1539	0	131
normalized size	1	1.	1.2	1.76	2.14	20.25	0.	1.72
time (sec)	N/A	0.309	0.184	0.046	1.278	2.837	0.	1.18

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	108	185	234	3087	0	192
normalized size	1	1.	1.11	1.91	2.41	31.82	0.	1.98
time (sec)	N/A	0.494	0.285	0.049	1.187	3.081	0.	1.187

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	73	92	491	0	235
normalized size	1	1.	0.89	1.33	1.67	8.93	0.	4.27
time (sec)	N/A	0.085	0.166	0.111	2.044	2.713	0.	1.188

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	250	953	0	3791	0	0
normalized size	1	1.	1.08	4.13	0.	16.41	0.	0.
time (sec)	N/A	0.541	2.446	0.155	0.	2.822	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	316	1186	0	5269	0	0
normalized size	1	1.	0.9	3.38	0.	15.01	0.	0.
time (sec)	N/A	0.867	2.302	0.134	0.	3.123	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	29	0	64	24	31	72	0	31
normalized size	1	0.	2.21	0.83	1.07	2.48	0.	1.07
time (sec)	N/A	0.029	0.025	0.021	1.237	2.447	0.	1.165

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	151	0	64	37	173	506	0	166
normalized size	1	0.	0.42	0.25	1.15	3.35	0.	1.1
time (sec)	N/A	0.022	0.251	0.022	1.799	2.512	0.	1.2

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	23	0	35	41	26	57	0	26
normalized size	1	0.	1.52	1.78	1.13	2.48	0.	1.13
time (sec)	N/A	0.016	0.187	0.035	1.914	1.839	0.	1.235

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	145	0	58	33	167	498	0	161
normalized size	1	0.	0.4	0.23	1.15	3.43	0.	1.11
time (sec)	N/A	0.007	0.165	0.012	1.808	2.148	0.	1.166

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	26	14	47	15	27
normalized size	1	1.	1.	2.17	1.17	3.92	1.25	2.25
time (sec)	N/A	0.014	0.025	0.003	1.219	1.969	0.297	1.157

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	A	B	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	147	0	59	42	169	597	0	163
normalized size	1	0.	0.4	0.29	1.15	4.06	0.	1.11
time (sec)	N/A	0.024	0.159	0.027	1.901	2.199	0.	1.185

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	20	0	40	44	26	55	0	22
normalized size	1	0.	2.	2.2	1.3	2.75	0.	1.1
time (sec)	N/A	0.024	0.153	0.027	1.621	2.095	0.	1.206

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	47	0	86	42	54	140	0	53
normalized size	1	0.	1.83	0.89	1.15	2.98	0.	1.13
time (sec)	N/A	0.068	0.104	0.019	1.541	1.885	0.	1.18

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	173	0	174	53	194	693	0	188
normalized size	1	0.	1.01	0.31	1.12	4.01	0.	1.09
time (sec)	N/A	0.048	0.649	0.022	1.778	2.146	0.	1.184

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	40	0	41	57	47	120	0	47
normalized size	1	0.	1.02	1.42	1.18	3.	0.	1.18
time (sec)	N/A	0.03	0.378	0.033	1.864	2.03	0.	1.239

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	B	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	165	0	146	47	186	680	0	180
normalized size	1	0.	0.88	0.28	1.13	4.12	0.	1.09
time (sec)	N/A	0.01	0.531	0.02	1.702	2.203	0.	1.453

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	24	35	28	68	12	26
normalized size	1	1.	1.71	2.5	2.	4.86	0.86	1.86
time (sec)	N/A	0.024	0.038	0.003	1.148	1.977	0.5	1.472

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	B	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	190	0	181	64	197	771	0	193
normalized size	1	0.	0.95	0.34	1.04	4.06	0.	1.02
time (sec)	N/A	0.045	0.736	0.026	1.931	2.145	0.	1.45

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	59	0	40	66	50	122	0	53
normalized size	1	0.	0.68	1.12	0.85	2.07	0.	0.9
time (sec)	N/A	0.053	0.355	0.029	1.819	1.886	0.	1.401

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	60	0	47	0	0	0	0	0
normalized size	1	0.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.09	0.036	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	79	0	168	0	0	0	0	0
normalized size	1	0.	2.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.364	0.027	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	176	0	218	0	0	0	0	0
normalized size	1	0.	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.817	0.033	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	79	0	259	0	0	0	0	0
normalized size	1	0.	3.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	1.907	0.106	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	99	0	126	0	0	0	0	0
normalized size	1	0.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	2.978	0.03	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	51	0	76	0	0	0	0	0
normalized size	1	0.	1.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	2.904	0.033	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	106	0	121	0	0	0	0	0
normalized size	1	0.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	2.904	0.029	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	158	0	177	0	0	0	0	0
normalized size	1	0.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	4.096	0.03	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	190	0	177	0	0	0	0	0
normalized size	1	0.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	3.17	0.029	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0
normalized size	1	0.	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	1.544	0.048	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0
normalized size	1	0.	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	1.775	0.046	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0
normalized size	1	0.	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	1.782	0.052	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	59	0	127	0	0	0	0	0
normalized size	1	0.	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	7.671	0.894	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	63	0	136	0	0	0	0	0
normalized size	1	0.	2.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	7.56	0.784	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	55	0	122	0	0	0	0	0
normalized size	1	0.	2.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	7.585	0.783	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	53	0	126	0	0	0	0	0
normalized size	1	0.	2.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	8.558	0.717	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	56	32	205	36	99
normalized size	1	1.	0.96	2.24	1.28	8.2	1.44	3.96
time (sec)	N/A	0.02	0.046	0.004	1.086	2.146	5.9	1.37

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	59	0	126	0	0	0	0	0
normalized size	1	0.	2.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	3.212	0.796	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	56	0	120	0	0	0	0	0
normalized size	1	0.	2.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	3.121	0.818	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	133	0	159	0	0	0	0	0
normalized size	1	0.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	8.162	0.809	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	137	0	169	0	0	0	0	0
normalized size	1	0.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	7.78	0.317	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	131	0	155	0	0	0	0	0
normalized size	1	0.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	7.733	0.098	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	127	0	163	0	0	0	0	0
normalized size	1	0.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	8.807	0.099	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	51	80	49	197	70	50
normalized size	1	1.	1.82	2.86	1.75	7.04	2.5	1.79
time (sec)	N/A	0.029	0.078	0.006	1.396	2.148	14.908	1.515

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	135	0	162	0	0	0	0	0
normalized size	1	0.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	3.416	0.102	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	136	0	159	0	0	0	0	0
normalized size	1	0.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	3.377	0.109	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	67	410	1841	75	170
normalized size	1	1.	1.	1.56	9.53	42.81	1.74	3.95
time (sec)	N/A	0.041	0.134	0.007	1.505	2.051	5.78	1.381

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	62	86	667	630	71	90
normalized size	1	1.	1.38	1.91	14.82	14.	1.58	2.
time (sec)	N/A	0.039	0.103	0.004	1.531	2.004	28.884	1.363

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	55	88	1119	5060	0	216
normalized size	1	1.	0.83	1.33	16.95	76.67	0.	3.27
time (sec)	N/A	0.057	0.208	0.006	1.757	2.324	0.	1.425

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	88	0	160	0	0	0	0	0
normalized size	1	0.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	13.584	1.214	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	169	0	516	0	0	0	0	0
normalized size	1	0.	3.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	16.539	0.218	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	307	0	606	0	0	0	0	0
normalized size	1	0.	1.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	16.76	0.218	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	115	0	387	0	0	0	0	0
normalized size	1	0.	3.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	3.824	0.088	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	135	0	174	0	0	0	0	0
normalized size	1	0.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	4.999	0.057	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	64	93	0	2091	0	0
normalized size	1	1.	0.88	1.27	0.	28.64	0.	0.
time (sec)	N/A	0.053	0.285	0.023	0.	2.235	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	57	92	0	1100	0	0
normalized size	1	1.	0.81	1.31	0.	15.71	0.	0.
time (sec)	N/A	0.052	0.133	0.013	0.	2.123	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	72	0	1000	66	0
normalized size	1	1.	1.	1.5	0.	20.83	1.38	0.
time (sec)	N/A	0.041	0.076	0.015	0.	2.115	3.235	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	0	999	66	0
normalized size	1	1.	1.	0.94	0.	21.26	1.4	0.
time (sec)	N/A	0.04	0.109	0.017	0.	2.069	7.105	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	44	93	0	2088	0	0
normalized size	1	1.	0.62	1.31	0.	29.41	0.	0.
time (sec)	N/A	0.052	0.158	0.017	0.	2.1	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	92	0	3644	0	0
normalized size	1	1.	0.64	1.28	0.	50.61	0.	0.
time (sec)	N/A	0.05	0.203	0.016	0.	2.216	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	136	149	0	24521	0	0
normalized size	1	1.	1.01	1.1	0.	181.64	0.	0.
time (sec)	N/A	0.358	1.305	0.168	0.	15.786	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	0	18436	0	0
normalized size	1	1.	1.	0.86	0.	175.58	0.	0.
time (sec)	N/A	0.225	0.074	0.057	0.	12.276	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	0	4811	0	0
normalized size	1	1.	1.	0.9	0.	82.95	0.	0.
time (sec)	N/A	0.12	0.032	0.075	0.	8.646	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	109	0	0	18421	0	0
normalized size	1	1.	1.03	0.	0.	173.78	0.	0.
time (sec)	N/A	0.249	0.328	0.218	0.	12.137	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	142	0	0	24602	0	0
normalized size	1	1.	0.78	0.	0.	134.44	0.	0.
time (sec)	N/A	0.337	0.576	0.193	0.	15.907	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	131	559	0	22316	0	0
normalized size	1	1.	0.99	4.23	0.	169.06	0.	0.
time (sec)	N/A	0.231	0.324	0.082	0.	20.853	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	76	143	127	1685	0	92
normalized size	1	1.	0.71	1.34	1.19	15.75	0.	0.86
time (sec)	N/A	0.069	0.132	0.013	1.575	2.24	0.	1.804

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	102	93	956	0	70
normalized size	1	1.	0.78	1.32	1.21	12.42	0.	0.91
time (sec)	N/A	0.05	0.089	0.012	1.583	2.111	0.	2.134

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	56	63	433	0	55
normalized size	1	1.	0.78	1.1	1.24	8.49	0.	1.08
time (sec)	N/A	0.035	0.088	0.007	1.592	1.986	0.	2.209

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	34	31	105	0	31
normalized size	1	1.	0.88	1.36	1.24	4.2	0.	1.24
time (sec)	N/A	0.015	0.014	0.01	1.615	1.921	0.	1.526

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	27	51	158	0	43
normalized size	1	1.	0.88	1.08	2.04	6.32	0.	1.72
time (sec)	N/A	0.017	0.017	0.011	1.039	2.046	0.	1.604

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	53	53	179	47	84	585	0	76
normalized size	1	1.	3.38	0.89	1.58	11.04	0.	1.43
time (sec)	N/A	0.038	1.721	0.01	1.042	2.067	0.	1.291

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	286	88	119	1291	0	97
normalized size	1	1.	3.53	1.09	1.47	15.94	0.	1.2
time (sec)	N/A	0.052	2.454	0.017	1.05	2.051	0.	1.227

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	123	149	2221	0	112
normalized size	1	1.	1.02	1.09	1.32	19.65	0.	0.99
time (sec)	N/A	0.072	10.105	0.014	1.166	2.117	0.	1.291

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	48	35	120	510	0	120
normalized size	1	1.	0.42	0.31	1.06	4.51	0.	1.06
time (sec)	N/A	0.087	0.043	0.059	1.576	2.295	0.	1.208

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	24	105	360	0	105
normalized size	1	1.	1.	0.25	1.11	3.79	0.	1.11
time (sec)	N/A	0.062	0.03	0.052	1.588	2.435	0.	1.275

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	36	30	154	0	31
normalized size	1	1.	1.	2.25	1.88	9.62	0.	1.94
time (sec)	N/A	0.014	0.012	0.063	1.532	2.389	0.	1.283

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	35	113	48	46	837	0	47
normalized size	1	1.	3.23	1.37	1.31	23.91	0.	1.34
time (sec)	N/A	0.029	1.466	0.072	1.544	2.361	0.	1.268

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	97	59	109	490	0	0
normalized size	1	1.	0.86	0.52	0.96	4.34	0.	0.
time (sec)	N/A	0.212	0.076	0.106	1.57	2.314	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	24	47	93	350	0	0
normalized size	1	1.	0.25	0.48	0.96	3.61	0.	0.
time (sec)	N/A	0.186	0.012	0.071	1.59	2.299	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	22	138	101	454	0	103
normalized size	1	1.	0.26	1.62	1.19	5.34	0.	1.21
time (sec)	N/A	0.122	0.015	0.076	1.605	2.16	0.	1.205

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	113	150	117	2221	0	119
normalized size	1	1.	1.05	1.39	1.08	20.56	0.	1.1
time (sec)	N/A	0.147	1.716	0.1	1.643	2.165	0.	1.29

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	51	36	0	4162	0	355
normalized size	1	1.	0.13	0.09	0.	10.9	0.	0.93
time (sec)	N/A	0.405	0.048	0.095	0.	2.643	0.	1.332

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	24	24	0	3330	0	339
normalized size	1	1.	0.07	0.07	0.	9.1	0.	0.93
time (sec)	N/A	0.244	0.012	0.059	0.	2.396	0.	1.448

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	22	56	131	447	0	132
normalized size	1	1.	0.19	0.48	1.13	3.85	0.	1.14
time (sec)	N/A	0.078	0.016	0.07	1.692	2.109	0.	1.304

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	113	68	147	637	0	149
normalized size	1	1.	0.84	0.51	1.1	4.75	0.	1.11
time (sec)	N/A	0.101	1.645	0.105	1.779	2.332	0.	1.506

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	81	70	0	1037	0	443
normalized size	1	1.	0.76	0.65	0.	9.69	0.	4.14
time (sec)	N/A	0.126	0.077	0.17	0.	2.258	0.	1.29

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	107	476	0	2920	0	616
normalized size	1	1.	0.7	3.13	0.	19.21	0.	4.05
time (sec)	N/A	0.176	0.139	0.194	0.	2.348	0.	1.271

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	205	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.188	4.129	0.105	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	169	0	0	0	0	0
normalized size	1	1.	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	3.257	0.068	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	141	0	0	0	0	0
normalized size	1	1.	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	1.808	0.045	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	134	0	0	0	0	0
normalized size	1	1.	2.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	2.388	0.059	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	145	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	3.047	0.073	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	185	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	3.313	0.113	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	133	324	196	3186	0	248
normalized size	1	1.	0.43	1.04	0.63	10.24	0.	0.8
time (sec)	N/A	0.902	0.217	0.277	1.704	2.118	0.	1.333

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	104	301	122	1175	0	174
normalized size	1	1.	0.54	1.56	0.63	6.09	0.	0.9
time (sec)	N/A	0.28	0.155	0.185	1.771	2.151	0.	1.274

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	218	47	132	0	81
normalized size	1	1.	0.61	2.63	0.57	1.59	0.	0.98
time (sec)	N/A	0.15	0.052	0.23	1.774	1.994	0.	1.23

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	213	76	196	0	119
normalized size	1	1.	0.61	2.57	0.92	2.36	0.	1.43
time (sec)	N/A	0.199	0.123	0.227	1.772	2.035	0.	1.302

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	334	298	151	1574	0	217
normalized size	1	1.	1.7	1.51	0.77	7.99	0.	1.1
time (sec)	N/A	0.865	7.586	0.193	1.752	2.007	0.	1.507

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	164	320	225	4176	0	290
normalized size	1	1.	0.51	1.	0.71	13.09	0.	0.91
time (sec)	N/A	1.77	10.657	0.197	1.682	2.216	0.	1.648

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	929	0	0
normalized size	1	1.	0.79	0.75	0.	5.92	0.	0.
time (sec)	N/A	0.395	0.214	0.026	0.	2.341	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	102	0	474	0	0
normalized size	1	1.	0.77	0.89	0.	4.12	0.	0.
time (sec)	N/A	0.266	0.141	0.023	0.	2.211	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	58	0	464	0	0
normalized size	1	1.	0.77	0.75	0.	6.03	0.	0.
time (sec)	N/A	0.146	0.125	0.017	0.	2.201	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	2.634	0.239	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	929	0	0
normalized size	1	1.	0.79	0.75	0.	5.92	0.	0.
time (sec)	N/A	0.373	0.263	0.023	0.	2.511	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	102	0	474	0	0
normalized size	1	1.	0.77	0.89	0.	4.12	0.	0.
time (sec)	N/A	0.25	0.158	0.022	0.	2.202	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	463	0	0
normalized size	1	1.	0.81	0.75	0.	6.01	0.	0.
time (sec)	N/A	0.139	0.104	0.02	0.	2.18	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	4.66	0.095	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [224] had the largest ratio of [1.5]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	2	1.	8	0.25
2	A	3	2	1.	8	0.25
3	A	3	2	1.	8	0.25
4	A	2	2	1.	8	0.25
5	A	2	2	1.	8	0.25
6	A	1	1	1.	6	0.167
7	A	1	1	1.	6	0.167
8	A	2	2	1.	8	0.25
9	A	2	2	1.	8	0.25
10	A	3	2	1.	8	0.25
11	A	3	2	1.	8	0.25
12	A	4	2	1.	8	0.25
13	A	7	6	1.	12	0.5
14	A	6	6	1.	12	0.5
15	A	6	6	1.	12	0.5
16	A	5	5	1.	12	0.417
17	A	5	5	1.	12	0.417
18	A	6	6	1.	12	0.5
19	A	6	6	1.	12	0.5
20	A	7	6	1.	12	0.5
21	A	9	9	1.	8	1.125
22	A	2	2	1.	8	0.25
23	A	2	2	1.	10	0.2
24	A	3	3	1.	10	0.3
25	A	2	2	1.	10	0.2
26	A	2	2	1.	10	0.2
27	A	4	3	1.	14	0.214
28	A	3	3	1.	14	0.214
29	A	2	2	1.	14	0.143
30	A	2	2	1.	14	0.143
31	A	3	3	1.	14	0.214
32	A	4	3	1.	14	0.214
33	A	7	7	1.	8	0.875
34	A	8	7	1.	10	0.7
35	A	7	7	1.	10	0.7

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	7	7	1.	10	0.7
37	A	5	3	1.	10	0.3
38	A	3	3	1.	10	0.3
39	A	3	3	1.	10	0.3
40	A	3	3	1.	12	0.25
41	A	5	3	1.	12	0.25
42	A	4	3	1.	12	0.25
43	A	3	3	1.	12	0.25
44	A	2	2	1.	12	0.167
45	A	2	2	1.	12	0.167
46	A	3	2	1.	12	0.167
47	A	4	2	1.	12	0.167
48	A	5	2	1.	12	0.167
49	A	6	2	1.	12	0.167
50	A	5	3	1.	8	0.375
51	A	4	3	1.	8	0.375
52	A	3	3	1.	8	0.375
53	A	2	2	1.	8	0.25
54	A	3	3	1.	8	0.375
55	A	4	3	1.	8	0.375
56	A	5	3	1.	8	0.375
57	A	5	4	1.	12	0.333
58	A	4	4	1.	12	0.333
59	A	3	3	1.	12	0.25
60	A	2	2	1.	12	0.167
61	A	2	2	1.	12	0.167
62	A	3	3	1.	12	0.25
63	A	4	4	1.	12	0.333
64	A	5	4	1.	12	0.333
65	A	2	2	1.	12	0.167
66	A	2	2	1.	12	0.167
67	A	5	4	1.	14	0.286
68	A	5	4	1.	14	0.286
69	A	5	4	1.	11	0.364
70	A	9	7	1.	11	0.636
71	A	5	4	1.	11	0.364
72	A	8	6	1.	9	0.667
73	A	8	7	1.	9	0.778
74	A	3	2	1.	11	0.182
75	A	8	7	1.	11	0.636
76	A	4	3	1.	11	0.273
77	A	9	8	1.	11	0.727
78	A	4	3	1.	11	0.273
79	A	10	8	1.	11	0.727

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	5	3	1.	13	0.231
81	A	10	9	1.	13	0.692
82	A	4	3	1.	13	0.231
83	A	6	6	1.	11	0.546
84	A	6	5	1.	11	0.454
85	A	3	2	1.	13	0.154
86	A	15	11	1.	13	0.846
87	A	3	2	1.	13	0.154
88	A	29	13	1.	13	1.
89	A	3	2	1.	13	0.154
90	A	6	5	1.	11	0.454
91	A	4	3	1.	11	0.273
92	A	3	2	1.	11	0.182
93	A	4	3	1.	11	0.273
94	A	2	2	1.	9	0.222
95	A	1	1	1.	9	0.111
96	A	2	2	1.	11	0.182
97	A	2	2	1.	11	0.182
98	A	2	1	1.	11	0.091
99	A	3	3	1.	11	0.273
100	A	3	2	1.	11	0.182
101	A	4	3	1.	11	0.273
102	A	3	2	1.	13	0.154
103	A	3	2	1.	13	0.154
104	A	3	2	1.	13	0.154
105	A	2	2	1.	13	0.154
106	A	2	2	1.	8	0.25
107	A	4	3	1.	13	0.231
108	A	5	4	1.	13	0.308
109	A	14	6	1.19	13	0.462
110	A	9	6	1.07	13	0.462
111	A	5	5	1.	13	0.385
112	A	2	2	1.	11	0.182
113	A	5	5	1.	11	0.454
114	A	9	6	1.	13	0.462
115	A	5	4	1.	11	0.364
116	A	4	4	1.	11	0.364
117	A	3	3	1.	11	0.273
118	A	3	2	1.	11	0.182
119	A	2	2	1.	9	0.222
120	A	2	2	1.	6	0.333
121	A	4	4	1.	9	0.444
122	A	4	4	1.	11	0.364
123	A	5	4	1.	11	0.364

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	6	4	1.	11	0.364
125	A	4	4	1.	11	0.364
126	A	3	3	1.	11	0.273
127	A	3	3	1.	11	0.273
128	A	4	4	1.	11	0.364
129	A	4	4	1.	13	0.308
130	A	3	3	1.	13	0.231
131	A	4	4	1.	13	0.308
132	A	4	4	1.	13	0.308
133	A	7	7	1.	13	0.538
134	A	6	6	1.	13	0.462
135	A	5	5	1.	13	0.385
136	A	4	4	1.	13	0.308
137	A	2	2	1.	11	0.182
138	A	2	2	1.	8	0.25
139	A	3	3	1.	11	0.273
140	A	4	4	1.	13	0.308
141	A	5	5	1.	13	0.385
142	A	6	6	1.	13	0.462
143	A	3	3	1.	14	0.214
144	A	9	6	1.	24	0.25
145	A	11	7	1.	26	0.269
146	F	0	0	N/A	0	N/A
147	F	0	0	N/A	0	N/A
148	F	0	0	N/A	0	N/A
149	F	0	0	N/A	0	N/A
150	A	2	1	1.	11	0.091
151	F	0	0	N/A	0	N/A
152	F	0	0	N/A	0	N/A
153	F	0	0	N/A	0	N/A
154	F	0	0	N/A	0	N/A
155	F	0	0	N/A	0	N/A
156	F	0	0	N/A	0	N/A
157	A	3	2	1.	13	0.154
158	F	0	0	N/A	0	N/A
159	F	0	0	N/A	0	N/A
160	F	0	0	N/A	0	N/A
161	F	0	0	N/A	0	N/A
162	F	0	0	N/A	0	N/A
163	F	0	0	N/A	0	N/A
164	F	0	0	N/A	0	N/A
165	F	0	0	N/A	0	N/A
166	F	0	0	N/A	0	N/A
167	F	0	0	N/A	0	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
168	F	0	0	N/A	0	N/A
169	F	0	0	N/A	0	N/A
170	F	0	0	N/A	0	N/A
171	F	0	0	N/A	0	N/A
172	F	0	0	N/A	0	N/A
173	F	0	0	N/A	0	N/A
174	F	0	0	N/A	0	N/A
175	F	0	0	N/A	0	N/A
176	A	2	1	1.	17	0.059
177	F	0	0	N/A	0	N/A
178	F	0	0	N/A	0	N/A
179	F	0	0	N/A	0	N/A
180	F	0	0	N/A	0	N/A
181	F	0	0	N/A	0	N/A
182	F	0	0	N/A	0	N/A
183	A	3	2	1.	19	0.105
184	F	0	0	N/A	0	N/A
185	F	0	0	N/A	0	N/A
186	A	3	2	1.	17	0.118
187	A	4	2	1.	17	0.118
188	A	4	2	1.	17	0.118
189	F	0	0	N/A	0	N/A
190	F	0	0	N/A	0	N/A
191	F	0	0	N/A	0	N/A
192	F	0	0	N/A	0	N/A
193	F	0	0	N/A	0	N/A
194	A	7	6	1.	19	0.316
195	A	7	6	1.	19	0.316
196	A	6	5	1.	19	0.263
197	A	6	5	1.	19	0.263
198	A	7	6	1.	19	0.316
199	A	7	6	1.	19	0.316
200	A	8	7	1.	23	0.304
201	A	7	6	1.	23	0.261
202	A	4	4	1.	21	0.19
203	A	8	5	1.	21	0.238
204	A	11	6	1.	23	0.261
205	A	8	7	1.	21	0.333
206	A	7	6	1.	16	0.375
207	A	7	6	1.	16	0.375
208	A	5	4	1.	16	0.25
209	A	3	3	1.	14	0.214
210	A	3	3	1.	14	0.214
211	A	5	4	1.	16	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	7	6	1.	16	0.375
213	A	7	6	1.	16	0.375
214	A	13	9	1.	10	0.9
215	A	11	8	1.	8	1.
216	A	5	5	1.	8	0.625
217	A	7	6	1.	10	0.6
218	A	14	9	1.	10	0.9
219	A	12	8	1.	8	1.
220	A	12	8	1.	8	1.
221	A	14	9	1.	10	0.9
222	A	23	9	1.	10	0.9
223	A	21	8	1.	8	1.
224	A	15	12	1.	8	1.5
225	A	17	13	1.	10	1.3
226	A	5	5	1.	14	0.357
227	A	7	6	1.	14	0.429
228	A	6	3	1.	18	0.167
229	A	5	3	1.	18	0.167
230	A	4	3	1.	16	0.188
231	A	4	3	1.	16	0.188
232	A	5	3	1.	18	0.167
233	A	6	3	1.	18	0.167
234	A	9	7	1.	25	0.28
235	A	8	7	1.	25	0.28
236	A	4	4	1.	25	0.16
237	A	4	4	1.	25	0.16
238	A	8	7	1.	25	0.28
239	A	9	7	1.	25	0.28
240	A	19	5	1.	9	0.556
241	A	13	5	1.	9	0.556
242	A	9	4	1.	7	0.571
243	A	0	0	0.	0	0.
244	A	19	5	1.	9	0.556
245	A	13	5	1.	9	0.556
246	A	9	4	1.	7	0.571
247	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int \tanh^6(a+bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^5(a+bx)}{5b}-\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh(a+bx)}{b}+x$$

```
[Out] x - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b) - Tanh[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0239146, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\tanh^5(a+bx)}{5b}-\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]^6,x]
```

```
[Out] x - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b) - Tanh[a + b*x]^5/(5*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^6(a+bx)\,dx &= -\frac{\tanh^5(a+bx)}{5b} + \int \tanh^4(a+bx)\,dx\\ &= -\frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b} + \int \tanh^2(a+bx)\,dx\\ &= -\frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b} + \int 1\,dx\\ &= x - \frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0188822, size = 53, normalized size = 1.23

$$-\frac{\tanh^5(a+bx)}{5b} - \frac{\tanh^3(a+bx)}{3b} + \frac{\tanh^{-1}(\tanh(a+bx))}{b} - \frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]^6,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b) - Tanh[a
 + b*x]^5/(5*b)
```

Maple [A] time = 0.005, size = 67, normalized size = 1.6

$$-\frac{(\tanh(bx+a))^5}{5b} - \frac{(\tanh(bx+a))^3}{3b} - \frac{\tanh(bx+a)}{b} - \frac{\ln(-1+\tanh(bx+a))}{2b} + \frac{\ln(1+\tanh(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(b*x+a)^6,x)
```

```
[Out] -1/5*tanh(b*x+a)^5/b-1/3*tanh(b*x+a)^3/b-tanh(b*x+a)/b-1/2/b*ln(-1+tanh(b*x
+a))+1/2*ln(1+tanh(b*x+a))/b
```

Maxima [B] time = 1.04313, size = 155, normalized size = 3.6

$$x + \frac{a}{b} - \frac{2\left(70\,e^{(-2\,bx-2\,a)} + 140\,e^{(-4\,bx-4\,a)} + 90\,e^{(-6\,bx-6\,a)} + 45\,e^{(-8\,bx-8\,a)} + 23\right)}{15\,b\left(5\,e^{(-2\,bx-2\,a)} + 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} + 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] x + a/b - 2/15*(70*e^(-2*b*x - 2*a) + 140*e^(-4*b*x - 4*a) + 90*e^(-6*b*x -
 6*a) + 45*e^(-8*b*x - 8*a) + 23)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4
*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

Fricas [B] time = 2.36573, size = 717, normalized size = 16.67

$$\frac{(15\,bx+23)\cosh(bx+a)^5 + 5\,(15\,bx+23)\cosh(bx+a)\sinh(bx+a)^4 - 23\,\sinh(bx+a)^5 + 5\,(15\,bx+23)\cosh(bx+ \ldots}{15\left(b\cosh(bx+a)^5 + 5\,b\,c \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/15*((15*b*x + 23)*cosh(b*x + a)^5 + 5*(15*b*x + 23)*cosh(b*x + a)*sinh(b*
x + a)^4 - 23*sinh(b*x + a)^5 + 5*(15*b*x + 23)*cosh(b*x + a)^3 - 5*(46*cos
h(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 5*(2*(15*b*x + 23)*cosh(b*x + a)^3 + 3*
(15*b*x + 23)*cosh(b*x + a))*sinh(b*x + a)^2 + 10*(15*b*x + 23)*cosh(b*x +
a) - 5*(23*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/(b*cos
h(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + 5*b*cosh(b*x + a)^3 + 5*
(2*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + 10*b*cosh(b*x +
 a))
```

Sympy [A] time = 0.653121, size = 39, normalized size = 0.91

$$\begin{cases} x - \frac{\tanh^{5}{(a+bx)}}{5b} - \frac{\tanh^{3}{(a+bx)}}{3b} - \frac{\tanh{(a+bx)}}{b} & \text{for } b \neq 0 \\ x \tanh^{6}{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)**6,x)
```

```
[Out] Piecewise((x - tanh(a + b*x)**5/(5*b) - tanh(a + b*x)**3/(3*b) - tanh(a + b
*x)/b, Ne(b, 0)), (x*tanh(a)**6, True))
```

Giac [A] time = 1.19926, size = 100, normalized size = 2.33

$$\frac{bx+a}{b} + \frac{2\left(45\,e^{(8\,bx+8\,a)} + 90\,e^{(6\,bx+6\,a)} + 140\,e^{(4\,bx+4\,a)} + 70\,e^{(2\,bx+2\,a)} + 23\right)}{15\,b\left(e^{(2\,bx+2\,a)} + 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] (b*x + a)/b + 2/15*(45*e^(8*b*x + 8*a) + 90*e^(6*b*x + 6*a) + 140*e^(4*b*x
+ 4*a) + 70*e^(2*b*x + 2*a) + 23)/(b*(e^(2*b*x + 2*a) + 1)^5)
```

3.2 $\int \tanh^5(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\tanh^4(a+bx)}{4b}-\frac{\tanh^2(a+bx)}{2b}+\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b) - Tanh[a + b*x]^4/(4*b)
```

Rubi [A] time = 0.0333051, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$-\frac{\tanh^4(a+bx)}{4b}-\frac{\tanh^2(a+bx)}{2b}+\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]^5,x]
```

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b) - Tanh[a + b*x]^4/(4*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^5(a+bx)\,dx &= -\frac{\tanh^4(a+bx)}{4b}+\int \tanh^3(a+bx)\,dx\\ &= -\frac{\tanh^2(a+bx)}{2b}-\frac{\tanh^4(a+bx)}{4b}+\int \tanh(a+bx)\,dx\\ &= \frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}-\frac{\tanh^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0864425, size = 37, normalized size = 0.88

$$\frac{-\tanh^4(a+bx)-2\tanh^2(a+bx)+4\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]^5,x]
```

```
[Out] (4*Log[Cosh[a + b*x]] - 2*Tanh[a + b*x]^2 - Tanh[a + b*x]^4)/(4*b)
```

Maple [A] time = 0.004, size = 56, normalized size = 1.3

$$-\frac{(\tanh(bx+a))^4}{4b}-\frac{(\tanh(bx+a))^2}{2b}-\frac{\ln(-1+\tanh(bx+a))}{2b}-\frac{\ln(1+\tanh(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(b*x+a)^5,x)
```

```
[Out] -1/4*tanh(b*x+a)^4/b-1/2*tanh(b*x+a)^2/b-1/2/b*ln(-1+tanh(b*x+a))-1/2*ln(1+
tanh(b*x+a))/b
```

Maxima [B] time = 1.54729, size = 138, normalized size = 3.29

$$x+\frac{a}{b}+\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{4\left(e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b\left(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-2*b*x - 2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-4*b*x - 4
*a) + e^(-6*b*x - 6*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) + 4*e^(
-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Fricas [B] time = 2.37979, size = 2642, normalized size = 62.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^8 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x
+ a)^8 + 4*(b*x - 1)*cosh(b*x + a)^6 + 4*(7*b*x*cosh(b*x + a)^2 + b*x - 1)*
sinh(b*x + a)^6 + 8*(7*b*x*cosh(b*x + a)^3 + 3*(b*x - 1)*cosh(b*x + a))*sin
h(b*x + a)^5 + 2*(3*b*x - 2)*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 +
30*(b*x - 1)*cosh(b*x + a)^2 + 3*b*x - 2)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b
*x + a)^5 + 10*(b*x - 1)*cosh(b*x + a)^3 + (3*b*x - 2)*cosh(b*x + a))*sinh(
b*x + a)^3 + 4*(b*x - 1)*cosh(b*x + a)^2 + 4*(7*b*x*cosh(b*x + a)^6 + 15*(b
*x - 1)*cosh(b*x + a)^4 + 3*(3*b*x - 2)*cosh(b*x + a)^2 + b*x - 1)*sinh(b*x
 + a)^2 + b*x - (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b
*x + a)^8 + 4*(7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 +
 8*(7*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x +
 a)^4 + 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4
*(7*cosh(b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a)^2 + 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cos
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(
b*x + a) - sinh(b*x + a))) + 8*(b*x*cosh(b*x + a)^7 + 3*(b*x - 1)*cosh(b*x
+ a)^5 + (3*b*x - 2)*cosh(b*x + a)^3 + (b*x - 1)*cosh(b*x + a))*sinh(b*x +
a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*x + a
```

```
)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 8
*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(b*x +
 a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)
^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sin
h(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 + 15*b*cosh(b*x
 + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a)^7 +
 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

Sympy [A] time = 0.47104, size = 42, normalized size = 1.

$$\begin{cases} x - \frac{\log(\tanh(a+bx)+1)}{b} - \frac{\tanh^4(a+bx)}{4b} - \frac{\tanh^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\tanh^5(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)**5,x)
```

```
[Out] Piecewise((x - log(tanh(a + b*x) + 1)/b - tanh(a + b*x)**4/(4*b) - tanh(a +
 b*x)**2/(2*b), Ne(b, 0)), (x*tanh(a)**5, True))
```

Giac [A] time = 1.18176, size = 99, normalized size = 2.36

$$-\frac{bx+a}{b} + \frac{\log\left(e^{(2bx+2a)}+1\right)}{b} + \frac{4\left(e^{(6bx+6a)}+e^{(4bx+4a)}+e^{(2bx+2a)}\right)}{b\left(e^{(2bx+2a)}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(e^(2*b*x + 2*a) + 1)/b + 4*(e^(6*b*x + 6*a) + e^(4*b*x +
 4*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) + 1)^4)
```

3.3 $\int \tanh^4(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh(a+bx)}{b}+x$$

```
[Out] x - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.015812, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]^4,x]
```

```
[Out] x - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^4(a+bx)\,dx &= -\frac{\tanh^3(a+bx)}{3b}+\int \tanh^2(a+bx)\,dx\\ &= -\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}+\int 1\,dx\\ &= x-\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0089919, size = 38, normalized size = 1.36

$$-\frac{\tanh^3(a+bx)}{3b}+\frac{\tanh^{-1}(\tanh(a+bx))}{b}-\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]^4,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Maple [B] time = 0.003, size = 54, normalized size = 1.9

$$-\frac{(\tanh(bx+a))^{3}}{3b}-\frac{\tanh(bx+a)}{b}-\frac{\ln(-1+\tanh(bx+a))}{2b}+\frac{\ln(1+\tanh(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(b*x+a)^4,x)
```

```
[Out] -1/3*tanh(b*x+a)^3/b-tanh(b*x+a)/b-1/2/b*ln(-1+tanh(b*x+a))+1/2*ln(1+tanh(b
*x+a))/b
```

Maxima [B] time = 1.05514, size = 96, normalized size = 3.43

$$x+\frac{a}{b}-\frac{4\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+2\right)}{3\,b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] x + a/b - 4/3*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + 2)/(b*(3*e^(-2*b*x
 - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

Fricas [B] time = 2.08288, size = 327, normalized size = 11.68

$$\frac{(3\,bx+4)\cosh(bx+a)^{3}+3\,(3\,bx+4)\cosh(bx+a)\sinh(bx+a)^{2}-12\,\cosh(bx+a)^{2}\sinh(bx+a)-4\,\sinh(bx+a)^{3}}{3\left(b\cosh(bx+a)^{3}+3\,b\cosh(bx+a)\sinh(bx+a)^{2}+3\,b\cosh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*x + 4)*cosh(b*x + a)^3 + 3*(3*b*x + 4)*cosh(b*x + a)*sinh(b*x + a
)^2 - 12*cosh(b*x + a)^2*sinh(b*x + a) - 4*sinh(b*x + a)^3 + 3*(3*b*x + 4)*
cosh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + 3*b
*cosh(b*x + a))
```

Sympy [A] time = 0.342054, size = 27, normalized size = 0.96

$$\begin{cases} x-\frac{\tanh^{3}(a+bx)}{3b}-\frac{\tanh(a+bx)}{b} & \text{for } b\neq 0\\ x\tanh^{4}(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)**4,x)
```

```
[Out] Piecewise((x - tanh(a + b*x)**3/(3*b) - tanh(a + b*x)/b, Ne(b, 0)), (x*tanh
(a)**4, True))
```

Giac [A] time = 1.19181, size = 70, normalized size = 2.5

$$\frac{bx+a}{b} + \frac{4\left(3\,e^{(4\,bx+4\,a)} + 3\,e^{(2\,bx+2\,a)} + 2\right)}{3\,b\left(e^{(2\,bx+2\,a)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] (b*x + a)/b + 4/3*(3*e^(4*b*x + 4*a) + 3*e^(2*b*x + 2*a) + 2)/(b*(e^(2*b*x
+ 2*a) + 1)^3)
```

3.4 $\int \tanh^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)

Rubi [A] time = 0.0171541, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$\frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Tanh[a + b*x]^3,x]

[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^3(a+bx)\,dx &= -\frac{\tanh^2(a+bx)}{2b}+\int \tanh(a+bx)\,dx\\ &= \frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0115058, size = 27, normalized size = 1.

$$\frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[a + b*x]^3,x]

[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)

Maple [A] time = 0.003, size = 43, normalized size = 1.6

$$-\frac{(\tanh(bx+a))^2}{2b}-\frac{\ln(-1+\tanh(bx+a))}{2b}-\frac{\ln(1+\tanh(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(b*x+a)^3,x)
```

```
[Out] -1/2*tanh(b*x+a)^2/b-1/2/b*ln(-1+tanh(b*x+a))-1/2*ln(1+tanh(b*x+a))/b
```

Maxima [B] time = 1.52498, size = 82, normalized size = 3.04

$$x+\frac{a}{b}+\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-2*b*x - 2*a) + 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x
- 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] time = 2.26226, size = 930, normalized size = 34.44

$$-\frac{bx\cosh(bx+a)^4+4\,bx\cosh(bx+a)\sinh(bx+a)^3+bx\sinh(bx+a)^4+2\,(bx-1)\cosh(bx+a)^2+2\left(3\,bx\cosh(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 + 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x - 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 + (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

Sympy [A] time = 0.242709, size = 31, normalized size = 1.15

$$\begin{cases} x-\frac{\log(\tanh(a+bx)+1)}{b}-\frac{\tanh^2(a+bx)}{2b} & \text{for } b\neq 0\\ x\tanh^3(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((x - log(tanh(a + b*x) + 1)/b - tanh(a + b*x)**2/(2*b), Ne(b, 0))
, (x*tanh(a)**3, True))
```

Giac [B] time = 1.1913, size = 73, normalized size = 2.7

$$-\frac{bx + a}{b} + \frac{\log\left(e^{(2\,bx+2\,a)} + 1\right)}{b} + \frac{2\,e^{(2\,bx+2\,a)}}{b\left(e^{(2\,bx+2\,a)} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(e^(2*b*x + 2*a) + 1)/b + 2*e^(2*b*x + 2*a)/(b*(e^(2*b*x
+ 2*a) + 1)^2)
```

3.5 $\int \tanh^2(a+bx)\,dx$

Optimal. Leaf size=13

$$x-\frac{\tanh(a+bx)}{b}$$

```
[Out] x - Tanh[a + b*x]/b
```

Rubi [A] time = 0.0085303, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$x-\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]^2,x]
```

```
[Out] x - Tanh[a + b*x]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^2(a+bx)\,dx &= -\frac{\tanh(a+bx)}{b}+\int 1\,dx\\ &= x-\frac{\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0070453, size = 23, normalized size = 1.77

$$\frac{\tanh^{-1}(\tanh(a+bx))}{b}-\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]^2,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b
```

Maple [B] time = 0.003, size = 41, normalized size = 3.2

$$-\frac{\tanh(bx+a)}{b}-\frac{\ln(-1+\tanh(bx+a))}{2b}+\frac{\ln(1+\tanh(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(b*x+a)^2,x)
```

```
[Out] -tanh(b*x+a)/b-1/2/b*ln(-1+tanh(b*x+a))+1/2*ln(1+tanh(b*x+a))/b
```

Maxima [A] time = 1.03595, size = 34, normalized size = 2.62

$$x + \frac{a}{b} - \frac{2}{b\left(e^{(-2\,bx-2\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x + a/b - 2/(b*(e^(-2*b*x - 2*a) + 1))
```

Fricas [B] time = 2.32793, size = 82, normalized size = 6.31

$$\frac{(bx+1)\cosh(bx+a) - \sinh(bx+a)}{b\cosh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + 1)*cosh(b*x + a) - sinh(b*x + a))/(b*cosh(b*x + a))
```

Sympy [A] time = 0.180094, size = 15, normalized size = 1.15

$$\begin{cases} x - \frac{\tanh(a+bx)}{b} & \text{for } b \neq 0 \\ x\tanh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)**2,x)
```

```
[Out] Piecewise((x - tanh(a + b*x)/b, Ne(b, 0)), (x*tanh(a)**2, True))
```

Giac [B] time = 1.17928, size = 38, normalized size = 2.92

$$\frac{bx+a}{b} + \frac{2}{b\left(e^{(2\,bx+2\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)/b + 2/(b*(e^(2*b*x + 2*a) + 1))
```

3.6 $\int \tanh(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] Log[Cosh[a + b*x]]/b
```

Rubi [A] time = 0.0057195, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x],x]

[Out] Log[Cosh[a + b*x]]/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] time = 0.0052227, size = 11, normalized size = 1.

$$\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x],x]

[Out] Log[Cosh[a + b*x]]/b
```

Maple [B] time = 0.003, size = 30, normalized size = 2.7

$$-\frac{\ln(-1+\tanh(bx+a))}{2b}-\frac{\ln(1+\tanh(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(b*x+a),x)
```

```
[Out] -1/2/b*ln(-1+tanh(b*x+a))-1/2*ln(1+tanh(b*x+a))/b
```

Maxima [A] time = 1.0177, size = 15, normalized size = 1.36

$$\frac{\log\left(\cosh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] log(cosh(b*x + a))/b
```

Fricas [B] time = 2.19036, size = 88, normalized size = 8.

$$-\frac{bx-\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x - log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [A] time = 0.151378, size = 17, normalized size = 1.55

$$\begin{cases} x-\frac{\log\left(\tanh\left(a+bx\right)+1\right)}{b} & \text{for}\ b\neq 0\\ x\tanh\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a),x)
```

```
[Out] Piecewise((x - log(tanh(a + b*x) + 1)/b, Ne(b, 0)), (x*tanh(a), True))
```

Giac [B] time = 1.22161, size = 36, normalized size = 3.27

$$-\frac{bx+a}{b}+\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(e^(2*b*x + 2*a) + 1)/b
```

3.7 $\int \coth(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] Log[Sinh[a + b*x]]/b
```

Rubi [A] time = 0.0059246, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}$$

Mathematica [A] time = 0.0094081, size = 19, normalized size = 1.73

$$\frac{\log(\tanh(a+bx))+\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x],x]
```

```
[Out] (Log[Cosh[a + b*x]] + Log[Tanh[a + b*x]])/b
```

Maple [B] time = 0.002, size = 30, normalized size = 2.7

$$-\frac{\ln(\coth(bx+a)-1)}{2b}-\frac{\ln(\coth(bx+a)+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a),x)
```

```
[Out] -1/2/b*ln(coth(b*x+a)-1)-1/2/b*ln(coth(b*x+a)+1)
```

Maxima [A] time = 1.02564, size = 15, normalized size = 1.36

$$\frac{\log(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sinh(b*x + a))/b
```

Fricas [B] time = 2.44702, size = 88, normalized size = 8.

$$-\frac{bx - \log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x - log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [A] time = 0.440701, size = 27, normalized size = 2.45

$$\begin{cases} x - \frac{\log(\tanh(a+bx)+1)}{b} + \frac{\log(\tanh(a+bx))}{b} & \text{for } b \neq 0 \\ x\coth(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a),x)
```

```
[Out] Piecewise((x - log(tanh(a + b*x) + 1)/b + log(tanh(a + b*x))/b, Ne(b, 0)),
(x*coth(a), True))
```

Giac [B] time = 1.18339, size = 38, normalized size = 3.45

$$-\frac{bx+a}{b} + \frac{\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(abs(e^(2*b*x + 2*a) - 1))/b
```

3.8 $\int \coth^2(a+bx)\,dx$

Optimal. Leaf size=13

$$x-\frac{\coth(a+bx)}{b}$$

```
[Out] x - Coth[a + b*x]/b
```

Rubi [A] time = 0.0081182, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$x-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2,x]
```

```
[Out] x - Coth[a + b*x]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\,dx &= -\frac{\coth(a+bx)}{b}+\int 1\,dx\\ &= x-\frac{\coth(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.0091073, size = 27, normalized size = 2.08

$$-\frac{\coth(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2,x]
```

```
[Out] -((Coth[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + b*x]^2])/b)
```

Maple [B] time = 0., size = 41, normalized size = 3.2

$$-\frac{\coth\left(bx+a\right)}{b}-\frac{\ln\left(\coth\left(bx+a\right)-1\right)}{2b}+\frac{\ln\left(\coth\left(bx+a\right)+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^2,x)
```

```
[Out] -coth(b*x+a)/b-1/2/b*ln(coth(b*x+a)-1)+1/2/b*ln(coth(b*x+a)+1)
```

Maxima [A] time = 1.04407, size = 34, normalized size = 2.62

$$x+\frac{a}{b}+\frac{2}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x + a/b + 2/(b*(e^(-2*b*x - 2*a) - 1))
```

Fricas [B] time = 2.27709, size = 82, normalized size = 6.31

$$\frac{(bx+1)\sinh\left(bx+a\right)-\cosh\left(bx+a\right)}{b\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + 1)*sinh(b*x + a) - cosh(b*x + a))/(b*sinh(b*x + a))
```

Sympy [A] time = 1.59867, size = 36, normalized size = 2.77

$$\begin{cases} \tilde{\infty}x & \text{for } a=\log\left(-e^{-bx}\right) \vee a=\log\left(e^{-bx}\right) \\ x\coth^{2}\left(a\right) & \text{for } b=0 \\ x-\frac{1}{b\tanh\left(a+bx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (x*coth(
a)**2, Eq(b, 0)), (x - 1/(b*tanh(a + b*x)), True))
```

Giac [B] time = 1.23787, size = 38, normalized size = 2.92

$$\frac{bx+a}{b}-\frac{2}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)/b - 2/(b*(e^(2*b*x + 2*a) - 1))
```

3.9 $\int \coth^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\sinh(a+bx))}{b}-\frac{\coth^2(a+bx)}{2b}$$

```
[Out] -Coth[a + b*x]^2/(2*b) + Log[Sinh[a + b*x]]/b
```

Rubi [A] time = 0.0175013, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$\frac{\log(\sinh(a+bx))}{b}-\frac{\coth^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^3,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) + Log[Sinh[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^3(a+bx)\,dx &= -\frac{\coth^2(a+bx)}{2b}+\int \coth(a+bx)\,dx\\ &= -\frac{\coth^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.068527, size = 34, normalized size = 1.26

$$-\frac{\coth^2(a+bx)-2\log(\tanh(a+bx))-2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^3,x]
```

```
[Out] -(Coth[a + b*x]^2 - 2*Log[Cosh[a + b*x]] - 2*Log[Tanh[a + b*x]])/(2*b)
```

Maple [A] time = 0.002, size = 43, normalized size = 1.6

$$-\frac{(\coth(bx+a))^2}{2b}-\frac{\ln(\coth(bx+a)-1)}{2b}-\frac{\ln(\coth(bx+a)+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^3,x)
```

```
[Out] -1/2*coth(b*x+a)^2/b-1/2/b*ln(coth(b*x+a)-1)-1/2/b*ln(coth(b*x+a)+1)
```

Maxima [B] time = 1.02922, size = 107, normalized size = 3.96

$$x+\frac{a}{b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-2*b*x -
 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] time = 2.38538, size = 930, normalized size = 34.44

$$-\frac{bx\cosh(bx+a)^4+4\,bx\cosh(bx+a)\sinh(bx+a)^3+bx\sinh(bx+a)^4-2\,(bx-1)\cosh(bx+a)^2+2\left(3\,bx\cosh(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x + 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 - (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

Sympy [A] time = 4.26156, size = 61, normalized size = 2.26

$$\begin{cases} x\coth^3(a) & \text{for } b=0 \\ \tilde{\infty}x & \text{for } a=\log\left(-e^{-bx}\right)\vee a=\log\left(e^{-bx}\right) \\ x-\frac{\log(\tanh(a+bx)+1)}{b}+\frac{\log(\tanh(a+bx))}{b}-\frac{1}{2b\tanh^2(a+bx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**3,x)
```

```
[Out] Piecewise((x*coth(a)**3, Eq(b, 0)), (zoo*x, Eq(a, log(exp(-b*x))) | Eq(a, l
og(-exp(-b*x)))), (x - log(tanh(a + b*x) + 1)/b + log(tanh(a + b*x))/b - 1/
(2*b*tanh(a + b*x)**2), True))
```

Giac [B] time = 1.22073, size = 74, normalized size = 2.74

$$-\frac{bx+a}{b}+\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}-\frac{2\,e^{(2\,bx+2\,a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(abs(e^(2*b*x + 2*a) - 1))/b - 2*e^(2*b*x + 2*a)/(b*(e^(2
*b*x + 2*a) - 1)^2)
```

3.10 $\int \coth^4(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\coth^3(a+bx)}{3b}-\frac{\coth(a+bx)}{b}+x$$

```
[Out] x - Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0161894, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\coth^3(a+bx)}{3b}-\frac{\coth(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^4,x]
```

```
[Out] x - Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \coth^4(a+bx)\,dx &= -\frac{\coth^3(a+bx)}{3b}+\int \coth^2(a+bx)\,dx\\ &= -\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\int 1\,dx\\ &= x-\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}\end{aligned}$$

Mathematica [C] time = 0.0098578, size = 31, normalized size = 1.11

$$-\frac{\coth^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^4,x]
```

```
[Out] -(Coth[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[a + b*x]^2])/(3*b)
```

Maple [B] time = 0.002, size = 54, normalized size = 1.9

$$-\frac{(\coth(bx+a))^3}{3b} - \frac{\coth(bx+a)}{b} - \frac{\ln(\coth(bx+a)-1)}{2b} + \frac{\ln(\coth(bx+a)+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^4,x)
```

```
[Out] -1/3*coth(b*x+a)^3/b-coth(b*x+a)/b-1/2/b*ln(coth(b*x+a)-1)+1/2/b*ln(coth(b*
x+a)+1)
```

Maxima [B] time = 1.04297, size = 96, normalized size = 3.43

$$x + \frac{a}{b} - \frac{4\left(3e^{(-2bx-2a)} - 3e^{(-4bx-4a)} - 2\right)}{3b\left(3e^{(-2bx-2a)} - 3e^{(-4bx-4a)} + e^{(-6bx-6a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] x + a/b - 4/3*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) - 2)/(b*(3*e^(-2*b*x
 - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) - 1))
```

Fricas [B] time = 2.11061, size = 286, normalized size = 10.21

$$\frac{(3bx+4)\sinh(bx+a)^3 - 4\cosh(bx+a)^3 - 12\cosh(bx+a)\sinh(bx+a)^2 + 3\left((3bx+4)\cosh(bx+a)^2 - 3bx - 4\right)\mathrm{si}}{3\left(b\sinh(bx+a)^3 + 3\left(b\cosh(bx+a)^2 - b\right)\sinh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*x + 4)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 - 12*cosh(b*x + a)*sin
h(b*x + a)^2 + 3*((3*b*x + 4)*cosh(b*x + a)^2 - 3*b*x - 4)*sinh(b*x + a))/(
b*sinh(b*x + a)^3 + 3*(b*cosh(b*x + a)^2 - b)*sinh(b*x + a))
```

Sympy [A] time = 9.25608, size = 49, normalized size = 1.75

$$\begin{cases} \tilde{\infty} x & \text{for } a = \log\left(-e^{-bx}\right) \vee a = \log\left(e^{-bx}\right) \\ x\coth^4(a) & \text{for } b = 0 \\ x - \frac{1}{b\tanh(a+bx)} - \frac{1}{3b\tanh^3(a+bx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**4,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (x*coth(
a)**4, Eq(b, 0)), (x - 1/(b*tanh(a + b*x)) - 1/(3*b*tanh(a + b*x)**3), True
```

```
))
```

Giac [A] time = 1.21794, size = 70, normalized size = 2.5

$$\frac{bx+a}{b} - \frac{4\left(3\,e^{(4\,bx+4\,a)} - 3\,e^{(2\,bx+2\,a)} + 2\right)}{3\,b\left(e^{(2\,bx+2\,a)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^4,x, algorithm="giac")
```

```
[Out] (b*x + a)/b - 4/3*(3*e^(4*b*x + 4*a) - 3*e^(2*b*x + 2*a) + 2)/(b*(e^(2*b*x
+ 2*a) - 1)^3)
```

3.11 $\int \coth^5(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\coth^4(a+bx)}{4b}-\frac{\coth^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] -Coth[a + b*x]^2/(2*b) - Coth[a + b*x]^4/(4*b) + Log[Sinh[a + b*x]]/b
```

Rubi [A] time = 0.0330967, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$-\frac{\coth^4(a+bx)}{4b}-\frac{\coth^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^5,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - Coth[a + b*x]^4/(4*b) + Log[Sinh[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^5(a+bx)\,dx &= -\frac{\coth^4(a+bx)}{4b}+\int \coth^3(a+bx)\,dx\\ &= -\frac{\coth^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\int \coth(a+bx)\,dx\\ &= -\frac{\coth^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{\log(\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.15924, size = 44, normalized size = 1.05

$$-\frac{\coth^4(a+bx)+2\coth^2(a+bx)-4\log(\tanh(a+bx))-4\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^5,x]
```

```
[Out] -(2*Coth[a + b*x]^2 + Coth[a + b*x]^4 - 4*Log[Cosh[a + b*x]] - 4*Log[Tanh[a
 + b*x]])/(4*b)
```

Maple [A] time = 0., size = 56, normalized size = 1.3

$$-\frac{(\coth(bx+a))^4}{4b} - \frac{(\coth(bx+a))^2}{2b} - \frac{\ln(\coth(bx+a)-1)}{2b} - \frac{\ln(\coth(bx+a)+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^5,x)
```

```
[Out] -1/4*coth(b*x+a)^4/b-1/2*coth(b*x+a)^2/b-1/2/b*ln(coth(b*x+a)-1)-1/2/b*ln(c
oth(b*x+a)+1)
```

Maxima [B] time = 1.04131, size = 165, normalized size = 3.93

$$x + \frac{a}{b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{4\left(e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)}\right)}{b\left(4\,e^{(-2\,bx-2\,a)} - 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} - e^{(-8\,bx-8\,a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 4*(e^(-2*b*x
 - 2*a) - e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4*e^(-2*b*x - 2*a) - 6*e^
(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) - 1))
```

Fricas [B] time = 2.39074, size = 2642, normalized size = 62.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^8 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x
+ a)^8 - 4*(b*x - 1)*cosh(b*x + a)^6 + 4*(7*b*x*cosh(b*x + a)^2 - b*x + 1)*
sinh(b*x + a)^6 + 8*(7*b*x*cosh(b*x + a)^3 - 3*(b*x - 1)*cosh(b*x + a))*sin
h(b*x + a)^5 + 2*(3*b*x - 2)*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 -
30*(b*x - 1)*cosh(b*x + a)^2 + 3*b*x - 2)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b
*x + a)^5 - 10*(b*x - 1)*cosh(b*x + a)^3 + (3*b*x - 2)*cosh(b*x + a))*sinh(
b*x + a)^3 - 4*(b*x - 1)*cosh(b*x + a)^2 + 4*(7*b*x*cosh(b*x + a)^6 - 15*(b
*x - 1)*cosh(b*x + a)^4 + 3*(3*b*x - 2)*cosh(b*x + a)^2 - b*x + 1)*sinh(b*x
 + a)^2 + b*x - (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b
*x + a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 +
 8*(7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x +
 a)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4
*(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x
+ a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cos
h(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(
b*x + a) - sinh(b*x + a))) + 8*(b*x*cosh(b*x + a)^7 - 3*(b*x - 1)*cosh(b*x
+ a)^5 + (3*b*x - 2)*cosh(b*x + a)^3 - (b*x - 1)*cosh(b*x + a))*sinh(b*x +
a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*x + a
```

```
)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 8
*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(b*x +
 a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)
^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sin
h(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 - 15*b*cosh(b*x
 + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a)^7 -
 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

Sympy [A] time = 19.7818, size = 75, normalized size = 1.79

$$\begin{cases} x\coth^5(a) & \text{for } b=0 \\ \tilde{\infty}x & \text{for } a=\log\left(-e^{-bx}\right) \vee a=\log\left(e^{-bx}\right) \\ x-\frac{\log(\tanh(a+bx)+1)}{b}+\frac{\log(\tanh(a+bx))}{b}-\frac{1}{2b\tanh^2(a+bx)}-\frac{1}{4b\tanh^4(a+bx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**5,x)
```

```
[Out] Piecewise((x*coth(a)**5, Eq(b, 0)), (zoo*x, Eq(a, log(exp(-b*x))) | Eq(a, l
og(-exp(-b*x)))), (x - log(tanh(a + b*x) + 1)/b + log(tanh(a + b*x))/b - 1/
(2*b*tanh(a + b*x)**2) - 1/(4*b*tanh(a + b*x)**4), True))
```

Giac [B] time = 1.20811, size = 103, normalized size = 2.45

$$-\frac{bx+a}{b}+\frac{\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{b}-\frac{4\left(e^{(6bx+6a)}-e^{(4bx+4a)}+e^{(2bx+2a)}\right)}{b\left(e^{(2bx+2a)}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -(b*x + a)/b + log(abs(e^(2*b*x + 2*a) - 1))/b - 4*(e^(6*b*x + 6*a) - e^(4*
b*x + 4*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) - 1)^4)
```

3.12 $\int \coth^6(a+bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\coth^5(a+bx)}{5b}-\frac{\coth^3(a+bx)}{3b}-\frac{\coth(a+bx)}{b}+x$$

```
[Out] x - Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b) - Coth[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0241105, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\coth^5(a+bx)}{5b}-\frac{\coth^3(a+bx)}{3b}-\frac{\coth(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^6,x]
```

```
[Out] x - Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b) - Coth[a + b*x]^5/(5*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \coth^6(a+bx)\,dx &= -\frac{\coth^5(a+bx)}{5b}+\int \coth^4(a+bx)\,dx\\ &= -\frac{\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}+\int \coth^2(a+bx)\,dx\\ &= -\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}+\int 1\,dx\\ &= x-\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}\end{aligned}$$

Mathematica [C] time = 0.0089742, size = 31, normalized size = 0.72

$$-\frac{\coth^5(a+bx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\tanh^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^6,x]
```

```
[Out] -(Coth[a + b*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[a + b*x]^2])/(5*b)
```

Maple [A] time = 0.002, size = 67, normalized size = 1.6

$$-\frac{(\coth(bx+a))^5}{5b} - \frac{(\coth(bx+a))^3}{3b} - \frac{\coth(bx+a)}{b} - \frac{\ln(\coth(bx+a)-1)}{2b} + \frac{\ln(\coth(bx+a)+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^6,x)
```

```
[Out] -1/5*coth(b*x+a)^5/b-1/3*coth(b*x+a)^3/b-coth(b*x+a)/b-1/2/b*ln(coth(b*x+a)
-1)+1/2/b*ln(coth(b*x+a)+1)
```

Maxima [B] time = 1.03757, size = 155, normalized size = 3.6

$$x + \frac{a}{b} - \frac{2\left(70\,e^{(-2\,bx-2\,a)} - 140\,e^{(-4\,bx-4\,a)} + 90\,e^{(-6\,bx-6\,a)} - 45\,e^{(-8\,bx-8\,a)} - 23\right)}{15\,b\left(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} - 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] x + a/b - 2/15*(70*e^(-2*b*x - 2*a) - 140*e^(-4*b*x - 4*a) + 90*e^(-6*b*x -
 6*a) - 45*e^(-8*b*x - 8*a) - 23)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4
*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1))
```

Fricas [B] time = 2.28793, size = 666, normalized size = 15.49

$$\frac{(15\,bx+23)\sinh(bx+a)^5 - 23\,\cosh(bx+a)^5 - 115\,\cosh(bx+a)\sinh(bx+a)^4 + 5\left(2\,(15\,bx+23)\cosh(bx+a)^2 - 15\right.}{15\left(b\sinh(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/15*((15*b*x + 23)*sinh(b*x + a)^5 - 23*cosh(b*x + a)^5 - 115*cosh(b*x + a
)*sinh(b*x + a)^4 + 5*(2*(15*b*x + 23)*cosh(b*x + a)^2 - 15*b*x - 23)*sinh(
b*x + a)^3 + 25*cosh(b*x + a)^3 - 5*(46*cosh(b*x + a)^3 - 15*cosh(b*x + a))
*sinh(b*x + a)^2 + 5*((15*b*x + 23)*cosh(b*x + a)^4 - 3*(15*b*x + 23)*cosh(
b*x + a)^2 + 30*b*x + 46)*sinh(b*x + a) - 50*cosh(b*x + a))/(b*sinh(b*x + a
)^5 + 5*(2*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + 5*(b*cosh(b*x + a)^4 -
3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [A] time = 44.8374, size = 63, normalized size = 1.47

$$\begin{cases} x\coth^6(a) & \text{for } b = 0 \\ \tilde{\infty} x & \text{for } a = \log\left(-e^{-bx}\right) \vee a = \log\left(e^{-bx}\right) \\ x - \frac{1}{b\tanh(a+bx)} - \frac{1}{3b\tanh^3(a+bx)} - \frac{1}{5b\tanh^5(a+bx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**6,x)
```

```
[Out] Piecewise((x*coth(a)**6, Eq(b, 0)), (zoo*x, Eq(a, log(exp(-b*x))) | Eq(a, l
og(-exp(-b*x)))), (x - 1/(b*tanh(a + b*x)) - 1/(3*b*tanh(a + b*x)**3) - 1/(
5*b*tanh(a + b*x)**5), True))
```

Giac [A] time = 1.16831, size = 100, normalized size = 2.33

$$\frac{bx+a}{b} - \frac{2\left(45\,e^{(8\,bx+8\,a)} - 90\,e^{(6\,bx+6\,a)} + 140\,e^{(4\,bx+4\,a)} - 70\,e^{(2\,bx+2\,a)} + 23\right)}{15\,b\left(e^{(2\,bx+2\,a)} - 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^6,x, algorithm="giac")
```

```
[Out] (b*x + a)/b - 2/15*(45*e^(8*b*x + 8*a) - 90*e^(6*b*x + 6*a) + 140*e^(4*b*x
+ 4*a) - 70*e^(2*b*x + 2*a) + 23)/(b*(e^(2*b*x + 2*a) - 1)^5)
```

3.13 $\int (b\tanh(c+dx))^{7/2}\,dx$

Optimal. Leaf size=97

$$\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}$$

```
[Out] (b^(7/2)*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d + (b^(7/2)*ArcTanh[Sqrt[b
*Tanh[c + d*x]]/Sqrt[b]])/d - (2*b^3*Sqrt[b*Tanh[c + d*x]])/d - (2*b*(b*Tan
h[c + d*x])^(5/2))/(5*d)
```

Rubi [A] time = 0.0698837, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3476, 329, 212, 206, 203}

$$\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tanh[c + d*x])^(7/2),x]
```

```
[Out] (b^(7/2)*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d + (b^(7/2)*ArcTanh[Sqrt[b
*Tanh[c + d*x]]/Sqrt[b]])/d - (2*b^3*Sqrt[b*Tanh[c + d*x]])/d - (2*b*(b*Tan
h[c + d*x])^(5/2))/(5*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (b\tanh(c+dx))^{7/2}\,dx &= -\frac{2b(b\tanh(c+dx))^{5/2}}{5d}+b^2\int (b\tanh(c+dx))^{3/2}\,dx\\ &= -\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}+b^4\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx\\ &= -\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}-\frac{b^5\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx,x,b\tanh(c+d\right.}{d}\\ &= -\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}-\frac{(2b^5)\operatorname{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\tanh(c+\right.}{d}\\ &= -\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}+\frac{b^4\operatorname{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{d}\\ &= \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}-\frac{2b(b\tanh}{5}\end{aligned}$$

Mathematica [A] time = 0.231609, size = 83, normalized size = 0.86

$$\frac{b^3\sqrt{b\tanh(c+dx)}\left(-2\tanh^{\frac{5}{2}}(c+dx)+5\tanh^{-1}\left(\sqrt{\tanh(c+dx)}\right)-10\sqrt{\tanh(c+dx)}+5\tan^{-1}\left(\sqrt{\tanh(c+dx)}\right)\right)}{5d\sqrt{\tanh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tanh[c + d*x])^(7/2),x]
```

```
[Out] (b^3*Sqrt[b*Tanh[c + d*x]]*(5*ArcTan[Sqrt[Tanh[c + d*x]]] + 5*ArcTanh[Sqrt[
Tanh[c + d*x]]] - 10*Sqrt[Tanh[c + d*x]] - 2*Tanh[c + d*x]^(5/2)))/(5*d*Sqr
t[Tanh[c + d*x]])
```

Maple [A] time = 0.032, size = 80, normalized size = 0.8

$$\frac{1}{d}b^{\frac{7}{2}}\arctan\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)+\frac{1}{d}b^{\frac{7}{2}}\mathrm{Artanh}\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)-2\,\frac{b^3\sqrt{b\tanh(dx+c)}}{d}-\frac{2\,b}{5\,d}(b\tanh(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(d*x+c))^(7/2),x)
```

```
[Out] b^(7/2)*arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*tanh(d*x
+c))^(1/2)/b^(1/2))/d-2*b^3*(b*tanh(d*x+c))^(1/2)/d-2/5*b*(b*tanh(d*x+c))^(
5/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tanh(dx+c))^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(d*x + c))^(7/2), x)
```

Fricas [B] time = 2.67289, size = 4136, normalized size = 42.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3
*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 +
 b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - 5*(b^3*
cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4
 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x +
 c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b)
*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x
 + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x + c)^4
+ 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*c
osh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(3*b^3*cosh(d*x + c)^
4 + 12*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 + 4*b^3*co
sh(d*x + c)^2 + 3*b^3 + 2*(9*b^3*cosh(d*x + c)^2 + 2*b^3)*sinh(d*x + c)^2 +
 4*(3*b^3*cosh(d*x + c)^3 + 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh
(d*x + c)/cosh(d*x + c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
 c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 +
d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)
+ d), -1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3
+ b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c
)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*si
nh(d*x + c))*sqrt(b)*arctan(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*
cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b))
 - 5*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(
d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 + b^3)*
sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c)
)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b
*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + s
inh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^2
+ 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sinh(
d*x + c)/cosh(d*x + c)) - b) + 16*(3*b^3*cosh(d*x + c)^4 + 12*b^3*cosh(d*x
+ c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 + 4*b^3*cosh(d*x + c)^2 + 3*b^
3 + 2*(9*b^3*cosh(d*x + c)^2 + 2*b^3)*sinh(d*x + c)^2 + 4*(3*b^3*cosh(d*x +
```

```
c)^3 + 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*x + c)/cosh(d*x +
c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x +
c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 +
4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tanh(c+dx))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*tanh(c + d*x))**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tanh(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*tanh(d*x + c))^(7/2), x)
```

3.14 $\int (b\tanh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=78

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\tanh(c+dx))^{3/2}}{3d}$$

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d) + (b^(5/2)*ArcTanh[Sqr
t[b*Tanh[c + d*x]]/Sqrt[b]])/d - (2*b*(b*Tanh[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.0487595, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3476, 329, 298, 203, 206}

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\tanh(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tanh[c + d*x])^(5/2),x]
```

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d) + (b^(5/2)*ArcTanh[Sqr
t[b*Tanh[c + d*x]]/Sqrt[b]])/d - (2*b*(b*Tanh[c + d*x])^(3/2))/(3*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (b\tanh(c+dx))^{5/2}\,dx &= -\frac{2b(b\tanh(c+dx))^{3/2}}{3d}+b^2\int \sqrt{b\tanh(c+dx)}\,dx \\
&= -\frac{2b(b\tanh(c+dx))^{3/2}}{3d}-\frac{b^3\,\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx,x,b\tanh(c+dx)\right)}{d} \\
&= -\frac{2b(b\tanh(c+dx))^{3/2}}{3d}-\frac{\left(2b^3\right)\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{d} \\
&= -\frac{2b(b\tanh(c+dx))^{3/2}}{3d}+\frac{b^3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{d}-\frac{b^3\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx
\end{aligned}
$$

$$
= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\tanh(c+dx))^{3/2}}{3d}
$$

Mathematica [A] time = 0.205455, size = 68, normalized size = 0.87

$$
-\frac{(b\tanh(c+dx))^{5/2}\left(2\tanh^{\frac{3}{2}}(c+dx)-3\tanh^{-1}\left(\sqrt{\tanh(c+dx)}\right)+3\tan^{-1}\left(\sqrt{\tanh(c+dx)}\right)\right)}{3d\tanh^{\frac{5}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tanh[c + d*x])^(5/2),x]
```

```
[Out] -((b*Tanh[c + d*x])^(5/2)*(3*ArcTan[Sqrt[Tanh[c + d*x]]] - 3*ArcTanh[Sqrt[T
anh[c + d*x]]] + 2*Tanh[c + d*x]^(3/2)))/(3*d*Tanh[c + d*x]^(5/2))
```

Maple [A] time = 0.018, size = 63, normalized size = 0.8

$$
-\frac{1}{d}b^{\frac{5}{2}}\arctan\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)+\frac{1}{d}b^{\frac{5}{2}}\text{Artanh}\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)-\frac{2b}{3d}(b\tanh(dx+c))^{\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(d*x+c))^(5/2),x)
```

```
[Out] -b^(5/2)*arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*tanh(d*
x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*tanh(d*x+c))^(3/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int (b\tanh(dx+c))^{\frac{5}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(d*x + c))^(5/2), x)
```

Fricas [B] time = 2.55349, size = 2653, normalized size = 34.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*si
nh(d*x + c)^2 + b^2)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/
(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 -
b)) - 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 + b^2)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*s
inh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c
)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4))
 + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*
x + c)^2 - b^2)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)))/(d*cosh(d*x + c)^2 + 2
*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d), 1/12*(6*(b^2*cosh(
d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)
*sqrt(b)*arctan(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + 3*(b^2*co
sh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b
^2)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 1
2*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2
*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 +
 sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^
2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sin
h(d*x + c)/cosh(d*x + c)) - b) - 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x +
c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(b*sinh(d*x + c)/cosh(d*x
 + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x +
 c)^2 + d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tanh (c + dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*tanh(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tanh(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tanh(d*x + c))^(5/2), x)
```

3.15 $\int (b\tanh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\tanh(c+dx)}}{d}$$

```
[Out] (b^(3/2)*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b
*Tanh[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Tanh[c + d*x]])/d
```

Rubi [A] time = 0.0505646, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3476, 329, 212, 206, 203}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\tanh(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tanh[c + d*x])^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b
*Tanh[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Tanh[c + d*x]])/d
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (b\tanh(c+dx))^{3/2}\,dx &= -\frac{2b\sqrt{b\tanh(c+dx)}}{d}+b^2\int\frac{1}{\sqrt{b\tanh(c+dx)}}\,dx\\
&= -\frac{2b\sqrt{b\tanh(c+dx)}}{d}-\frac{b^3\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}(-b^2+x^2)}\,dx,x,b\tanh(c+dx)\right)}{d}\\
&= -\frac{2b\sqrt{b\tanh(c+dx)}}{d}-\frac{(2b^3)\operatorname{Subst}\left(\int\frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{d}\\
&= -\frac{2b\sqrt{b\tanh(c+dx)}}{d}+\frac{b^2\operatorname{Subst}\left(\int\frac{1}{b-x^2}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{d}+\frac{b^2\operatorname{Subst}\left(\int\frac{1}{b+x^2}\,dx,x\right.}{d}\\
&= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\tanh(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.0848197, size = 61, normalized size = 0.81

$$\frac{(b\tanh(c+dx))^{3/2}\left(\tanh^{-1}\left(\sqrt{\tanh(c+dx)}\right)-2\sqrt{\tanh(c+dx)}+\tan^{-1}\left(\sqrt{\tanh(c+dx)}\right)\right)}{d\tanh^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tanh[c + d*x])^(3/2),x]
```

```
[Out] ((ArcTan[Sqrt[Tanh[c + d*x]]] + ArcTanh[Sqrt[Tanh[c + d*x]]] - 2*Sqrt[Tanh[
c + d*x]])*(b*Tanh[c + d*x])^(3/2))/(d*Tanh[c + d*x]^(3/2))
```

Maple [A] time = 0.017, size = 62, normalized size = 0.8

$$\frac{1}{d}b^{\frac{3}{2}}\arctan\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)+\frac{1}{d}b^{\frac{3}{2}}\operatorname{Artanh}\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)-2\frac{b\sqrt{b\tanh(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(d*x+c))^(3/2),x)
```

```
[Out] b^(3/2)*arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*tanh(d*x
+c))^(1/2)/b^(1/2))/d-2*b*(b*tanh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(b\tanh(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(d*x + c))^(3/2), x)
```

Fricas [B] time = 2.58247, size = 1755, normalized size = 23.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*b*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - sqrt
(-b)*b*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*b*sqrt(b*sinh(d*
x + c)/cosh(d*x + c)))/d, -1/4*(2*b^(3/2)*arctan(sqrt(b)*sqrt(b*sinh(d*x +
c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)) - b^(3/2)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)
^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)
*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c
)^2 + cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
)*sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - b) + 8*b*sqrt(b*sinh(d*x +
c)/cosh(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tanh (c + dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*tanh(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tanh (dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tanh(d*x + c))^(3/2), x)
```

3.16 $\int \sqrt{b\tanh(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}$$

```
[Out] -((Sqrt[b]*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d) + (Sqrt[b]*ArcTanh[Sqr
t[b*Tanh[c + d*x]]/Sqrt[b]])/d
```

Rubi [A] time = 0.0319265, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3476, 329, 298, 203, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tanh[c + d*x]],x]
```


```
[Out] -((Sqrt[b]*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d) + (Sqrt[b]*ArcTanh[Sqr
t[b*Tanh[c + d*x]]/Sqrt[b]])/d
```


Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{b\tanh(c+dx)}\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\tanh(c+dx)\right)}{d} \\
&= -\frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d} \\
&= \frac{b\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d} \\
&= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0380801, size = 51, normalized size = 0.88

$$
\frac{\sqrt{b\tanh(c+dx)}\left(\tanh^{-1}\left(\sqrt{\tanh(c+dx)}\right)-\tan^{-1}\left(\sqrt{\tanh(c+dx)}\right)\right)}{d\sqrt{\tanh(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tanh[c + d*x]],x]
```

```
[Out] ((-ArcTan[Sqrt[Tanh[c + d*x]]] + ArcTanh[Sqrt[Tanh[c + d*x]]])*Sqrt[b*Tanh[
c + d*x]])/(d*Sqrt[Tanh[c + d*x]])
```

Maple [A] time = 0.025, size = 47, normalized size = 0.8

$$
-\frac{1}{d}\arctan\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)\sqrt{b}+\frac{1}{d}\text{Artanh}\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)\sqrt{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(d*x+c))^(1/2),x)
```

```
[Out] -arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+arctanh((b*tanh(d*x+c))^(1
/2)/b^(1/2))*b^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \sqrt{b\tanh(dx+c)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(d*x + c)), x)
```

Fricas [B] time = 2.82232, size = 1635, normalized size = 28.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/d, 1/4*(2*sqrt(b)*arcta
n(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh
(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + sqrt(b)*log(2*b*cosh(d*
x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cos
h(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(
d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 +
cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) -
 b))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tanh{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tanh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tanh\left(dx + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tanh(d*x + c)), x)
```

3.17 $\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}+\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

```
[Out] ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Tanh[c +
 d*x]]/Sqrt[b]]/(Sqrt[b]*d)
```

Rubi [A] time = 0.0315493, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3476, 329, 212, 206, 203}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}+\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tanh[c + d*x]],x]
```

```
[Out] ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Tanh[c +
 d*x]]/Sqrt[b]]/(Sqrt[b]*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-b^2+x^2\right)}\,dx, x, b\tanh(c+dx)\right)}{d}$$
$$= -\frac{(2b)\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

Mathematica [A] time = 0.0336064, size = 49, normalized size = 0.86

$$\frac{\sqrt{\tanh(c+dx)}\left(\tanh^{-1}\left(\sqrt{\tanh(c+dx)}\right)+\tan^{-1}\left(\sqrt{\tanh(c+dx)}\right)\right)}{d\sqrt{b\tanh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tanh[c + d*x]],x]
```

```
[Out] ((ArcTan[Sqrt[Tanh[c + d*x]]] + ArcTanh[Sqrt[Tanh[c + d*x]]])*Sqrt[Tanh[c +
 d*x]])/(d*Sqrt[b*Tanh[c + d*x]])
```

Maple [A] time = 0.033, size = 46, normalized size = 0.8

$$\frac{1}{d}\arctan\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)\frac{1}{\sqrt{b}}+\frac{1}{d}\text{Artanh}\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tanh(d*x+c))^(1/2),x)
```

```
[Out] arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*tanh(d*x+c))^(1/
2)/b^(1/2))/d/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tanh(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*tanh(d*x + c)), x)
```

Fricas [B] time = 2.68511, size = 1647, normalized size = 28.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/(b*d), -1/4*(2*sqrt(b)*
arctan(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - sqrt(b)*log(2*b*co
sh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*si
nh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2
*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*
cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)
^3 + cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x +
c)) - b))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \tanh{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tanh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \tanh{(dx + c)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tanh(d*x + c)), x)
```

3.18 $\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\tanh(c+dx)}}$$

```
[Out] -(ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(b^(3/2)*d)) + ArcTanh[Sqrt[b*Tanh[
c + d*x]]/Sqrt[b]]/(b^(3/2)*d) - 2/(b*d*Sqrt[b*Tanh[c + d*x]])
```

Rubi [A] time = 0.048383, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3474, 3476, 329, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\tanh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tanh[c + d*x])^(-3/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(b^(3/2)*d)) + ArcTanh[Sqrt[b*Tanh[
c + d*x]]/Sqrt[b]]/(b^(3/2)*d) - 2/(b*d*Sqrt[b*Tanh[c + d*x]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx &= -\frac{2}{bd\sqrt{b\tanh(c+dx)}}+\frac{\int\sqrt{b\tanh(c+dx)}\,dx}{b^2}\\ &= -\frac{2}{bd\sqrt{b\tanh(c+dx)}}-\frac{\text{Subst}\left(\int\frac{\sqrt{x}}{-b^2+x^2}\,dx,x,b\tanh(c+dx)\right)}{bd}\\ &= -\frac{2}{bd\sqrt{b\tanh(c+dx)}}-\frac{2\,\text{Subst}\left(\int\frac{x^2}{-b^2+x^4}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{bd}\\ &= -\frac{2}{bd\sqrt{b\tanh(c+dx)}}+\frac{\text{Subst}\left(\int\frac{1}{b-x^2}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{bd}-\frac{\text{Subst}\left(\int\frac{1}{b+x^2}\,dx,x,\sqrt{b\tan}\right.}{bd}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\tanh(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.0736574, size = 36, normalized size = 0.46

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\tanh^2(c+dx)\right)}{bd\sqrt{b\tanh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tanh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Tanh[c + d*x]^2])/(b*d*Sqrt[b*Tanh[c +
d*x]])
```

Maple [A] time = 0.019, size = 65, normalized size = 0.8

$$-\frac{1}{d}\arctan\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{3}{2}}+\frac{1}{d}\text{Artanh}\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{3}{2}}-2\frac{1}{bd\sqrt{b\tanh(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tanh(d*x+c))^(3/2),x)
```

```
[Out] -arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*tanh(d*x+c))^(1
/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*tanh(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\tanh(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(d*x + c))^(-3/2), x)
```

Fricas [B] time = 2.70914, size = 2574, normalized size = 33.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 - 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1
)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b*sinh(d*x + c)/cosh(d*x
 + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*
d*sinh(d*x + c)^2 - b^2*d), 1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*sinh(d*x + c)
/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 - b)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*si
nh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh
(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*si
nh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 +
 cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))*sqr
t(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - b) - 8*(cosh(d*x + c)^2 + 2*cosh
(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b*sinh(d*x + c)/cosh(d*
x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2
*d*sinh(d*x + c)^2 - b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\tanh(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*tanh(c + d*x))**(-3/2), x)
```

Giac [B] time = 1.28834, size = 207, normalized size = 2.65

$$\frac{\frac{\pi+\log(|b|)+8}{\sqrt{b}d} - \frac{4\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}d} - \frac{2\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)}{\sqrt{b}d} + \frac{16}{\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}-\sqrt{b}\right)d}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + log(abs(b)) + 8)/(sqrt(b)*d) - 4*arctan(-(sqrt(b)*e^(2*d*x + 2*c
) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt(b))/(sqrt(b)*d) - 2*log(abs(-sqrt(b)*
e^(2*d*x + 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))/(sqrt(b)*d) + 16/((sqrt(b)*
e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b))*d))/b
```

3.19 $\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\tanh(c+dx))^{3/2}}$$

```
[Out] ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Tanh[c +
 d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Tanh[c + d*x])^(3/2))
```

Rubi [A] time = 0.0487133, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3474, 3476, 329, 212, 206, 203}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\tanh(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tanh[c + d*x])^(-5/2),x]
```

```
[Out] ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Tanh[c +
 d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Tanh[c + d*x])^(3/2))
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx &= -\frac{2}{3bd(b\tanh(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx}{b^2}\\ &= -\frac{2}{3bd(b\tanh(c+dx))^{3/2}}-\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx,x,b\tanh(c+dx)\right)}{bd}\\ &= -\frac{2}{3bd(b\tanh(c+dx))^{3/2}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{bd}\\ &= -\frac{2}{3bd(b\tanh(c+dx))^{3/2}}+\frac{\mathrm{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{b^2d}+\frac{\mathrm{Subst}\left(\int \frac{1}{b+x^2}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{b^2d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\tanh(c+dx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0768269, size = 38, normalized size = 0.48

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\tanh^2(c+dx)\right)}{3bd(b\tanh(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tanh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Tanh[c + d*x]^2])/(3*b*d*(b*Tanh[c + d*
x])^(3/2))
```

Maple [A] time = 0.02, size = 64, normalized size = 0.8

$$\frac{1}{d}\arctan\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{5}{2}}+\frac{1}{d}\mathrm{Artanh}\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{5}{2}}-\frac{2}{3\,bd}(b\tanh(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tanh(d*x+c))^(5/2),x)
```

```
[Out] arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*tanh(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*tanh(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \tanh (dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(d*x + c))^(-5/2), x)
```

Fricas [B] time = 2.55463, size = 3934, normalized size = 49.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*
sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 - b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sin
h(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2
- 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1
)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*s
inh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x +
c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x +
c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d
*x + c))*sinh(d*x + c) + 1)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)))/(b^3*d*cos
h(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4 - 2*b^3*d*cosh(d*x + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 - b^3*d)*s
inh(d*x + c)^2 + 4*(b^3*d*cosh(d*x + c)^3 - b^3*d*cosh(d*x + c))*sinh(d*x +
 c)), -1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*
x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 +
4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(
b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - 3*(cosh(d*x + c)^4 + 4*cosh(d*
x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d
*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*
x + c) + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x
+ c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x +
c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d
*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sq
rt(b*sinh(d*x + c)/cosh(d*x + c)) - b) + 8*(cosh(d*x + c)^4 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x +
 c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x +
c) + 1)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)))/(b^3*d*cosh(d*x + c)^4 + 4*b^3
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4 - 2*b^3*d*cosh(d*x
 + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 - b^3*d)*sinh(d*x + c)^2 + 4*(
```

```
b^3*d*cosh(d*x + c)^3 - b^3*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tanh\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*tanh(c + d*x))**(-5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.20 $\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{b^3d\sqrt{b\tanh(c+dx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{5bd(b\tanh(c+dx))^{5/2}}$$

```
[Out] -(ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(b^(7/2)*d)) + ArcTanh[Sqrt[b*Tanh[
c + d*x]]/Sqrt[b]]/(b^(7/2)*d) - 2/(5*b*d*(b*Tanh[c + d*x])^(5/2)) - 2/(b^3
*d*Sqrt[b*Tanh[c + d*x]])
```

Rubi [A] time = 0.0671875, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3474, 3476, 329, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{b^3d\sqrt{b\tanh(c+dx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{5bd(b\tanh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tanh[c + d*x])^(-7/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(b^(7/2)*d)) + ArcTanh[Sqrt[b*Tanh[
c + d*x]]/Sqrt[b]]/(b^(7/2)*d) - 2/(5*b*d*(b*Tanh[c + d*x])^(5/2)) - 2/(b^3
*d*Sqrt[b*Tanh[c + d*x]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx &= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}} + \frac{\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx}{b^2} \\
&= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\tanh(c+dx)}} + \frac{\int \sqrt{b\tanh(c+dx)}\,dx}{b^4} \\
&= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\tanh(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\tanh(c+dx)\right)}{b^3d} \\
&= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\tanh(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{b^3d} \\
&= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\tanh(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{b^3d} - \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} - \frac{2}{5bd(b\tanh(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\tanh(c+}}
\end{aligned}$$

Mathematica [C] time = 0.110415, size = 38, normalized size = 0.38

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};\tanh^2(c+dx)\right)}{5bd(b\tanh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tanh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, Tanh[c + d*x]^2])/(5*b*d*(b*Tanh[c + d
*x])^(5/2))
```

Maple [A] time = 0.02, size = 83, normalized size = 0.8

$$-\frac{1}{d}\arctan\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{7}{2}} + \frac{1}{d}\text{Artanh}\left(\sqrt{b\tanh(dx+c)}\frac{1}{\sqrt{b}}\right)b^{-\frac{7}{2}} - 2\frac{1}{b^3d\sqrt{b\tanh(dx+c)}} - \frac{2}{5bd}(b\tanh(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tanh(d*x+c))^(7/2),x)
```

```
[Out] -arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*tanh(d*x+c))^(1
/2)/b^(1/2))/b^(7/2)/d-2/b^3/d/(b*tanh(d*x+c))^(1/2)-2/5/b/d/(b*tanh(d*x+c)
```

```
)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\tanh(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(d*x + c))^(-7/2), x)
```

Fricas [B] time = 2.69335, size = 5796, normalized size = 57.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x +
c)^6 + 3*(5*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - 3*cosh(d*x + c)^4 + 4*(5
*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4
- 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x
+ c)^5 - 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) - 1)*sqrt(-b)*arc
tan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqr
t(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + 5*(cosh(d*x + c)^6 + 6*cosh
(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 - 1)*sin
h(d*x + c)^4 - 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 - 3*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 - 6*cosh(d*x + c)^2 + 1)*sinh(d*x +
c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 - 2*cosh(d*x + c)^3 + cosh(d*
x + c))*sinh(d*x + c) - 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x
+ c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh
(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4)) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sin
h(d*x + c)^6 + (45*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - cosh(d*x + c)^4 +
 4*(15*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)
^4 - 6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d
*x + c)^5 - 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 3)*sqrt(b*si
nh(d*x + c)/cosh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*
sinh(d*x + c)^5 + b^4*d*sinh(d*x + c)^6 - 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d
*cosh(d*x + c)^2 - b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 - b^4*d)*sinh(d*x + c
)^4 + 4*(5*b^4*d*cosh(d*x + c)^3 - 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 3*(5*b^4*d*cosh(d*x + c)^4 - 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c
)^2 + 6*(b^4*d*cosh(d*x + c)^5 - 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x +
 c))*sinh(d*x + c)), 1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x +
 c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - 3*cos
h(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*
(5*cosh(d*x + c)^4 - 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x +
```

```
c)^2 + 6*(cosh(d*x + c)^5 - 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c
) - 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d
*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + 5*(
cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*
cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x +
c)^3 - 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 - 6*cosh(d*x
 + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 - 2*c
osh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) - 1)*sqrt(b)*log(2*b*cosh(d*x
 + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x
 + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh
(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d
*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + c
osh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) -
b) - 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sinh(d*x
+ c)^6 + (45*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - cosh(d*x + c)^4 + 4*(15
*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)^4 - 6
*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d*x + c
)^5 - 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 3)*sqrt(b*sinh(d*x
 + c)/cosh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*sinh(d
*x + c)^5 + b^4*d*sinh(d*x + c)^6 - 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d*cosh(
d*x + c)^2 - b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 - b^4*d)*sinh(d*x + c)^4 +
4*(5*b^4*d*cosh(d*x + c)^3 - 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*
b^4*d*cosh(d*x + c)^4 - 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 +
6*(b^4*d*cosh(d*x + c)^5 - 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x + c))*s
inh(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\tanh (c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*tanh(c + d*x))**(-7/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tanh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.21 $\int \sqrt[3]{\tanh(8x)}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right)+\frac{1}{32}\log\left(\tanh^{\frac{4}{3}}(8x)+\tanh^{\frac{2}{3}}(8x)+1\right)-\frac{1}{16}\sqrt{3}\tan^{-1}\left(\frac{2\tanh^{\frac{2}{3}}(8x)+1}{\sqrt{3}}\right)$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Tanh[8*x]^(2/3))/Sqrt[3]])/16 - Log[1 - Tanh[8*x]^(
2/3)]/16 + Log[1 + Tanh[8*x]^(2/3) + Tanh[8*x]^(4/3)]/32
```

Rubi [A] time = 0.0597553, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3476, 329, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right)+\frac{1}{32}\log\left(\tanh^{\frac{4}{3}}(8x)+\tanh^{\frac{2}{3}}(8x)+1\right)-\frac{1}{16}\sqrt{3}\tan^{-1}\left(\frac{2\tanh^{\frac{2}{3}}(8x)+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[8*x]^(1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Tanh[8*x]^(2/3))/Sqrt[3]])/16 - Log[1 - Tanh[8*x]^(
2/3)]/16 + Log[1 + Tanh[8*x]^(2/3) + Tanh[8*x]^(4/3)]/32
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{\tanh(8x)}\,dx &= -\left(\frac{1}{8}\operatorname{Subst}\left(\int \frac{\sqrt[3]{x}}{-1+x^2}\,dx, x, \tanh(8x)\right)\right)\\
&= -\left(\frac{3}{8}\operatorname{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \sqrt[3]{\tanh(8x)}\right)\right)\\
&= -\left(\frac{3}{16}\operatorname{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \tanh^{\frac{2}{3}}(8x)\right)\right)\\
&= -\left(\frac{1}{16}\operatorname{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \tanh^{\frac{2}{3}}(8x)\right)\right) + \frac{1}{16}\operatorname{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \tanh^{\frac{2}{3}}(8x)\right)\\
&= -\frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right) + \frac{1}{32}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \tanh^{\frac{2}{3}}(8x)\right) - \frac{3}{32}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,d\right.\\
&= -\frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right) + \frac{1}{32}\log\left(1+\tanh^{\frac{2}{3}}(8x)+\tanh^{\frac{4}{3}}(8x)\right) + \frac{3}{16}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right.\\
&= -\frac{1}{16}\sqrt{3}\tan^{-1}\left(\frac{1+2\tanh^{\frac{2}{3}}(8x)}{\sqrt{3}}\right) - \frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right) + \frac{1}{32}\log\left(1+\tanh^{\frac{2}{3}}(8x)+\tanh^{\frac{4}{3}}(8x)\right)
\end{aligned}$$

Mathematica [C] time = 0.0258012, size = 26, normalized size = 0.38

$$\frac{3}{32}\tanh^{\frac{4}{3}}(8x)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\tanh^2(8x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[8*x]^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[2/3, 1, 5/3, Tanh[8*x]^2]*Tanh[8*x]^(4/3))/32
```

Maple [A] time = 0.023, size = 102, normalized size = 1.5

$$-\frac{1}{16}\ln\left(\sqrt[3]{\tanh\left(8x\right)}-1\right)+\frac{1}{32}\ln\left(\left(\tanh\left(8x\right)\right)^{\frac{2}{3}}+\sqrt[3]{\tanh\left(8x\right)}+1\right)+\frac{\sqrt{3}}{16}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[3]{\tanh\left(8x\right)}+1\right)\right)-\frac{1}{16}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(8*x)^(1/3),x)
```

```
[Out] -1/16*ln(tanh(8*x)^(1/3)-1)+1/32*ln(tanh(8*x)^(2/3)+tanh(8*x)^(1/3)+1)+1/16
*3^(1/2)*arctan(1/3*(2*tanh(8*x)^(1/3)+1)*3^(1/2))-1/16*ln(tanh(8*x)^(1/3)+
1)+1/32*ln(tanh(8*x)^(2/3)-tanh(8*x)^(1/3)+1)-1/16*3^(1/2)*arctan(1/3*(2*ta
nh(8*x)^(1/3)-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(8x\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(8*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tanh(8*x)^(1/3), x)
```

Fricas [B] time = 2.21668, size = 563, normalized size = 8.16

$$-\frac{1}{16}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{\sinh\left(8x\right)}{\cosh\left(8x\right)}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{16}\log\left(\left(\frac{\sinh\left(8x\right)}{\cosh\left(8x\right)}\right)^{\frac{2}{3}}-1\right)+\frac{1}{32}\log\left(\frac{\cosh\left(8x\right)^{2}+2\cosh\left(8x\right)\sinh}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(8*x)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(3)*arctan(2/3*sqrt(3)*(sinh(8*x)/cosh(8*x))^(2/3) + 1/3*sqrt(3))
 - 1/16*log((sinh(8*x)/cosh(8*x))^(2/3) - 1) + 1/32*log((cosh(8*x)^2 + 2*co
sh(8*x)*sinh(8*x) + sinh(8*x)^2 + (cosh(8*x)^2 + 2*cosh(8*x)*sinh(8*x) + si
nh(8*x)^2 + 1)*(sinh(8*x)/cosh(8*x))^(2/3) + (cosh(8*x)^2 + 2*cosh(8*x)*sin
h(8*x) + sinh(8*x)^2 - 1)*(sinh(8*x)/cosh(8*x))^(1/3) + 1)/(cosh(8*x)^2 + 2
*cosh(8*x)*sinh(8*x) + sinh(8*x)^2 + 1))
```

Sympy [A] time = 2.94131, size = 63, normalized size = 0.91

$$-\frac{\log\left(\tanh^{\frac{2}{3}}\left(8x\right)-1\right)}{16}+\frac{\log\left(\tanh^{\frac{4}{3}}\left(8x\right)+\tanh^{\frac{2}{3}}\left(8x\right)+1\right)}{32}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\left(\tanh^{\frac{2}{3}}\left(8x\right)+\frac{1}{2}\right)}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(8*x)**(1/3),x)
```

```
[Out] -log(tanh(8*x)**(2/3) - 1)/16 + log(tanh(8*x)**(4/3) + tanh(8*x)**(2/3) + 1
)/32 - sqrt(3)*atan(2*sqrt(3)*(tanh(8*x)**(2/3) + 1/2)/3)/16
```

Giac [B] time = 1.24303, size = 258, normalized size = 3.74

$$\frac{1}{16}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{e^{(16x)}-1}{e^{(16x)}+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{16}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{e^{(16x)}-1}{e^{(16x)}+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{32}\log\left(\left(\frac{e^{(16x)}-1}{e^{(16x)}+1}\right)^{\frac{2}{3}}+\left(\frac{e^{(16x)}\cdots}{e^{(16x)}\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(8*x)^(1/3),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(3)*arctan(1/3*sqrt(3)*(2*((e^(16*x) - 1)/(e^(16*x) + 1))^(1/3) +
1)) - 1/16*sqrt(3)*arctan(1/3*sqrt(3)*(2*((e^(16*x) - 1)/(e^(16*x) + 1))^(1
/3) - 1)) + 1/32*log(((e^(16*x) - 1)/(e^(16*x) + 1))^(2/3) + ((e^(16*x) - 1
)/(e^(16*x) + 1))^(1/3) + 1) + 1/32*log(((e^(16*x) - 1)/(e^(16*x) + 1))^(2/
3) - ((e^(16*x) - 1)/(e^(16*x) + 1))^(1/3) + 1) - 1/16*log(abs(((e^(16*x) -
 1)/(e^(16*x) + 1))^(1/3) + 1)) - 1/16*log(abs(((e^(16*x) - 1)/(e^(16*x) +
1))^(1/3) - 1))
```

3.22 $\int \tanh^n(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(a+bx)\right)}{b(n+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Tanh[a + b*x]^2]*Tanh[a + b*x]^
(1 + n))/(b*(1 + n))
```

Rubi [A] time = 0.0219585, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3476, 364}

$$\frac{\tanh^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Tanh[a + b*x]^2]*Tanh[a + b*x]^
(1 + n))/(b*(1 + n))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \tanh^n(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^n}{-1+x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\tanh^2(a+bx)\right)\tanh^{1+n}(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0447656, size = 45, normalized size = 1.05

$$\frac{\tanh^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;\tanh^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, Tanh[a + b*x]^2]*Tanh[a + b
*x]^(1 + n))/(b*(1 + n))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int (\tanh (bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(b*x+a)^n,x)
```

```
[Out] int(tanh(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh (bx + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(tanh(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\tanh (bx + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(tanh(b*x + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^n (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)**n,x)
```

```
[Out] Integral(tanh(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(tanh(b*x + a)^n, x)
```

3.23 $\int (b\tanh(c+dx))^n\,dx$

Optimal. Leaf size=48

$$\frac{(b\tanh(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(c+dx)\right)}{bd(n+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Tanh[c + d*x]^2]*(b*Tanh[c + d*
x])^(1 + n))/(b*d*(1 + n))
```

Rubi [A] time = 0.026378, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3476, 364}

$$\frac{(b\tanh(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\tanh^2(c+dx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tanh[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Tanh[c + d*x]^2]*(b*Tanh[c + d*
x])^(1 + n))/(b*d*(1 + n))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (b\tanh(c+dx))^n\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{x^n}{-b^2+x^2}\,dx,x,b\tanh(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\tanh^2(c+dx)\right)(b\tanh(c+dx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0415308, size = 51, normalized size = 1.06

$$\frac{\tanh(c+dx)(b\tanh(c+dx))^n\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;\tanh^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tanh[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, Tanh[c + d*x]^2]*Tanh[c + d
*x]*(b*Tanh[c + d*x])^n)/(d*(1 + n))
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int \left(b\tanh\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(d*x+c))^n,x)
```

```
[Out] int((b*tanh(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\tanh\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tanh(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh\left(c+dx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))**n,x)
```

```
[Out] Integral((b*tanh(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tanh (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tanh(d*x + c))^n, x)
```

3.24 $\int \left(a \tanh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=35

$$a \coth(x)\sqrt{a \tanh^2(x)} \log(\cosh(x)) - \frac{1}{2} a \tanh(x)\sqrt{a \tanh^2(x)}$$

```
[Out] a*Coth[x]*Log[Cosh[x]]*Sqrt[a*Tanh[x]^2] - (a*Tanh[x]*Sqrt[a*Tanh[x]^2])/2
```

Rubi [A] time = 0.020685, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 3475}

$$a \coth(x)\sqrt{a \tanh^2(x)} \log(\cosh(x)) - \frac{1}{2} a \tanh(x)\sqrt{a \tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Tanh[x]^2)^(3/2),x]

[Out] a*Coth[x]*Log[Cosh[x]]*Sqrt[a*Tanh[x]^2] - (a*Tanh[x]*Sqrt[a*Tanh[x]^2])/2
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a \tanh^2(x)\right)^{3/2} dx &= \left(a \coth(x)\sqrt{a \tanh^2(x)}\right) \int \tanh^3(x)\, dx \\
&= -\frac{1}{2} a \tanh(x)\sqrt{a \tanh^2(x)} + \left(a \coth(x)\sqrt{a \tanh^2(x)}\right) \int \tanh(x)\, dx \\
&= a \coth(x) \log(\cosh(x))\sqrt{a \tanh^2(x)} - \frac{1}{2} a \tanh(x)\sqrt{a \tanh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.0234227, size = 28, normalized size = 0.8

$$\frac{1}{2} a\sqrt{a \tanh^2(x)}(\text{csch}(x)\text{sech}(x) + 2\coth(x)\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a*(2*Coth[x]*Log[Cosh[x]] + Csch[x]*Sech[x])*Sqrt[a*Tanh[x]^2])/2
```

Maple [A] time = 0.025, size = 30, normalized size = 0.9

$$-\frac{(\tanh(x))^2+\ln(\tanh(x)-1)+\ln(1+\tanh(x))}{2\,(\tanh(x))^3}\left(a\,(\tanh(x))^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*tanh(x)^2)^(3/2),x)
```

```
[Out] -1/2*(a*tanh(x)^2)^(3/2)*(tanh(x)^2+ln(tanh(x)-1)+ln(1+tanh(x)))/tanh(x)^3
```

Maxima [A] time = 1.57375, size = 57, normalized size = 1.63

$$-a^{\frac{3}{2}}x-a^{\frac{3}{2}}\log\left(e^{(-2x)}+1\right)-\frac{2\,a^{\frac{3}{2}}e^{(-2x)}}{2\,e^{(-2x)}+e^{(-4x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -a^(3/2)*x - a^(3/2)*log(e^(-2*x) + 1) - 2*a^(3/2)*e^(-2*x)/(2*e^(-2*x) + e
^(-4*x) + 1)
```

Fricas [B] time = 2.42579, size = 1368, normalized size = 39.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(a*x*cosh(x)^4 + (a*x*e^(2*x) + a*x)*sinh(x)^4 + 4*(a*x*cosh(x)*e^(2*x) +
a*x*cosh(x))*sinh(x)^3 + 2*(a*x - a)*cosh(x)^2 + 2*(3*a*x*cosh(x)^2 + a*x +
 (3*a*x*cosh(x)^2 + a*x - a)*e^(2*x) - a)*sinh(x)^2 + a*x + (a*x*cosh(x)^4
+ 2*(a*x - a)*cosh(x)^2 + a*x)*e^(2*x) - (a*cosh(x)^4 + (a*e^(2*x) + a)*sin
h(x)^4 + 4*(a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(3
*a*cosh(x)^2 + (3*a*cosh(x)^2 + a)*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^4 +
2*a*cosh(x)^2 + a)*e^(2*x) + 4*(a*cosh(x)^3 + a*cosh(x) + (a*cosh(x)^3 + a*
cosh(x))*e^(2*x))*sinh(x) + a)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 4*(a*x*
cosh(x)^3 + (a*x - a)*cosh(x) + (a*x*cosh(x)^3 + (a*x - a)*cosh(x))*e^(2*x)
)*sinh(x))*sqrt((a*e^(4*x) - 2*a*e^(2*x) + a)/(e^(4*x) + 2*e^(2*x) + 1))/((
e^(2*x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^
3 - 2*(3*cosh(x)^2 - (3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 - 2*cosh(x)^2
 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) - 4*(cosh(x)^3 - (cosh(x)^3 + cosh
```

```
(x))*e^(2*x) + cosh(x))*sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\tanh^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((a*tanh(x)**2)**(3/2), x)
```

Giac [A] time = 1.20896, size = 70, normalized size = 2.

$$-\left(x\operatorname{sgn}\left(e^{(4x)}-1\right)-\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(e^{(4x)}-1\right)-\frac{2\,e^{(2x)}\operatorname{sgn}\left(e^{(4x)}-1\right)}{\left(e^{(2x)}+1\right)^2}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(x*sgn(e^(4*x) - 1) - log(e^(2*x) + 1)*sgn(e^(4*x) - 1) - 2*e^(2*x)*sgn(e^
(4*x) - 1)/(e^(2*x) + 1)^2)*a^(3/2)
```

3.25 $\int \sqrt{a \tanh^2(x)}\, dx$

Optimal. Leaf size=16

$$\coth(x)\sqrt{a \tanh^2(x)} \log(\cosh(x))$$

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[a*Tanh[x]^2]
```

Rubi [A] time = 0.0153141, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3658, 3475}

$$\coth(x)\sqrt{a \tanh^2(x)} \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Tanh[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[a*Tanh[x]^2]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a \tanh^2(x)}\, dx &= \left(\coth(x)\sqrt{a \tanh^2(x)}\right) \int \tanh(x)\, dx \\ &= \coth(x) \log(\cosh(x))\sqrt{a \tanh^2(x)} \end{aligned}$$

Mathematica [A] time = 0.0063562, size = 16, normalized size = 1.

$$\coth(x)\sqrt{a \tanh^2(x)} \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Tanh[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[a*Tanh[x]^2]
```

Maple [A] time = 0.033, size = 26, normalized size = 1.6

$$-\frac{\ln(\tanh(x)-1)+\ln(1+\tanh(x))}{2\tanh(x)}\sqrt{a(\tanh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*tanh(x)^2)^(1/2),x)
```

```
[Out] -1/2*(a*tanh(x)^2)^(1/2)*(ln(tanh(x)-1)+ln(1+tanh(x)))/tanh(x)
```

Maxima [A] time = 1.57406, size = 26, normalized size = 1.62

$$-\sqrt{a}x-\sqrt{a}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*x - sqrt(a)*log(e^(-2*x) + 1)
```

Fricas [B] time = 2.24951, size = 196, normalized size = 12.25

$$-\frac{\left(xe^{(2x)}-\left(e^{(2x)}+1\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+x\right)\sqrt{\frac{ae^{(4x)}-2ae^{(2x)}+a}{e^{(4x)}+2e^{(2x)}+1}}}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x*e^(2*x) - (e^(2*x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + x)*sqrt((a
*e^(4*x) - 2*a*e^(2*x) + a)/(e^(4*x) + 2*e^(2*x) + 1))/(e^(2*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\tanh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*tanh(x)**2), x)
```

Giac [B] time = 1.19594, size = 42, normalized size = 2.62

$$-\left(x\mathrm{sgn}\left(e^{(4x)}-1\right)-\log\left(e^{(2x)}+1\right)\mathrm{sgn}\left(e^{(4x)}-1\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(x*sgn(e^(4*x) - 1) - log(e^(2*x) + 1)*sgn(e^(4*x) - 1))*sqrt(a)
```

3.26 $\int \frac{1}{\sqrt{a\tanh^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{a\tanh^2(x)}}$$

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[a*Tanh[x]^2]
```

Rubi [A] time = 0.0154868, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3658, 3475}

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{a\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Tanh[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[a*Tanh[x]^2]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\tanh^2(x)}}\,dx &= \frac{\tanh(x)\int \coth(x)\,dx}{\sqrt{a\tanh^2(x)}}\\ &= \frac{\log(\sinh(x))\tanh(x)}{\sqrt{a\tanh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0077392, size = 16, normalized size = 1.

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{a\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Tanh[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[a*Tanh[x]^2]
```

Maple [A] time = 0.039, size = 29, normalized size = 1.8

$$-\frac{\tanh (x) (\ln (1+\tanh (x))-2 \ln (\tanh (x))+\ln (\tanh (x)-1))}{2} \frac{1}{\sqrt{a (\tanh (x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*tanh(x)^2)^(1/2),x)
```

```
[Out] -1/2*tanh(x)*(ln(1+tanh(x))-2*ln(tanh(x))+ln(tanh(x)-1))/(a*tanh(x)^2)^(1/2
)
```

Maxima [B] time = 1.59866, size = 42, normalized size = 2.62

$$-\frac{x}{\sqrt{a}}-\frac{\log \left(e^{(-x)}+1\right)}{\sqrt{a}}-\frac{\log \left(e^{(-x)}-1\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(a) - log(e^(-x) + 1)/sqrt(a) - log(e^(-x) - 1)/sqrt(a)
```

Fricas [B] time = 2.27023, size = 198, normalized size = 12.38

$$-\frac{\left(xe^{(2x)}-\left(e^{(2x)}+1\right)\log \left(\frac{2 \sinh(x)}{\cosh(x)-\sinh(x)}\right)+x\right)\sqrt{\frac{ae^{(4x)}-2ae^{(2x)}+a}{e^{(4x)}+2e^{(2x)}+1}}}{ae^{(2x)}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x*e^(2*x) - (e^(2*x) + 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) + x)*sqrt((a
*e^(4*x) - 2*a*e^(2*x) + a)/(e^(4*x) + 2*e^(2*x) + 1))/(a*e^(2*x) - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \tanh^2 (x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*tanh(x)**2), x)
```

Giac [B] time = 1.19243, size = 50, normalized size = 3.12

$$-\frac{x}{\sqrt{a}\,\mathrm{sgn}\left(e^{(4x)}-1\right)}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(e^{(4x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -x/(sqrt(a)*sgn(e^(4*x) - 1)) + log(abs(e^(2*x) - 1))/(sqrt(a)*sgn(e^(4*x)
- 1))
```

3.27 $\int \left(-\tanh^2(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{-\tanh^2(c+dx)}\tanh^3(c+dx)}{4d}-\frac{\sqrt{-\tanh^2(c+dx)}\tanh(c+dx)}{2d}+\frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d - (Tanh[c + d*x
]*Sqrt[-Tanh[c + d*x]^2])/(2*d) - (Tanh[c + d*x]^3*Sqrt[-Tanh[c + d*x]^2])/
(4*d)
```

Rubi [A] time = 0.0485525, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$-\frac{\sqrt{-\tanh^2(c+dx)}\tanh^3(c+dx)}{4d}-\frac{\sqrt{-\tanh^2(c+dx)}\tanh(c+dx)}{2d}+\frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(-Tanh[c + d*x]^2)^(5/2),x]
```


```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d - (Tanh[c + d*x
]*Sqrt[-Tanh[c + d*x]^2])/(2*d) - (Tanh[c + d*x]^3*Sqrt[-Tanh[c + d*x]^2])/
(4*d)
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(-\tanh^2(c+dx)\right)^{5/2} dx = \left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh^5(c+dx)\,dx$$
$$= -\frac{\tanh^3(c+dx)\sqrt{-\tanh^2(c+dx)}}{4d} + \left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh^3(c+dx$$
$$= -\frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d} - \frac{\tanh^3(c+dx)\sqrt{-\tanh^2(c+dx)}}{4d} + \left(\coth(c+dx)\sqrt{\right.$$
$$= \frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d} - \frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d} -$$

Mathematica [A] time = 0.29524, size = 56, normalized size = 0.64

$$\frac{\left(-\tanh^2(c+dx)\right)^{5/2}\coth(c+dx)\left(-2\coth^2(c+dx)+4\coth^4(c+dx)\log(\cosh(c+dx))-1\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Tanh[c + d*x]^2)^(5/2),x]
```

```
[Out] (Coth[c + d*x]*(-1 - 2*Coth[c + d*x]^2 + 4*Coth[c + d*x]^4*Log[Cosh[c + d*x
]])*(-Tanh[c + d*x]^2)^(5/2))/(4*d)
```

Maple [A] time = 0.026, size = 67, normalized size = 0.8

$$-\frac{(\tanh(dx+c))^4 + 2(\tanh(dx+c))^2 + 2\ln(\tanh(dx+c)-1) + 2\ln(\tanh(dx+c)+1)}{4d(\tanh(dx+c))^5}\left(-(\tanh(dx+c))^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-tanh(d*x+c)^2)^(5/2),x)
```

```
[Out] -1/4/d*(-tanh(d*x+c)^2)^(5/2)*(tanh(d*x+c)^4+2*tanh(d*x+c)^2+2*ln(tanh(d*x+
c)-1)+2*ln(tanh(d*x+c)+1))/tanh(d*x+c)^5
```

Maxima [C] time = 1.69352, size = 153, normalized size = 1.74

$$-\frac{i(dx+c)}{d} - \frac{i\log\left(e^{(-2dx-2c)}+1\right)}{d} - \frac{4ie^{(-2dx-2c)} + 4ie^{(-4dx-4c)} + 4ie^{(-6dx-6c)}}{d\left(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -I*(d*x + c)/d - I*log(e^(-2*d*x - 2*c) + 1)/d - (4*I*e^(-2*d*x - 2*c) + 4*
I*e^(-4*d*x - 4*c) + 4*I*e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4
*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))
```

Fricas [C] time = 2.33754, size = 467, normalized size = 5.31

$$\frac{-i\,dxe^{(8\,dx+8\,c)}-i\,dx+(-4i\,dx+4i)e^{(6\,dx+6\,c)}+(-6i\,dx+4i)e^{(4\,dx+4\,c)}+(-4i\,dx+4i)e^{(2\,dx+2\,c)}+\left(i\,e^{(8\,dx+8\,c)}+4i\,e^{(6\,dx+6\,c)}+\dots}{de^{(8\,dx+8\,c)}+4\,de^{(6\,dx+6\,c)}+6\,de^{(4\,dx+4\,c)}+4\,de^{(2\,dx+2\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (-I*d*x*e^(8*d*x + 8*c) - I*d*x + (-4*I*d*x + 4*I)*e^(6*d*x + 6*c) + (-6*I*
d*x + 4*I)*e^(4*d*x + 4*c) + (-4*I*d*x + 4*I)*e^(2*d*x + 2*c) + (I*e^(8*d*x
 + 8*c) + 4*I*e^(6*d*x + 6*c) + 6*I*e^(4*d*x + 4*c) + 4*I*e^(2*d*x + 2*c) +
 I)*log(e^(2*d*x + 2*c) + 1))/(d*e^(8*d*x + 8*c) + 4*d*e^(6*d*x + 6*c) + 6*
d*e^(4*d*x + 4*c) + 4*d*e^(2*d*x + 2*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\tanh^{2}{\left(c+dx\right)}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((-tanh(c + d*x)**2)**(5/2), x)
```

Giac [C] time = 1.3601, size = 192, normalized size = 2.18

$$\frac{i\,(dx+c)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)-i\,\log\left(e^{(2\,dx+2\,c)}+1\right)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)-\frac{4i\left(e^{(6\,dx+6\,c)}\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)+e^{(4\,dx+4\,c)}\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)\dots}{\left(e^{(2\,dx+2\,c)}+1\right)^{4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] (I*(d*x + c)*sgn(-e^(4*d*x + 4*c) + 1) - I*log(e^(2*d*x + 2*c) + 1)*sgn(-e^
(4*d*x + 4*c) + 1) - 4*I*(e^(6*d*x + 6*c)*sgn(-e^(4*d*x + 4*c) + 1) + e^(4*
d*x + 4*c)*sgn(-e^(4*d*x + 4*c) + 1) + e^(2*d*x + 2*c)*sgn(-e^(4*d*x + 4*c)
 + 1))/(e^(2*d*x + 2*c) + 1)^4)/d
```

3.28 $\int \left(-\tanh^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d}-\frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

```
[Out] -((Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d) + (Tanh[c +
d*x]*Sqrt[-Tanh[c + d*x]^2])/(2*d)
```

Rubi [A] time = 0.0322464, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d}-\frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(-Tanh[c + d*x]^2)^(3/2),x]
```

```
[Out] -((Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d) + (Tanh[c +
d*x]*Sqrt[-Tanh[c + d*x]^2])/(2*d)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(-\tanh^2(c+dx)\right)^{3/2}\,dx &= -\left(\left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh^3(c+dx)\,dx\right)\\ &= \frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d}-\left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh(c+dx)\,dx\\ &= -\frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d}+\frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d}\end{aligned}$$

Mathematica [A] time = 0.0941762, size = 46, normalized size = 0.77

$$\frac{\left(-\tanh^2(c+dx)\right)^{3/2}\coth(c+dx)\left(2\coth^2(c+dx)\log(\cosh(c+dx))-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Tanh[c + d*x]^2)^(3/2),x]
```

```
[Out] (Coth[c + d*x]*(-1 + 2*Coth[c + d*x]^2*Log[Cosh[c + d*x]])*(-Tanh[c + d*x]^
2)^(3/2))/(2*d)
```

Maple [A] time = 0.015, size = 53, normalized size = 0.9

$$-\frac{(\tanh(dx+c))^2+\ln(\tanh(dx+c)-1)+\ln(\tanh(dx+c)+1)}{2d(\tanh(dx+c))^3}\left(-(\tanh(dx+c))^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-tanh(d*x+c)^2)^(3/2),x)
```

```
[Out] -1/2/d*(-tanh(d*x+c)^2)^(3/2)*(tanh(d*x+c)^2+ln(tanh(d*x+c)-1)+ln(tanh(d*x+
c)+1))/tanh(d*x+c)^3
```

Maxima [C] time = 1.60142, size = 89, normalized size = 1.48

$$\frac{i(dx+c)}{d}+\frac{i\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2ie^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(d*x + c)/d + I*log(e^(-2*d*x - 2*c) + 1)/d + 2*I*e^(-2*d*x - 2*c)/(d*(2*
e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))
```

Fricas [C] time = 2.26999, size = 252, normalized size = 4.2

$$\frac{idxe^{(4dx+4c)}+idx+(2idx-2i)e^{(2dx+2c)}+\left(-ie^{(4dx+4c)}-2ie^{(2dx+2c)}-i\right)\log\left(e^{(2dx+2c)}+1\right)}{de^{(4dx+4c)}+2de^{(2dx+2c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (I*d*x*e^(4*d*x + 4*c) + I*d*x + (2*I*d*x - 2*I)*e^(2*d*x + 2*c) + (-I*e^(4
*d*x + 4*c) - 2*I*e^(2*d*x + 2*c) - I)*log(e^(2*d*x + 2*c) + 1))/(d*e^(4*d*
x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\tanh^2\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((-tanh(c + d*x)**2)**(3/2), x)
```

Giac [C] time = 1.29772, size = 124, normalized size = 2.07

$$\frac{-i\,(dx+c)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)+i\,\log\left(e^{(2\,dx+2\,c)}+1\right)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)+\frac{2i\,e^{(2\,dx+2\,c)}\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (-I*(d*x + c)*sgn(-e^(4*d*x + 4*c) + 1) + I*log(e^(2*d*x + 2*c) + 1)*sgn(-e
^(4*d*x + 4*c) + 1) + 2*I*e^(2*d*x + 2*c)*sgn(-e^(4*d*x + 4*c) + 1)/(e^(2*d
*x + 2*c) + 1)^2)/d
```

3.29 $\int \sqrt{-\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d
```

Rubi [A] time = 0.0176316, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Tanh[c + d*x]^2],x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-\tanh^2(c+dx)}\,dx &= \left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh(c+dx)\,dx\\ &= \frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.0330789, size = 31, normalized size = 1.

$$\frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Tanh[c + d*x]^2],x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d
```

Maple [A] time = 0.029, size = 45, normalized size = 1.5

$$-\frac{\ln(\tanh(dx+c)-1)+\ln(\tanh(dx+c)+1)}{2\,d\tanh(dx+c)}\sqrt{-(\tanh(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-tanh(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/2/d*(-tanh(d*x+c)^2)^(1/2)*(ln(tanh(d*x+c)-1)+ln(tanh(d*x+c)+1))/tanh(d*
x+c)
```

Maxima [C] time = 1.62238, size = 38, normalized size = 1.23

$$-\frac{i(dx+c)}{d}-\frac{i\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*(d*x + c)/d - I*log(e^(-2*d*x - 2*c) + 1)/d
```

Fricas [C] time = 2.34377, size = 55, normalized size = 1.77

$$\frac{-i\,dx+i\log\left(e^{(2\,dx+2\,c)}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*d*x + I*log(e^(2*d*x + 2*c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-tanh(c + d*x)**2), x)
```

Giac [C] time = 1.18398, size = 73, normalized size = 2.35

$$\frac{i\,(dx+c)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)-i\,\log\left(e^{(2\,dx+2\,c)}+1\right)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-tanh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (I*(d*x + c)*sgn(-e^(4*d*x + 4*c) + 1) - I*log(e^(2*d*x + 2*c) + 1)*sgn(-e^
(4*d*x + 4*c) + 1))/d
```

3.30 $\int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\, dx$

Optimal. Leaf size=31

$$\frac{\tanh(c+dx)\log(\sinh(c+dx))}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[Out] (Log[Sinh[c + d*x]]*Tanh[c + d*x])/(d*Sqrt[-Tanh[c + d*x]^2])
```

Rubi [A] time = 0.0174017, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\tanh(c+dx)\log(\sinh(c+dx))}{d\sqrt{-\tanh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Tanh[c + d*x]^2],x]
```

```
[Out] (Log[Sinh[c + d*x]]*Tanh[c + d*x])/(d*Sqrt[-Tanh[c + d*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\, dx &= \frac{\tanh(c+dx)\int \coth(c+dx)\, dx}{\sqrt{-\tanh^2(c+dx)}} \\ &= \frac{\log(\sinh(c+dx))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0792873, size = 39, normalized size = 1.26

$$\frac{\tanh(c+dx)(\log(\tanh(c+dx))+\log(\cosh(c+dx)))}{d\sqrt{-\tanh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Tanh[c + d*x]^2],x]
```

```
[Out] ((Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c + d*x])/(d*Sqrt[-Tanh[c +
 d*x]^2])
```

Maple [A] time = 0.027, size = 52, normalized size = 1.7

$$-\frac{\tanh(dx+c)\left(\ln(\tanh(dx+c)+1)-2\ln(\tanh(dx+c))+\ln(\tanh(dx+c)-1)\right)}{2d}\frac{1}{\sqrt{-(\tanh(dx+c))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-tanh(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/2/d*tanh(d*x+c)*(ln(tanh(d*x+c)+1)-2*ln(tanh(d*x+c))+ln(tanh(d*x+c)-1))/
(-tanh(d*x+c)^2)^(1/2)
```

Maxima [C] time = 1.57261, size = 61, normalized size = 1.97

$$\frac{i(dx+c)}{d}+\frac{i\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{i\log\left(e^{(-dx-c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] I*(d*x + c)/d + I*log(e^(-d*x - c) + 1)/d + I*log(e^(-d*x - c) - 1)/d
```

Fricas [C] time = 2.32129, size = 54, normalized size = 1.74

$$\frac{i\,dx-i\log\left(e^{(2\,dx+2\,c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (I*d*x - I*log(e^(2*d*x + 2*c) - 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-tanh(c + d*x)**2), x)
```

Giac [C] time = 1.22931, size = 85, normalized size = 2.74

$$\frac{\frac{-2 i dx - 2 i c}{\operatorname{sgn}\left(-e^{(4 dx + 4 c)} + 1\right)} + \frac{2 i \log\left(-i e^{(2 dx + 2 c)} + i\right)}{\operatorname{sgn}\left(-e^{(4 dx + 4 c)} + 1\right)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((-2*I*d*x - 2*I*c)/sgn(-e^(4*d*x + 4*c) + 1) + 2*I*log(-I*e^(2*d*x + 2
*c) + I)/sgn(-e^(4*d*x + 4*c) + 1))/d
```

3.31 $\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}-\frac{\tanh(c+dx)\log(\sinh(c+dx))}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[Out] Coth[c + d*x]/(2*d*Sqrt[-Tanh[c + d*x]^2]) - (Log[Sinh[c + d*x]]*Tanh[c + d
*x])/(d*Sqrt[-Tanh[c + d*x]^2])
```

Rubi [A] time = 0.0309542, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}-\frac{\tanh(c+dx)\log(\sinh(c+dx))}{d\sqrt{-\tanh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Tanh[c + d*x]^2)^(-3/2),x]
```

```
[Out] Coth[c + d*x]/(2*d*Sqrt[-Tanh[c + d*x]^2]) - (Log[Sinh[c + d*x]]*Tanh[c + d
*x])/(d*Sqrt[-Tanh[c + d*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx = -\frac{\tanh(c+dx)\int \coth^3(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}$$
$$= \frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}} - \frac{\tanh(c+dx)\int \coth(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}$$
$$= \frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}} - \frac{\log(\sinh(c+dx))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}}$$

Mathematica [A] time = 0.126368, size = 51, normalized size = 0.85

$$\frac{\coth(c+dx) - 2\tanh(c+dx)(\log(\tanh(c+dx)) + \log(\cosh(c+dx)))}{2d\sqrt{-\tanh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Tanh[c + d*x]^2)^(-3/2),x]
```

```
[Out] (Coth[c + d*x] - 2*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c + d*x])
/(2*d*Sqrt[-Tanh[c + d*x]^2])
```

Maple [A] time = 0.019, size = 79, normalized size = 1.3

$$-\frac{\tanh(dx+c)\left(\ln(\tanh(dx+c)+1)(\tanh(dx+c))^2 - 2\ln(\tanh(dx+c))(\tanh(dx+c))^2 + \ln(\tanh(dx+c)-1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-tanh(d*x+c)^2)^(3/2),x)
```

```
[Out] -1/2/d*tanh(d*x+c)*(ln(tanh(d*x+c)+1)*tanh(d*x+c)^2-2*ln(tanh(d*x+c))*tanh(
d*x+c)^2+ln(tanh(d*x+c)-1)*tanh(d*x+c)^2+1)/(-tanh(d*x+c)^2)^(3/2)
```

Maxima [C] time = 1.6302, size = 115, normalized size = 1.92

$$-\frac{i(dx+c)}{d} - \frac{i\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{i\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2i\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(d*x + c)/d - I*log(e^(-d*x - c) + 1)/d - I*log(e^(-d*x - c) - 1)/d - 2*
I*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))
```

Fricas [C] time = 2.27635, size = 252, normalized size = 4.2

$$\frac{-i\,dxe^{(4\,dx+4\,c)} - i\,dx + (2i\,dx - 2i)e^{(2\,dx+2\,c)} + \left(i\,e^{(4\,dx+4\,c)} - 2i\,e^{(2\,dx+2\,c)} + i\right)\log\left(e^{(2\,dx+2\,c)} - 1\right)}{de^{(4\,dx+4\,c)} - 2\,de^{(2\,dx+2\,c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*d*x*e^(4*d*x + 4*c) - I*d*x + (2*I*d*x - 2*I)*e^(2*d*x + 2*c) + (I*e^(4
*d*x + 4*c) - 2*I*e^(2*d*x + 2*c) + I)*log(e^(2*d*x + 2*c) - 1))/(d*e^(4*d*
x + 4*c) - 2*d*e^(2*d*x + 2*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\tanh^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((-tanh(c + d*x)**2)**(-3/2), x)
```

Giac [C] time = 1.26639, size = 151, normalized size = 2.52

$$\frac{i\log\left(ie^{(2\,dx+2\,c)}\right)}{2\,d\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}-\frac{i\log\left(-ie^{(2\,dx+2\,c)}+i\right)}{d\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}+\frac{2i\,e^{(2\,dx+2\,c)}}{d\left(e^{(2\,dx+2\,c)}-1\right)^2\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*I*log(I*e^(2*d*x + 2*c))/(d*sgn(-e^(4*d*x + 4*c) + 1)) - I*log(-I*e^(2*
d*x + 2*c) + I)/(d*sgn(-e^(4*d*x + 4*c) + 1)) + 2*I*e^(2*d*x + 2*c)/(d*(e^(
2*d*x + 2*c) - 1)^2*sgn(-e^(4*d*x + 4*c) + 1))
```

3.32 $\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}}-\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}+\frac{\tanh(c+dx)\log(\sinh(c+dx))}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[Out] -Coth[c + d*x]/(2*d*Sqrt[-Tanh[c + d*x]^2]) - Coth[c + d*x]^3/(4*d*Sqrt[-Ta
nh[c + d*x]^2]) + (Log[Sinh[c + d*x]]*Tanh[c + d*x])/(d*Sqrt[-Tanh[c + d*x]
^2])
```

Rubi [A] time = 0.0480801, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$-\frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}}-\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}+\frac{\tanh(c+dx)\log(\sinh(c+dx))}{d\sqrt{-\tanh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Tanh[c + d*x]^2)^(-5/2),x]
```

```
[Out] -Coth[c + d*x]/(2*d*Sqrt[-Tanh[c + d*x]^2]) - Coth[c + d*x]^3/(4*d*Sqrt[-Ta
nh[c + d*x]^2]) + (Log[Sinh[c + d*x]]*Tanh[c + d*x])/(d*Sqrt[-Tanh[c + d*x]
^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{5/2}}\,dx = \frac{\tanh(c+dx)\int \coth^5(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}$$
$$= -\frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}} + \frac{\tanh(c+dx)\int \coth^3(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}$$
$$= -\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}} - \frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}} + \frac{\tanh(c+dx)\int \coth(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}$$
$$= -\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}} - \frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}} + \frac{\log(\sinh(c+dx))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}}$$

Mathematica [A] time = 0.235704, size = 63, normalized size = 0.72

$$\frac{-\coth^3(c+dx) - 2\coth(c+dx) + 4\tanh(c+dx)(\log(\tanh(c+dx)) + \log(\cosh(c+dx)))}{4d\sqrt{-\tanh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Tanh[c + d*x]^2)^(-5/2),x]
```

```
[Out] (-2*Coth[c + d*x] - Coth[c + d*x]^3 + 4*(Log[Cosh[c + d*x]] + Log[Tanh[c +
d*x]])*Tanh[c + d*x])/(4*d*Sqrt[-Tanh[c + d*x]^2])
```

Maple [A] time = 0.017, size = 91, normalized size = 1.

$$-\frac{\tanh(dx+c)\left(2\ln(\tanh(dx+c)+1)(\tanh(dx+c))^4 - 4\ln(\tanh(dx+c))(\tanh(dx+c))^4 + 2\ln(\tanh(dx+c) - \right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-tanh(d*x+c)^2)^(5/2),x)
```

```
[Out] -1/4/d*tanh(d*x+c)*(2*ln(tanh(d*x+c)+1)*tanh(d*x+c)^4-4*ln(tanh(d*x+c))*tan
h(d*x+c)^4+2*ln(tanh(d*x+c)-1)*tanh(d*x+c)^4+2*tanh(d*x+c)^2+1)/(-tanh(d*x+
c)^2)^(5/2)
```

Maxima [C] time = 1.60669, size = 177, normalized size = 2.01

$$\frac{i(dx+c)}{d} + \frac{i\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{i\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{4i\,e^{(-2\,dx-2\,c)} - 4i\,e^{(-4\,dx-4\,c)} + 4i\,e^{(-6\,dx-6\,c)}}{d\left(4\,e^{(-2\,dx-2\,c)} - 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} - e^{(-8\,dx-8\,c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] I*(d*x + c)/d + I*log(e^(-d*x - c) + 1)/d + I*log(e^(-d*x - c) - 1)/d + (4*
I*e^(-2*d*x - 2*c) - 4*I*e^(-4*d*x - 4*c) + 4*I*e^(-6*d*x - 6*c))/(d*(4*e^(
```

```
-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c)
- 1))
```

Fricas [C] time = 2.32849, size = 466, normalized size = 5.3

$$\frac{i\,dxe^{(8\,dx+8\,c)} + i\,dx + (-4i\,dx + 4i)e^{(6\,dx+6\,c)} + (6i\,dx - 4i)e^{(4\,dx+4\,c)} + (-4i\,dx + 4i)e^{(2\,dx+2\,c)} + \left(-i\,e^{(8\,dx+8\,c)} + 4i\,e^{(6\,dx+6\,c)}\right.}{de^{(8\,dx+8\,c)} - 4\,de^{(6\,dx+6\,c)} + 6\,de^{(4\,dx+4\,c)} - 4\,de^{(2\,dx+2\,c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (I*d*x*e^(8*d*x + 8*c) + I*d*x + (-4*I*d*x + 4*I)*e^(6*d*x + 6*c) + (6*I*d*
x - 4*I)*e^(4*d*x + 4*c) + (-4*I*d*x + 4*I)*e^(2*d*x + 2*c) + (-I*e^(8*d*x
+ 8*c) + 4*I*e^(6*d*x + 6*c) - 6*I*e^(4*d*x + 4*c) + 4*I*e^(2*d*x + 2*c) -
I)*log(e^(2*d*x + 2*c) - 1))/(d*e^(8*d*x + 8*c) - 4*d*e^(6*d*x + 6*c) + 6*d
*e^(4*d*x + 4*c) - 4*d*e^(2*d*x + 2*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\tanh^{2}\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((-tanh(c + d*x)**2)**(-5/2), x)
```

Giac [C] time = 1.38535, size = 182, normalized size = 2.07

$$-\frac{i\,\log\left(i\,e^{(2\,dx+2\,c)}\right)}{2\,d\mathrm{sgn}\left(-e^{(4\,dx+4\,c)} + 1\right)} + \frac{i\,\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right)}{d\mathrm{sgn}\left(-e^{(4\,dx+4\,c)} + 1\right)} + \frac{-4i\,e^{(6\,dx+6\,c)} + 4i\,e^{(4\,dx+4\,c)} - 4i\,e^{(2\,dx+2\,c)}}{d\left(e^{(2\,dx+2\,c)} - 1\right)^{4}\mathrm{sgn}\left(-e^{(4\,dx+4\,c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-tanh(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*I*log(I*e^(2*d*x + 2*c))/(d*sgn(-e^(4*d*x + 4*c) + 1)) + I*log(abs(e^(
2*d*x + 2*c) - 1))/(d*sgn(-e^(4*d*x + 4*c) + 1)) + (-4*I*e^(6*d*x + 6*c) +
4*I*e^(4*d*x + 4*c) - 4*I*e^(2*d*x + 2*c))/(d*(e^(2*d*x + 2*c) - 1)^4*sgn(-
e^(4*d*x + 4*c) + 1))
```

3.33 $\int \sqrt{\tanh^3(x)}\, dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}+\frac{\sqrt{\tanh^3(x)}\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}-2\sqrt{\tanh^3(x)}\coth(x)$$

```
[Out] -2*Coth[x]*Sqrt[Tanh[x]^3] + (ArcTan[Sqrt[Tanh[x]]]*Sqrt[Tanh[x]^3])/Tanh[x
]^(3/2) + (ArcTanh[Sqrt[Tanh[x]]]*Sqrt[Tanh[x]^3])/Tanh[x]^(3/2)
```

Rubi [A] time = 0.0379271, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3658, 3473, 3476, 329, 212, 206, 203}

$$\frac{\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}+\frac{\sqrt{\tanh^3(x)}\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}-2\sqrt{\tanh^3(x)}\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tanh[x]^3],x]
```

```
[Out] -2*Coth[x]*Sqrt[Tanh[x]^3] + (ArcTan[Sqrt[Tanh[x]]]*Sqrt[Tanh[x]^3])/Tanh[x
]^(3/2) + (ArcTanh[Sqrt[Tanh[x]]]*Sqrt[Tanh[x]^3])/Tanh[x]^(3/2)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\tanh^3(x)}\,dx &= \frac{\sqrt{\tanh^3(x)}\int \tanh^{\frac{3}{2}}(x)\,dx}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} + \frac{\sqrt{\tanh^3(x)}\int \frac{1}{\sqrt{\tanh(x)}}\,dx}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} - \frac{\sqrt{\tanh^3(x)}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \tanh(x)\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} - \frac{\left(2\sqrt{\tanh^3(x)}\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} + \frac{\sqrt{\tanh^3(x)}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)} + \frac{\sqrt{\tanh^3(x)}\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} + \frac{\tan^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} + \frac{\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0382675, size = 38, normalized size = 0.67

$$\frac{\sqrt{\tanh^3(x)}\left(\tanh^{-1}\left(\sqrt{\tanh(x)}\right) - 2\sqrt{\tanh(x)} + \tan^{-1}\left(\sqrt{\tanh(x)}\right)\right)}{\tanh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tanh[x]^3],x]
```

```
[Out] ((ArcTan[Sqrt[Tanh[x]]] + ArcTanh[Sqrt[Tanh[x]]] - 2*Sqrt[Tanh[x]])*Sqrt[Ta
nh[x]^3])/Tanh[x]^(3/2)
```

Maple [A] time = 0.039, size = 43, normalized size = 0.8

$$-\frac{1}{2}\sqrt{(\tanh(x))^{3}}\left(4\sqrt{\tanh(x)}+\ln\left(\sqrt{\tanh(x)}-1\right)-\ln\left(\sqrt{\tanh(x)}+1\right)-2\arctan\left(\sqrt{\tanh(x)}\right)\right)(\tanh(x))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)^3)^(1/2),x)
```

```
[Out] -1/2*(tanh(x)^3)^(1/2)*(4*tanh(x)^(1/2)+ln(tanh(x)^(1/2)-1)-ln(tanh(x)^(1/2
)+1)-2*arctan(tanh(x)^(1/2)))/tanh(x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\tanh(x)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tanh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tanh(x)^3), x)
```

Fricas [B] time = 2.23298, size = 374, normalized size = 6.56

$$-2\sqrt{\frac{\sinh(x)}{\cosh(x)}}+\arctan\left(-\cosh(x)^{2}-2\cosh(x)\sinh(x)-\sinh(x)^{2}+\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}+1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tanh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sinh(x)/cosh(x)) + arctan(-cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^
2 + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(sinh(x)/cosh(x)))
- 1/2*log(-cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + (cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1)*sqrt(sinh(x)/cosh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\tanh^{3}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tanh(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(x)**3), x)
```

Giac [A] time = 1.21139, size = 74, normalized size = 1.3

$$\frac{4}{\sqrt{e^{(4x)}-1}-e^{(2x)}-1}+\arctan\left(\sqrt{e^{(4x)}-1}-e^{(2x)}\right)-\frac{1}{2}\log\left(-\sqrt{e^{(4x)}-1}+e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tanh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 4/(sqrt(e^(4*x) - 1) - e^(2*x) - 1) + arctan(sqrt(e^(4*x) - 1) - e^(2*x)) -
 1/2*log(-sqrt(e^(4*x) - 1) + e^(2*x))
```

3.34 $\int \left(a \tanh^3(x)\right)^{3/2} dx$

Optimal. Leaf size=86

$$-\frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)}-\frac{2}{3}a\sqrt{a\tanh^3(x)}+\frac{a\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}-\frac{a\sqrt{a\tanh^3(x)}\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}$$

```
[Out] (-2*a*Sqrt[a*Tanh[x]^3])/3 - (a*ArcTan[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Ta
nh[x]^(3/2) + (a*ArcTanh[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Tanh[x]^(3/2) -
(2*a*Tanh[x]^2*Sqrt[a*Tanh[x]^3])/7
```

Rubi [A] time = 0.0366665, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {3658, 3473, 3476, 329, 298, 203, 206}

$$-\frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)}-\frac{2}{3}a\sqrt{a\tanh^3(x)}+\frac{a\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}-\frac{a\sqrt{a\tanh^3(x)}\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Tanh[x]^3)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a*Tanh[x]^3])/3 - (a*ArcTan[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Ta
nh[x]^(3/2) + (a*ArcTanh[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Tanh[x]^(3/2) -
(2*a*Tanh[x]^2*Sqrt[a*Tanh[x]^3])/7
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a\tanh^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\tanh^3(x)}\right)\int \tanh^{\frac{9}{2}}(x)\,dx}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} + \frac{\left(a\sqrt{a\tanh^3(x)}\right)\int \tanh^{\frac{5}{2}}(x)\,dx}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} + \frac{\left(a\sqrt{a\tanh^3(x)}\right)\int \sqrt{\tanh(x)}\,dx}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} - \frac{\left(a\sqrt{a\tanh^3(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \tanh(\right.}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} - \frac{\left(2a\sqrt{a\tanh^3(x)}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\tan}\right.}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} + \frac{\left(a\sqrt{a\tanh^3(x)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh}\right.}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{a\tan^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} + \frac{a\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} -$$

Mathematica [A] time = 0.0590554, size = 55, normalized size = 0.64

$$-\frac{\left(a\tanh^3(x)\right)^{3/2}\left(6\tanh^{\frac{7}{2}}(x)+14\tanh^{\frac{3}{2}}(x)-21\tanh^{-1}\left(\sqrt{\tanh(x)}\right)+21\tan^{-1}\left(\sqrt{\tanh(x)}\right)\right)}{21\tanh^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Tanh[x]^3)^(3/2),x]
```

```
[Out] -((a*Tanh[x]^3)^(3/2)*(21*ArcTan[Sqrt[Tanh[x]]] - 21*ArcTanh[Sqrt[Tanh[x]]]
 + 14*Tanh[x]^(3/2) + 6*Tanh[x]^(7/2)))/(21*Tanh[x]^(9/2))
```

Maple [A] time = 0.03, size = 76, normalized size = 0.9

$$\frac{1}{21\,(\tanh(x))^3\,a^2}\left(a\,(\tanh(x))^3\right)^{\frac{3}{2}}\left(21\,a^{7/2}\mathrm{Artanh}\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)-21\,a^{7/2}\arctan\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)-6\,(a\tanh(x))^{7/2}-14$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*tanh(x)^3)^(3/2),x)
```

```
[Out] 1/21*(a*tanh(x)^3)^(3/2)*(21*a^(7/2)*arctanh((a*tanh(x))^(1/2)/a^(1/2))-21*
a^(7/2)*arctan((a*tanh(x))^(1/2)/a^(1/2))-6*(a*tanh(x))^(7/2)-14*a^2*(a*tan
h(x))^(3/2))/tanh(x)^3/(a*tanh(x))^(3/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\tanh(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*tanh(x)^3)^(3/2), x)
```

