Computer algebra independent integration tests

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/6.2.5-Hyperbolic-cosine-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [336]. This is test number [169].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (336)	% 0. (0)
Mathematica	% 99.7 (335)	% 0.3 (1)
Maple	% 87.2 (293)	% 12.8 (43)
Maxima	% 61.31 (206)	% 38.69 (130)
Fricas	% 84.23 (283)	% 15.77 (53)
Sympy	% 26.79 (90)	% 73.21 (246)
Giac	% 76.19 (256)	% 23.81 (80)

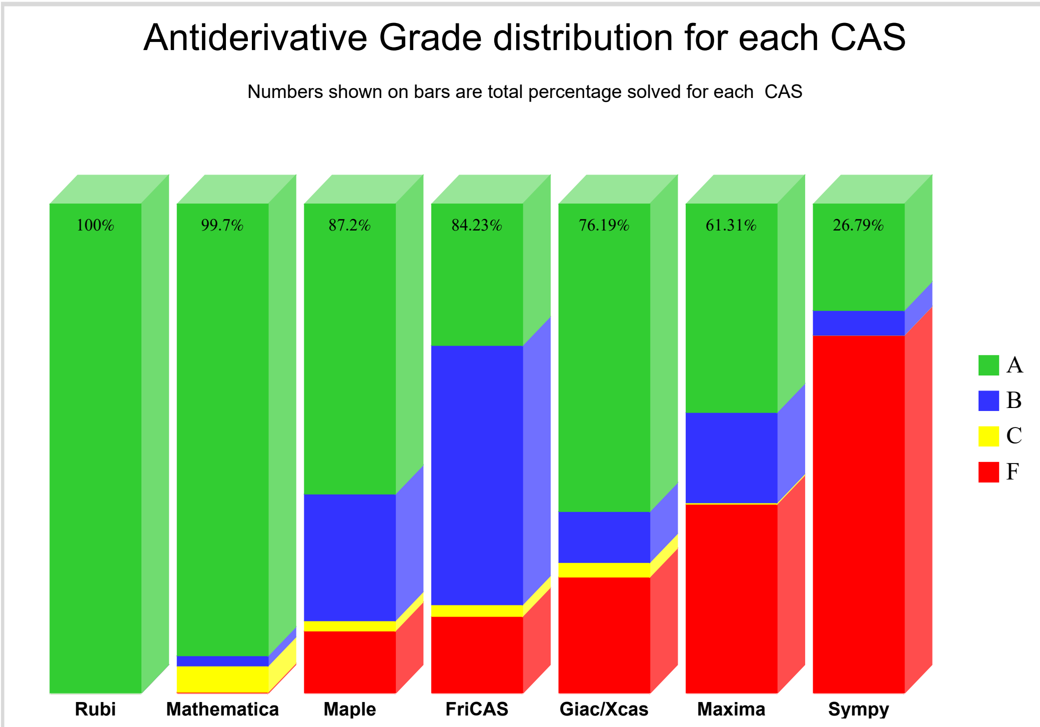
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

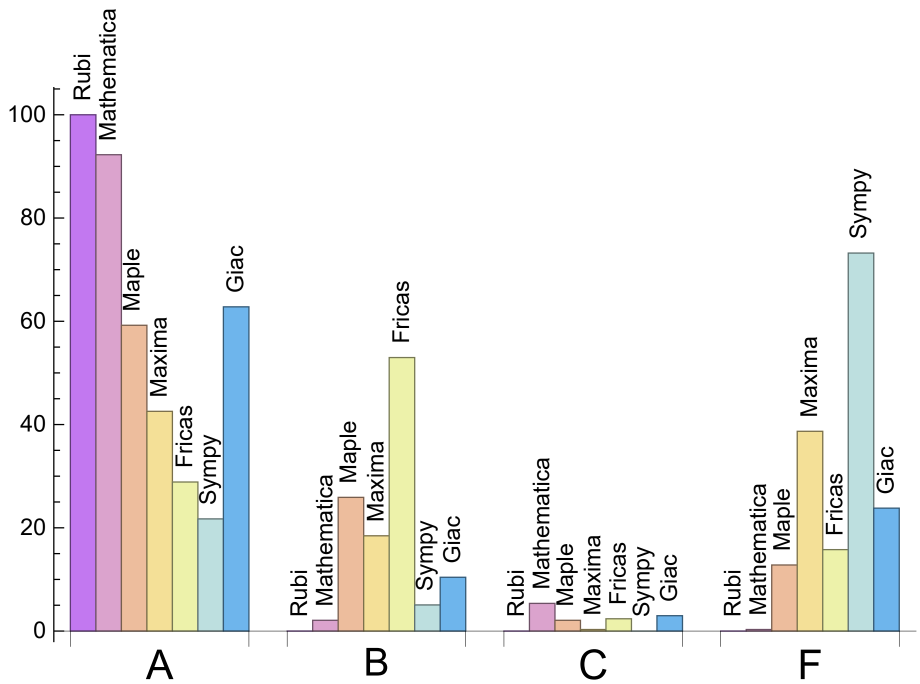
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	92.26	2.08	5.36	0.3
Maple	59.23	25.89	2.08	12.8
Maxima	42.56	18.45	0.3	38.69
Fricas	28.87	52.98	2.38	15.77
Sympy	21.73	5.06	0.	73.21
Giac	62.8	10.42	2.98	23.81

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	89.23	0.98	63.5	1.
Mathematica	0.97	99.53	0.99	55.	0.92
Maple	0.05	141.4	1.72	82.	1.16
Maxima	1.2	141.02	2.16	106.	1.68
Fricas	2.13	1306.38	13.32	562.	7.6
Sympy	11.26	123.01	2.72	46.	1.42
Giac	1.18	201.38	1.98	89.	1.55

1.4 list of integrals that has no closed form antiderivative

{216, 217, 221, 226, 227, 232, 233, 238}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {234, 258, 259, 319, 326, 327, 329, 331, 335, 336}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

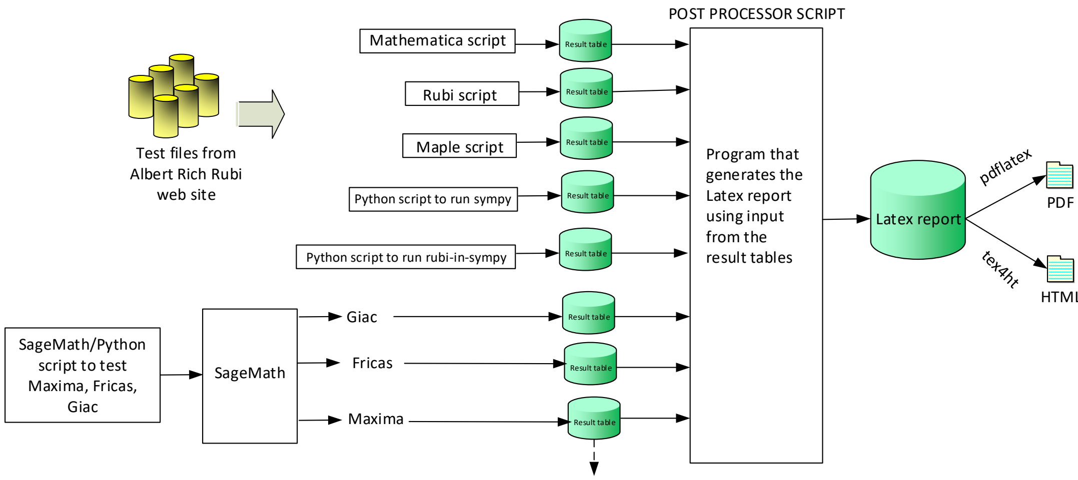
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 254, 255, 256, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 281, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 330, 331, 333, 334 }

B grade: { 1, 75, 247, 262, 325, 328, 329 }

C grade: { 9, 13, 17, 21, 130, 143, 210, 253, 257, 258, 275, 279, 280, 283, 284, 332, 335, 336 }

F grade: { 238}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 12, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 104, 105, 106, 111, 112, 113, 115, 117, 118, 121, 122, 123, 124, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 159, 160, 161, 162, 163, 164, 171, 172, 173, 174, 175, 176, 177, 178, 182, 183, 184, 185, 186, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 203, 204, 205, 207, 208, 209, 216, 217, 220, 221, 225, 226, 227, 232, 233, 238, 247, 248, 249, 250, 251, 256, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 278, 281, 282, 285, 286, 287, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334, 335, 336 }

B grade: { 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 40, 45, 46, 47, 54, 55, 61, 79, 80, 81, 82, 83, 84, 85, 101, 102, 103, 107, 108, 109, 110, 114, 116, 119, 120, 125, 126, 127, 135, 136, 137, 152, 153, 154, 155, 156, 157, 158, 165, 166, 167, 168, 169, 170, 179, 180, 181, 187, 189, 199, 202, 206, 210, 211, 212, 218, 219, 224, 230, 231, 236, 237, 252, 253, 254, 255, 257, 262, 263 }

C grade: { 275, 276, 279, 280, 283, 284, 302 }

F grade: { 23, 128, 129, 130, 131, 132, 133, 213, 214, 215, 222, 223, 228, 229, 234, 235, 239, 240, 241, 242, 243, 244, 245, 246, 258, 259, 260, 261, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 329, 330, 331, 332 }

2.1.4 Maxima

A grade: { 1, 2, 4, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 42, 43, 44, 62, 63, 64, 65, 66, 71, 72, 73, 74, 75, 76, 77, 78, 93, 97, 117, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 140, 141, 142, 143, 145, 146, 147, 151, 153, 155, 158, 159, 171, 172, 174, 180, 182, 183, 185, 191, 192, 193, 200, 201, 216, 217, 220, 221, 225, 226, 227, 232, 233, 238, 239, 240, 241, 242, 243, 245, 247, 248, 250, 264, 265, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 292, 293, 294, 295, 296, 297, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334

B grade: { 3, 5, 33, 34, 35, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 87, 88, 89, 90, 91, 92, 94, 95, 96, 98, 99, 100, 101, 102, 103, 138, 139, 144, 148, 149, 150, 152, 154, 156, 157, 160, 161, 162, 163, 164, 165, 167, 169, 176, 187, 188, 189, 190, 194, 195, 196, 237, 249, 251, 270, 272, 298 }

C grade: { 302 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 79, 80, 81, 82, 83, 84, 85, 86, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 128, 129, 130, 131, 132, 133, 166, 168, 170, 173, 175, 177, 178, 179, 181, 184, 186, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 222, 223, 224, 228, 229, 230, 231, 234, 235, 236, 244, 246, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 283, 284, 288, 289, 290, 291, 299, 329, 330, 331, 332, 335, 336 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 25, 26, 27, 28, 32, 36, 40, 44, 45, 50, 51, 57, 58, 62, 63, 64, 65, 66, 67, 71, 75, 93, 94, 97, 98, 101, 110, 114, 115, 117, 142, 143, 154, 155, 156, 157, 158, 159, 172, 182, 183, 192, 203, 204, 205, 206, 216, 217, 220, 221, 226, 227, 232, 233, 238, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 264, 266, 268, 274, 275, 279, 280, 292, 293, 294, 295, 299, 300, 301, 302, 303, 304, 312, 333, 334 }

B grade: { 24, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 46, 47, 48, 49, 52, 53, 54, 55, 56, 59, 60, 61, 68, 69, 70, 72, 73, 74, 76, 77, 78, 87, 88, 89, 90, 91, 92, 95, 96, 99, 100, 102, 103, 104, 105, 106, 111, 112, 113, 116, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208, 209, 210, 218, 219, 224, 225, 230, 231, 236, 237, 245, 246, 263, 265, 267, 269, 270, 271, 272, 273, 276, 277, 278, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 330, 335, 336 }

C grade: { 211, 212, 222, 223, 228, 229, 234, 235 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 79, 80, 81, 82, 83, 84, 85, 86, 107, 108, 109, 118, 119, 120, 128, 129, 130, 131, 132, 133, 213, 214, 215, 252, 253, 254, 255, 256, 257, 288, 289, 290, 291, 329, 331, 332 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 27, 32, 33, 34, 35, 36, 37, 38, 39, 57, 62, 63, 64, 65, 66, 67, 71, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 110, 114, 115, 117, 124, 140, 141, 142, 143, 146, 147, 148, 149, 159, 171, 199, 200, 201, 216, 221, 225, 226, 227, 233, 247, 249, 264, 265, 266, 267, 274, 278, 282, 286, 287, 299, 333, 334 }

B grade: { 24, 25, 26, 144, 145, 150, 151, 152, 153, 154, 155, 156, 157, 158, 273, 277, 281 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 68, 69, 70, 72, 73, 74, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 116, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 222, 223, 224, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 268, 269, 270, 271, 272, 275, 276, 279, 280, 283, 284, 285, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336 }

2.1.7 Giac

A grade: { 4, 5, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 110, 111, 114, 115, 117, 121, 122, 123, 124, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 216, 217, 220, 221, 225, 226, 227, 231, 232, 233, 237, 238, 239, 240, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334 }

B grade: { 1, 2, 3, 48, 49, 50, 66, 88, 89, 90, 91, 92, 105, 106, 112, 113, 116, 160, 162, 174, 176, 177, 180, 195, 208, 209, 241, 242, 243, 244, 245, 246, 247, 248, 249 }

C grade: { 40, 41, 101, 104, 285, 286, 287, 302, 306, 309 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 79, 80, 81, 82, 83, 84, 85, 86, 107, 108, 109, 118, 119, 120, 125, 128, 129, 130, 131, 132, 133, 197, 198, 210, 211, 212, 213, 214, 215, 218, 219, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 280, 288, 289, 290, 291, 329, 330, 331, 332, 335, 336 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	14	23	12	35
normalized size	1	1.	2.1	1.1	1.4	2.3	1.2	3.5
time (sec)	N/A	0.005	0.008	0.03	1.059	1.665	0.145	1.179

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	43	58	46	62
normalized size	1	1.	0.92	1.08	1.72	2.32	1.84	2.48
time (sec)	N/A	0.01	0.022	0.019	1.075	1.802	0.233	1.272

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	73	89	36	65
normalized size	1	1.	1.	0.88	2.81	3.42	1.38	2.5
time (sec)	N/A	0.012	0.006	0.06	1.045	1.704	0.487	1.295

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	39	81	134	95	92
normalized size	1	1.	0.72	0.85	1.76	2.91	2.07	2.
time (sec)	N/A	0.02	0.039	0.043	1.07	1.719	1.074	1.28

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	111	182	58	95
normalized size	1	1.	1.	0.8	2.71	4.44	1.41	2.32
time (sec)	N/A	0.013	0.013	0.036	1.029	1.696	2.107	1.267

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	49	116	248	139	122
normalized size	1	1.	0.64	0.73	1.73	3.7	2.07	1.82
time (sec)	N/A	0.033	0.039	0.035	1.074	1.758	4.16	1.269

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	55	201	0	0	0	0
normalized size	1	1.	0.8	2.91	0.	0.	0.	0.
time (sec)	N/A	0.033	0.107	0.114	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	188	0	0	0	0
normalized size	1	1.	0.96	4.09	0.	0.	0.	0.
time (sec)	N/A	0.02	0.051	0.037	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	81	174	0	0	0	0
normalized size	1	1.	1.76	3.78	0.	0.	0.	0.
time (sec)	N/A	0.019	0.094	0.033	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	0	0	0
normalized size	1	1.	1.	6.75	0.	0.	0.	0.
time (sec)	N/A	0.009	0.027	0.032	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	0	0	0
normalized size	1	1.	1.	6.75	0.	0.	0.	0.
time (sec)	N/A	0.009	0.026	0.036	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	103	0	0	0	0
normalized size	1	1.	1.	2.45	0.	0.	0.	0.
time (sec)	N/A	0.019	0.056	0.045	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	217	0	0	0	0
normalized size	1	1.	1.83	4.72	0.	0.	0.	0.
time (sec)	N/A	0.019	0.064	0.06	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	63	363	0	0	0	0
normalized size	1	1.	0.91	5.26	0.	0.	0.	0.
time (sec)	N/A	0.03	0.128	0.068	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	145	0	0	0	0
normalized size	1	1.	0.82	2.23	0.	0.	0.	0.
time (sec)	N/A	0.037	0.048	0.078	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	184	0	0	0	0
normalized size	1	1.	0.85	3.83	0.	0.	0.	0.
time (sec)	N/A	0.025	0.041	0.043	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	57	130	0	0	0	0
normalized size	1	1.	1.19	2.71	0.	0.	0.	0.
time (sec)	N/A	0.024	0.055	0.041	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	118	0	0	0	0
normalized size	1	1.	1.	4.37	0.	0.	0.	0.
time (sec)	N/A	0.015	0.009	0.043	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	100	0	0	0	0
normalized size	1	1.	1.	3.7	0.	0.	0.	0.
time (sec)	N/A	0.015	0.011	0.037	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	159	0	0	0	0
normalized size	1	1.	0.74	3.46	0.	0.	0.	0.
time (sec)	N/A	0.025	0.022	0.052	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	56	177	0	0	0	0
normalized size	1	1.	1.12	3.54	0.	0.	0.	0.
time (sec)	N/A	0.027	0.035	0.052	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	254	0	0	0	0
normalized size	1	1.	0.64	3.79	0.	0.	0.	0.
time (sec)	N/A	0.039	0.045	0.082	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.057	0.217	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	111	89	346	337	95
normalized size	1	1.	0.98	2.06	1.65	6.41	6.24	1.76
time (sec)	N/A	0.076	0.075	0.024	1.066	1.919	3.197	1.126

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	45	87	76	242	189	69
normalized size	1	1.	1.05	2.02	1.77	5.63	4.4	1.6
time (sec)	N/A	0.051	0.048	0.021	1.115	1.874	1.747	1.155

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	32	59	55	151	63	47
normalized size	1	1.	1.28	2.36	2.2	6.04	2.52	1.88
time (sec)	N/A	0.069	0.054	0.02	1.177	1.9	0.897	1.169

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	14	34	24	82	8	23
normalized size	1	1.	0.78	1.89	1.33	4.56	0.44	1.28
time (sec)	N/A	0.032	0.024	0.011	1.27	1.921	0.414	1.165

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	22	21	31	117	0	27
normalized size	1	1.	1.1	1.05	1.55	5.85	0.	1.35
time (sec)	N/A	0.042	0.024	0.017	1.819	1.79	0.	1.171

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	43	39	61	467	0	49
normalized size	1	1.	1.54	1.39	2.18	16.68	0.	1.75
time (sec)	N/A	0.068	0.08	0.022	1.793	1.8	0.	1.185

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	49	61	99	1099	0	65
normalized size	1	1.	1.14	1.42	2.3	25.56	0.	1.51
time (sec)	N/A	0.074	0.083	0.024	1.673	1.892	0.	1.145

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	60	81	136	2018	0	77
normalized size	1	1.	1.07	1.45	2.43	36.04	0.	1.38
time (sec)	N/A	0.077	0.176	0.024	1.667	1.957	0.	1.156

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	14	14	24	59	17	20
normalized size	1	1.	0.7	0.7	1.2	2.95	0.85	1.
time (sec)	N/A	0.01	0.014	0.01	1.165	1.731	0.592	1.163

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	34	30	122	319	36	34
normalized size	1	1.	0.72	0.64	2.6	6.79	0.77	0.72
time (sec)	N/A	0.022	0.029	0.01	1.071	1.793	1.347	1.157

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	44	43	277	486	51	49
normalized size	1	1.	0.63	0.61	3.96	6.94	0.73	0.7
time (sec)	N/A	0.036	0.055	0.009	1.055	1.797	3.491	1.147

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	54	56	491	984	68	63
normalized size	1	1.	0.58	0.6	5.28	10.58	0.73	0.68
time (sec)	N/A	0.054	0.077	0.01	1.096	1.75	9.119	1.171

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	14	16	24	58	32	20
normalized size	1	1.	0.61	0.7	1.04	2.52	1.39	0.87
time (sec)	N/A	0.011	0.023	0.01	1.066	1.825	0.712	1.182

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	31	32	122	319	48	34
normalized size	1	1.	0.61	0.63	2.39	6.25	0.94	0.67
time (sec)	N/A	0.025	0.027	0.015	1.099	1.858	1.618	1.197

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	41	45	277	485	65	49
normalized size	1	1.	0.54	0.59	3.64	6.38	0.86	0.64
time (sec)	N/A	0.04	0.051	0.013	1.081	1.798	3.947	1.166

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	51	58	491	984	82	63
normalized size	1	1.	0.5	0.57	4.86	9.74	0.81	0.62
time (sec)	N/A	0.057	0.071	0.014	1.087	1.745	9.601	1.139

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	34	92	154	228	0	76
normalized size	1	1.	0.67	1.8	3.02	4.47	0.	1.49
time (sec)	N/A	0.045	0.02	0.085	1.847	1.784	0.	1.222

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	40	0	308	0	134
normalized size	1	1.	0.66	0.75	0.	5.81	0.	2.53
time (sec)	N/A	0.049	0.028	0.053	0.	1.91	0.	1.181

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	71	73	163	856	0	142
normalized size	1	1.	0.8	0.82	1.83	9.62	0.	1.6
time (sec)	N/A	0.047	0.125	0.042	1.597	1.852	0.	1.195

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	109	385	0	101
normalized size	1	1.	0.93	0.98	1.85	6.53	0.	1.71
time (sec)	N/A	0.029	0.063	0.03	1.559	1.854	0.	1.115

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	54	123	0	47
normalized size	1	1.	1.12	1.65	2.08	4.73	0.	1.81
time (sec)	N/A	0.013	0.03	0.026	1.784	1.852	0.	1.155

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	103	116	456	0	28
normalized size	1	1.	0.87	2.24	2.52	9.91	0.	0.61
time (sec)	N/A	0.023	0.015	0.036	1.833	1.931	0.	1.198

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	144	230	632	0	99
normalized size	1	1.	0.82	1.87	2.99	8.21	0.	1.29
time (sec)	N/A	0.041	0.081	0.045	1.998	1.903	0.	1.253

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	91	178	338	1454	0	138
normalized size	1	1.	0.85	1.66	3.16	13.59	0.	1.29
time (sec)	N/A	0.062	0.263	0.046	1.89	1.964	0.	1.378

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	72	71	257	857	0	262
normalized size	1	1.	0.78	0.77	2.79	9.32	0.	2.85
time (sec)	N/A	0.052	0.133	0.042	1.619	1.803	0.	1.18

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	56	167	387	0	171
normalized size	1	1.	0.92	0.92	2.74	6.34	0.	2.8
time (sec)	N/A	0.031	0.091	0.036	1.607	1.882	0.	1.185

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	30	41	78	124	0	85
normalized size	1	1.	1.11	1.52	2.89	4.59	0.	3.15
time (sec)	N/A	0.014	0.032	0.034	1.592	1.749	0.	1.191

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	41	0	458	0	54
normalized size	1	1.	0.85	0.85	0.	9.54	0.	1.12
time (sec)	N/A	0.025	0.03	0.036	0.	1.906	0.	1.203

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	85	87	0	783	0	153
normalized size	1	1.	1.08	1.1	0.	9.91	0.	1.94
time (sec)	N/A	0.041	0.164	0.053	0.	1.929	0.	1.277

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	115	137	0	1605	0	227
normalized size	1	1.	1.05	1.25	0.	14.59	0.	2.06
time (sec)	N/A	0.063	0.194	0.053	0.	2.064	0.	1.379

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	99	264	0	3691	0	180
normalized size	1	1.	0.88	2.36	0.	32.96	0.	1.61
time (sec)	N/A	0.309	0.186	0.032	0.	2.113	0.	1.2

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	78	174	0	2083	0	124
normalized size	1	1.	0.92	2.05	0.	24.51	0.	1.46
time (sec)	N/A	0.169	0.127	0.023	0.	2.396	0.	1.237

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	94	0	1080	0	84
normalized size	1	1.	0.92	1.52	0.	17.42	0.	1.35
time (sec)	N/A	0.106	0.105	0.02	0.	2.253	0.	1.201

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	64	0	535	241	57
normalized size	1	1.	0.92	1.23	0.	10.29	4.63	1.1
time (sec)	N/A	0.053	0.045	0.014	0.	2.309	135.171	1.198

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	602	0	61
normalized size	1	1.	1.	0.94	0.	11.15	0.	1.13
time (sec)	N/A	0.067	0.051	0.02	0.	2.511	0.	1.188

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	73	0	1301	0	82
normalized size	1	1.	0.98	1.14	0.	20.33	0.	1.28
time (sec)	N/A	0.117	0.103	0.03	0.	2.569	0.	1.161

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	146	0	3337	0	120
normalized size	1	1.	0.94	1.68	0.	38.36	0.	1.38
time (sec)	N/A	0.3	0.199	0.027	0.	3.531	0.	1.199

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	101	239	0	5894	0	166
normalized size	1	1.	0.89	2.1	0.	51.7	0.	1.46
time (sec)	N/A	0.472	0.382	0.029	0.	3.904	0.	1.275

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	133	155	369	463	314	362
normalized size	1	1.	0.73	0.85	2.02	2.53	1.72	1.98
time (sec)	N/A	0.26	0.349	0.014	1.121	2.184	2.662	1.184

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	104	119	247	296	240	259
normalized size	1	1.	0.76	0.87	1.8	2.16	1.75	1.89
time (sec)	N/A	0.151	0.212	0.013	1.046	2.123	1.407	1.13

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	80	77	157	188	128	167
normalized size	1	1.	0.89	0.86	1.74	2.09	1.42	1.86
time (sec)	N/A	0.068	0.124	0.013	1.051	2.214	0.665	1.155

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	74	96	78	95
normalized size	1	1.	0.92	1.02	1.48	1.92	1.56	1.9
time (sec)	N/A	0.017	0.073	0.011	1.046	2.073	0.308	1.148

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	20	39	17	43
normalized size	1	1.	1.73	1.07	1.33	2.6	1.13	2.87
time (sec)	N/A	0.008	0.008	0.004	1.017	2.091	0.143	1.179

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	585	163	53
normalized size	1	1.	0.98	0.9	0.	11.94	3.33	1.08
time (sec)	N/A	0.035	0.048	0.01	0.	2.256	15.524	1.222

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	84	118	0	1787	0	138
normalized size	1	1.	0.98	1.37	0.	20.78	0.	1.6
time (sec)	N/A	0.084	0.207	0.02	0.	2.419	0.	1.232

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	113	186	0	5711	0	271
normalized size	1	1.	0.85	1.4	0.	42.94	0.	2.04
time (sec)	N/A	0.147	0.386	0.023	0.	2.739	0.	1.229

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	160	284	0	12951	0	451
normalized size	1	1.	0.87	1.54	0.	70.39	0.	2.45
time (sec)	N/A	0.253	0.998	0.024	0.	2.955	0.	1.237

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	18	26	80	24	22
normalized size	1	1.	1.05	0.82	1.18	3.64	1.09	1.
time (sec)	N/A	0.015	0.034	0.01	1.564	2.283	0.863	1.199

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	48	86	435	0	74
normalized size	1	1.	0.94	1.	1.79	9.06	0.	1.54
time (sec)	N/A	0.032	0.1	0.015	1.53	2.098	0.	1.156

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	79	146	1214	0	104
normalized size	1	1.	0.75	1.08	2.	16.63	0.	1.42
time (sec)	N/A	0.065	0.153	0.016	1.55	2.214	0.	1.131

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	65	110	205	2473	0	134
normalized size	1	1.	0.66	1.12	2.09	25.23	0.	1.37
time (sec)	N/A	0.097	0.245	0.016	1.575	2.489	0.	1.132

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	77	36	50	126	41	42
normalized size	1	1.	2.48	1.16	1.61	4.06	1.32	1.35
time (sec)	N/A	0.013	0.029	0.014	1.033	2.322	0.727	1.16

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	45	72	109	621	199	93
normalized size	1	1.	0.8	1.29	1.95	11.09	3.55	1.66
time (sec)	N/A	0.036	0.113	0.016	1.043	2.573	2.425	1.157

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	58	108	169	1661	445	123
normalized size	1	1.	0.72	1.33	2.09	20.51	5.49	1.52
time (sec)	N/A	0.063	0.188	0.017	1.033	2.173	5.735	1.151

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	68	144	228	3320	784	153
normalized size	1	1.	0.64	1.36	2.15	31.32	7.4	1.44
time (sec)	N/A	0.097	0.247	0.019	1.072	2.242	12.986	1.137

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	150	685	0	0	0	0
normalized size	1	1.	0.98	4.48	0.	0.	0.	0.
time (sec)	N/A	0.244	0.501	0.102	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	111	458	0	0	0	0
normalized size	1	1.	0.9	3.69	0.	0.	0.	0.
time (sec)	N/A	0.157	0.227	0.063	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	276	0	0	0	0
normalized size	1	1.	1.	4.52	0.	0.	0.	0.
time (sec)	N/A	0.039	0.085	0.062	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	146	0	0	0	0
normalized size	1	1.	1.	3.17	0.	0.	0.	0.
time (sec)	N/A	0.034	0.037	0.051	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	68	296	0	0	0	0
normalized size	1	1.	0.81	3.52	0.	0.	0.	0.
time (sec)	N/A	0.057	0.12	0.079	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	135	459	0	0	0	0
normalized size	1	1.	0.76	2.59	0.	0.	0.	0.
time (sec)	N/A	0.207	0.527	0.147	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	165	566	0	0	0	0
normalized size	1	1.	0.73	2.49	0.	0.	0.	0.
time (sec)	N/A	0.313	0.656	0.198	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	181	0	0	0	0
normalized size	1	1.	0.73	1.81	0.	0.	0.	0.
time (sec)	N/A	0.107	0.346	0.068	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	60	71	320	1553	0	207
normalized size	1	1.	0.64	0.76	3.4	16.52	0.	2.2
time (sec)	N/A	0.092	0.119	0.046	1.645	2.261	0.	1.248

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	46	57	220	805	0	153
normalized size	1	1.	0.68	0.84	3.24	11.84	0.	2.25
time (sec)	N/A	0.074	0.087	0.04	1.617	2.174	0.	1.154

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	122	317	0	93
normalized size	1	1.	0.78	0.98	3.05	7.92	0.	2.32
time (sec)	N/A	0.052	0.038	0.042	1.629	2.174	0.	1.114

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	61	69	389	1554	0	398
normalized size	1	1.	0.62	0.7	3.97	15.86	0.	4.06
time (sec)	N/A	0.104	0.135	0.05	1.673	2.254	0.	1.338