Fricas [B] time = 2.7242, size = 4099, normalized size = 47.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)
^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh
(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6
*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan((co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*sqrt(a*sinh(x)/cosh(x))/(
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)) - 21*(a*cosh(x)^6 + 6
*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*
sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(
5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)
^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)^3*sin
h(x) + 6*a*cosh(x)^2*sinh(x)^2 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*(c
osh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(a*sinh(x)/cosh(
x)) - 2*a)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cos
h(x)*sinh(x)^3 + sinh(x)^4)) + 16*(5*a*cosh(x)^6 + 30*a*cosh(x)*sinh(x)^5 +
 5*a*sinh(x)^6 - a*cosh(x)^4 + (75*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(25*a*cos
h(x)^3 - a*cosh(x))*sinh(x)^3 + a*cosh(x)^2 + (75*a*cosh(x)^4 - 6*a*cosh(x)
^2 + a)*sinh(x)^2 + 2*(15*a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x)
```

```
- 5*a)*sqrt(a*sinh(x)/cosh(x)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x
))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 +
6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1), -1/84*(42*(a*cosh(x)^6
+ 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 +
a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 +
3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh
(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*arctan((cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 + 1)*sqrt(a*sinh(x)/cosh(x))/sqrt(a)) - 21*(a*cosh(x)^6 + 6*
a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*s
inh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5
*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^
3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(2*a*cosh(x)^4 + 8*a*cosh(x)^3*sinh(
x) + 12*a*cosh(x)^2*sinh(x)^2 + 8*a*cosh(x)*sinh(x)^3 + 2*a*sinh(x)^4 + 2*(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 +
 cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))*sqrt(a)*sqrt(a*sinh(x)/cosh
(x)) - a) + 16*(5*a*cosh(x)^6 + 30*a*cosh(x)*sinh(x)^5 + 5*a*sinh(x)^6 - a*
cosh(x)^4 + (75*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(25*a*cosh(x)^3 - a*cosh(x))
*sinh(x)^3 + a*cosh(x)^2 + (75*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 +
 2*(15*a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - 5*a)*sqrt(a*sinh(
x)/cosh(x)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2
+ 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5
*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*co
sh(x)^3 + cosh(x))*sinh(x) + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \tanh^3(x)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)**3)**(3/2),x)
```

```
[Out] Integral((a*tanh(x)**3)**(3/2), x)
```

Giac [B] time = 1.30995, size = 462, normalized size = 5.37

$$-\frac{1}{42}\left(42\sqrt{a}\arctan\left(-\frac{\sqrt{a}e^{(2x)}-\sqrt{ae^{(4x)}-a}}{\sqrt{a}}\right)\operatorname{sgn}\left(e^{(4x)}-1\right)+21\sqrt{a}\log\left(\left|-\sqrt{a}e^{(2x)}+\sqrt{ae^{(4x)}-a}\right|\right)\operatorname{sgn}\left(e^{(4x)}-1\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/42*(42*sqrt(a)*arctan(-(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))/sqrt(a))*
sgn(e^(4*x) - 1) + 21*sqrt(a)*log(abs(-sqrt(a)*e^(2*x) + sqrt(a*e^(4*x) - a
)))*sgn(e^(4*x) - 1) + 16*(21*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^6*a*s
gn(e^(4*x) - 1) + 42*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^5*a^(3/2)*sgn(
e^(4*x) - 1) + 119*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^4*a^2*sgn(e^(4*x
) - 1) + 56*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^3*a^(5/2)*sgn(e^(4*x) -
 1) + 63*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^2*a^3*sgn(e^(4*x) - 1) + 1
4*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))*a^(7/2)*sgn(e^(4*x) - 1) + 5*a^4*
```

```
sgn(e^(4*x) - 1))/(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a) + sqrt(a))^7)*a
```

3.35 $\int \sqrt{a \tanh^3(x)}\, dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}+\frac{\sqrt{a\tanh^3(x)}\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}-2\coth(x)\sqrt{a\tanh^3(x)}$$

```
[Out] -2*Coth[x]*Sqrt[a*Tanh[x]^3] + (ArcTan[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Ta
nh[x]^(3/2) + (ArcTanh[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Tanh[x]^(3/2)
```

Rubi [A] time = 0.0296038, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {3658, 3473, 3476, 329, 212, 206, 203}

$$\frac{\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}+\frac{\sqrt{a\tanh^3(x)}\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}-2\coth(x)\sqrt{a\tanh^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Tanh[x]^3],x]
```

```
[Out] -2*Coth[x]*Sqrt[a*Tanh[x]^3] + (ArcTan[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Ta
nh[x]^(3/2) + (ArcTanh[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Tanh[x]^(3/2)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\tanh^3(x)}\,dx &= \frac{\sqrt{a\tanh^3(x)}\int \tanh^{\frac{3}{2}}(x)\,dx}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} + \frac{\sqrt{a\tanh^3(x)}\int \frac{1}{\sqrt{\tanh(x)}}\,dx}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} - \frac{\sqrt{a\tanh^3(x)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx, x, \tanh(x)\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} - \frac{\left(2\sqrt{a\tanh^3(x)}\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} + \frac{\sqrt{a\tanh^3(x)}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)} + \frac{\sqrt{a\tanh^3(x)}\,\text{Subst}\left(\int \cdots\right.}{\tanh^{\frac{3}{2}}\cdots}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} + \frac{\tan^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} + \frac{\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}
\end{aligned}$$

Mathematica [A] time = 0.0330763, size = 40, normalized size = 0.63

$$\frac{\sqrt{a\tanh^3(x)}\left(\tanh^{-1}\left(\sqrt{\tanh(x)}\right) - 2\sqrt{\tanh(x)} + \tan^{-1}\left(\sqrt{\tanh(x)}\right)\right)}{\tanh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Tanh[x]^3],x]
```

```
[Out] ((ArcTan[Sqrt[Tanh[x]]] + ArcTanh[Sqrt[Tanh[x]]] - 2*Sqrt[Tanh[x]])*Sqrt[a*
Tanh[x]^3])/Tanh[x]^(3/2)
```

Maple [A] time = 0.036, size = 62, normalized size = 1.

$$-\frac{1}{\tanh(x)}\sqrt{a(\tanh(x))^{3}}\left(2\sqrt{a\tanh(x)}-\sqrt{a}\,\mathrm{Artanh}\left(\sqrt{a\tanh(x)}\frac{1}{\sqrt{a}}\right)-\sqrt{a}\arctan\left(\sqrt{a\tanh(x)}\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{a\tanh(x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*tanh(x)^3)^(1/2),x)
```

```
[Out] -(a*tanh(x)^3)^(1/2)*(2*(a*tanh(x))^(1/2)-a^(1/2)*arctanh((a*tanh(x))^(1/2)
/a^(1/2))-a^(1/2)*arctan((a*tanh(x))^(1/2)/a^(1/2)))/tanh(x)/(a*tanh(x))^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\tanh(x)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*tanh(x)^3), x)
```

Fricas [B] time = 2.31753, size = 1237, normalized size = 19.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a)*arctan((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*
sqrt(a*sinh(x)/cosh(x))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 -
a)) + 1/4*sqrt(-a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)^3*sinh(x) + 6*a*cosh(x)^
2*sinh(x)^2 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(a*sinh(x)/cosh(x)) - 2*a)/(cosh(x)
^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + si
nh(x)^4)) - 2*sqrt(a*sinh(x)/cosh(x)), -1/2*sqrt(a)*arctan(sqrt(a)*sqrt(a*s
inh(x)/cosh(x))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)) + 1/
4*sqrt(a)*log(2*a*cosh(x)^4 + 8*a*cosh(x)^3*sinh(x) + 12*a*cosh(x)^2*sinh(x
)^2 + 8*a*cosh(x)*sinh(x)^3 + 2*a*sinh(x)^4 + 2*(cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^
3 + cosh(x))*sinh(x))*sqrt(a)*sqrt(a*sinh(x)/cosh(x)) - a) - 2*sqrt(a*sinh(
x)/cosh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\tanh^{3}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*tanh(x)**3), x)
```

Giac [B] time = 1.24178, size = 155, normalized size = 2.46

$$\sqrt{a}\arctan\left(-\frac{\sqrt{a}e^{(2x)}-\sqrt{ae^{(4x)}-a}}{\sqrt{a}}\right)\operatorname{sgn}\left(e^{(4x)}-1\right)-\frac{1}{2}\sqrt{a}\log\left(\left|-\sqrt{a}e^{(2x)}+\sqrt{ae^{(4x)}-a}\right|\right)\operatorname{sgn}\left(e^{(4x)}-1\right)-\frac{4\,a\operatorname{sgn}\left(\right.}{\sqrt{a}e^{(2x)}-\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a)*arctan(-(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))/sqrt(a))*sgn(e^(4*x
) - 1) - 1/2*sqrt(a)*log(abs(-sqrt(a)*e^(2*x) + sqrt(a*e^(4*x) - a)))*sgn(e
^(4*x) - 1) - 4*a*sgn(e^(4*x) - 1)/(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a) +
 sqrt(a))
```

3.36 $\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\tanh^{\frac{3}{2}}(x)}{\sqrt{a\tanh^3(x)}}-\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}}-\frac{\tanh^{\frac{3}{2}}(x)\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}}$$

```
[Out] (-2*Tanh[x])/Sqrt[a*Tanh[x]^3] - (ArcTan[Sqrt[Tanh[x]]]*Tanh[x]^(3/2))/Sqrt
[a*Tanh[x]^3] + (ArcTanh[Sqrt[Tanh[x]]]*Tanh[x]^(3/2))/Sqrt[a*Tanh[x]^3]
```

Rubi [A] time = 0.030095, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {3658, 3474, 3476, 329, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\tanh^{\frac{3}{2}}(x)}{\sqrt{a\tanh^3(x)}}-\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}}-\frac{\tanh^{\frac{3}{2}}(x)\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Tanh[x]^3],x]
```

```
[Out] (-2*Tanh[x])/Sqrt[a*Tanh[x]^3] - (ArcTan[Sqrt[Tanh[x]]]*Tanh[x]^(3/2))/Sqrt
[a*Tanh[x]^3] + (ArcTanh[Sqrt[Tanh[x]]]*Tanh[x]^(3/2))/Sqrt[a*Tanh[x]^3]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx &= \frac{\tanh^{\frac{3}{2}}(x)\int \frac{1}{\tanh^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\tanh^3(x)}}\\
&= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}}+\frac{\tanh^{\frac{3}{2}}(x)\int \sqrt{\tanh(x)}\,dx}{\sqrt{a\tanh^3(x)}}\\
&= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}}-\frac{\tanh^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx,x,\tanh(x)\right)}{\sqrt{a\tanh^3(x)}}\\
&= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}}-\frac{\left(2\tanh^{\frac{3}{2}}(x)\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}}\\
&= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}}+\frac{\tanh^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}}-\frac{\tanh^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}}\\
&= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}}-\frac{\tan^{-1}\left(\sqrt{\tanh(x)}\right)\tanh^{\frac{3}{2}}(x)}{\sqrt{a\tanh^3(x)}}+\frac{\tanh^{-1}\left(\sqrt{\tanh(x)}\right)\tanh^{\frac{3}{2}}(x)}{\sqrt{a\tanh^3(x)}}
\end{aligned}
$$

Mathematica [C] time = 0.0169666, size = 26, normalized size = 0.41

$$
-\frac{2\tanh(x)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\tanh^2(x)\right)}{\sqrt{a\tanh^3(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Tanh[x]^3],x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Tanh[x]^2]*Tanh[x])/Sqrt[a*Tanh[x]^3]
```

Maple [A] time = 0.04, size = 65, normalized size = 1.

$$-\tanh(x)\left(2a^{5/2}-\operatorname{Artanh}\left(\sqrt{a\tanh(x)}\frac{1}{\sqrt{a}}\right)a^{2}\sqrt{a\tanh(x)}+\arctan\left(\sqrt{a\tanh(x)}\frac{1}{\sqrt{a}}\right)a^{2}\sqrt{a\tanh(x)}\right)\frac{1}{\sqrt{a\,(\tanh(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*tanh(x)^3)^(1/2),x)
```

```
[Out] -tanh(x)*(2*a^(5/2)-arctanh((a*tanh(x))^(1/2)/a^(1/2))*a^2*(a*tanh(x))^(1/2
)+arctan((a*tanh(x))^(1/2)/a^(1/2))*a^2*(a*tanh(x))^(1/2))/(a*tanh(x)^3)^(1
/2)/a^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\tanh(x)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*tanh(x)^3), x)
```

Fricas [B] time = 2.45095, size = 1763, normalized size = 27.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*arctan((c
osh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*sqrt(a*sinh(x)/cosh(x))/
(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)) + (cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)^3*si
nh(x) + 6*a*cosh(x)^2*sinh(x)^2 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(a*sinh(x)/cosh
(x)) - 2*a)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4)) + 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(a*sinh(x)/cosh(x)))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 - a), -1/4*(2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)
*arctan((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a*sinh(x)/cosh
(x))/sqrt(a)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log
(2*a*cosh(x)^4 + 8*a*cosh(x)^3*sinh(x) + 12*a*cosh(x)^2*sinh(x)^2 + 8*a*cos
h(x)*sinh(x)^3 + 2*a*sinh(x)^4 + 2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*
sinh(x))*sqrt(a)*sqrt(a*sinh(x)/cosh(x)) - a) + 8*(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2 + 1)*sqrt(a*sinh(x)/cosh(x)))/(a*cosh(x)^2 + 2*a*cosh(x)*
sinh(x) + a*sinh(x)^2 - a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \tanh^3(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*tanh(x)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.37 $\int \left(a \tanh^4(x)\right)^{3/2} dx$

Optimal. Leaf size=69

$$-\frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)}-\frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)}+ax\coth^2(x)\sqrt{a\tanh^4(x)}-a\coth(x)\sqrt{a\tanh^4(x)}$$

```
[Out] -(a*Coth[x]*Sqrt[a*Tanh[x]^4]) + a*x*Coth[x]^2*Sqrt[a*Tanh[x]^4] - (a*Tanh[
x]*Sqrt[a*Tanh[x]^4])/3 - (a*Tanh[x]^3*Sqrt[a*Tanh[x]^4])/5
```

Rubi [A] time = 0.0262483, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 8}

$$-\frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)}-\frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)}+ax\coth^2(x)\sqrt{a\tanh^4(x)}-a\coth(x)\sqrt{a\tanh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Tanh[x]^4)^(3/2),x]
```

```
[Out] -(a*Coth[x]*Sqrt[a*Tanh[x]^4]) + a*x*Coth[x]^2*Sqrt[a*Tanh[x]^4] - (a*Tanh[
x]*Sqrt[a*Tanh[x]^4])/3 - (a*Tanh[x]^3*Sqrt[a*Tanh[x]^4])/5
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a\tanh^4(x)\right)^{3/2} dx &= \left(a\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int \tanh^6(x)\,dx\\
&= -\frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)}+\left(a\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int \tanh^4(x)\,dx\\
&= -\frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)}-\frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)}+\left(a\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int \tanh^2(
\\
&= -a\coth(x)\sqrt{a\tanh^4(x)}-\frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)}-\frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)}+\left(a\coth^2(x)
\\
&= -a\coth(x)\sqrt{a\tanh^4(x)}+ax\coth^2(x)\sqrt{a\tanh^4(x)}-\frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)}-\frac{1}{5}a\tanh^3(x
\end{aligned}$$

Mathematica [A] time = 0.147969, size = 56, normalized size = 0.81

$$\frac{1}{240}\coth(x)\mathrm{csch}^5(x)\left(a\tanh^4(x)\right)^{3/2}(-50\sinh(x)-25\sinh(3x)-23\sinh(5x)+150x\cosh(x)+75x\cosh(3x)+15x\cos$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Tanh[x]^4)^(3/2),x]
```

```
[Out] (Coth[x]*Csch[x]^5*(150*x*Cosh[x] + 75*x*Cosh[3*x] + 15*x*Cosh[5*x] - 50*Si
nh[x] - 25*Sinh[3*x] - 23*Sinh[5*x])*(a*Tanh[x]^4)^(3/2))/240
```

Maple [A] time = 0.026, size = 46, normalized size = 0.7

$$-\frac{6(\tanh(x))^5+10(\tanh(x))^3+15\ln(\tanh(x)-1)-15\ln(1+\tanh(x))+30\tanh(x)}{30(\tanh(x))^6}\left(a(\tanh(x))^4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*tanh(x)^4)^(3/2),x)
```

```
[Out] -1/30*(a*tanh(x)^4)^(3/2)*(6*tanh(x)^5+10*tanh(x)^3+15*ln(tanh(x)-1)-15*ln(
1+tanh(x))+30*tanh(x))/tanh(x)^6
```

Maxima [A] time = 1.6101, size = 111, normalized size = 1.61

$$a^{\frac{3}{2}}x-\frac{2\left(70\,a^{\frac{3}{2}}e^{(-2\,x)}+140\,a^{\frac{3}{2}}e^{(-4\,x)}+90\,a^{\frac{3}{2}}e^{(-6\,x)}+45\,a^{\frac{3}{2}}e^{(-8\,x)}+23\,a^{\frac{3}{2}}\right)}{15\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] a^(3/2)*x - 2/15*(70*a^(3/2)*e^(-2*x) + 140*a^(3/2)*e^(-4*x) + 90*a^(3/2)*e
^(-6*x) + 45*a^(3/2)*e^(-8*x) + 23*a^(3/2))/(5*e^(-2*x) + 10*e^(-4*x) + 10*
e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

Fricas [B] time = 2.52678, size = 6368, normalized size = 92.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*a*x*cosh(x)^10 + 15*(a*x*e^(4*x) + 2*a*x*e^(2*x) + a*x)*sinh(x)^10
 + 150*(a*x*cosh(x)*e^(4*x) + 2*a*x*cosh(x)*e^(2*x) + a*x*cosh(x))*sinh(x)^
9 + 15*(5*a*x + 6*a)*cosh(x)^8 + 15*(45*a*x*cosh(x)^2 + 5*a*x + (45*a*x*cos
h(x)^2 + 5*a*x + 6*a)*e^(4*x) + 2*(45*a*x*cosh(x)^2 + 5*a*x + 6*a)*e^(2*x)
+ 6*a)*sinh(x)^8 + 120*(15*a*x*cosh(x)^3 + (5*a*x + 6*a)*cosh(x) + (15*a*x*
cosh(x)^3 + (5*a*x + 6*a)*cosh(x))*e^(4*x) + 2*(15*a*x*cosh(x)^3 + (5*a*x +
```

```
 6*a)*cosh(x))*e^(2*x))*sinh(x)^7 + 30*(5*a*x + 6*a)*cosh(x)^6 + 30*(105*a*
x*cosh(x)^4 + 14*(5*a*x + 6*a)*cosh(x)^2 + 5*a*x + (105*a*x*cosh(x)^4 + 14*
(5*a*x + 6*a)*cosh(x)^2 + 5*a*x + 6*a)*e^(4*x) + 2*(105*a*x*cosh(x)^4 + 14*
(5*a*x + 6*a)*cosh(x)^2 + 5*a*x + 6*a)*e^(2*x) + 6*a)*sinh(x)^6 + 60*(63*a*
x*cosh(x)^5 + 14*(5*a*x + 6*a)*cosh(x)^3 + 3*(5*a*x + 6*a)*cosh(x) + (63*a*
x*cosh(x)^5 + 14*(5*a*x + 6*a)*cosh(x)^3 + 3*(5*a*x + 6*a)*cosh(x))*e^(4*x)
 + 2*(63*a*x*cosh(x)^5 + 14*(5*a*x + 6*a)*cosh(x)^3 + 3*(5*a*x + 6*a)*cosh(
x))*e^(2*x))*sinh(x)^5 + 10*(15*a*x + 28*a)*cosh(x)^4 + 10*(315*a*x*cosh(x)
^6 + 105*(5*a*x + 6*a)*cosh(x)^4 + 45*(5*a*x + 6*a)*cosh(x)^2 + 15*a*x + (3
15*a*x*cosh(x)^6 + 105*(5*a*x + 6*a)*cosh(x)^4 + 45*(5*a*x + 6*a)*cosh(x)^2
 + 15*a*x + 28*a)*e^(4*x) + 2*(315*a*x*cosh(x)^6 + 105*(5*a*x + 6*a)*cosh(x
)^4 + 45*(5*a*x + 6*a)*cosh(x)^2 + 15*a*x + 28*a)*e^(2*x) + 28*a)*sinh(x)^4
 + 40*(45*a*x*cosh(x)^7 + 21*(5*a*x + 6*a)*cosh(x)^5 + 15*(5*a*x + 6*a)*cos
h(x)^3 + (15*a*x + 28*a)*cosh(x) + (45*a*x*cosh(x)^7 + 21*(5*a*x + 6*a)*cos
h(x)^5 + 15*(5*a*x + 6*a)*cosh(x)^3 + (15*a*x + 28*a)*cosh(x))*e^(4*x) + 2*
(45*a*x*cosh(x)^7 + 21*(5*a*x + 6*a)*cosh(x)^5 + 15*(5*a*x + 6*a)*cosh(x)^3
 + (15*a*x + 28*a)*cosh(x))*e^(2*x))*sinh(x)^3 + 5*(15*a*x + 28*a)*cosh(x)^
2 + 5*(135*a*x*cosh(x)^8 + 84*(5*a*x + 6*a)*cosh(x)^6 + 90*(5*a*x + 6*a)*co
sh(x)^4 + 12*(15*a*x + 28*a)*cosh(x)^2 + 15*a*x + (135*a*x*cosh(x)^8 + 84*(
5*a*x + 6*a)*cosh(x)^6 + 90*(5*a*x + 6*a)*cosh(x)^4 + 12*(15*a*x + 28*a)*co
sh(x)^2 + 15*a*x + 28*a)*e^(4*x) + 2*(135*a*x*cosh(x)^8 + 84*(5*a*x + 6*a)*
cosh(x)^6 + 90*(5*a*x + 6*a)*cosh(x)^4 + 12*(15*a*x + 28*a)*cosh(x)^2 + 15*
a*x + 28*a)*e^(2*x) + 28*a)*sinh(x)^2 + 15*a*x + (15*a*x*cosh(x)^10 + 15*(5
*a*x + 6*a)*cosh(x)^8 + 30*(5*a*x + 6*a)*cosh(x)^6 + 10*(15*a*x + 28*a)*cos
h(x)^4 + 5*(15*a*x + 28*a)*cosh(x)^2 + 15*a*x + 46*a)*e^(4*x) + 2*(15*a*x*c
osh(x)^10 + 15*(5*a*x + 6*a)*cosh(x)^8 + 30*(5*a*x + 6*a)*cosh(x)^6 + 10*(1
5*a*x + 28*a)*cosh(x)^4 + 5*(15*a*x + 28*a)*cosh(x)^2 + 15*a*x + 46*a)*e^(2
*x) + 10*(15*a*x*cosh(x)^9 + 12*(5*a*x + 6*a)*cosh(x)^7 + 18*(5*a*x + 6*a)*
cosh(x)^5 + 4*(15*a*x + 28*a)*cosh(x)^3 + (15*a*x + 28*a)*cosh(x) + (15*a*x
*cosh(x)^9 + 12*(5*a*x + 6*a)*cosh(x)^7 + 18*(5*a*x + 6*a)*cosh(x)^5 + 4*(1
5*a*x + 28*a)*cosh(x)^3 + (15*a*x + 28*a)*cosh(x))*e^(4*x) + 2*(15*a*x*cosh
(x)^9 + 12*(5*a*x + 6*a)*cosh(x)^7 + 18*(5*a*x + 6*a)*cosh(x)^5 + 4*(15*a*x
 + 28*a)*cosh(x)^3 + (15*a*x + 28*a)*cosh(x))*e^(2*x))*sinh(x) + 46*a)*sqrt
((a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(e^(8*x) + 4*e^(
6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))/((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^10 +
cosh(x)^10 + 10*(cosh(x)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^9 +
 5*(9*cosh(x)^2 + (9*cosh(x)^2 + 1)*e^(4*x) - 2*(9*cosh(x)^2 + 1)*e^(2*x) +
 1)*sinh(x)^8 + 5*cosh(x)^8 + 40*(3*cosh(x)^3 + (3*cosh(x)^3 + cosh(x))*e^(
4*x) - 2*(3*cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x)^7 + 10*(21*cosh
(x)^4 + 14*cosh(x)^2 + (21*cosh(x)^4 + 14*cosh(x)^2 + 1)*e^(4*x) - 2*(21*co
sh(x)^4 + 14*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*c
osh(x)^5 + 70*cosh(x)^3 + (63*cosh(x)^5 + 70*cosh(x)^3 + 15*cosh(x))*e^(4*x
) - 2*(63*cosh(x)^5 + 70*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sinh
(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 + 15*cosh(x)^2 + (21*cosh(x)^6 + 35
*cosh(x)^4 + 15*cosh(x)^2 + 1)*e^(4*x) - 2*(21*cosh(x)^6 + 35*cosh(x)^4 + 1
5*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 +
7*cosh(x)^5 + 5*cosh(x)^3 + (3*cosh(x)^7 + 7*cosh(x)^5 + 5*cosh(x)^3 + cosh
(x))*e^(4*x) - 2*(3*cosh(x)^7 + 7*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*e^(2*x
) + cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 12*
cosh(x)^2 + (9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 12*cosh(x)^2 + 1)*
e^(4*x) - 2*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 12*cosh(x)^2 + 1)*
e^(2*x) + 1)*sinh(x)^2 + 5*cosh(x)^2 + (cosh(x)^10 + 5*cosh(x)^8 + 10*cosh(
x)^6 + 10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(4*x) - 2*(cosh(x)^10 + 5*cosh(x)^
8 + 10*cosh(x)^6 + 10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x) + 10*(cosh(x)^9
+ 4*cosh(x)^7 + 6*cosh(x)^5 + 4*cosh(x)^3 + (cosh(x)^9 + 4*cosh(x)^7 + 6*co
sh(x)^5 + 4*cosh(x)^3 + cosh(x))*e^(4*x) - 2*(cosh(x)^9 + 4*cosh(x)^7 + 6*c
osh(x)^5 + 4*cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\tanh^{4}(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)**4)**(3/2),x)
```

```
[Out] Integral((a*tanh(x)**4)**(3/2), x)
```

Giac [A] time = 1.18484, size = 61, normalized size = 0.88

$$\frac{1}{15}\,a^{\frac{3}{2}}\left(15\,x+\frac{2\left(45\,e^{(8\,x)}+90\,e^{(6\,x)}+140\,e^{(4\,x)}+70\,e^{(2\,x)}+23\right)}{\left(e^{(2\,x)}+1\right)^{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*a^(3/2)*(15*x + 2*(45*e^(8*x) + 90*e^(6*x) + 140*e^(4*x) + 70*e^(2*x)
+ 23)/(e^(2*x) + 1)^5)
```

3.38 $\int \sqrt{a\tanh^4(x)}\,dx$

Optimal. Leaf size=31

$$x\coth^2(x)\sqrt{a\tanh^4(x)} - \coth(x)\sqrt{a\tanh^4(x)}$$

```
[Out] -(Coth[x]*Sqrt[a*Tanh[x]^4]) + x*Coth[x]^2*Sqrt[a*Tanh[x]^4]
```

Rubi [A] time = 0.0137952, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 8}

$$x\coth^2(x)\sqrt{a\tanh^4(x)} - \coth(x)\sqrt{a\tanh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Tanh[x]^4],x]
```

```
[Out] -(Coth[x]*Sqrt[a*Tanh[x]^4]) + x*Coth[x]^2*Sqrt[a*Tanh[x]^4]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\tanh^4(x)}\,dx &= \left(\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int \tanh^2(x)\,dx\\ &= -\coth(x)\sqrt{a\tanh^4(x)} + \left(\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int 1\,dx\\ &= -\coth(x)\sqrt{a\tanh^4(x)} + x\coth^2(x)\sqrt{a\tanh^4(x)}\end{aligned}$$

Mathematica [A] time = 0.0168269, size = 19, normalized size = 0.61

$$\coth(x)(x\coth(x)-1)\sqrt{a\tanh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Tanh[x]^4],x]
```

```
[Out] Coth[x]*(-1 + x*Coth[x])*Sqrt[a*Tanh[x]^4]
```

Maple [A] time = 0.032, size = 32, normalized size = 1.

$$-\frac{2\tanh(x)+\ln(\tanh(x)-1)-\ln(1+\tanh(x))}{2\,(\tanh(x))^{2}}\sqrt{a\,(\tanh(x))^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*tanh(x)^4)^(1/2),x)
```

```
[Out] -1/2*(a*tanh(x)^4)^(1/2)*(2*tanh(x)+ln(tanh(x)-1)-ln(1+tanh(x)))/tanh(x)^2
```

Maxima [A] time = 1.60266, size = 26, normalized size = 0.84

$$\sqrt{a}x-\frac{2\sqrt{a}}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*x - 2*sqrt(a)/(e^(-2*x) + 1)
```

Fricas [B] time = 2.32083, size = 647, normalized size = 20.87

$$\frac{\left(x\cosh(x)^{2}+\left(xe^{(4x)}+2\,xe^{(2x)}+x\right)\sinh(x)^{2}+\left(x\cosh(x)^{2}+x+2\right)e^{(4x)}+2\left(x\cosh(x)^{2}+x+2\right)e^{(2x)}+2\left(x\cosh(x)\,e\right.\right.}{\left(e^{(4x)}-2\,e^{(2x)}+1\right)\sinh(x)^{2}+\cosh(x)^{2}+\left(\cosh(x)^{2}+1\right)e^{(4x)}-2\left(\cosh(x)^{2}+1\right)e^{(2x)}+2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x +
 2)*e^(4*x) + 2*(x*cosh(x)^2 + x + 2)*e^(2*x) + 2*(x*cosh(x)*e^(4*x) + 2*x*
cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x + 2)*sqrt((a*e^(8*x) - 4*a*e^(6*x)
 + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2
*x) + 1))/((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1
)*e^(4*x) - 2*(cosh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(4*x) - 2*cosh(x)*e^(2
*x) + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\tanh^{4}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*tanh(x)**4), x)
```

Giac [A] time = 1.19429, size = 22, normalized size = 0.71

$$\sqrt{a}\left(x+\frac{2}{e^{(2\,x)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a)*(x + 2/(e^(2*x) + 1))
```

3.39 $\int \frac{1}{\sqrt{a \tanh^4(x)}} \, dx$

Optimal. Leaf size=31

$$\frac{x \tanh^2(x)}{\sqrt{a \tanh^4(x)}} - \frac{\tanh(x)}{\sqrt{a \tanh^4(x)}}$$

```
[Out] -(Tanh[x]/Sqrt[a*Tanh[x]^4]) + (x*Tanh[x]^2)/Sqrt[a*Tanh[x]^4]
```

Rubi [A] time = 0.0138879, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 8}

$$\frac{x \tanh^2(x)}{\sqrt{a \tanh^4(x)}} - \frac{\tanh(x)}{\sqrt{a \tanh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Tanh[x]^4],x]
```

```
[Out] -(Tanh[x]/Sqrt[a*Tanh[x]^4]) + (x*Tanh[x]^2)/Sqrt[a*Tanh[x]^4]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a \tanh^4(x)}} \, dx &= \frac{\tanh^2(x) \int \coth^2(x) \, dx}{\sqrt{a \tanh^4(x)}} \\
&= -\frac{\tanh(x)}{\sqrt{a \tanh^4(x)}} + \frac{\tanh^2(x) \int 1 \, dx}{\sqrt{a \tanh^4(x)}} \\
&= -\frac{\tanh(x)}{\sqrt{a \tanh^4(x)}} + \frac{x \tanh^2(x)}{\sqrt{a \tanh^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0332763, size = 19, normalized size = 0.61

$$\frac{\tanh(x)(x\tanh(x)-1)}{\sqrt{a\tanh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Tanh[x]^4],x]
```

```
[Out] (Tanh[x]*(-1 + x*Tanh[x]))/Sqrt[a*Tanh[x]^4]
```

Maple [A] time = 0.039, size = 32, normalized size = 1.

$$\frac{\tanh(x)(\ln(1+\tanh(x))\tanh(x)-\ln(\tanh(x)-1)\tanh(x)-2)}{2}\frac{1}{\sqrt{a(\tanh(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*tanh(x)^4)^(1/2),x)
```

```
[Out] 1/2*tanh(x)*(ln(1+tanh(x))*tanh(x)-ln(tanh(x)-1)*tanh(x)-2)/(a*tanh(x)^4)^(
1/2)
```

Maxima [A] time = 1.64758, size = 31, normalized size = 1.

$$\frac{x}{\sqrt{a}}+\frac{2\sqrt{a}}{ae^{(-2x)}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(a) + 2*sqrt(a)/(a*e^(-2*x) - a)
```

Fricas [B] time = 2.13279, size = 668, normalized size = 21.55

$$\frac{\left(x\cosh(x)^2+\left(xe^{(4x)}+2xe^{(2x)}+x\right)\sinh(x)^2+\left(x\cosh(x)^2-x-2\right)e^{(4x)}+2\left(x\cosh(x)^2-x-2\right)e^{(2x)}+2\left(x\cosh(x\right.\right.}{a\cosh(x)^2+\left(ae^{(4x)}-2ae^{(2x)}+a\right)\sinh(x)^2+\left(a\cosh(x)^2-a\right)e^{(4x)}-2\left(a\cosh(x)^2-a\right)e^{(2x)}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 - x -
 2)*e^(4*x) + 2*(x*cosh(x)^2 - x - 2)*e^(2*x) + 2*(x*cosh(x)*e^(4*x) + 2*x*
cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) - x - 2)*sqrt((a*e^(8*x) - 4*a*e^(6*x)
 + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2
*x) + 1))/(a*cosh(x)^2 + (a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(
x)^2 - a)*e^(4*x) - 2*(a*cosh(x)^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) - 2*
```

```
a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x) - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \tanh^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*tanh(x)**4), x)
```

Giac [A] time = 1.26143, size = 26, normalized size = 0.84

$$\frac{x}{\sqrt{a}} - \frac{2}{\sqrt{a}\left(e^{(2x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] x/sqrt(a) - 2/(sqrt(a)*(e^(2*x) - 1))
```

3.40 $\int (b\tanh^m(c+dx))^n\,dx$

Optimal. Leaf size=57

$$\frac{\tanh(c+dx)\left(b\tanh^m(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\tanh^2(c+dx)\right)}{d(mn+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + m*n)/2, (3 + m*n)/2, Tanh[c + d*x]^2]*Tanh[c + d
*x]*(b*Tanh[c + d*x]^m)^n)/(d*(1 + m*n))
```

Rubi [A] time = 0.0393417, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3659, 3476, 364}

$$\frac{\tanh(c+dx)\left(b\tanh^m(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\tanh^2(c+dx)\right)}{d(mn+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tanh[c + d*x]^m)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m*n)/2, (3 + m*n)/2, Tanh[c + d*x]^2]*Tanh[c + d
*x]*(b*Tanh[c + d*x]^m)^n)/(d*(1 + m*n))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\tanh^m(c+dx)\right)^n\,dx = \left(\tanh^{-mn}(c+dx)\left(b\tanh^m(c+dx)\right)^n\right)\int \tanh^{mn}(c+dx)\,dx$$
$$= -\frac{\left(\tanh^{-mn}(c+dx)\left(b\tanh^m(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{mn}}{-1+x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+mn);\frac{1}{2}(3+mn);\tanh^2(c+dx)\right)\tanh(c+dx)\left(b\tanh^m(c+dx)\right)^n}{d(1+mn)}$$

Mathematica [A] time = 0.051644, size = 55, normalized size = 0.96

$$\frac{\tanh(c+dx)\left(b\tanh^m(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\tanh^2(c+dx)\right)}{dmn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tanh[c + d*x]^m)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m*n)/2, (3 + m*n)/2, Tanh[c + d*x]^2]*Tanh[c + d
*x]*(b*Tanh[c + d*x]^m)^n)/(d + d*m*n)
```

Maple [F] time = 3.984, size = 0, normalized size = 0.

$$\int \left(b\left(\tanh(dx+c)\right)^m\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(d*x+c)^m)^n,x)
```

```
[Out] int((b*tanh(d*x+c)^m)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh(dx+c)^m\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c)^m)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(d*x + c)^m)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tanh(dx+c)^m\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c)^m)^n,x, algorithm="fricas")
```

```
[Out] integral((b*tanh(d*x + c)^m)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh^{m}\left(c+dx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c)**m)**n,x)
```

```
[Out] Integral((b*tanh(c + d*x)**m)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tanh\left(dx+c\right)^{m}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(d*x+c)^m)^n,x, algorithm="giac")
```

```
[Out] integrate((b*tanh(d*x + c)^m)^n, x)
```

3.41 $\int (a + a\tanh(c + dx))^5\,dx$

Optimal. Leaf size=100

$$-\frac{8a^5\tanh(c+dx)}{d}-\frac{2a^2(a\tanh(c+dx)+a)^3}{3d}-\frac{2a\left(a^2\tanh(c+dx)+a^2\right)^2}{d}+\frac{16a^5\log(\cosh(c+dx))}{d}+16a^5x-\frac{a(a\tan$$

```
[Out] 16*a^5*x + (16*a^5*Log[Cosh[c + d*x]])/d - (8*a^5*Tanh[c + d*x])/d - (2*a^2
*(a + a*Tanh[c + d*x])^3)/(3*d) - (a*(a + a*Tanh[c + d*x])^4)/(4*d) - (2*a*
(a^2 + a^2*Tanh[c + d*x])^2)/d
```

Rubi [A] time = 0.0719517, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3478, 3477, 3475}

$$-\frac{8a^5\tanh(c+dx)}{d}-\frac{2a^2(a\tanh(c+dx)+a)^3}{3d}-\frac{2a\left(a^2\tanh(c+dx)+a^2\right)^2}{d}+\frac{16a^5\log(\cosh(c+dx))}{d}+16a^5x-\frac{a(a\tan$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tanh[c + d*x])^5,x]
```

```
[Out] 16*a^5*x + (16*a^5*Log[Cosh[c + d*x]])/d - (8*a^5*Tanh[c + d*x])/d - (2*a^2
*(a + a*Tanh[c + d*x])^3)/(3*d) - (a*(a + a*Tanh[c + d*x])^4)/(4*d) - (2*a*
(a^2 + a^2*Tanh[c + d*x])^2)/d
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\tanh(c + dx))^5 \, dx = -\frac{a(a + a\tanh(c + dx))^4}{4d} + (2a)\int (a + a\tanh(c + dx))^4 \, dx$$
$$= -\frac{2a^2(a + a\tanh(c + dx))^3}{3d} - \frac{a(a + a\tanh(c + dx))^4}{4d} + \left(4a^2\right)\int (a + a\tanh(c + dx))^3 \, dx$$
$$= -\frac{2a^3(a + a\tanh(c + dx))^2}{d} - \frac{2a^2(a + a\tanh(c + dx))^3}{3d} - \frac{a(a + a\tanh(c + dx))^4}{4d} + \left(8a^3$$
$$= 16a^5x - \frac{8a^5\tanh(c + dx)}{d} - \frac{2a^3(a + a\tanh(c + dx))^2}{d} - \frac{2a^2(a + a\tanh(c + dx))^3}{3d} - \frac{a(a$$
$$= 16a^5x + \frac{16a^5\log(\cosh(c + dx))}{d} - \frac{8a^5\tanh(c + dx)}{d} - \frac{2a^3(a + a\tanh(c + dx))^2}{d} - 2a^2($$

Mathematica [B] time = 1.60801, size = 202, normalized size = 2.02

$$\frac{a^5 \text{sech}(c)\text{sech}^4(c + dx)(-70\sinh(c + 2dx) + 30\sinh(3c + 2dx) - 25\sinh(3c + 4dx) + 48dx\cosh(3c + 2dx) + 18\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tanh[c + d*x])^5,x]
```

```
[Out] (a^5*Sech[c]*Sech[c + d*x]^4*(18*Cosh[3*c + 2*d*x] + 48*d*x*Cosh[3*c + 2*d*
x] + 12*d*x*Cosh[3*c + 4*d*x] + 12*d*x*Cosh[5*c + 4*d*x] + 48*Cosh[3*c + 2*
d*x]*Log[Cosh[c + d*x]] + 12*Cosh[3*c + 4*d*x]*Log[Cosh[c + d*x]] + 12*Cosh
[5*c + 4*d*x]*Log[Cosh[c + d*x]] + 6*Cosh[c + 2*d*x]*(3 + 8*d*x + 8*Log[Cos
h[c + d*x]]) + Cosh[c]*(33 + 72*d*x + 72*Log[Cosh[c + d*x]]) + 75*Sinh[c] -
 70*Sinh[c + 2*d*x] + 30*Sinh[3*c + 2*d*x] - 25*Sinh[3*c + 4*d*x]))/(12*d)
```

Maple [A] time = 0.006, size = 81, normalized size = 0.8

$$-\frac{a^5(\tanh(dx + c))^4}{4d} - \frac{5a^5(\tanh(dx + c))^3}{3d} - \frac{11a^5(\tanh(dx + c))^2}{2d} - 15\frac{a^5\tanh(dx + c)}{d} - 16\frac{a^5\ln(\tanh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tanh(d*x+c))^5,x)
```

```
[Out] -1/4/d*a^5*tanh(d*x+c)^4-5/3/d*a^5*tanh(d*x+c)^3-11/2/d*a^5*tanh(d*x+c)^2-1
5*a^5*tanh(d*x+c)/d-16/d*a^5*ln(tanh(d*x+c)-1)
```

Maxima [B] time = 1.59054, size = 408, normalized size = 4.08

$$\frac{5}{3}a^5\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1\right)}\right) + a^5\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)} + 1\right)}{d} + \frac{4\left(e^{(-2c}}{d\left(4e^{(-2dx-2c)} + 6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/3*a^5*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^5*(x
```

```
+ c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6
*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 10*a^5*(x + c/d + log(e^(-2*d*x - 2
*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
 1))) + 10*a^5*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^5*x + 5*a^5*log
(cosh(d*x + c))/d
```

Fricas [B] time = 2.35765, size = 2376, normalized size = 23.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 4/3*(48*a^5*cosh(d*x + c)^6 + 288*a^5*cosh(d*x + c)*sinh(d*x + c)^5 + 48*a^
5*sinh(d*x + c)^6 + 108*a^5*cosh(d*x + c)^4 + 88*a^5*cosh(d*x + c)^2 + 25*a
^5 + 36*(20*a^5*cosh(d*x + c)^2 + 3*a^5)*sinh(d*x + c)^4 + 48*(20*a^5*cosh(
d*x + c)^3 + 9*a^5*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(90*a^5*cosh(d*x + c)
^4 + 81*a^5*cosh(d*x + c)^2 + 11*a^5)*sinh(d*x + c)^2 + 12*(a^5*cosh(d*x +
c)^8 + 8*a^5*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*sinh(d*x + c)^8 + 4*a^5*co
sh(d*x + c)^6 + 6*a^5*cosh(d*x + c)^4 + 4*a^5*cosh(d*x + c)^2 + 4*(7*a^5*co
sh(d*x + c)^2 + a^5)*sinh(d*x + c)^6 + 8*(7*a^5*cosh(d*x + c)^3 + 3*a^5*cos
h(d*x + c))*sinh(d*x + c)^5 + a^5 + 2*(35*a^5*cosh(d*x + c)^4 + 30*a^5*cosh
(d*x + c)^2 + 3*a^5)*sinh(d*x + c)^4 + 8*(7*a^5*cosh(d*x + c)^5 + 10*a^5*co
sh(d*x + c)^3 + 3*a^5*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^5*cosh(d*x +
c)^6 + 15*a^5*cosh(d*x + c)^4 + 9*a^5*cosh(d*x + c)^2 + a^5)*sinh(d*x + c)^
2 + 8*(a^5*cosh(d*x + c)^7 + 3*a^5*cosh(d*x + c)^5 + 3*a^5*cosh(d*x + c)^3
+ a^5*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))) + 16*(18*a^5*cosh(d*x + c)^5 + 27*a^5*cosh(d*x + c)^3 + 11*a^
5*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh
(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x
 + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cos
h(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*
cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.615705, size = 95, normalized size = 0.95

$$\begin{cases} 32a^5x - \frac{16a^5\log(\tanh(c+dx)+1)}{d} - \frac{a^5\tanh^4(c+dx)}{4d} - \frac{5a^5\tanh^3(c+dx)}{3d} - \frac{11a^5\tanh^2(c+dx)}{2d} - \frac{15a^5\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\tanh(c)+a)^5 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))**5,x)
```

```
[Out] Piecewise((32*a**5*x - 16*a**5*log(tanh(c + d*x) + 1)/d - a**5*tanh(c + d*x
)**4/(4*d) - 5*a**5*tanh(c + d*x)**3/(3*d) - 11*a**5*tanh(c + d*x)**2/(2*d)
 - 15*a**5*tanh(c + d*x)/d, Ne(d, 0)), (x*(a*tanh(c) + a)**5, True))
```

Giac [A] time = 1.20261, size = 101, normalized size = 1.01

$$\frac{4}{3}a^5\left(\frac{12\log\left(e^{(2dx+2c)}+1\right)}{d}+\frac{48e^{(6dx+6c)}+108e^{(4dx+4c)}+88e^{(2dx+2c)}+25}{d\left(e^{(2dx+2c)}+1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 4/3*a^5*(12*log(e^(2*d*x + 2*c) + 1)/d + (48*e^(6*d*x + 6*c) + 108*e^(4*d*x
 + 4*c) + 88*e^(2*d*x + 2*c) + 25)/(d*(e^(2*d*x + 2*c) + 1)^4))
```

3.42 $\int (a+a\tanh(c+dx))^4\,dx$

Optimal. Leaf size=77

$$-\frac{4a^4\tanh(c+dx)}{d}-\frac{\left(a^2\tanh(c+dx)+a^2\right)^2}{d}+\frac{8a^4\log(\cosh(c+dx))}{d}+8a^4x-\frac{a(a\tanh(c+dx)+a)^3}{3d}$$

```
[Out] 8*a^4*x + (8*a^4*Log[Cosh[c + d*x]])/d - (4*a^4*Tanh[c + d*x])/d - (a*(a +
a*Tanh[c + d*x])^3)/(3*d) - (a^2 + a^2*Tanh[c + d*x])^2/d
```

Rubi [A] time = 0.0527818, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3478, 3477, 3475}

$$-\frac{4a^4\tanh(c+dx)}{d}-\frac{\left(a^2\tanh(c+dx)+a^2\right)^2}{d}+\frac{8a^4\log(\cosh(c+dx))}{d}+8a^4x-\frac{a(a\tanh(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tanh[c + d*x])^4,x]
```

```
[Out] 8*a^4*x + (8*a^4*Log[Cosh[c + d*x]])/d - (4*a^4*Tanh[c + d*x])/d - (a*(a +
a*Tanh[c + d*x])^3)/(3*d) - (a^2 + a^2*Tanh[c + d*x])^2/d
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\tanh(c+dx))^4\,dx=-\frac{a(a+a\tanh(c+dx))^3}{3d}+(2a)\int (a+a\tanh(c+dx))^3\,dx$$

$$=-\frac{a(a+a\tanh(c+dx))^3}{3d}-\frac{\left(a^2+a^2\tanh(c+dx)\right)^2}{d}+\left(4a^2\right)\int (a+a\tanh(c+dx))^2\,dx$$

$$=8a^4x-\frac{4a^4\tanh(c+dx)}{d}-\frac{a(a+a\tanh(c+dx))^3}{3d}-\frac{\left(a^2+a^2\tanh(c+dx)\right)^2}{d}+\left(8a^4\right)\int \text{ta}$$

$$=8a^4x+\frac{8a^4\log(\cosh(c+dx))}{d}-\frac{4a^4\tanh(c+dx)}{d}-\frac{a(a+a\tanh(c+dx))^3}{3d}-\frac{\left(a^2+a^2\tan\right.}{d}$$

Mathematica [B] time = 1.08826, size = 178, normalized size = 2.31

$$a^4\text{sech}(c)\text{sech}^3(c+dx)(\sinh(4dx)+\cosh(4dx))(12\sinh(2c+dx)-11\sinh(2c+3dx)+6dx\cosh(2c+3dx)+6dx\,\text{co}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tanh[c + d*x])^4,x]
```

```
[Out] (a^4*Sech[c]*Sech[c + d*x]^3*(Cosh[4*d*x] + Sinh[4*d*x])*(6*d*x*Cosh[2*c +
3*d*x] + 6*d*x*Cosh[4*c + 3*d*x] + 6*Cosh[2*c + 3*d*x]*Log[Cosh[c + d*x]] +
 6*Cosh[4*c + 3*d*x]*Log[Cosh[c + d*x]] + 6*Cosh[d*x]*(1 + 3*d*x + 3*Log[Co
sh[c + d*x]]) + 6*Cosh[2*c + d*x]*(1 + 3*d*x + 3*Log[Cosh[c + d*x]]) - 21*S
inh[d*x] + 12*Sinh[2*c + d*x] - 11*Sinh[2*c + 3*d*x]))/(6*d*(Cosh[d*x] + Si
nh[d*x])^4)
```

Maple [A] time = 0.003, size = 65, normalized size = 0.8

$$-\frac{a^4(\tanh(dx+c))^3}{3d}-2\frac{a^4(\tanh(dx+c))^2}{d}-7\frac{a^4\tanh(dx+c)}{d}-8\frac{a^4\ln(\tanh(dx+c)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tanh(d*x+c))^4,x)
```

```
[Out] -1/3/d*a^4*tanh(d*x+c)^3-2/d*a^4*tanh(d*x+c)^2-7*a^4*tanh(d*x+c)/d-8/d*a^4*
ln(tanh(d*x+c)-1)
```

Maxima [B] time = 1.65821, size = 265, normalized size = 3.44

$$\frac{1}{3}a^4\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+4a^4\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2c}}{d\left(2e^{(-2dx-2c)}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a^4*(
x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x
- 2*c) + e^(-4*d*x - 4*c) + 1))) + 6*a^4*(x + c/d - 2/(d*(e^(-2*d*x - 2*c)
+ 1))) + a^4*x + 4*a^4*log(cosh(d*x + c))/d
```

Fricas [B] time = 2.34099, size = 1470, normalized size = 19.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 4/3*(18*a^4*cosh(d*x + c)^4 + 72*a^4*cosh(d*x + c)*sinh(d*x + c)^3 + 18*a^4
*sinh(d*x + c)^4 + 27*a^4*cosh(d*x + c)^2 + 11*a^4 + 27*(4*a^4*cosh(d*x + c
)^2 + a^4)*sinh(d*x + c)^2 + 6*(a^4*cosh(d*x + c)^6 + 6*a^4*cosh(d*x + c)*s
inh(d*x + c)^5 + a^4*sinh(d*x + c)^6 + 3*a^4*cosh(d*x + c)^4 + 3*a^4*cosh(d
*x + c)^2 + 3*(5*a^4*cosh(d*x + c)^2 + a^4)*sinh(d*x + c)^4 + a^4 + 4*(5*a^
4*cosh(d*x + c)^3 + 3*a^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^4*cosh(d*
x + c)^4 + 6*a^4*cosh(d*x + c)^2 + a^4)*sinh(d*x + c)^2 + 6*(a^4*cosh(d*x +
 c)^5 + 2*a^4*cosh(d*x + c)^3 + a^4*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 18*(4*a^4*cosh(d*x + c)^3 + 3
*a^4*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*s
inh(d*x + c)^5 + d*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.422028, size = 76, normalized size = 0.99

$$\begin{cases} 16a^4x - \frac{8a^4\log(\tanh(c+dx)+1)}{d} - \frac{a^4\tanh^3(c+dx)}{3d} - \frac{2a^4\tanh^2(c+dx)}{d} - \frac{7a^4\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a\tanh(c)+a\right)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))**4,x)
```

```
[Out] Piecewise((16*a**4*x - 8*a**4*log(tanh(c + d*x) + 1)/d - a**4*tanh(c + d*x)
**3/(3*d) - 2*a**4*tanh(c + d*x)**2/d - 7*a**4*tanh(c + d*x)/d, Ne(d, 0)),
(x*(a*tanh(c) + a)**4, True))
```

Giac [A] time = 1.17487, size = 86, normalized size = 1.12

$$\frac{4}{3}a^4\left(\frac{6\log\left(e^{(2dx+2c)}+1\right)}{d} + \frac{18e^{(4dx+4c)} + 27e^{(2dx+2c)} + 11}{d\left(e^{(2dx+2c)}+1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 4/3*a^4*(6*log(e^(2*d*x + 2*c) + 1)/d + (18*e^(4*d*x + 4*c) + 27*e^(2*d*x +
 2*c) + 11)/(d*(e^(2*d*x + 2*c) + 1)^3))
```

3.43 $\int (a+a\tanh(c+dx))^3\,dx$

Optimal. Leaf size=56

$$-\frac{2a^3\tanh(c+dx)}{d}+\frac{4a^3\log(\cosh(c+dx))}{d}+4a^3x-\frac{a(a\tanh(c+dx)+a)^2}{2d}$$

```
[Out] 4*a^3*x + (4*a^3*Log[Cosh[c + d*x]])/d - (2*a^3*Tanh[c + d*x])/d - (a*(a +
a*Tanh[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.0362516, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3478, 3477, 3475}

$$-\frac{2a^3\tanh(c+dx)}{d}+\frac{4a^3\log(\cosh(c+dx))}{d}+4a^3x-\frac{a(a\tanh(c+dx)+a)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tanh[c + d*x])^3,x]
```

```
[Out] 4*a^3*x + (4*a^3*Log[Cosh[c + d*x]])/d - (2*a^3*Tanh[c + d*x])/d - (a*(a +
a*Tanh[c + d*x])^2)/(2*d)
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\tanh(c+dx))^3\,dx &= -\frac{a(a+a\tanh(c+dx))^2}{2d}+(2a)\int (a+a\tanh(c+dx))^2\,dx\\ &= 4a^3x-\frac{2a^3\tanh(c+dx)}{d}-\frac{a(a+a\tanh(c+dx))^2}{2d}+\left(4a^3\right)\int \tanh(c+dx)\,dx\\ &= 4a^3x+\frac{4a^3\log(\cosh(c+dx))}{d}-\frac{2a^3\tanh(c+dx)}{d}-\frac{a(a+a\tanh(c+dx))^2}{2d}\end{aligned}$$

Mathematica [A] time = 0.898455, size = 103, normalized size = 1.84

$$\frac{a^3\mathrm{sech}(c)\mathrm{sech}^2(c+dx)(-3\sinh(c+2dx)+2dx\cosh(3c+2dx)+2\cosh(3c+2dx)\log(\cosh(c+dx))+2\cosh(c+2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tanh[c + d*x])^3,x]
```

```
[Out] (a^3*Sech[c]*Sech[c + d*x]^2*(2*d*x*Cosh[3*c + 2*d*x] + 2*Cosh[3*c + 2*d*x]
*Log[Cosh[c + d*x]] + 2*Cosh[c + 2*d*x]*(d*x + Log[Cosh[c + d*x]]) + Cosh[c
]*(1 + 4*d*x + 4*Log[Cosh[c + d*x]]) + 3*Sinh[c] - 3*Sinh[c + 2*d*x]))/(2*d
)
```

Maple [A] time = 0.001, size = 49, normalized size = 0.9

$$-\frac{a^3 (\tanh(dx+c))^2}{2d} - 3\,\frac{a^3 \tanh(dx+c)}{d} - 4\,\frac{a^3 \ln(\tanh(dx+c)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tanh(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3*tanh(d*x+c)^2-3*a^3*tanh(d*x+c)/d-4/d*a^3*ln(tanh(d*x+c)-1)
```

Maxima [B] time = 1.61223, size = 157, normalized size = 2.8

$$a^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right) + 3\,a^3\left(x + \frac{c}{d} - \frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right) + a^3x + \frac{3\,a^3 \log(\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 3*a^3*(x + c/d - 2/(d*(e^(-2*d*x -
2*c) + 1))) + a^3*x + 3*a^3*log(cosh(d*x + c))/d
```

Fricas [B] time = 2.15635, size = 780, normalized size = 13.93

$$\frac{2\left(4\,a^3\cosh(dx+c)^2 + 8\,a^3\cosh(dx+c)\sinh(dx+c) + 4\,a^3\sinh(dx+c)^2 + 3\,a^3 + 2\left(a^3\cosh(dx+c)^4 + 4\,a^3\cosh(dx\right.\right.}{d\cosh(dx+c)^4 + 4\,d\cosh(dx+c)\sinh(dx+c)^3 + d\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 2*(4*a^3*cosh(d*x + c)^2 + 8*a^3*cosh(d*x + c)*sinh(d*x + c) + 4*a^3*sinh(d
*x + c)^2 + 3*a^3 + 2*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x +
 c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)^2 + a^3 + 2*(3*a^3*cosh(d
*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + a^3*cosh(d*x +
c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(d
*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 +
2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*co
```

```
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.280287, size = 61, normalized size = 1.09

$$\begin{cases} 8a^3x - \frac{4a^3\log(\tanh(c+dx)+1)}{d} - \frac{a^3\tanh^2(c+dx)}{2d} - \frac{3a^3\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\tanh(c)+a)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*x - 4*a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)*
*2/(2*d) - 3*a**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a*tanh(c) + a)**3, True))
```

Giac [A] time = 1.17114, size = 72, normalized size = 1.29

$$2a^3\left(\frac{2\log\left(e^{(2dx+2c)}+1\right)}{d} + \frac{4e^{(2dx+2c)}+3}{d\left(e^{(2dx+2c)}+1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*a^3*(2*log(e^(2*d*x + 2*c) + 1)/d + (4*e^(2*d*x + 2*c) + 3)/(d*(e^(2*d*x
+ 2*c) + 1)^2))
```

3.44 $\int (a + a\tanh(c + dx))^2 \, dx$

Optimal. Leaf size=36

$$-\frac{a^2 \tanh(c+dx)}{d} + \frac{2a^2 \log(\cosh(c+dx))}{d} + 2a^2 x$$

```
[Out] 2*a^2*x + (2*a^2*Log[Cosh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0206018, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3477, 3475}

$$-\frac{a^2 \tanh(c+dx)}{d} + \frac{2a^2 \log(\cosh(c+dx))}{d} + 2a^2 x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tanh[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (2*a^2*Log[Cosh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + a\tanh(c+dx))^2 \, dx &= 2a^2 x - \frac{a^2 \tanh(c+dx)}{d} + \left(2a^2\right) \int \tanh(c+dx) \, dx \\ &= 2a^2 x + \frac{2a^2 \log(\cosh(c+dx))}{d} - \frac{a^2 \tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.5045, size = 58, normalized size = 1.61

$$\frac{a^2 \text{sech}(c)\text{sech}(c+dx)(\cosh(dx)(\log(\cosh(c+dx)) + dx) + \cosh(2c+dx)(\log(\cosh(c+dx)) + dx) - \sinh(dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tanh[c + d*x])^2,x]
```

```
[Out] (a^2*Sech[c]*Sech[c + d*x]*(Cosh[d*x]*(d*x + Log[Cosh[c + d*x]]) + Cosh[2*c
 + d*x]*(d*x + Log[Cosh[c + d*x]]) - Sinh[d*x]))/d
```

Maple [A] time = 0.003, size = 33, normalized size = 0.9

$$-\frac{a^{2}\tanh\left(dx+c\right)}{d}-2\,\frac{a^{2}\ln\left(\tanh\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tanh(d*x+c))^2,x)
```

```
[Out] -a^2*tanh(d*x+c)/d-2*a^2/d*ln(tanh(d*x+c)-1)
```

Maxima [A] time = 1.07999, size = 68, normalized size = 1.89

$$a^{2}\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+a^{2}x+\frac{2\,a^{2}\log\left(\cosh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*x + 2*a^2*log(cosh(d*x +
 c))/d
```

Fricas [B] time = 2.19895, size = 308, normalized size = 8.56

$$\frac{2\left(a^{2}+\left(a^{2}\cosh\left(dx+c\right)^{2}+2\,a^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^{2}\sinh\left(dx+c\right)^{2}+a^{2}\right)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)}{d\cosh\left(dx+c\right)^{2}+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^{2}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2*(a^2 + (a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sin
h(d*x + c)^2 + a^2)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(
d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d
)
```

Sympy [A] time = 0.296032, size = 44, normalized size = 1.22

$$\begin{cases} 4a^{2}x-\frac{2a^{2}\log\left(\tanh\left(c+dx\right)+1\right)}{d}-\frac{a^{2}\tanh\left(c+dx\right)}{d} & \text{for}\: d\neq 0\\ x\left(a\tanh\left(c\right)+a\right)^{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))**2,x)
```

```
[Out] Piecewise((4*a**2*x - 2*a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)/
d, Ne(d, 0)), (x*(a*tanh(c) + a)**2, True))
```

Giac [A] time = 1.16192, size = 53, normalized size = 1.47

$$2\,a^2\left(\frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{d}+\frac{1}{d\left(e^{(2\,dx+2\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*a^2*(log(e^(2*d*x + 2*c) + 1)/d + 1/(d*(e^(2*d*x + 2*c) + 1)))
```

3.45 $\int \frac{1}{a+a\tanh(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{x}{2a}-\frac{1}{2d(a\tanh(c+dx)+a)}$$

```
[Out] x/(2*a) - 1/(2*d*(a + a*Tanh[c + d*x]))
```

Rubi [A] time = 0.0128619, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3479, 8}

$$\frac{x}{2a}-\frac{1}{2d(a\tanh(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tanh[c + d*x])^(-1),x]
```

```
[Out] x/(2*a) - 1/(2*d*(a + a*Tanh[c + d*x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\tanh(c+dx)}\,dx &= -\frac{1}{2d(a+a\tanh(c+dx))}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}-\frac{1}{2d(a+a\tanh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.100759, size = 39, normalized size = 1.39

$$\frac{(2dx+1)\tanh(c+dx)+2dx-1}{4ad(\tanh(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tanh[c + d*x])^(-1),x]
```

```
[Out] (-1 + 2*d*x + (1 + 2*d*x)*Tanh[c + d*x])/(4*a*d*(1 + Tanh[c + d*x]))
```

Maple [B] time = 0.017, size = 54, normalized size = 1.9

$$-\frac{1}{2da\left(\tanh\left(dx+c\right)+1\right)}+\frac{\ln\left(\tanh\left(dx+c\right)+1\right)}{4da}-\frac{\ln\left(\tanh\left(dx+c\right)-1\right)}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tanh(d*x+c)),x)
```

```
[Out] -1/2/d/a/(tanh(d*x+c)+1)+1/4/d/a*ln(tanh(d*x+c)+1)-1/4/d/a*ln(tanh(d*x+c)-1
)
```

Maxima [A] time = 1.08422, size = 42, normalized size = 1.5

$$\frac{dx+c}{2\,ad}-\frac{e^{\left(-2\,dx-2\,c\right)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(a*d) - 1/4*e^(-2*d*x - 2*c)/(a*d)
```

Fricas [B] time = 2.19028, size = 136, normalized size = 4.86

$$\frac{\left(2\,dx-1\right)\cosh\left(dx+c\right)+\left(2\,dx+1\right)\sinh\left(dx+c\right)}{4\,\left(ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*d*x - 1)*cosh(d*x + c) + (2*d*x + 1)*sinh(d*x + c))/(a*d*cosh(d*x +
 c) + a*d*sinh(d*x + c))
```

Sympy [A] time = 0.8272, size = 73, normalized size = 2.61

$$\begin{cases}\frac{dx\tanh\left(c+dx\right)}{2ad\tanh\left(c+dx\right)+2ad}+\frac{dx}{2ad\tanh\left(c+dx\right)+2ad}-\frac{1}{2ad\tanh\left(c+dx\right)+2ad} & \text{for}\ d\neq 0\\ \frac{x}{a\tanh\left(c\right)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c)),x)
```

```
[Out] Piecewise(((d*x*tanh(c + d*x)/(2*a*d*tanh(c + d*x) + 2*a*d) + d*x/(2*a*d*tan
h(c + d*x) + 2*a*d) - 1/(2*a*d*tanh(c + d*x) + 2*a*d), Ne(d, 0)), (x/(a*tan
h(c) + a), True))
```

Giac [A] time = 1.18865, size = 36, normalized size = 1.29

$$\frac{2\,dx + 2\,c - e^{(-2\,dx - 2\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*d*x + 2*c - e^(-2*d*x - 2*c))/(a*d)
```

3.46 $\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{4d\left(a^2\tanh(c+dx)+a^2\right)}+\frac{x}{4a^2}-\frac{1}{4d(a\tanh(c+dx)+a)^2}$$

```
[Out] x/(4*a^2) - 1/(4*d*(a + a*Tanh[c + d*x])^2) - 1/(4*d*(a^2 + a^2*Tanh[c + d*
x]))
```

Rubi [A] time = 0.0275757, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3479, 8}

$$-\frac{1}{4d\left(a^2\tanh(c+dx)+a^2\right)}+\frac{x}{4a^2}-\frac{1}{4d(a\tanh(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tanh[c + d*x])^(-2),x]
```

```
[Out] x/(4*a^2) - 1/(4*d*(a + a*Tanh[c + d*x])^2) - 1/(4*d*(a^2 + a^2*Tanh[c + d*
x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx &= -\frac{1}{4d(a+a\tanh(c+dx))^2}+\frac{\int \frac{1}{a+a\tanh(c+dx)}\,dx}{2a}\\ &= -\frac{1}{4d(a+a\tanh(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\tanh(c+dx)\right)}+\frac{\int 1\,dx}{4a^2}\\ &= \frac{x}{4a^2}-\frac{1}{4d(a+a\tanh(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\tanh(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.170535, size = 60, normalized size = 1.18

$$\frac{\operatorname{sech}^2(c+dx)((4dx+1)\sinh(2(c+dx))+(4dx-1)\cosh(2(c+dx))-4)}{16a^2d(\tanh(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tanh[c + d*x])^(-2),x]
```

```
[Out] (Sech[c + d*x]^2*(-4 + (-1 + 4*d*x)*Cosh[2*(c + d*x)] + (1 + 4*d*x)*Sinh[2*
(c + d*x)]))/(16*a^2*d*(1 + Tanh[c + d*x])^2)
```

Maple [A] time = 0.018, size = 72, normalized size = 1.4

$$-\frac{1}{4\,da^2\left(\tanh\left(dx+c\right)+1\right)^2}-\frac{1}{4\,da^2\left(\tanh\left(dx+c\right)+1\right)}+\frac{\ln\left(\tanh\left(dx+c\right)+1\right)}{8\,da^2}-\frac{\ln\left(\tanh\left(dx+c\right)-1\right)}{8\,da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tanh(d*x+c))^2,x)
```

```
[Out] -1/4/d/a^2/(tanh(d*x+c)+1)^2-1/4/d/a^2/(tanh(d*x+c)+1)+1/8/d/a^2*ln(tanh(d*
x+c)+1)-1/8/d/a^2*ln(tanh(d*x+c)-1)
```

Maxima [A] time = 1.08075, size = 58, normalized size = 1.14

$$\frac{dx+c}{4\,a^2d}-\frac{4\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(d*x + c)/(a^2*d) - 1/16*(4*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))/(a^2*d
)
```

Fricas [B] time = 2.22352, size = 270, normalized size = 5.29

$$\frac{(4\,dx-1)\cosh\left(dx+c\right)^2+2\,(4\,dx+1)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(4\,dx-1)\sinh\left(dx+c\right)^2-4}{16\left(a^2d\cosh\left(dx+c\right)^2+2\,a^2d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^2d\sinh\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*((4*d*x - 1)*cosh(d*x + c)^2 + 2*(4*d*x + 1)*cosh(d*x + c)*sinh(d*x +
c) + (4*d*x - 1)*sinh(d*x + c)^2 - 4)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh
(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2)
```

Sympy [A] time = 1.64479, size = 223, normalized size = 4.37

$$\begin{cases}\frac{dx\tanh^2(c+dx)}{4a^2d\tanh^2(c+dx)+8a^2d\tanh(c+dx)+4a^2d}+\frac{2dx\tanh(c+dx)}{4a^2d\tanh^2(c+dx)+8a^2d\tanh(c+dx)+4a^2d}+\frac{dx}{4a^2d\tanh^2(c+dx)+8a^2d\tanh(c+dx)+4a^2d}-\frac{}{4a^2d\tanh} \\ \frac{x}{(a\tanh(c)+a)^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))**2,x)
```

```
[Out] Piecewise((d*x*tanh(c + d*x)**2/(4*a**2*d*tanh(c + d*x)**2 + 8*a**2*d*tanh(
c + d*x) + 4*a**2*d) + 2*d*x*tanh(c + d*x)/(4*a**2*d*tanh(c + d*x)**2 + 8*a
**2*d*tanh(c + d*x) + 4*a**2*d) + d*x/(4*a**2*d*tanh(c + d*x)**2 + 8*a**2*d
*tanh(c + d*x) + 4*a**2*d) - tanh(c + d*x)/(4*a**2*d*tanh(c + d*x)**2 + 8*a
**2*d*tanh(c + d*x) + 4*a**2*d) - 2/(4*a**2*d*tanh(c + d*x)**2 + 8*a**2*d*t
anh(c + d*x) + 4*a**2*d), Ne(d, 0)), (x/(a*tanh(c) + a)**2, True))
```

Giac [A] time = 1.16991, size = 54, normalized size = 1.06

$$\frac{4\,dx - \left(4\,e^{(2\,dx+2\,c)} + 1\right)e^{(-4\,dx-4\,c)} + 4\,c}{16\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*d*x - (4*e^(2*d*x + 2*c) + 1)*e^(-4*d*x - 4*c) + 4*c)/(a^2*d)
```

3.47 $\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{8d\left(a^3\tanh(c+dx)+a^3\right)}+\frac{x}{8a^3}-\frac{1}{8ad(a\tanh(c+dx)+a)^2}-\frac{1}{6d(a\tanh(c+dx)+a)^3}$$

```
[Out] x/(8*a^3) - 1/(6*d*(a + a*Tanh[c + d*x])^3) - 1/(8*a*d*(a + a*Tanh[c + d*x]
)^2) - 1/(8*d*(a^3 + a^3*Tanh[c + d*x]))
```

Rubi [A] time = 0.0441102, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3479, 8}

$$-\frac{1}{8d\left(a^3\tanh(c+dx)+a^3\right)}+\frac{x}{8a^3}-\frac{1}{8ad(a\tanh(c+dx)+a)^2}-\frac{1}{6d(a\tanh(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tanh[c + d*x])^(-3),x]
```

```
[Out] x/(8*a^3) - 1/(6*d*(a + a*Tanh[c + d*x])^3) - 1/(8*a*d*(a + a*Tanh[c + d*x]
)^2) - 1/(8*d*(a^3 + a^3*Tanh[c + d*x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx &= -\frac{1}{6d(a+a\tanh(c+dx))^3}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx}{2a}\\ &= -\frac{1}{6d(a+a\tanh(c+dx))^3}-\frac{1}{8ad(a+a\tanh(c+dx))^2}+\frac{\int \frac{1}{a+a\tanh(c+dx)}\,dx}{4a^2}\\ &= -\frac{1}{6d(a+a\tanh(c+dx))^3}-\frac{1}{8ad(a+a\tanh(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\tanh(c+dx)\right)}+ \\ &= \frac{x}{8a^3}-\frac{1}{6d(a+a\tanh(c+dx))^3}-\frac{1}{8ad(a+a\tanh(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\tanh(c+dx)\right.}\end{aligned}$$

Mathematica [A] time = 0.24618, size = 83, normalized size = 1.14

$$\frac{\operatorname{sech}^3(c+dx)(-9\sinh(c+dx)+12dx\sinh(3(c+dx))+2\sinh(3(c+dx))-27\cosh(c+dx)+2(6dx-1)\cosh(3(c+}{96a^3d(\tanh(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tanh[c + d*x])^(-3),x]
```

```
[Out] (Sech[c + d*x]^3*(-27*Cosh[c + d*x] + 2*(-1 + 6*d*x)*Cosh[3*(c + d*x)] - 9*
Sinh[c + d*x] + 2*Sinh[3*(c + d*x)] + 12*d*x*Sinh[3*(c + d*x)]))/(96*a^3*d*
(1 + Tanh[c + d*x])^3)
```

Maple [A] time = 0.019, size = 90, normalized size = 1.2

$$-\frac{1}{6\,a^3d\,(\tanh(dx+c)+1)^3}-\frac{1}{8\,a^3d\,(\tanh(dx+c)+1)^2}-\frac{1}{8\,a^3d\,(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{16\,a^3d}-\frac{\ln(\tanh}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tanh(d*x+c))^3,x)
```

```
[Out] -1/6/a^3/d/(tanh(d*x+c)+1)^3-1/8/a^3/d/(tanh(d*x+c)+1)^2-1/8/a^3/d/(tanh(d*
x+c)+1)+1/16/a^3/d*ln(tanh(d*x+c)+1)-1/16/a^3/d*ln(tanh(d*x+c)-1)
```

Maxima [A] time = 1.13624, size = 76, normalized size = 1.04

$$\frac{dx+c}{8\,a^3d}-\frac{18\,e^{(-2\,dx-2\,c)}+9\,e^{(-4\,dx-4\,c)}+2\,e^{(-6\,dx-6\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x + c)/(a^3*d) - 1/96*(18*e^(-2*d*x - 2*c) + 9*e^(-4*d*x - 4*c) + 2*
e^(-6*d*x - 6*c))/(a^3*d)
```

Fricas [B] time = 2.11768, size = 428, normalized size = 5.86

$$\frac{2\,(6\,dx-1)\cosh(dx+c)^3+6\,(6\,dx-1)\cosh(dx+c)\sinh(dx+c)^2+2\,(6\,dx+1)\sinh(dx+c)^3+3\,\big(2\,(6\,dx+1)\cosh(}{96\,\big(a^3d\cosh(dx+c)^3+3\,a^3d\cosh(dx+c)^2\sinh(dx+c)+3\,a^3d\cosh(dx+c)\sinh(dx+c)^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(2*(6*d*x - 1)*cosh(d*x + c)^3 + 6*(6*d*x - 1)*cosh(d*x + c)*sinh(d*x
+ c)^2 + 2*(6*d*x + 1)*sinh(d*x + c)^3 + 3*(2*(6*d*x + 1)*cosh(d*x + c)^2 -
 3)*sinh(d*x + c) - 27*cosh(d*x + c))/(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh
(d*x + c)^2*sinh(d*x + c) + 3*a^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*d*s
inh(d*x + c)^3)
```

Sympy [A] time = 2.01707, size = 430, normalized size = 5.89

$$\begin{cases}\frac{3dx\tanh^3(c+dx)}{24a^3d\tanh^3(c+dx)+72a^3d\tanh^2(c+dx)+72a^3d\tanh(c+dx)+24a^3d}+\frac{9dx\tanh^2(c+dx)}{24a^3d\tanh^3(c+dx)+72a^3d\tanh^2(c+dx)+72a^3d\tanh(c+dx)+24a^3d}+\frac{}{24a^3d\tanh^3} \\ \frac{x}{(a\tanh(c)+a)^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))**3,x)
```

```
[Out] Piecewise((3*d*x*tanh(c + d*x)**3/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*t
anh(c + d*x)**2 + 72*a**3*d*tanh(c + d*x) + 24*a**3*d) + 9*d*x*tanh(c + d*x
)**2/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*tanh(c + d*x)**2 + 72*a**3*d*t
anh(c + d*x) + 24*a**3*d) + 9*d*x*tanh(c + d*x)/(24*a**3*d*tanh(c + d*x)**3
 + 72*a**3*d*tanh(c + d*x)**2 + 72*a**3*d*tanh(c + d*x) + 24*a**3*d) + 3*d*
x/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*tanh(c + d*x)**2 + 72*a**3*d*tanh
(c + d*x) + 24*a**3*d) - 3*tanh(c + d*x)**2/(24*a**3*d*tanh(c + d*x)**3 + 7
2*a**3*d*tanh(c + d*x)**2 + 72*a**3*d*tanh(c + d*x) + 24*a**3*d) - 9*tanh(c
 + d*x)/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*tanh(c + d*x)**2 + 72*a**3*
d*tanh(c + d*x) + 24*a**3*d) - 10/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*t
anh(c + d*x)**2 + 72*a**3*d*tanh(c + d*x) + 24*a**3*d), Ne(d, 0)), (x/(a*ta
nh(c) + a)**3, True))
```

Giac [A] time = 1.19553, size = 69, normalized size = 0.95

$$\frac{12\,dx - \left(18\,e^{(4\,dx+4\,c)} + 9\,e^{(2\,dx+2\,c)} + 2\right)e^{(-6\,dx-6\,c)} + 12\,c}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(12*d*x - (18*e^(4*d*x + 4*c) + 9*e^(2*d*x + 2*c) + 2)*e^(-6*d*x - 6*c
) + 12*c)/(a^3*d)
```

3.48 $\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{16d\left(a^4\tanh(c+dx)+a^4\right)}-\frac{1}{16d\left(a^2\tanh(c+dx)+a^2\right)^2}+\frac{x}{16a^4}-\frac{1}{12ad(a\tanh(c+dx)+a)^3}-\frac{1}{8d(a\tanh(c+dx)+$$

```
[Out] x/(16*a^4) - 1/(8*d*(a + a*Tanh[c + d*x])^4) - 1/(12*a*d*(a + a*Tanh[c + d*
x])^3) - 1/(16*d*(a^2 + a^2*Tanh[c + d*x])^2) - 1/(16*d*(a^4 + a^4*Tanh[c +
 d*x]))
```

Rubi [A] time = 0.0621694, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3479, 8}

$$-\frac{1}{16d\left(a^4\tanh(c+dx)+a^4\right)}-\frac{1}{16d\left(a^2\tanh(c+dx)+a^2\right)^2}+\frac{x}{16a^4}-\frac{1}{12ad(a\tanh(c+dx)+a)^3}-\frac{1}{8d(a\tanh(c+dx)+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tanh[c + d*x])^(-4),x]
```


```
[Out] x/(16*a^4) - 1/(8*d*(a + a*Tanh[c + d*x])^4) - 1/(12*a*d*(a + a*Tanh[c + d*
x])^3) - 1/(16*d*(a^2 + a^2*Tanh[c + d*x])^2) - 1/(16*d*(a^4 + a^4*Tanh[c +
 d*x]))
```


Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx = -\frac{1}{8d(a+a\tanh(c+dx))^4}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx}{2a}$$

$$= -\frac{1}{8d(a+a\tanh(c+dx))^4}-\frac{1}{12ad(a+a\tanh(c+dx))^3}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx}{4a^2}$$

$$= -\frac{1}{8d(a+a\tanh(c+dx))^4}-\frac{1}{12ad(a+a\tanh(c+dx))^3}-\frac{1}{16d\left(a^2+a^2\tanh(c+dx)\right)^2}+$$

$$= -\frac{1}{8d(a+a\tanh(c+dx))^4}-\frac{1}{12ad(a+a\tanh(c+dx))^3}-\frac{1}{16d\left(a^2+a^2\tanh(c+dx)\right)^2}-$$

$$= \frac{x}{16a^4}-\frac{1}{8d(a+a\tanh(c+dx))^4}-\frac{1}{12ad(a+a\tanh(c+dx))^3}-\frac{1}{16d\left(a^2+a^2\tanh(c+dx)\right.}$$

Mathematica [A] time = 0.249169, size = 88, normalized size = 0.92

$$\frac{\operatorname{sech}^4(c+dx)(-32\sinh(2(c+dx))+24dx\sinh(4(c+dx))+3\sinh(4(c+dx))-64\cosh(2(c+dx))+3(8dx-1)\cosh}{384a^4d(\tanh(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tanh[c + d*x])^(-4),x]
```

```
[Out] (Sech[c + d*x]^4*(-36 - 64*Cosh[2*(c + d*x)] + 3*(-1 + 8*d*x)*Cosh[4*(c + d
*x)] - 32*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)] + 24*d*x*Sinh[4*(c + d*x)
]))/(384*a^4*d*(1 + Tanh[c + d*x])^4)
```

Maple [A] time = 0.02, size = 108, normalized size = 1.1

$$-\frac{1}{8\,da^4\left(\tanh\left(dx+c\right)+1\right)^4}-\frac{1}{12\,da^4\left(\tanh\left(dx+c\right)+1\right)^3}-\frac{1}{16\,da^4\left(\tanh\left(dx+c\right)+1\right)^2}-\frac{1}{16\,da^4\left(\tanh\left(dx+c\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tanh(d*x+c))^4,x)
```

```
[Out] -1/8/d/a^4/(tanh(d*x+c)+1)^4-1/12/d/a^4/(tanh(d*x+c)+1)^3-1/16/d/a^4/(tanh(
d*x+c)+1)^2-1/16/d/a^4/(tanh(d*x+c)+1)+1/32/d/a^4*ln(tanh(d*x+c)+1)-1/32/d/
a^4*ln(tanh(d*x+c)-1)
```

Maxima [A] time = 1.07848, size = 90, normalized size = 0.94

$$\frac{dx+c}{16\,a^4d}-\frac{48\,e^{(-2\,dx-2\,c)}+36\,e^{(-4\,dx-4\,c)}+16\,e^{(-6\,dx-6\,c)}+3\,e^{(-8\,dx-8\,c)}}{384\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/16*(d*x + c)/(a^4*d) - 1/384*(48*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 16*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c))/(a^4*d)
```

Fricas [B] time = 2.15626, size = 594, normalized size = 6.19

$$\frac{3\left(8\,dx-1\right)\cosh\left(dx+c\right)^4+12\left(8\,dx+1\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+3\left(8\,dx-1\right)\sinh\left(dx+c\right)^4+2\left(9\left(8\,dx-1\right)\mathrm{co}\right.}{384\left(a^4d\cosh\left(dx+c\right)^4+4\,a^4d\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)+6\,a^4d\cosh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(3*(8*d*x - 1)*cosh(d*x + c)^4 + 12*(8*d*x + 1)*cosh(d*x + c)*sinh(d*
x + c)^3 + 3*(8*d*x - 1)*sinh(d*x + c)^4 + 2*(9*(8*d*x - 1)*cosh(d*x + c)^2
 - 32)*sinh(d*x + c)^2 - 64*cosh(d*x + c)^2 + 4*(3*(8*d*x + 1)*cosh(d*x + c
)^3 - 16*cosh(d*x + c))*sinh(d*x + c) - 36)/(a^4*d*cosh(d*x + c)^4 + 4*a^4*
```

```
d*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^4*d*cosh(d*x + c)^2*sinh(d*x + c)^2 +
 4*a^4*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^4*d*sinh(d*x + c)^4)
```

Sympy [A] time = 2.60847, size = 694, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))**4,x)
```

```
[Out] Piecewise((3*d*x*tanh(c + d*x)**4/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*
tanh(c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) +
 48*a**4*d) + 12*d*x*tanh(c + d*x)**3/(48*a**4*d*tanh(c + d*x)**4 + 192*a**
4*d*tanh(c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*
x) + 48*a**4*d) + 18*d*x*tanh(c + d*x)**2/(48*a**4*d*tanh(c + d*x)**4 + 192
*a**4*d*tanh(c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c
+ d*x) + 48*a**4*d) + 12*d*x*tanh(c + d*x)/(48*a**4*d*tanh(c + d*x)**4 + 19
2*a**4*d*tanh(c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c
 + d*x) + 48*a**4*d) + 3*d*x/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(
c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a
**4*d) - 3*tanh(c + d*x)**3/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(c
 + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a*
*4*d) - 12*tanh(c + d*x)**2/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(c
 + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a*
*4*d) - 19*tanh(c + d*x)/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(c +
d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a**4*
d) - 16/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(c + d*x)**3 + 288*a**
4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a**4*d), Ne(d, 0)), (x
/(a*tanh(c) + a)**4, True))
```

Giac [A] time = 1.18606, size = 84, normalized size = 0.88

$$\frac{24\,dx - \left(48\,e^{(6\,dx+6\,c)} + 36\,e^{(4\,dx+4\,c)} + 16\,e^{(2\,dx+2\,c)} + 3\right)e^{(-8\,dx-8\,c)} + 24\,c}{384\,a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/384*(24*d*x - (48*e^(6*d*x + 6*c) + 36*e^(4*d*x + 4*c) + 16*e^(2*d*x + 2*
c) + 3)*e^(-8*d*x - 8*c) + 24*c)/(a^4*d)
```

3.49 $\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx$

Optimal. Leaf size=121

$$-\frac{1}{32d\left(a^5\tanh(c+dx)+a^5\right)}-\frac{1}{32ad\left(a^2\tanh(c+dx)+a^2\right)^2}-\frac{1}{24a^2d(a\tanh(c+dx)+a)^3}+\frac{x}{32a^5}-\frac{1}{16ad(a\tanh(c$$

```
[Out] x/(32*a^5) - 1/(10*d*(a + a*Tanh[c + d*x])^5) - 1/(16*a*d*(a + a*Tanh[c + d
*x])^4) - 1/(24*a^2*d*(a + a*Tanh[c + d*x])^3) - 1/(32*a*d*(a^2 + a^2*Tanh[
c + d*x])^2) - 1/(32*d*(a^5 + a^5*Tanh[c + d*x]))
```

Rubi [A] time = 0.082235, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3479, 8}

$$-\frac{1}{32d\left(a^5\tanh(c+dx)+a^5\right)}-\frac{1}{32ad\left(a^2\tanh(c+dx)+a^2\right)^2}-\frac{1}{24a^2d(a\tanh(c+dx)+a)^3}+\frac{x}{32a^5}-\frac{1}{16ad(a\tanh(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tanh[c + d*x])^(-5),x]
```

```
[Out] x/(32*a^5) - 1/(10*d*(a + a*Tanh[c + d*x])^5) - 1/(16*a*d*(a + a*Tanh[c + d
*x])^4) - 1/(24*a^2*d*(a + a*Tanh[c + d*x])^3) - 1/(32*a*d*(a^2 + a^2*Tanh[
c + d*x])^2) - 1/(32*d*(a^5 + a^5*Tanh[c + d*x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx &= -\frac{1}{10d(a+a\tanh(c+dx))^5}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx}{2a}\\
&= -\frac{1}{10d(a+a\tanh(c+dx))^5}-\frac{1}{16ad(a+a\tanh(c+dx))^4}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx}{4a^2}\\
&= -\frac{1}{10d(a+a\tanh(c+dx))^5}-\frac{1}{16ad(a+a\tanh(c+dx))^4}-\frac{1}{24a^2d(a+a\tanh(c+dx))^3}\\
&= -\frac{1}{10d(a+a\tanh(c+dx))^5}-\frac{1}{16ad(a+a\tanh(c+dx))^4}-\frac{1}{24a^2d(a+a\tanh(c+dx))^3}\\
&= -\frac{1}{10d(a+a\tanh(c+dx))^5}-\frac{1}{16ad(a+a\tanh(c+dx))^4}-\frac{1}{24a^2d(a+a\tanh(c+dx))^3}\\
&= \frac{x}{32a^5}-\frac{1}{10d(a+a\tanh(c+dx))^5}-\frac{1}{16ad(a+a\tanh(c+dx))^4}-\frac{1}{24a^2d(a+a\tanh(c+
\end{aligned}$$

Mathematica [A] time = 0.275754, size = 109, normalized size = 0.9

$$\frac{\operatorname{sech}^5(c+dx)(-100\sinh(c+dx)-225\sinh(3(c+dx))+120dx\sinh(5(c+dx))+12\sinh(5(c+dx))-500\cosh(c+dx)}{3840a^5d(\tanh(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tanh[c + d*x])^(-5),x]
```

```
[Out] (Sech[c + d*x]^5*(-500*Cosh[c + d*x] - 375*Cosh[3*(c + d*x)] - 12*Cosh[5*(c
 + d*x)] + 120*d*x*Cosh[5*(c + d*x)] - 100*Sinh[c + d*x] - 225*Sinh[3*(c +
d*x)] + 12*Sinh[5*(c + d*x)] + 120*d*x*Sinh[5*(c + d*x)]))/(3840*a^5*d*(1 +
 Tanh[c + d*x])^5)
```

Maple [A] time = 0.027, size = 126, normalized size = 1.

$$-\frac{1}{10\,da^5\,(\tanh(dx+c)+1)^5}-\frac{1}{16\,da^5\,(\tanh(dx+c)+1)^4}-\frac{1}{24\,da^5\,(\tanh(dx+c)+1)^3}-\frac{1}{32\,da^5\,(\tanh(dx+c)+1)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tanh(d*x+c))^5,x)
```

```
[Out] -1/10/d/a^5/(tanh(d*x+c)+1)^5-1/16/d/a^5/(tanh(d*x+c)+1)^4-1/24/d/a^5/(tanh
(d*x+c)+1)^3-1/32/d/a^5/(tanh(d*x+c)+1)^2-1/32/d/a^5/(tanh(d*x+c)+1)+1/64/d
/a^5*ln(tanh(d*x+c)+1)-1/64/d/a^5*ln(tanh(d*x+c)-1)
```

Maxima [A] time = 1.09581, size = 105, normalized size = 0.87

$$\frac{dx+c}{32\,a^5d}-\frac{300\,e^{(-2\,dx-2\,c)}+300\,e^{(-4\,dx-4\,c)}+200\,e^{(-6\,dx-6\,c)}+75\,e^{(-8\,dx-8\,c)}+12\,e^{(-10\,dx-10\,c)}}{3840\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/32*(d*x + c)/(a^5*d) - 1/3840*(300*e^(-2*d*x - 2*c) + 300*e^(-4*d*x - 4*c
) + 200*e^(-6*d*x - 6*c) + 75*e^(-8*d*x - 8*c) + 12*e^(-10*d*x - 10*c))/(a^
5*d)
```

Fricas [B] time = 2.19692, size = 798, normalized size = 6.6

$$\frac{12\,(10\,dx-1)\cosh(dx+c)^5+60\,(10\,dx-1)\cosh(dx+c)\sinh(dx+c)^4+12\,(10\,dx+1)\sinh(dx+c)^5+15\,\big(8\,(10\,dx+}{3840\,\big(a^5d\cosh(dx+c)^5+5\,a^5d\cosh(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3840*(12*(10*d*x - 1)*cosh(d*x + c)^5 + 60*(10*d*x - 1)*cosh(d*x + c)*sin
h(d*x + c)^4 + 12*(10*d*x + 1)*sinh(d*x + c)^5 + 15*(8*(10*d*x + 1)*cosh(d*
```

```
x + c)^2 - 15)*sinh(d*x + c)^3 - 375*cosh(d*x + c)^3 + 15*(8*(10*d*x - 1)*c
osh(d*x + c)^3 - 75*cosh(d*x + c))*sinh(d*x + c)^2 + 5*(12*(10*d*x + 1)*cos
h(d*x + c)^4 - 135*cosh(d*x + c)^2 - 20)*sinh(d*x + c) - 500*cosh(d*x + c))
/(a^5*d*cosh(d*x + c)^5 + 5*a^5*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^5*d*
cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^5*d*cosh(d*x + c)^2*sinh(d*x + c)^3
+ 5*a^5*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^5*d*sinh(d*x + c)^5)
```

Sympy [A] time = 4.17607, size = 1018, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))**5,x)
```

```
[Out] Piecewise((15*d*x*tanh(c + d*x)**5/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5
*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d
*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) + 75*d*x*tanh(c + d*x)**4/
(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*t
anh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x)
+ 480*a**5*d) + 150*d*x*tanh(c + d*x)**3/(480*a**5*d*tanh(c + d*x)**5 + 240
0*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh
(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) + 150*d*x*tanh(c + d
*x)**2/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a
**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c
+ d*x) + 480*a**5*d) + 75*d*x*tanh(c + d*x)/(480*a**5*d*tanh(c + d*x)**5 +
2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*t
anh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) + 15*d*x/(480*a**
5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c +
d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a*
*5*d) - 15*tanh(c + d*x)**4/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh
(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2
+ 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) - 75*tanh(c + d*x)**3/(480*a**5*d
*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x
)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*
d) - 155*tanh(c + d*x)**2/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c
 + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 +
2400*a**5*d*tanh(c + d*x) + 480*a**5*d) - 175*tanh(c + d*x)/(480*a**5*d*tan
h(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3
 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) -
 128/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**
5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c +
d*x) + 480*a**5*d), Ne(d, 0)), (x/(a*tanh(c) + a)**5, True))
```

Giac [A] time = 1.20414, size = 99, normalized size = 0.82

$$\frac{120\,dx - \left(300\,e^{(8\,dx+8\,c)} + 300\,e^{(6\,dx+6\,c)} + 200\,e^{(4\,dx+4\,c)} + 75\,e^{(2\,dx+2\,c)} + 12\right)e^{(-10\,dx-10\,c)} + 120\,c}{3840\,a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tanh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/3840*(120*d*x - (300*e^(8*d*x + 8*c) + 300*e^(6*d*x + 6*c) + 200*e^(4*d*x
 + 4*c) + 75*e^(2*d*x + 2*c) + 12)*e^(-10*d*x - 10*c) + 120*c)/(a^5*d)
```

3.50 $\int (1+\tanh(x))^{7/2}\,dx$

Optimal. Leaf size=57

$$8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{5}(\tanh(x)+1)^{5/2}-\frac{4}{3}(\tanh(x)+1)^{3/2}-8\sqrt{\tanh(x)+1}$$

```
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 8*Sqrt[1 + Tanh[x]] - (4*(1
+ Tanh[x])^(3/2))/3 - (2*(1 + Tanh[x])^(5/2))/5
```

Rubi [A] time = 0.0409798, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3478, 3480, 206}

$$8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{5}(\tanh(x)+1)^{5/2}-\frac{4}{3}(\tanh(x)+1)^{3/2}-8\sqrt{\tanh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x])^(7/2),x]
```

```
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 8*Sqrt[1 + Tanh[x]] - (4*(1
+ Tanh[x])^(3/2))/3 - (2*(1 + Tanh[x])^(5/2))/5
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (1+\tanh(x))^{7/2}\,dx &= -\frac{2}{5}(1+\tanh(x))^{5/2}+2\int (1+\tanh(x))^{5/2}\,dx\\
&= -\frac{4}{3}(1+\tanh(x))^{3/2}-\frac{2}{5}(1+\tanh(x))^{5/2}+4\int (1+\tanh(x))^{3/2}\,dx\\
&= -8\sqrt{1+\tanh(x)}-\frac{4}{3}(1+\tanh(x))^{3/2}-\frac{2}{5}(1+\tanh(x))^{5/2}+8\int \sqrt{1+\tanh(x)}\,dx\\
&= -8\sqrt{1+\tanh(x)}-\frac{4}{3}(1+\tanh(x))^{3/2}-\frac{2}{5}(1+\tanh(x))^{5/2}+16\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tan
\end{aligned}$$

$$= 8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-8\sqrt{1+\tanh(x)}-\frac{4}{3}(1+\tanh(x))^{3/2}-\frac{2}{5}(1+\tanh(x))^{5/2}$$

Mathematica [A] time = 0.200731, size = 65, normalized size = 1.14

$$\frac{\cosh^3(x)\left(8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)(\tanh(x)+1)^3-\frac{2}{15}(\tanh(x)+1)^{7/2}\left(16\tanh(x)-3\text{sech}^2(x)+76\right)\right)}{(\sinh(x)+\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x])^(7/2),x]
```

```
[Out] (Cosh[x]^3*(8*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]*(1 + Tanh[x])^3 -
(2*(1 + Tanh[x])^(7/2)*(76 - 3*Sech[x]^2 + 16*Tanh[x]))/15))/(Cosh[x] + Sin
h[x])^3
```

Maple [A] time = 0.019, size = 43, normalized size = 0.8

$$8\,\text{Artanh}\left(1/2\,\sqrt{1+\tanh(x)}\sqrt{2}\right)\sqrt{2}-8\,\sqrt{1+\tanh(x)}-\frac{4}{3}\,(1+\tanh(x))^{\frac{3}{2}}-\frac{2}{5}\,(1+\tanh(x))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x))^(7/2),x)
```

```
[Out] 8*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-8*(1+tanh(x))^(1/2)-4/3*(1
+tanh(x))^(3/2)-2/5*(1+tanh(x))^(5/2)
```

Maxima [A] time = 1.63342, size = 112, normalized size = 1.96

$$-4\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2\,x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2\,x)}+1}}}\right)-\frac{8\,\sqrt{2}}{\sqrt{e^{(-2\,x)}+1}}-\frac{8\,\sqrt{2}}{3\left(e^{(-2\,x)}+1\right)^{\frac{3}{2}}}-\frac{8\,\sqrt{2}}{5\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] -4*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)/s
qrt(e^(-2*x) + 1))) - 8*sqrt(2)/sqrt(e^(-2*x) + 1) - 8/3*sqrt(2)/(e^(-2*x)
+ 1)^(3/2) - 8/5*sqrt(2)/(e^(-2*x) + 1)^(5/2)
```

Fricas [B] time = 2.34353, size = 1470, normalized size = 25.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/15*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^5 + 115*sqrt(2)*cosh(x)*sinh(x)^4 + 23
*sqrt(2)*sinh(x)^5 + 5*(46*sqrt(2)*cosh(x)^2 + 7*sqrt(2))*sinh(x)^3 + 35*sq
rt(2)*cosh(x)^3 + 5*(46*sqrt(2)*cosh(x)^3 + 21*sqrt(2)*cosh(x))*sinh(x)^2 +
 5*(23*sqrt(2)*cosh(x)^4 + 21*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x) + 15*s
```

```
qrt(2)*cosh(x))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^6 +
 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 +
 sqrt(2))*sinh(x)^4 + 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 + 3*sqrt
(2)*cosh(x))*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 + 6*sqrt(2)*cosh(x)^2 + sqr
t(2))*sinh(x)^2 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 + 2*sqrt(2)*co
sh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(
cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) -
 2*sinh(x)^2 - 1))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh
(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3
+ 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5
+ 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2774, size = 189, normalized size = 3.32

$$\frac{4}{15}\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{4}-135\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{3}+170\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{2}-100\sqrt{e^{(4x)}+e^{(2x)}}+\cdots}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(7/2),x, algorithm="giac")
```

```
[Out] 4/15*sqrt(2)*(2*(45*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^4 - 135*(sqrt(e^(4*
x) + e^(2*x)) - e^(2*x))^3 + 170*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 10
0*sqrt(e^(4*x) + e^(2*x)) + 100*e^(2*x) + 23)/(sqrt(e^(4*x) + e^(2*x)) - e^
(2*x) - 1)^5 - 15*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

3.51 $\int (1+\tanh(x))^{5/2}\,dx$

Optimal. Leaf size=45

$$4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\tanh(x)+1)^{3/2}-4\sqrt{\tanh(x)+1}$$

```
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 4*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(3/2))/3
```

Rubi [A] time = 0.0308033, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3478, 3480, 206}

$$4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\tanh(x)+1)^{3/2}-4\sqrt{\tanh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x])^(5/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 4*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(3/2))/3
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (1+\tanh(x))^{5/2}\,dx &= -\frac{2}{3}(1+\tanh(x))^{3/2}+2\int (1+\tanh(x))^{3/2}\,dx\\ &= -4\sqrt{1+\tanh(x)}-\frac{2}{3}(1+\tanh(x))^{3/2}+4\int \sqrt{1+\tanh(x)}\,dx\\ &= -4\sqrt{1+\tanh(x)}-\frac{2}{3}(1+\tanh(x))^{3/2}+8\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\ &= 4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-4\sqrt{1+\tanh(x)}-\frac{2}{3}(1+\tanh(x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.164421, size = 39, normalized size = 0.87

$$4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}\sqrt{\tanh(x)+1}(\tanh(x)+7)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x])^(5/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*Sqrt[1 + Tanh[x]]*(7 + Ta
nh[x]))/3
```

Maple [A] time = 0.013, size = 35, normalized size = 0.8

$$4\,\mathrm{Artanh}\left(1/2\,\sqrt{1+\tanh(x)}\sqrt{2}\right)\sqrt{2}-4\,\sqrt{1+\tanh(x)}-\frac{2}{3}\,(1+\tanh(x))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x))^(5/2),x)
```

```
[Out] 4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-4*(1+tanh(x))^(1/2)-2/3*(1
+tanh(x))^(3/2)
```

Maxima [B] time = 1.58942, size = 95, normalized size = 2.11

$$-2\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2\,x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2\,x)}+1}}}\right)-\frac{4\,\sqrt{2}}{\sqrt{e^{(-2\,x)}+1}}-\frac{4\,\sqrt{2}}{3\left(e^{(-2\,x)}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)/s
qrt(e^(-2*x) + 1))) - 4*sqrt(2)/sqrt(e^(-2*x) + 1) - 4/3*sqrt(2)/(e^(-2*x)
+ 1)^(3/2)
```

Fricas [B] time = 2.32679, size = 871, normalized size = 19.36

$$-\frac{2\left(2\,\sqrt{2}\left(4\,\sqrt{2}\cosh(x)^{3}+12\,\sqrt{2}\cosh(x)\sinh(x)^{2}+4\,\sqrt{2}\sinh(x)^{3}+3\left(4\,\sqrt{2}\cosh(x)^{2}+\sqrt{2}\right)\sinh(x)+3\,\sqrt{2}\cosh(x)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^3 + 12*sqrt(2)*cosh(x)*sinh(x)^2 + 4*sqr
t(2)*sinh(x)^3 + 3*(4*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 3*sqrt(2)*cosh
(x))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
```

```
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*si
nh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
 sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (\tanh(x) + 1)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))**(5/2),x)
```

```
[Out] Integral((tanh(x) + 1)**(5/2), x)
```

Giac [B] time = 1.1742, size = 130, normalized size = 2.89

$$\frac{2}{3}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{2}-9\sqrt{e^{(4x)}+e^{(2x)}}+9e^{(2x)}+4\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right)^{3}}-3\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(2*(6*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 9*sqrt(e^(4*x) +
e^(2*x)) + 9*e^(2*x) + 4)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1)^3 - 3*log
(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

3.52 $\int (1+\tanh(x))^{3/2}\,dx$

Optimal. Leaf size=33

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-2\sqrt{\tanh(x)+1}$$

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Rubi [A] time = 0.0211645, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3478, 3480, 206}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-2\sqrt{\tanh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (1+\tanh(x))^{3/2}\,dx &= -2\sqrt{1+\tanh(x)}+2\int \sqrt{1+\tanh(x)}\,dx\\ &= -2\sqrt{1+\tanh(x)}+4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\ &= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-2\sqrt{1+\tanh(x)}\end{aligned}$$

Mathematica [A] time = 0.0665317, size = 33, normalized size = 1.

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-2\sqrt{\tanh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Maple [A] time = 0.013, size = 27, normalized size = 0.8

$$2\,\mathrm{Artanh}\left(1/2\,\sqrt{1+\tanh(x)}\sqrt{2}\right)\sqrt{2}-2\,\sqrt{1+\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x))^(3/2),x)
```

```
[Out] 2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)
```

Maxima [B] time = 1.56577, size = 77, normalized size = 2.33

$$-\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right)-\frac{2\sqrt{2}}{\sqrt{e^{(-2x)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)/sqr
t(e^(-2*x) + 1))) - 2*sqrt(2)/sqrt(e^(-2*x) + 1)
```

Fricas [B] time = 2.25509, size = 451, normalized size = 13.67

$$-\frac{2\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh(x)^{2}+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^{2}+\sqrt{2}\right)\log}{\cosh(x)^{2}+2\cosh(x)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(cosh(x)/(cosh(x) - sin
h(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
 + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(\tanh(x)+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))**(3/2),x)
```

```
[Out] Integral((tanh(x) + 1)**(3/2), x)
```

Giac [A] time = 1.17263, size = 70, normalized size = 2.12

$$\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1}-\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(2/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1) - log(-2*sqrt(e^(4*x) +
e^(2*x)) + 2*e^(2*x) + 1))
```

3.53 $\int \sqrt{1+\tanh(x)}\,dx$

Optimal. Leaf size=21

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)$$

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]
```

Rubi [A] time = 0.0114613, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3480, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Tanh[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\tanh(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0450765, size = 21, normalized size = 1.

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Tanh[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]
```

Maple [A] time = 0.032, size = 17, normalized size = 0.8

$$\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x))^(1/2),x)
```

```
[Out] arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.60106, size = 58, normalized size = 2.76

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)
/sqrt(e^(-2*x) + 1)))
```

Fricas [B] time = 2.21971, size = 182, normalized size = 8.67

$$\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{2}\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))-2\cosh(x)^2-4\cosh(x)\sinh(x)-2\sinh(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(x) + 1), x)
```

Giac [A] time = 1.20234, size = 36, normalized size = 1.71

$$-\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1)
```

3.54 $\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\tanh(x)+1}}$$

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Tanh[x]]
```

Rubi [A] time = 0.0227825, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3479, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\tanh(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Tanh[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Tanh[x]]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx &= -\frac{1}{\sqrt{1+\tanh(x)}}+\frac{1}{2}\int \sqrt{1+\tanh(x)}\,dx\\
&= -\frac{1}{\sqrt{1+\tanh(x)}}+\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\tanh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0719634, size = 32, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\tanh(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Tanh[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Tanh[x]]
```

Maple [A] time = 0.033, size = 27, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right)-\frac{1}{\sqrt{1+\tanh(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tanh(x))^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+tanh(x))^(1/2)
```

Maxima [B] time = 1.58565, size = 77, normalized size = 2.41

$$-\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right)-\frac{1}{2}\sqrt{2}\sqrt{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)
/sqrt(e^(-2*x) + 1))) - 1/2*sqrt(2)*sqrt(e^(-2*x) + 1)
```

Fricas [B] time = 2.29071, size = 301, normalized size = 9.41

$$\frac{\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(-2\sqrt{2}\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))-2\cosh(x)^2-4\cosh(x)\sinh(x)-2\,\mathrm{s}\right.}{4(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(
x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*si
nh(x)^2 - 1) - 4*sqrt(cosh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\tanh(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(tanh(x) + 1), x)
```

Giac [A] time = 1.18187, size = 68, normalized size = 2.12

$$-\frac{1}{4}\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}}+\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(2/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x)) + log(-2*sqrt(e^(4*x) +
 e^(2*x)) + 2*e^(2*x) + 1))
```

3.55 $\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{2\sqrt{\tanh(x)+1}}-\frac{1}{3(\tanh(x)+1)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Tanh[x])^(3/2))
- 1/(2*Sqrt[1 + Tanh[x]])
```

Rubi [A] time = 0.0307678, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3479, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{2\sqrt{\tanh(x)+1}}-\frac{1}{3(\tanh(x)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x])^(-3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Tanh[x])^(3/2))
- 1/(2*Sqrt[1 + Tanh[x]])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx &= -\frac{1}{3(1+\tanh(x))^{3/2}}+\frac{1}{2}\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx\\
&= -\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}+\frac{1}{4}\int \sqrt{1+\tanh(x)}\,dx\\
&= -\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.133154, size = 53, normalized size = 1.08

$$\frac{1}{12}\left(3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2(\cosh(x)-\sinh(x))(3\sinh(x)+5\cosh(x))}{\sqrt{\tanh(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x])^(-3/2),x]
```

```
[Out] (3*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*(Cosh[x] - Sinh[x])*(5*C
osh[x] + 3*Sinh[x]))/Sqrt[1 + Tanh[x]])/12
```

Maple [A] time = 0.014, size = 35, normalized size = 0.7

$$\frac{\sqrt{2}}{4}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right)-\frac{1}{2}\frac{1}{\sqrt{1+\tanh(x)}}-\frac{1}{3}(1+\tanh(x))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tanh(x))^(3/2),x)
```

```
[Out] 1/4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/2/(1+tanh(x))^(1/2)-1/
3/(1+tanh(x))^(3/2)
```

Maxima [B] time = 1.60729, size = 93, normalized size = 1.9

$$-\frac{1}{12}\sqrt{2}\left(\frac{3}{e^{(-2x)}+1}+1\right)\left(e^{(-2x)}+1\right)^{\frac{3}{2}}-\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(2)*(3/(e^(-2*x) + 1) + 1)*(e^(-2*x) + 1)^(3/2) - 1/8*sqrt(2)*log
(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)/sqrt(e^(-2*x) +
 1)))
```

Fricas [B] time = 2.3397, size = 579, normalized size = 11.82

$$-\frac{2\sqrt{2}\left(4\sqrt{2}\cosh(x)^2+8\sqrt{2}\cosh(x)\sinh(x)+4\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)\right.}{24\left(\cosh(x)^3+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^2 + 8*sqrt(2)*cosh(x)*sinh(x) + 4*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
```

```
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*
cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\tanh(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))**(3/2),x)
```

```
[Out] Integral((tanh(x) + 1)**(-3/2), x)
```

Giac [B] time = 1.20401, size = 130, normalized size = 2.65

$$-\frac{1}{24}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{2}-3\sqrt{e^{(4x)}+e^{(2x)}}+3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{3}}+3\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(2*(6*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 3*sqrt(e^(4*x)
+ e^(2*x)) + 3*e^(2*x) + 1)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^3 + 3*log(-
2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1) - 8)
```

3.56 $\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{4\sqrt{\tanh(x)+1}}-\frac{1}{6(\tanh(x)+1)^{3/2}}-\frac{1}{5(\tanh(x)+1)^{5/2}}$$

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Tanh[x])^(5/2))
- 1/(6*(1 + Tanh[x])^(3/2)) - 1/(4*Sqrt[1 + Tanh[x]])
```

Rubi [A] time = 0.0414043, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3479, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{4\sqrt{\tanh(x)+1}}-\frac{1}{6(\tanh(x)+1)^{3/2}}-\frac{1}{5(\tanh(x)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x])^(-5/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Tanh[x])^(5/2))
- 1/(6*(1 + Tanh[x])^(3/2)) - 1/(4*Sqrt[1 + Tanh[x]])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx &= -\frac{1}{5(1+\tanh(x))^{5/2}}+\frac{1}{2}\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx\\ &= -\frac{1}{5(1+\tanh(x))^{5/2}}-\frac{1}{6(1+\tanh(x))^{3/2}}+\frac{1}{4}\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx\\ &= -\frac{1}{5(1+\tanh(x))^{5/2}}-\frac{1}{6(1+\tanh(x))^{3/2}}-\frac{1}{4\sqrt{1+\tanh(x)}}+\frac{1}{8}\int \sqrt{1+\tanh(x)}\,dx\\ &= -\frac{1}{5(1+\tanh(x))^{5/2}}-\frac{1}{6(1+\tanh(x))^{3/2}}-\frac{1}{4\sqrt{1+\tanh(x)}}+\frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\right.\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{5(1+\tanh(x))^{5/2}}-\frac{1}{6(1+\tanh(x))^{3/2}}-\frac{1}{4\sqrt{1+\tanh(x)}}\end{aligned}$$

Mathematica [A] time = 0.229374, size = 62, normalized size = 1.02

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{(\sinh(2x)-\cosh(2x))(20\sinh(2x)+26\cosh(2x)+11)}{60\sqrt{\tanh(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x])^(-5/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(4*Sqrt[2]) + ((-Cosh[2*x] + Sinh[2*x])*
(11 + 26*Cosh[2*x] + 20*Sinh[2*x]))/(60*Sqrt[1 + Tanh[x]])
```

Maple [A] time = 0.017, size = 43, normalized size = 0.7

$$\frac{\sqrt{2}}{8}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right)-\frac{1}{4}\frac{1}{\sqrt{1+\tanh(x)}}-\frac{1}{5}(1+\tanh(x))^{-\frac{5}{2}}-\frac{1}{6}(1+\tanh(x))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tanh(x))^(5/2),x)
```

```
[Out] 1/8*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/4/(1+tanh(x))^(1/2)-1/
5/(1+tanh(x))^(5/2)-1/6/(1+tanh(x))^(3/2)
```

Maxima [A] time = 1.64044, size = 107, normalized size = 1.75

$$-\frac{1}{120}\sqrt{2}\left(\frac{5}{e^{(-2x)}+1}+\frac{15}{\left(e^{(-2x)}+1\right)^2}+3\right)\left(e^{(-2x)}+1\right)^{\frac{5}{2}}-\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/120*sqrt(2)*(5/(e^(-2*x) + 1) + 15/(e^(-2*x) + 1)^2 + 3)*(e^(-2*x) + 1)^
(5/2) - 1/16*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) +
```

```
sqrt(2)/sqrt(e^(-2*x) + 1)))
```

Fricas [B] time = 2.22798, size = 921, normalized size = 15.1

$$-\frac{2\sqrt{2}\left(23\sqrt{2}\cosh(x)^4+92\sqrt{2}\cosh(x)\sinh(x)^3+23\sqrt{2}\sinh(x)^4+\left(138\sqrt{2}\cosh(x)^2+11\sqrt{2}\right)\sinh(x)^2+11\sqrt{2}\cosh\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/240*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^4 + 92*sqrt(2)*cosh(x)*sinh(x)^3 + 23
*sqrt(2)*sinh(x)^4 + (138*sqrt(2)*cosh(x)^2 + 11*sqrt(2))*sinh(x)^2 + 11*sq
rt(2)*cosh(x)^2 + 2*(46*sqrt(2)*cosh(x)^3 + 11*sqrt(2)*cosh(x))*sinh(x) + 3
*sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^5 + 5*sqr
t(2)*cosh(x)^4*sinh(x) + 10*sqrt(2)*cosh(x)^3*sinh(x)^2 + 10*sqrt(2)*cosh(x
)^2*sinh(x)^3 + 5*sqrt(2)*cosh(x)*sinh(x)^4 + sqrt(2)*sinh(x)^5)*log(-2*sqr
t(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 -
4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10
*cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(
x)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\tanh(x)+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))**(5/2),x)
```

```
[Out] Integral((tanh(x) + 1)**(-5/2), x)
```

Giac [B] time = 1.24633, size = 189, normalized size = 3.1

$$-\frac{1}{240}\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^4-45\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^3+35\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^2-15\sqrt{e^{(4x)}+e^{(2x)}}+1\cdots}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(2)*(2*(45*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^4 - 45*(sqrt(e^(4
*x) + e^(2*x)) - e^(2*x))^3 + 35*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 15
*sqrt(e^(4*x) + e^(2*x)) + 15*e^(2*x) + 3)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*
x))^5 + 15*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1) - 46)
```

3.57 $\int (a+b\tanh(c+dx))^5\,dx$

Optimal. Leaf size=142

$$-\frac{b\left(3a^2+b^2\right)(a+b\tanh(c+dx))^2}{2d}-\frac{4ab^2\left(a^2+b^2\right)\tanh(c+dx)}{d}+\frac{b\left(10a^2b^2+5a^4+b^4\right)\log(\cosh(c+dx))}{d}+ax\left(1\right.$$

```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x + (b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Cosh[c +
 d*x]])/d - (4*a*b^2*(a^2 + b^2)*Tanh[c + d*x])/d - (b*(3*a^2 + b^2)*(a + b
*Tanh[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Tanh[c + d*x])^3)/(3*d) - (b*(a +
b*Tanh[c + d*x])^4)/(4*d)
```

Rubi [A] time = 0.211294, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3482, 3528, 3525, 3475}

$$-\frac{b\left(3a^2+b^2\right)(a+b\tanh(c+dx))^2}{2d}-\frac{4ab^2\left(a^2+b^2\right)\tanh(c+dx)}{d}+\frac{b\left(10a^2b^2+5a^4+b^4\right)\log(\cosh(c+dx))}{d}+ax\left(1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x])^5,x]
```



```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x + (b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Cosh[c +
 d*x]])/d - (4*a*b^2*(a^2 + b^2)*Tanh[c + d*x])/d - (b*(3*a^2 + b^2)*(a + b
*Tanh[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Tanh[c + d*x])^3)/(3*d) - (b*(a +
b*Tanh[c + d*x])^4)/(4*d)
```


Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\tanh(c+dx))^5\,dx &= -\frac{b(a+b\tanh(c+dx))^4}{4d} + \int (a+b\tanh(c+dx))^3\left(a^2+b^2+2ab\tanh(c+dx)\right)\,dx \\
&= -\frac{2ab(a+b\tanh(c+dx))^3}{3d} - \frac{b(a+b\tanh(c+dx))^4}{4d} + \int (a+b\tanh(c+dx))^2\left(a\left(a^2+3b\right.\right. \\
&= -\frac{b\left(3a^2+b^2\right)(a+b\tanh(c+dx))^2}{2d} - \frac{2ab(a+b\tanh(c+dx))^3}{3d} - \frac{b(a+b\tanh(c+dx))^4}{4d} + \\
&= a\left(a^4+10a^2b^2+5b^4\right)x - \frac{4ab^2\left(a^2+b^2\right)\tanh(c+dx)}{d} - \frac{b\left(3a^2+b^2\right)(a+b\tanh(c+dx))^2}{2d} \\
&= a\left(a^4+10a^2b^2+5b^4\right)x + \frac{b\left(5a^4+10a^2b^2+b^4\right)\log(\cosh(c+dx))}{d} - \frac{4ab^2\left(a^2+b^2\right)\tanh(c}{d}
\end{aligned}
$$

Mathematica [A] time = 0.640478, size = 114, normalized size = 0.8

$$
-\frac{6b^3\left(10a^2+b^2\right)\tanh^2(c+dx)+60ab^2\left(2a^2+b^2\right)\tanh(c+dx)+20ab^4\tanh^3(c+dx)-6(a-b)^5\log(\tanh(c+dx)+1}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x])^5,x]
```

```
[Out] -(6*(a + b)^5*Log[1 - Tanh[c + d*x]] - 6*(a - b)^5*Log[1 + Tanh[c + d*x]] +
 60*a*b^2*(2*a^2 + b^2)*Tanh[c + d*x] + 6*b^3*(10*a^2 + b^2)*Tanh[c + d*x]^
2 + 20*a*b^4*Tanh[c + d*x]^3 + 3*b^5*Tanh[c + d*x]^4)/(12*d)
```

Maple [B] time = 0.005, size = 322, normalized size = 2.3

$$
\frac{\ln(\tanh(dx+c)+1)\,a^5}{2d} - \frac{5\,\ln(\tanh(dx+c)+1)\,a^4b}{2d} + 5\,\frac{\ln(\tanh(dx+c)+1)\,a^3b^2}{d} - 5\,\frac{\ln(\tanh(dx+c)+1)\,a^2b^3}{d} +
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c))^5,x)
```

```
[Out] 1/2/d*ln(tanh(d*x+c)+1)*a^5-5/2/d*ln(tanh(d*x+c)+1)*a^4*b+5/d*ln(tanh(d*x+c
)+1)*a^3*b^2-5/d*ln(tanh(d*x+c)+1)*a^2*b^3+5/2/d*ln(tanh(d*x+c)+1)*a*b^4-1/
2/d*ln(tanh(d*x+c)+1)*b^5-1/2/d*a^5*ln(tanh(d*x+c)-1)-5/2/d*ln(tanh(d*x+c)-
1)*a^4*b-5/d*ln(tanh(d*x+c)-1)*a^3*b^2-5/d*ln(tanh(d*x+c)-1)*a^2*b^3-5/2/d*
ln(tanh(d*x+c)-1)*a*b^4-1/2/d*ln(tanh(d*x+c)-1)*b^5-1/2/d*tanh(d*x+c)^2*b^5
-1/4/d*b^5*tanh(d*x+c)^4-5/3/d*tanh(d*x+c)^3*a*b^4-5/d*tanh(d*x+c)^2*a^2*b^
3-5/d*a*b^4*tanh(d*x+c)-10/d*a^3*b^2*tanh(d*x+c)
```

Maxima [B] time = 1.6546, size = 419, normalized size = 2.95

$$
\frac{5}{3}\,ab^4\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right)+b^5\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-}\right.}{d\left(4\,e^{(-2\,dx-2\,c)}+6\,e^{(}\right.}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/3*a*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d
*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + b^5*(
x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4
*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(
-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 10*a^2*b^3*(x + c/d + log(e^(-2*d
*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x -
4*c) + 1))) + 10*a^3*b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^5*x +
 5*a^4*b*log(cosh(d*x + c))/d
```

Fricas [B] time = 2.61619, size = 6413, normalized size = 45.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d
*x + c)^8 + 24*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5)*d*x*sinh(d*x + c)^8 + 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^
4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*co
sh(d*x + c)^6 + 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + 7*(a^5 - 5*a^4*
b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^2 + (a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*sinh(d*x + c)^6 + 24
*(7*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x
+ c)^3 + 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3
*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 60
*a^3*b^2 + 40*a*b^4 + 6*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^
5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^4
 + 6*(35*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh
(d*x + c)^4 + 30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b
 + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x + 30*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 24*(7*(a^5 - 5*a^4*b + 10*a^3*b^2
 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^5 + 10*(5*a^3*b^2 + 5*a^2*
b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
b^5)*d*x)*cosh(d*x + c)^3 + (30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3
*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
- b^5)*d*x + 4*(45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5 + 3*(a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2 + 4*(21*
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)
^6 + 45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5 + 45*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^4 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b
^4 - b^5)*d*x + 9*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*
a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 3*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^8 + 8*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a^4*b + 10*a^2*b^3 +
 b^5)*sinh(d*x + c)^8 + 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^6 + 4*
(5*a^4*b + 10*a^2*b^3 + b^5 + 7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^3 + 3*
(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*a^4*b + 10*
a^2*b^3 + b^5 + 6*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 2*(15*a^4*
b + 30*a^2*b^3 + 3*b^5 + 35*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 +
30*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*
```

```
a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 10*(5*a^4*b + 10*a^2*b^3 + b^5)
*cosh(d*x + c)^3 + 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 4*(7*(5*a^4*b + 10
*a^2*b^3 + b^5)*cosh(d*x + c)^6 + 5*a^4*b + 10*a^2*b^3 + b^5 + 15*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 9*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
7 + 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 3*(5*a^4*b + 10*a^2*b^
3 + b^5)*cosh(d*x + c)^3 + (5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh
(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(3*(a^5
 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^7 +
 9*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 1
0*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^5 + 3*(30*a^3*b^2 + 20*a^2*b^
3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5)*d*x)*cosh(d*x + c)^3 + (45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5
+ 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^
7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*s
inh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 +
3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)
^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d
*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x +
 c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.757339, size = 211, normalized size = 1.49

$$\begin{cases} a^5x + 5a^4bx - \frac{5a^4b\log(\tanh(c+dx)+1)}{d} + 10a^3b^2x - \frac{10a^3b^2\tanh(c+dx)}{d} + 10a^2b^3x - \frac{10a^2b^3\log(\tanh(c+dx)+1)}{d} - \frac{5a^2b^3\tanh^2(c+dx)}{d} + 5 \\ x(a+b\tanh(c))^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*x - 5*a**4*b*log(tanh(c + d*x) + 1)/d + 10*a**
3*b**2*x - 10*a**3*b**2*tanh(c + d*x)/d + 10*a**2*b**3*x - 10*a**2*b**3*log
(tanh(c + d*x) + 1)/d - 5*a**2*b**3*tanh(c + d*x)**2/d + 5*a*b**4*x - 5*a*b
**4*tanh(c + d*x)**3/(3*d) - 5*a*b**4*tanh(c + d*x)/d + b**5*x - b**5*log(t
anh(c + d*x) + 1)/d - b**5*tanh(c + d*x)**4/(4*d) - b**5*tanh(c + d*x)**2/(
2*d), Ne(d, 0)), (x*(a + b*tanh(c))**5, True))
```

Giac [A] time = 1.2168, size = 305, normalized size = 2.15

$$\frac{\left(a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5\right)(dx+c)}{d} + \frac{\left(5a^4b + 10a^2b^3 + b^5\right)\log\left(e^{(2dx+2c)} + 1\right)}{d} + \frac{4\left(15a^3b^2 + 10ab^4 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(d*x + c)/d + (5*
a^4*b + 10*a^2*b^3 + b^5)*log(e^(2*d*x + 2*c) + 1)/d + 4/3*(15*a^3*b^2 + 10
*a*b^4 + 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5)*e^(6*d*x + 6*c) + 3*(15*
```

```
a^3*b^2 + 10*a^2*b^3 + 10*a*b^4 + b^5)*e^(4*d*x + 4*c) + (45*a^3*b^2 + 15*a
^2*b^3 + 25*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/(d*(e^(2*d*x + 2*c) + 1)^4)
```

3.58 $\int (a+b\tanh(c+dx))^4\,dx$

Optimal. Leaf size=101

$$-\frac{b^2\left(3a^2+b^2\right)\tanh(c+dx)}{d}+\frac{4ab\left(a^2+b^2\right)\log(\cosh(c+dx))}{d}+x\left(6a^2b^2+a^4+b^4\right)-\frac{b(a+b\tanh(c+dx))^3}{3d}-ab(a+$$

```
[Out] (a^4 + 6*a^2*b^2 + b^4)*x + (4*a*b*(a^2 + b^2)*Log[Cosh[c + d*x]])/d - (b^2
*(3*a^2 + b^2)*Tanh[c + d*x])/d - (a*b*(a + b*Tanh[c + d*x])^2)/d - (b*(a +
 b*Tanh[c + d*x])^3)/(3*d)
```

Rubi [A] time = 0.124212, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3482, 3528, 3525, 3475}

$$-\frac{b^2\left(3a^2+b^2\right)\tanh(c+dx)}{d}+\frac{4ab\left(a^2+b^2\right)\log(\cosh(c+dx))}{d}+x\left(6a^2b^2+a^4+b^4\right)-\frac{b(a+b\tanh(c+dx))^3}{3d}-ab(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x])^4,x]
```


```
[Out] (a^4 + 6*a^2*b^2 + b^4)*x + (4*a*b*(a^2 + b^2)*Log[Cosh[c + d*x]])/d - (b^2
*(3*a^2 + b^2)*Tanh[c + d*x])/d - (a*b*(a + b*Tanh[c + d*x])^2)/d - (b*(a +
 b*Tanh[c + d*x])^3)/(3*d)
```


Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\tanh(c+dx))^4\,dx = -\frac{b(a+b\tanh(c+dx))^3}{3d} + \int (a+b\tanh(c+dx))^2\left(a^2+b^2+2ab\tanh(c+dx)\right)dx$$
$$= -\frac{ab(a+b\tanh(c+dx))^2}{d} - \frac{b(a+b\tanh(c+dx))^3}{3d} + \int (a+b\tanh(c+dx))\left(a\left(a^2+3b\right.\right.$$
$$= \left(a^4+6a^2b^2+b^4\right)x - \frac{b^2\left(3a^2+b^2\right)\tanh(c+dx)}{d} - \frac{ab(a+b\tanh(c+dx))^2}{d} - \frac{b(a+b\tanh}{}$$
$$= \left(a^4+6a^2b^2+b^4\right)x + \frac{4ab\left(a^2+b^2\right)\log(\cosh(c+dx))}{d} - \frac{b^2\left(3a^2+b^2\right)\tanh(c+dx)}{d} - \frac{a}{}$$

Mathematica [A] time = 0.333722, size = 91, normalized size = 0.9

$$-\frac{6b^2\left(6a^2+b^2\right)\tanh(c+dx)+12ab^3\tanh^2(c+dx)-3(a-b)^4\log(\tanh(c+dx)+1)+3(a+b)^4\log(1-\tanh(c+d}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x])^4,x]
```

```
[Out] -(3*(a + b)^4*Log[1 - Tanh[c + d*x]] - 3*(a - b)^4*Log[1 + Tanh[c + d*x]] +
 6*b^2*(6*a^2 + b^2)*Tanh[c + d*x] + 12*a*b^3*Tanh[c + d*x]^2 + 2*b^4*Tanh[
c + d*x]^3)/(6*d)
```

Maple [B] time = 0.006, size = 246, normalized size = 2.4

$$-\frac{b^4(\tanh(dx+c))^3}{3d} - 2\frac{(\tanh(dx+c))^2ab^3}{d} - 6\frac{a^2\tanh(dx+c)b^2}{d} - \frac{\tanh(dx+c)b^4}{d} - \frac{a^4\ln(\tanh(dx+c)-1)}{2d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c))^4,x)
```

```
[Out] -1/3/d*b^4*tanh(d*x+c)^3-2/d*tanh(d*x+c)^2*a*b^3-6/d*tanh(d*x+c)*a^2*b^2-1/
d*tanh(d*x+c)*b^4-1/2/d*a^4*ln(tanh(d*x+c)-1)-2/d*ln(tanh(d*x+c)-1)*a^3*b-3
/d*ln(tanh(d*x+c)-1)*a^2*b^2-2/d*ln(tanh(d*x+c)-1)*a*b^3-1/2/d*ln(tanh(d*x+
c)-1)*b^4+1/2/d*ln(tanh(d*x+c)+1)*a^4-2/d*ln(tanh(d*x+c)+1)*a^3*b+3/d*ln(ta
nh(d*x+c)+1)*a^2*b^2-2/d*ln(tanh(d*x+c)+1)*a*b^3+1/2/d*ln(tanh(d*x+c)+1)*b^
4
```

Maxima [B] time = 1.87088, size = 271, normalized size = 2.68

$$\frac{1}{3}b^4\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+4ab^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2}}{d\left(2e^{(-2dx-2c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a*b^3
*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*
```

```
x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 6*a^2*b^2*(x + c/d - 2/(d*(e^(-2*d*x -
 2*c) + 1))) + a^4*x + 4*a^3*b*log(cosh(d*x + c))/d
```

Fricas [B] time = 2.4622, size = 3318, normalized size = 32.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^6 + 18
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c
)^5 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*sinh(d*x + c)^6 + 3
*(12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x)*cosh(d*x + c)^4 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4
)*d*x*cosh(d*x + c)^2 + 12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*sinh(d*x + c)^4 + 36*a^2*b^2 + 8*b^4 + 12*(5
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 + (12*a^2*
b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x + 3*(24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x*cosh(d*x + c)^4 + 24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x + 6*(12*a^2*b^2 + 8*a*b^3 + 4*b^4
+ 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 12*((a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(a^3*b + a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^3*b + a*b^3)*sinh(d*x + c)^6 + 3*(a^3*b + a*b^
3)*cosh(d*x + c)^4 + 3*(a^3*b + a*b^3 + 5*(a^3*b + a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + a^3*b + a*b^3 + 4*(5*(a^3*b + a*b^3)*cosh(d*x + c)^3 + 3*
(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3*b + a*b^3)*cosh(d*x
 + c)^2 + 3*(5*(a^3*b + a*b^3)*cosh(d*x + c)^4 + a^3*b + a*b^3 + 6*(a^3*b +
 a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((a^3*b + a*b^3)*cosh(d*x + c)
^5 + 2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*cosh(d*x + c))*sin
h(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^5 + 2*(12*a^2*b^
2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*co
sh(d*x + c)^3 + (24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^
2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^4
 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 3
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x +
 c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 + 2
*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.526812, size = 144, normalized size = 1.43

$$\begin{cases} a^4x + 4a^3bx - \frac{4a^3b\log(\tanh(c+dx)+1)}{d} + 6a^2b^2x - \frac{6a^2b^2\tanh(c+dx)}{d} + 4ab^3x - \frac{4ab^3\log(\tanh(c+dx)+1)}{d} - \frac{2ab^3\tanh^2(c+dx)}{d} + b^4x - \cdots \\ x(a+b\tanh(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*x - 4*a**3*b*log(tanh(c + d*x) + 1)/d + 6*a**2
*b**2*x - 6*a**2*b**2*tanh(c + d*x)/d + 4*a*b**3*x - 4*a*b**3*log(tanh(c +
d*x) + 1)/d - 2*a*b**3*tanh(c + d*x)**2/d + b**4*x - b**4*tanh(c + d*x)**3/
(3*d) - b**4*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c))**4, True))
```

<u>Giac</u> [A] time = 1.19948, size = 209, normalized size = 2.07

$$\frac{\left(a^4 - 4\,a^3b + 6\,a^2b^2 - 4\,ab^3 + b^4\right)(dx + c)}{d} + \frac{4\left(a^3b + ab^3\right)\log\left(e^{(2\,dx+2\,c)} + 1\right)}{d} + \frac{4\left(9\,a^2b^2 + 2\,b^4 + 3\left(3\,a^2b^2 + 2\,ab^3 + \cdots\right.\right.}{3\,d\left(e^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(d*x + c)/d + 4*(a^3*b + a*b^3)
*log(e^(2*d*x + 2*c) + 1)/d + 4/3*(9*a^2*b^2 + 2*b^4 + 3*(3*a^2*b^2 + 2*a*b
^3 + b^4)*e^(4*d*x + 4*c) + 3*(6*a^2*b^2 + 2*a*b^3 + b^4)*e^(2*d*x + 2*c))/
(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.59 $\int (a+b\tanh(c+dx))^3\,dx$

Optimal. Leaf size=69

$$\frac{b\left(3a^2+b^2\right)\log(\cosh(c+dx))}{d}+ax\left(a^2+3b^2\right)-\frac{2ab^2\tanh(c+dx)}{d}-\frac{b(a+b\tanh(c+dx))^2}{2d}$$

```
[Out] a*(a^2 + 3*b^2)*x + (b*(3*a^2 + b^2)*Log[Cosh[c + d*x]])/d - (2*a*b^2*Tanh[
c + d*x])/d - (b*(a + b*Tanh[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.0645049, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3482, 3525, 3475}

$$\frac{b\left(3a^2+b^2\right)\log(\cosh(c+dx))}{d}+ax\left(a^2+3b^2\right)-\frac{2ab^2\tanh(c+dx)}{d}-\frac{b(a+b\tanh(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x])^3,x]
```

```
[Out] a*(a^2 + 3*b^2)*x + (b*(3*a^2 + b^2)*Log[Cosh[c + d*x]])/d - (2*a*b^2*Tanh[
c + d*x])/d - (b*(a + b*Tanh[c + d*x])^2)/(2*d)
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\tanh(c+dx))^3\,dx &= -\frac{b(a+b\tanh(c+dx))^2}{2d}+\int (a+b\tanh(c+dx))\left(a^2+b^2+2ab\tanh(c+dx)\right)dx\\ &= a\left(a^2+3b^2\right)x-\frac{2ab^2\tanh(c+dx)}{d}-\frac{b(a+b\tanh(c+dx))^2}{2d}+\left(b\left(3a^2+b^2\right)\right)\int \tanh(c+\\ &= a\left(a^2+3b^2\right)x+\frac{b\left(3a^2+b^2\right)\log(\cosh(c+dx))}{d}-\frac{2ab^2\tanh(c+dx)}{d}-\frac{b(a+b\tanh(c+dx}{2d}\end{aligned}$$

Mathematica [A] time = 0.327111, size = 67, normalized size = 0.97

$$-\frac{6ab^2\tanh(c+dx)+(a-b)^3(-\log(\tanh(c+dx)+1))+(a+b)^3\log(1-\tanh(c+dx))+b^3\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x])^3,x]
```

```
[Out] -((a + b)^3*Log[1 - Tanh[c + d*x]] - (a - b)^3*Log[1 + Tanh[c + d*x]] + 6*a
*b^2*Tanh[c + d*x] + b^3*Tanh[c + d*x]^2)/(2*d)
```

Maple [B] time = 0.004, size = 173, normalized size = 2.5

$$-\frac{b^3 (\tanh(dx+c))^2}{2d} - 3\frac{ab^2 \tanh(dx+c)}{d} - \frac{a^3 \ln(\tanh(dx+c)-1)}{2d} - \frac{3\ln(\tanh(dx+c)-1)a^2b}{2d} - \frac{3\ln(\tanh(dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c))^3,x)
```

```
[Out] -1/2/d*b^3*tanh(d*x+c)^2-3*a*b^2*tanh(d*x+c)/d-1/2/d*a^3*ln(tanh(d*x+c)-1)-
3/2/d*ln(tanh(d*x+c)-1)*a^2*b-3/2/d*ln(tanh(d*x+c)-1)*a*b^2-1/2/d*ln(tanh(d
*x+c)-1)*b^3+1/2/d*ln(tanh(d*x+c)+1)*a^3-3/2/d*ln(tanh(d*x+c)+1)*a^2*b+3/2/
d*ln(tanh(d*x+c)+1)*a*b^2-1/2/d*ln(tanh(d*x+c)+1)*b^3
```

Maxima [A] time = 1.78289, size = 159, normalized size = 2.3

$$b^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right) + 3\,ab^2\left(x + \frac{c}{d} - \frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right) + a^3x + \frac{3\,a^2b\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 3*a*b^2*(x + c/d - 2/(d*(e^(-2*d*x
- 2*c) + 1))) + a^3*x + 3*a^2*b*log(cosh(d*x + c))/d
```

Fricas [B] time = 2.29269, size = 1563, normalized size = 22.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*x*sinh(d*x + c)^4 + 6*a*b^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x
+ 2*(3*a*b^2 + b^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 +
 2*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^2 + 3*a*b^2 + b^3 +
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((3*a^2*b + b^3)*co
sh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b
+ b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3 + 2*(3*a^2*b + b^3)*cosh(d*x + c)^2
+ 2*(3*a^2*b + b^3 + 3*(3*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
```

```
*((3*a^2*b + b^3)*cosh(d*x + c)^3 + (3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x
 + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^3 + (3*a*b^2 + b^3 + (a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4
*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2
+ 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*co
sh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] time = 0.341366, size = 100, normalized size = 1.45

$$\begin{cases} a^3x + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} + 3ab^2x - \frac{3ab^2\tanh(c+dx)}{d} + b^3x - \frac{b^3\log(\tanh(c+dx)+1)}{d} - \frac{b^3\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a+b\tanh(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d + 3*a*b*
*2*x - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*log(tanh(c + d*x) + 1)/d -
b**3*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c))**3, True))
```

Giac [A] time = 1.23742, size = 138, normalized size = 2.

$$\frac{\left(a^3 - 3\,a^2b + 3\,ab^2 - b^3\right)(dx + c)}{d} + \frac{\left(3\,a^2b + b^3\right)\log\left(e^{(2\,dx+2\,c)} + 1\right)}{d} + \frac{2\left(3\,ab^2 + \left(3\,ab^2 + b^3\right)e^{(2\,dx+2\,c)}\right)}{d\left(e^{(2\,dx+2\,c)} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(d*x + c)/d + (3*a^2*b + b^3)*log(e^(2*d*x
+ 2*c) + 1)/d + 2*(3*a*b^2 + (3*a*b^2 + b^3)*e^(2*d*x + 2*c))/(d*(e^(2*d*x
+ 2*c) + 1)^2)
```

3.60 $\int (a+b\tanh(c+dx))^2\,dx$

Optimal. Leaf size=38

$$x\left(a^2+b^2\right)+\frac{2ab\log(\cosh(c+dx))}{d}-\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] (a^2 + b^2)*x + (2*a*b*Log[Cosh[c + d*x]])/d - (b^2*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0226947, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3477, 3475}

$$x\left(a^2+b^2\right)+\frac{2ab\log(\cosh(c+dx))}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x])^2,x]
```

```
[Out] (a^2 + b^2)*x + (2*a*b*Log[Cosh[c + d*x]])/d - (b^2*Tanh[c + d*x])/d
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\tanh(c+dx))^2\,dx &= \left(a^2+b^2\right)x-\frac{b^2\tanh(c+dx)}{d}+(2ab)\int \tanh(c+dx)\,dx\\ &= \left(a^2+b^2\right)x+\frac{2ab\log(\cosh(c+dx))}{d}-\frac{b^2\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0951131, size = 54, normalized size = 1.42

$$\frac{(a-b)^2\log(\tanh(c+dx)+1)-(a+b)^2\log(1-\tanh(c+dx))-2b^2\tanh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x])^2,x]
```

```
[Out] (-((a + b)^2*Log[1 - Tanh[c + d*x]]) + (a - b)^2*Log[1 + Tanh[c + d*x]] - 2
*b^2*Tanh[c + d*x])/(2*d)
```

Maple [B] time = 0.004, size = 116, normalized size = 3.1

$$-\frac{b^2\tanh(dx+c)}{d}-\frac{a^2\ln(\tanh(dx+c)-1)}{2d}-\frac{\ln(\tanh(dx+c)-1)\,ab}{d}-\frac{\ln(\tanh(dx+c)-1)\,b^2}{2d}+\frac{\ln(\tanh(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c))^2,x)
```

```
[Out] -b^2*tanh(d*x+c)/d-1/2*a^2/d*ln(tanh(d*x+c)-1)-1/d*ln(tanh(d*x+c)-1)*a*b-1/
2/d*ln(tanh(d*x+c)-1)*b^2+1/2/d*ln(tanh(d*x+c)+1)*a^2-1/d*ln(tanh(d*x+c)+1)
*a*b+1/2/d*ln(tanh(d*x+c)+1)*b^2
```

Maxima [A] time = 1.12727, size = 66, normalized size = 1.74

$$b^2\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+a^2x+\frac{2\,ab\log(\cosh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*x + 2*a*b*log(cosh(d*x +
 c))/d
```

Fricas [B] time = 2.21507, size = 535, normalized size = 14.08

$$\frac{\left(a^2-2\,ab+b^2\right)dx\cosh(dx+c)^2+2\left(a^2-2\,ab+b^2\right)dx\cosh(dx+c)\sinh(dx+c)+\left(a^2-2\,ab+b^2\right)dx\sinh(dx+c)^2+}{d\cosh(dx+c)^2+2\,d\cosh(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - 2*a*b + b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c) + (a^2 - 2*a*b + b^2)*d*x*sinh(d*x + c)^2 + (a^2 - 2*
a*b + b^2)*d*x + 2*b^2 + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(
d*x + c) + a*b*sinh(d*x + c)^2 + a*b)*log(2*cosh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*s
inh(d*x + c)^2 + d)
```

Sympy [A] time = 0.224003, size = 54, normalized size = 1.42

$$\begin{cases} a^2x+2abx-\frac{2ab\log(\tanh(c+dx)+1)}{d}+b^2x-\frac{b^2\tanh(c+dx)}{d} & \text{for}\ d\neq 0 \\ x\left(a+b\tanh(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d + b**2*x - b**
2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c))**2, True))
```

Giac [A] time = 1.23799, size = 82, normalized size = 2.16

$$\frac{2\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)}{d}+\frac{\left(a^2-2\,ab+b^2\right)(dx+c)}{d}+\frac{2\,b^2}{d\left(e^{(2\,dx+2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*a*b*log(e^(2*d*x + 2*c) + 1)/d + (a^2 - 2*a*b + b^2)*(d*x + c)/d + 2*b^2/
(d*(e^(2*d*x + 2*c) + 1))
```

3.61 $\int \frac{1}{a+b\tanh(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\left(a^2-b^2\right)}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[c + d*x] + b*Sinh[c + d*x]])/((a^2 - b^2)
*d)
```

Rubi [A] time = 0.0538593, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3484, 3530}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[c + d*x] + b*Sinh[c + d*x]])/((a^2 - b^2)
*d)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\tanh(c+dx)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(c+dx)+b\sinh(c+dx))}{\left(a^2-b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.0821919, size = 64, normalized size = 1.28

$$\frac{(b-a)\log(1-\tanh(c+dx))+(a+b)\log(\tanh(c+dx)+1)-2b\log(a+b\tanh(c+dx))}{2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x])^(-1),x]
```

```
[Out] ((-a + b)*Log[1 - Tanh[c + d*x]] + (a + b)*Log[1 + Tanh[c + d*x]] - 2*b*Log
[a + b*Tanh[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A] time = 0.019, size = 76, normalized size = 1.5

$$\frac{\ln(\tanh(dx+c)+1)}{d(2a-2b)} - \frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)} - \frac{b\ln(a+b\tanh(dx+c))}{d(a-b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c)),x)
```

```
[Out] 1/d/(2*a-2*b)*ln(tanh(d*x+c)+1)-1/d/(2*b+2*a)*ln(tanh(d*x+c)-1)-1/d*b/(a-b)
/(a+b)*ln(a+b*tanh(d*x+c))
```

Maxima [A] time = 1.17669, size = 76, normalized size = 1.52

$$-\frac{b\log\left(-(a-b)e^{(-2\,dx-2\,c)}-a-b\right)}{\left(a^2-b^2\right)d} + \frac{dx+c}{(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*d*x - 2*c) - a - b)/((a^2 - b^2)*d) + (d*x + c)/((a +
 b)*d)
```

Fricas [A] time = 2.32217, size = 149, normalized size = 2.98

$$\frac{(a+b)dx - b\log\left(\frac{2\,(a\cosh(dx+c)+b\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a + b)*d*x - b*log(2*(a*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 - b^2)*d)
```

Sympy [A] time = 2.36528, size = 224, normalized size = 4.48

$$\begin{cases} \frac{\tilde{\infty}x}{\tanh(c)} & \text{for}\ a=0 \wedge b=0 \wedge d=0 \\ -\frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)-2bd} + \frac{dx}{2bd\tanh(c+dx)-2bd} + \frac{1}{2bd\tanh(c+dx)-2bd} & \text{for}\ a=-b \\ \frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)+2bd} + \frac{dx}{2bd\tanh(c+dx)+2bd} - \frac{1}{2bd\tanh(c+dx)+2bd} & \text{for}\ a=b \\ \frac{x}{a+b\tanh(c)} & \text{for}\ d=0 \\ \frac{x}{a} & \text{for}\ b=0 \\ \frac{adx}{a^2d-b^2d} - \frac{bdx}{a^2d-b^2d} - \frac{b\log\left(\frac{a}{b}+\tanh(c+dx)\right)}{a^2d-b^2d} + \frac{b\log(\tanh(c+dx)+1)}{a^2d-b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*x*tanh(c + d
*x)/(2*b*d*tanh(c + d*x) - 2*b*d) + d*x/(2*b*d*tanh(c + d*x) - 2*b*d) + 1/(
2*b*d*tanh(c + d*x) - 2*b*d), Eq(a, -b)), (d*x*tanh(c + d*x)/(2*b*d*tanh(c
+ d*x) + 2*b*d) + d*x/(2*b*d*tanh(c + d*x) + 2*b*d) - 1/(2*b*d*tanh(c + d*x
) + 2*b*d), Eq(a, b)), (x/(a + b*tanh(c)), Eq(d, 0)), (x/a, Eq(b, 0)), (a*d
*x/(a**2*d - b**2*d) - b*d*x/(a**2*d - b**2*d) - b*log(a/b + tanh(c + d*x))
/(a**2*d - b**2*d) + b*log(tanh(c + d*x) + 1)/(a**2*d - b**2*d), True))
```

Giac [A] time = 1.24107, size = 85, normalized size = 1.7

$$-\frac{b\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b\right|\right)}{a^2d-b^2d}+\frac{dx+c}{ad-bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b))/(a^2*d - b^2*d)
+ (d*x + c)/(a*d - b*d)
```

3.62 $\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{b}{d\left(a^2-b^2\right)(a+b\tanh(c+dx))}-\frac{2ab\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{x\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}$$

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - (2*a*b*Log[a*Cosh[c + d*x] + b*Sinh[c + d*x
]])/((a^2 - b^2)^2*d) + b/((a^2 - b^2)*d*(a + b*Tanh[c + d*x]))
```

Rubi [A] time = 0.0903209, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3483, 3531, 3530}

$$\frac{b}{d\left(a^2-b^2\right)(a+b\tanh(c+dx))}-\frac{2ab\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{x\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x])^(-2),x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - (2*a*b*Log[a*Cosh[c + d*x] + b*Sinh[c + d*x
]])/((a^2 - b^2)^2*d) + b/((a^2 - b^2)*d*(a + b*Tanh[c + d*x]))
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx = \frac{b}{\left(a^2-b^2\right)d(a+b\tanh(c+dx))} + \frac{\int \frac{a-b\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2} + \frac{b}{\left(a^2-b^2\right)d(a+b\tanh(c+dx))} - \frac{(2iab)\int \frac{-ib-ia\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{\left(a^2-b^2\right)^2}$$

$$= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2} - \frac{2ab\log(a\cosh(c+dx)+b\sinh(c+dx))}{\left(a^2-b^2\right)^2 d} + \frac{b}{\left(a^2-b^2\right)d(a+b\tanh(c+dx))}$$

Mathematica [A] time = 1.05945, size = 94, normalized size = 1.11

$$\frac{\frac{2b\left(\frac{a^2-b^2}{a+b\tanh(c+dx)}-2a\log(a+b\tanh(c+dx))\right)}{\left(a^2-b^2\right)^2} - \frac{\log(1-\tanh(c+dx))}{(a+b)^2} + \frac{\log(\tanh(c+dx)+1)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x])^(-2),x]
```

```
[Out] (-(Log[1 - Tanh[c + d*x]]/(a + b)^2) + Log[1 + Tanh[c + d*x]]/(a - b)^2 + (
2*b*(-2*a*Log[a + b*Tanh[c + d*x]] + (a^2 - b^2)/(a + b*Tanh[c + d*x])))/(a
^2 - b^2)^2)/(2*d)
```

Maple [A] time = 0.027, size = 101, normalized size = 1.2

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a-b)^2} - \frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^2} + \frac{b}{d(a-b)(a+b)(a+b\tanh(dx+c))} - 2\frac{ab\ln(a+b\tanh(dx+c))}{d(a+b)^2(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c))^2,x)
```

```
[Out] 1/2/d/(a-b)^2*ln(tanh(d*x+c)+1)-1/2/d/(a+b)^2*ln(tanh(d*x+c)-1)+1/d*b/(a-b)
/(a+b)/(a+b*tanh(d*x+c))-2/d*a*b/(a+b)^2/(a-b)^2*ln(a+b*tanh(d*x+c))
```

Maxima [A] time = 1.15772, size = 171, normalized size = 2.01

$$-\frac{2ab\log\left(-(a-b)e^{(-2dx-2c)}-a-b\right)}{\left(a^4-2a^2b^2+b^4\right)d} - \frac{2b^2}{\left(a^4-2a^2b^2+b^4+\left(a^4-2a^3b+2ab^3-b^4\right)e^{(-2dx-2c)}\right)d} + \frac{dx+c}{\left(a^2+2ab+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*d*x - 2*c) - a - b)/((a^4 - 2*a^2*b^2 + b^4)*d) -
 2*b^2/((a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*d*x
- 2*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

Fricas [B] time = 2.39271, size = 980, normalized size = 11.53

$$\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)dx\cosh\left(dx+c\right)^2+2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a^3+3\,a^2b+\right.}{\left(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5\right)d\cosh\left(dx+c\right)^2+2\left(a^5+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + 2*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x*sinh(d*x + c)^2 + 2*a*b^2 - 2*b^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*
x - 2*(a^2*b - a*b^2 + (a^2*b + a*b^2)*cosh(d*x + c)^2 + 2*(a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c) + (a^2*b + a*b^2)*sinh(d*x + c)^2)*log(2*(a*cos
h(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))))/((a^5 + a^4
*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)^2 + 2*(a^5 + a^4*
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a
^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*sinh(d*x + c)^2 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*d)
```

Sympy [A] time = 32.4244, size = 3599, normalized size = 42.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (d*x*tanh(c +
 d*x)**2/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d) -
2*d*x*tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2*d*tanh(c + d*x) + 4
*b**2*d) + d*x/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2
*d) - tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2*d*tanh(c + d*x) + 4
*b**2*d) + 2/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d
), Eq(a, -b)), (d*x*tanh(c + d*x)**2/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*
tanh(c + d*x) + 4*b**2*d) + 2*d*x*tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2
+ 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + d*x/(4*b**2*d*tanh(c + d*x)**2 + 8*b
**2*d*tanh(c + d*x) + 4*b**2*d) - tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2
+ 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - 2/(4*b**2*d*tanh(c + d*x)**2 + 8*b**
2*d*tanh(c + d*x) + 4*b**2*d), Eq(a, b)), (d*x*exp(4*c)*exp(4*d*x)*tanh(c +
 d*x)**2/(4*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(4*c)
*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*exp(4*c)*exp(4*d*x) + 8*b**2*d*exp(2*c
)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(2*c)*exp(2*d*x) + 4*b**2*d*tan
h(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - 2*d*x*exp(4*c)*exp(4*d
*x)*tanh(c + d*x)/(4*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x)**2 - 8*b**2*d
*exp(4*c)*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*exp(4*c)*exp(4*d*x) + 8*b**2*
d*exp(2*c)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(2*c)*exp(2*d*x) + 4*b
**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + d*x*exp(4*c)*
exp(4*d*x)/(4*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(4*
c)*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*exp(4*c)*exp(4*d*x) + 8*b**2*d*exp(2
*c)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(2*c)*exp(2*d*x) + 4*b**2*d*t
anh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - 4*d*x*exp(2*c)*exp(2
*d*x)*tanh(c + d*x)**2/(4*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x)**2 - 8*b
**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*exp(4*c)*exp(4*d*x) + 8*
b**2*d*exp(2*c)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(2*c)*exp(2*d*x)
+ 4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + 4*d*x*ex
```

```
p(2*c)*exp(2*d*x)/(4*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x)**2 - 8*b**2*d
*exp(4*c)*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*exp(4*c)*exp(4*d*x) + 8*b**2*
d*exp(2*c)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(2*c)*exp(2*d*x) + 4*b
**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + d*x*tanh(c +
d*x)**2/(4*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(4*c)*
exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*exp(4*c)*exp(4*d*x) + 8*b**2*d*exp(2*c)
*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(2*c)*exp(2*d*x) + 4*b**2*d*tanh
(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + 2*d*x*tanh(c + d*x)/(4*
b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(4*c)*exp(4*d*x)*
tanh(c + d*x) + 4*b**2*d*exp(4*c)*exp(4*d*x) + 8*b**2*d*exp(2*c)*exp(2*d*x)
*tanh(c + d*x)**2 - 8*b**2*d*exp(2*c)*exp(2*d*x) + 4*b**2*d*tanh(c + d*x)**
2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + d*x/(4*b**2*d*exp(4*c)*exp(4*d*x)*
tanh(c + d*x)**2 - 8*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*ex
p(4*c)*exp(4*d*x) + 8*b**2*d*exp(2*c)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*
d*exp(2*c)*exp(2*d*x) + 4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x)
+ 4*b**2*d) - exp(4*c)*exp(4*d*x)*tanh(c + d*x)/(4*b**2*d*exp(4*c)*exp(4*d*
x)*tanh(c + d*x)**2 - 8*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d
*exp(4*c)*exp(4*d*x) + 8*b**2*d*exp(2*c)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b*
*2*d*exp(2*c)*exp(2*d*x) + 4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*
x) + 4*b**2*d) + 2*exp(4*c)*exp(4*d*x)/(4*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c
 + d*x)**2 - 8*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*exp(4*c)
*exp(4*d*x) + 8*b**2*d*exp(2*c)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(
2*c)*exp(2*d*x) + 4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b*
*2*d) + 4*exp(2*c)*exp(2*d*x)*tanh(c + d*x)/(4*b**2*d*exp(4*c)*exp(4*d*x)*t
anh(c + d*x)**2 - 8*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*exp
(4*c)*exp(4*d*x) + 8*b**2*d*exp(2*c)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*d
*exp(2*c)*exp(2*d*x) + 4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) +
 4*b**2*d) - tanh(c + d*x)/(4*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x)**2 -
 8*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x) + 4*b**2*d*exp(4*c)*exp(4*d*x)
+ 8*b**2*d*exp(2*c)*exp(2*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(2*c)*exp(2*d
*x) + 4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - 2/(4
*b**2*d*exp(4*c)*exp(4*d*x)*tanh(c + d*x)**2 - 8*b**2*d*exp(4*c)*exp(4*d*x)
*tanh(c + d*x) + 4*b**2*d*exp(4*c)*exp(4*d*x) + 8*b**2*d*exp(2*c)*exp(2*d*x
)*tanh(c + d*x)**2 - 8*b**2*d*exp(2*c)*exp(2*d*x) + 4*b**2*d*tanh(c + d*x)*
*2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d), Eq(a, -(b*exp(2*c)*exp(2*d*x) - b)
/(exp(2*c)*exp(2*d*x) + 1))), (x/(a + b*tanh(c))**2, Eq(d, 0)), (x/a**2, Eq
(b, 0)), (a**3*d*x/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**
2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) + a**2*b*d*x*tanh
(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*
tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) - 2*a**2*b*d*x/(a**5*d + a
**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**
4*d + b**5*d*tanh(c + d*x)) - 2*a**2*b*log(a/b + tanh(c + d*x))/(a**5*d + a
**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**
4*d + b**5*d*tanh(c + d*x)) + 2*a**2*b*log(tanh(c + d*x) + 1)/(a**5*d + a**
4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*
d + b**5*d*tanh(c + d*x)) + a**2*b/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**
3*b**2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) -
 2*a*b**2*d*x*tanh(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*
d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) + a*b**2
*d*x/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(
c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) - 2*a*b**2*log(a/b + tanh(c + d
*x))*tanh(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**
2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) + 2*a*b**2*log(ta
nh(c + d*x) + 1)*tanh(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b*
*2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) + b**
3*d*x*tanh(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a*
*2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) - b**3/(a**5*d +
 a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b
**4*d + b**5*d*tanh(c + d*x)), True))
```

Giac [A] time = 1.21512, size = 188, normalized size = 2.21

$$-\frac{2\,ab\log\left(\left|-ae^{\left(2\,dx+2\,c\right)}-be^{\left(2\,dx+2\,c\right)}-a+b\right|\right)}{a^4d-2\,a^2b^2d+b^4d}+\frac{dx+c}{a^2d-2\,abd+b^2d}+\frac{2\,\left(ab^2-b^3\right)}{\left(ae^{\left(2\,dx+2\,c\right)}+be^{\left(2\,dx+2\,c\right)}+a-b\right)\left(a+b\right)^2\left(a-b\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(-a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - a + b))/(a^4*d - 2*
a^2*b^2*d + b^4*d) + (d*x + c)/(a^2*d - 2*a*b*d + b^2*d) + 2*(a*b^2 - b^3)/
((a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)*(a + b)^2*(a - b)^2*d)
```

3.63 $\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$\frac{2ab}{d\left(a^2-b^2\right)^2(a+b\tanh(c+dx))}+\frac{b}{2d\left(a^2-b^2\right)(a+b\tanh(c+dx))^2}-\frac{b\left(3a^2+b^2\right)\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\left(a^2-b^2\right)^3}$$

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 - (b*(3*a^2 + b^2)*Log[a*Cosh[c + d*x] +
b*Sinh[c + d*x]])/((a^2 - b^2)^3*d) + b/(2*(a^2 - b^2)*d*(a + b*Tanh[c + d*
x])^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Tanh[c + d*x]))
```

Rubi [A] time = 0.177583, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3483, 3529, 3531, 3530}

$$\frac{2ab}{d\left(a^2-b^2\right)^2(a+b\tanh(c+dx))}+\frac{b}{2d\left(a^2-b^2\right)(a+b\tanh(c+dx))^2}-\frac{b\left(3a^2+b^2\right)\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x])^(-3),x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 - (b*(3*a^2 + b^2)*Log[a*Cosh[c + d*x] +
b*Sinh[c + d*x]])/((a^2 - b^2)^3*d) + b/(2*(a^2 - b^2)*d*(a + b*Tanh[c + d*
x])^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Tanh[c + d*x]))
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx = \frac{b}{2\left(a^2-b^2\right)d(a+b\tanh(c+dx))^2}+\frac{\int \frac{a-b\tanh(c+dx)}{(a+b\tanh(c+dx))^2}\,dx}{a^2-b^2}$$

$$= \frac{b}{2\left(a^2-b^2\right)d(a+b\tanh(c+dx))^2}+\frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\tanh(c+dx))}+\frac{\int \frac{a^2+b^2-2ab\tanh}{a+b\tanh(c}}{\left(a^2-b^2\right.}$$

$$= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3}+\frac{b}{2\left(a^2-b^2\right)d(a+b\tanh(c+dx))^2}+\frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\tanh(c+dx))}$$

$$= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3}-\frac{b\left(3a^2+b^2\right)\log(a\cosh(c+dx)+b\sinh(c+dx))}{\left(a^2-b^2\right)^3 d}+\frac{b}{2\left(a^2-b^2\right)d(a+b}$$

Mathematica [A] time = 2.29119, size = 122, normalized size = 0.95

$$\frac{\frac{b\left(\frac{\left(a^2-b^2\right)\left(5a^2+4ab\tanh(c+dx)-b^2\right)}{(a+b\tanh(c+dx))^2}-2\left(3a^2+b^2\right)\log(a+b\tanh(c+dx))\right)}{\left(a^2-b^2\right)^3}-\frac{\log(1-\tanh(c+dx))}{(a+b)^3}+\frac{\log(\tanh(c+dx)+1)}{(a-b)^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x])^(-3),x]
```

```
[Out] (-(Log[1 - Tanh[c + d*x]]/(a + b)^3) + Log[1 + Tanh[c + d*x]]/(a - b)^3 + (
b*(-2*(3*a^2 + b^2)*Log[a + b*Tanh[c + d*x]] + ((a^2 - b^2)*(5*a^2 - b^2 +
4*a*b*Tanh[c + d*x]))/(a + b*Tanh[c + d*x])^2))/(a^2 - b^2)^3)/(2*d)
```

Maple [A] time = 0.03, size = 166, normalized size = 1.3

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a-b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^3}+\frac{b}{2d(a-b)(a+b)(a+b\tanh(dx+c))^2}+2\frac{ab}{d(a+b)^2(a-b)^2(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c))^3,x)
```

```
[Out] 1/2/d/(a-b)^3*ln(tanh(d*x+c)+1)-1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)+1/2/d*b/(a-
b)/(a+b)/(a+b*tanh(d*x+c))^2+2/d*a*b/(a+b)^2/(a-b)^2/(a+b*tanh(d*x+c))-3/d*
b/(a+b)^3/(a-b)^3*ln(a+b*tanh(d*x+c))*a^2-1/d*b^3/(a+b)^3/(a-b)^3*ln(a+b*ta
nh(d*x+c))
```

Maxima [B] time = 1.42882, size = 439, normalized size = 3.4

$$-\frac{\left(3a^2b+b^3\right)\log\left(-(a-b)e^{(-2dx-2c)}-a-b\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)d}-\frac{}{\left(a^7+a^6b-3a^5b^2-3a^4b^3+3a^3b^4+3a^2b^5-ab^6-b^7+2\left(a^7-a^6b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b + b^3)*log(-(a - b)*e^(-2*d*x - 2*c) - a - b)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*d) - 2*(3*a^2*b^2 + 3*a*b^3 + (3*a^2*b^2 - 2*a*b^3 - b^4)
*e^(-2*d*x - 2*c))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7 + 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*e^(-2*d*x - 2*c) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^
4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*e^(-4*d*x - 4*c))*d) + (d*x +
c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] time = 2.57486, size = 3182, normalized size = 24.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c
)^4 + 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*x*sinh(d*x + c)^4 + 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + (a^5 + a
^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*x + 2*(3*a^3*b^2 - a^2*b^3 -
3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x
)*cosh(d*x + c)^2 + 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 + 3*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^2 + (a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*sinh(d*x + c)^2 - (3*
a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
 + 2*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^4*b + 6*a^3*b^2 + 4*
a^2*b^3 + 2*a*b^4 + b^5)*sinh(d*x + c)^4 + 2*(3*a^4*b - 2*a^2*b^3 - b^5)*co
sh(d*x + c)^2 + 2*(3*a^4*b - 2*a^2*b^3 - b^5 + 3*(3*a^4*b + 6*a^3*b^2 + 4*a
^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^3 + (3*a^4*b - 2*a^2*b^3
 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c) + b*sinh(d*x +
 c))/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^3 + (3*a^3*b^2 - a^2*b^3 - 3*a*
b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c))/((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3
*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^4 + 4*(a^8 + 2*a^7*b - 2*
a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c
)*sinh(d*x + c)^3 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*
a^2*b^6 - 2*a*b^7 - b^8)*d*sinh(d*x + c)^4 + 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*d*cosh(d*x + c)^2 + 2*(3*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6
*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^2 + (a^8
- 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sinh(d*x + c)^2 + (a^8 - 2*a^
7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)*d + 4*
((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 -
 b^8)*d*cosh(d*x + c)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d
*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25006, size = 288, normalized size = 2.23

$$-\frac{\left(3a^2b+b^3\right)\log\left(\left|-ae^{(2dx+2c)}-be^{(2dx+2c)}-a+b\right|\right)}{a^6d-3a^4b^2d+3a^2b^4d-b^6d}+\frac{dx+c}{a^3d-3a^2bd+3ab^2d-b^3d}+\frac{2\left(\left(3a^2b^2-4ab^3+b^4\right)e^{(2dx+2c)}\cdots\right.}{\left(ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b + b^3)*log(abs(-a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - a + b))/(
a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d) + (d*x + c)/(a^3*d - 3*a^2*b*d +
 3*a*b^2*d - b^3*d) + 2*((3*a^2*b^2 - 4*a*b^3 + b^4)*e^(2*d*x + 2*c) + 3*(a
^3*b^2 - 2*a^2*b^3 + a*b^4)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c
) + a - b)^2*(a + b)^2*(a - b)^3*d)
```

3.64 $\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx$

Optimal. Leaf size=169

$$\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a+b\tanh(c+dx))}+\frac{ab}{d\left(a^2-b^2\right)^2(a+b\tanh(c+dx))^2}+\frac{b}{3d\left(a^2-b^2\right)(a+b\tanh(c+dx))^3}-\frac{4ab\left(a^2+b\right.}{}$$

```
[Out] ((a^4 + 6*a^2*b^2 + b^4)*x)/(a^2 - b^2)^4 - (4*a*b*(a^2 + b^2)*Log[a*Cosh[c
 + d*x] + b*Sinh[c + d*x]])/((a^2 - b^2)^4*d) + b/(3*(a^2 - b^2)*d*(a + b*T
anh[c + d*x])^3) + (a*b)/((a^2 - b^2)^2*d*(a + b*Tanh[c + d*x])^2) + (b*(3*
a^2 + b^2))/((a^2 - b^2)^3*d*(a + b*Tanh[c + d*x]))
```

Rubi [A] time = 0.268928, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3483, 3529, 3531, 3530}

$$\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a+b\tanh(c+dx))}+\frac{ab}{d\left(a^2-b^2\right)^2(a+b\tanh(c+dx))^2}+\frac{b}{3d\left(a^2-b^2\right)(a+b\tanh(c+dx))^3}-\frac{4ab\left(a^2+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x])^(-4),x]
```


```
[Out] ((a^4 + 6*a^2*b^2 + b^4)*x)/(a^2 - b^2)^4 - (4*a*b*(a^2 + b^2)*Log[a*Cosh[c
 + d*x] + b*Sinh[c + d*x]])/((a^2 - b^2)^4*d) + b/(3*(a^2 - b^2)*d*(a + b*T
anh[c + d*x])^3) + (a*b)/((a^2 - b^2)^2*d*(a + b*Tanh[c + d*x])^2) + (b*(3*
a^2 + b^2))/((a^2 - b^2)^3*d*(a + b*Tanh[c + d*x]))
```


Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
```

```
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx = \frac{b}{3\left(a^2-b^2\right)d(a+b\tanh(c+dx))^3}+\frac{\int \frac{a-b\tanh(c+dx)}{(a+b\tanh(c+dx))^3}\,dx}{a^2-b^2}$$

$$= \frac{b}{3\left(a^2-b^2\right)d(a+b\tanh(c+dx))^3}+\frac{ab}{\left(a^2-b^2\right)^2 d(a+b\tanh(c+dx))^2}+\frac{\int \frac{a^2+b^2-2ab\tan}{(a+b\tanh(c}}{\left(a^2-b\right.}$$

$$= \frac{b}{3\left(a^2-b^2\right)d(a+b\tanh(c+dx))^3}+\frac{ab}{\left(a^2-b^2\right)^2 d(a+b\tanh(c+dx))^2}+\frac{b(3}{\left(a^2-b^2\right)^3 d(a}$$

$$= \frac{\left(a^4+6a^2b^2+b^4\right)x}{\left(a^2-b^2\right)^4}+\frac{b}{3\left(a^2-b^2\right)d(a+b\tanh(c+dx))^3}+\frac{ab}{\left(a^2-b^2\right)^2 d(a+b\tanh(c+}$$

$$= \frac{\left(a^4+6a^2b^2+b^4\right)x}{\left(a^2-b^2\right)^4}-\frac{4ab\left(a^2+b^2\right)\log(a\cosh(c+dx)+b\sinh(c+dx))}{\left(a^2-b^2\right)^4 d}+\frac{}{3\left(a^2-b^2\right)}$$

Mathematica [A] time = 3.25703, size = 160, normalized size = 0.95

$$\frac{\frac{2b\left(\frac{\left(a^2-b^2\right)\left(3b^2\left(3a^2+b^2\right)\tanh^2(c+dx)+3ab\left(7a^2+b^2\right)\tanh(c+dx)-2a^2b^2+13a^4+b^4\right)}{(a+b\tanh(c+dx))^3}-12a\left(a^2+b^2\right)\log(a+b\tanh(c+dx))\right)}{\left(a^2-b^2\right)^4}-\frac{3\log(1-\tanh(c+dx))}{(a+b)^4}+\frac{3\log(\tanh(c+dx}{(a-b)^4}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x])^(-4),x]
```

```
[Out] ((-3*Log[1 - Tanh[c + d*x]])/(a + b)^4 + (3*Log[1 + Tanh[c + d*x]])/(a - b)
^4 + (2*b*(-12*a*(a^2 + b^2)*Log[a + b*Tanh[c + d*x]] + ((a^2 - b^2)*(13*a^
4 - 2*a^2*b^2 + b^4 + 3*a*b*(7*a^2 + b^2)*Tanh[c + d*x] + 3*b^2*(3*a^2 + b^
2)*Tanh[c + d*x]^2))/(a + b*Tanh[c + d*x])^3))/(a^2 - b^2)^4)/(6*d)
```

Maple [A] time = 0.035, size = 230, normalized size = 1.4

$$\frac{\ln(\tanh(dx+c)+1)}{2d(a-b)^4}-\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^4}+\frac{b}{3d(a-b)(a+b)(a+b\tanh(dx+c))^3}+\frac{ab}{d(a+b)^2(a-b)^2(a+b\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c))^4,x)
```

```
[Out] 1/2/d/(a-b)^4*ln(tanh(d*x+c)+1)-1/2/d/(a+b)^4*ln(tanh(d*x+c)-1)+1/3/d*b/(a-
b)/(a+b)/(a+b*tanh(d*x+c))^3+1/d*a*b/(a+b)^2/(a-b)^2/(a+b*tanh(d*x+c))^2+3/
d*b/(a+b)^3/(a-b)^3/(a+b*tanh(d*x+c))*a^2+1/d*b^3/(a+b)^3/(a-b)^3/(a+b*tanh
(d*x+c))-4/d*b*a^3/(a+b)^4/(a-b)^4*ln(a+b*tanh(d*x+c))-4/d*b^3*a/(a+b)^4/(a
-b)^4*ln(a+b*tanh(d*x+c))
```

Maxima [B] time = 1.4165, size = 709, normalized size = 4.2

$$-\frac{4\left(a^{3}b+ab^{3}\right)\log\left(-(a-b)e^{(-2\,dx-2\,c)}-a-b\right)}{\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)d}-\frac{\cdots}{3\left(a^{10}+2\,a^{9}b-3\,a^{8}b^{2}-8\,a^{7}b^{3}+2\,a^{6}b^{4}+12\,a^{5}b^{5}+2\,a^{4}b^{6}-8\,a^{3}b^{7}-3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -4*(a^3*b + a*b^3)*log(-(a - b)*e^(-2*d*x - 2*c) - a - b)/((a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d) - 4/3*(9*a^4*b^2 + 18*a^3*b^3 + 11*a^2*b
^4 + 4*a*b^5 + 2*b^6 + 3*(6*a^4*b^2 + 2*a^3*b^3 - 5*a^2*b^4 - 2*a*b^5 - b^6
)*e^(-2*d*x - 2*c) + 3*(3*a^4*b^2 - 4*a^3*b^3 + b^6)*e^(-4*d*x - 4*c))/((a^
10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 -
 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10 + 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4
- 10*a^4*b^6 + 5*a^2*b^8 - b^10)*e^(-2*d*x - 2*c) + 3*(a^10 - 2*a^9*b - 3*a
^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2
*b^8 - 2*a*b^9 + b^10)*e^(-4*d*x - 4*c) + (a^10 - 4*a^9*b + 3*a^8*b^2 + 8*a
^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^10)*
e^(-6*d*x - 6*c))*d) + (d*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d)
```

Fricas [B] time = 2.8968, size = 8128, normalized size = 48.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 +
 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^6 + 18*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a
^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)*sinh(d*
x + c)^5 + 3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2
*b^5 + 7*a*b^6 + b^7)*d*x*sinh(d*x + c)^6 + 36*a^5*b^2 - 108*a^4*b^3 + 116*
a^3*b^4 - 60*a^2*b^5 + 24*a*b^6 - 8*b^7 + 3*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^
3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*
b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^4 + 3*(12*a^5*b^2 + 4*a
^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 15*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35
*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^2 + 3
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 -
 b^7)*d*x)*sinh(d*x + c)^4 + 12*(5*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3
 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^3 + (12*a^5*b
^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^
2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7)*d*x + 3*(24*a^5*b^2 - 32*a^4*b^3 - 12*a^3*b^4 + 28*a^2
*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^
4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c)^2 + 3*(24*a^5*b^2 - 32*a^4*
b^3 - 12*a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 15*(a^7 + 7*a^6*b + 21*a
^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x
 + c)^4 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*
a*b^6 + b^7)*d*x + 6*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7
 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b
^6 - b^7)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 12*(a^6*b - 3*a^5*b^2 + 4
```

```
*a^4*b^3 - 4*a^3*b^4 + 3*a^2*b^5 - a*b^6 + (a^6*b + 3*a^5*b^2 + 4*a^4*b^3 +
 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^6 + 6*(a^6*b + 3*a^5*b^2 + 4*
a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)*sinh(d*x + c)^5 + (a
^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*sinh(d*x + c)
^6 + 3*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^4 + 3*(a^6*b + a^5
*b^2 - a^2*b^5 - a*b^6 + 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a
^2*b^5 + a*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^6*b + 3*a^5*b^2
+ 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^3 + 3*(a^6*b + a
^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^6*b - a^5*b
^2 - a^2*b^5 + a*b^6)*cosh(d*x + c)^2 + 3*(a^6*b - a^5*b^2 - a^2*b^5 + a*b^
6 + 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(
d*x + c)^4 + 6*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 6*((a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^
6)*cosh(d*x + c)^5 + 2*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^3
+ (a^6*b - a^5*b^2 - a^2*b^5 + a*b^6)*cosh(d*x + c))*sinh(d*x + c))*log(2*(
a*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*
(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^
6 + b^7)*d*x*cosh(d*x + c)^5 + 2*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a
*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2
*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^3 + (24*a^5*b^2 - 32*a^4*b^3 - 12*
a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^
4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c))*sinh(d*x +
 c))/((a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14
*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x +
 c)^6 + 6*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5
+ 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b
^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10
+ b^11)*d*sinh(d*x + c)^6 + 3*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a
^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b
^10 - b^11)*d*cosh(d*x + c)^4 + 3*(5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^
3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9
+ 3*a*b^10 + b^11)*d*cosh(d*x + c)^2 + (a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b
^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*
b^9 - a*b^10 - b^11)*d)*sinh(d*x + c)^4 + 3*(a^11 - a^10*b - 5*a^9*b^2 + 5*
a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5
*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^11 + 3*a^10*b - a^9*b
^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*
b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^3 + 3*(a^11 + a^10*b - 5*a
^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*
a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5
*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^
4 + 6*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10
*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x +
 c)^2 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 -
10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d)*sinh(d*
x + c)^2 + (a^11 - 3*a^10*b - a^9*b^2 + 11*a^8*b^3 - 6*a^7*b^4 - 14*a^6*b^5
 + 14*a^5*b^6 + 6*a^4*b^7 - 11*a^3*b^8 + a^2*b^9 + 3*a*b^10 - b^11)*d + 6*(
(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b
^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^5
+ 2*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a
^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c
)^3 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10
*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x +
 c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27223, size = 427, normalized size = 2.53

$$-\frac{4\left(a^{3}b+ab^{3}\right)\log\left(\left|-ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^{8}d-4\,a^{6}b^{2}d+6\,a^{4}b^{4}d-4\,a^{2}b^{6}d+b^{8}d}+\frac{dx+c}{a^{4}d-4\,a^{3}bd+6\,a^{2}b^{2}d-4\,ab^{3}d+b^{4}d}+\frac{4\left(3\left(3\,a^{4}b^{2}-2\,a^{3}b^{3}-2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4*(a^3*b + a*b^3)*log(abs(-a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - a + b))
/(a^8*d - 4*a^6*b^2*d + 6*a^4*b^4*d - 4*a^2*b^6*d + b^8*d) + (d*x + c)/(a^4
*d - 4*a^3*b*d + 6*a^2*b^2*d - 4*a*b^3*d + b^4*d) + 4/3*(3*(3*a^4*b^2 - 2*a
^3*b^3 - 2*a^2*b^4 + 2*a*b^5 - b^6)*e^(4*d*x + 4*c) + 3*(6*a^4*b^2 - 14*a^3
*b^3 + 11*a^2*b^4 - 4*a*b^5 + b^6)*e^(2*d*x + 2*c) + (9*a^5*b^2 - 27*a^4*b^
3 + 29*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6 - 2*b^7)/(a + b))/((a*e^(2*d*x + 2*c)
 + b*e^(2*d*x + 2*c) + a - b)^3*(a + b)^3*(a - b)^4*d)
```

3.65 $\int \frac{1}{4+6\tanh(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{3\log(3\sinh(c+dx)+2\cosh(c+dx))}{10d}-\frac{x}{5}$$

```
[Out] -x/5 + (3*Log[2*Cosh[c + d*x] + 3*Sinh[c + d*x]])/(10*d)
```

Rubi [A] time = 0.0440098, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3484, 3530}

$$\frac{3\log(3\sinh(c+dx)+2\cosh(c+dx))}{10d}-\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 6*Tanh[c + d*x])^(-1),x]
```

```
[Out] -x/5 + (3*Log[2*Cosh[c + d*x] + 3*Sinh[c + d*x]])/(10*d)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+6\tanh(c+dx)}\,dx &= -\frac{x}{5}+\frac{3}{10}i\int \frac{-6i-4i\tanh(c+dx)}{4+6\tanh(c+dx)}\,dx\\ &= -\frac{x}{5}+\frac{3\log(2\cosh(c+dx)+3\sinh(c+dx))}{10d}\end{aligned}$$

Mathematica [A] time = 0.0333346, size = 53, normalized size = 1.71

$$-\frac{\log(1-\tanh(c+dx))}{20d}-\frac{\log(\tanh(c+dx)+1)}{4d}+\frac{3\log(3\tanh(c+dx)+2)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 6*Tanh[c + d*x])^(-1),x]
```

```
[Out] -Log[1 - Tanh[c + d*x]]/(20*d) - Log[1 + Tanh[c + d*x]]/(4*d) + (3*Log[2 +
3*Tanh[c + d*x]])/(10*d)
```

Maple [A] time = 0.017, size = 46, normalized size = 1.5

$$-\frac{\ln (\tanh (dx+c)+1)}{4d}-\frac{\ln (\tanh (dx+c)-1)}{20d}+\frac{3\ln (2+3\tanh (dx+c))}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+6*tanh(d*x+c)),x)
```

```
[Out] -1/4/d*ln(tanh(d*x+c)+1)-1/20/d*ln(tanh(d*x+c)-1)+3/10/d*ln(2+3*tanh(d*x+c)
)
```

Maxima [A] time = 1.11251, size = 38, normalized size = 1.23

$$\frac{dx+c}{10d}+\frac{3\log \left(e^{(-2dx-2c)}-5\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*tanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/10*(d*x + c)/d + 3/10*log(e^(-2*d*x - 2*c) - 5)/d
```

Fricas [A] time = 2.25843, size = 130, normalized size = 4.19

$$-\frac{5dx-3\log \left(\frac{2(2\cosh(dx+c)+3\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*tanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/10*(5*d*x - 3*log(2*(2*cosh(d*x + c) + 3*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/d
```

Sympy [A] time = 0.634291, size = 42, normalized size = 1.35

$$\begin{cases} \frac{x}{10}+\frac{3\log \left(\tanh (c+dx)+\frac{2}{3}\right)}{10d}-\frac{3\log (\tanh (c+dx)+1)}{10d} & \text{for } d\neq 0 \\ \frac{x}{6\tanh (c)+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*tanh(d*x+c)),x)
```

```
[Out] Piecewise((x/10 + 3*log(tanh(c + d*x) + 2/3)/(10*d) - 3*log(tanh(c + d*x) +
 1)/(10*d), Ne(d, 0)), (x/(6*tanh(c) + 4), True))
```

Giac [A] time = 1.18147, size = 42, normalized size = 1.35

$$-\frac{dx+c}{2d}+\frac{3\log\left(\left|5e^{(2dx+2c)}-1\right|\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*tanh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)/d + 3/10*log(abs(5*e^(2*d*x + 2*c) - 1))/d
```

3.66 $\int \frac{1}{4-6\tanh(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{3\log(2\cosh(c+dx)-3\sinh(c+dx))}{10d}-\frac{x}{5}$$

```
[Out] -x/5 - (3*Log[2*Cosh[c + d*x] - 3*Sinh[c + d*x]])/(10*d)
```

Rubi [A] time = 0.0422853, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3484, 3530}

$$-\frac{3\log(2\cosh(c+dx)-3\sinh(c+dx))}{10d}-\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 6*Tanh[c + d*x])^(-1),x]
```

```
[Out] -x/5 - (3*Log[2*Cosh[c + d*x] - 3*Sinh[c + d*x]])/(10*d)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-6\tanh(c+dx)}\,dx &= -\frac{x}{5}-\frac{3}{10}i\int \frac{6i-4i\tanh(c+dx)}{4-6\tanh(c+dx)}\,dx\\ &= -\frac{x}{5}-\frac{3\log(2\cosh(c+dx)-3\sinh(c+dx))}{10d}\end{aligned}$$

Mathematica [A] time = 0.0316149, size = 53, normalized size = 1.71

$$-\frac{3\log(2-3\tanh(c+dx))}{10d}+\frac{\log(1-\tanh(c+dx))}{4d}+\frac{\log(\tanh(c+dx)+1)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 6*Tanh[c + d*x])^(-1),x]
```

```
[Out] (-3*Log[2 - 3*Tanh[c + d*x]])/(10*d) + Log[1 - Tanh[c + d*x]]/(4*d) + Log[1
 + Tanh[c + d*x]]/(20*d)
```

Maple [A] time = 0.017, size = 46, normalized size = 1.5

$$-\frac{3\ln\left(-2+3\tanh\left(dx+c\right)\right)}{10d}+\frac{\ln\left(\tanh\left(dx+c\right)+1\right)}{20d}+\frac{\ln\left(\tanh\left(dx+c\right)-1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-6*tanh(d*x+c)),x)
```

```
[Out] -3/10/d*ln(-2+3*tanh(d*x+c))+1/20/d*ln(tanh(d*x+c)+1)+1/4/d*ln(tanh(d*x+c)-
1)
```

Maxima [A] time = 1.11217, size = 39, normalized size = 1.26

$$-\frac{1}{2}x-\frac{c}{2d}-\frac{3\log\left(5e^{(-2dx-2c)}-1\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*tanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*c/d - 3/10*log(5*e^(-2*d*x - 2*c) - 1)/d
```

Fricas [A] time = 2.26511, size = 127, normalized size = 4.1

$$\frac{dx-3\log\left(-\frac{2(2\cosh(dx+c)-3\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*tanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/10*(d*x - 3*log(-2*(2*cosh(d*x + c) - 3*sinh(d*x + c))/(cosh(d*x + c) - s
inh(d*x + c))))/d
```

Sympy [A] time = 0.571761, size = 42, normalized size = 1.35

$$\begin{cases}-\frac{x}{2}-\frac{3\log\left(\tanh\left(c+dx\right)-\frac{2}{3}\right)}{10d}+\frac{3\log\left(\tanh\left(c+dx\right)+1\right)}{10d} & \text{for } d\neq 0\\ \frac{x}{4-6\tanh\left(c\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*tanh(d*x+c)),x)
```

```
[Out] Piecewise((-x/2 - 3*log(tanh(c + d*x) - 2/3)/(10*d) + 3*log(tanh(c + d*x) +
 1)/(10*d), Ne(d, 0)), (x/(4 - 6*tanh(c)), True))
```

Giac [A] time = 1.18794, size = 39, normalized size = 1.26

$$\frac{dx+c}{10\,d}-\frac{3\,\log\left(\left|e^{(2\,dx+2\,c)}-5\right|\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*tanh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10*(d*x + c)/d - 3/10*log(abs(e^(2*d*x + 2*c) - 5))/d
```

3.67 $\int \sqrt{a+b\tanh(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]])/d
```

Rubi [A] time = 0.0723861, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3485, 700, 1130, 207}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tanh[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]])/d
```

Rule 3485

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\tanh(c+dx)}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{-b^2+x^2}\,dx, x, b\tanh(c+dx)\right)}{d}$$

$$= -\frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d}$$

$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{a+\right.}}{d}$$

$$= -\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Mathematica [A] time = 0.0880251, size = 74, normalized size = 1.

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tanh[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]])/d
```

Maple [A] time = 0.052, size = 63, normalized size = 0.9

$$\frac{1}{d}\text{Artanh}\left(\sqrt{a+b\tanh(dx+c)}\frac{1}{\sqrt{a+b}}\right)\sqrt{a+b} - \frac{1}{d}\sqrt{-a+b}\arctan\left(\sqrt{a+b\tanh(dx+c)}\frac{1}{\sqrt{-a+b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c))^(1/2),x)
```

```
[Out] arctanh((a+b*tanh(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d-1/d*(-a+b)^(1/2)
*arctan((a+b*tanh(d*x+c))^(1/2)/(-a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.9119, size = 5682, normalized size = 76.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh(d*x + c
)^4 + 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^2 + a^2 + a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*x + c)^4
 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + (2*a
 + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)
^2 + 2*(2*(a + b)*cosh(d*x + c)^3 + (2*a + b)*cosh(d*x + c))*sinh(d*x + c)
+ a)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c)) +
8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + a*b)*cosh(d*x + c))*sinh(d*
x + c)) + sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 + 4*(a^2 - a*
b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*si
nh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x
 + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + (2*a - b)*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 + (2*
a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((a*cosh(d*x +
 c) + b*sinh(d*x + c))/cosh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 +
2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x +
 c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 + a)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2)) -
sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 + 4*(a^2 - a*b)*cosh(d*
x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c
)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh
(d*x + c)^3 + a*sinh(d*x + c)^4 + (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x
 + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b*si
nh(d*x + c))/cosh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 + 2*(a^2 - a
*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a - b)*sqrt(-a +
b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2 - b^2)*cos
h(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh
(d*x + c)^2 + a^2 - 2*a*b + b^2)) - sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a
^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 + 2*a^2 - b^
2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a + b)*sinh(d*x + c)^4 + (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x +
 c)^2 + 2*a + b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 + (2*a + b)
*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b*si
nh(d*x + c))/cosh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/d, -1/2*(sqrt(-a + b)*arctan(-(a*cos
h(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a - b)
*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2
 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2
- b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + b^2)) + sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 + a)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
```

```
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2)))/d
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \tanh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tanh (dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tanh(d*x + c) + a), x)
```

3.68 $\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.0686009, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3485, 708, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tanh[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 3485

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}(-b^2+x^2)}\,dx, x, b\tanh(c+dx)\right)}{d}$$
$$= -\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

Mathematica [A] time = 0.0638881, size = 74, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tanh[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Maple [A] time = 0.039, size = 62, normalized size = 0.8

$$\frac{1}{d}\text{Artanh}\left(\sqrt{a+b\tanh(dx+c)}\frac{1}{\sqrt{a+b}}\right)\frac{1}{\sqrt{a+b}} + \frac{1}{d}\arctan\left(\sqrt{a+b\tanh(dx+c)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c))^(1/2),x)
```

```
[Out] arctanh((a+b*tanh(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/d/(-a+b)^(1/2)
*arctan((a+b*tanh(d*x+c))^(1/2)/(-a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.84483, size = 5844, normalized size = 78.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh
(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 + (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 + 2*a + b)*sinh(
d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 + (2*a + b)*cosh(d*x + c))*sinh(d
*x + c) + a)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x
+ c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c)) + (a + b)*sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 +
4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)
^4 + 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 + 2
*a^2 - 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + (2*a - b)*cosh(
d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 2*(2*a*cosh(
d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sq
rt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c)) + 4*((2*a^2 - b^2)*co
sh(d*x + c)^3 + 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a^2 - b^2)*d), -1/4*(
2*(a - b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
 + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a)*sqrt(-a - b)*sqrt((a*cos
h(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b +
 b^2)*sinh(d*x + c)^2 + a^2 - b^2)) - (a + b)*sqrt(a - b)*log(((2*a^2 - b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2
- b^2)*sinh(d*x + c)^4 + 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)
*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 -
 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 + (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x +
 c)^2 + 2*(2*a*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a
 - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c)) +
 4*((2*a^2 - b^2)*cosh(d*x + c)^3 + 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x +
 c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a
^2 - b^2)*d), -1/4*(2*(a + b)*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a
*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a - b)*sqrt(-a + b)*sqrt
((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2 - b^2)*cosh(d*x +
 c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x +
c)^2 + a^2 - 2*a*b + b^2)) - sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(
a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 + 2*a^2 - b
^2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x
+ c)^2 + 2*a + b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 + (2*a + b
)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b*s
inh(d*x + c))/cosh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/((a^2 - b^2)*d), -1/2*((a + b)*sqrt
(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*s
inh(d*x + c)^2 + a - b)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c
))/cosh(d*x + c))/((a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c
)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + b^2)) + (a -
b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*s
inh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a)*sqrt(-a - b)*sqrt((a*cosh(d*x +
```

```
 c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^2 + a^2 - b^2)))/((a^2 - b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \tanh{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tanh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \tanh\left(dx + c\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tanh(d*x + c) + a), x)
```

3.69 $\int \frac{\sinh^4(x)}{1+\tanh(x)} dx$

Optimal. Leaf size=60

$$\frac{x}{16}-\frac{1}{8(1-\tanh(x))}-\frac{3}{16(\tanh(x)+1)}+\frac{1}{32(1-\tanh(x))^2}+\frac{5}{32(\tanh(x)+1)^2}-\frac{1}{24(\tanh(x)+1)^3}$$

```
[Out] x/16 + 1/(32*(1 - Tanh[x])^2) - 1/(8*(1 - Tanh[x])) - 1/(24*(1 + Tanh[x])^3
) + 5/(32*(1 + Tanh[x])^2) - 3/(16*(1 + Tanh[x]))
```

Rubi [A] time = 0.0724269, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3516, 848, 88, 207}

$$\frac{x}{16}-\frac{1}{8(1-\tanh(x))}-\frac{3}{16(\tanh(x)+1)}+\frac{1}{32(1-\tanh(x))^2}+\frac{5}{32(\tanh(x)+1)^2}-\frac{1}{24(\tanh(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] x/16 + 1/(32*(1 - Tanh[x])^2) - 1/(8*(1 - Tanh[x])) - 1/(24*(1 + Tanh[x])^3
) + 5/(32*(1 + Tanh[x])^2) - 3/(16*(1 + Tanh[x]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx = -\text{Subst}\left(\int \frac{x^4}{(1+x)\left(-1+x^2\right)^3}\,dx, x, \tanh(x)\right)$$
$$= -\text{Subst}\left(\int \frac{x^4}{(-1+x)^3(1+x)^4}\,dx, x, \tanh(x)\right)$$
$$= -\text{Subst}\left(\int\left(\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}-\frac{1}{8(1+x)^4}+\frac{5}{16(1+x)^3}-\frac{3}{16(1+x)^2}+\frac{1}{16\left(-1+x^2\right)}\right)dx,\right.$$
$$= \frac{1}{32(1-\tanh(x))^2}-\frac{1}{8(1-\tanh(x))}-\frac{1}{24(1+\tanh(x))^3}+\frac{5}{32(1+\tanh(x))^2}-\frac{3}{16(1+\tanh(x))}-$$
$$= \frac{x}{16}+\frac{1}{32(1-\tanh(x))^2}-\frac{1}{8(1-\tanh(x))}-\frac{1}{24(1+\tanh(x))^3}+\frac{5}{32(1+\tanh(x))^2}-\frac{3}{16(1+\tanh(x}$$

Mathematica [A] time = 0.0763446, size = 42, normalized size = 0.7

$$\frac{1}{192}(12x-3\sinh(2x)-3\sinh(4x)+\sinh(6x)-15\cosh(2x)+6\cosh(4x)-\cosh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (12*x - 15*Cosh[2*x] + 6*Cosh[4*x] - Cosh[6*x] - 3*Sinh[2*x] - 3*Sinh[4*x]
+ Sinh[6*x])/192
```

Maple [B] time = 0.031, size = 98, normalized size = 1.6

$$-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-6}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5}-\frac{7}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}+\frac{1}{12}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{16}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(1+tanh(x)),x)
```

```
[Out] -1/3/(tanh(1/2*x)+1)^6+1/(tanh(1/2*x)+1)^5-7/8/(tanh(1/2*x)+1)^4+1/12/(tanh
(1/2*x)+1)^3+1/8/(tanh(1/2*x)+1)^2+1/16*ln(tanh(1/2*x)+1)+1/8/(tanh(1/2*x)-
1)^4+1/4/(tanh(1/2*x)-1)^3-1/8/(tanh(1/2*x)-1)-1/16*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.12109, size = 49, normalized size = 0.82

$$-\frac{1}{128}\left(6\,e^{(-2\,x)}-1\right)e^{(4\,x)}+\frac{1}{16}\,x-\frac{1}{32}\,e^{(-2\,x)}+\frac{3}{128}\,e^{(-4\,x)}-\frac{1}{192}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] -1/128*(6*e^(-2*x) - 1)*e^(4*x) + 1/16*x - 1/32*e^(-2*x) + 3/128*e^(-4*x) -
 1/192*e^(-6*x)
```

Fricas [B] time = 2.24885, size = 319, normalized size = 5.32

$$\frac{\cosh(x)^5 + 5\cosh(x)\sinh(x)^4 + 5\sinh(x)^5 + \left(50\cosh(x)^2 - 27\right)\sinh(x)^3 - 9\cosh(x)^3 + \left(10\cosh(x)^3 - 27\cos\right.}{384\left(\cosh(x) + \sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + 5*sinh(x)^5 + (50*cosh(x)^2 - 27)*
sinh(x)^3 - 9*cosh(x)^3 + (10*cosh(x)^3 - 27*cosh(x))*sinh(x)^2 + 12*(2*x -
 1)*cosh(x) + (25*cosh(x)^4 - 81*cosh(x)^2 + 24*x + 12)*sinh(x))/(cosh(x) +
 sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^4(x)}{\tanh(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(1+tanh(x)),x)
```

```
[Out] Integral(sinh(x)**4/(tanh(x) + 1), x)
```

Giac [A] time = 1.21964, size = 57, normalized size = 0.95

$$-\frac{1}{384}\left(22\,e^{(6\,x)} + 12\,e^{(4\,x)} - 9\,e^{(2\,x)} + 2\right)e^{(-6\,x)} + \frac{1}{16}\,x + \frac{1}{128}\,e^{(4\,x)} - \frac{3}{64}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -1/384*(22*e^(6*x) + 12*e^(4*x) - 9*e^(2*x) + 2)*e^(-6*x) + 1/16*x + 1/128*
e^(4*x) - 3/64*e^(2*x)
```

3.70 $\int \frac{\sinh^3(x)}{1+\tanh(x)} \, dx$

Optimal. Leaf size=25

$$-\frac{\sinh^5(x)}{5}+\frac{\cosh^5(x)}{5}-\frac{\cosh^3(x)}{3}$$

```
[Out] -Cosh[x]^3/3 + Cosh[x]^5/5 - Sinh[x]^5/5
```

Rubi [A] time = 0.166771, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3518, 3108, 3107, 2565, 14, 2564, 30}

$$-\frac{\sinh^5(x)}{5}+\frac{\cosh^5(x)}{5}-\frac{\cosh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 + Tanh[x]),x]
```

```
[Out] -Cosh[x]^3/3 + Cosh[x]^5/5 - Sinh[x]^5/5
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^3(x)}{1+\tanh(x)}\,dx &= \int \frac{\cosh(x)\sinh^3(x)}{\cosh(x)+\sinh(x)}\,dx \\
&= i\int \cosh(x)(-i\cosh(x)+i\sinh(x))\sinh^3(x)\,dx \\
&= -\int \left(-\cosh^2(x)\sinh^3(x)+\cosh(x)\sinh^4(x)\right)\,dx \\
&= \int \cosh^2(x)\sinh^3(x)\,dx - \int \cosh(x)\sinh^4(x)\,dx \\
&= i\,\text{Subst}\left(\int x^4\,dx, x, i\sinh(x)\right) - \text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \cosh(x)\right) \\
&= -\frac{1}{5}\sinh^5(x) - \text{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, \cosh(x)\right) \\
&= -\frac{1}{3}\cosh^3(x) + \frac{\cosh^5(x)}{5} - \frac{\sinh^5(x)}{5}
\end{aligned}
$$

Mathematica [A] time = 0.0593123, size = 34, normalized size = 1.36

$$\frac{1}{120}(\cosh(x)-\sinh(x))(-10\sinh(2x)+\sinh(4x)-20\cosh(2x)+4\cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Tanh[x]),x]
```

```
[Out] ((Cosh[x] - Sinh[x])*(-20*Cosh[2*x] + 4*Cosh[4*x] - 10*Sinh[2*x] + Sinh[4*x
]))/120
```

Maple [B] time = 0.027, size = 72, normalized size = 2.9

$$\frac{2}{5}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5} - \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4} + \frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} - \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{1}{6}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-3} - \frac{1}{4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1+tanh(x)),x)
```

```
[Out] 2/5/(tanh(1/2*x)+1)^5-1/(tanh(1/2*x)+1)^4+2/3/(tanh(1/2*x)+1)^3-1/8/(tanh(1
/2*x)+1)-1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2+1/8/(tanh(1/2*x)-1)
```

Maxima [A] time = 1.18165, size = 36, normalized size = 1.44

$$-\frac{1}{48}\left(6e^{(-2x)}-1\right)e^{(3x)}-\frac{1}{24}e^{(-3x)}+\frac{1}{80}e^{(-5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] -1/48*(6*e^(-2*x) - 1)*e^(3*x) - 1/24*e^(-3*x) + 1/80*e^(-5*x)
```

Fricas [B] time = 2.19843, size = 200, normalized size = 8.

$$\frac{\cosh(x)^4+\cosh(x)\sinh(x)^3+\sinh(x)^4+\left(6\cosh(x)^2-5\right)\sinh(x)^2-5\cosh(x)^2+\left(\cosh(x)^3-5\cosh(x)\right)\sinh(x)}{30\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(x)^4 + cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 5)*sinh(x)
^2 - 5*cosh(x)^2 + (cosh(x)^3 - 5*cosh(x))*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] time = 1.59728, size = 134, normalized size = 5.36

$$\frac{3\sinh^3(x)\tanh(x)}{15\tanh(x)+15}-\frac{3\sinh^3(x)}{15\tanh(x)+15}+\frac{6\sinh^2(x)\cosh(x)\tanh(x)}{15\tanh(x)+15}+\frac{9\sinh^2(x)\cosh(x)}{15\tanh(x)+15}-\frac{6\sinh(x)\cosh^2(x)\tanh(x)}{15\tanh(x)+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+tanh(x)),x)
```

```
[Out] 3*sinh(x)**3*tanh(x)/(15*tanh(x) + 15) - 3*sinh(x)**3/(15*tanh(x) + 15) + 6
*sinh(x)**2*cosh(x)*tanh(x)/(15*tanh(x) + 15) + 9*sinh(x)**2*cosh(x)/(15*ta
nh(x) + 15) - 6*sinh(x)*cosh(x)**2*tanh(x)/(15*tanh(x) + 15) + 6*sinh(x)*co
sh(x)**2/(15*tanh(x) + 15) - 8*cosh(x)**3*tanh(x)/(15*tanh(x) + 15) - 2*cos
h(x)**3/(15*tanh(x) + 15)
```

Giac [A] time = 1.19442, size = 34, normalized size = 1.36

$$-\frac{1}{240}\left(10e^{(2x)}-3\right)e^{(-5x)}+\frac{1}{48}e^{(3x)}-\frac{1}{8}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -1/240*(10*e^(2*x) - 3)*e^(-5*x) + 1/48*e^(3*x) - 1/8*e^x
```

3.71 $\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{x}{8}+\frac{1}{8(1-\tanh(x))}+\frac{1}{4(\tanh(x)+1)}-\frac{1}{8(\tanh(x)+1)^2}$$

```
[Out] -x/8 + 1/(8*(1 - Tanh[x])) - 1/(8*(1 + Tanh[x])^2) + 1/(4*(1 + Tanh[x]))
```

Rubi [A] time = 0.0587289, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3516, 848, 88, 207}

$$-\frac{x}{8}+\frac{1}{8(1-\tanh(x))}+\frac{1}{4(\tanh(x)+1)}-\frac{1}{8(\tanh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 + Tanh[x]),x]
```

```
[Out] -x/8 + 1/(8*(1 - Tanh[x])) - 1/(8*(1 + Tanh[x])^2) + 1/(4*(1 + Tanh[x]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{(1+x)\left(-1+x^2\right)^2}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \frac{x^2}{(-1+x)^2(1+x)^3}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}-\frac{1}{4(1+x)^2}+\frac{1}{8\left(-1+x^2\right)}\right)dx, x, \tanh(x)\right)\\ &= \frac{1}{8(1-\tanh(x))}-\frac{1}{8(1+\tanh(x))^2}+\frac{1}{4(1+\tanh(x))}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)\\ &= -\frac{x}{8}+\frac{1}{8(1-\tanh(x))}-\frac{1}{8(1+\tanh(x))^2}+\frac{1}{4(1+\tanh(x))}\end{aligned}$$

Mathematica [A] time = 0.0371192, size = 24, normalized size = 0.63

$$\frac{1}{32}(-4x+\sinh(4x)+4\cosh(2x)-\cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 + Tanh[x]),x]
```

```
[Out] (-4*x + 4*Cosh[2*x] - Cosh[4*x] + Sinh[4*x])/32
```

Maple [B] time = 0.027, size = 68, normalized size = 1.8

$$-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{8}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-2}+\frac{1}{4}\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1+tanh(x)),x)
```

```
[Out] -1/2/(tanh(1/2*x)+1)^4+1/(tanh(1/2*x)+1)^3-1/2/(tanh(1/2*x)+1)^2-1/8*ln(tan
h(1/2*x)+1)+1/4/(tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)+1/8*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.13915, size = 30, normalized size = 0.79

$$-\frac{1}{8}x+\frac{1}{16}e^{(2x)}+\frac{1}{16}e^{(-2x)}-\frac{1}{32}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*x + 1/16*e^(2*x) + 1/16*e^(-2*x) - 1/32*e^(-4*x)
```

Fricas [A] time = 2.31414, size = 176, normalized size = 4.63

$$\frac{\cosh(x)^3+3\cosh(x)\sinh(x)^2+3\sinh(x)^3-2(2x-1)\cosh(x)+\left(9\cosh(x)^2-4x-2\right)\sinh(x)}{32(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*sinh(x)^3 - 2*(2*x - 1)*cosh(x) +
 (9*cosh(x)^2 - 4*x - 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+tanh(x)),x)
```

```
[Out] Integral(sinh(x)**2/(tanh(x) + 1), x)
```

Giac [A] time = 1.22711, size = 41, normalized size = 1.08

$$\frac{1}{32}\left(3\,e^{(4\,x)}+2\,e^{(2\,x)}-1\right)e^{(-4\,x)}-\frac{1}{8}\,x+\frac{1}{16}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/32*(3*e^(4*x) + 2*e^(2*x) - 1)*e^(-4*x) - 1/8*x + 1/16*e^(2*x)
```

3.72 $\int \frac{\sinh(x)}{1+\tanh(x)} dx$

Optimal. Leaf size=17

$$\frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

```
[Out] Cosh[x]^3/3 - Sinh[x]^3/3
```

Rubi [A] time = 0.110752, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3518, 3108, 3107, 2565, 30, 2564}

$$\frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Tanh[x]),x]

[Out] Cosh[x]^3/3 - Sinh[x]^3/3
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{1+\tanh(x)}\,dx &= \int \frac{\cosh(x)\sinh(x)}{\cosh(x)+\sinh(x)}\,dx \\
&= i\int \cosh(x)(-i\cosh(x)+i\sinh(x))\sinh(x)\,dx \\
&= \int \left(\cosh^2(x)\sinh(x)-\cosh(x)\sinh^2(x)\right)\,dx \\
&= \int \cosh^2(x)\sinh(x)\,dx-\int \cosh(x)\sinh^2(x)\,dx \\
&= -\left(i\operatorname{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)\right)+\operatorname{Subst}\left(\int x^2\,dx,x,\cosh(x)\right) \\
&= \frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0162827, size = 19, normalized size = 1.12

$$\frac{1}{12}\left(-4\sinh^3(x)+3\cosh(x)+\cosh(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Tanh[x]),x]
```

```
[Out] (3*Cosh[x] + Cosh[3*x] - 4*Sinh[x]^3)/12
```

Maple [B] time = 0.023, size = 42, normalized size = 2.5

$$\frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+tanh(x)),x)
```

```
[Out] 2/3/(tanh(1/2*x)+1)^3-1/(tanh(1/2*x)+1)^2+1/2/(tanh(1/2*x)+1)-1/2/(tanh(1/2
*x)-1)
```

Maxima [A] time = 1.15552, size = 15, normalized size = 0.88

$$\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

Fricas [A] time = 2.4385, size = 90, normalized size = 5.29

$$\frac{\cosh(x)^2 + \cosh(x)\sinh(x) + \sinh(x)^2}{3(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.436478, size = 48, normalized size = 2.82

$$\frac{\sinh(x)\tanh(x)}{3\tanh(x) + 3} - \frac{\sinh(x)}{3\tanh(x) + 3} + \frac{2\cosh(x)\tanh(x)}{3\tanh(x) + 3} + \frac{\cosh(x)}{3\tanh(x) + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+tanh(x)),x)
```

```
[Out] sinh(x)*tanh(x)/(3*tanh(x) + 3) - sinh(x)/(3*tanh(x) + 3) + 2*cosh(x)*tanh(
x)/(3*tanh(x) + 3) + cosh(x)/(3*tanh(x) + 3)
```

Giac [A] time = 1.18694, size = 15, normalized size = 0.88

$$\frac{1}{12}e^{(-3x)} + \frac{1}{4}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

3.73 $\int \frac{\textbf{csch}(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=12

$$-\sinh(x)+\cosh(x)-\tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]] + Cosh[x] - Sinh[x]
```

Rubi [A] time = 0.10985, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3518, 3108, 3107, 2637, 2592, 321, 206}

$$-\sinh(x)+\cosh(x)-\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(1 + Tanh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + Cosh[x] - Sinh[x]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{1+\tanh(x)}\,dx &= \int \frac{\coth(x)}{\cosh(x)+\sinh(x)}\,dx\\ &= i\int \coth(x)(-i\cosh(x)+i\sinh(x))\,dx\\ &= -\int (\cosh(x)-\cosh(x)\coth(x))\,dx\\ &= -\int \cosh(x)\,dx+\int \cosh(x)\coth(x)\,dx\\ &= -\sinh(x)-\operatorname{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cosh(x)\right)\\ &= \cosh(x)-\sinh(x)-\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cosh(x)\right)\\ &= -\tanh^{-1}(\cosh(x))+\cosh(x)-\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0212519, size = 14, normalized size = 1.17

$$-\sinh(x)+\cosh(x)+\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(1 + Tanh[x]),x]
```

```
[Out] Cosh[x] + Log[Tanh[x/2]] - Sinh[x]
```

Maple [A] time = 0.023, size = 17, normalized size = 1.4

$$\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\,\left(\tanh\left(x/2\right)+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(1+tanh(x)),x)
```

```
[Out] ln(tanh(1/2*x))+2/(tanh(1/2*x)+1)
```

Maxima [A] time = 1.14267, size = 28, normalized size = 2.33

$$e^{(-x)}-\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [B] time = 2.63148, size = 167, normalized size = 13.92

$$-\frac{(\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)+1)-(\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)-1)-1}{\cosh(x)+\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -((cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) - (cosh(x) + sinh(x))*log(
cosh(x) + sinh(x) - 1) - 1)/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)/(tanh(x) + 1), x)
```

Giac [A] time = 1.28094, size = 24, normalized size = 2.

$$e^{(-x)}-\log(e^x+1)+\log(|e^x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] e^(-x) - log(e^x + 1) + log(abs(e^x - 1))
```

3.74 $\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=15

$$-\coth(x)-\log(\tanh(x))+\log(\tanh(x)+1)$$

```
[Out] -Coth[x] - Log[Tanh[x]] + Log[1 + Tanh[x]]
```

Rubi [A] time = 0.0388343, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3516, 44}

$$-\coth(x)-\log(\tanh(x))+\log(\tanh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(1 + Tanh[x]),x]
```

```
[Out] -Coth[x] - Log[Tanh[x]] + Log[1 + Tanh[x]]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, \tanh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx, x, \tanh(x)\right)\\
&= -\coth(x)-\log(\tanh(x))+\log(1+\tanh(x))
\end{aligned}$$

Mathematica [A] time = 0.0285427, size = 11, normalized size = 0.73

$$x-\coth(x)-\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(1 + Tanh[x]),x]
```

```
[Out] x - Coth[x] - Log[Sinh[x]]
```

Maple [B] time = 0.026, size = 32, normalized size = 2.1

$$-\frac{1}{2}\tanh\left(\frac{x}{2}\right)+2\ln\left(\tanh\left(x/2\right)+1\right)-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(1+tanh(x)),x)
```

```
[Out] -1/2*tanh(1/2*x)+2*ln(tanh(1/2*x)+1)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))
```

Maxima [A] time = 1.12086, size = 39, normalized size = 2.6

$$\frac{2}{e^{(-2x)}-1}-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] time = 2.63251, size = 267, normalized size = 17.8

$$\frac{2x\cosh(x)^2+4x\cosh(x)\sinh(x)+2x\sinh(x)^2-\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] (2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 2*x - 2)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^2(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)**2/(tanh(x) + 1), x)
```

<u>Giac</u> [A] time = 1.17582, size = 39, normalized size = 2.6

$$2x + \frac{e^{(2x)} - 3}{e^{(2x)} - 1} - \log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 2*x + (e^(2*x) - 3)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

3.75 $\int \frac{\operatorname{csch}^3(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=18

$$\operatorname{csch}(x)-\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\operatorname{csch}(x)$$

```
[Out] -ArcTanh[Cosh[x]]/2 + Csch[x] - (Coth[x]*Csch[x])/2
```

Rubi [A] time = 0.160599, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3518, 3108, 3107, 2606, 8, 2611, 3770}

$$\operatorname{csch}(x)-\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(1 + Tanh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/2 + Csch[x] - (Coth[x]*Csch[x])/2
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{1+\tanh(x)}\,dx &= \int \frac{\coth(x)\operatorname{csch}^2(x)}{\cosh(x)+\sinh(x)}\,dx \\
&= i\int \coth(x)\operatorname{csch}^2(x)(-i\cosh(x)+i\sinh(x))\,dx \\
&= \int \left(-\coth(x)\operatorname{csch}(x)+\coth^2(x)\operatorname{csch}(x)\right)\,dx \\
&= -\int \coth(x)\operatorname{csch}(x)\,dx+\int \coth^2(x)\operatorname{csch}(x)\,dx \\
&= -\frac{1}{2}\coth(x)\operatorname{csch}(x)+i\,\mathrm{Subst}\left(\int 1\,dx,x,-i\operatorname{csch}(x)\right)+\frac{1}{2}\int \operatorname{csch}(x)\,dx \\
&= -\frac{1}{2}\tanh^{-1}(\cosh(x))+\operatorname{csch}(x)-\frac{1}{2}\coth(x)\operatorname{csch}(x)
\end{aligned}$$

Mathematica [A] time = 0.0787912, size = 20, normalized size = 1.11

$$\frac{1}{2}\left(\log\left(\tanh\left(\frac{x}{2}\right)\right)-(\coth(x)-2)\operatorname{csch}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (-((-2 + Coth[x])*Csch[x]) + Log[Tanh[x/2]])/2
```

Maple [B] time = 0.027, size = 39, normalized size = 2.2

$$\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{2}\tanh\left(\frac{x}{2}\right)-\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(1+tanh(x)),x)
```

```
[Out] 1/8*tanh(1/2*x)^2-1/2*tanh(1/2*x)-1/8/tanh(1/2*x)^2+1/2/tanh(1/2*x)+1/2*ln(
tanh(1/2*x))
```

Maxima [B] time = 1.11479, size = 65, normalized size = 3.61

$$-\frac{e^{(-x)}-3\,e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) - 3*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - 1/2*log(e^(-x) + 1) +
1/2*log(e^(-x) - 1)
```

Fricas [B] time = 2.64399, size = 733, normalized size = 40.72

$$\frac{2\cosh(x)^3 + 6\cosh(x)\sinh(x)^2 + 2\sinh(x)^3 - \left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 1\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(x)^3 + 6*cosh(x)*sinh(x)^2 + 2*sinh(x)^3 - (cosh(x)^4 + 4*cosh(
x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*
(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + (cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(
x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*
(cosh(x)^2 - 1)*sinh(x) - 6*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sin
h(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(
x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(x)}{\tanh(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)**3/(tanh(x) + 1), x)
```

Giac [B] time = 1.21324, size = 46, normalized size = 2.56

$$\frac{e^{(3x)} - 3e^x}{\left(e^{(2x)} - 1\right)^2} - \frac{1}{2}\log\left(e^x + 1\right) + \frac{1}{2}\log\left(|e^x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] (e^(3*x) - 3*e^x)/(e^(2*x) - 1)^2 - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1)
)
```

3.76 $\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\coth^2(x)}{2}-\frac{\coth^3(x)}{3}$$

```
[Out] Coth[x]^2/2 - Coth[x]^3/3
```

Rubi [A] time = 0.044374, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3516, 848, 43}

$$\frac{\coth^2(x)}{2}-\frac{\coth^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(1 + Tanh[x]),x]
```

```
[Out] Coth[x]^2/2 - Coth[x]^3/3
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{-1+x^2}{x^4(1+x)}\,dx,x,\tanh(x)\right)\\
&= -\operatorname{Subst}\left(\int \frac{-1+x}{x^4}\,dx,x,\tanh(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x^3}\right)dx,x,\tanh(x)\right)\\
&= \frac{\coth^2(x)}{2}-\frac{\coth^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0477782, size = 20, normalized size = 1.18

$$-\frac{1}{6}\text{csch}(x)(2\cosh(x)+(2\coth(x)-3)\text{csch}(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(1 + Tanh[x]),x]
```

```
[Out] -(Csch[x]*(2*Cosh[x] + (-3 + 2*Coth[x])*Csch[x]))/6
```

Maple [B] time = 0.03, size = 48, normalized size = 2.8

$$-\frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-\frac{1}{8}\tanh\left(\frac{x}{2}\right)+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}-\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(1+tanh(x)),x)
```

```
[Out] -1/24*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2-1/8*tanh(1/2*x)+1/8/tanh(1/2*x)^2-1/8
/tanh(1/2*x)-1/24/tanh(1/2*x)^3
```

Maxima [B] time = 1.16533, size = 101, normalized size = 5.94

$$-\frac{2\,e^{(-2\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}+\frac{4\,e^{(-4\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}+\frac{2}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 4*e^(-4*x)/(3*e^(-2*
x) - 3*e^(-4*x) + e^(-6*x) - 1) + 2/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) -
 1)
```

Fricas [B] time = 2.28356, size = 286, normalized size = 16.82

$$-\frac{4\,(2\,\cosh(x)+\sinh(x))}{3\left(\cosh(x)^{5}+5\,\cosh(x)\sinh(x)^{4}+\sinh(x)^{5}+\left(10\,\cosh(x)^{2}-3\right)\sinh(x)^{3}-3\,\cosh(x)^{3}+\left(10\,\cosh(x)^{3}-9\,\cosh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 - 3)*sinh(x)^3 - 3*cosh(x)^3 + (10*cosh(x)^3 - 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 - 9*cosh(x)^2 + 4)*sinh(x) + 2*cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)**4/(tanh(x) + 1), x)
```

Giac [A] time = 1.22717, size = 24, normalized size = 1.41

$$-\frac{2\left(3\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) + 1)/(e^(2*x) - 1)^3
```

3.77 $\int \frac{\mathrm{csch}^5(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=34

$$\frac{\mathrm{csch}^3(x)}{3}+\frac{1}{8}\tanh^{-1}(\cosh(x))-\frac{1}{4}\coth(x)\mathrm{csch}^3(x)-\frac{1}{8}\coth(x)\mathrm{csch}(x)$$

```
[Out] ArcTanh[Cosh[x]]/8 - (Coth[x]*Csch[x])/8 + Csch[x]^3/3 - (Coth[x]*Csch[x]^3
)/4
```

Rubi [A] time = 0.191537, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3518, 3108, 3107, 2606, 30, 2611, 3768, 3770}

$$\frac{\mathrm{csch}^3(x)}{3}+\frac{1}{8}\tanh^{-1}(\cosh(x))-\frac{1}{4}\coth(x)\mathrm{csch}^3(x)-\frac{1}{8}\coth(x)\mathrm{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^5/(1 + Tanh[x]),x]
```

```
[Out] ArcTanh[Cosh[x]]/8 - (Coth[x]*Csch[x])/8 + Csch[x]^3/3 - (Coth[x]*Csch[x]^3
)/4
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^5(x)}{1+\tanh(x)}\,dx &= \int \frac{\coth(x)\operatorname{csch}^4(x)}{\cosh(x)+\sinh(x)}\,dx \\
&= i\int \coth(x)\operatorname{csch}^4(x)(-i\cosh(x)+i\sinh(x))\,dx \\
&= -\int \left(\coth(x)\operatorname{csch}^3(x)-\coth^2(x)\operatorname{csch}^3(x)\right)dx \\
&= -\int \coth(x)\operatorname{csch}^3(x)\,dx+\int \coth^2(x)\operatorname{csch}^3(x)\,dx \\
&= -\frac{1}{4}\coth(x)\operatorname{csch}^3(x)-i\,\mathrm{Subst}\left(\int x^2\,dx,x,-i\operatorname{csch}(x)\right)+\frac{1}{4}\int \operatorname{csch}^3(x)\,dx \\
&= -\frac{1}{8}\coth(x)\operatorname{csch}(x)+\frac{\operatorname{csch}^3(x)}{3}-\frac{1}{4}\coth(x)\operatorname{csch}^3(x)-\frac{1}{8}\int \operatorname{csch}(x)\,dx \\
&= \frac{1}{8}\tanh^{-1}(\cosh(x))-\frac{1}{8}\coth(x)\operatorname{csch}(x)+\frac{\operatorname{csch}^3(x)}{3}-\frac{1}{4}\coth(x)\operatorname{csch}^3(x)
\end{aligned}
$$

Mathematica [A] time = 0.118608, size = 49, normalized size = 1.44

$$-\frac{1}{192}\operatorname{csch}^4(x)\left(42\cosh(x)+6\cosh(3x)+2\sinh(x)\left(-9\sinh(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)+3\sinh(3x)\log\left(\tanh\left(\frac{x}{2}\right)\right)-32\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^5/(1 + Tanh[x]),x]
```

```
[Out] -(Csch[x]^4*(42*Cosh[x] + 6*Cosh[3*x] + 2*Sinh[x]*(-32 - 9*Log[Tanh[x/2]]*S
inh[x] + 3*Log[Tanh[x/2]]*Sinh[3*x])))/192
```

Maple [B] time = 0.033, size = 55, normalized size = 1.6

$$\frac{1}{64}\left(\tanh\left(\frac{x}{2}\right)\right)^4-\frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{1}{8}\tanh\left(\frac{x}{2}\right)-\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+\frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}-\frac{1}{8}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{64}\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^5/(1+tanh(x)),x)
```

```
[Out] 1/64*tanh(1/2*x)^4-1/24*tanh(1/2*x)^3+1/8*tanh(1/2*x)-1/8/tanh(1/2*x)+1/24/
tanh(1/2*x)^3-1/8*ln(tanh(1/2*x))-1/64/tanh(1/2*x)^4
```

Maxima [B] time = 1.07946, size = 100, normalized size = 2.94

$$\frac{3\,e^{(-x)} - 11\,e^{(-3\,x)} + 53\,e^{(-5\,x)} + 3\,e^{(-7\,x)}}{12\,\left(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1\right)} + \frac{1}{8}\,\log\left(e^{(-x)} + 1\right) - \frac{1}{8}\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*e^(-x) - 11*e^(-3*x) + 53*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^
(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 1/8*log(e^(-x) + 1) - 1/8*log(e^(-x)
- 1)
```

Fricas [B] time = 2.18859, size = 2136, normalized size = 62.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(6*cosh(x)^7 + 42*cosh(x)*sinh(x)^6 + 6*sinh(x)^7 + 2*(63*cosh(x)^2 -
 11)*sinh(x)^5 - 22*cosh(x)^5 + 10*(21*cosh(x)^3 - 11*cosh(x))*sinh(x)^4 +
2*(105*cosh(x)^4 - 110*cosh(x)^2 + 53)*sinh(x)^3 + 106*cosh(x)^3 + 2*(63*co
sh(x)^5 - 110*cosh(x)^3 + 159*cosh(x))*sinh(x)^2 - 3*(cosh(x)^8 + 8*cosh(x)
*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7
*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sin
h(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3
 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2
 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cos
h(x) + sinh(x) + 1) + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7
*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(
x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*c
osh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x
)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5
 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(21*c
osh(x)^6 - 55*cosh(x)^4 + 159*cosh(x)^2 + 3)*sinh(x) + 6*cosh(x))/(cosh(x)^
8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cos
h(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(
x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(
x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2
- 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x)
 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^5(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**5/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)**5/(tanh(x) + 1), x)
```

Giac [A] time = 1.32025, size = 66, normalized size = 1.94

$$-\frac{3\,e^{(7\,x)}-11\,e^{(5\,x)}+53\,e^{(3\,x)}+3\,e^{x}}{12\,\left(e^{(2\,x)}-1\right)^{4}}+\frac{1}{8}\,\log\left(e^{x}+1\right)-\frac{1}{8}\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -1/12*(3*e^(7*x) - 11*e^(5*x) + 53*e^(3*x) + 3*e^x)/(e^(2*x) - 1)^4 + 1/8*l
og(e^x + 1) - 1/8*log(abs(e^x - 1))
```

3.78 $\int \frac{\text{csch}^6(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{5}\coth^5(x)+\frac{\coth^4(x)}{4}+\frac{\coth^3(x)}{3}-\frac{\coth^2(x)}{2}$$

```
[Out] -Coth[x]^2/2 + Coth[x]^3/3 + Coth[x]^4/4 - Coth[x]^5/5
```

Rubi [A] time = 0.0536724, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3516, 848, 75}

$$-\frac{1}{5}\coth^5(x)+\frac{\coth^4(x)}{4}+\frac{\coth^3(x)}{3}-\frac{\coth^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^6/(1 + Tanh[x]),x]
```

```
[Out] -Coth[x]^2/2 + Coth[x]^3/3 + Coth[x]^4/4 - Coth[x]^5/5
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{csch}^6(x)}{1+\tanh(x)}\,dx &= \text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6(1+x)}\,dx,x,\tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{(-1+x)^2(1+x)}{x^6}\,dx,x,\tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^6}-\frac{1}{x^5}-\frac{1}{x^4}+\frac{1}{x^3}\right)dx,x,\tanh(x)\right)\\
&= -\frac{1}{2}\coth^2(x)+\frac{\coth^3(x)}{3}+\frac{\coth^4(x)}{4}-\frac{\coth^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.0507342, size = 27, normalized size = 0.82

$$\frac{1}{120}\text{csch}^5(x)(30\sinh(x) - 20\cosh(x) - 5\cosh(3x) + \cosh(5x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^6/(1 + Tanh[x]),x]
```

```
[Out] (Csch[x]^5*(-20*Cosh[x] - 5*Cosh[3*x] + Cosh[5*x] + 30*Sinh[x]))/120
```

Maple [B] time = 0.033, size = 80, normalized size = 2.4

$$-\frac{1}{160}\left(\tanh\left(\frac{x}{2}\right)\right)^5 + \frac{1}{64}\left(\tanh\left(\frac{x}{2}\right)\right)^4 + \frac{1}{96}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{1}{16}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{1}{16}\tanh\left(\frac{x}{2}\right) - \frac{1}{16}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} + \frac{1}{16}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^6/(1+tanh(x)),x)
```

```
[Out] -1/160*tanh(1/2*x)^5+1/64*tanh(1/2*x)^4+1/96*tanh(1/2*x)^3-1/16*tanh(1/2*x)
^2+1/16*tanh(1/2*x)-1/16/tanh(1/2*x)^2+1/16/tanh(1/2*x)+1/96/tanh(1/2*x)^3-
1/160/tanh(1/2*x)^5+1/64/tanh(1/2*x)^4
```

Maxima [B] time = 1.10543, size = 201, normalized size = 6.09

$$\frac{4\,e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1\right)} - \frac{8\,e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1\right)} + \frac{}{5\,e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 4/3*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*
x) - 1) - 8/3*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
 + e^(-10*x) - 1) + 8*e^(-6*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*
e^(-8*x) + e^(-10*x) - 1) - 4/15/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) -
5*e^(-8*x) + e^(-10*x) - 1)
```

Fricas [B] time = 1.98475, size = 626, normalized size = 18.97

$$-\frac{}{15\left(\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + \left(28\cosh(x)^2 - 5\right)\sinh(x)^6 - 5\cosh(x)^6 + 2\left(28\cosh(x)^3 - 15\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -4/15*(19*cosh(x)^2 + 42*cosh(x)*sinh(x) + 19*sinh(x)^2 + 5)/(cosh(x)^8 + 8
*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 5)*sinh(x)^6 - 5*cosh(x)^6
 + 2*(28*cosh(x)^3 - 15*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 15*cosh(x)^2
```

```
 + 2)*sinh(x)^4 + 10*cosh(x)^4 + 4*(14*cosh(x)^5 - 25*cosh(x)^3 + 10*cosh(x
))*sinh(x)^3 + (28*cosh(x)^6 - 75*cosh(x)^4 + 60*cosh(x)^2 - 11)*sinh(x)^2
- 11*cosh(x)^2 + 2*(4*cosh(x)^7 - 15*cosh(x)^5 + 20*cosh(x)^3 - 9*cosh(x))*
sinh(x) + 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^6(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**6/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)**6/(tanh(x) + 1), x)
```

Giac [A] time = 1.21643, size = 32, normalized size = 0.97

$$-\frac{4\left(20\,e^{(4\,x)}+5\,e^{(2\,x)}-1\right)}{15\left(e^{(2\,x)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -4/15*(20*e^(4*x) + 5*e^(2*x) - 1)/(e^(2*x) - 1)^5
```

3.79 $\int \frac{\mathbf{csch}^7(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=44

$$\frac{\operatorname{csch}^5(x)}{5}-\frac{1}{16}\tanh^{-1}(\cosh(x))-\frac{1}{6}\coth(x)\operatorname{csch}^5(x)-\frac{1}{24}\coth(x)\operatorname{csch}^3(x)+\frac{1}{16}\coth(x)\operatorname{csch}(x)$$

```
[Out] -ArcTanh[Cosh[x]]/16 + (Coth[x]*Csch[x])/16 - (Coth[x]*Csch[x]^3)/24 + Csch
[x]^5/5 - (Coth[x]*Csch[x]^5)/6
```

Rubi [A] time = 0.210609, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3518, 3108, 3107, 2606, 30, 2611, 3768, 3770}

$$\frac{\operatorname{csch}^5(x)}{5}-\frac{1}{16}\tanh^{-1}(\cosh(x))-\frac{1}{6}\coth(x)\operatorname{csch}^5(x)-\frac{1}{24}\coth(x)\operatorname{csch}^3(x)+\frac{1}{16}\coth(x)\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^7/(1 + Tanh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/16 + (Coth[x]*Csch[x])/16 - (Coth[x]*Csch[x]^3)/24 + Csch
[x]^5/5 - (Coth[x]*Csch[x]^5)/6
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^7(x)}{1+\tanh(x)}\,dx &= \int \frac{\coth(x)\operatorname{csch}^6(x)}{\cosh(x)+\sinh(x)}\,dx\\
&= i\int \coth(x)\operatorname{csch}^6(x)(-i\cosh(x)+i\sinh(x))\,dx\\
&= \int \left(-\coth(x)\operatorname{csch}^5(x)+\coth^2(x)\operatorname{csch}^5(x)\right)dx\\
&= -\int \coth(x)\operatorname{csch}^5(x)\,dx+\int \coth^2(x)\operatorname{csch}^5(x)\,dx\\
&= -\frac{1}{6}\coth(x)\operatorname{csch}^5(x)+i\,\mathrm{Subst}\left(\int x^4\,dx,x,-i\operatorname{csch}(x)\right)+\frac{1}{6}\int \operatorname{csch}^5(x)\,dx\\
&= -\frac{1}{24}\coth(x)\operatorname{csch}^3(x)+\frac{\operatorname{csch}^5(x)}{5}-\frac{1}{6}\coth(x)\operatorname{csch}^5(x)-\frac{1}{8}\int \operatorname{csch}^3(x)\,dx\\
&= \frac{1}{16}\coth(x)\operatorname{csch}(x)-\frac{1}{24}\coth(x)\operatorname{csch}^3(x)+\frac{\operatorname{csch}^5(x)}{5}-\frac{1}{6}\coth(x)\operatorname{csch}^5(x)+\frac{1}{16}\int \operatorname{csch}(x)\,dx\\
&= -\frac{1}{16}\tanh^{-1}(\cosh(x))+\frac{1}{16}\coth(x)\operatorname{csch}(x)-\frac{1}{24}\coth(x)\operatorname{csch}^3(x)+\frac{\operatorname{csch}^5(x)}{5}-\frac{1}{6}\coth(x)\operatorname{csch}^5
\end{aligned}
$$

Mathematica [A] time = 0.263901, size = 68, normalized size = 1.55

$$
\frac{\operatorname{csch}^6(x)\left(-1140\cosh(x)-170\cosh(3x)+30\cosh(5x)+6\sinh(x)\left(50\sinh(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)-25\sinh(3x)\log\left(\tan\right.\right.\right.}{7680}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^7/(1 + Tanh[x]),x]
```

```
[Out] (Csch[x]^6*(-1140*Cosh[x] - 170*Cosh[3*x] + 30*Cosh[5*x] + 6*Sinh[x]*(256 +
 50*Log[Tanh[x/2]]*Sinh[x] - 25*Log[Tanh[x/2]]*Sinh[3*x] + 5*Log[Tanh[x/2]]
*Sinh[5*x])))/7680
```

Maple [B] time = 0.033, size = 103, normalized size = 2.3

$$\frac{1}{384}\left(\tanh\left(\frac{x}{2}\right)\right)^{6}-\frac{1}{160}\left(\tanh\left(\frac{x}{2}\right)\right)^{5}-\frac{1}{128}\left(\tanh\left(\frac{x}{2}\right)\right)^{4}+\frac{1}{32}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}-\frac{1}{128}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-\frac{1}{16}\tanh\left(\frac{x}{2}\right)+\frac{1}{128}\left(t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^7/(1+tanh(x)),x)
```

```
[Out] 1/384*tanh(1/2*x)^6-1/160*tanh(1/2*x)^5-1/128*tanh(1/2*x)^4+1/32*tanh(1/2*x
)^3-1/128*tanh(1/2*x)^2-1/16*tanh(1/2*x)+1/128/tanh(1/2*x)^2-1/384/tanh(1/2
*x)^6+1/16/tanh(1/2*x)-1/32/tanh(1/2*x)^3+1/160/tanh(1/2*x)^5+1/16*ln(tanh(
1/2*x))+1/128/tanh(1/2*x)^4
```

Maxima [B] time = 1.21299, size = 132, normalized size = 3.

$$-\frac{15\,e^{(-x)}-85\,e^{(-3\,x)}+198\,e^{(-5\,x)}-1338\,e^{(-7\,x)}-85\,e^{(-9\,x)}+15\,e^{(-11\,x)}}{120\left(6\,e^{(-2\,x)}-15\,e^{(-4\,x)}+20\,e^{(-6\,x)}-15\,e^{(-8\,x)}+6\,e^{(-10\,x)}-e^{(-12\,x)}-1\right)}-\frac{1}{16}\log\left(e^{(-x)}+1\right)+\frac{1}{16}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^7/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] -1/120*(15*e^(-x) - 85*e^(-3*x) + 198*e^(-5*x) - 1338*e^(-7*x) - 85*e^(-9*x
) + 15*e^(-11*x))/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6
*e^(-10*x) - e^(-12*x) - 1) - 1/16*log(e^(-x) + 1) + 1/16*log(e^(-x) - 1)
```

Fricas [B] time = 2.31769, size = 4358, normalized size = 99.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^7/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/240*(30*cosh(x)^11 + 330*cosh(x)*sinh(x)^10 + 30*sinh(x)^11 + 10*(165*cos
h(x)^2 - 17)*sinh(x)^9 - 170*cosh(x)^9 + 90*(55*cosh(x)^3 - 17*cosh(x))*sin
h(x)^8 + 36*(275*cosh(x)^4 - 170*cosh(x)^2 + 11)*sinh(x)^7 + 396*cosh(x)^7
+ 84*(165*cosh(x)^5 - 170*cosh(x)^3 + 33*cosh(x))*sinh(x)^6 + 12*(1155*cosh
(x)^6 - 1785*cosh(x)^4 + 693*cosh(x)^2 - 223)*sinh(x)^5 - 2676*cosh(x)^5 +
60*(165*cosh(x)^7 - 357*cosh(x)^5 + 231*cosh(x)^3 - 223*cosh(x))*sinh(x)^4
+ 10*(495*cosh(x)^8 - 1428*cosh(x)^6 + 1386*cosh(x)^4 - 2676*cosh(x)^2 - 17
)*sinh(x)^3 - 170*cosh(x)^3 + 6*(275*cosh(x)^9 - 1020*cosh(x)^7 + 1386*cosh
(x)^5 - 4460*cosh(x)^3 - 85*cosh(x))*sinh(x)^2 - 15*(cosh(x)^12 + 12*cosh(x
)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10
+ 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2
 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x
))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x
)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cos
h(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(
x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*c
osh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh
(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(
x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(
```

```
x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 15*(cosh(x)^12 +
12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*co
sh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*
cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 +
 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 -
5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^
3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 -
 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)
^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10
- 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2
- 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5
+ 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(55*co
sh(x)^10 - 255*cosh(x)^8 + 462*cosh(x)^6 - 2230*cosh(x)^4 - 85*cosh(x)^2 +
5)*sinh(x) + 30*cosh(x))/(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 +
 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh
(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x
)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(
x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33
*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*co
sh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*co
sh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3
*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*c
osh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*c
osh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^7(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**7/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)**7/(tanh(x) + 1), x)
```

Giac [A] time = 1.28759, size = 82, normalized size = 1.86

$$\frac{15\,e^{(11\,x)}-85\,e^{(9\,x)}+198\,e^{(7\,x)}-1338\,e^{(5\,x)}-85\,e^{(3\,x)}+15\,e^{x}}{120\left(e^{(2\,x)}-1\right)^{6}}-\frac{1}{16}\log\left(e^{x}+1\right)+\frac{1}{16}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^7/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/120*(15*e^(11*x) - 85*e^(9*x) + 198*e^(7*x) - 1338*e^(5*x) - 85*e^(3*x) +
 15*e^x)/(e^(2*x) - 1)^6 - 1/16*log(e^x + 1) + 1/16*log(abs(e^x - 1))
```

3.80 $\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=147

$$-\frac{a^4 b \log(a+b\tanh(x))}{\left(a^2-b^2\right)^3}-\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)}+\frac{\cosh^2(x)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\tanh(x)\right)}{8\left(a^2-b^2\right)^2}-\frac{a(3a+b)\log(1}{16(a+}$$

```
[Out] -(a*(3*a + b)*Log[1 - Tanh[x]])/(16*(a + b)^3) + (a*(3*a - b)*Log[1 + Tanh[
x]])/(16*(a - b)^3) - (a^4*b*Log[a + b*Tanh[x]])/(a^2 - b^2)^3 - (Cosh[x]^4
*(b - a*Tanh[x]))/(4*(a^2 - b^2)) + (Cosh[x]^2*(4*b*(2*a^2 - b^2) - a*(5*a^
2 - b^2)*Tanh[x]))/(8*(a^2 - b^2)^2)
```

Rubi [A] time = 0.345582, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3516, 1647, 801}

$$-\frac{a^4 b \log(a+b\tanh(x))}{\left(a^2-b^2\right)^3}-\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)}+\frac{\cosh^2(x)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\tanh(x)\right)}{8\left(a^2-b^2\right)^2}-\frac{a(3a+b)\log(1}{16(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Tanh[x]),x]
```


```
[Out] -(a*(3*a + b)*Log[1 - Tanh[x]])/(16*(a + b)^3) + (a*(3*a - b)*Log[1 + Tanh[
x]])/(16*(a - b)^3) - (a^4*b*Log[a + b*Tanh[x]])/(a^2 - b^2)^3 - (Cosh[x]^4
*(b - a*Tanh[x]))/(4*(a^2 - b^2)) + (Cosh[x]^2*(4*b*(2*a^2 - b^2) - a*(5*a^
2 - b^2)*Tanh[x]))/(8*(a^2 - b^2)^2)
```


Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx = -\left(b\,\text{Subst}\left(\int \frac{x^4}{(a+x)\left(-b^2+x^2\right)^3}\,dx, x, b\tanh(x)\right)\right)$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2-b^2}-\frac{3ab^4x}{a^2-b^2}+4b^2x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\tanh(x)\right)}{4b}$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)} + \frac{\cosh^2(x)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\tanh(x)\right)}{8\left(a^2-b^2\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4(3}{(a^2-}}{}\right.}{}$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)} + \frac{\cosh^2(x)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\tanh(x)\right)}{8\left(a^2-b^2\right)^2} - \frac{\text{Subst}\left(\int\left(-\frac{a}{2(}\right.\right.}{}$$

$$= -\frac{a(3a+b)\log(1-\tanh(x))}{16(a+b)^3} + \frac{a(3a-b)\log(1+\tanh(x))}{16(a-b)^3} - \frac{a^4b\log(a+b\tanh(x))}{\left(a^2-b^2\right)^3} - \frac{\cosh^4(x}{4}$$

Mathematica [A] time = 0.539949, size = 144, normalized size = 0.98

$$\frac{24a^3b^2x-8a^3\left(a^2-b^2\right)\sinh(2x)-2a^3b^2\sinh(4x)+4b\left(-4a^2b^2+3a^4+b^4\right)\cosh(2x)-b\left(a^2-b^2\right)^2\cosh(4x)-32a^4b}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (12*a^5*x + 24*a^3*b^2*x - 4*a*b^4*x + 4*b*(3*a^4 - 4*a^2*b^2 + b^4)*Cosh[2
*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*a^4*b*Log[a*Cosh[x] + b*Sinh[x]] - 8*a
^3*(a^2 - b^2)*Sinh[2*x] + a^5*Sinh[4*x] - 2*a^3*b^2*Sinh[4*x] + a*b^4*Sinh
[4*x])/(32*(a - b)^3*(a + b)^3)
```

Maple [B] time = 0.046, size = 320, normalized size = 2.2

$$-8\frac{1}{(32a-32b)(\tanh(x/2)+1)^4}+32\frac{1}{(64a-64b)(\tanh(x/2)+1)^3}+\frac{a}{8(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{b}{8(a-b)^2}\left(\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*tanh(x)),x)
```

```
[Out] -8/(32*a-32*b)/(tanh(1/2*x)+1)^4+32/(64*a-64*b)/(tanh(1/2*x)+1)^3+1/8/(a-b)
^2/(tanh(1/2*x)+1)^2*a+1/8/(a-b)^2/(tanh(1/2*x)+1)^2*b-3/8/(a-b)^2/(tanh(1/
2*x)+1)*a+1/8/(a-b)^2/(tanh(1/2*x)+1)*b+3/8*a^2/(a-b)^3*ln(tanh(1/2*x)+1)-1
/8*a/(a-b)^3*ln(tanh(1/2*x)+1)*b-a^4*b/(a-b)^3/(a+b)^3*ln(a*tanh(1/2*x)^2+2
*tanh(1/2*x)*b+a)+8/(32*a+32*b)/(tanh(1/2*x)-1)^4+32/(64*a+64*b)/(tanh(1/2*
x)-1)^3-1/8/(a+b)^2/(tanh(1/2*x)-1)^2*a+1/8/(a+b)^2/(tanh(1/2*x)-1)^2*b-3/8
/(a+b)^2/(tanh(1/2*x)-1)*a-1/8/(a+b)^2/(tanh(1/2*x)-1)*b-3/8*a^2/(a+b)^3*ln
(tanh(1/2*x)-1)-1/8*a/(a+b)^3*ln(tanh(1/2*x)-1)*b
```

Maxima [A] time = 1.16127, size = 220, normalized size = 1.5

$$-\frac{a^4 b \log\left(-(a-b)e^{(-2x)} - a - b\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{\left(3a^2 + ab\right)x}{8\left(a^3 + 3a^2b + 3ab^2 + b^3\right)} - \frac{\left(4(2a+b)e^{(-2x)} - a - b\right)e^{(4x)}}{64\left(a^2 + 2ab + b^2\right)} + \frac{4(2a-b)e^{(-2x)} - (a -}{64\left(a^2 - 2ab + b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -a^4*b*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
 1/8*(3*a^2 + a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*(2*a + b)*e^
(-2*x) - a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*(2*a - b)*e^(-2*x) -
(a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Fricas [B] time = 2.54086, size = 2747, normalized size = 18.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(2*a^5 - 3*a^4*
b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^6 - 4*(2*a^5 - 3*a^4*b - 2*a^3*b^2
 + 4*a^2*b^3 - b^5 - 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^4
+ 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(2
*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^
4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 30*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*
a^2*b^3 - b^5)*cosh(x)^2 + 4*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x)*sinh(
x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 -
 10*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^3 + 4*(3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x))*sinh(x)^3 + 4*(2*a^5 + 3*a^4*b - 2*
a^3*b^2 - 4*a^2*b^3 + b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 2*a^5 + 3*a^4*b - 2*a^3*b^2 - 4*a^2*b^3 +
b^5 - 15*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^4 + 12*(3*
a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^4*b*cosh(
x)^4 + 4*a^4*b*cosh(x)^3*sinh(x) + 6*a^4*b*cosh(x)^2*sinh(x)^2 + 4*a^4*b*co
sh(x)*sinh(x)^3 + a^4*b*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) -
 sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^7 - 3*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^5 + 4*(3*a^5
 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^3 + (2*a^5 + 3*a^4*b - 2*a^3*b^2
- 4*a^2*b^3 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*si
nh(x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*tanh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18694, size = 289, normalized size = 1.97

$$-\frac{a^4 b \log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{\left(3a^2 - ab\right)x}{8\left(a^3 - 3a^2b + 3ab^2 - b^3\right)} - \frac{\left(18a^2e^{(4x)} - 6abe^{(4x)} - 8a^2e^{(2x)} + 12abe^{(2x)} - 4b^2\right.}{64\left(a^3 - 3a^2b + 3ab^2 - b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -a^4*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + 1/8*(3*a^2 - a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/64*(18*a^
2*e^(4*x) - 6*a*b*e^(4*x) - 8*a^2*e^(2*x) + 12*a*b*e^(2*x) - 4*b^2*e^(2*x)
+ a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(
4*x) + b*e^(4*x) - 8*a*e^(2*x) - 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

3.81 $\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=137

$$-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{a\cosh(x)}{a^2-b^2}-\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a^3b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

```
[Out] -((a^3*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3)
/(3*(a^2 - b^2)) + (a^2*b*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```

Rubi [A] time = 0.278773, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3518, 3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{a\cosh(x)}{a^2-b^2}-\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a^3b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Tanh[x]),x]
```


```
[Out] -((a^3*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3)
/(3*(a^2 - b^2)) + (a^2*b*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```


Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx = \int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \frac{a\int \sinh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh(x)\sinh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$
$$= \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{\left(a^3b\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^2\right)\int \sinh(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int \left(1-x^2\right)dx, x, \mathrm{co}\right.}{a^2-b^2}$$
$$= -\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{b\sinh^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(ia^3b\right)\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x}\right.}{}$$
$$= -\frac{a^3b\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{b\sinh}{3\left(a^2\right.}$$

Mathematica [A] time = 1.14782, size = 180, normalized size = 1.31

$$\frac{-3a\sqrt{a-b}\sqrt{a+b}\left(3a^2+b^2\right)\cosh(x)+a\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)\cosh(3x)+b\left(3\sqrt{a-b}\sqrt{a+b}\left(5a^2-b^2\right)\sinh(x)-\sqrt{\right.}{12(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (-3*a*Sqrt[a - b]*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] + a*Sqrt[a - b]*Sqrt[a
+ b]*(a^2 - b^2)*Cosh[3*x] + b*(-24*a^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a -
b]*Sqrt[a + b])] + 3*Sqrt[a - b]*Sqrt[a + b]*(5*a^2 - b^2)*Sinh[x] - Sqrt[a
 - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/2))
```

Maple [A] time = 0.042, size = 166, normalized size = 1.2

$$-8\,\frac{1}{(16\,a-16\,b)\,(\tanh(x/2)+1)^2}+\frac{16}{48\,a-48\,b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{a}{2\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-2\,\frac{a^3 b}{(a-b)^2\,(a+b)^2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*tanh(x)),x)
```

```
[Out] -8/(16*a-16*b)/(tanh(1/2*x)+1)^2+16/3/(tanh(1/2*x)+1)^3/(16*a-16*b)-1/2/(a-
b)^2/(tanh(1/2*x)+1)*a-2*a^3*b/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-16/3/(tanh(1/2*x)-1)^3/(16*a+16*b)-8/
(16*a+16*b)/(tanh(1/2*x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.45245, size = 4208, normalized size = 30.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
 a*b^4 - b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
```

```
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))
 + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*
a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5
*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^
2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(
x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 - 5*(a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(3*a^5 - 5*
a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5
+ 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cos
h(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*a^5 - 5*a^4*b
 - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*tanh(x)), x)
```

Giac [A] time = 1.21709, size = 220, normalized size = 1.61

$$-\frac{2\,a^3 b \arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-9\,a^2e^x-12\,abe^x-3\,b}{24\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2 -
2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^x
- 12*a*b*e^x - 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.82 $\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=84

$$\frac{a^2b\log(a+b\tanh(x))}{\left(a^2-b^2\right)^2}-\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)}+\frac{a\log(1-\tanh(x))}{4(a+b)^2}-\frac{a\log(\tanh(x)+1)}{4(a-b)^2}$$

```
[Out] (a*Log[1 - Tanh[x]])/(4*(a + b)^2) - (a*Log[1 + Tanh[x]])/(4*(a - b)^2) + (
a^2*b*Log[a + b*Tanh[x]])/(a^2 - b^2)^2 - (Cosh[x]^2*(b - a*Tanh[x]))/(2*(a
^2 - b^2))
```

Rubi [A] time = 0.161791, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3516, 1647, 801}

$$\frac{a^2b\log(a+b\tanh(x))}{\left(a^2-b^2\right)^2}-\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)}+\frac{a\log(1-\tanh(x))}{4(a+b)^2}-\frac{a\log(\tanh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (a*Log[1 - Tanh[x]])/(4*(a + b)^2) - (a*Log[1 + Tanh[x]])/(4*(a - b)^2) + (
a^2*b*Log[a + b*Tanh[x]])/(a^2 - b^2)^2 - (Cosh[x]^2*(b - a*Tanh[x]))/(2*(a
^2 - b^2))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx = b\,\text{Subst}\left(\int \frac{x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\tanh(x)\right)$$

$$= -\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{a^2b^2}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}}{(a+x)\left(-b^2+x^2\right)}\,dx, x, b\tanh(x)\right)}{2b}$$

$$= -\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \left(-\frac{ab}{2(a+b)^2(b-x)} + \frac{2a^2b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{ab}{2(a-b)^2(b+x)}\right) dx, x, b\tanh(\right.}{2b}$$

$$= \frac{a\log(1-\tanh(x))}{4(a+b)^2} - \frac{a\log(1+\tanh(x))}{4(a-b)^2} + \frac{a^2b\log(a+b\tanh(x))}{\left(a^2-b^2\right)^2} - \frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.221769, size = 73, normalized size = 0.87

$$\frac{\left(b^3-a^2b\right)\cosh(2x) + a\left(-2x\left(a^2+b^2\right)+\left(a^2-b^2\right)\sinh(2x)+4ab\log(a\cosh(x)+b\sinh(x))\right)}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] ((-(a^2*b) + b^3)*Cosh[2*x] + a*(-2*(a^2 + b^2)*x + 4*a*b*Log[a*Cosh[x] + b
*Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Maple [A] time = 0.039, size = 145, normalized size = 1.7

$$-4\,\frac{1}{(8\,a-8\,b)\,(\tanh(x/2)+1)^2} + 8\,\frac{1}{(16\,a-16\,b)\,(\tanh(x/2)+1)} - \frac{a}{2\,(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{a^2b}{(a-b)^2\,(a+b)^2}\ln\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*tanh(x)),x)
```

```
[Out] -4/(8*a-8*b)/(tanh(1/2*x)+1)^2+8/(16*a-16*b)/(tanh(1/2*x)+1)-1/2*a/(a-b)^2*
ln(tanh(1/2*x)+1)+a^2*b/(a-b)^2/(a+b)^2*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+
a)+4/(8*a+8*b)/(tanh(1/2*x)-1)^2+8/(16*a+16*b)/(tanh(1/2*x)-1)+1/2*a/(a+b)^
2*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.16775, size = 112, normalized size = 1.33

$$\frac{a^2b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} - \frac{ax}{2\left(a^2+2\,ab+b^2\right)} + \frac{e^{(2\,x)}}{8\,(a+b)} - \frac{e^{(-2\,x)}}{8\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] a^2*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*a*x/(a^2
 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Fricas [B] time = 2.63341, size = 826, normalized size = 9.83

$$\frac{(a^3 - a^2b - ab^2 + b^3)\cosh(x)^4 + 4(a^3 - a^2b - ab^2 + b^3)\cosh(x)\sinh(x)^3 + (a^3 - a^2b - ab^2 + b^3)\sinh(x)^4 - 4(a^3 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 + 2*a^2*
b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 + 8*(a^2*b*co
sh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(a*cosh(x) + b*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2
*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*tanh(x)), x)
```

Giac [A] time = 1.21264, size = 136, normalized size = 1.62

$$\frac{a^2b\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^4 - 2a^2b^2 + b^4} - \frac{ax}{2(a^2 - 2ab + b^2)} + \frac{\left(2ae^{(2x)} - a + b\right)e^{(-2x)}}{8(a^2 - 2ab + b^2)} + \frac{e^{(2x)}}{8(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] a^2*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/2
*a*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a*b
+ b^2) + 1/8*e^(2*x)/(a + b)
```

3.83 $\int \frac{\sinh(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=72

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}+\frac{ab\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] (a*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (
a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rubi [A] time = 0.116433, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3518, 3109, 2637, 2638, 3074, 206}

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}+\frac{ab\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Tanh[x]),x]
```



```
[Out] (a*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (
a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```


Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{a+b\tanh(x)}\,dx &= \int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx \\
&= \frac{a\int \sinh(x)\,dx}{a^2-b^2} - \frac{b\int \cosh(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} \\
&= \frac{a\cosh(x)}{a^2-b^2} - \frac{b\sinh(x)}{a^2-b^2} + \frac{(iab)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2} \\
&= \frac{ab\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a\cosh(x)}{a^2-b^2} - \frac{b\sinh(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.220985, size = 79, normalized size = 1.1

$$\frac{b\sinh(x)}{b^2-a^2} + \frac{a\cosh(x)}{a^2-b^2} + \frac{2ab\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Tanh[x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*
(a + b)^(3/2)) + (a*Cosh[x])/(a^2 - b^2) + (b*Sinh[x])/(-a^2 + b^2)
```

Maple [A] time = 0.03, size = 92, normalized size = 1.3

$$2\,\frac{ab}{(a+b)\,(a-b)\,\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tanh\,(x/2)+2\,b}{\sqrt{a^2-b^2}}\right) - 4\,\frac{1}{(4\,a+4\,b)\,(\tanh\,(x/2)-1)} + 4\,\frac{1}{(4\,a-4\,b)\,(\tanh\,(x/2)\cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*tanh(x)),x)
```

```
[Out] 2*a*b/(a+b)/(a-b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2
)^(1/2))-4/(4*a+4*b)/(tanh(1/2*x)-1)+4/(4*a-4*b)/(tanh(1/2*x)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.51874, size = 1103, normalized size = 15.32

$$\left[\frac{a^3 + a^2b - ab^2 - b^3 + \left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)^2 + 2\left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)\sinh\left(x\right) + \left(a^3 - a^2b - ab^2 + b^3\right)}{2\left(\left(a^4 - 2a^2b^2 + b^4\right)\cosh\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2
*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*
sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cos
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)
*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x
) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
2*a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b -
a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a
^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^
2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(x\right)}{a + b\tanh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*tanh(x)),x)
```

```
[Out] Integral(sinh(x)/(a + b*tanh(x)), x)
```

Giac [A] time = 1.18728, size = 81, normalized size = 1.12

$$\frac{2\,ab\arctan\left(\frac{ae^x + be^x}{\sqrt{a^2 - b^2}}\right)}{\left(a^2 - b^2\right)^{\frac{3}{2}}} + \frac{e^{(-x)}}{2\,(a - b)} + \frac{e^x}{2\,(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 2*a*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 1/2*e^(-x
)/(a - b) + 1/2*e^x/(a + b)
```

3.84 $\int \frac{\operatorname{csch}(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=52

$$-\frac{b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cosh(x))}{a}$$

```
[Out] -((b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]))
- ArcTanh[Cosh[x]]/a
```

Rubi [A] time = 0.142876, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3518, 3110, 3770, 3074, 206}

$$-\frac{b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Tanh[x]),x]
```

```
[Out] -((b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]))
- ArcTanh[Cosh[x]]/a
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}(x)}{a+b\tanh(x)}\,dx &= \int \frac{\coth(x)}{a\cosh(x)+b\sinh(x)}\,dx\\
&= i\int \left(-\frac{i\operatorname{csch}(x)}{a}+\frac{ib}{a(a\cosh(x)+b\sinh(x))}\right)dx\\
&= \frac{\int \operatorname{csch}(x)\,dx}{a}-\frac{b\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a}\\
&= -\frac{\tanh^{-1}(\cosh(x))}{a}-\frac{(ib)\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a}\\
&= -\frac{b\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cosh(x))}{a}
\end{aligned}
$$

Mathematica [A] time = 0.0903689, size = 59, normalized size = 1.13

$$
\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2b\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Tanh[x]),x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(Sqrt[a - b]*Sq
rt[a + b]) + Log[Tanh[x/2]])/a
```

Maple [A] time = 0.033, size = 53, normalized size = 1.

$$
-2\,\frac{b}{a\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tanh(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*tanh(x)),x)
```

```
[Out] -2/a*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/
a*ln(tanh(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45129, size = 675, normalized size = 12.98

$$\left[-\frac{\sqrt{-a^2+b^2}b\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\,\sqrt{-a^2+b^2}(\cosh(x)+\sinh(x))-a+b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+a-b}\right)+\left(a^2-b^2\right)\log\left(\cosh\left(x\right)+\right.}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a^2 + b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) +
(a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b
)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + (a^
2 - b^2)*log(cosh(x) + sinh(x) + 1) - (a^2 - b^2)*log(cosh(x) + sinh(x) - 1
))/(a^3 - a*b^2), (2*sqrt(a^2 - b^2)*b*arctan(sqrt(a^2 - b^2)/((a + b)*cosh
(x) + (a + b)*sinh(x))) - (a^2 - b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 - b
^2)*log(cosh(x) + sinh(x) - 1))/(a^3 - a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{a+b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*tanh(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*tanh(x)), x)
```

Giac [A] time = 1.21293, size = 81, normalized size = 1.56

$$-\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a}-\frac{\log\left(e^x+1\right)}{a}+\frac{\log\left(|e^x-1|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a) - log(e^x
+ 1)/a + log(abs(e^x - 1))/a
```

3.85 $\int \frac{\text{csch}^2(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=29

$$-\frac{b\log(\tanh(x))}{a^2}+\frac{b\log(a+b\tanh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[Out] -(Coth[x]/a) - (b*Log[Tanh[x]])/a^2 + (b*Log[a + b*Tanh[x]])/a^2
```

Rubi [A] time = 0.0541423, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3516, 44}

$$-\frac{b\log(\tanh(x))}{a^2}+\frac{b\log(a+b\tanh(x))}{a^2}-\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] -(Coth[x]/a) - (b*Log[Tanh[x]])/a^2 + (b*Log[a + b*Tanh[x]])/a^2
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{csch}^2(x)}{a+b\tanh(x)}\,dx &= b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx,x,b\tanh(x)\right)\\
&= b\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx,x,b\tanh(x)\right)\\
&= -\frac{\coth(x)}{a}-\frac{b\log(\tanh(x))}{a^2}+\frac{b\log(a+b\tanh(x))}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0834297, size = 28, normalized size = 0.97

$$-\frac{-b\log(a\cosh(x)+b\sinh(x))+a\coth(x)+b\log(\sinh(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] -((a*Coth[x] + b*Log[Sinh[x]] - b*Log[a*Cosh[x] + b*Sinh[x]])/a^2)
```

Maple [A] time = 0.039, size = 56, normalized size = 1.9

$$-\frac{1}{2a}\tanh\left(\frac{x}{2}\right)+\frac{b}{a^2}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2+2\tanh(x/2)\,b+a\right)-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{b}{a^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*tanh(x)),x)
```

```
[Out] -1/2/a*tanh(1/2*x)+1/a^2*b*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)-1/2/a/tanh
(1/2*x)-1/a^2*b*ln(tanh(1/2*x))
```

Maxima [B] time = 1.04015, size = 88, normalized size = 3.03

$$\frac{b\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2}-\frac{b\log\left(e^{(-x)}+1\right)}{a^2}-\frac{b\log\left(e^{(-x)}-1\right)}{a^2}+\frac{2}{ae^{(-2x)}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] b*log(-(a - b)*e^(-2*x) - a - b)/a^2 - b*log(e^(-x) + 1)/a^2 - b*log(e^(-x)
 - 1)/a^2 + 2/(a*e^(-2*x) - a)
```

Fricas [B] time = 2.25994, size = 360, normalized size = 12.41

$$\frac{\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2-b\right)\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+\cdots\right.}{a^2\cosh(x)^2+2\,a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] ((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(2*(a*cosh(x) + b
*sinh(x))/(cosh(x) - sinh(x))) - (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 - b)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 2*a)/(a^2*cosh(x)^2 + 2*a^
2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*tanh(x)), x)
```

Giac [B] time = 1.19888, size = 105, normalized size = 3.62

$$\frac{\left(ab+b^2\right)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^3+a^2b}-\frac{b\log\left(\left|e^{(2x)}-1\right|\right)}{a^2}+\frac{be^{(2x)}-2a-b}{a^2\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] (a*b + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^3 + a^2*b) - b*log(a
bs(e^(2*x) - 1))/a^2 + (b*e^(2*x) - 2*a - b)/(a^2*(e^(2*x) - 1))
```

3.86 $\int \frac{\mathrm{csch}^3(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=82

$$-\frac{b^2\tanh^{-1}(\cosh(x))}{a^3}+\frac{b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}+\frac{b\,\mathrm{csch}(x)}{a^2}+\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\mathrm{csch}(x)}{2a}$$

```
[Out] (b*Sqrt[a^2 - b^2]*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 + A
rcTanh[Cosh[x]]/(2*a) - (b^2*ArcTanh[Cosh[x]])/a^3 + (b*Csch[x])/a^2 - (Cot
h[x]*Csch[x])/(2*a)
```

Rubi [A] time = 0.293225, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {3518, 3110, 3768, 3770, 2621, 321, 207, 2622, 3104, 3074, 206}

$$-\frac{b^2\tanh^{-1}(\cosh(x))}{a^3}+\frac{b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}+\frac{b\,\mathrm{csch}(x)}{a^2}+\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\mathrm{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (b*Sqrt[a^2 - b^2]*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 + A
rcTanh[Cosh[x]]/(2*a) - (b^2*ArcTanh[Cosh[x]])/a^3 + (b*Csch[x])/a^2 - (Cot
h[x]*Csch[x])/(2*a)
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{a+b\tanh(x)}\,dx = \int \frac{\coth(x)\operatorname{csch}^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= -\left(i\int\left(\frac{i\operatorname{csch}^3(x)}{a} - \frac{ib\operatorname{csch}^2(x)\operatorname{sech}(x)}{a^2} + \frac{ib^2\operatorname{csch}(x)\operatorname{sech}^2(x)}{a^3} - \frac{ib^3\operatorname{sech}^2(x)}{a^3(a\cosh(x)+b\sinh(x))}\right) d$$
$$= \frac{\int \operatorname{csch}^3(x)\,dx}{a} - \frac{b\int \operatorname{csch}^2(x)\operatorname{sech}(x)\,dx}{a^2} + \frac{b^2\int \operatorname{csch}(x)\operatorname{sech}^2(x)\,dx}{a^3} - \frac{b^3\int \frac{\operatorname{sech}^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^3}$$
$$= -\frac{\coth(x)\operatorname{csch}(x)}{2a} - \frac{b^2\operatorname{sech}(x)}{a^3} - \frac{\int \operatorname{csch}(x)\,dx}{2a} + \frac{(ib)\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(x)\right)}{a^2} - \frac{b\int s}{}$$
$$= -\frac{b\tan^{-1}(\sinh(x))}{a^2} + \frac{\tanh^{-1}(\cosh(x))}{2a} + \frac{b\operatorname{csch}(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{(ib)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,d\right.}{a^2}$$
$$= \frac{b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3} + \frac{\tanh^{-1}(\cosh(x))}{2a} - \frac{b^2\tanh^{-1}(\cosh(x))}{a^3} + \frac{b\operatorname{csch}(x)}{a^2} - \frac{c}{}$$

Mathematica [A] time = 0.345669, size = 123, normalized size = 1.5

$$-\frac{a^2\operatorname{csch}^2\left(\frac{x}{2}\right) + a^2\operatorname{sech}^2\left(\frac{x}{2}\right) + 4a^2\log\left(\tanh\left(\frac{x}{2}\right)\right) + 4ab\tanh\left(\frac{x}{2}\right) - 4ab\coth\left(\frac{x}{2}\right) - 16b\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+}}\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] -(-16*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[
a + b])] - 4*a*b*Coth[x/2] + a^2*Csch[x/2]^2 + 4*a^2*Log[Tanh[x/2]] - 8*b^2
*Log[Tanh[x/2]] + a^2*Sech[x/2]^2 + 4*a*b*Tanh[x/2])/(8*a^3)
```

Maple [A] time = 0.042, size = 110, normalized size = 1.3

$$\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^2 - \frac{b}{2a^2}\tanh\left(\frac{x}{2}\right) + 2\frac{b\sqrt{a^2-b^2}}{a^3}\arctan\left(1/2\,\frac{2a\tanh(x/2)+2b}{\sqrt{a^2-b^2}}\right) - \frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} - \frac{1}{2a}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*tanh(x)),x)
```

```
[Out] 1/8/a*tanh(1/2*x)^2-1/2/a^2*tanh(1/2*x)*b+2*b*(a^2-b^2)^(1/2)/a^3*arctan(1/
2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/8/a/tanh(1/2*x)^2-1/2/a*ln(tanh(
1/2*x))+1/a^3*ln(tanh(1/2*x))*b^2+1/2*b/a^2/tanh(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78036, size = 3212, normalized size = 39.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2 - 2*a*b)*cosh(x)^3 + 6*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + 2*(a
^2 - 2*a*b)*sinh(x)^3 - 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^
4 - 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x)^3 - b*co
sh(x))*sinh(x) + b)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cos
h(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) -
 a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a - b)) + 2*(a^2 + 2*a*b)*cosh(x) - ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2
*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*b^2)*sinh(x)^4 - 2*(a^2 - 2*b^2)*cosh(x)
^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 - a^2 + 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 +
4*((a^2 - 2*b^2)*cosh(x)^3 - (a^2 - 2*b^2)*cosh(x))*sinh(x))*log(cosh(x) +
sinh(x) + 1) + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3
 + (a^2 - 2*b^2)*sinh(x)^4 - 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)
*cosh(x)^2 - a^2 + 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x
)^3 - (a^2 - 2*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(3*(a^
2 - 2*a*b)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)
*sinh(x)^3 + a^3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a
^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x)), -1/2*(2*(a^2 - 2*
a*b)*cosh(x)^3 + 6*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + 2*(a^2 - 2*a*b)*sinh(x
)^3 + 4*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*b*cosh(x)^2
+ 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x)^3 - b*cosh(x))*sinh(x) + b
)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)
)) + 2*(a^2 + 2*a*b)*cosh(x) - ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*c
osh(x)*sinh(x)^3 + (a^2 - 2*b^2)*sinh(x)^4 - 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*
(3*(a^2 - 2*b^2)*cosh(x)^2 - a^2 + 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2
 - 2*b^2)*cosh(x)^3 - (a^2 - 2*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
 + 1) + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2
 - 2*b^2)*sinh(x)^4 - 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x
)^2 - a^2 + 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 - (
a^2 - 2*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(3*(a^2 - 2*a
*b)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x
)^3 + a^3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sin
h(x)^2 + 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*tanh(x)), x)
```

Giac [A] time = 1.18418, size = 169, normalized size = 2.06

$$\frac{\left(a^2-2\,b^2\right)\log\left(e^x+1\right)}{2\,a^3}-\frac{\left(a^2-2\,b^2\right)\log\left(|e^x-1|\right)}{2\,a^3}+\frac{2\left(a^2b-b^3\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^3}-\frac{ae^{(3\,x)}-2\,be^{(3\,x)}+ae^x+2\,be^x}{a^2\left(e^{(2\,x)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 1/2*(a^2 - 2*b^2)*log(e^x + 1)/a^3 - 1/2*(a^2 - 2*b^2)*log(abs(e^x - 1))/a^
3 + 2*(a^2*b - b^3)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2
)*a^3) - (a*e^(3*x) - 2*b*e^(3*x) + a*e^x + 2*b*e^x)/(a^2*(e^(2*x) - 1)^2)
```

3.87 $\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=78

$$\frac{(a^2-b^2)\coth(x)}{a^3}+\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}-\frac{b(a^2-b^2)\log(a+b\tanh(x))}{a^4}+\frac{b\coth^2(x)}{2a^2}-\frac{\coth^3(x)}{3a}$$

```
[Out] ((a^2 - b^2)*Coth[x])/a^3 + (b*Coth[x]^2)/(2*a^2) - Coth[x]^3/(3*a) + (b*(a
^2 - b^2)*Log[Tanh[x]])/a^4 - (b*(a^2 - b^2)*Log[a + b*Tanh[x]])/a^4
```

Rubi [A] time = 0.0997105, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3516, 894}

$$\frac{(a^2-b^2)\coth(x)}{a^3}+\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}-\frac{b(a^2-b^2)\log(a+b\tanh(x))}{a^4}+\frac{b\coth^2(x)}{2a^2}-\frac{\coth^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] ((a^2 - b^2)*Coth[x])/a^3 + (b*Coth[x]^2)/(2*a^2) - Coth[x]^3/(3*a) + (b*(a
^2 - b^2)*Log[Tanh[x]])/a^4 - (b*(a^2 - b^2)*Log[a + b*Tanh[x]])/a^4
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx &= -\left(b\,\text{Subst}\left(\int \frac{-b^2+x^2}{x^4(a+x)}\,dx,x,b\tanh(x)\right)\right)\\ &= -\left(b\,\text{Subst}\left(\int \left(-\frac{b^2}{ax^4}+\frac{b^2}{a^2x^3}+\frac{a^2-b^2}{a^3x^2}+\frac{-a^2+b^2}{a^4x}+\frac{a^2-b^2}{a^4(a+x)}\right)dx,x,b\tanh(x)\right)\right)\\ &= \frac{(a^2-b^2)\coth(x)}{a^3}+\frac{b\coth^2(x)}{2a^2}-\frac{\coth^3(x)}{3a}+\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}-\frac{b(a^2-b^2)\log(a+b\tanh(x))}{a^4}\end{aligned}$$

Mathematica [A] time = 0.239268, size = 70, normalized size = 0.9

$$\frac{-2\coth(x)\left(a^3\operatorname{csch}^2(x)-2a^3+3ab^2\right)+6b(a^2-b^2)(\log(\sinh(x))-\log(a\cosh(x)+b\sinh(x)))+3a^2b\operatorname{csch}^2(x)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (3*a^2*b*Csch[x]^2 - 2*Coth[x]*(-2*a^3 + 3*a*b^2 + a^3*Csch[x]^2) + 6*b*(a^
2 - b^2)*(Log[Sinh[x]] - Log[a*Cosh[x] + b*Sinh[x]]))/(6*a^4)
```

Maple [B] time = 0.045, size = 166, normalized size = 2.1

$$-\frac{1}{24\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}+\frac{b}{8\,a^{2}}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\frac{3}{8\,a}\tanh\left(\frac{x}{2}\right)-\frac{b^{2}}{2\,a^{3}}\tanh\left(\frac{x}{2}\right)-\frac{b}{a^{2}}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+2\tanh\left(x/2\right)b+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*tanh(x)),x)
```

```
[Out] -1/24/a*tanh(1/2*x)^3+1/8/a^2*b*tanh(1/2*x)^2+3/8/a*tanh(1/2*x)-1/2/a^3*b^2
*tanh(1/2*x)-1/a^2*b*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)+1/a^4*b^3*ln(a*t
anh(1/2*x)^2+2*tanh(1/2*x)*b+a)-1/24/a/tanh(1/2*x)^3+3/8/a/tanh(1/2*x)-1/2/
a^3/tanh(1/2*x)*b^2+1/8/a^2*b/tanh(1/2*x)^2+1/a^2*b*ln(tanh(1/2*x))-1/a^4*b
^3*ln(tanh(1/2*x))
```

Maxima [B] time = 1.12194, size = 217, normalized size = 2.78

$$-\frac{2\left(2\,a^{2}-3\,b^{2}-3\left(2\,a^{2}-ab-2\,b^{2}\right)e^{(-2\,x)}-3\left(ab+b^{2}\right)e^{(-4\,x)}\right)}{3\left(3\,a^{3}e^{(-2\,x)}-3\,a^{3}e^{(-4\,x)}+a^{3}e^{(-6\,x)}-a^{3}\right)}-\frac{\left(a^{2}b-b^{3}\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^{4}}+\frac{\left(a^{2}b-b^{3}\right)\mathrm{lo}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -2/3*(2*a^2 - 3*b^2 - 3*(2*a^2 - a*b - 2*b^2)*e^(-2*x) - 3*(a*b + b^2)*e^(-
4*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-4*x) + a^3*e^(-6*x) - a^3) - (a^2*b - b^3
)*log(-(a - b)*e^(-2*x) - a - b)/a^4 + (a^2*b - b^3)*log(e^(-x) + 1)/a^4 +
(a^2*b - b^3)*log(e^(-x) - 1)/a^4
```

Fricas [B] time = 2.46359, size = 2186, normalized size = 28.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^2*b - a*b^2)*cosh(x)^4 + 24*(a^2*b - a*b^2)*cosh(x)*sinh(x)^3 + 6
*(a^2*b - a*b^2)*sinh(x)^4 + 4*a^3 - 6*a*b^2 - 6*(2*a^3 + a^2*b - 2*a*b^2)*
cosh(x)^2 - 6*(2*a^3 + a^2*b - 2*a*b^2 - 6*(a^2*b - a*b^2)*cosh(x)^2)*sinh(
x)^2 - 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^
2*b - b^3)*sinh(x)^6 - 3*(a^2*b - b^3)*cosh(x)^4 - 3*(a^2*b - b^3 - 5*(a^2*
b - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 - 3*(a^2*b - b
^3)*cosh(x))*sinh(x)^3 - a^2*b + b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^
```

```
2*b - b^3)*cosh(x)^4 + a^2*b - b^3 - 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 +
 6*((a^2*b - b^3)*cosh(x)^5 - 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cos
h(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 3*((a^2
*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^2*b - b^3)*sin
h(x)^6 - 3*(a^2*b - b^3)*cosh(x)^4 - 3*(a^2*b - b^3 - 5*(a^2*b - b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 - 3*(a^2*b - b^3)*cosh(x))*s
inh(x)^3 - a^2*b + b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^2*b - b^3)*cos
h(x)^4 + a^2*b - b^3 - 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^2*b - b
^3)*cosh(x)^5 - 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x))
*log(2*sinh(x)/(cosh(x) - sinh(x))) + 12*(2*(a^2*b - a*b^2)*cosh(x)^3 - (2*
a^3 + a^2*b - 2*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sin
h(x)^5 + a^4*sinh(x)^6 - 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(
x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(5*a^4*cosh(x)^3 - 3*a^4*cosh(x))*sinh(x)^3
 + 3*(5*a^4*cosh(x)^4 - 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5
 - 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^4(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*tanh(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*tanh(x)), x)
```

Giac [B] time = 1.2955, size = 273, normalized size = 3.5

$$-\frac{\left(a^3b + a^2b^2 - ab^3 - b^4\right)\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^5 + a^4b} + \frac{\left(a^2b - b^3\right)\log\left(\left|e^{(2x)} - 1\right|\right)}{a^4} - \frac{11\,a^2be^{(6x)} - 11\,b^3e^{(6x)} - 45\,a^2be^{(4x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -(a^3*b + a^2*b^2 - a*b^3 - b^4)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a
^5 + a^4*b) + (a^2*b - b^3)*log(abs(e^(2*x) - 1))/a^4 - 1/6*(11*a^2*b*e^(6*
x) - 11*b^3*e^(6*x) - 45*a^2*b*e^(4*x) + 12*a*b^2*e^(4*x) + 33*b^3*e^(4*x)
+ 24*a^3*e^(2*x) + 45*a^2*b*e^(2*x) - 24*a*b^2*e^(2*x) - 33*b^3*e^(2*x) - 8
*a^3 - 11*a^2*b + 12*a*b^2 + 11*b^3)/(a^4*(e^(2*x) - 1)^3)
```

3.88 $\int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=255

$$\frac{3b^3\operatorname{csch}(x)}{2a^4}+\frac{b^4\operatorname{sech}(x)}{a^5}+\frac{b^2\left(a^2-b^2\right)\operatorname{sech}(x)}{a^5}-\frac{3b^2\operatorname{sech}(x)}{2a^3}+\frac{b^3\tan^{-1}(\sinh(x))}{a^4}+\frac{b\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{a^4}-$$

```
[Out] -((b*ArcTan[Sinh[x]])/a^2) + (b^3*ArcTan[Sinh[x]])/a^4 + (b*(a^2 - b^2)*Arc
Tan[Sinh[x]])/a^4 - (b*(a^2 - b^2)^(3/2)*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqr
t[a^2 - b^2]])/a^5 - (3*ArcTanh[Cosh[x]])/(8*a) + (3*b^2*ArcTanh[Cosh[x]])/
(2*a^3) - (b^4*ArcTanh[Cosh[x]])/a^5 - (b*Csch[x])/a^2 + (3*b^3*Csch[x])/(2
*a^4) + (3*Coth[x]*Csch[x])/(8*a) + (b*Csch[x]^3)/(3*a^2) - (Coth[x]*Csch[x
]^3)/(4*a) - (3*b^2*Sech[x])/(2*a^3) + (b^4*Sech[x])/a^5 + (b^2*(a^2 - b^2)
*Sech[x])/a^5 - (b^2*Csch[x]^2*Sech[x])/(2*a^3) - (b^3*Csch[x]*Sech[x]^2)/(
2*a^4) - (b^3*Sech[x]*Tanh[x])/(2*a^4)
```

Rubi [A] time = 0.547651, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 13, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {3518, 3110, 3768, 3770, 2621, 302, 207, 2622, 288, 321, 3104, 3074, 206}

$$\frac{3b^3\operatorname{csch}(x)}{2a^4}+\frac{b^4\operatorname{sech}(x)}{a^5}+\frac{b^2\left(a^2-b^2\right)\operatorname{sech}(x)}{a^5}-\frac{3b^2\operatorname{sech}(x)}{2a^3}+\frac{b^3\tan^{-1}(\sinh(x))}{a^4}+\frac{b\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{a^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^5/(a + b*Tanh[x]),x]
```


```
[Out] -((b*ArcTan[Sinh[x]])/a^2) + (b^3*ArcTan[Sinh[x]])/a^4 + (b*(a^2 - b^2)*Arc
Tan[Sinh[x]])/a^4 - (b*(a^2 - b^2)^(3/2)*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqr
t[a^2 - b^2]])/a^5 - (3*ArcTanh[Cosh[x]])/(8*a) + (3*b^2*ArcTanh[Cosh[x]])/
(2*a^3) - (b^4*ArcTanh[Cosh[x]])/a^5 - (b*Csch[x])/a^2 + (3*b^3*Csch[x])/(2
*a^4) + (3*Coth[x]*Csch[x])/(8*a) + (b*Csch[x]^3)/(3*a^2) - (Coth[x]*Csch[x
]^3)/(4*a) - (3*b^2*Sech[x])/(2*a^3) + (b^4*Sech[x])/a^5 + (b^2*(a^2 - b^2)
*Sech[x])/a^5 - (b^2*Csch[x]^2*Sech[x])/(2*a^3) - (b^3*Csch[x]*Sech[x]^2)/(
2*a^4) - (b^3*Sech[x]*Tanh[x])/(2*a^4)
```


Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

IntegerQ[2*n]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx = \int \frac{\coth(x)\operatorname{csch}^4(x)}{a\cosh(x)+b\sinh(x)}\,dx$$

$$= i\int\left(-\frac{i\operatorname{csch}^5(x)}{a}+\frac{ib\operatorname{csch}^4(x)\operatorname{sech}(x)}{a^2}-\frac{ib^2\operatorname{csch}^3(x)\operatorname{sech}^2(x)}{a^3}+\frac{ib^3\operatorname{csch}^2(x)\operatorname{sech}^3(x)}{a^4}-\frac{ib^4\operatorname{csc}\right.$$

$$= \frac{\int \operatorname{csch}^5(x)\,dx}{a}-\frac{b\int \operatorname{csch}^4(x)\operatorname{sech}(x)\,dx}{a^2}+\frac{b^2\int \operatorname{csch}^3(x)\operatorname{sech}^2(x)\,dx}{a^3}-\frac{b^3\int \operatorname{csch}^2(x)\operatorname{sech}^3(x)}{a^4}$$

$$= -\frac{\coth(x)\operatorname{csch}^3(x)}{4a}-\frac{b^4\operatorname{sech}^3(x)}{3a^5}-\frac{3\int \operatorname{csch}^3(x)\,dx}{4a}-\frac{(ib)\operatorname{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,x,-i\operatorname{csch}(x)\right)}{a^2}-$$

$$= \frac{3\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{b^2\left(a^2-b^2\right)\operatorname{sech}(x)}{a^5}-\frac{b^2\operatorname{csch}^2(x)\operatorname{sech}(x)}{2a^3}-\frac{b^3\operatorname{csch}(x}{2}$$

$$= -\frac{b^3\tan^{-1}(\sinh(x))}{2a^4}+\frac{b\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{a^4}-\frac{3\tanh^{-1}(\cosh(x))}{8a}-\frac{b\operatorname{csch}(x)}{a^2}+\frac{3b^3\operatorname{csch}}{2a^4}$$

$$= -\frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{b^3\tan^{-1}(\sinh(x))}{a^4}+\frac{b\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{a^4}-\frac{b\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b}{\cdot}\right.}{a^5}$$