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	47	55	269	807	0	286
normalized size	1	1.	0.66	0.77	3.79	11.37	0.	4.03
time (sec)	N/A	0.08	0.097	0.049	1.605	2.154	0.	1.254

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	32	39	147	319	0	166
normalized size	1	1.	0.73	0.89	3.34	7.25	0.	3.77
time (sec)	N/A	0.058	0.051	0.052	1.638	2.147	0.	1.233

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	23	34	35	96	15	23
normalized size	1	1.	1.28	1.89	1.94	5.33	0.83	1.28
time (sec)	N/A	0.036	0.058	0.01	1.04	2.147	0.411	1.162

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	34	174	165	36	41
normalized size	1	1.	0.71	0.97	4.97	4.71	1.03	1.17
time (sec)	N/A	0.038	0.05	0.007	1.037	2.076	0.857	1.213

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	42	38	355	427	46	62
normalized size	1	1.	0.75	0.68	6.34	7.62	0.82	1.11
time (sec)	N/A	0.048	0.076	0.009	1.044	2.108	1.819	1.177

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	57	55	606	586	78	81
normalized size	1	1.	0.76	0.73	8.08	7.81	1.04	1.08
time (sec)	N/A	0.059	0.092	0.009	1.075	2.097	4.364	1.197

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	35	37	36	97	15	22
normalized size	1	1.	1.75	1.85	1.8	4.85	0.75	1.1
time (sec)	N/A	0.04	0.054	0.016	1.033	2.172	0.66	1.163

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	26	177	163	36	43
normalized size	1	1.	0.68	0.7	4.78	4.41	0.97	1.16
time (sec)	N/A	0.041	0.049	0.013	1.048	2.061	1.204	1.164

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	39	360	425	46	62
normalized size	1	1.	0.7	0.65	6.	7.08	0.77	1.03
time (sec)	N/A	0.055	0.073	0.013	1.052	2.032	2.334	1.179

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	57	56	609	585	78	81
normalized size	1	1.	0.7	0.69	7.52	7.22	0.96	1.
time (sec)	N/A	0.066	0.088	0.013	1.066	2.084	5.146	1.19

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	41	128	235	244	0	100
normalized size	1	1.	0.73	2.29	4.2	4.36	0.	1.79
time (sec)	N/A	0.066	0.034	0.053	1.946	2.168	0.	1.236

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	44	159	405	581	0	105
normalized size	1	1.	0.68	2.45	6.23	8.94	0.	1.62
time (sec)	N/A	0.068	0.077	0.062	2.036	2.184	0.	1.336

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	57	209	576	1453	0	159
normalized size	1	1.	0.61	2.25	6.19	15.62	0.	1.71
time (sec)	N/A	0.089	0.154	0.065	2.072	2.25	0.	1.415

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	40	63	0	324	0	157
normalized size	1	1.	0.7	1.11	0.	5.68	0.	2.75
time (sec)	N/A	0.065	0.054	0.066	0.	2.117	0.	1.198

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	71	83	0	680	0	150
normalized size	1	1.	1.09	1.28	0.	10.46	0.	2.31
time (sec)	N/A	0.073	0.149	0.07	0.	2.197	0.	1.228

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	108	118	0	1553	0	255
normalized size	1	1.	1.15	1.26	0.	16.52	0.	2.71
time (sec)	N/A	0.096	0.36	0.07	0.	2.249	0.	1.34

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	203	1365	0	0	0	0
normalized size	1	1.	0.87	5.86	0.	0.	0.	0.
time (sec)	N/A	0.454	0.563	0.168	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	124	973	0	0	0	0
normalized size	1	1.	0.69	5.38	0.	0.	0.	0.
time (sec)	N/A	0.322	0.639	0.105	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	123	605	0	0	0	0
normalized size	1	1.	0.89	4.38	0.	0.	0.	0.
time (sec)	N/A	0.206	0.333	0.094	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	103	0	574	403	68
normalized size	1	1.	0.98	1.72	0.	9.57	6.72	1.13
time (sec)	N/A	0.068	0.091	0.016	0.	2.276	154.892	1.18

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	81	108	0	1914	0	144
normalized size	1	1.	0.99	1.32	0.	23.34	0.	1.76
time (sec)	N/A	0.078	0.167	0.018	0.	2.3	0.	1.157

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	134	207	0	7004	0	336
normalized size	1	1.	0.99	1.53	0.	51.88	0.	2.49
time (sec)	N/A	0.169	0.395	0.023	0.	2.672	0.	1.203

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	196	342	0	17173	0	612
normalized size	1	1.	0.99	1.74	0.	87.17	0.	3.11
time (sec)	N/A	0.348	0.754	0.025	0.	3.395	0.	1.199

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	107	0	491	170	77
normalized size	1	1.	1.	1.91	0.	8.77	3.04	1.38
time (sec)	N/A	0.078	0.069	0.021	0.	2.283	154.659	1.207

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	9	3	8
normalized size	1	1.	1.	1.17	0.	1.5	0.5	1.33
time (sec)	N/A	0.001	0.	0.	0.	2.018	0.389	1.163

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	29	0	159	0	35
normalized size	1	1.	1.	2.64	0.	14.45	0.	3.18
time (sec)	N/A	0.029	0.052	0.017	0.	2.089	0.	1.184

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	32	46	139	44	50
normalized size	1	1.	0.67	0.89	1.28	3.86	1.22	1.39
time (sec)	N/A	0.05	0.072	0.016	1.532	2.247	1.077	1.195

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	80	218	0	0	0	0
normalized size	1	1.	0.74	2.02	0.	0.	0.	0.
time (sec)	N/A	0.112	0.443	0.096	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	133	483	0	0	0	0
normalized size	1	1.	0.88	3.18	0.	0.	0.	0.
time (sec)	N/A	0.205	0.357	0.197	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	172	797	0	0	0	0
normalized size	1	1.	0.74	3.45	0.	0.	0.	0.
time (sec)	N/A	0.35	0.841	0.346	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	42	38	96	2461	0	107
normalized size	1	1.	0.58	0.53	1.33	34.18	0.	1.49
time (sec)	N/A	0.055	0.026	0.04	1.521	2.286	0.	1.25

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	72	1462	0	82
normalized size	1	1.	0.68	0.6	1.36	27.58	0.	1.55
time (sec)	N/A	0.036	0.016	0.037	1.564	2.208	0.	1.167

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	47	672	0	39
normalized size	1	1.	0.76	0.71	1.38	19.76	0.	1.15
time (sec)	N/A	0.023	0.009	0.037	1.597	2.424	0.	1.143

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	23	216	19	19
normalized size	1	1.	1.	1.15	1.77	16.62	1.46	1.46
time (sec)	N/A	0.012	0.004	0.028	1.598	2.4	0.618	1.156

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	55	11	552	0	0
normalized size	1	1.	1.31	3.44	0.69	34.5	0.	0.
time (sec)	N/A	0.015	0.007	0.044	1.693	2.184	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	82	55	878	0	76
normalized size	1	1.	0.74	1.95	1.31	20.9	0.	1.81
time (sec)	N/A	0.025	0.014	0.059	1.773	2.188	0.	1.155

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	40	102	101	2461	0	100
normalized size	1	1.	0.66	1.67	1.66	40.34	0.	1.64
time (sec)	N/A	0.039	0.035	0.062	1.802	2.335	0.	1.145

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	65	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.115	0.075	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.071	0.051	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	59	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.051	0.05	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	36	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.021	0.056	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	48	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.059	0.052	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	61	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.1	0.052	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	53	177	135	4963	0	154
normalized size	1	1.	0.4	1.34	1.02	37.6	0.	1.17
time (sec)	N/A	0.053	0.116	0.138	1.722	2.139	0.	1.163

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	38	131	84	2068	0	70
normalized size	1	1.	0.49	1.68	1.08	26.51	0.	0.9
time (sec)	N/A	0.035	0.064	0.111	1.652	2.117	0.	1.152

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	25	89	36	551	0	38
normalized size	1	1.	0.69	2.47	1.	15.31	0.	1.06
time (sec)	N/A	0.016	0.014	0.107	1.633	1.973	0.	1.165

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	56	22	338	0	18
normalized size	1	1.	1.	3.73	1.47	22.53	0.	1.2
time (sec)	N/A	0.017	0.005	0.085	1.724	1.881	0.	1.161

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	30	80	223	3163	0	47
normalized size	1	1.	0.45	1.19	3.33	47.21	0.	0.7
time (sec)	N/A	0.025	0.027	0.096	1.582	1.972	0.	1.183

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	47	96	617	9072	0	72
normalized size	1	1.	0.4	0.82	5.27	77.54	0.	0.62
time (sec)	N/A	0.035	0.048	0.095	1.652	2.494	0.	1.153

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	11	122	7	14
normalized size	1	1.	1.5	1.12	1.38	15.25	0.88	1.75
time (sec)	N/A	0.021	0.011	0.004	1.068	2.105	0.432	1.156

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	11	122	7	14
normalized size	1	1.	1.5	1.12	1.38	15.25	0.88	1.75
time (sec)	N/A	0.022	0.01	0.005	1.034	2.066	0.44	1.143

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	24	16	77	7	14
normalized size	1	1.	1.5	2.	1.33	6.42	0.58	1.17
time (sec)	N/A	0.031	0.006	0.014	1.083	2.195	0.734	1.225

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	24	26	16	77	7	14
normalized size	1	1.	1.71	1.86	1.14	5.5	0.5	1.
time (sec)	N/A	0.031	0.01	0.016	1.049	2.153	1.426	1.136

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	13	11	31	197	70	28
normalized size	1	1.	1.3	1.1	3.1	19.7	7.	2.8
time (sec)	N/A	0.038	0.018	0.012	1.13	2.221	0.658	1.162

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	13	11	31	198	70	30
normalized size	1	1.	1.08	0.92	2.58	16.5	5.83	2.5
time (sec)	N/A	0.037	0.017	0.015	1.029	2.096	0.643	1.12

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	11	197	15	16
normalized size	1	1.	1.2	0.9	1.1	19.7	1.5	1.6
time (sec)	N/A	0.021	0.009	0.004	1.063	2.045	0.818	1.185

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	11	196	14	16
normalized size	1	1.	1.	0.92	0.92	16.33	1.17	1.33
time (sec)	N/A	0.021	0.01	0.005	1.062	2.132	0.842	1.144

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	66	127	7	22
normalized size	1	1.	0.86	0.64	4.71	9.07	0.5	1.57
time (sec)	N/A	0.032	0.026	0.011	1.066	2.112	1.506	1.167

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	9	66	126	8	22
normalized size	1	1.	0.75	0.56	4.12	7.88	0.5	1.38
time (sec)	N/A	0.033	0.029	0.013	1.105	2.056	2.576	1.156

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	20	15	42	336	194	28
normalized size	1	1.	1.43	1.07	3.	24.	13.86	2.
time (sec)	N/A	0.04	0.009	0.014	1.156	1.866	0.915	1.149

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	27	17	47	335	194	27
normalized size	1	1.	1.35	0.85	2.35	16.75	9.7	1.35
time (sec)	N/A	0.04	0.012	0.018	1.158	1.889	0.95	1.137

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	208	138	340	1253	122
normalized size	1	1.	0.89	3.65	2.42	5.96	21.98	2.14
time (sec)	N/A	0.059	0.067	0.04	1.112	1.999	22.958	1.178

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	27	107	113	313	218	101
normalized size	1	1.	0.59	2.33	2.46	6.8	4.74	2.2
time (sec)	N/A	0.065	0.03	0.034	1.099	1.815	12.796	1.208

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	39	156	105	188	692	89
normalized size	1	1.	0.89	3.55	2.39	4.27	15.73	2.02
time (sec)	N/A	0.054	0.052	0.033	1.109	1.892	7.642	1.255

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	21	87	81	165	104	69
normalized size	1	1.	0.64	2.64	2.45	5.	3.15	2.09
time (sec)	N/A	0.056	0.02	0.027	1.146	1.799	4.058	1.205

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	103	73	89	294	54
normalized size	1	1.	0.81	3.32	2.35	2.87	9.48	1.74
time (sec)	N/A	0.046	0.035	0.023	1.113	1.862	2.149	1.279

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	13	47	49	58	27	36
normalized size	1	1.	0.68	2.47	2.58	3.05	1.42	1.89
time (sec)	N/A	0.043	0.012	0.02	1.112	1.88	1.106	1.247

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	51	31	24	46	23
normalized size	1	1.	1.31	3.92	2.38	1.85	3.54	1.77
time (sec)	N/A	0.039	0.008	0.017	1.104	1.827	0.555	1.187

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	12	12	15	53	7	23
normalized size	1	1.	1.33	1.33	1.67	5.89	0.78	2.56
time (sec)	N/A	0.024	0.005	0.006	1.06	1.967	0.157	1.132

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	23	63	397	0	70
normalized size	1	1.	1.83	1.	2.74	17.26	0.	3.04
time (sec)	N/A	0.052	0.027	0.016	1.017	1.837	0.	1.189

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	29	80	292	0	47
normalized size	1	1.	1.25	1.21	3.33	12.17	0.	1.96
time (sec)	N/A	0.048	0.046	0.016	1.01	1.855	0.	1.173

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	60	45	139	2066	0	127
normalized size	1	1.	1.22	0.92	2.84	42.16	0.	2.59
time (sec)	N/A	0.081	0.129	0.02	1.061	1.951	0.	1.188

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	38	45	315	809	0	80
normalized size	1	1.	1.03	1.22	8.51	21.86	0.	2.16
time (sec)	N/A	0.051	0.053	0.02	1.034	1.76	0.	1.132

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	89	67	209	5257	0	157
normalized size	1	1.	1.14	0.86	2.68	67.4	0.	2.01
time (sec)	N/A	0.106	0.256	0.022	1.027	2.085	0.	1.161

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	144	1039	419	5296	0	309
normalized size	1	1.	1.03	7.42	2.99	37.83	0.	2.21
time (sec)	N/A	0.167	0.175	0.034	1.065	2.157	0.	1.132

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	679	0	7443	0	359
normalized size	1	1.	1.	4.41	0.	48.33	0.	2.33
time (sec)	N/A	0.425	0.226	0.028	0.	2.344	0.	1.184

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	84	599	240	2228	0	167
normalized size	1	1.	1.01	7.22	2.89	26.84	0.	2.01
time (sec)	N/A	0.107	0.107	0.026	1.05	1.935	0.	1.224

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	95	338	0	2843	0	197
normalized size	1	1.	0.91	3.25	0.	27.34	0.	1.89
time (sec)	N/A	0.241	0.18	0.021	0.	2.084	0.	1.165

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	283	113	629	0	76
normalized size	1	1.	1.	7.08	2.82	15.72	0.	1.9
time (sec)	N/A	0.069	0.053	0.02	1.024	1.942	0.	1.184

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	129	0	805	0	92
normalized size	1	1.	0.92	2.19	0.	13.64	0.	1.56
time (sec)	N/A	0.112	0.078	0.019	0.	2.067	0.	1.126

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	72	14	26
normalized size	1	1.	1.	1.09	1.36	6.55	1.27	2.36
time (sec)	N/A	0.027	0.015	0.006	0.999	1.913	0.415	1.164

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	37	52	80	181	0	90
normalized size	1	1.	0.7	0.98	1.51	3.42	0.	1.7
time (sec)	N/A	0.077	0.065	0.016	1.036	2.019	0.	1.2

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	77	78	0	1172	0	103
normalized size	1	1.	1.15	1.16	0.	17.49	0.	1.54
time (sec)	N/A	0.09	0.198	0.019	0.	2.065	0.	1.164

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	100	97	208	2072	0	242
normalized size	1	1.	1.1	1.07	2.29	22.77	0.	2.66
time (sec)	N/A	0.159	0.269	0.023	1.105	2.259	0.	1.159

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	141	127	0	5405	0	211
normalized size	1	1.	1.28	1.15	0.	49.14	0.	1.92
time (sec)	N/A	0.246	0.534	0.024	0.	2.152	0.	1.194

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	148	191	470	8195	0	456
normalized size	1	1.	0.98	1.26	3.11	54.27	0.	3.02
time (sec)	N/A	0.254	0.861	0.026	1.149	2.762	0.	1.16

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	201	213	0	15046	0	409
normalized size	1	1.	1.26	1.34	0.	94.63	0.	2.57
time (sec)	N/A	0.477	1.714	0.026	0.	2.785	0.	1.199

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	61	99	0	1663	0	92
normalized size	1	1.	0.91	1.48	0.	24.82	0.	1.37
time (sec)	N/A	0.104	0.099	0.023	0.	2.103	0.	1.166

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	100	315	0	5057	0	194
normalized size	1	1.	0.88	2.79	0.	44.75	0.	1.72
time (sec)	N/A	0.406	0.412	0.032	0.	2.735	0.	1.208

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	140	130	1166	0	155
normalized size	1	1.	0.81	2.46	2.28	20.46	0.	2.72
time (sec)	N/A	0.099	0.093	0.03	1.569	2.006	0.	1.156

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	108	0	984	0	90
normalized size	1	1.	1.	1.77	0.	16.13	0.	1.48
time (sec)	N/A	0.232	0.105	0.022	0.	2.173	0.	1.147

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	45	116	0	45
normalized size	1	1.	1.	1.05	2.25	5.8	0.	2.25
time (sec)	N/A	0.042	0.009	0.014	1.594	1.957	0.	1.233

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	53	80	182	0	90
normalized size	1	1.	0.7	0.98	1.48	3.37	0.	1.67
time (sec)	N/A	0.07	0.07	0.02	1.033	2.1	0.	1.229

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	78	0	1172	0	103
normalized size	1	1.	1.	1.01	0.	15.22	0.	1.34
time (sec)	N/A	0.094	0.201	0.024	0.	1.962	0.	1.194

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	101	97	211	2071	0	240
normalized size	1	1.	1.07	1.03	2.24	22.03	0.	2.55
time (sec)	N/A	0.199	0.21	0.029	1.094	2.242	0.	1.196

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	127	0	5677	0	232
normalized size	1	1.	0.96	0.93	0.	41.44	0.	1.69
time (sec)	N/A	0.196	0.507	0.029	0.	2.302	0.	1.21

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	58	115	120	2522	0	78
normalized size	1	1.	1.26	2.5	2.61	54.83	0.	1.7
time (sec)	N/A	0.093	0.085	0.06	1.56	1.888	0.	1.181

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	25	30	301	562	0	65
normalized size	1	1.	0.83	1.	10.03	18.73	0.	2.17
time (sec)	N/A	0.086	0.027	0.051	1.08	1.837	0.	1.194

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	71	77	1031	0	53
normalized size	1	1.	1.39	2.15	2.33	31.24	0.	1.61
time (sec)	N/A	0.08	0.059	0.036	1.559	1.736	0.	1.182

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	18	95	221	0	30
normalized size	1	1.	0.89	0.95	5.	11.63	0.	1.58
time (sec)	N/A	0.066	0.021	0.026	1.071	1.822	0.	1.156

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	18	31	31	185	0	30
normalized size	1	1.	1.2	2.07	2.07	12.33	0.	2.
time (sec)	N/A	0.049	0.045	0.02	1.543	1.924	0.	1.169

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	12	19	32	96	0	30
normalized size	1	1.	0.67	1.06	1.78	5.33	0.	1.67
time (sec)	N/A	0.04	0.017	0.016	1.054	1.905	0.	1.248

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	42	23	65	397	0	70
normalized size	1	1.	1.27	0.7	1.97	12.03	0.	2.12
time (sec)	N/A	0.065	0.037	0.019	1.072	1.969	0.	1.229

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	25	29	163	292	0	47
normalized size	1	1.	0.83	0.97	5.43	9.73	0.	1.57
time (sec)	N/A	0.079	0.048	0.021	1.017	1.766	0.	1.188

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	60	45	139	2071	0	127
normalized size	1	1.	1.3	0.98	3.02	45.02	0.	2.76
time (sec)	N/A	0.107	0.099	0.027	1.086	2.01	0.	1.175

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	45	633	728	0	80
normalized size	1	1.	1.	1.1	15.44	17.76	0.	1.95
time (sec)	N/A	0.081	0.073	0.028	1.067	1.851	0.	1.314

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	0	1162	0	0
normalized size	1	1.	1.	0.81	0.	31.41	0.	0.
time (sec)	N/A	0.063	0.02	0.013	0.	4.114	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	1099	0	0
normalized size	1	1.	1.	0.79	0.	45.79	0.	0.
time (sec)	N/A	0.057	0.011	0.016	0.	2.048	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	137	0	717	741	81
normalized size	1	1.	0.98	2.45	0.	12.8	13.23	1.45
time (sec)	N/A	0.128	0.082	0.017	0.	1.92	138.828	1.219

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	28	26	169	20	30
normalized size	1	1.	1.06	1.56	1.44	9.39	1.11	1.67
time (sec)	N/A	0.077	0.032	0.011	1.055	2.052	0.409	1.155

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	19	36	27	167	31	30
normalized size	1	1.	0.79	1.5	1.12	6.96	1.29	1.25
time (sec)	N/A	0.086	0.044	0.017	1.061	1.79	0.665	1.16

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	125	0	806	0	89
normalized size	1	1.	0.94	1.92	0.	12.4	0.	1.37
time (sec)	N/A	0.148	0.143	0.029	0.	2.095	0.	1.193

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	81	139	0	842	0	122
normalized size	1	1.	0.81	1.39	0.	8.42	0.	1.22
time (sec)	N/A	0.166	0.244	0.026	0.	14.019	0.	1.176

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	63	89	0	643	0	72
normalized size	1	1.	1.02	1.44	0.	10.37	0.	1.16
time (sec)	N/A	0.133	0.114	0.03	0.	4.647	0.	1.21

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	81	138	0	840	0	122
normalized size	1	1.	0.82	1.39	0.	8.48	0.	1.23
time (sec)	N/A	0.305	0.188	0.025	0.	22.681	0.	1.237

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	81	276	0	948	0	135
normalized size	1	1.	0.94	3.21	0.	11.02	0.	1.57
time (sec)	N/A	0.154	0.234	0.036	0.	2.04	0.	1.226

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	115	144	0	2384	0	225
normalized size	1	1.	0.95	1.19	0.	19.7	0.	1.86
time (sec)	N/A	0.176	0.409	0.036	0.	2.224	0.	1.217

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	175	273	0	8046	0	539
normalized size	1	1.	0.94	1.46	0.	43.03	0.	2.88
time (sec)	N/A	0.264	0.785	0.042	0.	2.981	0.	1.176

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	245	459	0	19134	0	946
normalized size	1	1.	0.94	1.77	0.	73.59	0.	3.64
time (sec)	N/A	0.448	2.407	0.046	0.	3.965	0.	1.3

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	536	487	0	1968	0	0
normalized size	1	1.	2.81	2.55	0.	10.3	0.	0.
time (sec)	N/A	0.377	0.562	0.046	0.	1.954	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	221	686	0	2967	0	0
normalized size	1	1.	0.76	2.36	0.	10.2	0.	0.
time (sec)	N/A	0.567	0.717	0.038	0.	2.121	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	295	889	0	3956	0	0
normalized size	1	1.	0.75	2.27	0.	10.12	0.	0.
time (sec)	N/A	0.599	0.61	0.039	0.	2.363	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.077	0.133	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.035	0.083	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.033	0.023	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	5.706	0.028	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	23.486	0.035	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	138	0	1202	0	0
normalized size	1	1.	0.98	2.3	0.	20.03	0.	0.
time (sec)	N/A	0.057	0.125	0.069	0.	1.94	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	231	0	3969	0	0
normalized size	1	1.	1.	2.66	0.	45.62	0.	0.
time (sec)	N/A	0.089	0.244	0.083	0.	2.058	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	57	204	0	57
normalized size	1	1.	0.87	1.	1.21	4.34	0.	1.21
time (sec)	N/A	0.462	0.106	0.075	1.313	1.829	0.	1.171

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	5.098	0.072	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	326	0	0	1585	0	0
normalized size	1	1.	1.	0.	0.	4.85	0.	0.
time (sec)	N/A	0.479	0.033	0.227	0.	2.091	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	244	0	0	1245	0	0
normalized size	1	1.	1.	0.	0.	5.08	0.	0.
time (sec)	N/A	0.39	0.021	0.172	0.	1.995	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	160	368	0	902	0	0
normalized size	1	1.	0.99	2.29	0.	5.6	0.	0.
time (sec)	N/A	0.242	0.012	0.047	0.	1.979	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	104	41	42
normalized size	1	1.	1.	1.06	1.33	5.78	2.28	2.33
time (sec)	N/A	0.032	0.037	0.004	1.005	1.863	1.033	1.321

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	13.026	0.127	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	23.641	0.102	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	386	0	0	2889	0	0
normalized size	1	1.	0.78	0.	0.	5.84	0.	0.
time (sec)	N/A	0.839	1.392	0.117	0.	2.217	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	293	0	0	2303	0	0
normalized size	1	1.	0.79	0.	0.	6.22	0.	0.
time (sec)	N/A	0.704	1.197	0.109	0.	2.137	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	187	862	0	1666	0	0
normalized size	1	1.	0.77	3.53	0.	6.83	0.	0.
time (sec)	N/A	0.418	0.949	0.056	0.	2.113	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	69	177	0	1085	0	131
normalized size	1	1.	0.95	2.42	0.	14.86	0.	1.79
time (sec)	N/A	0.123	0.18	0.009	0.	2.03	0.	1.243

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	108.57	0.076	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	31.047	0.054	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	586	586	1082	0	0	4741	0	0
normalized size	1	1.	1.85	0.	0.	8.09	0.	0.
time (sec)	N/A	0.686	11.851	0.205	0.	2.472	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	432	831	0	0	3811	0	0
normalized size	1	1.	1.92	0.	0.	8.82	0.	0.
time (sec)	N/A	0.563	8.913	0.17	0.	2.247	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	414	860	0	2828	0	0
normalized size	1	1.	1.44	2.99	0.	9.82	0.	0.
time (sec)	N/A	0.336	2.926	0.066	0.	2.12	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	55	415	176	848	0	123
normalized size	1	1.	0.9	6.8	2.89	13.9	0.	2.02
time (sec)	N/A	0.073	0.102	0.026	1.057	1.931	0.	1.176

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	180.	0.148	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	41	0	69	123	0	63
normalized size	1	1.	0.76	0.	1.28	2.28	0.	1.17
time (sec)	N/A	0.012	0.06	0.034	1.099	1.787	0.	1.196

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	0	90	258	0	228
normalized size	1	1.	0.64	0.	1.02	2.93	0.	2.59
time (sec)	N/A	0.02	0.088	0.097	1.112	1.921	0.	1.277

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	117	0	155	536	0	898
normalized size	1	1.	0.79	0.	1.04	3.6	0.	6.03
time (sec)	N/A	0.039	0.521	0.115	1.198	1.875	0.	1.36

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	167	0	174	801	0	1049
normalized size	1	1.	0.87	0.	0.91	4.19	0.	5.49
time (sec)	N/A	0.051	0.416	0.116	1.221	1.807	0.	1.294

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	0	86	305	0	317
normalized size	1	1.	0.74	0.	1.18	4.18	0.	4.34
time (sec)	N/A	0.023	0.122	0.038	1.12	1.814	0.	1.182

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	87	0	0	726	0	1025
normalized size	1	1.	0.72	0.	0.	6.05	0.	8.54
time (sec)	N/A	0.048	0.272	0.1	0.	1.976	0.	1.297

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	197	292	0	186	1547	0	4354
normalized size	1	0.97	1.44	0.	0.92	7.62	0.	21.45
time (sec)	N/A	0.083	1.339	0.128	1.219	1.931	0.	1.51

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	260	311	0	0	2880	0	9288
normalized size	1	0.98	1.17	0.	0.	10.83	0.	34.92
time (sec)	N/A	0.128	3.378	0.138	0.	2.074	0.	1.73

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	24	53	41	57
normalized size	1	1.	2.06	1.06	1.33	2.94	2.28	3.17
time (sec)	N/A	0.016	0.011	0.007	1.049	1.919	1.188	1.136

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	66	122	0	108
normalized size	1	1.	0.92	1.33	1.69	3.13	0.	2.77
time (sec)	N/A	0.031	0.027	0.014	1.056	1.989	0.	1.196

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	116	167	87	109
normalized size	1	1.	1.	0.86	2.76	3.98	2.07	2.6
time (sec)	N/A	0.032	0.009	0.014	1.028	1.918	16.149	1.208

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	84	126	270	0	154
normalized size	1	1.	0.7	1.15	1.73	3.7	0.	2.11
time (sec)	N/A	0.047	0.046	0.015	1.046	1.876	0.	1.268

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	51	176	333	0	157
normalized size	1	1.	1.	0.78	2.71	5.12	0.	2.42
time (sec)	N/A	0.037	0.018	0.017	1.04	1.946	0.	1.192

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	62	256	0	0	0	0
normalized size	1	1.	0.93	3.82	0.	0.	0.	0.
time (sec)	N/A	0.044	0.056	0.085	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	114	237	0	0	0	0
normalized size	1	1.	1.7	3.54	0.	0.	0.	0.
time (sec)	N/A	0.044	0.122	0.09	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	0	0	0
normalized size	1	1.	1.	6.54	0.	0.	0.	0.
time (sec)	N/A	0.028	0.021	0.07	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	0	0	0
normalized size	1	1.	1.	6.54	0.	0.	0.	0.
time (sec)	N/A	0.028	0.02	0.069	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	141	0	0	0	0
normalized size	1	1.	0.92	2.24	0.	0.	0.	0.
time (sec)	N/A	0.042	0.057	0.098	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	122	295	0	0	0	0
normalized size	1	1.	1.82	4.4	0.	0.	0.	0.
time (sec)	N/A	0.042	0.085	0.089	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	206	85	0	0	475	0	0
normalized size	1	1.	0.41	0.	0.	2.31	0.	0.
time (sec)	N/A	0.154	0.459	0.334	0.	1.863	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	74	0	0	360	0	0
normalized size	1	1.	0.73	0.	0.	3.53	0.	0.
time (sec)	N/A	0.082	0.318	0.151	0.	1.797	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	61	0	0	167	0	0
normalized size	1	1.	1.45	0.	0.	3.98	0.	0.
time (sec)	N/A	0.05	0.149	0.149	0.	1.8	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	121	0	0	312	0	0
normalized size	1	1.	1.2	0.	0.	3.09	0.	0.
time (sec)	N/A	0.075	0.252	0.148	0.	1.874	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	373	347	0	344	0	0
normalized size	1	1.	3.69	3.44	0.	3.41	0.	0.
time (sec)	N/A	0.177	0.352	0.04	0.	1.902	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	111	358	0	774	0	0
normalized size	1	1.	1.04	3.35	0.	7.23	0.	0.
time (sec)	N/A	0.194	0.285	0.091	0.	1.837	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	62	77	92	325	139	81
normalized size	1	1.	0.75	0.93	1.11	3.92	1.67	0.98
time (sec)	N/A	0.039	0.042	0.011	1.065	1.793	82.59	1.326