Mathematica [A] time = 0.846593, size = 296, normalized size = 1.16

$$\frac{-16ab\left(7a^2-6b^2\right)\coth\left(\frac{x}{2}\right)+6a^2\left(3a^2-4b^2\right)\operatorname{csch}^2\left(\frac{x}{2}\right)-24a^2b^2\operatorname{sech}^2\left(\frac{x}{2}\right)-288a^2b^2\log\left(\tanh\left(\frac{x}{2}\right)\right)+112a^3b\tanh\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] (-384*a^2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*S
qrt[a + b])] + 384*Sqrt[a - b]*b^3*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sq
rt[a - b]*Sqrt[a + b])] - 16*a*b*(7*a^2 - 6*b^2)*Coth[x/2] + 6*a^2*(3*a^2 -
 4*b^2)*Csch[x/2]^2 + 72*a^4*Log[Tanh[x/2]] - 288*a^2*b^2*Log[Tanh[x/2]] +
192*b^4*Log[Tanh[x/2]] + 18*a^4*Sech[x/2]^2 - 24*a^2*b^2*Sech[x/2]^2 + 3*a^
4*Sech[x/2]^4 + 64*a^3*b*Csch[x]^3*Sinh[x/2]^4 + a^3*Csch[x/2]^4*(-3*a + 4*
b*Sinh[x]) + 112*a^3*b*Tanh[x/2] - 96*a*b^3*Tanh[x/2])/(192*a^5)
```

Maple [A] time = 0.047, size = 311, normalized size = 1.2

$$\frac{1}{64a}\left(\tanh\left(\frac{x}{2}\right)\right)^4-\frac{b}{24a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^3-\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{b^2}{8a^3}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{5b}{8a^2}\tanh\left(\frac{x}{2}\right)-\frac{b^3}{2a^4}\tanh\left(\frac{x}{2}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^5/(a+b*tanh(x)),x)
```

```
[Out] 1/64/a*tanh(1/2*x)^4-1/24/a^2*tanh(1/2*x)^3*b-1/8/a*tanh(1/2*x)^2+1/8/a^3*t
anh(1/2*x)^2*b^2+5/8/a^2*tanh(1/2*x)*b-1/2/a^4*tanh(1/2*x)*b^3-2/a*b/(a^2-b
^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))+4*b^3/a^3/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-2*b^5/a^5/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/64/a/tanh(1
/2*x)^4+1/8/a/tanh(1/2*x)^2-1/8/a^3/tanh(1/2*x)^2*b^2+3/8/a*ln(tanh(1/2*x))
-3/2/a^3*ln(tanh(1/2*x))*b^2+1/a^5*ln(tanh(1/2*x))*b^4+1/24/a^2*b/tanh(1/2*
x)^3-5/8*b/a^2/tanh(1/2*x)+1/2*b^3/a^4/tanh(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.80162, size = 12964, normalized size = 50.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*(6*(3*a^4 - 8*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cosh(x)^7 + 42*(3*a^4 - 8*
a^3*b - 4*a^2*b^2 + 8*a*b^3)*cosh(x)*sinh(x)^6 + 6*(3*a^4 - 8*a^3*b - 4*a^2
*b^2 + 8*a*b^3)*sinh(x)^7 - 2*(33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*b^3)*
cosh(x)^5 - 2*(33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*b^3 - 63*(3*a^4 - 8*a
^3*b - 4*a^2*b^2 + 8*a*b^3)*cosh(x)^2)*sinh(x)^5 + 10*(21*(3*a^4 - 8*a^3*b
- 4*a^2*b^2 + 8*a*b^3)*cosh(x)^3 - (33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*
b^3)*cosh(x))*sinh(x)^4 - 2*(33*a^4 + 104*a^3*b - 12*a^2*b^2 - 72*a*b^3)*co
sh(x)^3 + 2*(105*(3*a^4 - 8*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cosh(x)^4 - 33*a^4
 - 104*a^3*b + 12*a^2*b^2 + 72*a*b^3 - 10*(33*a^4 - 104*a^3*b - 12*a^2*b^2
+ 72*a*b^3)*cosh(x)^2)*sinh(x)^3 + 2*(63*(3*a^4 - 8*a^3*b - 4*a^2*b^2 + 8*a
*b^3)*cosh(x)^5 - 10*(33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*b^3)*cosh(x)^3
 - 3*(33*a^4 + 104*a^3*b - 12*a^2*b^2 - 72*a*b^3)*cosh(x))*sinh(x)^2 - 24*(
(a^2*b - b^3)*cosh(x)^8 + 8*(a^2*b - b^3)*cosh(x)*sinh(x)^7 + (a^2*b - b^3)
*sinh(x)^8 - 4*(a^2*b - b^3)*cosh(x)^6 - 4*(a^2*b - b^3 - 7*(a^2*b - b^3)*c
osh(x)^2)*sinh(x)^6 + 8*(7*(a^2*b - b^3)*cosh(x)^3 - 3*(a^2*b - b^3)*cosh(x
))*sinh(x)^5 + 6*(a^2*b - b^3)*cosh(x)^4 + 2*(35*(a^2*b - b^3)*cosh(x)^4 +
3*a^2*b - 3*b^3 - 30*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^2*b - b^3
)*cosh(x)^5 - 10*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*sinh(x)
^3 + a^2*b - b^3 - 4*(a^2*b - b^3)*cosh(x)^2 + 4*(7*(a^2*b - b^3)*cosh(x)^6
 - 15*(a^2*b - b^3)*cosh(x)^4 - a^2*b + b^3 + 9*(a^2*b - b^3)*cosh(x)^2)*si
nh(x)^2 + 8*((a^2*b - b^3)*cosh(x)^7 - 3*(a^2*b - b^3)*cosh(x)^5 + 3*(a^2*b
 - b^3)*cosh(x)^3 - (a^2*b - b^3)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log(((
a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-
```

```
a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 6*(3*a^4 + 8*a^3*b - 4*a^2*b^2
- 8*a*b^3)*cosh(x) - 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^8 + 8*(3*a^4 -
 12*a^2*b^2 + 8*b^4)*cosh(x)*sinh(x)^7 + (3*a^4 - 12*a^2*b^2 + 8*b^4)*sinh(
x)^8 - 4*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^6 - 4*(3*a^4 - 12*a^2*b^2 + 8
*b^4 - 7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^4 -
12*a^2*b^2 + 8*b^4)*cosh(x)^3 - 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x))*sin
h(x)^5 + 6*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^4 + 2*(35*(3*a^4 - 12*a^2*b
^2 + 8*b^4)*cosh(x)^4 + 9*a^4 - 36*a^2*b^2 + 24*b^4 - 30*(3*a^4 - 12*a^2*b^
2 + 8*b^4)*cosh(x)^2)*sinh(x)^4 + 3*a^4 - 12*a^2*b^2 + 8*b^4 + 8*(7*(3*a^4
- 12*a^2*b^2 + 8*b^4)*cosh(x)^5 - 10*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^3
 + 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x))*sinh(x)^3 - 4*(3*a^4 - 12*a^2*b^
2 + 8*b^4)*cosh(x)^2 + 4*(7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^6 - 15*(3*
a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^4 - 3*a^4 + 12*a^2*b^2 - 8*b^4 + 9*(3*a^4
 - 12*a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^4 - 12*a^2*b^2 + 8*b^
4)*cosh(x)^7 - 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^5 + 3*(3*a^4 - 12*a^2
*b^2 + 8*b^4)*cosh(x)^3 - (3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x))*sinh(x))*lo
g(cosh(x) + sinh(x) + 1) + 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^8 + 8*(3
*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)*sinh(x)^7 + (3*a^4 - 12*a^2*b^2 + 8*b^4)
*sinh(x)^8 - 4*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^6 - 4*(3*a^4 - 12*a^2*b
^2 + 8*b^4 - 7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*
a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^3 - 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x
))*sinh(x)^5 + 6*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^4 + 2*(35*(3*a^4 - 12
*a^2*b^2 + 8*b^4)*cosh(x)^4 + 9*a^4 - 36*a^2*b^2 + 24*b^4 - 30*(3*a^4 - 12*
a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^4 + 3*a^4 - 12*a^2*b^2 + 8*b^4 + 8*(7*(
3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^5 - 10*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos
h(x)^3 + 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x))*sinh(x)^3 - 4*(3*a^4 - 12*
a^2*b^2 + 8*b^4)*cosh(x)^2 + 4*(7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^6 -
15*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^4 - 3*a^4 + 12*a^2*b^2 - 8*b^4 + 9*
(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^4 - 12*a^2*b^2
+ 8*b^4)*cosh(x)^7 - 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^5 + 3*(3*a^4 -
12*a^2*b^2 + 8*b^4)*cosh(x)^3 - (3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x))*sinh(
x))*log(cosh(x) + sinh(x) - 1) + 2*(21*(3*a^4 - 8*a^3*b - 4*a^2*b^2 + 8*a*b
^3)*cosh(x)^6 - 5*(33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*b^3)*cosh(x)^4 +
9*a^4 + 24*a^3*b - 12*a^2*b^2 - 24*a*b^3 - 3*(33*a^4 + 104*a^3*b - 12*a^2*b
^2 - 72*a*b^3)*cosh(x)^2)*sinh(x))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7
 + a^5*sinh(x)^8 - 4*a^5*cosh(x)^6 + 6*a^5*cosh(x)^4 - 4*a^5*cosh(x)^2 + 4*
(7*a^5*cosh(x)^2 - a^5)*sinh(x)^6 + 8*(7*a^5*cosh(x)^3 - 3*a^5*cosh(x))*sin
h(x)^5 + a^5 + 2*(35*a^5*cosh(x)^4 - 30*a^5*cosh(x)^2 + 3*a^5)*sinh(x)^4 +
8*(7*a^5*cosh(x)^5 - 10*a^5*cosh(x)^3 + 3*a^5*cosh(x))*sinh(x)^3 + 4*(7*a^5
*cosh(x)^6 - 15*a^5*cosh(x)^4 + 9*a^5*cosh(x)^2 - a^5)*sinh(x)^2 + 8*(a^5*c
osh(x)^7 - 3*a^5*cosh(x)^5 + 3*a^5*cosh(x)^3 - a^5*cosh(x))*sinh(x)), 1/24*
(6*(3*a^4 - 8*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cosh(x)^7 + 42*(3*a^4 - 8*a^3*b
- 4*a^2*b^2 + 8*a*b^3)*cosh(x)*sinh(x)^6 + 6*(3*a^4 - 8*a^3*b - 4*a^2*b^2 +
 8*a*b^3)*sinh(x)^7 - 2*(33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*b^3)*cosh(x
)^5 - 2*(33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*b^3 - 63*(3*a^4 - 8*a^3*b -
 4*a^2*b^2 + 8*a*b^3)*cosh(x)^2)*sinh(x)^5 + 10*(21*(3*a^4 - 8*a^3*b - 4*a^
2*b^2 + 8*a*b^3)*cosh(x)^3 - (33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*b^3)*c
osh(x))*sinh(x)^4 - 2*(33*a^4 + 104*a^3*b - 12*a^2*b^2 - 72*a*b^3)*cosh(x)^
3 + 2*(105*(3*a^4 - 8*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cosh(x)^4 - 33*a^4 - 104
*a^3*b + 12*a^2*b^2 + 72*a*b^3 - 10*(33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a
*b^3)*cosh(x)^2)*sinh(x)^3 + 2*(63*(3*a^4 - 8*a^3*b - 4*a^2*b^2 + 8*a*b^3)*
cosh(x)^5 - 10*(33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*b^3)*cosh(x)^3 - 3*(
33*a^4 + 104*a^3*b - 12*a^2*b^2 - 72*a*b^3)*cosh(x))*sinh(x)^2 + 48*((a^2*b
 - b^3)*cosh(x)^8 + 8*(a^2*b - b^3)*cosh(x)*sinh(x)^7 + (a^2*b - b^3)*sinh(
x)^8 - 4*(a^2*b - b^3)*cosh(x)^6 - 4*(a^2*b - b^3 - 7*(a^2*b - b^3)*cosh(x)
^2)*sinh(x)^6 + 8*(7*(a^2*b - b^3)*cosh(x)^3 - 3*(a^2*b - b^3)*cosh(x))*sin
h(x)^5 + 6*(a^2*b - b^3)*cosh(x)^4 + 2*(35*(a^2*b - b^3)*cosh(x)^4 + 3*a^2*
b - 3*b^3 - 30*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^2*b - b^3)*cosh
```

```
(x)^5 - 10*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*sinh(x)^3 + a
^2*b - b^3 - 4*(a^2*b - b^3)*cosh(x)^2 + 4*(7*(a^2*b - b^3)*cosh(x)^6 - 15*
(a^2*b - b^3)*cosh(x)^4 - a^2*b + b^3 + 9*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^
2 + 8*((a^2*b - b^3)*cosh(x)^7 - 3*(a^2*b - b^3)*cosh(x)^5 + 3*(a^2*b - b^3
)*cosh(x)^3 - (a^2*b - b^3)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a
^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*(3*a^4 + 8*a^3*b - 4*a^2
*b^2 - 8*a*b^3)*cosh(x) - 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^8 + 8*(3*
a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)*sinh(x)^7 + (3*a^4 - 12*a^2*b^2 + 8*b^4)*
sinh(x)^8 - 4*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^6 - 4*(3*a^4 - 12*a^2*b^
2 + 8*b^4 - 7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a
^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^3 - 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)
)*sinh(x)^5 + 6*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^4 + 2*(35*(3*a^4 - 12*
a^2*b^2 + 8*b^4)*cosh(x)^4 + 9*a^4 - 36*a^2*b^2 + 24*b^4 - 30*(3*a^4 - 12*a
^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^4 + 3*a^4 - 12*a^2*b^2 + 8*b^4 + 8*(7*(3
*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^5 - 10*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh
(x)^3 + 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x))*sinh(x)^3 - 4*(3*a^4 - 12*a
^2*b^2 + 8*b^4)*cosh(x)^2 + 4*(7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^6 - 1
5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^4 - 3*a^4 + 12*a^2*b^2 - 8*b^4 + 9*(
3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^4 - 12*a^2*b^2 +
 8*b^4)*cosh(x)^7 - 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^5 + 3*(3*a^4 - 1
2*a^2*b^2 + 8*b^4)*cosh(x)^3 - (3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x))*sinh(x
))*log(cosh(x) + sinh(x) + 1) + 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^8 +
 8*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)*sinh(x)^7 + (3*a^4 - 12*a^2*b^2 + 8
*b^4)*sinh(x)^8 - 4*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^6 - 4*(3*a^4 - 12*
a^2*b^2 + 8*b^4 - 7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(
7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^3 - 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*c
osh(x))*sinh(x)^5 + 6*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^4 + 2*(35*(3*a^4
 - 12*a^2*b^2 + 8*b^4)*cosh(x)^4 + 9*a^4 - 36*a^2*b^2 + 24*b^4 - 30*(3*a^4
- 12*a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^4 + 3*a^4 - 12*a^2*b^2 + 8*b^4 + 8
*(7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^5 - 10*(3*a^4 - 12*a^2*b^2 + 8*b^4
)*cosh(x)^3 + 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x))*sinh(x)^3 - 4*(3*a^4
- 12*a^2*b^2 + 8*b^4)*cosh(x)^2 + 4*(7*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)
^6 - 15*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^4 - 3*a^4 + 12*a^2*b^2 - 8*b^4
 + 9*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^4 - 12*a^2
*b^2 + 8*b^4)*cosh(x)^7 - 3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^5 + 3*(3*a
^4 - 12*a^2*b^2 + 8*b^4)*cosh(x)^3 - (3*a^4 - 12*a^2*b^2 + 8*b^4)*cosh(x))*
sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(21*(3*a^4 - 8*a^3*b - 4*a^2*b^2 +
8*a*b^3)*cosh(x)^6 - 5*(33*a^4 - 104*a^3*b - 12*a^2*b^2 + 72*a*b^3)*cosh(x)
^4 + 9*a^4 + 24*a^3*b - 12*a^2*b^2 - 24*a*b^3 - 3*(33*a^4 + 104*a^3*b - 12*
a^2*b^2 - 72*a*b^3)*cosh(x)^2)*sinh(x))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh
(x)^7 + a^5*sinh(x)^8 - 4*a^5*cosh(x)^6 + 6*a^5*cosh(x)^4 - 4*a^5*cosh(x)^2
 + 4*(7*a^5*cosh(x)^2 - a^5)*sinh(x)^6 + 8*(7*a^5*cosh(x)^3 - 3*a^5*cosh(x)
)*sinh(x)^5 + a^5 + 2*(35*a^5*cosh(x)^4 - 30*a^5*cosh(x)^2 + 3*a^5)*sinh(x)
^4 + 8*(7*a^5*cosh(x)^5 - 10*a^5*cosh(x)^3 + 3*a^5*cosh(x))*sinh(x)^3 + 4*(
7*a^5*cosh(x)^6 - 15*a^5*cosh(x)^4 + 9*a^5*cosh(x)^2 - a^5)*sinh(x)^2 + 8*(
a^5*cosh(x)^7 - 3*a^5*cosh(x)^5 + 3*a^5*cosh(x)^3 - a^5*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**5/(a+b*tanh(x)),x)
```

```
[Out] Integral(csch(x)**5/(a + b*tanh(x)), x)
```

Giac [A] time = 1.25187, size = 369, normalized size = 1.45

$$-\frac{\left(3\,a^{4}-12\,a^{2}b^{2}+8\,b^{4}\right)\log\left(e^{x}+1\right)}{8\,a^{5}}+\frac{\left(3\,a^{4}-12\,a^{2}b^{2}+8\,b^{4}\right)\log\left(\left|e^{x}-1\right|\right)}{8\,a^{5}}-\frac{2\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}a^{5}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -1/8*(3*a^4 - 12*a^2*b^2 + 8*b^4)*log(e^x + 1)/a^5 + 1/8*(3*a^4 - 12*a^2*b^
2 + 8*b^4)*log(abs(e^x - 1))/a^5 - 2*(a^4*b - 2*a^2*b^3 + b^5)*arctan((a*e^
x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5) + 1/12*(9*a^3*e^(7*x) - 2
4*a^2*b*e^(7*x) - 12*a*b^2*e^(7*x) + 24*b^3*e^(7*x) - 33*a^3*e^(5*x) + 104*
a^2*b*e^(5*x) + 12*a*b^2*e^(5*x) - 72*b^3*e^(5*x) - 33*a^3*e^(3*x) - 104*a^
2*b*e^(3*x) + 12*a*b^2*e^(3*x) + 72*b^3*e^(3*x) + 9*a^3*e^x + 24*a^2*b*e^x
- 12*a*b^2*e^x - 24*b^3*e^x)/(a^4*(e^(2*x) - 1)^4)
```

3.89 $\int \frac{\mathrm{csch}^6(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=130

$$\frac{(2a^2-b^2)\coth^3(x)}{3a^3}-\frac{b(2a^2-b^2)\coth^2(x)}{2a^4}-\frac{(a^2-b^2)^2\coth(x)}{a^5}-\frac{b(a^2-b^2)^2\log(\tanh(x))}{a^6}+\frac{b(a^2-b^2)^2\log(a+b\,\mathrm{ta}}{a^6}$$

```
[Out] -(((a^2 - b^2)^2*Coth[x])/a^5) - (b*(2*a^2 - b^2)*Coth[x]^2)/(2*a^4) + ((2*
a^2 - b^2)*Coth[x]^3)/(3*a^3) + (b*Coth[x]^4)/(4*a^2) - Coth[x]^5/(5*a) - (
b*(a^2 - b^2)^2*Log[Tanh[x]])/a^6 + (b*(a^2 - b^2)^2*Log[a + b*Tanh[x]])/a^
6
```

Rubi [A] time = 0.154467, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3516, 894}

$$\frac{(2a^2-b^2)\coth^3(x)}{3a^3}-\frac{b(2a^2-b^2)\coth^2(x)}{2a^4}-\frac{(a^2-b^2)^2\coth(x)}{a^5}-\frac{b(a^2-b^2)^2\log(\tanh(x))}{a^6}+\frac{b(a^2-b^2)^2\log(a+b\,\mathrm{ta}}{a^6}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^6/(a + b*Tanh[x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Coth[x])/a^5) - (b*(2*a^2 - b^2)*Coth[x]^2)/(2*a^4) + ((2*
a^2 - b^2)*Coth[x]^3)/(3*a^3) + (b*Coth[x]^4)/(4*a^2) - Coth[x]^5/(5*a) - (
b*(a^2 - b^2)^2*Log[Tanh[x]])/a^6 + (b*(a^2 - b^2)^2*Log[a + b*Tanh[x]])/a^
6
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\mathrm{csch}^6(x)}{a+b\tanh(x)}\,dx &= b\,\mathrm{Subst}\left(\int \frac{(-b^2+x^2)^2}{x^6(a+x)}\,dx, x, b\tanh(x)\right)\\ &= b\,\mathrm{Subst}\left(\int \left(\frac{b^4}{ax^6}-\frac{b^4}{a^2x^5}+\frac{-2a^2b^2+b^4}{a^3x^4}+\frac{2a^2b^2-b^4}{a^4x^3}+\frac{(a^2-b^2)^2}{a^5x^2}-\frac{(a^2-b^2)^2}{a^6x}+\frac{(a^2-b^2)^2}{a^6(a+x)}\right) dx\right.\\ &= -\frac{(a^2-b^2)^2\coth(x)}{a^5}-\frac{b(2a^2-b^2)\coth^2(x)}{2a^4}+\frac{(2a^2-b^2)\coth^3(x)}{3a^3}+\frac{b\coth^4(x)}{4a^2}-\frac{\coth^5(x)}{5a}-\frac{b}{}\end{aligned}$$

Mathematica [A] time = 0.582419, size = 119, normalized size = 0.92

$$\frac{15b\left(-2a^2\left(a^2-b^2\right)\text{csch}^2(x)-4\left(a^2-b^2\right)^2\left(\log(\sinh(x))-\log(a\cosh(x)+b\sinh(x))\right)+a^4\text{csch}^4(x)\right)-4\coth(x)\left(\left(5\right.\right.}{60a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^6/(a + b*Tanh[x]),x]
```

```
[Out] (-4*Coth[x]*(8*a^5 - 25*a^3*b^2 + 15*a*b^4 + (-4*a^5 + 5*a^3*b^2)*Csch[x]^2
 + 3*a^5*Csch[x]^4) + 15*b*(-2*a^2*(a^2 - b^2)*Csch[x]^2 + a^4*Csch[x]^4 -
4*(a^2 - b^2)^2*(Log[Sinh[x]] - Log[a*Cosh[x] + b*Sinh[x]])))/(60*a^6)
```

Maple [B] time = 0.051, size = 333, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^6/(a+b*tanh(x)),x)
```

```
[Out] -1/160/a*tanh(1/2*x)^5+1/64/a^2*b*tanh(1/2*x)^4+5/96/a*tanh(1/2*x)^3-1/24/a
^3*tanh(1/2*x)^3*b^2-3/16/a^2*b*tanh(1/2*x)^2+1/8/a^4*tanh(1/2*x)^2*b^3-5/1
6/a*tanh(1/2*x)+7/8/a^3*b^2*tanh(1/2*x)-1/2/a^5*tanh(1/2*x)*b^4+1/a^2*b*ln(
a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)-2/a^4*b^3*ln(a*tanh(1/2*x)^2+2*tanh(1/2*
x)*b+a)+1/a^6*b^5*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)-1/160/a/tanh(1/2*x)
^5+5/96/a/tanh(1/2*x)^3-1/24/a^3/tanh(1/2*x)^3*b^2-5/16/a/tanh(1/2*x)+7/8/a
^3/tanh(1/2*x)*b^2-1/2/a^5/tanh(1/2*x)*b^4+1/64/a^2*b/tanh(1/2*x)^4-3/16/a^
2*b/tanh(1/2*x)^2+1/8/a^4*b^3/tanh(1/2*x)^2-1/a^2*b*ln(tanh(1/2*x))+2/a^4*b
^3*ln(tanh(1/2*x))-1/a^6*b^5*ln(tanh(1/2*x))
```

Maxima [B] time = 1.111, size = 416, normalized size = 3.2

$$\frac{2\left(8\,a^4-25\,a^2b^2+15\,b^4-5\left(8\,a^4-3\,a^3b-22\,a^2b^2+3\,ab^3+12\,b^4\right)e^{(-2\,x)}+5\left(16\,a^4-15\,a^3b-32\,a^2b^2+9\,ab^3+18\,b^4\right.\right.}{15\left(5\,a^5e^{(-2\,x)}-10\,a^5e^{(-4\,x)}+10\,a^5e^{(-6\,x)}-5\,a^5e^{(-8\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(8*a^4 - 25*a^2*b^2 + 15*b^4 - 5*(8*a^4 - 3*a^3*b - 22*a^2*b^2 + 3*a*b
^3 + 12*b^4)*e^(-2*x) + 5*(16*a^4 - 15*a^3*b - 32*a^2*b^2 + 9*a*b^3 + 18*b^
4)*e^(-4*x) + 15*(5*a^3*b + 6*a^2*b^2 - 3*a*b^3 - 4*b^4)*e^(-6*x) - 15*(a^3
*b + a^2*b^2 - a*b^3 - b^4)*e^(-8*x))/(5*a^5*e^(-2*x) - 10*a^5*e^(-4*x) + 1
0*a^5*e^(-6*x) - 5*a^5*e^(-8*x) + a^5*e^(-10*x) - a^5) + (a^4*b - 2*a^2*b^3
 + b^5)*log(-(a - b)*e^(-2*x) - a - b)/a^6 - (a^4*b - 2*a^2*b^3 + b^5)*log(
e^(-x) + 1)/a^6 - (a^4*b - 2*a^2*b^3 + b^5)*log(e^(-x) - 1)/a^6
```

Fricas [B] time = 2.74749, size = 7144, normalized size = 54.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(30*(a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*cosh(x)^8 + 240*(a^4*b - a^3*
b^2 - a^2*b^3 + a*b^4)*cosh(x)*sinh(x)^7 + 30*(a^4*b - a^3*b^2 - a^2*b^3 +
a*b^4)*sinh(x)^8 - 30*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4)*cosh(x)^6
 - 30*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4 - 28*(a^4*b - a^3*b^2 - a^
2*b^3 + a*b^4)*cosh(x)^2)*sinh(x)^6 + 60*(28*(a^4*b - a^3*b^2 - a^2*b^3 + a
*b^4)*cosh(x)^3 - 3*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4)*cosh(x))*si
nh(x)^5 + 16*a^5 - 50*a^3*b^2 + 30*a*b^4 + 10*(16*a^5 + 15*a^4*b - 32*a^3*b
^2 - 9*a^2*b^3 + 18*a*b^4)*cosh(x)^4 + 10*(16*a^5 + 15*a^4*b - 32*a^3*b^2 -
 9*a^2*b^3 + 18*a*b^4 + 210*(a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*cosh(x)^4 -
 45*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(
42*(a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*cosh(x)^5 - 15*(5*a^4*b - 6*a^3*b^2
- 3*a^2*b^3 + 4*a*b^4)*cosh(x)^3 + (16*a^5 + 15*a^4*b - 32*a^3*b^2 - 9*a^2*
b^3 + 18*a*b^4)*cosh(x))*sinh(x)^3 - 10*(8*a^5 + 3*a^4*b - 22*a^3*b^2 - 3*a
^2*b^3 + 12*a*b^4)*cosh(x)^2 + 10*(84*(a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*c
osh(x)^6 - 8*a^5 - 3*a^4*b + 22*a^3*b^2 + 3*a^2*b^3 - 12*a*b^4 - 45*(5*a^4*
b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4)*cosh(x)^4 + 6*(16*a^5 + 15*a^4*b - 32*
a^3*b^2 - 9*a^2*b^3 + 18*a*b^4)*cosh(x)^2)*sinh(x)^2 - 15*((a^4*b - 2*a^2*b
^3 + b^5)*cosh(x)^10 + 10*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)*sinh(x)^9 + (a^
4*b - 2*a^2*b^3 + b^5)*sinh(x)^10 - 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^8 -
 5*(a^4*b - 2*a^2*b^3 + b^5 - 9*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x
)^8 + 40*(3*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 - (a^4*b - 2*a^2*b^3 + b^5)
*cosh(x))*sinh(x)^7 + 10*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^6 + 10*(a^4*b -
2*a^2*b^3 + b^5 + 21*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^4 - 14*(a^4*b - 2*a^
2*b^3 + b^5)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)
^5 - 70*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 + 15*(a^4*b - 2*a^2*b^3 + b^5)*
cosh(x))*sinh(x)^5 - a^4*b + 2*a^2*b^3 - b^5 - 10*(a^4*b - 2*a^2*b^3 + b^5)
*cosh(x)^4 + 10*(21*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^6 - a^4*b + 2*a^2*b^3
 - b^5 - 35*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^4 + 15*(a^4*b - 2*a^2*b^3 + b
^5)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^7 - 7*(a
^4*b - 2*a^2*b^3 + b^5)*cosh(x)^5 + 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 -
 (a^4*b - 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 + 5*(a^4*b - 2*a^2*b^3 + b^5)
*cosh(x)^2 + 5*(9*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^8 - 28*(a^4*b - 2*a^2*b
^3 + b^5)*cosh(x)^6 + a^4*b - 2*a^2*b^3 + b^5 + 30*(a^4*b - 2*a^2*b^3 + b^5
)*cosh(x)^4 - 12*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^2 + 10*((a^4*
b - 2*a^2*b^3 + b^5)*cosh(x)^9 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^7 + 6*
(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^5 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3
 + (a^4*b - 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 15*((a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^10 + 10*(a^4
*b - 2*a^2*b^3 + b^5)*cosh(x)*sinh(x)^9 + (a^4*b - 2*a^2*b^3 + b^5)*sinh(x)
^10 - 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^8 - 5*(a^4*b - 2*a^2*b^3 + b^5 -
9*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^4*b - 2*a^2*b^3
 + b^5)*cosh(x)^3 - (a^4*b - 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^7 + 10*(a^4*
b - 2*a^2*b^3 + b^5)*cosh(x)^6 + 10*(a^4*b - 2*a^2*b^3 + b^5 + 21*(a^4*b -
2*a^2*b^3 + b^5)*cosh(x)^4 - 14*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x
)^6 + 4*(63*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^5 - 70*(a^4*b - 2*a^2*b^3 + b
^5)*cosh(x)^3 + 15*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^5 - a^4*b + 2
*a^2*b^3 - b^5 - 10*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^4 + 10*(21*(a^4*b - 2
*a^2*b^3 + b^5)*cosh(x)^6 - a^4*b + 2*a^2*b^3 - b^5 - 35*(a^4*b - 2*a^2*b^3
 + b^5)*cosh(x)^4 + 15*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^4 + 40*
(3*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^7 - 7*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x
)^5 + 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 - (a^4*b - 2*a^2*b^3 + b^5)*cos
h(x))*sinh(x)^3 + 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2 + 5*(9*(a^4*b - 2*a
^2*b^3 + b^5)*cosh(x)^8 - 28*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^6 + a^4*b -
2*a^2*b^3 + b^5 + 30*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^4 - 12*(a^4*b - 2*a^
2*b^3 + b^5)*cosh(x)^2)*sinh(x)^2 + 10*((a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^9
```

```
 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^7 + 6*(a^4*b - 2*a^2*b^3 + b^5)*cosh
(x)^5 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 + (a^4*b - 2*a^2*b^3 + b^5)*c
osh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 20*(12*(a^4*b - a^3*b
^2 - a^2*b^3 + a*b^4)*cosh(x)^7 - 9*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*
b^4)*cosh(x)^5 + 2*(16*a^5 + 15*a^4*b - 32*a^3*b^2 - 9*a^2*b^3 + 18*a*b^4)*
cosh(x)^3 - (8*a^5 + 3*a^4*b - 22*a^3*b^2 - 3*a^2*b^3 + 12*a*b^4)*cosh(x))*
sinh(x))/(a^6*cosh(x)^10 + 10*a^6*cosh(x)*sinh(x)^9 + a^6*sinh(x)^10 - 5*a^
6*cosh(x)^8 + 10*a^6*cosh(x)^6 - 10*a^6*cosh(x)^4 + 5*(9*a^6*cosh(x)^2 - a^
6)*sinh(x)^8 + 5*a^6*cosh(x)^2 + 40*(3*a^6*cosh(x)^3 - a^6*cosh(x))*sinh(x)
^7 + 10*(21*a^6*cosh(x)^4 - 14*a^6*cosh(x)^2 + a^6)*sinh(x)^6 - a^6 + 4*(63
*a^6*cosh(x)^5 - 70*a^6*cosh(x)^3 + 15*a^6*cosh(x))*sinh(x)^5 + 10*(21*a^6*
cosh(x)^6 - 35*a^6*cosh(x)^4 + 15*a^6*cosh(x)^2 - a^6)*sinh(x)^4 + 40*(3*a^
6*cosh(x)^7 - 7*a^6*cosh(x)^5 + 5*a^6*cosh(x)^3 - a^6*cosh(x))*sinh(x)^3 +
5*(9*a^6*cosh(x)^8 - 28*a^6*cosh(x)^6 + 30*a^6*cosh(x)^4 - 12*a^6*cosh(x)^2
 + a^6)*sinh(x)^2 + 10*(a^6*cosh(x)^9 - 4*a^6*cosh(x)^7 + 6*a^6*cosh(x)^5 -
 4*a^6*cosh(x)^3 + a^6*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^6(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**6/(a+b*tanh(x)),x)
```

```
[Out] Integral(csch(x)**6/(a + b*tanh(x)), x)
```

Giac [B] time = 1.21859, size = 556, normalized size = 4.28

$$\frac{\left(a^5b + a^4b^2 - 2\,a^3b^3 - 2\,a^2b^4 + ab^5 + b^6\right)\log\left(\left|ae^{(2\,x)} + be^{(2\,x)} + a - b\right|\right)}{a^7 + a^6b} - \frac{\left(a^4b - 2\,a^2b^3 + b^5\right)\log\left(\left|e^{(2\,x)} - 1\right|\right)}{a^6} + \frac{137\,a^4 b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] (a^5*b + a^4*b^2 - 2*a^3*b^3 - 2*a^2*b^4 + a*b^5 + b^6)*log(abs(a*e^(2*x) +
 b*e^(2*x) + a - b))/(a^7 + a^6*b) - (a^4*b - 2*a^2*b^3 + b^5)*log(abs(e^(2
*x) - 1))/a^6 + 1/60*(137*a^4*b*e^(10*x) - 274*a^2*b^3*e^(10*x) + 137*b^5*e
^(10*x) - 805*a^4*b*e^(8*x) + 120*a^3*b^2*e^(8*x) + 1490*a^2*b^3*e^(8*x) -
120*a*b^4*e^(8*x) - 685*b^5*e^(8*x) + 1970*a^4*b*e^(6*x) - 720*a^3*b^2*e^(6
*x) - 3100*a^2*b^3*e^(6*x) + 480*a*b^4*e^(6*x) + 1370*b^5*e^(6*x) - 640*a^5
*e^(4*x) - 1970*a^4*b*e^(4*x) + 1280*a^3*b^2*e^(4*x) + 3100*a^2*b^3*e^(4*x)
 - 720*a*b^4*e^(4*x) - 1370*b^5*e^(4*x) + 320*a^5*e^(2*x) + 805*a^4*b*e^(2*
x) - 880*a^3*b^2*e^(2*x) - 1490*a^2*b^3*e^(2*x) + 480*a*b^4*e^(2*x) + 685*b
^5*e^(2*x) - 64*a^5 - 137*a^4*b + 200*a^3*b^2 + 274*a^2*b^3 - 120*a*b^4 - 1
37*b^5)/(a^6*(e^(2*x) - 1)^5)
```

3.90 $\int \frac{\operatorname{csch}(x)}{i+\tanh(x)}\,dx$

Optimal. Leaf size=33

$$i\tanh^{-1}(\cosh(x))-\frac{i\tanh^{-1}\left(\frac{\cosh(x)+i\sinh(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] I*ArcTanh[Cosh[x]] - (I*ArcTanh[(Cosh[x] + I*Sinh[x])/Sqrt[2]])/Sqrt[2]
```

Rubi [A] time = 0.102202, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3518, 3110, 3770, 3074, 206}

$$i\tanh^{-1}(\cosh(x))-\frac{i\tanh^{-1}\left(\frac{\cosh(x)+i\sinh(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(I + Tanh[x]),x]
```

```
[Out] I*ArcTanh[Cosh[x]] - (I*ArcTanh[(Cosh[x] + I*Sinh[x])/Sqrt[2]])/Sqrt[2]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{i+\tanh(x)}\,dx = \int \frac{\coth(x)}{i\cosh(x)+\sinh(x)}\,dx$$
$$= i\int \left(-\operatorname{csch}(x) - \frac{i}{\cosh(x)-i\sinh(x)}\right)dx$$
$$= -(i\int \operatorname{csch}(x)\,dx) + \int \frac{1}{\cosh(x)-i\sinh(x)}\,dx$$
$$= i\tanh^{-1}(\cosh(x)) + i\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\cosh(x)-i\sinh(x)\right)$$
$$= i\tanh^{-1}(\cosh(x)) - \frac{i\tanh^{-1}\left(\frac{\cosh(x)+i\sinh(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0676641, size = 46, normalized size = 1.39

$$-i\left(\sqrt{2}\tanh^{-1}\left(\frac{1+i\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right) - \log\left(\cosh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(I + Tanh[x]),x]
```

```
[Out] (-I)*(Sqrt[2]*ArcTanh[(1 + I*Tanh[x/2])/Sqrt[2]] - Log[Cosh[x/2]] + Log[Sin
h[x/2]])
```

Maple [A] time = 0.036, size = 29, normalized size = 0.9

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}}{4}\left(2\,\tanh\left(x/2\right)-2\,i\right)\right) - i\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+tanh(x)),x)
```

```
[Out] 2^(1/2)*arctan(1/4*(2*tanh(1/2*x)-2*I)*2^(1/2))-I*ln(tanh(1/2*x))
```

Maxima [A] time = 1.56259, size = 46, normalized size = 1.39

$$-\sqrt{2}\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}e^{(-x)}\right) + i\,\log\left(e^{(-x)}+1\right) - i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+tanh(x)),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arctan((1/2*I + 1/2)*sqrt(2)*e^(-x)) + I*log(e^(-x) + 1) - I*log(e
^(-x) - 1)
```

Fricas [A] time = 2.32153, size = 180, normalized size = 5.45

$$-\frac{1}{2}i\sqrt{2}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}+e^x\right)+\frac{1}{2}i\sqrt{2}\log\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}+e^x\right)+i\log\left(e^x+1\right)-i\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(2)*log(-(1/2*I - 1/2)*sqrt(2) + e^x) + 1/2*I*sqrt(2)*log((1/2*I
 - 1/2)*sqrt(2) + e^x) + I*log(e^x + 1) - I*log(e^x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}(x)}{\tanh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+tanh(x)),x)
```

```
[Out] Integral(csch(x)/(tanh(x) + I), x)
```

Giac [A] time = 1.22636, size = 38, normalized size = 1.15

$$\sqrt{2}\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}e^x\right)+i\log\left(e^x+1\right)-i\log\left(|e^x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+tanh(x)),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(-(1/2*I - 1/2)*sqrt(2)*e^x) + I*log(e^x + 1) - I*log(abs(e^x
 - 1))
```

3.91 $\int \frac{\cosh^4(x)}{1+\tanh(x)} \, dx$

Optimal. Leaf size=60

$$\frac{5x}{16}+\frac{1}{8(1-\tanh(x))}-\frac{3}{16(\tanh(x)+1)}+\frac{1}{32(1-\tanh(x))^2}-\frac{3}{32(\tanh(x)+1)^2}-\frac{1}{24(\tanh(x)+1)^3}$$

```
[Out] (5*x)/16 + 1/(32*(1 - Tanh[x])^2) + 1/(8*(1 - Tanh[x])) - 1/(24*(1 + Tanh[x
])^3) - 3/(32*(1 + Tanh[x])^2) - 3/(16*(1 + Tanh[x]))
```

Rubi [A] time = 0.0611005, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3487, 44, 207}

$$\frac{5x}{16}+\frac{1}{8(1-\tanh(x))}-\frac{3}{16(\tanh(x)+1)}+\frac{1}{32(1-\tanh(x))^2}-\frac{3}{32(\tanh(x)+1)^2}-\frac{1}{24(\tanh(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (5*x)/16 + 1/(32*(1 - Tanh[x])^2) + 1/(8*(1 - Tanh[x])) - 1/(24*(1 + Tanh[x
])^3) - 3/(32*(1 + Tanh[x])^2) - 3/(16*(1 + Tanh[x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(x)}{1+\tanh(x)} \, dx &= \text{Subst}\left(\int \frac{1}{(1-x)^3(1+x)^4} \, dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}+\frac{1}{8(1+x)^4}+\frac{3}{16(1+x)^3}+\frac{3}{16(1+x)^2}-\frac{5}{16\left(-1+x^2\right)}\right)\right.\\
&= \frac{1}{32(1-\tanh(x))^2}+\frac{1}{8(1-\tanh(x))}-\frac{1}{24(1+\tanh(x))^3}-\frac{3}{32(1+\tanh(x))^2}-\frac{3}{16(1+\tanh(x))}\\
&= \frac{5x}{16}+\frac{1}{32(1-\tanh(x))^2}+\frac{1}{8(1-\tanh(x))}-\frac{1}{24(1+\tanh(x))^3}-\frac{3}{32(1+\tanh(x))^2}-\frac{3}{16(1+\tan}
\end{aligned}$$

Mathematica [A] time = 0.0448211, size = 42, normalized size = 0.7

$$\frac{1}{192}(60x + 45\sinh(2x) + 9\sinh(4x) + \sinh(6x) - 15\cosh(2x) - 6\cosh(4x) - \cosh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (60*x - 15*Cosh[2*x] - 6*Cosh[4*x] - Cosh[6*x] + 45*Sinh[2*x] + 9*Sinh[4*x]
 + Sinh[6*x])/192
```

Maple [B] time = 0.033, size = 116, normalized size = 1.9

$$-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-6}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5}-\frac{15}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}+\frac{25}{12}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{15}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(1+tanh(x)),x)
```

```
[Out] -1/3/(tanh(1/2*x)+1)^6+1/(tanh(1/2*x)+1)^5-15/8/(tanh(1/2*x)+1)^4+25/12/(ta
nh(1/2*x)+1)^3-15/8/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)+5/16*ln(tanh(1/2*x)
+1)+1/8/(tanh(1/2*x)-1)^4+1/4/(tanh(1/2*x)-1)^3+1/2/(tanh(1/2*x)-1)^2+3/8/(
tanh(1/2*x)-1)-5/16*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.11433, size = 49, normalized size = 0.82

$$\frac{1}{128}\left(10\,e^{(-2\,x)}+1\right)e^{(4\,x)}+\frac{5}{16}\,x-\frac{5}{32}\,e^{(-2\,x)}-\frac{5}{128}\,e^{(-4\,x)}-\frac{1}{192}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/128*(10*e^(-2*x) + 1)*e^(4*x) + 5/16*x - 5/32*e^(-2*x) - 5/128*e^(-4*x) -
 1/192*e^(-6*x)
```

Fricas [B] time = 2.19796, size = 323, normalized size = 5.38

$$\frac{\cosh(x)^5+5\,\cosh(x)\sinh(x)^4+5\,\sinh(x)^5+5\left(10\,\cosh(x)^2+9\right)\sinh(x)^3+15\,\cosh(x)^3+5\left(2\,\cosh(x)^3+9\,\cosh\right.}{384\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + 5*sinh(x)^5 + 5*(10*cosh(x)^2 + 9)
*sinh(x)^3 + 15*cosh(x)^3 + 5*(2*cosh(x)^3 + 9*cosh(x))*sinh(x)^2 + 60*(2*x
 - 1)*cosh(x) + 5*(5*cosh(x)^4 + 27*cosh(x)^2 + 24*x + 12)*sinh(x))/(cosh(x
) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^4(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(1+tanh(x)),x)
```

```
[Out] Integral(cosh(x)**4/(tanh(x) + 1), x)
```

Giac [A] time = 1.25069, size = 57, normalized size = 0.95

$$-\frac{1}{384}\left(110\,e^{(6\,x)}+60\,e^{(4\,x)}+15\,e^{(2\,x)}+2\right)e^{(-6\,x)}+\frac{5}{16}\,x+\frac{1}{128}\,e^{(4\,x)}+\frac{5}{64}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -1/384*(110*e^(6*x) + 60*e^(4*x) + 15*e^(2*x) + 2)*e^(-6*x) + 5/16*x + 1/12
8*e^(4*x) + 5/64*e^(2*x)
```

3.92 $\int \frac{\cosh^3(x)}{1+\tanh(x)} \, dx$

Optimal. Leaf size=29

$$\frac{4\sinh^3(x)}{15}+\frac{4\sinh(x)}{5}-\frac{\cosh^3(x)}{5(\tanh(x)+1)}$$

```
[Out] (4*Sinh[x])/5 + (4*Sinh[x]^3)/15 - Cosh[x]^3/(5*(1 + Tanh[x]))
```

Rubi [A] time = 0.0420266, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3502, 2633}

$$\frac{4\sinh^3(x)}{15}+\frac{4\sinh(x)}{5}-\frac{\cosh^3(x)}{5(\tanh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (4*Sinh[x])/5 + (4*Sinh[x]^3)/15 - Cosh[x]^3/(5*(1 + Tanh[x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{1+\tanh(x)} \, dx &= -\frac{\cosh^3(x)}{5(1+\tanh(x))}+\frac{4}{5}\int \cosh^3(x) \, dx \\
&= -\frac{\cosh^3(x)}{5(1+\tanh(x))}+\frac{4}{5} i \operatorname{Subst}\left(\int \left(1-x^2\right) \, dx, x, -i \sinh(x)\right) \\
&= \frac{4\sinh(x)}{5}+\frac{4\sinh^3(x)}{15}-\frac{\cosh^3(x)}{5(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] time = 0.0465145, size = 36, normalized size = 1.24

$$\frac{\operatorname{sech}(x)(40\sinh(2x)+4\sinh(4x)+20\cosh(2x)+\cosh(4x)-45)}{120(\tanh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (Sech[x]*(-45 + 20*Cosh[2*x] + Cosh[4*x] + 40*Sinh[2*x] + 4*Sinh[4*x]))/(12
0*(1 + Tanh[x]))
```

Maple [B] time = 0.032, size = 80, normalized size = 2.8

$$-\frac{2}{5}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}-\frac{5}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{3}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{11}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{6}\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1+tanh(x)),x)
```

```
[Out] -2/5/(tanh(1/2*x)+1)^5+1/(tanh(1/2*x)+1)^4-5/3/(tanh(1/2*x)+1)^3+3/2/(tanh(
1/2*x)+1)^2-11/8/(tanh(1/2*x)+1)-1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^
2-5/8/(tanh(1/2*x)-1)
```

Maxima [A] time = 1.06333, size = 45, normalized size = 1.55

$$\frac{1}{48}\left(12\,e^{(-2\,x)}+1\right)e^{(3\,x)}-\frac{3}{8}\,e^{(-x)}-\frac{1}{12}\,e^{(-3\,x)}-\frac{1}{80}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/48*(12*e^(-2*x) + 1)*e^(3*x) - 3/8*e^(-x) - 1/12*e^(-3*x) - 1/80*e^(-5*x)
```

Fricas [B] time = 2.06148, size = 221, normalized size = 7.62

$$\frac{\cosh(x)^4+16\,\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\,\cosh(x)^2+10\right)\sinh(x)^2+20\,\cosh(x)^2+16\left(\cosh(x)^3+5\,\mathrm{cosh}\right.}{120\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/120*(cosh(x)^4 + 16*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 10)*
sinh(x)^2 + 20*cosh(x)^2 + 16*(cosh(x)^3 + 5*cosh(x))*sinh(x) - 45)/(cosh(x
) + sinh(x))
```

Sympy [B] time = 1.59062, size = 134, normalized size = 4.62

$$-\frac{8\sinh^3(x)\tanh(x)}{15\tanh(x)+15}-\frac{2\sinh^3(x)}{15\tanh(x)+15}-\frac{6\sinh^2(x)\cosh(x)\tanh(x)}{15\tanh(x)+15}+\frac{6\sinh^2(x)\cosh(x)}{15\tanh(x)+15}+\frac{6\sinh(x)\cosh^2(x)}{15\tanh(x)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1+tanh(x)),x)
```

```
[Out] -8*sinh(x)**3*tanh(x)/(15*tanh(x) + 15) - 2*sinh(x)**3/(15*tanh(x) + 15) -
6*sinh(x)**2*cosh(x)*tanh(x)/(15*tanh(x) + 15) + 6*sinh(x)**2*cosh(x)/(15*t
anh(x) + 15) + 6*sinh(x)*cosh(x)**2*tanh(x)/(15*tanh(x) + 15) + 9*sinh(x)*c
osh(x)**2/(15*tanh(x) + 15) + 3*cosh(x)**3*tanh(x)/(15*tanh(x) + 15) - 3*co
sh(x)**3/(15*tanh(x) + 15)
```

Giac [A] time = 1.17814, size = 42, normalized size = 1.45

$$-\frac{1}{240}\left(90\,e^{(4\,x)}+20\,e^{(2\,x)}+3\right)e^{(-5\,x)}+\frac{1}{48}\,e^{(3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -1/240*(90*e^(4*x) + 20*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) + 1/4*e^x
```

3.93 $\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=38

$$\frac{3x}{8}+\frac{1}{8(1-\tanh(x))}-\frac{1}{4(\tanh(x)+1)}-\frac{1}{8(\tanh(x)+1)^2}$$

```
[Out] (3*x)/8 + 1/(8*(1 - Tanh[x])) - 1/(8*(1 + Tanh[x])^2) - 1/(4*(1 + Tanh[x]))
```

Rubi [A] time = 0.0492305, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3487, 44, 207}

$$\frac{3x}{8}+\frac{1}{8(1-\tanh(x))}-\frac{1}{4(\tanh(x)+1)}-\frac{1}{8(\tanh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 + Tanh[x]),x]
```

```
[Out] (3*x)/8 + 1/(8*(1 - Tanh[x])) - 1/(8*(1 + Tanh[x])^2) - 1/(4*(1 + Tanh[x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x)^2(1+x)^3}\,dx,x,\tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}+\frac{1}{4(1+x)^2}-\frac{3}{8\left(-1+x^2\right)}\right)dx,x,\tanh(x)\right)\\
&= \frac{1}{8(1-\tanh(x))}-\frac{1}{8(1+\tanh(x))^2}-\frac{1}{4(1+\tanh(x))}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\tanh(x)\right)\\
&= \frac{3x}{8}+\frac{1}{8(1-\tanh(x))}-\frac{1}{8(1+\tanh(x))^2}-\frac{1}{4(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] time = 0.0299549, size = 30, normalized size = 0.79

$$\frac{1}{32}(12x + 8\sinh(2x) + \sinh(4x) - 4\cosh(2x) - \cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 + Tanh[x]),x]
```

```
[Out] (12*x - 4*Cosh[2*x] - Cosh[4*x] + 8*Sinh[2*x] + Sinh[4*x])/32
```

Maple [B] time = 0.032, size = 76, normalized size = 2.

$$-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{3}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{3}{8}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{4}\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1+tanh(x)),x)
```

```
[Out] -1/2/(tanh(1/2*x)+1)^4+1/(tanh(1/2*x)+1)^3-3/2/(tanh(1/2*x)+1)^2+1/(tanh(1/
2*x)+1)+3/8*ln(tanh(1/2*x)+1)+1/4/(tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)-3/8
*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.01783, size = 30, normalized size = 0.79

$$\frac{3}{8}x + \frac{1}{16}e^{(2x)} - \frac{3}{16}e^{(-2x)} - \frac{1}{32}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/16*e^(2*x) - 3/16*e^(-2*x) - 1/32*e^(-4*x)
```

Fricas [A] time = 2.14854, size = 178, normalized size = 4.68

$$\frac{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + 3\sinh(x)^3 + 6(2x-1)\cosh(x) + 3\left(3\cosh(x)^2 + 4x + 2\right)\sinh(x)}{32(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*sinh(x)^3 + 6*(2*x - 1)*cosh(x) +
 3*(3*cosh(x)^2 + 4*x + 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1+tanh(x)),x)
```

```
[Out] Integral(cosh(x)**2/(tanh(x) + 1), x)
```

Giac [A] time = 1.20796, size = 41, normalized size = 1.08

$$-\frac{1}{32}\left(9e^{(4x)} + 6e^{(2x)} + 1\right)e^{(-4x)} + \frac{3}{8}x + \frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -1/32*(9*e^(4*x) + 6*e^(2*x) + 1)*e^(-4*x) + 3/8*x + 1/16*e^(2*x)
```

3.94 $\int \frac{\cosh(x)}{1+\tanh(x)} \, dx$

Optimal. Leaf size=19

$$\frac{2\sinh(x)}{3} - \frac{\cosh(x)}{3(\tanh(x)+1)}$$

```
[Out] (2*Sinh[x])/3 - Cosh[x]/(3*(1 + Tanh[x]))
```

Rubi [A] time = 0.0300766, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3502, 2637}

$$\frac{2\sinh(x)}{3} - \frac{\cosh(x)}{3(\tanh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(1 + Tanh[x]),x]

[Out] (2*Sinh[x])/3 - Cosh[x]/(3*(1 + Tanh[x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{1+\tanh(x)} \, dx &= -\frac{\cosh(x)}{3(1+\tanh(x))} + \frac{2}{3}\int \cosh(x) \, dx \\ &= \frac{2\sinh(x)}{3} - \frac{\cosh(x)}{3(1+\tanh(x))}\end{aligned}$$

Mathematica [A] time = 0.0268828, size = 23, normalized size = 1.21

$$\frac{1}{12}(9\sinh(x) + \sinh(3x) - 3\cosh(x) - \cosh(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(1 + Tanh[x]),x]

[Out] (-3*Cosh[x] - Cosh[3*x] + 9*Sinh[x] + Sinh[3*x])/12
```

Maple [B] time = 0.027, size = 40, normalized size = 2.1

$$-\frac{2}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{3}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1+tanh(x)),x)
```

```
[Out] -2/3/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2-3/2/(tanh(1/2*x)+1)-1/2/(tanh(1/
2*x)-1)
```

Maxima [A] time = 1.00966, size = 23, normalized size = 1.21

$$-\frac{1}{2}\,e^{(-x)}-\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x) - 1/12*e^(-3*x) + 1/4*e^x
```

Fricas [A] time = 2.21826, size = 99, normalized size = 5.21

$$\frac{\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-3}{6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(cosh(x)^2 + 4*cosh(x)*sinh(x) + sinh(x)^2 - 3)/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.435763, size = 48, normalized size = 2.53

$$\frac{2\sinh\left(x\right)\tanh\left(x\right)}{3\tanh\left(x\right)+3}+\frac{\sinh\left(x\right)}{3\tanh\left(x\right)+3}+\frac{\cosh\left(x\right)\tanh\left(x\right)}{3\tanh\left(x\right)+3}-\frac{\cosh\left(x\right)}{3\tanh\left(x\right)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+tanh(x)),x)
```

```
[Out] 2*sinh(x)*tanh(x)/(3*tanh(x) + 3) + sinh(x)/(3*tanh(x) + 3) + cosh(x)*tanh(
x)/(3*tanh(x) + 3) - cosh(x)/(3*tanh(x) + 3)
```

Giac [A] time = 1.25906, size = 26, normalized size = 1.37

$$-\frac{1}{12}\left(6\,e^{(2\,x)}+1\right)e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -1/12*(6*e^(2*x) + 1)*e^(-3*x) + 1/4*e^x
```

3.95 $\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=10

$$-\frac{\operatorname{sech}(x)}{\tanh(x)+1}$$

```
[Out] -(Sech[x]/(1 + Tanh[x]))
```

Rubi [A] time = 0.019921, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3488}

$$-\frac{\operatorname{sech}(x)}{\tanh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(1 + Tanh[x]),x]
```

```
[Out] -(Sech[x]/(1 + Tanh[x]))
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx=-\frac{\operatorname{sech}(x)}{1+\tanh(x)}$$

Mathematica [A] time = 0.0032273, size = 7, normalized size = 0.7

$$\sinh(x)-\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(1 + Tanh[x]),x]
```

```
[Out] -Cosh[x] + Sinh[x]
```

Maple [A] time = 0.003, size = 11, normalized size = 1.1

$$-\frac{\operatorname{sech}(x)}{1+\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(1+tanh(x)),x)

[Out] -sech(x)/(1+tanh(x))
```

Maxima [A] time = 1.17869, size = 8, normalized size = 0.8

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+tanh(x)),x, algorithm="maxima")

[Out] -e^(-x)
```

Fricas [A] time = 2.23128, size = 32, normalized size = 3.2

$$-\frac{1}{\cosh(x) + \sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+tanh(x)),x, algorithm="fricas")

[Out] -1/(cosh(x) + sinh(x))
```

Sympy [A] time = 0.381505, size = 8, normalized size = 0.8

$$-\frac{\operatorname{sech}(x)}{\tanh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+tanh(x)),x)

[Out] -sech(x)/(tanh(x) + 1)
```

Giac [A] time = 1.19, size = 8, normalized size = 0.8

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+tanh(x)),x, algorithm="giac")

[Out] -e^(-x)
```

3.96 $\int \frac{\operatorname{sech}^2(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=5

$$\log(\tanh(x)+1)$$

```
[Out] Log[1 + Tanh[x]]
```

Rubi [A] time = 0.0334008, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3487, 31}

$$\log(\tanh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(1 + Tanh[x]),x]

[Out] Log[1 + Tanh[x]]
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^2(x)}{1+\tanh(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tanh(x)\right)\\ &= \log(1+\tanh(x)) \end{aligned}$$

Mathematica [A] time = 0.0038613, size = 7, normalized size = 1.4

$$x-\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(1 + Tanh[x]),x]

[Out] x - Log[Cosh[x]]
```

Maple [A] time = 0.018, size = 6, normalized size = 1.2

$$\ln(1+\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1+tanh(x)),x)
```

```
[Out] ln(1+tanh(x))
```

Maxima [A] time = 1.16682, size = 7, normalized size = 1.4

$$\log\left(\tanh\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] log(tanh(x) + 1)
```

Fricas [B] time = 2.31621, size = 57, normalized size = 11.4

$$2\,x-\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 2*x - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{2}\left(x\right)}{\tanh\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1+tanh(x)),x)
```

```
[Out] Integral(sech(x)**2/(tanh(x) + 1), x)
```

Giac [B] time = 1.26671, size = 18, normalized size = 3.6

$$2\,x-\log\left(e^{\left(2\,x\right)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 2*x - log(e^(2*x) + 1)
```

3.97 $\int \frac{\text{sech}^3(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=6

$$\text{sech}(x)+\tan^{-1}(\sinh(x))$$

```
[Out] ArcTan[Sinh[x]] + Sech[x]
```

Rubi [A] time = 0.035025, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3501, 3770}

$$\text{sech}(x)+\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(1 + Tanh[x]),x]
```

```
[Out] ArcTan[Sinh[x]] + Sech[x]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^3(x)}{1+\tanh(x)}\,dx &= \text{sech}(x)+\int \text{sech}(x)\,dx\\ &= \tan^{-1}(\sinh(x))+\text{sech}(x)\end{aligned}$$

Mathematica [A] time = 0.0245494, size = 12, normalized size = 2.

$$\text{sech}(x)+2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(1 + Tanh[x]),x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] + Sech[x]
```

Maple [B] time = 0.022, size = 21, normalized size = 3.5

$$2\left((\tanh(x/2))^2+1\right)^{-1}+2\arctan(\tanh(x/2))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(1+tanh(x)),x)
```

```
[Out] 2/(tanh(1/2*x)^2+1)+2*arctan(tanh(1/2*x))
```

Maxima [B] time = 1.58321, size = 30, normalized size = 5.

$$\frac{2\,e^{(-x)}}{e^{(-2\,x)}+1}-2\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(e^(-2*x) + 1) - 2*arctan(e^(-x))
```

Fricas [B] time = 2.08175, size = 198, normalized size = 33.

$$\frac{2\left(\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\arctan(\cosh(x)+\sinh(x))+\cosh(x)+\sinh(x)\right)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(cosh(x) + sinh(x)
) + cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^3(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(1+tanh(x)),x)
```

```
[Out] Integral(sech(x)**3/(tanh(x) + 1), x)
```

Giac [B] time = 1.22233, size = 24, normalized size = 4.

$$\frac{2\,e^x}{e^{(2\,x)}+1}+2\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 2*e^x/(e^(2*x) + 1) + 2*arctan(e^x)
```

3.98 $\int \frac{\text{sech}^4(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=11

$$\tanh(x)-\frac{\tanh^2(x)}{2}$$

```
[Out] Tanh[x] - Tanh[x]^2/2
```

Rubi [A] time = 0.032956, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3487}

$$\tanh(x)-\frac{\tanh^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(1 + Tanh[x]),x]
```

```
[Out] Tanh[x] - Tanh[x]^2/2
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^4(x)}{1+\tanh(x)}\,dx &= \text{Subst}\left(\int (1-x)\,dx,x,\tanh(x)\right)\\ &= \tanh(x)-\frac{\tanh^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0236339, size = 11, normalized size = 1.

$$\tanh(x)+\frac{\text{sech}^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(1 + Tanh[x]),x]
```

```
[Out] Sech[x]^2/2 + Tanh[x]
```

Maple [B] time = 0.025, size = 34, normalized size = 3.1

$$-2\,\frac{-(\tanh(x/2))^3+(\tanh(x/2))^2-\tanh(x/2)}{\left((\tanh(x/2))^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(1+tanh(x)),x)
```

```
[Out] -2*(-tanh(1/2*x)^3+tanh(1/2*x)^2-tanh(1/2*x))/(tanh(1/2*x)^2+1)^2
```

Maxima [B] time = 1.1833, size = 50, normalized size = 4.55

$$\frac{4\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+\frac{2}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 4*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2/(2*e^(-2*x) + e^(-4*x) + 1)
```

Fricas [B] time = 2.02179, size = 181, normalized size = 16.45

$$-\frac{2}{\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\,\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -2/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(
x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{4}\left(x\right)}{\tanh\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(1+tanh(x)),x)
```

```
[Out] Integral(sech(x)**4/(tanh(x) + 1), x)
```

Giac [A] time = 1.2545, size = 14, normalized size = 1.27

$$-\frac{2}{\left(e^{(2\,x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -2/(e^(2*x) + 1)^2
```

3.99 $\int \frac{\operatorname{sech}^5(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=24

$$\frac{\operatorname{sech}^3(x)}{3}+\frac{1}{2}\tan^{-1}(\sinh(x))+\frac{1}{2}\tanh(x)\operatorname{sech}(x)$$

```
[Out] ArcTan[Sinh[x]]/2 + Sech[x]^3/3 + (Sech[x]*Tanh[x])/2
```

Rubi [A] time = 0.0422554, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3501, 3768, 3770}

$$\frac{\operatorname{sech}^3(x)}{3}+\frac{1}{2}\tan^{-1}(\sinh(x))+\frac{1}{2}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(1 + Tanh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/2 + Sech[x]^3/3 + (Sech[x]*Tanh[x])/2
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^5(x)}{1+\tanh(x)}\,dx &= \frac{\operatorname{sech}^3(x)}{3}+\int \operatorname{sech}^3(x)\,dx\\ &= \frac{\operatorname{sech}^3(x)}{3}+\frac{1}{2}\operatorname{sech}(x)\tanh(x)+\frac{1}{2}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{2}\tan^{-1}(\sinh(x))+\frac{\operatorname{sech}^3(x)}{3}+\frac{1}{2}\operatorname{sech}(x)\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0270483, size = 24, normalized size = 1.

$$\frac{\operatorname{sech}^3(x)}{3}+\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(1 + Tanh[x]),x]
```

```
[Out] ArcTan[Tanh[x/2]] + Sech[x]^3/3 + (Sech[x]*Tanh[x])/2
```

Maple [B] time = 0.026, size = 41, normalized size = 1.7

$$2\,\frac{-1/2\,(\tanh(x/2))^5+(\tanh(x/2))^4+1/2\,\tanh(x/2)+1/3}{\left((\tanh(x/2))^2+1\right)^3}+\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(1+tanh(x)),x)
```

```
[Out] 2*(-1/2*tanh(1/2*x)^5+tanh(1/2*x)^4+1/2*tanh(1/2*x)+1/3)/(tanh(1/2*x)^2+1)^
3+arctan(tanh(1/2*x))
```

Maxima [B] time = 1.63739, size = 66, normalized size = 2.75

$$\frac{3\,e^{(-x)}+8\,e^{(-3\,x)}-3\,e^{(-5\,x)}}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}-\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*e^(-x) + 8*e^(-3*x) - 3*e^(-5*x))/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x
) + 1) - arctan(e^(-x))
```

Fricas [B] time = 2.21472, size = 968, normalized size = 40.33

$$\frac{3\,\cosh(x)^5+15\,\cosh(x)\sinh(x)^4+3\,\sinh(x)^5+2\left(15\,\cosh(x)^2+4\right)\sinh(x)^3+8\,\cosh(x)^3+6\left(5\,\cosh(x)^3+4\,\cdots\right.}{3\left(\mathrm{co}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*cosh(x)^5 + 15*cosh(x)*sinh(x)^4 + 3*sinh(x)^5 + 2*(15*cosh(x)^2 + 4
)*sinh(x)^3 + 8*cosh(x)^3 + 6*(5*cosh(x)^3 + 4*cosh(x))*sinh(x)^2 + 3*(cosh
(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3
*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cos
h(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))
*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(5*cosh(x)^4 + 8*cosh(x)^2 - 1)
```

```
*sinh(x) - 3*cosh(x))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*c
osh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)
^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)
^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^5(x)}{\tanh(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(1+tanh(x)),x)
```

```
[Out] Integral(sech(x)**5/(tanh(x) + 1), x)
```

Giac [A] time = 1.22936, size = 42, normalized size = 1.75

$$\frac{3\, e^{(5\, x)} + 8\, e^{(3\, x)} - 3\, e^{x}}{3\, \left(e^{(2\, x)} + 1\right)^{3}} + \arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/3*(3*e^(5*x) + 8*e^(3*x) - 3*e^x)/(e^(2*x) + 1)^3 + arctan(e^x)
```

3.100 $\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=25

$$\frac{1}{4}(1-\tanh(x))^4-\frac{2}{3}(1-\tanh(x))^3$$

```
[Out] (-2*(1 - Tanh[x])^3)/3 + (1 - Tanh[x])^4/4
```

Rubi [A] time = 0.0405961, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3487, 43}

$$\frac{1}{4}(1-\tanh(x))^4-\frac{2}{3}(1-\tanh(x))^3$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^6/(1 + Tanh[x]),x]
```

```
[Out] (-2*(1 - Tanh[x])^3)/3 + (1 - Tanh[x])^4/4
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx &= \operatorname{Subst}\left(\int (1-x)^2(1+x)\,dx, x, \tanh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(2(1-x)^2-(1-x)^3\right)\,dx, x, \tanh(x)\right)\\
&= -\frac{2}{3}(1-\tanh(x))^3+\frac{1}{4}(1-\tanh(x))^4
\end{aligned}$$

Mathematica [A] time = 0.0298958, size = 20, normalized size = 0.8

$$\frac{1}{12}(4\sinh(2x)+\sinh(4x)+3)\operatorname{sech}^4(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^6/(1 + Tanh[x]),x]
```

```
[Out] (Sech[x]^4*(3 + 4*Sinh[2*x] + Sinh[4*x]))/12
```

Maple [B] time = 0.024, size = 56, normalized size = 2.2

$$-2\,\frac{-\left(\tanh\left(x/2\right)\right)^{7}+\left(\tanh\left(x/2\right)\right)^{6}-5/3\,\left(\tanh\left(x/2\right)\right)^{5}-5/3\,\left(\tanh\left(x/2\right)\right)^{3}+\left(\tanh\left(x/2\right)\right)^{2}-\tanh\left(x/2\right)}{\left(\left(\tanh\left(x/2\right)\right)^{2}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^6/(1+tanh(x)),x)
```

```
[Out] -2*(-tanh(1/2*x)^7+tanh(1/2*x)^6-5/3*tanh(1/2*x)^5-5/3*tanh(1/2*x)^3+tanh(1
/2*x)^2-tanh(1/2*x))/(tanh(1/2*x)^2+1)^4
```

Maxima [B] time = 1.11806, size = 126, normalized size = 5.04

$$\frac{16\,e^{(-2\,x)}}{3\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)}+\frac{8\,e^{(-4\,x)}}{4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1}+\frac{4}{3\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 8*e^(
-4*x)/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 4/3/(4*e^(-2*
x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1)
```

Fricas [B] time = 2.15348, size = 467, normalized size = 18.68

$$-\frac{}{3\left(\cosh\left(x\right)^{7}+7\,\cosh\left(x\right)\sinh\left(x\right)^{6}+\sinh\left(x\right)^{7}+\left(21\,\cosh\left(x\right)^{2}+4\right)\sinh\left(x\right)^{5}+4\,\cosh\left(x\right)^{5}+5\left(7\,\cosh\left(x\right)^{3}+4\,\cosh\left(\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(5*cosh(x) + 3*sinh(x))/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 +
 (21*cosh(x)^2 + 4)*sinh(x)^5 + 4*cosh(x)^5 + 5*(7*cosh(x)^3 + 4*cosh(x))*s
inh(x)^4 + (35*cosh(x)^4 + 40*cosh(x)^2 + 6)*sinh(x)^3 + 6*cosh(x)^3 + (21*
cosh(x)^5 + 40*cosh(x)^3 + 18*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 + 20*cosh(x
)^4 + 18*cosh(x)^2 + 3)*sinh(x) + 5*cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{6}\left(x\right)}{\tanh\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**6/(1+tanh(x)),x)
```

```
[Out] Integral(sech(x)**6/(tanh(x) + 1), x)
```

Giac [A] time = 1.20393, size = 24, normalized size = 0.96

$$-\frac{4\left(4\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -4/3*(4*e^(2*x) + 1)/(e^(2*x) + 1)^4
```

3.101 $\int \frac{\text{sech}^7(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=34

$$\frac{\text{sech}^5(x)}{5}+\frac{3}{8}\tan^{-1}(\sinh(x))+\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{8}\tanh(x)\text{sech}(x)$$

```
[Out] (3*ArcTan[Sinh[x]])/8 + Sech[x]^5/5 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Ta
nh[x])/4
```

Rubi [A] time = 0.0483828, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3501, 3768, 3770}

$$\frac{\text{sech}^5(x)}{5}+\frac{3}{8}\tan^{-1}(\sinh(x))+\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{8}\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^7/(1 + Tanh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 + Sech[x]^5/5 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Ta
nh[x])/4
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^7(x)}{1+\tanh(x)}\,dx &= \frac{\operatorname{sech}^5(x)}{5} + \int \operatorname{sech}^5(x)\,dx \\
&= \frac{\operatorname{sech}^5(x)}{5} + \frac{1}{4}\operatorname{sech}^3(x)\tanh(x) + \frac{3}{4}\int \operatorname{sech}^3(x)\,dx \\
&= \frac{\operatorname{sech}^5(x)}{5} + \frac{3}{8}\operatorname{sech}(x)\tanh(x) + \frac{1}{4}\operatorname{sech}^3(x)\tanh(x) + \frac{3}{8}\int \operatorname{sech}(x)\,dx \\
&= \frac{3}{8}\tan^{-1}(\sinh(x)) + \frac{\operatorname{sech}^5(x)}{5} + \frac{3}{8}\operatorname{sech}(x)\tanh(x) + \frac{1}{4}\operatorname{sech}^3(x)\tanh(x)
\end{aligned}
$$

Mathematica [A] time = 0.0326179, size = 34, normalized size = 1.

$$\frac{1}{40}\left(8\operatorname{sech}^5(x) + 30\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + 10\tanh(x)\operatorname{sech}^3(x) + 15\tanh(x)\operatorname{sech}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^7/(1 + Tanh[x]),x]
```

```
[Out] (30*ArcTan[Tanh[x/2]] + 8*Sech[x]^5 + 15*Sech[x]*Tanh[x] + 10*Sech[x]^3*Tan
h[x])/40
```

Maple [B] time = 0.028, size = 67, normalized size = 2.

$$2\,\frac{-5/8\,(\tanh(x/2))^9 + (\tanh(x/2))^8 - 1/4\,(\tanh(x/2))^7 + 2\,(\tanh(x/2))^4 + 1/4\,(\tanh(x/2))^3 + 5/8\,\tanh(x/2) + 1/5}{\left((\tanh(x/2))^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^7/(1+tanh(x)),x)
```

```
[Out] 2*(-5/8*tanh(1/2*x)^9+tanh(1/2*x)^8-1/4*tanh(1/2*x)^7+2*tanh(1/2*x)^4+1/4*t
anh(1/2*x)^3+5/8*tanh(1/2*x)+1/5)/(tanh(1/2*x)^2+1)^5+3/4*arctan(tanh(1/2*x
))
```

Maxima [B] time = 1.74522, size = 99, normalized size = 2.91

$$\frac{15\,e^{(-x)} + 70\,e^{(-3\,x)} + 128\,e^{(-5\,x)} - 70\,e^{(-7\,x)} - 15\,e^{(-9\,x)}}{20\left(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1\right)} - \frac{3}{4}\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/20*(15*e^(-x) + 70*e^(-3*x) + 128*e^(-5*x) - 70*e^(-7*x) - 15*e^(-9*x))/(
5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) - 3/4*
arctan(e^(-x))
```

Fricas [B] time = 2.15577, size = 2298, normalized size = 67.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/20*(15*cosh(x)^9 + 135*cosh(x)*sinh(x)^8 + 15*sinh(x)^9 + 10*(54*cosh(x)^
2 + 7)*sinh(x)^7 + 70*cosh(x)^7 + 70*(18*cosh(x)^3 + 7*cosh(x))*sinh(x)^6 +
 2*(945*cosh(x)^4 + 735*cosh(x)^2 + 64)*sinh(x)^5 + 128*cosh(x)^5 + 10*(189
*cosh(x)^5 + 245*cosh(x)^3 + 64*cosh(x))*sinh(x)^4 + 10*(126*cosh(x)^6 + 24
5*cosh(x)^4 + 128*cosh(x)^2 - 7)*sinh(x)^3 - 70*cosh(x)^3 + 10*(54*cosh(x)^
7 + 147*cosh(x)^5 + 128*cosh(x)^3 - 21*cosh(x))*sinh(x)^2 + 15*(cosh(x)^10
+ 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 1)*sinh(x)^8 + 5*cos
h(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*sinh(x)^7 + 10*(21*cosh(x)^4 + 14*cosh(
x)^2 + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*cosh(x)^5 + 70*cosh(x)^3 + 15*co
sh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh
(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^5 + 5*cosh(x)^3 + cosh(x
))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 12*cosh(x)^2
+ 1)*sinh(x)^2 + 5*cosh(x)^2 + 10*(cosh(x)^9 + 4*cosh(x)^7 + 6*cosh(x)^5 +
4*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*(27*cosh(
x)^8 + 98*cosh(x)^6 + 128*cosh(x)^4 - 42*cosh(x)^2 - 3)*sinh(x) - 15*cosh(x
))/(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 1)*si
nh(x)^8 + 5*cosh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*sinh(x)^7 + 10*(21*cosh(
x)^4 + 14*cosh(x)^2 + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*cosh(x)^5 + 70*co
sh(x)^3 + 15*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 + 15*cosh
(x)^2 + 1)*sinh(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^5 + 5*cos
h(x)^3 + cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4
+ 12*cosh(x)^2 + 1)*sinh(x)^2 + 5*cosh(x)^2 + 10*(cosh(x)^9 + 4*cosh(x)^7 +
 6*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^7(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**7/(1+tanh(x)),x)
```

```
[Out] Integral(sech(x)**7/(tanh(x) + 1), x)
```

Giac [A] time = 1.22723, size = 61, normalized size = 1.79

$$\frac{15\,e^{(9\,x)}+70\,e^{(7\,x)}+128\,e^{(5\,x)}-70\,e^{(3\,x)}-15\,e^{x}}{20\,\left(e^{(2\,x)}+1\right)^{5}}+\frac{3}{4}\,\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/20*(15*e^(9*x) + 70*e^(7*x) + 128*e^(5*x) - 70*e^(3*x) - 15*e^x)/(e^(2*x)
 + 1)^5 + 3/4*arctan(e^x)
```

3.102 $\int \frac{\text{sech}^8(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=140

$$-\frac{\left(a^2-3b^2\right)\tanh^4(x)}{4b^3}+\frac{a\left(a^2-3b^2\right)\tanh^3(x)}{3b^4}-\frac{\left(-3a^2b^2+a^4+3b^4\right)\tanh^2(x)}{2b^5}+\frac{a\left(-3a^2b^2+a^4+3b^4\right)\tanh(x)}{b^6}-\frac{(a}{}$$

```
[Out] -(((a^2 - b^2)^3*Log[a + b*Tanh[x]])/b^7) + (a*(a^4 - 3*a^2*b^2 + 3*b^4)*Ta
nh[x])/b^6 - ((a^4 - 3*a^2*b^2 + 3*b^4)*Tanh[x]^2)/(2*b^5) + (a*(a^2 - 3*b^
2)*Tanh[x]^3)/(3*b^4) - ((a^2 - 3*b^2)*Tanh[x]^4)/(4*b^3) + (a*Tanh[x]^5)/(
5*b^2) - Tanh[x]^6/(6*b)
```

Rubi [A] time = 0.158316, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 697}

$$-\frac{\left(a^2-3b^2\right)\tanh^4(x)}{4b^3}+\frac{a\left(a^2-3b^2\right)\tanh^3(x)}{3b^4}-\frac{\left(-3a^2b^2+a^4+3b^4\right)\tanh^2(x)}{2b^5}+\frac{a\left(-3a^2b^2+a^4+3b^4\right)\tanh(x)}{b^6}-\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^8/(a + b*Tanh[x]),x]
```

```
[Out] -(((a^2 - b^2)^3*Log[a + b*Tanh[x]])/b^7) + (a*(a^4 - 3*a^2*b^2 + 3*b^4)*Ta
nh[x])/b^6 - ((a^4 - 3*a^2*b^2 + 3*b^4)*Tanh[x]^2)/(2*b^5) + (a*(a^2 - 3*b^
2)*Tanh[x]^3)/(3*b^4) - ((a^2 - 3*b^2)*Tanh[x]^4)/(4*b^3) + (a*Tanh[x]^5)/(
5*b^2) - Tanh[x]^6/(6*b)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^8(x)}{a+b\tanh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{b^2}\right)^3}{a+x}\,dx,x,b\tanh(x)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^5-3a^3b^2+3ab^4}{b^6}-\frac{\left(a^4-3a^2b^2+3b^4\right)x}{b^6}+\frac{a\left(a^2-3b^2\right)x^2}{b^6}+\frac{\left(-a^2+3b^2\right)x^3}{b^6}+\frac{ax^4}{b^6}-\frac{x^5}{b^6}+\frac{\left(-a^2+b^2\right)^3}{b^6(a+x)}\right)dx,x\right.}{b}\\ &= -\frac{\left(a^2-b^2\right)^3\log(a+b\tanh(x))}{b^7}+\frac{a\left(a^4-3a^2b^2+3b^4\right)\tanh(x)}{b^6}-\frac{\left(a^4-3a^2b^2+3b^4\right)\tanh^2(x)}{2b^5}\end{aligned}$$

Mathematica [A] time = 0.550502, size = 137, normalized size = 0.98

$$\frac{4ab\left(-40a^2b^2+15a^4+33b^4\right)\tanh(x)+3b^4\text{sech}^4(x)\left(-5a^2+4ab\tanh(x)+5b^2\right)+2b^2\text{sech}^2(x)\left(15\left(a^2-b^2\right)^2-2ab\left(5a\right.\right.}{60b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^8/(a + b*Tanh[x]),x]
```

```
[Out] (60*(a^2 - b^2)^3*(Log[Cosh[x]] - Log[a*Cosh[x] + b*Sinh[x]]) + 10*b^6*Sech
[x]^6 + 4*a*b*(15*a^4 - 40*a^2*b^2 + 33*b^4)*Tanh[x] + 3*b^4*Sech[x]^4*(-5*
a^2 + 5*b^2 + 4*a*b*Tanh[x]) + 2*b^2*Sech[x]^2*(15*(a^2 - b^2)^2 - 2*a*b*(5
*a^2 - 9*b^2)*Tanh[x]))/(60*b^7)
```

Maple [B] time = 0.056, size = 925, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^8/(a+b*tanh(x)),x)
```

```
[Out] 20/b^6/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^7*a^5-52/b^4/(tanh(1/2*x)^2+1)^6*tan
h(1/2*x)^7*a^3+212/5/b^2/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^7*a-12/b^5/(tanh(1
/2*x)^2+1)^6*tanh(1/2*x)^6*a^4+28/b^3/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^6*a^2
+20/b^6/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^5*a^5-52/b^4/(tanh(1/2*x)^2+1)^6*ta
nh(1/2*x)^5*a^3+212/5/b^2/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^5*a+10/b^6/(tanh(
1/2*x)^2+1)^6*tanh(1/2*x)^3*a^5-82/3/b^4/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^3*
a^3+22/b^2/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^3*a+2/b^6/(tanh(1/2*x)^2+1)^6*ta
nh(1/2*x)^11*a^5-6/b^4/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^11*a^3+6/b^2/(tanh(1
/2*x)^2+1)^6*tanh(1/2*x)^11*a-2/b^5/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^10*a^4+
6/b^3/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^10*a^2-8/b^5/(tanh(1/2*x)^2+1)^6*tanh
(1/2*x)^8*a^4-3/b^5*ln(tanh(1/2*x)^2+1)*a^4+1/b^7*ln(tanh(1/2*x)^2+1)*a^6-1
/b^7*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)*a^6+3/b^5*ln(a*tanh(1/2*x)^2+2*t
anh(1/2*x)*b+a)*a^4-3/b^3*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)*a^2-68/3/b/
(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^6-6/b/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^10-12
/b/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^8-12/b/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^4
-6/b/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^2+3/b^3*ln(tanh(1/2*x)^2+1)*a^2+20/b^3
/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^8*a^2-8/b^5/(tanh(1/2*x)^2+1)^6*tanh(1/2*x
)^4*a^4+20/b^3/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^4*a^2-2/b^5/(tanh(1/2*x)^2+1
)^6*tanh(1/2*x)^2*a^4+6/b^3/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^2*a^2+2/b^6/(ta
nh(1/2*x)^2+1)^6*tanh(1/2*x)*a^5-6/b^4/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)*a^3+
6/b^2/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)*a+10/b^6/(tanh(1/2*x)^2+1)^6*tanh(1/2
*x)^9*a^5-82/3/b^4/(tanh(1/2*x)^2+1)^6*tanh(1/2*x)^9*a^3+22/b^2/(tanh(1/2*x
)^2+1)^6*tanh(1/2*x)^9*a+1/b*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)-1/b*ln(t
anh(1/2*x)^2+1)
```

Maxima [B] time = 1.73043, size = 521, normalized size = 3.72

$$\frac{2\left(15\,a^5-40\,a^3b^2+33\,ab^4+3\left(25\,a^5+5\,a^4b-70\,a^3b^2-10\,a^2b^3+61\,ab^4+5\,b^5\right)e^{(-2\,x)}+30\left(5\,a^5+2\,a^4b-14\,a^3b^2-5\,a^2\right.\right.}{15\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^8/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*a^5 - 40*a^3*b^2 + 33*a*b^4 + 3*(25*a^5 + 5*a^4*b - 70*a^3*b^2 - 1
0*a^2*b^3 + 61*a*b^4 + 5*b^5)*e^(-2*x) + 30*(5*a^5 + 2*a^4*b - 14*a^3*b^2 -
 5*a^2*b^3 + 13*a*b^4 + 3*b^5)*e^(-4*x) + 10*(15*a^5 + 9*a^4*b - 40*a^3*b^2
 - 24*a^2*b^3 + 33*a*b^4 + 23*b^5)*e^(-6*x) + 15*(5*a^5 + 4*a^4*b - 12*a^3*
b^2 - 10*a^2*b^3 + 7*a*b^4 + 6*b^5)*e^(-8*x) + 15*(a^5 + a^4*b - 2*a^3*b^2
- 2*a^2*b^3 + a*b^4 + b^5)*e^(-10*x))/(6*b^6*e^(-2*x) + 15*b^6*e^(-4*x) + 2
0*b^6*e^(-6*x) + 15*b^6*e^(-8*x) + 6*b^6*e^(-10*x) + b^6*e^(-12*x) + b^6) -
 (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(-(a - b)*e^(-2*x) - a - b)/b^7 + (
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(e^(-2*x) + 1)/b^7
```

Fricas [B] time = 3.1661, size = 12186, normalized size = 87.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^8/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(30*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*cosh(x)^1
0 + 300*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*cosh(x)*sin
h(x)^9 + 30*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*sinh(x)
^10 + 30*(5*a^5*b - 4*a^4*b^2 - 12*a^3*b^3 + 10*a^2*b^4 + 7*a*b^5 - 6*b^6)*
cosh(x)^8 + 30*(5*a^5*b - 4*a^4*b^2 - 12*a^3*b^3 + 10*a^2*b^4 + 7*a*b^5 - 6
*b^6 + 45*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*cosh(x)^2
)*sinh(x)^8 + 240*(15*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^
6)*cosh(x)^3 + (5*a^5*b - 4*a^4*b^2 - 12*a^3*b^3 + 10*a^2*b^4 + 7*a*b^5 - 6
*b^6)*cosh(x))*sinh(x)^7 + 20*(15*a^5*b - 9*a^4*b^2 - 40*a^3*b^3 + 24*a^2*b
^4 + 33*a*b^5 - 23*b^6)*cosh(x)^6 + 20*(15*a^5*b - 9*a^4*b^2 - 40*a^3*b^3 +
 24*a^2*b^4 + 33*a*b^5 - 23*b^6 + 315*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*
b^4 + a*b^5 - b^6)*cosh(x)^4 + 42*(5*a^5*b - 4*a^4*b^2 - 12*a^3*b^3 + 10*a^
2*b^4 + 7*a*b^5 - 6*b^6)*cosh(x)^2)*sinh(x)^6 + 30*a^5*b - 80*a^3*b^3 + 66*
a*b^5 + 120*(63*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*cos
h(x)^5 + 14*(5*a^5*b - 4*a^4*b^2 - 12*a^3*b^3 + 10*a^2*b^4 + 7*a*b^5 - 6*b^
6)*cosh(x)^3 + (15*a^5*b - 9*a^4*b^2 - 40*a^3*b^3 + 24*a^2*b^4 + 33*a*b^5 -
 23*b^6)*cosh(x))*sinh(x)^5 + 60*(5*a^5*b - 2*a^4*b^2 - 14*a^3*b^3 + 5*a^2*
b^4 + 13*a*b^5 - 3*b^6)*cosh(x)^4 + 60*(105*(a^5*b - a^4*b^2 - 2*a^3*b^3 +
2*a^2*b^4 + a*b^5 - b^6)*cosh(x)^6 + 5*a^5*b - 2*a^4*b^2 - 14*a^3*b^3 + 5*a
^2*b^4 + 13*a*b^5 - 3*b^6 + 35*(5*a^5*b - 4*a^4*b^2 - 12*a^3*b^3 + 10*a^2*b
^4 + 7*a*b^5 - 6*b^6)*cosh(x)^4 + 5*(15*a^5*b - 9*a^4*b^2 - 40*a^3*b^3 + 24
*a^2*b^4 + 33*a*b^5 - 23*b^6)*cosh(x)^2)*sinh(x)^4 + 80*(45*(a^5*b - a^4*b^
2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*cosh(x)^7 + 21*(5*a^5*b - 4*a^4*b^
2 - 12*a^3*b^3 + 10*a^2*b^4 + 7*a*b^5 - 6*b^6)*cosh(x)^5 + 5*(15*a^5*b - 9*
a^4*b^2 - 40*a^3*b^3 + 24*a^2*b^4 + 33*a*b^5 - 23*b^6)*cosh(x)^3 + 3*(5*a^5
*b - 2*a^4*b^2 - 14*a^3*b^3 + 5*a^2*b^4 + 13*a*b^5 - 3*b^6)*cosh(x))*sinh(x
)^3 + 6*(25*a^5*b - 5*a^4*b^2 - 70*a^3*b^3 + 10*a^2*b^4 + 61*a*b^5 - 5*b^6)
*cosh(x)^2 + 6*(225*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)
*cosh(x)^8 + 140*(5*a^5*b - 4*a^4*b^2 - 12*a^3*b^3 + 10*a^2*b^4 + 7*a*b^5 -
 6*b^6)*cosh(x)^6 + 25*a^5*b - 5*a^4*b^2 - 70*a^3*b^3 + 10*a^2*b^4 + 61*a*b
^5 - 5*b^6 + 50*(15*a^5*b - 9*a^4*b^2 - 40*a^3*b^3 + 24*a^2*b^4 + 33*a*b^5
- 23*b^6)*cosh(x)^4 + 60*(5*a^5*b - 2*a^4*b^2 - 14*a^3*b^3 + 5*a^2*b^4 + 13
*a*b^5 - 3*b^6)*cosh(x)^2)*sinh(x)^2 + 15*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^12 + 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^11
+ (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^12 + 6*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*cosh(x)^10 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 11*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^10 + 20*(11*(a^6 - 3*a^4*b
```

^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
h(x))*sinh(x)^9 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 15*(a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 33*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)^4 + 18*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^8 + 24*
(33*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 30*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^3 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*
sinh(x)^7 + 20*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 4*(231*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 5*a^6 - 15*a^4*b^2 + 15*a^2*b^4
- 5*b^6 + 315*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 105*(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6 + 24*(33*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 63*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*cosh(x)^3 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^5
+ 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 15*(33*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 84*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
h(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x)^4 + 20*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh
(x)^4 + 20*(11*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^9 + 36*(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 42*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)^5 + 20*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 6*(a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^2 + 6*(11*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^
10 + 45*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 70*(a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*cosh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 50*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x)^2)*sinh(x)^2 + 12*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh
(x)^11 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^9 + 10*(a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^5 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh
(x) - sinh(x))) - 15*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^12 + 12*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^11 + (a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*sinh(x)^12 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)
^10 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 11*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x)^2)*sinh(x)^10 + 20*(11*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
cosh(x)^3 + 3(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^9 + 15*
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 15*(a^6 - 3*a^4*b^2 + 3*a^2
b^4 - b^6 + 33(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 18*(a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^8 + 24*(33*(a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*cosh(x)^5 + 30*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(
x)^3 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^7 + 20*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 4*(231*(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*cosh(x)^6 + 5*a^6 - 15*a^4*b^2 + 15*a^2*b^4 - 5*b^6 + 315*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 105*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*cosh(x)^2)*sinh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 24*(33*(a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 63*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x)^5 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 5*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^5 + 15*(a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*cosh(x)^4 + 15*(33*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
cosh(x)^8 + 84*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 +
20*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 20*(11*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^9 + 36*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*cosh(x)^7 + 42*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 20*(a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x))*sinh(x)^3 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^
2 + 6*(11*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^10 + 45*(a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 50*(a^6 - 3*a^4*b^2 + 3*a^2*

```
b^4 - b^6)*cosh(x)^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*si
nh(x)^2 + 12*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^11 + 5*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^9 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*cosh(x)^7 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 5*(a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 12*(25*(a^5*b - a^4
*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*cosh(x)^9 + 20*(5*a^5*b - 4*a^4
*b^2 - 12*a^3*b^3 + 10*a^2*b^4 + 7*a*b^5 - 6*b^6)*cosh(x)^7 + 10*(15*a^5*b
- 9*a^4*b^2 - 40*a^3*b^3 + 24*a^2*b^4 + 33*a*b^5 - 23*b^6)*cosh(x)^5 + 20*(
5*a^5*b - 2*a^4*b^2 - 14*a^3*b^3 + 5*a^2*b^4 + 13*a*b^5 - 3*b^6)*cosh(x)^3
+ (25*a^5*b - 5*a^4*b^2 - 70*a^3*b^3 + 10*a^2*b^4 + 61*a*b^5 - 5*b^6)*cosh(
x))*sinh(x))/(b^7*cosh(x)^12 + 12*b^7*cosh(x)*sinh(x)^11 + b^7*sinh(x)^12 +
 6*b^7*cosh(x)^10 + 15*b^7*cosh(x)^8 + 20*b^7*cosh(x)^6 + 15*b^7*cosh(x)^4
+ 6*(11*b^7*cosh(x)^2 + b^7)*sinh(x)^10 + 20*(11*b^7*cosh(x)^3 + 3*b^7*cosh
(x))*sinh(x)^9 + 6*b^7*cosh(x)^2 + 15*(33*b^7*cosh(x)^4 + 18*b^7*cosh(x)^2
+ b^7)*sinh(x)^8 + 24*(33*b^7*cosh(x)^5 + 30*b^7*cosh(x)^3 + 5*b^7*cosh(x))
*sinh(x)^7 + b^7 + 4*(231*b^7*cosh(x)^6 + 315*b^7*cosh(x)^4 + 105*b^7*cosh(
x)^2 + 5*b^7)*sinh(x)^6 + 24*(33*b^7*cosh(x)^7 + 63*b^7*cosh(x)^5 + 35*b^7*
cosh(x)^3 + 5*b^7*cosh(x))*sinh(x)^5 + 15*(33*b^7*cosh(x)^8 + 84*b^7*cosh(x
)^6 + 70*b^7*cosh(x)^4 + 20*b^7*cosh(x)^2 + b^7)*sinh(x)^4 + 20*(11*b^7*cos
h(x)^9 + 36*b^7*cosh(x)^7 + 42*b^7*cosh(x)^5 + 20*b^7*cosh(x)^3 + 3*b^7*cos
h(x))*sinh(x)^3 + 6*(11*b^7*cosh(x)^10 + 45*b^7*cosh(x)^8 + 70*b^7*cosh(x)^
6 + 50*b^7*cosh(x)^4 + 15*b^7*cosh(x)^2 + b^7)*sinh(x)^2 + 12*(b^7*cosh(x)^
11 + 5*b^7*cosh(x)^9 + 10*b^7*cosh(x)^7 + 10*b^7*cosh(x)^5 + 5*b^7*cosh(x)^
3 + b^7*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**8/(a+b*tanh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23068, size = 801, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^8/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7
)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b^7 + b^8) + (a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*log(e^(2*x) + 1)/b^7 - 1/60*(147*a^6*e^(12*x) - 441*a^4*
b^2*e^(12*x) + 441*a^2*b^4*e^(12*x) - 147*b^6*e^(12*x) + 882*a^6*e^(10*x) +
 120*a^5*b*e^(10*x) - 2766*a^4*b^2*e^(10*x) - 240*a^3*b^3*e^(10*x) + 2886*a
^2*b^4*e^(10*x) + 120*a*b^5*e^(10*x) - 1002*b^6*e^(10*x) + 2205*a^6*e^(8*x)
 + 600*a^5*b*e^(8*x) - 7095*a^4*b^2*e^(8*x) - 1440*a^3*b^3*e^(8*x) + 7815*a
^2*b^4*e^(8*x) + 840*a*b^5*e^(8*x) - 2925*b^6*e^(8*x) + 2940*a^6*e^(6*x) +
1200*a^5*b*e^(6*x) - 9540*a^4*b^2*e^(6*x) - 3200*a^3*b^3*e^(6*x) + 10740*a^
2*b^4*e^(6*x) + 2640*a*b^5*e^(6*x) - 4780*b^6*e^(6*x) + 2205*a^6*e^(4*x) +
```

```
1200*a^5*b*e^(4*x) - 7095*a^4*b^2*e^(4*x) - 3360*a^3*b^3*e^(4*x) + 7815*a^2
*b^4*e^(4*x) + 3120*a*b^5*e^(4*x) - 2925*b^6*e^(4*x) + 882*a^6*e^(2*x) + 60
0*a^5*b*e^(2*x) - 2766*a^4*b^2*e^(2*x) - 1680*a^3*b^3*e^(2*x) + 2886*a^2*b^
4*e^(2*x) + 1464*a*b^5*e^(2*x) - 1002*b^6*e^(2*x) + 147*a^6 + 120*a^5*b - 4
41*a^4*b^2 - 320*a^3*b^3 + 441*a^2*b^4 + 264*a*b^5 - 147*b^6)/(b^7*(e^(2*x)
 + 1)^6)
```

3.103 $\int \frac{\mathrm{sech}^6(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=83

$$\frac{(a^2-2b^2)\tanh^2(x)}{2b^3}-\frac{a(a^2-2b^2)\tanh(x)}{b^4}+\frac{(a^2-b^2)^2\log(a+b\tanh(x))}{b^5}-\frac{a\tanh^3(x)}{3b^2}+\frac{\tanh^4(x)}{4b}$$

```
[Out] ((a^2 - b^2)^2*Log[a + b*Tanh[x]])/b^5 - (a*(a^2 - 2*b^2)*Tanh[x])/b^4 + ((
a^2 - 2*b^2)*Tanh[x]^2)/(2*b^3) - (a*Tanh[x]^3)/(3*b^2) + Tanh[x]^4/(4*b)
```

Rubi [A] time = 0.111035, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 697}

$$\frac{(a^2-2b^2)\tanh^2(x)}{2b^3}-\frac{a(a^2-2b^2)\tanh(x)}{b^4}+\frac{(a^2-b^2)^2\log(a+b\tanh(x))}{b^5}-\frac{a\tanh^3(x)}{3b^2}+\frac{\tanh^4(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^6/(a + b*Tanh[x]),x]
```

```
[Out] ((a^2 - b^2)^2*Log[a + b*Tanh[x]])/b^5 - (a*(a^2 - 2*b^2)*Tanh[x])/b^4 + ((
a^2 - 2*b^2)*Tanh[x]^2)/(2*b^3) - (a*Tanh[x]^3)/(3*b^2) + Tanh[x]^4/(4*b)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\mathrm{sech}^6(x)}{a+b\tanh(x)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\left(1-\frac{x^2}{b^2}\right)^2}{a+x}\,dx,x,b\tanh(x)\right)}{b}\\ &= \frac{\mathrm{Subst}\left(\int \left(\frac{-a^3+2ab^2}{b^4}-\frac{(-a^2+2b^2)x}{b^4}-\frac{ax^2}{b^4}+\frac{x^3}{b^4}+\frac{(-a^2+b^2)^2}{b^4(a+x)}\right)dx,x,b\tanh(x)\right)}{b}\\ &= \frac{(a^2-b^2)^2\log(a+b\tanh(x))}{b^5}-\frac{a(a^2-2b^2)\tanh(x)}{b^4}+\frac{(a^2-2b^2)\tanh^2(x)}{2b^3}-\frac{a\tanh^3(x)}{3b^2}+\frac{\mathrm{t}}{}\end{aligned}$$

Mathematica [A] time = 0.347151, size = 92, normalized size = 1.11

$$\frac{\mathrm{sech}^2(x)\left(-6a^2b^2+4ab^3\tanh(x)+6b^4\right)-4\left(ab\left(3a^2-5b^2\right)\tanh(x)+3\left(a^2-b^2\right)^2(\log(\cosh(x))-\log(a\cosh(x)+b\right.}{12b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^6/(a + b*Tanh[x]),x]
```

```
[Out] (3*b^4*Sech[x]^4 + Sech[x]^2*(-6*a^2*b^2 + 6*b^4 + 4*a*b^3*Tanh[x]) - 4*(3*
(a^2 - b^2)^2*(Log[Cosh[x]] - Log[a*Cosh[x] + b*Sinh[x]]) + a*b*(3*a^2 - 5*
b^2)*Tanh[x]))/(12*b^5)
```

Maple [B] time = 0.046, size = 438, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^6/(a+b*tanh(x)),x)
```

```
[Out] 1/b^5*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)*a^4-2/b^3*ln(a*tanh(1/2*x)^2+2*
tanh(1/2*x)*b+a)*a^2+1/b*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)-2/b^4/(tanh(
1/2*x)^2+1)^4*tanh(1/2*x)^7*a^3+4/b^2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a+2
/b^3/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^2-4/b/(tanh(1/2*x)^2+1)^4*tanh(1/2
*x)^6-6/b^4/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^3+28/3/b^2/(tanh(1/2*x)^2+1
)^4*tanh(1/2*x)^5*a+4/b^3/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a^2-4/b/(tanh(1
/2*x)^2+1)^4*tanh(1/2*x)^4-6/b^4/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^3+28/3
/b^2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a+2/b^3/(tanh(1/2*x)^2+1)^4*tanh(1/2
*x)^2*a^2-4/b/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2-2/b^4/(tanh(1/2*x)^2+1)^4*t
anh(1/2*x)*a^3+4/b^2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a-1/b^5*ln(tanh(1/2*x)
^2+1)*a^4+2/b^3*ln(tanh(1/2*x)^2+1)*a^2-1/b*ln(tanh(1/2*x)^2+1)
```

Maxima [B] time = 1.61512, size = 275, normalized size = 3.31

$$-\frac{2\left(3a^3 - 5ab^2 + \left(9a^3 + 3a^2b - 17ab^2 - 3b^3\right)e^{(-2x)} + 3\left(3a^3 + 2a^2b - 5ab^2 - 4b^3\right)e^{(-4x)} + 3\left(a^3 + a^2b - ab^2 - b^3\right)e^{(-6x}}{3\left(4b^4e^{(-2x)} + 6b^4e^{(-4x)} + 4b^4e^{(-6x)} + b^4e^{(-8x)} + b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a^3 - 5*a*b^2 + (9*a^3 + 3*a^2*b - 17*a*b^2 - 3*b^3)*e^(-2*x) + 3*(
3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*e^(-4*x) + 3*(a^3 + a^2*b - a*b^2 - b^3)
*e^(-6*x))/(4*b^4*e^(-2*x) + 6*b^4*e^(-4*x) + 4*b^4*e^(-6*x) + b^4*e^(-8*x)
 + b^4) + (a^4 - 2*a^2*b^2 + b^4)*log(-(a - b)*e^(-2*x) - a - b)/b^5 - (a^4
 - 2*a^2*b^2 + b^4)*log(e^(-2*x) + 1)/b^5
```

Fricas [B] time = 2.64347, size = 4520, normalized size = 54.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^6 + 36*(a^3*b - a^2*b^2 - a*
b^3 + b^4)*cosh(x)*sinh(x)^5 + 6*(a^3*b - a^2*b^2 - a*b^3 + b^4)*sinh(x)^6
+ 6*(3*a^3*b - 2*a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(x)^4 + 6*(3*a^3*b - 2*a^2*
b^2 - 5*a*b^3 + 4*b^4 + 15*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^2)*sinh(
x)^4 + 6*a^3*b - 10*a*b^3 + 24*(5*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^3
 + (3*a^3*b - 2*a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(x))*sinh(x)^3 + 2*(9*a^3*b
- 3*a^2*b^2 - 17*a*b^3 + 3*b^4)*cosh(x)^2 + 2*(45*(a^3*b - a^2*b^2 - a*b^3
+ b^4)*cosh(x)^4 + 9*a^3*b - 3*a^2*b^2 - 17*a*b^3 + 3*b^4 + 18*(3*a^3*b - 2
*a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(x)^2)*sinh(x)^2 + 3*((a^4 - 2*a^2*b^2 + b^
4)*cosh(x)^8 + 8*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b
^2 + b^4)*sinh(x)^8 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*
b^2 + b^4 + 7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6
*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^
4 + 3*a^4 - 6*a^2*b^2 + 3*b^4 + 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(
x)^4 + a^4 - 2*a^2*b^2 + b^4 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*
(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh
(x)^3 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*
cosh(x)^6 + 15*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
9*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)
*cosh(x)^7 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4
)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*cosh(x) +
b*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*
(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)
^8 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*b^2 + b^4 + 7*(a^
4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*co
sh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^2
 + b^4)*cosh(x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^2
*b^2 + 3*b^4 + 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^
2*b^2 + b^4 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*(a^4 - 2*a^2*b^2
+ b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 -
2*a^2*b^2 + b^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 15*(a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 3*(a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^
4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) +
 4*(9*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^5 + 6*(3*a^3*b - 2*a^2*b^2 -
5*a*b^3 + 4*b^4)*cosh(x)^3 + (9*a^3*b - 3*a^2*b^2 - 17*a*b^3 + 3*b^4)*cosh(
x))*sinh(x))/(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sinh(x)^8 + 4*b
^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b^5*cosh(x)^2 + b^5
)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^5 + b^5 + 2*(35*b
^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*cosh(x)^5 + 1
0*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cosh(x)^6 + 15*b^5*co
sh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(x)^7 + 3*b^5*cosh(
x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**6/(a+b*tanh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25654, size = 427, normalized size = 5.14

$$\frac{\left(a^{5}+a^{4}b-2\,a^{3}b^{2}-2\,a^{2}b^{3}+ab^{4}+b^{5}\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{ab^{5}+b^{6}}-\frac{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log\left(e^{(2\,x)}+1\right)}{b^{5}}+\frac{25\,a^{4}e^{(8\,x)}-50}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*log(abs(a*e^(2*x) + b*e
^(2*x) + a - b))/(a*b^5 + b^6) - (a^4 - 2*a^2*b^2 + b^4)*log(e^(2*x) + 1)/b
^5 + 1/12*(25*a^4*e^(8*x) - 50*a^2*b^2*e^(8*x) + 25*b^4*e^(8*x) + 100*a^4*e
^(6*x) + 24*a^3*b*e^(6*x) - 224*a^2*b^2*e^(6*x) - 24*a*b^3*e^(6*x) + 124*b^
4*e^(6*x) + 150*a^4*e^(4*x) + 72*a^3*b*e^(4*x) - 348*a^2*b^2*e^(4*x) - 120*
a*b^3*e^(4*x) + 246*b^4*e^(4*x) + 100*a^4*e^(2*x) + 72*a^3*b*e^(2*x) - 224*
a^2*b^2*e^(2*x) - 136*a*b^3*e^(2*x) + 124*b^4*e^(2*x) + 25*a^4 + 24*a^3*b -
 50*a^2*b^2 - 40*a*b^3 + 25*b^4)/(b^5*(e^(2*x) + 1)^4)
```

3.104 $\int \frac{\text{sech}^4(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{\left(a^2-b^2\right)\log(a+b\tanh(x))}{b^3}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}$$

```
[Out] -(((a^2 - b^2)*Log[a + b*Tanh[x]])/b^3) + (a*Tanh[x])/b^2 - Tanh[x]^2/(2*b)
```

Rubi [A] time = 0.06617, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 697}

$$-\frac{\left(a^2-b^2\right)\log(a+b\tanh(x))}{b^3}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] -(((a^2 - b^2)*Log[a + b*Tanh[x]])/b^3) + (a*Tanh[x])/b^2 - Tanh[x]^2/(2*b)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^4(x)}{a+b\tanh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{b^2}}{a+x}\,dx,x,b\tanh(x)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{b^2}-\frac{x}{b^2}+\frac{-a^2+b^2}{b^2(a+x)}\right)dx,x,b\tanh(x)\right)}{b}\\ &= -\frac{\left(a^2-b^2\right)\log(a+b\tanh(x))}{b^3}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}\end{aligned}$$

Mathematica [A] time = 0.157174, size = 49, normalized size = 1.22

$$\frac{2\left(a^2-b^2\right)(\log(\cosh(x))-\log(a\cosh(x)+b\sinh(x)))+2ab\tanh(x)+b^2\text{sech}^2(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (2*(a^2 - b^2)*(Log[Cosh[x]] - Log[a*Cosh[x] + b*Sinh[x]]) + b^2*Sech[x]^2
+ 2*a*b*Tanh[x])/(2*b^3)
```

Maple [B] time = 0.039, size = 143, normalized size = 3.6

$$-\frac{a^2}{b^3}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2+2\tanh(x/2)\,b+a\right)+\frac{1}{b}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2+2\tanh(x/2)\,b+a\right)+2\frac{a(\tanh(x/2))^3}{b^2\left((\tanh(x/2))^2+1\right)^2}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*tanh(x)),x)
```

```
[Out] -1/b^3*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)*a^2+1/b*ln(a*tanh(1/2*x)^2+2*t
anh(1/2*x)*b+a)+2/b^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a-2/b/(tanh(1/2*x)^
2+1)^2*tanh(1/2*x)^2+2/b^2/(tanh(1/2*x)^2+1)^2*a*tanh(1/2*x)+1/b^3*ln(tanh(
1/2*x)^2+1)*a^2-1/b*ln(tanh(1/2*x)^2+1)
```

Maxima [B] time = 1.6785, size = 120, normalized size = 3.

$$\frac{2\left((a+b)e^{(-2x)}+a\right)}{2b^2e^{(-2x)}+b^2e^{(-4x)}+b^2}-\frac{\left(a^2-b^2\right)\log\left(-(a-b)e^{(-2x)}-a-b\right)}{b^3}+\frac{\left(a^2-b^2\right)\log\left(e^{(-2x)}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] 2*((a + b)*e^(-2*x) + a)/(2*b^2*e^(-2*x) + b^2*e^(-4*x) + b^2) - (a^2 - b^2
)*log(-(a - b)*e^(-2*x) - a - b)/b^3 + (a^2 - b^2)*log(e^(-2*x) + 1)/b^3
```

Fricas [B] time = 2.43367, size = 1088, normalized size = 27.2

$$-\frac{2\left(ab-b^2\right)\cosh(x)^2+4\left(ab-b^2\right)\cosh(x)\sinh(x)+2\left(ab-b^2\right)\sinh(x)^2+2ab+\left(\left(a^2-b^2\right)\cosh(x)^4+4\left(a^2-b^2\right)\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -(2*(a*b - b^2)*cosh(x)^2 + 4*(a*b - b^2)*cosh(x)*sinh(x) + 2*(a*b - b^2)*s
inh(x)^2 + 2*a*b + ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3
 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(
x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 -
 b^2)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x)))
- ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*si
nh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)
*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*si
nh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)*s
inh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3
```

```
)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*tanh(x)), x)
```

Giac [B] time = 1.20692, size = 140, normalized size = 3.5

$$-\frac{\left(a^3+a^2b-ab^2-b^3\right)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{ab^3+b^4}+\frac{\left(a^2-b^2\right)\log\left(e^{(2x)}+1\right)}{b^3}-\frac{2\left(ab+\left(ab-b^2\right)e^{(2x)}\right)}{b^3\left(e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + a^2*b - a*b^2 - b^3)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b^3
 + b^4) + (a^2 - b^2)*log(e^(2*x) + 1)/b^3 - 2*(a*b + (a*b - b^2)*e^(2*x))/
(b^3*(e^(2*x) + 1)^2)
```

3.105 $\int \frac{\text{sech}^2(x)}{a+b\tanh(x)} dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\tanh(x))}{b}$$

[Out] Log[a + b*Tanh[x]]/b

Rubi [A] time = 0.041569, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$\frac{\log(a+b\tanh(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/(a + b*Tanh[x]),x]

[Out] Log[a + b*Tanh[x]]/b

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^2(x)}{a+b\tanh(x)} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b\tanh(x)\right)}{b}\\ &= \frac{\log(a+b\tanh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0441283, size = 20, normalized size = 1.82

$$\frac{\log(a\cosh(x)+b\sinh(x))-\log(\cosh(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^2/(a + b*Tanh[x]),x]

[Out] (-Log[Cosh[x]] + Log[a*Cosh[x] + b*Sinh[x]])/b

Maple [A] time = 0.023, size = 12, normalized size = 1.1

$$\frac{\ln\left(a+b\tanh\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*tanh(x)),x)
```

```
[Out] ln(a+b*tanh(x))/b
```

Maxima [A] time = 1.16568, size = 15, normalized size = 1.36

$$\frac{\log\left(b\tanh\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] log(b*tanh(x) + a)/b
```

Fricas [B] time = 2.31094, size = 126, normalized size = 11.45

$$\frac{\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] (log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x
) - sinh(x))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^{2}\left(x\right)}{a+b\tanh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*tanh(x)), x)
```

Giac [B] time = 1.20997, size = 61, normalized size = 5.55

$$\frac{(a+b)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{ab+b^{2}}-\frac{\log\left(e^{(2\,x)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] (a + b)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b + b^2) - log(e^(2*x) +
 1)/b
```

3.106 $\int \frac{1}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0458598, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3484, 3530}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\tanh(x)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.0446429, size = 29, normalized size = 0.74

$$\frac{ax-b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] time = 0.011, size = 55, normalized size = 1.4

$$\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2b+2a}-\frac{b\ln(a+b\tanh(x))}{(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)),x)
```

```
[Out] 1/(2*a-2*b)*ln(1+tanh(x))-1/(2*b+2*a)*ln(tanh(x)-1)-b/(a-b)/(a+b)*ln(a+b*ta
nh(x))
```

Maxima [A] time = 1.14036, size = 55, normalized size = 1.41

$$-\frac{b\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A] time = 2.35943, size = 108, normalized size = 2.77

$$\frac{(a+b)x-b\log\left(\frac{2(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [A] time = 0.74283, size = 146, normalized size = 3.74

$$\begin{cases} \tilde{\infty}\left(x-\log(\tanh(x)+1)+\log(\tanh(x))\right) & \text{for } a=0\wedge b=0\\ \frac{x}{a} & \text{for } b=0\\ -\frac{x\tanh(x)}{2b\tanh(x)-2b}+\frac{x}{2b\tanh(x)-2b}+\frac{1}{2b\tanh(x)-2b} & \text{for } a=-b\\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}-\frac{1}{2b\tanh(x)+2b} & \text{for } a=b\\ \frac{ax}{a^2-b^2}-\frac{bx}{a^2-b^2}-\frac{b\log\left(\frac{a}{b}+\tanh(x)\right)}{a^2-b^2}+\frac{b\log(\tanh(x)+1)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 (x/a, Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) +
 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b
*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2) - b*
x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) + 1)/(
a**2 - b**2), True))
```

Giac [A] time = 1.21395, size = 58, normalized size = 1.49

$$-\frac{b\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^2 - b^2} + \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

3.107 $\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=91

$$\frac{b^3\log(a+b\tanh(x))}{\left(a^2-b^2\right)^2}-\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)}-\frac{(a+2b)\log(1-\tanh(x))}{4(a+b)^2}+\frac{(a-2b)\log(\tanh(x)+1)}{4(a-b)^2}$$

```
[Out] -((a + 2*b)*Log[1 - Tanh[x]])/(4*(a + b)^2) + ((a - 2*b)*Log[1 + Tanh[x]])/
(4*(a - b)^2) + (b^3*Log[a + b*Tanh[x]])/(a^2 - b^2)^2 - (Cosh[x]^2*(b - a*
Tanh[x]))/(2*(a^2 - b^2))
```

Rubi [A] time = 0.1455, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3506, 741, 801}

$$\frac{b^3\log(a+b\tanh(x))}{\left(a^2-b^2\right)^2}-\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)}-\frac{(a+2b)\log(1-\tanh(x))}{4(a+b)^2}+\frac{(a-2b)\log(\tanh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] -((a + 2*b)*Log[1 - Tanh[x]])/(4*(a + b)^2) + ((a - 2*b)*Log[1 + Tanh[x]])/
(4*(a - b)^2) + (b^3*Log[a + b*Tanh[x]])/(a^2 - b^2)^2 - (Cosh[x]^2*(b - a*
Tanh[x]))/(2*(a^2 - b^2))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}dx, x, b\tanh(x)\right)}{b}$$
$$= -\frac{\cosh^2(x)(b-a\tanh(x))}{2(a^2-b^2)} + \frac{b\,\text{Subst}\left(\int \frac{-2+\frac{a^2}{b^2}+\frac{ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)}dx, x, b\tanh(x)\right)}{2(a^2-b^2)}$$
$$= -\frac{\cosh^2(x)(b-a\tanh(x))}{2(a^2-b^2)} + \frac{b\,\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right)dx, x, b\tan\right.}{2(a^2-b^2)}$$
$$= -\frac{(a+2b)\log(1-\tanh(x))}{4(a+b)^2} + \frac{(a-2b)\log(1+\tanh(x))}{4(a-b)^2} + \frac{b^3\log(a+b\tanh(x))}{(a^2-b^2)^2} - \frac{\cosh^2(x)(b}{2(a^2}$$

Mathematica [A] time = 0.112653, size = 75, normalized size = 0.82

$$\frac{a(a^2-b^2)\sinh(2x) + (b^3-a^2b)\cosh(2x) + 2a^3x - 6ab^2x + 4b^3\log(a\cosh(x)+b\sinh(x))}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (2*a^3*x - 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] + 4*b^3*Log[a*Cosh[x] + b
*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Maple [B] time = 0.046, size = 175, normalized size = 1.9

$$-\frac{1}{2a-2b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + 2\frac{1}{(4a-4b)(\tanh(x/2)+1)} + \frac{a}{2(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{b}{(a-b)^2}\ln\left(\tanh\left(\frac{x}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*tanh(x)),x)
```

```
[Out] -1/(2*a-2*b)/(tanh(1/2*x)+1)^2+2/(4*a-4*b)/(tanh(1/2*x)+1)+1/2*a/(a-b)^2*ln
(tanh(1/2*x)+1)-1/(a-b)^2*ln(tanh(1/2*x)+1)*b+b^3/(a-b)^2/(a+b)^2*ln(a*tanh
(1/2*x)^2+2*tanh(1/2*x)*b+a)+1/(2*b+2*a)/(tanh(1/2*x)-1)^2+2/(4*a+4*b)/(tan
h(1/2*x)-1)-1/2*a/(a+b)^2*ln(tanh(1/2*x)-1)-1/(a+b)^2*ln(tanh(1/2*x)-1)*b
```

Maxima [A] time = 1.22641, size = 116, normalized size = 1.27

$$\frac{b^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4} + \frac{(a+2b)x}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a + 2*b)*
x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Fricas [B] time = 2.30382, size = 818, normalized size = 8.99

$$\frac{\left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)^4 + 4\left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)\sinh\left(x\right)^3 + \left(a^3 - a^2b - ab^2 + b^3\right)\sinh\left(x\right)^4 + 4\left(a^3 - 3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 - 3*a*b^
2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 + 8*(b^3*cosh
(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2*(a^3
- 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2\left(x\right)}{a + b\tanh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*tanh(x)), x)
```

Giac [A] time = 1.21046, size = 150, normalized size = 1.65

$$\frac{b^3\log\left(\left|ae^{(2\,x)} + be^{(2\,x)} + a - b\right|\right)}{a^4 - 2\,a^2b^2 + b^4} + \frac{(a - 2\,b)x}{2\left(a^2 - 2\,ab + b^2\right)} - \frac{\left(2\,ae^{(2\,x)} - 4\,be^{(2\,x)} + a - b\right)e^{(-2\,x)}}{8\left(a^2 - 2\,ab + b^2\right)} + \frac{e^{(2\,x)}}{8\,(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(
a - 2*b)*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) + a - b)*e^
(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

3.108 $\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=155

$$-\frac{b^5\log(a+b\tanh(x))}{\left(a^2-b^2\right)^3}-\frac{\left(3a^2+9ab+8b^2\right)\log(1-\tanh(x))}{16(a+b)^3}+\frac{\left(3a^2-9ab+8b^2\right)\log(\tanh(x)+1)}{16(a-b)^3}-\frac{\cosh^4(x)(b-a}{4\left(a^2-\right.}$$

```
[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Tanh[x]])/(16*(a + b)^3) + ((3*a^2 - 9*a*
b + 8*b^2)*Log[1 + Tanh[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Tanh[x]])/(a^2
 - b^2)^3 - (Cosh[x]^4*(b - a*Tanh[x]))/(4*(a^2 - b^2)) + (Cosh[x]^2*(4*b^3
 - a*(7 - (3*a^2)/b^2)*b^2*Tanh[x]))/(8*(a^2 - b^2)^2)
```

Rubi [A] time = 0.236122, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3506, 741, 823, 801}

$$-\frac{b^5\log(a+b\tanh(x))}{\left(a^2-b^2\right)^3}-\frac{\left(3a^2+9ab+8b^2\right)\log(1-\tanh(x))}{16(a+b)^3}+\frac{\left(3a^2-9ab+8b^2\right)\log(\tanh(x)+1)}{16(a-b)^3}-\frac{\cosh^4(x)(b-a}{4\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Tanh[x]])/(16*(a + b)^3) + ((3*a^2 - 9*a*
b + 8*b^2)*Log[1 + Tanh[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Tanh[x]])/(a^2
 - b^2)^3 - (Cosh[x]^4*(b - a*Tanh[x]))/(4*(a^2 - b^2)) + (Cosh[x]^2*(4*b^3
 - a*(7 - (3*a^2)/b^2)*b^2*Tanh[x]))/(8*(a^2 - b^2)^2)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^3}\,dx, x, b\tanh(x)\right)}{b}$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)} + \frac{b\,\text{Subst}\left(\int \frac{-4+\frac{3a^2}{b^2}+\frac{3ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\tanh(x)\right)}{4\left(a^2-b^2\right)}$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)} + \frac{\cosh^2(x)\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\tanh(x)\right)}{8\left(a^2-b^2\right)^2} - \frac{b^5\,\text{Subst}\left(\int \frac{-\frac{3a^4-7a^2b^2+8b^4}{b^6}+\cdots}{(a+x)\left(1-\cdots\right)}\cdots\right)}{8\left(a^2-\cdots\right)}$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)} + \frac{\cosh^2(x)\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\tanh(x)\right)}{8\left(a^2-b^2\right)^2} - \frac{b^5\,\text{Subst}\left(\int\left(-\frac{(a-b)^2\left(3a^2+9\cdots\right)}{2b^5(a+b)(\cdots}\cdots\right)\right)}{\cdots}$$

$$= -\frac{\left(3a^2+9ab+8b^2\right)\log(1-\tanh(x))}{16(a+b)^3} + \frac{\left(3a^2-9ab+8b^2\right)\log(1+\tanh(x))}{16(a-b)^3} - \frac{b^5\log(a+b\tanh(\cdots}{\left(a^2-b^2\right)^3}$$

Mathematica [A] time = 0.194281, size = 156, normalized size = 1.01

$$\frac{-40a^3b^2x-24a^3b^2\sinh(2x)-2a^3b^2\sinh(4x)-4b\left(-4a^2b^2+a^4+3b^4\right)\cosh(2x)-b\left(a^2-b^2\right)^2\cosh(4x)+12a^5x+8a^5\,\text{s}\cdots}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (12*a^5*x - 40*a^3*b^2*x + 60*a*b^4*x - 4*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Cosh[
2*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*b^5*Log[a*Cosh[x] + b*Sinh[x]] + 8*a^
5*Sinh[2*x] - 24*a^3*b^2*Sinh[2*x] + 16*a*b^4*Sinh[2*x] + a^5*Sinh[4*x] - 2
*a^3*b^2*Sinh[4*x] + a*b^4*Sinh[4*x])/(32*(a - b)^3*(a + b)^3)
```

Maple [B] time = 0.049, size = 354, normalized size = 2.3

$$-\frac{1}{4a-4b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}+2\frac{1}{(4a-4b)(\tanh(x/2)+1)^3}+\frac{5a}{8(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{7b}{8(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*tanh(x)),x)
```

```
[Out] -1/2/(2*a-2*b)/(tanh(1/2*x)+1)^4+2/(4*a-4*b)/(tanh(1/2*x)+1)^3+5/8/(a-b)^2/
(tanh(1/2*x)+1)*a-7/8/(a-b)^2/(tanh(1/2*x)+1)*b-7/8/(a-b)^2/(tanh(1/2*x)+1)
^2*a+9/8/(a-b)^2/(tanh(1/2*x)+1)^2*b+3/8*a^2/(a-b)^3*ln(tanh(1/2*x)+1)-9/8*
a/(a-b)^3*ln(tanh(1/2*x)+1)*b+1/(a-b)^3*ln(tanh(1/2*x)+1)*b^2-b^5/(a-b)^3/(
a+b)^3*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)+1/2/(2*b+2*a)/(tanh(1/2*x)-1)^
4+2/(4*a+4*b)/(tanh(1/2*x)-1)^3+7/8/(a+b)^2/(tanh(1/2*x)-1)^2*a+9/8/(a+b)^2
/(tanh(1/2*x)-1)^2*b+5/8/(a+b)^2/(tanh(1/2*x)-1)*a+7/8/(a+b)^2/(tanh(1/2*x)
-1)*b-3/8*a^2/(a+b)^3*ln(tanh(1/2*x)-1)-9/8*a/(a+b)^3*ln(tanh(1/2*x)-1)*b-1
/(a+b)^3*ln(tanh(1/2*x)-1)*b^2
```

Maxima [A] time = 1.16831, size = 223, normalized size = 1.44

$$-\frac{b^{5}\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}}+\frac{\left(3a^{2}+9ab+8b^{2}\right)x}{8\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)}+\frac{\left(4(2a+3b)e^{(-2x)}+a+b\right)e^{(4x)}}{64\left(a^{2}+2ab+b^{2}\right)}-\frac{4(2a-3b)e^{(-2x)}+}{64\left(a^{2}-2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1
/8*(3*a^2 + 9*a*b + 8*b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*(2*a
 + 3*b)*e^(-2*x) + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*(2*a - 3*b)
*e^(-2*x) + (a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Fricas [B] time = 2.39283, size = 2882, normalized size = 18.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(2*a^5 - a^4*b
- 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^6 + 4*(2*a^5 - a^4*b - 6
*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5 + 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 + 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^
5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 +
 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 30*(
2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^2 + 4*(3*a
^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 10*(2*a^5 - a^4*b - 6*a^3*b^2
+ 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^3 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x))*sinh(x)^3 - 4*(2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 +
 4*a*b^4 + 3*b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^6 - 2*a^5 - a^4*b + 6*a^3*b^2 + 4*a^2*b^3 - 4*a*b^4 - 3
*b^5 + 15*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^4 + 12*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^2)*sinh(x)^2 - 64
*(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b
```

```
^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x
) - sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^7 + 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^5 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^3 - (2*a^5 + a^4*b
 - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^
2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*tanh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24017, size = 306, normalized size = 1.97

$$-\frac{b^5\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{\left(3a^2-9ab+8b^2\right)x}{8\left(a^3-3a^2b+3ab^2-b^3\right)}-\frac{\left(18a^2e^{(4x)}-54abe^{(4x)}+48b^2e^{(4x)}+8a^2e^{(2x)}-20abe^{(\cdots}{64\left(a^3-3a^2b+3ab^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6) + 1/8*(3*a^2 - 9*a*b + 8*b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/6
4*(18*a^2*e^(4*x) - 54*a*b*e^(4*x) + 48*b^2*e^(4*x) + 8*a^2*e^(2*x) - 20*a*
b*e^(2*x) + 12*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3
*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x) + 8*a*e^(2*x) + 12*b*e^(2*x))/(
a^2 + 2*a*b + b^2)
```

3.109 $\int \frac{\text{sech}^7(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=157

$$-\frac{\left(a^2-b^2\right)\text{sech}^3(x)}{3b^3}+\frac{\left(a^2-b^2\right)^2\text{sech}(x)}{b^5}+\frac{a\left(-20a^2b^2+8a^4+15b^4\right)\tan^{-1}(\sinh(x))}{8b^6}-\frac{a\left(4a^2-7b^2\right)\tanh(x)\text{sech}(x}{8b^4}$$

```
[Out] (a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/(8*b^6) - ((a^2 - b^2)^(5
/2)*ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/b^6 + ((a^2 - b^2)^2
*Sech[x])/b^5 - ((a^2 - b^2)*Sech[x]^3)/(3*b^3) + Sech[x]^5/(5*b) - (a*(4*a
^2 - 7*b^2)*Sech[x]*Tanh[x])/(8*b^4) + (a*Sech[x]^3*Tanh[x])/(4*b^2)
```

Rubi [A] time = 0.272871, antiderivative size = 187, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3510, 3486, 3768, 3770, 3509, 206}

$$-\frac{\left(a^2-b^2\right)\text{sech}^3(x)}{3b^3}+\frac{\left(a^2-b^2\right)^2\text{sech}(x)}{b^5}+\frac{a\left(a^2-b^2\right)^2\tan^{-1}(\sinh(x))}{b^6}-\frac{a\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{2b^4}-\frac{a\left(a^2-b^2\right)\text{ta}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^7/(a + b*Tanh[x]),x]
```

```
[Out] (3*a*ArcTan[Sinh[x]])/(8*b^2) - (a*(a^2 - b^2)*ArcTan[Sinh[x]])/(2*b^4) + (
a*(a^2 - b^2)^2*ArcTan[Sinh[x]])/b^6 - ((a^2 - b^2)^(5/2)*ArcTan[(Cosh[x]*(
b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/b^6 + ((a^2 - b^2)^2*Sech[x])/b^5 - ((a^2
 - b^2)*Sech[x]^3)/(3*b^3) + Sech[x]^5/(5*b) + (3*a*Sech[x]*Tanh[x])/(8*b^2
) - (a*(a^2 - b^2)*Sech[x]*Tanh[x])/(2*b^4) + (a*Sech[x]^3*Tanh[x])/(4*b^2)
```

Rule 3510

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Dist[d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[(d^2*(a^2 + b^2))/b^2, Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^7(x)}{a+b\tanh(x)}\,dx &= \frac{\int \operatorname{sech}^5(x)(a-b\tanh(x))\,dx}{b^2} - \frac{\left(a^2-b^2\right)\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx}{b^2}\\ &= \frac{\operatorname{sech}^5(x)}{5b} + \frac{a\int \operatorname{sech}^5(x)\,dx}{b^2} - \frac{\left(a^2-b^2\right)\int \operatorname{sech}^3(x)(a-b\tanh(x))\,dx}{b^4} + \frac{\left(a^2-b^2\right)^2\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx}{b^4}\\ &= -\frac{\left(a^2-b^2\right)\operatorname{sech}^3(x)}{3b^3} + \frac{\operatorname{sech}^5(x)}{5b} + \frac{a\operatorname{sech}^3(x)\tanh(x)}{4b^2} + \frac{(3a)\int \operatorname{sech}^3(x)\,dx}{4b^2} - \frac{\left(a\left(a^2-b^2\right)\right)\int \operatorname{sec}}{b^4}\\ &= \frac{\left(a^2-b^2\right)^2\operatorname{sech}(x)}{b^5} - \frac{\left(a^2-b^2\right)\operatorname{sech}^3(x)}{3b^3} + \frac{\operatorname{sech}^5(x)}{5b} + \frac{3a\operatorname{sech}(x)\tanh(x)}{8b^2} - \frac{a\left(a^2-b^2\right)\operatorname{sech}(x)\,\mathrm{t}}{2b^4}\\ &= \frac{3a\tan^{-1}(\sinh(x))}{8b^2} - \frac{a\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{2b^4} + \frac{a\left(a^2-b^2\right)^2\tan^{-1}(\sinh(x))}{b^6} - \frac{\left(a^2-b^2\right)^{5/2}\tan^{-}}{}\end{aligned}$$

Mathematica [A] time = 0.496309, size = 166, normalized size = 1.06

$$\frac{30\left(a\left(-20a^2b^2+8a^4+15b^4\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-8\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)^2\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)\right)+10b^3\operatorname{sech}^3(x)\left(-4a^2+\right.}{120b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^7/(a + b*Tanh[x]),x]
```

```
[Out] (30*(a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[Tanh[x/2]] - 8*Sqrt[a - b]*Sqrt
[a + b]*(a^2 - b^2)^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])
+ 24*b^5*Sech[x]^5 + 10*b^3*Sech[x]^3*(-4*a^2 + 4*b^2 + 3*a*b*Tanh[x]) + 15
*b*Sech[x]*(8*(a^2 - b^2)^2 + (-4*a^3*b + 7*a*b^3)*Tanh[x]))/(120*b^6)
```

Maple [B] time = 0.053, size = 715, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^7/(a+b*tanh(x)),x)
```

```
[Out] -14/3/b^3/(tanh(1/2*x)^2+1)^5*a^2-5/b^4*arctan(tanh(1/2*x))*a^3+15/4/b^2*ar
ctan(tanh(1/2*x))*a+2/b^6*arctan(tanh(1/2*x))*a^5+6/b/(tanh(1/2*x)^2+1)^5*t
```

```
anh(1/2*x)^8+12/b/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^6+56/3/b/(tanh(1/2*x)^2+1
)^5*tanh(1/2*x)^4+28/3/b/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^2+2/b^5/(tanh(1/2*
x)^2+1)^5*a^4+46/15/b/(tanh(1/2*x)^2+1)^5+2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/b^4/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^9*
a^3-9/4/b^2/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^9*a+2/b^5/(tanh(1/2*x)^2+1)^5*t
anh(1/2*x)^8*a^4-6/b^3/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^8*a^2+2/b^4/(tanh(1/
2*x)^2+1)^5*tanh(1/2*x)^7*a^3-5/2/b^2/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^7*a+8
/b^5/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^6*a^4-20/b^3/(tanh(1/2*x)^2+1)^5*tanh(
1/2*x)^6*a^2+12/b^5/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^4*a^4-80/3/b^3/(tanh(1/
2*x)^2+1)^5*tanh(1/2*x)^4*a^2-2/b^4/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^3*a^3+5
/2/b^2/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^3*a+8/b^5/(tanh(1/2*x)^2+1)^5*tanh(1
/2*x)^2*a^4-52/3/b^3/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^2*a^2-1/b^4/(tanh(1/2*
x)^2+1)^5*tanh(1/2*x)*a^3+9/4/b^2/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)*a-2/b^6/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))*a^6+6/b^4/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))*a^4-6/b^2
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.48934, size = 16338, normalized size = 104.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^9 +
135*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)*sinh(x)^8
+ 15*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*sinh(x)^9 + 10*(4
8*a^4*b - 12*a^3*b^2 - 112*a^2*b^3 + 33*a*b^4 + 64*b^5)*cosh(x)^7 + 10*(48*
a^4*b - 12*a^3*b^2 - 112*a^2*b^3 + 33*a*b^4 + 64*b^5 + 54*(8*a^4*b - 4*a^3*
b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^7 + 70*(18*(8*a^4*b
- 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^3 + (48*a^4*b - 12*a^3*
b^2 - 112*a^2*b^3 + 33*a*b^4 + 64*b^5)*cosh(x))*sinh(x)^6 + 16*(45*a^4*b -
110*a^2*b^3 + 89*b^5)*cosh(x)^5 + 2*(360*a^4*b - 880*a^2*b^3 + 712*b^5 + 94
5*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^4 + 105*(48*
a^4*b - 12*a^3*b^2 - 112*a^2*b^3 + 33*a*b^4 + 64*b^5)*cosh(x)^2)*sinh(x)^5
+ 10*(189*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^5 +
35*(48*a^4*b - 12*a^3*b^2 - 112*a^2*b^3 + 33*a*b^4 + 64*b^5)*cosh(x)^3 + 8*
(45*a^4*b - 110*a^2*b^3 + 89*b^5)*cosh(x))*sinh(x)^4 + 10*(48*a^4*b + 12*a^
3*b^2 - 112*a^2*b^3 - 33*a*b^4 + 64*b^5)*cosh(x)^3 + 10*(126*(8*a^4*b - 4*a
^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^6 + 48*a^4*b + 12*a^3*b^2 -
112*a^2*b^3 - 33*a*b^4 + 64*b^5 + 35*(48*a^4*b - 12*a^3*b^2 - 112*a^2*b^3 +
 33*a*b^4 + 64*b^5)*cosh(x)^4 + 16*(45*a^4*b - 110*a^2*b^3 + 89*b^5)*cosh(x
)^2)*sinh(x)^3 + 10*(54*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5
)*cosh(x)^7 + 21*(48*a^4*b - 12*a^3*b^2 - 112*a^2*b^3 + 33*a*b^4 + 64*b^5)*
```

cosh(x)^5 + 16*(45*a^4*b - 110*a^2*b^3 + 89*b^5)*cosh(x)^3 + 3*(48*a^4*b + 12*a^3*b^2 - 112*a^2*b^3 - 33*a*b^4 + 64*b^5)*cosh(x))*sinh(x)^2 + 60*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^10 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^9 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^10 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 5*(a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^7 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 10*(21*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 14*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 70*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 15*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 10*(21*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 15*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 40*(3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 + 5*(9*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 28*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 12*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^9 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 15*((8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^10 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^9 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)*sinh(x)^10 + 5*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 5*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 9*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^7 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 21*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 14*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 70*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 10*(21*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 35*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 + 7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 5*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 5*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 5*(9*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 28*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 30*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 12*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 10*((8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^9 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 + 6*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 15*(8*a^4*b + 4*a^3*b^2 - 16*a^2*b^3 - 7*a*b^4 + 8*b^5)*cosh(x) + 5*(27*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^8 + 14*(48*a^4*b - 12*a^3*b^2 - 112*a^2*b^3 + 33*a*b^4 + 64*b^5)*cosh(x)^6 + 24*a^4*b + 12*a^3*b^2 - 48*a^2*b^3 - 21*a*b^4 + 24*b^5 + 16*(45*a^4*b - 110*a^2*b^3 + 89*b^5)*cosh(x)^4 + 6*(48*a^4*b + 12*a^3*b^2 - 112*a^2*b^3 - 33*a*b^4 + 64*b^5)*cosh(x)^2)*sinh(x))/(b^6*cosh(x)^10 + 10*b^6*cosh(x)*sinh(x)^9 + b^6*sinh(x)^10 + 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^6 + 10*b^6*cosh(x)^4 + 5*(9*b^6*cosh(x)^2 + b^6)*sinh(x)^8 + 5*b^6*cosh(x)^2 + 40*(3*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 + 14*b^6*cosh(x)^2 + b^6)*sinh(x)^6 + b^6 + 4*(63*b^6*cosh(x)^5 + 70*b^6*cosh(x)^3 + 15*b^6*cosh(x))*sinh(x)^5 + 10*(21*b^6*cosh(x)^6 + 35*b^6*cosh(x)^4 + 15*b^6*cosh(x)^2 + b^6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 + 7*b^6*cosh(x)^5 + 5*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x)^3

```
+ 5*(9*b^6*cosh(x)^8 + 28*b^6*cosh(x)^6 + 30*b^6*cosh(x)^4 + 12*b^6*cosh(x)
^2 + b^6)*sinh(x)^2 + 10*(b^6*cosh(x)^9 + 4*b^6*cosh(x)^7 + 6*b^6*cosh(x)^5
 + 4*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x)), 1/60*(15*(8*a^4*b - 4*a^3*b^2 -
 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^9 + 135*(8*a^4*b - 4*a^3*b^2 - 16*a^
2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)*sinh(x)^8 + 15*(8*a^4*b - 4*a^3*b^2 - 16*a
^2*b^3 + 7*a*b^4 + 8*b^5)*sinh(x)^9 + 10*(48*a^4*b - 12*a^3*b^2 - 112*a^2*b
^3 + 33*a*b^4 + 64*b^5)*cosh(x)^7 + 10*(48*a^4*b - 12*a^3*b^2 - 112*a^2*b^3
 + 33*a*b^4 + 64*b^5 + 54*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b
^5)*cosh(x)^2)*sinh(x)^7 + 70*(18*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b
^4 + 8*b^5)*cosh(x)^3 + (48*a^4*b - 12*a^3*b^2 - 112*a^2*b^3 + 33*a*b^4 + 6
4*b^5)*cosh(x))*sinh(x)^6 + 16*(45*a^4*b - 110*a^2*b^3 + 89*b^5)*cosh(x)^5
+ 2*(360*a^4*b - 880*a^2*b^3 + 712*b^5 + 945*(8*a^4*b - 4*a^3*b^2 - 16*a^2*
b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^4 + 105*(48*a^4*b - 12*a^3*b^2 - 112*a^2*b^3
 + 33*a*b^4 + 64*b^5)*cosh(x)^2)*sinh(x)^5 + 10*(189*(8*a^4*b - 4*a^3*b^2 -
 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^5 + 35*(48*a^4*b - 12*a^3*b^2 - 112*
a^2*b^3 + 33*a*b^4 + 64*b^5)*cosh(x)^3 + 8*(45*a^4*b - 110*a^2*b^3 + 89*b^5
)*cosh(x))*sinh(x)^4 + 10*(48*a^4*b + 12*a^3*b^2 - 112*a^2*b^3 - 33*a*b^4 +
 64*b^5)*cosh(x)^3 + 10*(126*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 +
8*b^5)*cosh(x)^6 + 48*a^4*b + 12*a^3*b^2 - 112*a^2*b^3 - 33*a*b^4 + 64*b^5
+ 35*(48*a^4*b - 12*a^3*b^2 - 112*a^2*b^3 + 33*a*b^4 + 64*b^5)*cosh(x)^4 +
16*(45*a^4*b - 110*a^2*b^3 + 89*b^5)*cosh(x)^2)*sinh(x)^3 + 10*(54*(8*a^4*b
 - 4*a^3*b^2 - 16*a^2*b^3 + 7*a*b^4 + 8*b^5)*cosh(x)^7 + 21*(48*a^4*b - 12*
a^3*b^2 - 112*a^2*b^3 + 33*a*b^4 + 64*b^5)*cosh(x)^5 + 16*(45*a^4*b - 110*a
^2*b^3 + 89*b^5)*cosh(x)^3 + 3*(48*a^4*b + 12*a^3*b^2 - 112*a^2*b^3 - 33*a*
b^4 + 64*b^5)*cosh(x))*sinh(x)^2 + 120*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^10
+ 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^9 + (a^4 - 2*a^2*b^2 + b^4)*si
nh(x)^10 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 5*(a^4 - 2*a^2*b^2 + b^4 +
 9*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^7 + 10*(a^4 - 2*a
^2*b^2 + b^4)*cosh(x)^6 + 10*(21*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 -
2*a^2*b^2 + b^4 + 14*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 70*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 1
5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 10*(a^4 - 2*a^2*b^2 + b^4)*c
osh(x)^4 + 10*(21*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 35*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 15*(a^4 - 2*a^2*b^2 + b^4)*cosh(x
)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 40*(3*(a^4 - 2*a^2*b^2 + b^4)*cosh
(x)^7 + 7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 5*(a^4 - 2*a^2*b^2 + b^4)*cos
h(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 5*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^2 + 5*(9*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 28*(a^4 - 2*a^2*b
^2 + b^4)*cosh(x)^6 + 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^
2 + b^4 + 12*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^4 - 2*a^
2*b^2 + b^4)*cosh(x)^9 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 6*(a^4 - 2*a
^2*b^2 + b^4)*cosh(x)^5 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^
2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a +
 b)*cosh(x) + (a + b)*sinh(x))) + 15*((8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(
x)^10 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^9 + (8*a^5 - 20*
a^3*b^2 + 15*a*b^4)*sinh(x)^10 + 5*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^
8 + 5*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 9*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*co
sh(x)^2)*sinh(x)^8 + 40*(3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a
^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^7 + 10*(8*a^5 - 20*a^3*b^2 + 1
5*a*b^4)*cosh(x)^6 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 21*(8*a^5 - 20*a^3
*b^2 + 15*a*b^4)*cosh(x)^4 + 14*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*
sinh(x)^6 + 4*(63*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 70*(8*a^5 - 2
0*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x
))*sinh(x)^5 + 8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 10*(8*a^5 - 20*a^3*b^2 + 15*
a*b^4)*cosh(x)^4 + 10*(21*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5
 - 20*a^3*b^2 + 15*a*b^4 + 35*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 1
5*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(3*(8*a^5 - 20*
a^3*b^2 + 15*a*b^4)*cosh(x)^7 + 7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5
```

```
 + 5*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a
*b^4)*cosh(x))*sinh(x)^3 + 5*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 5*
(9*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 28*(8*a^5 - 20*a^3*b^2 + 15*
a*b^4)*cosh(x)^6 + 8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 30*(8*a^5 - 20*a^3*b^2 +
 15*a*b^4)*cosh(x)^4 + 12*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x
)^2 + 10*((8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^9 + 4*(8*a^5 - 20*a^3*b^2
 + 15*a*b^4)*cosh(x)^7 + 6*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 4*(8
*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)*c
osh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 15*(8*a^4*b + 4*a^3*b^2 - 16*a
^2*b^3 - 7*a*b^4 + 8*b^5)*cosh(x) + 5*(27*(8*a^4*b - 4*a^3*b^2 - 16*a^2*b^3
 + 7*a*b^4 + 8*b^5)*cosh(x)^8 + 14*(48*a^4*b - 12*a^3*b^2 - 112*a^2*b^3 + 3
3*a*b^4 + 64*b^5)*cosh(x)^6 + 24*a^4*b + 12*a^3*b^2 - 48*a^2*b^3 - 21*a*b^4
 + 24*b^5 + 16*(45*a^4*b - 110*a^2*b^3 + 89*b^5)*cosh(x)^4 + 6*(48*a^4*b +
12*a^3*b^2 - 112*a^2*b^3 - 33*a*b^4 + 64*b^5)*cosh(x)^2)*sinh(x))/(b^6*cosh
(x)^10 + 10*b^6*cosh(x)*sinh(x)^9 + b^6*sinh(x)^10 + 5*b^6*cosh(x)^8 + 10*b
^6*cosh(x)^6 + 10*b^6*cosh(x)^4 + 5*(9*b^6*cosh(x)^2 + b^6)*sinh(x)^8 + 5*b
^6*cosh(x)^2 + 40*(3*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*co
sh(x)^4 + 14*b^6*cosh(x)^2 + b^6)*sinh(x)^6 + b^6 + 4*(63*b^6*cosh(x)^5 + 7
0*b^6*cosh(x)^3 + 15*b^6*cosh(x))*sinh(x)^5 + 10*(21*b^6*cosh(x)^6 + 35*b^6
*cosh(x)^4 + 15*b^6*cosh(x)^2 + b^6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 + 7*b^
6*cosh(x)^5 + 5*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8
 + 28*b^6*cosh(x)^6 + 30*b^6*cosh(x)^4 + 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2
+ 10*(b^6*cosh(x)^9 + 4*b^6*cosh(x)^7 + 6*b^6*cosh(x)^5 + 4*b^6*cosh(x)^3 +
 b^6*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**7/(a+b*tanh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23608, size = 440, normalized size = 2.8

$$\frac{\left(8\,a^5-20\,a^3b^2+15\,ab^4\right)\arctan\left(e^x\right)}{4\,b^6}-\frac{2\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^6}+\frac{120\,a^4e^{(9\,x)}-60\,a^3be^{(9\,x)}-240\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 1/4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*arctan(e^x)/b^6 - 2*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b
^6) + 1/60*(120*a^4*e^(9*x) - 60*a^3*b*e^(9*x) - 240*a^2*b^2*e^(9*x) + 105*
a*b^3*e^(9*x) + 120*b^4*e^(9*x) + 480*a^4*e^(7*x) - 120*a^3*b*e^(7*x) - 112
0*a^2*b^2*e^(7*x) + 330*a*b^3*e^(7*x) + 640*b^4*e^(7*x) + 720*a^4*e^(5*x) -
 1760*a^2*b^2*e^(5*x) + 1424*b^4*e^(5*x) + 480*a^4*e^(3*x) + 120*a^3*b*e^(3
*x) - 1120*a^2*b^2*e^(3*x) - 330*a*b^3*e^(3*x) + 640*b^4*e^(3*x) + 120*a^4*
e^x + 60*a^3*b*e^x - 240*a^2*b^2*e^x - 105*a*b^3*e^x + 120*b^4*e^x)/(b^5*(e
^(2*x) + 1)^5)
```

3.110 $\int \frac{\text{sech}^5(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=102

$$-\frac{(a^2-b^2)\,\text{sech}(x)}{b^3}-\frac{a\,(2a^2-3b^2)\tan^{-1}(\sinh(x))}{2b^4}+\frac{(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^4}+\frac{a\tanh(x)\text{sech}(x)}{2b^2}+\text{se}$$

```
[Out] -(a*(2*a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^4) + ((a^2 - b^2)^(3/2)*ArcTan[(C
osh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/b^4 - ((a^2 - b^2)*Sech[x])/b^3 +
 Sech[x]^3/(3*b) + (a*Sech[x]*Tanh[x])/(2*b^2)
```

Rubi [A] time = 0.166317, antiderivative size = 109, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3510, 3486, 3768, 3770, 3509, 206}

$$-\frac{(a^2-b^2)\,\text{sech}(x)}{b^3}-\frac{a\,(a^2-b^2)\tan^{-1}(\sinh(x))}{b^4}+\frac{(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^4}+\frac{a\tan^{-1}(\sinh(x))}{2b^2}+a\tan$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] (a*ArcTan[Sinh[x]])/(2*b^2) - (a*(a^2 - b^2)*ArcTan[Sinh[x]])/b^4 + ((a^2 -
 b^2)^(3/2)*ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/b^4 - ((a^2
- b^2)*Sech[x])/b^3 + Sech[x]^3/(3*b) + (a*Sech[x]*Tanh[x])/(2*b^2)
```

Rule 3510

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Dist[d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[(d^2*(a^2 + b^2))/b^2, Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx = \frac{\int \operatorname{sech}^3(x)(a-b\tanh(x))\,dx}{b^2} - \frac{(a^2-b^2)\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx}{b^2}$$
$$= \frac{\operatorname{sech}^3(x)}{3b} + \frac{a\int \operatorname{sech}^3(x)\,dx}{b^2} - \frac{(a^2-b^2)\int \operatorname{sech}(x)(a-b\tanh(x))\,dx}{b^4} + \frac{(a^2-b^2)^2\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx}{b^4}$$
$$= -\frac{(a^2-b^2)\operatorname{sech}(x)}{b^3} + \frac{\operatorname{sech}^3(x)}{3b} + \frac{a\operatorname{sech}(x)\tanh(x)}{2b^2} + \frac{a\int \operatorname{sech}(x)\,dx}{2b^2} - \frac{(a(a^2-b^2))\int \operatorname{sech}(x)\,d}{b^4}$$
$$= \frac{a\tan^{-1}(\sinh(x))}{2b^2} - \frac{a(a^2-b^2)\tan^{-1}(\sinh(x))}{b^4} + \frac{(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{b^4} - \frac{(a^2-b^2}{}$$

Mathematica [A] time = 0.259356, size = 116, normalized size = 1.14

$$\frac{-6\left(a(2a^2-3b^2)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+2\sqrt{a-b}\sqrt{a+b}\,(b^2-a^2)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)\right)+3b\operatorname{sech}(x)\left(-2a^2+ab\tanh(x)+2b\right.}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] (-6*(a*(2*a^2 - 3*b^2)*ArcTan[Tanh[x/2]] + 2*Sqrt[a - b]*Sqrt[a + b]*(-a^2
+ b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]) + 2*b^3*Sech[x]
^3 + 3*b*Sech[x]*(-2*a^2 + 2*b^2 + a*b*Tanh[x]))/(6*b^4)
```

Maple [B] time = 0.042, size = 316, normalized size = 3.1

$$2\frac{a^4}{b^4\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tanh(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)-4\frac{a^2}{b^2\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tanh(x/2)+2\,b}{\sqrt{a^2-b^2}}\right)+2\frac{1}{\sqrt{a^2-b^2}}\arctan\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(a+b*tanh(x)),x)
```

```
[Out] 2/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))*a^4
-4/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))*a^
2+2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/b^2
/(tanh(1/2*x)^2+1)^3*a*tanh(1/2*x)^5-2/b^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^
4*a^2+4/b/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^4-4/b^3/(tanh(1/2*x)^2+1)^3*tanh(
1/2*x)^2*a^2+4/b/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^2+1/b^2/(tanh(1/2*x)^2+1)^
```

```
3*tanh(1/2*x)*a-2/b^3/(tanh(1/2*x)^2+1)^3*a^2+8/3/b/(tanh(1/2*x)^2+1)^3-2/b
^4*arctan(tanh(1/2*x))*a^3+3/b^2*arctan(tanh(1/2*x))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.1628, size = 5131, normalized size = 50.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^5 + 15*(2*a^2*b - a*b^2 - 2*b^3)
*cosh(x)*sinh(x)^4 + 3*(2*a^2*b - a*b^2 - 2*b^3)*sinh(x)^5 + 4*(3*a^2*b - 5
*b^3)*cosh(x)^3 + 2*(6*a^2*b - 10*b^3 + 15*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x
)^2)*sinh(x)^3 + 6*(5*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^3 + 2*(3*a^2*b - 5*
b^3)*cosh(x))*sinh(x)^2 + 3*((a^2 - b^2)*cosh(x)^6 + 6*(a^2 - b^2)*cosh(x)*
sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4 + 3*(5*(a^2 - b
^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*cosh(x)^3 + 3*(a^2
- b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(5*(a^2 - b^2)*cosh
(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 6*((a^
2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x)
)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 3*((2*
a^3 - 3*a*b^2)*cosh(x)^6 + 6*(2*a^3 - 3*a*b^2)*cosh(x)*sinh(x)^5 + (2*a^3 -
 3*a*b^2)*sinh(x)^6 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^4 + 3*(2*a^3 - 3*a*b^2 +
5*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(2*a^3 - 3*a*b^2)*cosh(x)^3
 + 3*(2*a^3 - 3*a*b^2)*cosh(x))*sinh(x)^3 + 2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*
a*b^2)*cosh(x)^2 + 3*(5*(2*a^3 - 3*a*b^2)*cosh(x)^4 + 2*a^3 - 3*a*b^2 + 6*(
2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((2*a^3 - 3*a*b^2)*cosh(x)^5 + 2*
(2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*sinh(x))*arctan(co
sh(x) + sinh(x)) + 3*(2*a^2*b + a*b^2 - 2*b^3)*cosh(x) + 3*(5*(2*a^2*b - a*
b^2 - 2*b^3)*cosh(x)^4 + 2*a^2*b + a*b^2 - 2*b^3 + 4*(3*a^2*b - 5*b^3)*cosh
(x)^2)*sinh(x))/(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 +
3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*sinh(x)^4 + b
^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 + 6
*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(x)^3 + b^4*
cosh(x))*sinh(x)), -1/3*(3*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^5 + 15*(2*a^2*
b - a*b^2 - 2*b^3)*cosh(x)*sinh(x)^4 + 3*(2*a^2*b - a*b^2 - 2*b^3)*sinh(x)^
5 + 4*(3*a^2*b - 5*b^3)*cosh(x)^3 + 2*(6*a^2*b - 10*b^3 + 15*(2*a^2*b - a*b
^2 - 2*b^3)*cosh(x)^2)*sinh(x)^3 + 6*(5*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^3
 + 2*(3*a^2*b - 5*b^3)*cosh(x))*sinh(x)^2 + 6*((a^2 - b^2)*cosh(x)^6 + 6*(a
^2 - b^2)*cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)
^4 + 3*(5*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*c
osh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(
```

```
5*(a^2 - b^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 +
a^2 - b^2 + 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2
)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x)
 + (a + b)*sinh(x))) + 3*((2*a^3 - 3*a*b^2)*cosh(x)^6 + 6*(2*a^3 - 3*a*b^2)
*cosh(x)*sinh(x)^5 + (2*a^3 - 3*a*b^2)*sinh(x)^6 + 3*(2*a^3 - 3*a*b^2)*cosh
(x)^4 + 3*(2*a^3 - 3*a*b^2 + 5*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(
5*(2*a^3 - 3*a*b^2)*cosh(x)^3 + 3*(2*a^3 - 3*a*b^2)*cosh(x))*sinh(x)^3 + 2*
a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2 + 3*(5*(2*a^3 - 3*a*b^2)*cosh
(x)^4 + 2*a^3 - 3*a*b^2 + 6*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((2*
a^3 - 3*a*b^2)*cosh(x)^5 + 2*(2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2
)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(2*a^2*b + a*b^2 - 2*b^3)
*cosh(x) + 3*(5*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^4 + 2*a^2*b + a*b^2 - 2*b
^3 + 4*(3*a^2*b - 5*b^3)*cosh(x)^2)*sinh(x))/(b^4*cosh(x)^6 + 6*b^4*cosh(x)
*sinh(x)^5 + b^4*sinh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*c
osh(x)^2 + b^4)*sinh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(
x)^3 + 3*(5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(
x)^5 + 2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(a+b*tanh(x)),x)

[Out] Timed out
```

Giac [A] time = 1.26496, size = 205, normalized size = 2.01

$$-\frac{\left(2\,a^3-3\,ab^2\right)\arctan\left(e^x\right)}{b^4}+\frac{2\left(a^4-2\,a^2b^2+b^4\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^4}-\frac{6\,a^2e^{(5\,x)}-3\,abe^{(5\,x)}-6\,b^2e^{(5\,x)}+12\,a^2e^{(3\,x)}-20\,b^2e^{\cdots}}{3\,b^3\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*tanh(x)),x, algorithm="giac")

[Out] -(2*a^3 - 3*a*b^2)*arctan(e^x)/b^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((a*e^
x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^4) - 1/3*(6*a^2*e^(5*x) - 3*
a*b*e^(5*x) - 6*b^2*e^(5*x) + 12*a^2*e^(3*x) - 20*b^2*e^(3*x) + 6*a^2*e^x +
 3*a*b*e^x - 6*b^2*e^x)/(b^3*(e^(2*x) + 1)^3)
```

3.111 $\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{\sqrt{a^2-b^2}\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\operatorname{sech}(x)}{b}$$

```
[Out] (a*ArcTan[Sinh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTan[(Cosh[x]*(b + a*Tanh[x]))
/Sqrt[a^2 - b^2]])/b^2 + Sech[x]/b
```

Rubi [A] time = 0.0881527, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3510, 3486, 3770, 3509, 206}

$$-\frac{\sqrt{a^2-b^2}\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\operatorname{sech}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (a*ArcTan[Sinh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTan[(Cosh[x]*(b + a*Tanh[x]))
/Sqrt[a^2 - b^2]])/b^2 + Sech[x]/b
```

Rule 3510

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Dist[d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[(d^2*(a^2 + b^2))/b^2, Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx = \frac{\int \operatorname{sech}(x)(a-b\tanh(x))\,dx}{b^2} - \frac{\left(a^2-b^2\right)\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx}{b^2}$$
$$= \frac{\operatorname{sech}(x)}{b} + \frac{a\int \operatorname{sech}(x)\,dx}{b^2} - \frac{\left(i\left(a^2-b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, \cosh(x)(-ib-ia\tanh(x))\right)}{b^2}$$
$$= \frac{a\tan^{-1}(\sinh(x))}{b^2} - \frac{\sqrt{a^2-b^2}\tan^{-1}\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{b^2} + \frac{\operatorname{sech}(x)}{b}$$

Mathematica [A] time = 0.084032, size = 65, normalized size = 1.16

$$\frac{-2\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)+2a\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+b\operatorname{sech}(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (2*a*ArcTan[Tanh[x/2]] - 2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])
/(Sqrt[a - b]*Sqrt[a + b])] + b*Sech[x])/b^2
```

Maple [B] time = 0.03, size = 110, normalized size = 2.

$$-2\frac{a^2}{b^2\sqrt{a^2-b^2}}\arctan\left(1/2\frac{2a\tanh(x/2)+2b}{\sqrt{a^2-b^2}}\right)+2\frac{1}{\sqrt{a^2-b^2}}\arctan\left(1/2\frac{2a\tanh(x/2)+2b}{\sqrt{a^2-b^2}}\right)+2\frac{1}{b\left((\tanh(x/2))^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*tanh(x)),x)
```

```
[Out] -2/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))*a^
2+2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))+2/b/(
tanh(1/2*x)^2+1)+2/b^2*arctan(tanh(1/2*x))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56028, size = 1000, normalized size = 17.86

$$\left[\frac{\sqrt{-a^2+b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\sqrt{-a^2+b^2}(}{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(}\right.}{b^2\cosh(x)^2+2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(((a
+ b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^
2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x
)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 2*(a*cosh(x)^2 + 2*a*cosh(x)*sinh
(x) + a*sinh(x)^2 + a)*arctan(cosh(x) + sinh(x)) + 2*b*cosh(x) + 2*b*sinh(x
))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 + b^2), 2*(sqrt(a
^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(sqrt(a^2 -
 b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + (a*cosh(x)^2 + 2*a*cosh(x)*sin
h(x) + a*sinh(x)^2 + a)*arctan(cosh(x) + sinh(x)) + b*cosh(x) + b*sinh(x))/
(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 + b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*tanh(x)), x)
```

Giac [A] time = 1.24752, size = 85, normalized size = 1.52

$$\frac{2\,a\arctan\left(e^x\right)}{b^2}-\frac{2\sqrt{a^2-b^2}\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{2\,e^x}{b\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 2*a*arctan(e^x)/b^2 - 2*sqrt(a^2 - b^2)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b
^2))/b^2 + 2*e^x/(b*(e^(2*x) + 1))
```

3.112 $\int \frac{\text{sech}(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=37

$$\frac{\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2]
```

Rubi [A] time = 0.0318675, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3509, 206}

$$\frac{\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Tanh[x]),x]
```

```
[Out] ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}(x)}{a+b\tanh(x)}\,dx &= i\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, \cosh(x)(-ib-ia\tanh(x))\right)\\ &= \frac{\tan^{-1}\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] time = 0.0288495, size = 46, normalized size = 1.24

$$\frac{2\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Tanh[x]),x]
```

```
[Out] (2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(Sqrt[a - b]*Sqrt[a
 + b])
```

Maple [A] time = 0.018, size = 39, normalized size = 1.1

$$2\,{\frac {1}{\sqrt {{a}^{2}-{b}^{2}}}\arctan \left(1/2\,{\frac {2\,a\tanh \left(x/2 \right) +2\,b}{\sqrt {{a}^{2}-{b}^{2}}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*tanh(x)),x)
```

```
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16479, size = 423, normalized size = 11.43

$$\left[-\frac{\sqrt{-a^{2}+b^{2}}\log\left(\frac{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}-2\,\sqrt{-a^{2}+b^{2}}(\cosh(x)+\sinh(x))-a+b}{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}+a-b}\right)}{a^{2}-b^{2}},\, -\frac{2\,\arctan\left(\frac{\sqrt{a^{2}-b^{2}}}{(a+b)\cosh(x)+(a\cdots}\right.}{\sqrt{a^{2}-b^{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))/(a^2 - b
^2), -2*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/sqrt(a^
2 - b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}{\left (x \right)}}{a + b \tanh{\left (x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*tanh(x)), x)
```

Giac [A] time = 1.29313, size = 47, normalized size = 1.27

$$\frac{2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

3.113 $\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=73

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{b^2\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -((b^2*ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)
) - (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rubi [A] time = 0.0900475, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3511, 3486, 2637, 3509, 206}

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{b^2\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Tanh[x]),x]
```

```
[Out] -((b^2*ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)
) - (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 3511

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx &= \frac{\int \cosh(x)(a-b\tanh(x))\,dx}{a^2-b^2}-\frac{b^2\int \frac{\mathrm{sech}(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\
&= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\int \cosh(x)\,dx}{a^2-b^2}-\frac{\left(ib^2\right)\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,\cosh(x)(-ib-ia\tanh(x))\right)}{a^2-b^2}\\
&= -\frac{b^2\tan^{-1}\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] time = 0.224738, size = 80, normalized size = 1.1

$$
\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{b^2-a^2}-\frac{2b^2\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Tanh[x]),x]
```

```
[Out] (-2*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)
*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Maple [A] time = 0.038, size = 93, normalized size = 1.3

$$
-2\,\frac{b^2}{(a+b)\,(a-b)\,\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tanh\left(x/2\right)+2\,b}{\sqrt{a^2-b^2}}\right)-2\,\frac{1}{(2\,b+2\,a)\left(\tanh\left(x/2\right)-1\right)}-2\,\frac{1}{(2\,a-2\,b)\left(\tanh\left(x/2\right)+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*tanh(x)),x)
```

```
[Out] -2*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^
2)^(1/2))-2/(2*b+2*a)/(tanh(1/2*x)-1)-2/(2*a-2*b)/(tanh(1/2*x)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49567, size = 1106, normalized size = 15.15

$$\left[-\frac{a^3 + a^2b - ab^2 - b^3 - \left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)^2 - 2\left(a^3 - a^2b - ab^2 + b^3\right)\cosh\left(x\right)\sinh\left(x\right) - \left(a^3 - a^2b - ab^2 + \right.}{2\left(\left(a^4 - 2a^2b^2 + b^4\right)c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b
- a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) -
(a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(
a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(x\right)}{a + b\tanh\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*tanh(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*tanh(x)), x)
```

Giac [A] time = 1.21068, size = 82, normalized size = 1.12

$$-\frac{2\,b^2\arctan\left(\frac{ae^x + be^x}{\sqrt{a^2 - b^2}}\right)}{\left(a^2 - b^2\right)^{\frac{3}{2}}} - \frac{e^{(-x)}}{2\left(a - b\right)} + \frac{e^x}{2\left(a + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(-
x)/(a - b) + 1/2*e^x/(a + b)
```

3.114 $\int \frac{\cosh^3(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=132

$$\frac{a\sinh^3(x)}{3(a^2-b^2)}-\frac{ab^2\sinh(x)}{(a^2-b^2)^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh^3(x)}{3(a^2-b^2)}+\frac{b^3\cosh(x)}{(a^2-b^2)^2}+\frac{b^4\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

```
[Out] (b^4*ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) +
 (b^3*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) - (a*b^2*Sinh[
x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^2))
```

Rubi [A] time = 0.15572, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3511, 3486, 2633, 2637, 3509, 206}

$$\frac{a\sinh^3(x)}{3(a^2-b^2)}-\frac{ab^2\sinh(x)}{(a^2-b^2)^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh^3(x)}{3(a^2-b^2)}+\frac{b^3\cosh(x)}{(a^2-b^2)^2}+\frac{b^4\tan^{-1}\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (b^4*ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) +
 (b^3*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) - (a*b^2*Sinh[
x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^2))
```

Rule 3511

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\tanh(x)}\,dx = \frac{\int \cosh^3(x)(a-b\tanh(x))\,dx}{a^2-b^2} - \frac{b^2\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}$$
$$= -\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)} - \frac{b^2\int \cosh(x)(a-b\tanh(x))\,dx}{\left(a^2-b^2\right)^2} + \frac{b^4\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{a\int \cosh^3(x)\,dx}{a^2-b^2}$$
$$= \frac{b^3\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{b\cosh^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(ab^2\right)\int \cosh(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(ib^4\right)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, \cosh(x)(-ib-i\ldots\right.}{\left(a^2-b^2\right)^2}$$
$$= \frac{b^4\tan^{-1}\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b^3\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{b\cosh^3(x)}{3\left(a^2-b^2\right)} - \frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh^3(\ldots}{3\left(a^2-b\ldots\right.}$$

Mathematica [A] time = 0.45916, size = 258, normalized size = 1.95

$$\frac{-21a^2b^2\sqrt{a-b}\sinh(x) - a^2b^2\sqrt{a-b}\sinh(3x) - 3b\sqrt{a-b}\left(a^2b+a^3-5ab^2-5b^3\right)\cosh(x) + 9a^4\sqrt{a-b}\sinh(x) + a^4\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (24*b^4*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] - 3
*Sqrt[a - b]*b*(a^3 + a^2*b - 5*a*b^2 - 5*b^3)*Cosh[x] - (a - b)^(3/2)*b*(a
 + b)^2*Cosh[3*x] + 9*a^4*Sqrt[a - b]*Sinh[x] + 9*a^3*Sqrt[a - b]*b*Sinh[x]
 - 21*a^2*Sqrt[a - b]*b^2*Sinh[x] - 21*a*Sqrt[a - b]*b^3*Sinh[x] + a^4*Sqrt
[a - b]*Sinh[3*x] + a^3*Sqrt[a - b]*b*Sinh[3*x] - a^2*Sqrt[a - b]*b^2*Sinh[
3*x] - a*Sqrt[a - b]*b^3*Sinh[3*x])/(12*(a - b)^(5/2)*(a + b)^3)
```

Maple [A] time = 0.044, size = 198, normalized size = 1.5

$$-\frac{2}{6a-6b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} + \frac{1}{2a-2b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \frac{a}{(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{3b}{2(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*tanh(x)),x)
```

```
[Out] -2/3/(tanh(1/2*x)+1)^3/(2*a-2*b)+1/(2*a-2*b)/(tanh(1/2*x)+1)^2-1/(a-b)^2/(t
anh(1/2*x)+1)*a+3/2/(a-b)^2/(tanh(1/2*x)+1)*b+2*b^4/(a-b)^2/(a+b)^2/(a^2-b^
```

```
2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-2/3/(tanh(1/2*x)
-1)^3/(2*b+2*a)-1/(2*b+2*a)/(tanh(1/2*x)-1)^2-1/(a+b)^2/(tanh(1/2*x)-1)*a-3
/2/(a+b)^2/(tanh(1/2*x)-1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56586, size = 4251, normalized size = 32.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*
a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b
^3 + 7*a*b^4 - 5*b^5)*cosh(x)^4 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3
 + 7*a*b^4 - 5*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^3 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^
5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4
 + 5*b^5)*cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 +
 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 -
6*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sin
h(x)^2 - 24*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x
)^2 + b^4*sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x))
- a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
 + a - b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^5 + 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^3
 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*sinh
(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
cosh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b
- 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2
+ 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a
*b^4 - 5*b^5)*cosh(x)^4 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b
^4 - 5*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^
2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cos
h(x)^3 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(
x))*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5
)*cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5 -
 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5
```

```
- a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sinh(x)^2 -
48*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^
4*sinh(x)^3)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a +
b)*sinh(x))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 + 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x
)^3 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*s
inh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*tanh(x)), x)
```

Giac [A] time = 1.26922, size = 219, normalized size = 1.66

$$\frac{2\,b^4\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{\left(9\,ae^{(2\,x)}-15\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}+9\,a^2e^x+24\,abe^x+15}{24\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt
(a^2 - b^2)) - 1/24*(9*a*e^(2*x) - 15*b*e^(2*x) + a - b)*e^(-3*x)/(a^2 - 2*
a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 9*a^2*e^x +
24*a*b*e^x + 15*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.115 $\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=43

$$\frac{5x}{2}+\frac{\tanh^4(x)}{2(\tanh(x)+1)}-\frac{5\tanh^3(x)}{6}+\tanh^2(x)-\frac{5\tanh(x)}{2}-2\log(\cosh(x))$$

```
[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Ta
nh[x]^4/(2*(1 + Tanh[x]))
```

Rubi [A] time = 0.0893573, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3550, 3528, 3525, 3475}

$$\frac{5x}{2}+\frac{\tanh^4(x)}{2(\tanh(x)+1)}-\frac{5\tanh^3(x)}{6}+\tanh^2(x)-\frac{5\tanh(x)}{2}-2\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(1 + Tanh[x]),x]
```

```
[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Ta
nh[x]^4/(2*(1 + Tanh[x]))
```

Rule 3550

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(c + d*Tan[e + f*x])^(n - 1))/
(2*a*f*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx &= \frac{\tanh^4(x)}{2(1+\tanh(x))} - \frac{1}{2}\int (4-5\tanh(x))\tanh^3(x)\,dx \\ &= -\frac{5}{6}\tanh^3(x) + \frac{\tanh^4(x)}{2(1+\tanh(x))} + \frac{1}{2}i\int (-5i+4i\tanh(x))\tanh^2(x)\,dx \\ &= \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^4(x)}{2(1+\tanh(x))} + \frac{1}{2}\int \tanh(x)(-4+5\tanh(x))\,dx \\ &= \frac{5x}{2} - \frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^4(x)}{2(1+\tanh(x))} - 2\int \tanh(x)\,dx \\ &= \frac{5x}{2} - 2\log(\cosh(x)) - \frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^4(x)}{2(1+\tanh(x))}\end{aligned}$$

Mathematica [A] time = 0.09463, size = 40, normalized size = 0.93

$$\frac{1}{12}\left(30x - 3\sinh(2x) + 3\cosh(2x) - 28\tanh(x) - 24\log(\cosh(x)) + (4\tanh(x)-6)\mathbf{sech}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(1 + Tanh[x]),x]
```

```
[Out] (30*x + 3*Cosh[2*x] - 24*Log[Cosh[x]] - 3*Sinh[2*x] - 28*Tanh[x] + Sech[x]^
2*(-6 + 4*Tanh[x]))/12
```

Maple [A] time = 0.023, size = 40, normalized size = 0.9

$$-\frac{(\tanh(x))^3}{3} + \frac{(\tanh(x))^2}{2} - 2\tanh(x) + \frac{1}{2+2\tanh(x)} + \frac{9\ln(1+\tanh(x))}{4} - \frac{\ln(\tanh(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(1+tanh(x)),x)
```

```
[Out] -1/3*tanh(x)^3+1/2*tanh(x)^2-2*tanh(x)+1/2/(1+tanh(x))+9/4*ln(1+tanh(x))-1/
4*ln(tanh(x)-1)
```

Maxima [A] time = 1.64636, size = 74, normalized size = 1.72

$$\frac{1}{2}x - \frac{2\left(15e^{(-2x)} + 12e^{(-4x)} + 7\right)}{3\left(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1\right)} + \frac{1}{4}e^{(-2x)} - 2\log\left(e^{(-2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/3*(15*e^(-2*x) + 12*e^(-4*x) + 7)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-
6*x) + 1) + 1/4*e^(-2*x) - 2*log(e^(-2*x) + 1)
```

Fricas [B] time = 2.36756, size = 1897, normalized size = 44.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 + 3*(54*x +
 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 + 54*x + 17)*sinh(x)^6 + 18*(168*x*cosh
(x)^3 + (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420*x*
cosh(x)^4 + 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*cosh(
x)^5 + 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 + (54*x +
65)*cosh(x)^2 + (1512*x*cosh(x)^6 + 45*(54*x + 17)*cosh(x)^4 + 486*(2*x + 1
)*cosh(x)^2 + 54*x + 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^3 +
9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cosh(x)
^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh(x)^
7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) -
sinh(x))) + 2*(216*x*cosh(x)^7 + 9*(54*x + 17)*cosh(x)^5 + 162*(2*x + 1)*co
sh(x)^3 + (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^
7 + sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^
3 + 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cos
h(x)^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh
(x)^7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [B] time = 0.636991, size = 104, normalized size = 2.42

$$\frac{3x\tanh(x)}{6\tanh(x)+6}+\frac{3x}{6\tanh(x)+6}+\frac{12\log(\tanh(x)+1)\tanh(x)}{6\tanh(x)+6}+\frac{12\log(\tanh(x)+1)}{6\tanh(x)+6}-\frac{2\tanh^4(x)}{6\tanh(x)+6}+\frac{\tanh^3(x}{6\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(1+tanh(x)),x)
```

```
[Out] 3*x*tanh(x)/(6*tanh(x) + 6) + 3*x/(6*tanh(x) + 6) + 12*log(tanh(x) + 1)*tan
h(x)/(6*tanh(x) + 6) + 12*log(tanh(x) + 1)/(6*tanh(x) + 6) - 2*tanh(x)**4/(
6*tanh(x) + 6) + tanh(x)**3/(6*tanh(x) + 6) - 9*tanh(x)**2/(6*tanh(x) + 6)
+ 15/(6*tanh(x) + 6)
```

Giac [A] time = 1.23858, size = 63, normalized size = 1.47

$$\frac{9}{2}x+\frac{\left(51e^{(6x)}+81e^{(4x)}+65e^{(2x)}+3\right)e^{(-2x)}}{12\left(e^{(2x)}+1\right)^3}-2\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 9/2*x + 1/12*(51*e^(6*x) + 81*e^(4*x) + 65*e^(2*x) + 3)*e^(-2*x)/(e^(2*x) +
 1)^3 - 2*log(e^(2*x) + 1)
```

3.116 $\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{3x}{2}+\frac{\tanh^3(x)}{2(\tanh(x)+1)}-\tanh^2(x)+\frac{3\tanh(x)}{2}+2\log(\cosh(x))$$

```
[Out] (-3*x)/2 + 2*Log[Cosh[x]] + (3*Tanh[x])/2 - Tanh[x]^2 + Tanh[x]^3/(2*(1 + T
anh[x]))
```

Rubi [A] time = 0.0657397, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3550, 3528, 3525, 3475}

$$-\frac{3x}{2}+\frac{\tanh^3(x)}{2(\tanh(x)+1)}-\tanh^2(x)+\frac{3\tanh(x)}{2}+2\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (-3*x)/2 + 2*Log[Cosh[x]] + (3*Tanh[x])/2 - Tanh[x]^2 + Tanh[x]^3/(2*(1 + T
anh[x]))
```

Rule 3550

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(c + d*Tan[e + f*x])^(n - 1))/
(2*a*f*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx = \frac{\tanh^3(x)}{2(1+\tanh(x))} - \frac{1}{2}\int (3-4\tanh(x))\tanh^2(x)\,dx$$
$$= -\tanh^2(x) + \frac{\tanh^3(x)}{2(1+\tanh(x))} + \frac{1}{2}i\int (-4i+3i\tanh(x))\tanh(x)\,dx$$
$$= -\frac{3x}{2} + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^3(x)}{2(1+\tanh(x))} + 2\int \tanh(x)\,dx$$
$$= -\frac{3x}{2} + 2\log(\cosh(x)) + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^3(x)}{2(1+\tanh(x))}$$

Mathematica [A] time = 0.0516263, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(-6x+\sinh(2x)-\cosh(2x)+4\tanh(x)+2\text{sech}^2(x)+8\log(\cosh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (-6*x - Cosh[2*x] + 8*Log[Cosh[x]] + 2*Sech[x]^2 + Sinh[2*x] + 4*Tanh[x])/4
```

Maple [A] time = 0.019, size = 32, normalized size = 0.9

$$-\frac{(\tanh(x))^2}{2} + \tanh(x) - \frac{1}{2+2\tanh(x)} - \frac{7\ln(1+\tanh(x))}{4} - \frac{\ln(\tanh(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(1+tanh(x)),x)
```

```
[Out] -1/2*tanh(x)^2+tanh(x)-1/2/(1+tanh(x))-7/4*ln(1+tanh(x))-1/4*ln(tanh(x)-1)
```

Maxima [A] time = 1.76215, size = 58, normalized size = 1.57

$$\frac{1}{2}x + \frac{2\left(2e^{(-2x)}+1\right)}{2e^{(-2x)}+e^{(-4x)}+1} - \frac{1}{4}e^{(-2x)} + 2\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) + 1)/(2*e^(-2*x) + e^(-4*x) + 1) - 1/4*e^(-2*x) + 2*l
og(e^(-2*x) + 1)
```

Fricas [B] time = 2.32173, size = 1168, normalized size = 31.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 + (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 + 28*x + 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 + (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 +
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh
(x)^3 + 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 + (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh(x)^3 + 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [B] time = 0.538901, size = 85, normalized size = 2.3

$$\frac{x\tanh(x)}{2\tanh(x)+2}+\frac{x}{2\tanh(x)+2}-\frac{4\log(\tanh(x)+1)\tanh(x)}{2\tanh(x)+2}-\frac{4\log(\tanh(x)+1)}{2\tanh(x)+2}-\frac{\tanh^3(x)}{2\tanh(x)+2}+\frac{\tanh^2(x)}{2\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(1+tanh(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 4*log(tanh(x) + 1)*tanh(x)/
(2*tanh(x) + 2) - 4*log(tanh(x) + 1)/(2*tanh(x) + 2) - tanh(x)**3/(2*tanh(x
) + 2) + tanh(x)**2/(2*tanh(x) + 2) - 3/(2*tanh(x) + 2)
```

Giac [A] time = 1.27143, size = 53, normalized size = 1.43

$$-\frac{7}{2}x-\frac{\left(e^{(4x)}+10e^{(2x)}+1\right)e^{(-2x)}}{4\left(e^{(2x)}+1\right)^2}+2\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -7/2*x - 1/4*(e^(4*x) + 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1)^2 + 2*log(e^
(2*x) + 1)
```

3.117 $\int \frac{\tanh^3(x)}{1+\tanh(x)} dx$

Optimal. Leaf size=31

$$\frac{3x}{2} + \frac{\tanh^2(x)}{2(\tanh(x)+1)} - \frac{3\tanh(x)}{2} - \log(\cosh(x))$$

```
[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]^2/(2*(1 + Tanh[x]))
```

Rubi [A] time = 0.0512713, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3550, 3525, 3475}

$$\frac{3x}{2} + \frac{\tanh^2(x)}{2(\tanh(x)+1)} - \frac{3\tanh(x)}{2} - \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]^2/(2*(1 + Tanh[x]))
```

Rule 3550

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(c + d*Tan[e + f*x])^(n - 1))/
(2*a*f*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{1+\tanh(x)} dx &= \frac{\tanh^2(x)}{2(1+\tanh(x))} - \frac{1}{2}\int (2-3\tanh(x))\tanh(x)\, dx \\
&= \frac{3x}{2} - \frac{3\tanh(x)}{2} + \frac{\tanh^2(x)}{2(1+\tanh(x))} - \int \tanh(x)\, dx \\
&= \frac{3x}{2} - \log(\cosh(x)) - \frac{3\tanh(x)}{2} + \frac{\tanh^2(x)}{2(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] time = 0.0453772, size = 27, normalized size = 0.87

$$\frac{1}{4}(6x-\sinh(2x)+\cosh(2x)-4\tanh(x)-4\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (6*x + Cosh[2*x] - 4*Log[Cosh[x]] - Sinh[2*x] - 4*Tanh[x])/4
```

Maple [A] time = 0.017, size = 28, normalized size = 0.9

$$-\tanh(x)+\frac{1}{2+2\tanh(x)}+\frac{5\ln(1+\tanh(x))}{4}-\frac{\ln(\tanh(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(1+tanh(x)),x)
```

```
[Out] -tanh(x)+1/2/(1+tanh(x))+5/4*ln(1+tanh(x))-1/4*ln(tanh(x)-1)
```

Maxima [A] time = 1.92124, size = 39, normalized size = 1.26

$$\frac{1}{2}x-\frac{2}{e^{(-2x)}+1}+\frac{1}{4}e^{(-2x)}-\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/(e^(-2*x) + 1) + 1/4*e^(-2*x) - log(e^(-2*x) + 1)
```

Fricas [B] time = 2.28245, size = 632, normalized size = 20.39

$$\frac{10\,x\cosh(x)^4+40\,x\cosh(x)\sinh(x)^3+10\,x\sinh(x)^4+(10\,x+9)\cosh(x)^2+\left(60\,x\cosh(x)^2+10\,x+9\right)\sinh(x)^2-\cdots}{4\left(\cosh(x)^4+4\cosh\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 + (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 + 10*x + 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*co
sh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 + (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + c
osh(x))*sinh(x))
```

Sympy [B] time = 0.470005, size = 75, normalized size = 2.42

$$\frac{x\tanh(x)}{2\tanh(x)+2}+\frac{x}{2\tanh(x)+2}+\frac{2\log(\tanh(x)+1)\tanh(x)}{2\tanh(x)+2}+\frac{2\log(\tanh(x)+1)}{2\tanh(x)+2}-\frac{2\tanh^2(x)}{2\tanh(x)+2}+\frac{3}{2\tanh(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(1+tanh(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 2*log(tanh(x) + 1)*tanh(x)/
(2*tanh(x) + 2) + 2*log(tanh(x) + 1)/(2*tanh(x) + 2) - 2*tanh(x)**2/(2*tanh
(x) + 2) + 3/(2*tanh(x) + 2)
```

Giac [A] time = 1.19976, size = 47, normalized size = 1.52

$$\frac{5}{2}x+\frac{\left(9e^{(2x)}+1\right)e^{(-2x)}}{4\left(e^{(2x)}+1\right)}-\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 5/2*x + 1/4*(9*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1) - log(e^(2*x) + 1)
```

3.118 $\int \frac{\tanh^2(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{x}{2}-\frac{1}{2(\tanh(x)+1)}+\log(\cosh(x))$$

```
[Out] -x/2 + Log[Cosh[x]] - 1/(2*(1 + Tanh[x]))
```

Rubi [A] time = 0.0375599, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3540, 3475}

$$-\frac{x}{2}-\frac{1}{2(\tanh(x)+1)}+\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(1 + Tanh[x]),x]
```

```
[Out] -x/2 + Log[Cosh[x]] - 1/(2*(1 + Tanh[x]))
```

Rule 3540

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(a*c + b*d)^2*(a + b*Tan[e + f*x])^m)
/(2*a^3*f*m), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*
c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{1+\tanh(x)}\,dx &= -\frac{1}{2(1+\tanh(x))}-\frac{1}{2}\int (1-2\tanh(x))\,dx\\ &= -\frac{x}{2}-\frac{1}{2(1+\tanh(x))}+\int \tanh(x)\,dx\\ &= -\frac{x}{2}+\log(\cosh(x))-\frac{1}{2(1+\tanh(x))}\end{aligned}$$

Mathematica [A] time = 0.0276375, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x+\sinh(2x)-\cosh(2x)+4\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(1 + Tanh[x]),x]
```

```
[Out] (-2*x - Cosh[2*x] + 4*Log[Cosh[x]] + Sinh[2*x])/4
```

Maple [A] time = 0.016, size = 24, normalized size = 1.3

$$-\frac{1}{2+2\,\tanh\left(x\right)}-\frac{3\,\ln\left(1+\tanh\left(x\right)\right)}{4}-\frac{\ln\left(\tanh\left(x\right)-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(1+tanh(x)),x)
```

```
[Out] -1/2/(1+tanh(x))-3/4*ln(1+tanh(x))-1/4*ln(tanh(x)-1)
```

Maxima [A] time = 1.84957, size = 23, normalized size = 1.21

$$\frac{1}{2}\,x-\frac{1}{4}\,e^{(-2\,x)}+\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*e^(-2*x) + log(e^(-2*x) + 1)
```

Fricas [B] time = 2.32245, size = 259, normalized size = 13.63

$$-\frac{6\,x\cosh\left(x\right)^{2}+12\,x\cosh\left(x\right)\sinh\left(x\right)+6\,x\sinh\left(x\right)^{2}-4\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right.}{4\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] time = 0.428343, size = 61, normalized size = 3.21

$$\frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}-\frac{2\log\left(\tanh\left(x\right)+1\right)\tanh\left(x\right)}{2\tanh\left(x\right)+2}-\frac{2\log\left(\tanh\left(x\right)+1\right)}{2\tanh\left(x\right)+2}-\frac{1}{2\tanh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(1+tanh(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 2*log(tanh(x) + 1)*tanh(x)/
(2*tanh(x) + 2) - 2*log(tanh(x) + 1)/(2*tanh(x) + 2) - 1/(2*tanh(x) + 2)
```

Giac [A] time = 1.20933, size = 23, normalized size = 1.21

$$-\frac{3}{2}x - \frac{1}{4}e^{(-2x)} + \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -3/2*x - 1/4*e^(-2*x) + log(e^(2*x) + 1)
```

3.119 $\int \frac{\tanh(x)}{1+\tanh(x)} \, dx$

Optimal. Leaf size=16

$$\frac{x}{2} + \frac{1}{2(\tanh(x)+1)}$$

```
[Out] x/2 + 1/(2*(1 + Tanh[x]))
```

Rubi [A] time = 0.0198789, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3526, 8}

$$\frac{x}{2} + \frac{1}{2(\tanh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(1 + Tanh[x]),x]

[Out] x/2 + 1/(2*(1 + Tanh[x]))
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh(x)}{1+\tanh(x)} \, dx &= \frac{1}{2(1+\tanh(x))} + \frac{\int 1 \, dx}{2} \\ &= \frac{x}{2} + \frac{1}{2(1+\tanh(x))} \end{aligned}$$

Mathematica [A] time = 0.0260664, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x - \sinh(2x) + \cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(1 + Tanh[x]),x]

[Out] (2*x + Cosh[2*x] - Sinh[2*x])/4
```

Maple [A] time = 0.016, size = 24, normalized size = 1.5

$$\frac{1}{2+2\,\tanh\left(x\right)}+\frac{\ln\left(1+\tanh\left(x\right)\right)}{4}-\frac{\ln\left(\tanh\left(x\right)-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1+tanh(x)),x)
```

```
[Out] 1/2/(1+tanh(x))+1/4*ln(1+tanh(x))-1/4*ln(tanh(x)-1)
```

Maxima [A] time = 1.26689, size = 14, normalized size = 0.88

$$\frac{1}{2}\,x+\frac{1}{4}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

Fricas [B] time = 2.21052, size = 88, normalized size = 5.5

$$\frac{\left(2\,x+1\right)\cosh\left(x\right)+\left(2\,x-1\right)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.387998, size = 27, normalized size = 1.69

$$\frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}+\frac{1}{2\tanh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Giac [A] time = 1.20642, size = 14, normalized size = 0.88

$$\frac{1}{2}\,x+\frac{1}{4}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

3.120 $\int \frac{1}{1+\tanh(x)} dx$

Optimal. Leaf size=16

$$\frac{x}{2} - \frac{1}{2(\tanh(x)+1)}$$

```
[Out] x/2 - 1/(2*(1 + Tanh[x]))
```

Rubi [A] time = 0.0084776, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{2} - \frac{1}{2(\tanh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Tanh[x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\tanh(x)} dx &= -\frac{1}{2(1+\tanh(x))} + \frac{\int 1 \, dx}{2} \\ &= \frac{x}{2} - \frac{1}{2(1+\tanh(x))} \end{aligned}$$

Mathematica [A] time = 0.0147839, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x + \sinh(2x) - \cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x])^(-1),x]
```

```
[Out] (2*x - Cosh[2*x] + Sinh[2*x])/4
```

Maple [A] time = 0.014, size = 24, normalized size = 1.5

$$-\frac{1}{2+2\tanh(x)}+\frac{\ln(1+\tanh(x))}{4}-\frac{\ln(\tanh(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tanh(x)),x)
```

```
[Out] -1/2/(1+tanh(x))+1/4*ln(1+tanh(x))-1/4*ln(tanh(x)-1)
```

Maxima [A] time = 1.18072, size = 14, normalized size = 0.88

$$\frac{1}{2}x-\frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*e^(-2*x)
```

Fricas [B] time = 2.1037, size = 88, normalized size = 5.5

$$\frac{(2x-1)\cosh(x)+(2x+1)\sinh(x)}{4(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*x - 1)*cosh(x) + (2*x + 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.396384, size = 27, normalized size = 1.69

$$\frac{x\tanh(x)}{2\tanh(x)+2}+\frac{x}{2\tanh(x)+2}-\frac{1}{2\tanh(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 1/(2*tanh(x) + 2)
```

Giac [A] time = 1.19661, size = 14, normalized size = 0.88

$$\frac{1}{2}x-\frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*e^(-2*x)
```

3.121 $\int \frac{\coth(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{x}{2}+\frac{1}{2(\tanh(x)+1)}+\log(\sinh(x))$$

```
[Out] -x/2 + Log[Sinh[x]] + 1/(2*(1 + Tanh[x]))
```

Rubi [A] time = 0.0404755, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3551, 3479, 8, 3475}

$$-\frac{x}{2}+\frac{1}{2(\tanh(x)+1)}+\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(1 + Tanh[x]),x]
```

```
[Out] -x/2 + Log[Sinh[x]] + 1/(2*(1 + Tanh[x]))
```

Rule 3551

```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Tan[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{1+\tanh(x)}\,dx &= \int \coth(x)\,dx-\int \frac{1}{1+\tanh(x)}\,dx\\
&= \log(\sinh(x))+\frac{1}{2(1+\tanh(x))}-\frac{\int 1\,dx}{2}\\
&= -\frac{x}{2}+\log(\sinh(x))+\frac{1}{2(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] time = 0.0281768, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x - \sinh(2x) + \cosh(2x) + 4\log(\sinh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 + Tanh[x]),x]
```

```
[Out] (-2*x + Cosh[2*x] + 4*Log[Sinh[x]] - Sinh[2*x])/4
```

Maple [B] time = 0.027, size = 43, normalized size = 2.3

$$-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{3}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+tanh(x)),x)
```

```
[Out] -1/(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)^2-3/2*ln(tanh(1/2*x)+1)+ln(tanh(1/2*x)
)-1/2*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.27018, size = 32, normalized size = 1.68

$$\frac{1}{2}x + \frac{1}{4}e^{(-2x)} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*e^(-2*x) + log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [B] time = 2.37501, size = 259, normalized size = 13.63

$$-\frac{6\,x\cosh(x)^2 + 12\,x\cosh(x)\sinh(x) + 6\,x\sinh(x)^2 - 4\left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2\right)\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right) - 1}{4\left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+tanh(x)),x)
```

```
[Out] Integral(coth(x)/(tanh(x) + 1), x)
```

Giac [A] time = 1.20533, size = 24, normalized size = 1.26

$$-\frac{3}{2}x+\frac{1}{4}e^{(-2x)}+\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -3/2*x + 1/4*e^(-2*x) + log(abs(e^(2*x) - 1))
```

3.122 $\int \frac{\coth^2(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=29

$$\frac{3x}{2}-\frac{3\coth(x)}{2}-\log(\sinh(x))+\frac{\coth(x)}{2(\tanh(x)+1)}$$

```
[Out] (3*x)/2 - (3*Coth[x])/2 - Log[Sinh[x]] + Coth[x]/(2*(1 + Tanh[x]))
```

Rubi [A] time = 0.0676766, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3552, 3529, 3531, 3475}

$$\frac{3x}{2}-\frac{3\coth(x)}{2}-\log(\sinh(x))+\frac{\coth(x)}{2(\tanh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(1 + Tanh[x]),x]
```

```
[Out] (3*x)/2 - (3*Coth[x])/2 - Log[Sinh[x]] + Coth[x]/(2*(1 + Tanh[x]))
```

Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c
 - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Ta
n[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^2(x)}{1+\tanh(x)}\,dx = \frac{\coth(x)}{2(1+\tanh(x))} - \frac{1}{2}\int \coth^2(x)(-3+2\tanh(x))\,dx$$
$$= -\frac{3\coth(x)}{2} + \frac{\coth(x)}{2(1+\tanh(x))} - \frac{1}{2}i\int \coth(x)(-2i+3i\tanh(x))\,dx$$
$$= \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{\coth(x)}{2(1+\tanh(x))} - \int \coth(x)\,dx$$
$$= \frac{3x}{2} - \frac{3\coth(x)}{2} - \log(\sinh(x)) + \frac{\coth(x)}{2(1+\tanh(x))}$$

Mathematica [A] time = 0.0433458, size = 27, normalized size = 0.93

$$\frac{1}{4}(6x+\sinh(2x)-\cosh(2x)-4\coth(x)-4\log(\sinh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(1 + Tanh[x]),x]
```

```
[Out] (6*x - Cosh[2*x] - 4*Coth[x] - 4*Log[Sinh[x]] + Sinh[2*x])/4
```

Maple [B] time = 0.029, size = 59, normalized size = 2.

$$-\frac{1}{2}\tanh\left(\frac{x}{2}\right)-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{5}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{2}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1+tanh(x)),x)
```

```
[Out] -1/2*tanh(1/2*x)-1/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)+5/2*ln(tanh(1/2*x)+1
)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))-1/2*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.21798, size = 51, normalized size = 1.76

$$\frac{1}{2}x+\frac{2}{e^{(-2x)}-1}-\frac{1}{4}e^{(-2x)}-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2/(e^(-2*x) - 1) - 1/4*e^(-2*x) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] time = 2.25379, size = 632, normalized size = 21.79

$$\frac{10\,x\cosh(x)^4+40\,x\cosh(x)\sinh(x)^3+10\,x\sinh(x)^4-(10\,x+9)\cosh(x)^2+\left(60\,x\cosh(x)^2-10\,x-9\right)\sinh(x)^2-4\left(\right.}{4\left(\cosh(x)^4+4\,\cosh(x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 - (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 - 10*x - 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*co
sh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 - (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - c
osh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(1+tanh(x)),x)
```

```
[Out] Integral(coth(x)**2/(tanh(x) + 1), x)
```

Giac [A] time = 1.20825, size = 49, normalized size = 1.69

$$\frac{5}{2}\,x - \frac{\left(9\,e^{(2\,x)} - 1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)} - 1\right)} - \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 5/2*x - 1/4*(9*e^(2*x) - 1)*e^(-2*x)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

3.123 $\int \frac{\coth^3(x)}{1+\tanh(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{3x}{2}-\coth^2(x)+\frac{3\coth(x)}{2}+2\log(\sinh(x))+\frac{\coth^2(x)}{2(\tanh(x)+1)}$$

```
[Out] (-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + 2*Log[Sinh[x]] + Coth[x]^2/(2*(1 + T
anh[x]))
```

Rubi [A] time = 0.0888637, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3552, 3529, 3531, 3475}

$$-\frac{3x}{2}-\coth^2(x)+\frac{3\coth(x)}{2}+2\log(\sinh(x))+\frac{\coth^2(x)}{2(\tanh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + 2*Log[Sinh[x]] + Coth[x]^2/(2*(1 + T
anh[x]))
```

Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c
 - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Ta
n[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^3(x)}{1+\tanh(x)}\,dx = \frac{\coth^2(x)}{2(1+\tanh(x))} - \frac{1}{2}\int \coth^3(x)(-4+3\tanh(x))\,dx$$
$$= -\coth^2(x) + \frac{\coth^2(x)}{2(1+\tanh(x))} - \frac{1}{2}i\int \coth^2(x)(-3i+4i\tanh(x))\,dx$$
$$= \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^2(x)}{2(1+\tanh(x))} + \frac{1}{2}\int \coth(x)(4-3\tanh(x))\,dx$$
$$= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^2(x)}{2(1+\tanh(x))} + 2\int \coth(x)\,dx$$
$$= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + 2\log(\sinh(x)) + \frac{\coth^2(x)}{2(1+\tanh(x))}$$

Mathematica [A] time = 0.0589725, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(-6x - \sinh(2x) + \cosh(2x) + 4\coth(x) - 2\mathrm{csch}^2(x) + 8\log(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (-6*x + Cosh[2*x] + 4*Coth[x] - 2*Csch[x]^2 + 8*Log[Sinh[x]] - Sinh[2*x])/4
```

Maple [B] time = 0.032, size = 75, normalized size = 2.

$$-\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{1}{2}\tanh\left(\frac{x}{2}\right) + \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{7}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(1+tanh(x)),x)
```

```
[Out] -1/8*tanh(1/2*x)^2+1/2*tanh(1/2*x)+1/(tanh(1/2*x)+1)^2-1/(tanh(1/2*x)+1)-7/
2*ln(tanh(1/2*x)+1)-1/8/tanh(1/2*x)^2+1/2/tanh(1/2*x)+2*ln(tanh(1/2*x))-1/2
*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.39836, size = 73, normalized size = 1.97

$$\frac{1}{2}x + \frac{2\left(2e^{(-2x)}-1\right)}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{1}{4}e^{(-2x)} + 2\log\left(e^{(-x)}+1\right) + 2\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) - 1)/(2*e^(-2*x) - e^(-4*x) - 1) + 1/4*e^(-2*x) + 2*l
og(e^(-x) + 1) + 2*log(e^(-x) - 1)
```

Fricas [B] time = 2.21779, size = 1168, normalized size = 31.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 - (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 - 28*x - 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 - (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 -
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh
(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*sinh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 - (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) - 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(1+tanh(x)),x)
```

```
[Out] Integral(coth(x)**3/(tanh(x) + 1), x)
```

Giac [A] time = 1.18321, size = 54, normalized size = 1.46

$$-\frac{7}{2}\,x+\frac{\left(e^{(4\,x)}-10\,e^{(2\,x)}+1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}-1\right)^2}+2\,\log\left(\left|e^{(2\,x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -7/2*x + 1/4*(e^(4*x) - 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) - 1)^2 + 2*log(ab
s(e^(2*x) - 1))
```

3.124 $\int \frac{\coth^4(x)}{1+\tanh(x)} \, dx$

Optimal. Leaf size=43

$$\frac{5x}{2}-\frac{5\coth^3(x)}{6}+\coth^2(x)-\frac{5\coth(x)}{2}-2\log(\sinh(x))+\frac{\coth^3(x)}{2(\tanh(x)+1)}$$

```
[Out] (5*x)/2 - (5*Coth[x])/2 + Coth[x]^2 - (5*Coth[x]^3)/6 - 2*Log[Sinh[x]] + Co
th[x]^3/(2*(1 + Tanh[x]))
```

Rubi [A] time = 0.10075, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3552, 3529, 3531, 3475}

$$\frac{5x}{2}-\frac{5\coth^3(x)}{6}+\coth^2(x)-\frac{5\coth(x)}{2}-2\log(\sinh(x))+\frac{\coth^3(x)}{2(\tanh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (5*x)/2 - (5*Coth[x])/2 + Coth[x]^2 - (5*Coth[x]^3)/6 - 2*Log[Sinh[x]] + Co
th[x]^3/(2*(1 + Tanh[x]))
```

Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c
 - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Ta
n[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx = \frac{\coth^3(x)}{2(1+\tanh(x))} - \frac{1}{2}\int \coth^4(x)(-5+4\tanh(x))\,dx$$
$$= -\frac{5}{6}\coth^3(x) + \frac{\coth^3(x)}{2(1+\tanh(x))} - \frac{1}{2}i\int \coth^3(x)(-4i+5i\tanh(x))\,dx$$
$$= \coth^2(x) - \frac{5\coth^3(x)}{6} + \frac{\coth^3(x)}{2(1+\tanh(x))} + \frac{1}{2}\int \coth^2(x)(5-4\tanh(x))\,dx$$
$$= -\frac{5\coth(x)}{2} + \coth^2(x) - \frac{5\coth^3(x)}{6} + \frac{\coth^3(x)}{2(1+\tanh(x))} + \frac{1}{2}i\int \coth(x)(4i-5i\tanh(x))\,dx$$
$$= \frac{5x}{2} - \frac{5\coth(x)}{2} + \coth^2(x) - \frac{5\coth^3(x)}{6} + \frac{\coth^3(x)}{2(1+\tanh(x))} - 2\int \coth(x)\,dx$$
$$= \frac{5x}{2} - \frac{5\coth(x)}{2} + \coth^2(x) - \frac{5\coth^3(x)}{6} - 2\log(\sinh(x)) + \frac{\coth^3(x)}{2(1+\tanh(x))}$$

Mathematica [A] time = 0.127994, size = 42, normalized size = 0.98

$$\frac{1}{12}\left(-3\cosh(2x) - 4\coth(x)\left(\text{csch}^2(x)+7\right) + 3\left(10x + \sinh(2x) + 2\text{csch}^2(x) - 8\log(\sinh(x))\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (-3*Cosh[2*x] - 4*Coth[x]*(7 + Csch[x]^2) + 3*(10*x + 2*Csch[x]^2 - 8*Log[S
inh[x]] + Sinh[2*x]))/12
```

Maple [B] time = 0.036, size = 91, normalized size = 2.1

$$-\frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^3 + \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2 - \frac{9}{8}\tanh\left(\frac{x}{2}\right) - \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{9}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{24}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(1+tanh(x)),x)
```

```
[Out] -1/24*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2-9/8*tanh(1/2*x)-1/(tanh(1/2*x)+1)^2+1
/(tanh(1/2*x)+1)+9/2*ln(tanh(1/2*x)+1)-1/24/tanh(1/2*x)^3+1/8/tanh(1/2*x)^2
-9/8/tanh(1/2*x)-2*ln(tanh(1/2*x))-1/2*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.14226, size = 86, normalized size = 2.

$$\frac{1}{2}x - \frac{2\left(15e^{(-2x)} - 12e^{(-4x)} - 7\right)}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)} - \frac{1}{4}e^{(-2x)} - 2\log\left(e^{(-x)}+1\right) - 2\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/3*(15*e^(-2*x) - 12*e^(-4*x) - 7)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-
6*x) - 1) - 1/4*e^(-2*x) - 2*log(e^(-x) + 1) - 2*log(e^(-x) - 1)
```

Fricas [B] time = 2.21132, size = 1897, normalized size = 44.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 - 3*(54*x +
 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 - 54*x - 17)*sinh(x)^6 + 18*(168*x*cosh
(x)^3 - (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420*x*
cosh(x)^4 - 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*cosh(
x)^5 - 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 - (54*x +
65)*cosh(x)^2 + (1512*x*cosh(x)^6 - 45*(54*x + 17)*cosh(x)^4 + 486*(2*x + 1
)*cosh(x)^2 - 54*x - 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 -
9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 - 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cosh(x)
^6 - 45*cosh(x)^4 + 18*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(4*cosh(x)^
7 - 9*cosh(x)^5 + 6*cosh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) -
sinh(x))) + 2*(216*x*cosh(x)^7 - 9*(54*x + 17)*cosh(x)^5 + 162*(2*x + 1)*co
sh(x)^3 - (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^
7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^
3 - 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 - 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cos
h(x)^6 - 45*cosh(x)^4 + 18*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(4*cosh
(x)^7 - 9*cosh(x)^5 + 6*cosh(x)^3 - cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(x)}{\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(1+tanh(x)),x)
```

```
[Out] Integral(coth(x)**4/(tanh(x) + 1), x)
```

Giac [A] time = 1.27094, size = 65, normalized size = 1.51

$$\frac{9}{2}\,x - \frac{\left(51\,e^{(6\,x)} - 81\,e^{(4\,x)} + 65\,e^{(2\,x)} - 3\right)e^{(-2\,x)}}{12\,\left(e^{(2\,x)} - 1\right)^3} - 2\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 9/2*x - 1/12*(51*e^(6*x) - 81*e^(4*x) + 65*e^(2*x) - 3)*e^(-2*x)/(e^(2*x) -
 1)^3 - 2*log(abs(e^(2*x) - 1))
```

3.125 $\int \tanh(x)(1+\tanh(x))^{3/2}\,dx$

Optimal. Leaf size=45

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\tanh(x)+1)^{3/2}-2\sqrt{\tanh(x)+1}$$

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(3/2))/3
```

Rubi [A] time = 0.0478134, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3527, 3478, 3480, 206}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\tanh(x)+1)^{3/2}-2\sqrt{\tanh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(3/2))/3
```

Rule 3527

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \tanh(x)(1+\tanh(x))^{3/2}\,dx = -\frac{2}{3}(1+\tanh(x))^{3/2} + \int (1+\tanh(x))^{3/2}\,dx$$
$$= -2\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2} + 2\int \sqrt{1+\tanh(x)}\,dx$$
$$= -2\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2} + 4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right)$$
$$= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2}$$

Mathematica [A] time = 0.17986, size = 39, normalized size = 0.87

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right) - \frac{2}{3}\sqrt{\tanh(x)+1}(\tanh(x)+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*Sqrt[1 + Tanh[x]]*(4 + Ta
nh[x]))/3
```

Maple [A] time = 0.011, size = 35, normalized size = 0.8

$$2\,\text{Artanh}\left(1/2\,\sqrt{1+\tanh(x)}\sqrt{2}\right)\sqrt{2} - 2\,\sqrt{1+\tanh(x)} - \frac{2}{3}\,(1+\tanh(x))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(1+tanh(x))^(3/2),x)
```

```
[Out] 2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)-2/3*(1
+tanh(x))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,\sqrt{2}}{3\left(e^{(-2\,x)}+1\right)^{\frac{3}{2}}} + \int \frac{2\,\sqrt{2}e^{(-x)}}{\left(e^{(-x)}+e^{(-3\,x)}\right)\left(e^{(-2\,x)}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(1+tanh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(2)/(e^(-2*x) + 1)^(3/2) + integrate(2*sqrt(2)*e^(-x)/((e^(-x) + e
^(-3*x))*(e^(-2*x) + 1)^(3/2)), x)
```

Fricas [B] time = 2.33998, size = 871, normalized size = 19.36

$$-\frac{2\,\sqrt{2}\left(5\,\sqrt{2}\cosh(x)^3 + 15\,\sqrt{2}\cosh(x)\sinh(x)^2 + 5\,\sqrt{2}\sinh(x)^3 + 3\left(5\,\sqrt{2}\cosh(x)^2 + \sqrt{2}\right)\sinh(x) + 3\,\sqrt{2}\cosh(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x)*sinh(x)^2 + 5*sqr
t(2)*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 3*sqrt(2)*cosh
(x))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*si
nh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
 sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [A] time = 11.3527, size = 82, normalized size = 1.82

$$-\frac{2\left(\tanh\left(x\right)+1\right)^{\frac{3}{2}}}{3}-2\sqrt{\tanh\left(x\right)+1}-4\left(\begin{cases}-\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{\tanh\left(x\right)+1}}{2}\right)}{2} & \text{for}\ \tanh\left(x\right)+1>2\\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh\left(x\right)+1}}{2}\right)}{2} & \text{for}\ \tanh\left(x\right)+1<2\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(1+tanh(x))**(3/2),x)
```

```
[Out] -2*(tanh(x) + 1)**(3/2)/3 - 2*sqrt(tanh(x) + 1) - 4*Piecewise((-sqrt(2)*aco
th(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 > 2), (-sqrt(2)*atanh(sqrt(2
)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 < 2))
```

Giac [B] time = 1.35467, size = 130, normalized size = 2.89

$$\frac{1}{3}\sqrt{2}\left(\frac{2\left(9\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{2}-12\sqrt{e^{(4x)}+e^{(2x)}}+12e^{(2x)}+5\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right)^{3}}-3\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(1+tanh(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(2*(9*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 12*sqrt(e^(4*x) +
 e^(2*x)) + 12*e^(2*x) + 5)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1)^3 - 3*l
og(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

3.126 $\int \tanh(x)\sqrt{1+\tanh(x)}\,dx$

Optimal. Leaf size=32

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-2\sqrt{\tanh(x)+1}$$

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Rubi [A] time = 0.0352135, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3527, 3480, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-2\sqrt{\tanh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*Sqrt[1 + Tanh[x]],x]

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Rule 3527

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \tanh(x)\sqrt{1+\tanh(x)}\,dx &= -2\sqrt{1+\tanh(x)}+\int \sqrt{1+\tanh(x)}\,dx\\ &= -2\sqrt{1+\tanh(x)}+2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-2\sqrt{1+\tanh(x)}\end{aligned}$$

Mathematica [A] time = 0.0441599, size = 32, normalized size = 1.

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-2\sqrt{\tanh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*Sqrt[1 + Tanh[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Maple [A] time = 0.026, size = 26, normalized size = 0.8

$$\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right)\sqrt{2}-2\sqrt{1+\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x))^(1/2)*tanh(x),x)
```

```
[Out] arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}+\int\frac{\sqrt{2}e^{(-x)}}{\left(e^{(-x)}+e^{(-3x)}\right)\sqrt{e^{(-2x)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] -sqrt(2)/sqrt(e^(-2*x) + 1) + integrate(sqrt(2)*e^(-x)/((e^(-x) + e^(-3*x))
*sqrt(e^(-2*x) + 1)), x)
```

Fricas [B] time = 2.29843, size = 456, normalized size = 14.25

$$-\frac{4\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh(x)^{2}+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^{2}+\sqrt{2}\right)\log\left(-\right.}{2\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+s\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(cosh(x)/(cosh(x) -
 sinh(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
 sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [A] time = 2.13934, size = 70, normalized size = 2.19

$$-2\sqrt{\tanh(x)+1}-2\left(\begin{cases}-\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for}\ \tanh(x)+1>2\\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for}\ \tanh(x)+1<2\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))**(1/2)*tanh(x),x)
```

```
[Out] -2*sqrt(tanh(x) + 1) - 2*Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(tanh(x) + 1
)/2)/2, tanh(x) + 1 > 2), (-sqrt(2)*atanh(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, t
anh(x) + 1 < 2))
```

Giac [B] time = 1.26433, size = 72, normalized size = 2.25

$$\frac{1}{2}\sqrt{2}\left(\frac{4}{\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1}-\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(4/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1) - log(-2*sqrt(e^(4*x
) + e^(2*x)) + 2*e^(2*x) + 1))
```

3.127 $\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{\tanh(x)+1}}$$

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Tanh[x]]
```

Rubi [A] time = 0.0365176, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3526, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{\tanh(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[1 + Tanh[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Tanh[x]]
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx &= \frac{1}{\sqrt{1+\tanh(x)}}+\frac{1}{2}\int \sqrt{1+\tanh(x)}\,dx\\
&= \frac{1}{\sqrt{1+\tanh(x)}}+\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{1+\tanh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0539926, size = 30, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{\tanh(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[1 + Tanh[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Tanh[x]]
```

Maple [A] time = 0.043, size = 25, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right)+\frac{1}{\sqrt{1+\tanh(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1+tanh(x))^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+tanh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\sqrt{2}\sqrt{e^{(-2x)}+1}+\int\frac{e^{(-x)}}{\frac{\sqrt{2}e^{(-x)}}{\sqrt{e^{(-2x)}+1}}+\frac{\sqrt{2}e^{(-3x)}}{\sqrt{e^{(-2x)}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*sqrt(e^(-2*x) + 1) + integrate(e^(-x)/(sqrt(2)*e^(-x)/sqrt(e^(-
2*x) + 1) + sqrt(2)*e^(-3*x)/sqrt(e^(-2*x) + 1)), x)
```

Fricas [B] time = 2.15684, size = 301, normalized size = 10.03

$$\frac{\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(-2\sqrt{2}\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))-2\cosh(x)^2-4\cosh(x)\sinh(x)-2\,\mathrm{s}\right.}{4(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(
x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*si
nh(x)^2 - 1) + 4*sqrt(cosh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [A] time = 2.40823, size = 66, normalized size = 2.2

$$-\begin{cases} -\frac{\sqrt{2}\,\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for } \tanh(x)+1>2 \\ -\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for } \tanh(x)+1<2 \end{cases} + \frac{1}{\sqrt{\tanh(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x))**(1/2),x)
```

```
[Out] -Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 > 2)
, (-sqrt(2)*atanh(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 < 2)) + 1/sqr
t(tanh(x) + 1)
```

Giac [B] time = 1.26666, size = 78, normalized size = 2.6

$$-\frac{1}{4}\,\sqrt{2}\log\left(-2\,\sqrt{e^{(4\,x)}+e^{(2\,x)}}+2\,e^{(2\,x)}+1\right)-\frac{1}{2}\,\sqrt{2}+\frac{\sqrt{2}}{2\left(\sqrt{e^{(4\,x)}+e^{(2\,x)}}-e^{(2\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1) - 1/2*sqrt(2)
+ 1/2*sqrt(2)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))
```

3.128 $\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}} \, dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{2\sqrt{\tanh(x)+1}} + \frac{1}{3(\tanh(x)+1)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Tanh[x])^(3/2))
- 1/(2*Sqrt[1 + Tanh[x]])
```

Rubi [A] time = 0.0488047, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3526, 3479, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{2\sqrt{\tanh(x)+1}} + \frac{1}{3(\tanh(x)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(1 + Tanh[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Tanh[x])^(3/2))
- 1/(2*Sqrt[1 + Tanh[x]])
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx &= \frac{1}{3(1+\tanh(x))^{3/2}}+\frac{1}{2}\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx \\
&= \frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}+\frac{1}{4}\int \sqrt{1+\tanh(x)}\,dx \\
&= \frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right) \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.11948, size = 51, normalized size = 1.04

$$
\frac{1}{12}\left(3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2(\cosh(x)-\sinh(x))(3\sinh(x)+\cosh(x))}{\sqrt{\tanh(x)+1}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(1 + Tanh[x])^(3/2),x]
```

```
[Out] (3*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*(Cosh[x] - Sinh[x])*(Cos
h[x] + 3*Sinh[x]))/Sqrt[1 + Tanh[x]])/12
```

Maple [A] time = 0.016, size = 35, normalized size = 0.7

$$
\frac{\sqrt{2}}{4}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right)-\frac{1}{2}\frac{1}{\sqrt{1+\tanh(x)}}+\frac{1}{3}(1+\tanh(x))^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1+tanh(x))^(3/2),x)
```

```
[Out] 1/4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/2/(1+tanh(x))^(1/2)+1/
3/(1+tanh(x))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\frac{1}{12}\sqrt{2}\left(e^{(-2x)}+1\right)^{\frac{3}{2}}+\int \frac{e^{(-x)}}{2\left(\frac{\sqrt{2}e^{(-x)}}{\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}+\frac{\sqrt{2}e^{(-3x)}}{\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}\right)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*(e^(-2*x) + 1)^(3/2) + integrate(1/2*e^(-x)/(sqrt(2)*e^(-x)/(e
^(-2*x) + 1)^(3/2) + sqrt(2)*e^(-3*x)/(e^(-2*x) + 1)^(3/2)), x)
```

Fricas [B] time = 2.24738, size = 579, normalized size = 11.82

$$-\frac{2\sqrt{2}\left(2\sqrt{2}\cosh(x)^2+4\sqrt{2}\cosh(x)\sinh(x)+2\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh\right.}{24\left(\cosh(x)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(2*sqrt(2)*cosh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x) + 2*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*
cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [A] time = 10.0321, size = 82, normalized size = 1.67

$$-\frac{\begin{cases} -\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for}\ \tanh(x)+1>2 \\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for}\ \tanh(x)+1<2 \end{cases}}{2}-\frac{1}{2\sqrt{\tanh(x)+1}}+\frac{1}{3(\tanh(x)+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x))**(3/2),x)
```

```
[Out] -Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 > 2)
, (-sqrt(2)*atanh(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 < 2))/2 - 1/(
2*sqrt(tanh(x) + 1)) + 1/(3*(tanh(x) + 1)**(3/2))
```

Giac [B] time = 1.26935, size = 100, normalized size = 2.04

$$-\frac{1}{24}\sqrt{2}\left(\frac{2\left(3\sqrt{e^{(4x)}+e^{(2x)}}-3e^{(2x)}-1\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^3}+3\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+tanh(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(2*(3*sqrt(e^(4*x) + e^(2*x)) - 3*e^(2*x) - 1)/(sqrt(e^(4*x)
+ e^(2*x)) - e^(2*x))^3 + 3*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1)
 - 4)
```

3.129 $\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx$

Optimal. Leaf size=45

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{5}(\tanh(x)+1)^{5/2}-2\sqrt{\tanh(x)+1}$$

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(5/2))/5
```

Rubi [A] time = 0.0581242, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3543, 3478, 3480, 206}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{5}(\tanh(x)+1)^{5/2}-2\sqrt{\tanh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2*(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(5/2))/5
```

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx &= -\frac{2}{5}(1+\tanh(x))^{5/2}+\int (1+\tanh(x))^{3/2}\,dx\\
&= -2\sqrt{1+\tanh(x)}-\frac{2}{5}(1+\tanh(x))^{5/2}+2\int \sqrt{1+\tanh(x)}\,dx\\
&= -2\sqrt{1+\tanh(x)}-\frac{2}{5}(1+\tanh(x))^{5/2}+4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-2\sqrt{1+\tanh(x)}-\frac{2}{5}(1+\tanh(x))^{5/2}
\end{aligned}
$$

Mathematica [A] time = 0.197637, size = 53, normalized size = 1.18

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{1}{5}\sqrt{\tanh(x)+1}\text{sech}^2(x)(2\sinh(2x)+7\cosh(2x)+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2*(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (Sech[x]^2*(5 + 7*Cosh[2*x]
+ 2*Sinh[2*x])*Sqrt[1 + Tanh[x]])/5
```

Maple [A] time = 0.019, size = 35, normalized size = 0.8

$$2\,\text{Artanh}\left(1/2\,\sqrt{1+\tanh(x)}\sqrt{2}\right)\sqrt{2}-2\,\sqrt{1+\tanh(x)}-\frac{2}{5}\,(1+\tanh(x))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(1+tanh(x))^(3/2),x)
```

```
[Out] 2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)-2/5*(1
+tanh(x))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\tanh(x)+1)^{\frac{3}{2}}\tanh(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(1+tanh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tanh(x) + 1)^(3/2)*tanh(x)^2, x)
```

Fricas [B] time = 2.34753, size = 1450, normalized size = 32.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*sqrt(2)*(9*sqrt(2)*cosh(x)^5 + 45*sqrt(2)*cosh(x)*sinh(x)^4 + 9*sqr
t(2)*sinh(x)^5 + 10*(9*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^3 + 10*sqrt(2)*
cosh(x)^3 + 30*(3*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x)^2 + 5*(9*sqr
t(2)*cosh(x)^4 + 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 5*sqrt(2)*cosh(x)
)*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 5*(sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh
(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(
x)^4 + 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*si
nh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 + 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2
 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 + 2*sqrt(2)*cosh(x)^3 + sqrt(
2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(
x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 -
1))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh
(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^
4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 +
 cosh(x))*sinh(x) + 1)
```

Sympy [A] time = 17.3534, size = 82, normalized size = 1.82

$$-\frac{2\left(\tanh(x)+1\right)^{\frac{5}{2}}}{5}-2\sqrt{\tanh(x)+1}-4\left(\begin{cases}-\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for } \tanh(x)+1>2\\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for } \tanh(x)+1<2\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2*(1+tanh(x))**(3/2),x)
```

```
[Out] -2*(tanh(x) + 1)**(5/2)/5 - 2*sqrt(tanh(x) + 1) - 4*Piecewise((-sqrt(2)*aco
th(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 > 2), (-sqrt(2)*atanh(sqrt(2
)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 < 2))
```

Giac [B] time = 1.29067, size = 189, normalized size = 4.2

$$\frac{1}{5}\sqrt{2}\left(\frac{2\left(25\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{4}-60\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{3}+70\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{2}-40\sqrt{e^{(4x)}+e^{(2x)}}+40e^{(}}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(1+tanh(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(2)*(2*(25*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^4 - 60*(sqrt(e^(4*x)
 + e^(2*x)) - e^(2*x))^3 + 70*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 40*sq
rt(e^(4*x) + e^(2*x)) + 40*e^(2*x) + 9)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x)
- 1)^5 - 5*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

3.130 $\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx$

Optimal. Leaf size=34

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\tanh(x)+1)^{3/2}$$

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*(1 + Tanh[x])^(3/2))/3
```

Rubi [A] time = 0.0441479, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3543, 3480, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\tanh(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2*Sqrt[1 + Tanh[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*(1 + Tanh[x])^(3/2))/3
```

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx &= -\frac{2}{3}(1+\tanh(x))^{3/2}+\int \sqrt{1+\tanh(x)}\,dx\\ &= -\frac{2}{3}(1+\tanh(x))^{3/2}+2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\tanh(x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.067778, size = 34, normalized size = 1.

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\tanh(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2*Sqrt[1 + Tanh[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*(1 + Tanh[x])^(3/2))/3
```

Maple [A] time = 0.034, size = 26, normalized size = 0.8

$$\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right)\sqrt{2}-\frac{2}{3}(1+\tanh(x))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x))^(1/2)*tanh(x)^2,x)
```

```
[Out] arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2/3*(1+tanh(x))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\tanh(x)+1}\tanh(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tanh(x) + 1)*tanh(x)^2, x)
```

Fricas [B] time = 2.20589, size = 818, normalized size = 24.06

$$-\frac{8\sqrt{2}\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)^2\sinh(x)+3\sqrt{2}\cosh(x)\sinh(x)^2+\sqrt{2}\sinh(x)^3\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(8*sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2
)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*sqrt(cosh(x)/(cosh(x) - sinh(x)))
- 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqrt
(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cos
h(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sin
h(x) - 2*sinh(x)^2 - 1))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(
3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x)
+ 1)
```

Sympy [A] time = 2.99924, size = 71, normalized size = 2.09

$$-\frac{2\left(\tanh\left(x\right)+1\right)^{\frac{3}{2}}}{3}-2\left(\begin{cases}-\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{\tanh\left(x\right)+1}}{2}\right)}{2} & \text{for}\ \tanh\left(x\right)+1>2\\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh\left(x\right)+1}}{2}\right)}{2} & \text{for}\ \tanh\left(x\right)+1<2\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))**(1/2)*tanh(x)**2,x)
```

```
[Out] -2*(tanh(x) + 1)**(3/2)/3 - 2*Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(tanh(x
) + 1)/2)/2, tanh(x) + 1 > 2), (-sqrt(2)*atanh(sqrt(2)*sqrt(tanh(x) + 1)/2)
/2, tanh(x) + 1 < 2))
```

Giac [B] time = 1.22259, size = 130, normalized size = 3.82

$$\frac{1}{6}\sqrt{2}\left(\frac{8\left(3\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{2}-3\sqrt{e^{(4x)}+e^{(2x)}}+3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right)^{3}}-3\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(8*(3*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 3*sqrt(e^(4*x) +
e^(2*x)) + 3*e^(2*x) + 1)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1)^3 - 3*log
(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

3.131 $\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}} dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}} - 2\sqrt{\tanh(x)+1} - \frac{1}{\sqrt{\tanh(x)+1}}$$

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Tanh[x]] - 2*Sqrt[1
 + Tanh[x]]
```

Rubi [A] time = 0.0558897, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3543, 3479, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}} - 2\sqrt{\tanh(x)+1} - \frac{1}{\sqrt{\tanh(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/Sqrt[1 + Tanh[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Tanh[x]] - 2*Sqrt[1
 + Tanh[x]]
```

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx = -2\sqrt{1+\tanh(x)} + \int \frac{1}{\sqrt{1+\tanh(x)}}\,dx$$
$$= -\frac{1}{\sqrt{1+\tanh(x)}} - 2\sqrt{1+\tanh(x)} + \frac{1}{2}\int \sqrt{1+\tanh(x)}\,dx$$
$$= -\frac{1}{\sqrt{1+\tanh(x)}} - 2\sqrt{1+\tanh(x)} + \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{1+\tanh(x)}} - 2\sqrt{1+\tanh(x)}$$

Mathematica [A] time = 0.0877899, size = 37, normalized size = 0.88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{-2\tanh(x)-3}{\sqrt{\tanh(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/Sqrt[1 + Tanh[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] + (-3 - 2*Tanh[x])/Sqrt[1 + Tanh
[x]]
```

Maple [A] time = 0.039, size = 35, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right) - \frac{1}{\sqrt{1+\tanh(x)}} - 2\sqrt{1+\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(1+tanh(x))^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+tanh(x))^(1/2)-2*(1
+tanh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{\sqrt{\tanh(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/sqrt(tanh(x) + 1), x)
```

Fricas [B] time = 2.34995, size = 639, normalized size = 15.21

$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh(x)^2+10\sqrt{2}\cosh(x)\sinh(x)+5\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)\cdots\right.}{4\left(\cosh(x)^3+3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^2 + 10*sqrt(2)*cosh(x)*sinh(x) + 5*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - (sqrt(2)*cosh(x
)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^
2 + sqrt(2))*sinh(x) + sqrt(2)*cosh(x))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sin
h(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 +
1)*sinh(x) + cosh(x))
```

Sympy [A] time = 3.40278, size = 78, normalized size = 1.86

$$-2\sqrt{\tanh(x)+1}-\begin{cases}-\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for}\ \tanh(x)+1>2\\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for}\ \tanh(x)+1<2\end{cases}-\frac{1}{\sqrt{\tanh(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(1+tanh(x))**(1/2),x)
```

```
[Out] -2*sqrt(tanh(x) + 1) - Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(tanh(x) + 1)/
2)/2, tanh(x) + 1 > 2), (-sqrt(2)*atanh(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, tan
h(x) + 1 < 2)) - 1/sqrt(tanh(x) + 1)
```

Giac [A] time = 1.25964, size = 73, normalized size = 1.74

$$-\frac{1}{4}\sqrt{2}\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)-\frac{5\sqrt{2}e^{(2x)}+\sqrt{2}}{2\sqrt{e^{(4x)}+e^{(2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1) - 1/2*(5*sqrt(
2)*e^(2*x) + sqrt(2))/sqrt(e^(4*x) + e^(2*x))
```

3.132 $\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{3}{2\sqrt{\tanh(x)+1}}-\frac{1}{3(\tanh(x)+1)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Tanh[x])^(3/2))
+ 3/(2*Sqrt[1 + Tanh[x]])
```

Rubi [A] time = 0.079619, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3540, 3526, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{3}{2\sqrt{\tanh(x)+1}}-\frac{1}{3(\tanh(x)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(1 + Tanh[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Tanh[x])^(3/2))
+ 3/(2*Sqrt[1 + Tanh[x]])
```

Rule 3540

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(a*c + b*d)^2*(a + b*Tan[e + f*x])^m)
/(2*a^3*f*m), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*
c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx = -\frac{1}{3(1+\tanh(x))^{3/2}} - \frac{1}{2}\int \frac{1-2\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx$$
$$= -\frac{1}{3(1+\tanh(x))^{3/2}} + \frac{3}{2\sqrt{1+\tanh(x)}} + \frac{1}{4}\int \sqrt{1+\tanh(x)}\,dx$$
$$= -\frac{1}{3(1+\tanh(x))^{3/2}} + \frac{3}{2\sqrt{1+\tanh(x)}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{3(1+\tanh(x))^{3/2}} + \frac{3}{2\sqrt{1+\tanh(x)}}$$

Mathematica [A] time = 0.095627, size = 53, normalized size = 1.08

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{(\cosh(x)-\sinh(x))(9\sinh(x)+7\cosh(x))}{6\sqrt{\tanh(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(1 + Tanh[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) + ((Cosh[x] - Sinh[x])*(7*Co
sh[x] + 9*Sinh[x]))/(6*Sqrt[1 + Tanh[x]])
```

Maple [A] time = 0.021, size = 35, normalized size = 0.7

$$\frac{\sqrt{2}}{4}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(x)}\right) + \frac{3}{2}\frac{1}{\sqrt{1+\tanh(x)}} - \frac{1}{3}(1+\tanh(x))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(1+tanh(x))^(3/2),x)
```

```
[Out] 1/4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)+3/2/(1+tanh(x))^(1/2)-1/
3/(1+tanh(x))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^2}{(\tanh(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(tanh(x) + 1)^(3/2), x)
```

Fricas [B] time = 2.26444, size = 579, normalized size = 11.82

$$\frac{2\sqrt{2}\left(8\sqrt{2}\cosh(x)^2+16\sqrt{2}\cosh(x)\sinh(x)+8\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}+3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh\right.}{24\left(\cosh(x)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(2)*(8*sqrt(2)*cosh(x)^2 + 16*sqrt(2)*cosh(x)*sinh(x) + 8*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*
cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [A] time = 11.4247, size = 82, normalized size = 1.67

$$-\frac{\begin{cases}-\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for}\ \tanh(x)+1>2\\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} & \text{for}\ \tanh(x)+1<2\end{cases}}{2}+\frac{3}{2\sqrt{\tanh(x)+1}}-\frac{1}{3(\tanh(x)+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(1+tanh(x))**(3/2),x)
```

```
[Out] -Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 > 2)
, (-sqrt(2)*atanh(sqrt(2)*sqrt(tanh(x) + 1)/2)/2, tanh(x) + 1 < 2))/2 + 3/(
2*sqrt(tanh(x) + 1)) - 1/(3*(tanh(x) + 1)**(3/2))
```

Giac [B] time = 1.26838, size = 136, normalized size = 2.78

$$-\frac{1}{8}\sqrt{2}\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)-\frac{2}{3}\sqrt{2}+\frac{\sqrt{2}\left(6\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^2+3\sqrt{e^{(4x)}+e^{(2x)}}-3e^{(2x)}-1\right)}{12\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1) - 2/3*sqrt(2)
+ 1/12*sqrt(2)*(6*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 + 3*sqrt(e^(4*x) +
e^(2*x)) - 3*e^(2*x) - 1)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^3
```

3.133 $\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{bx}{a^2-b^2}-\frac{(a^2+b^2)\tanh(x)}{b^3}+\frac{a^5\log(a+b\tanh(x))}{b^4(a^2-b^2)}+\frac{a\log(\cosh(x))}{a^2-b^2}+\frac{a\tanh^2(x)}{2b^2}-\frac{\tanh^3(x)}{3b}$$

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[Cosh[x]])/(a^2 - b^2) + (a^5*Log[a + b*Tanh[x
]])/(b^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/b^3 + (a*Tanh[x]^2)/(2*b^2) -
 Tanh[x]^3/(3*b)
```

Rubi [A] time = 0.366946, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3566, 3647, 3648, 3626, 3617, 31, 3475}

$$-\frac{bx}{a^2-b^2}-\frac{(a^2+b^2)\tanh(x)}{b^3}+\frac{a^5\log(a+b\tanh(x))}{b^4(a^2-b^2)}+\frac{a\log(\cosh(x))}{a^2-b^2}+\frac{a\tanh^2(x)}{2b^2}-\frac{\tanh^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[Cosh[x]])/(a^2 - b^2) + (a^5*Log[a + b*Tanh[x
]])/(b^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/b^3 + (a*Tanh[x]^2)/(2*b^2) -
 Tanh[x]^3/(3*b)
```

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
 Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m + n +
 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx = -\frac{\tanh^3(x)}{3b} - \frac{\int \frac{\tanh^2(x)\left(-3a-3b\tanh(x)+3a\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{3b}$$

$$= \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh^3(x)}{3b} - \frac{\int \frac{\tanh(x)\left(6a^2-6\left(a^2+b^2\right)\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{6b^2}$$

$$= -\frac{\left(a^2+b^2\right)\tanh(x)}{b^3} + \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh^3(x)}{3b} - \frac{\int \frac{-6a\left(a^2+b^2\right)-6b^3\tanh(x)+6a\left(a^2+b^2\right)\tanh^2(x)}{a+b\tanh(x)}\,dx}{6b^3}$$

$$= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\tanh(x)}{b^3} + \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh^3(x)}{3b} + \frac{a\int \tanh(x)\,dx}{a^2-b^2} + \frac{a^5\int \frac{1-\tanh^2(x)}{a+b\tanh(x)}\,dx}{b^3\left(a^2-b^2\right)}$$

$$= -\frac{bx}{a^2-b^2} + \frac{a\log(\cosh(x))}{a^2-b^2} - \frac{\left(a^2+b^2\right)\tanh(x)}{b^3} + \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh^3(x)}{3b} + \frac{a^5\,\mathrm{Subst}\left(\int \frac{1}{a+x}\right.}{b^4\left(a^2\right.}$$

$$= -\frac{bx}{a^2-b^2} + \frac{a\log(\cosh(x))}{a^2-b^2} + \frac{a^5\log(a+b\tanh(x))}{b^4\left(a^2-b^2\right)} - \frac{\left(a^2+b^2\right)\tanh(x)}{b^3} + \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh}{3b}$$

Mathematica [A] time = 0.453001, size = 105, normalized size = 1.12

$$-\frac{2b\left(a^2b^2+3a^4-4b^4\right)\tanh(x)+6a\left(a^4-b^4\right)\log(\cosh(x))+b^2\left(b^2-a^2\right)\text{sech}^2(x)(2b\tanh(x)-3a)-6a^5\log(a\cosh(x)}{6b^4(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] -(6*b^5*x + 6*a*(a^4 - b^4)*Log[Cosh[x]] - 6*a^5*Log[a*Cosh[x] + b*Sinh[x]]
 + 2*b*(3*a^4 + a^2*b^2 - 4*b^4)*Tanh[x] + b^2*(-a^2 + b^2)*Sech[x]^2*(-3*a
 + 2*b*Tanh[x]))/(6*(a - b)*b^4*(a + b))
```

Maple [A] time = 0.023, size = 96, normalized size = 1.

$$-\frac{(\tanh(x))^3}{3b}+\frac{a(\tanh(x))^2}{2b^2}-\frac{a^2\tanh(x)}{b^3}-\frac{\tanh(x)}{b}-\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2b+2a}+\frac{a^5\ln(a+b\tanh(x))}{b^4(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*tanh(x)),x)
```

```
[Out] -1/3*tanh(x)^3/b+1/2*a*tanh(x)^2/b^2-1/b^3*a^2*tanh(x)-tanh(x)/b-1/(2*a-2*b
)*ln(1+tanh(x))-1/(2*b+2*a)*ln(tanh(x)-1)+1/b^4*a^5/(a+b)/(a-b)*ln(a+b*tanh
(x))
```

Maxima [A] time = 1.76491, size = 203, normalized size = 2.16

$$\frac{a^5\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2b^4-b^6}-\frac{2\left(3a^2+4b^2+3\left(2a^2+ab+2b^2\right)e^{(-2x)}+3\left(a^2+ab+2b^2\right)e^{(-4x)}\right)}{3\left(3b^3e^{(-2x)}+3b^3e^{(-4x)}+b^3e^{(-6x)}+b^3\right)}+\frac{x}{a+b}-\frac{\left(a^3+ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] a^5*log(-(a - b)*e^(-2*x) - a - b)/(a^2*b^4 - b^6) - 2/3*(3*a^2 + 4*b^2 + 3
*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(a^2 + a*b + 2*b^2)*e^(-4*x))/(3*b^3*e^
(-2*x) + 3*b^3*e^(-4*x) + b^3*e^(-6*x) + b^3) + x/(a + b) - (a^3 + a*b^2)*l
og(e^(-2*x) + 1)/b^4
```

Fricas [B] time = 2.62615, size = 3089, normalized size = 32.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a*b^4 + b^5)*x*cosh(x)^6 + 18*(a*b^4 + b^5)*x*cosh(x)*sinh(x)^5 +
3*(a*b^4 + b^5)*x*sinh(x)^6 - 6*a^4*b - 2*a^2*b^3 + 8*b^5 - 3*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^4 - 3*(2
```

```
*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 15*(a*b^4 + b^5)*x*cosh(
x)^2 - 3*(a*b^4 + b^5)*x)*sinh(x)^4 + 12*(5*(a*b^4 + b^5)*x*cosh(x)^3 - (2*
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x
))*sinh(x)^3 - 3*(4*a^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x
)*cosh(x)^2 + 3*(15*(a*b^4 + b^5)*x*cosh(x)^4 - 4*a^4*b + 2*a^3*b^2 - 2*a*b
^4 + 4*b^5 - 6*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^
4 + b^5)*x)*cosh(x)^2 + 3*(a*b^4 + b^5)*x)*sinh(x)^2 + 3*(a*b^4 + b^5)*x -
3*(a^5*cosh(x)^6 + 6*a^5*cosh(x)*sinh(x)^5 + a^5*sinh(x)^6 + 3*a^5*cosh(x)^
4 + 3*a^5*cosh(x)^2 + a^5 + 3*(5*a^5*cosh(x)^2 + a^5)*sinh(x)^4 + 4*(5*a^5*
cosh(x)^3 + 3*a^5*cosh(x))*sinh(x)^3 + 3*(5*a^5*cosh(x)^4 + 6*a^5*cosh(x)^2
 + a^5)*sinh(x)^2 + 6*(a^5*cosh(x)^5 + 2*a^5*cosh(x)^3 + a^5*cosh(x))*sinh(
x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 3*((a^5 - a*b^4)*c
osh(x)^6 + 6*(a^5 - a*b^4)*cosh(x)*sinh(x)^5 + (a^5 - a*b^4)*sinh(x)^6 + a^
5 - a*b^4 + 3*(a^5 - a*b^4)*cosh(x)^4 + 3*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a*b^4)*cosh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)^2 + 3*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*
cosh(x)^4 + 6*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*((a^5 - a*b^4)*cosh(x)
^5 + 2*(a^5 - a*b^4)*cosh(x)^3 + (a^5 - a*b^4)*cosh(x))*sinh(x))*log(2*cosh
(x)/(cosh(x) - sinh(x))) + 6*(3*(a*b^4 + b^5)*x*cosh(x)^5 - 2*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^3 - (4*a
^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x))*sinh(x))/(
(a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^2*b^4
- b^6)*sinh(x)^6 + a^2*b^4 - b^6 + 3*(a^2*b^4 - b^6)*cosh(x)^4 + 3*(a^2*b^4
 - b^6 + 5*(a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b^4 - b^6)*cosh
(x)^3 + 3*(a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^2*b^4 - b^6)*cosh(x)^2
+ 3*(a^2*b^4 - b^6 + 5*(a^2*b^4 - b^6)*cosh(x)^4 + 6*(a^2*b^4 - b^6)*cosh(x
)^2)*sinh(x)^2 + 6*((a^2*b^4 - b^6)*cosh(x)^5 + 2*(a^2*b^4 - b^6)*cosh(x)^3
 + (a^2*b^4 - b^6)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21017, size = 192, normalized size = 2.04

$$\frac{a^5 \log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^2b^4 - b^6} - \frac{x}{a-b} - \frac{\left(a^3 + ab^2\right)\log\left(e^{(2x)} + 1\right)}{b^4} + \frac{2\left(3a^2b + 4b^3 + 3\left(a^2b - ab^2 + 2b^3\right)e^{(4x)} + 3(\cdots\right.}{3b^4\left(e^{(2x)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] a^5*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2*b^4 - b^6) - x/(a - b) - (
a^3 + a*b^2)*log(e^(2*x) + 1)/b^4 + 2/3*(3*a^2*b + 4*b^3 + 3*(a^2*b - a*b^2
 + 2*b^3)*e^(4*x) + 3*(2*a^2*b - a*b^2 + 2*b^3)*e^(2*x))/(b^4*(e^(2*x) + 1)
^3)
```

3.134 $\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=76

$$\frac{ax}{a^2-b^2}-\frac{a^4\log(a+b\tanh(x))}{b^3\left(a^2-b^2\right)}-\frac{b\log(\cosh(x))}{a^2-b^2}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/(a^2 - b^2) - (a^4*Log[a + b*Tanh[x]])
/(b^3*(a^2 - b^2)) + (a*Tanh[x])/b^2 - Tanh[x]^2/(2*b)
```

Rubi [A] time = 0.213464, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3566, 3647, 3627, 3617, 31, 3475}

$$\frac{ax}{a^2-b^2}-\frac{a^4\log(a+b\tanh(x))}{b^3\left(a^2-b^2\right)}-\frac{b\log(\cosh(x))}{a^2-b^2}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/(a^2 - b^2) - (a^4*Log[a + b*Tanh[x]])
/(b^3*(a^2 - b^2)) + (a*Tanh[x])/b^2 - Tanh[x]^2/(2*b)
```

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3627

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*(A - C)*x)/(a^2 + b^2), x] + (Dist[(a^2*C +
A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] -
 Dist[(b*(A - C))/(a^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e,
```

```
f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx &= -\frac{\tanh^2(x)}{2b}-\frac{\int \frac{\tanh(x)\left(-2a-2b\tanh(x)+2a\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{2b}\\
&= \frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}-\frac{\int \frac{2a^2-2\left(a^2+b^2\right)\tanh^2(x)}{a+b\tanh(x)}\,dx}{2b^2}\\
&= \frac{ax}{a^2-b^2}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}-\frac{a^4\int \frac{1-\tanh^2(x)}{a+b\tanh(x)}\,dx}{b^2\left(a^2-b^2\right)}-\frac{b\int \tanh(x)\,dx}{a^2-b^2}\\
&= \frac{ax}{a^2-b^2}-\frac{b\log(\cosh(x))}{a^2-b^2}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}-\frac{a^4\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\tanh(x)\right)}{b^3\left(a^2-b^2\right)}\\
&= \frac{ax}{a^2-b^2}-\frac{b\log(\cosh(x))}{a^2-b^2}-\frac{a^4\log(a+b\tanh(x))}{b^3\left(a^2-b^2\right)}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.273711, size = 88, normalized size = 1.16

$$
\frac{b^2\left(a^2-b^2\right)\operatorname{sech}^2(x)+2\left(ab\left(a^2-b^2\right)\tanh(x)+\left(a^4-b^4\right)\log(\cosh(x))+a^4(-\log(a\cosh(x)+b\sinh(x)))+ab^3x\right)}{2b^3(a-b)(a+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (b^2*(a^2 - b^2)*Sech[x]^2 + 2*(a*b^3*x + (a^4 - b^4)*Log[Cosh[x]] - a^4*Lo
g[a*Cosh[x] + b*Sinh[x]] + a*b*(a^2 - b^2)*Tanh[x]))/(2*(a - b)*b^3*(a + b)
)
```

Maple [A] time = 0.022, size = 76, normalized size = 1.

$$
-\frac{(\tanh(x))^2}{2b}+\frac{a\tanh(x)}{b^2}+\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2b+2a}-\frac{a^4\ln(a+b\tanh(x))}{b^3(a+b)(a-b)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*tanh(x)),x)
```

```
[Out] -1/2*tanh(x)^2/b+a*tanh(x)/b^2+1/(2*a-2*b)*ln(1+tanh(x))-1/(2*b+2*a)*ln(tan
h(x)-1)-1/b^3*a^4/(a+b)/(a-b)*ln(a+b*tanh(x))
```

Maxima [A] time = 1.83016, size = 135, normalized size = 1.78

$$-\frac{a^4\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2b^3-b^5}+\frac{2\left((a+b)e^{(-2x)}+a\right)}{2b^2e^{(-2x)}+b^2e^{(-4x)}+b^2}+\frac{x}{a+b}+\frac{\left(a^2+b^2\right)\log\left(e^{(-2x)}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -a^4*log(-(a - b)*e^(-2*x) - a - b)/(a^2*b^3 - b^5) + 2*((a + b)*e^(-2*x) +
 a)/(2*b^2*e^(-2*x) + b^2*e^(-4*x) + b^2) + x/(a + b) + (a^2 + b^2)*log(e^(
-2*x) + 1)/b^3
```

Fricas [B] time = 2.51416, size = 1538, normalized size = 20.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] ((a*b^3 + b^4)*x*cosh(x)^4 + 4*(a*b^3 + b^4)*x*cosh(x)*sinh(x)^3 + (a*b^3 +
 b^4)*x*sinh(x)^4 - 2*a^3*b + 2*a*b^3 - 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 -
(a*b^3 + b^4)*x)*cosh(x)^2 - 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 - 3*(a*b^3 +
b^4)*x*cosh(x)^2 - (a*b^3 + b^4)*x)*sinh(x)^2 + (a*b^3 + b^4)*x - (a^4*cosh
(x)^4 + 4*a^4*cosh(x)*sinh(x)^3 + a^4*sinh(x)^4 + 2*a^4*cosh(x)^2 + a^4 + 2
*(3*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 4*(a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x
))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + ((a^4 - b^4)*cosh(x
)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4 +
 2*(a^4 - b^4)*cosh(x)^2 + 2*(a^4 - b^4 + 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)^
2 + 4*((a^4 - b^4)*cosh(x)^3 + (a^4 - b^4)*cosh(x))*sinh(x))*log(2*cosh(x)/
(cosh(x) - sinh(x))) + 4*((a*b^3 + b^4)*x*cosh(x)^3 - (a^3*b - a^2*b^2 - a*
b^3 + b^4 - (a*b^3 + b^4)*x)*cosh(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 -
b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh
(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5 + 3*(a^2*b^3 - b^5)*
cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3 + (a^2*b^3 - b^5)*cosh(
x))*sinh(x))
```

Sympy [A] time = 105.921, size = 442, normalized size = 5.82

$$\begin{cases} \tilde{\infty}\left(x - \log\left(\tanh\left(x\right) + 1\right) - \frac{\tanh^{2}\left(x\right)}{2}\right) \\ \frac{x - \frac{\tanh^{3}\left(x\right)}{3} - \tanh\left(x\right)}{a} \\ \frac{7x\tanh\left(x\right)}{2b\tanh\left(x\right) - 2b} - \frac{7x}{2b\tanh\left(x\right) - 2b} - \frac{4\log\left(\tanh\left(x\right) + 1\right)\tanh\left(x\right)}{2b\tanh\left(x\right) - 2b} + \frac{4\log\left(\tanh\left(x\right) + 1\right)}{2b\tanh\left(x\right) - 2b} - \frac{\tanh^{3}\left(x\right)}{2b\tanh\left(x\right) - 2b} - \frac{\tanh^{2}\left(x\right)}{2b\tanh\left(x\right) - 2b} + \frac{3}{2b\tanh\left(x\right) - 2b} \\ \frac{x\tanh\left(x\right)}{2b\tanh\left(x\right) + 2b} + \frac{x}{2b\tanh\left(x\right) + 2b} - \frac{4\log\left(\tanh\left(x\right) + 1\right)\tanh\left(x\right)}{2b\tanh\left(x\right) + 2b} - \frac{4\log\left(\tanh\left(x\right) + 1\right)}{2b\tanh\left(x\right) + 2b} - \frac{\tanh^{3}\left(x\right)}{2b\tanh\left(x\right) + 2b} + \frac{\tanh^{2}\left(x\right)}{2b\tanh\left(x\right) + 2b} - \frac{3}{2b\tanh\left(x\right) + 2b} \\ -\frac{2a^{4}\log\left(\frac{a}{b} + \tanh\left(x\right)\right)}{2a^{2}b^{3} - 2b^{5}} + \frac{2a^{3}b\tanh\left(x\right)}{2a^{2}b^{3} - 2b^{5}} - \frac{a^{2}b^{2}\tanh^{2}\left(x\right)}{2a^{2}b^{3} - 2b^{5}} + \frac{2ab^{3}x}{2a^{2}b^{3} - 2b^{5}} - \frac{2ab^{3}\tanh\left(x\right)}{2a^{2}b^{3} - 2b^{5}} - \frac{2b^{4}x}{2a^{2}b^{3} - 2b^{5}} + \frac{2b^{4}\log\left(\tanh\left(x\right) + 1\right)}{2a^{2}b^{3} - 2b^{5}} + \frac{b^{4}\tanh^{2}\left(x\right)}{2a^{2}b^{3} - 2b^{5}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) - tanh(x)**2/2), Eq(a, 0) & Eq(b, 0)),
 ((x - tanh(x)**3/3 - tanh(x))/a, Eq(b, 0)), (7*x*tanh(x)/(2*b*tanh(x) - 2*
b) - 7*x/(2*b*tanh(x) - 2*b) - 4*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) - 2*
b) + 4*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b) - tanh(x)**3/(2*b*tanh(x) - 2*b
) - tanh(x)**2/(2*b*tanh(x) - 2*b) + 3/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*
tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 4*log(tanh(x) + 1)*ta
nh(x)/(2*b*tanh(x) + 2*b) - 4*log(tanh(x) + 1)/(2*b*tanh(x) + 2*b) - tanh(x
)**3/(2*b*tanh(x) + 2*b) + tanh(x)**2/(2*b*tanh(x) + 2*b) - 3/(2*b*tanh(x)
+ 2*b), Eq(a, b)), (-2*a**4*log(a/b + tanh(x))/(2*a**2*b**3 - 2*b**5) + 2*a
**3*b*tanh(x)/(2*a**2*b**3 - 2*b**5) - a**2*b**2*tanh(x)**2/(2*a**2*b**3 -
2*b**5) + 2*a*b**3*x/(2*a**2*b**3 - 2*b**5) - 2*a*b**3*tanh(x)/(2*a**2*b**3
 - 2*b**5) - 2*b**4*x/(2*a**2*b**3 - 2*b**5) + 2*b**4*log(tanh(x) + 1)/(2*a
**2*b**3 - 2*b**5) + b**4*tanh(x)**2/(2*a**2*b**3 - 2*b**5), True))
```

Giac [A] time = 1.31051, size = 132, normalized size = 1.74

$$-\frac{a^{4}\log\left(\left|ae^{\left(2x\right)} + be^{\left(2x\right)} + a - b\right|\right)}{a^{2}b^{3} - b^{5}} + \frac{x}{a - b} + \frac{\left(a^{2} + b^{2}\right)\log\left(e^{\left(2x\right)} + 1\right)}{b^{3}} - \frac{2\left(ab + \left(ab - b^{2}\right)e^{\left(2x\right)}\right)}{b^{3}\left(e^{\left(2x\right)} + 1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -a^4*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2*b^3 - b^5) + x/(a - b) +
(a^2 + b^2)*log(e^(2*x) + 1)/b^3 - 2*(a*b + (a*b - b^2)*e^(2*x))/(b^3*(e^(2
*x) + 1)^2)
```

3.135 $\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=64

$$-\frac{bx}{a^2-b^2}+\frac{a^3\log(a+b\tanh(x))}{b^2\left(a^2-b^2\right)}+\frac{a\log(\cosh(x))}{a^2-b^2}-\frac{\tanh(x)}{b}$$

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[Cosh[x]])/(a^2 - b^2) + (a^3*Log[a + b*Tanh[x
]])/(b^2*(a^2 - b^2)) - Tanh[x]/b
```

Rubi [A] time = 0.128603, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3566, 3626, 3617, 31, 3475}

$$-\frac{bx}{a^2-b^2}+\frac{a^3\log(a+b\tanh(x))}{b^2\left(a^2-b^2\right)}+\frac{a\log(\cosh(x))}{a^2-b^2}-\frac{\tanh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[Cosh[x]])/(a^2 - b^2) + (a^3*Log[a + b*Tanh[x
]])/(b^2*(a^2 - b^2)) - Tanh[x]/b
```

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx &= -\frac{\tanh(x)}{b}-\frac{\int \frac{-a-b\tanh(x)+a\tanh^2(x)}{a+b\tanh(x)}\,dx}{b}\\
&= -\frac{bx}{a^2-b^2}-\frac{\tanh(x)}{b}+\frac{a\int \tanh(x)\,dx}{a^2-b^2}+\frac{a^3\int \frac{1-\tanh^2(x)}{a+b\tanh(x)}\,dx}{b\left(a^2-b^2\right)}\\
&= -\frac{bx}{a^2-b^2}+\frac{a\log(\cosh(x))}{a^2-b^2}-\frac{\tanh(x)}{b}+\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\tanh(x)\right)}{b^2\left(a^2-b^2\right)}\\
&= -\frac{bx}{a^2-b^2}+\frac{a\log(\cosh(x))}{a^2-b^2}+\frac{a^3\log(a+b\tanh(x))}{b^2\left(a^2-b^2\right)}-\frac{\tanh(x)}{b}
\end{aligned}$$

Mathematica [A] time = 0.120322, size = 65, normalized size = 1.02

$$\frac{\left(b^3-a^2b\right)\tanh(x)+\left(ab^2-a^3\right)\log(\cosh(x))+a^3\log(a\cosh(x)+b\sinh(x))-b^3x}{b^2(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (-(b^3*x) + (-a^3 + a*b^2)*Log[Cosh[x]] + a^3*Log[a*Cosh[x] + b*Sinh[x]] +
(-(a^2*b) + b^3)*Tanh[x])/((a - b)*b^2*(a + b))
```

Maple [A] time = 0.02, size = 67, normalized size = 1.1

$$-\frac{\tanh(x)}{b}-\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2b+2a}+\frac{a^3\ln(a+b\tanh(x))}{b^2(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*tanh(x)),x)
```

```
[Out] -tanh(x)/b-1/(2*a-2*b)*ln(1+tanh(x))-1/(2*b+2*a)*ln(tanh(x)-1)+1/b^2*a^3/(a
+b)/(a-b)*ln(a+b*tanh(x))
```

Maxima [A] time = 1.8459, size = 96, normalized size = 1.5

$$\frac{a^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2b^2-b^4}+\frac{x}{a+b}-\frac{a\log\left(e^{(-2x)}+1\right)}{b^2}-\frac{2}{be^{(-2x)}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] a^3*log(-(a - b)*e^(-2*x) - a - b)/(a^2*b^2 - b^4) + x/(a + b) - a*log(e^(-
2*x) + 1)/b^2 - 2/(b*e^(-2*x) + b)
```

Fricas [B] time = 2.49475, size = 662, normalized size = 10.34

$$-\frac{\left(ab^2+b^3\right)x\cosh\left(x\right)^2+2\left(ab^2+b^3\right)x\cosh\left(x\right)\sinh\left(x\right)+\left(ab^2+b^3\right)x\sinh\left(x\right)^2-2\,a^2b+2\,b^3+\left(ab^2+b^3\right)x-\left(a^3\cosh\left(\right.\right.}{a^2b^2-b^4+\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -((a*b^2 + b^3)*x*cosh(x)^2 + 2*(a*b^2 + b^3)*x*cosh(x)*sinh(x) + (a*b^2 +
b^3)*x*sinh(x)^2 - 2*a^2*b + 2*b^3 + (a*b^2 + b^3)*x - (a^3*cosh(x)^2 + 2*a
^3*cosh(x)*sinh(x) + a^3*sinh(x)^2 + a^3)*log(2*(a*cosh(x) + b*sinh(x))/(co
sh(x) - sinh(x))) + (a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 - a*b^2
)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(
x))))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^4)*cosh(x
)*sinh(x) + (a^2*b^2 - b^4)*sinh(x)^2)
```

Sympy [A] time = 1.3596, size = 330, normalized size = 5.16

$$\begin{cases} \tilde{\infty}\left(x-\tanh\left(x\right)\right) & \text{for}\ a=0\wedge b=0 \\ \frac{5x\tanh\left(x\right)}{2b\tanh\left(x\right)-2b}-\frac{5x}{2b\tanh\left(x\right)-2b}-\frac{2\log\left(\tanh\left(x\right)+1\right)\tanh\left(x\right)}{2b\tanh\left(x\right)-2b}+\frac{2\log\left(\tanh\left(x\right)+1\right)}{2b\tanh\left(x\right)-2b}-\frac{2\tanh^2\left(x\right)}{2b\tanh\left(x\right)-2b}+\frac{3}{2b\tanh\left(x\right)-2b} & \text{for}\ a=-b \\ \frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)+2b}+\frac{x}{2b\tanh\left(x\right)+2b}+\frac{2\log\left(\tanh\left(x\right)+1\right)\tanh\left(x\right)}{2b\tanh\left(x\right)+2b}+\frac{2\log\left(\tanh\left(x\right)+1\right)}{2b\tanh\left(x\right)+2b}-\frac{2\tanh^2\left(x\right)}{2b\tanh\left(x\right)+2b}+\frac{3}{2b\tanh\left(x\right)+2b} & \text{for}\ a=b \\ \frac{x-\log\left(\tanh\left(x\right)+1\right)-\frac{\tanh^2\left(x\right)}{2}}{a} & \text{for}\ b=0 \\ \frac{a^3\log\left(\frac{a}{b}+\tanh\left(x\right)\right)}{a^2b^2-b^4}-\frac{a^2b\tanh\left(x\right)}{a^2b^2-b^4}+\frac{ab^2x}{a^2b^2-b^4}-\frac{ab^2\log\left(\tanh\left(x\right)+1\right)}{a^2b^2-b^4}-\frac{b^3x}{a^2b^2-b^4}+\frac{b^3\tanh\left(x\right)}{a^2b^2-b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - tanh(x)), Eq(a, 0) & Eq(b, 0)), (5*x*tanh(x)/(2*b*tanh(
x) - 2*b) - 5*x/(2*b*tanh(x) - 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(
x) - 2*b) + 2*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b) - 2*tanh(x)**2/(2*b*tanh
(x) - 2*b) + 3/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2
*b) + x/(2*b*tanh(x) + 2*b) + 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) + 2*b
) + 2*log(tanh(x) + 1)/(2*b*tanh(x) + 2*b) - 2*tanh(x)**2/(2*b*tanh(x) + 2*
b) + 3/(2*b*tanh(x) + 2*b), Eq(a, b)), ((x - log(tanh(x) + 1) - tanh(x)**2/
2)/a, Eq(b, 0)), (a**3*log(a/b + tanh(x))/(a**2*b**2 - b**4) - a**2*b*tanh(
x)/(a**2*b**2 - b**4) + a*b**2*x/(a**2*b**2 - b**4) - a*b**2*log(tanh(x) +
1)/(a**2*b**2 - b**4) - b**3*x/(a**2*b**2 - b**4) + b**3*tanh(x)/(a**2*b**2
 - b**4), True))
```

Giac [A] time = 1.20419, size = 101, normalized size = 1.58

$$\frac{a^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^2b^2-b^4}-\frac{x}{a-b}-\frac{a\log\left(e^{(2x)}+1\right)}{b^2}+\frac{2}{b\left(e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] a^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2*b^2 - b^4) - x/(a - b) - a
*log(e^(2*x) + 1)/b^2 + 2/(b*(e^(2*x) + 1))
```

3.136 $\int \frac{\tanh^2(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=63

$$\frac{a^3x}{b^2\left(a^2-b^2\right)}-\frac{a^2\log(a\cosh(x)+b\sinh(x))}{b\left(a^2-b^2\right)}-\frac{ax}{b^2}+\frac{\log(\cosh(x))}{b}$$

```
[Out] -((a*x)/b^2) + (a^3*x)/(b^2*(a^2 - b^2)) + Log[Cosh[x]]/b - (a^2*Log[a*Cosh
[x] + b*Sinh[x]])/(b*(a^2 - b^2))
```

Rubi [A] time = 0.0918364, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3541, 3475, 3484, 3530}

$$\frac{a^3x}{b^2\left(a^2-b^2\right)}-\frac{a^2\log(a\cosh(x)+b\sinh(x))}{b\left(a^2-b^2\right)}-\frac{ax}{b^2}+\frac{\log(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] -((a*x)/b^2) + (a^3*x)/(b^2*(a^2 - b^2)) + Log[Cosh[x]]/b - (a^2*Log[a*Cosh
[x] + b*Sinh[x]])/(b*(a^2 - b^2))
```

Rule 3541

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(d*(2*b*c - a*d)*x)/b^2, x] + (Dist[d^2/b, In
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{a+b\tanh(x)}\,dx = -\frac{ax}{b^2} + \frac{a^2\int \frac{1}{a+b\tanh(x)}\,dx}{b^2} + \frac{\int \tanh(x)\,dx}{b}$$
$$= -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\cosh(x))}{b} - \frac{(ia^2)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{b(a^2-b^2)}$$
$$= -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\cosh(x))}{b} - \frac{a^2\log(a\cosh(x)+b\sinh(x))}{b(a^2-b^2)}$$

Mathematica [A] time = 0.074753, size = 49, normalized size = 0.78

$$\frac{-a^2\log(a\cosh(x)+b\sinh(x)) + a^2\log(\cosh(x)) + abx - b^2\log(\cosh(x))}{a^2b-b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (a*b*x + a^2*Log[Cosh[x]] - b^2*Log[Cosh[x]] - a^2*Log[a*Cosh[x] + b*Sinh[x
]])/(a^2*b - b^3)
```

Maple [A] time = 0.017, size = 60, normalized size = 1.

$$\frac{\ln(1+\tanh(x))}{2a-2b} - \frac{\ln(\tanh(x)-1)}{2b+2a} - \frac{a^2\ln(a+b\tanh(x))}{(a+b)(a-b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*tanh(x)),x)
```

```
[Out] 1/(2*a-2*b)*ln(1+tanh(x))-1/(2*b+2*a)*ln(tanh(x)-1)-a^2/(a+b)/(a-b)/b*ln(a+
b*tanh(x))
```

Maxima [A] time = 1.8147, size = 76, normalized size = 1.21

$$-\frac{a^2\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2b-b^3} + \frac{x}{a+b} + \frac{\log\left(e^{(-2x)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -a^2*log(-(a - b)*e^(-2*x) - a - b)/(a^2*b - b^3) + x/(a + b) + log(e^(-2*x
) + 1)/b
```

Fricas [A] time = 2.28739, size = 186, normalized size = 2.95

$$-\frac{a^2\log\left(\frac{2(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right) - (ab+b^2)x - (a^2-b^2)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2b-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -(a^2*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - (a*b + b^2)*x -
(a^2 - b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
```

Sympy [A] time = 0.871377, size = 243, normalized size = 3.86

$$\begin{cases} \tilde{\infty}\left(x - \log\left(\tanh\left(x\right) + 1\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{3x\tanh(x)}{2b\tanh(x)-2b} - \frac{3x}{2b\tanh(x)-2b} - \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)-2b} + \frac{2\log(\tanh(x)+1)}{2b\tanh(x)-2b} + \frac{1}{2b\tanh(x)-2b} & \text{for } a = -b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} - \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)+2b} - \frac{2\log(\tanh(x)+1)}{2b\tanh(x)+2b} - \frac{1}{2b\tanh(x)+2b} & \text{for } a = b \\ \frac{x-\tanh(x)}{a} & \text{for } b = 0 \\ -\frac{a^2\log\left(\frac{a}{b}+\tanh(x)\right)}{a^2b-b^3} + \frac{abx}{a^2b-b^3} - \frac{b^2x}{a^2b-b^3} + \frac{b^2\log(\tanh(x)+1)}{a^2b-b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1)), Eq(a, 0) & Eq(b, 0)), (3*x*tanh(x)/(
2*b*tanh(x) - 2*b) - 3*x/(2*b*tanh(x) - 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(
2*b*tanh(x) - 2*b) + 2*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b) + 1/(2*b*tanh(x
) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b
) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) + 2*b) - 2*log(tanh(x) + 1)/(2*
b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), ((x - tanh(x))/a, Eq(b
, 0)), (-a**2*log(a/b + tanh(x))/(a**2*b - b**3) + a*b*x/(a**2*b - b**3) -
b**2*x/(a**2*b - b**3) + b**2*log(tanh(x) + 1)/(a**2*b - b**3), True))
```

Giac [A] time = 1.18135, size = 78, normalized size = 1.24

$$-\frac{a^2\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^2b - b^3} + \frac{x}{a - b} + \frac{\log\left(e^{(2x)} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2*b - b^3) + x/(a - b) + lo
g(e^(2*x) + 1)/b
```

3.137 $\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0593377, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3531, 3530}

$$\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx&=-\frac{bx}{a^2-b^2}+\frac{(ia)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\&=-\frac{bx}{a^2-b^2}+\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.0537865, size = 29, normalized size = 0.74

$$\frac{a\log(a\cosh(x)+b\sinh(x))-bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]),x]
```

```
[Out] (-(b*x) + a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] time = 0.019, size = 55, normalized size = 1.4

$$-\frac{\ln\left(1+\tanh\left(x\right)\right)}{2\,a-2\,b}-\frac{\ln\left(\tanh\left(x\right)-1\right)}{2\,b+2\,a}+\frac{a\ln\left(a+b\tanh\left(x\right)\right)}{\left(a+b\right)\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)),x)
```

```
[Out] -1/(2*a-2*b)*ln(1+tanh(x))-1/(2*b+2*a)*ln(tanh(x)-1)+a/(a+b)/(a-b)*ln(a+b*t
anh(x))
```

Maxima [A] time = 1.27725, size = 54, normalized size = 1.38

$$\frac{a\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] a*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A] time = 2.16439, size = 109, normalized size = 2.79

$$-\frac{(a+b)x-a\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -((a + b)*x - a*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [A] time = 0.701834, size = 141, normalized size = 3.62

$$\begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ \frac{x-\log\left(\tanh\left(x\right)+1\right)}{a} & \text{for } b=0 \\ \frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)-2b}-\frac{x}{2b\tanh\left(x\right)-2b}+\frac{1}{2b\tanh\left(x\right)-2b} & \text{for } a=-b \\ \frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)+2b}+\frac{x}{2b\tanh\left(x\right)+2b}+\frac{1}{2b\tanh\left(x\right)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2}+\frac{a\log\left(\frac{a}{b}+\tanh\left(x\right)\right)}{a^2-b^2}-\frac{a\log\left(\tanh\left(x\right)+1\right)}{a^2-b^2}-\frac{bx}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1))/a, Eq(b, 0)
), (x*tanh(x)/(2*b*tanh(x) - 2*b) - x/(2*b*tanh(x) - 2*b) + 1/(2*b*tanh(x)
- 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b)
+ 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2) + a*log(a/b + tanh(x
))/(a**2 - b**2) - a*log(tanh(x) + 1)/(a**2 - b**2) - b*x/(a**2 - b**2), Tr
ue))
```

Giac [A] time = 1.19049, size = 58, normalized size = 1.49

$$\frac{a\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^2 - b^2} - \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) - x/(a - b)
```

3.138 $\int \frac{1}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0460166, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3484, 3530}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\tanh(x)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.0411584, size = 29, normalized size = 0.74

$$\frac{ax-b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] time = 0., size = 55, normalized size = 1.4

$$\frac{\ln\left(1+\tanh\left(x\right)\right)}{2\,a-2\,b}-\frac{\ln\left(\tanh\left(x\right)-1\right)}{2\,b+2\,a}-\frac{b\ln\left(a+b\tanh\left(x\right)\right)}{\left(a+b\right)\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)),x)
```

```
[Out] 1/(2*a-2*b)*ln(1+tanh(x))-1/(2*b+2*a)*ln(tanh(x)-1)-b/(a-b)/(a+b)*ln(a+b*ta
nh(x))
```

Maxima [A] time = 1.1422, size = 55, normalized size = 1.41

$$-\frac{b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A] time = 2.17193, size = 108, normalized size = 2.77

$$\frac{(a+b)x-b\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [A] time = 0.687084, size = 146, normalized size = 3.74

$$\begin{cases} \tilde{\infty}\left(x-\log\left(\tanh\left(x\right)+1\right)+\log\left(\tanh\left(x\right)\right)\right) & \text{for}\ a=0\wedge b=0\\ \frac{x}{a} & \text{for}\ b=0\\ -\frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)-2b}+\frac{x}{2b\tanh\left(x\right)-2b}+\frac{1}{2b\tanh\left(x\right)-2b} & \text{for}\ a=-b\\ \frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)+2b}+\frac{x}{2b\tanh\left(x\right)+2b}-\frac{1}{2b\tanh\left(x\right)+2b} & \text{for}\ a=b\\ \frac{ax}{a^2-b^2}-\frac{bx}{a^2-b^2}-\frac{b\log\left(\frac{a}{b}+\tanh\left(x\right)\right)}{a^2-b^2}+\frac{b\log\left(\tanh\left(x\right)+1\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 (x/a, Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) +
 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b
*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2) - b*
x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) + 1)/(
a**2 - b**2), True))
```

Giac [A] time = 1.16697, size = 58, normalized size = 1.49

$$-\frac{b\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^2 - b^2} + \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

3.139 $\int \frac{\coth(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{bx}{a^2-b^2}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{a\left(a^2-b^2\right)}+\frac{\log(\sinh(x))}{a}$$

```
[Out] -((b*x)/(a^2 - b^2)) + Log[Sinh[x]]/a + (b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a
*(a^2 - b^2))
```

Rubi [A] time = 0.0788969, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3571, 3530, 3475}

$$-\frac{bx}{a^2-b^2}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{a\left(a^2-b^2\right)}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + Log[Sinh[x]]/a + (b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a
*(a^2 - b^2))
```

Rule 3571

```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[((a*c - b*d)*x)/((a^2 + b^2)*(c^2 + d^2)), x
] + (Dist[b^2/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Ta
n[e + f*x]), x], x] - Dist[d^2/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e
+ f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+b\tanh(x)}\,dx &= -\frac{bx}{a^2-b^2}+\frac{\int \coth(x)\,dx}{a}+\frac{\left(ib^2\right)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a\left(a^2-b^2\right)}\\ &= -\frac{bx}{a^2-b^2}+\frac{\log(\sinh(x))}{a}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{a\left(a^2-b^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0769372, size = 46, normalized size = 0.9

$$\frac{(a^2-b^2)\log(\sinh(x))+b(b\log(a\cosh(x)+b\sinh(x))-ax)}{a^3-ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Tanh[x]),x]
```

```
[Out] ((a^2 - b^2)*Log[Sinh[x]] + b*(-(a*x) + b*Log[a*Cosh[x] + b*Sinh[x]]))/(a^3
 - a*b^2)
```

Maple [A] time = 0.039, size = 78, normalized size = 1.5

$$-\frac{1}{a-b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{b^2}{(a+b)(a-b)a}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2+2\tanh(x/2)b+a\right)+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{a+b}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*tanh(x)),x)
```

```
[Out] -1/(a-b)*ln(tanh(1/2*x)+1)+b^2/(a+b)/(a-b)/a*ln(a*tanh(1/2*x)^2+2*tanh(1/2*
x)*b+a)+1/a*ln(tanh(1/2*x))-1/(a+b)*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.261, size = 88, normalized size = 1.73

$$\frac{b^2\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^3-ab^2}+\frac{x}{a+b}+\frac{\log\left(e^{(-x)}+1\right)}{a}+\frac{\log\left(e^{(-x)}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^3 - a*b^2) + x/(a + b) + log(e^(-x) +
 1)/a + log(e^(-x) - 1)/a
```

Fricas [A] time = 2.35667, size = 185, normalized size = 3.63

$$\frac{b^2\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\left(a^2+ab\right)x+\left(a^2-b^2\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a^3-ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] (b^2*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - (a^2 + a*b)*x + (
a^2 - b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*tanh(x)), x)
```

Giac [A] time = 1.18033, size = 78, normalized size = 1.53

$$\frac{b^2\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^3 - ab^2} - \frac{x}{a-b} + \frac{\log\left(\left|e^{(2x)} - 1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^3 - a*b^2) - x/(a - b) + log
(abs(e^(2*x) - 1))/a
```

3.140 $\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{ax}{a^2-b^2}-\frac{b^3\log(a\cosh(x)+b\sinh(x))}{a^2\left(a^2-b^2\right)}-\frac{b\log(\sinh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[Out] (a*x)/(a^2 - b^2) - Coth[x]/a - (b*Log[Sinh[x]])/a^2 - (b^3*Log[a*Cosh[x] +
 b*Sinh[x]])/(a^2*(a^2 - b^2))
```

Rubi [A] time = 0.182009, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3569, 3651, 3530, 3475}

$$\frac{ax}{a^2-b^2}-\frac{b^3\log(a\cosh(x)+b\sinh(x))}{a^2\left(a^2-b^2\right)}-\frac{b\log(\sinh(x))}{a^2}-\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - Coth[x]/a - (b*Log[Sinh[x]])/a^2 - (b^3*Log[a*Cosh[x] +
 b*Sinh[x]])/(a^2*(a^2 - b^2))
```

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx = -\frac{\coth(x)}{a} - \frac{i\int \frac{\coth(x)\left(-ib+ia\tanh(x)+ib\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{a}$$

$$= \frac{ax}{a^2-b^2} - \frac{\coth(x)}{a} - \frac{b\int \coth(x)\,dx}{a^2} - \frac{\left(ib^3\right)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{ax}{a^2-b^2} - \frac{\coth(x)}{a} - \frac{b\log(\sinh(x))}{a^2} - \frac{b^3\log(a\cosh(x)+b\sinh(x))}{a^2\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.122652, size = 64, normalized size = 1.07

$$\frac{\left(ab^2-a^3\right)\coth(x)+\left(b^3-a^2b\right)\log(\sinh(x))+a^3x-b^3\log(a\cosh(x)+b\sinh(x))}{a^4-a^2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (a^3*x + (-a^3 + a*b^2)*Coth[x] + (-(a^2*b) + b^3)*Log[Sinh[x]] - b^3*Log[a
*Cosh[x] + b*Sinh[x]])/(a^4 - a^2*b^2)
```

Maple [A] time = 0.042, size = 100, normalized size = 1.7

$$-\frac{1}{2a}\tanh\left(\frac{x}{2}\right)+\frac{1}{a-b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{b^3}{a^2(a+b)(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2+2\tanh(x/2)\,b+a\right)-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*tanh(x)),x)
```

```
[Out] -1/2/a*tanh(1/2*x)+1/(a-b)*ln(tanh(1/2*x)+1)-1/a^2*b^3/(a+b)/(a-b)*ln(a*tan
h(1/2*x)^2+2*tanh(1/2*x)*b+a)-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))-1/(
a+b)*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.25858, size = 116, normalized size = 1.93

$$-\frac{b^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-a^2b^2}+\frac{x}{a+b}-\frac{b\log\left(e^{(-x)}+1\right)}{a^2}-\frac{b\log\left(e^{(-x)}-1\right)}{a^2}+\frac{2}{ae^{(-2x)}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - a^2*b^2) + x/(a + b) - b*log(e^(
-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + 2/(a*e^(-2*x) - a)
```

Fricas [B] time = 2.48095, size = 662, normalized size = 11.03

$$-\frac{\left(a^{3}+a^{2}b\right)x\cosh\left(x\right)^{2}+2\left(a^{3}+a^{2}b\right)x\cosh\left(x\right)\sinh\left(x\right)+\left(a^{3}+a^{2}b\right)x\sinh\left(x\right)^{2}-2a^{3}+2ab^{2}-\left(a^{3}+a^{2}b\right)x-\left(b^{3}\cosh\left(\right.\right.}{a^{4}-a^{2}b^{2}-\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -((a^3 + a^2*b)*x*cosh(x)^2 + 2*(a^3 + a^2*b)*x*cosh(x)*sinh(x) + (a^3 + a^
2*b)*x*sinh(x)^2 - 2*a^3 + 2*a*b^2 - (a^3 + a^2*b)*x - (b^3*cosh(x)^2 + 2*b
^3*cosh(x)*sinh(x) + b^3*sinh(x)^2 - b^3)*log(2*(a*cosh(x) + b*sinh(x))/(co
sh(x) - sinh(x))) + (a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - 2*(a^2*b - b^3
)*cosh(x)*sinh(x) - (a^2*b - b^3)*sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(
x))))/(a^4 - a^2*b^2 - (a^4 - a^2*b^2)*cosh(x)^2 - 2*(a^4 - a^2*b^2)*cosh(x
)*sinh(x) - (a^4 - a^2*b^2)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^{2}\left(x\right)}{a+b\tanh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*tanh(x)), x)
```

Giac [A] time = 1.15227, size = 101, normalized size = 1.68

$$-\frac{b^{3}\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^{4}-a^{2}b^{2}}+\frac{x}{a-b}-\frac{b\log\left(\left|e^{(2x)}-1\right|\right)}{a^{2}}-\frac{2}{a\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - a^2*b^2) + x/(a - b) -
b*log(abs(e^(2*x) - 1))/a^2 - 2/(a*(e^(2*x) - 1))
```

3.141 $\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=76

$$-\frac{bx}{a^2-b^2}+\frac{\left(a^2+b^2\right)\log(\sinh(x))}{a^3}+\frac{b^4\log(a\cosh(x)+b\sinh(x))}{a^3\left(a^2-b^2\right)}+\frac{b\coth(x)}{a^2}-\frac{\coth^2(x)}{2a}$$

```
[Out] -((b*x)/(a^2 - b^2)) + (b*Coth[x])/a^2 - Coth[x]^2/(2*a) + ((a^2 + b^2)*Log
[Sinh[x]])/a^3 + (b^4*Log[a*Cosh[x] + b*Sinh[x]])/(a^3*(a^2 - b^2))
```

Rubi [A] time = 0.309112, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3569, 3649, 3652, 3530, 3475}

$$-\frac{bx}{a^2-b^2}+\frac{\left(a^2+b^2\right)\log(\sinh(x))}{a^3}+\frac{b^4\log(a\cosh(x)+b\sinh(x))}{a^3\left(a^2-b^2\right)}+\frac{b\coth(x)}{a^2}-\frac{\coth^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (b*Coth[x])/a^2 - Coth[x]^2/(2*a) + ((a^2 + b^2)*Log
[Sinh[x]])/a^3 + (b^4*Log[a*Cosh[x] + b*Sinh[x]])/(a^3*(a^2 - b^2))
```

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3652

```
Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[((a*
(A*c - c*C) - b*(A*d - C*d))*x)/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^
```

```
2 + a^2*C)/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
 + f*x]), x], x] - Dist[(c^2*C + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d -
 c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx &= -\frac{\coth^2(x)}{2a}-\frac{i\int \frac{\coth^2(x)\left(-2ib+2ia\tanh(x)+2ib\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{2a}\\
&= \frac{b\coth(x)}{a^2}-\frac{\coth^2(x)}{2a}-\frac{\int \frac{\coth(x)\left(-2\left(a^2+b^2\right)+2b^2\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{2a^2}\\
&= -\frac{bx}{a^2-b^2}+\frac{b\coth(x)}{a^2}-\frac{\coth^2(x)}{2a}+\frac{\left(ib^4\right)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^3\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\int \coth(x)\,dx}{a^3}\\
&= -\frac{bx}{a^2-b^2}+\frac{b\coth(x)}{a^2}-\frac{\coth^2(x)}{2a}+\frac{\left(a^2+b^2\right)\log(\sinh(x))}{a^3}+\frac{b^4\log(a\cosh(x)+b\sinh(x))}{a^3\left(a^2-b^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.184197, size = 91, normalized size = 1.2

$$\frac{2ab\left(a^2-b^2\right)\coth(x)+\left(a^2b^2-a^4\right)\text{csch}^2(x)-2a^3bx+2a^4\log(\sinh(x))+2b^4\log(a\cosh(x)+b\sinh(x))-2b^4\log(\sinh}{2a^3(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (-2*a^3*b*x + 2*a*b*(a^2 - b^2)*Coth[x] + (-a^4 + a^2*b^2)*Csch[x]^2 + 2*a^
4*Log[Sinh[x]] - 2*b^4*Log[Sinh[x]] + 2*b^4*Log[a*Cosh[x] + b*Sinh[x]])/(2*
a^3*(a - b)*(a + b))
```

Maple [A] time = 0.046, size = 134, normalized size = 1.8

$$-\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{b}{2a^2}\tanh\left(\frac{x}{2}\right)-\frac{1}{a-b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{b^4}{a^3(a+b)(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2+2\tanh(x/2)b+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*tanh(x)),x)
```

```
[Out] -1/8/a*tanh(1/2*x)^2+1/2/a^2*tanh(1/2*x)*b-1/(a-b)*ln(tanh(1/2*x)+1)+1/a^3*
b^4/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b+a)-1/8/a/tanh(1/2*x)^2+1
/a*ln(tanh(1/2*x))+1/a^3*ln(tanh(1/2*x))*b^2+1/2*b/a^2/tanh(1/2*x)-1/(a+b)*
ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.27848, size = 163, normalized size = 2.14

$$\frac{b^4 \log\left(-(a-b)e^{(-2x)} - a - b\right)}{a^5 - a^3b^2} + \frac{2\left((a+b)e^{(-2x)} - b\right)}{2\,a^2e^{(-2x)} - a^2e^{(-4x)} - a^2} + \frac{x}{a+b} + \frac{\left(a^2+b^2\right)\log\left(e^{(-x)}+1\right)}{a^3} + \frac{\left(a^2+b^2\right)\log\left(e^{(-x)} -\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-(a - b)*e^(-2*x) - a - b)/(a^5 - a^3*b^2) + 2*((a + b)*e^(-2*x) -
b)/(2*a^2*e^(-2*x) - a^2*e^(-4*x) - a^2) + x/(a + b) + (a^2 + b^2)*log(e^(-
x) + 1)/a^3 + (a^2 + b^2)*log(e^(-x) - 1)/a^3
```

Fricas [B] time = 2.83733, size = 1539, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + a^3*b)*x*cosh(x)^4 + 4*(a^4 + a^3*b)*x*cosh(x)*sinh(x)^3 + (a^4 +
a^3*b)*x*sinh(x)^4 + 2*a^3*b - 2*a*b^3 + 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 -
 (a^4 + a^3*b)*x)*cosh(x)^2 + 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 + 3*(a^4 + a
^3*b)*x*cosh(x)^2 - (a^4 + a^3*b)*x)*sinh(x)^2 + (a^4 + a^3*b)*x - (b^4*cos
h(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 - 2*b^4*cosh(x)^2 + b^4 +
2*(3*b^4*cosh(x)^2 - b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - b^4*cosh(x))*sinh(
x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - ((a^4 - b^4)*cosh(
x)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4
- 2*(a^4 - b^4)*cosh(x)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)
^2 + 4*((a^4 - b^4)*cosh(x)^3 - (a^4 - b^4)*cosh(x))*sinh(x))*log(2*sinh(x)
/(cosh(x) - sinh(x))) + 4*((a^4 + a^3*b)*x*cosh(x)^3 + (a^4 - a^3*b - a^2*b
^2 + a*b^3 - (a^4 + a^3*b)*x)*cosh(x))*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3
*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sin
h(x)^4 - 2*(a^5 - a^3*b^2)*cosh(x)^2 - 2*(a^5 - a^3*b^2 - 3*(a^5 - a^3*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 - (a^5 - a^3*b^2)*cosh
(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*tanh(x)), x)
```

Giac [A] time = 1.17969, size = 131, normalized size = 1.72

$$\frac{b^4\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^5 - a^3b^2} - \frac{x}{a-b} + \frac{\left(a^2 + b^2\right)\log\left(\left|e^{(2x)} - 1\right|\right)}{a^3} - \frac{2\left(ab + \left(a^2 - ab\right)e^{(2x)}\right)}{a^3\left(e^{(2x)} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^5 - a^3*b^2) - x/(a - b) + (
a^2 + b^2)*log(abs(e^(2*x) - 1))/a^3 - 2*(a*b + (a^2 - a*b)*e^(2*x))/(a^3*(
e^(2*x) - 1)^2)
```

3.142 $\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=97

$$\frac{ax}{a^2-b^2}-\frac{(a^2+b^2)\coth(x)}{a^3}-\frac{b(a^2+b^2)\log(\sinh(x))}{a^4}-\frac{b^5\log(a\cosh(x)+b\sinh(x))}{a^4(a^2-b^2)}+\frac{b\coth^2(x)}{2a^2}-\frac{\coth^3(x)}{3a}$$

```
[Out] (a*x)/(a^2 - b^2) - ((a^2 + b^2)*Coth[x])/a^3 + (b*Coth[x]^2)/(2*a^2) - Cot
h[x]^3/(3*a) - (b*(a^2 + b^2)*Log[Sinh[x]])/a^4 - (b^5*Log[a*Cosh[x] + b*Si
nh[x]])/(a^4*(a^2 - b^2))
```

Rubi [A] time = 0.494199, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3569, 3649, 3650, 3651, 3530, 3475}

$$\frac{ax}{a^2-b^2}-\frac{(a^2+b^2)\coth(x)}{a^3}-\frac{b(a^2+b^2)\log(\sinh(x))}{a^4}-\frac{b^5\log(a\cosh(x)+b\sinh(x))}{a^4(a^2-b^2)}+\frac{b\coth^2(x)}{2a^2}-\frac{\coth^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - ((a^2 + b^2)*Coth[x])/a^3 + (b*Coth[x]^2)/(2*a^2) - Cot
h[x]^3/(3*a) - (b*(a^2 + b^2)*Log[Sinh[x]])/a^4 - (b^5*Log[a*Cosh[x] + b*Si
nh[x]])/(a^4*(a^2 - b^2))
```

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n
+ 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] && !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx &= -\frac{\coth^3(x)}{3a} - \frac{i\int \frac{\coth^3(x)\left(-3ib+3ia\tanh(x)+3ib\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{3a} \\
&= \frac{b\coth^2(x)}{2a^2} - \frac{\coth^3(x)}{3a} - \frac{\int \frac{\coth^2(x)\left(-6\left(a^2+b^2\right)+6b^2\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{6a^2} \\
&= -\frac{\left(a^2+b^2\right)\coth(x)}{a^3} + \frac{b\coth^2(x)}{2a^2} - \frac{\coth^3(x)}{3a} + \frac{i\int \frac{\coth(x)\left(6ib\left(a^2+b^2\right)-6ia^3\tanh(x)-6ib\left(a^2+b^2\right)\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{6a^3} \\
&= \frac{ax}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{a^3} + \frac{b\coth^2(x)}{2a^2} - \frac{\coth^3(x)}{3a} - \frac{\left(ib^5\right)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^4\left(a^2-b^2\right)} - \frac{\left(b\left(a^2+b^2\right)\right)\int}{a^4} \\
&= \frac{ax}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{a^3} + \frac{b\coth^2(x)}{2a^2} - \frac{\coth^3(x)}{3a} - \frac{b\left(a^2+b^2\right)\log(\sinh(x))}{a^4} - \frac{b^5\log(a\cosh(}{a^4\left(a^2\right.}
\end{aligned}$$

Mathematica [A] time = 0.2852, size = 108, normalized size = 1.11

$$\frac{3a^2b\left(a^2-b^2\right)\operatorname{csch}^2(x)+6\left(b^5-a^4b\right)\log(\sinh(x))-2a\left(a^2-b^2\right)\coth(x)\left(a^2\operatorname{csch}^2(x)+4a^2+3b^2\right)+6a^5x-6b^5\log(a\,\mathrm{c}}{6a^4(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (6*a^5*x + 3*a^2*b*(a^2 - b^2)*Csch[x]^2 - 2*a*(a^2 - b^2)*Coth[x]*(4*a^2 +
 3*b^2 + a^2*Csch[x]^2) + 6*(-(a^4*b) + b^5)*Log[Sinh[x]] - 6*b^5*Log[a*Cos
h[x] + b*Sinh[x]])/(6*a^4*(a - b)*(a + b))
```

Maple [A] time = 0.049, size = 185, normalized size = 1.9

$$-\frac{1}{24a}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{b}{8a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{5}{8a}\tanh\left(\frac{x}{2}\right)-\frac{b^2}{2a^3}\tanh\left(\frac{x}{2}\right)+\frac{1}{a-b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{b^5}{a^4(a+b)(a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*tanh(x)),x)
```

```
[Out] -1/24/a*tanh(1/2*x)^3+1/8/a^2*b*tanh(1/2*x)^2-5/8/a*tanh(1/2*x)-1/2/a^3*b^2
*tanh(1/2*x)+1/(a-b)*ln(tanh(1/2*x)+1)-1/a^4*b^5/(a+b)/(a-b)*ln(a*tanh(1/2*
x)^2+2*tanh(1/2*x)*b+a)-1/24/a/tanh(1/2*x)^3-5/8/a/tanh(1/2*x)-1/2/a^3/tanh
(1/2*x)*b^2+1/8/a^2*b/tanh(1/2*x)^2-1/a^2*b*ln(tanh(1/2*x))-1/a^4*b^3*ln(ta
nh(1/2*x))-1/(a+b)*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.18695, size = 234, normalized size = 2.41

$$-\frac{b^5\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-a^4b^2}+\frac{2\left(4a^2+3b^2-3\left(2a^2+ab+2b^2\right)e^{(-2x)}+3\left(2a^2+ab+b^2\right)e^{(-4x)}\right)}{3\left(3a^3e^{(-2x)}-3a^3e^{(-4x)}+a^3e^{(-6x)}-a^3\right)}+\frac{x}{a+b}-\frac{(a^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - a^4*b^2) + 2/3*(4*a^2 + 3*b^2 -
3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(2*a^2 + a*b + b^2)*e^(-4*x))/(3*a^3*e
^(-2*x) - 3*a^3*e^(-4*x) + a^3*e^(-6*x) - a^3) + x/(a + b) - (a^2*b + b^3)*
log(e^(-x) + 1)/a^4 - (a^2*b + b^3)*log(e^(-x) - 1)/a^4
```

Fricas [B] time = 3.0814, size = 3087, normalized size = 31.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 + a^4*b)*x*cosh(x)^6 + 18*(a^5 + a^4*b)*x*cosh(x)*sinh(x)^5 + 3
*(a^5 + a^4*b)*x*sinh(x)^6 - 8*a^5 + 2*a^3*b^2 + 6*a*b^4 - 3*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^4 - 3*(4*
a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - 15*(a^5 + a^4*b)*x*cosh(x
)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^4 + 12*(5*(a^5 + a^4*b)*x*cosh(x)^3 - (4*a
^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)
)*sinh(x)^3 + 3*(4*a^5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)
*cosh(x)^2 + 3*(15*(a^5 + a^4*b)*x*cosh(x)^4 + 4*a^5 - 2*a^4*b + 2*a^2*b^3
```

```
- 4*a*b^4 - 6*(4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 +
 a^4*b)*x)*cosh(x)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^2 - 3*(a^5 + a^4*b)*x - 3
*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 - 3*b^5*cosh(x)^4
 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*cosh(x)^2 - b^5)*sinh(x)^4 + 4*(5*b^5*c
osh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 - 6*b^5*cosh(x)^2
+ b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 - 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x
))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^4*b - b^5)*co
sh(x)^6 + 6*(a^4*b - b^5)*cosh(x)*sinh(x)^5 + (a^4*b - b^5)*sinh(x)^6 - a^4
*b + b^5 - 3*(a^4*b - b^5)*cosh(x)^4 - 3*(a^4*b - b^5 - 5*(a^4*b - b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - b^5)*cosh(x)^3 - 3*(a^4*b - b^5)*cosh(x))
*sinh(x)^3 + 3*(a^4*b - b^5)*cosh(x)^2 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*c
osh(x)^4 - 6*(a^4*b - b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b - b^5)*cosh(x)^
5 - 2*(a^4*b - b^5)*cosh(x)^3 + (a^4*b - b^5)*cosh(x))*sinh(x))*log(2*sinh(
x)/(cosh(x) - sinh(x))) + 6*(3*(a^5 + a^4*b)*x*cosh(x)^5 - 2*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^3 + (4*a^
5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x))*sinh(x))/((
a^6 - a^4*b^2)*cosh(x)^6 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4
*b^2)*sinh(x)^6 - a^6 + a^4*b^2 - 3*(a^6 - a^4*b^2)*cosh(x)^4 - 3*(a^6 - a^
4*b^2 - 5*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(
x)^3 - 3*(a^6 - a^4*b^2)*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 +
 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^4 - 6*(a^6 - a^4*b^2)*cosh(x)
^2)*sinh(x)^2 + 6*((a^6 - a^4*b^2)*cosh(x)^5 - 2*(a^6 - a^4*b^2)*cosh(x)^3
+ (a^6 - a^4*b^2)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(x)}{a + b\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*tanh(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*tanh(x)), x)
```

Giac [A] time = 1.18652, size = 192, normalized size = 1.98

$$-\frac{b^5\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^6 - a^4b^2} + \frac{x}{a-b} - \frac{\left(a^2b + b^3\right)\log\left(\left|e^{(2x)} - 1\right|\right)}{a^4} - \frac{2\left(4a^3 + 3ab^2 + 3\left(2a^3 - a^2b + ab^2\right)e^{(4x)} - 3\left(2\right.\right.}{3a^4\left(e^{(2x)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - a^4*b^2) + x/(a - b) -
(a^2*b + b^3)*log(abs(e^(2*x) - 1))/a^4 - 2/3*(4*a^3 + 3*a*b^2 + 3*(2*a^3 -
 a^2*b + a*b^2)*e^(4*x) - 3*(2*a^3 - a^2*b + 2*a*b^2)*e^(2*x))/(a^4*(e^(2*x
) - 1)^3)
```