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	67	72	263	182	77
normalized size	1	1.	0.82	1.18	1.26	4.61	3.19	1.35
time (sec)	N/A	0.037	0.033	0.01	1.017	1.682	21.157	1.289

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	39	49	54	154	78	46
normalized size	1	1.	0.8	1.	1.1	3.14	1.59	0.94
time (sec)	N/A	0.029	0.02	0.01	1.184	2.012	5.695	1.227

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	32	131	80	30
normalized size	1	1.	1.	1.61	1.39	5.7	3.48	1.3
time (sec)	N/A	0.015	0.011	0.007	1.113	1.796	1.256	1.199

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	22	76	0	22
normalized size	1	1.	1.	1.12	1.29	4.47	0.	1.29
time (sec)	N/A	0.017	0.012	0.006	1.593	1.783	0.	1.161

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	36	45	50	304	0	50
normalized size	1	1.	0.9	1.12	1.25	7.6	0.	1.25
time (sec)	N/A	0.029	0.061	0.011	1.652	1.739	0.	1.308

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	30	92	238	0	42
normalized size	1	1.	1.	1.03	3.17	8.21	0.	1.45
time (sec)	N/A	0.027	0.018	0.037	1.066	1.66	0.	1.281

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	64	83	112	1424	0	82
normalized size	1	1.	0.67	0.87	1.18	14.99	0.	0.86
time (sec)	N/A	0.047	0.083	0.04	1.567	1.841	0.	1.252

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	44	61	232	635	0	57
normalized size	1	1.	0.73	1.02	3.87	10.58	0.	0.95
time (sec)	N/A	0.048	0.034	0.035	1.041	1.781	0.	1.22

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	23	170	42	23
normalized size	1	1.	1.	1.31	0.88	6.54	1.62	0.88
time (sec)	N/A	0.02	0.013	0.019	1.062	1.826	0.814	1.238

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	22	18	95	20	18
normalized size	1	1.	0.84	1.16	0.95	5.	1.05	0.95
time (sec)	N/A	0.011	0.008	0.007	1.004	1.745	0.302	1.258

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	24	25	103	354	0	103
normalized size	1	1.	0.26	0.27	1.12	3.85	0.	1.12
time (sec)	N/A	0.064	0.01	0.041	1.562	1.964	0.	1.168

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	36	119	494	0	119
normalized size	1	1.	0.95	0.32	1.07	4.45	0.	1.07
time (sec)	N/A	0.075	0.085	0.043	1.562	1.939	0.	1.221

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	23	240	42	23
normalized size	1	1.	1.	1.31	0.88	9.23	1.62	0.88
time (sec)	N/A	0.019	0.015	0.016	1.047	1.789	0.803	1.21

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	26	18	130	20	18
normalized size	1	1.	0.84	1.37	0.95	6.84	1.05	0.95
time (sec)	N/A	0.012	0.009	0.018	1.056	1.901	0.302	1.223

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	24	79	96	288	0	59
normalized size	1	1.	0.44	1.44	1.75	5.24	0.	1.07
time (sec)	N/A	0.058	0.011	0.04	1.566	2.064	0.	1.209

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	34	59	107	474	0	0
normalized size	1	1.	0.31	0.54	0.97	4.31	0.	0.
time (sec)	N/A	0.205	0.02	0.053	1.599	2.084	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	23	311	42	23
normalized size	1	1.	1.	1.31	0.88	11.96	1.62	0.88
time (sec)	N/A	0.019	0.015	0.013	1.034	1.906	0.816	1.332

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	26	18	163	20	18
normalized size	1	1.	1.	1.37	0.95	8.58	1.05	0.95
time (sec)	N/A	0.011	0.009	0.008	1.04	1.865	0.304	1.262

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	24	25	0	3322	0	336
normalized size	1	1.	0.06	0.07	0.	8.95	0.	0.91
time (sec)	N/A	0.321	0.01	0.036	0.	2.22	0.	1.357

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	34	36	0	4143	0	352
normalized size	1	1.	0.09	0.09	0.	10.93	0.	0.93
time (sec)	N/A	0.319	0.02	0.046	0.	2.112	0.	1.254

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	159	326	181	5434	0	1673
normalized size	1	1.	0.79	1.61	0.9	26.9	0.	8.28
time (sec)	N/A	0.078	0.657	0.065	1.088	2.032	0.	1.466

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	85	143	127	1783	604	1219
normalized size	1	1.	0.64	1.08	0.96	13.51	4.58	9.23
time (sec)	N/A	0.053	0.225	0.037	1.089	2.526	110.87	1.453

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	50	74	85	643	316	825
normalized size	1	1.	0.67	0.99	1.13	8.57	4.21	11.
time (sec)	N/A	0.018	0.104	0.017	1.05	2.519	12.015	1.371

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.019	0.016	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.016	0.023	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	96	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.217	0.029	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	101	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.185	0.033	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	106	0	151	562	0	136
normalized size	1	1.	0.42	0.	0.6	2.25	0.	0.54
time (sec)	N/A	0.229	0.097	180.	1.096	2.195	0.	1.212

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	78	0	100	323	0	99
normalized size	1	1.	0.48	0.	0.62	1.99	0.	0.61
time (sec)	N/A	0.124	0.107	180.	1.151	2.018	0.	1.291

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	48	0	39	163	0	31
normalized size	1	1.	0.65	0.	0.53	2.2	0.	0.42
time (sec)	N/A	0.099	0.038	180.	1.151	1.726	0.	1.228

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	28	97	0	27
normalized size	1	1.	0.95	0.	0.64	2.2	0.	0.61
time (sec)	N/A	0.113	0.054	180.	1.714	1.826	0.	1.233

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	46	0	113	302	0	51
normalized size	1	1.	0.82	0.	2.02	5.39	0.	0.91
time (sec)	N/A	0.129	0.072	180.	1.068	1.793	0.	1.251

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	72	0	282	797	0	69
normalized size	1	1.	0.51	0.	2.	5.65	0.	0.49
time (sec)	N/A	0.197	0.069	180.	1.056	1.923	0.	1.208

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	84	0	521	1516	0	86
normalized size	1	1.	0.44	0.	2.73	7.94	0.	0.45
time (sec)	N/A	0.259	0.079	180.	1.082	1.858	0.	1.269

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	28	62	0	135	99	43
normalized size	1	1.	0.68	1.51	0.	3.29	2.41	1.05
time (sec)	N/A	0.013	0.046	0.016	0.	1.888	1.428	1.232

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	79	72	88	286	0	99
normalized size	1	1.	0.93	0.85	1.04	3.36	0.	1.16
time (sec)	N/A	0.081	0.081	0.124	1.1	1.853	0.	1.31

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	97	109	367	0	123
normalized size	1	1.	0.9	0.96	1.08	3.63	0.	1.22
time (sec)	N/A	0.125	0.146	0.099	1.09	1.755	0.	1.258

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	61	130	0	61
normalized size	1	1.	0.78	0.8	0.94	2.	0.	0.94
time (sec)	N/A	0.064	0.067	0.069	1.061	1.925	0.	1.204

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	71	48	63	234	0	66
normalized size	1	1.	1.09	0.74	0.97	3.6	0.	1.02
time (sec)	N/A	0.079	0.096	0.072	1.083	1.886	0.	1.277

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	122	105	120	466	0	136
normalized size	1	1.	1.06	0.91	1.04	4.05	0.	1.18
time (sec)	N/A	0.159	0.353	0.139	1.078	1.822	0.	1.228

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	102	100	122	591	0	143
normalized size	1	1.	0.93	0.91	1.11	5.37	0.	1.3
time (sec)	N/A	0.148	0.122	0.122	1.083	1.872	0.	1.234

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	171	826	0	479
normalized size	1	1.	1.01	0.85	1.16	5.58	0.	3.24
time (sec)	N/A	0.191	0.677	0.172	1.574	1.959	0.	1.3

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	286	207	270	1287	0	301
normalized size	1	1.	1.2	0.87	1.13	5.38	0.	1.26
time (sec)	N/A	0.286	0.41	0.208	1.665	2.098	0.	1.302

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	123	126	138	699	0	181
normalized size	1	1.	1.07	1.1	1.2	6.08	0.	1.57
time (sec)	N/A	0.22	0.267	0.098	1.028	1.972	0.	1.229

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	193	956	0	525
normalized size	1	1.	1.37	0.98	1.2	5.94	0.	3.26
time (sec)	N/A	0.273	0.626	0.127	1.536	1.925	0.	1.294

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	353	265	308	1519	0	385
normalized size	1	1.	1.37	1.03	1.2	5.91	0.	1.5
time (sec)	N/A	0.466	0.77	0.171	1.565	2.102	0.	1.283

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	142	598	0	178
normalized size	1	1.	0.78	0.88	1.07	4.5	0.	1.34
time (sec)	N/A	0.194	0.151	0.097	1.064	1.935	0.	1.284

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	177	678	0	203
normalized size	1	1.	0.81	0.86	1.1	4.21	0.	1.26
time (sec)	N/A	0.22	0.225	0.113	1.048	2.018	0.	1.37

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	285	1202	0	356
normalized size	1	1.	0.79	0.86	1.05	4.44	0.	1.31
time (sec)	N/A	0.338	0.448	0.14	1.067	2.023	0.	1.348

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	75	70	93	412	0	101
normalized size	1	1.	0.93	0.86	1.15	5.09	0.	1.25
time (sec)	N/A	0.158	0.317	0.073	1.035	1.887	0.	1.317

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	135	720	0	144
normalized size	1	1.	1.4	0.79	1.05	5.62	0.	1.12
time (sec)	N/A	0.203	0.542	0.091	1.103	2.018	0.	1.358

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	270	144	193	1301	0	209
normalized size	1	1.	1.58	0.84	1.13	7.61	0.	1.22
time (sec)	N/A	0.297	1.196	0.127	1.128	2.045	0.	1.34

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	165	147	171	855	0	232
normalized size	1	1.	1.18	1.05	1.22	6.11	0.	1.66
time (sec)	N/A	0.311	0.652	0.157	1.073	2.011	0.	1.3

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	217	1119	0	267
normalized size	1	1.	1.41	0.97	1.19	6.11	0.	1.46
time (sec)	N/A	0.332	1.427	0.168	1.1	1.917	0.	1.24

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	300	300	478	302	355	2219	0	475
normalized size	1	1.	1.59	1.01	1.18	7.4	0.	1.58
time (sec)	N/A	0.581	5.745	0.236	1.136	2.154	0.	1.388

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	134	156	174	717	0	228
normalized size	1	1.	0.88	1.02	1.14	4.69	0.	1.49
time (sec)	N/A	0.291	0.309	0.106	1.089	2.029	0.	1.309

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	183	211	250	934	0	304
normalized size	1	1.	0.84	0.96	1.14	4.26	0.	1.39
time (sec)	N/A	0.369	0.522	0.171	1.081	1.978	0.	1.247

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	262	316	355	1434	0	463
normalized size	1	1.	0.83	1.	1.13	4.55	0.	1.47
time (sec)	N/A	0.457	0.957	0.159	1.109	1.991	0.	1.307

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	185	160	188	878	0	244
normalized size	1	1.	1.2	1.04	1.22	5.7	0.	1.58
time (sec)	N/A	0.321	0.658	0.164	1.075	1.914	0.	1.268

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	269	1269	0	323
normalized size	1	1.	1.14	0.96	1.2	5.64	0.	1.44
time (sec)	N/A	0.35	2.2	0.213	1.079	2.075	0.	1.295

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	2511	326	387	2249	0	498
normalized size	1	1.	7.77	1.01	1.2	6.96	0.	1.54
time (sec)	N/A	0.512	6.498	0.233	1.11	2.088	0.	1.267

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	251	186	204	975	0	282
normalized size	1	1.	1.56	1.16	1.27	6.06	0.	1.75
time (sec)	N/A	0.426	1.494	0.124	1.081	1.997	0.	1.254

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	290	1382	0	369
normalized size	1	1.	1.42	1.04	1.21	5.78	0.	1.54
time (sec)	N/A	0.506	5.978	0.149	1.149	1.982	0.	1.332

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	425	2454	0	582
normalized size	1	1.	8.69	1.12	1.24	7.13	0.	1.69
time (sec)	N/A	0.734	6.643	0.197	1.133	2.187	0.	1.295

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	20	20	46	0	0	0	0	0
normalized size	1	1.	2.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.346	0.07	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	16	0	0	374	0	0
normalized size	1	1.	0.67	0.	0.	15.58	0.	0.
time (sec)	N/A	0.051	0.075	0.069	0.	1.798	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	64	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.584	0.093	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	76	0	0	0	0	0
normalized size	1	1.	2.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.18	0.063	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	49	80	41	49
normalized size	1	1.	0.87	0.83	1.63	2.67	1.37	1.63
time (sec)	N/A	0.038	0.064	0.007	1.049	1.835	0.252	1.237

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	52	109	184	85	101
normalized size	1	1.	0.91	0.93	1.95	3.29	1.52	1.8
time (sec)	N/A	0.075	0.088	0.007	1.043	1.805	0.896	1.213

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	624	0	0
normalized size	1	1.	0.85	1.	0.	2.93	0.	0.
time (sec)	N/A	0.523	0.281	0.039	0.	1.85	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	248	370	0	1445	0	0
normalized size	1	1.	0.92	1.37	0.	5.33	0.	0.
time (sec)	N/A	0.751	0.467	0.04	0.	1.982	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [279] had the largest ratio of [1.125]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167
2	A	2	2	1.	8	0.25
3	A	2	1	1.	8	0.125
4	A	3	2	1.	8	0.25
5	A	2	1	1.	8	0.125
6	A	4	2	1.	8	0.25
7	A	3	2	1.	10	0.2
8	A	2	2	1.	10	0.2

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	2	2	1.	10	0.2
10	A	1	1	1.	10	0.1
11	A	1	1	1.	10	0.1
12	A	2	2	1.	10	0.2
13	A	2	2	1.	10	0.2
14	A	3	2	1.	10	0.2
15	A	4	3	1.	8	0.375
16	A	3	3	1.	8	0.375
17	A	3	3	1.	8	0.375
18	A	2	2	1.	8	0.25
19	A	2	2	1.	8	0.25
20	A	3	3	1.	8	0.375
21	A	3	3	1.	8	0.375
22	A	4	3	1.	8	0.375
23	A	1	1	1.	10	0.1
24	A	6	5	1.	13	0.385
25	A	2	2	1.	13	0.154
26	A	4	4	1.	13	0.308
27	A	2	2	1.	11	0.182
28	A	3	3	1.	11	0.273
29	A	5	5	1.	13	0.385
30	A	6	6	1.	13	0.462
31	A	6	5	1.	13	0.385
32	A	1	1	1.	10	0.1
33	A	2	2	1.	10	0.2
34	A	3	2	1.	10	0.2
35	A	4	2	1.	10	0.2
36	A	1	1	1.	12	0.083
37	A	2	2	1.	12	0.167
38	A	3	2	1.	12	0.167
39	A	4	2	1.	12	0.167
40	A	3	3	1.	13	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	3	3	1.	14	0.214
42	A	3	2	1.	14	0.143
43	A	2	2	1.	14	0.143
44	A	1	1	1.	14	0.071
45	A	2	2	1.	14	0.143
46	A	3	3	1.	14	0.214
47	A	4	3	1.	14	0.214
48	A	3	2	1.	15	0.133
49	A	2	2	1.	15	0.133
50	A	1	1	1.	15	0.067
51	A	2	2	1.	15	0.133
52	A	3	3	1.	15	0.2
53	A	4	3	1.	15	0.2
54	A	6	6	1.	13	0.462
55	A	5	5	1.	13	0.385
56	A	5	5	1.	13	0.385
57	A	3	3	1.	11	0.273
58	A	4	4	1.	11	0.364
59	A	6	6	1.	13	0.462
60	A	6	6	1.	13	0.462
61	A	7	6	1.	13	0.462
62	A	4	3	1.	12	0.25
63	A	3	3	1.	12	0.25
64	A	2	2	1.	12	0.167
65	A	1	1	1.	12	0.083
66	A	2	1	1.	10	0.1
67	A	2	2	1.	12	0.167
68	A	4	4	1.	12	0.333
69	A	5	5	1.	12	0.417
70	A	6	5	1.	12	0.417
71	A	2	2	1.	12	0.167
72	A	4	4	1.	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	5	5	1.	12	0.417
74	A	6	5	1.	12	0.417
75	A	1	1	1.	12	0.083
76	A	3	3	1.	12	0.25
77	A	4	4	1.	12	0.333
78	A	5	4	1.	12	0.333
79	A	7	7	1.	10	0.7
80	A	6	6	1.	10	0.6
81	A	2	2	1.	14	0.143
82	A	2	2	1.	10	0.2
83	A	4	4	1.	10	0.4
84	A	7	7	1.	10	0.7
85	A	8	7	1.	10	0.7
86	A	5	5	1.	13	0.385
87	A	4	3	1.	17	0.176
88	A	3	3	1.	17	0.176
89	A	2	2	1.	17	0.118
90	A	4	3	1.	18	0.167
91	A	3	3	1.	18	0.167
92	A	2	2	1.	18	0.111
93	A	2	2	1.	13	0.154
94	A	2	2	1.	13	0.154
95	A	3	3	1.	13	0.231
96	A	4	3	1.	13	0.231
97	A	2	2	1.	15	0.133
98	A	2	2	1.	15	0.133
99	A	3	3	1.	15	0.2
100	A	4	3	1.	15	0.2
101	A	3	3	1.	17	0.176
102	A	3	3	1.	17	0.176
103	A	4	4	1.	17	0.235
104	A	3	3	1.	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	3	3	1.	18	0.167
106	A	4	4	1.	18	0.222
107	A	8	6	1.	17	0.353
108	A	7	6	1.	17	0.353
109	A	6	6	1.	17	0.353
110	A	3	3	1.	15	0.2
111	A	4	4	1.	15	0.267
112	A	5	4	1.	15	0.267
113	A	6	4	1.	15	0.267
114	A	3	3	1.	20	0.15
115	A	2	2	1.	20	0.1
116	A	2	2	1.	15	0.133
117	A	2	2	1.	13	0.154
118	A	5	5	1.	17	0.294
119	A	6	6	1.	17	0.353
120	A	7	6	1.	17	0.353
121	A	5	3	1.	10	0.3
122	A	4	3	1.	10	0.3
123	A	3	3	1.	10	0.3
124	A	2	2	1.	10	0.2
125	A	2	2	1.	10	0.2
126	A	3	3	1.	10	0.3
127	A	4	3	1.	10	0.3
128	A	6	3	1.	10	0.3
129	A	4	3	1.	10	0.3
130	A	3	3	1.	10	0.3
131	A	3	3	1.	10	0.3
132	A	4	3	1.	10	0.3
133	A	6	3	1.	10	0.3
134	A	7	3	1.	10	0.3
135	A	5	3	1.	10	0.3
136	A	3	3	1.	10	0.3

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
137	A	3	3	1.	10	0.3
138	A	3	2	1.	10	0.2
139	A	3	2	1.	10	0.2
140	A	2	2	1.	9	0.222
141	A	2	2	1.	11	0.182
142	A	2	2	1.	11	0.182
143	A	2	2	1.	13	0.154
144	A	3	2	1.	11	0.182
145	A	3	2	1.	13	0.154
146	A	2	2	1.	9	0.222
147	A	2	2	1.	11	0.182
148	A	1	1	1.	11	0.091
149	A	1	1	1.	13	0.077
150	A	3	2	1.	11	0.182
151	A	3	2	1.	13	0.154
152	A	5	3	1.	13	0.231
153	A	3	2	1.	13	0.154
154	A	4	3	1.	13	0.231
155	A	3	2	1.	13	0.154
156	A	3	3	1.	13	0.231
157	A	2	1	1.	13	0.077
158	A	2	2	1.	13	0.154
159	A	2	2	1.	11	0.182
160	A	4	3	1.	11	0.273
161	A	3	3	1.	13	0.231
162	A	4	3	1.	13	0.231
163	A	3	2	1.	13	0.154
164	A	4	3	1.	13	0.231
165	A	3	2	1.	13	0.154
166	A	6	5	1.	13	0.385
167	A	3	2	1.	13	0.154
168	A	5	5	1.	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
169	A	3	2	1.	13	0.154
170	A	4	4	1.	13	0.308
171	A	2	2	1.	11	0.182
172	A	6	4	1.	11	0.364
173	A	4	4	1.	13	0.308
174	A	4	3	1.	13	0.231
175	A	5	5	1.	13	0.385
176	A	5	4	1.	13	0.308
177	A	6	5	1.	13	0.385
178	A	4	4	1.	13	0.308
179	A	6	6	1.	13	0.462
180	A	3	2	1.	13	0.154
181	A	6	6	1.	13	0.462
182	A	4	4	1.	11	0.364
183	A	3	2	1.	11	0.182
184	A	7	6	1.	13	0.462
185	A	4	3	1.	13	0.231
186	A	12	8	1.	13	0.615
187	A	6	5	1.	13	0.385
188	A	5	4	1.	13	0.308
189	A	5	5	1.	13	0.385
190	A	5	4	1.	13	0.308
191	A	4	4	1.	13	0.308
192	A	4	4	1.	11	0.364
193	A	5	5	1.	11	0.454
194	A	5	4	1.	13	0.308
195	A	6	5	1.	13	0.385
196	A	6	5	1.	13	0.385
197	A	4	4	1.	13	0.308
198	A	3	3	1.	13	0.231
199	A	6	5	1.	15	0.333
200	A	5	4	1.	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	4	1.	15	0.267
202	A	8	7	1.	15	0.467
203	A	7	5	1.	15	0.333
204	A	5	5	1.	15	0.333
205	A	11	8	1.	15	0.533
206	A	6	6	1.	31	0.194
207	A	7	7	1.	31	0.226
208	A	8	7	1.	31	0.226
209	A	9	7	1.	31	0.226
210	A	9	6	1.	12	0.5
211	A	11	7	1.	14	0.5
212	A	13	8	1.	14	0.571
213	A	5	3	1.	36	0.083
214	A	4	3	1.	36	0.083
215	A	2	2	1.	34	0.059
216	A	0	0	0.	0	0.
217	A	0	0	0.	0	0.
218	A	3	3	1.	12	0.25
219	A	5	5	1.	12	0.417
220	A	13	5	1.	20	0.25
221	A	0	0	0.	0	0.
222	A	11	6	1.	22	0.273
223	A	9	5	1.	22	0.227
224	A	7	4	1.	20	0.2
225	A	2	2	1.	19	0.105
226	A	0	0	0.	0	0.
227	A	0	0	0.	0	0.
228	A	18	11	1.	24	0.458
229	A	15	10	1.	24	0.417
230	A	12	9	1.	22	0.409
231	A	4	4	1.	21	0.19
232	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
233	A	0	0	0.	0	0.
234	A	21	14	1.	24	0.583
235	A	16	11	1.	24	0.458
236	A	13	10	1.	22	0.454
237	A	3	2	1.	21	0.095
238	A	0	0	0.	0	0.
239	A	1	1	1.	11	0.091
240	A	2	2	1.	13	0.154
241	A	2	2	1.	13	0.154
242	A	3	2	1.	13	0.154
243	A	1	1	1.	15	0.067
244	A	2	2	1.	17	0.118
245	A	2	2	0.97	17	0.118
246	A	3	2	0.98	17	0.118
247	A	2	1	1.	15	0.067
248	A	3	2	1.	17	0.118
249	A	3	1	1.	17	0.059
250	A	4	2	1.	17	0.118
251	A	3	1	1.	17	0.059
252	A	3	2	1.	19	0.105
253	A	3	2	1.	19	0.105
254	A	2	1	1.	19	0.053
255	A	2	1	1.	19	0.053
256	A	3	2	1.	19	0.105
257	A	3	2	1.	19	0.105
258	A	8	8	1.	18	0.444
259	A	6	6	1.	18	0.333
260	A	3	3	1.	18	0.167
261	A	4	4	1.	18	0.222
262	A	5	5	1.	14	0.357
263	A	6	6	1.	16	0.375
264	A	4	3	1.	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	5	4	1.	16	0.25
266	A	4	3	1.	16	0.188
267	A	4	3	1.	14	0.214
268	A	3	3	1.	14	0.214
269	A	4	4	1.	16	0.25
270	A	3	3	1.	16	0.188
271	A	6	5	1.	16	0.312
272	A	5	4	1.	16	0.25
273	A	4	3	1.	10	0.3
274	A	4	3	1.	8	0.375
275	A	11	8	1.	8	1.
276	A	12	9	1.	10	0.9
277	A	4	3	1.	10	0.3
278	A	4	3	1.	8	0.375
279	A	9	9	1.	8	1.125
280	A	13	9	1.	10	0.9
281	A	4	3	1.	10	0.3
282	A	4	3	1.	8	0.375
283	A	21	9	1.	8	1.125
284	A	22	9	1.	10	0.9
285	A	2	2	1.	18	0.111
286	A	2	2	1.	18	0.111
287	A	1	1	1.	16	0.062
288	A	1	1	1.	16	0.062
289	A	1	1	1.	18	0.056
290	A	2	2	1.	18	0.111
291	A	2	2	1.	18	0.111
292	A	6	5	1.	25	0.2
293	A	6	5	1.	25	0.2
294	A	5	4	1.	25	0.16
295	A	4	4	1.	25	0.16
296	A	4	4	1.	25	0.16

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	6	5	1.	25	0.2
298	A	6	5	1.	25	0.2
299	A	1	1	1.	10	0.1
300	A	6	4	1.	12	0.333
301	A	6	4	1.	15	0.267
302	A	6	3	1.	12	0.25
303	A	4	2	1.	14	0.143
304	A	6	3	1.	17	0.176
305	A	8	5	1.	16	0.312
306	A	9	6	1.	18	0.333
307	A	14	5	1.	18	0.278
308	A	8	5	1.	19	0.263
309	A	9	6	1.	21	0.286
310	A	14	5	1.	21	0.238
311	A	8	4	1.	16	0.25
312	A	9	4	1.	18	0.222
313	A	14	4	1.	18	0.222
314	A	6	4	1.	18	0.222
315	A	7	4	1.	20	0.2
316	A	10	4	1.	20	0.2
317	A	8	5	1.	21	0.238
318	A	9	5	1.	23	0.217
319	A	14	5	1.	23	0.217
320	A	8	4	1.	19	0.21
321	A	10	4	1.	21	0.19
322	A	14	4	1.	21	0.19
323	A	8	5	1.	21	0.238
324	A	10	5	1.	23	0.217
325	A	14	5	1.	23	0.217
326	A	8	5	1.	24	0.208
327	A	10	5	1.	26	0.192
328	A	14	5	1.	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	2	1	1.	17	0.059
330	A	2	1	1.	20	0.05
331	A	3	1	1.	20	0.05
332	A	3	2	1.	21	0.095
333	A	6	5	1.	6	0.833
334	A	9	6	1.	6	1.
335	A	8	4	1.	16	0.25
336	A	8	4	1.	19	0.21

Chapter 3

Listing of integrals

3.1 $\int \cosh(a + bx)\, dx$

Optimal. Leaf size=10

$$\frac{\sinh(a + bx)}{b}$$

```
[Out] Sinh[a + b*x]/b
```

Rubi [A] time = 0.0048472, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2637}

$$\frac{\sinh(a + bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x],x]
```

```
[Out] Sinh[a + b*x]/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\,dx = \frac{\sinh(a+bx)}{b}$$

Mathematica [B] time = 0.0079008, size = 21, normalized size = 2.1

$$\frac{\sinh(a)\cosh(bx)}{b} + \frac{\cosh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x],x]
```

```
[Out] (Cosh[b*x]*Sinh[a])/b + (Cosh[a]*Sinh[b*x])/b
```

Maple [A] time = 0.03, size = 11, normalized size = 1.1

$$\frac{\sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a),x)
```

```
[Out] sinh(b*x+a)/b
```

Maxima [A] time = 1.05949, size = 14, normalized size = 1.4

$$\frac{\sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] sinh(b*x + a)/b
```

Fricas [A] time = 1.66529, size = 23, normalized size = 2.3

$$\frac{\sinh\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] sinh(b*x + a)/b
```

Sympy [A] time = 0.145025, size = 12, normalized size = 1.2

$$\begin{cases} \frac{\sinh\left(a+bx\right)}{b} & \text{for } b \neq 0 \\ x\cosh\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x)
```

```
[Out] Piecewise((sinh(a + b*x)/b, Ne(b, 0)), (x*cosh(a), True))
```

Giac [B] time = 1.17878, size = 35, normalized size = 3.5

$$\frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

3.2 $\int \cosh^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}$$

```
[Out] x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0096989, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2,x]
```

```
[Out] x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0222999, size = 23, normalized size = 0.92

$$\frac{2(a+bx)+\sinh(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.019, size = 27, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 1.07541, size = 43, normalized size = 1.72

$$\frac{1}{2}x+\frac{e^{(2bx+2a)}}{8b}-\frac{e^{(-2bx-2a)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Fricas [A] time = 1.80165, size = 58, normalized size = 2.32

$$\frac{bx+\cosh(bx+a)\sinh(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + cosh(b*x + a)*sinh(b*x + a))/b
```

Sympy [A] time = 0.233346, size = 46, normalized size = 1.84

$$\begin{cases} -\frac{x\sinh^2(a+bx)}{2} + \frac{x\cosh^2(a+bx)}{2} + \frac{\sinh(a+bx)\cosh(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cos
h(a + b*x)/(2*b), Ne(b, 0)), (x*cosh(a)**2, True))
```

Giac [B] time = 1.2718, size = 62, normalized size = 2.48

$$\frac{4\,bx - \left(2\,e^{(2\,bx+2\,a)} + 1\right)e^{(-2\,bx-2\,a)} + 4\,a + e^{(2\,bx+2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*b*x - (2*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 4*a + e^(2*b*x + 2*
a))/b
```

3.3 $\int \cosh^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0117305, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0062334, size = 26, normalized size = 1.

$$\frac{\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Maple [A] time = 0.06, size = 23, normalized size = 0.9

$$\frac{\sinh(bx+a)}{b}\left(\frac{2}{3}+\frac{(\cosh(bx+a))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3,x)
```

```
[Out] 1/b*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)
```