3.143 $\int \frac{x\,\mathrm{sech}^2(x)}{(a+b\tanh(x))^2}\,dx$

Optimal. Leaf size=55

$$\frac{ax}{b\left(a^2-b^2\right)}-\frac{\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{x}{b(a+b\tanh(x))}$$

```
[Out] (a*x)/(b*(a^2 - b^2)) - Log[a*Cosh[x] + b*Sinh[x]]/(a^2 - b^2) - x/(b*(a +
b*Tanh[x]))
```

Rubi [A] time = 0.0854264, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5466, 3484, 3530}

$$\frac{ax}{b\left(a^2-b^2\right)}-\frac{\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{x}{b(a+b\tanh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sech[x]^2)/(a + b*Tanh[x])^2,x]
```

```
[Out] (a*x)/(b*(a^2 - b^2)) - Log[a*Cosh[x] + b*Sinh[x]]/(a^2 - b^2) - x/(b*(a +
b*Tanh[x]))
```

Rule 5466

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^2*((a_) + (b_.)*Tan
h[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Tanh[c
+ d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x
)^(m - 1)*(a + b*Tanh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{x\operatorname{sech}^2(x)}{(a+b\tanh(x))^2}\,dx = -\frac{x}{b(a+b\tanh(x))} + \frac{\int \frac{1}{a+b\tanh(x)}\,dx}{b}$$
$$= \frac{ax}{b\left(a^2-b^2\right)} - \frac{x}{b(a+b\tanh(x))} - \frac{i\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}$$
$$= \frac{ax}{b\left(a^2-b^2\right)} - \frac{\log(a\cosh(x)+b\sinh(x))}{a^2-b^2} - \frac{x}{b(a+b\tanh(x))}$$

Mathematica [A] time = 0.165549, size = 49, normalized size = 0.89

$$\frac{bx-a\log(a\cosh(x)+b\sinh(x))}{a^3-ab^2} + \frac{x\sinh(x)}{a^2\cosh(x)+ab\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sech[x]^2)/(a + b*Tanh[x])^2,x]
```

```
[Out] (b*x - a*Log[a*Cosh[x] + b*Sinh[x]])/(a^3 - a*b^2) + (x*Sinh[x])/(a^2*Cosh[
x] + a*b*Sinh[x])
```

Maple [A] time = 0.111, size = 73, normalized size = 1.3

$$2\,\frac{x}{a^2-b^2} - 2\,\frac{x}{\left(a\mathrm{e}^{2\,x}+b\mathrm{e}^{2\,x}+a-b\right)(a+b)} - \frac{1}{a^2-b^2}\ln\left(\mathrm{e}^{2\,x}+\frac{a-b}{a+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(x)^2/(a+b*tanh(x))^2,x)
```

```
[Out] 2/(a^2-b^2)*x-2*x/(a*exp(2*x)+b*exp(2*x)+a-b)/(a+b)-1/(a^2-b^2)*ln(exp(2*x)
+(a-b)/(a+b))
```

Maxima [A] time = 2.04363, size = 92, normalized size = 1.67

$$\frac{2\,xe^{(2\,x)}}{a^2-2\,ab+b^2+\left(a^2-b^2\right)e^{(2\,x)}} - \frac{\log\left(\frac{(a+b)e^{(2\,x)}+a-b}{a+b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(x)^2/(a+b*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x*e^(2*x)/(a^2 - 2*a*b + b^2 + (a^2 - b^2)*e^(2*x)) - log(((a + b)*e^(2*x
) + a - b)/(a + b))/(a^2 - b^2)
```

Fricas [B] time = 2.71255, size = 491, normalized size = 8.93

$$\frac{2\,(a+b)x\cosh(x)^2+4\,(a+b)x\cosh(x)\sinh(x)+2\,(a+b)x\sinh(x)^2-\left((a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)\ldots\right.}{a^3-a^2b-ab^2+b^3+\left(a^3+a^2b-ab^2-b^3\right)\cosh(x)^2+2\left(a^3+a^2b-ab^2-b^3\right)\cosh(x)\sinh(x)+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(x)^2/(a+b*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*(a + b)*x*cosh(x)^2 + 4*(a + b)*x*cosh(x)*sinh(x) + 2*(a + b)*x*sinh(x)^
2 - ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a
- b)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^3 - a^2*b - a*b
^2 + b^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 -
 b^3)*cosh(x)*sinh(x) + (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{sech}^2(x)}{(a + b\tanh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(x)**2/(a+b*tanh(x))**2,x)
```

```
[Out] Integral(x*sech(x)**2/(a + b*tanh(x))**2, x)
```

Giac [B] time = 1.1877, size = 235, normalized size = 4.27

$$\frac{2\,axe^{(2\,x)} + 2\,bxe^{(2\,x)} - ae^{(2\,x)}\log\left(-ae^{(2\,x)} - be^{(2\,x)} - a + b\right) - be^{(2\,x)}\log\left(-ae^{(2\,x)} - be^{(2\,x)} - a + b\right) - a\log\left(-ae^{(2\,x)} - be^{(2}\right.}{a^3e^{(2\,x)} + a^2be^{(2\,x)} - ab^2e^{(2\,x)} - b^3e^{(2\,x)} + a^3 - a^2b - ab^2 + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(x)^2/(a+b*tanh(x))^2,x, algorithm="giac")
```

```
[Out] (2*a*x*e^(2*x) + 2*b*x*e^(2*x) - a*e^(2*x)*log(-a*e^(2*x) - b*e^(2*x) - a +
 b) - b*e^(2*x)*log(-a*e^(2*x) - b*e^(2*x) - a + b) - a*log(-a*e^(2*x) - b*
e^(2*x) - a + b) + b*log(-a*e^(2*x) - b*e^(2*x) - a + b))/(a^3*e^(2*x) + a^
2*b*e^(2*x) - a*b^2*e^(2*x) - b^3*e^(2*x) + a^3 - a^2*b - a*b^2 + b^3)
```

3.144 $\int \frac{x\mathbf{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=231

$$\frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}\right)}{4\sqrt{-a}\sqrt{b}d^2}-\frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}\right)}{4\sqrt{-a}\sqrt{b}d^2}+\frac{x\log\left(\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}+1\right)}{2\sqrt{-a}\sqrt{b}d}-\frac{x\log\left(\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}+1\right)}{2\sqrt{-a}\sqrt{b}d}$$

```
[Out] (x*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a - 2*Sqrt[-a]*Sqrt[b] - b)])/(2*Sqrt
[-a]*Sqrt[b]*d) - (x*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*Sqrt
[b] - b)])/(2*Sqrt[-a]*Sqrt[b]*d) + PolyLog[2, -(((a + b)*E^(2*c + 2*d*x))/
(a - 2*Sqrt[-a]*Sqrt[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^2) - PolyLog[2, -(((a
+ b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*Sqrt[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^
2)
```

Rubi [A] time = 0.540997, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5632, 3320, 2264, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}\right)}{4\sqrt{-a}\sqrt{b}d^2}-\frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}\right)}{4\sqrt{-a}\sqrt{b}d^2}+\frac{x\log\left(\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}+1\right)}{2\sqrt{-a}\sqrt{b}d}-\frac{x\log\left(\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}+1\right)}{2\sqrt{-a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sech[c + d*x]^2)/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (x*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a - 2*Sqrt[-a]*Sqrt[b] - b)])/(2*Sqrt
[-a]*Sqrt[b]*d) - (x*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*Sqrt
[b] - b)])/(2*Sqrt[-a]*Sqrt[b]*d) + PolyLog[2, -(((a + b)*E^(2*c + 2*d*x))/
(a - 2*Sqrt[-a]*Sqrt[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^2) - PolyLog[2, -(((a
+ b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*Sqrt[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^
2)
```

Rule 5632

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sech[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*T
anh[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b - c + (
b + c)*Cosh[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m
, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= 2\int \frac{x}{a-b+(a+b)\cosh(2c+2dx)}\,dx\\
&= 4\int \frac{e^{2c+2dx}x}{a+b+2(a-b)e^{2c+2dx}+(a+b)e^{2(2c+2dx)}}\,dx\\
&= \frac{(2(a+b))\int \frac{e^{2c+2dx}x}{2(a-b)-4\sqrt{-a}\sqrt{b}+2(a+b)e^{2c+2dx}}\,dx}{\sqrt{-a}\sqrt{b}} - \frac{(2(a+b))\int \frac{e^{2c+2dx}x}{2(a-b)+4\sqrt{-a}\sqrt{b}+2(a+b)e^{2c+2dx}}\,dx}{\sqrt{-a}\sqrt{b}}\\
&= \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{\int \log\left(1+\frac{2(a+b)e^{2c+2dx}}{2(a-b)-4\sqrt{-a}\sqrt{b}}\right)dx}{2\sqrt{-a}\sqrt{b}d} + \frac{\int \log
\end{aligned}$$

$$\begin{aligned}
&= \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{2(a+b)x}{2(a-b)-4\sqrt{-a}\sqrt{b}}\right)}{x}\,dx, x, e^{2c-}\right.}{4\sqrt{-a}\sqrt{b}d^2}\\
&= \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} + \frac{\operatorname{Li}_2\left(-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{4\sqrt{-a}\sqrt{b}d^2} - \frac{\operatorname{Li}_2\left(-\frac{(a+b)e^{2c+2d}}{a+2\sqrt{-a}\sqrt{b}}\right.}{4\sqrt{-a}\sqrt{b}d^2}
\end{aligned}$$

Mathematica [A] time = 2.44641, size = 250, normalized size = 1.08

$$\frac{\sqrt{a}\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2(c+dx)}}{-2\sqrt{-a}\sqrt{b}+a-b}\right)-\sqrt{a}\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2(c+dx)}}{2\sqrt{-a}\sqrt{b}+a-b}\right)+2\sqrt{a}(c+dx)\log\left(\frac{(a+b)e^{2(c+dx)}}{-2\sqrt{-a}\sqrt{b}+a-b}+1\right)-2\sqrt{a}(c+dx)\operatorname{lo}}{4\sqrt{-a^2}\sqrt{b}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sech[c + d*x]^2)/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-4*Sqrt[-a]*c*ArcTan[(a - b + (a + b)*E^(2*(c + d*x)))/(2*Sqrt[a]*Sqrt[b])
] + 2*Sqrt[a]*(c + d*x)*Log[1 + ((a + b)*E^(2*(c + d*x)))/(a - 2*Sqrt[-a]*S
qrt[b] - b)] - 2*Sqrt[a]*(c + d*x)*Log[1 + ((a + b)*E^(2*(c + d*x)))/(a + 2
*Sqrt[-a]*Sqrt[b] - b)] + Sqrt[a]*PolyLog[2, -(((a + b)*E^(2*(c + d*x)))/(a
```

```
- 2*Sqrt[-a]*Sqrt[b] - b))] - Sqrt[a]*PolyLog[2, -(((a + b)*E^(2*(c + d*x)
))/(a + 2*Sqrt[-a]*Sqrt[b] - b))])/(4*Sqrt[-a^2]*Sqrt[b]*d^2)
```

Maple [B] time = 0.155, size = 953, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 1/2/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*c^2+1/4/d^2/(-a*b)^(1/2)/(-2*(
-a*b)^(1/2)-a+b)*polylog(2,(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*a+1/
2/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*
b)^(1/2)-a+b))*a*c-1/2/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*ex
p(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*b*c-1/d/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a
+b)*a*c*x+1/d/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*c*x+1/2/d/(-a*b)^(1/2)/(
-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*a*x-1
/2/d/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b
)^(1/2)-a+b))*b*x-1/d/(-a*b)^(1/2)*c*x+1/d/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)
*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*x+1/2/d/(-a*b)^(1/2)*ln(1-(a+b)*exp(
2*d*x+2*c)/(2*(-a*b)^(1/2)-a+b))*x-2/d/(-2*(-a*b)^(1/2)-a+b)*c*x+1/d^2/(-2*
(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*c+1/2/d^
2/(-a*b)^(1/2)*ln(1-(a+b)*exp(2*d*x+2*c)/(2*(-a*b)^(1/2)-a+b))*c-1/d^2*c/(a
*b)^(1/2)*arctan(1/4*(2*(a+b)*exp(2*d*x+2*c)+2*a-2*b)/(a*b)^(1/2))-1/2/(-a*
b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*x^2+1/2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)
*b*x^2-1/(-2*(-a*b)^(1/2)-a+b)*x^2-1/2/(-a*b)^(1/2)*x^2-1/d^2/(-2*(-a*b)^(1
/2)-a+b)*c^2-1/2/d^2/(-a*b)^(1/2)*c^2+1/2/d^2/(-2*(-a*b)^(1/2)-a+b)*polylog
(2,(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))+1/4/d^2/(-a*b)^(1/2)*polylog
(2,(a+b)*exp(2*d*x+2*c)/(2*(-a*b)^(1/2)-a+b))-1/4/d^2/(-a*b)^(1/2)/(-2*(-a*
b)^(1/2)-a+b)*polylog(2,(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*b-1/2/d
^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{sech}(dx+c)^2}{b \tanh(dx+c)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(x*sech(d*x + c)^2/(b*tanh(d*x + c)^2 + a), x)
```

Fricas [B] time = 2.82195, size = 3791, normalized size = 16.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*c*log(2*sqrt(-(2*(a + b)*sqrt(
-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x
+ c)) + (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*c*log(-2*sqrt(-(2*(a + b)*sq
rt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d
*x + c)) - (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*c*log(2*sqrt((2*(a + b)*s
qrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(
d*x + c)) - (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*c*log(-2*sqrt((2*(a + b)
*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) + 2*cosh(d*x + c) + 2*sin
h(d*x + c)) - (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*dilog(-(((a - b)*cosh(
d*x + c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(
d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 +
 2*a*b + b^2)) + a - b)/(a + b)) + a + b)/(a + b) + 1) - (a + b)*sqrt(-a*b/
(a^2 + 2*a*b + b^2))*dilog((((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c)
- 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b
+ b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b))
- a - b)/(a + b) + 1) + (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*dilog(-(((a
- b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x + c) + (a
+ b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a
*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) + a + b)/(a + b) + 1) + (a + b)*s
qrt(-a*b/(a^2 + 2*a*b + b^2))*dilog((((a - b)*cosh(d*x + c) + (a - b)*sinh(
d*x + c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2
 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(
a + b)) - a - b)/(a + b) + 1) - ((a + b)*d*x + (a + b)*c)*sqrt(-a*b/(a^2 +
2*a*b + b^2))*log((((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) - 2*((a +
 b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*
sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b)) + a + b)/
(a + b)) - ((a + b)*d*x + (a + b)*c)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*log(-((
(a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) +
(a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqr
t(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b)) - a - b)/(a + b)) + ((a + b)*
d*x + (a + b)*c)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*log((((a - b)*cosh(d*x + c)
 + (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c)
)*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b +
b^2)) - a + b)/(a + b)) + a + b)/(a + b)) + ((a + b)*d*x + (a + b)*c)*sqrt(
-a*b/(a^2 + 2*a*b + b^2))*log(-(((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x +
 c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*
a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b
)) - a - b)/(a + b)))/(a*b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,\mathrm{sech}^2\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(x*sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,\mathrm{sech}\left(dx+c\right)^2}{b\tanh\left(dx+c\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(x*sech(d*x + c)^2/(b*tanh(d*x + c)^2 + a), x)
```

3.145 $\int \frac{x^2\mathbf{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=351

$$\frac{x\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{x\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{\text{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}\right)}{4\sqrt{-a}\sqrt{b}d^3}+\frac{\text{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}\right)}{4\sqrt{-a}\sqrt{b}d^3}$$

```
[Out] (x^2*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a - 2*Sqrt[-a]*Sqrt[b] - b)])/(2*Sq
rt[-a]*Sqrt[b]*d) - (x^2*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*
Sqrt[b] - b)])/(2*Sqrt[-a]*Sqrt[b]*d) + (x*PolyLog[2, -(((a + b)*E^(2*c + 2
*d*x))/(a - 2*Sqrt[-a]*Sqrt[b] - b))])/(2*Sqrt[-a]*Sqrt[b]*d^2) - (x*PolyLo
g[2, -(((a + b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*Sqrt[b] - b))])/(2*Sqrt[-a
]*Sqrt[b]*d^2) - PolyLog[3, -(((a + b)*E^(2*c + 2*d*x))/(a - 2*Sqrt[-a]*Sqr
t[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^3) + PolyLog[3, -(((a + b)*E^(2*c + 2*d*x
))/(a + 2*Sqrt[-a]*Sqrt[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^3)
```

Rubi [A] time = 0.866671, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5632, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{x\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{\text{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}\right)}{4\sqrt{-a}\sqrt{b}d^3}+\frac{\text{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}\right)}{4\sqrt{-a}\sqrt{b}d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sech[c + d*x]^2)/(a + b*Tanh[c + d*x]^2),x]
```


```
[Out] (x^2*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a - 2*Sqrt[-a]*Sqrt[b] - b)])/(2*Sq
rt[-a]*Sqrt[b]*d) - (x^2*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*
Sqrt[b] - b)])/(2*Sqrt[-a]*Sqrt[b]*d) + (x*PolyLog[2, -(((a + b)*E^(2*c + 2
*d*x))/(a - 2*Sqrt[-a]*Sqrt[b] - b))])/(2*Sqrt[-a]*Sqrt[b]*d^2) - (x*PolyLo
g[2, -(((a + b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*Sqrt[b] - b))])/(2*Sqrt[-a
]*Sqrt[b]*d^2) - PolyLog[3, -(((a + b)*E^(2*c + 2*d*x))/(a - 2*Sqrt[-a]*Sqr
t[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^3) + PolyLog[3, -(((a + b)*E^(2*c + 2*d*x
))/(a + 2*Sqrt[-a]*Sqrt[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^3)
```


Rule 5632

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sech[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*T
anh[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b - c + (
b + c)*Cosh[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m
, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = 2\int \frac{x^2}{a-b+(a+b)\cosh(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{2c+2dx}x^2}{a+b+2(a-b)e^{2c+2dx}+(a+b)e^{2(2c+2dx)}}\,dx$$

$$= \frac{(2(a+b))\int \frac{e^{2c+2dx}x^2}{2(a-b)-4\sqrt{-a}\sqrt{b}+2(a+b)e^{2c+2dx}}\,dx}{\sqrt{-a}\sqrt{b}} - \frac{(2(a+b))\int \frac{e^{2c+2dx}x^2}{2(a-b)+4\sqrt{-a}\sqrt{b}+2(a+b)e^{2c+2dx}}\,dx}{\sqrt{-a}\sqrt{b}}$$

$$= \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{\int x\log\left(1+\frac{2(a+b)e^{2c+2dx}}{2(a-b)-4\sqrt{-a}\sqrt{b}}\right)dx}{\sqrt{-a}\sqrt{b}d} + \frac{\int}{}$$

$$= \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} + \frac{x\operatorname{Li}_2\left(-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2} - \frac{x\operatorname{Li}_2\left(-\frac{(a+b}{a+2\sqrt{}}\right.}{2\sqrt{-a}\sqrt{}}$$

$$= \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} + \frac{x\operatorname{Li}_2\left(-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2} - \frac{x\operatorname{Li}_2\left(-\frac{(a+b}{a+2\sqrt{}}\right.}{2\sqrt{-a}\sqrt{}}$$

$$= \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} + \frac{x\operatorname{Li}_2\left(-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2} - \frac{x\operatorname{Li}_2\left(-\frac{(a+b}{a+2\sqrt{}}\right.}{2\sqrt{-a}\sqrt{}}$$

Mathematica [C] time = 2.30237, size = 316, normalized size = 0.9

$$\frac{i\left(2dx\operatorname{PolyLog}\left(2,-\frac{\left(\sqrt{a}-i\sqrt{b}\right)e^{2(c+dx)}}{\sqrt{a}+i\sqrt{b}}\right)-2dx\operatorname{PolyLog}\left(2,-\frac{\left(\sqrt{a}+i\sqrt{b}\right)e^{2(c+dx)}}{\sqrt{a}-i\sqrt{b}}\right)-\operatorname{PolyLog}\left(3,-\frac{\left(\sqrt{a}-i\sqrt{b}\right)e^{2(c+dx)}}{\sqrt{a}+i\sqrt{b}}\right)+\operatorname{PolyLog}\left(\right.\right.}{4\sqrt{a}\sqrt{b}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sech[c + d*x]^2)/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((I/4)*(2*d^2*x^2*Log[1 + ((Sqrt[a] - I*Sqrt[b])*E^(2*(c + d*x)))/(Sqrt[a]
+ I*Sqrt[b])] - 2*d^2*x^2*Log[1 + ((Sqrt[a] + I*Sqrt[b])*E^(2*(c + d*x)))/(
Sqrt[a] - I*Sqrt[b])] + 2*d*x*PolyLog[2, -(((Sqrt[a] - I*Sqrt[b])*E^(2*(c +
 d*x)))/(Sqrt[a] + I*Sqrt[b]))] - 2*d*x*PolyLog[2, -(((Sqrt[a] + I*Sqrt[b])
*E^(2*(c + d*x)))/(Sqrt[a] - I*Sqrt[b]))] - PolyLog[3, -(((Sqrt[a] - I*Sqrt
[b])*E^(2*(c + d*x)))/(Sqrt[a] + I*Sqrt[b]))] + PolyLog[3, -(((Sqrt[a] + I*
Sqrt[b])*E^(2*(c + d*x)))/(Sqrt[a] - I*Sqrt[b]))]))/(Sqrt[a]*Sqrt[b]*d^3)
```

Maple [B] time = 0.134, size = 1186, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] -2/3/d^3*c^3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b-1/4/d^3/(-a*b)^(1/2)/(-2*
(-a*b)^(1/2)-a+b)*a*polylog(3,(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))+1
/4/d^3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*polylog(3,(a+b)*exp(2*d*x+2*c)/
(-2*(-a*b)^(1/2)-a+b))+1/2/d^3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*ln(1-(a
```

```
+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*c^2-1/2/d^3/(-a*b)^(1/2)/(-2*(-a*
b)^(1/2)-a+b)*a*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*c^2-1/2/d^
2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*polylog(2,(a+b)*exp(2*d*x+2*c)/(-2*(
-a*b)^(1/2)-a+b))*x+1/2/d/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*ln(1-(a+b)*e
xp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*x^2+1/d^2*c^2/(-a*b)^(1/2)/(-2*(-a*b)^
(1/2)-a+b)*a*x+1/2/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*polylog(2,(a+b)
*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*x-1/2/d/(-a*b)^(1/2)/(-2*(-a*b)^(1/2
)-a+b)*b*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*x^2-1/d^2*c^2/(-a
*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*x-1/3/(-a*b)^(1/2)*x^3-2/3/(-2*(-a*b)^(1/
2)-a+b)*x^3+2/3/d^3*c^3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a-1/2/d^3/(-2*(-
a*b)^(1/2)-a+b)*polylog(3,(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))+2/3/d
^3*c^3/(-a*b)^(1/2)+4/3/d^3*c^3/(-2*(-a*b)^(1/2)-a+b)-1/4/d^3/(-a*b)^(1/2)*
polylog(3,(a+b)*exp(2*d*x+2*c)/(2*(-a*b)^(1/2)-a+b))-1/3/(-a*b)^(1/2)/(-2*(
-a*b)^(1/2)-a+b)*a*x^3+1/d^3*c^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*exp(2*d*x+
2*c)+2*a-2*b)/(a*b)^(1/2))+1/d^2*c^2/(-a*b)^(1/2)*x+2/d^2*c^2/(-2*(-a*b)^(1
/2)-a+b)*x-1/d^3/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)
^(1/2)-a+b))*c^2-1/2/d^3/(-a*b)^(1/2)*ln(1-(a+b)*exp(2*d*x+2*c)/(2*(-a*b)^(
1/2)-a+b))*c^2+1/2/d^2/(-a*b)^(1/2)*polylog(2,(a+b)*exp(2*d*x+2*c)/(2*(-a*b
)^(1/2)-a+b))*x+1/d/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a
*b)^(1/2)-a+b))*x^2+1/d^2/(-2*(-a*b)^(1/2)-a+b)*polylog(2,(a+b)*exp(2*d*x+2
*c)/(-2*(-a*b)^(1/2)-a+b))*x+1/2/d/(-a*b)^(1/2)*ln(1-(a+b)*exp(2*d*x+2*c)/(
2*(-a*b)^(1/2)-a+b))*x^2+1/3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{sech}(dx+c)^2}{b\tanh(dx+c)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sech(d*x + c)^2/(b*tanh(d*x + c)^2 + a), x)
```

Fricas [C] time = 3.12261, size = 5269, normalized size = 15.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*d*x*dilog(-(((a - b)*cosh(d*x
 + c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x
 + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*
a*b + b^2)) + a - b)/(a + b)) + a + b)/(a + b) + 1) + 2*(a + b)*sqrt(-a*b/(
a^2 + 2*a*b + b^2))*d*x*dilog((((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x +
c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a
*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b
)) - a - b)/(a + b) + 1) - 2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*d*x*dil
og(-(((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x +
 c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b
)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) + a + b)/(a + b) + 1) -
2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*d*x*dilog((((a - b)*cosh(d*x + c)
+ (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))
```

```
*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b
^2)) - a + b)/(a + b)) - a - b)/(a + b) + 1) + (a + b)*sqrt(-a*b/(a^2 + 2*a
*b + b^2))*c^2*log(2*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a -
b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x + c)) + (a + b)*sqrt(-a*b/(a^2 +
 2*a*b + b^2))*c^2*log(-2*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) +
 a - b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x + c)) - (a + b)*sqrt(-a*b/(
a^2 + 2*a*b + b^2))*c^2*log(2*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)
) - a + b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x + c)) - (a + b)*sqrt(-a*
b/(a^2 + 2*a*b + b^2))*c^2*log(-2*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b +
b^2)) - a + b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x + c)) + ((a + b)*d^2
*x^2 - (a + b)*c^2)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*log((((a - b)*cosh(d*x +
 c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x +
 c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*
b + b^2)) + a - b)/(a + b)) + a + b)/(a + b)) + ((a + b)*d^2*x^2 - (a + b)*
c^2)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*log(-(((a - b)*cosh(d*x + c) + (a - b)*
sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b
/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a
- b)/(a + b)) - a - b)/(a + b)) - ((a + b)*d^2*x^2 - (a + b)*c^2)*sqrt(-a*b
/(a^2 + 2*a*b + b^2))*log((((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) +
 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b +
 b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) +
a + b)/(a + b)) - ((a + b)*d^2*x^2 - (a + b)*c^2)*sqrt(-a*b/(a^2 + 2*a*b +
b^2))*log(-(((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) + 2*((a + b)*cos
h(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2
*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) - a - b)/(a + b))
 - 2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*polylog(3, ((a - b)*cosh(d*x +
c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x +
c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b
 + b^2)) + a - b)/(a + b))/(a + b)) - 2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^
2))*polylog(3, -((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) - 2*((a + b)
*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqr
t(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b))/(a + b)) + 2
*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*polylog(3, ((a - b)*cosh(d*x + c) +
 (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*
sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^
2)) - a + b)/(a + b))/(a + b)) + 2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*p
olylog(3, -((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) + 2*((a + b)*cosh
(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*
(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b))/(a + b)))/(a*b*d^3
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(x**2*sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{sech}(dx+c)^2}{b\tanh(dx+c)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(d*x + c)^2/(b*tanh(d*x + c)^2 + a), x)
```

3.146 $\int x^3 \tanh(a+2\log(x))\,dx$

Optimal. Leaf size=29

$$\frac{x^4}{4}-\frac{1}{2}e^{-2a}\log\left(e^{2a}x^4+1\right)$$

```
[Out] x^4/4 - Log[1 + E^(2*a)*x^4]/(2*E^(2*a))
```

Rubi [F] time = 0.0291397, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3 \tanh(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Tanh[a + 2*Log[x]],x]

[Out] Defer[Int][x^3*Tanh[a + 2*Log[x]], x]
```

Rubi steps

$$\int x^3 \tanh(a+2\log(x))\,dx=\int x^3 \tanh(a+2\log(x))\,dx$$

Mathematica [B] time = 0.0251789, size = 64, normalized size = 2.21

$$-\frac{1}{2}\cosh(2a)\log\left(x^4\sinh(a)+x^4\cosh(a)-\sinh(a)+\cosh(a)\right)+\frac{1}{2}\sinh(2a)\log\left(x^4\sinh(a)+x^4\cosh(a)-\sinh(a)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[a + 2*Log[x]],x]

[Out] x^4/4 - (Cosh[2*a]*Log[Cosh[a] + x^4*Cosh[a] - Sinh[a] + x^4*Sinh[a]])/2 +
(Log[Cosh[a] + x^4*Cosh[a] - Sinh[a] + x^4*Sinh[a]]*Sinh[2*a])/2
```

Maple [A] time = 0.021, size = 24, normalized size = 0.8

$$\frac{x^4}{4}-\frac{e^{-2a}\ln\left(1+e^{2a}x^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tanh(a+2*ln(x)),x)

[Out] 1/4*x^4-1/2*exp(-2*a)*ln(1+exp(2*a)*x^4)
```

Maxima [A] time = 1.23674, size = 31, normalized size = 1.07

$$\frac{1}{4}\,x^4-\frac{1}{2}\,e^{(-2\,a)}\log\left(x^4e^{(2\,a)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/2*e^(-2*a)*log(x^4*e^(2*a) + 1)
```

Fricas [A] time = 2.44727, size = 72, normalized size = 2.48

$$\frac{1}{4}\left(x^4e^{(2\,a)}-2\,\log\left(x^4e^{(2\,a)}+1\right)\right)e^{(-2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^4*e^(2*a) - 2*log(x^4*e^(2*a) + 1))*e^(-2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\tanh\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tanh(a+2*ln(x)),x)
```

```
[Out] Integral(x**3*tanh(a + 2*log(x)), x)
```

Giac [A] time = 1.16495, size = 31, normalized size = 1.07

$$\frac{1}{4}\,x^4-\frac{1}{2}\,e^{(-2\,a)}\log\left(x^4e^{(2\,a)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/2*e^(-2*a)*log(x^4*e^(2*a) + 1)
```

3.147 $\int x^2 \tanh(a+2\log(x))\,dx$

Optimal. Leaf size=151

$$-\frac{e^{-3a/2}\log\left(e^a x^2-\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}}+\frac{e^{-3a/2}\log\left(e^a x^2+\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}}+\frac{e^{-3a/2}\tan^{-1}\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}-\frac{e^{-3a/2}\tan^{-1}\left(\sqrt{2}e^{a/2}x\right.}{\sqrt{2}}$$

```
[Out] x^3/3 + ArcTan[1 - Sqrt[2]*E^(a/2)*x]/(Sqrt[2]*E^((3*a)/2)) - ArcTan[1 + Sq
rt[2]*E^(a/2)*x]/(Sqrt[2]*E^((3*a)/2)) - Log[1 - Sqrt[2]*E^(a/2)*x + E^a*x^
2]/(2*Sqrt[2]*E^((3*a)/2)) + Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2]/(2*Sqrt[2
]*E^((3*a)/2))
```

Rubi [F] time = 0.0216819, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \tanh(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Tanh[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][x^2*Tanh[a + 2*Log[x]], x]
```

Rubi steps

$$\int x^2 \tanh(a+2\log(x))\,dx=\int x^2 \tanh(a+2\log(x))\,dx$$

Mathematica [C] time = 0.251108, size = 64, normalized size = 0.42

$$\frac{1}{6}\left(3(\cosh(2a)-\sinh(2a))\text{RootSum}\left[\#1^4\sinh(a)+\#1^4\cosh(a)-\sinh(a)+\cosh(a)\&,\frac{\log(x)-\log(x-\#1)}{\#1}\&\right]+2x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tanh[a + 2*Log[x]],x]
```

```
[Out] (2*x^3 + 3*RootSum[Cosh[a] - Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Log
[x] - Log[x - #1])/#1 & ]*(Cosh[2*a] - Sinh[2*a]))/6
```

Maple [C] time = 0.022, size = 37, normalized size = 0.3

$$\frac{x^3}{3}-\frac{e^{-2a}}{2}\sum_{\_R=\text{RootOf}\left(e^{2a}\_Z^4+1\right)}\frac{\ln(x-\_R)}{\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tanh(a+2*ln(x)),x)
```

```
[Out] 1/3*x^3-1/2*exp(-2*a)*sum(1/_R*ln(x-_R),_R=RootOf(exp(2*a)*_Z^4+1))
```

Maxima [A] time = 1.79862, size = 173, normalized size = 1.15

$$\frac{1}{3}x^3 - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2xe^a + \sqrt{2}e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2xe^a - \sqrt{2}e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a + sqrt(2)*e^(1/2*a))*e^(-
1/2*a))*e^(-3/2*a) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a - sqrt(2)*e^(1
/2*a))*e^(-1/2*a))*e^(-3/2*a) + 1/4*sqrt(2)*e^(-3/2*a)*log(x^2*e^a + sqrt(2
)*x*e^(1/2*a) + 1) - 1/4*sqrt(2)*e^(-3/2*a)*log(x^2*e^a - sqrt(2)*x*e^(1/2*
a) + 1)
```

Fricas [A] time = 2.51188, size = 506, normalized size = 3.35

$$\frac{1}{3}x^3 + \sqrt{2}\arctan\left(-\sqrt{2}xe^{\left(\frac{1}{2}a\right)} + \sqrt{2}\sqrt{\sqrt{2}xe^{\left(-\frac{1}{2}a\right)} + x^2 + e^{(-a)}}e^{\left(\frac{1}{2}a\right)} - 1\right)e^{\left(-\frac{3}{2}a\right)} + \sqrt{2}\arctan\left(-\sqrt{2}xe^{\left(\frac{1}{2}a\right)} + \sqrt{2}\sqrt{-\sqrt{2}xe^{\left(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + sqrt(2)*arctan(-sqrt(2)*x*e^(1/2*a) + sqrt(2)*sqrt(sqrt(2)*x*e^(-
1/2*a) + x^2 + e^(-a))*e^(1/2*a) - 1)*e^(-3/2*a) + sqrt(2)*arctan(-sqrt(2)*
x*e^(1/2*a) + sqrt(2)*sqrt(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a))*e^(1/2*a)
+ 1)*e^(-3/2*a) + 1/4*sqrt(2)*e^(-3/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e
^(-a)) - 1/4*sqrt(2)*e^(-3/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \tanh\left(a + 2\log(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tanh(a+2*ln(x)),x)
```

```
[Out] Integral(x**2*tanh(a + 2*log(x)), x)
```

Giac [A] time = 1.2002, size = 166, normalized size = 1.1

$$\frac{1}{3}x^3 - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)} + 2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)} - 2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{1}{4}\sqrt{2}e^{\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*
a))*e^(-3/2*a) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)
*e^(1/2*a))*e^(-3/2*a) + 1/4*sqrt(2)*e^(-3/2*a)*log(sqrt(2)*x*e^(-1/2*a) +
x^2 + e^(-a)) - 1/4*sqrt(2)*e^(-3/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^
(-a))
```

3.148 $\int x \tanh(a+2\log(x))\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-e^{-a}\tan^{-1}\left(e^a x^2\right)$$

```
[Out] x^2/2 - ArcTan[E^a*x^2]/E^a
```

Rubi [F] time = 0.0160746, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \tanh(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Tanh[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][x*Tanh[a + 2*Log[x]], x]
```

Rubi steps

$$\int x \tanh(a+2\log(x))\,dx=\int x \tanh(a+2\log(x))\,dx$$

Mathematica [A] time = 0.186758, size = 35, normalized size = 1.52

$$-\cosh(a)\tan^{-1}\left(x^2(\sinh(a)+\cosh(a))\right)+\sinh(a)\tan^{-1}\left(x^2(\sinh(a)+\cosh(a))\right)+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tanh[a + 2*Log[x]],x]
```

```
[Out] x^2/2 - ArcTan[x^2*(Cosh[a] + Sinh[a])]*Cosh[a] + ArcTan[x^2*(Cosh[a] + Sin
h[a])]*Sinh[a]
```

Maple [C] time = 0.035, size = 41, normalized size = 1.8

$$\frac{x^2}{2}+\frac{i}{2}\mathrm{e}^{-a}\ln\left(\mathrm{e}^a x^2-i\right)-\frac{i}{2}\mathrm{e}^{-a}\ln\left(\mathrm{e}^a x^2+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tanh(a+2*ln(x)),x)
```

```
[Out] 1/2*x^2+1/2*I*exp(-a)*ln(exp(a)*x^2-I)-1/2*I*exp(-a)*ln(exp(a)*x^2+I)
```

Maxima [A] time = 1.91379, size = 26, normalized size = 1.13

$$\frac{1}{2}x^2 - \arctan\left(x^2e^a\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - arctan(x^2*e^a)*e^(-a)
```

Fricas [A] time = 1.83915, size = 57, normalized size = 2.48

$$\frac{1}{2}\left(x^2e^a - 2\arctan\left(x^2e^a\right)\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*e^a - 2*arctan(x^2*e^a))*e^(-a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\tanh\left(a + 2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(a+2*ln(x)),x)
```

```
[Out] Integral(x*tanh(a + 2*log(x)), x)
```

Giac [A] time = 1.23499, size = 26, normalized size = 1.13

$$\frac{1}{2}x^2 - \arctan\left(x^2e^a\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - arctan(x^2*e^a)*e^(-a)
```

3.149 $\int \tanh(a+2\log(x))\,dx$

Optimal. Leaf size=145

$$\frac{e^{-a/2}\log\left(e^a x^2-\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}}-\frac{e^{-a/2}\log\left(e^a x^2+\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}}+\frac{e^{-a/2}\tan^{-1}\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}-\frac{e^{-a/2}\tan^{-1}\left(\sqrt{2}e^{a/2}x+1\right)}{\sqrt{2}}+$$

```
[Out] x + ArcTan[1 - Sqrt[2]*E^(a/2)*x]/(Sqrt[2]*E^(a/2)) - ArcTan[1 + Sqrt[2]*E^
(a/2)*x]/(Sqrt[2]*E^(a/2)) + Log[1 - Sqrt[2]*E^(a/2)*x + E^a*x^2]/(2*Sqrt[2
]*E^(a/2)) - Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2]/(2*Sqrt[2]*E^(a/2))
```

Rubi [F] time = 0.0073467, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][Tanh[a + 2*Log[x]], x]
```

Rubi steps

$$\int \tanh(a+2\log(x))\,dx=\int \tanh(a+2\log(x))\,dx$$

Mathematica [C] time = 0.165275, size = 58, normalized size = 0.4

$$\frac{1}{2}(\cosh(2a)-\sinh(2a))\text{RootSum}\left[\#1^4\sinh(a)+\#1^4\cosh(a)-\sinh(a)+\cosh(a)\&,\frac{\log(x)-\log(x-\#1)}{\#1^3}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + 2*Log[x]],x]
```

```
[Out] x + (RootSum[Cosh[a] - Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Log[x] -
Log[x - #1])/#1^3 & ]*(Cosh[2*a] - Sinh[2*a]))/2
```

Maple [C] time = 0.012, size = 33, normalized size = 0.2

$$x-\frac{e^{-2a}}{2}\sum_{\_R=\text{RootOf}\left(e^{2a}\_Z^4+1\right)}\frac{\ln(x-\_R)}{\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+2*ln(x)),x)
```

```
[Out] x-1/2*exp(-2*a)*sum(1/_R^3*ln(x-_R),_R=RootOf(exp(2*a)*_Z^4+1))
```

Maxima [A] time = 1.80798, size = 167, normalized size = 1.15

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,xe^{a}+\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,xe^{a}-\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}-\frac{1}{4}\sqrt{2}e^{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a + sqrt(2)*e^(1/2*a))*e^(-1/2*a))*e
^(-1/2*a) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a - sqrt(2)*e^(1/2*a))*e^
(-1/2*a))*e^(-1/2*a) - 1/4*sqrt(2)*e^(-1/2*a)*log(x^2*e^a + sqrt(2)*x*e^(1/
2*a) + 1) + 1/4*sqrt(2)*e^(-1/2*a)*log(x^2*e^a - sqrt(2)*x*e^(1/2*a) + 1) +
 x
```

Fricas [A] time = 2.14815, size = 498, normalized size = 3.43

$$\sqrt{2}\arctan\left(-\sqrt{2}xe^{\left(\frac{1}{2}\,a\right)}+\sqrt{2}\sqrt{\sqrt{2}xe^{\left(-\frac{1}{2}\,a\right)}+x^{2}+e^{(-a)}}e^{\left(\frac{1}{2}\,a\right)}-1\right)e^{\left(-\frac{1}{2}\,a\right)}+\sqrt{2}\arctan\left(-\sqrt{2}xe^{\left(\frac{1}{2}\,a\right)}+\sqrt{2}\sqrt{-\sqrt{2}xe^{\left(-\frac{1}{2}\,a\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(-sqrt(2)*x*e^(1/2*a) + sqrt(2)*sqrt(sqrt(2)*x*e^(-1/2*a) + x
^2 + e^(-a))*e^(1/2*a) - 1)*e^(-1/2*a) + sqrt(2)*arctan(-sqrt(2)*x*e^(1/2*a
) + sqrt(2)*sqrt(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a))*e^(1/2*a) + 1)*e^(-1
/2*a) - 1/4*sqrt(2)*e^(-1/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) + 1
/4*sqrt(2)*e^(-1/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) + x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*ln(x)),x)
```

```
[Out] Integral(tanh(a + 2*log(x)), x)
```

Giac [A] time = 1.1658, size = 161, normalized size = 1.11

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}\,a\right)}+2\,x\right)e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}\,a\right)}-2\,x\right)e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}-\frac{1}{4}\sqrt{2}e^{\left(-\frac{1}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*a))*e^(-1
/2*a) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)*e^(1/2*a
))*e^(-1/2*a) - 1/4*sqrt(2)*e^(-1/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(
-a)) + 1/4*sqrt(2)*e^(-1/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) + x
```

3.150 $\int \frac{\tanh(a+2\log(x))}{x}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log(\cosh(a+2\log(x)))$$

```
[Out] Log[Cosh[a + 2*Log[x]]]/2
```

Rubi [A] time = 0.0135338, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3475}

$$\frac{1}{2}\log(\cosh(a+2\log(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + 2*Log[x]]/x,x]

[Out] Log[Cosh[a + 2*Log[x]]]/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(a+2\log(x))}{x}\,dx &= \text{Subst}\left(\int \tanh(a+2x)\,dx, x, \log(x)\right)\\ &= \frac{1}{2}\log(\cosh(a+2\log(x)))\end{aligned}$$

Mathematica [A] time = 0.0247089, size = 12, normalized size = 1.

$$\frac{1}{2}\log(\cosh(a+2\log(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + 2*Log[x]]/x,x]

[Out] Log[Cosh[a + 2*Log[x]]]/2
```

Maple [B] time = 0.003, size = 26, normalized size = 2.2

$$-\frac{\ln(\tanh(a+2\ln(x))-1)}{4}-\frac{\ln(\tanh(a+2\ln(x))+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+2*ln(x))/x,x)
```

```
[Out] -1/4*ln(tanh(a+2*ln(x))-1)-1/4*ln(tanh(a+2*ln(x))+1)
```

Maxima [A] time = 1.21895, size = 14, normalized size = 1.17

$$\frac{1}{2}\,\log\left(\cosh\left(a+2\,\log\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(cosh(a + 2*log(x)))
```

Fricas [A] time = 1.96916, size = 47, normalized size = 3.92

$$\frac{1}{2}\,\log\left(x^{4}e^{(2\,a)}+1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))/x,x, algorithm="fricas")
```

```
[Out] 1/2*log(x^4*e^(2*a) + 1) - log(x)
```

Sympy [A] time = 0.297289, size = 15, normalized size = 1.25

$$\log\left(x\right)-\frac{\log\left(\tanh\left(a+2\log\left(x\right)\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*ln(x))/x,x)
```

```
[Out] log(x) - log(tanh(a + 2*log(x)) + 1)/2
```

Giac [A] time = 1.15682, size = 27, normalized size = 2.25

$$\frac{1}{2}\,\log\left(x^{4}e^{(2\,a)}+1\right)-\frac{1}{4}\,\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))/x,x, algorithm="giac")
```

```
[Out] 1/2*log(x^4*e^(2*a) + 1) - 1/4*log(x^4)
```

3.151 $\int \frac{\tanh(a+2\log(x))}{x^2}\,dx$

Optimal. Leaf size=147

$$\frac{e^{a/2}\log\left(e^a x^2-\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}}-\frac{e^{a/2}\log\left(e^a x^2+\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}}-\frac{e^{a/2}\tan^{-1}\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}+\frac{e^{a/2}\tan^{-1}\left(\sqrt{2}e^{a/2}x+1\right)}{\sqrt{2}}+\frac{1}{x}$$

```
[Out] x^(-1) - (E^(a/2)*ArcTan[1 - Sqrt[2]*E^(a/2)*x])/Sqrt[2] + (E^(a/2)*ArcTan[
1 + Sqrt[2]*E^(a/2)*x])/Sqrt[2] + (E^(a/2)*Log[1 - Sqrt[2]*E^(a/2)*x + E^a*
x^2])/(2*Sqrt[2]) - (E^(a/2)*Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2])/(2*Sqrt[
2])
```

Rubi [F] time = 0.0237645, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + 2*Log[x]]/x^2,x]
```

```
[Out] Defer[Int][Tanh[a + 2*Log[x]]/x^2, x]
```

Rubi steps

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx=\int \frac{\tanh(a+2\log(x))}{x^2}\,dx$$

Mathematica [C] time = 0.159208, size = 59, normalized size = 0.4

$$\frac{2-x(\sinh(a)+\cosh(a))^2\text{RootSum}\left[-\#1^4\sinh(a)+\#1^4\cosh(a)+\sinh(a)+\cosh(a)\&,\frac{\log\left(\frac{1}{x}-\#1\right)+\log(x)}{\#1^3}\&\right]}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + 2*Log[x]]/x^2,x]
```

```
[Out] (2 - x*RootSum[Cosh[a] + Sinh[a] + Cosh[a]*#1^4 - Sinh[a]*#1^4 & , (Log[x]
+ Log[x^(-1) - #1])/#1^3 & ]*(Cosh[a] + Sinh[a])^2)/(2*x)
```

Maple [C] time = 0.027, size = 42, normalized size = 0.3

$$x^{-1}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^4+e^{2a}\right)}\_R\ln\left(\left(5\_R^4+4e^{2a}\right)x-\_R^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+2*ln(x))/x^2,x)
```

```
[Out] 1/x+1/2*sum(_R*ln((5*_R^4+4*exp(2*a))*x-_R^3),_R=RootOf(_Z^4+exp(2*a)))
```

Maxima [A] time = 1.90063, size = 169, normalized size = 1.15

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(\frac{1}{2}a\right)}+\frac{2}{x}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(\frac{1}{2}a\right)}-\frac{2}{x}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{4}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\log\Bigg(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(1/2*a) + 2/x)*e^(-1/2*a))*e^(1/
2*a) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(1/2*a) - 2/x)*e^(-1/2*a)
)*e^(1/2*a) - 1/4*sqrt(2)*e^(1/2*a)*log(sqrt(2)*e^(1/2*a)/x + 1/x^2 + e^a)
+ 1/4*sqrt(2)*e^(1/2*a)*log(-sqrt(2)*e^(1/2*a)/x + 1/x^2 + e^a) + 1/x
```

Fricas [B] time = 2.19939, size = 597, normalized size = 4.06

$$-\frac{4\sqrt{2}x\arctan\left(-\left(\sqrt{2}xe^{\left(\frac{5}{2}a\right)}-\sqrt{2}\sqrt{x^2e^{(4a)}+\sqrt{2}xe^{\left(\frac{7}{2}a\right)}+e^{(3a)}}e^{\left(\frac{1}{2}a\right)}+e^{(2a)}\right)e^{(-2a)}\right)e^{\left(\frac{1}{2}a\right)}+4\sqrt{2}x\arctan\left(-\left(\sqrt{2}xe^{\left(\frac{5}{2}a\right)}-\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*x*arctan(-(sqrt(2)*x*e^(5/2*a) - sqrt(2)*sqrt(x^2*e^(4*a) +
 sqrt(2)*x*e^(7/2*a) + e^(3*a))*e^(1/2*a) + e^(2*a))*e^(-2*a))*e^(1/2*a) +
4*sqrt(2)*x*arctan(-(sqrt(2)*x*e^(5/2*a) - sqrt(2)*sqrt(x^2*e^(4*a) - sqrt(
2)*x*e^(7/2*a) + e^(3*a))*e^(1/2*a) - e^(2*a))*e^(-2*a))*e^(1/2*a) + sqrt(2
)*x*e^(1/2*a)*log(x^2*e^(4*a) + sqrt(2)*x*e^(7/2*a) + e^(3*a)) - sqrt(2)*x*
e^(1/2*a)*log(x^2*e^(4*a) - sqrt(2)*x*e^(7/2*a) + e^(3*a)) - 4)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*ln(x))/x**2,x)
```

```
[Out] Integral(tanh(a + 2*log(x))/x**2, x)
```

Giac [A] time = 1.18468, size = 163, normalized size = 1.11

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}+2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}-2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{4}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\log\Big(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*a))*e^(1/2
*a) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)*e^(1/2*a))
*e^(1/2*a) - 1/4*sqrt(2)*e^(1/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a))
 + 1/4*sqrt(2)*e^(1/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) + 1/x
```

3.152 $\int \frac{\tanh(a+2\log(x))}{x^3} dx$

Optimal. Leaf size=20

$$e^a \tan^{-1}\left(e^a x^2\right) + \frac{1}{2x^2}$$

[Out] 1/(2*x^2) + E^a*ArcTan[E^a*x^2]

Rubi [F] time = 0.0244611, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh(a+2\log(x))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Tanh[a + 2*Log[x]]/x^3,x]

[Out] Defer[Int][Tanh[a + 2*Log[x]]/x^3, x]

Rubi steps

$$\int \frac{\tanh(a+2\log(x))}{x^3} dx = \int \frac{\tanh(a+2\log(x))}{x^3} dx$$

Mathematica [A] time = 0.153145, size = 40, normalized size = 2.

$$\cosh(a)\left(-\tan^{-1}\left(\frac{\cosh(a)-\sinh(a)}{x^2}\right)\right) - \sinh(a)\tan^{-1}\left(\frac{\cosh(a)-\sinh(a)}{x^2}\right) + \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[a + 2*Log[x]]/x^3,x]

[Out] 1/(2*x^2) - ArcTan[(Cosh[a] - Sinh[a])/x^2]*Cosh[a] - ArcTan[(Cosh[a] - Sin h[a])/x^2]*Sinh[a]

Maple [C] time = 0.027, size = 44, normalized size = 2.2

$$\frac{1}{2x^2} + \frac{\sum_{\_R=\mathrm{RootOf}(e^{2a}+\_Z^2)} \_R \ln\left(\left(4e^{2a}+5\_R^2\right)x^2 - \_R\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(a+2*ln(x))/x^3,x)

[Out] 1/2/x^2+1/2*sum(_R*ln((4*exp(2*a)+5*_R^2)*x^2-_R),_R=RootOf(exp(2*a)+_Z^2))

Maxima [A] time = 1.6208, size = 26, normalized size = 1.3

$$-\arctan\left(\frac{e^{(-a)}}{x^2}\right)e^a + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))/x^3,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-a)/x^2)*e^a + 1/2/x^2
```

Fricas [A] time = 2.09518, size = 55, normalized size = 2.75

$$\frac{2x^2\arctan\left(x^2e^a\right)e^a + 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*arctan(x^2*e^a)*e^a + 1)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(a + 2\log\left(x\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*ln(x))/x**3,x)
```

```
[Out] Integral(tanh(a + 2*log(x))/x**3, x)
```

Giac [A] time = 1.20641, size = 22, normalized size = 1.1

$$\arctan\left(x^2e^a\right)e^a + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))/x^3,x, algorithm="giac")
```

```
[Out] arctan(x^2*e^a)*e^a + 1/2/x^2
```

3.153 $\int x^3 \tanh^2(a + 2\log(x))\,dx$

Optimal. Leaf size=47

$$-\frac{e^{-2a}}{e^{2a}x^4+1} - e^{-2a}\log\left(e^{2a}x^4+1\right) + \frac{x^4}{4}$$

```
[Out] x^4/4 - 1/(E^(2*a)*(1 + E^(2*a)*x^4)) - Log[1 + E^(2*a)*x^4]/E^(2*a)
```

Rubi [F] time = 0.0678357, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3 \tanh^2(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][x^3*Tanh[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int x^3 \tanh^2(a + 2\log(x))\,dx = \int x^3 \tanh^2(a + 2\log(x))\,dx$$

Mathematica [A] time = 0.10442, size = 86, normalized size = 1.83

$$\frac{\sinh(3a) - \cosh(3a)}{\left(x^4-1\right)\sinh(a) + \left(x^4+1\right)\cosh(a)} - \cosh(2a)\log\left(\left(x^4-1\right)\sinh(a) + \left(x^4+1\right)\cosh(a)\right) + \sinh(2a)\log\left(\left(x^4-1\right)\sinh(a)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] x^4/4 - Cosh[2*a]*Log[(1 + x^4)*Cosh[a] + (-1 + x^4)*Sinh[a]] + Log[(1 + x^
4)*Cosh[a] + (-1 + x^4)*Sinh[a]]*Sinh[2*a] + (-Cosh[3*a] + Sinh[3*a])/((1 +
 x^4)*Cosh[a] + (-1 + x^4)*Sinh[a])
```

Maple [A] time = 0.019, size = 42, normalized size = 0.9

$$\frac{x^4}{4} - \frac{e^{-2a}}{1+e^{2a}x^4} - e^{-2a}\ln\left(1+e^{2a}x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tanh(a+2*ln(x))^2,x)
```

```
[Out] 1/4*x^4-exp(-2*a)/(1+exp(2*a)*x^4)-exp(-2*a)*ln(1+exp(2*a)*x^4)
```

Maxima [A] time = 1.54095, size = 54, normalized size = 1.15

$$\frac{1}{4}x^4 - e^{(-2a)}\log\left(x^4e^{(2a)} + 1\right) - \frac{1}{x^4e^{(4a)} + e^{(2a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - e^(-2*a)*log(x^4*e^(2*a) + 1) - 1/(x^4*e^(4*a) + e^(2*a))
```

Fricas [A] time = 1.88471, size = 140, normalized size = 2.98

$$\frac{x^8e^{(4a)} + x^4e^{(2a)} - 4\left(x^4e^{(2a)} + 1\right)\log\left(x^4e^{(2a)} + 1\right) - 4}{4\left(x^4e^{(4a)} + e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^8*e^(4*a) + x^4*e^(2*a) - 4*(x^4*e^(2*a) + 1)*log(x^4*e^(2*a) + 1) -
 4)/(x^4*e^(4*a) + e^(2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\tanh^2\left(a + 2\log(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tanh(a+2*ln(x))**2,x)
```

```
[Out] Integral(x**3*tanh(a + 2*log(x))**2, x)
```

Giac [A] time = 1.1802, size = 53, normalized size = 1.13

$$\frac{1}{4}x^4 + \frac{x^4}{x^4e^{(2a)} + 1} - e^{(-2a)}\log\left(x^4e^{(2a)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + x^4/(x^4*e^(2*a) + 1) - e^(-2*a)*log(x^4*e^(2*a) + 1)
```

3.154 $\int x^2 \tanh^2(a + 2\log(x))\,dx$

Optimal. Leaf size=173

$$\frac{x^3}{e^{2a}x^4+1} - \frac{3e^{-3a/2}\log\left(e^a x^2 - \sqrt{2}e^{a/2}x + 1\right)}{4\sqrt{2}} + \frac{3e^{-3a/2}\log\left(e^a x^2 + \sqrt{2}e^{a/2}x + 1\right)}{4\sqrt{2}} + \frac{3e^{-3a/2}\tan^{-1}\left(1 - \sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{3e^{-3a/2}\text{ta}}{}$$

```
[Out] x^3/3 + x^3/(1 + E^(2*a)*x^4) + (3*ArcTan[1 - Sqrt[2]*E^(a/2)*x])/(2*Sqrt[2
]*E^((3*a)/2)) - (3*ArcTan[1 + Sqrt[2]*E^(a/2)*x])/(2*Sqrt[2]*E^((3*a)/2))
- (3*Log[1 - Sqrt[2]*E^(a/2)*x + E^a*x^2])/(4*Sqrt[2]*E^((3*a)/2)) + (3*Log
[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2])/(4*Sqrt[2]*E^((3*a)/2))
```

Rubi [F] time = 0.0479177, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \tanh^2(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][x^2*Tanh[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int x^2 \tanh^2(a + 2\log(x))\,dx = \int x^2 \tanh^2(a + 2\log(x))\,dx$$

Mathematica [A] time = 0.649469, size = 174, normalized size = 1.01

$$\frac{1}{12}\left(\frac{12x^3}{e^{2a}x^4+1} + 9(-1)^{3/4}e^{-3a/2}\log\left(\sqrt[4]{-1}e^{-3a/2} - e^{-a}x\right) + 9\sqrt[4]{-1}e^{-3a/2}\log\left((-1)^{3/4}e^{-3a/2} - e^{-a}x\right) - 9(-1)^{3/4}e^{-3a/2}\log\left(e^{-a}x + \right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] (4*x^3 + (12*x^3)/(1 + E^(2*a)*x^4) + (9*(-1)^(3/4)*Log[(-1)^(1/4)/E^((3*a)
/2) - x/E^a])/E^((3*a)/2) + (9*(-1)^(1/4)*Log[(-1)^(3/4)/E^((3*a)/2) - x/E^
a])/E^((3*a)/2) - (9*(-1)^(3/4)*Log[(-1)^(1/4)/E^((3*a)/2) + x/E^a])/E^((3*
a)/2) - (9*(-1)^(1/4)*Log[(-1)^(3/4)/E^((3*a)/2) + x/E^a])/E^((3*a)/2))/12
```

Maple [C] time = 0.022, size = 53, normalized size = 0.3

$$\frac{x^3}{3} + \frac{x^3}{1+e^{2a}x^4} - \frac{3\,e^{-2a}}{4}\sum_{\_R=\text{RootOf}\left(e^{2a}\_Z^4+1\right)}\frac{\ln(x-\_R)}{\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tanh(a+2*ln(x))^2,x)
```

```
[Out] 1/3*x^3+x^3/(1+exp(2*a)*x^4)-3/4*exp(-2*a)*sum(1/_R*ln(x-_R),_R=RootOf(exp(
2*a)*_Z^4+1))
```

Maxima [A] time = 1.77848, size = 194, normalized size = 1.12

$$\frac{1}{3}x^3 - \frac{3}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2xe^a + \sqrt{2}e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} - \frac{3}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2xe^a - \sqrt{2}e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 3/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a + sqrt(2)*e^(1/2*a))*e^(-
1/2*a))*e^(-3/2*a) - 3/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a - sqrt(2)*e^(1
/2*a))*e^(-1/2*a))*e^(-3/2*a) + 3/8*sqrt(2)*e^(-3/2*a)*log(x^2*e^a + sqrt(2
)*x*e^(1/2*a) + 1) - 3/8*sqrt(2)*e^(-3/2*a)*log(x^2*e^a - sqrt(2)*x*e^(1/2*
a) + 1) + x^3/(x^4*e^(2*a) + 1)
```

Fricas [A] time = 2.1458, size = 693, normalized size = 4.01

$$8x^7e^{(2a)} + 32x^3 + 36\left(\sqrt{2}x^4e^{(2a)} + \sqrt{2}\right)\arctan\left(-\sqrt{2}xe^{\left(\frac{1}{2}a\right)} + \sqrt{2}\sqrt{\sqrt{2}xe^{\left(-\frac{1}{2}a\right)} + x^2 + e^{(-a)}}e^{\left(\frac{1}{2}a\right)} - 1\right)e^{\left(-\frac{3}{2}a\right)} + 36\left(\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(8*x^7*e^(2*a) + 32*x^3 + 36*(sqrt(2)*x^4*e^(2*a) + sqrt(2))*arctan(-s
qrt(2)*x*e^(1/2*a) + sqrt(2)*sqrt(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a))*e^(1
/2*a) - 1)*e^(-3/2*a) + 36*(sqrt(2)*x^4*e^(2*a) + sqrt(2))*arctan(-sqrt(2)*
x*e^(1/2*a) + sqrt(2)*sqrt(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a))*e^(1/2*a)
+ 1)*e^(-3/2*a) + 9*(sqrt(2)*x^4*e^(2*a) + sqrt(2))*e^(-3/2*a)*log(sqrt(2)*
x*e^(-1/2*a) + x^2 + e^(-a)) - 9*(sqrt(2)*x^4*e^(2*a) + sqrt(2))*e^(-3/2*a)
*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)))/(x^4*e^(2*a) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\tanh^2(a + 2\log(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tanh(a+2*ln(x))**2,x)
```

```
[Out] Integral(x**2*tanh(a + 2*log(x))**2, x)
```

Giac [A] time = 1.18396, size = 188, normalized size = 1.09

$$\frac{1}{3}x^3 - \frac{3}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)} + 2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} - \frac{3}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)} - 2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{3}{8}\sqrt{2}e^{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 3/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*
a))*e^(-3/2*a) - 3/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)
*e^(1/2*a))*e^(-3/2*a) + 3/8*sqrt(2)*e^(-3/2*a)*log(sqrt(2)*x*e^(-1/2*a) +
x^2 + e^(-a)) - 3/8*sqrt(2)*e^(-3/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^
(-a)) + x^3/(x^4*e^(2*a) + 1)
```

3.155 $\int x \tanh^2(a + 2\log(x))\,dx$

Optimal. Leaf size=40

$$\frac{x^2}{e^{2a}x^4+1} - e^{-a}\tan^{-1}\left(e^a x^2\right) + \frac{x^2}{2}$$

[Out] x^2/2 + x^2/(1 + E^(2*a)*x^4) - ArcTan[E^a*x^2]/E^a

Rubi [F] time = 0.0302817, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \tanh^2(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

[In] Int[x*Tanh[a + 2*Log[x]]^2,x]

[Out] Defer[Int][x*Tanh[a + 2*Log[x]]^2, x]

Rubi steps

$$\int x \tanh^2(a + 2\log(x))\,dx = \int x \tanh^2(a + 2\log(x))\,dx$$

Mathematica [A] time = 0.377913, size = 41, normalized size = 1.02

$$\frac{x^2}{e^{2(a+2\log(x))}+1} - e^{-a}\tan^{-1}\left(e^a x^2\right) + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Tanh[a + 2*Log[x]]^2,x]

[Out] x^2/2 + x^2/(1 + E^(2*(a + 2*Log[x]))) - ArcTan[E^a*x^2]/E^a

---

Maple [C] time = 0.033, size = 57, normalized size = 1.4

$$\frac{x^2}{2} + \frac{x^2}{1+e^{2a}x^4} + \frac{i}{2}e^{-a}\ln\left(e^a x^2 - i\right) - \frac{i}{2}e^{-a}\ln\left(e^a x^2 + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*tanh(a+2*ln(x))^2,x)

[Out] 1/2*x^2+x^2/(1+exp(2*a)*x^4)+1/2*I*exp(-a)*ln(exp(a)*x^2-I)-1/2*I*exp(-a)*l
n(exp(a)*x^2+I)

Maxima [A] time = 1.86376, size = 47, normalized size = 1.18

$$\frac{1}{2}\,x^2 - \arctan\left(x^2 e^a\right) e^{(-a)} + \frac{x^2}{x^4 e^{(2\,a)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - arctan(x^2*e^a)*e^(-a) + x^2/(x^4*e^(2*a) + 1)
```

Fricas [A] time = 2.02991, size = 120, normalized size = 3.

$$\frac{x^6 e^{(3\,a)} + 3\,x^2 e^a - 2\left(x^4 e^{(2\,a)} + 1\right)\arctan\left(x^2 e^a\right)}{2\left(x^4 e^{(3\,a)} + e^a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^6*e^(3*a) + 3*x^2*e^a - 2*(x^4*e^(2*a) + 1)*arctan(x^2*e^a))/(x^4*e^
(3*a) + e^a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \tanh^2(a + 2\log(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(a+2*ln(x))**2,x)
```

```
[Out] Integral(x*tanh(a + 2*log(x))**2, x)
```

Giac [A] time = 1.23874, size = 47, normalized size = 1.18

$$\frac{1}{2}\,x^2 - \arctan\left(x^2 e^a\right) e^{(-a)} + \frac{x^2}{x^4 e^{(2\,a)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - arctan(x^2*e^a)*e^(-a) + x^2/(x^4*e^(2*a) + 1)
```

3.156 $\int \tanh^2(a + 2\log(x))\,dx$

Optimal. Leaf size=165

$$\frac{x}{e^{2a}x^4+1}+\frac{e^{-a/2}\log\left(e^a x^2-\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}}-\frac{e^{-a/2}\log\left(e^a x^2+\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}}+\frac{e^{-a/2}\tan^{-1}\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}-\frac{e^{-a/2}\tan^{-1}\left(\right.}{2\sqrt{}}$$

```
[Out] x + x/(1 + E^(2*a)*x^4) + ArcTan[1 - Sqrt[2]*E^(a/2)*x]/(2*Sqrt[2]*E^(a/2))
 - ArcTan[1 + Sqrt[2]*E^(a/2)*x]/(2*Sqrt[2]*E^(a/2)) + Log[1 - Sqrt[2]*E^(a
/2)*x + E^a*x^2]/(4*Sqrt[2]*E^(a/2)) - Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2]
/(4*Sqrt[2]*E^(a/2))
```

Rubi [F] time = 0.0100514, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^2(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][Tanh[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int \tanh^2(a+2\log(x))\,dx=\int \tanh^2(a+2\log(x))\,dx$$

Mathematica [A] time = 0.531036, size = 146, normalized size = 0.88

$$\frac{1}{4}\left(\frac{4x}{e^{2a}x^4+1}+\sqrt[4]{-1}e^{-a/2}\log\left(\sqrt[4]{-1}e^{-a/2}-x\right)+(-1)^{3/4}e^{-a/2}\log\left((-1)^{3/4}e^{-a/2}-x\right)-\sqrt[4]{-1}e^{-a/2}\log\left(\sqrt[4]{-1}e^{-a/2}+x\right)-(-1)^{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] (4*x + (4*x)/(1 + E^(2*a)*x^4) + ((-1)^(1/4)*Log[(-1)^(1/4)/E^(a/2) - x])/E
^(a/2) + ((-1)^(3/4)*Log[(-1)^(3/4)/E^(a/2) - x])/E^(a/2) - ((-1)^(1/4)*Log
[(-1)^(1/4)/E^(a/2) + x])/E^(a/2) - ((-1)^(3/4)*Log[(-1)^(3/4)/E^(a/2) + x]
)/E^(a/2))/4
```

Maple [C] time = 0.02, size = 47, normalized size = 0.3

$$x+\frac{x}{1+e^{2a}x^4}-\frac{e^{-2a}}{4}\sum_{\_R=\mathrm{RootOf}\left(e^{2a}\_Z^4+1\right)}\frac{\ln(x-\_R)}{\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+2*ln(x))^2,x)
```

```
[Out] x+x/(1+exp(2*a)*x^4)-1/4*exp(-2*a)*sum(1/_R^3*ln(x-_R),_R=RootOf(exp(2*a)*_
Z^4+1))
```

Maxima [A] time = 1.70158, size = 186, normalized size = 1.13

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2xe^{a}+\sqrt{2}e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2xe^{a}-\sqrt{2}e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}-\frac{1}{8}\sqrt{2}e^{\left(-\frac{1}{2}a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a + sqrt(2)*e^(1/2*a))*e^(-1/2*a))*e
^(-1/2*a) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a - sqrt(2)*e^(1/2*a))*e^
(-1/2*a))*e^(-1/2*a) - 1/8*sqrt(2)*e^(-1/2*a)*log(x^2*e^a + sqrt(2)*x*e^(1/
2*a) + 1) + 1/8*sqrt(2)*e^(-1/2*a)*log(x^2*e^a - sqrt(2)*x*e^(1/2*a) + 1) +
 x + x/(x^4*e^(2*a) + 1)
```

Fricas [B] time = 2.20292, size = 680, normalized size = 4.12

$$8x^{5}e^{(2a)}+4\left(\sqrt{2}x^{4}e^{(2a)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}xe^{\left(\frac{1}{2}a\right)}+\sqrt{2}\sqrt{\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^{2}+e^{(-a)}}e^{\left(\frac{1}{2}a\right)}-1\right)e^{\left(-\frac{1}{2}a\right)}+4\left(\sqrt{2}x^{4}e^{(2a)}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*x^5*e^(2*a) + 4*(sqrt(2)*x^4*e^(2*a) + sqrt(2))*arctan(-sqrt(2)*x*e^
(1/2*a) + sqrt(2)*sqrt(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a))*e^(1/2*a) - 1)*
e^(-1/2*a) + 4*(sqrt(2)*x^4*e^(2*a) + sqrt(2))*arctan(-sqrt(2)*x*e^(1/2*a)
+ sqrt(2)*sqrt(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a))*e^(1/2*a) + 1)*e^(-1/2
*a) - (sqrt(2)*x^4*e^(2*a) + sqrt(2))*e^(-1/2*a)*log(sqrt(2)*x*e^(-1/2*a) +
 x^2 + e^(-a)) + (sqrt(2)*x^4*e^(2*a) + sqrt(2))*e^(-1/2*a)*log(-sqrt(2)*x*
e^(-1/2*a) + x^2 + e^(-a)) + 16*x)/(x^4*e^(2*a) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^{2}\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*ln(x))**2,x)
```

```
[Out] Integral(tanh(a + 2*log(x))**2, x)
```

Giac [A] time = 1.45328, size = 180, normalized size = 1.09

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}+2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}-2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}-\frac{1}{8}\sqrt{2}e^{\left(-\frac{1}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*a))*e^(-1
/2*a) - 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)*e^(1/2*a
))*e^(-1/2*a) - 1/8*sqrt(2)*e^(-1/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(
-a)) + 1/8*sqrt(2)*e^(-1/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) + x
 + x/(x^4*e^(2*a) + 1)
```

3.157 $\int \frac{\tanh^2(a+2\log(x))}{x}\,dx$

Optimal. Leaf size=14

$$\log(x) - \frac{1}{2}\tanh(a+2\log(x))$$

```
[Out] Log[x] - Tanh[a + 2*Log[x]]/2
```

Rubi [A] time = 0.023689, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3473, 8}

$$\log(x) - \frac{1}{2}\tanh(a+2\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + 2*Log[x]]^2/x,x]

[Out] Log[x] - Tanh[a + 2*Log[x]]/2
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(a+2\log(x))}{x}\,dx &= \text{Subst}\left(\int \tanh^2(a+2x)\,dx, x, \log(x)\right)\\
&= -\frac{1}{2}\tanh(a+2\log(x)) + \text{Subst}\left(\int 1\,dx, x, \log(x)\right)\\
&= \log(x) - \frac{1}{2}\tanh(a+2\log(x))
\end{aligned}$$

Mathematica [A] time = 0.0377657, size = 24, normalized size = 1.71

$$\frac{1}{2}\tanh^{-1}(\tanh(a+2\log(x))) - \frac{1}{2}\tanh(a+2\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + 2*Log[x]]^2/x,x]

[Out] ArcTanh[Tanh[a + 2*Log[x]]]/2 - Tanh[a + 2*Log[x]]/2
```

Maple [B] time = 0.003, size = 35, normalized size = 2.5

$$-\frac{\tanh\left(a+2\ln\left(x\right)\right)}{2}-\frac{\ln\left(\tanh\left(a+2\ln\left(x\right)\right)-1\right)}{4}+\frac{\ln\left(\tanh\left(a+2\ln\left(x\right)\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+2*ln(x))^2/x,x)
```

```
[Out] -1/2*tanh(a+2*ln(x))-1/4*ln(tanh(a+2*ln(x))-1)+1/4*ln(tanh(a+2*ln(x))+1)
```

Maxima [A] time = 1.1484, size = 28, normalized size = 2.

$$\frac{1}{2}\,a-\frac{1}{e^{\left(-2\,a-4\,\log\left(x\right)\right)}+1}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*a - 1/(e^(-2*a - 4*log(x)) + 1) + log(x)
```

Fricas [B] time = 1.97699, size = 68, normalized size = 4.86

$$\frac{\left(x^{4}e^{\left(2\,a\right)}+1\right)\log\left(x\right)+1}{x^{4}e^{\left(2\,a\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2/x,x, algorithm="fricas")
```

```
[Out] ((x^4*e^(2*a) + 1)*log(x) + 1)/(x^4*e^(2*a) + 1)
```

Sympy [A] time = 0.499783, size = 12, normalized size = 0.86

$$\log\left(x\right)-\frac{\tanh\left(a+2\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*ln(x))**2/x,x)
```

```
[Out] log(x) - tanh(a + 2*log(x))/2
```

Giac [A] time = 1.4718, size = 26, normalized size = 1.86

$$\frac{1}{x^{4}e^{\left(2\,a\right)}+1}+\frac{1}{4}\,\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2/x,x, algorithm="giac")
```

```
[Out] 1/(x^4*e^(2*a) + 1) + 1/4*log(x^4)
```

3.158 $\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx$

Optimal. Leaf size=190

$$-\frac{2e^{2a}x^3}{e^{2a}x^4+1}-\frac{1}{x\left(e^{2a}x^4+1\right)}-\frac{e^{a/2}\log\left(e^ax^2-\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}}+\frac{e^{a/2}\log\left(e^ax^2+\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}}+\frac{e^{a/2}\tan^{-1}\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}$$

```
[Out] -(1/(x*(1 + E^(2*a)*x^4))) - (2*E^(2*a)*x^3)/(1 + E^(2*a)*x^4) + (E^(a/2)*A
rcTan[1 - Sqrt[2]*E^(a/2)*x])/(2*Sqrt[2]) - (E^(a/2)*ArcTan[1 + Sqrt[2]*E^(
a/2)*x])/(2*Sqrt[2]) - (E^(a/2)*Log[1 - Sqrt[2]*E^(a/2)*x + E^a*x^2])/(4*Sq
rt[2]) + (E^(a/2)*Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2])/(4*Sqrt[2])
```

Rubi [F] time = 0.044585, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] Defer[Int][Tanh[a + 2*Log[x]]^2/x^2, x]
```

Rubi steps

$$\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx=\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx$$

Mathematica [A] time = 0.735635, size = 181, normalized size = 0.95

$$\frac{1}{4}\left(-\frac{4}{\frac{e^{-2a}}{x^3}+x}+(-1)^{3/4}e^{a/2}\log\left(\frac{e^{-2a}\left(\sqrt[4]{-1}-e^{a/2}x\right)}{x^4}\right)+\sqrt[4]{-1}e^{a/2}\log\left(\frac{e^{-2a}\left((-1)^{3/4}-e^{a/2}x\right)}{x^4}\right)-(-1)^{3/4}e^{a/2}\log\left(\frac{e^{-2a}\left(e^{a/2}x\right.}{x^4}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] (-4/x - 4/(1/(E^(2*a)*x^3) + x) + (-1)^(3/4)*E^(a/2)*Log[((-1)^(1/4) - E^(a
/2)*x)/(E^(2*a)*x^4)] + (-1)^(1/4)*E^(a/2)*Log[((-1)^(3/4) - E^(a/2)*x)/(E^
(2*a)*x^4)] - (-1)^(3/4)*E^(a/2)*Log[((-1)^(1/4) + E^(a/2)*x)/(E^(2*a)*x^4)
] - (-1)^(1/4)*E^(a/2)*Log[((-1)^(3/4) + E^(a/2)*x)/(E^(2*a)*x^4)])/4
```

Maple [C] time = 0.026, size = 64, normalized size = 0.3

$$\frac{-2\,e^{2\,a}x^4-1}{x\left(1+e^{2\,a}x^4\right)}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4+e^{2\,a})}\_R\ln\left(\left(5\_R^4+4\,e^{2\,a}\right)x+\_R^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+2*ln(x))^2/x^2,x)
```

```
[Out] (-2*exp(2*a)*x^4-1)/x/(1+exp(2*a)*x^4)+1/4*sum(_R*ln((5*_R^4+4*exp(2*a))*x+
_R^3),_R=RootOf(_Z^4+exp(2*a)))
```

Maxima [A] time = 1.93076, size = 197, normalized size = 1.04

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(\frac{1}{2}a\right)}+\frac{2}{x}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(\frac{1}{2}a\right)}-\frac{2}{x}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}+\frac{1}{8}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\sqrt{2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(1/2*a) + 2/x)*e^(-1/2*a))*e^(1/2
*a) + 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(1/2*a) - 2/x)*e^(-1/2*a))
*e^(1/2*a) + 1/8*sqrt(2)*e^(1/2*a)*log(sqrt(2)*e^(1/2*a)/x + 1/x^2 + e^a) -
 1/8*sqrt(2)*e^(1/2*a)*log(-sqrt(2)*e^(1/2*a)/x + 1/x^2 + e^a) - 1/x - e^(2
*a)/(x*(1/x^4 + e^(2*a)))
```

Fricas [B] time = 2.14456, size = 771, normalized size = 4.06

$$-\frac{16\,x^{4}e^{(2\,a)}-4\left(\sqrt{2}x^{5}e^{(2\,a)}+\sqrt{2}x\right)\arctan\left(-\left(\sqrt{2}xe^{\left(\frac{5}{2}a\right)}-\sqrt{2}\sqrt{x^{2}e^{(4\,a)}+\sqrt{2}xe^{\left(\frac{7}{2}a\right)}+e^{(3\,a)}}e^{\left(\frac{1}{2}a\right)}+e^{(2\,a)}\right)e^{(-2\,a)}\right)e^{\left(\frac{1}{2}a\right)}-4\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(16*x^4*e^(2*a) - 4*(sqrt(2)*x^5*e^(2*a) + sqrt(2)*x)*arctan(-(sqrt(2)
*x*e^(5/2*a) - sqrt(2)*sqrt(x^2*e^(4*a) + sqrt(2)*x*e^(7/2*a) + e^(3*a))*e^
(1/2*a) + e^(2*a))*e^(-2*a))*e^(1/2*a) - 4*(sqrt(2)*x^5*e^(2*a) + sqrt(2)*x
)*arctan(-(sqrt(2)*x*e^(5/2*a) - sqrt(2)*sqrt(x^2*e^(4*a) - sqrt(2)*x*e^(7/
2*a) + e^(3*a))*e^(1/2*a) - e^(2*a))*e^(-2*a))*e^(1/2*a) - (sqrt(2)*x^5*e^(
2*a) + sqrt(2)*x)*e^(1/2*a)*log(x^2*e^(4*a) + sqrt(2)*x*e^(7/2*a) + e^(3*a)
) + (sqrt(2)*x^5*e^(2*a) + sqrt(2)*x)*e^(1/2*a)*log(x^2*e^(4*a) - sqrt(2)*x
*e^(7/2*a) + e^(3*a)) + 8)/(x^5*e^(2*a) + x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^{2}\left(a+2\log\left(x\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*ln(x))**2/x**2,x)
```

```
[Out] Integral(tanh(a + 2*log(x))**2/x**2, x)
```

Giac [A] time = 1.4497, size = 193, normalized size = 1.02

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}+2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}-2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}+\frac{1}{8}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*a))*e^(1/
2*a) - 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)*e^(1/2*a)
)*e^(1/2*a) + 1/8*sqrt(2)*e^(1/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)
) - 1/8*sqrt(2)*e^(1/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) - (2*x^
4*e^(2*a) + 1)/(x^5*e^(2*a) + x)
```

3.159 $\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx$

Optimal. Leaf size=59

$$-\frac{3e^{2a}x^2}{2\left(e^{2a}x^4+1\right)}-\frac{1}{2x^2\left(e^{2a}x^4+1\right)}-e^a\tan^{-1}\left(e^ax^2\right)$$

```
[Out] -1/(2*x^2*(1 + E^(2*a)*x^4)) - (3*E^(2*a)*x^2)/(2*(1 + E^(2*a)*x^4)) - E^a*
ArcTan[E^a*x^2]
```

Rubi [F] time = 0.0534242, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] Defer[Int][Tanh[a + 2*Log[x]]^2/x^3, x]
```

Rubi steps

$$\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx=\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx$$

Mathematica [A] time = 0.355313, size = 40, normalized size = 0.68

$$\frac{-\frac{2}{e^{-2(a+2\log(x))}+1}-1}{2x^2}+e^a\tan^{-1}\left(\frac{e^{-a}}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] (-1 - 2/(1 + E^(-2*(a + 2*Log[x]))))/(2*x^2) + E^a*ArcTan[1/(E^a*x^2)]
```

Maple [C] time = 0.029, size = 66, normalized size = 1.1

$$\frac{1}{x^2\left(1+e^{2a}x^4\right)}\left(-\frac{3\,e^{2a}x^4}{2}-\frac{1}{2}\right)+\frac{\sum_{\_R=\mathrm{RootOf}(e^{2a}+\_Z^2)}\_R\,\ln\left(\left(-4\,e^{2a}-5\,\_R^2\right)x^2-\_R\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+2*ln(x))^2/x^3,x)
```

```
[Out] (-3/2*exp(2*a)*x^4-1/2)/x^2/(1+exp(2*a)*x^4)+1/2*sum(_R*ln((-4*exp(2*a)-5*_
R^2)*x^2-_R),_R=RootOf(exp(2*a)+_Z^2))
```

Maxima [A] time = 1.81886, size = 50, normalized size = 0.85

$$\arctan\left(\frac{e^{(-a)}}{x^2}\right)e^a - \frac{1}{2\,x^2} - \frac{e^{(2\,a)}}{x^2\left(\frac{1}{x^4} + e^{(2\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2/x^3,x, algorithm="maxima")
```

```
[Out] arctan(e^(-a)/x^2)*e^a - 1/2/x^2 - e^(2*a)/(x^2*(1/x^4 + e^(2*a)))
```

Fricas [A] time = 1.88606, size = 122, normalized size = 2.07

$$-\frac{3\,x^4e^{(2\,a)} + 2\left(x^6e^{(3\,a)} + x^2e^a\right)\arctan\left(x^2e^a\right) + 1}{2\left(x^6e^{(2\,a)} + x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4*e^(2*a) + 2*(x^6*e^(3*a) + x^2*e^a)*arctan(x^2*e^a) + 1)/(x^6*e
^(2*a) + x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2\left(a + 2\log\left(x\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*ln(x))**2/x**3,x)
```

```
[Out] Integral(tanh(a + 2*log(x))**2/x**3, x)
```

Giac [A] time = 1.40139, size = 53, normalized size = 0.9

$$-\arctan\left(x^2e^a\right)e^a - \frac{3\,x^4e^{(2\,a)} + 1}{2\left(x^6e^{(2\,a)} + x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^2/x^3,x, algorithm="giac")
```

```
[Out] -arctan(x^2*e^a)*e^a - 1/2*(3*x^4*e^(2*a) + 1)/(x^6*e^(2*a) + x^2)
```

3.160 $\int (ex)^m \tanh(a+2\log(x))\,dx$

Optimal. Leaf size=60

$$\frac{(ex)^{m+1}}{e(m+1)}-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-e^{2a}x^4\right)}{e(m+1)}$$

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/4
, (5 + m)/4, -(E^(2*a)*x^4)])/(e*(1 + m))
```

Rubi [F] time = 0.0391039, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \tanh(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Tanh[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][(e*x)^m*Tanh[a + 2*Log[x]], x]
```

Rubi steps

$$\int (ex)^m \tanh(a+2\log(x))\,dx=\int (ex)^m \tanh(a+2\log(x))\,dx$$

Mathematica [A] time = 0.0895244, size = 47, normalized size = 0.78

$$-\frac{x(ex)^m\left(2\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-x^4(\cosh(2a)+\sinh(2a))\right)-1\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tanh[a + 2*Log[x]],x]
```

```
[Out] -((x*(e*x)^m*(-1 + 2*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, -(x^4*(Cosh
[2*a] + Sinh[2*a]))]))/(1 + m))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (ex)^m \tanh(a+2\ln(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tanh(a+2*ln(x)),x)
```

```
[Out] int((e*x)^m*tanh(a+2*ln(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh (a + 2 \log (x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*tanh(a + 2*log(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \tanh (a + 2 \log (x)), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*tanh(a + 2*log(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh (a + 2 \log (x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tanh(a+2*ln(x)),x)
```

```
[Out] Integral((e*x)**m*tanh(a + 2*log(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh (a + 2 \log (x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+2*log(x)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tanh(a + 2*log(x)), x)
```

3.161 $\int (ex)^m \tanh^2(a+2\log(x))\,dx$

Optimal. Leaf size=79

$$-\frac{(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-e^{2a}x^4\right)}{e}+\frac{(ex)^{m+1}}{e\left(e^{2a}x^4+1\right)}+\frac{(ex)^{m+1}}{e(m+1)}$$

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) + (e*x)^(1 + m)/(e*(1 + E^(2*a)*x^4)) - ((e*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, -(E^(2*a)*x^4)])/e
```

Rubi [F] time = 0.0691203, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][(e*x)^m*Tanh[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx=\int (ex)^m \tanh^2(a+2\log(x))\,dx$$

Mathematica [B] time = 0.363646, size = 168, normalized size = 2.13

$$\frac{x(ex)^m\left(\frac{x^4(\sinh(a)+\cosh(a))\left((m+5)x^4(\sinh(a)+\cosh(a))\,{}_2F_1\left(2,\frac{m+9}{4};\frac{m+13}{4};-x^4(\cosh(2a)+\sinh(2a))\right)-2(m+9)(\cosh(a)-\sinh(a))\,{}_2F_1\left(2,\frac{m+5}{4};\frac{m+9}{4};-x^4(\cosh(2\cdots\right.\right.}{(m+5)(m+9)}\cdots\right.}{(\cosh(a)-\sinh(a))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] (x*(e*x)^m*((x^4*(Cosh[a] + Sinh[a])*(-2*(9 + m)*Hypergeometric2F1[2, (5 +
m)/4, (9 + m)/4, -(x^4*(Cosh[2*a] + Sinh[2*a]))]*(Cosh[a] - Sinh[a]) + (5 +
 m)*x^4*Hypergeometric2F1[2, (9 + m)/4, (13 + m)/4, -(x^4*(Cosh[2*a] + Sinh
[2*a]))]*(Cosh[a] + Sinh[a])))/((5 + m)*(9 + m)) + (Hypergeometric2F1[2, (1
 + m)/4, (5 + m)/4, -(x^4*(Cosh[2*a] + Sinh[2*a]))]*(Cosh[2*a] - Sinh[2*a])
)/(1 + m)))/(Cosh[a] - Sinh[a])^2
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int (ex)^m\left(\tanh\left(a+2\ln(x)\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tanh(a+2*ln(x))^2,x)
```

```
[Out] int((e*x)^m*tanh(a+2*ln(x))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh (a+2 \log (x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*tanh(a + 2*log(x))^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \tanh (a+2 \log (x))^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*tanh(a + 2*log(x))^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh^2 (a+2\log (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tanh(a+2*ln(x))**2,x)
```

```
[Out] Integral((e*x)**m*tanh(a + 2*log(x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh (a+2 \log (x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tanh(a + 2*log(x))^2, x)
```

3.162 $\int (ex)^m \tanh^3(a+2\log(x))\,dx$

Optimal. Leaf size=176

$$-\frac{(m^2+2m+9)(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-e^{2a}x^4\right)}{4e(m+1)}-\frac{e^{-2a}\left(e^{4a}(m+5)x^4+e^{2a}(3-m)\right)(ex)^{m+1}}{8e\left(e^{2a}x^4+1\right)}-\frac{\left(1-e^{2a}x^4\right)^2(ex)^{m+1}}{4e\left(e^{2a}x^4+1\right)^2}+\frac{(\ldots}{\ldots}$$

```
[Out] ((3 + m)*(5 + m)*(e*x)^(1 + m))/(8*e*(1 + m)) - ((e*x)^(1 + m)*(1 - E^(2*a)
*x^4)^2)/(4*e*(1 + E^(2*a)*x^4)^2) - ((e*x)^(1 + m)*(E^(2*a)*(3 - m) + E^(4
*a)*(5 + m)*x^4))/(8*e*E^(2*a)*(1 + E^(2*a)*x^4)) - ((9 + 2*m + m^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, -(E^(2*a)*x^4)])/(4*e*(1
 + m))
```

Rubi [F] time = 0.0739888, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Tanh[a + 2*Log[x]]^3,x]
```

```
[Out] Defer[Int][(e*x)^m*Tanh[a + 2*Log[x]]^3, x]
```

Rubi steps

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx=\int (ex)^m \tanh^3(a+2\log(x))\,dx$$

Mathematica [A] time = 0.816673, size = 218, normalized size = 1.24

$$\frac{x(ex)^m\left(\frac{x^8(\sinh(2a)+\cosh(2a))\left((m+9)x^4(\sinh(a)+\cosh(a))\,{}_2F_1\left(3,\frac{m+13}{4};\frac{m+17}{4};-x^4(\cosh(2a)+\sinh(2a))\right)-3(m+13)(\cosh(a)-\sinh(a))\,{}_2F_1\left(3,\frac{m+9}{4};\frac{m+13}{4};-x^4(\cosh\ldots\right)\right)}{(m+9)(m+13)}\ldots\right.}{(\cosh(a)-\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tanh[a + 2*Log[x]]^3,x]
```

```
[Out] (x*(e*x)^m*((3*x^4*Hypergeometric2F1[3, (5 + m)/4, (9 + m)/4, -(x^4*(Cosh[2
*a] + Sinh[2*a]))]*(Cosh[a] - Sinh[a]))/(5 + m) - (Hypergeometric2F1[3, (1
+ m)/4, (5 + m)/4, -(x^4*(Cosh[2*a] + Sinh[2*a]))]*(Cosh[a] - Sinh[a])^3)/(
1 + m) + (x^8*(-3*(13 + m)*Hypergeometric2F1[3, (9 + m)/4, (13 + m)/4, -(x^
4*(Cosh[2*a] + Sinh[2*a]))]*(Cosh[a] - Sinh[a]) + (9 + m)*x^4*Hypergeometri
c2F1[3, (13 + m)/4, (17 + m)/4, -(x^4*(Cosh[2*a] + Sinh[2*a]))]*(Cosh[a] +
Sinh[a]))*(Cosh[2*a] + Sinh[2*a]))/((9 + m)*(13 + m))))/(Cosh[a] - Sinh[a])
^3
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (ex)^m (\tanh (a+2\ln (x)))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tanh(a+2*ln(x))^3,x)
```

```
[Out] int((e*x)^m*tanh(a+2*ln(x))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh (a+2\,\log (x))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*tanh(a + 2*log(x))^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \tanh (a+2\,\log (x))^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*tanh(a + 2*log(x))^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh^3 (a+2\log (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tanh(a+2*ln(x))**3,x)
```

```
[Out] Integral((e*x)**m*tanh(a + 2*log(x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh (a+2\,\log (x))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tanh(a + 2*log(x))^3, x)
```

3.163 $\int \tanh^p(a+b\log(x))\,dx$

Optimal. Leaf size=79

$$x\left(1-e^{2a}x^{2b}\right)^{-p}\left(e^{2a}x^{2b}-1\right)^{p}F_1\left(\frac{1}{2b};-p,p;\frac{1}{2}\left(2+\frac{1}{b}\right);e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

```
[Out] (x*(-1 + E^(2*a)*x^(2*b))^p*AppellF1[1/(2*b), -p, p, (2 + b^(-1))/2, E^(2*a
)*x^(2*b), -(E^(2*a)*x^(2*b))])/(1 - E^(2*a)*x^(2*b))^p
```

Rubi [F] time = 0.0229634, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^p(a+b\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*Log[x]]^p,x]
```

```
[Out] Defer[Int][Tanh[a + b*Log[x]]^p, x]
```

Rubi steps

$$\int \tanh^p(a+b\log(x))\,dx=\int \tanh^p(a+b\log(x))\,dx$$

Mathematica [B] time = 1.90654, size = 259, normalized size = 3.28

$$\frac{(2b+1)x\left(\frac{e^{2a}x^{2b}-1}{e^{2a}x^{2b}+1}\right)^{p}F_1\left(\frac{1}{2b};-p,p;1+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{-2e^{2a}bpx^{2b}F_1\left(1+\frac{1}{2b};1-p,p;2+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)-2e^{2a}bpx^{2b}F_1\left(1+\frac{1}{2b};-p,p+1;2+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)+(2b+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[a + b*Log[x]]^p,x]
```

```
[Out] ((1 + 2*b)*x*((-1 + E^(2*a)*x^(2*b))/(1 + E^(2*a)*x^(2*b)))^p*AppellF1[1/(2
*b), -p, p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(-2*b*E^(2*a
)*p*x^(2*b)*AppellF1[1 + 1/(2*b), 1 - p, p, 2 + 1/(2*b), E^(2*a)*x^(2*b), -
(E^(2*a)*x^(2*b))] - 2*b*E^(2*a)*p*x^(2*b)*AppellF1[1 + 1/(2*b), -p, 1 + p,
 2 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))] + (1 + 2*b)*AppellF1[1/(
2*b), -p, p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int (\tanh(a+b\ln(x)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+b*ln(x))^p,x)
```

```
[Out] int(tanh(a+b*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(b\log\left(x\right) + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tanh(b*log(x) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\tanh\left(b\log\left(x\right) + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(b*log(x) + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^p\left(a + b\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*ln(x))**p,x)
```

```
[Out] Integral(tanh(a + b*log(x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(b\log\left(x\right) + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(tanh(b*log(x) + a)^p, x)
```

3.164 $\int (ex)^m \tanh^p(a+b\log(x))\,dx$

Optimal. Leaf size=99

$$\frac{(ex)^{m+1}\left(1-e^{2a}x^{2b}\right)^{-p}\left(e^{2a}x^{2b}-1\right)^p F_1\left(\frac{m+1}{2b};-p,p;\frac{m+1}{2b}+1;e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(m+1)}$$

```
[Out] ((e*x)^(1 + m)*(-1 + E^(2*a)*x^(2*b))^p*AppellF1[(1 + m)/(2*b), -p, p, 1 +
(1 + m)/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(e*(1 + m)*(1 - E^(2*a
)*x^(2*b))^p)
```

Rubi [F] time = 0.11413, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \tanh^p(a+b\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Tanh[a + b*Log[x]]^p,x]

[Out] Defer[Int][(e*x)^m*Tanh[a + b*Log[x]]^p, x]
```

Rubi steps

$$\int (ex)^m \tanh^p(a+b\log(x))\,dx = \int (ex)^m \tanh^p(a+b\log(x))\,dx$$

Mathematica [A] time = 2.97782, size = 126, normalized size = 1.27

$$\frac{x(ex)^m\left(1-e^{2a}x^{2b}\right)^{-p}\left(\frac{e^{2a}x^{2b}-1}{e^{2a}x^{2b}+1}\right)^p\left(e^{2a}x^{2b}+1\right)^p F_1\left(\frac{m+1}{2b};-p,p;\frac{m+1}{2b}+1;e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Tanh[a + b*Log[x]]^p,x]

[Out] (x*(e*x)^m*((-1 + E^(2*a)*x^(2*b))/(1 + E^(2*a)*x^(2*b)))^p*(1 + E^(2*a)*x^
(2*b))^p*AppellF1[(1 + m)/(2*b), -p, p, 1 + (1 + m)/(2*b), E^(2*a)*x^(2*b),
 -(E^(2*a)*x^(2*b))])/((1 + m)*(1 - E^(2*a)*x^(2*b))^p)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int (ex)^m (\tanh(a+b\ln(x)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tanh(a+b*ln(x))^p,x)
```

```
[Out] int((e*x)^m*tanh(a+b*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh (b\log (x) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+b*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*tanh(b*log(x) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \tanh (b\log (x) + a)^p , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+b*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*tanh(b*log(x) + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh^p (a + b\log (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tanh(a+b*ln(x))**p,x)
```

```
[Out] Integral((e*x)**m*tanh(a + b*log(x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh (b\log (x) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(a+b*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tanh(b*log(x) + a)^p, x)
```

3.165 $\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx$

Optimal. Leaf size=51

$$\frac{e^{-2a}2^{-p}\left(e^{2a}x-1\right)^{p+1}\,{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(1-e^{2a}x\right)\right)}{p+1}$$

```
[Out] ((-1 + E^(2*a)*x)^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 - E^(2*a)*x
)/2])/(2^p*E^(2*a)*(1 + p))
```

Rubi [F] time = 0.0468423, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + Log[x]/2]^p,x]
```

```
[Out] Defer[Int][Tanh[(2*a + Log[x])/2]^p, x]
```

Rubi steps

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \tanh^p\left(\frac{1}{2}(2a+\log(x))\right)\,dx$$

Mathematica [A] time = 2.90375, size = 76, normalized size = 1.49

$$\frac{e^{-2a}2^{-p}\left(\frac{e^{2a}x-1}{e^{2a}x+1}\right)^{p+1}\left(e^{2a}x+1\right)^{p+1}\,{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(1-e^{2a}x\right)\right)}{p+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[a + Log[x]/2]^p,x]
```

```
[Out] (((-1 + E^(2*a)*x)/(1 + E^(2*a)*x))^(1 + p)*(1 + E^(2*a)*x)^(1 + p)*Hyperge
ometric2F1[p, 1 + p, 2 + p, (1 - E^(2*a)*x)/2])/(2^p*E^(2*a)*(1 + p))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \left(\tanh\left(a+\frac{\ln(x)}{2}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+1/2*ln(x))^p,x)
```

```
[Out] int(tanh(a+1/2*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a + \frac{1}{2}\log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tanh(a + 1/2*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\tanh\left(a + \frac{1}{2}\log(x)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(a + 1/2*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^p\left(a + \frac{\log(x)}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/2*ln(x))**p,x)
```

```
[Out] Integral(tanh(a + log(x)/2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a + \frac{1}{2}\log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/2*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(tanh(a + 1/2*log(x))^p, x)
```

3.166 $\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx$

Optimal. Leaf size=106

$$e^{-4a}\left(e^{2a}\sqrt{x}-1\right)^{p+1}\left(e^{2a}\sqrt{x}+1\right)^{1-p}-\frac{e^{-4a}2^{1-p}p\left(e^{2a}\sqrt{x}-1\right)^{p+1}{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(1-e^{2a}\sqrt{x}\right)\right)}{p+1}$$

```
[Out] ((-1 + E^(2*a)*Sqrt[x])^(1 + p)*(1 + E^(2*a)*Sqrt[x])^(1 - p))/E^(4*a) - (2
^(1 - p)*p*(-1 + E^(2*a)*Sqrt[x])^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p
, (1 - E^(2*a)*Sqrt[x])/2])/(E^(4*a)*(1 + p))
```

Rubi [F] time = 0.0527003, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + Log[x]/4]^p,x]
```

```
[Out] Defer[Int][Tanh[(4*a + Log[x])/4]^p, x]
```

Rubi steps

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)dx=\int \tanh^p\left(\frac{1}{4}(4a+\log(x))\right)dx$$

Mathematica [A] time = 2.90407, size = 121, normalized size = 1.14

$$\frac{e^{-4a}\left(e^{2a}\sqrt{x}-1\right)\left(\frac{e^{2a}\sqrt{x}-1}{2e^{2a}\sqrt{x}+2}\right)^p\left(2^p(p+1)\left(e^{2a}\sqrt{x}+1\right)-2p\left(e^{2a}\sqrt{x}+1\right)^p{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(1-e^{2a}\sqrt{x}\right)\right)\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + Log[x]/4]^p,x]
```

```
[Out] ((-1 + E^(2*a)*Sqrt[x])*((-1 + E^(2*a)*Sqrt[x])/(2 + 2*E^(2*a)*Sqrt[x]))^p*
(2^p*(1 + p)*(1 + E^(2*a)*Sqrt[x]) - 2*p*(1 + E^(2*a)*Sqrt[x])^p*Hypergeome
tric2F1[p, 1 + p, 2 + p, (1 - E^(2*a)*Sqrt[x])/2]))/(E^(4*a)*(1 + p))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int\left(\tanh\left(a+\frac{\ln(x)}{4}\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+1/4*ln(x))^p,x)
```

```
[Out] int(tanh(a+1/4*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a + \frac{1}{4}\log\left(x\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/4*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tanh(a + 1/4*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\tanh\left(a + \frac{1}{4}\log\left(x\right)\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/4*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(a + 1/4*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^{p}\left(a + \frac{\log\left(x\right)}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/4*ln(x))**p,x)
```

```
[Out] Integral(tanh(a + log(x)/4)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a + \frac{1}{4}\log\left(x\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/4*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(tanh(a + 1/4*log(x))^p, x)
```

3.167 $\int \tanh^p\left(a+\frac{\log(x)}{6}\right)\,dx$

Optimal. Leaf size=158

$$\frac{e^{-6a}2^{-p}\left(2p^2+1\right)\left(e^{2a}\sqrt[3]{x}-1\right)^{p+1}\,{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(1-e^{2a}\sqrt[3]{x}\right)\right)}{p+1}-e^{-6a}p\left(e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(e^{2a}\sqrt[3]{x}+1\right)^{1-p}+e^{-4a}\sqrt[3]{x}\left(e^{2a}\right.$$

```
[Out] -((p*(-1 + E^(2*a)*x^(1/3))^(1 + p)*(1 + E^(2*a)*x^(1/3))^(1 - p))/E^(6*a))
 + ((-1 + E^(2*a)*x^(1/3))^(1 + p)*(1 + E^(2*a)*x^(1/3))^(1 - p)*x^(1/3))/E
^(4*a) + ((1 + 2*p^2)*(-1 + E^(2*a)*x^(1/3))^(1 + p)*Hypergeometric2F1[p, 1
 + p, 2 + p, (1 - E^(2*a)*x^(1/3))/2])/(2^p*E^(6*a)*(1 + p))
```

Rubi [F] time = 0.0519714, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^p\left(a+\frac{\log(x)}{6}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + Log[x]/6]^p,x]
```

```
[Out] Defer[Int][Tanh[(6*a + Log[x])/6]^p, x]
```

Rubi steps

$$\int \tanh^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \tanh^p\left(\frac{1}{6}(6a+\log(x))\right)\,dx$$

Mathematica [C] time = 4.09641, size = 177, normalized size = 1.12

$$\frac{4x\left(\frac{e^{2a}\sqrt[3]{x}-1}{e^{2a}\sqrt[3]{x}+1}\right)^p F_1\left(3;-p,p;4;e^{2a}\sqrt[3]{x},-e^{2a}\sqrt[3]{x}\right)}{4F_1\left(3;-p,p;4;e^{2a}\sqrt[3]{x},-e^{2a}\sqrt[3]{x}\right)-e^{2a}p\sqrt[3]{x}\left(F_1\left(4;1-p,p;5;e^{2a}\sqrt[3]{x},-e^{2a}\sqrt[3]{x}\right)+F_1\left(4;-p,p+1;5;e^{2a}\sqrt[3]{x},-e^{2a}\sqrt[3]{x}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[a + Log[x]/6]^p,x]
```

```
[Out] (4*((-1 + E^(2*a)*x^(1/3))/(1 + E^(2*a)*x^(1/3)))^p*x*AppellF1[3, -p, p, 4,
 E^(2*a)*x^(1/3), -(E^(2*a)*x^(1/3))])/(4*AppellF1[3, -p, p, 4, E^(2*a)*x^(
1/3), -(E^(2*a)*x^(1/3))] - E^(2*a)*p*x^(1/3)*(AppellF1[4, 1 - p, p, 5, E^(
2*a)*x^(1/3), -(E^(2*a)*x^(1/3))] + AppellF1[4, -p, 1 + p, 5, E^(2*a)*x^(1/
3), -(E^(2*a)*x^(1/3))]))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \left(\tanh\left(a+\frac{\ln(x)}{6}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+1/6*ln(x))^p,x)
```

```
[Out] int(tanh(a+1/6*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a + \frac{1}{6}\log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/6*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tanh(a + 1/6*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\tanh\left(a + \frac{1}{6}\log(x)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/6*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(a + 1/6*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^p\left(a + \frac{\log(x)}{6}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/6*ln(x))**p,x)
```

```
[Out] Integral(tanh(a + log(x)/6)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a + \frac{1}{6}\log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/6*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(tanh(a + 1/6*log(x))^p, x)
```

3.168 $\int \tanh^p\left(a+\frac{\log(x)}{8}\right)\,dx$

Optimal. Leaf size=190

$$-\frac{e^{-8a}2^{2-p}p\left(p^2+2\right)\left(e^{2a}\sqrt[4]{x}-1\right)^{p+1}{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(1-e^{2a}\sqrt[4]{x}\right)\right)}{3(p+1)}+\frac{1}{3}e^{-12a}\left(e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(e^{4a}\left(2p^2+3\right)-2e^{6a}p\sqrt[4]{x}\right.$$

```
[Out] ((-1 + E^(2*a)*x^(1/4))^(1 + p)*(1 + E^(2*a)*x^(1/4))^(1 - p)*(E^(4*a)*(3 +
 2*p^2) - 2*E^(6*a)*p*x^(1/4)))/(3*E^(12*a)) + ((-1 + E^(2*a)*x^(1/4))^(1 +
 p)*(1 + E^(2*a)*x^(1/4))^(1 - p)*Sqrt[x])/E^(4*a) - (2^(2 - p)*p*(2 + p^2)
*(-1 + E^(2*a)*x^(1/4))^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 - E^(
2*a)*x^(1/4))/2])/(3*E^(8*a)*(1 + p))
```

Rubi [F] time = 0.0519402, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^p\left(a+\frac{\log(x)}{8}\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + Log[x]/8]^p,x]
```

```
[Out] Defer[Int][Tanh[(8*a + Log[x])/8]^p, x]
```

Rubi steps

$$\int \tanh^p\left(a+\frac{\log(x)}{8}\right)dx=\int \tanh^p\left(\frac{1}{8}(8a+\log(x))\right)dx$$

Mathematica [C] time = 3.17013, size = 177, normalized size = 0.93

$$\frac{5x\left(\frac{e^{2a}\sqrt[4]{x}-1}{e^{2a}\sqrt[4]{x}+1}\right)^p F_1\left(4;-p,p;5;e^{2a}\sqrt[4]{x},-e^{2a}\sqrt[4]{x}\right)}{5F_1\left(4;-p,p;5;e^{2a}\sqrt[4]{x},-e^{2a}\sqrt[4]{x}\right)-e^{2a}p\sqrt[4]{x}\left(F_1\left(5;1-p,p;6;e^{2a}\sqrt[4]{x},-e^{2a}\sqrt[4]{x}\right)+F_1\left(5;-p,p+1;6;e^{2a}\sqrt[4]{x},-e^{2a}\sqrt[4]{x}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[a + Log[x]/8]^p,x]
```

```
[Out] (5*((-1 + E^(2*a)*x^(1/4))/(1 + E^(2*a)*x^(1/4)))^p*x*AppellF1[4, -p, p, 5,
 E^(2*a)*x^(1/4), -(E^(2*a)*x^(1/4))])/(5*AppellF1[4, -p, p, 5, E^(2*a)*x^(
1/4), -(E^(2*a)*x^(1/4))] - E^(2*a)*p*x^(1/4)*(AppellF1[5, 1 - p, p, 6, E^(
2*a)*x^(1/4), -(E^(2*a)*x^(1/4))] + AppellF1[5, -p, 1 + p, 6, E^(2*a)*x^(1/
4), -(E^(2*a)*x^(1/4))]))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \left(\tanh\left(a+\frac{\ln(x)}{8}\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+1/8*ln(x))^p,x)
```

```
[Out] int(tanh(a+1/8*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a + \frac{1}{8}\log\left(x\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/8*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tanh(a + 1/8*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\tanh\left(a + \frac{1}{8}\log\left(x\right)\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/8*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(a + 1/8*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^{p}\left(a + \frac{\log\left(x\right)}{8}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/8*ln(x))**p,x)
```

```
[Out] Integral(tanh(a + log(x)/8)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a + \frac{1}{8}\log\left(x\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+1/8*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(tanh(a + 1/8*log(x))^p, x)
```

3.169 $\int \tanh^p(a+\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(1-e^{2a}x^2\right)^{-p}\left(e^{2a}x^2-1\right)^p F_1\left(\frac{1}{2};-p,p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)$$

```
[Out] (x*(-1 + E^(2*a)*x^2)^p*AppellF1[1/2, -p, p, 3/2, E^(2*a)*x^2, -(E^(2*a)*x^
2)])/(1 - E^(2*a)*x^2)^p
```

Rubi [F] time = 0.0158417, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^p(a+\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + Log[x]]^p,x]
```

```
[Out] Defer[Int][Tanh[a + Log[x]]^p, x]
```

Rubi steps

$$\int \tanh^p(a+\log(x))\,dx=\int \tanh^p(a+\log(x))\,dx$$

Mathematica [B] time = 1.54428, size = 171, normalized size = 2.8

$$\frac{3x\left(\frac{e^{2a}x^2-1}{e^{2a}x^2+1}\right)^p F_1\left(\frac{1}{2};-p,p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)}{3F_1\left(\frac{1}{2};-p,p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)-2e^{2a}px^2\left(F_1\left(\frac{3}{2};1-p,p;\frac{5}{2};e^{2a}x^2,-e^{2a}x^2\right)+F_1\left(\frac{3}{2};-p,p+1;\frac{5}{2};e^{2a}x^2,-e^{2a}x^2\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[a + Log[x]]^p,x]
```

```
[Out] (3*x*((-1 + E^(2*a)*x^2)/(1 + E^(2*a)*x^2))^p*AppellF1[1/2, -p, p, 3/2, E^(
2*a)*x^2, -(E^(2*a)*x^2)])/(3*AppellF1[1/2, -p, p, 3/2, E^(2*a)*x^2, -(E^(2
*a)*x^2)] - 2*E^(2*a)*p*x^2*(AppellF1[3/2, 1 - p, p, 5/2, E^(2*a)*x^2, -(E^
(2*a)*x^2)] + AppellF1[3/2, -p, 1 + p, 5/2, E^(2*a)*x^2, -(E^(2*a)*x^2)]))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int (\tanh(a+\ln(x)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+ln(x))^p,x)
```

```
[Out] int(tanh(a+ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh(a + \log(x))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tanh(a + log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\tanh(a + \log(x))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(a + log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^p(a + \log(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+ln(x))**p,x)
```

```
[Out] Integral(tanh(a + log(x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh(a + \log(x))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(tanh(a + log(x))^p, x)
```

3.170 $\int \tanh^p(a + 2\log(x))\, dx$

Optimal. Leaf size=61

$$x\left(1-e^{2a}x^4\right)^{-p}\left(e^{2a}x^4-1\right)^p F_1\left(\frac{1}{4};-p,p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)$$

```
[Out] (x*(-1 + E^(2*a)*x^4)^p*AppellF1[1/4, -p, p, 5/4, E^(2*a)*x^4, -(E^(2*a)*x^
4)])/(1 - E^(2*a)*x^4)^p
```

Rubi [F] time = 0.0108366, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^p(a+2\log(x))\, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + 2*Log[x]]^p,x]
```

```
[Out] Defer[Int][Tanh[a + 2*Log[x]]^p, x]
```

Rubi steps

$$\int \tanh^p(a+2\log(x))\, dx = \int \tanh^p(a+2\log(x))\, dx$$

Mathematica [B] time = 1.77489, size = 171, normalized size = 2.8

$$\frac{5x\left(\frac{e^{2a}x^4-1}{e^{2a}x^4+1}\right)^p F_1\left(\frac{1}{4};-p,p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)}{5F_1\left(\frac{1}{4};-p,p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)-4e^{2a}px^4\left(F_1\left(\frac{5}{4};1-p,p;\frac{9}{4};e^{2a}x^4,-e^{2a}x^4\right)+F_1\left(\frac{5}{4};-p,p+1;\frac{9}{4};e^{2a}x^4,-e^{2a}x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[a + 2*Log[x]]^p,x]
```

```
[Out] (5*x*((-1 + E^(2*a)*x^4)/(1 + E^(2*a)*x^4))^p*AppellF1[1/4, -p, p, 5/4, E^(
2*a)*x^4, -(E^(2*a)*x^4)])/(5*AppellF1[1/4, -p, p, 5/4, E^(2*a)*x^4, -(E^(2
*a)*x^4)] - 4*E^(2*a)*p*x^4*(AppellF1[5/4, 1 - p, p, 9/4, E^(2*a)*x^4, -(E^
(2*a)*x^4)] + AppellF1[5/4, -p, 1 + p, 9/4, E^(2*a)*x^4, -(E^(2*a)*x^4)]))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (\tanh(a+2\ln(x)))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+2*ln(x))^p,x)
```

```
[Out] int(tanh(a+2*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a+2\,\log\left(x\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tanh(a + 2*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\tanh\left(a+2\,\log\left(x\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(a + 2*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^{p}\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*ln(x))**p,x)
```

```
[Out] Integral(tanh(a + 2*log(x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(a+2\,\log\left(x\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+2*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(tanh(a + 2*log(x))^p, x)
```

3.171 $\int \tanh^p(a+3\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(1-e^{2a}x^6\right)^{-p}\left(e^{2a}x^6-1\right)^p F_1\left(\frac{1}{6};-p,p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)$$

```
[Out] (x*(-1 + E^(2*a)*x^6)^p*AppellF1[1/6, -p, p, 7/6, E^(2*a)*x^6, -(E^(2*a)*x^
6)])/(1 - E^(2*a)*x^6)^p
```

Rubi [F] time = 0.0182731, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^p(a+3\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + 3*Log[x]]^p,x]
```

```
[Out] Defer[Int][Tanh[a + 3*Log[x]]^p, x]
```

Rubi steps

$$\int \tanh^p(a+3\log(x))\,dx=\int \tanh^p(a+3\log(x))\,dx$$

Mathematica [B] time = 1.78222, size = 171, normalized size = 2.8

$$\frac{7x\left(\frac{e^{2a}x^6-1}{e^{2a}x^6+1}\right)^p F_1\left(\frac{1}{6};-p,p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)}{7F_1\left(\frac{1}{6};-p,p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)-6e^{2a}px^6\left(F_1\left(\frac{7}{6};1-p,p;\frac{13}{6};e^{2a}x^6,-e^{2a}x^6\right)+F_1\left(\frac{7}{6};-p,p+1;\frac{13}{6};e^{2a}x^6,-e^{2a}x^6\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[a + 3*Log[x]]^p,x]
```

```
[Out] (7*x*((-1 + E^(2*a)*x^6)/(1 + E^(2*a)*x^6))^p*AppellF1[1/6, -p, p, 7/6, E^(
2*a)*x^6, -(E^(2*a)*x^6)])/(7*AppellF1[1/6, -p, p, 7/6, E^(2*a)*x^6, -(E^(2
*a)*x^6)] - 6*E^(2*a)*p*x^6*(AppellF1[7/6, 1 - p, p, 13/6, E^(2*a)*x^6, -(E
^(2*a)*x^6)] + AppellF1[7/6, -p, 1 + p, 13/6, E^(2*a)*x^6, -(E^(2*a)*x^6)])
)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int (\tanh(a+3\ln(x)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+3*ln(x))^p,x)
```

```
[Out] int(tanh(a+3*ln(x))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh \left(a+3 \log \left(x\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+3*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(tanh(a + 3*log(x))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\tanh \left(a+3 \log \left(x\right)\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+3*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(a + 3*log(x))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^{p}{\left(a+3\log{\left(x \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+3*ln(x))**p,x)
```

```
[Out] Integral(tanh(a + 3*log(x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh \left(a+3 \log \left(x\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+3*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(tanh(a + 3*log(x))^p, x)
```

3.172 $\int x^3 \tanh(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=59

$$\frac{x^4}{4}-\frac{1}{2}x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};-e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] x^4/4 - (x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), -(E^(2*a*d)*(c*
x^n)^(2*b*d))])/2
```

Rubi [F] time = 0.0415971, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3\tanh(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][x^3*Tanh[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int x^3\tanh(d(a+b\log(cx^n)))\,dx=\int x^3\tanh(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] time = 7.6711, size = 127, normalized size = 2.15

$$\frac{x^4\left(2e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{2}{bdn};2+\frac{2}{bdn};-e^{2d(a+b\log(cx^n))}\right)-(bdn+2)\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};-e^{2d(a+b\log(cx^n))}\right)\right)}{4bdn+8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^4*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 +
2/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - (2 + b*d*n)*Hypergeometric2F1[1,
2/(b*d*n), 1 + 2/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]))/(8 + 4*b*d*n)
```

Maple [F] time = 0.894, size = 0, normalized size = 0.

$$\int x^3\tanh(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tanh(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^3*tanh(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^4 - 2\int \frac{x^3}{c^{2\,bd}e^{(2\,bd\log(x^n)+2\,ad)} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 2*integrate(x^3/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*d) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\tanh\left(bd\log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^3*tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tanh(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\tanh\left((b\log\left(cx^n\right) + a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^3*tanh((b*log(c*x^n) + a)*d), x)
```

3.173 $\int x^2 \tanh(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=63

$$\frac{x^3}{3}-\frac{2}{3}x^3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};-e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] x^3/3 - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), -(E^(2*a*
d)*(c*x^n)^(2*b*d))])/3
```

Rubi [F] time = 0.0296561, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\tanh(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Tanh[d*(a + b*Log[c*x^n])],x]

[Out] Defer[Int][x^2*Tanh[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int x^2\tanh(d(a+b\log(cx^n)))\,dx=\int x^2\tanh(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] time = 7.56023, size = 136, normalized size = 2.16

$$\frac{x^3\left(3e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{3}{2bdn};2+\frac{3}{2bdn};-e^{2d(a+b\log(cx^n))}\right)-(2bdn+3)\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};-e^{2d(a+b\log(cx^n))}\right)\right)}{6bdn+9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tanh[d*(a + b*Log[c*x^n])],x]

[Out] (x^3*(3*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n), 2
+ 3/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - (3 + 2*b*d*n)*Hypergeometric2
F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]))/(9 + 6*b
*d*n)
```

Maple [F] time = 0.784, size = 0, normalized size = 0.

$$\int x^2\tanh(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tanh(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*tanh(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3 - 2\int \frac{x^2}{c^{2\,bd}e^{(2\,bd\log(x^n)+2\,ad)} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 2*integrate(x^2/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*d) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2\tanh\left(bd\log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\tanh\left(ad + bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tanh(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*tanh(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\tanh\left(\left(b\log\left(cx^n\right) + a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*tanh((b*log(c*x^n) + a)*d), x)
```

3.174 $\int x \tanh(d(a + b\log(cx^n)))\,dx$

Optimal. Leaf size=55

$$\frac{x^2}{2} - x^2\,{}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; -e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] x^2/2 - x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), -(E^(2*a*d)*(c*x
^n)^(2*b*d))]
```

Rubi [F] time = 0.0240282, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \tanh(d(a + b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][x*Tanh[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int x \tanh(d(a + b\log(cx^n)))\,dx = \int x \tanh(d(a + b\log(cx^n)))\,dx$$

Mathematica [B] time = 7.58517, size = 122, normalized size = 2.22

$$\frac{x^2\left(e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1, 1 + \frac{1}{bdn}; 2 + \frac{1}{bdn}; -e^{2d(a+b\log(cx^n))}\right) - (bdn + 1)\,{}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; -e^{2d(a+b\log(cx^n))}\right)\right)}{2bdn + 2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 + 1/
(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - (1 + b*d*n)*Hypergeometric2F1[1, 1/
(b*d*n), 1 + 1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]))/(2 + 2*b*d*n)
```

Maple [F] time = 0.783, size = 0, normalized size = 0.

$$\int x \tanh(d(a + b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tanh(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*tanh(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2 - 2\int \frac{x}{c^{2\,bd}e^{(2\,bd\log(x^n)+2\,ad)} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*integrate(x/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*d) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\tanh\left(bd\log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x*tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\tanh\left(ad + bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*tanh(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\tanh\left((b\log\left(cx^n\right) + a)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x*tanh((b*log(c*x^n) + a)*d), x)
```

3.175 $\int \tanh(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=53

$$x-2x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};-e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] x - 2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), -(E^(2*a*d)*(c*x
^n)^(2*b*d))]
```

Rubi [F] time = 0.0112735, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][Tanh[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int \tanh(d(a+b\log(cx^n)))\,dx=\int \tanh(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] time = 8.55814, size = 126, normalized size = 2.38

$$\frac{xe^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{1}{2bdn};2+\frac{1}{2bdn};-e^{2d(a+b\log(cx^n))}\right)}{2bdn+1}-x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};-e^{2d(a+b\log(cx^n))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (E^(2*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 + 1/(
2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))])/(1 + 2*b*d*n) - x*Hypergeometric2F1
[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]
```

Maple [F] time = 0.717, size = 0, normalized size = 0.

$$\int \tanh(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x - 2\int \frac{1}{c^{2\,bd}e^{\left(2\,bd\log\left(x^{n}\right)+2\,ad\right)} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] x - 2*integrate(1/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*d) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\tanh\left(bd\log\left(cx^{n}\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(d\left(a + b\log\left(cx^{n}\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(tanh(d*(a + b*log(c*x**n))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left(\left(b\log\left(cx^{n}\right) + a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(tanh((b*log(c*x^n) + a)*d), x)
```

3.176 $\int \frac{\tanh(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=25

$$\frac{\log\left(\cosh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

[Out] Log[Cosh[a*d + b*d*Log[c*x^n]]]/(b*d*n)

Rubi [A] time = 0.0204699, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3475}

$$\frac{\log\left(\cosh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

[In] Int[Tanh[d*(a + b*Log[c*x^n])]/x,x]

[Out] Log[Cosh[a*d + b*d*Log[c*x^n]]]/(b*d*n)

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \tanh(d(a+bx))\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\log\left(\cosh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}\end{aligned}$$

Mathematica [A] time = 0.0460165, size = 24, normalized size = 0.96

$$\frac{\log\left(\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]/x,x]

[Out] Log[Cosh[d*(a + b*Log[c*x^n])]]/(b*d*n)

Maple [B] time = 0.004, size = 56, normalized size = 2.2

$$-\frac{\ln\left(\tanh\left(d\left(a+b\ln\left(cx^n\right)\right)\right)-1\right)}{2\,dbn}-\frac{\ln\left(\tanh\left(d\left(a+b\ln\left(cx^n\right)\right)\right)+1\right)}{2\,dbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] -1/2/b/d/n*ln(tanh(d*(a+b*ln(c*x^n)))-1)-1/2/b/d/n*ln(tanh(d*(a+b*ln(c*x^n)
))+1)
```

Maxima [A] time = 1.08618, size = 32, normalized size = 1.28

$$\frac{\log\left(\cosh\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] log(cosh((b*log(c*x^n) + a)*d))/(b*d*n)
```

Fricas [B] time = 2.14591, size = 205, normalized size = 8.2

$$-\frac{bdn\log\left(x\right)-\log\left(\frac{2\cosh(bdn\log(x)+bd\log(c)+ad)}{\cosh(bdn\log(x)+bd\log(c)+ad)-\sinh(bdn\log(x)+bd\log(c)+ad)}\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] -(b*d*n*log(x) - log(2*cosh(b*d*n*log(x) + b*d*log(c) + a*d)/(cosh(b*d*n*lo
g(x) + b*d*log(c) + a*d) - sinh(b*d*n*log(x) + b*d*log(c) + a*d))))/(b*d*n)
```

Sympy [A] time = 5.89987, size = 36, normalized size = 1.44

$$-\frac{\log\left(bdn\tanh^{2}\left(ad+bd\log\left(cx^{n}\right)\right)-bdn\right)}{2bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] -log(b*d*n*tanh(a*d + b*d*log(c*x**n))**2 - b*d*n)/(2*b*d*n)
```

Giac [B] time = 1.37024, size = 99, normalized size = 3.96

$$\frac{\log\left(2\,x^{2\,bdn}|c|^{2\,bd}\cos\left(\pi bd\mathrm{sgn}\left(c\right)-\pi bd\right)e^{(2\,ad)}+x^{4\,bdn}|c|^{4\,bd}e^{(4\,ad)}+1\right)}{2\,bdn}-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] 1/2*log(2*x^(2*b*d*n)*abs(c)^(2*b*d)*cos(pi*b*d*sgn(c) - pi*b*d)*e^(2*a*d)
+ x^(4*b*d*n)*abs(c)^(4*b*d)*e^(4*a*d) + 1)/(b*d*n) - log(x)
```

3.177 $\int \frac{\tanh(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=59

$$\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};-e^{2ad}\left(cx^n\right)^{2bd}\right)}{x}-\frac{1}{x}$$

```
[Out] -x^(-1) + (2*Hypergeometric2F1[1, -1/(2*b*d*n), 1 - 1/(2*b*d*n), -(E^(2*a*d
)*(c*x^n)^(2*b*d))])/x
```

Rubi [F] time = 0.0294655, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] Defer[Int][Tanh[d*(a + b*Log[c*x^n])]/x^2, x]
```

Rubi steps

$$\int \frac{\tanh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx=\int \frac{\tanh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Mathematica [B] time = 3.21226, size = 126, normalized size = 2.14

$$\frac{\frac{e^{2d\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(1,1-\frac{1}{2bdn};2-\frac{1}{2bdn};-e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)}{2bdn-1}+{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};-e^{2d(a+b\log(cx^n))}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2
*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))])/(-1 + 2*b*d*n) + Hypergeometric2F1[1
, -1/(2*b*d*n), 1 - 1/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))])/x
```

Maple [F] time = 0.796, size = 0, normalized size = 0.

$$\int \frac{\tanh\left(d\left(a+b\ln\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x} - 2\int \frac{1}{c^{2\,bd}x^2e^{(2\,bd\log(x^n)+2\,ad)} + x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] -1/x - 2*integrate(1/(c^(2*b*d)*x^2*e^(2*b*d*log(x^n) + 2*a*d) + x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh\left(bd\log\left(cx^n\right) + ad\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(tanh(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(ad + bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(tanh(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(\left(b\log\left(cx^n\right) + a\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(tanh((b*log(c*x^n) + a)*d)/x^2, x)
```

3.178 $\int \frac{\tanh(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=56

$$\frac{{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};-e^{2ad}\left(cx^n\right)^{2bd}\right)}{x^2}-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) + Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), -(E^(2*a*d)*
(c*x^n)^(2*b*d))]/x^2
```

Rubi [F] time = 0.0286536, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] Defer[Int][Tanh[d*(a + b*Log[c*x^n])]/x^3, x]
```

Rubi steps

$$\int \frac{\tanh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx=\int \frac{\tanh\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Mathematica [B] time = 3.12066, size = 120, normalized size = 2.14

$$\frac{\frac{e^{2d\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(1,1-\frac{1}{bdn};2-\frac{1}{bdn};-e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)}{bdn-1}+{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};-e^{2d(a+b\log(cx^n))}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*d
*n), -E^(2*d*(a + b*Log[c*x^n]))])/(-1 + b*d*n) + Hypergeometric2F1[1, -(1/
(b*d*n)), 1 - 1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))])/(2*x^2)
```

Maple [F] time = 0.818, size = 0, normalized size = 0.

$$\int \frac{\tanh\left(d\left(a+b\ln\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2\,x^2} - 2\int \frac{1}{c^{2\,bd}x^3e^{(2\,bd\log(x^n)+2\,ad)} + x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - 2*integrate(1/(c^(2*b*d)*x^3*e^(2*b*d*log(x^n) + 2*a*d) + x^3),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh\left(bd\log\left(cx^n\right) + ad\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(tanh(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(ad + bd\log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(tanh(a*d + b*d*log(c*x**n))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(\left(b\log\left(cx^n\right) + a\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(tanh((b*log(c*x^n) + a)*d)/x^3, x)
```

3.179 $\int x^3 \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=133

$$-\frac{2x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};-e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x^4\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(e^{2ad}(cx^n)^{2bd}+1\right)}+\frac{1}{4}x^4\left(\frac{4}{bdn}+1\right)$$

```
[Out] ((1 + 4/(b*d*n))*x^4)/4 + (x^4*(1 - E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 +
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2
/(b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n)
```

Rubi [F] time = 0.0857137, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3 \tanh^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][x^3*Tanh[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int x^3 \tanh^2(d(a+b\log(cx^n)))\,dx=\int x^3 \tanh^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 8.16189, size = 159, normalized size = 1.2

$$\frac{x^4\left(8e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{2}{bdn};2+\frac{2}{bdn};-e^{2d(a+b\log(cx^n))}\right)+(bdn+2)\left(-4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};-e^{2d(a+b\log(cx^n))}\right)-4\tanh\right.\right.}{4bdn(bdn+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^4*(8*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 +
2/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(b*d*n - 4*Hypergeome
tric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - 4*Tanh[
d*(a + b*Log[c*x^n])])))/(4*b*d*n*(2 + b*d*n))
```

Maple [F] time = 0.809, size = 0, normalized size = 0.

$$\int x^3 (\tanh(d(a+b\ln(cx^n))))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^3*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2\,bd}dnx^4e^{(2\,bd\log(x^n)+2\,ad)}+(bdn+8)x^4}{4\left(bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}+bdn\right)}-8\int\frac{x^3}{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}+bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*c^(2*b*d)*d*n*x^4*e^(2*b*d*log(x^n) + 2*a*d) + (b*d*n + 8)*x^4)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) - 8*integrate(x^3/(b*c^(2*
b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\tanh\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tanh(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\tanh\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*tanh((b*log(c*x^n) + a)*d)^2, x)
```

3.180 $\int x^2 \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=137

$$-\frac{2x^3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};-e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x^3\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(e^{2ad}(cx^n)^{2bd}+1\right)}+\frac{1}{3}x^3\left(\frac{3}{bdn}+1\right)$$

```
[Out] ((1 + 3/(b*d*n))*x^3)/3 + (x^3*(1 - E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 +
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 +
 3/(2*b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n)
```

Rubi [F] time = 0.0603137, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\tanh^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][x^2*Tanh[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int x^2\tanh^2(d(a+b\log(cx^n)))\,dx=\int x^2\tanh^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 7.78012, size = 169, normalized size = 1.23

$$\frac{x^3\left(9e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{3}{2bdn};2+\frac{3}{2bdn};-e^{2d(a+b\log(cx^n))}\right)+(2bdn+3)\left(-3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};-e^{2d(a+b\log(cx^n))}\right)-3\right.\right.}{3bdn(2bdn+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^3*(9*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n), 2
+ 3/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(b*d*n - 3*Hype
rgeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]
 - 3*Tanh[d*(a + b*Log[c*x^n])])))/(3*b*d*n*(3 + 2*b*d*n))
```

Maple [F] time = 0.317, size = 0, normalized size = 0.

$$\int x^2\left(\tanh(d(a+b\ln(cx^n)))\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^2*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2\,bd}dnx^3e^{(2\,bd\log(x^n)+2\,ad)} + (bdn+6)x^3}{3\left(bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)} + bdn\right)} - 6\int \frac{x^2}{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)} + bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c^(2*b*d)*d*n*x^3*e^(2*b*d*log(x^n) + 2*a*d) + (b*d*n + 6)*x^3)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) - 6*integrate(x^2/(b*c^(2*
b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\tanh\left(bd\log\left(cx^n\right) + ad\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\tanh^2\left(ad + bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*tanh(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x**2*tanh(a*d + b*d*log(c*x**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\tanh\left(\left(b\log\left(cx^n\right) + a\right)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*tanh((b*log(c*x^n) + a)*d)^2, x)
```

3.181 $\int x \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=131

$$-\frac{2x^2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};-e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdn}+\frac{x^2\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdn\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}+\frac{1}{2}x^2\left(\frac{2}{bdn}+1\right)$$

```
[Out] ((1 + 2/(b*d*n))*x^2)/2 + (x^2*(1 - E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 +
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1
/(b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n)
```

Rubi [F] time = 0.0427686, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\tanh^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][x*Tanh[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int x\tanh^2(d(a+b\log(cx^n)))\,dx=\int x\tanh^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 7.73318, size = 155, normalized size = 1.18

$$\frac{x^2\left(2e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{1}{bdn};2+\frac{1}{bdn};-e^{2d(a+b\log(cx^n))}\right)+(bdn+1)\left(-2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};-e^{2d(a+b\log(cx^n))}\right)-2\tanh\right.\right.}{2bdn(bdn+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^2*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(b*d*n - 2*Hypergeome
tric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - 2*Tanh[
d*(a + b*Log[c*x^n])])))/(2*b*d*n*(1 + b*d*n))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x\left(\tanh(d(a+b\ln(cx^n)))\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2\,bd}dnx^2e^{(2\,bd\log(x^n)+2\,ad)}+(bdn+4)x^2}{2\left(bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}+bdn\right)}-4\int\frac{x}{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}+bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c^(2*b*d)*d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) + (b*d*n + 4)*x^2)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) - 4*integrate(x/(b*c^(2*b*
d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\tanh\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x*tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\tanh^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x*tanh(a*d + b*d*log(c*x**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\tanh\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x*tanh((b*log(c*x^n) + a)*d)^2, x)
```

3.182 $\int \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=127

$$-\frac{2x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};-e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(e^{2ad}(cx^n)^{2bd}+1\right)}+x\left(\frac{1}{bdn}+1\right)$$

```
[Out] (1 + 1/(b*d*n))*x + (x*(1 - E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 + E^(2*a*
d)*(c*x^n)^(2*b*d))) - (2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*
n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n)
```

Rubi [F] time = 0.0138581, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][Tanh[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int \tanh^2(d(a+b\log(cx^n)))\,dx=\int \tanh^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 8.8066, size = 163, normalized size = 1.28

$$\frac{x\left(e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{1}{2bdn};2+\frac{1}{2bdn};-e^{2d(a+b\log(cx^n))}\right)+(2bdn+1)\left(-{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};-e^{2d(a+b\log(cx^n))}\right)-\tanh\right.\right.}{bdn(2bdn+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x*(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 + 1/
(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (1 + 2*b*d*n)*(b*d*n - Hypergeome
tric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - Tan
h[d*(a + b*Log[c*x^n])])))/(b*d*n*(1 + 2*b*d*n))
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (\tanh(d(a+b\ln(cx^n))))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bc^{2\,bd}dnxe^{\left(2\,bd\log\left(x^{n}\right)+2\,ad\right)}+(bdn+2)x}{bc^{2\,bd}dne^{\left(2\,bd\log\left(x^{n}\right)+2\,ad\right)}+bdn}-2\int\frac{1}{bc^{2\,bd}dne^{\left(2\,bd\log\left(x^{n}\right)+2\,ad\right)}+bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] (b*c^(2*b*d)*d*n*x*e^(2*b*d*log(x^n) + 2*a*d) + (b*d*n + 2)*x)/(b*c^(2*b*d)
*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) - 2*integrate(1/(b*c^(2*b*d)*d*n*e
^(2*b*d*log(x^n) + 2*a*d) + b*d*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\tanh\left(bd\log\left(cx^{n}\right)+ad\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\tanh^{2}\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(tanh(d*(a + b*log(c*x**n)))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\tanh\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(tanh((b*log(c*x^n) + a)*d)^2, x)
```

3.183 $\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=28

$$\log(x)-\frac{\tanh(ad+bd\log(cx^n))}{bdn}$$

```
[Out] Log[x] - Tanh[a*d + b*d*Log[c*x^n]]/(b*d*n)
```

Rubi [A] time = 0.0290263, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3473, 8}

$$\log(x)-\frac{\tanh(ad+bd\log(cx^n))}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] Log[x] - Tanh[a*d + b*d*Log[c*x^n]]/(b*d*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \tanh^2(d(a+bx))\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\tanh(ad+bd\log(cx^n))}{bdn}+\frac{\operatorname{Subst}\left(\int 1\,dx,x,\log(cx^n)\right)}{n}\\ &= \log(x)-\frac{\tanh(ad+bd\log(cx^n))}{bdn}\end{aligned}$$

Mathematica [A] time = 0.0777238, size = 51, normalized size = 1.82

$$\frac{\tanh^{-1}(\tanh(ad+bd\log(cx^n)))}{bdn}-\frac{\tanh(ad+bd\log(cx^n))}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] ArcTanh[Tanh[a*d + b*d*Log[c*x^n]]]/(b*d*n) - Tanh[a*d + b*d*Log[c*x^n]]/(b
*d*n)
```

Maple [B] time = 0.006, size = 80, normalized size = 2.9

$$-\frac{\tanh\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)}{dbn}-\frac{\ln\left(\tanh\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)-1\right)}{2\,dbn}+\frac{\ln\left(\tanh\left(d\left(a+b\ln\left(cx^{n}\right)\right)\right)+1\right)}{2\,dbn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^2/x,x)
```

```
[Out] -1/b/d/n*tanh(d*(a+b*ln(c*x^n)))-1/2/b/d/n*ln(tanh(d*(a+b*ln(c*x^n)))-1)+1/
2/b/d/n*ln(tanh(d*(a+b*ln(c*x^n)))+1)
```

Maxima [A] time = 1.39626, size = 49, normalized size = 1.75

$$\frac{2}{bc^{2\,bd}dne^{\left(2\,bd\log\left(x^{n}\right)+2\,ad\right)}+bdn}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

```
[Out] 2/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) + log(x)
```

Fricas [B] time = 2.14826, size = 197, normalized size = 7.04

$$\frac{\left(bdn\log\left(x\right)+1\right)\cosh\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)-\sinh\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)}{bdn\cosh\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

```
[Out] ((b*d*n*log(x) + 1)*cosh(b*d*n*log(x) + b*d*log(c) + a*d) - sinh(b*d*n*log(
x) + b*d*log(c) + a*d))/(b*d*n*cosh(b*d*n*log(x) + b*d*log(c) + a*d))
```

Sympy [B] time = 14.9082, size = 70, normalized size = 2.5

$$-\frac{\log\left(\tanh\left(ad+bd\log\left(cx^{n}\right)\right)-1\right)}{2bdn}+\frac{\log\left(\tanh\left(ad+bd\log\left(cx^{n}\right)\right)+1\right)}{2bdn}-\frac{\tanh\left(ad+bd\log\left(cx^{n}\right)\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))**2/x,x)
```

```
[Out] -log(tanh(a*d + b*d*log(c*x**n)) - 1)/(2*b*d*n) + log(tanh(a*d + b*d*log(c*
x**n)) + 1)/(2*b*d*n) - tanh(a*d + b*d*log(c*x**n))/(b*d*n)
```

Giac [A] time = 1.51496, size = 50, normalized size = 1.79

$$\frac{2}{\left(c^{2\,bd}x^{2\,bdn}e^{(2\,ad)}+1\right)bdn}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

```
[Out] 2/((c^(2*b*d)*x^(2*b*d*n)*e^(2*a*d) + 1)*b*d*n) + log(x)
```

3.184 $\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=135

$$-\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};-e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdnx}+\frac{1-e^{2ad}\left(cx^n\right)^{2bd}}{bdnx\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}-\frac{1-\frac{1}{bdn}}{x}$$

```
[Out] -((1 - 1/(b*d*n))/x) + (1 - E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x*(1 + E^(2*a
*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -1/(2*b*d*n), 1 - 1/(2*b*d*
n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n*x)
```

Rubi [F] time = 0.0523026, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] Defer[Int][Tanh[d*(a + b*Log[c*x^n])]^2/x^2, x]
```

Rubi steps

$$\int \frac{\tanh^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx=\int \frac{\tanh^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Mathematica [A] time = 3.41635, size = 162, normalized size = 1.2

$$-\frac{e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{2bdn};2-\frac{1}{2bdn};-e^{2d(a+b\log(cx^n))}\right)+(2bdn-1)\left({}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};-e^{2d(a+b\log(cx^n))}\right)+\tan\right.}{bdnx(2bdn-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] -((E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(
2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (-1 + 2*b*d*n)*(b*d*n + Hypergeome
tric2F1[1, -1/(2*b*d*n), 1 - 1/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + Ta
nh[d*(a + b*Log[c*x^n])]))/(b*d*n*(-1 + 2*b*d*n)*x))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{\left(\tanh\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}+bdn-2}{bc^{2\,bd}dnxe^{(2\,bd\log(x^n)+2\,ad)}+bdnx}+2\int\frac{1}{bc^{2\,bd}dnx^2e^{(2\,bd\log(x^n)+2\,ad)}+bdnx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n - 2)/(b*c^(2*b*d)*d*n*
x*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n*x) + 2*integrate(1/(b*c^(2*b*d)*d*n*x^
2*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\tanh\left(bd\log\left(cx^n\right)+ad\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(tanh(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh^2\left(ad+bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

```
[Out] Integral(tanh(a*d + b*d*log(c*x**n))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(tanh((b*log(c*x^n) + a)*d)^2/x^2, x)
```

3.185 $\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=136

$$-\frac{2\,{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};-e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdnx^2}+\frac{1-e^{2ad}\left(cx^n\right)^{2bd}}{bdnx^2\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}+\frac{2-bdn}{2bdnx^2}$$

```
[Out] (2 - b*d*n)/(2*b*d*n*x^2) + (1 - E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x^2*(1 +
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/
(b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n*x^2)
```

Rubi [F] time = 0.0519386, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] Defer[Int][Tanh[d*(a + b*Log[c*x^n])]^2/x^3, x]
```

Rubi steps

$$\int \frac{\tanh^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx=\int \frac{\tanh^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Mathematica [A] time = 3.37692, size = 159, normalized size = 1.17

$$-\frac{2e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{bdn};2-\frac{1}{bdn};-e^{2d(a+b\log(cx^n))}\right)+(bdn-1)\left(2\,{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};-e^{2d(a+b\log(cx^n))}\right)+2\tanh\right.}{2bdnx^2(bdn-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] -(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b
*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (-1 + b*d*n)*(b*d*n + 2*Hypergeometri
c2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + 2*Tanh[
d*(a + b*Log[c*x^n])]))/(2*b*d*n*(-1 + b*d*n)*x^2)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{\left(\tanh\left(d\left(a+b\ln\left(cx^n\right)\right)\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)} + bdn - 4}{2\left(bc^{2\,bd}dnx^2e^{(2\,bd\log(x^n)+2\,ad)} + bdnx^2\right)} + 4\int \frac{1}{bc^{2\,bd}dnx^3e^{(2\,bd\log(x^n)+2\,ad)} + bdnx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n - 4)/(b*c^(2*b*d)*
d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n*x^2) + 4*integrate(1/(b*c^(2*b*d
)*d*n*x^3*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh\left(bd\log\left(cx^n\right) + ad\right)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(tanh(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2\left(ad + bd\log\left(cx^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

```
[Out] Integral(tanh(a*d + b*d*log(c*x**n))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(\left(b\log\left(cx^n\right) + a\right)d\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(tanh((b*log(c*x^n) + a)*d)^2/x^3, x)
```

3.186 $\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=43

$$\frac{\log(\cosh(a+b\log(cx^n)))}{bn}-\frac{\tanh^2(a+b\log(cx^n))}{2bn}$$

```
[Out] Log[Cosh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]^2/(2*b*n)
```

Rubi [A] time = 0.0406062, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3473, 3475}

$$\frac{\log(\cosh(a+b\log(cx^n)))}{bn}-\frac{\tanh^2(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Log[Cosh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]^2/(2*b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \tanh^3(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\tanh^2(a+b\log(cx^n))}{2bn}+\frac{\text{Subst}\left(\int \tanh(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{\log(\cosh(a+b\log(cx^n)))}{bn}-\frac{\tanh^2(a+b\log(cx^n))}{2bn}\end{aligned}$$

Mathematica [A] time = 0.133635, size = 43, normalized size = 1.

$$\frac{\log(\cosh(a+b\log(cx^n)))}{bn}-\frac{\tanh^2(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Log[Cosh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]^2/(2*b*n)
```

Maple [A] time = 0.007, size = 67, normalized size = 1.6

$$-\frac{(\tanh(a+b\ln(cx^n)))^2}{2bn}-\frac{\ln(\tanh(a+b\ln(cx^n))-1)}{2bn}-\frac{\ln(\tanh(a+b\ln(cx^n))+1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] -1/2*tanh(a+b*ln(c*x^n))^2/b/n-1/2/n/b*ln(tanh(a+b*ln(c*x^n))-1)-1/2/n/b*ln
(tanh(a+b*ln(c*x^n))+1)
```

Maxima [B] time = 1.50507, size = 410, normalized size = 9.53

$$\frac{4\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+3}{4\left(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)}-\frac{2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+3}{4\left(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)}+\frac{}{4\left(bc^{4\,b}ne^{(4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/4*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 3)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4
*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 1/4*(2*c^(2*b)*e^(2*b*l
og(x^n) + 2*a) + 3)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(
2*b*log(x^n) + 2*a) + b*n) + 3/4*(2*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*
c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b
*n) - 3/4/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^
n) + 2*a) + b*n) + log((c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)*e^(-2*a)/c^(2*b
))/(b*n) - log(x)
```

Fricas [B] time = 2.05109, size = 1841, normalized size = 42.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) + 4*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^4 + 2*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^2 + b*n*log(x) + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) + b*n
*log(x) - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c
) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) +
 a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
 + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*cosh(b*n*log(x) + b*log(c
) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)))
 + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3*log(x) + (b*n*log(x) - 1)*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*
```

```
log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*l
og(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 + 2*b*n*cos
h(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) +
b*log(c) + a)^3 + b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*
log(c) + a))
```

Sympy [A] time = 5.77996, size = 75, normalized size = 1.74

$$\begin{cases} \log(x)\tanh^3(a) & \text{for } b=0 \wedge n=0 \\ \log(x)\tanh^3(a+b\log(c)) & \text{for } n=0 \\ \log(x)\tanh^3(a) & \text{for } b=0 \\ \log(x) - \frac{\log(\tanh(a+bn\log(x)+b\log(c))+1)}{bn} - \frac{\tanh^2(a+bn\log(x)+b\log(c))}{2bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*tanh(a)**3, Eq(b, 0) & Eq(n, 0)), (log(x)*tanh(a + b*log(
c))**3, Eq(n, 0)), (log(x)*tanh(a)**3, Eq(b, 0)), (log(x) - log(tanh(a + b*
n*log(x) + b*log(c)) + 1)/(b*n) - tanh(a + b*n*log(x) + b*log(c))**2/(2*b*n
), True))
```

Giac [B] time = 1.38078, size = 170, normalized size = 3.95

$$\frac{\log\left(2\,x^{2\,bn}|c|^{2\,b}\cos\left(\pi b\operatorname{sgn}(c) - \pi b\right)e^{(2\,a)} + x^{4\,bn}|c|^{4\,b}e^{(4\,a)} + 1\right)}{2\,bn} - \frac{3\,c^{4\,b}x^{4\,bn}e^{(4\,a)} + 2\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 3}{2\left(c^{2\,b}x^{2\,bn}e^{(2\,a)} + 1\right)^2 bn} - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/2*log(2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b*n
)*abs(c)^(4*b)*e^(4*a) + 1)/(b*n) - 1/2*(3*c^(4*b)*x^(4*b*n)*e^(4*a) + 2*c^
(2*b)*x^(2*b*n)*e^(2*a) + 3)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^2*b*n) - log(
x)
```

3.187 $\int \frac{\tanh^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}+\log(x)$$

```
[Out] Log[x] - Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rubi [A] time = 0.0394355, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3473, 8}

$$-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Log[x] - Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \tanh^4(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\text{Subst}\left(\int \tanh^2(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\text{Subst}\left(\int 1\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \log(x)-\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}\end{aligned}$$

Mathematica [A] time = 0.102834, size = 62, normalized size = 1.38

$$-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\tanh^{-1}\left(\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] ArcTanh[Tanh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh
[a + b*Log[c*x^n]]^3/(3*b*n)
```

Maple [A] time = 0.004, size = 86, normalized size = 1.9

$$-\frac{(\tanh(a+b\ln(cx^n)))^3}{3bn}-\frac{\tanh(a+b\ln(cx^n))}{bn}-\frac{\ln(\tanh(a+b\ln(cx^n))-1)}{2bn}+\frac{\ln(\tanh(a+b\ln(cx^n))+1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] -1/3*tanh(a+b*ln(c*x^n))^3/b/n-tanh(a+b*ln(c*x^n))/b/n-1/2/n/b*ln(tanh(a+b*
ln(c*x^n))-1)+1/2/n/b*ln(tanh(a+b*ln(c*x^n))+1)
```

Maxima [B] time = 1.53108, size = 667, normalized size = 14.82

$$\frac{18\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)}+27\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+11}{12\left(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}+3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)}+\frac{6\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)}+1}{12\left(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}+3\,bc^{4\,b}ne^{(4\,b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/12*(18*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 27*c^(2*b)*e^(2*b*log(x^n) + 2*a)
 + 11)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n)
+ 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/12*(6*c^(4*b)*e^(4
*b*log(x^n) + 4*a) + 15*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 11)/(b*c^(6*b)*n*e
^(6*b*log(x^n) + 6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*
n*e^(2*b*log(x^n) + 2*a) + b*n) + 2/3*(3*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 3
*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) +
3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a)
 + b*n) - 1/2*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*lo
g(x^n) + 6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b
*log(x^n) + 2*a) + b*n) + 2/3/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 3*b*c^(
4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
 + log(x)
```

Fricas [B] time = 2.00437, size = 630, normalized size = 14.

$$\frac{(3\,bn\log(x)+4)\cosh(bn\log(x)+b\log(c)+a)^3+3\,(3\,bn\log(x)+4)\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(}{3\left(bn\cosh(bn\log(x)+b\log(c)+a)^3+3\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*n*log(x) + 4)*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(3*b*n*log(x)
 + 4)*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - 1
2*cosh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) - 4*sin
h(b*n*log(x) + b*log(c) + a)^3 + 3*(3*b*n*log(x) + 4)*cosh(b*n*log(x) + b*l
```

```
og(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n*cosh(b*n*log(x) +
 b*log(c) + a))
```

Sympy [A] time = 28.8841, size = 71, normalized size = 1.58

$$\begin{cases} \log(x)\tanh^4(a) & \text{for } b = 0 \wedge n = 0 \\ \log(x)\tanh^4(a + b\log(c)) & \text{for } n = 0 \\ \log(x)\tanh^4(a) & \text{for } b = 0 \\ \log(x) - \frac{\tanh^3(a+bn\log(x)+b\log(c))}{3bn} - \frac{\tanh(a+bn\log(x)+b\log(c))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Piecewise((log(x)*tanh(a)**4, Eq(b, 0) & Eq(n, 0)), (log(x)*tanh(a + b*log(
c))**4, Eq(n, 0)), (log(x)*tanh(a)**4, Eq(b, 0)), (log(x) - tanh(a + b*n*lo
g(x) + b*log(c))**3/(3*b*n) - tanh(a + b*n*log(x) + b*log(c))/(b*n), True))
```

Giac [A] time = 1.3631, size = 90, normalized size = 2.

$$\frac{4\left(3\,c^{4\,b}x^{4\,bn}e^{(4\,a)} + 3\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 2\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)} + 1\right)^{3}bn} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] 4/3*(3*c^(4*b)*x^(4*b*n)*e^(4*a) + 3*c^(2*b)*x^(2*b*n)*e^(2*a) + 2)/((c^(2*
b)*x^(2*b*n)*e^(2*a) + 1)^3*b*n) + log(x)
```

3.188 $\int \frac{\tanh^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=66

$$-\frac{\tanh^4(a+b\log(cx^n))}{4bn}-\frac{\tanh^2(a+b\log(cx^n))}{2bn}+\frac{\log(\cosh(a+b\log(cx^n)))}{bn}$$

```
[Out] Log[Cosh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]^2/(2*b*n) - Tanh
[a + b*Log[c*x^n]]^4/(4*b*n)
```

Rubi [A] time = 0.0570094, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3473, 3475}

$$-\frac{\tanh^4(a+b\log(cx^n))}{4bn}-\frac{\tanh^2(a+b\log(cx^n))}{2bn}+\frac{\log(\cosh(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Log[Cosh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]^2/(2*b*n) - Tanh
[a + b*Log[c*x^n]]^4/(4*b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \tanh^5(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\tanh^4(a+b\log(cx^n))}{4bn}+\frac{\text{Subst}\left(\int \tanh^3(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\tanh^2(a+b\log(cx^n))}{2bn}-\frac{\tanh^4(a+b\log(cx^n))}{4bn}+\frac{\text{Subst}\left(\int \tanh(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\log(\cosh(a+b\log(cx^n)))}{bn}-\frac{\tanh^2(a+b\log(cx^n))}{2bn}-\frac{\tanh^4(a+b\log(cx^n))}{4bn}\end{aligned}$$

Mathematica [A] time = 0.208275, size = 55, normalized size = 0.83

$$\frac{-\tanh^4(a+b\log(cx^n))-2\tanh^2(a+b\log(cx^n))+4\log(\cosh(a+b\log(cx^n)))}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (4*Log[Cosh[a + b*Log[c*x^n]]] - 2*Tanh[a + b*Log[c*x^n]]^2 - Tanh[a + b*Lo
g[c*x^n]]^4)/(4*b*n)
```

Maple [A] time = 0.006, size = 88, normalized size = 1.3

$$-\frac{(\tanh(a+b\ln(cx^n)))^4}{4bn}-\frac{(\tanh(a+b\ln(cx^n)))^2}{2bn}-\frac{\ln(\tanh(a+b\ln(cx^n))-1)}{2bn}-\frac{\ln(\tanh(a+b\ln(cx^n))+1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] -1/4*tanh(a+b*ln(c*x^n))^4/b/n-1/2*tanh(a+b*ln(c*x^n))^2/b/n-1/2/n/b*ln(tan
h(a+b*ln(c*x^n))-1)-1/2/n/b*ln(tanh(a+b*ln(c*x^n))+1)
```

Maxima [B] time = 1.75699, size = 1119, normalized size = 16.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/24*(48*c^(6*b)*e^(6*b*log(x^n) + 6*a) + 108*c^(4*b)*e^(4*b*log(x^n) + 4*a
) + 88*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) +
8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n)
 + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 1/24*(12*c^(6*b)*e^
(6*b*log(x^n) + 6*a) + 42*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 52*c^(2*b)*e^(2*
b*log(x^n) + 2*a) + 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n
*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b
)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 5/8*(4*c^(6*b)*e^(6*b*log(x^n) + 6*a) +
 6*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 4*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(
b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) +
 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a
) + b*n) - 5/12*(6*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 4*c^(2*b)*e^(2*b*log(x^
n) + 2*a) + 1)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*l
og(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*
b*log(x^n) + 2*a) + b*n) + 5/12*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c
^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*
b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) +
 b*n) - 5/8/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(
x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*l
og(x^n) + 2*a) + b*n) + log((c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)*e^(-2*a)/c
^(2*b))/(b*n) - log(x)
```

Fricas [B] time = 2.32373, size = 5060, normalized size = 76.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^8*log(x) + 8*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^8 + 4*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) + b*n*log(x) - 1)*
sinh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a
)^3*log(x) + 3*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*l
og(x) + b*log(c) + a)^5 + 2*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) +
 a)^4 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) + 30*(b*n*log(x)
 - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n*log(x) - 2)*sinh(b*n*log(x)
 + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5*log(x) + 10
*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^3 + (3*b*n*log(x) - 2)*co
sh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(b*n*l
og(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n*log(x) + 4*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^6*log(x) + 15*(b*n*log(x) - 1)*cosh(b*n*log(x) +
b*log(c) + a)^4 + 3*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) + a)^2 +
b*n*log(x) - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*lo
g(c) + a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c)
+ a)^7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c
) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^6 + 4*cosh(b*n*log(x) + b*log
(c) + a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b
*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b
*log(c) + a)^4 + 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x)
+ b*log(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log
(x) + b*log(c) + a)^5 + 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*l
og(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*lo
g(x) + b*log(c) + a)^6 + 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*
log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*
n*log(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 + 3*cosh(
b*n*log(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b
*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*cosh(
b*n*log(x) + b*log(c) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(
x) + b*log(c) + a))) + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7*log(x) + 3*
(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^5 + (3*b*n*log(x) - 2)*cos
h(b*n*log(x) + b*log(c) + a)^3 + (b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c
) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) +
a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^7 + b*n*sinh(b*n*log(x) + b*log(c) + a)^8 + 4*b*n*cosh(b*n*log(x) + b*log
(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log
(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*
cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) +
a)^4 + 30*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) +
b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^5 + 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*
(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 + 15*b*n*cosh(b*n*log(x) + b*log(c
) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 + 3*b*n*c
osh(b*n*log(x) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3
+ b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42526, size = 216, normalized size = 3.27

$$\frac{\log\left(2\,x^{2\,bn}|c|^{2\,b}\cos\left(\pi b\operatorname{sgn}\left(c\right)-\pi b\right)e^{(2\,a)}+x^{4\,bn}|c|^{4\,b}e^{(4\,a)}+1\right)}{2\,bn}-\frac{25\,c^{8\,b}x^{8\,bn}e^{(8\,a)}+52\,c^{6\,b}x^{6\,bn}e^{(6\,a)}+102\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+52}{12\,\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^{4}bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/2*log(2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b*n
)*abs(c)^(4*b)*e^(4*a) + 1)/(b*n) - 1/12*(25*c^(8*b)*x^(8*b*n)*e^(8*a) + 52
*c^(6*b)*x^(6*b*n)*e^(6*a) + 102*c^(4*b)*x^(4*b*n)*e^(4*a) + 52*c^(2*b)*x^(
2*b*n)*e^(2*a) + 25)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^4*b*n) - log(x)
```

3.189 $\int (ex)^m \tanh (d (a + b \log (cx^n))) \, dx$

Optimal. Leaf size=88

$$\frac{(ex)^{m+1}}{e(m+1)} - \frac{2(ex)^{m+1} {}_2F_1\left(1, \frac{m+1}{2bdn}; \frac{m+1}{2bdn} + 1; -e^{2ad} (cx^n)^{2bd}\right)}{e(m+1)}$$

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(
2*b*d*n), 1 + (1 + m)/(2*b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(e*(1 + m))
```

Rubi [F] time = 0.0464648, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \tanh (d (a + b \log (cx^n))) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][(e*x)^m*Tanh[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int (ex)^m \tanh (d (a + b \log (cx^n))) \, dx = \int (ex)^m \tanh (d (a + b \log (cx^n))) \, dx$$

Mathematica [A] time = 13.584, size = 160, normalized size = 1.82

$$\frac{x(ex)^m \left(\frac{(m+1)e^{2ad}(cx^n)^{2bd} {}_2F_1\left(1, \frac{m+2bdn+1}{2bdn}; \frac{m+4bdn+1}{2bdn}; -e^{2ad}(cx^n)^{2bd}\right)}{2bdn+m+1} - {}_2F_1\left(1, \frac{m+1}{2bdn}; \frac{m+1}{2bdn} + 1; -e^{2d(a+b\log(cx^n))}\right)\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x*(e*x)^m*(-Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n),
 -E^(2*d*(a + b*Log[c*x^n]))] + (E^(2*a*d)*(1 + m)*(c*x^n)^(2*b*d)*Hypergeo
metric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), -(E^
(2*a*d)*(c*x^n)^(2*b*d))])/(1 + m + 2*b*d*n)))/(1 + m)
```

Maple [F] time = 1.214, size = 0, normalized size = 0.

$$\int (ex)^m \tanh (d (a + b \ln (cx^n))) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tanh(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*tanh(d*(a+b*ln(c*x^n))),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^m x x^m}{m+1} - 2\,e^m \int \frac{x^m}{c^{2\,bd} e^{(2\,bd \log(x^n) + 2\,ad)} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] e^m*x*x^m/(m + 1) - 2*e^m*integrate(x^m/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*
d) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \tanh\left(bd \log\left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh\left(ad + bd \log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tanh(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*tanh(a*d + b*d*log(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh\left(\left(b \log\left(cx^n\right) + a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d), x)
```

3.190 $\int (ex)^m \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=169

$$-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;-e^{2ad}(cx^n)^{2bd}\right)}{bden}+\frac{(ex)^{m+1}\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bden\left(e^{2ad}(cx^n)^{2bd}+1\right)}+\frac{(ex)^{m+1}(bdn+m+1)}{bde(m+1)n}$$

```
[Out] ((1 + m + b*d*n)*(e*x)^(1 + m))/(b*d*e*(1 + m)*n) + ((e*x)^(1 + m)*(1 - E^(
2*a*d)*(c*x^n)^(2*b*d)))/(b*d*e*n*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*(e*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), -
(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*e*n)
```

Rubi [F] time = 0.0769626, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \tanh^2(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int (ex)^m \tanh^2(d(a+b\log(cx^n)))\,dx=\int (ex)^m \tanh^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] time = 16.5395, size = 516, normalized size = 3.05

$$(m+1)x^{-m}(ex)^m\text{sech}(d(a+b(\log(cx^n)-n\log(x))))\left(\frac{x^{m+1}\sinh(bdn\log(x))\text{sech}(d(a+b\log(cx^n)))}{m+1}-\frac{\cosh(d(a+b(\log(cx^n)-n\log(x))))\,e}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x*(e*x)^m)/(1 + m) - (x*(e*x)^m*Sech[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]
*Sech[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x
]])/(b*d*n) + ((1 + m)*(e*x)^m*Sech[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(
(x^(1 + m)*Sech[d*(a + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/(1 + m) - (Cosh[d
*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(E^((a + 2*a*m + b*(1 + m)*n*Log[x] +
b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Hypergeome
tric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), -E^(2*d*(a + b*Log[c*x
^n]))] - E^((a*(1 + 2*m + 2*b*d*n))/(b*n) + (1 + m + 2*b*d*n)*Log[x] + ((1
+ 2*m + 2*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1
, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), -E^(2*d*(a + b*
Log[c*x^n]))] + E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x
]) + Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Tanh[d*(a + b*Log[c*x^n])]))/(E^
```

```
(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*(1 + m + 2*b
*d*n))))/(b*d*n*x^m)
```

Maple [F] time = 0.218, size = 0, normalized size = 0.

$$\int (ex)^m (\tanh (d (a + b \ln (cx^n))))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\,e^m(m+1)\int \frac{x^m}{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)} + bdn}\,dx + \frac{bc^{2\,bd}de^m nxe^{(2\,bd\log(x^n)+2\,ad+m\log(x))} + (bde^m n + 2\,e^m(m+1))xx^m}{(mn+n)bc^{2\,bd}de^{(2\,bd\log(x^n)+2\,ad)} + (mn+n)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -2*e^m*(m + 1)*integrate(x^m/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) +
b*d*n), x) + (b*c^(2*b*d)*d*e^m*n*x*e^(2*b*d*log(x^n) + 2*a*d + m*log(x)) +
 (b*d*e^m*n + 2*e^m*(m + 1))*x*x^m)/((m*n + n)*b*c^(2*b*d)*d*e^(2*b*d*log(x
^n) + 2*a*d) + (m*n + n)*b*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex)^m \tanh (bd\log (cx^n) + ad)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tanh(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d)^2, x)
```

3.191 $\int (ex)^m \tanh^3(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=307

$$-\frac{(ex)^{m+1}\left(2b^2d^2n^2+m^2+2m+1\right){}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;-e^{2ad}\left(cx^n\right)^{2bd}\right)}{b^2d^2e(m+1)n^2}+\frac{e^{-2ad}(ex)^{m+1}\left(\frac{e^{2ad}(-2bdn+m+1)}{n}-\frac{e^{4ad}(2bdn+m+1)(c\right.}{n}}{2b^2d^2en\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}$$

```
[Out] ((1 + m + b*d*n)*(1 + m + 2*b*d*n)*(e*x)^(1 + m))/(2*b^2*d^2*e*(1 + m)*n^2)
 - ((e*x)^(1 + m)*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^2)/(2*b*d*e*n*(1 + E^(2*a
*d)*(c*x^n)^(2*b*d))^2) + ((e*x)^(1 + m)*((E^(2*a*d)*(1 + m - 2*b*d*n))/n -
 (E^(4*a*d)*(1 + m + 2*b*d*n)*(c*x^n)^(2*b*d))/n))/(2*b^2*d^2*e*E^(2*a*d)*n
*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), -(E^
(2*a*d)*(c*x^n)^(2*b*d))])/(b^2*d^2*e*(1 + m)*n^2)
```

Rubi [F] time = 0.071333, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \tanh^3(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^3,x]

[Out] Defer[Int][(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^3, x]
```

Rubi steps

$$\int (ex)^m \tanh^3(d(a+b\log(cx^n)))\,dx=\int (ex)^m \tanh^3(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 16.7604, size = 606, normalized size = 1.97

$$x^{-m}(ex)^m\left(2b^2d^2n^2+m^2+2m+1\right)\operatorname{sech}(d(a+b(\log(cx^n)-n\log(x))))\left(\frac{x^{m+1}\sinh(bdn\log(x))\operatorname{sech}(d(a+b\log(cx^n)))}{m+1}-\frac{\cosh(d(a+b(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^3,x]

[Out] (x*(e*x)^m*Sech[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*
b*d*n) - ((1 + m)*x*(e*x)^m*Sech[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sech
[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x]])/(
2*b^2*d^2*n^2) + ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Sech[d*(a + b*(-(
n*Log[x]) + Log[c*x^n]))]*((x^(1 + m)*Sech[d*(a + b*Log[c*x^n])]*Sinh[b*d*n
*Log[x]])/(1 + m) - (Cosh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(E^((a + 2*
a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(
1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d
```

```
*n), -E^(2*d*(a + b*Log[c*x^n]))] - E^((a*(1 + 2*m + 2*b*d*n))/(b*n) + (1 +
 m + 2*b*d*n)*Log[x] + ((1 + 2*m + 2*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*
(1 + m)*Hypergeometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)
/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + E^((a + 2*a*m + b*(1 + m)*n*Log[
x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Tanh[
d*(a + b*Log[c*x^n])]))/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/
(b*n))*(1 + m)*(1 + m + 2*b*d*n))))/(2*b^2*d^2*n^2*x^m) + (x*(e*x)^m*Tanh[d
*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m)
```

Maple [F] time = 0.218, size = 0, normalized size = 0.

$$\int (ex)^m (\tanh (d (a + b \ln (cx^n))))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(2\,b^2d^2e^mn^2 + \left(m^2 + 2\,m + 1\right)e^m\right)\int \frac{x^m}{b^2c^{2\,bd}d^2n^2e^{(2\,bd\log(x^n) + 2\,ad)} + b^2d^2n^2}\,dx + \frac{b^2c^{4\,bd}d^2e^mn^2xe^{(4\,bd\log(x^n) + 4\,ad + m\log(x))}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] -(2*b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*integrate(x^m/(b^2*c^(2*b*d)*d^2
*n^2*e^(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2), x) + (b^2*c^(4*b*d)*d^2*e^m
*n^2*x*e^(4*b*d*log(x^n) + 4*a*d + m*log(x)) + (b^2*d^2*e^m*n^2 + (m^2 + 2*
m + 1)*e^m)*x*x^m + (2*b^2*c^(2*b*d)*d^2*e^m*n^2*e^(2*a*d) + 2*(m*n + n)*b*
c^(2*b*d)*d*e^m*e^(2*a*d) + (m^2 + 2*m + 1)*c^(2*b*d)*e^m*e^(2*a*d))*x*e^(2
*b*d*log(x^n) + m*log(x)))/((m*n^2 + n^2)*b^2*c^(4*b*d)*d^2*e^(4*b*d*log(x^
n) + 4*a*d) + 2*(m*n^2 + n^2)*b^2*c^(2*b*d)*d^2*e^(2*b*d*log(x^n) + 2*a*d)
+ (m*n^2 + n^2)*b^2*d^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex)^m \tanh (bd \log (cx^n) + ad)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*tanh(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tanh(d*(a+b*ln(c*x**n)))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh ((b \log (cx^n) + a)d)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d)^3, x)
```

3.192 $\int \tanh^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=115

$$x\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(e^{2ad}(cx^n)^{2bd}-1\right)^p F_1\left(\frac{1}{2bdn};-p,p;1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] (x*(-1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[1/(2*b*d*n), -p, p, 1 + 1/(2
*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(1 - E^(
2*a*d)*(c*x^n)^(2*b*d))^p
```

Rubi [F] time = 0.0143694, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \tanh^p(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] Defer[Int][Tanh[d*(a + b*Log[c*x^n])]^p, x]
```

Rubi steps

$$\int \tanh^p(d(a+b\log(cx^n)))\,dx=\int \tanh^p(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] time = 3.82438, size = 387, normalized size = 3.37

$$\frac{x(2bdn+1)\left(\frac{e^{2ad}(cx^n)^{2bd}-1}{e^{2ad}(cx^n)^{2bd}+1}\right)^p F_1\left(\frac{1}{2bdn};-p,p;1\right.}{-2bdnpe^{2ad}(cx^n)^{2bd}F_1\left(1+\frac{1}{2bdn};1-p,p;2+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)-2bdnpe^{2ad}(cx^n)^{2bd}F_1\left(1+\frac{1}{2bdn};-p,p\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((1 + 2*b*d*n)*x*((-1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(1 + E^(2*a*d)*(c*x^n)^(
2*b*d)))^p*AppellF1[1/(2*b*d*n), -p, p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^
(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(-2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*
d)*AppellF1[1 + 1/(2*b*d*n), 1 - p, p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(
2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] - 2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d)
*AppellF1[1 + 1/(2*b*d*n), -p, 1 + p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2
*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + (1 + 2*b*d*n)*AppellF1[1/(2*b*d*n),
-p, p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b
*d))])
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int (\tanh(d(a+b\ln(cx^n))))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left((b\log\left(cx^{n}\right)+a)d\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate(tanh((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\tanh\left(bd\log\left(cx^{n}\right)+ad\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh^{p}\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Integral(tanh(d*(a + b*log(c*x**n)))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \tanh\left((b\log\left(cx^{n}\right)+a)d\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] integrate(tanh((b*log(c*x^n) + a)*d)^p, x)
```

3.193 $\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=135

$$\frac{(ex)^{m+1}\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)^{-p}\left(e^{2ad}\left(cx^n\right)^{2bd}-1\right)^p F_1\left(\frac{m+1}{2bdn};-p,p;\frac{m+1}{2bdn}+1;e^{2ad}\left(cx^n\right)^{2bd},-e^{2ad}\left(cx^n\right)^{2bd}\right)}{e(m+1)}$$

```
[Out] ((e*x)^(1 + m)*(-1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[(1 + m)/(2*b*d*n
), -p, p, 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*
x^n)^(2*b*d))])/(e*(1 + m)*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rubi [F] time = 0.0988439, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] Defer[Int][(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^p, x]
```

Rubi steps

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx=\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx$$

Mathematica [A] time = 4.99858, size = 174, normalized size = 1.29

$$\frac{x(ex)^m\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)^{-p}\left(\frac{e^{2ad}(cx^n)^{2bd}-1}{e^{2ad}(cx^n)^{2bd}+1}\right)^p\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)^p F_1\left(\frac{m+1}{2bdn};-p,p;\frac{m+1}{2bdn}+1;e^{2ad}\left(cx^n\right)^{2bd},-e^{2ad}\left(cx^n\right)^{2bd}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*((-1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(1 + E^(2*a*d)*(c*x^n)^(2*b*d)
))^p*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[(1 + m)/(2*b*d*n), -p, p, 1
 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d
))])/((1 + m)*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int (ex)^m \left(\tanh(d(a+b\ln(cx^n)))\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh((b\log(cx^n) + a)d)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \tanh(bd\log(cx^n) + ad)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*tanh(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*tanh(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tanh((b\log(cx^n) + a)d)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d)^p, x)
```

3.194 $\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=73

$$-\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}+\frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}-\frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n) - (2*Tanh[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Rubi [A] time = 0.0527211, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3473, 3476, 329, 298, 203, 206}

$$-\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}+\frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}-\frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n) - (2*Tanh[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \tanh^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \sqrt{\tanh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \tanh(a+b\log(cx^n))\right)}{bn}$$
$$= -\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$
$$= -\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2\tanh^{\frac{3}{2}}(a+b}{3bn}$$

Mathematica [A] time = 0.285213, size = 64, normalized size = 0.88

$$-\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n)) - 3\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right) + 3\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -(3*ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]] - 3*ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]] + 2*Tanh[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Maple [A] time = 0.023, size = 93, normalized size = 1.3

$$-\frac{2}{3\,bn}(\tanh(a+b\ln(cx^n)))^{\frac{3}{2}} - \frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}-1\right) + \frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}+1\right) - \frac{1}{bn}\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+b*ln(c*x^n))^(5/2)/x,x)
```

```
[Out] -2/3*tanh(a+b*ln(c*x^n))^(3/2)/b/n-1/2/b/n*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)+
1/2/b/n*ln(tanh(a+b*ln(c*x^n))^(1/2)+1)-arctan(tanh(a+b*ln(c*x^n))^(1/2))/b
/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh \left(b \log \left(c x^{n}\right)+a\right)^{\frac{5}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(tanh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [B] time = 2.23465, size = 2091, normalized size = 28.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1
)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 +
(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh
(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(s
inh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) + 4*cosh(b
*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh
(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(
x) + b*log(c) + a)^2 + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b
*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x)
 + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x)
 + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(
c) + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log
(c) + a))) + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
 a) + 4*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) +
 a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
 sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a
)/cosh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a
)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
 + b*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.195 $\int \frac{\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n) - (2*Sqrt[Tanh[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] time = 0.0523155, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3473, 3476, 329, 212, 206, 203}

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n) - (2*Sqrt[Tanh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \tanh^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tanh(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \tanh(a+b\log(cx^n))\right)}{bn}\\
&= -\frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn}-\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}\\
&= -\frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}+\frac{\text{Subst}\left(\int\right.}{}\\
&= \frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}-\frac{2\sqrt{\tanh(a+b\log}}{bn}
\end{aligned}
$$

Mathematica [A] time = 0.133162, size = 57, normalized size = 0.81

$$
\frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)-2\sqrt{\tanh(a+b\log(cx^n))}+\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Tanh[a + b*Log[c*x^n]]
]] - 2*Sqrt[Tanh[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A] time = 0.013, size = 92, normalized size = 1.3

$$
-2\frac{\sqrt{\tanh(a+b\ln(cx^n))}}{bn}-\frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}-1\right)+\frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}+1\right)+\frac{1}{bn}\arctan\left(\sqrt{\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] -2*tanh(a+b*ln(c*x^n))^(1/2)/b/n-1/2/b/n*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)+1/
2/b/n*ln(tanh(a+b*ln(c*x^n))^(1/2)+1)+arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\tanh(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(tanh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [B] time = 2.12309, size = 1100, normalized size = 15.71

$$-\frac{4\sqrt{\frac{\sinh(bn\log(x)+b\log(c)+a)}{\cosh(bn\log(x)+b\log(c)+a)}} - 2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a
)) - 2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*s
qrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) + log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n*
log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.196 $\int \frac{\sqrt{\tanh(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n)
```

Rubi [A] time = 0.04067, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3476, 329, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tanh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\tanh(a+b\log(cx^n))}}{x}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\tanh(a+bx)}\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \tanh(a+b\log(cx^n))\right)}{bn} \\
&= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(a+b\,\text{lo}}}{bn} \\
&= -\frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.075704, size = 48, normalized size = 1.

$$\frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tanh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n)
```

Maple [A] time = 0.015, size = 72, normalized size = 1.5

$$-\frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}-1\right) + \frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}+1\right) - \frac{1}{bn}\arctan\left(\sqrt{\tanh(a+b\ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] -1/2/b/n*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)+1/2/b/n*ln(tanh(a+b*ln(c*x^n))^(1/
2)+1)-arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tanh(b\log(cx^n)+a)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tanh(b*log(c*x^n) + a))/x, x)
```

Fricas [B] time = 2.11472, size = 1000, normalized size = 20.83

$$-\frac{2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a
)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*
sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) + lo
g(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b
*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n
*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [A] time = 3.23495, size = 66, normalized size = 1.38

$$-\frac{\log\left(\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}-1\right)}{2bn}+\frac{\log\left(\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}+1\right)}{2bn}-\frac{\operatorname{atan}\left(\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] -log(sqrt(tanh(a + b*log(c*x**n))) - 1)/(2*b*n) + log(sqrt(tanh(a + b*log(c
*x**n))) + 1)/(2*b*n) - atan(sqrt(tanh(a + b*log(c*x**n))))/(b*n)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.197 $\int \frac{1}{x\sqrt{\tanh(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n)
```

Rubi [A] time = 0.0398733, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3476, 329, 212, 206, 203}

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Tanh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{\tanh(a+b\log(cx^n))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tanh(a+bx)}}\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \tanh(a+b\log(cx^n))\right)}{bn} \\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(a+b\log}\right.}{bn} \\
&= \frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}
\end{aligned}
$$

Mathematica [A] time = 0.108694, size = 47, normalized size = 1.

$$\frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Tanh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n)
```

Maple [A] time = 0.017, size = 44, normalized size = 0.9

$$\frac{1}{bn}\arctan\left(\sqrt{\tanh(a+b\ln(cx^n))}\right) + \frac{1}{bn}\text{Artanh}\left(\sqrt{\tanh(a+b\ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/tanh(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(tanh(a+b*ln(c*x^n))^(1/2))/b/
n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\tanh(b\log(cx^n)+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(tanh(b*log(c*x^n) + a))), x)
```

Fricas [B] time = 2.06853, size = 999, normalized size = 21.26

$$\frac{2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*s
qrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) - log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n*
log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [A] time = 7.10488, size = 66, normalized size = 1.4

$$-\frac{\log\left(\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}-1\right)}{2bn}+\frac{\log\left(\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}+1\right)}{2bn}+\frac{\operatorname{atan}\left(\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] -log(sqrt(tanh(a + b*log(c*x**n))) - 1)/(2*b*n) + log(sqrt(tanh(a + b*log(c
*x**n))) + 1)/(2*b*n) + atan(sqrt(tanh(a + b*log(c*x**n))))/(b*n)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{\tanh\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(tanh(b*log(c*x^n) + a))), x)
```

3.198 $\int \frac{1}{x \tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2}{bn\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}-\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Tanh[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.0517119, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3474, 3476, 329, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2}{bn\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}-\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Tanh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Tanh[a + b*Log[c*x^n]]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\tanh^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\tanh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \tanh(a+b\log(cx^n))\right)}{bn}\\
&= -\frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}\\
&= -\frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \text{Sub}\\
&= -\frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{}{bn\sqrt{\tanh}}
\end{aligned}$$

Mathematica [C] time = 0.15784, size = 44, normalized size = 0.62

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\tanh^2(a+b\log(cx^n))\right)}{bn\sqrt{\tanh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Tanh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Tanh[a + b*Log[c*x^n]]^2])/(b*n*Sqrt[Ta
nh[a + b*Log[c*x^n]]])
```

Maple [A] time = 0.017, size = 93, normalized size = 1.3

$$\frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}+1\right)-2\,\frac{1}{bn\sqrt{\tanh(a+b\ln(cx^n))}}-\frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}-1\right)-\frac{1}{bn}\,\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/tanh(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 1/2/b/n*ln(tanh(a+b*ln(c*x^n))^(1/2)+1)-2/b/n/tanh(a+b*ln(c*x^n))^(1/2)-1/2
/b/n*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)-arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \tanh\left(b \log\left(cx^{n}\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*tanh(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [B] time = 2.10027, size = 2088, normalized size = 29.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1
)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 +
(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh
(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(s
inh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) + 4*cosh(b
*n*log(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b
*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x)
 + b*log(c) + a)^2 - 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n
*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) +
 b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) +
 b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
 + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c
) + a))) + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
) + 4*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a
)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + s
inh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)/
cosh(b*n*log(x) + b*log(c) + a)) - 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^
2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
 b*n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \tanh^{\frac{3}{2}}{\left(a + b \log{\left(cx^{n} \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*tanh(a + b*log(c*x**n))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \tanh \left(b \log \left(c x^{n}\right)+a\right)^{\frac{3}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*tanh(b*log(c*x^n) + a)^(3/2)), x)
```

3.199 $\int \frac{1}{x \tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=72

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2}{3bn\tanh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n) - 2/(3*b*n*Tanh[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] time = 0.0500082, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3474, 3476, 329, 212, 206, 203}

$$\frac{\tanh^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2}{3bn\tanh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{\tan^{-1}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Tanh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n) - 2/(3*b*n*Tanh[a + b*Log[c*x^n]]^(3/2))
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\tanh^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tanh(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \tanh(a+b\log(cx^n))\right)}{bn}$$
$$= -\frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$
$$= -\frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\text{Su}}{}$$
$$= \frac{\tan^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{}{3bn\tanh^{\frac{3}{2}}(}$$

Mathematica [C] time = 0.202918, size = 46, normalized size = 0.64

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\tanh^2(a+b\log(cx^n))\right)}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Tanh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Tanh[a + b*Log[c*x^n]]^2])/(3*b*n*Tanh[
a + b*Log[c*x^n]]^(3/2))
```

Maple [A] time = 0.016, size = 92, normalized size = 1.3

$$\frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}+1\right) - \frac{2}{3\,bn}(\tanh(a+b\ln(cx^n)))^{-\frac{3}{2}} - \frac{1}{2\,bn}\ln\left(\sqrt{\tanh(a+b\ln(cx^n))}-1\right) + \frac{1}{bn}\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/tanh(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] 1/2/b/n*ln(tanh(a+b*ln(c*x^n))^(1/2)+1)-2/3/b/n/tanh(a+b*ln(c*x^n))^(3/2)-1
/2/b/n*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)+arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/
n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\tanh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*tanh(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [B] time = 2.21552, size = 3644, normalized size = 50.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*n*log(x) + b*log(c) + a)^4 + 16*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*sinh(b*n*log(x) + b*log(c) + a)^4
 + 8*(3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 - 6*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4
 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c)
 + a)^3 - cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)
+ 1)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2
 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*s
inh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqr
t(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) - 8*cos
h(b*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*c
osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sin
h(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(c
osh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a) + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*lo
g(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*
log(c) + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b
*log(c) + a))) + 16*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) +
b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 4*(cosh(b*n*log(x) + b*log
(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
 a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c)
 + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(
c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*sqrt(sinh(b*n*log(x) + b*log(
c) + a)/cosh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(
```

```
c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cosh(b*n*log(x) +
b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*
n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3
 - b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\tanh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*tanh(b*log(c*x^n) + a)^(5/2)), x)
```

3.200 $\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$

Optimal. Leaf size=135

$$\frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4c^{3/2}}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c}$$

```
[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c*
Tanh[x]^4)])])/(4*c^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt
[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c]) - Sqr
t[a + b*Tanh[x]^2 + c*Tanh[x]^4]/(2*c)
```

Rubi [A] time = 0.358366, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3700, 1251, 1653, 843, 621, 206, 724}

$$\frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4c^{3/2}}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c*
Tanh[x]^4)])])/(4*c^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt
[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c]) - Sqr
t[a + b*Tanh[x]^2 + c*Tanh[x]^4]/(2*c)
```

Rule 3700

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
```

```
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx &= -\text{Subst}\left(\int \frac{x^5}{\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\right)\\
&= -\frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c} - \frac{\text{Subst}\left(\int \frac{\frac{b}{2}+\frac{1}{2}(b-2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)}{2c}\\
&= -\frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\text{t}\right.\\
&= -\frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c} - \frac{(b-2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\tanh}{\sqrt{a+b\tanh^2(x)+c}}\right.}{2c}\\
&= \frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4c^{3/2}} + \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\,\text{ta}}}\right.}{2\sqrt{a+b+c}}
\end{aligned}
$$

Mathematica [A] time = 1.30541, size = 136, normalized size = 1.01

$$\frac{1}{4}\left(\frac{(2c-b)\tanh^{-1}\left(\frac{-b-2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{c^{3/2}}+\frac{2\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{\sqrt{a+b+c}}-\frac{2\sqrt{a+b\tanh^2(x)+c\tanh^4}}{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] (((-b + 2*c)*ArcTanh[(-b - 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 +
 c*Tanh[x]^4])])/c^(3/2) + (2*ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sq
rt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])])/Sqrt[a + b + c] - (2*S
qrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])/c)/4
```

Maple [A] time = 0.168, size = 149, normalized size = 1.1

$$-\frac{1}{2c}\sqrt{a+b(\tanh(x))^2+c(\tanh(x))^4}+\frac{b}{4}\ln\left(\left(\frac{b}{2}+c(\tanh(x))^2\right)\frac{1}{\sqrt{c}}+\sqrt{a+b(\tanh(x))^2+c(\tanh(x))^4}\right)c^{-\frac{3}{2}}-\frac{1}{2}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
```

```
[Out] -1/2*(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)/c+1/4*b/c^(3/2)*ln((1/2*b+c*tanh(x)^
2)/c^(1/2)+(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))-1/2*ln((1/2*b+c*tanh(x)^2)/c^
(1/2)+(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))/c^(1/2)+1/2/(a+b+c)^(1/2)*arctanh(
1/2*(b*tanh(x)^2+2*c*tanh(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*tanh(x)^2+c*tanh(x
)^4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh(x)^5}{\sqrt{c\tanh(x)^4+b\tanh(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(tanh(x)^5/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Fricas [B] time = 15.7863, size = 24521, normalized size = 181.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*(((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a +
 b)*c - 2*c^2)*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x
)^4 + 2*(a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*
a + b)*c - 2*c^2)*cosh(x)^2 + a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^2 +
a*b + b^2 - (2*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh
(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(c)*log(((b
^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cos
h(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 + 4*(b^2 + 4*a*c -
 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 + b^2 + 4*
a*c - 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 + 3*(
b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c
^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 + 30*(b^2 + 4
*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(
7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 + 10*(b^2 + 4*a*c - 8*c^2)*cosh(x
)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 + 4*(b^2 + 4*a*
c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 + 15*(b
^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x
)^2 + b^2 + 4*a*c - 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(
b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 + 2*(b - 2*c)*cosh(x)^2 +
2*(3*(b + 2*c)*cosh(x)^2 + b - 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b
 - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4 + (
a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2
*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2*b)*
c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 + 3*(b^2 + 4*a*c - 8
*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 + (b^2 + 4*a
*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8
+ 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))
*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 +
8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*
cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cos
h(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*(c^2*cosh(x)^4 + 4*c^2*co
sh(x)*sinh(x)^3 + c^2*sinh(x)^4 + 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 + c^
2)*sinh(x)^2 + c^2 + 4*(c^2*cosh(x)^3 + c^2*cosh(x))*sinh(x))*sqrt(a + b +
c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^
6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b -
 b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2
 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x
)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*
b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*s
inh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*
a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(
x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a
 - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
+ 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c
 - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 + (a^2
```

```
+ a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 4*sqrt(2)*((a + b)
*c + c^2)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*c
osh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*
c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sin
h(x)^3 + sinh(x)^4)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3
)*cosh(x)*sinh(x)^3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 + 2
*((a + b)*c^2 + c^3)*cosh(x)^2 + 2*((a + b)*c^2 + c^3 + 3*((a + b)*c^2 + c^
3)*cosh(x)^2)*sinh(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 + ((a + b)*c^2 +
 c^3)*cosh(x))*sinh(x)), -1/8*(4*(c^2*cosh(x)^4 + 4*c^2*cosh(x)*sinh(x)^3 +
 c^2*sinh(x)^4 + 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 + c^2)*sinh(x)^2 + c^
2 + 4*(c^2*cosh(x)^3 + c^2*cosh(x))*sinh(x))*sqrt(-a - b - c)*arctan(sqrt(2
)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*si
nh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a - c)*sinh(x)
^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(
-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b +
3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 +
 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6
 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c
 - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3
+ 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2
*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2
 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
 + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b -
 b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2
)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(
a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 +
3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c
+ 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) + ((a*b + b
^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a + b)*c - 2*c^2)*c
osh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^4 + 2*(a*b + b
^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a + b)*c - 2*c^2
)*cosh(x)^2 + a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^2 + a*b + b^2 - (2*a
 + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^3 + (a*b + b
^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(c)*log(((b^2 + 4*(a + 2*b)
*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)*sinh(x)^7 +
 (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 + 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^
6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 + b^2 + 4*a*c - 8*c^2)*sin
h(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 + 3*(b^2 + 4*a*c - 8*
c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^4 +
2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 + 30*(b^2 + 4*a*c - 8*c^2)*co
sh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(7*(b^2 + 4*(a +
2*b)*c + 8*c^2)*cosh(x)^5 + 10*(b^2 + 4*a*c - 8*c^2)*cosh(x)^3 + (3*b^2 + 4
*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 + 4*(b^2 + 4*a*c - 8*c^2)*cosh(
x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 + 15*(b^2 + 4*a*c - 8*c
^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^2 + b^2 + 4*a*
c - 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)
*sinh(x)^3 + (b + 2*c)*sinh(x)^4 + 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*c
osh(x)^2 + b - 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b - 2*c)*cosh(x))
*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(
x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x
)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2*b)*c + 8*c^2 + 8*((
```

```
b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 + 3*(b^2 + 4*a*c - 8*c^2)*cosh(x)^5
+ (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 + (b^2 + 4*a*c - 8*c^2)*cosh
(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2
 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(
35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 +
 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*co
sh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(
x)^3 + cosh(x))*sinh(x) + 1)) + 4*sqrt(2)*((a + b)*c + c^2)*sqrt(((a + b +
c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(((
a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3)*cosh(x)*sinh(x)^3 + ((a
 + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 + 2*((a + b)*c^2 + c^3)*cosh
(x)^2 + 2*((a + b)*c^2 + c^3 + 3*((a + b)*c^2 + c^3)*cosh(x)^2)*sinh(x)^2 +
 4*(((a + b)*c^2 + c^3)*cosh(x)^3 + ((a + b)*c^2 + c^3)*cosh(x))*sinh(x)),
-1/4*(((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a +
b)*c - 2*c^2)*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)
^4 + 2*(a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a
 + b)*c - 2*c^2)*cosh(x)^2 + a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^2 + a
*b + b^2 - (2*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(
x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(-c)*arctan(
1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c
)*sinh(x)^4 + 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 + b - 2*c)*s
inh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*s
qrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cos
h(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)
/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(
x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*cosh(
x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 + 4*(a*c - c^2)*cosh(x)^6 + 4*(7
*((a + b)*c + c^2)*cosh(x)^2 + a*c - c^2)*sinh(x)^6 + 8*(7*((a + b)*c + c^2
)*cosh(x)^3 + 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 + 30*(a*c - c^2)*cosh(x)^2 + (3
*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 + 10*(a*c -
 c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a*c - c^2)*
cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 + 15*(a*c - c^2)*cosh(x)^4 + 3
*((3*a - b)*c + 3*c^2)*cosh(x)^2 + a*c - c^2)*sinh(x)^2 + (a + b)*c + c^2 +
 8*(((a + b)*c + c^2)*cosh(x)^7 + 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*c +
3*c^2)*cosh(x)^3 + (a*c - c^2)*cosh(x))*sinh(x))) - (c^2*cosh(x)^4 + 4*c^2*
cosh(x)*sinh(x)^3 + c^2*sinh(x)^4 + 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 +
c^2)*sinh(x)^2 + c^2 + 4*(c^2*cosh(x)^3 + c^2*cosh(x))*sinh(x))*sqrt(a + b
+ c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a
^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh
(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 +
a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(
3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sin
h(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 +
 a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^
2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
```

```
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b
*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 + (a^
2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 2*sqrt(2)*((a +
b)*c + c^2)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b +
3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c
^3)*cosh(x)*sinh(x)^3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 +
 2*((a + b)*c^2 + c^3)*cosh(x)^2 + 2*((a + b)*c^2 + c^3 + 3*((a + b)*c^2 +
c^3)*cosh(x)^2)*sinh(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 + ((a + b)*c^2
 + c^3)*cosh(x))*sinh(x)), -1/4*(2*(c^2*cosh(x)^4 + 4*c^2*cosh(x)*sinh(x)^3
 + c^2*sinh(x)^4 + 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 + c^2)*sinh(x)^2 +
c^2 + 4*(c^2*cosh(x)^3 + c^2*cosh(x))*sinh(x))*sqrt(-a - b - c)*arctan(sqrt
(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*
sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a - c)*sinh(
x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a + b + c)*sqr
t(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a -
c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b
+ 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)
*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8
 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a
*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)
^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b
*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 +
 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b
^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)
*c + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b
 - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c
^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15
*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c
 + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7
+ 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*
c + 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) + ((a*b +
 b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a + b)*c - 2*c^2)
*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^4 + 2*(a*b +
 b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a + b)*c - 2*c
^2)*cosh(x)^2 + a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^2 + a*b + b^2 - (2
*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^3 + (a*b +
 b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(-c)*arctan(1/2*sqrt(2)*(
(b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 +
 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 + b - 2*c)*sinh(x)^2 + 4*
((b + 2*c)*cosh(x)^3 + (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(-c)*sqrt(
((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3
*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 -
 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x
)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7
+ ((a + b)*c + c^2)*sinh(x)^8 + 4*(a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c +
 c^2)*cosh(x)^2 + a*c - c^2)*sinh(x)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 +
 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(
35*((a + b)*c + c^2)*cosh(x)^4 + 30*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3
*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 + 10*(a*c - c^2)*cosh(x)
^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a*c - c^2)*cosh(x)^2 + 4
*(7*((a + b)*c + c^2)*cosh(x)^6 + 15*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c
 + 3*c^2)*cosh(x)^2 + a*c - c^2)*sinh(x)^2 + (a + b)*c + c^2 + 8*(((a + b)*
```

```
c + c^2)*cosh(x)^7 + 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x
)^3 + (a*c - c^2)*cosh(x))*sinh(x))) + 2*sqrt(2)*((a + b)*c + c^2)*sqrt(((a
 + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a
 + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*
cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4
)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3)*cosh(x)*sinh(x)^
3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 + 2*((a + b)*c^2 + c^
3)*cosh(x)^2 + 2*((a + b)*c^2 + c^3 + 3*((a + b)*c^2 + c^3)*cosh(x)^2)*sinh
(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 + ((a + b)*c^2 + c^3)*cosh(x))*sin
h(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{\sqrt{a + b\tanh^2(x) + c\tanh^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(tanh(x)**5/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^5}{\sqrt{c\tanh(x)^4 + b\tanh(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^5/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

3.201 $\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$

Optimal. Leaf size=105

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{c}}$$

```
[Out] -ArcTanh[(b + 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4]
)]/(2*Sqrt[c]) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Rubi [A] time = 0.225196, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3700, 1251, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -ArcTanh[(b + 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4]
)]/(2*Sqrt[c]) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 3700

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+ \cdots}}\right.\\
&= \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)+\text{Subst}\left(\int \frac{}{4a+4b \cdots}\right.\\
&= \frac{\tanh^{-1}\left(\frac{-b-2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}
\end{aligned}$$

Mathematica [A] time = 0.0739012, size = 105, normalized size = 1.

$$\frac{1}{2}\left(\frac{\tanh^{-1}\left(\frac{-b-2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{\sqrt{a+b+c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] (ArcTanh[(-b - 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4
])]/Sqrt[c] + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sq
rt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/Sqrt[a + b + c])/2
```

Maple [A] time = 0.057, size = 90, normalized size = 0.9

$$-\frac{1}{2}\ln\left(\left(\frac{b}{2}+c(\tanh(x))^2\right)\frac{1}{\sqrt{c}}+\sqrt{a+b(\tanh(x))^2+c(\tanh(x))^4}\right)\frac{1}{\sqrt{c}}+\frac{1}{2}\text{Artanh}\left(\frac{b(\tanh(x))^2+2c(\tanh(x))^2 \cdots}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
```

```
[Out] -1/2*ln((1/2*b+c*tanh(x)^2)/c^(1/2)+(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))/c^(1
/2)+1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*tanh(x)^2+2*c*tanh(x)^2+2*a+b)/(a+b+c)
^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)^3}{\sqrt{c\tanh(x)^4 + b\tanh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(tanh(x)^3/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Fricas [B] time = 12.2761, size = 18436, normalized size = 175.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((a + b + c)*sqrt(c)*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*
(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*
c^2)*sinh(x)^8 + 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b
)*c + 8*c^2)*cosh(x)^2 + b^2 + 4*a*c - 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a
+ 2*b)*c + 8*c^2)*cosh(x)^3 + 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 +
2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c
 + 8*c^2)*cosh(x)^4 + 30*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a -
 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5
+ 10*(b^2 + 4*a*c - 8*c^2)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*c
osh(x))*sinh(x)^3 + 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a +
2*b)*c + 8*c^2)*cosh(x)^6 + 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 +
 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^2 + b^2 + 4*a*c - 8*c^2)*sinh(x)^2 - 4*s
qrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sin
h(x)^4 + 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 + b - 2*c)*sinh(x
)^2 + 4*((b + 2*c)*cosh(x)^3 + (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c
)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2
 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh
(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4)) + b^2 + 4*(a + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^
2)*cosh(x)^7 + 3*(b^2 + 4*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c
 + 24*c^2)*cosh(x)^3 + (b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 +
 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x
)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^
2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))
```

```
*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4
*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) +
1)) + sqrt(a + b + c)*c*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x
)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 +
2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh
(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b
- b*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(
x)^3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*
(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a +
 b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*
c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c
^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a
*b - b*c - c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 +
(a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 +
2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7
 + 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c
^2)*cosh(x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*co
sh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))
)/((a + b)*c + c^2), -1/4*(2*sqrt(-a - b - c)*c*arctan(sqrt(2)*((a + b + c)
*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a
 - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b
 + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sq
rt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2
*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^
4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sin
h(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4
 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)
*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x
)^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a
+ b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 +
4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*
c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x
)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b -
 b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(
x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) - (a + b + c)*sqrt(c)*log
(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)
*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 + 4*(b^2 + 4*a
*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 + b^2
+ 4*a*c - 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 +
 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c +
24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 + 30*(b^2
 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 +
 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 + 10*(b^2 + 4*a*c - 8*c^2)*co
sh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 + 4*(b^2 +
```

```
4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 + 1
5*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*co
sh(x)^2 + b^2 + 4*a*c - 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 +
 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 + 2*(b - 2*c)*cosh(x)^
2 + 2*(3*(b + 2*c)*cosh(x)^2 + b - 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3
+ (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4
 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2
 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) +
 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2
*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 + 3*(b^2 + 4*a*c
 - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 + (b^2 +
 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x
)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh
(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^
4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 +
 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3
*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)))/((a + b)*c + c^2), 1/4*(
2*(a + b + c)*sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c
)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 + 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b
 + 2*c)*cosh(x)^2 + b - 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b - 2*c)
*cosh(x))*sinh(x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b
+ c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2
*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)
^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)
^8 + 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 +
4*(a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c + c^2)*cosh(x)^2 + a*c - c^2)*sin
h(x)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 + 3*(a*c - c^2)*cosh(x))*sinh(x)^
5 + 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 +
 30*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*
c + c^2)*cosh(x)^5 + 10*(a*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(
x))*sinh(x)^3 + 4*(a*c - c^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6
 + 15*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 + a*c - c^2)
*sinh(x)^2 + (a + b)*c + c^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 + 3*(a*c - c^
2)*cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x)^3 + (a*c - c^2)*cosh(x))*sinh(
x))) + sqrt(a + b + c)*c*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(
x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cos
h(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b
 - b*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh
(x)^3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2
*(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a
+ b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*co
sh(x)^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)
*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c -
c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 +
a*b - b*c - c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)
*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a
+ b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*
cosh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 +
 (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 +
 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
7 + 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*
c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*c
osh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
))/((a + b)*c + c^2), -1/2*(sqrt(-a - b - c)*c*arctan(sqrt(2)*((a + b + c)*
```

```
cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a
- c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b
+ c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqr
t(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*
(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4
 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh
(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
 b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)
^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b
- b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4
+ 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*
c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)
^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a +
 b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c
 - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)
^2 + a^2 + a*b - b*c - c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c
^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b -
b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x
)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) - (a + b + c)*sqrt(-c)*arc
tan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b +
 2*c)*sinh(x)^4 + 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 + b - 2*
c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b - 2*c)*cosh(x))*sinh(x) + b + 2*
c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b +
3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*c
osh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 + 4*(a*c - c^2)*cosh(x)^6 +
4*(7*((a + b)*c + c^2)*cosh(x)^2 + a*c - c^2)*sinh(x)^6 + 8*(7*((a + b)*c +
 c^2)*cosh(x)^3 + 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2
)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 + 30*(a*c - c^2)*cosh(x)^2
+ (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 + 10*(a
*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a*c - c
^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 + 15*(a*c - c^2)*cosh(x)^4
 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 + a*c - c^2)*sinh(x)^2 + (a + b)*c + c
^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 + 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*
c + 3*c^2)*cosh(x)^3 + (a*c - c^2)*cosh(x))*sinh(x))))/((a + b)*c + c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\tanh^2(x) + c\tanh^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(tanh(x)**3/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.202 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[
x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Rubi [A] time = 0.120054, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3700, 1247, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[
x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 3700

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = -\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\right)$$
$$= \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Mathematica [A] time = 0.0321828, size = 58, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[
x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Maple [A] time = 0.075, size = 52, normalized size = 0.9

$$\frac{1}{2}\text{Artanh}\left(\frac{b(\tanh(x))^2+2c(\tanh(x))^2+2a+b}{2}\frac{1}{\sqrt{a+b+c}}\frac{1}{\sqrt{a+b(\tanh(x))^2+c(\tanh(x))^4}}\right)\frac{1}{\sqrt{a+b+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
```

```
[Out] 1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*tanh(x)^2+2*c*tanh(x)^2+2*a+b)/(a+b+c)^(1/
2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{c\tanh(x)^4+b\tanh(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Fricas [B] time = 8.64591, size = 4811, normalized size = 82.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a
^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh
(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 +
a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(
3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sin
h(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 +
 a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^
2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b
*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 + (a^
2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sqrt(a + b + c), -
1/2*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 +
 (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 +
 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4
*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh
(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^
2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(
7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b*c -
 c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*si
nh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2
*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a
^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2
)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*
a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c
^2)*cosh(x))*sinh(x)))/(a + b + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.203 $\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$

Optimal. Leaf size=106

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4]
)]/(2*Sqrt[a]) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Rubi [A] time = 0.248959, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3700, 1251, 960, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4]
)]/(2*Sqrt[a]) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 3700

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a-bx+cx^2}}+\frac{1}{x\sqrt{a-bx+cx^2}}\right)dx, x, -\tanh^2(x)\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+\cdots}}\right.\\
&= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)-\text{Subst}\left(\int \frac{1}{4a+4b+\cdots}\right.\\
&= -\frac{\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{-2a-b+(-b-2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}
\end{aligned}$$

Mathematica [A] time = 0.327901, size = 109, normalized size = 1.03

$$-\frac{\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{-2a-(b+2c)\tanh^2(x)-b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4]
)]/(2*Sqrt[a]) - ArcTanh[(-2*a - b - (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c
]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Maple [F] time = 0.218, size = 0, normalized size = 0.

$$\int \coth(x)\frac{1}{\sqrt{a+b\left(\tanh(x)\right)^2+c\left(\tanh(x)\right)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
```

```
[Out] int(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{c\tanh(x)^4 + b\tanh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Fricas [B] time = 12.1371, size = 18421, normalized size = 173.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b + c)*sqrt(a)*log(((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*
(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*
a*c)*sinh(x)^8 + 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b +
b^2 + 4*a*c)*cosh(x)^2 + 8*a^2 - b^2 - 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a
*b + b^2 + 4*a*c)*cosh(x)^3 + 3*(8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 +
2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2
+ 4*a*c)*cosh(x)^4 + 30*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b +
3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 +
10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh
(x))*sinh(x)^3 + 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b +
b^2 + 4*a*c)*cosh(x)^6 + 15*(8*a^2 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8
*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 + 8*a^2 - b^2 - 4*a*c)*sinh(x)^2 - 4*sqrt(
2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)
^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2
+ 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sq
rt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2
*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^
4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sin
h(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*c
osh(x)^7 + 3*(8*a^2 - b^2 - 4*a*c)*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12
*a*c)*cosh(x)^3 + (8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*co
sh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 +
 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3
)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh
(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh
(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) +
 sqrt(a + b + c)*a*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 +
 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6
 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c
 - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3
+ 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a +
b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
```

```
h(x)^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3
*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2
+ 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*c
osh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b -
b*c - c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(
x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x
))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a +
b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a -
 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*
(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*c
osh(x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(a^
2 + a*b + a*c), -1/4*(2*a*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh
(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)
*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((
a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(
a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4
*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^
4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*
c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^6 +
 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b - b*
c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^
2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30
*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 +
 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*
c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c
^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 +
 a^2 + a*b - b*c - c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 +
 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c
- c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3
+ (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) - (a + b + c)*sqrt(a)*log(((8*
a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh
(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 + 4*(8*a^2 - b^2 -
4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 + 8*a^2 - b
^2 - 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 + 3*(8
*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c
)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 + 30*(8*a^2 - b
^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(
8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 + 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3
 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 + 4*(8*a^2 - b^2 -
4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 + 15*(8*a^2
 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 +
 8*a^2 - b^2 - 4*a*c)*sinh(x)^2 - 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a +
 b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*
(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 + (2*a -
b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b
 + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a -
2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4
```

```
*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 + 3*(8*a^2 - b^2 - 4*a*c)
*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 + (8*a^2 - b^2 - 4
*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7
*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(
x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*c
osh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x
)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5
 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)))/(a^2 + a*b + a*c), 1/4*(2*sqrt(-a)
*(a + b + c)*arctan(1/2*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*
sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*co
sh(x)^2 + 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*
sinh(x) + 2*a + b)*sqrt(-a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(
x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x
)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + a*b + a*c)*cosh(x)^8 + 8*(a^
2 + a*b + a*c)*cosh(x)*sinh(x)^7 + (a^2 + a*b + a*c)*sinh(x)^8 + 4*(a^2 - a
*c)*cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^2 + a^2 - a*c)*sinh(x)^6 + 8
*(7*(a^2 + a*b + a*c)*cosh(x)^3 + 3*(a^2 - a*c)*cosh(x))*sinh(x)^5 + 2*(3*a
^2 - a*b + 3*a*c)*cosh(x)^4 + 2*(35*(a^2 + a*b + a*c)*cosh(x)^4 + 30*(a^2 -
 a*c)*cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*sinh(x)^4 + 8*(7*(a^2 + a*b + a*c)*c
osh(x)^5 + 10*(a^2 - a*c)*cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*cosh(x))*sinh(x
)^3 + 4*(a^2 - a*c)*cosh(x)^2 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^6 + 15*(a^2
- a*c)*cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*cosh(x)^2 + a^2 - a*c)*sinh(x)^2
 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*cosh(x)^7 + 3*(a^2 - a*c)*cosh(x)
^5 + (3*a^2 - a*b + 3*a*c)*cosh(x)^3 + (a^2 - a*c)*cosh(x))*sinh(x))) + sqr
t(a + b + c)*a*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c
^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*
(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)
^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3
*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 +
10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2
)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*
a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(
x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c
- c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*s
inh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*c
osh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*s
inh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b +
c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c
)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2
*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a +
 b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2
 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(
x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*s
inh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(a^2 +
a*b + a*c), 1/2*(sqrt(-a)*(a + b + c)*arctan(1/2*sqrt(2)*((2*a + b)*cosh(x)
^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh
(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x
)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a)*sqrt(((a + b + c)*cosh
(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh
(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh
(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + a*b
 + a*c)*cosh(x)^8 + 8*(a^2 + a*b + a*c)*cosh(x)*sinh(x)^7 + (a^2 + a*b + a*
c)*sinh(x)^8 + 4*(a^2 - a*c)*cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^2 +
 a^2 - a*c)*sinh(x)^6 + 8*(7*(a^2 + a*b + a*c)*cosh(x)^3 + 3*(a^2 - a*c)*co
```

```
sh(x))*sinh(x)^5 + 2*(3*a^2 - a*b + 3*a*c)*cosh(x)^4 + 2*(35*(a^2 + a*b + a
*c)*cosh(x)^4 + 30*(a^2 - a*c)*cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*sinh(x)^4 +
 8*(7*(a^2 + a*b + a*c)*cosh(x)^5 + 10*(a^2 - a*c)*cosh(x)^3 + (3*a^2 - a*b
 + 3*a*c)*cosh(x))*sinh(x)^3 + 4*(a^2 - a*c)*cosh(x)^2 + 4*(7*(a^2 + a*b +
a*c)*cosh(x)^6 + 15*(a^2 - a*c)*cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*cosh(x)
^2 + a^2 - a*c)*sinh(x)^2 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*cosh(x)^
7 + 3*(a^2 - a*c)*cosh(x)^5 + (3*a^2 - a*b + 3*a*c)*cosh(x)^3 + (a^2 - a*c)
*cosh(x))*sinh(x))) - a*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x
)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*c
osh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*c
osh(x)^3 + (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a
+ b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a
+ b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*c
osh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
)/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^6 + 8
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b - b*c
- c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)
*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(
a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*
c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 1
0*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c
+ 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2
)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 + a
^2 + a*b - b*c - c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8
*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c -
c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 +
(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))))/(a^2 + a*b + a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a + b\tanh^2(x) + c\tanh^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{c\tanh(x)^4 + b\tanh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

3.204 $\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$

Optimal. Leaf size=183

$$\frac{b\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4a^{3/2}}-\frac{\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}-$$

```
[Out] -ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4]
)]/(2*Sqrt[a]) + (b*ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[
x]^2 + c*Tanh[x]^4])])/(4*a^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2
)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b +
c]) - (Coth[x]^2*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])/(2*a)
```

Rubi [A] time = 0.337185, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3700, 1251, 960, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4a^{3/2}}-\frac{\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}-$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```


```
[Out] -ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4]
)]/(2*Sqrt[a]) + (b*ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[
x]^2 + c*Tanh[x]^4])])/(4*a^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2
)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b +
c]) - (Coth[x]^2*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])/(2*a)
```


Rule 3700

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol]
 :> Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
```

```
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx &= -\mathrm{Subst}\left(\int \frac{1}{x^3\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{1}{x^2\sqrt{a-bx+cx^2}} - \frac{1}{x\sqrt{a-bx+cx^2}} + \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\right) dx\right.\right.\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\right) + \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-bx+c}}\right.\\
&= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2a} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(\right.}{4a}\\
&= -\frac{\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}} + \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} -\\
&= -\frac{\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}} + \frac{b\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4a^{3/2}} + \mathrm{ta}
\end{aligned}$$

Mathematica [A] time = 0.575876, size = 142, normalized size = 0.78

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4a^{3/2}} + \frac{\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)+c\,\mathrm{t}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -((2*a - b)*ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2 + c
*Tanh[x]^4])])/(4*a^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqr
t[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c]) - (C
oth[x]^2*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])/(2*a)
```

Maple [F] time = 0.193, size = 0, normalized size = 0.

$$\int (\coth (x))^3 \frac{1}{\sqrt{a + b (\tanh (x))^2 + c (\tanh (x))^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth (x)^3}{\sqrt{c \tanh (x)^4 + b \tanh (x)^2 + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(coth(x)^3/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Fricas [B] time = 15.9068, size = 24602, normalized size = 134.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*(((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2
+ (2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x
)^4 - 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b
^2 + (2*a - b)*c)*cosh(x)^2 - 2*a^2 - a*b + b^2 - (2*a - b)*c)*sinh(x)^2 +
2*a^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh
(x)^3 - (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(a)*log(((8
*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cos
h(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 + 4*(8*a^2 - b^2 -
 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 + 8*a^2 -
b^2 - 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 + 3*(
8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*
```

```
c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 + 30*(8*a^2 -
b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*
(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 + 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^
3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 + 4*(8*a^2 - b^2 -
 4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 + 15*(8*a^
2 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2
+ 8*a^2 - b^2 - 4*a*c)*sinh(x)^2 + 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a
+ b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3
*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 + (2*a -
 b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a +
b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a -
 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 +
4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 + 3*(8*a^2 - b^2 - 4*a*c
)*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 + (8*a^2 - b^2 -
4*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(
7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh
(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*
cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(
x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^
5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) - 2*(a^2*cosh(x)^4 + 4*a^2*cosh(x)
*sinh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*si
nh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b + c)*lo
g(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^6 + 8
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b - b*c
- c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)
^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2 + a*
b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x)^4 +
 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b
*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*sinh(x
)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 +
 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^2
+ sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b
 + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a - c)
*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a + b +
c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*
(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a
 - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*co
sh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*(
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c - c^
2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 + (a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 4*sqrt(2)*(a^2 + a*b +
a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x
)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(c
osh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^
3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cosh(x)^4 + 4*(a^3 + a^2*b + a^2*c)
*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*sinh(x)^4 + a^3 + a^2*b + a^2*c
- 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 - 2*(a^3 + a^2*b + a^2*c - 3*(a^3 + a^2
*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b + a^2*c)*cosh(x)^3 - (a^
3 + a^2*b + a^2*c)*cosh(x))*sinh(x)), -1/8*(4*(a^2*cosh(x)^4 + 4*a^2*cosh(x
)*sinh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*s
inh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b - c)*
arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a
```

```
 - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^
4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2
 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
- 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 +
 (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c
^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^
2 + a*b - b*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*
a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2
*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2
+ 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*
a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*
b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*
(3*a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^2 + a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)
*cosh(x)^7 + 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2
*(3*a + b)*c + 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))
) + ((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2 + (
2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^4
 - 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2
+ (2*a - b)*c)*cosh(x)^2 - 2*a^2 - a*b + b^2 - (2*a - b)*c)*sinh(x)^2 + 2*a
^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)
^3 - (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(a)*log(((8*a^
2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x
)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 + 4*(8*a^2 - b^2 - 4*
a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 + 8*a^2 - b^2
 - 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 + 3*(8*a
^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*
cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 + 30*(8*a^2 - b^2
 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(8*
a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 + 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3 +
 (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 + 4*(8*a^2 - b^2 - 4*
a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 + 15*(8*a^2 -
 b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 + 8
*a^2 - b^2 - 4*a*c)*sinh(x)^2 + 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b
)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2
*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 + (2*a - b)
*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b +
 c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*
c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4*a
*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 + 3*(8*a^2 - b^2 - 4*a*c)*c
osh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 + (8*a^2 - b^2 - 4*a
*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*c
osh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)
^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cos
h(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^
4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 +
 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 4*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((
a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(
a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4
*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^
4)))/((a^3 + a^2*b + a^2*c)*cosh(x)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sin
h(x)^3 + (a^3 + a^2*b + a^2*c)*sinh(x)^4 + a^3 + a^2*b + a^2*c - 2*(a^3 + a
^2*b + a^2*c)*cosh(x)^2 - 2*(a^3 + a^2*b + a^2*c - 3*(a^3 + a^2*b + a^2*c)*
```

```
cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b + a^2*c)*cosh(x)^3 - (a^3 + a^2*b +
a^2*c)*cosh(x))*sinh(x)), 1/4*(((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4
 + 4*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b
^2 + (2*a - b)*c)*sinh(x)^4 - 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2
 + 2*(3*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 - 2*a^2 - a*b + b^2 - (
2*a - b)*c)*sinh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b -
 b^2 + (2*a - b)*c)*cosh(x)^3 - (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*
sinh(x))*sqrt(-a)*arctan(1/2*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cos
h(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a +
b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh
(x))*sinh(x) + 2*a + b)*sqrt(-a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*
sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*s
inh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*si
nh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + a*b + a*c)*cosh(x)^8 +
8*(a^2 + a*b + a*c)*cosh(x)*sinh(x)^7 + (a^2 + a*b + a*c)*sinh(x)^8 + 4*(a^
2 - a*c)*cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^2 + a^2 - a*c)*sinh(x)^
6 + 8*(7*(a^2 + a*b + a*c)*cosh(x)^3 + 3*(a^2 - a*c)*cosh(x))*sinh(x)^5 + 2
*(3*a^2 - a*b + 3*a*c)*cosh(x)^4 + 2*(35*(a^2 + a*b + a*c)*cosh(x)^4 + 30*(
a^2 - a*c)*cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*sinh(x)^4 + 8*(7*(a^2 + a*b + a
*c)*cosh(x)^5 + 10*(a^2 - a*c)*cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*cosh(x))*s
inh(x)^3 + 4*(a^2 - a*c)*cosh(x)^2 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^6 + 15*
(a^2 - a*c)*cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*cosh(x)^2 + a^2 - a*c)*sinh
(x)^2 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*cosh(x)^7 + 3*(a^2 - a*c)*co
sh(x)^5 + (3*a^2 - a*b + 3*a*c)*cosh(x)^3 + (a^2 - a*c)*cosh(x))*sinh(x)))
+ (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^
2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh
(x))*sinh(x))*sqrt(a + b + c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*
cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2
)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2
+ a*b - b*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)
*cosh(x)^3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*
b + 2*(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
 + c^2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b +
2*(a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a
 + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 +
 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b
*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a
^2 + a*b - b*c - c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b
 + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(
3*(a + b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a
- c)*cosh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x
)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x
)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x
) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^7 + 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4
+ 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4)) - 2*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b +
 c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*
c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cos
h(x)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*
sinh(x)^4 + a^3 + a^2*b + a^2*c - 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 - 2*(a^
3 + a^2*b + a^2*c - 3*(a^3 + a^2*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
+ a^2*b + a^2*c)*cosh(x)^3 - (a^3 + a^2*b + a^2*c)*cosh(x))*sinh(x)), 1/4*(
((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2 + (2*a
```

```
- b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^4 - 2
*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2 + (2
*a - b)*c)*cosh(x)^2 - 2*a^2 - a*b + b^2 - (2*a - b)*c)*sinh(x)^2 + 2*a^2 +
 a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^3 -
 (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(-a)*arctan(1/2*sq
rt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh
(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)
^2 + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a
)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2
 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh
(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4))/((a^2 + a*b + a*c)*cosh(x)^8 + 8*(a^2 + a*b + a*c)*cosh(x)*sin
h(x)^7 + (a^2 + a*b + a*c)*sinh(x)^8 + 4*(a^2 - a*c)*cosh(x)^6 + 4*(7*(a^2
+ a*b + a*c)*cosh(x)^2 + a^2 - a*c)*sinh(x)^6 + 8*(7*(a^2 + a*b + a*c)*cosh
(x)^3 + 3*(a^2 - a*c)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - a*b + 3*a*c)*cosh(x)^
4 + 2*(35*(a^2 + a*b + a*c)*cosh(x)^4 + 30*(a^2 - a*c)*cosh(x)^2 + 3*a^2 -
a*b + 3*a*c)*sinh(x)^4 + 8*(7*(a^2 + a*b + a*c)*cosh(x)^5 + 10*(a^2 - a*c)*
cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*cosh(x))*sinh(x)^3 + 4*(a^2 - a*c)*cosh(x
)^2 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^6 + 15*(a^2 - a*c)*cosh(x)^4 + 3*(3*a^
2 - a*b + 3*a*c)*cosh(x)^2 + a^2 - a*c)*sinh(x)^2 + a^2 + a*b + a*c + 8*((a
^2 + a*b + a*c)*cosh(x)^7 + 3*(a^2 - a*c)*cosh(x)^5 + (3*a^2 - a*b + 3*a*c)
*cosh(x)^3 + (a^2 - a*c)*cosh(x))*sinh(x))) - 2*(a^2*cosh(x)^4 + 4*a^2*cosh
(x)*sinh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)
*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b - c
)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 +
 a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x
)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)
^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^
2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c
^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7
 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c -
 c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 +
a^2 + a*b - b*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2)*cosh(x)^3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 +
2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 +
 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^
2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (
3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4*(a^2 +
a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*c
osh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 +
2*(3*a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^2 + a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^7 + 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 +
 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x
))) - 2*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)
*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*
sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*s
inh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cosh(x
)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*sin
h(x)^4 + a^3 + a^2*b + a^2*c - 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 - 2*(a^3 +
 a^2*b + a^2*c - 3*(a^3 + a^2*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a
^2*b + a^2*c)*cosh(x)^3 - (a^3 + a^2*b + a^2*c)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{\sqrt{a + b\tanh^2(x) + c\tanh^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(coth(x)**3/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.205 $\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx$

Optimal. Leaf size=132

$$-\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)-\frac{1}{2}$$

```
[Out] -((b + 2*c)*ArcTanh[(b + 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c
*Tanh[x]^4])])/(4*Sqrt[c]) + (Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*
Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])])/2 - Sq
rt[a + b*Tanh[x]^2 + c*Tanh[x]^4]/2
```

Rubi [A] time = 0.231061, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3700, 1247, 734, 843, 621, 206, 724}

$$-\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)-\frac{1}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```


```
[Out] -((b + 2*c)*ArcTanh[(b + 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c
*Tanh[x]^4])])/(4*Sqrt[c]) + (Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*
Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])])/2 - Sq
rt[a + b*Tanh[x]^2 + c*Tanh[x]^4]/2
```


Rule 3700

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx &= -\text{Subst}\left(\int \frac{x\sqrt{a-bx^2+cx^4}}{1+x^2}\,dx, x, i\tanh(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a-bx+cx^2}}{1+x}\,dx, x, -\tanh^2(x)\right)\right)\\
&= -\frac{1}{2}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}+\frac{1}{4}\text{Subst}\left(\int \frac{-2a-b+(b+2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,dx\right.\\
&= -\frac{1}{2}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}+\frac{1}{2}(-a-b-c)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-}}\right.\\
&= -\frac{1}{2}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}+(a+b+c)\,\text{Subst}\left(\int \frac{1}{4a+4b+4c-}\right.\\
&= -\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\tanh^{-1}\left(\frac{}{2\sqrt{a}}\right.
\end{aligned}$$

Mathematica [A] time = 0.32414, size = 131, normalized size = 0.99

$$\frac{1}{4}\left(\frac{(b+2c)\tanh^{-1}\left(\frac{-b-2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{\sqrt{c}}+2\sqrt{a+b+c}\tanh^{-1}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)-2\sqrt{a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] (((b + 2*c)*ArcTanh[(-b - 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 +
c*Tanh[x]^4])]])/Sqrt[c] + 2*Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*Ta
nh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])] - 2*Sqrt[
a + b*Tanh[x]^2 + c*Tanh[x]^4])/4
```

Maple [C] time = 0.082, size = 559, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*tanh(x),x)
```

```
[Out] -1/2*(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)-1/8*(-b-c)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*tanh(x)^2)^(1/2)*(4+2*(b+(-
4*a*c+b^2)^(1/2))/a*tanh(x)^2)^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*Elli
pticF(1/2*tanh(x)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*ln((b+2*c*tanh(x)^2)/c^(1/2)+2*(a+b*tan
h(x)^2+c*tanh(x)^4)^(1/2))*c^(1/2)-1/4*ln((b+2*c*tanh(x)^2)/c^(1/2)+2*(a+b*
tanh(x)^2+c*tanh(x)^4)^(1/2))/c^(1/2)*b+1/2*a/(a+b+c)^(1/2)*arctanh(1/2*(b*
tanh(x)^2+2*c*tanh(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1
/2))+1/2*b/(a+b+c)^(1/2)*arctanh(1/2*(b*tanh(x)^2+2*c*tanh(x)^2+2*a+b)/(a+b
+c)^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))+1/2*c/(a+b+c)^(1/2)*arctanh(1/
2*(b*tanh(x)^2+2*c*tanh(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^
4)^(1/2))-1/8*(b+c)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*tanh(x)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*tanh(x)^2)
^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*EllipticF(1/2*tanh(x)*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \tanh(x)^4 + b \tanh(x)^2 + a} \tanh(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a)*tanh(x), x)
```

Fricas [B] time = 20.8531, size = 22316, normalized size = 169.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/8*(((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh
(x)^4 + 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 + b + 2*c)*sinh(x)
^2 + 4*((b + 2*c)*cosh(x)^3 + (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)
*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*
c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 + 4*(b^2 +
 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 +
b^2 + 4*a*c - 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)
^3 + 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*
c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 + 30*
(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)
^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 + 10*(b^2 + 4*a*c - 8*c^2
)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 + 4*(b^
2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6
 + 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2
)*cosh(x)^2 + b^2 + 4*a*c - 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)
^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 + 2*(b - 2*c)*cosh
(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 + b - 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x
)^3 + (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(
x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(
x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(
x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a
 + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 + 3*(b^2 + 4
*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 + (b
^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + si
nh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*
cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh
(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)
^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7
 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*(c*cosh(x)^4 + 4*
c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 + 2*c*cosh(x)^2 + 2*(3*c*cosh(x)^2 + c)*s
inh(x)^2 + 4*(c*cosh(x)^3 + c*cosh(x))*sinh(x) + c)*sqrt(a + b + c)*log(((a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^6 + 8*(7*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b - b*c - c^2
)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^4 +
2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2 + a*b - b
*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b*c -
c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 +
 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*
c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*
b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^2 + sqr
t(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)
*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a - c)*sinh
(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a + b + c)*sq
rt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a -
c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b
+ 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)
*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2
+ 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c - c^2)*co
sh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*
c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*s
inh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 4*sqrt(2)*c*sqrt(((a + b + c
)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c
)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^
3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(c*c
osh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 + 2*c*cosh(x)^2 + 2*(3*c*cos
```

```
h(x)^2 + c)*sinh(x)^2 + 4*(c*cosh(x)^3 + c*cosh(x))*sinh(x) + c), -1/8*(4*(
c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 + 2*c*cosh(x)^2 + 2*(3*c*
cosh(x)^2 + c)*sinh(x)^2 + 4*(c*cosh(x)^3 + c*cosh(x))*sinh(x) + c)*sqrt(-a
 - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sin
h(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cos
h(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sin
h(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c
)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)
*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*
sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*s
inh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b
 - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh
(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a +
 b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(
3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^2)*cosh(x)^
2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*
a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b*c - c^2)*cosh(
x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 + 4
*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b
- b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^2
 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b
)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b
 - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c^2)*cosh(x)
)*sinh(x))) - ((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2
*c)*sinh(x)^4 + 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 + b + 2*c)
*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)
*sqrt(c)*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b
)*c + 8*c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 +
4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh
(x)^2 + b^2 + 4*a*c - 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)
*cosh(x)^3 + 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a
 - 2*b)*c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)
^4 + 30*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)
*sinh(x)^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 + 10*(b^2 + 4*a*c
 - 8*c^2)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3
 + 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*c
osh(x)^6 + 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c
+ 24*c^2)*cosh(x)^2 + b^2 + 4*a*c - 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)
*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 + 2*(b - 2
*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 + b - 2*c)*sinh(x)^2 + 4*((b + 2*c
)*cosh(x)^3 + (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b +
c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^
2 + 4*(a + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 + 3*
(b^2 + 4*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x
)^3 + (b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x
)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x
)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4
+ 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7
*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(c
osh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 4*sqrt(2)*c
*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2
+ 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(
x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
sinh(x)^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 + 2*c*cosh(
```

```
x)^2 + 2*(3*c*cosh(x)^2 + c)*sinh(x)^2 + 4*(c*cosh(x)^3 + c*cosh(x))*sinh(x
) + c), 1/4*(((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*
c)*sinh(x)^4 + 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 + b + 2*c)*
sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)*
sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh
(x)^3 + (b + 2*c)*sinh(x)^4 + 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x
)^2 + b - 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b - 2*c)*cosh(x))*sinh
(x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b
)*c + c^2)*cosh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 + 4*(a*c - c^2)*
cosh(x)^6 + 4*(7*((a + b)*c + c^2)*cosh(x)^2 + a*c - c^2)*sinh(x)^6 + 8*(7*
((a + b)*c + c^2)*cosh(x)^3 + 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a -
b)*c + 3*c^2)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 + 30*(a*c - c^2
)*cosh(x)^2 + (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(
x)^5 + 10*(a*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3
+ 4*(a*c - c^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 + 15*(a*c - c^
2)*cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 + a*c - c^2)*sinh(x)^2 + (
a + b)*c + c^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 + 3*(a*c - c^2)*cosh(x)^5 +
 ((3*a - b)*c + 3*c^2)*cosh(x)^3 + (a*c - c^2)*cosh(x))*sinh(x))) + (c*cosh
(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 + 2*c*cosh(x)^2 + 2*(3*c*cosh(x
)^2 + c)*sinh(x)^2 + 4*(c*cosh(x)^3 + c*cosh(x))*sinh(x) + c)*sqrt(a + b +
c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^
6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b -
 b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2
 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x
)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*
b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*s
inh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*
a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(
x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + a
 - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
+ 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c
 - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 + (a^2
+ a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 2*sqrt(2)*c*sqrt((
(a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*
(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 -
4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)
^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 + 2*c*cosh(x)^2 +
2*(3*c*cosh(x)^2 + c)*sinh(x)^2 + 4*(c*cosh(x)^3 + c*cosh(x))*sinh(x) + c),
 -1/4*(2*(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 + 2*c*cosh(x)^2
 + 2*(3*c*cosh(x)^2 + c)*sinh(x)^2 + 4*(c*cosh(x)^3 + c*cosh(x))*sinh(x) +
c)*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*c
osh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a +
b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*co
sh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 +
(a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 +
```

```
2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)
*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4*
(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
 c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(
x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^2
)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b*c -
c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sin
h(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^
2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)
*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a
^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c^
2)*cosh(x))*sinh(x))) - ((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^
3 + (b + 2*c)*sinh(x)^4 + 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2
+ b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b + 2*c)*cosh(x))*sinh(x)
+ b + 2*c)*sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*c
osh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 + 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b +
2*c)*cosh(x)^2 + b - 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 + (b - 2*c)*co
sh(x))*sinh(x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c
)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)
*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*
sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8
+ 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 + 4*(
a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c + c^2)*cosh(x)^2 + a*c - c^2)*sinh(x
)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 + 3*(a*c - c^2)*cosh(x))*sinh(x)^5 +
 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 + 30
*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c +
 c^2)*cosh(x)^5 + 10*(a*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 + 4*(a*c - c^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 + 1
5*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 + a*c - c^2)*si
nh(x)^2 + (a + b)*c + c^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 + 3*(a*c - c^2)*
cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x)^3 + (a*c - c^2)*cosh(x))*sinh(x))
) + 2*sqrt(2)*c*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 + 4*(a
- c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^2 + 3*a -
b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(
x)*sinh(x)^3 + sinh(x)^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x
)^4 + 2*c*cosh(x)^2 + 2*(3*c*cosh(x)^2 + c)*sinh(x)^2 + 4*(c*cosh(x)^3 + c*
cosh(x))*sinh(x) + c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\tanh^2(x) + c\tanh^4(x)}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2+c*tanh(x)**4)**(1/2)*tanh(x),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2 + c*tanh(x)**4)*tanh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \tanh(x)^4 + b \tanh(x)^2 + a} \tanh(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a)*tanh(x), x)
```

3.206 $\int e^{a+bx}\tanh^4(a+bx)\,dx$

Optimal. Leaf size=107

$$\frac{e^{a+bx}}{b}+\frac{5e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{14e^{a+bx}}{3b\left(e^{2a+2bx}+1\right)^2}+\frac{8e^{a+bx}}{3b\left(e^{2a+2bx}+1\right)^3}-\frac{3\tan^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b + (8*E^(a + b*x))/(3*b*(1 + E^(2*a + 2*b*x))^3) - (14*E^(a +
b*x))/(3*b*(1 + E^(2*a + 2*b*x))^2) + (5*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*
x))) - (3*ArcTan[E^(a + b*x)])/b
```

Rubi [A] time = 0.0685635, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 390, 1258, 1157, 385, 203}

$$\frac{e^{a+bx}}{b}+\frac{5e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{14e^{a+bx}}{3b\left(e^{2a+2bx}+1\right)^2}+\frac{8e^{a+bx}}{3b\left(e^{2a+2bx}+1\right)^3}-\frac{3\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Tanh[a + b*x]^4,x]
```

```
[Out] E^(a + b*x)/b + (8*E^(a + b*x))/(3*b*(1 + E^(2*a + 2*b*x))^3) - (14*E^(a +
b*x))/(3*b*(1 + E^(2*a + 2*b*x))^2) + (5*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*
x))) - (3*ArcTan[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1258

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sy
mbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e
^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^
2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + c*x^4)
^p - (-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)],
 x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[
m/2, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\tanh^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(1+x^2\right)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{8x^2\left(1+x^4\right)}{\left(1+x^2\right)^4}\right)dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}-\frac{8\,\text{Subst}\left(\int \frac{x^2\left(1+x^4\right)}{\left(1+x^2\right)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1+e^{2a+2bx}\right)^3}+\frac{4\,\text{Subst}\left(\int \frac{-2+6x^2-6x^4}{\left(1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{3b}\\
&= \frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{14e^{a+bx}}{3b\left(1+e^{2a+2bx}\right)^2}-\frac{\text{Subst}\left(\int \frac{-6+24x^2}{\left(1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{3b}\\
&= \frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{14e^{a+bx}}{3b\left(1+e^{2a+2bx}\right)^2}+\frac{5e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}-\frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{14e^{a+bx}}{3b\left(1+e^{2a+2bx}\right)^2}+\frac{5e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}-\frac{3\tan^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.131919, size = 76, normalized size = 0.71

$$
\frac{e^{a+bx}\left(25e^{2(a+bx)}+24e^{4(a+bx)}+3e^{6(a+bx)}+12\right)}{3b\left(e^{2(a+bx)}+1\right)^3}-\frac{3\tan^{-1}\left(e^{a+bx}\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Tanh[a + b*x]^4,x]
```

```
[Out] (E^(a + b*x)*(12 + 25*E^(2*(a + b*x)) + 24*E^(4*(a + b*x)) + 3*E^(6*(a + b*
x))))/(3*b*(1 + E^(2*(a + b*x)))^3) - (3*ArcTan[E^(a + b*x)])/b
```

Maple [A] time = 0.013, size = 143, normalized size = 1.3

$$\frac{(\sinh(bx+a))^4}{b(\cosh(bx+a))^3}+\frac{4(\sinh(bx+a))^2}{3b(\cosh(bx+a))^3}-\frac{8(\sinh(bx+a))^2}{3b\cosh(bx+a)}+\frac{8\cosh(bx+a)}{3b}+\frac{(\sinh(bx+a))^3}{b(\cosh(bx+a))^2}+3\frac{\sinh(bx}{b(\cosh(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*tanh(b*x+a)^4,x)
```

```
[Out] 1/b*sinh(b*x+a)^4/cosh(b*x+a)^3+4/3/b*sinh(b*x+a)^2/cosh(b*x+a)^3-8/3/b*sin
h(b*x+a)^2/cosh(b*x+a)+8/3/b*cosh(b*x+a)+1/b*sinh(b*x+a)^3/cosh(b*x+a)^2+3/
b*sinh(b*x+a)/cosh(b*x+a)^2-3/2/b*sech(b*x+a)*tanh(b*x+a)-3*arctan(exp(b*x+
a))/b
```

Maxima [A] time = 1.57505, size = 127, normalized size = 1.19

$$-\frac{3\arctan\left(e^{(bx+a)}\right)}{b}+\frac{e^{(bx+a)}}{b}+\frac{15\,e^{(5\,bx+5\,a)}+16\,e^{(3\,bx+3\,a)}+9\,e^{(bx+a)}}{3\,b\left(e^{(6\,bx+6\,a)}+3\,e^{(4\,bx+4\,a)}+3\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -3*arctan(e^(b*x + a))/b + e^(b*x + a)/b + 1/3*(15*e^(5*b*x + 5*a) + 16*e^(
3*b*x + 3*a) + 9*e^(b*x + a))/(b*(e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) + 3*e
^(2*b*x + 2*a) + 1))
```

Fricas [B] time = 2.24032, size = 1685, normalized size = 15.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + 3*(21*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^5 + 24*cosh(b*x + a)^5 + 15*(
7*cosh(b*x + a)^3 + 8*cosh(b*x + a))*sinh(b*x + a)^4 + 5*(21*cosh(b*x + a)^
4 + 48*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 25*cosh(b*x + a)^3 + 3*(21*co
sh(b*x + a)^5 + 80*cosh(b*x + a)^3 + 25*cosh(b*x + a))*sinh(b*x + a)^2 - 9*
(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*
x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*
cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) +
sinh(b*x + a)) + 3*(7*cosh(b*x + a)^6 + 40*cosh(b*x + a)^4 + 25*cosh(b*x +
a)^2 + 4)*sinh(b*x + a) + 12*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b
*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*
```

```
cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*
x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*
b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*
x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\tanh^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)**4,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*tanh(a + b*x)**4, x)
```

Giac [A] time = 1.80442, size = 92, normalized size = 0.86

$$\frac{\frac{15\,e^{(5\,bx+5\,a)}+16\,e^{(3\,bx+3\,a)}+9\,e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^{3}}-9\,\arctan\left(e^{(bx+a)}\right)+3\,e^{(bx+a)}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*((15*e^(5*b*x + 5*a) + 16*e^(3*b*x + 3*a) + 9*e^(b*x + a))/(e^(2*b*x +
2*a) + 1)^3 - 9*arctan(e^(b*x + a)) + 3*e^(b*x + a))/b
```

3.207 $\int e^{a+bx} \tanh^3(a+bx)\,dx$

Optimal. Leaf size=77

$$\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)^2}-\frac{3\tan^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 + E^(2*a + 2*b*x))) - (3*ArcTan[E^(a + b*x)])/b
```

Rubi [A] time = 0.0498214, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 390, 1158, 12, 288, 203}

$$\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)^2}-\frac{3\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Tanh[a + b*x]^3,x]
```

```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 + E^(2*a + 2*b*x))) - (3*ArcTan[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\tanh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{2\left(1+3x^4\right)}{\left(1+x^2\right)^3}\right)dx, x, e^{a+bx}\right)}{b} \\
&= \frac{e^{a+bx}}{b} - \frac{2\,\text{Subst}\left(\int \frac{1+3x^4}{\left(1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{\text{Subst}\left(\int -\frac{12x^2}{\left(1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{2b} \\
&= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} - \frac{6\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} - \frac{3\tan^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0887836, size = 60, normalized size = 0.78

$$
\frac{e^{a+bx}\left(5e^{2(a+bx)}+e^{4(a+bx)}+2\right)}{b\left(e^{2(a+bx)}+1\right)^2} - \frac{3\tan^{-1}\left(e^{a+bx}\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Tanh[a + b*x]^3,x]
```

```
[Out] (E^(a + b*x)*(2 + 5*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(b*(1 + E^(2*(a + b
*x)))^2) - (3*ArcTan[E^(a + b*x)])/b
```

Maple [A] time = 0.012, size = 102, normalized size = 1.3

$$
\frac{(\sinh(bx+a))^3}{b(\cosh(bx+a))^2} + 3\frac{\sinh(bx+a)}{b(\cosh(bx+a))^2} - \frac{3\,\text{sech}(bx+a)\tanh(bx+a)}{2b} - 3\frac{\arctan\left(e^{bx+a}\right)}{b} - \frac{(\sinh(bx+a))^2}{b\cosh(bx+a)} + 2\,\text{cosh}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*tanh(b*x+a)^3,x)
```

```
[Out] 1/b*sinh(b*x+a)^3/cosh(b*x+a)^2+3/b*sinh(b*x+a)/cosh(b*x+a)^2-3/2/b*sech(b*
x+a)*tanh(b*x+a)-3*arctan(exp(b*x+a))/b-1/b*sinh(b*x+a)^2/cosh(b*x+a)+2/b*c
osh(b*x+a)
```

Maxima [A] time = 1.58349, size = 93, normalized size = 1.21

$$-\frac{3\,\arctan\left(e^{(bx+a)}\right)}{b}+\frac{e^{(bx+a)}}{b}+\frac{3\,e^{(3\,bx+3\,a)}+e^{(bx+a)}}{b\left(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -3*arctan(e^(b*x + a))/b + e^(b*x + a)/b + (3*e^(3*b*x + 3*a) + e^(b*x + a)
)/(b*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1))
```

Fricas [B] time = 2.11078, size = 956, normalized size = 12.42

$$\frac{\cosh(bx+a)^5+5\,\cosh(bx+a)\sinh(bx+a)^4+\sinh(bx+a)^5+5\left(2\,\cosh(bx+a)^2+1\right)\sinh(bx+a)^3+5\,\cosh(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 5*(2
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 5*cosh(b*x + a)^3 + 5*(2*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^4 + 4*cosh(b*x
+ a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x
+ a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (5*cosh(b*x + a)^4 + 15*c
osh(b*x + a)^2 + 2)*sinh(b*x + a) + 2*cosh(b*x + a))/(b*cosh(b*x + a)^4 + 4
*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2
+ 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*co
sh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\tanh^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)**3,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*tanh(a + b*x)**3, x)
```

Giac [A] time = 2.13441, size = 70, normalized size = 0.91

$$\frac{\frac{3\,e^{(3\,bx+3\,a)}+e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^{2}}-3\,\arctan\left(e^{(bx+a)}\right)+e^{(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] ((3*e^(3*b*x + 3*a) + e^(b*x + a))/(e^(2*b*x + 2*a) + 1)^2 - 3*arctan(e^(b*
x + a)) + e^(b*x + a))/b
```

3.208 $\int e^{a+bx} \tanh^2(a+bx)\,dx$

Optimal. Leaf size=51

$$\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))) - (2*ArcTan[E^(a
+ b*x)])/b
```

Rubi [A] time = 0.0349531, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 390, 288, 203}

$$\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Tanh[a + b*x]^2,x]
```

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))) - (2*ArcTan[E^(a
+ b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{a+bx}\tanh^2(a+bx)\,dx=\frac{\text{Subst}\left(\int\frac{(1-x^2)^2}{(1+x^2)^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{\text{Subst}\left(\int\left(1-\frac{4x^2}{(1+x^2)^2}\right)dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}-\frac{4\,\text{Subst}\left(\int\frac{x^2}{(1+x^2)^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}-\frac{2\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}-\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [A] time = 0.0875275, size = 40, normalized size = 0.78

$$\frac{e^{a+bx}\left(\frac{2}{e^{2(a+bx)}+1}+1\right)-2\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Tanh[a + b*x]^2,x]
```

```
[Out] (E^(a + b*x)*(1 + 2/(1 + E^(2*(a + b*x)))) - 2*ArcTan[E^(a + b*x)])/b
```

Maple [A] time = 0.007, size = 56, normalized size = 1.1

$$-\frac{(\sinh(bx+a))^2}{b\cosh(bx+a)}+2\,\frac{\cosh(bx+a)}{b}+\frac{\sinh(bx+a)}{b}-2\,\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*tanh(b*x+a)^2,x)
```

```
[Out] -1/b*sinh(b*x+a)^2/cosh(b*x+a)+2/b*cosh(b*x+a)+1/b*sinh(b*x+a)-2*arctan(exp
(b*x+a))/b
```

Maxima [A] time = 1.59225, size = 63, normalized size = 1.24

$$-\frac{2\arctan\left(e^{(bx+a)}\right)}{b}+\frac{e^{(bx+a)}}{b}+\frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(b*x + a))/b + e^(b*x + a)/b + 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a
) + 1))
```

Fricas [B] time = 1.98645, size = 433, normalized size = 8.49

$$\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - 2\left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \right.}{b\cosh(bx+a)^2 + 2b\cosh(bx+a)\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - 2*(c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arcta
n(cosh(b*x + a) + sinh(b*x + a)) + 3*(cosh(b*x + a)^2 + 1)*sinh(b*x + a) +
3*cosh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*s
inh(b*x + a)^2 + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\tanh^{2}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*tanh(a + b*x)**2, x)
```

Giac [A] time = 2.20872, size = 55, normalized size = 1.08

$$\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}+1} - 2\arctan\left(e^{(bx+a)}\right) + e^{(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (2*e^(b*x + a)/(e^(2*b*x + 2*a) + 1) - 2*arctan(e^(b*x + a)) + e^(b*x + a))
/b
```

3.209 $\int e^{a+bx} \tanh(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{e^{a+bx}}{b}-\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b - (2*ArcTan[E^(a + b*x)])/b
```

Rubi [A] time = 0.0148009, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 388, 203}

$$\frac{e^{a+bx}}{b}-\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Tanh[a + b*x],x]
```

```
[Out] E^(a + b*x)/b - (2*ArcTan[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\tanh(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{1+x^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{e^{a+bx}}{b}-\frac{2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{e^{a+bx}}{b}-\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0144078, size = 22, normalized size = 0.88

$$\frac{e^{a+bx} - 2\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Tanh[a + b*x],x]
```

```
[Out] (E^(a + b*x) - 2*ArcTan[E^(a + b*x)])/b
```

Maple [A] time = 0.01, size = 34, normalized size = 1.4

$$\frac{\sinh(bx+a)}{b} - 2\,\frac{\arctan\left(e^{bx+a}\right)}{b} + \frac{\cosh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*tanh(b*x+a),x)
```

```
[Out] 1/b*sinh(b*x+a)-2*arctan(exp(b*x+a))/b+1/b*cosh(b*x+a)
```

Maxima [A] time = 1.61474, size = 31, normalized size = 1.24

$$-\frac{2\arctan\left(e^{(bx+a)}\right)}{b} + \frac{e^{(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(b*x + a))/b + e^(b*x + a)/b
```

Fricas [A] time = 1.92056, size = 105, normalized size = 4.2

$$-\frac{2\arctan\left(\cosh(bx+a) + \sinh(bx+a)\right) - \cosh(bx+a) - \sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*arctan(cosh(b*x + a) + sinh(b*x + a)) - cosh(b*x + a) - sinh(b*x + a))/
b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\tanh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a),x)
```

```
[Out] exp(a)*Integral(exp(b*x)*tanh(a + b*x), x)
```

Giac [A] time = 1.52603, size = 31, normalized size = 1.24

$$-\frac{2\arctan\left(e^{(bx+a)}\right)-e^{(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -(2*arctan(e^(b*x + a)) - e^(b*x + a))/b
```

3.210 $\int e^{a+bx} \coth(a+bx) \, dx$

Optimal. Leaf size=25

$$\frac{e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0168253, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 388, 206}

$$\frac{e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x],x]
```

```
[Out] E^(a + b*x)/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\coth(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx,x,e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}-\frac{2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0165427, size = 22, normalized size = 0.88

$$\frac{e^{a+bx}-2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x],x]
```

```
[Out] (E^(a + b*x) - 2*ArcTanh[E^(a + b*x)])/b
```

Maple [A] time = 0.011, size = 27, normalized size = 1.1

$$\frac{\sinh(bx+a)+\cosh(bx+a)-2\,\mathrm{Artanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*coth(b*x+a),x)
```

```
[Out] 1/b*(sinh(b*x+a)+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Maxima [A] time = 1.03853, size = 51, normalized size = 2.04

$$\frac{e^{(bx+a)}}{b}-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Fricas [B] time = 2.04564, size = 158, normalized size = 6.32

$$\frac{\cosh(bx+a)-\log(\cosh(bx+a)+\sinh(bx+a)+1)+\log(\cosh(bx+a)+\sinh(bx+a)-1)+\sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + sinh(b*x + a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\coth(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a),x)
```

```
[Out] exp(a)*Integral(exp(b*x)*coth(a + b*x), x)
```

Giac [A] time = 1.60445, size = 43, normalized size = 1.72

$$\frac{e^{(bx+a)} - \log\left(e^{(bx+a)} + 1\right) + \log\left(\left|e^{(bx+a)} - 1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a),x, algorithm="giac")
```

```
[Out] (e^(b*x + a) - log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

3.211 $\int e^{a+bx} \coth^2(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a
 + b*x)])/b
```

Rubi [A] time = 0.0377712, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 390, 288, 206}

$$\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a
 + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{4x^2}{(1-x^2)^2}\right)dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{e^{a+bx}}{b} + \frac{4\,\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [C] time = 1.72062, size = 179, normalized size = 3.38

$$\frac{e^{a+bx}\left(\frac{4}{105}\left(e^{a+bx}+e^{3(a+bx)}\right)^2 \text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2\right\},\left\{1,1,\frac{9}{2}\right\},e^{2(a+bx)}\right)+\frac{1}{48}e^{-4(a+bx)}\left(-713e^{2(a+bx)}-181e^{4}\right.\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^2,x]
```

```
[Out] (E^(a + b*x)*((-375 - 713*E^(2*(a + b*x)) - 181*E^(4*(a + b*x)) + 61*E^(6*(
a + b*x)) + (3*(125 + 196*E^(2*(a + b*x)) - 14*E^(4*(a + b*x)) - 52*E^(6*(a
 + b*x)) + E^(8*(a + b*x)))*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a +
b*x))])/(48*E^(4*(a + b*x))) + (4*(E^(a + b*x) + E^(3*(a + b*x)))^2*Hyperge
ometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*(a + b*x))])/105))/b
```

Maple [A] time = 0.01, size = 47, normalized size = 0.9

$$\frac{1}{b}\left(\cosh(bx+a) - 2\,\text{Artanh}\left(e^{bx+a}\right) - \frac{(\cosh(bx+a))^2}{\sinh(bx+a)} + 2\sinh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*coth(b*x+a)^2,x)
```

```
[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a))-cosh(b*x+a)^2/sinh(b*x+a)+2*sinh(b*x
+a))
```

Maxima [A] time = 1.04158, size = 84, normalized size = 1.58

$$\frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b} - \frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x
+ a)/(b*(e^(2*b*x + 2*a) - 1))
```

Fricas [B] time = 2.06683, size = 585, normalized size = 11.04

$$\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - \left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cos
h(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(
cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^2 +
2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\coth^{2}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*coth(a + b*x)**2, x)
```

Giac [A] time = 1.29135, size = 76, normalized size = 1.43

$$-\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1} - e^{(bx+a)} + \log\left(e^{(bx+a)}+1\right) - \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(b*x + a) + log(e^(b*x + a) + 1)
- log(abs(e^(b*x + a) - 1)))/b
```

3.212 $\int e^{a+bx} \coth^3(a+bx)\,dx$

Optimal. Leaf size=81

$$\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 - E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0519935, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 390, 1158, 12, 288, 207}

$$\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^3,x]
```


```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 - E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```


Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\coth^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(-1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{2\left(1+3x^4\right)}{\left(-1+x^2\right)^3}\right)dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}+\frac{2\,\text{Subst}\left(\int \frac{1+3x^4}{\left(-1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{\text{Subst}\left(\int \frac{12x^2}{\left(-1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6\,\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [C] time = 2.45359, size = 286, normalized size = 3.53

$$
-\frac{e^{-5(a+bx)}\left(256e^{8(a+bx)}\left(e^{2(a+bx)}+1\right)^3\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2,2,2\right\},\left\{1,1,1,1,\frac{11}{2}\right\},e^{2(a+bx)}\right)+384e^{8(a+bx)}\left(5e^{2(a
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^3,x]
```

```
[Out] -(-21*(252105 + 507305*E^(2*(a + b*x)) + 173916*E^(4*(a + b*x)) - 154296*E^
(6*(a + b*x)) - 73885*E^(8*(a + b*x)) + 4887*E^(10*(a + b*x))) - (315*(-168
07 - 28218*E^(2*(a + b*x)) + 1173*E^(4*(a + b*x)) + 17748*E^(6*(a + b*x)) +
 4299*E^(8*(a + b*x)) - 1434*E^(10*(a + b*x)) + 7*E^(12*(a + b*x)))*ArcTanh
[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 384*E^(8*(a + b*x))*(1 + E
^(2*(a + b*x)))^2*(7 + 5*E^(2*(a + b*x)))*HypergeometricPFQ[{3/2, 2, 2, 2,
2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*(1 + E^(2*(a +
```

```
b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, E^(2*(
a + b*x))])/(60480*b*E^(5*(a + b*x)))
```

Maple [A] time = 0.017, size = 88, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{(\cosh(bx+a))^2}{\sinh(bx+a)}+2\sinh(bx+a)+\frac{(\cosh(bx+a))^3}{(\sinh(bx+a))^2}-3\frac{\cosh(bx+a)}{(\sinh(bx+a))^2}+\frac{3\operatorname{csch}(bx+a)\coth(bx+a)}{2}-3\,\mathrm{A}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*coth(b*x+a)^3,x)
```

```
[Out] 1/b*(-cosh(b*x+a)^2/sinh(b*x+a)+2*sinh(b*x+a)+cosh(b*x+a)^3/sinh(b*x+a)^2-3
/sinh(b*x+a)^2*cosh(b*x+a)+3/2*csch(b*x+a)*coth(b*x+a)-3*arctanh(exp(b*x+a)
))
```

Maxima [A] time = 1.04991, size = 119, normalized size = 1.47

$$\frac{e^{(bx+a)}}{b}-\frac{3\log\left(e^{(bx+a)}+1\right)}{2b}+\frac{3\log\left(e^{(bx+a)}-1\right)}{2b}-\frac{3e^{(3bx+3a)}-e^{(bx+a)}}{b\left(e^{(4bx+4a)}-2e^{(2bx+2a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] e^(b*x + a)/b - 3/2*log(e^(b*x + a) + 1)/b + 3/2*log(e^(b*x + a) - 1)/b - (
3*e^(3*b*x + 3*a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) +
1))
```

Fricas [B] time = 2.05096, size = 1291, normalized size = 15.94

$$\frac{2\cosh(bx+a)^5+10\cosh(bx+a)\sinh(bx+a)^4+2\sinh(bx+a)^5+10\left(2\cosh(bx+a)^2-1\right)\sinh(bx+a)^3-10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*sinh(b*x + a)
^5 + 10*(2*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 10*(
2*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^4 +
 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 -
 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh
(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(5
*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 2)*sinh(b*x + a) + 4*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
```

```
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22713, size = 97, normalized size = 1.2

$$-\frac{\frac{2\left(3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-2\,e^{(bx+a)}+3\,\log\left(e^{(bx+a)}+1\right)-3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - 2*e^(b*
x + a) + 3*log(e^(b*x + a) + 1) - 3*log(abs(e^(b*x + a) - 1)))/b
```

3.213 $\int e^{a+bx}\coth^4(a+bx)\,dx$

Optimal. Leaf size=113

$$\frac{e^{a+bx}}{b}+\frac{5e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2}+\frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] E^(a + b*x)/b + (8*E^(a + b*x))/(3*b*(1 - E^(2*a + 2*b*x))^3) - (14*E^(a +
b*x))/(3*b*(1 - E^(2*a + 2*b*x))^2) + (5*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*
x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0716714, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 390, 1258, 1157, 385, 206}

$$\frac{e^{a+bx}}{b}+\frac{5e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2}+\frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^4,x]
```

```
[Out] E^(a + b*x)/b + (8*E^(a + b*x))/(3*b*(1 - E^(2*a + 2*b*x))^3) - (14*E^(a +
b*x))/(3*b*(1 - E^(2*a + 2*b*x))^2) + (5*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*
x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1258

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sy
mbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e
^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^
2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + c*x^4)
^p - (-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)],
 x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[
m/2, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\coth^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{\left(1-x^2\right)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{8x^2\left(1+x^4\right)}{\left(1-x^2\right)^4}\right)dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}+\frac{8\,\text{Subst}\left(\int \frac{x^2\left(1+x^4\right)}{\left(1-x^2\right)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3}+\frac{4\,\text{Subst}\left(\int \frac{-2-6x^2-6x^4}{\left(1-x^2\right)^3}\,dx, x, e^{a+bx}\right)}{3b}\\
&= \frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2}-\frac{\text{Subst}\left(\int \frac{-6-24x^2}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{3b}\\
&= \frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2}+\frac{5e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2}+\frac{5e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 10.1054, size = 115, normalized size = 1.02

$$
\frac{-24e^{a+bx}+50e^{3(a+bx)}-48e^{5(a+bx)}+6e^{7(a+bx)}+9\left(e^{2(a+bx)}-1\right)^3\log\left(1-e^{a+bx}\right)-9\left(e^{2(a+bx)}-1\right)^3\log\left(e^{a+bx}+1\right)}{6b\left(e^{2(a+bx)}-1\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^4,x]
```

```
[Out] (-24*E^(a + b*x) + 50*E^(3*(a + b*x)) - 48*E^(5*(a + b*x)) + 6*E^(7*(a + b*
x)) + 9*(-1 + E^(2*(a + b*x)))^3*Log[1 - E^(a + b*x)] - 9*(-1 + E^(2*(a + b
*x)))^3*Log[1 + E^(a + b*x)])/(6*b*(-1 + E^(2*(a + b*x)))^3)
```

Maple [A] time = 0.014, size = 123, normalized size = 1.1

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^3}{(\sinh(bx+a))^2}-3\frac{\cosh(bx+a)}{(\sinh(bx+a))^2}+\frac{3\operatorname{csch}(bx+a)\coth(bx+a)}{2}-3\operatorname{Artanh}\left(e^{bx+a}\right)+\frac{(\cosh(bx+a))^4}{(\sinh(bx+a))^3}-\frac{4(}{3(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*coth(b*x+a)^4,x)
```

```
[Out] 1/b*(cosh(b*x+a)^3/sinh(b*x+a)^2-3/sinh(b*x+a)^2*cosh(b*x+a)+3/2*csch(b*x+a
)*coth(b*x+a)-3*arctanh(exp(b*x+a))+cosh(b*x+a)^4/sinh(b*x+a)^3-4/3/sinh(b*
x+a)^3*cosh(b*x+a)^2-8/3*cosh(b*x+a)^2/sinh(b*x+a)+8/3*sinh(b*x+a))
```

Maxima [A] time = 1.1656, size = 149, normalized size = 1.32

$$\frac{e^{(bx+a)}}{b}-\frac{3\log\left(e^{(bx+a)}+1\right)}{2b}+\frac{3\log\left(e^{(bx+a)}-1\right)}{2b}-\frac{15e^{(5bx+5a)}-16e^{(3bx+3a)}+9e^{(bx+a)}}{3b\left(e^{(6bx+6a)}-3e^{(4bx+4a)}+3e^{(2bx+2a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] e^(b*x + a)/b - 3/2*log(e^(b*x + a) + 1)/b + 3/2*log(e^(b*x + a) - 1)/b - 1
/3*(15*e^(5*b*x + 5*a) - 16*e^(3*b*x + 3*a) + 9*e^(b*x + a))/(b*(e^(6*b*x +
 6*a) - 3*e^(4*b*x + 4*a) + 3*e^(2*b*x + 2*a) - 1))
```

Fricas [B] time = 2.11681, size = 2221, normalized size = 19.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*cosh(b*x + a)^7 + 42*cosh(b*x + a)*sinh(b*x + a)^6 + 6*sinh(b*x + a)
^7 + 6*(21*cosh(b*x + a)^2 - 8)*sinh(b*x + a)^5 - 48*cosh(b*x + a)^5 + 30*(
7*cosh(b*x + a)^3 - 8*cosh(b*x + a))*sinh(b*x + a)^4 + 10*(21*cosh(b*x + a)
^4 - 48*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 50*cosh(b*x + a)^3 + 6*(21*c
osh(b*x + a)^5 - 80*cosh(b*x + a)^3 + 25*cosh(b*x + a))*sinh(b*x + a)^2 - 9
*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x
+ a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b
*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2
*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + si
nh(b*x + a) + 1) + 9*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + s
inh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a
)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b
```

```
*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*
(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*lo
g(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(7*cosh(b*x + a)^6 - 40*cosh(b*x +
 a)^4 + 25*cosh(b*x + a)^2 - 4)*sinh(b*x + a) - 24*cosh(b*x + a))/(b*cosh(b
*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 3*b*cos
h(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*
x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*
b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*
x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29063, size = 112, normalized size = 0.99

$$-\frac{\frac{2\left(15\,e^{(5\,bx+5\,a)}-16\,e^{(3\,bx+3\,a)}+9\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^{3}}-6\,e^{(bx+a)}+9\,\log\left(e^{(bx+a)}+1\right)-9\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*(15*e^(5*b*x + 5*a) - 16*e^(3*b*x + 3*a) + 9*e^(b*x + a))/(e^(2*b*x
 + 2*a) - 1)^3 - 6*e^(b*x + a) + 9*log(e^(b*x + a) + 1) - 9*log(abs(e^(b*x
+ a) - 1)))/b
```

3.214 $\int e^x \tanh^2(2x)\,dx$

Optimal. Leaf size=113

$$e^x+\frac{e^x}{e^{4x}+1}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}$$

```
[Out] E^x + E^x/(1 + E^(4*x)) + ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) - ArcTan[1 +
Sqrt[2]*E^x]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) - Log
[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rubi [A] time = 0.0871351, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2282, 390, 288, 211, 1165, 628, 1162, 617, 204}

$$e^x+\frac{e^x}{e^{4x}+1}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Tanh[2*x]^2,x]
```

```
[Out] E^x + E^x/(1 + E^(4*x)) + ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) - ArcTan[1 +
Sqrt[2]*E^x]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) - Log
[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^x \tanh^2(2x)\,dx = \text{Subst}\left(\int \frac{(1-x^4)^2}{(1+x^4)^2}\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{4x^4}{(1+x^4)^2}\right)dx, x, e^x\right)$$
$$= e^x - 4\,\text{Subst}\left(\int \frac{x^4}{(1+x^4)^2}\,dx, x, e^x\right)$$
$$= e^x + \frac{e^x}{1+e^{4x}} - \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= e^x + \frac{e^x}{1+e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= e^x + \frac{e^x}{1+e^{4x}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{\text{Sul}}{}$$
$$= e^x + \frac{e^x}{1+e^{4x}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{2\sqrt{2}} +$$
$$= e^x + \frac{e^x}{1+e^{4x}} + \frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+\right.}{4\sqrt{2}}$$

Mathematica [C] time = 0.0428853, size = 48, normalized size = 0.42

$$\frac{1}{4}\text{RootSum}\left[\#1^4+1\&, \frac{x-\log\left(e^x-\#1\right)}{\#1^3}\&\right] + e^x + \frac{e^x}{e^{4x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Tanh[2*x]^2,x]
```

```
[Out] E^x + E^x/(1 + E^(4*x)) + RootSum[1 + #1^4 & , (x - Log[E^x - #1])/#1^3 & ]
/4
```

Maple [C] time = 0.059, size = 35, normalized size = 0.3

$$e^x + \frac{e^x}{1+e^{4x}} + \sum_{\_R=\text{RootOf}\left(256\_Z^4+1\right)} \_R\ln\left(e^x - 4\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*tanh(2*x)^2,x)
```

```
[Out] exp(x)+exp(x)/(1+exp(4*x))+sum(_R*ln(exp(x)-4*_R),_R=RootOf(256*_Z^4+1))
```

Maxima [A] time = 1.57587, size = 120, normalized size = 1.06

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{1}{8}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + e^x/(e^(4*x) + 1) + e^x
```

Fricas [B] time = 2.29514, size = 510, normalized size = 4.51

$$\frac{4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)+4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+4}\right.}{8\left(e^{(4x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(
2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*
e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - (sqrt(2)*e^(4
*x) + sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*e^(4*x) + sqrt
(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 8*e^(5*x) + 16*e^x)/(e^(4*x) + 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\tanh^{2}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(2*x)**2,x)
```

```
[Out] Integral(exp(x)*tanh(2*x)**2, x)
```

Giac [A] time = 1.2077, size = 120, normalized size = 1.06

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(2*x)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + e^x/(e^(4*x) + 1) + e^x
```

3.215 $\int e^x \tanh(2x)\,dx$

Optimal. Leaf size=95

$$e^x+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}$$

```
[Out] E^x + ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2] + L
og[1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/
(2*Sqrt[2])
```

Rubi [A] time = 0.0620859, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2282, 388, 211, 1165, 628, 1162, 617, 204}

$$e^x+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Tanh[2*x],x]
```

```
[Out] E^x + ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2] + L
og[1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/
(2*Sqrt[2])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^x \tanh(2x)\,dx &= \text{Subst}\left(\int \frac{-1+x^4}{1+x^4}\,dx, x, e^x\right)\\ &= e^x - 2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\ &= e^x - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)\\ &= e^x - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-}\right.}{2\sqrt{2}}\\ &= e^x + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-}\right.}{}\\ &= e^x + \frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}e^x\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0297731, size = 95, normalized size = 1.

$$e^x + \frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}} - \frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Tanh[2*x],x]
```

```
[Out] E^x + ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2] + L
og[1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/
```

```
(2*Sqrt[2])
```

Maple [C] time = 0.052, size = 24, normalized size = 0.3

$$e^x + \sum_{\_R=\text{RootOf}\left(16\_Z^4+1\right)} \_R \ln\left(e^x - 2\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*tanh(2*x),x)
```

```
[Out] exp(x)+sum(_R*ln(exp(x)-2*_R),_R=RootOf(16*_Z^4+1))
```

Maxima [A] time = 1.5883, size = 105, normalized size = 1.11

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + e^x
```

Fricas [A] time = 2.43529, size = 360, normalized size = 3.79

$$\sqrt{2}\arctan\left(-\sqrt{2}e^x + \sqrt{2}\sqrt{\sqrt{2}e^x + e^{(2x)} + 1} - 1\right) + \sqrt{2}\arctan\left(-\sqrt{2}e^x + \frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^x + 4e^{(2x)} + 4} + 1\right) - \frac{1}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(2*x),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1)
+ sqrt(2)*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x)
 + 4) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1/4*sqrt(2)*l
og(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + e^x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \tanh(2x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(2*x),x)
```

```
[Out] Integral(exp(x)*tanh(2*x), x)
```

Giac [A] time = 1.27455, size = 105, normalized size = 1.11

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + e^x
```

3.216 $\int e^x \coth(2x)\,dx$

Optimal. Leaf size=16

$$e^x - \tan^{-1}(e^x) - \tanh^{-1}(e^x)$$

```
[Out] E^x - ArcTan[E^x] - ArcTanh[E^x]
```

Rubi [A] time = 0.0141921, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2282, 388, 212, 206, 203}

$$e^x - \tan^{-1}(e^x) - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x],x]
```

```
[Out] E^x - ArcTan[E^x] - ArcTanh[E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^x \coth(2x)\,dx &= \text{Subst}\left(\int \frac{-1-x^4}{1-x^4}\,dx, x, e^x\right)\\ &= e^x - 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)\\ &= e^x - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\ &= e^x - \tan^{-1}(e^x) - \tanh^{-1}(e^x)\end{aligned}$$

Mathematica [A] time = 0.0119652, size = 16, normalized size = 1.

$$e^x - \tan^{-1}(e^x) - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Coth[2*x],x]
```

```
[Out] E^x - ArcTan[E^x] - ArcTanh[E^x]
```

Maple [C] time = 0.063, size = 36, normalized size = 2.3

$$e^x + \frac{\ln(e^x-1)}{2} + \frac{i}{2}\ln(e^x - i) - \frac{i}{2}\ln(e^x + i) - \frac{\ln(e^x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x),x)
```

```
[Out] exp(x)+1/2*ln(exp(x)-1)+1/2*I*ln(exp(x)-I)-1/2*I*ln(exp(x)+I)-1/2*ln(exp(x)
+1)
```

Maxima [A] time = 1.53196, size = 30, normalized size = 1.88

$$-\arctan(e^x) + e^x - \frac{1}{2}\log(e^x+1) + \frac{1}{2}\log(e^x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x),x, algorithm="maxima")
```

```
[Out] -arctan(e^x) + e^x - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Fricas [B] time = 2.38938, size = 154, normalized size = 9.62

$$-\arctan(\cosh(x)+\sinh(x)) + \cosh(x) - \frac{1}{2}\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}\log(\cosh(x)+\sinh(x)-1) + \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x),x, algorithm="fricas")
```

```
[Out] -arctan(cosh(x) + sinh(x)) + cosh(x) - 1/2*log(cosh(x) + sinh(x) + 1) + 1/2
*log(cosh(x) + sinh(x) - 1) + sinh(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \coth(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x),x)
```

```
[Out] Integral(exp(x)*coth(2*x), x)
```

Giac [A] time = 1.28272, size = 31, normalized size = 1.94

$$-\arctan(e^x) + e^x - \frac{1}{2}\log(e^x + 1) + \frac{1}{2}\log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x),x, algorithm="giac")
```

```
[Out] -arctan(e^x) + e^x - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.217 $\int e^x \coth^2(2x)\,dx$

Optimal. Leaf size=35

$$e^x + \frac{e^x}{1-e^{4x}} - \frac{1}{2}\tan^{-1}(e^x) - \frac{1}{2}\tanh^{-1}(e^x)$$

```
[Out] E^x + E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rubi [A] time = 0.028648, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2282, 390, 288, 212, 206, 203}

$$e^x + \frac{e^x}{1-e^{4x}} - \frac{1}{2}\tan^{-1}(e^x) - \frac{1}{2}\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x]^2,x]
```

```
[Out] E^x + E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^x \coth^2(2x)\,dx &= \text{Subst}\left(\int \frac{(1+x^4)^2}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{4x^4}{(1-x^4)^2}\right)dx, x, e^x\right)\\
&= e^x + 4\,\text{Subst}\left(\int \frac{x^4}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{1-e^{4x}} - \text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{1-e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{1-e^{4x}} - \frac{1}{2}\tan^{-1}(e^x) - \frac{1}{2}\tanh^{-1}(e^x)
\end{aligned}$$

Mathematica [C] time = 1.46632, size = 113, normalized size = 3.23

$$\frac{16}{585}e^{5x}\left(e^{4x}+1\right)^2 \text{HypergeometricPFQ}\left(\left\{\frac{5}{4},2,2,2\right\},\left\{1,1,\frac{17}{4}\right\},e^{4x}\right)+\frac{1}{640}e^{-7x}\left(5\left(1208e^{4x}+102e^{8x}-248e^{12x}+e\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^x*Coth[2*x]^2,x]
```

```
[Out] (-3645 - 6769*E^(4*x) - 1483*E^(8*x) + 681*E^(12*x) + 5*(729 + 1208*E^(4*x)
 + 102*E^(8*x) - 248*E^(12*x) + E^(16*x))*Hypergeometric2F1[1/4, 1, 5/4, E^
(4*x)])/(640*E^(7*x)) + (16*E^(5*x)*(1 + E^(4*x))^2*HypergeometricPFQ[{5/4,
 2, 2, 2}, {1, 1, 17/4}, E^(4*x)])/585
```

Maple [C] time = 0.072, size = 48, normalized size = 1.4

$$e^x - \frac{e^x}{e^{4x}-1} + \frac{i}{4}\ln(e^x - i) - \frac{i}{4}\ln(e^x + i) + \frac{\ln(e^x-1)}{4} - \frac{\ln(e^x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x)^2,x)
```

```
[Out] exp(x)-exp(x)/(exp(4*x)-1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)+1/4*ln(exp
(x)-1)-1/4*ln(exp(x)+1)
```

Maxima [A] time = 1.54437, size = 46, normalized size = 1.31

$$-\frac{e^x}{e^{(4x)}-1}-\frac{1}{2}\arctan\left(e^x\right)+e^x-\frac{1}{4}\log\left(e^x+1\right)+\frac{1}{4}\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2,x, algorithm="maxima")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) + e^x - 1/4*log(e^x + 1) + 1/4*log(e^x
 - 1)
```

Fricas [B] time = 2.36093, size = 837, normalized size = 23.91

$$4\cosh(x)^5+40\cosh(x)^3\sinh(x)^2+40\cosh(x)^2\sinh(x)^3+20\cosh(x)\sinh(x)^4+4\sinh(x)^5-2\left(\cosh(x)^4+4\text{ co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*cosh(x)^5 + 40*cosh(x)^3*sinh(x)^2 + 40*cosh(x)^2*sinh(x)^3 + 20*cos
h(x)*sinh(x)^4 + 4*sinh(x)^5 - 2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh
(x)) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sinh(x) + 1) + (cosh(x)^4 + 4*cos
h(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 -
1)*log(cosh(x) + sinh(x) - 1) + 4*(5*cosh(x)^4 - 2)*sinh(x) - 8*cosh(x))/(c
osh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^
3 + sinh(x)^4 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x\coth^2(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)**2,x)
```

```
[Out] Integral(exp(x)*coth(2*x)**2, x)
```

Giac [A] time = 1.26826, size = 47, normalized size = 1.34

$$-\frac{e^x}{e^{(4x)}-1}-\frac{1}{2}\arctan\left(e^x\right)+e^x-\frac{1}{4}\log\left(e^x+1\right)+\frac{1}{4}\log\left(\left|e^x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2,x, algorithm="giac")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) + e^x - 1/4*log(e^x + 1) + 1/4*log(abs
(e^x - 1))
```

3.218 $\int e^x \tanh^2(3x)\,dx$

Optimal. Leaf size=113

$$e^x+\frac{2e^x}{3\left(e^{6x}+1\right)}+\frac{\log\left(-\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}}-\frac{\log\left(\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}}-\frac{2}{9}\tan^{-1}\left(e^x\right)+\frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right)-\frac{1}{9}\tan^{-1}\left(2e^x\right.$$

```
[Out] E^x + (2*E^x)/(3*(1 + E^(6*x))) - (2*ArcTan[E^x])/9 + ArcTan[Sqrt[3] - 2*E^
x]/9 - ArcTan[Sqrt[3] + 2*E^x]/9 + Log[1 - Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3
]) - Log[1 + Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3])
```

Rubi [A] time = 0.212323, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2282, 390, 288, 209, 634, 618, 204, 628, 203}

$$e^x+\frac{2e^x}{3\left(e^{6x}+1\right)}+\frac{\log\left(-\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}}-\frac{\log\left(\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}}-\frac{2}{9}\tan^{-1}\left(e^x\right)+\frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right)-\frac{1}{9}\tan^{-1}\left(2e^x\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Tanh[3*x]^2,x]
```

```
[Out] E^x + (2*E^x)/(3*(1 + E^(6*x))) - (2*ArcTan[E^x])/9 + ArcTan[Sqrt[3] - 2*E^
x]/9 - ArcTan[Sqrt[3] + 2*E^x]/9 + Log[1 - Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3
]) - Log[1 + Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
```

```
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^x \tanh^2(3x)\,dx = \text{Subst}\left(\int \frac{\left(1-x^6\right)^2}{\left(1+x^6\right)^2}\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{4x^6}{\left(1+x^6\right)^2}\right)dx, x, e^x\right)$$
$$= e^x - 4\,\text{Subst}\left(\int \frac{x^6}{\left(1+x^6\right)^2}\,dx, x, e^x\right)$$
$$= e^x + \frac{2e^x}{3\left(1+e^{6x}\right)} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^6}\,dx, x, e^x\right)$$
$$= e^x + \frac{2e^x}{3\left(1+e^{6x}\right)} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - \frac{2}{9}\text{Subst}\left(\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right) - \frac{2}{9}\text{Subs}$$
$$= e^x + \frac{2e^x}{3\left(1+e^{6x}\right)} - \frac{2}{9}\tan^{-1}\left(e^x\right) - \frac{1}{18}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right) - \frac{1}{18}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x}\right.$$
$$= e^x + \frac{2e^x}{3\left(1+e^{6x}\right)} - \frac{2}{9}\tan^{-1}\left(e^x\right) + \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} - \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} + \frac{1}{9}\text{Subst}\left(\int \frac{}{-1}\right.$$
$$= e^x + \frac{2e^x}{3\left(1+e^{6x}\right)} - \frac{2}{9}\tan^{-1}\left(e^x\right) + \frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right) - \frac{1}{9}\tan^{-1}\left(\sqrt{3}+2e^x\right) + \frac{\log\left(1-\sqrt{3}e^x+e^{2}\right.}{6\sqrt{3}}$$

Mathematica [C] time = 0.0759737, size = 97, normalized size = 0.86

$$-\frac{1}{9}\text{RootSum}\left[\#1^4-\#1^2+1\&,\frac{\#1^2x-\#1^2\log\left(e^x-\#1\right)+2\log\left(e^x-\#1\right)-2x}{2\#1^3-\#1}\&\right]+e^x+\frac{2e^x}{3\left(e^{6x}+1\right)}-\frac{2}{9}\tan^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Tanh[3*x]^2,x]
```

```
[Out] E^x + (2*E^x)/(3*(1 + E^(6*x))) - (2*ArcTan[E^x])/9 - RootSum[1 - #1^2 + #1
^4 & , (-2*x + 2*Log[E^x - #1] + x*#1^2 - Log[E^x - #1]*#1^2)/(-#1 + 2*#1^3
) & ]/9
```

Maple [C] time = 0.106, size = 59, normalized size = 0.5

$$e^x + \frac{2\,e^x}{3+3\,e^{6x}} + \sum_{\_R=\text{RootOf}\left(6561\_Z^4-81\_Z^2+1\right)} \_R\ln\left(e^x-9\_R\right) + \frac{i}{9}\ln\left(e^x-i\right) - \frac{i}{9}\ln\left(e^x+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*tanh(3*x)^2,x)
```

```
[Out] exp(x)+2/3*exp(x)/(1+exp(6*x))+sum(_R*ln(exp(x)-9*_R),_R=RootOf(6561*_Z^4-8
1*_Z^2+1))+1/9*I*ln(exp(x)-I)-1/9*I*ln(exp(x)+I)
```

Maxima [A] time = 1.57048, size = 109, normalized size = 0.96

$$-\frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)+\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)+\frac{2e^{x}}{3\left(e^{(6x)}+1\right)}-\frac{1}{9}\arctan\left(\sqrt{3}+2e^{x}\right)-\frac{1}{9}\arctan\left(-\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(3*x)^2,x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/18*sqrt(3)*log(-sqrt(3)*e^
x + e^(2*x) + 1) + 2/3*e^x/(e^(6*x) + 1) - 1/9*arctan(sqrt(3) + 2*e^x) - 1/
9*arctan(-sqrt(3) + 2*e^x) - 2/9*arctan(e^x) + e^x
```

Fricas [A] time = 2.31428, size = 490, normalized size = 4.34

$$\frac{4\left(e^{(6x)}+1\right)\arctan\left(\sqrt{3}+\sqrt{-4\sqrt{3}e^{x}+4e^{(2x)}+4}-2e^{x}\right)+4\left(e^{(6x)}+1\right)\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}e^{x}+e^{(2x)}+1}-2e^{x}\right)-4}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(3*x)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(4*(e^(6*x) + 1)*arctan(sqrt(3) + sqrt(-4*sqrt(3)*e^x + 4*e^(2*x) + 4)
 - 2*e^x) + 4*(e^(6*x) + 1)*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*e^x + e^(2*x)
+ 1) - 2*e^x) - 4*(e^(6*x) + 1)*arctan(e^x) - (sqrt(3)*e^(6*x) + sqrt(3))*l
og(4*sqrt(3)*e^x + 4*e^(2*x) + 4) + (sqrt(3)*e^(6*x) + sqrt(3))*log(-4*sqrt
(3)*e^x + 4*e^(2*x) + 4) + 18*e^(7*x) + 30*e^x)/(e^(6*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\tanh^{2}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(3*x)**2,x)
```

```
[Out] Integral(exp(x)*tanh(3*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\tanh\left(3x\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(3*x)^2,x, algorithm="giac")
```

```
[Out] integrate(e^x*tanh(3*x)^2, x)
```

3.219 $\int e^x \tanh(3x)\,dx$

Optimal. Leaf size=97

$$e^x+\frac{\log\left(-\sqrt{3}e^x+e^{2x}+1\right)}{2\sqrt{3}}-\frac{\log\left(\sqrt{3}e^x+e^{2x}+1\right)}{2\sqrt{3}}-\frac{2}{3}\tan^{-1}\left(e^x\right)+\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2e^x\right)-\frac{1}{3}\tan^{-1}\left(2e^x+\sqrt{3}\right)$$

```
[Out] E^x - (2*ArcTan[E^x])/3 + ArcTan[Sqrt[3] - 2*E^x]/3 - ArcTan[Sqrt[3] + 2*E^
x]/3 + Log[1 - Sqrt[3]*E^x + E^(2*x)]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*E^x + E
^(2*x)]/(2*Sqrt[3])
```

Rubi [A] time = 0.186301, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2282, 388, 209, 634, 618, 204, 628, 203}

$$e^x+\frac{\log\left(-\sqrt{3}e^x+e^{2x}+1\right)}{2\sqrt{3}}-\frac{\log\left(\sqrt{3}e^x+e^{2x}+1\right)}{2\sqrt{3}}-\frac{2}{3}\tan^{-1}\left(e^x\right)+\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2e^x\right)-\frac{1}{3}\tan^{-1}\left(2e^x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Tanh[3*x],x]
```

```
[Out] E^x - (2*ArcTan[E^x])/3 + ArcTan[Sqrt[3] - 2*E^x]/3 - ArcTan[Sqrt[3] + 2*E^
x]/3 + Log[1 - Sqrt[3]*E^x + E^(2*x)]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*E^x + E
^(2*x)]/(2*Sqrt[3])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^x \tanh(3x)\,dx &= \text{Subst}\left(\int \frac{-1+x^6}{1+x^6}\,dx, x, e^x\right)\\
&= e^x - 2\,\text{Subst}\left(\int \frac{1}{1+x^6}\,dx, x, e^x\right)\\
&= e^x - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1+\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x\right.\\
&= e^x - \frac{2}{3}\tan^{-1}(e^x) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, e^x\right) + \frac{\text{Su}}{}\\
&= e^x - \frac{2}{3}\tan^{-1}(e^x) + \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{2\sqrt{3}} - \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{2\sqrt{3}} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+\right.\\
&= e^x - \frac{2}{3}\tan^{-1}(e^x) + \frac{1}{3}\tan^{-1}\left(\sqrt{3}-2e^x\right) - \frac{1}{3}\tan^{-1}\left(\sqrt{3}+2e^x\right) + \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{2\sqrt{3}} - \frac{\log\left(1+\sqrt{3}e\right.}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.0117698, size = 24, normalized size = 0.25

$$e^x - 2e^x\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-e^{6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Tanh[3*x],x]
```

```
[Out] E^x - 2*E^x*Hypergeometric2F1[1/6, 1, 7/6, -E^(6*x)]
```

Maple [C] time = 0.071, size = 47, normalized size = 0.5

$$e^{x} + \frac{i}{3}\ln\left(e^{x} - i\right) - \frac{i}{3}\ln\left(e^{x} + i\right) + \sum_{\_R=\mathrm{RootOf}\left(81\_Z^{4}-9\_Z^{2}+1\right)} \_R\ln\left(e^{x} - 3\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*tanh(3*x),x)
```

```
[Out] exp(x)+1/3*I*ln(exp(x)-I)-1/3*I*ln(exp(x)+I)+sum(_R*ln(exp(x)-3*_R),_R=Root
Of(81*_Z^4-9*_Z^2+1))
```

Maxima [A] time = 1.58983, size = 93, normalized size = 0.96

$$-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}e^{x} + e^{(2x)} + 1\right) + \frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}e^{x} + e^{(2x)} + 1\right) - \frac{1}{3}\arctan\left(\sqrt{3} + 2e^{x}\right) - \frac{1}{3}\arctan\left(-\sqrt{3} + 2e^{x}\right) - \frac{2}{3}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(3*x),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*sqrt(3)*log(-sqrt(3)*e^x
+ e^(2*x) + 1) - 1/3*arctan(sqrt(3) + 2*e^x) - 1/3*arctan(-sqrt(3) + 2*e^x)
 - 2/3*arctan(e^x) + e^x
```

Fricas [A] time = 2.29901, size = 350, normalized size = 3.61

$$-\frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}e^{x} + 4e^{(2x)} + 4\right) + \frac{1}{6}\sqrt{3}\log\left(-4\sqrt{3}e^{x} + 4e^{(2x)} + 4\right) + \frac{2}{3}\arctan\left(\sqrt{3} + \sqrt{-4\sqrt{3}e^{x} + 4e^{(2x)} + 4} - 2e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(3*x),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*log(4*sqrt(3)*e^x + 4*e^(2*x) + 4) + 1/6*sqrt(3)*log(-4*sqrt(3
)*e^x + 4*e^(2*x) + 4) + 2/3*arctan(sqrt(3) + sqrt(-4*sqrt(3)*e^x + 4*e^(2*
x) + 4) - 2*e^x) + 2/3*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*e^x + e^(2*x) + 1)
- 2*e^x) - 2/3*arctan(e^x) + e^x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\tanh(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(3*x),x)
```

```
[Out] Integral(exp(x)*tanh(3*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \tanh(3x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(3*x),x, algorithm="giac")
```

```
[Out] integrate(e^x*tanh(3*x), x)
```

3.220 $\int e^x \coth(3x)\, dx$

Optimal. Leaf size=85

$$e^x+\frac{1}{6}\log\left(-e^x+e^{2x}+1\right)-\frac{1}{6}\log\left(e^x+e^{2x}+1\right)+\frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(e^x\right)$$

```
[Out] E^x + ArcTan[(1 - 2*E^x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*E^x)/Sqrt[3]]/Sqr
t[3] - (2*ArcTanh[E^x])/3 + Log[1 - E^x + E^(2*x)]/6 - Log[1 + E^x + E^(2*x
)]/6
```

Rubi [A] time = 0.122385, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2282, 388, 210, 634, 618, 204, 628, 206}

$$e^x+\frac{1}{6}\log\left(-e^x+e^{2x}+1\right)-\frac{1}{6}\log\left(e^x+e^{2x}+1\right)+\frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[3*x],x]
```

```
[Out] E^x + ArcTan[(1 - 2*E^x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*E^x)/Sqrt[3]]/Sqr
t[3] - (2*ArcTanh[E^x])/3 + Log[1 - E^x + E^(2*x)]/6 - Log[1 + E^x + E^(2*x
)]/6
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^x \coth(3x)\,dx &= \text{Subst}\left(\int \frac{-1-x^6}{1-x^6}\,dx, x, e^x\right)\\
&= e^x - 2\,\text{Subst}\left(\int \frac{1}{1-x^6}\,dx, x, e^x\right)\\
&= e^x - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, e^x\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx, x\right.\\
&= e^x - \frac{2}{3}\tanh^{-1}(e^x) + \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, e^x\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, e^x\right) - \frac{1}{2}\,\text{Subst}\\
&= e^x - \frac{2}{3}\tanh^{-1}(e^x) + \frac{1}{6}\log\left(1-e^x+e^{2x}\right) - \frac{1}{6}\log\left(1+e^x+e^{2x}\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2e^x\right)\\
&= e^x - \frac{\tan^{-1}\left(\frac{-1+2e^x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2e^x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\tanh^{-1}(e^x) + \frac{1}{6}\log\left(1-e^x+e^{2x}\right) - \frac{1}{6}\log\left(1+e^x+e^{2x}\right)
\end{aligned}$$

Mathematica [C] time = 0.0151578, size = 22, normalized size = 0.26

$$e^x - 2e^x\,{}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; e^{6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Coth[3*x],x]
```

```
[Out] E^x - 2*E^x*Hypergeometric2F1[1/6, 1, 7/6, E^(6*x)]
```

Maple [C] time = 0.076, size = 138, normalized size = 1.6

$$e^{x}+\frac{1}{6}\ln\left(e^{x}-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)+\frac{i}{6}\ln\left(e^{x}-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}+\frac{1}{6}\ln\left(e^{x}-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\frac{i}{6}\ln\left(e^{x}-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}+\frac{\ln\left(e^{x}-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(3*x),x)
```

```
[Out] exp(x)+1/6*ln(exp(x)-1/2-1/2*I*3^(1/2))+1/6*I*ln(exp(x)-1/2-1/2*I*3^(1/2))*
3^(1/2)+1/6*ln(exp(x)-1/2+1/2*I*3^(1/2))-1/6*I*ln(exp(x)-1/2+1/2*I*3^(1/2))
*3^(1/2)+1/3*ln(exp(x)-1)-1/3*ln(exp(x)+1)-1/6*ln(exp(x)+1/2-1/2*I*3^(1/2))
+1/6*I*ln(exp(x)+1/2-1/2*I*3^(1/2))*3^(1/2)-1/6*ln(exp(x)+1/2+1/2*I*3^(1/2)
)-1/6*I*ln(exp(x)+1/2+1/2*I*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.60532, size = 101, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}-1)\right)+e^{x}-\frac{1}{6}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{6}\log\left(e^{(2x)}-e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) + e^x - 1/6*log(e^(2*x) + e^x + 1) + 1/6*log(e^(2*x) - e^x
+ 1) - 1/3*log(e^x + 1) + 1/3*log(e^x - 1)
```

Fricas [A] time = 2.15978, size = 454, normalized size = 5.34

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\cosh(x)+\frac{2}{3}\sqrt{3}\sinh(x)+\frac{1}{3}\sqrt{3}\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\cosh(x)+\frac{2}{3}\sqrt{3}\sinh(x)-\frac{1}{3}\sqrt{3}\right)+\text{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) + 1/3*sqrt(3)
) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) - 1/3*sqrt
(3)) + cosh(x) - 1/6*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) + 1/6*log((2*
cosh(x) - 1)/(cosh(x) - sinh(x))) - 1/3*log(cosh(x) + sinh(x) + 1) + 1/3*lo
g(cosh(x) + sinh(x) - 1) + sinh(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\coth(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(3*x),x)
```

```
[Out] Integral(exp(x)*coth(3*x), x)
```

<u>Giac</u> [A] time = 1.20479, size = 103, normalized size = 1.21

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}-1)\right)+e^{x}-\frac{1}{6}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{6}\log\left(e^{(2x)}-e^{x}+1\right)-\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(3*x),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) + e^x - 1/6*log(e^(2*x) + e^x + 1) + 1/6*log(e^(2*x) - e^x
+ 1) - 1/3*log(e^x + 1) + 1/3*log(abs(e^x - 1))
```

3.221 $\int e^x \coth^2(3x)\,dx$

Optimal. Leaf size=108

$$e^x+\frac{2e^x}{3\left(1-e^{6x}\right)}+\frac{1}{18}\log\left(-e^x+e^{2x}+1\right)-\frac{1}{18}\log\left(e^x+e^{2x}+1\right)+\frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\tanh^{-1}\left(e^x\right)$$

```
[Out] E^x + (2*E^x)/(3*(1 - E^(6*x))) + ArcTan[(1 - 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) -
 ArcTan[(1 + 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - (2*ArcTanh[E^x])/9 + Log[1 - E^x
 + E^(2*x)]/18 - Log[1 + E^x + E^(2*x)]/18
```

Rubi [A] time = 0.146851, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2282, 390, 288, 210, 634, 618, 204, 628, 206}

$$e^x+\frac{2e^x}{3\left(1-e^{6x}\right)}+\frac{1}{18}\log\left(-e^x+e^{2x}+1\right)-\frac{1}{18}\log\left(e^x+e^{2x}+1\right)+\frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[3*x]^2,x]
```

```
[Out] E^x + (2*E^x)/(3*(1 - E^(6*x))) + ArcTan[(1 - 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) -
 ArcTan[(1 + 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - (2*ArcTanh[E^x])/9 + Log[1 - E^x
 + E^(2*x)]/18 - Log[1 + E^x + E^(2*x)]/18
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
```

```
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^x \coth^2(3x)\,dx = \text{Subst}\left(\int \frac{(1+x^6)^2}{(1-x^6)^2}\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(1+\frac{4x^6}{(1-x^6)^2}\right)dx, x, e^x\right)$$
$$= e^x + 4\,\text{Subst}\left(\int \frac{x^6}{(1-x^6)^2}\,dx, x, e^x\right)$$
$$= e^x + \frac{2e^x}{3(1-e^{6x})} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^6}\,dx, x, e^x\right)$$
$$= e^x + \frac{2e^x}{3(1-e^{6x})} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{2}{9}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, e^x\right) - \frac{2}{9}\text{Subst}\Bigg(\int$$
$$= e^x + \frac{2e^x}{3(1-e^{6x})} - \frac{2}{9}\tanh^{-1}(e^x) + \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, e^x\right) - \frac{1}{18}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\right.$$
$$= e^x + \frac{2e^x}{3(1-e^{6x})} - \frac{2}{9}\tanh^{-1}(e^x) + \frac{1}{18}\log(1-e^x+e^{2x}) - \frac{1}{18}\log(1+e^x+e^{2x}) + \frac{1}{3}\text{Subst}\Bigg(\int$$
$$= e^x + \frac{2e^x}{3(1-e^{6x})} + \frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2}{9}\tanh^{-1}(e^x) + \frac{1}{18}\log(1-e^x+e^{2x}) - \frac{1}{18}\text{lo}$$

Mathematica [C] time = 1.71621, size = 113, normalized size = 1.05

$$\frac{36e^{7x}(e^{6x}+1)^2\,\text{HypergeometricPFQ}\left(\left\{\frac{7}{6},2,2,2\right\},\left\{1,1,\frac{25}{6}\right\},e^{6x}\right)}{1729} + \frac{e^{-11x}\left(7\left(3708e^{6x}+538e^{12x}-684e^{18x}+e^{24x}+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^x*Coth[3*x]^2,x]
```

```
[Out] (-15379 - 28153*E^(6*x) - 5633*E^(12*x) + 3109*E^(18*x) + 7*(2197 + 3708*E^
(6*x) + 538*E^(12*x) - 684*E^(18*x) + E^(24*x))*Hypergeometric2F1[1/6, 1, 7
/6, E^(6*x)])/(3024*E^(11*x)) + (36*E^(7*x)*(1 + E^(6*x))^2*HypergeometricP
FQ[{7/6, 2, 2, 2}, {1, 1, 25/6}, E^(6*x)])/1729
```

Maple [C] time = 0.1, size = 150, normalized size = 1.4

$$e^x - \frac{2\,e^x}{3\,e^{6x}-3} + \frac{1}{18}\ln\left(e^x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) + \frac{i}{18}\ln\left(e^x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3} + \frac{1}{18}\ln\left(e^x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right) - \frac{i}{18}\ln\left(e^x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(3*x)^2,x)
```

```
[Out] exp(x)-2/3*exp(x)/(exp(6*x)-1)+1/18*ln(exp(x)-1/2-1/2*I*3^(1/2))+1/18*I*ln(
exp(x)-1/2-1/2*I*3^(1/2))*3^(1/2)+1/18*ln(exp(x)-1/2+1/2*I*3^(1/2))-1/18*I*
ln(exp(x)-1/2+1/2*I*3^(1/2))*3^(1/2)+1/9*ln(exp(x)-1)-1/9*ln(exp(x)+1)-1/18
*ln(exp(x)+1/2-1/2*I*3^(1/2))+1/18*I*ln(exp(x)+1/2-1/2*I*3^(1/2))*3^(1/2)-1
/18*ln(exp(x)+1/2+1/2*I*3^(1/2))-1/18*I*ln(exp(x)+1/2+1/2*I*3^(1/2))*3^(1/2
)
```

Maxima [A] time = 1.64295, size = 117, normalized size = 1.08

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}+1)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}-1)\right)-\frac{2e^{x}}{3\left(e^{(6x)}-1\right)}+e^{x}-\frac{1}{18}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{18}\log\left(e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(3*x)^2,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) + e^x - 1/18*log(e^(2*x) + e^x + 1)
 + 1/18*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(e^x - 1)
```

Fricas [B] time = 2.16464, size = 2221, normalized size = 20.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(3*x)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*cosh(x)^7 + 378*cosh(x)^5*sinh(x)^2 + 630*cosh(x)^4*sinh(x)^3 + 63
0*cosh(x)^3*sinh(x)^4 + 378*cosh(x)^2*sinh(x)^5 + 126*cosh(x)*sinh(x)^6 + 1
8*sinh(x)^7 - 2*(sqrt(3)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(
3)*cosh(x)^4*sinh(x)^2 + 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x
)^2*sinh(x)^4 + 6*sqrt(3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*
arctan(2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) + 1/3*sqrt(3)) - 2*(sqrt(3
)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh(x)^4*sinh(x)^2
+ 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sinh(x)^4 + 6*sqrt(
3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(2/3*sqrt(3)*cosh
(x) + 2/3*sqrt(3)*sinh(x) - 1/3*sqrt(3)) - (cosh(x)^6 + 6*cosh(x)^5*sinh(x)
 + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4
 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log((2*cosh(x) + 1)/(cosh(x) - sinh
(x))) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
- 1)*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(cosh(x)^6 + 6*cosh(x)^5*
sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*si
nh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cosh(x) + sinh(x) + 1) +
 2*(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*
log(cosh(x) + sinh(x) - 1) + 6*(21*cosh(x)^6 - 5)*sinh(x) - 30*cosh(x))/(co
sh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(
x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\coth^{2}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(3*x)**2,x)
```

```
[Out] Integral(exp(x)*coth(3*x)**2, x)
```

Giac [A] time = 1.28956, size = 119, normalized size = 1.1

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right)-\frac{2e^x}{3\left(e^{(6x)}-1\right)}+e^x-\frac{1}{18}\log\left(e^{(2x)}+e^x+1\right)+\frac{1}{18}\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(3*x)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) + e^x - 1/18*log(e^(2*x) + e^x + 1)
 + 1/18*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(abs(e^x - 1))
```

3.222 $\int e^x \tanh^2(4x) \, dx$

Optimal. Leaf size=382

$$e^x + \frac{e^x}{2\left(e^{8x}+1\right)} + \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(-\sqrt{2-\sqrt{2}}e^x + e^{2x} + 1\right) - \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}e^x + e^{2x} + 1\right) + \frac{1}{32}\sqrt{2+\sqrt{2}}\log($$

```
[Out] E^x + E^x/(2*(1 + E^(8*x))) + ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + S
qrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqr
t[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E
^x)/Sqrt[2 + Sqrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]
] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) + (Sqrt[2 - Sqrt[2]
]*Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 - Sqrt[2]]*Log[1 +
 Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 +
 Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]
*E^x + E^(2*x)])/32
```

Rubi [A] time = 0.40506, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2282, 390, 288, 213, 1169, 634, 618, 204, 628}

$$e^x + \frac{e^x}{2\left(e^{8x}+1\right)} + \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(-\sqrt{2-\sqrt{2}}e^x + e^{2x} + 1\right) - \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}e^x + e^{2x} + 1\right) + \frac{1}{32}\sqrt{2+\sqrt{2}}\log($$

Antiderivative was successfully verified.