Maxima [B] time = 1.04511, size = 73, normalized size = 2.81

$$\frac{e^{(3\,bx+3\,a)}}{24\,b}+\frac{3\,e^{(bx+a)}}{8\,b}-\frac{3\,e^{(-bx-a)}}{8\,b}-\frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

Fricas [A] time = 1.70444, size = 89, normalized size = 3.42

$$\frac{\sinh(bx+a)^3+3\left(\cosh(bx+a)^2+3\right)\sinh(bx+a)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

Sympy [A] time = 0.486892, size = 36, normalized size = 1.38

$$\begin{cases} -\frac{2\sinh^3(a+bx)}{3b} + \frac{\sinh(a+bx)\cosh^2(a+bx)}{b} & \text{for } b \neq 0 \\ x\cosh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*sinh(a + b*x)**3/(3*b) + sinh(a + b*x)*cosh(a + b*x)**2/b, Ne
(b, 0)), (x*cosh(a)**3, True))
```

Giac [B] time = 1.2953, size = 65, normalized size = 2.5

$$-\frac{\left(9\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}-e^{(3\,bx+3\,a)}-9\,e^{(bx+a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - 9*e^(b*
x + a))/b
```

3.4 $\int \cosh^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

```
[Out] (3*x)/8 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a +
 b*x])/(4*b)
```

Rubi [A] time = 0.0199424, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a +
 b*x])/(4*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cosh^4(a+bx)\,dx &= \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} + \frac{3}{4}\int \cosh^2(a+bx)\,dx \\
&= \frac{3\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} + \frac{3\int 1\,dx}{8} \\
&= \frac{3x}{8} + \frac{3\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.0393123, size = 33, normalized size = 0.72

$$\frac{12(a+bx)+8\sinh(2(a+bx))+\sinh(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) + 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

Maple [A] time = 0.043, size = 39, normalized size = 0.9

$$\frac{1}{b}\left(\left(\frac{(\cosh(bx+a))^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4,x)
```

```
[Out] 1/b*((1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/8*b*x+3/8*a)
```

Maxima [A] time = 1.07024, size = 81, normalized size = 1.76

$$\frac{3}{8}x+\frac{e^{(4bx+4a)}}{64b}+\frac{e^{(2bx+2a)}}{8b}-\frac{e^{(-2bx-2a)}}{8b}-\frac{e^{(-4bx-4a)}}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Fricas [A] time = 1.71935, size = 134, normalized size = 2.91

$$\frac{\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+3\,bx+\left(\cosh\left(bx+a\right)^{3}+4\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 + 4*cosh(b*x
+ a))*sinh(b*x + a))/b
```

Sympy [A] time = 1.07407, size = 95, normalized size = 2.07

$$\begin{cases} \frac{3x\sinh^{4}(a+bx)}{8}-\frac{3x\sinh^{2}(a+bx)\cosh^{2}(a+bx)}{4}+\frac{3x\cosh^{4}(a+bx)}{8}-\frac{3\sinh^{3}(a+bx)\cosh(a+bx)}{8b}+\frac{5\sinh(a+bx)\cosh^{3}(a+bx)}{8b} & \text{for } b\neq 0 \\ x\cosh^{4}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/4
 + 3*x*cosh(a + b*x)**4/8 - 3*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + 5*sinh
(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*cosh(a)**4, True))
```

Giac [A] time = 1.2801, size = 92, normalized size = 2.

$$\frac{24\,bx-\left(18\,e^{(4\,bx+4\,a)}+8\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}+24\,a+e^{(4\,bx+4\,a)}+8\,e^{(2\,bx+2\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/64*(24*b*x - (18*e^(4*b*x + 4*a) + 8*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4*a
) + 24*a + e^(4*b*x + 4*a) + 8*e^(2*b*x + 2*a))/b
```

3.5 $\int \cosh^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\sinh^5(a+bx)}{5b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] Sinh[a + b*x]/b + (2*Sinh[a + b*x]^3)/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0132193, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\sinh^5(a+bx)}{5b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^5,x]
```

```
[Out] Sinh[a + b*x]/b + (2*Sinh[a + b*x]^3)/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^5(a+bx)\,dx&=\frac{i\,\text{Subst}\left(\int\left(1-2x^2+x^4\right)dx,x,-i\sinh(a+bx)\right)}{b}\\&=\frac{\sinh(a+bx)}{b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0134485, size = 41, normalized size = 1.

$$\frac{\sinh^5(a+bx)}{5b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^5,x]
```

```
[Out] Sinh[a + b*x]/b + (2*Sinh[a + b*x]^3)/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Maple [A] time = 0.036, size = 33, normalized size = 0.8

$$\frac{\sinh(bx+a)}{b}\left(\frac{8}{15}+\frac{(\cosh(bx+a))^4}{5}+\frac{4(\cosh(bx+a))^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^5,x)
```

```
[Out] 1/b*(8/15+1/5*cosh(b*x+a)^4+4/15*cosh(b*x+a)^2)*sinh(b*x+a)
```

Maxima [B] time = 1.02851, size = 111, normalized size = 2.71

$$\frac{e^{(5\,bx+5\,a)}}{160\,b}+\frac{5\,e^{(3\,bx+3\,a)}}{96\,b}+\frac{5\,e^{(bx+a)}}{16\,b}-\frac{5\,e^{(-bx-a)}}{16\,b}-\frac{5\,e^{(-3\,bx-3\,a)}}{96\,b}-\frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

Fricas [A] time = 1.69626, size = 182, normalized size = 4.44

$$\frac{3\,\sinh(bx+a)^5+5\left(6\,\cosh(bx+a)^2+5\right)\sinh(bx+a)^3+15\left(\cosh(bx+a)^4+5\,\cosh(bx+a)^2+10\right)\sinh(bx+a)}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*(
cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b
```

Sympy [A] time = 2.10741, size = 58, normalized size = 1.41

$$\begin{cases} \frac{8\sinh^5(a+bx)}{15b} - \frac{4\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{\sinh(a+bx)\cosh^4(a+bx)}{b} & \text{for } b \neq 0 \\ x\cosh^5(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**5,x)
```

```
[Out] Piecewise((8*sinh(a + b*x)**5/(15*b) - 4*sinh(a + b*x)**3*cosh(a + b*x)**2/
(3*b) + sinh(a + b*x)*cosh(a + b*x)**4/b, Ne(b, 0)), (x*cosh(a)**5, True))
```

Giac [A] time = 1.26741, size = 95, normalized size = 2.32

$$-\frac{\left(150\,e^{(4\,bx+4\,a)} + 25\,e^{(2\,bx+2\,a)} + 3\right)e^{(-5\,bx-5\,a)} - 3\,e^{(5\,bx+5\,a)} - 25\,e^{(3\,bx+3\,a)} - 150\,e^{(bx+a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/480*((150*e^(4*b*x + 4*a) + 25*e^(2*b*x + 2*a) + 3)*e^(-5*b*x - 5*a) - 3
*e^(5*b*x + 5*a) - 25*e^(3*b*x + 3*a) - 150*e^(b*x + a))/b
```

3.6 $\int \cosh^6(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}+\frac{5x}{16}$$

```
[Out] (5*x)/16 + (5*Cosh[a + b*x]*Sinh[a + b*x])/(16*b) + (5*Cosh[a + b*x]^3*Sinh
[a + b*x])/(24*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b)
```

Rubi [A] time = 0.0329988, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}+\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cosh[a + b*x]*Sinh[a + b*x])/(16*b) + (5*Cosh[a + b*x]^3*Sinh
[a + b*x])/(24*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh^6(a+bx)\,dx = \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} + \frac{5}{6}\int \cosh^4(a+bx)\,dx$$
$$= \frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} + \frac{5}{8}\int \cosh^2(a+bx)\,dx$$
$$= \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} + \frac{5\int 1}{16}$$
$$= \frac{5x}{16} + \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}$$

Mathematica [A] time = 0.038896, size = 43, normalized size = 0.64

$$\frac{45\sinh(2(a+bx)) + 9\sinh(4(a+bx)) + \sinh(6(a+bx)) + 60a + 60bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^6,x]
```

```
[Out] (60*a + 60*b*x + 45*Sinh[2*(a + b*x)] + 9*Sinh[4*(a + b*x)] + Sinh[6*(a + b
*x)])/(192*b)
```

Maple [A] time = 0.035, size = 49, normalized size = 0.7

$$\frac{1}{b}\left(\left(\frac{(\cosh(bx+a))^5}{6} + \frac{5\,(\cosh(bx+a))^3}{24} + \frac{5\,\cosh(bx+a)}{16}\right)\sinh(bx+a) + \frac{5\,bx}{16} + \frac{5\,a}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^6,x)
```

```
[Out] 1/b*((1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*cosh(b*x+a))*sinh(b*x+a)+5/
16*b*x+5/16*a)
```

Maxima [A] time = 1.07407, size = 116, normalized size = 1.73

$$\frac{\left(9\,e^{(-2\,bx-2\,a)} + 45\,e^{(-4\,bx-4\,a)} + 1\right)e^{(6\,bx+6\,a)}}{384\,b} + \frac{5\,(bx+a)}{16\,b} - \frac{45\,e^{(-2\,bx-2\,a)} + 9\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/384*(9*e^(-2*b*x - 2*a) + 45*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b + 5/
16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) + 9*e^(-4*b*x - 4*a) + e^(-6*b*
x - 6*a))/b
```

Fricas [A] time = 1.75843, size = 248, normalized size = 3.7

$$\frac{3\cosh(bx+a)\sinh(bx+a)^5 + 2\left(5\cosh(bx+a)^3 + 9\cosh(bx+a)\right)\sinh(bx+a)^3 + 30\,bx + 3\left(\cosh(bx+a)^5 + 6\right.}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 9*cosh(b*x +
 a))*sinh(b*x + a)^3 + 30*b*x + 3*(cosh(b*x + a)^5 + 6*cosh(b*x + a)^3 + 15
*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [A] time = 4.15961, size = 139, normalized size = 2.07

$$\begin{cases} -\frac{5x\sinh^6(a+bx)}{16} + \frac{15x\sinh^4(a+bx)\cosh^2(a+bx)}{16} - \frac{15x\sinh^2(a+bx)\cosh^4(a+bx)}{16} + \frac{5x\cosh^6(a+bx)}{16} + \frac{5\sinh^5(a+bx)\cosh(a+bx)}{16b} - \frac{5\sinh^3(a+b \\ x\cosh^6(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**6,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**6/16 + 15*x*sinh(a + b*x)**4*cosh(a + b*x)**
2/16 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + 5*x*cosh(a + b*x)**6/16
+ 5*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a + b*x
)**3/(6*b) + 11*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*cosh(a
)**6, True))
```

Giac [A] time = 1.26928, size = 122, normalized size = 1.82

$$\frac{120\,bx - \left(110\,e^{(6\,bx+6\,a)} + 45\,e^{(4\,bx+4\,a)} + 9\,e^{(2\,bx+2\,a)} + 1\right)e^{(-6\,bx-6\,a)} + 120\,a + e^{(6\,bx+6\,a)} + 9\,e^{(4\,bx+4\,a)} + 45\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 1/384*(120*b*x - (110*e^(6*b*x + 6*a) + 45*e^(4*b*x + 4*a) + 9*e^(2*b*x + 2
*a) + 1)*e^(-6*b*x - 6*a) + 120*a + e^(6*b*x + 6*a) + 9*e^(4*b*x + 4*a) + 4
5*e^(2*b*x + 2*a))/b
```

3.7 $\int \cosh^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{10i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{21b}+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{10\sinh(a+bx)\sqrt{\cosh(a+bx)}}{21b}$$

```
[Out] (((-10*I)/21)*EllipticF[(I/2)*(a + b*x), 2])/b + (10*Sqrt[Cosh[a + b*x]]*Si
nh[a + b*x])/(21*b) + (2*Cosh[a + b*x]^(5/2)*Sinh[a + b*x])/(7*b)
```

Rubi [A] time = 0.0328581, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2641}

$$-\frac{10iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{21b}+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{10\sinh(a+bx)\sqrt{\cosh(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I/2)*(a + b*x), 2])/b + (10*Sqrt[Cosh[a + b*x]]*Si
nh[a + b*x])/(21*b) + (2*Cosh[a + b*x]^(5/2)*Sinh[a + b*x])/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b} + \frac{5}{7}\int \cosh^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{10\sqrt{\cosh(a+bx)}\sinh(a+bx)}{21b} + \frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b} + \frac{5}{21}\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$$
$$= -\frac{10iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{21b} + \frac{10\sqrt{\cosh(a+bx)}\sinh(a+bx)}{21b} + \frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b}$$

Mathematica [A] time = 0.106611, size = 55, normalized size = 0.8

$$\frac{(23\sinh(a+bx)+3\sinh(3(a+bx)))\sqrt{\cosh(a+bx)}-20i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(7/2),x]
```

```
[Out] ((-20*I)*EllipticF[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*(23*Sinh[a + b
*x] + 3*Sinh[3*(a + b*x)]))/(42*b)
```

Maple [B] time = 0.114, size = 201, normalized size = 2.9

$$\frac{2}{21\,b}\sqrt{\left(2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^2-1\right)\left(\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(48\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^9-120\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^7+128\,(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(7/2),x)
```

```
[Out] 2/21*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(48*cosh(1/2
*b*x+1/2*a)^9-120*cosh(1/2*b*x+1/2*a)^7+128*cosh(1/2*b*x+1/2*a)^5-72*cosh(1
/2*b*x+1/2*a)^3+5*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+
1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+16*cosh(1/2*b*x+1/2*a))/(2*
sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*c
osh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh{(bx+a)}^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh{(bx+a)}^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh{(bx+a)}^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(7/2), x)
```

3.8 $\int \cosh^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Cosh[a + b*x]^(3/2)*Sinh[
a + b*x])/(5*b)
```

Rubi [A] time = 0.0196099, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2639}

$$\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Cosh[a + b*x]^(3/2)*Sinh[
a + b*x])/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = \frac{2\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{5b} + \frac{3}{5}\int \sqrt{\cosh(a+bx)}\,dx$$
$$= -\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b} + \frac{2\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{5b}$$

Mathematica [A] time = 0.050679, size = 44, normalized size = 0.96

$$\frac{\sinh(2(a+bx))\sqrt{\cosh(a+bx)} - 6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(5/2),x]
```

```
[Out] ((-6*I)*EllipticE[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*Sinh[2*(a + b*x
)])/(5*b)
```

Maple [B] time = 0.037, size = 188, normalized size = 4.1

$$\frac{2}{5b}\sqrt{\left(2\,(\cosh(1/2\,bx+a/2))^2-1\right)\left(\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(8\,(\cosh(1/2\,bx+a/2))^7-16\,(\cosh(1/2\,bx+a/2))^5+10\,(\cosh(1/$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(5/2),x)
```

```
[Out] 2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2*b
*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1/2*
b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b
*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/
b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh(bx+a)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(5/2), x)
```

3.9 $\int \cosh^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sqrt[Cosh[a + b*x]]*Sinh[
a + b*x])/(3*b)
```

Rubi [A] time = 0.0194128, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2635, 2641}

$$\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(3/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sqrt[Cosh[a + b*x]]*Sinh[
a + b*x])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \frac{2\sqrt{\cosh(a+bx)}\sinh(a+bx)}{3b} + \frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$$
$$= -\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b} + \frac{2\sqrt{\cosh(a+bx)}\sinh(a+bx)}{3b}$$

Mathematica [C] time = 0.0939686, size = 81, normalized size = 1.76

$$\frac{2\sqrt{\sinh(2(a+bx))+\cosh(2(a+bx))+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+bx))-\sinh(2(a+bx))\right)+\sinh(2(a+bx))}{3b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(3/2),x]
```

```
[Out] (Sinh[2*(a + b*x)] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(a + b*x)]
- Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]])/(3*b*
Sqrt[Cosh[a + b*x]])
```

Maple [B] time = 0.033, size = 174, normalized size = 3.8

$$\frac{2}{3b}\sqrt{\left(2\,(\cosh(1/2\,bx+a/2))^2-1\right)\left(\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(4\,(\cosh(1/2\,bx+a/2))^5-6\,(\cosh(1/2\,bx+a/2))^3+\sqrt{-\left(\sinh\left(\frac{bx}{2}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(3/2),x)
```

```
[Out] 2/3*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2*b
*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh
(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh(1/
2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/
2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cosh(bx+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(3/2), x)
```

3.10 $\int \sqrt{\cosh(a+bx)}\,dx$

Optimal. Leaf size=20

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

Rubi [A] time = 0.0089143, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2639}

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cosh(a+bx)}\,dx=-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Mathematica [A] time = 0.0269927, size = 20, normalized size = 1.

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

Maple [B] time = 0.032, size = 135, normalized size = 6.8

$$-2\,\frac{\sqrt{\left(2\,(\cosh(1/2\,bx+a/2))^2-1\right)(\sinh(1/2\,bx+a/2))^2}\sqrt{-(\sinh(1/2\,bx+a/2))^2}\sqrt{-2\,(\cosh(1/2\,bx+a/2))^2+1}\mathrm{Elliptic}}{\sqrt{2\,(\sinh(1/2\,bx+a/2))^4+(\sinh(1/2\,bx+a/2))^2}\sinh(1/2\,bx+a/2)\sqrt{2\,(\cosh(1/2\,bx+a/2))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(1/2),x)
```

```
[Out] -2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x
+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b*x+
1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(
1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cosh(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\cosh(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cosh(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cosh{(a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(cosh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cosh\left(bx + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cosh(b*x + a)), x)
```

3.11 $\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

Rubi [A] time = 0.0086202, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2641}

$$-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Mathematica [A] time = 0.0257158, size = 20, normalized size = 1.

$$-\frac{2i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

Maple [B] time = 0.036, size = 135, normalized size = 6.8

$$2\,\frac{\sqrt{\left(2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^{2}+1}\mathrm{Elliptic}}{\sqrt{2\,\left(\sinh\left(1/2\,bx+a/2\right)\right)^{4}+\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sinh\left(1/2\,bx+a/2\right)\sqrt{2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(1/2),x)
```

```
[Out] 2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x+
1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4
+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/sinh(1
/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cosh(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{\cosh\left(bx+a\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(cosh(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cosh\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cosh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cosh(b*x + a)), x)
```

3.12 $\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=42

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
+ b*x]])
```

Rubi [A] time = 0.0185497, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2639}

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
+ b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}} - \int \sqrt{\cosh(a+bx)}\,dx$$
$$= \frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b} + \frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}$$

Mathematica [A] time = 0.0563266, size = 42, normalized size = 1.

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}} + \frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
+ b*x]])
```

Maple [A] time = 0.045, size = 103, normalized size = 2.5

$$2\,\frac{\sqrt{-2\,\left(\sinh\left(1/2\,bx+a/2\right)\right)^2-1}\sqrt{-\left(\sinh\left(1/2\,bx+a/2\right)\right)^2}\mathrm{EllipticE}\left(\cosh\left(1/2\,bx+a/2\right),\sqrt{2}\right)+2\,\cosh\left(1/2\,bx+a/2\right)(\mathrm{si}}{\sinh\left(1/2\,bx+a/2\right)\sqrt{2\,\left(\cosh\left(1/2\,bx+a/2\right)\right)^2-1}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(3/2),x)
```

```
[Out] 2*((-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*Ellipt
icE(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^
2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\cosh\left(bx+a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(-3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(-3/2), x)
```

3.13 $\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{2i\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(3*b*Cosh[
a + b*x]^(3/2))
```

Rubi [A] time = 0.0191879, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2641}

$$\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(-5/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(3*b*Cosh[
a + b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)} + \frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$$
$$= -\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b} + \frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

Mathematica [C] time = 0.0636724, size = 84, normalized size = 1.83

$$\frac{2\left(\cosh(a+bx)\sqrt{\sinh(2(a+bx))+\cosh(2(a+bx))+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+bx))-\sinh(2(a+bx))\right)+\sinh(a+b\right.}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(-5/2),x]
```

```
[Out] (2*(Sinh[a + b*x] + Cosh[a + b*x]*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*
(a + b*x)] - Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*
x)]]))/(3*b*Cosh[a + b*x]^(3/2))
```

Maple [B] time = 0.06, size = 217, normalized size = 4.7

$$\frac{2}{3\,b}\left(2\,\sqrt{-\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}}\sqrt{-2\,\left(\sinh\left(1/2\,bx+a/2\right)\right)^{2}-1}\mathrm{EllipticF}\left(\cosh\left(1/2\,bx+a/2\right),\sqrt{2}\right)\left(\sinh\left(1/2\,bx+a/2\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(5/2),x)
```

```
[Out] 2/3*(2*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*El
lipticF(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^2+(-sinh(1/2*b*x+1
/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cosh(1/2*b*x+1/
2*a),2^(1/2))+2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2)*((2*cosh(1/2*b*x
+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(3/2)/sinh(1/2*b*x+1/2*a)/
b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\cosh(bx+a)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(-5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(-5/2), x)
```

3.14 $\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=69

$$\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

```
[Out] (((6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(5*b*Cosh[a
 + b*x]^(5/2)) + (6*Sinh[a + b*x])/(5*b*Sqrt[Cosh[a + b*x]])
```

Rubi [A] time = 0.03007, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2636, 2639}

$$\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(-7/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(5*b*Cosh[a
 + b*x]^(5/2)) + (6*Sinh[a + b*x])/(5*b*Sqrt[Cosh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{3}{5}\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}} - \frac{3}{5}\int \sqrt{\cosh(a+bx)}\,dx$$
$$= \frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b} + \frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

Mathematica [A] time = 0.127899, size = 63, normalized size = 0.91

$$\frac{3\sinh(2(a+bx)) + 2\tanh(a+bx) + 6i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(-7/2),x]
```

```
[Out] ((6*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*(a + b*x), 2] + 3*Sinh[2*(a + b*
x)] + 2*Tanh[a + b*x])/(5*b*Cosh[a + b*x]^(3/2))
```

Maple [B] time = 0.068, size = 363, normalized size = 5.3

$$\frac{2}{5b}\sqrt{\left(2\,(\cosh(1/2\,bx+a/2))^2-1\right)\left(\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(12\sqrt{-2\,(\sinh(1/2\,bx+a/2))^2-1}\sqrt{-(\sinh(1/2\,bx+a/2))^2}\mathrm{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(7/2),x)
```

```
[Out] 2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(8*sinh(1/2*b
*x+1/2*a)^6+12*sinh(1/2*b*x+1/2*a)^4+6*sinh(1/2*b*x+1/2*a)^2+1)/sinh(1/2*b*
x+1/2*a)^3*(12*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*(-sinh(1/2*b*x+1/2*a)^2)^
(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^4+24*sinh(
1/2*b*x+1/2*a)^6*cosh(1/2*b*x+1/2*a)+12*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*
(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(
```

```
1/2*b*x+1/2*a)^2+24*sinh(1/2*b*x+1/2*a)^4*cosh(1/2*b*x+1/2*a)+3*(-2*sinh(1/
2*b*x+1/2*a)^2-1)^(1/2)*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cosh(1/2*b
*x+1/2*a),2^(1/2))+8*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2)*(2*sinh(1/2
*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2
)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh{(bx+a)}^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\cosh{(bx+a)}^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(-7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(-7/2), x)
```

3.15 $\int (a\cosh(x))^{7/2}\,dx$

Optimal. Leaf size=65

$$-\frac{10ia^4\sqrt{\cosh(x)}\text{EllipticF}\left(\frac{ix}{2},2\right)}{21\sqrt{a\cosh(x)}}+\frac{10}{21}a^3\sinh(x)\sqrt{a\cosh(x)}+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

```
[Out] (((-10*I)/21)*a^4*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (1
0*a^3*Sqrt[a*Cosh[x]]*Sinh[x])/21 + (2*a*(a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rubi [A] time = 0.0374672, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2635, 2642, 2641}

$$\frac{10}{21}a^3\sinh(x)\sqrt{a\cosh(x)}-\frac{10ia^4\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)}{21\sqrt{a\cosh(x)}}+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*a^4*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (1
0*a^3*Sqrt[a*Cosh[x]]*Sinh[x])/21 + (2*a*(a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x))^{7/2}\,dx &= \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x) + \frac{1}{7}\left(5a^2\right)\int (a\cosh(x))^{3/2}\,dx\\ &= \frac{10}{21}a^3\sqrt{a\cosh(x)}\sinh(x) + \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x) + \frac{1}{21}\left(5a^4\right)\int \frac{1}{\sqrt{a\cosh(x)}}\,dx\\ &= \frac{10}{21}a^3\sqrt{a\cosh(x)}\sinh(x) + \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x) + \frac{\left(5a^4\sqrt{\cosh(x)}\right)\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{21\sqrt{a\cosh(x)}}\\ &= -\frac{10ia^4\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)}{21\sqrt{a\cosh(x)}} + \frac{10}{21}a^3\sqrt{a\cosh(x)}\sinh(x) + \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0477263, size = 53, normalized size = 0.82

$$\frac{a^3\sqrt{a\cosh(x)}\left((23\sinh(x)+3\sinh(3x))\sqrt{\cosh(x)}-20i\text{EllipticF}\left(\frac{ix}{2},2\right)\right)}{42\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(7/2),x]
```

```
[Out] (a^3*Sqrt[a*Cosh[x]]*((-20*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(23*Sin
h[x] + 3*Sinh[3*x])))/(42*Sqrt[Cosh[x]])
```

Maple [B] time = 0.078, size = 145, normalized size = 2.2

$$\frac{a^4}{21}\sqrt{a\left(2\left(\cosh(x/2)\right)^2-1\right)\left(\sinh\left(\frac{x}{2}\right)\right)^2}\left(96\left(\cosh(x/2)\right)^9-240\left(\cosh(x/2)\right)^7+256\left(\cosh(x/2)\right)^5+5\sqrt{2}\sqrt{-2\left(\cosh(x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(7/2),x)
```

```
[Out] 1/21*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^4*(96*cosh(1/2*x)^9-240*
cosh(1/2*x)^7+256*cosh(1/2*x)^5+5*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(1/2)*(-sinh
(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*2^(1/2),1/2*2^(1/2))-144*cosh(1/2*x)
^3+32*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a
*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cosh(x))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\cosh(x)}a^3\cosh(x)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))*a^3*cosh(x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \cosh (x))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(7/2), x)
```

3.16 $\int (a\cosh(x))^{5/2}\,dx$

Optimal. Leaf size=48

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}-\frac{6ia^2E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5\sqrt{\cosh(x)}}$$

```
[Out] (((-6*I)/5)*a^2*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]] + (2*a
*(a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rubi [A] time = 0.0252784, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2635, 2640, 2639}

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}-\frac{6ia^2E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(5/2),x]
```

```
[Out] (((-6*I)/5)*a^2*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]] + (2*a
*(a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x))^{5/2}\,dx &= \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x)+\frac{1}{5}\left(3a^2\right)\int \sqrt{a\cosh(x)}\,dx\\ &= \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x)+\frac{\left(3a^2\sqrt{a\cosh(x)}\right)\int \sqrt{\cosh(x)}\,dx}{5\sqrt{\cosh(x)}}\\ &= -\frac{6ia^2\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{5\sqrt{\cosh(x)}}+\frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0413884, size = 41, normalized size = 0.85

$$\frac{2(a\cosh(x))^{5/2}\left(\sinh(x)\cosh^{\frac{3}{2}}(x)-3iE\left(\frac{ix}{2}\middle|2\right)\right)}{5\cosh^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(5/2),x]
```

```
[Out] (2*(a*Cosh[x])^(5/2)*((-3*I)*EllipticE[(I/2)*x, 2] + Cosh[x]^(3/2)*Sinh[x])
)/(5*Cosh[x]^(5/2))
```

Maple [B] time = 0.043, size = 184, normalized size = 3.8

$$\frac{a^3}{5}\sqrt{a\left(2\,(\cosh(x/2))^2-1\right)\left(\sinh\left(\frac{x}{2}\right)\right)^2}\left(16\,\cosh(x/2)\,(\sinh(x/2))^6+16\,(\sinh(x/2))^4\cosh(x/2)+3\sqrt{2}\sqrt{-2\,(\sinh(x/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(5/2),x)
```

```
[Out] 1/5*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^3*(16*cosh(1/2*x)*sinh(1/
2*x)^6+16*sinh(1/2*x)^4*cosh(1/2*x)+3*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-
```

```
sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*2^(1/2),1/2*2^(1/2))-6*2^(1/2)*(
-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*2^(1
/2),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh(1/2*
x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cosh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\cosh(x)}a^2\cosh(x)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))*a^2*cosh(x)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cosh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(5/2), x)
```

3.17 $\int (a\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=48

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}-\frac{2ia^2\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\sqrt{a\cosh(x)}}$$

```
[Out] (((-2*I)/3)*a^2*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (2*a
*Sqrt[a*Cosh[x]]*Sinh[x])/3
```

Rubi [A] time = 0.0243887, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2635, 2642, 2641}

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}-\frac{2ia^2\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)}{3\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*a^2*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (2*a
*Sqrt[a*Cosh[x]]*Sinh[x])/3
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x))^{3/2}\,dx &= \frac{2}{3}a\sqrt{a\cosh(x)}\sinh(x)+\frac{1}{3}a^2\int\frac{1}{\sqrt{a\cosh(x)}}\,dx\\ &= \frac{2}{3}a\sqrt{a\cosh(x)}\sinh(x)+\frac{\left(a^2\sqrt{\cosh(x)}\right)\int\frac{1}{\sqrt{\cosh(x)}}\,dx}{3\sqrt{a\cosh(x)}}\\ &= -\frac{2ia^2\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)}{3\sqrt{a\cosh(x)}}+\frac{2}{3}a\sqrt{a\cosh(x)}\sinh(x)\end{aligned}$$

Mathematica [C] time = 0.0553999, size = 57, normalized size = 1.19

$$\frac{2}{3}(a\cosh(x))^{3/2}\left(\operatorname{sech}^2(x)\sqrt{\sinh(2x)+\cosh(2x)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2x)-\sinh(2x)\right)+\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(3/2),x]
```

```
[Out] (2*(a*Cosh[x])^(3/2)*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*
x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

Maple [B] time = 0.041, size = 130, normalized size = 2.7

$$\frac{a^2}{3}\sqrt{a\left(2\,(\cosh(x/2))^2-1\right)\left(\sinh\left(\frac{x}{2}\right)\right)^2}\left(8\,(\sinh(x/2))^4\cosh(x/2)+\sqrt{2}\sqrt{-2\,(\sinh(x/2))^2-1}\sqrt{-\left(\sinh\left(\frac{x}{2}\right)\right)^2}\,\mathrm{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(3/2),x)
```

```
[Out] 1/3*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^2*(8*sinh(1/2*x)^4*cosh(1
/2*x)+2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(c
```

```
osh(1/2*x)*2^(1/2),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2
*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cosh{(x)})^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\cosh{(x)}}a\cosh{(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))*a*cosh(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \cosh(x))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(3/2), x)
```

3.18 $\int \sqrt{a \cosh(x)}\,dx$

Optimal. Leaf size=27

$$-\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{\sqrt{\cosh(x)}}$$

```
[Out] ((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

Rubi [A] time = 0.0150689, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2640, 2639}

$$-\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\cosh(x)}\,dx = \frac{\sqrt{a\cosh(x)}\int\sqrt{\cosh(x)}\,dx}{\sqrt{\cosh(x)}}$$
$$= -\frac{2i\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{\cosh(x)}}$$

Mathematica [A] time = 0.0092948, size = 27, normalized size = 1.

$$-\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

Maple [B] time = 0.043, size = 118, normalized size = 4.4

$$a\sqrt{2}\sqrt{a\left(2\,(\cosh(x/2))^2-1\right)\left(\sinh\left(\frac{x}{2}\right)\right)^2}\sqrt{-2\,(\cosh(x/2))^2+1}\sqrt{-\left(\sinh\left(\frac{x}{2}\right)\right)^2}\left(\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{2},\frac{\sqrt{2}}{2}\right)-2\,\mathrm{Ell}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(1/2),x)
```

```
[Out] (a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a*2^(1/2)*(-2*cosh(1/2*x)^2+1)^
(1/2)*(-sinh(1/2*x)^2)^(1/2)*(EllipticF(cosh(1/2*x)*2^(1/2),1/2*2^(1/2))-2*
EllipticE(cosh(1/2*x)*2^(1/2),1/2*2^(1/2)))/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)
^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\cosh(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x)), x)
```

3.19 $\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$

Optimal. Leaf size=27

$$-\frac{2i\sqrt{\cosh(x)}\text{EllipticF}\left(\frac{ix}{2},2\right)}{\sqrt{a\cosh(x)}}$$

```
[Out] ((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

Rubi [A] time = 0.014622, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2642, 2641}

$$-\frac{2i\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = \frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{\sqrt{a\cosh(x)}}$$
$$= -\frac{2i\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}$$

Mathematica [A] time = 0.0113846, size = 27, normalized size = 1.

$$-\frac{2i\sqrt{\cosh(x)}\text{EllipticF}\left(\frac{ix}{2},2\right)}{\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

Maple [B] time = 0.037, size = 100, normalized size = 3.7

$$\sqrt{2}\sqrt{a\left(2\,(\cosh(x/2))^2-1\right)\left(\sinh\left(\frac{x}{2}\right)\right)^2}\sqrt{-2\,(\cosh(x/2))^2+1}\sqrt{-\left(\sinh\left(\frac{x}{2}\right)\right)^2}\,\text{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{2},\frac{\sqrt{2}}{2}\right)\frac{}{\sqrt{a\left(2\,(\sin\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(1/2),x)
```

```
[Out] (a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(1
/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)*Ellipt
icF(cosh(1/2*x)*2^(1/2),1/2*2^(1/2))/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*cosh(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)}}{a\cosh(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))/(a*cosh(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cosh(x)), x)
```

3.20 $\int \frac{1}{(a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}+\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{a^2\sqrt{\cosh(x)}}$$

```
[Out] ((2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[Cosh[x]]) + (2*Sinh
[x])/(a*Sqrt[a*Cosh[x]])
```

Rubi [A] time = 0.0253687, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2636, 2640, 2639}

$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}+\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{a^2\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(-3/2),x]
```

```
[Out] ((2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[Cosh[x]]) + (2*Sinh
[x])/(a*Sqrt[a*Cosh[x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\cosh(x))^{3/2}}\,dx &= \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}} - \frac{\int \sqrt{a\cosh(x)}\,dx}{a^2}\\ &= \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}} - \frac{\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}\,dx}{a^2\sqrt{\cosh(x)}}\\ &= \frac{2i\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{a^2\sqrt{\cosh(x)}} + \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.0216983, size = 34, normalized size = 0.74

$$\frac{2\cosh(x)\left(\sinh(x) + i\sqrt{\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)\right)}{(a\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(-3/2),x]
```

```
[Out] (2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/(a*Cosh[x])^(
3/2)
```

Maple [B] time = 0.052, size = 159, normalized size = 3.5

$$-\frac{1}{a}\sqrt{2\,(\sinh(x/2))^4\,a + \left(\sinh\left(\frac{x}{2}\right)\right)^2 a}\left(\sqrt{2}\sqrt{-2\,(\sinh(x/2))^2 - 1}\sqrt{-\left(\sinh\left(\frac{x}{2}\right)\right)^2}\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{2},\frac{\sqrt{2}}{2}\right) - 2\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(3/2),x)
```

```
[Out] -1/a*(2*sinh(1/2*x)^4*a+sinh(1/2*x)^2*a)^(1/2)*(2^(1/2)*(-2*sinh(1/2*x)^2-1
)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*2^(1/2),1/2*2^(1/2))-2
```

```
*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1
/2*x)*2^(1/2),1/2*2^(1/2))-4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4
+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)}}{a^2\cosh(x)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))/(a^2*cosh(x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(-3/2), x)
```

3.21 $\int \frac{1}{(a\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}-\frac{2i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3a^2\sqrt{a\cosh(x)}}$$

```
[Out] (((-2*I)/3)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]]) + (2
*Sinh[x])/(3*a*(a*Cosh[x])^(3/2))
```

Rubi [A] time = 0.0267797, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2636, 2642, 2641}

$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}-\frac{2i\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)}{3a^2\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(-5/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]]) + (2
*Sinh[x])/(3*a*(a*Cosh[x])^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\cosh(x))^{5/2}}\,dx &= \frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}+\frac{\int \frac{1}{\sqrt{a\cosh(x)}}\,dx}{3a^2}\\ &= \frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}+\frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3a^2\sqrt{a\cosh(x)}}\\ &= -\frac{2i\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)}{3a^2\sqrt{a\cosh(x)}}+\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0347288, size = 56, normalized size = 1.12

$$\frac{2\left(\sqrt{\sinh(2x)+\cosh(2x)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2x)-\sinh(2x)\right)+\tanh(x)\right)}{3a^2\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(-5/2),x]
```

```
[Out] (2*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*x]]*Sqrt[1 + Cosh[
2*x] + Sinh[2*x]] + Tanh[x]))/(3*a^2*Sqrt[a*Cosh[x]])
```

Maple [B] time = 0.052, size = 177, normalized size = 3.5

$$\frac{1}{3\,a^2}\left(2\,\sqrt{2}\sqrt{-2\,(\sinh(x/2))^2-1}\sqrt{-(\sinh(x/2))^2}\mathrm{EllipticF}\left(\cosh(x/2)\sqrt{2},1/2\,\sqrt{2}\right)(\sinh(x/2))^2+\sqrt{2}\sqrt{-2\,(\sinh(x/2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(5/2),x)
```

```
[Out] 1/3*(2*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(
cosh(1/2*x)*2^(1/2),1/2*2^(1/2))*sinh(1/2*x)^2+2^(1/2)*(-2*sinh(1/2*x)^2-1)
```

```
^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*2^(1/2),1/2*2^(1/2))+4*
sinh(1/2*x)^2*cosh(1/2*x))/a^2*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)/
(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(2*cosh(1/2*x)^2-1)/sinh(1/2*x)/(
a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)}}{a^3\cosh(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))/(a^3*cosh(x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(-5/2), x)
```

3.22 $\int \frac{1}{(a\cosh(x))^{7/2}}\,dx$

Optimal. Leaf size=67

$$\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}+\frac{6iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5a^4\sqrt{\cosh(x)}}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

```
[Out] (((6*I)/5)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^4*Sqrt[Cosh[x]]) + (2*
Sinh[x])/(5*a*(a*Cosh[x])^(5/2)) + (6*Sinh[x])/(5*a^3*Sqrt[a*Cosh[x]])
```

Rubi [A] time = 0.0387202, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2636, 2640, 2639}

$$\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}+\frac{6iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5a^4\sqrt{\cosh(x)}}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(-7/2),x]
```

```
[Out] (((6*I)/5)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^4*Sqrt[Cosh[x]]) + (2*
Sinh[x])/(5*a*(a*Cosh[x])^(5/2)) + (6*Sinh[x])/(5*a^3*Sqrt[a*Cosh[x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\cosh(x))^{7/2}}\,dx &= \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{3\int \frac{1}{(a\cosh(x))^{3/2}}\,dx}{5a^2}\\ &= \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}-\frac{3\int \sqrt{a\cosh(x)}\,dx}{5a^4}\\ &= \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}-\frac{\left(3\sqrt{a\cosh(x)}\right)\int \sqrt{\cosh(x)}\,dx}{5a^4\sqrt{\cosh(x)}}\\ &= \frac{6i\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{5a^4\sqrt{\cosh(x)}}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.0446776, size = 43, normalized size = 0.64

$$\frac{2\left(\tanh(x)+3i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)+3\sinh(x)\cosh(x)\right)}{5a^2(a\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(-7/2),x]
```

```
[Out] (2*((3*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 3*Cosh[x]*Sinh[x] + Tanh[x]
))/(5*a^2*(a*Cosh[x])^(3/2))
```

Maple [B] time = 0.082, size = 254, normalized size = 3.8

$$2\,\frac{\sqrt{a\left(2\,(\cosh(x/2))^2-1\right)(\sinh(x/2))^2}}{a^3\sinh(x/2)\sqrt{a\left(2\,(\cosh(x/2))^2-1\right)}}\left(1/20\,\frac{\cosh(x/2)\sqrt{a\left(2\,(\sinh(x/2))^4+(\sinh(x/2))^2\right)}}{a\left((\cosh(x/2))^2-1/2\right)^3}+6/5\,\frac{(\sinh(x/2))^2}{\sqrt{a\left(2\,(\cosh(x/2))^2\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(7/2),x)
```

```
[Out] 2*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)/a^3*(1/20*cosh(1/2*x)/a*(a*(2
*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(cosh(1/2*x)^2-1/2)^3+6/5*sinh(1/2*x)^
2*cosh(1/2*x)/(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)+3/10*2^(1/2)*(-2*
cosh(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*
x)^2))^(1/2)*EllipticF(cosh(1/2*x)*2^(1/2),1/2*2^(1/2))-3/5*2^(1/2)*(-2*cos
h(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^
2))^(1/2)*(EllipticF(cosh(1/2*x)*2^(1/2),1/2*2^(1/2))-EllipticE(cosh(1/2*x)
*2^(1/2),1/2*2^(1/2))))/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\cosh(x)}}{a^4\cosh(x)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))/(a^4*cosh(x)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(x))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(-7/2), x)
```

3.23 $\int (b\cosh(c+dx))^n\,dx$

Optimal. Leaf size=71

$$-\frac{\sinh(c+dx)(b\cosh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(c+dx)\right)}{bd(n+1)\sqrt{-\sinh^2(c+dx)}}$$

```
[Out] -(((b*Cosh[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, C
osh[c + d*x]^2]*Sinh[c + d*x])/(b*d*(1 + n)*Sqrt[-Sinh[c + d*x]^2]))
```

Rubi [A] time = 0.01884, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$-\frac{\sinh(c+dx)(b\cosh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(c+dx)\right)}{bd(n+1)\sqrt{-\sinh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cosh[c + d*x])^n,x]
```

```
[Out] -(((b*Cosh[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, C
osh[c + d*x]^2]*Sinh[c + d*x])/(b*d*(1 + n)*Sqrt[-Sinh[c + d*x]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cosh(c+dx))^n\,dx = -\frac{(b\cosh(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cosh^2(c+dx)\right)\sinh(c+dx)}{bd(1+n)\sqrt{-\sinh^2(c+dx)}}$$

Mathematica [A] time = 0.0573724, size = 65, normalized size = 0.92

$$\frac{\sqrt{-\sinh^2(c+dx)}\coth(c+dx)(b\cosh(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cosh[c + d*x])^n,x]
```

```
[Out] ((b*Cosh[c + d*x])^n*Coth[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cosh[c + d*x]^2]*Sqrt[-Sinh[c + d*x]^2])/(d*(1 + n))
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int (b\cosh(dx+c))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(d*x+c))^n,x)
```

```
[Out] int((b*cosh(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(dx+c))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cosh(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))**n,x)
```

```
[Out] Integral((b*cosh(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cosh(d*x + c))^n, x)
```

3.24 $\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3\sinh(x)\cosh(x)}{2a}$$

```
[Out] (-3*x)/(2*a) + (4*Sinh[x])/a - (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^3*Sinh[
x])/(a + a*Cosh[x]) + (4*Sinh[x]^3)/(3*a)
```

Rubi [A] time = 0.0758203, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2767, 2748, 2635, 8, 2633}

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sinh[x])/a - (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^3*Sinh[
x])/(a + a*Cosh[x]) + (4*Sinh[x]^3)/(3*a)
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx &= -\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}-\frac{\int \cosh^2(x)(3a-4a\cosh(x))\,dx}{a^2}\\ &= -\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}-\frac{3\int \cosh^2(x)\,dx}{a}+\frac{4\int \cosh^3(x)\,dx}{a}\\ &= -\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}+\frac{(4i)\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(x)\right)}{a}-\frac{3\int 1\,dx}{2a}\\ &= -\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}+\frac{4\sinh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.0752781, size = 53, normalized size = 0.98

$$\frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)-36x\cosh\left(\frac{x}{2}\right)\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*x
)/2] + Sinh[(7*x)/2]))/(24*a)
```

Maple [B] time = 0.024, size = 111, normalized size = 2.1

$$\frac{1}{a}\tanh\left(\frac{x}{2}\right)-\frac{1}{3a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{5}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{3}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{3a}\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)-1/3/a/(tanh(1/2*x)+1)^3+1/a/(tanh(1/2*x)+1)^2-5/2/a/(tanh(1
/2*x)+1)-3/2/a*ln(tanh(1/2*x)+1)-1/3/a/(tanh(1/2*x)-1)^3-1/a/(tanh(1/2*x)-1
)^2-5/2/a/(tanh(1/2*x)-1)+3/2/a*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.06566, size = 89, normalized size = 1.65

$$-\frac{3x}{2a}-\frac{21e^{(-x)}-3e^{(-2x)}+e^{(-3x)}}{24a}-\frac{2e^{(-x)}-18e^{(-2x)}-69e^{(-3x)}-1}{24\left(ae^{(-3x)}+ae^{(-4x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 18
*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

Fricas [B] time = 1.91861, size = 346, normalized size = 6.41

$$\frac{\cosh(x)^4+(4\cosh(x)-1)\sinh(x)^3+\sinh(x)^4-3\cosh(x)^3+\left(6\cosh(x)^2-9\cosh(x)+20\right)\sinh(x)^2-3(12x-1}{24(a\cosh(x)+a\sinh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6*
cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)^2
 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x - 69
```

```
)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [B] time = 3.19746, size = 337, normalized size = 6.24

$$-\frac{9x\tanh^{6}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}+\frac{27x\tanh^{4}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}-\frac{}{6a\tanh^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+a*cosh(x)),x)
```

```
[Out] -9*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2
 - 6*a) + 27*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*ta
nh(x/2)**2 - 6*a) - 27*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) + 9*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 1
8*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)**7/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)
**4 + 18*a*tanh(x/2)**2 - 6*a) - 48*tanh(x/2)**5/(6*a*tanh(x/2)**6 - 18*a*t
anh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 50*tanh(x/2)**3/(6*a*tanh(x/2)**6
- 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) - 24*tanh(x/2)/(6*a*tanh(x/2
)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

Giac [A] time = 1.12639, size = 95, normalized size = 1.76

$$-\frac{3\,x}{2\,a}-\frac{\left(69\,e^{(3\,x)}+18\,e^{(2\,x)}-2\,e^{x}+1\right)e^{(-3\,x)}}{24\,a(e^{x}+1)}+\frac{a^{2}e^{(3\,x)}-3\,a^{2}e^{(2\,x)}+21\,a^{2}e^{x}}{24\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1)
) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

3.25 $\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=43

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}+\frac{3\sinh(x)\cosh(x)}{2a}$$

```
[Out] (3*x)/(2*a) - (2*Sinh[x])/a + (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^2*Sinh[x
])/(a + a*Cosh[x])
```

Rubi [A] time = 0.0512014, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2767, 2734}

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}+\frac{3\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sinh[x])/a + (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^2*Sinh[x
])/(a + a*Cosh[x])
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx &= -\frac{\cosh^2(x)\sinh(x)}{a+a\cosh(x)}-\frac{\int \cosh(x)(2a-3a\cosh(x))\,dx}{a^2}\\
&= \frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\cosh(x)}
\end{aligned}
$$

Mathematica [A] time = 0.048308, size = 45, normalized size = 1.05

$$
\frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(-12\sinh\left(\frac{x}{2}\right)-3\sinh\left(\frac{3x}{2}\right)+\sinh\left(\frac{5x}{2}\right)+12x\cosh\left(\frac{x}{2}\right)\right)}{8a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2]
))/(8*a)
```

Maple [B] time = 0.021, size = 87, normalized size = 2.

$$
-\frac{1}{a}\tanh\left(\frac{x}{2}\right)-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{3}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{3}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-2}+\frac{3}{2a}\left(\tan\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+a*cosh(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)-1/2/a/(tanh(1/2*x)+1)^2+3/2/a/(tanh(1/2*x)+1)+3/2/a*ln(tan
h(1/2*x)+1)+1/2/a/(tanh(1/2*x)-1)^2+3/2/a/(tanh(1/2*x)-1)-3/2/a*ln(tanh(1/2
*x)-1)
```

Maxima [A] time = 1.1151, size = 76, normalized size = 1.77

$$
\frac{3\,x}{2\,a}+\frac{4\,e^{(-x)}-e^{(-2\,x)}}{8\,a}-\frac{3\,e^{(-x)}+20\,e^{(-2\,x)}-1}{8\left(ae^{(-2\,x)}+ae^{(-3\,x)}\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(a
*e^(-2*x) + a*e^(-3*x))
```

Fricas [A] time = 1.87436, size = 242, normalized size = 5.63

$$\frac{\cosh(x)^3 + (3\cosh(x) - 4)\sinh(x)^2 + \sinh(x)^3 + (12x - 1)\cosh(x) - 4\cosh(x)^2 + \left(3\cosh(x)^2 + 12x - 4\cosh(x)\right.}{8\left(a\cosh(x) + a\sinh(x) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x)
 - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20)/
(a*cosh(x) + a*sinh(x) + a)
```

Sympy [B] time = 1.7469, size = 189, normalized size = 4.4

$$\frac{3x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{6x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} + \frac{3x}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{}{2a\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*cosh(x)),x)
```

```
[Out] 3*x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 6*x*tanh(x/2
)**2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + 3*x/(2*a*tanh(x/2)**4 -
4*a*tanh(x/2)**2 + 2*a) - 2*tanh(x/2)**5/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)*
*2 + 2*a) + 10*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 4
*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

Giac [A] time = 1.15494, size = 69, normalized size = 1.6

$$\frac{3\,x}{2\,a}+\frac{\left(20\,e^{(2\,x)}+3\,e^{x}-1\right)e^{(-2\,x)}}{8\,a(e^{x}+1)}+\frac{ae^{(2\,x)}-4\,ae^{x}}{8\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(2
*x) - 4*a*e^x)/a^2
```

3.26 $\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=25

$$-\frac{x}{a}+\frac{\sinh(x)}{a}+\frac{\sinh(x)}{a(\cosh(x)+1)}$$

```
[Out] -(x/a) + Sinh[x]/a + Sinh[x]/(a*(1 + Cosh[x]))
```

Rubi [A] time = 0.0693868, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2746, 12, 2735, 2648}

$$-\frac{x}{a}+\frac{\sinh(x)}{a}+\frac{\sinh(x)}{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -(x/a) + Sinh[x]/a + Sinh[x]/(a*(1 + Cosh[x]))
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx &= \frac{\sinh(x)}{a} - \frac{\int \frac{a\cosh(x)}{a+a\cosh(x)}\,dx}{a}\\
&= \frac{\sinh(x)}{a} - \int \frac{\cosh(x)}{a+a\cosh(x)}\,dx\\
&= -\frac{x}{a} + \frac{\sinh(x)}{a} + \int \frac{1}{a+a\cosh(x)}\,dx\\
&= -\frac{x}{a} + \frac{\sinh(x)}{a} + \frac{\sinh(x)}{a+a\cosh(x)}
\end{aligned}$$

Mathematica [A] time = 0.0544024, size = 32, normalized size = 1.28

$$\frac{-2x+3\tanh\left(\frac{x}{2}\right)+\sinh\left(\frac{3x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

Maple [B] time = 0.02, size = 59, normalized size = 2.4

$$\frac{1}{a}\tanh\left(\frac{x}{2}\right)-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+a*cosh(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)-1/a/(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)+1)-1/a/(tanh(1/2*x)-
1)+1/a*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.17736, size = 55, normalized size = 2.2

$$-\frac{x}{a}+\frac{5\,e^{(-x)}+1}{2\left(ae^{(-x)}+ae^{(-2\,x)}\right)}-\frac{e^{(-x)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

Fricas [A] time = 1.90001, size = 151, normalized size = 6.04

$$-\frac{2\,x\cosh\left(x\right)-\cosh\left(x\right)^{2}+2\,\left(x-\cosh\left(x\right)-1\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}+2\,x+5}{2\,\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 + 2
*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [B] time = 0.896657, size = 63, normalized size = 2.52

$$-\frac{x\tanh^{2}\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right)-a}+\frac{x}{a\tanh^{2}\left(\frac{x}{2}\right)-a}+\frac{\tanh^{3}\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right)-a}-\frac{3\tanh\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right)-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*cosh(x)),x)
```

```
[Out] -x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) + tanh(x/2)**
3/(a*tanh(x/2)**2 - a) - 3*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

Giac [A] time = 1.16933, size = 47, normalized size = 1.88

$$-\frac{x}{a} - \frac{(5\,e^{x} + 1)e^{(-x)}}{2\,a(e^{x} + 1)} + \frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

3.27 $\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{x}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

```
[Out] x/a - Sinh[x]/(a + a*Cosh[x])
```

Rubi [A] time = 0.0315345, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2735, 2648}

$$\frac{x}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + a*Cosh[x]),x]
```

```
[Out] x/a - Sinh[x]/(a + a*Cosh[x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\cosh(x)}\,dx\\ &= \frac{x}{a}-\frac{\sinh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.0239296, size = 14, normalized size = 0.78

$$\frac{x-\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + a*Cosh[x]),x]
```

```
[Out] (x - Tanh[x/2])/a
```

Maple [A] time = 0.011, size = 34, normalized size = 1.9

$$-\frac{1}{a}\tanh\left(\frac{x}{2}\right)-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*cosh(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)
```

Maxima [A] time = 1.26951, size = 24, normalized size = 1.33

$$\frac{x}{a}-\frac{2}{ae^{(-x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] x/a - 2/(a*e^(-x) + a)
```

Fricas [A] time = 1.92144, size = 82, normalized size = 4.56

$$\frac{x\cosh(x)+x\sinh(x)+x+2}{a\cosh(x)+a\sinh(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) + x + 2)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [A] time = 0.414297, size = 8, normalized size = 0.44

$$\frac{x}{a} - \frac{\tanh\left(\frac{x}{2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x)
```

```
[Out] x/a - tanh(x/2)/a
```

Giac [A] time = 1.16483, size = 23, normalized size = 1.28

$$\frac{x}{a} + \frac{2}{a(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] x/a + 2/(a*(e^x + 1))
```

3.28 $\int \frac{\mathbf{sech}(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

```
[Out] ArcTan[Sinh[x]]/a - Sinh[x]/(a + a*Cosh[x])
```

Rubi [A] time = 0.0415903, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2747, 3770, 2648}

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + a*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - Sinh[x]/(a + a*Cosh[x])
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx = \frac{\int \operatorname{sech}(x)\,dx}{a} - \int \frac{1}{a+a\cosh(x)}\,dx$$
$$= \frac{\tan^{-1}(\sinh(x))}{a} - \frac{\sinh(x)}{a+a\cosh(x)}$$

Mathematica [A] time = 0.0235633, size = 22, normalized size = 1.1

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - \tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + a*Cosh[x]),x]
```

```
[Out] (2*ArcTan[Tanh[x/2]] - Tanh[x/2])/a
```

Maple [A] time = 0.017, size = 21, normalized size = 1.1

$$-\frac{1}{a}\tanh\left(\frac{x}{2}\right) + 2\,\frac{\arctan\left(\tanh\left(x/2\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+a*cosh(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)+2/a*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.81931, size = 31, normalized size = 1.55

$$-\frac{2\,\arctan\left(e^{(-x)}\right)}{a} - \frac{2}{ae^{(-x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

Fricas [A] time = 1.78969, size = 117, normalized size = 5.85

$$\frac{2\left(\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1\right)}{a\cosh\left(x\right)+a\sinh\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*si
nh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.17131, size = 27, normalized size = 1.35

$$\frac{2\,\arctan\left(e^{x}\right)}{a}+\frac{2}{a(e^{x}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

3.29 $\int \frac{\text{sech}^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh(x)}{a}-\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a\cosh(x)+a}$$

```
[Out] -(ArcTan[Sinh[x]]/a) + (2*Tanh[x])/a - Tanh[x]/(a + a*Cosh[x])
```

Rubi [A] time = 0.0683189, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 8, 3770}

$$\frac{2\tanh(x)}{a}-\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -(ArcTan[Sinh[x]]/a) + (2*Tanh[x])/a - Tanh[x]/(a + a*Cosh[x])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx &= -\frac{\tanh(x)}{a+a\cosh(x)} - \frac{\int (-2a+a\cosh(x))\operatorname{sech}^2(x)\,dx}{a^2} \\
&= -\frac{\tanh(x)}{a+a\cosh(x)} - \frac{\int \operatorname{sech}(x)\,dx}{a} + \frac{2\int \operatorname{sech}^2(x)\,dx}{a} \\
&= -\frac{\tan^{-1}(\sinh(x))}{a} - \frac{\tanh(x)}{a+a\cosh(x)} + \frac{(2i)\operatorname{Subst}(\int 1\,dx, x, -i\tanh(x))}{a} \\
&= -\frac{\tan^{-1}(\sinh(x))}{a} + \frac{2\tanh(x)}{a} - \frac{\tanh(x)}{a+a\cosh(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0797092, size = 43, normalized size = 1.54

$$
\frac{2\cosh\left(\frac{x}{2}\right)\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(\tanh(x)-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)\right)}{a(\cosh(x)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (2*Cosh[x/2]*(Sinh[x/2] + Cosh[x/2]*(-2*ArcTan[Tanh[x/2]] + Tanh[x])))/(a*(
1 + Cosh[x]))
```

Maple [A] time = 0.022, size = 39, normalized size = 1.4

$$
\frac{1}{a}\tanh\left(\frac{x}{2}\right)+2\,\frac{\tanh(x/2)}{a\left((\tanh(x/2))^2+1\right)}-2\,\frac{\arctan(\tanh(x/2))}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+a*cosh(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)+2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2/a*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.79331, size = 61, normalized size = 2.18

$$\frac{2\left(e^{(-x)} + e^{(-2x)} + 2\right)}{ae^{(-x)} + ae^{(-2x)} + ae^{(-3x)} + a} + \frac{2\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arct
an(e^(-x))/a
```

Fricas [B] time = 1.79978, size = 467, normalized size = 16.68

$$-\frac{2\left(\left(\cosh(x)^3 + (3\cosh(x)+1)\sinh(x)^2 + \sinh(x)^3 + \cosh(x)^2 + \left(3\cosh(x)^2 + 2\cosh(x)+1\right)\sinh(x) + \cosh(x)\right.\right.}{a\cosh(x)^3 + a\sinh(x)^3 + a\cosh(x)^2 + (3a\cosh(x)+a)\sinh(x)^2 + a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*cos
h(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) +
cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x)^3
 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (3
*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^2(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)**2/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.18519, size = 49, normalized size = 1.75

$$-\frac{2\arctan\left(e^{x}\right)}{a}-\frac{2\left(e^{(2x)}+e^{x}+2\right)}{a\left(e^{(3x)}+e^{(2x)}+e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

3.30 $\int \frac{\text{sech}^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=43

$$-\frac{2\tanh(x)}{a}+\frac{3\tan^{-1}(\sinh(x))}{2a}+\frac{3\tanh(x)\text{sech}(x)}{2a}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

```
[Out] (3*ArcTan[Sinh[x]])/(2*a) - (2*Tanh[x])/a + (3*Sech[x]*Tanh[x])/(2*a) - (Se
ch[x]*Tanh[x])/(a + a*Cosh[x])
```

Rubi [A] time = 0.0744833, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$-\frac{2\tanh(x)}{a}+\frac{3\tan^{-1}(\sinh(x))}{2a}+\frac{3\tanh(x)\text{sech}(x)}{2a}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(2*a) - (2*Tanh[x])/a + (3*Sech[x]*Tanh[x])/(2*a) - (Se
ch[x]*Tanh[x])/(a + a*Cosh[x])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx &= -\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}-\frac{\int(-3a+2a\cosh(x))\operatorname{sech}^3(x)\,dx}{a^2}\\
&= -\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}-\frac{2\int \operatorname{sech}^2(x)\,dx}{a}+\frac{3\int \operatorname{sech}^3(x)\,dx}{a}\\
&= \frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}-\frac{(2i)\operatorname{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)}{a}+\frac{3\int \operatorname{sech}(x)\,dx}{2a}\\
&= \frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.0830855, size = 49, normalized size = 1.14

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\cosh\left(\frac{x}{2}\right)\left(6\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)(\operatorname{sech}(x)-2)\right)-2\sinh\left(\frac{x}{2}\right)\right)}{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]*(-2*Sinh[x/2] + Cosh[x/2]*(6*ArcTan[Tanh[x/2]] + (-2 + Sech[x])*
Tanh[x])))/(a*(1 + Cosh[x]))
```