```
[In] Int[E^x*Tanh[4*x]^2,x]
```

```
[Out] E^x + E^x/(2*(1 + E^(8*x))) + ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + S
qrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqr
t[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E
^x)/Sqrt[2 + Sqrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]
] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) + (Sqrt[2 - Sqrt[2]
]*Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 - Sqrt[2]]*Log[1 +
 Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 +
 Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]
*E^x + E^(2*x)])/32
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^x \tanh^2(4x)\,dx = \text{Subst}\left(\int \frac{\left(1-x^8\right)^2}{\left(1+x^8\right)^2}\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{4x^8}{\left(1+x^8\right)^2}\right)dx, x, e^x\right)$$
$$= e^x - 4\,\text{Subst}\left(\int \frac{x^8}{\left(1+x^8\right)^2}\,dx, x, e^x\right)$$
$$= e^x + \frac{e^x}{2\left(1+e^{8x}\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^8}\,dx, x, e^x\right)$$
$$= e^x + \frac{e^x}{2\left(1+e^{8x}\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}}$$
$$= e^x + \frac{e^x}{2\left(1+e^{8x}\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} -$$
$$= e^x + \frac{e^x}{2\left(1+e^{8x}\right)} - \frac{1}{16}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right) - \frac{1}{16}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{S}$$
$$= e^x + \frac{e^x}{2\left(1+e^{8x}\right)} + \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) - \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)$$
$$= e^x + \frac{e^x}{2\left(1+e^{8x}\right)} + \frac{1}{16}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{16}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{16}$$

Mathematica [C] time = 0.0480806, size = 51, normalized size = 0.13

$$\frac{1}{16}\text{RootSum}\left[\#1^8+1\&, \frac{x-\log\left(e^x-\#1\right)}{\#1^7}\&\right] + e^x + \frac{e^x}{2\left(e^{8x}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Tanh[4*x]^2,x]
```

```
[Out] E^x + E^x/(2*(1 + E^(8*x))) + RootSum[1 + #1^8 & , (x - Log[E^x - #1])/#1^7
 & ]/16
```

Maple [C] time = 0.095, size = 36, normalized size = 0.1

$$e^x + \frac{e^x}{2+2\,e^{8x}} + \sum_{\_R=\text{RootOf}\left(4294967296\,\_Z^8+1\right)} \_R\,\ln\left(e^x - 16\,\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*tanh(4*x)^2,x)
```

```
[Out] exp(x)+1/2*exp(x)/(1+exp(8*x))+sum(_R*ln(exp(x)-16*_R),_R=RootOf(4294967296
*_Z^8+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,e^{(9\,x)}+3\,e^{x}}{2\,\left(e^{(8\,x)}+1\right)}-\int\frac{e^{x}}{2\,\left(e^{(8\,x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(4*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*e^(9*x) + 3*e^x)/(e^(8*x) + 1) - integrate(1/2*e^x/(e^(8*x) + 1), x)
```

Fricas [B] time = 2.64269, size = 4162, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(4*x)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(8*sqrt(-sqrt(2) + 2)*(e^(8*x) + 1)*arctan((2*sqrt(sqrt(sqrt(2) + 2)*
e^x + e^(2*x) + 1) - sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) + 8*sqr
t(-sqrt(2) + 2)*(e^(8*x) + 1)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*e^x + e^(2*
x) + 1) + sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) - 2*sqrt(-sqrt(2)
+ 2)*(e^(8*x) + 1)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 2*sqrt(-sqrt
(2) + 2)*(e^(8*x) + 1)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 8*(sqrt
(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*arctan((2*sqrt(sqrt(-sqrt(2) + 2
)*e^x + e^(2*x) + 1) - sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2)) + 8*(
sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*arctan((2*sqrt(-sqrt(-sqrt(2
) + 2)*e^x + e^(2*x) + 1) + sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2))
+ 4*(sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-
sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan(-(2*sqrt(2)*e^x - sqrt(2)*
sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4
*e^(2*x) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2)
+ sqrt(-sqrt(2) + 2))) + 4*(sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e^
(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan(-(2
*sqrt(2)*e^x - sqrt(2)*sqrt(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sq
rt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) +
 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 4*(sqrt(2)*sqrt(sqrt(2) +
2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(
sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2) +
 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2)
+ 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 4*(s
qrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2
) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(-2
*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*
x) + 4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt
(-sqrt(2) + 2))) - (sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e^(8*x) +
sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(2*sqrt(2)*sqrt
(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - (sq
rt(2)*sqrt(sqrt(2) + 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2)
 + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*
sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*sqrt(sqrt(2) + 2
)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(s
qrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2)
 + 2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*
```

```
e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(-2*s
qrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x)
 + 4) - 2*(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*log(sqrt(sqrt(2)
+ 2)*e^x + e^(2*x) + 1) + 2*(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))
*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 128*e^(9*x) + 192*e^x)/(e^(8*x
) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \tanh^2(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(4*x)**2,x)
```

```
[Out] Integral(exp(x)*tanh(4*x)**2, x)
```

Giac [A] time = 1.33246, size = 355, normalized size = 0.93

$$-\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^x}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^x}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{16}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+}}{\sqrt{\sqrt{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(4*x)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) +
 2)) - 1/16*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sq
rt(2) + 2)) - 1/16*sqrt(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sq
rt(sqrt(2) + 2)) - 1/16*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e
^x)/sqrt(sqrt(2) + 2)) - 1/32*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x +
 e^(2*x) + 1) + 1/32*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x)
 + 1) - 1/32*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) +
 1/32*sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/2*e
^x/(e^(8*x) + 1) + e^x
```

3.223 $\int e^x \tanh(4x)\,dx$

Optimal. Leaf size=366

$$e^x + \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(-\sqrt{2-\sqrt{2}}e^x + e^{2x} + 1\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}e^x + e^{2x} + 1\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(-\sqrt{2+\sqrt{2}}e^x\right.$$

```
[Out] E^x + ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 -
Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[
2*(2 + Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/(
2*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt
[2]]]/(2*Sqrt[2*(2 + Sqrt[2])]) + (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[
2]]*E^x + E^(2*x)])/8 - (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*E^x +
E^(2*x)])/8 + (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/
8 - (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/8
```

Rubi [A] time = 0.244167, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2282, 388, 213, 1169, 634, 618, 204, 628}

$$e^x + \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(-\sqrt{2-\sqrt{2}}e^x + e^{2x} + 1\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}e^x + e^{2x} + 1\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(-\sqrt{2+\sqrt{2}}e^x\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Tanh[4*x],x]
```

```
[Out] E^x + ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 -
Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[
2*(2 + Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/(
2*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt
[2]]]/(2*Sqrt[2*(2 + Sqrt[2])]) + (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[
2]]*E^x + E^(2*x)])/8 - (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*E^x +
E^(2*x)])/8 + (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/
8 - (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^x \tanh(4x)\,dx = \text{Subst}\left(\int \frac{-1+x^8}{1+x^8}\,dx, x, e^x\right)$$
$$= e^x - 2\,\text{Subst}\left(\int \frac{1}{1+x^8}\,dx, x, e^x\right)$$
$$= e^x - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}}$$
$$= e^x - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{}}{}\right.}{2}$$
$$= e^x - \frac{1}{4}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right) - \frac{1}{4}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{}{1+}\right.$$
$$= e^x + \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}$$
$$= e^x + \frac{1}{4}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\tan^{-}$$

Mathematica [C] time = 0.0117819, size = 24, normalized size = 0.07

$$e^x - 2e^x\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-e^{8x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Tanh[4*x],x]
```

```
[Out] E^x - 2*E^x*Hypergeometric2F1[1/8, 1, 9/8, -E^(8*x)]
```

Maple [C] time = 0.059, size = 24, normalized size = 0.1

$$e^x + \sum_{\_R=\text{RootOf}\left(65536\_Z^8+1\right)} \_R\ln\left(e^x - 4\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*tanh(4*x),x)
```

```
[Out] exp(x)+sum(_R*ln(exp(x)-4*_R),_R=RootOf(65536*_Z^8+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e^x - 2\int \frac{e^x}{e^{(8x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(4*x),x, algorithm="maxima")
```

```
[Out] e^x - 2*integrate(e^x/(e^(8*x) + 1), x)
```

Fricas [B] time = 2.39636, size = 3330, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqr
t(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-s
qrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/
(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2)
+ sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*e^x - sqrt(2)*sqrt(-2*sqrt
(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) +
4) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqr
t(2) + 2))) - 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*
arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sq
rt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) + sqrt(-s
qrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 1/8*(sqrt(2)*sqrt(
sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*
sqrt(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x +
4*e^(2*x) + 4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2)
 - sqrt(-sqrt(2) + 2))) - 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-s
qrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2)
+ 2)*e^x + 4*e^(2*x) + 4) - 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(
-sqrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2
) + 2)*e^x + 4*e^(2*x) + 4) + 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqr
t(-sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqr
t(2) + 2)*e^x + 4*e^(2*x) + 4) + 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*
sqrt(-sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-
sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + 1/4*sqrt(-sqrt(2) + 2)*arctan((2*sqrt(s
qrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) - sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt
(2) + 2)) + 1/4*sqrt(-sqrt(2) + 2)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*e^x +
e^(2*x) + 1) + sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(sq
rt(2) + 2)*arctan((2*sqrt(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - sqrt(-sqr
t(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2)) + 1/4*sqrt(sqrt(2) + 2)*arctan((2*sqr
t(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt
(sqrt(2) + 2)) - 1/16*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2*x)
 + 1) + 1/16*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) -
1/16*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/16*sq
rt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + e^x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x} \tanh(4x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(4*x),x)
```

```
[Out] Integral(exp(x)*tanh(4*x), x)
```

Giac [A] time = 1.44803, size = 339, normalized size = 0.93

$$-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+\ldots}}{\sqrt{\sqrt{2}\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tanh(4*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) +
2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt
(2) + 2)) - 1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt(
sqrt(2) + 2)) - 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^x)/
sqrt(sqrt(2) + 2)) - 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2
*x) + 1) + 1/8*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1)
- 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sq
rt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + e^x
```

3.224 $\int e^x \coth(4x)\,dx$

Optimal. Leaf size=116

$$e^x+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(e^x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(e\right.$$

```
[Out] E^x - ArcTan[E^x]/2 + ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) - ArcTan[1 + Sqrt
[2]*E^x]/(2*Sqrt[2]) - ArcTanh[E^x]/2 + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*S
qrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rubi [A] time = 0.0777028, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.5, Rules used = {2282, 388, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$e^x+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(e^x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(e\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[4*x],x]
```

```
[Out] E^x - ArcTan[E^x]/2 + ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) - ArcTan[1 + Sqrt
[2]*E^x]/(2*Sqrt[2]) - ArcTanh[E^x]/2 + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*S
qrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^x \coth(4x)\,dx = \text{Subst}\left(\int \frac{-1-x^8}{1-x^8}\,dx, x, e^x\right)$$
$$= e^x - 2\,\text{Subst}\left(\int \frac{1}{1-x^8}\,dx, x, e^x\right)$$
$$= e^x - \text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= e^x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) - \frac{1}{2}$$
$$= e^x - \frac{1}{2}\tan^{-1}(e^x) - \frac{1}{2}\tanh^{-1}(e^x) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\right.$$
$$= e^x - \frac{1}{2}\tan^{-1}(e^x) - \frac{1}{2}\tanh^{-1}(e^x) + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.}{2\sqrt{2}}$$
$$= e^x - \frac{1}{2}\tan^{-1}(e^x) + \frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{1}{2}\tanh^{-1}(e^x) + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} -$$

Mathematica [C] time = 0.0164104, size = 22, normalized size = 0.19

$$e^x - 2e^x\,{}_2F_1\left(\frac{1}{8}, 1; \frac{9}{8}; e^{8x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Coth[4*x],x]
```

```
[Out] E^x - 2*E^x*Hypergeometric2F1[1/8, 1, 9/8, E^(8*x)]
```

Maple [C] time = 0.07, size = 56, normalized size = 0.5

$$e^x - \frac{\ln(e^x+1)}{4} + \frac{\ln(e^x-1)}{4} + \frac{i}{4}\ln(e^x-i) - \frac{i}{4}\ln(e^x+i) + \sum_{\_R=\text{RootOf}\left(256\_Z^4+1\right)} \_R\ln(e^x - 4\_R)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(4*x),x)
```

```
[Out] exp(x)-1/4*ln(exp(x)+1)+1/4*ln(exp(x)-1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)
+I)+sum(_R*ln(exp(x)-4*_R),_R=RootOf(256*_Z^4+1))
```

Maxima [A] time = 1.69206, size = 131, normalized size = 1.13

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(4*x),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) + e^x - 1/4*
log(e^x + 1) + 1/4*log(e^x - 1)
```

Fricas [A] time = 2.10871, size = 447, normalized size = 3.85

$$\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+4e^{(2x)}+4}+1\right)-\frac{1}{8}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(4*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) -
 1) + 1/2*sqrt(2)*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4
*e^(2*x) + 4) + 1) - 1/8*sqrt(2)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1/8*s
qrt(2)*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 1/2*arctan(e^x) + e^x - 1/4*lo
g(e^x + 1) + 1/4*log(e^x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\coth(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(4*x),x)
```

```
[Out] Integral(exp(x)*coth(4*x), x)
```

Giac [A] time = 1.3042, size = 132, normalized size = 1.14

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(4*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) + e^x - 1/4*
log(e^x + 1) + 1/4*log(abs(e^x - 1))
```

3.225 $\int e^x \coth^2(4x)\,dx$

Optimal. Leaf size=134

$$e^x+\frac{e^x}{2\left(1-e^{8x}\right)}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{1}{8}\tan^{-1}\left(e^x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}$$

```
[Out] E^x + E^x/(2*(1 - E^(8*x))) - ArcTan[E^x]/8 + ArcTan[1 - Sqrt[2]*E^x]/(8*Sq
rt[2]) - ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - ArcTanh[E^x]/8 + Log[1 - Sqr
t[2]*E^x + E^(2*x)]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[
2])
```

Rubi [A] time = 0.101267, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.3, Rules used = {2282, 390, 288, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$e^x+\frac{e^x}{2\left(1-e^{8x}\right)}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{1}{8}\tan^{-1}\left(e^x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[4*x]^2,x]
```


```
[Out] E^x + E^x/(2*(1 - E^(8*x))) - ArcTan[E^x]/8 + ArcTan[1 - Sqrt[2]*E^x]/(8*Sq
rt[2]) - ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - ArcTanh[E^x]/8 + Log[1 - Sqr
t[2]*E^x + E^(2*x)]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[
2])
```


Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
```

```
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^x \coth^2(4x)\,dx &= \text{Subst}\left(\int \frac{(1+x^8)^2}{(1-x^8)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{4x^8}{(1-x^8)^2}\right)dx, x, e^x\right)\\
&= e^x + 4\,\text{Subst}\left(\int \frac{x^8}{(1-x^8)^2}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^8}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1-x}{1+x}\right.\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\tan^{-1}(e^x) - \frac{1}{8}\tanh^{-1}(e^x) - \frac{1}{16}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{1}{16}\text{Subst}\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\tan^{-1}(e^x) - \frac{1}{8}\tanh^{-1}(e^x) + \frac{\log(1-\sqrt{2}e^x+e^{2x})}{16\sqrt{2}} - \frac{\log(1+\sqrt{2}e^x+e^{2x})}{16\sqrt{2}} - \frac{\text{Su}}{}\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\tan^{-1}(e^x) + \frac{\tan^{-1}(1-\sqrt{2}e^x)}{8\sqrt{2}} - \frac{\tan^{-1}(1+\sqrt{2}e^x)}{8\sqrt{2}} - \frac{1}{8}\tanh^{-1}(e^x) + \frac{\log(1-}{16}
\end{aligned}$$

Mathematica [C] time = 1.64459, size = 113, normalized size = 0.84

$$\frac{64e^{9x}\left(e^{8x}+1\right)^2 \text{HypergeometricPFQ}\left(\left\{\frac{9}{8},2,2,2\right\},\left\{1,1,\frac{33}{8}\right\},e^{8x}\right)}{3825} + \frac{e^{-15x}\left(9\left(8368e^{8x}+1486e^{16x}-1456e^{24x}+e^{32x}+4\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^x*Coth[4*x]^2,x]
```

```
[Out] (-44217 - 80225*E^(8*x) - 15127*E^(16*x) + 9361*E^(24*x) + 9*(4913 + 8368*E
^(8*x) + 1486*E^(16*x) - 1456*E^(24*x) + E^(32*x))*Hypergeometric2F1[1/8, 1
, 9/8, E^(8*x)])/(9216*E^(15*x)) + (64*E^(9*x)*(1 + E^(8*x))^2*Hypergeometr
icPFQ[{9/8, 2, 2, 2}, {1, 1, 33/8}, E^(8*x)])/3825
```

Maple [C] time = 0.105, size = 68, normalized size = 0.5

$$e^x - \frac{e^x}{2e^{8x}-2} + \frac{\ln(e^x-1)}{16} + \sum_{\_R=\text{RootOf}(65536\_Z^4+1)} \_R\ln(e^x-16\_R) + \frac{i}{16}\ln(e^x-i) - \frac{i}{16}\ln(e^x+i) - \frac{\ln(e^x+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(4*x)^2,x)
```

```
[Out] exp(x)-1/2*exp(x)/(exp(8*x)-1)+1/16*ln(exp(x)-1)+sum(_R*ln(exp(x)-16*_R),_R
=RootOf(65536*_Z^4+1))+1/16*I*ln(exp(x)-I)-1/16*I*ln(exp(x)+I)-1/16*ln(exp(
x)+1)
```

Maxima [A] time = 1.77914, size = 147, normalized size = 1.1

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{32}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(4*x)^2,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) + e^x - 1/16*log(e^x + 1) + 1/16*log(e^x - 1)
```

Fricas [B] time = 2.33155, size = 637, normalized size = 4.75

$$4\left(\sqrt{2}e^{(8x)}-\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)+4\left(\sqrt{2}e^{(8x)}-\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(4*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(sqrt(2)*e^(8*x) - sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt
(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(8*x) - sqrt(2))*arctan(-sqrt(2)
*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - 4*(e^(8*x) -
 1)*arctan(e^x) - (sqrt(2)*e^(8*x) - sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x)
 + 4) + (sqrt(2)*e^(8*x) - sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 2
*(e^(8*x) - 1)*log(e^x + 1) + 2*(e^(8*x) - 1)*log(e^x - 1) + 32*e^(9*x) - 4
8*e^x)/(e^(8*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\coth^{2}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(4*x)**2,x)
```

```
[Out] Integral(exp(x)*coth(4*x)**2, x)
```

Giac [A] time = 1.50573, size = 149, normalized size = 1.11

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{32}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(4*x)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) + e^x - 1/16*log(e^x + 1) + 1/16*log(abs(e^x - 1))
```

3.226 $\int \frac{e^x}{a-\tanh(2x)}\,dx$

Optimal. Leaf size=107

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{(1-a)\sqrt{a+1}\sqrt[4]{1-a^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{(1-a)\sqrt{a+1}\sqrt[4]{1-a^2}}-\frac{e^x}{1-a}$$

```
[Out] -(E^x/(1 - a)) + ArcTan[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)]/((1 - a)*Sqrt[1
+ a]*(1 - a^2)^(1/4)) + ArcTanh[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)]/((1 - a)
*Sqrt[1 + a]*(1 - a^2)^(1/4))
```

Rubi [A] time = 0.125877, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2282, 388, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{(1-a)\sqrt{a+1}\sqrt[4]{1-a^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{(1-a)\sqrt{a+1}\sqrt[4]{1-a^2}}-\frac{e^x}{1-a}$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(a - Tanh[2*x]),x]
```

```
[Out] -(E^x/(1 - a)) + ArcTan[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)]/((1 - a)*Sqrt[1
+ a]*(1 - a^2)^(1/4)) + ArcTanh[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)]/((1 - a)
*Sqrt[1 + a]*(1 - a^2)^(1/4))
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e^x}{a-\tanh(2x)}\,dx = \text{Subst}\left(\int \frac{1+x^4}{1+a-(1-a)x^4}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1-a}+\frac{2\,\text{Subst}\left(\int \frac{1}{1+a+(-1+a)x^4}\,dx, x, e^x\right)}{1-a}$$
$$= -\frac{e^x}{1-a}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+a}-\sqrt{1-a}x^2}\,dx, x, e^x\right)}{(1-a)\sqrt{1+a}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+a}+\sqrt{1-a}x^2}\,dx, x, e^x\right)}{(1-a)\sqrt{1+a}}$$
$$= -\frac{e^x}{1-a}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{(1-a)\sqrt{1+a}\sqrt[4]{1-a^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{(1-a)\sqrt{1+a}\sqrt[4]{1-a^2}}$$

Mathematica [A] time = 0.076932, size = 81, normalized size = 0.76

$$\frac{-\sqrt[4]{1-a}(a+1)^{3/4}e^x+\tan^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{(1-a)^{5/4}(a+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(a - Tanh[2*x]),x]
```

```
[Out] (-((1 - a)^(1/4)*(1 + a)^(3/4)*E^x) + ArcTan[((1 - a)^(1/4)*E^x)/(1 + a)^(1
/4)] + ArcTanh[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)])/((1 - a)^(5/4)*(1 + a)^(
3/4))
```

Maple [C] time = 0.17, size = 70, normalized size = 0.7

$$\frac{e^x}{-1+a}+\sum_{\_R=\text{RootOf}\left(1+\left(16a^8-32a^7-32a^6+96a^5-96a^3+32a^2+32a-16\right)\_Z^4\right)}\_R\ln\left(e^x+\left(-2a^2+2\right)\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(a-tanh(2*x)),x)
```

```
[Out] exp(x)/(-1+a)+sum(_R*ln(exp(x)+(-2*a^2+2)*_R),_R=RootOf(1+(16*a^8-32*a^7-32
*a^6+96*a^5-96*a^3+32*a^2+32*a-16)*_Z^4))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a-tanh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25756, size = 1037, normalized size = 9.69

$$-\frac{4\,(a-1)\left(-\frac{1}{a^8-2\,a^7-2\,a^6+6\,a^5-6\,a^3+2\,a^2+2\,a-1}\right)^{\frac{1}{4}}\arctan\left(-\left(a^6-2\,a^5-a^4+4\,a^3-a^2-2\,a+1\right)\left(-\frac{1}{a^8-2\,a^7-2\,a^6+6\,a^5-6\,a^3+2\,a^2+2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a-tanh(2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(a - 1)*(-1/(a^8 - 2*a^7 - 2*a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1)
)^(1/4)*arctan(-(a^6 - 2*a^5 - a^4 + 4*a^3 - a^2 - 2*a + 1)*(-1/(a^8 - 2*a^
7 - 2*a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(3/4)*e^x + (a^6 - 2*a^5 - a^
4 + 4*a^3 - a^2 - 2*a + 1)*sqrt((a^4 - 2*a^2 + 1)*sqrt(-1/(a^8 - 2*a^7 - 2*
a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1)) + e^(2*x))*(-1/(a^8 - 2*a^7 - 2*a^6
 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(3/4)) + (a - 1)*(-1/(a^8 - 2*a^7 - 2*
a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(1/4)*log((a^2 - 1)*(-1/(a^8 - 2*a^
7 - 2*a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(1/4) + e^x) - (a - 1)*(-1/(a
^8 - 2*a^7 - 2*a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(1/4)*log(-(a^2 - 1)
*(-1/(a^8 - 2*a^7 - 2*a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(1/4) + e^x)
- 2*e^x)/(a - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{a-\tanh(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a-tanh(2*x)),x)
```

```
[Out] Integral(exp(x)/(a - tanh(2*x)), x)
```

Giac [B] time = 1.29012, size = 443, normalized size = 4.14

$$-\frac{\left(a^4-2\,a^3+2\,a-1\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}+2\,e^x\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3-\sqrt{2}a^2-\sqrt{2}a+\sqrt{2}}-\frac{\left(a^4-2\,a^3+2\,a-1\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}-2\,e^x\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3-\sqrt{2}a^2-\sqrt{2}a+\sqrt{2}}-\frac{\left(a^4-2\,a^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a-tanh(2*x)),x, algorithm="giac")
```

```
[Out] -(a^4 - 2*a^3 + 2*a - 1)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*((a + 1)/(a - 1)
)^(1/4) + 2*e^x)/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^3 - sqrt(2)*a^2 - sqrt
```

```
(2)*a + sqrt(2)) - (a^4 - 2*a^3 + 2*a - 1)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(
2)*((a + 1)/(a - 1))^(1/4) - 2*e^x)/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^3 -
 sqrt(2)*a^2 - sqrt(2)*a + sqrt(2)) - 1/2*(a^4 - 2*a^3 + 2*a - 1)^(1/4)*log
(sqrt(2)*((a + 1)/(a - 1))^(1/4)*e^x + sqrt((a + 1)/(a - 1)) + e^(2*x))/(sq
rt(2)*a^3 - sqrt(2)*a^2 - sqrt(2)*a + sqrt(2)) + 1/2*(a^4 - 2*a^3 + 2*a - 1
)^(1/4)*log(-sqrt(2)*((a + 1)/(a - 1))^(1/4)*e^x + sqrt((a + 1)/(a - 1)) +
e^(2*x))/(sqrt(2)*a^3 - sqrt(2)*a^2 - sqrt(2)*a + sqrt(2)) + e^x/(a - 1)
```

3.227 $\int \frac{e^x}{(a-\tanh(2x))^2}\,dx$

Optimal. Leaf size=152

$$-\frac{(4a+1)\tan^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{2(1-a)^2(a+1)^{3/2}\sqrt[4]{1-a^2}}-\frac{(4a+1)\tanh^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{2(1-a)^2(a+1)^{3/2}\sqrt[4]{1-a^2}}+\frac{e^x}{(1-a)^2}+\frac{e^x}{(1-a)^2(a+1)\left((a-1)e^{4x}+a+1\right)}$$

```
[Out] E^x/(1 - a)^2 + E^x/((1 - a)^2*(1 + a)*(1 + a + (-1 + a)*E^(4*x))) - ((1 +
4*a)*ArcTan[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)])/(2*(1 - a)^2*(1 + a)^(3/2)*
(1 - a^2)^(1/4)) - ((1 + 4*a)*ArcTanh[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)])/(
2*(1 - a)^2*(1 + a)^(3/2)*(1 - a^2)^(1/4))
```

Rubi [A] time = 0.1764, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2282, 390, 385, 212, 208, 205}

$$-\frac{(4a+1)\tan^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{2(1-a)^2(a+1)^{3/2}\sqrt[4]{1-a^2}}-\frac{(4a+1)\tanh^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{2(1-a)^2(a+1)^{3/2}\sqrt[4]{1-a^2}}+\frac{e^x}{(1-a)^2}+\frac{e^x}{(1-a)^2(a+1)\left((a-1)e^{4x}+a+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(a - Tanh[2*x])^2,x]
```


```
[Out] E^x/(1 - a)^2 + E^x/((1 - a)^2*(1 + a)*(1 + a + (-1 + a)*E^(4*x))) - ((1 +
4*a)*ArcTan[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)])/(2*(1 - a)^2*(1 + a)^(3/2)*
(1 - a^2)^(1/4)) - ((1 + 4*a)*ArcTanh[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)])/(
2*(1 - a)^2*(1 + a)^(3/2)*(1 - a^2)^(1/4))
```


Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{(a-\tanh(2x))^2}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^4\right)^2}{\left(1+a-(1-a)x^4\right)^2}\,dx,x,e^x\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{(-1+a)^2}-\frac{4\left(a-(1-a)x^4\right)}{(-1+a)^2\left(1+a+(-1+a)x^4\right)^2}\right)dx,x,e^x\right)\\
&= \frac{e^x}{(1-a)^2}-\frac{4\,\text{Subst}\left(\int \frac{a-(1-a)x^4}{\left(1+a+(-1+a)x^4\right)^2}\,dx,x,e^x\right)}{(1-a)^2}\\
&= \frac{e^x}{(1-a)^2}+\frac{e^x}{(1-a)^2(1+a)\left(1+a-(1-a)e^{4x}\right)}-\frac{(1+4a)\,\text{Subst}\left(\int \frac{1}{1+a+(-1+a)x^4}\,dx,x,e^x\right)}{(1-a)^2(1+a)}\\
&= \frac{e^x}{(1-a)^2}+\frac{e^x}{(1-a)^2(1+a)\left(1+a-(1-a)e^{4x}\right)}-\frac{(1+4a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+a}-\sqrt{1-a}x^2}\,dx,x,e^x\right)}{2(1-a)^2(1+a)^{3/2}}-\frac{(1+\cdots}{\cdots}\\
&= \frac{e^x}{(1-a)^2}+\frac{e^x}{(1-a)^2(1+a)\left(1+a-(1-a)e^{4x}\right)}-\frac{(1+4a)\tan^{-1}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{2(1-a)^2(1+a)^{3/2}\sqrt[4]{1-a^2}}-\frac{(1+4a)\tanh^{-1}\left(\cdots\right.}{2(1-a)^2(1+a)^{3/2}\cdots}
\end{aligned}$$

Mathematica [C] time = 0.139498, size = 107, normalized size = 0.7

$$\frac{(4a+1)\text{RootSum}\left[\#1^4a-\#1^4+a+1\&,\frac{x-\log\left(e^x-\#1\right)}{\#1^3}\&\right]+\frac{4(a-1)e^x\left(a^2\left(e^{4x}+1\right)+2a-e^{4x}+2\right)}{ae^{4x}+a-e^{4x}+1}}{4(a-1)^3(a+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(a - Tanh[2*x])^2,x]
```

```
[Out] ((4*(-1 + a)*E^x*(2 + 2*a - E^(4*x) + a^2*(1 + E^(4*x))))/(1 + a - E^(4*x)
+ a*E^(4*x)) + (1 + 4*a)*RootSum[1 + a - #1^4 + a*#1^4 & , (x - Log[E^x - #
1])/#1^3 & ])/(4*(-1 + a)^3*(1 + a))
```

Maple [C] time = 0.194, size = 476, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(a-tanh(2*x))^2,x)
```

```
[Out] -2/(-1+a)^2/(tanh(1/2*x)-1)+1/(-1+a)^2/(a*tanh(1/2*x)^4-4*tanh(1/2*x)^3+6*a
*tanh(1/2*x)^2-4*tanh(1/2*x)+a)/(1+a)*tanh(1/2*x)^3-2/(-1+a)^2/(a*tanh(1/2*
x)^4-4*tanh(1/2*x)^3+6*a*tanh(1/2*x)^2-4*tanh(1/2*x)+a)/a/(1+a)*tanh(1/2*x)
^3+3/(-1+a)^2/(a*tanh(1/2*x)^4-4*tanh(1/2*x)^3+6*a*tanh(1/2*x)^2-4*tanh(1/2
*x)+a)/(1+a)*tanh(1/2*x)^2-2/(-1+a)^2/(a*tanh(1/2*x)^4-4*tanh(1/2*x)^3+6*a*
tanh(1/2*x)^2-4*tanh(1/2*x)+a)/a/(1+a)*tanh(1/2*x)-1/(-1+a)^2/(a*tanh(1/2*x
)^4-4*tanh(1/2*x)^3+6*a*tanh(1/2*x)^2-4*tanh(1/2*x)+a)/(1+a)*tanh(1/2*x)+1/
(-1+a)^2/(a*tanh(1/2*x)^4-4*tanh(1/2*x)^3+6*a*tanh(1/2*x)^2-4*tanh(1/2*x)+a
)/(1+a)-1/4/(-1+a)^2/(1+a)*sum((_R^2-2*_R+1)/(_R^3*a-3*_R^2+3*_R*a-1)*ln(ta
nh(1/2*x)-_R),_R=RootOf(_Z^4*a-4*_Z^3+6*_Z^2*a-4*_Z+a))-1/(-1+a)^2/(1+a)*su
m((_R^2-2*_R+1)/(_R^3*a-3*_R^2+3*_R*a-1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^4*
a-4*_Z^3+6*_Z^2*a-4*_Z+a))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a-tanh(2*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34791, size = 2920, normalized size = 19.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a-tanh(2*x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^4 - 2*a^2 + (a^4 - 2*a^3 + 2*a - 1)*e^(4*x) + 1)*(-(256*a^4 + 25
6*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12 - 42
*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 + 6*a
^2 + 2*a - 1))^(1/4)*arctan(-((4*a^13 - 7*a^12 - 18*a^11 + 36*a^10 + 30*a^9
 - 75*a^8 - 20*a^7 + 80*a^6 - 45*a^4 + 6*a^3 + 12*a^2 - 2*a - 1)*(-(256*a^4
 + 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12
 - 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3
+ 6*a^2 + 2*a - 1))^(3/4)*e^x - (a^12 - 2*a^11 - 4*a^10 + 10*a^9 + 5*a^8 -
20*a^7 + 20*a^5 - 5*a^4 - 10*a^3 + 4*a^2 + 2*a - 1)*sqrt((16*a^2 + 8*a + 1)
*e^(2*x) + (a^8 - 4*a^6 + 6*a^4 - 4*a^2 + 1)*sqrt(-(256*a^4 + 256*a^3 + 96*
a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12 - 42*a^11 - 14*
a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 + 6*a^2 + 2*a -
1)))*(-(256*a^4 + 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14
*a^13 + 14*a^12 - 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 1
4*a^4 - 14*a^3 + 6*a^2 + 2*a - 1))^(3/4))/(256*a^4 + 256*a^3 + 96*a^2 + 16*
a + 1)) + (a^4 - 2*a^2 + (a^4 - 2*a^3 + 2*a - 1)*e^(4*x) + 1)*(-(256*a^4 +
256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12 -
42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 + 6
*a^2 + 2*a - 1))^(1/4)*log((4*a + 1)*e^x + (a^4 - 2*a^2 + 1)*(-(256*a^4 + 2
```

```
56*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12 - 4
2*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 + 6*
a^2 + 2*a - 1))^(1/4)) - (a^4 - 2*a^2 + (a^4 - 2*a^3 + 2*a - 1)*e^(4*x) + 1
)*(-(256*a^4 + 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^
13 + 14*a^12 - 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a
^4 - 14*a^3 + 6*a^2 + 2*a - 1))^(1/4)*log((4*a + 1)*e^x - (a^4 - 2*a^2 + 1)
*(-(256*a^4 + 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^1
3 + 14*a^12 - 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^
4 - 14*a^3 + 6*a^2 + 2*a - 1))^(1/4)) - 4*(a^2 - 1)*e^(5*x) - 4*(a^2 + 2*a
+ 2)*e^x)/(a^4 - 2*a^2 + (a^4 - 2*a^3 + 2*a - 1)*e^(4*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{(a-\tanh(2x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a-tanh(2*x))**2,x)
```

```
[Out] Integral(exp(x)/(a - tanh(2*x))**2, x)
```

Giac [B] time = 1.27103, size = 616, normalized size = 4.05

$$-\frac{\left(a^4-2a^3+2a-1\right)^{\frac{1}{4}}(4a+1)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}+2e^x\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^5-\sqrt{2}a^4-2\sqrt{2}a^3+2\sqrt{2}a^2+\sqrt{2}a-\sqrt{2}\right)}-\frac{\left(a^4-2a^3+2a-1\right)^{\frac{1}{4}}(4a+1)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}-2e^x\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^5-\sqrt{2}a^4-2\sqrt{2}a^3+2\sqrt{2}a^2+\sqrt{2}a-\sqrt{2}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a-tanh(2*x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^4 - 2*a^3 + 2*a - 1)^(1/4)*(4*a + 1)*arctan(1/2*sqrt(2)*(sqrt(2)*((
a + 1)/(a - 1))^(1/4) + 2*e^x)/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^5 - sqrt
(2)*a^4 - 2*sqrt(2)*a^3 + 2*sqrt(2)*a^2 + sqrt(2)*a - sqrt(2)) - 1/2*(a^4 -
 2*a^3 + 2*a - 1)^(1/4)*(4*a + 1)*arctan(-1/2*sqrt(2)*(sqrt(2)*((a + 1)/(a
- 1))^(1/4) - 2*e^x)/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^5 - sqrt(2)*a^4 -
2*sqrt(2)*a^3 + 2*sqrt(2)*a^2 + sqrt(2)*a - sqrt(2)) - 1/4*(a^4 - 2*a^3 + 2
*a - 1)^(1/4)*(4*a + 1)*log(sqrt(2)*((a + 1)/(a - 1))^(1/4)*e^x + sqrt((a +
 1)/(a - 1)) + e^(2*x))/(sqrt(2)*a^5 - sqrt(2)*a^4 - 2*sqrt(2)*a^3 + 2*sqrt
(2)*a^2 + sqrt(2)*a - sqrt(2)) + 1/4*(a^4 - 2*a^3 + 2*a - 1)^(1/4)*(4*a + 1
)*log(-sqrt(2)*((a + 1)/(a - 1))^(1/4)*e^x + sqrt((a + 1)/(a - 1)) + e^(2*x
))/(sqrt(2)*a^5 - sqrt(2)*a^4 - 2*sqrt(2)*a^3 + 2*sqrt(2)*a^2 + sqrt(2)*a -
 sqrt(2)) + e^x/(a^2 - 2*a + 1) + e^x/((a^3 - a^2 - a + 1)*(a*e^(4*x) + a -
 e^(4*x) + 1))
```

3.228 $\int e^{c(a+bx)}\tanh^3(d+ex)\,dx$

Optimal. Leaf size=167

$$-\frac{6e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}+\frac{12e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}-\frac{8e^{c(a+bx)}\,{}_2F_1\left(3,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}+$$

```
[Out] E^(c*(a + b*x))/(b*c) - (6*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c) + (12*E^(c*(a + b*x))*Hypergeom
etric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c) - (8*E^(
c*(a + b*x))*Hypergeometric2F1[3, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d +
e*x))])/(b*c)
```

Rubi [A] time = 0.188268, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5484, 2194, 2251}

$$-\frac{6e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}+\frac{12e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}-\frac{8e^{c(a+bx)}\,{}_2F_1\left(3,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Tanh[d + e*x]^3,x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (6*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c) + (12*E^(c*(a + b*x))*Hypergeom
etric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c) - (8*E^(
c*(a + b*x))*Hypergeometric2F1[3, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d +
e*x))])/(b*c)
```

Rule 5484

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(-1 + E^(2*(d + e*x)))^n)/(1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^3(d+ex)\,dx = \int\left(e^{c(a+bx)} - \frac{8e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^3} + \frac{12e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^2} - \frac{6e^{c(a+bx)}}{1+e^{2(d+ex)}}\right)dx$$
$$= -\left(6\int\frac{e^{c(a+bx)}}{1+e^{2(d+ex)}}dx\right) - 8\int\frac{e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^3}dx + 12\int\frac{e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^2}dx + \int e^{c(a+bx)}\,dx$$
$$= \frac{e^{c(a+bx)}}{bc} - \frac{6e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};1+\frac{bc}{2e};-e^{2(d+ex)}\right)}{bc} + \frac{12e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};1+\frac{bc}{2e};-e^{2(d+ex)}\right)}{bc} - \frac{8e}{}$$

Mathematica [A] time = 4.12859, size = 205, normalized size = 1.23

$$\frac{1}{2}e^{ac}\left(\frac{2e^{2d}\left(b^2c^2+2e^2\right)\left(\frac{e^{x(bc+2e)}\,{}_2F_1\left(1,\frac{bc}{2e}+1;\frac{bc}{2e}+2;-e^{2(d+ex)}\right)}{bc+2e} - \frac{e^{bcx}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}\right)}{\left(e^{2d}+1\right)e^2} - \frac{bc\,\text{sech}(d)e^{bcx}\sinh(ex)\text{sech}(d+ex)}{e^2} + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Tanh[d + e*x]^3,x]
```

```
[Out] (E^(a*c)*((2*(b^2*c^2 + 2*e^2)*E^(2*d)*((E^((b*c + 2*e)*x)*Hypergeometric2F
1[1, 1 + (b*c)/(2*e), 2 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c + 2*e) - (E^
(b*c*x)*Hypergeometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))
])/(b*c)))/(e^2*(1 + E^(2*d))) + (E^(b*c*x)*Sech[d + e*x]^2)/e - (b*c*E^(b*
c*x)*Sech[d]*Sech[d + e*x]*Sinh[e*x])/e^2 + (2*E^(b*c*x)*Tanh[d])/(b*c)))/2
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left(\tanh(ex+d)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*tanh(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*tanh(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 48*(b^2*c^2*e*e^(a*c) + 2*e^3*e^(a*c))*integrate(e^(b*c*x)/(b^3*c^3 - 12*b^
2*c^2*e + 44*b*c*e^2 - 48*e^3 + (b^3*c^3*e^(8*d) - 12*b^2*c^2*e*e^(8*d) + 4
4*b*c*e^2*e^(8*d) - 48*e^3*e^(8*d))*e^(8*e*x) + 4*(b^3*c^3*e^(6*d) - 12*b^2
*c^2*e*e^(6*d) + 44*b*c*e^2*e^(6*d) - 48*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^
3*e^(4*d) - 12*b^2*c^2*e*e^(4*d) + 44*b*c*e^2*e^(4*d) - 48*e^3*e^(4*d))*e^(
```

```
4*e*x) + 4*(b^3*c^3*e^(2*d) - 12*b^2*c^2*e*e^(2*d) + 44*b*c*e^2*e^(2*d) - 4
8*e^3*e^(2*d))*e^(2*e*x)), x) - (b^3*c^3*e^(a*c) + 36*b^2*c^2*e*e^(a*c) + 4
4*b*c*e^2*e^(a*c) + 48*e^3*e^(a*c) - (b^3*c^3*e^(a*c + 6*d) - 12*b^2*c^2*e*
e^(a*c + 6*d) + 44*b*c*e^2*e^(a*c + 6*d) - 48*e^3*e^(a*c + 6*d))*e^(6*e*x)
+ 3*(b^3*c^3*e^(a*c + 4*d) - 8*b^2*c^2*e*e^(a*c + 4*d) + 4*b*c*e^2*e^(a*c +
 4*d) + 48*e^3*e^(a*c + 4*d))*e^(4*e*x) - 3*(b^3*c^3*e^(a*c + 2*d) - 28*b*c
*e^2*e^(a*c + 2*d) - 48*e^3*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^4*c^4 -
12*b^3*c^3*e + 44*b^2*c^2*e^2 - 48*b*c*e^3 + (b^4*c^4*e^(6*d) - 12*b^3*c^3*
e*e^(6*d) + 44*b^2*c^2*e^2*e^(6*d) - 48*b*c*e^3*e^(6*d))*e^(6*e*x) + 3*(b^4
*c^4*e^(4*d) - 12*b^3*c^3*e*e^(4*d) + 44*b^2*c^2*e^2*e^(4*d) - 48*b*c*e^3*e
^(4*d))*e^(4*e*x) + 3*(b^4*c^4*e^(2*d) - 12*b^3*c^3*e*e^(2*d) + 44*b^2*c^2*
e^2*e^(2*d) - 48*b*c*e^3*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{(bcx+ac)}\tanh\left(ex+d\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tanh(e*x + d)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{((bx+a)c)}\tanh\left(ex+d\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(e^((b*x + a)*c)*tanh(e*x + d)^3, x)
```

3.229 $\int e^{c(a+bx)} \tanh^2(d+ex)\,dx$

Optimal. Leaf size=117

$$-\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}+\frac{e^{c(a+bx)}}{bc}$$

```
[Out] E^(c*(a + b*x))/(b*c) - (4*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c) + (4*E^(c*(a + b*x))*Hypergeome
tric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c)
```

Rubi [A] time = 0.126796, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5484, 2194, 2251}

$$-\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}+\frac{e^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Tanh[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (4*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c) + (4*E^(c*(a + b*x))*Hypergeome
tric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c)
```

Rule 5484

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(-1 + E^(2*(d + e*x)))^n)/(1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^2(d+ex)\,dx = \int \left(e^{c(a+bx)} + \frac{4e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^2} - \frac{4e^{c(a+bx)}}{1+e^{2(d+ex)}}\right)dx$$
$$= 4\int \frac{e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^2}\,dx - 4\int \frac{e^{c(a+bx)}}{1+e^{2(d+ex)}}\,dx + \int e^{c(a+bx)}\,dx$$
$$= \frac{e^{c(a+bx)}}{bc} - \frac{4e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};1+\frac{bc}{2e};-e^{2(d+ex)}\right)}{bc} + \frac{4e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};1+\frac{bc}{2e};-e^{2(d+ex)}\right)}{bc}$$

Mathematica [A] time = 3.25664, size = 169, normalized size = 1.44

$$\frac{e^{c(a+bx)}\left(2b^2c^2e^{2(d+ex)}\,{}_2F_1\left(1,\frac{bc}{2e}+1;\frac{bc}{2e}+2;-e^{2(d+ex)}\right) - (bc+2e)\left(2bce^{2d}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right) - \left(e^{2d}+1\right)(e-bcs\right.\right.}{bc\left(e^{2d}+1\right)e(bc+2e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Tanh[d + e*x]^2,x]
```

```
[Out] (E^(c*(a + b*x))*(2*b^2*c^2*E^(2*(d + e*x))*Hypergeometric2F1[1, 1 + (b*c)/
(2*e), 2 + (b*c)/(2*e), -E^(2*(d + e*x))] - (b*c + 2*e)*(2*b*c*E^(2*d)*Hype
rgeometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))] - (1 + E^(
2*d))*(e - b*c*Sech[d]*Sech[d + e*x]*Sinh[e*x]))))/(b*c*e*(b*c + 2*e)*(1 +
E^(2*d)))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left(\tanh(ex+d)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*tanh(e*x+d)^2,x)
```

```
[Out] int(exp(c*(b*x+a))*tanh(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-16\,bce\int \frac{e^{(bcx+ac)}}{b^2c^2 - 6\,bce + 8\,e^2 + \left(b^2c^2e^{(6\,d)} - 6\,bcee^{(6\,d)} + 8\,e^2e^{(6\,d)}\right)e^{(6\,ex)} + 3\left(b^2c^2e^{(4\,d)} - 6\,bcee^{(4\,d)} + 8\,e^2e^{(4\,d)}\right)e^{(4\,ex)} + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -16*b*c*e*integrate(e^(b*c*x + a*c)/(b^2*c^2 - 6*b*c*e + 8*e^2 + (b^2*c^2*e
^(6*d) - 6*b*c*e*e^(6*d) + 8*e^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d) -
6*b*c*e*e^(4*d) + 8*e^2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d) - 6*b*c*e*e
^(2*d) + 8*e^2*e^(2*d))*e^(2*e*x)), x) + (b^2*c^2*e^(a*c) + 10*b*c*e*e^(a*c
) + 8*e^2*e^(a*c) + (b^2*c^2*e^(a*c + 4*d) - 6*b*c*e*e^(a*c + 4*d) + 8*e^2*
e^(a*c + 4*d))*e^(4*e*x) - 2*(b^2*c^2*e^(a*c + 2*d) - 2*b*c*e*e^(a*c + 2*d)
```

```
 - 8*e^2*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^3*c^3 - 6*b^2*c^2*e + 8*b*c
*e^2 + (b^3*c^3*e^(4*d) - 6*b^2*c^2*e*e^(4*d) + 8*b*c*e^2*e^(4*d))*e^(4*e*x
) + 2*(b^3*c^3*e^(2*d) - 6*b^2*c^2*e*e^(2*d) + 8*b*c*e^2*e^(2*d))*e^(2*e*x)
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{(bcx+ac)}\tanh\left(ex+d\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tanh(e*x + d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int e^{bcx}\tanh^2\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)**2,x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*tanh(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{((bx+a)c)}\tanh\left(ex+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(e^((b*x + a)*c)*tanh(e*x + d)^2, x)
```

3.230 $\int e^{c(a+bx)}\tanh(d+ex)\,dx$

Optimal. Leaf size=67

$$\frac{e^{c(a+bx)}}{bc}-\frac{2e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}$$

```
[Out] E^(c*(a + b*x))/(b*c) - (2*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c)
```

Rubi [A] time = 0.0708198, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5484, 2194, 2251}

$$\frac{e^{c(a+bx)}}{bc}-\frac{2e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Tanh[d + e*x],x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (2*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c)
```

Rule 5484

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(-1 + E^(2*(d + e*x)))^n)/(1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\tanh(d+ex)\,dx &= \int\left(e^{c(a+bx)}-\frac{2e^{c(a+bx)}}{1+e^{2(d+ex)}}\right)dx\\ &= -\left(2\int\frac{e^{c(a+bx)}}{1+e^{2(d+ex)}}\,dx\right)+\int e^{c(a+bx)}\,dx\\ &= \frac{e^{c(a+bx)}}{bc}-\frac{2e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};1+\frac{bc}{2e};-e^{2(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [B] time = 1.80816, size = 141, normalized size = 2.1

$$\frac{e^{c(a+bx)}\left(2bce^{2(d+ex)}\,{}_2F_1\left(1,\frac{bc}{2e}+1;\frac{bc}{2e}+2;-e^{2(d+ex)}\right)-(bc+2e)\left(2e^{2d}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;-e^{2(d+ex)}\right)-e^{2d}+1\right)\right)}{bc\left(e^{2d}+1\right)(bc+2e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Tanh[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*(2*b*c*E^(2*(d + e*x))*Hypergeometric2F1[1, 1 + (b*c)/(2*e
), 2 + (b*c)/(2*e), -E^(2*(d + e*x))] - (b*c + 2*e)*(1 - E^(2*d) + 2*E^(2*d
)*Hypergeometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))])))/(
b*c*(b*c + 2*e)*(1 + E^(2*d)))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\tanh(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*tanh(e*x+d),x)
```

```
[Out] int(exp(c*(b*x+a))*tanh(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4\,e\int\frac{e^{(bcx+ac)}}{bc+\left(bce^{(4\,d)}-2\,ee^{(4\,d)}\right)e^{(4\,ex)}+2\left(bce^{(2\,d)}-2\,ee^{(2\,d)}\right)e^{(2\,ex)}-2\,e}\,dx-\frac{\left(bce^{(ac)}+2\,ee^{(ac)}-\left(bce^{(ac+2\,d)}-2\,ee^{(ac+2\,d)}\right)e^{(2\,ex}}{b^2c^2-2\,bce+\left(b^2c^2e^{(2\,d)}-2\,bcee^{(2\,d)}\right)e^{(2\,ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d),x, algorithm="maxima")
```

```
[Out] 4*e*integrate(e^(b*c*x + a*c)/(b*c + (b*c*e^(4*d) - 2*e*e^(4*d))*e^(4*e*x)
+ 2*(b*c*e^(2*d) - 2*e*e^(2*d))*e^(2*e*x) - 2*e), x) - (b*c*e^(a*c) + 2*e*e
^(a*c) - (b*c*e^(a*c + 2*d) - 2*e*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^2*
c^2 - 2*b*c*e + (b^2*c^2*e^(2*d) - 2*b*c*e*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{(bcx+ac)}\tanh(ex+d)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tanh(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int e^{bcx}\tanh(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*tanh(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{((bx+a)c)}\tanh(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(e^((b*x + a)*c)*tanh(e*x + d), x)
```

3.231 $\int e^{c(a+bx)} \coth(d+ex)\,dx$

Optimal. Leaf size=65

$$\frac{e^{c(a+bx)}}{bc}-\frac{2e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}$$

```
[Out] E^(c*(a + b*x))/(b*c) - (2*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c)
```

Rubi [A] time = 0.070891, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5485, 2194, 2251}

$$\frac{e^{c(a+bx)}}{bc}-\frac{2e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Coth[d + e*x],x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (2*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c)
```

Rule 5485

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/(-1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\coth(d+ex)\,dx &= \int \left(e^{c(a+bx)}+\frac{2e^{c(a+bx)}}{-1+e^{2(d+ex)}}\right)dx\\ &= 2\int \frac{e^{c(a+bx)}}{-1+e^{2(d+ex)}}\,dx+\int e^{c(a+bx)}\,dx\\ &= \frac{e^{c(a+bx)}}{bc}-\frac{2e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};1+\frac{bc}{2e};e^{2(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [B] time = 2.38843, size = 134, normalized size = 2.06

$$\frac{e^{c(a+bx)}\left(2bce^{2(d+ex)}\,{}_2F_1\left(1,\frac{bc}{2e}+1;\frac{bc}{2e}+2;e^{2(d+ex)}\right)+(bc+2e)\left(-2e^{2d}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)+e^{2d}+1\right)\right)}{bc\left(e^{2d}-1\right)(bc+2e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Coth[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*(2*b*c*E^(2*(d + e*x))*Hypergeometric2F1[1, 1 + (b*c)/(2*e
), 2 + (b*c)/(2*e), E^(2*(d + e*x))] + (b*c + 2*e)*(1 + E^(2*d) - 2*E^(2*d)
*Hypergeometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))])))/(b*
c*(b*c + 2*e)*(-1 + E^(2*d)))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\coth(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*coth(e*x+d),x)
```

```
[Out] int(exp(c*(b*x+a))*coth(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4\,e\int\frac{e^{(bcx+ac)}}{bc+\left(bce^{(4\,d)}-2\,ee^{(4\,d)}\right)e^{(4\,ex)}-2\left(bce^{(2\,d)}-2\,ee^{(2\,d)}\right)e^{(2\,ex)}-2\,e}\,dx-\frac{\left(bce^{(ac)}+2\,ee^{(ac)}+\left(bce^{(ac+2\,d)}-2\,ee^{(ac+2\,d)}\right)e^{(}\right.}{b^2c^2-2\,bce-\left(b^2c^2e^{(2\,d)}-2\,bcee^{(2\,d)}\right)e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d),x, algorithm="maxima")
```

```
[Out] 4*e*integrate(e^(b*c*x + a*c)/(b*c + (b*c*e^(4*d) - 2*e*e^(4*d))*e^(4*e*x)
- 2*(b*c*e^(2*d) - 2*e*e^(2*d))*e^(2*e*x) - 2*e), x) - (b*c*e^(a*c) + 2*e*e
^(a*c) + (b*c*e^(a*c + 2*d) - 2*e*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^2*
c^2 - 2*b*c*e - (b^2*c^2*e^(2*d) - 2*b*c*e*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth(ex+d)\,e^{(bcx+ac)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(coth(e*x + d)*e^(b*c*x + a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int e^{bcx}\coth\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*coth(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\coth\left(ex+d\right)e^{\left(\left(bx+a\right)c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(coth(e*x + d)*e^((b*x + a)*c), x)
```

3.232 $\int e^{c(a+bx)}\coth^2(d+ex)\,dx$

Optimal. Leaf size=113

$$-\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}+\frac{e^{c(a+bx)}}{bc}$$

```
[Out] E^(c*(a + b*x))/(b*c) - (4*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c) + (4*E^(c*(a + b*x))*Hypergeomet
ric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c)
```

Rubi [A] time = 0.125741, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5485, 2194, 2251}

$$-\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}+\frac{e^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Coth[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (4*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c) + (4*E^(c*(a + b*x))*Hypergeomet
ric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c)
```

Rule 5485

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/(-1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\coth^2(d+ex)\,dx=\int\left(e^{c(a+bx)}+\frac{4e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^2}+\frac{4e^{c(a+bx)}}{-1+e^{2(d+ex)}}\right)dx$$
$$=4\int\frac{e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^2}dx+4\int\frac{e^{c(a+bx)}}{-1+e^{2(d+ex)}}dx+\int e^{c(a+bx)}dx$$
$$=\frac{e^{c(a+bx)}}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};1+\frac{bc}{2e};e^{2(d+ex)}\right)}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};1+\frac{bc}{2e};e^{2(d+ex)}\right)}{bc}$$

Mathematica [A] time = 3.04745, size = 145, normalized size = 1.28

$$e^{c(a+bx)}\left(\frac{2e^{2d}\left(bce^{2ex}\,{}_2F_1\left(1,\frac{bc}{2e}+1;\frac{bc}{2e}+2;e^{2(d+ex)}\right)-(bc+2e)\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)\right)}{\left(e^{2d}-1\right)e(bc+2e)}+\frac{1}{bc}+\frac{\text{csch}(d)\sinh(ex)\text{csch}(d}{e}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Coth[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))*(1/(b*c) + (2*E^(2*d)*(b*c*E^(2*e*x)*Hypergeometric2F1[1, 1
 + (b*c)/(2*e), 2 + (b*c)/(2*e), E^(2*(d + e*x))] - (b*c + 2*e)*Hypergeomet
ric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))]))/(e*(b*c + 2*e)*(
-1 + E^(2*d))) + (Csch[d]*Csch[d + e*x]*Sinh[e*x])/e)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left(\coth(ex+d)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*coth(e*x+d)^2,x)
```

```
[Out] int(exp(c*(b*x+a))*coth(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16\,bce\int-\frac{e^{(bcx+ac)}}{b^2c^2-6\,bce+8\,e^2-\left(b^2c^2e^{(6\,d)}-6\,bcee^{(6\,d)}+8\,e^2e^{(6\,d)}\right)e^{(6\,ex)}+3\left(b^2c^2e^{(4\,d)}-6\,bcee^{(4\,d)}+8\,e^2e^{(4\,d)}\right)e^{(4\,ex)}-3\left(b^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 16*b*c*e*integrate(-e^(b*c*x + a*c)/(b^2*c^2 - 6*b*c*e + 8*e^2 - (b^2*c^2*e
^(6*d) - 6*b*c*e*e^(6*d) + 8*e^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d) -
6*b*c*e*e^(4*d) + 8*e^2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d) - 6*b*c*e*e
^(2*d) + 8*e^2*e^(2*d))*e^(2*e*x)), x) + (b^2*c^2*e^(a*c) + 10*b*c*e*e^(a*c
) + 8*e^2*e^(a*c) + (b^2*c^2*e^(a*c + 4*d) - 6*b*c*e*e^(a*c + 4*d) + 8*e^2*
e^(a*c + 4*d))*e^(4*e*x) + 2*(b^2*c^2*e^(a*c + 2*d) - 2*b*c*e*e^(a*c + 2*d)
```

```
 - 8*e^2*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^3*c^3 - 6*b^2*c^2*e + 8*b*c
*e^2 + (b^3*c^3*e^(4*d) - 6*b^2*c^2*e*e^(4*d) + 8*b*c*e^2*e^(4*d))*e^(4*e*x
) - 2*(b^3*c^3*e^(2*d) - 6*b^2*c^2*e*e^(2*d) + 8*b*c*e^2*e^(2*d))*e^(2*e*x)
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth\left(ex+d\right)^{2} e^{\left(bcx+ac\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(coth(e*x + d)^2*e^(b*c*x + a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac} \int e^{bcx} \coth^{2}\left(d+ex\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)**2,x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*coth(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(ex+d\right)^{2} e^{\left((bx+a)c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(coth(e*x + d)^2*e^((b*x + a)*c), x)
```

3.233 $\int e^{c(a+bx)}\coth^3(d+ex)\,dx$

Optimal. Leaf size=161

$$-\frac{6e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}+\frac{12e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}-\frac{8e^{c(a+bx)}\,{}_2F_1\left(3,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}+\frac{e^{c(a+bx)}}{bc}$$

```
[Out] E^(c*(a + b*x))/(b*c) - (6*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c) + (12*E^(c*(a + b*x))*Hypergeome
tric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c) - (8*E^(c*
(a + b*x))*Hypergeometric2F1[3, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x
))])/(b*c)
```

Rubi [A] time = 0.175779, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5485, 2194, 2251}

$$-\frac{6e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}+\frac{12e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}-\frac{8e^{c(a+bx)}\,{}_2F_1\left(3,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)}{bc}+\frac{e^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Coth[d + e*x]^3,x]
```


```
[Out] E^(c*(a + b*x))/(b*c) - (6*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c) + (12*E^(c*(a + b*x))*Hypergeome
tric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c) - (8*E^(c*
(a + b*x))*Hypergeometric2F1[3, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x
))])/(b*c)
```


Rule 5485

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/(-1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\coth^3(d+ex)\,dx = \int\left(e^{c(a+bx)} + \frac{8e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^3} + \frac{12e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^2} + \frac{6e^{c(a+bx)}}{-1+e^{2(d+ex)}}\right)dx$$

$$= 6\int\frac{e^{c(a+bx)}}{-1+e^{2(d+ex)}}dx + 8\int\frac{e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^3}dx + 12\int\frac{e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^2}dx + \int e^{c(a+bx)}\,d$$

$$= \frac{e^{c(a+bx)}}{bc} - \frac{6e^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc}{2e};1+\frac{bc}{2e};e^{2(d+ex)}\right)}{bc} + \frac{12e^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc}{2e};1+\frac{bc}{2e};e^{2(d+ex)}\right)}{bc} - \frac{8e}{}$$

Mathematica [A] time = 3.31338, size = 185, normalized size = 1.15

$$\frac{1}{2}e^{c(a+bx)}\left(\frac{2e^{2d}\left(b^2c^2+2e^2\right)\left(bce^{2ex}\,{}_2F_1\left(1,\frac{bc}{2e}+1;\frac{bc}{2e}+2;e^{2(d+ex)}\right)-(bc+2e)\,{}_2F_1\left(1,\frac{bc}{2e};\frac{bc}{2e}+1;e^{2(d+ex)}\right)\right)}{bc\left(e^{2d}-1\right)e^2(bc+2e)} + \frac{bc\operatorname{csch}(d)\,\mathrm{si}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Coth[d + e*x]^3,x]
```

```
[Out] (E^(c*(a + b*x))*((2*Coth[d])/(b*c) - Csch[d + e*x]^2/e + (2*(b^2*c^2 + 2*e
^2)*E^(2*d)*(b*c*E^(2*e*x)*Hypergeometric2F1[1, 1 + (b*c)/(2*e), 2 + (b*c)/
(2*e), E^(2*(d + e*x))] - (b*c + 2*e)*Hypergeometric2F1[1, (b*c)/(2*e), 1 +
 (b*c)/(2*e), E^(2*(d + e*x))]))/(b*c*e^2*(b*c + 2*e)*(-1 + E^(2*d))) + (b*
c*Csch[d]*Csch[d + e*x]*Sinh[e*x])/e^2))/2
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left(\coth(ex+d)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*coth(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*coth(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 48*(b^2*c^2*e*e^(a*c) + 2*e^3*e^(a*c))*integrate(e^(b*c*x)/(b^3*c^3 - 12*b^
2*c^2*e + 44*b*c*e^2 - 48*e^3 + (b^3*c^3*e^(8*d) - 12*b^2*c^2*e*e^(8*d) + 4
4*b*c*e^2*e^(8*d) - 48*e^3*e^(8*d))*e^(8*e*x) - 4*(b^3*c^3*e^(6*d) - 12*b^2
*c^2*e*e^(6*d) + 44*b*c*e^2*e^(6*d) - 48*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^
3*e^(4*d) - 12*b^2*c^2*e*e^(4*d) + 44*b*c*e^2*e^(4*d) - 48*e^3*e^(4*d))*e^(
4*e*x) - 4*(b^3*c^3*e^(2*d) - 12*b^2*c^2*e*e^(2*d) + 44*b*c*e^2*e^(2*d) - 4
8*e^3*e^(2*d))*e^(2*e*x)), x) - (b^3*c^3*e^(a*c) + 36*b^2*c^2*e*e^(a*c) + 4
```

```
4*b*c*e^2*e^(a*c) + 48*e^3*e^(a*c) + (b^3*c^3*e^(a*c + 6*d) - 12*b^2*c^2*e*
e^(a*c + 6*d) + 44*b*c*e^2*e^(a*c + 6*d) - 48*e^3*e^(a*c + 6*d))*e^(6*e*x)
+ 3*(b^3*c^3*e^(a*c + 4*d) - 8*b^2*c^2*e*e^(a*c + 4*d) + 4*b*c*e^2*e^(a*c +
 4*d) + 48*e^3*e^(a*c + 4*d))*e^(4*e*x) + 3*(b^3*c^3*e^(a*c + 2*d) - 28*b*c
*e^2*e^(a*c + 2*d) - 48*e^3*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^4*c^4 -
12*b^3*c^3*e + 44*b^2*c^2*e^2 - 48*b*c*e^3 - (b^4*c^4*e^(6*d) - 12*b^3*c^3*
e*e^(6*d) + 44*b^2*c^2*e^2*e^(6*d) - 48*b*c*e^3*e^(6*d))*e^(6*e*x) + 3*(b^4
*c^4*e^(4*d) - 12*b^3*c^3*e*e^(4*d) + 44*b^2*c^2*e^2*e^(4*d) - 48*b*c*e^3*e
^(4*d))*e^(4*e*x) - 3*(b^4*c^4*e^(2*d) - 12*b^3*c^3*e*e^(2*d) + 44*b^2*c^2*
e^2*e^(2*d) - 48*b*c*e^3*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\coth\left(ex+d\right)^{3} e^{(bcx+ac)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(coth(e*x + d)^3*e^(b*c*x + a*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \coth\left(ex+d\right)^{3} e^{((bx+a)c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(coth(e*x + d)^3*e^((b*x + a)*c), x)
```

3.234 $\int e^{c(a+bx)} \tanh^2(ac+bcx)^{5/2} \, dx$

Optimal. Leaf size=311

$$\frac{e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}+\frac{25e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{4bc\left(e^{2c(a+bx)}+1\right)}-\frac{55e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx}}{6bc\left(e^{2c(a+bx)}+\right.}$$

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c) - (4*E^
(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c
*(a + b*x)))^4) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c
*x]^2])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x))*Coth[a*c +
b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2) + (25*E
^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(4*b*c*(1 + E^(
2*c*(a + b*x)))) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x]*Sqrt[Tanh[
a*c + b*c*x]^2])/(4*b*c)
```

Rubi [A] time = 0.902018, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6720, 2282, 390, 1814, 1157, 385, 203}

$$\frac{e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}+\frac{25e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{4bc\left(e^{2c(a+bx)}+1\right)}-\frac{55e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx}}{6bc\left(e^{2c(a+bx)}+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Tanh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c) - (4*E^
(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c
*(a + b*x)))^4) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c
*x]^2])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x))*Coth[a*c +
b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2) + (25*E
^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(4*b*c*(1 + E^(
2*c*(a + b*x)))) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x]*Sqrt[Tanh[
a*c + b*c*x]^2])/(4*b*c)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{5/2}\,dx = \left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\int e^{c(a+bx)}\tanh^5(ac+bcx)\,dx$$

$$= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{(-1+x^2)^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \left(1-\frac{2(1+10x^4+5x^8)}{(1+x^2)^5}\right)dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{\left(2\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{S}}{bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+b}}{bc\left(1+e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+b}}{bc\left(1+e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+b}}{bc\left(1+e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+b}}{bc\left(1+e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+b}}{bc\left(1+e^{2c(a+bx)}\right)^4}$$

Mathematica [A] time = 0.217265, size = 133, normalized size = 0.43

$$\frac{\left(e^{c(a+bx)}\left(157e^{2c(a+bx)}+187e^{4c(a+bx)}+123e^{6c(a+bx)}+12e^{8c(a+bx)}+33\right)-45\left(e^{2c(a+bx)}+1\right)^4\tan^{-1}\left(e^{c(a+bx)}\right)\right)\sqrt{\tanh^2(c(a}}{12bc\left(e^{2c(a+bx)}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Tanh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(33 + 157*E^(2*c*(a + b*x)) + 187*E^(4*c*(a + b*x)) + 123
*E^(6*c*(a + b*x)) + 12*E^(8*c*(a + b*x))) - 45*(1 + E^(2*c*(a + b*x)))^4*A
rcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)]*Sqrt[Tanh[c*(a + b*x)]^2])/(12*b*
c*(1 + E^(2*c*(a + b*x)))^4)
```

Maple [C] time = 0.277, size = 324, normalized size = 1.

$$\frac{\left(1+e^{2c(bx+a)}\right)e^{c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)cb}\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}+\frac{e^{c(bx+a)}\left(75e^{6c(bx+a)}+115e^{4c(bx+a)}+109e^{2c(bx+a)}+21\right)}{\left(12e^{2c(bx+a)}-12\right)\left(1+e^{2c(bx+a)}\right)^3cb}\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 1/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))*((exp(2*c*(b*x+a))-1)^2/(1+exp(
2*c*(b*x+a)))^2)^(1/2)*exp(c*(b*x+a))/c/b+1/12/(exp(2*c*(b*x+a))-1)/(1+exp(
2*c*(b*x+a)))^3*((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)*exp(c
*(b*x+a))*(75*exp(6*c*(b*x+a))+115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))+21
)/c/b+15/8*I/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))*((exp(2*c*(b*x+a))-1
)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-I)-15/8*I/(exp(2*c*
(b*x+a))-1)*(1+exp(2*c*(b*x+a)))*((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)
))^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+I)
```

Maxima [A] time = 1.70385, size = 196, normalized size = 0.63

$$-\frac{15\arctan\left(e^{(bcx+ac)}\right)}{4\,bc}+\frac{12\,e^{(9\,bcx+9\,ac)}+123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}+157\,e^{(3\,bcx+3\,ac)}+33\,e^{(bcx+ac)}}{12\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/4*arctan(e^(b*c*x + a*c))/(b*c) + 1/12*(12*e^(9*b*c*x + 9*a*c) + 123*e^
(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x + 5*a*c) + 157*e^(3*b*c*x + 3*a*c) + 33*
e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4
*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 2.1185, size = 3186, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*cosh(b*c*x + a*c)^9 + 108*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
12*sinh(b*c*x + a*c)^9 + 3*(144*cosh(b*c*x + a*c)^2 + 41)*sinh(b*c*x + a*c)
^7 + 123*cosh(b*c*x + a*c)^7 + 21*(48*cosh(b*c*x + a*c)^3 + 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + (1512*cosh(b*c*x + a*c)^4 + 2583*cosh(b*c*x +
a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 187*cosh(b*c*x + a*c)^5 + (1512*cosh(b*
c*x + a*c)^5 + 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b*c*x
 + a*c)^4 + (1008*cosh(b*c*x + a*c)^6 + 4305*cosh(b*c*x + a*c)^4 + 1870*cos
h(b*c*x + a*c)^2 + 157)*sinh(b*c*x + a*c)^3 + 157*cosh(b*c*x + a*c)^3 + (43
2*cosh(b*c*x + a*c)^7 + 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x + a*c)^3
 + 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*c)^8 + 8
*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*
c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^6 + 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b
*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x
 + a*c)^4 + 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x
+ a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x
 + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 + 15*cosh(b*c*x + a
*c)^4 + 9*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 4*cosh(b*c*x + a*c
)^2 + 8*(cosh(b*c*x + a*c)^7 + 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^
3 + cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + si
nh(b*c*x + a*c)) + (108*cosh(b*c*x + a*c)^8 + 861*cosh(b*c*x + a*c)^6 + 935
*cosh(b*c*x + a*c)^4 + 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 33
```

```
*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh
(b*c*x + a*c)^7 + b*c*sinh(b*c*x + a*c)^8 + 4*b*c*cosh(b*c*x + a*c)^6 + 4*(
7*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a
*c)^4 + 8*(7*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^5 + 2*(35*b*c*cosh(b*c*x + a*c)^4 + 30*b*c*cosh(b*c*x + a*c)^2 + 3*b
*c)*sinh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x +
 a*c)^5 + 10*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*b*c*cosh(b*c*x + a*c)^6 + 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*
c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x
+ a*c)^7 + 3*b*c*cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh
(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33277, size = 248, normalized size = 0.8

$$-\frac{45 \arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2bcx+2ac)}-1\right)-12\, e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2bcx+2ac)}-1\right)-\frac{75\, e^{(7bcx+7ac)}\operatorname{sgn}\left(e^{(2bcx+2ac)}-1\right)+115\, e^{(5bcx+5ac)}\operatorname{sg}}{}}{12\, bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(45*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - 12*e^(b*c*
x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (75*e^(7*b*c*x + 7*a*c)*sgn(e^(2*b*
c*x + 2*a*c) - 1) + 115*e^(5*b*c*x + 5*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) +
109*e^(3*b*c*x + 3*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 21*e^(b*c*x + a*c)*s
gn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) + 1)^4)/(b*c)
```

3.235 $\int e^{c(a+bx)}\tanh^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=193

$$\frac{e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}+\frac{3e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)}-\frac{2e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c) - (2*E^
(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c
*(a + b*x)))^2) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*
x]^2])/(b*c*(1 + E^(2*c*(a + b*x)))) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c
+ b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c)
```

Rubi [A] time = 0.279721, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6720, 2282, 390, 1158, 12, 288, 203}

$$\frac{e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}+\frac{3e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)}-\frac{2e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Tanh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c) - (2*E^
(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c
*(a + b*x)))^2) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*
x]^2])/(b*c*(1 + E^(2*c*(a + b*x)))) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c
+ b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{3/2}\,dx = \left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\int e^{c(a+bx)}\tanh^3(ac+bcx)\,dx$$

$$= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \left(1-\frac{2\left(1+3x^4\right)}{\left(1+x^2\right)^3}\right)dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{\left(2\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{S}}{bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+b}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+b}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+b}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+b}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

Mathematica [A] time = 0.155257, size = 104, normalized size = 0.54

$$\frac{\left(e^{c(a+bx)}\left(5e^{2c(a+bx)}+e^{4c(a+bx)}+2\right)-3\left(e^{2c(a+bx)}+1\right)^2\tan^{-1}\left(e^{c(a+bx)}\right)\right)\sqrt{\tanh^2(c(a+bx))}\coth(c(a+bx))}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Tanh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(2 + 5*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))) - 3*(1 + E^
(2*c*(a + b*x)))^2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)]*Sqrt[Tanh[c*(
a + b*x)]^2])/(b*c*(1 + E^(2*c*(a + b*x)))^2)
```

Maple [C] time = 0.185, size = 301, normalized size = 1.6

$$\frac{\left(1+e^{2c(bx+a)}\right)e^{c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)cb}\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}+\frac{e^{c(bx+a)}\left(3e^{2c(bx+a)}+1\right)}{\left(e^{2c(bx+a)}-1\right)\left(1+e^{2c(bx+a)}\right)cb}\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}+\frac{\frac{3i}{2}\left(1+e^{2c(bx+a)}\right)\ln\left(e^{c(bx}\right.}{\left(e^{2c(bx+a)}-1\right)cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 1/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))*((exp(2*c*(b*x+a))-1)^2/(1+exp(
2*c*(b*x+a)))^2)^(1/2)*exp(c*(b*x+a))/c/b+1/(exp(2*c*(b*x+a))-1)/(1+exp(2*c
*(b*x+a)))*((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)*exp(c*(b*x
+a))*(3*exp(2*c*(b*x+a))+1)/c/b+3/2*I/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+
a)))*((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/c/b*ln(exp(c*(b*
x+a))-I)-3/2*I/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))*((exp(2*c*(b*x+a))
-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+I)
```

Maxima [A] time = 1.77111, size = 122, normalized size = 0.63

$$-\frac{3\arctan\left(e^{(bcx+ac)}\right)}{bc}+\frac{e^{(5bcx+5ac)}+5e^{(3bcx+3ac)}+2e^{(bcx+ac)}}{bc\left(e^{(4bcx+4ac)}+2e^{(2bcx+2ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3*arctan(e^(b*c*x + a*c))/(b*c) + (e^(5*b*c*x + 5*a*c) + 5*e^(3*b*c*x + 3*
a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c)
 + 1))
```

Fricas [B] time = 2.15092, size = 1175, normalized size = 6.09

$$\frac{\cosh(bcx+ac)^5+5\cosh(bcx+ac)\sinh(bcx+ac)^4+\sinh(bcx+ac)^5+5\left(2\cosh(bcx+ac)^2+1\right)\sinh(bcx+ac)^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + sinh(b*c*x
 + a*c)^5 + 5*(2*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^3 + 5*cosh(b*c*
x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a
*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 2
*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*sinh(b*c
*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + (5*cosh(b*c*
x + a*c)^4 + 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 2*cosh(b*c*x +
 a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
^3 + b*c*sinh(b*c*x + a*c)^4 + 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*
c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3
+ b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27395, size = 174, normalized size = 0.9

$$-\frac{3\arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2bcx+2ac)}-1\right)-e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2bcx+2ac)}-1\right)-\frac{3e^{(3bcx+3ac)}\operatorname{sgn}\left(e^{(2bcx+2ac)}-1\right)+e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2bcx+2ac)}-1\right)}{\left(e^{(2bcx+2ac)}+1\right)^2}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*x + a*c)*
sgn(e^(2*b*c*x + 2*a*c) - 1) - (3*e^(3*b*c*x + 3*a*c)*sgn(e^(2*b*c*x + 2*a*
c) - 1) + e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c
) + 1)^2)/(b*c)
```

3.236 $\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}-\frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}$$

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c) - (2*Ar
cTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c)
```

Rubi [A] time = 0.149856, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 388, 203}

$$\frac{e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}-\frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Tanh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c) - (2*Ar
cTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx = \left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\int e^{c(a+bx)}\tanh(ac+bcx)\,dx$$
$$= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{-1+x^2}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{\left(2\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{S}}{bc}$$
$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\coth(ac+bcx)\sqrt{\tanh}}{bc}$$

Mathematica [A] time = 0.0522581, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)} - 2\tan^{-1}\left(e^{c(a+bx)}\right)\right)\sqrt{\tanh^2(c(a+bx))}\coth(c(a+bx))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Tanh[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)]*Sqrt[Tanh[
c*(a + b*x)]^2])/(b*c)
```

Maple [C] time = 0.23, size = 218, normalized size = 2.6

$$\frac{\left(1+e^{2c(bx+a)}\right)e^{c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)cb}\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}} + \frac{i\left(1+e^{2c(bx+a)}\right)\ln\left(e^{c(bx+a)}-i\right)}{\left(e^{2c(bx+a)}-1\right)cb}\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}} - \frac{i\left(1+e^{2c(bx+a)}\right)\ln\left(e^{c(b}\right.}{\left(e^{2c(bx+a)}-1\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))*((exp(2*c*(b*x+a))-1)^2/(1+exp(
2*c*(b*x+a)))^2)^(1/2)*exp(c*(b*x+a))/c/b+I*((exp(2*c*(b*x+a))-1)^2/(1+exp(
2*c*(b*x+a)))^2)^(1/2)/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp
(c*(b*x+a))-I)-I*((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(exp
(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp(c*(b*x+a))+I)
```

Maxima [A] time = 1.77358, size = 47, normalized size = 0.57

$$-\frac{2\arctan\left(e^{(bcx+ac)}\right)}{bc} + \frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(b*c*x + a*c))/(b*c) + e^(b*c*x + a*c)/(b*c)
```

Fricas [A] time = 1.99386, size = 132, normalized size = 1.59

$$-\frac{2\arctan\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23014, size = 81, normalized size = 0.98

$$-\frac{2\arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*x + a*c)*
sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c)
```

3.237 $\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

```
[Out] (E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2]) - (2*Ar
cTanh[E^(c*(a + b*x))]*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2])
```

Rubi [A] time = 0.199188, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 388, 206}

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Tanh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2]) - (2*Ar
cTanh[E^(c*(a + b*x))]*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx = \frac{\tanh(ac+bcx)\int e^{c(a+bx)}\coth(ac+bcx)\,dx}{\sqrt{\tanh^2(ac+bcx)}}$$
$$= \frac{\tanh(ac+bcx)\,\text{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$
$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{(2\tanh(ac+bcx))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$
$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

Mathematica [A] time = 0.123081, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)} - 2\tanh^{-1}\left(e^{c(a+bx)}\right)\right)\tanh(c(a+bx))}{bc\sqrt{\tanh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Tanh[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTanh[E^(c*(a + b*x))])*Tanh[c*(a + b*x)])/(b*c*Sqr
t[Tanh[c*(a + b*x)]^2])
```

Maple [B] time = 0.227, size = 213, normalized size = 2.6

$$\frac{\left(e^{2c(bx+a)}-1\right)e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}} + \frac{\left(e^{2c(bx+a)}-1\right)\ln\left(e^{c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}} - \frac{\left(e^{2c(bx+a)}-1\right)\ln\left(e^{c(bx+a)}+1\right)}{\left(1+e^{2c(bx+a)}\right)cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(1+exp(2*c*(b*x+a))
)*(exp(2*c*(b*x+a))-1)*exp(c*(b*x+a))/c/b+1/((exp(2*c*(b*x+a))-1)^2/(1+exp(
2*c*(b*x+a)))^2)^(1/2)/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)/c/b*ln(exp
(c*(b*x+a))-1)-1/((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(1+e
xp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)/c/b*ln(exp(c*(b*x+a))+1)
```

Maxima [A] time = 1.77196, size = 76, normalized size = 0.92

$$\frac{e^{(bcx+ac)}}{bc} - \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(b*c*x + a*c)/(b*c) - log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c
) - 1)/(b*c)
```

Fricas [A] time = 2.03521, size = 196, normalized size = 2.36

$$\frac{\cosh\left(bcx+ac\right)-\log\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)+1\right)+\log\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)-1\right)+\sinh\left(bcx+ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c) - log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) + log(c
osh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + sinh(b*c*x + a*c))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\tanh^2\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(tanh(a*c + b*c*x)**2), x)
```

Giac [A] time = 1.30181, size = 119, normalized size = 1.43

$$\frac{e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-\log\left(e^{(bcx+ac)}+1\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+\log\left(\left|e^{(bcx+ac)}-1\right|\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - log(e^(b*c*x + a*c) + 1)*sg
n(e^(2*b*c*x + 2*a*c) - 1) + log(abs(e^(b*c*x + a*c) - 1))*sgn(e^(2*b*c*x +
 2*a*c) - 1))/(b*c)
```

3.238 $\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{3\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}+\frac{3e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\tanh}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{}}$$

```
[Out] (E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2]) - (2*E^
(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt[Tanh[a
*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a
 + b*x)))*Sqrt[Tanh[a*c + b*c*x]^2]) - (3*ArcTanh[E^(c*(a + b*x))]*Tanh[a*c
 + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2])
```

Rubi [A] time = 0.86523, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6720, 2282, 390, 1158, 12, 288, 207}

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{3\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}+\frac{3e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\tanh}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Tanh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2]) - (2*E^
(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt[Tanh[a
*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a
 + b*x)))*Sqrt[Tanh[a*c + b*c*x]^2]) - (3*ArcTanh[E^(c*(a + b*x))]*Tanh[a*c
 + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx = \frac{\tanh(ac+bcx)\int e^{c(a+bx)}\coth^3(ac+bcx)\,dx}{\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{\tanh(ac+bcx)\,\text{Subst}\left(\int \frac{(1+x^2)^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{\tanh(ac+bcx)\,\text{Subst}\left(\int \left(1+\frac{2(1+3x^4)}{(-1+x^2)^3}\right)dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} + \frac{(2\tanh(ac+bcx))\,\text{Subst}\left(\int \frac{1+3x^4}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} + \frac{\tanh(ac+bcx)\,\text{Subst}\left(\int -\ldots\right.}{2bc\sqrt{\tanh^2(ac\ldots}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} + \frac{(6\tanh(ac+bcx))\,\text{Subst}\left(\ldots\right.}{bc\sqrt{\tanh^2(a\ldots}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\tanh(ac+bc\ldots}{bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac\ldots}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\tanh(ac+bc\ldots}{bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac\ldots}}$$

Mathematica [C] time = 7.5859, size = 334, normalized size = 1.7

$$-\frac{e^{-5c(a+bx)}\tanh^3(c(a+bx))\left(256e^{8c(a+bx)}\left(e^{2c(a+bx)}+1\right)^3 \text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2,2,2\right\},\left\{1,1,1,1,\frac{11}{2}\right\},e^{2c(a+bx)}\ldots\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))/(Tanh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -((-21*(252105 + 507305*E^(2*c*(a + b*x)) + 173916*E^(4*c*(a + b*x)) - 1542
96*E^(6*c*(a + b*x)) - 73885*E^(8*c*(a + b*x)) + 4887*E^(10*c*(a + b*x))) -
 (315*(-16807 - 28218*E^(2*c*(a + b*x)) + 1173*E^(4*c*(a + b*x)) + 17748*E^
(6*c*(a + b*x)) + 4299*E^(8*c*(a + b*x)) - 1434*E^(10*c*(a + b*x)) + 7*E^(1
2*c*(a + b*x)))*ArcTanh[Sqrt[E^(2*c*(a + b*x))]])/Sqrt[E^(2*c*(a + b*x))] +
 384*E^(8*c*(a + b*x))*(1 + E^(2*c*(a + b*x)))^2*(7 + 5*E^(2*c*(a + b*x)))*
HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*c*(a + b*x))] +
256*E^(8*c*(a + b*x))*(1 + E^(2*c*(a + b*x)))^3*HypergeometricPFQ[{3/2, 2,
2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, E^(2*c*(a + b*x))])*Tanh[c*(a + b*x)]^3)/(
60480*b*c*E^(5*c*(a + b*x))*(Tanh[c*(a + b*x)]^2)^(3/2))
```

Maple [A] time = 0.193, size = 298, normalized size = 1.5

$$\frac{\left(e^{2c(bx+a)}-1\right)e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}}-\frac{e^{c(bx+a)}\left(3e^{2c(bx+a)}-1\right)}{\left(e^{2c(bx+a)}-1\right)\left(1+e^{2c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}}+\frac{\left(3e^{2c(bx+a)}-3\right)\ln\left(e^{c(bx+a)}-\right.}{\left(2+2e^{2c(bx+a)}\right)cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 1/((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(1+exp(2*c*(b*x+a))
)*(exp(2*c*(b*x+a))-1)*exp(c*(b*x+a))/c/b-1/(exp(2*c*(b*x+a))-1)/(1+exp(2*c
*(b*x+a)))/((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)*exp(c*(b*x
+a))*(3*exp(2*c*(b*x+a))-1)/c/b+3/2/((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x
+a)))^2)^(1/2)/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)/c/b*ln(exp(c*(b*x+
a))-1)-3/2/((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(1+exp(2*c
*(b*x+a)))*(exp(2*c*(b*x+a))-1)/c/b*ln(exp(c*(b*x+a))+1)
```

Maxima [A] time = 1.75246, size = 151, normalized size = 0.77

$$-\frac{3\log\left(e^{(bcx+ac)}+1\right)}{2bc}+\frac{3\log\left(e^{(bcx+ac)}-1\right)}{2bc}+\frac{e^{(5bcx+5ac)}-5e^{(3bcx+3ac)}+2e^{(bcx+ac)}}{bc\left(e^{(4bcx+4ac)}-2e^{(2bcx+2ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(e^(b*c*x + a*c) + 1)/(b*c) + 3/2*log(e^(b*c*x + a*c) - 1)/(b*c) +
(e^(5*b*c*x + 5*a*c) - 5*e^(3*b*c*x + 3*a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(
4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 2.00662, size = 1574, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + 2*s
inh(b*c*x + a*c)^5 + 10*(2*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^3 - 1
0*cosh(b*c*x + a*c)^3 + 10*(2*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x
 + a*c)^3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x
+ a*c)^2 - 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*
c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) +
 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + sinh(b*
c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 2*cosh(b
*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*
c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + 2*(5*cosh(b*c*x +
a*c)^4 - 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 4*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
 b*c*sinh(b*c*x + a*c)^4 - 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*c*x
```

```
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3 - b*
c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.50692, size = 217, normalized size = 1.1

$$\frac{2\,e^{(bcx+ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-3\,\log\left(e^{(bcx+ac)}+1\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+3\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-\frac{2(3}{}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 3*log(e^(b*c*x + a*c)
 + 1)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 3*log(abs(e^(b*c*x + a*c) - 1))*sgn(e^
(2*b*c*x + 2*a*c) - 1) - 2*(3*e^(3*b*c*x + 3*a*c)*sgn(e^(2*b*c*x + 2*a*c) -
 1) - e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) -
1)^2)/(b*c)
```

3.239 $\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}} dx$

Optimal. Leaf size=319

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{15\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{4bc\sqrt{\tanh^2(ac+bcx)}}+\frac{25e^{c(a+bx)}\tanh(ac+bcx)}{4bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}}-\frac{55e^{c(a+b}}{6bc\left(1-e^{2c(a+}\right.}$$

```
[Out] (E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2]) - (4*E^
(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4*Sqrt[Tanh[a
*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c
*(a + b*x)))^3*Sqrt[Tanh[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Tanh[a*c +
b*c*x])/(6*b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt[Tanh[a*c + b*c*x]^2]) + (25*E
^(c*(a + b*x))*Tanh[a*c + b*c*x])/(4*b*c*(1 - E^(2*c*(a + b*x)))*Sqrt[Tanh[
a*c + b*c*x]^2]) - (15*ArcTanh[E^(c*(a + b*x))]*Tanh[a*c + b*c*x])/(4*b*c*S
qrt[Tanh[a*c + b*c*x]^2])
```

Rubi [A] time = 1.77048, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {6720, 2282, 390, 1814, 1157, 385, 207}

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{15\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{4bc\sqrt{\tanh^2(ac+bcx)}}+\frac{25e^{c(a+bx)}\tanh(ac+bcx)}{4bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}}-\frac{55e^{c(a+b}}{6bc\left(1-e^{2c(a+}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Tanh[a*c + b*c*x]^2)^(5/2),x]
```


```
[Out] (E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2]) - (4*E^
(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4*Sqrt[Tanh[a
*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c
*(a + b*x)))^3*Sqrt[Tanh[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Tanh[a*c +
b*c*x])/(6*b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt[Tanh[a*c + b*c*x]^2]) + (25*E
^(c*(a + b*x))*Tanh[a*c + b*c*x])/(4*b*c*(1 - E^(2*c*(a + b*x)))*Sqrt[Tanh[
a*c + b*c*x]^2]) - (15*ArcTanh[E^(c*(a + b*x))]*Tanh[a*c + b*c*x])/(4*b*c*S
qrt[Tanh[a*c + b*c*x]^2])
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx = \frac{\tanh(ac+bcx)\int e^{c(a+bx)}\coth^5(ac+bcx)\,dx}{\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{\tanh(ac+bcx)\,\text{Subst}\left(\int \frac{(1+x^2)^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{\tanh(ac+bcx)\,\text{Subst}\left(\int \left(1+\frac{2(1+10x^4+5x^8)}{(-1+x^2)^5}\right)dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} + \frac{(2\tanh(ac+bcx))\,\text{Subst}\left(\int \frac{1+10x^4+5x^8}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\tanh^2(ac+bcx)}} + \frac{\tanh(ac+bcx)\,\text{Subst}\big(\int}{4bc\sqrt{\text{ta}}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\tanh^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\tanh(ac}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\text{tan}}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\tanh^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\tanh(ac}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\text{tan}}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\tanh^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\tanh(ac}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\text{tan}}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\tanh^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\tanh(ac}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\text{tan}}}$$

<u>**Mathematica**</u> **[A]** time = 10.6569, size = 164, normalized size = 0.51

$$\frac{\left(66e^{c(a+bx)} - 314e^{3c(a+bx)} + 374e^{5c(a+bx)} - 246e^{7c(a+bx)} + 24e^{9c(a+bx)} + 45\left(e^{2c(a+bx)}-1\right)^4\log\left(1-e^{c(a+bx)}\right) - 45\left(e^{2c(a+bx)}\right.}{24bc\left(e^{2c(a+bx)}-1\right)^4\sqrt{\tanh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Tanh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((66*E^(c*(a + b*x)) - 314*E^(3*c*(a + b*x)) + 374*E^(5*c*(a + b*x)) - 246*
E^(7*c*(a + b*x)) + 24*E^(9*c*(a + b*x)) + 45*(-1 + E^(2*c*(a + b*x)))^4*Lo
g[1 - E^(c*(a + b*x))] - 45*(-1 + E^(2*c*(a + b*x)))^4*Log[1 + E^(c*(a + b*
x))])*Tanh[c*(a + b*x)])/(24*b*c*(-1 + E^(2*c*(a + b*x)))^4*Sqrt[Tanh[c*(a
+ b*x)]^2])
```

Maple [A] time = 0.197, size = 320, normalized size = 1.

$$\frac{\left(e^{2c(bx+a)}-1\right)e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}}-\frac{e^{c(bx+a)}\left(75\,e^{6c(bx+a)}-115\,e^{4c(bx+a)}+109\,e^{2c(bx+a)}-21\right)}{12\left(e^{2c(bx+a)}-1\right)^3\left(1+e^{2c(bx+a)}\right)cb}\frac{1}{\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}}+\frac{\left(15\,e^{2c(b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 1/((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(1+exp(2*c*(b*x+a))
)*(exp(2*c*(b*x+a))-1)*exp(c*(b*x+a))/c/b-1/12/(exp(2*c*(b*x+a))-1)^3/(1+ex
p(2*c*(b*x+a)))/((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)*exp(c
*(b*x+a))*(75*exp(6*c*(b*x+a))-115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))-21
)/c/b+15/8/((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(1+exp(2*c
*(b*x+a)))*(exp(2*c*(b*x+a))-1)/c/b*ln(exp(c*(b*x+a))-1)-15/8/((exp(2*c*(b*
x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x
+a))-1)/c/b*ln(exp(c*(b*x+a))+1)
```

Maxima [A] time = 1.68155, size = 225, normalized size = 0.71

$$-\frac{15\,\log\left(e^{(bcx+ac)}+1\right)}{8\,bc}+\frac{15\,\log\left(e^{(bcx+ac)}-1\right)}{8\,bc}+\frac{12\,e^{(9\,bcx+9\,ac)}-123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}-157\,e^{(3\,bcx+3\,ac)}+33\,e^{(bc}}{12\,bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(b*c*x + a*c) + 1)/(b*c) + 15/8*log(e^(b*c*x + a*c) - 1)/(b*c)
+ 1/12*(12*e^(9*b*c*x + 9*a*c) - 123*e^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x +
 5*a*c) - 157*e^(3*b*c*x + 3*a*c) + 33*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x +
8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a
*c) + 1))
```

Fricas [B] time = 2.21561, size = 4176, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*cosh(b*c*x + a*c)^9 + 216*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
24*sinh(b*c*x + a*c)^9 + 6*(144*cosh(b*c*x + a*c)^2 - 41)*sinh(b*c*x + a*c)
^7 - 246*cosh(b*c*x + a*c)^7 + 42*(48*cosh(b*c*x + a*c)^3 - 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + 2*(1512*cosh(b*c*x + a*c)^4 - 2583*cosh(b*c*x
+ a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 374*cosh(b*c*x + a*c)^5 + 2*(1512*cos
h(b*c*x + a*c)^5 - 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b
*c*x + a*c)^4 + 2*(1008*cosh(b*c*x + a*c)^6 - 4305*cosh(b*c*x + a*c)^4 + 18
70*cosh(b*c*x + a*c)^2 - 157)*sinh(b*c*x + a*c)^3 - 314*cosh(b*c*x + a*c)^3
 + 2*(432*cosh(b*c*x + a*c)^7 - 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x
+ a*c)^3 - 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*
c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7
```

```
*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(
7*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*co
sh(b*c*x + a*c)^4 - 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cos
h(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b
*c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c
*x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x
 + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c)
 + sinh(b*c*x + a*c) + 1) + 45*(cosh(b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*c*x + a*c)^2 - 1)*si
nh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 - 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x + a*c)^4 - 30*cosh(
b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh
(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x
 + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x
+ a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c
))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) +
2*(108*cosh(b*c*x + a*c)^8 - 861*cosh(b*c*x + a*c)^6 + 935*cosh(b*c*x + a*c
)^4 - 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 66*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 +
 b*c*sinh(b*c*x + a*c)^8 - 4*b*c*cosh(b*c*x + a*c)^6 + 4*(7*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)^4 + 8*(7*b*c*
cosh(b*c*x + a*c)^3 - 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*
b*c*cosh(b*c*x + a*c)^4 - 30*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x +
a*c)^4 - 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x + a*c)^5 - 10*b*c*
cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*b
*c*cosh(b*c*x + a*c)^6 - 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*c*cosh(b*c*x + a*
c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x + a*c)^7 - 3*b*c*
cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.64773, size = 290, normalized size = 0.91

$$\frac{24\,e^{(bcx+ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-45\,\log\left(e^{(bcx+ac)}+1\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+45\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right.}{24\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 45*log(e^(b*c*x + a
*c) + 1)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 45*log(abs(e^(b*c*x + a*c) - 1))*sg
```

```
n(e^(2*b*c*x + 2*a*c) - 1) - 2*(75*e^(7*b*c*x + 7*a*c)*sgn(e^(2*b*c*x + 2*a
*c) - 1) - 115*e^(5*b*c*x + 5*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 109*e^(3*
b*c*x + 3*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 21*e^(b*c*x + a*c)*sgn(e^(2*b
*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) - 1)^4)/(b*c)
```

3.240 $\int \sin^3(\tanh(a+bx))\,dx$

Optimal. Leaf size=157

$$\frac{\sin(3)\text{CosIntegral}(3-3\tanh(a+bx))}{8b}+\frac{\sin(3)\text{CosIntegral}(3\tanh(a+bx)+3)}{8b}-\frac{3\sin(1)\text{CosIntegral}(1-\tanh}{8b}$$

```
[Out] (-3*CosIntegral[1 - Tanh[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Tanh[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Tanh[a + b*x]]*Sin[3])/(8*b) +
 (CosIntegral[3 + 3*Tanh[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Tanh[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Tanh[a + b*x]])/(8*b) +
 (3*Cos[1]*SinIntegral[1 + Tanh[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Tanh[a + b*x]])/(8*b)
```

Rubi [A] time = 0.39518, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\sin(3)\text{CosIntegral}(3-3\tanh(a+bx))}{8b}+\frac{\sin(3)\text{CosIntegral}(3\tanh(a+bx)+3)}{8b}-\frac{3\sin(1)\text{CosIntegral}(1-\tanh}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Tanh[a + b*x]]^3,x]
```

```
[Out] (-3*CosIntegral[1 - Tanh[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Tanh[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Tanh[a + b*x]]*Sin[3])/(8*b) +
 (CosIntegral[3 + 3*Tanh[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Tanh[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Tanh[a + b*x]])/(8*b) +
 (3*Cos[1]*SinIntegral[1 + Tanh[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Tanh[a + b*x]])/(8*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin^3(\tanh(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sin^3(x)}{2(-1+x)} + \frac{\sin^3(x)}{2(1+x)}\right)dx, x, \tanh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(-1+x)} - \frac{\sin(3x)}{4(-1+x)}\right)dx, x, \tanh(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(1+x)} - \frac{\sin(3x)}{4(1+x)}\right)dx, x, \text{tanh}}{2b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{\text{Subst}\left(\int \frac{\sin(3x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{3\,\text{Subst}\left(\int \right.}{}$$
$$= \frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\sin(1-x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \tanh(a\right.}{8b}$$
$$= -\frac{3\text{Ci}(1-\tanh(a+bx))\sin(1)}{8b} - \frac{3\text{Ci}(1+\tanh(a+bx))\sin(1)}{8b} + \frac{\text{Ci}(3-3\tanh(a+bx))\sin}{8b}$$

Mathematica [A] time = 0.214449, size = 124, normalized size = 0.79

$$2\sin(3)\text{CosIntegral}(3-3\tanh(a+bx)) + 2\sin(3)\text{CosIntegral}(3\tanh(a+bx)+3) - 6\sin(1)\text{CosIntegral}(1-\tanh($$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Tanh[a + b*x]]^3,x]
```

```
[Out] (-6*CosIntegral[1 - Tanh[a + b*x]]*Sin[1] - 6*CosIntegral[1 + Tanh[a + b*x]
]*Sin[1] + 2*CosIntegral[3 - 3*Tanh[a + b*x]]*Sin[3] + 2*CosIntegral[3 + 3*
Tanh[a + b*x]]*Sin[3] - 2*Cos[3]*SinIntegral[3 - 3*Tanh[a + b*x]] + 6*Cos[1
]*SinIntegral[1 - Tanh[a + b*x]] + 6*Cos[1]*SinIntegral[1 + Tanh[a + b*x]]
- 2*Cos[3]*SinIntegral[3 + 3*Tanh[a + b*x]])/(16*b)
```

Maple [A] time = 0.026, size = 118, normalized size = 0.8

$$\frac{1}{b}\left(-\frac{\text{Si}(3+3\tanh(bx+a))\cos(3)}{8} + \frac{\text{Ci}(3+3\tanh(bx+a))\sin(3)}{8} + \frac{\text{Si}(-3+3\tanh(bx+a))\cos(3)}{8} + \frac{\text{Ci}(-3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(tanh(b*x+a))^3,x)
```

```
[Out] 1/b*(-1/8*Si(3+3*tanh(b*x+a))*cos(3)+1/8*Ci(3+3*tanh(b*x+a))*sin(3)+1/8*Si(
-3+3*tanh(b*x+a))*cos(3)+1/8*Ci(-3+3*tanh(b*x+a))*sin(3)+3/8*Si(1+tanh(b*x+
a))*cos(1)-3/8*Ci(1+tanh(b*x+a))*sin(1)-3/8*Si(-1+tanh(b*x+a))*cos(1)-3/8*C
i(-1+tanh(b*x+a))*sin(1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\tanh\left(bx+a\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(tanh(b*x + a))^3, x)
```

Fricas [B] time = 2.34124, size = 929, normalized size = 5.92

$$\frac{\operatorname{Ci}\left(\frac{6\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}+1}\right)\sin\left(3\right)+\operatorname{Ci}\left(-\frac{6\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}+1}\right)\sin\left(3\right)+\operatorname{Ci}\left(\frac{6}{e^{\left(2\,bx+2\,a\right)}+1}\right)\sin\left(3\right)+\operatorname{Ci}\left(-\frac{6}{e^{\left(2\,bx+2\,a\right)}+1}\right)\sin\left(3\right)-3\,\operatorname{Ci}\left(\frac{2\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}+1}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(cos_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1))*sin(3) + cos_in
tegral(-6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1))*sin(3) + cos_integral(6/(e
^(2*b*x + 2*a) + 1))*sin(3) + cos_integral(-6/(e^(2*b*x + 2*a) + 1))*sin(3)
 - 3*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1))*sin(1) - 3*cos_i
ntegral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1))*sin(1) - 3*cos_integral(2
/(e^(2*b*x + 2*a) + 1))*sin(1) - 3*cos_integral(-2/(e^(2*b*x + 2*a) + 1))*s
in(1) - 2*cos(3)*sin_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) + 6*
cos(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) - 2*cos(3)*sin
_integral(6/(e^(2*b*x + 2*a) + 1)) + 6*cos(1)*sin_integral(2/(e^(2*b*x + 2*
a) + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{3}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a))**3,x)
```

```
[Out] Integral(sin(tanh(a + b*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\tanh\left(bx+a\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(tanh(b*x + a))^3, x)
```

3.241 $\int \sin^2(\tanh(a+bx))\,dx$

Optimal. Leaf size=115

$$\frac{\cos(2)\text{CosIntegral}(2-2\tanh(a+bx))}{4b}-\frac{\cos(2)\text{CosIntegral}(2\tanh(a+bx)+2)}{4b}+\frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b}-\text{si}$$

```
[Out] (Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2
*Tanh[a + b*x]])/(4*b) - Log[1 - Tanh[a + b*x]]/(4*b) + Log[1 + Tanh[a + b*
x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Tanh[a + b*x]])/(4*b) - (Sin[2]*SinIn
tegral[2 + 2*Tanh[a + b*x]])/(4*b)
```

Rubi [A] time = 0.265657, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\cos(2)\text{CosIntegral}(2-2\tanh(a+bx))}{4b}-\frac{\cos(2)\text{CosIntegral}(2\tanh(a+bx)+2)}{4b}+\frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b}-\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Tanh[a + b*x]]^2,x]
```

```
[Out] (Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2
*Tanh[a + b*x]])/(4*b) - Log[1 - Tanh[a + b*x]]/(4*b) + Log[1 + Tanh[a + b*
x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Tanh[a + b*x]])/(4*b) - (Sin[2]*SinIn
tegral[2 + 2*Tanh[a + b*x]])/(4*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin^2(\tanh(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sin^2(x)}{2(-1+x)}+\frac{\sin^2(x)}{2(1+x)}\right)dx, x, \tanh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}-\frac{\cos(2x)}{2(-1+x)}\right)dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}-\frac{\cos(2x)}{2(1+x)}\right)dx, x, \text{ta}\right.}{2b}$$
$$= -\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{4b}$$
$$= -\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b}+\frac{\cos(2)\,\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx, x, \tanh\right.}{4b}$$
$$= \frac{\cos(2)\text{Ci}(2-2\tanh(a+bx))}{4b}-\frac{\cos(2)\text{Ci}(2+2\tanh(a+bx))}{4b}-\frac{\log(1-\tanh(a+bx))}{4b}+$$

Mathematica [A] time = 0.140706, size = 88, normalized size = 0.77

$$\frac{\cos(2)\text{CosIntegral}(2-2\tanh(a+bx))-\cos(2)\text{CosIntegral}(2(\tanh(a+bx)+1))+\sin(2)\text{Si}(2-2\tanh(a+bx))-}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Tanh[a + b*x]]^2,x]
```

```
[Out] (Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]] - Cos[2]*CosIntegral[2*(1 + Tanh[a
 + b*x])] - Log[1 - Tanh[a + b*x]] + Log[1 + Tanh[a + b*x]] + Sin[2]*SinInt
egral[2 - 2*Tanh[a + b*x]] - Sin[2]*SinIntegral[2*(1 + Tanh[a + b*x])])/(4*
b)
```

Maple [A] time = 0.023, size = 102, normalized size = 0.9

$$-\frac{\ln(-1+\tanh(bx+a))}{4b}+\frac{\ln(1+\tanh(bx+a))}{4b}-\frac{\text{Si}(2+2\tanh(bx+a))\sin(2)}{4b}-\frac{\text{Ci}(2+2\tanh(bx+a))\cos(2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(tanh(b*x+a))^2,x)
```

```
[Out] -1/4/b*ln(-1+tanh(b*x+a))+1/4*ln(1+tanh(b*x+a))/b-1/4*Si(2+2*tanh(b*x+a))*s
in(2)/b-1/4*Ci(2+2*tanh(b*x+a))*cos(2)/b-1/4*Si(-2+2*tanh(b*x+a))*sin(2)/b+
1/4/b*Ci(-2+2*tanh(b*x+a))*cos(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x-\frac{1}{2}\int \cos\left(\frac{2\left(e^{(2bx+2a)}-1\right)}{e^{(2bx+2a)}+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/2*integrate(cos(2*(e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)), x
)
```

Fricas [A] time = 2.21134, size = 474, normalized size = 4.12

$$\frac{4\,bx - \cos(2)\,\mathrm{Ci}\left(\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right) - \cos(2)\,\mathrm{Ci}\left(-\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right) + \cos(2)\,\mathrm{Ci}\left(\frac{4}{e^{(2\,bx+2\,a)}+1}\right) + \cos(2)\,\mathrm{Ci}\left(-\frac{4}{e^{(2\,bx+2\,a)}+1}\right) - 2\,\sin(2)\,\mathrm{Si}\left(\right.}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x - cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) -
 cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) + cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) + 1)) + cos(2)*cos_integral(-4/(e^(2*b*x + 2*a
) + 1)) - 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) +
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^2(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a))**2,x)
```

```
[Out] Integral(sin(tanh(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\tanh(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(tanh(b*x + a))^2, x)
```

3.242 $\int \sin(\tanh(a+bx))\,dx$

Optimal. Leaf size=77

$$-\frac{\sin(1)\text{CosIntegral}(1-\tanh(a+bx))}{2b}-\frac{\sin(1)\text{CosIntegral}(\tanh(a+bx)+1)}{2b}+\frac{\cos(1)\text{Si}(1-\tanh(a+bx))}{2b}+\text{cos}$$

```
[Out] -(CosIntegral[1 - Tanh[a + b*x]]*Sin[1])/(2*b) - (CosIntegral[1 + Tanh[a +
b*x]]*Sin[1])/(2*b) + (Cos[1]*SinIntegral[1 - Tanh[a + b*x]])/(2*b) + (Cos[
1]*SinIntegral[1 + Tanh[a + b*x]])/(2*b)
```

Rubi [A] time = 0.145559, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3333, 3303, 3299, 3302}

$$-\frac{\sin(1)\text{CosIntegral}(1-\tanh(a+bx))}{2b}-\frac{\sin(1)\text{CosIntegral}(\tanh(a+bx)+1)}{2b}+\frac{\cos(1)\text{Si}(1-\tanh(a+bx))}{2b}+\text{cos}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Tanh[a + b*x]],x]
```

```
[Out] -(CosIntegral[1 - Tanh[a + b*x]]*Sin[1])/(2*b) - (CosIntegral[1 + Tanh[a +
b*x]]*Sin[1])/(2*b) + (Cos[1]*SinIntegral[1 - Tanh[a + b*x]])/(2*b) + (Cos[
1]*SinIntegral[1 + Tanh[a + b*x]])/(2*b)
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin(\tanh(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{2(1-x)}+\frac{\sin(x)}{2(1+x)}\right)dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x}\,dx, x, \tanh(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}$$
$$= -\frac{\cos(1)\,\text{Subst}\left(\int \frac{\sin(1-x)}{1-x}\,dx, x, \tanh(a+bx)\right)}{2b} + \frac{\cos(1)\,\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{}$$
$$= -\frac{\mathbf{Ci}(1-\tanh(a+bx))\sin(1)}{2b} - \frac{\mathbf{Ci}(1+\tanh(a+bx))\sin(1)}{2b} + \frac{\cos(1)\mathbf{Si}(1-\tanh(a+bx))}{2b} +$$

Mathematica [A] time = 0.125437, size = 59, normalized size = 0.77

$$-\frac{\sin(1)\mathbf{CosIntegral}(1-\tanh(a+bx)) + \sin(1)\mathbf{CosIntegral}(\tanh(a+bx)+1) - \cos(1)(\mathbf{Si}(1-\tanh(a+bx)) + \mathbf{Si}(\tan}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Tanh[a + b*x]],x]
```

```
[Out] -(CosIntegral[1 - Tanh[a + b*x]]*Sin[1] + CosIntegral[1 + Tanh[a + b*x]]*Si
n[1] - Cos[1]*(SinIntegral[1 - Tanh[a + b*x]] + SinIntegral[1 + Tanh[a + b*
x]]))/(2*b)
```

Maple [A] time = 0.017, size = 58, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\text{Si}(1+\tanh(bx+a))\cos(1)}{2} - \frac{\text{Ci}(1+\tanh(bx+a))\sin(1)}{2} - \frac{\text{Si}(-1+\tanh(bx+a))\cos(1)}{2} - \frac{\text{Ci}(-1+\tanh(bx+}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(tanh(b*x+a)),x)
```

```
[Out] 1/b*(1/2*Si(1+tanh(b*x+a))*cos(1)-1/2*Ci(1+tanh(b*x+a))*sin(1)-1/2*Si(-1+ta
nh(b*x+a))*cos(1)-1/2*Ci(-1+tanh(b*x+a))*sin(1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(sin(tanh(b*x + a)), x)
```

Fricas [B] time = 2.20098, size = 464, normalized size = 6.03

$$-\frac{\operatorname{Ci}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right)\sin(1)+\operatorname{Ci}\left(-\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right)\sin(1)+\operatorname{Ci}\left(\frac{2}{e^{(2\,bx+2\,a)}+1}\right)\sin(1)+\operatorname{Ci}\left(-\frac{2}{e^{(2\,bx+2\,a)}+1}\right)\sin(1)-2\,\cos(1)\operatorname{Si}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/4*(cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1))*sin(1) + cos_in
tegral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1))*sin(1) + cos_integral(2/(e
^(2*b*x + 2*a) + 1))*sin(1) + cos_integral(-2/(e^(2*b*x + 2*a) + 1))*sin(1)
 - 2*cos(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) - 2*cos(1
)*sin_integral(2/(e^(2*b*x + 2*a) + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a)),x)
```

```
[Out] Integral(sin(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(sin(tanh(b*x + a)), x)
```

3.243 $\int \csc(\tanh(a+bx))\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}\text{Unintegrable}\left(\frac{\text{sech}^2(a+bx)\csc(\tanh(a+bx))}{\tanh(a+bx)+1},x\right)-\frac{1}{2}\text{Unintegrable}\left(\frac{\text{sech}^2(a+bx)\csc(\tanh(a+bx))}{\tanh(a+bx)-1},x\right)$$

```
[Out] -Unintegrable[(Csc[Tanh[a + b*x]]*Sech[a + b*x]^2)/(-1 + Tanh[a + b*x]), x]
/2 + Unintegrable[(Csc[Tanh[a + b*x]]*Sech[a + b*x]^2)/(1 + Tanh[a + b*x]),
 x]/2
```

Rubi [A] time = 0.0768353, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \csc(\tanh(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[Tanh[a + b*x]],x]
```

```
[Out] -Defer[Subst][Defer[Int][Csc[x]/(-1 + x), x], x, Tanh[a + b*x]]/(2*b) + Def
er[Subst][Defer[Int][Csc[x]/(1 + x), x], x, Tanh[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}
\int \csc(\tanh(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\csc(x)}{1-x^2}\,dx,x,\tanh(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\csc(x)}{2(-1+x)}+\frac{\csc(x)}{2(1+x)}\right)dx,x,\tanh(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\csc(x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\csc(x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 2.63444, size = 0, normalized size = 0.

$$\int \csc(\tanh(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[Tanh[a + b*x]],x]
```

```
[Out] Integrate[Csc[Tanh[a + b*x]], x]
```

Maple [A] time = 0.239, size = 0, normalized size = 0.

$$\int \csc(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(tanh(b*x+a)),x)
```

```
[Out] int(csc(tanh(b*x+a)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \csc(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(csc(tanh(b*x + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\csc\left(\tanh\left(bx+a\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(csc(tanh(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \csc(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(csc(tanh(b*x + a)), x)
```

3.244 $\int \cos^3(\tanh(a+bx))\,dx$

Optimal. Leaf size=157

$$-\frac{\cos(3)\text{CosIntegral}(3-3\tanh(a+bx))}{8b}-\frac{3\cos(1)\text{CosIntegral}(1-\tanh(a+bx))}{8b}+\frac{3\cos(1)\text{CosIntegral}(\tanh(a+b}{8b}$$

```
[Out] -(Cos[3]*CosIntegral[3 - 3*Tanh[a + b*x]])/(8*b) - (3*Cos[1]*CosIntegral[1
- Tanh[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Tanh[a + b*x]])/(8*b) +
 (Cos[3]*CosIntegral[3 + 3*Tanh[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 -
3*Tanh[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Tanh[a + b*x]])/(8*b) +
 (3*Sin[1]*SinIntegral[1 + Tanh[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 +
3*Tanh[a + b*x]])/(8*b)
```

Rubi [A] time = 0.372814, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$-\frac{\cos(3)\text{CosIntegral}(3-3\tanh(a+bx))}{8b}-\frac{3\cos(1)\text{CosIntegral}(1-\tanh(a+bx))}{8b}+\frac{3\cos(1)\text{CosIntegral}(\tanh(a+b}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Tanh[a + b*x]]^3,x]
```

```
[Out] -(Cos[3]*CosIntegral[3 - 3*Tanh[a + b*x]])/(8*b) - (3*Cos[1]*CosIntegral[1
- Tanh[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Tanh[a + b*x]])/(8*b) +
 (Cos[3]*CosIntegral[3 + 3*Tanh[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 -
3*Tanh[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Tanh[a + b*x]])/(8*b) +
 (3*Sin[1]*SinIntegral[1 + Tanh[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 +
3*Tanh[a + b*x]])/(8*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \cos^3(\tanh(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\cos^3(x)}{2(-1+x)} + \frac{\cos^3(x)}{2(1+x)}\right) dx, x, \tanh(a+bx)\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(-1+x)} + \frac{\cos(3x)}{4(-1+x)}\right) dx, x, \tanh(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(1+x)} + \frac{\cos(3x)}{4(1+x)}\right) dx, x, \ldots\right.}{2b}$$

$$= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{\text{Subst}\left(\int \frac{\cos(3x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{3\,\text{Subs}\ldots}{}$$

$$= -\frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\cos(1-x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \text{ta}\ldots\right.}{8b}$$

$$= -\frac{\cos(3)\text{Ci}(3-3\tanh(a+bx))}{8b} - \frac{3\cos(1)\text{Ci}(1-\tanh(a+bx))}{8b} + \frac{3\cos(1)\text{Ci}(1+\tanh(a\ldots}{8b}$$

Mathematica [A] time = 0.262534, size = 124, normalized size = 0.79

$$-2\cos(3)\text{CosIntegral}(3-3\tanh(a+bx)) - 6\cos(1)\text{CosIntegral}(1-\tanh(a+bx)) + 6\cos(1)\text{CosIntegral}(\tanh(\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*Cos[3]*CosIntegral[3 - 3*Tanh[a + b*x]] - 6*Cos[1]*CosIntegral[1 - Tanh
[a + b*x]] + 6*Cos[1]*CosIntegral[1 + Tanh[a + b*x]] + 2*Cos[3]*CosIntegral
[3 + 3*Tanh[a + b*x]] - 2*Sin[3]*SinIntegral[3 - 3*Tanh[a + b*x]] - 6*Sin[1
]*SinIntegral[1 - Tanh[a + b*x]] + 6*Sin[1]*SinIntegral[1 + Tanh[a + b*x]]
+ 2*Sin[3]*SinIntegral[3 + 3*Tanh[a + b*x]])/(16*b)
```

Maple [A] time = 0.023, size = 118, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\text{Si}(3+3\tanh(bx+a))\sin(3)}{8} + \frac{\text{Ci}(3+3\tanh(bx+a))\cos(3)}{8} + \frac{\text{Si}(-3+3\tanh(bx+a))\sin(3)}{8} - \frac{\text{Ci}(-3+\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(tanh(b*x+a))^3,x)
```

```
[Out] 1/b*(1/8*Si(3+3*tanh(b*x+a))*sin(3)+1/8*Ci(3+3*tanh(b*x+a))*cos(3)+1/8*Si(-
3+3*tanh(b*x+a))*sin(3)-1/8*Ci(-3+3*tanh(b*x+a))*cos(3)+3/8*Si(1+tanh(b*x+a
))*sin(1)+3/8*Ci(1+tanh(b*x+a))*cos(1)+3/8*Si(-1+tanh(b*x+a))*sin(1)-3/8*Ci
(-1+tanh(b*x+a))*cos(1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\tanh\left(bx+a\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(tanh(b*x + a))^3, x)
```

Fricas [B] time = 2.51146, size = 929, normalized size = 5.92

$$\cos\left(3\right)\operatorname{Ci}\left(\frac{6\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right)+3\,\cos\left(1\right)\operatorname{Ci}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right)+3\,\cos\left(1\right)\operatorname{Ci}\left(-\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right)+\cos\left(3\right)\operatorname{Ci}\left(-\frac{6\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right)-\cos\left(3\right)\operatorname{Ci}\left(\frac{}{e^{(2\,b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(cos(3)*cos_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) + 3*cos(
1)*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) + 3*cos(1)*cos_int
egral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) + cos(3)*cos_integral(-6*e^
(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) - cos(3)*cos_integral(6/(e^(2*b*x + 2*
a) + 1)) - 3*cos(1)*cos_integral(2/(e^(2*b*x + 2*a) + 1)) - 3*cos(1)*cos_in
tegral(-2/(e^(2*b*x + 2*a) + 1)) - cos(3)*cos_integral(-6/(e^(2*b*x + 2*a)
+ 1)) + 2*sin(3)*sin_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) + 6*
sin(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) - 2*sin(3)*sin
_integral(6/(e^(2*b*x + 2*a) + 1)) - 6*sin(1)*sin_integral(2/(e^(2*b*x + 2*
a) + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^3\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a))**3,x)
```

```
[Out] Integral(cos(tanh(a + b*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\tanh\left(bx+a\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(tanh(b*x + a))^3, x)
```

3.245 $\int \cos^2(\tanh(a+bx))\,dx$

Optimal. Leaf size=115

$$-\frac{\cos(2)\text{CosIntegral}(2-2\tanh(a+bx))}{4b}+\frac{\cos(2)\text{CosIntegral}(2\tanh(a+bx)+2)}{4b}-\frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b}$$

```
[Out] -(Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]])/(4*b) + (Cos[2]*CosIntegral[2 +
2*Tanh[a + b*x]])/(4*b) - Log[1 - Tanh[a + b*x]]/(4*b) + Log[1 + Tanh[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Tanh[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Tanh[a + b*x]])/(4*b)
```

Rubi [A] time = 0.249996, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$-\frac{\cos(2)\text{CosIntegral}(2-2\tanh(a+bx))}{4b}+\frac{\cos(2)\text{CosIntegral}(2\tanh(a+bx)+2)}{4b}-\frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Tanh[a + b*x]]^2,x]
```

```
[Out] -(Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]])/(4*b) + (Cos[2]*CosIntegral[2 +
2*Tanh[a + b*x]])/(4*b) - Log[1 - Tanh[a + b*x]]/(4*b) + Log[1 + Tanh[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Tanh[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Tanh[a + b*x]])/(4*b)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \cos^2(\tanh(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1-x^2}\,dx,x,\tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\cos^2(x)}{2(-1+x)}+\frac{\cos^2(x)}{2(1+x)}\right)dx,x,\tanh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}+\frac{\cos(2x)}{2(-1+x)}\right)dx,x,\tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}+\frac{\cos(2x)}{2(1+x)}\right)dx,x,\tanh\right.}{2b}$$
$$= -\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{4b}+$$
$$= -\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b}-\frac{\cos(2)\,\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx,x,\tanh(a+\right.}{4b}$$
$$= -\frac{\cos(2)\text{Ci}(2-2\tanh(a+bx))}{4b}+\frac{\cos(2)\text{Ci}(2+2\tanh(a+bx))}{4b}-\frac{\log(1-\tanh(a+bx))}{4b}+$$

Mathematica [A] time = 0.158163, size = 88, normalized size = 0.77

$$\frac{-\cos(2)\text{CosIntegral}(2-2\tanh(a+bx))+\cos(2)\text{CosIntegral}(2(\tanh(a+bx)+1))-\sin(2)\text{Si}(2-2\tanh(a+bx))+}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Tanh[a + b*x]]^2,x]
```

```
[Out] (-(Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]]) + Cos[2]*CosIntegral[2*(1 + Tan
h[a + b*x])] - Log[1 - Tanh[a + b*x]] + Log[1 + Tanh[a + b*x]] - Sin[2]*Sin
Integral[2 - 2*Tanh[a + b*x]] + Sin[2]*SinIntegral[2*(1 + Tanh[a + b*x])])/
(4*b)
```

Maple [A] time = 0.022, size = 102, normalized size = 0.9

$$\frac{\text{Si}(2+2\tanh(bx+a))\sin(2)}{4b}+\frac{\text{Ci}(2+2\tanh(bx+a))\cos(2)}{4b}+\frac{\text{Si}(-2+2\tanh(bx+a))\sin(2)}{4b}-\frac{\text{Ci}(-2+2\tanh}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(tanh(b*x+a))^2,x)
```

```
[Out] 1/4*Si(2+2*tanh(b*x+a))*sin(2)/b+1/4*Ci(2+2*tanh(b*x+a))*cos(2)/b+1/4*Si(-2
+2*tanh(b*x+a))*sin(2)/b-1/4/b*Ci(-2+2*tanh(b*x+a))*cos(2)-1/4/b*ln(-1+tanh
(b*x+a))+1/4*ln(1+tanh(b*x+a))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x+\frac{1}{2}\int \cos\left(\frac{2\left(e^{(2bx+2a)}-1\right)}{e^{(2bx+2a)}+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*integrate(cos(2*(e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)), x
)
```

Fricas [A] time = 2.20169, size = 474, normalized size = 4.12

$$\frac{4\,bx + \cos\left(2\right)\operatorname{Ci}\left(\frac{4\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}+1}\right) + \cos\left(2\right)\operatorname{Ci}\left(-\frac{4\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}+1}\right) - \cos\left(2\right)\operatorname{Ci}\left(\frac{4}{e^{\left(2\,bx+2\,a\right)}+1}\right) - \cos\left(2\right)\operatorname{Ci}\left(-\frac{4}{e^{\left(2\,bx+2\,a\right)}+1}\right) + 2\,\sin\left(2\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x + cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) +
 cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) - cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) + 1)) - cos(2)*cos_integral(-4/(e^(2*b*x + 2*a
) + 1)) + 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) -
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^{2}\left(\tanh\left(a + bx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a))**2,x)
```

```
[Out] Integral(cos(tanh(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\left(\tanh\left(bx + a\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(tanh(b*x + a))^2, x)
```

3.246 $\int \cos(\tanh(a+bx))\,dx$

Optimal. Leaf size=77

$$-\frac{\cos(1)\text{CosIntegral}(1-\tanh(a+bx))}{2b}+\frac{\cos(1)\text{CosIntegral}(\tanh(a+bx)+1)}{2b}-\frac{\sin(1)\text{Si}(1-\tanh(a+bx))}{2b}+\frac{\sin(1)}{}$$

```
[Out] -(Cos[1]*CosIntegral[1 - Tanh[a + b*x]])/(2*b) + (Cos[1]*CosIntegral[1 + Ta
nh[a + b*x]])/(2*b) - (Sin[1]*SinIntegral[1 - Tanh[a + b*x]])/(2*b) + (Sin[
1]*SinIntegral[1 + Tanh[a + b*x]])/(2*b)
```

Rubi [A] time = 0.138608, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3334, 3303, 3299, 3302}

$$-\frac{\cos(1)\text{CosIntegral}(1-\tanh(a+bx))}{2b}+\frac{\cos(1)\text{CosIntegral}(\tanh(a+bx)+1)}{2b}-\frac{\sin(1)\text{Si}(1-\tanh(a+bx))}{2b}+\frac{\sin(1)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Tanh[a + b*x]],x]
```


```
[Out] -(Cos[1]*CosIntegral[1 - Tanh[a + b*x]])/(2*b) + (Cos[1]*CosIntegral[1 + Ta
nh[a + b*x]])/(2*b) - (Sin[1]*SinIntegral[1 - Tanh[a + b*x]])/(2*b) + (Sin[
1]*SinIntegral[1 + Tanh[a + b*x]])/(2*b)
```


Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \cos(\tanh(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x^2}\,dx,x,\tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{2(1-x)}+\frac{\cos(x)}{2(1+x)}\right)\,dx,x,\tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x}\,dx,x,\tanh(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cos(x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{2b}$$
$$= \frac{\cos(1)\,\text{Subst}\left(\int \frac{\cos(1-x)}{1-x}\,dx,x,\tanh(a+bx)\right)}{2b} + \frac{\cos(1)\,\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx,x,\tanh(a+b$$
$$= -\frac{\cos(1)\text{Ci}(1-\tanh(a+bx))}{2b} + \frac{\cos(1)\text{Ci}(1+\tanh(a+bx))}{2b} - \frac{\sin(1)\text{Si}(1-\tanh(a+bx))}{2b}$$

Mathematica [A] time = 0.103914, size = 62, normalized size = 0.81

$$\frac{-\cos(1)\text{CosIntegral}(1-\tanh(a+bx))+\cos(1)\text{CosIntegral}(\tanh(a+bx)+1)-\sin(1)\text{Si}(1-\tanh(a+bx))+\sin(}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Tanh[a + b*x]],x]
```

```
[Out] (-(Cos[1]*CosIntegral[1 - Tanh[a + b*x]]) + Cos[1]*CosIntegral[1 + Tanh[a +
 b*x]] - Sin[1]*SinIntegral[1 - Tanh[a + b*x]] + Sin[1]*SinIntegral[1 + Tan
h[a + b*x]])/(2*b)
```

Maple [A] time = 0.02, size = 58, normalized size = 0.8

$$\frac{1}{b}\left(\frac{\text{Si}\,(1+\tanh\,(bx+a))\sin\,(1)}{2} + \frac{\text{Ci}\,(1+\tanh\,(bx+a))\cos\,(1)}{2} + \frac{\text{Si}\,(-1+\tanh\,(bx+a))\sin\,(1)}{2} - \frac{\text{Ci}\,(-1+\tanh\,(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(tanh(b*x+a)),x)
```

```
[Out] 1/b*(1/2*Si(1+tanh(b*x+a))*sin(1)+1/2*Ci(1+tanh(b*x+a))*cos(1)+1/2*Si(-1+ta
nh(b*x+a))*sin(1)-1/2*Ci(-1+tanh(b*x+a))*cos(1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos\,(\tanh\,(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(cos(tanh(b*x + a)), x)
```

Fricas [B] time = 2.18029, size = 463, normalized size = 6.01

$$\frac{\cos(1)\,\mathrm{Ci}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right)+\cos(1)\,\mathrm{Ci}\left(-\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}+1}\right)-\cos(1)\,\mathrm{Ci}\left(\frac{2}{e^{(2\,bx+2\,a)}+1}\right)-\cos(1)\,\mathrm{Ci}\left(-\frac{2}{e^{(2\,bx+2\,a)}+1}\right)+2\,\sin(1)\,\mathrm{Si}\left(\frac{2\,e^{(2\,bx+2}}{e^{(2\,bx+2\,a}}\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(cos(1)*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) + cos(1)*
cos_integral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) - cos(1)*cos_integra
l(2/(e^(2*b*x + 2*a) + 1)) - cos(1)*cos_integral(-2/(e^(2*b*x + 2*a) + 1))
+ 2*sin(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)) - 2*sin(1)
*sin_integral(2/(e^(2*b*x + 2*a) + 1)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a)),x)
```

```
[Out] Integral(cos(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(cos(tanh(b*x + a)), x)
```

3.247 $\int \sec(\tanh(a+bx))\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}\text{Unintegrable}\left(\frac{\text{sech}^2(a+bx)\sec(\tanh(a+bx))}{\tanh(a+bx)+1},x\right)-\frac{1}{2}\text{Unintegrable}\left(\frac{\text{sech}^2(a+bx)\sec(\tanh(a+bx))}{\tanh(a+bx)-1},x\right)$$

```
[Out] -Unintegrable[(Sec[Tanh[a + b*x]]*Sech[a + b*x]^2)/(-1 + Tanh[a + b*x]), x]
/2 + Unintegrable[(Sec[Tanh[a + b*x]]*Sech[a + b*x]^2)/(1 + Tanh[a + b*x]),
 x]/2
```

Rubi [A] time = 0.0771812, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec(\tanh(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[Tanh[a + b*x]],x]
```

```
[Out] -Defer[Subst][Defer[Int][Sec[x]/(-1 + x), x], x, Tanh[a + b*x]]/(2*b) + Def
er[Subst][Defer[Int][Sec[x]/(1 + x), x], x, Tanh[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}\int \sec(\tanh(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\sec(x)}{1-x^2}\,dx,x,\tanh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{\sec(x)}{2(-1+x)}+\frac{\sec(x)}{2(1+x)}\right)dx,x,\tanh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \frac{\sec(x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sec(x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{2b}\end{aligned}$$

Mathematica [A] time = 4.66012, size = 0, normalized size = 0.

$$\int \sec(\tanh(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[Tanh[a + b*x]],x]
```

```
[Out] Integrate[Sec[Tanh[a + b*x]], x]
```

Maple [A] time = 0.095, size = 0, normalized size = 0.

$$\int \sec(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(tanh(b*x+a)),x)
```

```
[Out] int(sec(tanh(b*x+a)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sec(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(sec(tanh(b*x + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sec\left(\tanh\left(bx+a\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(sec(tanh(b*x + a)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sec(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(tanh(b*x+a)),x)
```

```
[Out] Integral(sec(tanh(a + b*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sec(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(sec(tanh(b*x + a)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```