Maple [A] time = 0.024, size = 61, normalized size = 1.4

$$-\frac{1}{a}\tanh\left(\frac{x}{2}\right)-3\frac{\left(\tanh(x/2)\right)^{3}}{a\left(\left(\tanh(x/2)\right)^{2}+1\right)^{2}}-\frac{1}{a}\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-2}+3\frac{\arctan\left(\tanh(x/2)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+a*cosh(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)-3/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-1/a/(tanh(1/2*x)^2+1
)^2*tanh(1/2*x)+3/a*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.67315, size = 99, normalized size = 2.3

$$-\frac{e^{(-x)}+5\,e^{(-2\,x)}+3\,e^{(-3\,x)}+3\,e^{(-4\,x)}+4}{ae^{(-x)}+2\,ae^{(-2\,x)}+2\,ae^{(-3\,x)}+ae^{(-4\,x)}+ae^{(-5\,x)}+a}-\frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2*
x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

Fricas [B] time = 1.89154, size = 1099, normalized size = 25.56

$$\frac{3\cosh(x)^{4}+3\left(4\cosh(x)+1\right)\sinh(x)^{3}+3\sinh(x)^{4}+3\cosh(x)^{3}+\left(18\cosh(x)^{2}+9\cosh(x)+5\right)\sinh(x)^{2}+3\left(\cdots\right.}{a\cosh(x)^{5}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] (3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (1
8*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*sin
h(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^3
+ 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 2
*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)*sinh
(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh(x)^3
+ 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5 + a*sin
h(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3 + 2*(5*a
*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*cosh(x)^3
+ 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*cosh(x)^4 +
 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^3(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)**3/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.14482, size = 65, normalized size = 1.51

$$\frac{3\,\arctan\left(e^{x}\right)}{a} + \frac{e^{(3\,x)} + 2\,e^{(2\,x)} - e^{x} + 2}{a\left(e^{(2\,x)} + 1\right)^{2}} + \frac{2}{a\left(e^{x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/(
a*(e^x + 1))
```

3.31 $\int \frac{\mathbf{sech}^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{4\tanh^3(x)}{3a}+\frac{4\tanh(x)}{a}-\frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{3\tanh(x)\mathbf{sech}(x)}{2a}-\frac{\tanh(x)\mathbf{sech}^2(x)}{a\cosh(x)+a}$$

```
[Out] (-3*ArcTan[Sinh[x]])/(2*a) + (4*Tanh[x])/a - (3*Sech[x]*Tanh[x])/(2*a) - (S
ech[x]^2*Tanh[x])/(a + a*Cosh[x]) - (4*Tanh[x]^3)/(3*a)
```

Rubi [A] time = 0.0769904, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 3768, 3770}

$$-\frac{4\tanh^3(x)}{3a}+\frac{4\tanh(x)}{a}-\frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{3\tanh(x)\mathbf{sech}(x)}{2a}-\frac{\tanh(x)\mathbf{sech}^2(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (-3*ArcTan[Sinh[x]])/(2*a) + (4*Tanh[x])/a - (3*Sech[x]*Tanh[x])/(2*a) - (S
ech[x]^2*Tanh[x])/(a + a*Cosh[x]) - (4*Tanh[x]^3)/(3*a)
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{\int(-4a+3a\cosh(x))\operatorname{sech}^4(x)\,dx}{a^2}\\ &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{3\int \operatorname{sech}^3(x)\,dx}{a}+\frac{4\int \operatorname{sech}^4(x)\,dx}{a}\\ &= -\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}+\frac{(4i)\operatorname{Subst}\left(\int\left(1+x^2\right)\,dx,x,-i\tanh(x)\right)}{a}-\frac{3\int \operatorname{sech}(x)}{2a}\\ &= -\frac{3\tan^{-1}(\sinh(x))}{2a}+\frac{4\tanh(x)}{a}-\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{4\tanh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.175914, size = 60, normalized size = 1.07

$$\frac{\cosh\left(\frac{x}{2}\right)\left(6\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(\tanh(x)\left(2\operatorname{sech}^2(x)-3\operatorname{sech}(x)+10\right)-18\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)\right)}{3a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]*(6*Sinh[x/2] + Cosh[x/2]*(-18*ArcTan[Tanh[x/2]] + (10 - 3*Sech[x
] + 2*Sech[x]^2)*Tanh[x])))/(3*a*(1 + Cosh[x]))
```

Maple [A] time = 0.024, size = 81, normalized size = 1.5

$$\frac{1}{a}\tanh\left(\frac{x}{2}\right)+5\frac{(\tanh(x/2))^5}{a\left((\tanh(x/2))^2+1\right)^3}+\frac{16}{3a}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-3}+3\frac{\tanh(x/2)}{a\left((\tanh(x/2))^2+1\right)^3}-3\frac{\arctan(\text{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)+5/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5+16/3/a/(tanh(1/2*x)^2
+1)^3*tanh(1/2*x)^3+3/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)-3/a*arctan(tanh(1/2
*x))
```

Maxima [A] time = 1.66707, size = 136, normalized size = 2.43

$$\frac{7\,e^{(-x)}+39\,e^{(-2\,x)}+24\,e^{(-3\,x)}+24\,e^{(-4\,x)}+9\,e^{(-5\,x)}+9\,e^{(-6\,x)}+16}{3\left(ae^{(-x)}+3\,ae^{(-2\,x)}+3\,ae^{(-3\,x)}+3\,ae^{(-4\,x)}+3\,ae^{(-5\,x)}+ae^{(-6\,x)}+ae^{(-7\,x)}+a\right)}+\frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(7*e^(-x) + 39*e^(-2*x) + 24*e^(-3*x) + 24*e^(-4*x) + 9*e^(-5*x) + 9*e^
(-6*x) + 16)/(a*e^(-x) + 3*a*e^(-2*x) + 3*a*e^(-3*x) + 3*a*e^(-4*x) + 3*a*e
^(-5*x) + a*e^(-6*x) + a*e^(-7*x) + a) + 3*arctan(e^(-x))/a
```

Fricas [B] time = 1.95651, size = 2018, normalized size = 36.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(9*cosh(x)^6 + 9*(6*cosh(x) + 1)*sinh(x)^5 + 9*sinh(x)^6 + 9*cosh(x)^5
 + 3*(45*cosh(x)^2 + 15*cosh(x) + 8)*sinh(x)^4 + 24*cosh(x)^4 + 6*(30*cosh(
```

```
x)^3 + 15*cosh(x)^2 + 16*cosh(x) + 4)*sinh(x)^3 + 24*cosh(x)^3 + 3*(45*cosh
(x)^4 + 30*cosh(x)^3 + 48*cosh(x)^2 + 24*cosh(x) + 13)*sinh(x)^2 + 9*(cosh(
x)^7 + (7*cosh(x) + 1)*sinh(x)^6 + sinh(x)^7 + cosh(x)^6 + 3*(7*cosh(x)^2 +
 2*cosh(x) + 1)*sinh(x)^5 + 3*cosh(x)^5 + (35*cosh(x)^3 + 15*cosh(x)^2 + 15
*cosh(x) + 3)*sinh(x)^4 + 3*cosh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^3 + 30*c
osh(x)^2 + 12*cosh(x) + 3)*sinh(x)^3 + 3*cosh(x)^3 + 3*(7*cosh(x)^5 + 5*cos
h(x)^4 + 10*cosh(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 3*cosh(x)^
2 + (7*cosh(x)^6 + 6*cosh(x)^5 + 15*cosh(x)^4 + 12*cosh(x)^3 + 9*cosh(x)^2
+ 6*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 39*cosh
(x)^2 + (54*cosh(x)^5 + 45*cosh(x)^4 + 96*cosh(x)^3 + 72*cosh(x)^2 + 78*cos
h(x) + 7)*sinh(x) + 7*cosh(x) + 16)/(a*cosh(x)^7 + a*sinh(x)^7 + a*cosh(x)^
6 + (7*a*cosh(x) + a)*sinh(x)^6 + 3*a*cosh(x)^5 + 3*(7*a*cosh(x)^2 + 2*a*co
sh(x) + a)*sinh(x)^5 + 3*a*cosh(x)^4 + (35*a*cosh(x)^3 + 15*a*cosh(x)^2 + 1
5*a*cosh(x) + 3*a)*sinh(x)^4 + 3*a*cosh(x)^3 + (35*a*cosh(x)^4 + 20*a*cosh(
x)^3 + 30*a*cosh(x)^2 + 12*a*cosh(x) + 3*a)*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(
7*a*cosh(x)^5 + 5*a*cosh(x)^4 + 10*a*cosh(x)^3 + 6*a*cosh(x)^2 + 3*a*cosh(x
) + a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 6*a*cosh(x)^5 + 15*a*cosh(x
)^4 + 12*a*cosh(x)^3 + 9*a*cosh(x)^2 + 6*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^4(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)**4/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.15633, size = 77, normalized size = 1.38

$$-\frac{3\arctan\left(e^{x}\right)}{a}-\frac{2}{a(e^{x}+1)}-\frac{3\,e^{(5\,x)}+6\,e^{(4\,x)}+24\,e^{(2\,x)}-3\,e^{x}+10}{3\,a\left(e^{(2\,x)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -3*arctan(e^x)/a - 2/(a*(e^x + 1)) - 1/3*(3*e^(5*x) + 6*e^(4*x) + 24*e^(2*x
) - 3*e^x + 10)/(a*(e^(2*x) + 1)^3)
```

3.32 $\int \frac{1}{1+\cosh(c+dx)}\,dx$

Optimal. Leaf size=20

$$\frac{\sinh(c+dx)}{d(\cosh(c+dx)+1)}$$

```
[Out] Sinh[c + d*x]/(d*(1 + Cosh[c + d*x]))
```

Rubi [A] time = 0.0100075, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2648}

$$\frac{\sinh(c+dx)}{d(\cosh(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-1),x]
```

```
[Out] Sinh[c + d*x]/(d*(1 + Cosh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = \frac{\sinh(c+dx)}{d(1+\cosh(c+dx))}$$

Mathematica [A] time = 0.0141596, size = 14, normalized size = 0.7

$$\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-1),x]
```

```
[Out] Tanh[(c + d*x)/2]/d
```

Maple [A] time = 0.01, size = 14, normalized size = 0.7

$$\frac{1}{d}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c)),x)
```

```
[Out] 1/d*tanh(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.16488, size = 24, normalized size = 1.2

$$\frac{2}{d\left(e^{(-dx-c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/(d*(e^(-d*x - c) + 1))
```

Fricas [A] time = 1.73141, size = 59, normalized size = 2.95

$$-\frac{2}{d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/(d*cosh(d*x + c) + d*sinh(d*x + c) + d)
```

Sympy [A] time = 0.592076, size = 17, normalized size = 0.85

$$\begin{cases} \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} & \text{for } d \neq 0 \\ \frac{x}{\cosh(c)+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x)
```

```
[Out] Piecewise((tanh(c/2 + d*x/2)/d, Ne(d, 0)), (x/(cosh(c) + 1), True))
```

Giac [A] time = 1.16337, size = 20, normalized size = 1.

$$-\frac{2}{d\left(e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/(d*(e^(d*x + c) + 1))
```

3.33 $\int \frac{1}{(1+\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}$$

```
[Out] Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x])^2) + Sinh[c + d*x]/(3*d*(1 + Cosh[c
+ d*x]))
```

Rubi [A] time = 0.0223655, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2650, 2648}

$$\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-2),x]
```

```
[Out] Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x])^2) + Sinh[c + d*x]/(3*d*(1 + Cosh[c
+ d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = \frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))^2} + \frac{1}{3}\int \frac{1}{1+\cosh(c+dx)}\,dx$$
$$= \frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))^2} + \frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))}$$

Mathematica [A] time = 0.0287307, size = 34, normalized size = 0.72

$$\frac{4\sinh(c+dx)+\sinh(2(c+dx))}{6d(\cosh(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-2),x]
```

```
[Out] (4*Sinh[c + d*x] + Sinh[2*(c + d*x)])/(6*d*(1 + Cosh[c + d*x])^2)
```

Maple [A] time = 0.01, size = 30, normalized size = 0.6

$$\frac{1}{d}\left(-\frac{1}{6}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/6*tanh(1/2*d*x+1/2*c)^3+1/2*tanh(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.07127, size = 122, normalized size = 2.6

$$\frac{2\,e^{(-dx-c)}}{d\left(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1\right)}+\frac{2}{3\,d\left(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) +
 1)) + 2/3/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) + 1))
```

Fricas [B] time = 1.79341, size = 319, normalized size = 6.79

$$-\frac{2\,(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)+1)}{3\,\left(d\cosh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)^{2}+3\,(d\cosh\left(dx+c\right)+d)\sinh\left(dx+c\right)^{2}+3\,d\cosh\left(dx+c\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1)/(d*cosh(d*x + c)^3 + d*sinh(d*
x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) + d)*sinh(d*x + c)^2 +
3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c) + d)*sinh(d*x
+ c) + d)
```

Sympy [A] time = 1.34671, size = 36, normalized size = 0.77

$$\begin{cases} -\frac{\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} & \text{for}\ d\neq 0 \\ \frac{x}{(\cosh\left(c\right)+1)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((-tanh(c/2 + d*x/2)**3/(6*d) + tanh(c/2 + d*x/2)/(2*d), Ne(d, 0))
, (x/(cosh(c) + 1)**2, True))
```

Giac [A] time = 1.15745, size = 34, normalized size = 0.72

$$-\frac{2\left(3\,e^{(dx+c)}+1\right)}{3\,d\left(e^{(dx+c)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^3)
```

3.34 $\int \frac{1}{(1+\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=70

$$\frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)}+\frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)^2}+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}$$

```
[Out] Sinh[c + d*x]/(5*d*(1 + Cosh[c + d*x])^3) + (2*Sinh[c + d*x])/(15*d*(1 + Co
sh[c + d*x])^2) + (2*Sinh[c + d*x])/(15*d*(1 + Cosh[c + d*x]))
```

Rubi [A] time = 0.036479, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2650, 2648}

$$\frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)}+\frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)^2}+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-3),x]
```

```
[Out] Sinh[c + d*x]/(5*d*(1 + Cosh[c + d*x])^3) + (2*Sinh[c + d*x])/(15*d*(1 + Co
sh[c + d*x])^2) + (2*Sinh[c + d*x])/(15*d*(1 + Cosh[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3} + \frac{2}{5}\int \frac{1}{(1+\cosh(c+dx))^2}\,dx$$
$$= \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))^2} + \frac{2}{15}\int \frac{1}{1+\cosh(c+dx)}\,dx$$
$$= \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))^2} + \frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))}$$

Mathematica [A] time = 0.055126, size = 44, normalized size = 0.63

$$\frac{15\sinh(c+dx) + 6\sinh(2(c+dx)) + \sinh(3(c+dx))}{30d(\cosh(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-3),x]
```

```
[Out] (15*Sinh[c + d*x] + 6*Sinh[2*(c + d*x)] + Sinh[3*(c + d*x)])/(30*d*(1 + Cos
h[c + d*x])^3)
```

Maple [A] time = 0.009, size = 43, normalized size = 0.6

$$\frac{1}{d}\left(\frac{1}{20}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{1}{6}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{1}{4}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/20*tanh(1/2*d*x+1/2*c)^5-1/6*tanh(1/2*d*x+1/2*c)^3+1/4*tanh(1/2*d*x+
1/2*c))
```

Maxima [B] time = 1.05483, size = 277, normalized size = 3.96

$$\frac{4\,e^{(-dx-c)}}{3\,d\left(5\,e^{(-dx-c)} + 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)} + 5\,e^{(-4\,dx-4\,c)} + e^{(-5\,dx-5\,c)} + 1\right)} + \frac{8\,e^{(-2\,dx-2\,c)}}{3\,d\left(5\,e^{(-dx-c)} + 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) + 1)) + 8/3*e^(-2*d*x - 2*c)/(
d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*d*x
 - 4*c) + e^(-5*d*x - 5*c) + 1)) + 4/15/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x -
 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) + 1))
```

Fricas [B] time = 1.79728, size = 486, normalized size = 6.94

$$-\frac{}{15\left(d\cosh(dx+c)^4 + d\sinh(dx+c)^4 + 5\,d\cosh(dx+c)^3 + (4\,d\cosh(dx+c) + 5\,d)\sinh(dx+c)^3 + 10\,d\cosh(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) + 5)/(d*cosh(d*x + c)^4 + d*sinh(
d*x + c)^4 + 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^
3 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 + 15*d*cosh(d*x + c) + 10*d
)*sinh(d*x + c)^2 + 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 + 15*d*cosh(d
*x + c)^2 + 20*d*cosh(d*x + c) + 9*d)*sinh(d*x + c) + 5*d)
```

Sympy [A] time = 3.49056, size = 51, normalized size = 0.73

$$\begin{cases} \frac{\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20d} - \frac{\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d} + \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d} & \text{for}\: d \neq 0 \\ \frac{x}{(\cosh(c)+1)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((tanh(c/2 + d*x/2)**5/(20*d) - tanh(c/2 + d*x/2)**3/(6*d) + tanh(
c/2 + d*x/2)/(4*d), Ne(d, 0)), (x/(cosh(c) + 1)**3, True))
```

Giac [A] time = 1.14721, size = 49, normalized size = 0.7

$$-\frac{4\left(10\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}+1\right)}{15\,d\left(e^{(dx+c)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -4/15*(10*e^(2*d*x + 2*c) + 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^5)
```

3.35 $\int \frac{1}{(1+\cosh(c+dx))^4} \, dx$

Optimal. Leaf size=93

$$\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)}+\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)^2}+\frac{3\sinh(c+dx)}{35d(\cosh(c+dx)+1)^3}+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

```
[Out] Sinh[c + d*x]/(7*d*(1 + Cosh[c + d*x])^4) + (3*Sinh[c + d*x])/(35*d*(1 + Co
sh[c + d*x])^3) + (2*Sinh[c + d*x])/(35*d*(1 + Cosh[c + d*x])^2) + (2*Sinh[
c + d*x])/(35*d*(1 + Cosh[c + d*x]))
```

Rubi [A] time = 0.0535939, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2650, 2648}

$$\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)}+\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)^2}+\frac{3\sinh(c+dx)}{35d(\cosh(c+dx)+1)^3}+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-4),x]
```

```
[Out] Sinh[c + d*x]/(7*d*(1 + Cosh[c + d*x])^4) + (3*Sinh[c + d*x])/(35*d*(1 + Co
sh[c + d*x])^3) + (2*Sinh[c + d*x])/(35*d*(1 + Cosh[c + d*x])^2) + (2*Sinh[
c + d*x])/(35*d*(1 + Cosh[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3}{7}\int \frac{1}{(1+\cosh(c+dx))^3}\,dx$$
$$= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{6}{35}\int \frac{1}{(1+\cosh(c+dx))^2}\,dx$$
$$= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))^2} + \frac{2}{35}\int \frac{1}{1+\cos}$$
$$= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))^2} + \frac{2\sinh(c}{35d(1+\cosh}$$

Mathematica [A] time = 0.0770106, size = 54, normalized size = 0.58

$$\frac{56\sinh(c+dx)+28\sinh(2(c+dx))+8\sinh(3(c+dx))+\sinh(4(c+dx))}{140d(\cosh(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-4),x]
```

```
[Out] (56*Sinh[c + d*x] + 28*Sinh[2*(c + d*x)] + 8*Sinh[3*(c + d*x)] + Sinh[4*(c
+ d*x)])/(140*d*(1 + Cosh[c + d*x])^4)
```

Maple [A] time = 0.01, size = 56, normalized size = 0.6

$$\frac{1}{d}\left(-\frac{1}{56}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3}{40}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{8}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{1}{8}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c))^4,x)
```

```
[Out] 1/d*(-1/56*tanh(1/2*d*x+1/2*c)^7+3/40*tanh(1/2*d*x+1/2*c)^5-1/8*tanh(1/2*d*
x+1/2*c)^3+1/8*tanh(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.09567, size = 491, normalized size = 5.28

$$\frac{4e^{(-dx-c)}}{5d\left(7e^{(-dx-c)}+21e^{(-2dx-2c)}+35e^{(-3dx-3c)}+35e^{(-4dx-4c)}+21e^{(-5dx-5c)}+7e^{(-6dx-6c)}+e^{(-7dx-7c)}+1\right)} + \frac{}{5d\left(7e^{(-dx-c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x -
3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(
-7*d*x - 7*c) + 1)) + 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) + 21*e^(-2*d
*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c)
 + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1)) + 4*e^(-3*d*x - 3*c)/(d*(7*e
^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*
c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1)) + 4/
35/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-
4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c)
+ 1))
```

Fricas [B] time = 1.7497, size = 984, normalized size = 10.58

$$-\frac{}{35\left(d\cosh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{6}+7\,d\cosh\left(dx+c\right)^{5}+\left(6\,d\cosh\left(dx+c\right)+7\,d\right)\sinh\left(dx+c\right)^{5}+21\,d\cosh\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) + 2)*sinh(d*x + c) + 35*sin
h(d*x + c)^2 + 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)^6
 + 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) + 7*d)*sinh(d*x + c)^5 + 21*d*c
osh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 + 35*d*cosh(d*x + c) + 21*d)*sinh(d*
x + c)^4 + 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 + 70*d*cosh(d*x + c
)^2 + 84*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)^2 + (
15*d*cosh(d*x + c)^4 + 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2 + 105*d
*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 + 22*d*cosh(d*x + c) + (6*d*cosh(d*x
 + c)^5 + 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 105*d*cosh(d*x + c)
^2 + 70*d*cosh(d*x + c) + 20*d)*sinh(d*x + c) + 7*d)
```

Sympy [A] time = 9.11852, size = 68, normalized size = 0.73

$$\begin{cases} -\frac{\tanh^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56d}+\frac{3\tanh^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40d}-\frac{\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d} & \text{for}\: d\neq 0 \\ \frac{x}{\left(\cosh\left(c\right)+1\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((-tanh(c/2 + d*x/2)**7/(56*d) + 3*tanh(c/2 + d*x/2)**5/(40*d) - t
anh(c/2 + d*x/2)**3/(8*d) + tanh(c/2 + d*x/2)/(8*d), Ne(d, 0)), (x/(cosh(c)
 + 1)**4, True))
```

Giac [A] time = 1.17115, size = 63, normalized size = 0.68

$$-\frac{4\left(35\,e^{(3\,dx+3\,c)}+21\,e^{(2\,dx+2\,c)}+7\,e^{(dx+c)}+1\right)}{35\,d\left(e^{(dx+c)}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) + 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) + 1)/(d*(e^(
d*x + c) + 1)^7)
```

3.36 $\int \frac{1}{1-\cosh(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

```
[Out] -(Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])))
```

Rubi [A] time = 0.0109634, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2648}

$$-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-1),x]
```

```
[Out] -(Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = -\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

Mathematica [A] time = 0.0229011, size = 14, normalized size = 0.61

$$\frac{\coth\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-1),x]
```

```
[Out] Coth[(c + d*x)/2]/d
```

Maple [A] time = 0.01, size = 16, normalized size = 0.7

$$\frac{1}{d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c)),x)
```

```
[Out] 1/d/tanh(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.06642, size = 24, normalized size = 1.04

$$-\frac{2}{d\left(e^{(-dx-c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/(d*(e^(-d*x - c) - 1))
```

Fricas [A] time = 1.82494, size = 58, normalized size = 2.52

$$\frac{2}{d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/(d*cosh(d*x + c) + d*sinh(d*x + c) - d)
```

Sympy [A] time = 0.712213, size = 32, normalized size = 1.39

$$\begin{cases} \tilde{\infty}x & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ \frac{x}{1-\cosh(c)} & \text{for } d = 0 \\ \frac{1}{d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x/
(1 - cosh(c)), Eq(d, 0)), (1/(d*tanh(c/2 + d*x/2)), True))
```

Giac [A] time = 1.18187, size = 20, normalized size = 0.87

$$\frac{2}{d\left(e^{(dx+c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(d*(e^(d*x + c) - 1))
```

3.37 $\int \frac{1}{(1-\cosh(c+dx))^2} dx$

Optimal. Leaf size=51

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}$$

```
[Out] -Sinh[c + d*x]/(3*d*(1 - Cosh[c + d*x])^2) - Sinh[c + d*x]/(3*d*(1 - Cosh[c
 + d*x]))
```

Rubi [A] time = 0.0248098, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-2),x]
```

```
[Out] -Sinh[c + d*x]/(3*d*(1 - Cosh[c + d*x])^2) - Sinh[c + d*x]/(3*d*(1 - Cosh[c
 + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = -\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2} + \frac{1}{3}\int \frac{1}{1-\cosh(c+dx)}\,dx$$
$$= -\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2} - \frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}$$

Mathematica [A] time = 0.027212, size = 31, normalized size = 0.61

$$\frac{\sinh(c+dx)(\cosh(c+dx)-2)}{3d(\cosh(c+dx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-2),x]
```

```
[Out] ((-2 + Cosh[c + d*x])*Sinh[c + d*x])/(3*d*(-1 + Cosh[c + d*x])^2)
```

Maple [A] time = 0.015, size = 32, normalized size = 0.6

$$\frac{1}{d}\left(\frac{1}{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1} - \frac{1}{6}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2/tanh(1/2*d*x+1/2*c)-1/6/tanh(1/2*d*x+1/2*c)^3)
```

Maxima [B] time = 1.09933, size = 122, normalized size = 2.39

$$\frac{2\,e^{(-dx-c)}}{d\left(3\,e^{(-dx-c)} - 3\,e^{(-2\,dx-2\,c)} + e^{(-3\,dx-3\,c)} - 1\right)} - \frac{2}{3\,d\left(3\,e^{(-dx-c)} - 3\,e^{(-2\,dx-2\,c)} + e^{(-3\,dx-3\,c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) -
 1)) - 2/3/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) - 1))
```

Fricas [B] time = 1.85765, size = 319, normalized size = 6.25

$$-\frac{2\,(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)-1)}{3\,\left(d\cosh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{3}-3\,d\cosh\left(dx+c\right)^{2}+3\,(d\cosh\left(dx+c\right)-d)\sinh\left(dx+c\right)^{2}+3\,d\cosh\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) - 1)/(d*cosh(d*x + c)^3 + d*sinh(d*
x + c)^3 - 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) - d)*sinh(d*x + c)^2 +
3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 - 2*d*cosh(d*x + c) + d)*sinh(d*x
+ c) - d)
```

Sympy [A] time = 1.61781, size = 48, normalized size = 0.94

$$\begin{cases} \tilde{\infty}x & \text{for } c=0\wedge d=0 \\ \frac{x}{(1-\cosh(c))^{2}} & \text{for } d=0 \\ \tilde{\infty}x & \text{for } c=-dx \\ \frac{1}{2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{1}{6d\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(c, 0) & Eq(d, 0)), (x/(1 - cosh(c))**2, Eq(d, 0)), (zo
o*x, Eq(c, -d*x)), (1/(2*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 + d*x/2)**3
), True))
```

Giac [A] time = 1.19703, size = 34, normalized size = 0.67

$$-\frac{2\left(3\,e^{(dx+c)}-1\right)}{3\,d\left(e^{(dx+c)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(d*x + c) - 1)/(d*(e^(d*x + c) - 1)^3)
```

3.38 $\int \frac{1}{(1-\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=76

$$-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2}-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

```
[Out] -Sinh[c + d*x]/(5*d*(1 - Cosh[c + d*x])^3) - (2*Sinh[c + d*x])/(15*d*(1 - C
osh[c + d*x])^2) - (2*Sinh[c + d*x])/(15*d*(1 - Cosh[c + d*x]))
```

Rubi [A] time = 0.0403379, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2}-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-3),x]
```

```
[Out] -Sinh[c + d*x]/(5*d*(1 - Cosh[c + d*x])^3) - (2*Sinh[c + d*x])/(15*d*(1 - C
osh[c + d*x])^2) - (2*Sinh[c + d*x])/(15*d*(1 - Cosh[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3} + \frac{2}{5}\int \frac{1}{(1-\cosh(c+dx))^2}\,dx$$
$$= -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2} + \frac{2}{15}\int \frac{1}{1-\cosh(c+dx)}\,dx$$
$$= -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}$$

Mathematica [A] time = 0.0506645, size = 41, normalized size = 0.54

$$\frac{\sinh(c+dx)(-6\cosh(c+dx)+\cosh(2(c+dx))+8)}{15d(\cosh(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-3),x]
```

```
[Out] ((8 - 6*Cosh[c + d*x] + Cosh[2*(c + d*x)])*Sinh[c + d*x])/(15*d*(-1 + Cosh[
c + d*x])^3)
```

Maple [A] time = 0.013, size = 45, normalized size = 0.6

$$\frac{1}{d}\left(\frac{1}{4}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1} - \frac{1}{6}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3} + \frac{1}{20}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/4/tanh(1/2*d*x+1/2*c)-1/6/tanh(1/2*d*x+1/2*c)^3+1/20/tanh(1/2*d*x+1/
2*c)^5)
```

Maxima [B] time = 1.0811, size = 277, normalized size = 3.64

$$\frac{4\,e^{(-dx-c)}}{3\,d\left(5\,e^{(-dx-c)} - 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)} - 5\,e^{(-4\,dx-4\,c)} + e^{(-5\,dx-5\,c)} - 1\right)} - \frac{8\,e^{(-2\,dx-2\,c)}}{3\,d\left(5\,e^{(-dx-c)} - 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) - 1)) - 8/3*e^(-2*d*x - 2*c)/(
d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*d*x
 - 4*c) + e^(-5*d*x - 5*c) - 1)) - 4/15/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x -
 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) - 1))
```

Fricas [B] time = 1.79807, size = 485, normalized size = 6.38

$$\frac{}{15\left(d\cosh\left(dx+c\right)^{4}+d\sinh\left(dx+c\right)^{4}-5\,d\cosh\left(dx+c\right)^{3}+\left(4\,d\cosh\left(dx+c\right)-5\,d\right)\sinh\left(dx+c\right)^{3}+10\,d\cosh\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) - 5)/(d*cosh(d*x + c)^4 + d*sinh(d
*x + c)^4 - 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) - 5*d)*sinh(d*x + c)^3
 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 - 15*d*cosh(d*x + c) + 10*d)
*sinh(d*x + c)^2 - 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 - 15*d*cosh(d*
x + c)^2 + 20*d*cosh(d*x + c) - 9*d)*sinh(d*x + c) + 5*d)
```

Sympy [A] time = 3.94714, size = 65, normalized size = 0.86

$$\begin{cases} \tilde{\infty}x & \text{for } c = 0 \wedge d = 0 \\ \frac{x}{\left(1-\cosh\left(c\right)\right)^{3}} & \text{for } d = 0 \\ \tilde{\infty}x & \text{for } c = -dx \\ \frac{1}{4d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{1}{6d\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{1}{20d\tanh^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((zoo*x, Eq(c, 0) & Eq(d, 0)), (x/(1 - cosh(c))**3, Eq(d, 0)), (zo
o*x, Eq(c, -d*x)), (1/(4*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 + d*x/2)**3
```

```
) + 1/(20*d*tanh(c/2 + d*x/2)**5), True))
```

Giac [A] time = 1.16561, size = 49, normalized size = 0.64

$$\frac{4\left(10\,e^{(2\,dx+2\,c)}-5\,e^{(dx+c)}+1\right)}{15\,d\left(e^{(dx+c)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 4/15*(10*e^(2*d*x + 2*c) - 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) - 1)^5)
```

3.39 $\int \frac{1}{(1-\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=101

$$-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2}-\frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3}-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

```
[Out] -Sinh[c + d*x]/(7*d*(1 - Cosh[c + d*x])^4) - (3*Sinh[c + d*x])/(35*d*(1 - C
osh[c + d*x])^3) - (2*Sinh[c + d*x])/(35*d*(1 - Cosh[c + d*x])^2) - (2*Sinh
[c + d*x])/(35*d*(1 - Cosh[c + d*x]))
```

Rubi [A] time = 0.0571527, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2}-\frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3}-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-4),x]
```

```
[Out] -Sinh[c + d*x]/(7*d*(1 - Cosh[c + d*x])^4) - (3*Sinh[c + d*x])/(35*d*(1 - C
osh[c + d*x])^3) - (2*Sinh[c + d*x])/(35*d*(1 - Cosh[c + d*x])^2) - (2*Sinh
[c + d*x])/(35*d*(1 - Cosh[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} + \frac{3}{7}\int \frac{1}{(1-\cosh(c+dx))^3}\,dx$$
$$= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} - \frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3} + \frac{6}{35}\int \frac{1}{(1-\cosh(c+dx))^2}\,dx$$
$$= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} - \frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2} + \frac{2}{35}\int \frac{1}{1-\mathrm{c}}$$
$$= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} - \frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2} - \frac{2\sinh(}{35d(1-\cos}$$

Mathematica [A] time = 0.0711323, size = 51, normalized size = 0.5

$$\frac{\sinh(c+dx)(29\cosh(c+dx)-8\cosh(2(c+dx))+\cosh(3(c+dx))-32)}{70d(\cosh(c+dx)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-4),x]
```

```
[Out] ((-32 + 29*Cosh[c + d*x] - 8*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)])*Sinh[c
+ d*x])/(70*d*(-1 + Cosh[c + d*x])^4)
```

Maple [A] time = 0.014, size = 58, normalized size = 0.6

$$\frac{1}{d}\left(\frac{1}{8}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{1}{8}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}-\frac{1}{56}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-7}+\frac{3}{40}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/8/tanh(1/2*d*x+1/2*c)-1/8/tanh(1/2*d*x+1/2*c)^3-1/56/tanh(1/2*d*x+1/
2*c)^7+3/40/tanh(1/2*d*x+1/2*c)^5)
```

Maxima [B] time = 1.08714, size = 491, normalized size = 4.86

$$\frac{4\,e^{(-dx-c)}}{5\,d\left(7\,e^{(-dx-c)}-21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}-35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7\,c)}-1\right)}-\frac{}{5\,d\left(7\,e^{(-dx-c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x -
3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(
-7*d*x - 7*c) - 1)) - 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) - 21*e^(-2*d
*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c)
 - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1)) + 4*e^(-3*d*x - 3*c)/(d*(7*e
^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*x - 4*
c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1)) - 4/
35/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-
4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c)
- 1))
```

Fricas [B] time = 1.74523, size = 984, normalized size = 9.74

$$-\frac{}{35\left(d\cosh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{6}-7\,d\cosh\left(dx+c\right)^{5}+\left(6\,d\cosh\left(dx+c\right)-7\,d\right)\sinh\left(dx+c\right)^{5}+21\,d\cosh\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) - 2)*sinh(d*x + c) + 35*sin
h(d*x + c)^2 - 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)^6
 - 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) - 7*d)*sinh(d*x + c)^5 + 21*d*c
osh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 - 35*d*cosh(d*x + c) + 21*d)*sinh(d*
x + c)^4 - 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 - 70*d*cosh(d*x + c
)^2 + 84*d*cosh(d*x + c) - 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)^2 + (
15*d*cosh(d*x + c)^4 - 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2 - 105*d
*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 - 22*d*cosh(d*x + c) + (6*d*cosh(d*x
 + c)^5 - 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 - 105*d*cosh(d*x + c)
^2 + 70*d*cosh(d*x + c) - 20*d)*sinh(d*x + c) + 7*d)
```

Sympy [A] time = 9.60131, size = 82, normalized size = 0.81

$$\begin{cases} \tilde{\infty}x & \text{for } c = 0 \wedge d = 0 \\ \frac{x}{(1-\cosh(c))^4} & \text{for } d = 0 \\ \tilde{\infty}x & \text{for } c = -dx \\ \frac{1}{8d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{8d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} + \frac{3}{40d\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{56d\tanh^7\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((zoo*x, Eq(c, 0) & Eq(d, 0)), (x/(1 - cosh(c))**4, Eq(d, 0)), (zo
o*x, Eq(c, -d*x)), (1/(8*d*tanh(c/2 + d*x/2)) - 1/(8*d*tanh(c/2 + d*x/2)**3
) + 3/(40*d*tanh(c/2 + d*x/2)**5) - 1/(56*d*tanh(c/2 + d*x/2)**7), True))
```

Giac [A] time = 1.13852, size = 63, normalized size = 0.62

$$-\frac{4\left(35\,e^{(3\,dx+3\,c)} - 21\,e^{(2\,dx+2\,c)} + 7\,e^{(dx+c)} - 1\right)}{35\,d\left(e^{(dx+c)} - 1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) - 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) - 1)/(d*(e^(
d*x + c) - 1)^7)
```

3.40 $\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

Optimal. Leaf size=51

$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/Sqrt[a]
) + (2*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Rubi [A] time = 0.0452953, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2751, 2649, 206}

$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/Sqrt[a]
) + (2*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx &= \frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}} - \int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx\\ &= \frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}} - 2i\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right)\\ &= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}} + \frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.0200889, size = 34, normalized size = 0.67

$$-\frac{2\cosh\left(\frac{x}{2}\right)\left(\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)-2\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(\cosh(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] (-2*Cosh[x/2]*(ArcTan[Sinh[x/2]] - 2*Sinh[x/2]))/Sqrt[a*(1 + Cosh[x])]
```

Maple [B] time = 0.085, size = 92, normalized size = 1.8

$$\frac{\sqrt{2}}{a}\cosh\left(\frac{x}{2}\right)\sqrt{\left(\sinh\left(\frac{x}{2}\right)\right)^2 a}\left(\ln\left(2\,\frac{\sqrt{(\sinh(x/2))^2 a}\sqrt{-a}-a}{\cosh(x/2)}\right)a+2\sqrt{(\sinh(x/2))^2 a}\sqrt{-a}\right)\frac{1}{\sqrt{-a}}\left(\sinh\left(\frac{x}{2}\right)\right)^{-1}\frac{1}{\sqrt{a\left(\cos\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*cosh(x))^(1/2),x)
```

```
[Out] cosh(1/2*x)*(sinh(1/2*x)^2*a)^(1/2)*(ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1
/2)*(-a)^(1/2)-a))*a+2*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/a/(-a)^(1/2)/sin
```

```
h(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [B] time = 1.8469, size = 154, normalized size = 3.02

$$-\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)+\frac{1}{3}\sqrt{2}\left(\frac{3\arctan\left(e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{2e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}e^{(-x)}+\sqrt{a}}\right)+\frac{3\sqrt{2}\sqrt{a}e^{\left(\frac{3}{2}x\right)}-\sqrt{2}\sqrt{a}e^{\left(-\right.}}{3\left(ae^{x}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) +
1/3*sqrt(2)*(3*arctan(e^(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/2*
x)/(sqrt(a)*e^(-x) + sqrt(a))) + 1/3*(3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt(2)
*sqrt(a)*e^(-1/2*x))/(a*e^x + a)
```

Fricas [A] time = 1.78399, size = 228, normalized size = 4.47

$$\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)-\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\left(\cosh(x)+\sinh(x)\right)}{\sqrt{a}}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) - 1) - sqrt(2)*
sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + sin
h(x))/sqrt(a)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(x\right)}{\sqrt{a\left(\cosh\left(x\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(a*(cosh(x) + 1)), x)
```

Giac [C] time = 1.2217, size = 76, normalized size = 1.49

$$-\frac{1}{4}\sqrt{2}\left(\frac{8\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}} - \frac{4e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}} + \frac{4e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}} - \frac{-8i\sqrt{-a}\arctan\left(-i\right) + 8\sqrt{-a}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(8*arctan(e^(1/2*x))/sqrt(a) - 4*e^(1/2*x)/sqrt(a) + 4*e^(-1/2
*x)/sqrt(a) - (-8*I*sqrt(-a)*arctan(-I) + 8*sqrt(-a))/a)
```

3.41 $\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

Optimal. Leaf size=53

$$\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/Sqrt[a]
) + (2*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Rubi [A] time = 0.0493212, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2751, 2649, 206}

$$\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/Sqrt[a]
) + (2*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx &= \frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}+\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx\\ &= \frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}+2i\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right)\\ &= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.0281173, size = 35, normalized size = 0.66

$$\frac{2\sinh\left(\frac{x}{2}\right)\left(2\cosh\left(\frac{x}{2}\right)+\log\left(\tanh\left(\frac{x}{4}\right)\right)\right)}{\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] (2*(2*Cosh[x/2] + Log[Tanh[x/4]])*Sinh[x/2])/Sqrt[a - a*Cosh[x]]
```

Maple [A] time = 0.053, size = 40, normalized size = 0.8

$$\sinh\left(\frac{x}{2}\right)\left(4\cosh(x/2)+\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)-\ln\left(1+\cosh\left(\frac{x}{2}\right)\right)\right)\frac{1}{\sqrt{-2\left(\sinh(x/2)\right)^2 a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a-a*cosh(x))^(1/2),x)
```

```
[Out] sinh(1/2*x)*(4*cosh(1/2*x)+ln(-1+cosh(1/2*x))-ln(1+cosh(1/2*x)))/(-2*sinh(1
/2*x)^2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\sqrt{-a\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/sqrt(-a*cosh(x) + a), x)
```

Fricas [B] time = 1.90982, size = 308, normalized size = 5.81

$$\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right)-2\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(x)+\sinh(x)+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x)))
*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x) + sinh(x)
 - 1)) - 2*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) + 1))/
a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\sqrt{-a(\cosh(x)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(-a*(cosh(x) - 1)), x)
```

Giac [C] time = 1.18088, size = 134, normalized size = 2.53

$$\frac{1}{4}\sqrt{2}\left(\frac{\left(8i\sqrt{-a}\arctan\left(-i\right)-8\sqrt{-a}\right)\operatorname{sgn}\left(-e^{x}+1\right)}{a}-\frac{8\arctan\left(\frac{\sqrt{-ae^{x}}}{\sqrt{a}}\right)}{\sqrt{a}\operatorname{sgn}\left(-e^{x}+1\right)}-\frac{4}{\sqrt{-ae^{x}}\operatorname{sgn}\left(-e^{x}+1\right)}+\frac{4\sqrt{-ae^{x}}}{a\operatorname{sgn}\left(-e^{x}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((8*I*sqrt(-a)*arctan(-I) - 8*sqrt(-a))*sgn(-e^x + 1)/a - 8*arc
tan(sqrt(-a*e^x)/sqrt(a))/(sqrt(a)*sgn(-e^x + 1)) - 4/(sqrt(-a*e^x)*sgn(-e^
x + 1)) + 4*sqrt(-a*e^x)/(a*sgn(-e^x + 1)))
```

3.42 $\int (a + a\cosh(c + dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{64a^3\sinh(c+dx)}{15d\sqrt{a\cosh(c+dx)+a}}+\frac{16a^2\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{15d}+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (64*a^3*Sinh[c + d*x])/(15*d*Sqrt[a + a*Cosh[c + d*x]]) + (16*a^2*Sqrt[a +
a*Cosh[c + d*x]]*Sinh[c + d*x])/(15*d) + (2*a*(a + a*Cosh[c + d*x])^(3/2)*S
inh[c + d*x])/(5*d)
```

Rubi [A] time = 0.0468435, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{64a^3\sinh(c+dx)}{15d\sqrt{a\cosh(c+dx)+a}}+\frac{16a^2\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{15d}+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sinh[c + d*x])/(15*d*Sqrt[a + a*Cosh[c + d*x]]) + (16*a^2*Sqrt[a +
a*Cosh[c + d*x]]*Sinh[c + d*x])/(15*d) + (2*a*(a + a*Cosh[c + d*x])^(3/2)*S
inh[c + d*x])/(5*d)
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\cosh(c+dx))^{5/2}\,dx = \frac{2a(a+a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d} + \frac{1}{5}(8a)\int (a+a\cosh(c+dx))^{3/2}\,dx$$
$$= \frac{16a^2\sqrt{a+a\cosh(c+dx)}\sinh(c+dx)}{15d} + \frac{2a(a+a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d} + \frac{1}{15}(3$$
$$= \frac{64a^3\sinh(c+dx)}{15d\sqrt{a+a\cosh(c+dx)}} + \frac{16a^2\sqrt{a+a\cosh(c+dx)}\sinh(c+dx)}{15d} + \frac{2a(a+a\cosh(c+d}{5a}$$

Mathematica [A] time = 0.12481, size = 71, normalized size = 0.8

$$\frac{a^2\left(150\sinh\left(\frac{1}{2}(c+dx)\right)+25\sinh\left(\frac{3}{2}(c+dx)\right)+3\sinh\left(\frac{5}{2}(c+dx)\right)\right)\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cosh(c+dx)+1)}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(150*Sinh[(c + d*x)/2] +
 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2]))/(30*d)
```

Maple [A] time = 0.042, size = 73, normalized size = 0.8

$$\frac{8a^3\sqrt{2}}{15d}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\left(\cosh(1/2\,dx+c/2)\right)^4+4\left(\cosh(1/2\,dx+c/2)\right)^2+8\right)\frac{1}{\sqrt{a\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(5/2),x)
```

```
[Out] 8/15*a^3*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(3*cosh(1/2*d*x+1/2*c)^4+4
*cosh(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 1.59718, size = 163, normalized size = 1.83

$$\frac{\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{5}{2}dx+\frac{5}{2}c\right)}}{20d}+\frac{5\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{3}{2}dx+\frac{3}{2}c\right)}}{12d}+\frac{5\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}-\frac{5\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{2d}-\frac{5\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{3}{2}dx-\frac{3}{2}c\right)}}{12d}-\frac{\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{5}{2}dx-\frac{5}{2}c\right)}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/20*sqrt(2)*a^(5/2)*e^(5/2*d*x + 5/2*c)/d + 5/12*sqrt(2)*a^(5/2)*e^(3/2*d*x + 3/2*c)/d + 5/2*sqrt(2)*a^(5/2)*e^(1/2*d*x + 1/2*c)/d - 5/2*sqrt(2)*a^(5/2)*e^(-1/2*d*x - 1/2*c)/d - 5/12*sqrt(2)*a^(5/2)*e^(-3/2*d*x - 3/2*c)/d - 1/20*sqrt(2)*a^(5/2)*e^(-5/2*d*x - 5/2*c)/d

---

Fricas [B] time = 1.8515, size = 856, normalized size = 9.62

$$\sqrt{\tfrac{1}{2}}\left(3\,a^2\cosh\left(dx+c\right)^5+3\,a^2\sinh\left(dx+c\right)^5+25\,a^2\cosh\left(dx+c\right)^4+150\,a^2\cosh\left(dx+c\right)^3+5\left(3\,a^2\cosh\left(dx+c\right)+5\,a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 + 25*a^2*cosh(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) + 5*a^2)*sinh(d*x + c)^4 - 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 + 10*a^2*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a^2*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c)^2 - 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 + 20*a^2*cosh(d*x + c)^3 + 90*a^2*cosh(d*x + c)^2 - 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cosh(d*x+c))**(5/2),x)

[Out] Timed out

Giac [A] time = 1.19484, size = 142, normalized size = 1.6

$$-\frac{\sqrt{2}\left(\left(150\,a^{\frac{5}{2}}e^{\left(2\,dx+\frac{5}{2}\,c\right)}+25\,a^{\frac{5}{2}}e^{\left(dx+\frac{3}{2}\,c\right)}+3\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,c\right)}\right)e^{\left(-\frac{5}{2}\,dx-3\,c\right)}-\left(3\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}\,dx+\frac{35}{2}\,c\right)}+25\,a^{\frac{5}{2}}e^{\left(\frac{3}{2}\,dx+\frac{33}{2}\,c\right)}+150\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,dx+\frac{31}{2}\,c\right)}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*((150*a^(5/2)*e^(2*d*x + 5/2*c) + 25*a^(5/2)*e^(d*x + 3/2*c)
+ 3*a^(5/2)*e^(1/2*c))*e^(-5/2*d*x - 3*c) - (3*a^(5/2)*e^(5/2*d*x + 35/2*c)
 + 25*a^(5/2)*e^(3/2*d*x + 33/2*c) + 150*a^(5/2)*e^(1/2*d*x + 31/2*c))*e^(-
15*c))/d
```

3.43 $\int (a + a\cosh(c + dx))^{3/2} \, dx$

Optimal. Leaf size=59

$$\frac{8a^2 \sinh(c + dx)}{3d\sqrt{a\cosh(c + dx) + a}} + \frac{2a\sinh(c + dx)\sqrt{a\cosh(c + dx) + a}}{3d}$$

```
[Out] (8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a + a*Cosh[c + d*x]]) + (2*a*Sqrt[a + a*Cos
h[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Rubi [A] time = 0.0287345, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{8a^2 \sinh(c + dx)}{3d\sqrt{a\cosh(c + dx) + a}} + \frac{2a\sinh(c + dx)\sqrt{a\cosh(c + dx) + a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a + a*Cosh[c + d*x]]) + (2*a*Sqrt[a + a*Cos
h[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\cosh(c+dx))^{3/2}\,dx = \frac{2a\sqrt{a+a\cosh(c+dx)}\sinh(c+dx)}{3d} + \frac{1}{3}(4a)\int \sqrt{a+a\cosh(c+dx)}\,dx$$
$$= \frac{8a^2\sinh(c+dx)}{3d\sqrt{a+a\cosh(c+dx)}} + \frac{2a\sqrt{a+a\cosh(c+dx)}\sinh(c+dx)}{3d}$$

Mathematica [A] time = 0.0633739, size = 55, normalized size = 0.93

$$\frac{a\left(9\sinh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{3}{2}(c+dx)\right)\right)\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cosh(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(9*Sinh[(c + d*x)/2] + Sin
h[(3*(c + d*x))/2]))/(3*d)
```

Maple [A] time = 0.03, size = 58, normalized size = 1.

$$\frac{4\,a^2\sqrt{2}}{3\,d}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\right)\frac{1}{\sqrt{a\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(3/2),x)
```

```
[Out] 4/3*a^2*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2+2)*2
^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A] time = 1.55873, size = 109, normalized size = 1.85

$$\frac{\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}}{6\,d}+\frac{3\,\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}-\frac{3\,\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{2\,d}-\frac{\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,dx-\frac{3}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*a^(3/2)*e^(3/2*d*x + 3/2*c)/d + 3/2*sqrt(2)*a^(3/2)*e^(1/2*d*x
+ 1/2*c)/d - 3/2*sqrt(2)*a^(3/2)*e^(-1/2*d*x - 1/2*c)/d - 1/6*sqrt(2)*a^(3/
2)*e^(-3/2*d*x - 3/2*c)/d
```

Fricas [B] time = 1.85382, size = 385, normalized size = 6.53

$$\frac{\sqrt{\frac{1}{2}}\left(a\cosh\left(dx+c\right)^{3}+a\sinh\left(dx+c\right)^{3}+9\,a\cosh\left(dx+c\right)^{2}+3\left(a\cosh\left(dx+c\right)+3\,a\right)\sinh\left(dx+c\right)^{2}-9\,a\cosh\left(dx+c\right)\right.}{3\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 + 9*a*cosh(d*x + c)^2
+ 3*(a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*cosh
(d*x + c)^2 + 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) - a)*sqrt(a/(cosh(d*x
+ c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cosh\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cosh(c + d*x) + a)**(3/2), x)
```

Giac [A] time = 1.11523, size = 101, normalized size = 1.71

$$-\frac{\sqrt{2}\left(\left(9\,a^{\frac{3}{2}}e^{\left(dx+\frac{3}{2}\,c\right)}+a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,c\right)}\right)e^{\left(-\frac{3}{2}\,dx-2\,c\right)}-\left(a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,dx+\frac{15}{2}\,c\right)}+9\,a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,dx+\frac{13}{2}\,c\right)}\right)e^{\left(-6\,c\right)}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((9*a^(3/2)*e^(d*x + 3/2*c) + a^(3/2)*e^(1/2*c))*e^(-3/2*d*x -
 2*c) - (a^(3/2)*e^(3/2*d*x + 15/2*c) + 9*a^(3/2)*e^(1/2*d*x + 13/2*c))*e^(
-6*c))/d
```

3.44 $\int \sqrt{a+a\cosh(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sinh(c+dx)}{d\sqrt{a\cosh(c+dx)+a}}$$

```
[Out] (2*a*Sinh[c + d*x])/(d*Sqrt[a + a*Cosh[c + d*x]])
```

Rubi [A] time = 0.013316, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2646}

$$\frac{2a\sinh(c+dx)}{d\sqrt{a\cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*a*Sinh[c + d*x])/(d*Sqrt[a + a*Cosh[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cosh(c+dx)}\,dx=\frac{2a\sinh(c+dx)}{d\sqrt{a+a\cosh(c+dx)}}$$

Mathematica [A] time = 0.0302019, size = 29, normalized size = 1.12

$$\frac{2\tanh\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cosh(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[c + d*x])]*Tanh[(c + d*x)/2])/d
```

Maple [A] time = 0.026, size = 43, normalized size = 1.7

$$2\,{\frac{a\cosh \left(1/2\,dx+c/2 \right) \sinh \left(1/2\,dx+c/2 \right) \sqrt{2}}{\sqrt{a \left(\cosh \left(1/2\,dx+c/2 \right) \right) ^{2}}d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] 2*a*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

Maxima [A] time = 1.78412, size = 54, normalized size = 2.08

$$\frac{\sqrt{2}\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}-\frac{\sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*sqrt(a)*e^(1/2*d*x + 1/2*c)/d - sqrt(2)*sqrt(a)*e^(-1/2*d*x - 1/2*c
)/d
```

Fricas [A] time = 1.85192, size = 123, normalized size = 4.73

$$\frac{2\,\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d
*x + c) - 1)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cosh\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cosh(c + d*x) + a), x)
```

Giac [A] time = 1.15514, size = 47, normalized size = 1.81

$$\frac{\sqrt{2}\left(\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(a)*e^(1/2*d*x + 1/2*c) - sqrt(a)*e^(-1/2*d*x - 1/2*c))/d
```

3.45 $\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])
])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0225107, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2649, 206}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])
])/(Sqrt[a]*d)
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{(2i)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(c+dx)}{\sqrt{a+a\cosh(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.0154185, size = 40, normalized size = 0.87

$$\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)\tan^{-1}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cosh(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*ArcTan[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2])/(d*Sqrt[a*(1 + Cosh[c + d*x
])])
```

Maple [B] time = 0.036, size = 103, normalized size = 2.2

$$-\frac{\sqrt{2}}{d}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a}\ln\left(2\,\frac{\sqrt{(\sinh(1/2\,dx+c/2))^2 a}\sqrt{-a}-a}{\cosh(1/2\,dx+c/2)}\right)\frac{1}{\sqrt{-a}}\left(\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}\frac{1}{\sqrt{a\left(\cosh\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] -cosh(1/2*d*x+1/2*c)*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/(-a)^(1/2)*ln(2/cosh(1
/2*d*x+1/2*c)*((sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*(-a)^(1/2)-a))/sinh(1/2*d*x+
1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 1.83332, size = 116, normalized size = 2.52

$$2\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{\sqrt{a}d}+\frac{e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\left(\sqrt{a}e^{(dx+c)}+\sqrt{a}\right)d}\right)-\frac{2\sqrt{2}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\sqrt{a}de^{(dx+c)}+\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*(arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d) + e^(1/2*d*x + 1/2*c)/((
sqrt(a)*e^(d*x + c) + sqrt(a))*d)) - 2*sqrt(2)*e^(1/2*d*x + 1/2*c)/(sqrt(a)
*d*e^(d*x + c) + sqrt(a)*d)
```

Fricas [A] time = 1.93087, size = 456, normalized size = 9.91

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\sqrt{-\frac{1}{a}}(\cosh(dx+c)+\sinh(dx+c))+\cosh(dx+c)+\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)+1}\right)}{d}, \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\cdots}}\cdots}{\cdots}\right)}{\sqrt{a}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(
d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) + cosh(d*x + c) + sin
h(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) + 1))/d, 2*sqrt(2)*arctan(sq
rt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + si
nh(d*x + c))/sqrt(a))/(sqrt(a)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(c + d*x) + a), x)
```

Giac [A] time = 1.19836, size = 28, normalized size = 0.61

$$\frac{2\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d)
```

3.46 $\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])]/(2*Sqrt
[2]*a^(3/2)*d) + Sinh[c + d*x]/(2*d*(a + a*Cosh[c + d*x])^(3/2))
```

Rubi [A] time = 0.0414038, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])]/(2*Sqrt
[2]*a^(3/2)*d) + Sinh[c + d*x]/(2*d*(a + a*Cosh[c + d*x])^(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx &= \frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx}{4a}\\ &= \frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}}+\frac{i\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{ia\sinh(c+dx)}{\sqrt{a+a\cosh(c+dx)}}\right)}{2ad}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0813707, size = 63, normalized size = 0.82

$$\frac{\cosh^2\left(\frac{1}{2}(c+dx)\right)\left(\tanh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\tan^{-1}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d(a(\cosh(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] (Cosh[(c + d*x)/2]^2*(ArcTan[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2] + Tanh[(c
 + d*x)/2]))/(d*(a*(1 + Cosh[c + d*x]))^(3/2))
```

Maple [B] time = 0.045, size = 144, normalized size = 1.9

$$-\frac{\sqrt{2}}{4\,a^2d}\sqrt{\left(\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a}\left(\ln\left(2\,\frac{\sqrt{(\sinh(1/2\,dx+c/2))^2\,a}\sqrt{-a}-a}{\cosh(1/2\,dx+c/2)}\right)a\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\sqrt{\left(\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a\sqrt{-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(d*x+c))^(3/2),x)
```

```
[Out] -1/4*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*(ln(2/cosh(1/2*d*x+1/2*c)*((sinh(1/2*d
*x+1/2*c)^2*a)^(1/2)*(-a)^(1/2)-a))*a*cosh(1/2*d*x+1/2*c)^2-(sinh(1/2*d*x+1
/2*c)^2*a)^(1/2)*(-a)^(1/2))/a^2/cosh(1/2*d*x+1/2*c)/(-a)^(1/2)/sinh(1/2*d*
x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 1.99841, size = 230, normalized size = 2.99

$$\frac{1}{6}\sqrt{2}\left(\frac{3e^{\left(\frac{5}{2}dx+\frac{5}{2}c\right)}+8e^{\left(\frac{3}{2}dx+\frac{3}{2}c\right)}-3e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\left(a^{\frac{3}{2}}e^{(3dx+3c)}+3a^{\frac{3}{2}}e^{(2dx+2c)}+3a^{\frac{3}{2}}e^{(dx+c)}+a^{\frac{3}{2}}\right)d}+\frac{3\arctan\left(e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{a^{\frac{3}{2}}d}\right)-\frac{4\sqrt{2}e^{\left(\frac{3}{2}dx+\frac{3}{2}c\right)}}{3\left(a^{\frac{3}{2}}de^{(3dx+3c)}+3a^{\frac{3}{2}}de^{(2dx+2c)}+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*((3*e^(5/2*d*x + 5/2*c) + 8*e^(3/2*d*x + 3/2*c) - 3*e^(1/2*d*x
+ 1/2*c))/((a^(3/2)*e^(3*d*x + 3*c) + 3*a^(3/2)*e^(2*d*x + 2*c) + 3*a^(3/2)
*e^(d*x + c) + a^(3/2))*d) + 3*arctan(e^(1/2*d*x + 1/2*c))/(a^(3/2)*d)) - 4
/3*sqrt(2)*e^(3/2*d*x + 3/2*c)/(a^(3/2)*d*e^(3*d*x + 3*c) + 3*a^(3/2)*d*e^(
2*d*x + 2*c) + 3*a^(3/2)*d*e^(d*x + c) + a^(3/2)*d)
```

Fricas [B] time = 1.90273, size = 632, normalized size = 8.21

$$-\frac{\sqrt{2}\left(\cosh(dx+c)^2+2(\cosh(dx+c)+1)\sinh(dx+c)+\sinh(dx+c)^2+2\cosh(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{c}\right.}{2\left(a^2d\cosh(dx+c)^2+a^2d\sinh(dx+c)^2+2a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh
(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a)
*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/a) - 2*sqrt(1/2)*(cosh(d*x + c)^2
+ (2*cosh(d*x + c) - 1)*sinh(d*x + c) + sinh(d*x + c)^2 - cosh(d*x + c))*sq
rt(a/(cosh(d*x + c) + sinh(d*x + c))))/(a^2*d*cosh(d*x + c)^2 + a^2*d*sinh(
d*x + c)^2 + 2*a^2*d*cosh(d*x + c) + a^2*d + 2*(a^2*d*cosh(d*x + c) + a^2*d
```

```
)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cosh(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cosh(c + d*x) + a)**(-3/2), x)
```

Giac [A] time = 1.25308, size = 99, normalized size = 1.29

$$\frac{\frac{\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{\sqrt{a}d}+\frac{\sqrt{2}\left(a^{\frac{3}{2}}e^{\left(\frac{3}{2}dx+\frac{3}{2}c\right)}-a^{\frac{3}{2}}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{\left(ae^{(dx+c)}+a\right)^{2}d}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d) + sqrt(2)*(a^(3/2)*e^(
3/2*d*x + 3/2*c) - a^(3/2)*e^(1/2*d*x + 1/2*c))/((a*e^(d*x + c) + a)^2*d))/
a
```

3.47 $\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sinh(c+dx)}{16ad(a\cosh(c+dx)+a)^{3/2}}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$

```
[Out] (3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Sinh[c + d*x]/(4*d*(a + a*Cosh[c + d*x])^(5/2)) + (3*
Sinh[c + d*x])/(16*a*d*(a + a*Cosh[c + d*x])^(3/2))
```

Rubi [A] time = 0.0624466, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sinh(c+dx)}{16ad(a\cosh(c+dx)+a)^{3/2}}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Sinh[c + d*x]/(4*d*(a + a*Cosh[c + d*x])^(5/2)) + (3*
Sinh[c + d*x])/(16*a*d*(a + a*Cosh[c + d*x])^(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx}{32a^2}$$

$$= \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{i\cdots}{\sqrt{a\cdots}}\right.}{16a^2d}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}}$$

Mathematica [A] time = 0.263268, size = 91, normalized size = 0.85

$$\frac{\cosh^5\left(\frac{1}{2}(c+dx)\right)\left(32\sinh^5\left(\frac{1}{2}(c+dx)\right)\mathrm{csch}^4(c+dx)+3\left(\tan^{-1}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+\tanh\left(\frac{1}{2}(c+dx)\right)\mathrm{sech}\left(\frac{1}{2}(c+dx\right.\right.\right.}{4d(a(\cosh(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] (Cosh[(c + d*x)/2]^5*(32*Csch[c + d*x]^4*Sinh[(c + d*x)/2]^5 + 3*(ArcTan[Si
nh[(c + d*x)/2]] + Sech[(c + d*x)/2]*Tanh[(c + d*x)/2])))/(4*d*(a*(1 + Cosh
[c + d*x]))^(5/2))
```

Maple [B] time = 0.046, size = 178, normalized size = 1.7

$$-\frac{\sqrt{2}}{32a^3d}\sqrt{\left(\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a}\left(3\ln\left(2\frac{\sqrt{(\sinh(1/2\,dx+c/2))^2a}\sqrt{-a}-a}{\cosh(1/2\,dx+c/2)}\right)a(\cosh(1/2\,dx+c/2))^4-3\sqrt{(\sinh(1/2\,dx+c\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(d*x+c))^(5/2),x)
```

```
[Out] -1/32*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*(3*ln(2/cosh(1/2*d*x+1/2*c)*((sinh(1/
2*d*x+1/2*c)^2*a)^(1/2)*(-a)^(1/2)-a))*a*cosh(1/2*d*x+1/2*c)^4-3*(sinh(1/2*
d*x+1/2*c)^2*a)^(1/2)*cosh(1/2*d*x+1/2*c)^2*(-a)^(1/2)-2*(sinh(1/2*d*x+1/2*
c)^2*a)^(1/2)*(-a)^(1/2))/a^3/cosh(1/2*d*x+1/2*c)^3/(-a)^(1/2)/sinh(1/2*d*x
+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] time = 1.89003, size = 338, normalized size = 3.16

$$\frac{1}{80}\sqrt{2}\left(\frac{15\,e^{\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}+70\,e^{\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}+128\,e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}-70\,e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-15\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(a^{\frac{5}{2}}e^{(5\,dx+5\,c)}+5\,a^{\frac{5}{2}}e^{(4\,dx+4\,c)}+10\,a^{\frac{5}{2}}e^{(3\,dx+3\,c)}+10\,a^{\frac{5}{2}}e^{(2\,dx+2\,c)}+5\,a^{\frac{5}{2}}e^{(dx+c)}+a^{\frac{5}{2}}\right)d}+\frac{15\,\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{a^{\frac{5}{2}}d}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/80*sqrt(2)*((15*e^(9/2*d*x + 9/2*c) + 70*e^(7/2*d*x + 7/2*c) + 128*e^(5/2
*d*x + 5/2*c) - 70*e^(3/2*d*x + 3/2*c) - 15*e^(1/2*d*x + 1/2*c))/((a^(5/2)*
e^(5*d*x + 5*c) + 5*a^(5/2)*e^(4*d*x + 4*c) + 10*a^(5/2)*e^(3*d*x + 3*c) +
10*a^(5/2)*e^(2*d*x + 2*c) + 5*a^(5/2)*e^(d*x + c) + a^(5/2))*d) + 15*arcta
n(e^(1/2*d*x + 1/2*c))/(a^(5/2)*d)) - 8/5*sqrt(2)*e^(5/2*d*x + 5/2*c)/(a^(5
/2)*d*e^(5*d*x + 5*c) + 5*a^(5/2)*d*e^(4*d*x + 4*c) + 10*a^(5/2)*d*e^(3*d*x
 + 3*c) + 10*a^(5/2)*d*e^(2*d*x + 2*c) + 5*a^(5/2)*d*e^(d*x + c) + a^(5/2)*
d)
```

Fricas [B] time = 1.96441, size = 1454, normalized size = 13.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 +
 sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c)
```

```
 + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x
 + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)*sqrt(a)
*arctan(sqrt(2)*sqrt(1/2)*sqrt(a)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/a
) - 2*sqrt(1/2)*(3*cosh(d*x + c)^4 + (12*cosh(d*x + c) + 11)*sinh(d*x + c)^
3 + 3*sinh(d*x + c)^4 + 11*cosh(d*x + c)^3 + (18*cosh(d*x + c)^2 + 33*cosh(
d*x + c) - 11)*sinh(d*x + c)^2 - 11*cosh(d*x + c)^2 + (12*cosh(d*x + c)^3 +
 33*cosh(d*x + c)^2 - 22*cosh(d*x + c) - 3)*sinh(d*x + c) - 3*cosh(d*x + c)
)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c))))/(a^3*d*cosh(d*x + c)^4 + a^3*d*s
inh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3 + 6*a^3*d*cosh(d*x + c)^2 + 4*a^3*
d*cosh(d*x + c) + a^3*d + 4*(a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^3 +
 6*(a^3*d*cosh(d*x + c)^2 + 2*a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^2
+ 4*(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c)^2 + 3*a^3*d*cosh(d*x + c
) + a^3*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37774, size = 138, normalized size = 1.29

$$\frac{3\,\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{16\,a^{\frac{5}{2}}d}+\frac{\sqrt{2}\left(3\,a^{\frac{7}{2}}e^{\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}+11\,a^{\frac{7}{2}}e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}-11\,a^{\frac{7}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-3\,a^{\frac{7}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{16\left(ae^{(dx+c)}+a\right)^{4}a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*arctan(e^(1/2*d*x + 1/2*c))/(a^(5/2)*d) + 1/16*sqrt(2)*(3*a^(7
/2)*e^(7/2*d*x + 7/2*c) + 11*a^(7/2)*e^(5/2*d*x + 5/2*c) - 11*a^(7/2)*e^(3/
2*d*x + 3/2*c) - 3*a^(7/2)*e^(1/2*d*x + 1/2*c))/((a*e^(d*x + c) + a)^4*a^2*
d)
```

3.48 $\int (a - a\cosh(c + dx))^{5/2}\,dx$

Optimal. Leaf size=92

$$-\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}}-\frac{16a^2\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{15d}-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

```
[Out] (-64*a^3*Sinh[c + d*x])/(15*d*Sqrt[a - a*Cosh[c + d*x]]) - (16*a^2*Sqrt[a -
 a*Cosh[c + d*x]]*Sinh[c + d*x])/(15*d) - (2*a*(a - a*Cosh[c + d*x])^(3/2)*
Sinh[c + d*x])/(5*d)
```

Rubi [A] time = 0.0520059, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2647, 2646}

$$-\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}}-\frac{16a^2\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{15d}-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(5/2),x]
```


```
[Out] (-64*a^3*Sinh[c + d*x])/(15*d*Sqrt[a - a*Cosh[c + d*x]]) - (16*a^2*Sqrt[a -
 a*Cosh[c + d*x]]*Sinh[c + d*x])/(15*d) - (2*a*(a - a*Cosh[c + d*x])^(3/2)*
Sinh[c + d*x])/(5*d)
```


Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a-a\cosh(c+dx))^{5/2}\,dx = -\frac{2a(a-a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d} + \frac{1}{5}(8a)\int (a-a\cosh(c+dx))^{3/2}\,dx$$
$$= -\frac{16a^2\sqrt{a-a\cosh(c+dx)}\sinh(c+dx)}{15d} - \frac{2a(a-a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d} + \frac{1}{15}\big(32$$
$$= -\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}} - \frac{16a^2\sqrt{a-a\cosh(c+dx)}\sinh(c+dx)}{15d} - \frac{2a(a-a\cosh(c+dx}{5d}$$

Mathematica [A] time = 0.132919, size = 72, normalized size = 0.78

$$\frac{a^2\left(150\cosh\left(\frac{1}{2}(c+dx)\right)-25\cosh\left(\frac{3}{2}(c+dx)\right)+3\cosh\left(\frac{5}{2}(c+dx)\right)\right)\text{csch}\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cosh(c+dx)}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a - a*Cosh[c + d*x]]*(150*Cosh[(c + d*x)/2] - 25*Cosh[(3*(c + d*x
))/2] + 3*Cosh[(5*(c + d*x))/2])*Csch[(c + d*x)/2])/(30*d)
```

Maple [A] time = 0.042, size = 71, normalized size = 0.8

$$-\frac{16\,a^3}{15\,d}\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\,(\sinh(1/2\,dx+c/2))^4-4\,(\sinh(1/2\,dx+c/2))^2+8\right)\frac{1}{\sqrt{-2\,(\sinh(1/2\,dx+c/2))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(d*x+c))^(5/2),x)
```

```
[Out] -16/15*sinh(1/2*d*x+1/2*c)*a^3*cosh(1/2*d*x+1/2*c)*(3*sinh(1/2*d*x+1/2*c)^4
-4*sinh(1/2*d*x+1/2*c)^2+8)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 1.61936, size = 257, normalized size = 2.79

$$\frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-dx-c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-2\,dx-2\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-3\,dx-3\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} + \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-4\,dx-4\,c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{\sqrt{2}a^{\frac{5}{2}}e^{(-5\,dx-5\,c)}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{\sqrt{2}a^{\frac{5}{2}}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 5/12*sqrt(2)*a^(5/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*a
^(5/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*a^(5/2)*e^(
-3*d*x - 3*c)/(d*(-e^(-d*x - c))^(5/2)) + 5/12*sqrt(2)*a^(5/2)*e^(-4*d*x -
4*c)/(d*(-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)*e^(-5*d*x - 5*c)/(d*(
-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)/(d*(-e^(-d*x - c))^(5/2))
```

Fricas [B] time = 1.8034, size = 857, normalized size = 9.32

$$\frac{\sqrt{\frac{1}{2}}\left(3\,a^2\cosh\left(dx+c\right)^5+3\,a^2\sinh\left(dx+c\right)^5-25\,a^2\cosh\left(dx+c\right)^4+150\,a^2\cosh\left(dx+c\right)^3+5\left(3\,a^2\cosh\left(dx+c\right)-5\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 - 25*a^2*cosh
(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) - 5*a^2)*sin
h(d*x + c)^4 + 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 - 10*a^2
*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a^2*c
osh(d*x + c)^3 - 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) + 5*a^2)*sinh
(d*x + c)^2 + 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(d*x + c)^3 + 9
0*a^2*cosh(d*x + c)^2 + 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))*sqrt(-
a/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*s
inh(d*x + c) + d*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18001, size = 262, normalized size = 2.85

$$-\frac{\sqrt{2}\left(3\sqrt{-ae^{(dx+c)}}a^2e^{(2\,dx+2\,c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)-25\sqrt{-ae^{(dx+c)}}a^2e^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)+150\sqrt{-ae^{(dx+c)}}a^2\mathrm{sgn}\left(-e^{(dx+c)}\right.\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(3*sqrt(-a*e^(d*x + c))*a^2*e^(2*d*x + 2*c)*sgn(-e^(d*x + c)
+ 1) - 25*sqrt(-a*e^(d*x + c))*a^2*e^(d*x + c)*sgn(-e^(d*x + c) + 1) + 150*
sqrt(-a*e^(d*x + c))*a^2*sgn(-e^(d*x + c) + 1) - (150*a^5*e^(2*d*x + 2*c)*s
gn(-e^(d*x + c) + 1) - 25*a^5*e^(d*x + c)*sgn(-e^(d*x + c) + 1) + 3*a^5*sgn
(-e^(d*x + c) + 1))*e^(-2*d*x - 2*c)/(sqrt(-a*e^(d*x + c))*a^2))/d
```

3.49 $\int (a-a\cosh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=61

$$-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}$$

```
[Out] (-8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a - a*Cosh[c + d*x]]) - (2*a*Sqrt[a - a*Co
sh[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Rubi [A] time = 0.031382, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2647, 2646}

$$-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a - a*Cosh[c + d*x]]) - (2*a*Sqrt[a - a*Co
sh[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a-a\cosh(c+dx))^{3/2}\,dx = -\frac{2a\sqrt{a-a\cosh(c+dx)}\sinh(c+dx)}{3d} + \frac{1}{3}(4a)\int \sqrt{a-a\cosh(c+dx)}\,dx$$
$$= -\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}} - \frac{2a\sqrt{a-a\cosh(c+dx)}\sinh(c+dx)}{3d}$$

Mathematica [A] time = 0.09078, size = 56, normalized size = 0.92

$$-\frac{a\left(\cosh\left(\frac{3}{2}(c+dx)\right)-9\cosh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cosh(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] -(a*Sqrt[a - a*Cosh[c + d*x]]*(-9*Cosh[(c + d*x)/2] + Cosh[(3*(c + d*x))/2]
)*Csch[(c + d*x)/2])/(3*d)
```

Maple [A] time = 0.036, size = 56, normalized size = 0.9

$$\frac{8\,a^2}{3\,d}\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-3\right)\frac{1}{\sqrt{-2\,\left(\sinh\left(1/2\,dx+c/2\right)\right)^2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(d*x+c))^(3/2),x)
```

```
[Out] 8/3*sinh(1/2*d*x+1/2*c)*a^2*cosh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2-3)/(
-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 1.60667, size = 167, normalized size = 2.74

$$\frac{3\sqrt{2}a^{\frac{3}{2}}e^{(-dx-c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}+\frac{3\sqrt{2}a^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}e^{(-3\,dx-3\,c)}}{6\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}}{6\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(2)*a^(3/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(3/2)) + 3/2*sqrt(2)*a^
(3/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)*e^(-
3*d*x - 3*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)/(d*(-e^(-d*x -
 c))^(3/2))
```

Fricas [B] time = 1.8818, size = 387, normalized size = 6.34

$$-\frac{\sqrt{\frac{1}{2}}\left(a\cosh\left(dx+c\right)^{3}+a\sinh\left(dx+c\right)^{3}-9\,a\cosh\left(dx+c\right)^{2}+3\left(a\cosh\left(dx+c\right)-3\,a\right)\sinh\left(dx+c\right)^{2}-9\,a\cosh\left(dx+\right.}{3\left(d\cosh\left(dx+c\right)+d\sinh\left(dx-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 - 9*a*cosh(d*x + c)^2
 + 3*(a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*cos
h(d*x + c)^2 - 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) + a)*sqrt(-a/(cosh(d*
x + c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a\cosh\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((-a*cosh(c + d*x) + a)**(3/2), x)
```

Giac [B] time = 1.18524, size = 171, normalized size = 2.8

$$\frac{\sqrt{2}\left(\sqrt{-ae^{(dx+c)}}ae^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)-9\sqrt{-ae^{(dx+c)}}a\mathrm{sgn}\left(-e^{(dx+c)}+1\right)+\frac{\left(9\,a^{3}e^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)-a^{3}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)\right)e^{(-dx-c)}}{\sqrt{-ae^{(dx+c)}}a}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(sqrt(-a*e^(d*x + c))*a*e^(d*x + c)*sgn(-e^(d*x + c) + 1) - 9*s
qrt(-a*e^(d*x + c))*a*sgn(-e^(d*x + c) + 1) + (9*a^3*e^(d*x + c)*sgn(-e^(d*
x + c) + 1) - a^3*sgn(-e^(d*x + c) + 1))*e^(-d*x - c)/(sqrt(-a*e^(d*x + c))
*a))/d
```

3.50 $\int \sqrt{a-a\cosh(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

```
[Out] (-2*a*Sinh[c + d*x])/(d*Sqrt[a - a*Cosh[c + d*x]])
```

Rubi [A] time = 0.0144789, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2646}

$$-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cosh[c + d*x]],x]
```

```
[Out] (-2*a*Sinh[c + d*x])/(d*Sqrt[a - a*Cosh[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a-a\cosh(c+dx)}\,dx = -\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

Mathematica [A] time = 0.0321064, size = 30, normalized size = 1.11

$$\frac{2\coth\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cosh(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a - a*Cosh[c + d*x]]*Coth[(c + d*x)/2])/d
```

Maple [A] time = 0.034, size = 41, normalized size = 1.5

$$-4\,{\frac{\sinh \left(1/2\,dx+c/2 \right) a\cosh \left(1/2\,dx+c/2 \right) }{\sqrt{-2\, \left(\sinh \left(1/2\,dx+c/2 \right) \right) ^{2}a}d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(d*x+c))^(1/2),x)
```

```
[Out] -4*sinh(1/2*d*x+1/2*c)*a*cosh(1/2*d*x+1/2*c)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^(
1/2)/d
```

Maxima [B] time = 1.59228, size = 78, normalized size = 2.89

$$-\frac{\sqrt{2}\sqrt{a}e^{\left(-dx-c\right)}}{d\sqrt{-e^{\left(-dx-c\right)}}} - \frac{\sqrt{2}\sqrt{a}}{d\sqrt{-e^{\left(-dx-c\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*sqrt(a)*e^(-d*x - c)/(d*sqrt(-e^(-d*x - c))) - sqrt(2)*sqrt(a)/(d*
sqrt(-e^(-d*x - c)))
```

Fricas [A] time = 1.74937, size = 124, normalized size = 4.59

$$\frac{2\,\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(
d*x + c) + 1)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\cosh\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*cosh(c + d*x) + a), x)
```

Giac [B] time = 1.19146, size = 85, normalized size = 3.15

$$-\frac{\sqrt{2}\left(\sqrt{-ae^{(dx+c)}}a\operatorname{sgn}\left(-e^{(dx+c)}+1\right)-\frac{a^2\operatorname{sgn}\left(-e^{(dx+c)}+1\right)}{\sqrt{-ae^{(dx+c)}}}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(-a*e^(d*x + c))*a*sgn(-e^(d*x + c) + 1) - a^2*sgn(-e^(d*x +
c) + 1)/sqrt(-a*e^(d*x + c)))/(a*d)
```

3.51 $\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]
])])/(Sqrt[a]*d))
```

Rubi [A] time = 0.0254998, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2649, 206}

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Cosh[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]
])])/(Sqrt[a]*d))
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx = \frac{(2i)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.0300706, size = 41, normalized size = 0.85

$$\frac{2\sinh\left(\frac{1}{2}(c+dx)\right)\log\left(\tanh\left(\frac{1}{4}(c+dx)\right)\right)}{d\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Cosh[c + d*x]],x]
```

```
[Out] (2*Log[Tanh[(c + d*x)/4]]*Sinh[(c + d*x)/2])/(d*Sqrt[a - a*Cosh[c + d*x]])
```

Maple [A] time = 0.036, size = 41, normalized size = 0.9

$$-2\,\frac{\sinh(1/2\,dx+c/2)\,\mathrm{Artanh}(\cosh(1/2\,dx+c/2))}{\sqrt{-2\,(\sinh(1/2\,dx+c/2))^2\,a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cosh(d*x+c))^(1/2),x)
```

```
[Out] -2*sinh(1/2*d*x+1/2*c)*arctanh(cosh(1/2*d*x+1/2*c))/(-2*sinh(1/2*d*x+1/2*c)
^2*a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\cosh(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-a*cosh(d*x + c) + a), x)
```

Fricas [A] time = 1.9059, size = 458, normalized size = 9.54

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\sqrt{-\frac{1}{a}}(\cosh(dx+c)+\sinh(dx+c))-\cosh(dx+c)-\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)-1}\right)}{d}, \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}(\cosh(dx+c)+\sinh(dx+c))}{\sqrt{a}}\right)}{\sqrt{a}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(
d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) - cosh(d*x + c) - sin
h(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) - 1))/d, 2*sqrt(2)*arctan(sq
rt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + s
inh(d*x + c))/sqrt(a))/(sqrt(a)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\cosh\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*cosh(c + d*x) + a), x)
```

Giac [A] time = 1.20268, size = 54, normalized size = 1.12

$$-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{\sqrt{a}d\operatorname{sgn}\left(-e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(sqrt(a)*d*sgn(-e^(d*x + c)
 + 1))
```

3.52 $\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

```
[Out] -ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])]/(2*Sqr
t[2]*a^(3/2)*d) - Sinh[c + d*x]/(2*d*(a - a*Cosh[c + d*x])^(3/2))
```

Rubi [A] time = 0.0414287, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2650, 2649, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] -ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])]/(2*Sqr
t[2]*a^(3/2)*d) - Sinh[c + d*x]/(2*d*(a - a*Cosh[c + d*x])^(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = -\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}} + \frac{i\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}\right)}{2ad}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

Mathematica [A] time = 0.164053, size = 85, normalized size = 1.08

$$\frac{\sinh^3\left(\frac{1}{2}(c+dx)\right)\left(\text{csch}^2\left(\frac{1}{4}(c+dx)\right)+\text{sech}^2\left(\frac{1}{4}(c+dx)\right)+4\log\left(\tanh\left(\frac{1}{4}(c+dx)\right)\right)\right)}{4ad(\cosh(c+dx)-1)\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] ((Csch[(c + d*x)/4]^2 + 4*Log[Tanh[(c + d*x)/4]] + Sech[(c + d*x)/4]^2)*Sin
h[(c + d*x)/2]^3)/(4*a*d*(-1 + Cosh[c + d*x])*Sqrt[a - a*Cosh[c + d*x]])
```

Maple [A] time = 0.053, size = 87, normalized size = 1.1

$$-\frac{1}{4\,ad}\left(-2\,\cosh(1/2\,dx+c/2)+\left(-\ln\left(-1+\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\ln\left(1+\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\left(\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)\left(\sinh\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cosh(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a*(-2*cosh(1/2*d*x+1/2*c)+(-ln(-1+cosh(1/2*d*x+1/2*c))+ln(1+cosh(1/2*d
*x+1/2*c)))*sinh(1/2*d*x+1/2*c)^2)/sinh(1/2*d*x+1/2*c)/(-2*sinh(1/2*d*x+1/2
*c)^2*a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-a\cosh(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*cosh(d*x + c) + a)^(-3/2), x)
```

Fricas [B] time = 1.92949, size = 783, normalized size = 9.91

$$-\frac{\sqrt{2}\left(\cosh(dx+c)^2+2(\cosh(dx+c)-1)\sinh(dx+c)+\sinh(dx+c)^2-2\cosh(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-a}\sqrt{-\cdots}}{\cdots}\right.}{4\left(a^2d\cosh(dx+c)^2+a^2d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sinh(d*x + c) + sinh
(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(
-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x + c)
) + a*cosh(d*x + c) + a*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c) -
 1)) + 4*sqrt(1/2)*(cosh(d*x + c)^2 + (2*cosh(d*x + c) + 1)*sinh(d*x + c) +
 sinh(d*x + c)^2 + cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c))))
/(a^2*d*cosh(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 - 2*a^2*d*cosh(d*x + c) + a
^2*d + 2*(a^2*d*cosh(d*x + c) - a^2*d)*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-a\cosh(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((-a*cosh(c + d*x) + a)**(-3/2), x)
```

Giac [A] time = 1.27652, size = 153, normalized size = 1.94

$$-\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{\sqrt{a}d\operatorname{sgn}\left(-e^{(dx+c)}+1\right)}-\frac{\sqrt{2}\left(\sqrt{-ae^{(dx+c)}}ae^{(dx+c)}+\sqrt{-ae^{(dx+c)}}a\right)}{\left(ae^{(dx+c)}-a\right)^{2}d\operatorname{sgn}\left(-e^{(dx+c)}+1\right)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(sqrt(a)*d*sgn(-e^(d*x +
 c) + 1)) - sqrt(2)*(sqrt(-a*e^(d*x + c))*a*e^(d*x + c) + sqrt(-a*e^(d*x +
c))*a)/((a*e^(d*x + c) - a)^2*d*sgn(-e^(d*x + c) + 1)))/a
```

3.53 $\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

```
[Out] (-3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - Sinh[c + d*x]/(4*d*(a - a*Cosh[c + d*x])^(5/2)) - (3
*Sinh[c + d*x])/(16*a*d*(a - a*Cosh[c + d*x])^(3/2))
```

Rubi [A] time = 0.062987, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2650, 2649, 206}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(-5/2),x]
```


```
[Out] (-3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - Sinh[c + d*x]/(4*d*(a - a*Cosh[c + d*x])^(5/2)) - (3
*Sinh[c + d*x])/(16*a*d*(a - a*Cosh[c + d*x])^(3/2))
```


Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = -\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx}{32a^2}$$

$$= -\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}} + \frac{(3i)\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\sqrt{}\right.}{16a^2d}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}}$$

Mathematica [A] time = 0.193563, size = 115, normalized size = 1.05

$$\frac{\sinh^5\left(\frac{1}{2}(c+dx)\right)\left(-\text{csch}^4\left(\frac{1}{4}(c+dx)\right)+6\text{csch}^2\left(\frac{1}{4}(c+dx)\right)+\text{sech}^4\left(\frac{1}{4}(c+dx)\right)+6\text{sech}^2\left(\frac{1}{4}(c+dx)\right)+24\log\left(\tanh\right.\right.}{32a^2d(\cosh(c+dx)-1)^2\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] ((6*Csch[(c + d*x)/4]^2 - Csch[(c + d*x)/4]^4 + 24*Log[Tanh[(c + d*x)/4]] +
 6*Sech[(c + d*x)/4]^2 + Sech[(c + d*x)/4]^4)*Sinh[(c + d*x)/2]^5)/(32*a^2*
d*(-1 + Cosh[c + d*x])^2*Sqrt[a - a*Cosh[c + d*x]])
```

Maple [A] time = 0.053, size = 137, normalized size = 1.3

$$-\frac{1}{32a^2d}\left(-6\left(\sinh(1/2\,dx+c/2)\right)^2\cosh(1/2\,dx+c/2)+4\cosh(1/2\,dx+c/2)+\left(3\ln(1+\cosh(1/2\,dx+c/2))-3\ln(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cosh(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^2*(-6*sinh(1/2*d*x+1/2*c)^2*cosh(1/2*d*x+1/2*c)+4*cosh(1/2*d*x+1/2*
c)+(3*ln(1+cosh(1/2*d*x+1/2*c))-3*ln(-1+cosh(1/2*d*x+1/2*c)))*sinh(1/2*d*x+
1/2*c)^4)/(1+cosh(1/2*d*x+1/2*c))/(-1+cosh(1/2*d*x+1/2*c))/sinh(1/2*d*x+1/2
*c)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-a\cosh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*cosh(d*x + c) + a)^(-5/2), x)
```

Fricas [B] time = 2.06401, size = 1605, normalized size = 14.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 +
 sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c)
 + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x
 + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)*sqrt(-a
)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)
))*(cosh(d*x + c) + sinh(d*x + c)) + a*cosh(d*x + c) + a*sinh(d*x + c) + a)
/(cosh(d*x + c) + sinh(d*x + c) - 1)) + 4*sqrt(1/2)*(3*cosh(d*x + c)^4 + (1
2*cosh(d*x + c) - 11)*sinh(d*x + c)^3 + 3*sinh(d*x + c)^4 - 11*cosh(d*x + c
)^3 + (18*cosh(d*x + c)^2 - 33*cosh(d*x + c) - 11)*sinh(d*x + c)^2 - 11*cos
h(d*x + c)^2 + (12*cosh(d*x + c)^3 - 33*cosh(d*x + c)^2 - 22*cosh(d*x + c)
+ 3)*sinh(d*x + c) + 3*cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c
```

```
))))/(a^3*d*cosh(d*x + c)^4 + a^3*d*sinh(d*x + c)^4 - 4*a^3*d*cosh(d*x + c)
^3 + 6*a^3*d*cosh(d*x + c)^2 - 4*a^3*d*cosh(d*x + c) + a^3*d + 4*(a^3*d*cos
h(d*x + c) - a^3*d)*sinh(d*x + c)^3 + 6*(a^3*d*cosh(d*x + c)^2 - 2*a^3*d*co
sh(d*x + c) + a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 - 3*a^3*d*c
osh(d*x + c)^2 + 3*a^3*d*cosh(d*x + c) - a^3*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37888, size = 227, normalized size = 2.06

$$-\frac{3\,\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}+\frac{\sqrt{2}\left(3\,\sqrt{-ae^{(dx+c)}}a^{3}e^{(3\,dx+3\,c)}-11\,\sqrt{-ae^{(dx+c)}}a^{3}e^{(2\,dx+2\,c)}-11\,\sqrt{-ae^{(dx+c)}}a^{3}e^{(dx+c)}+3\,\sqrt{-ae^{(dx}}\right.}{16\left(ae^{(dx+c)}-a\right)^{4}a^{2}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(a^(5/2)*d*sgn(-e^(d*x +
 c) + 1)) + 1/16*sqrt(2)*(3*sqrt(-a*e^(d*x + c))*a^3*e^(3*d*x + 3*c) - 11*s
qrt(-a*e^(d*x + c))*a^3*e^(2*d*x + 2*c) - 11*sqrt(-a*e^(d*x + c))*a^3*e^(d*
x + c) + 3*sqrt(-a*e^(d*x + c))*a^3)/((a*e^(d*x + c) - a)^4*a^2*d*sgn(-e^(d
*x + c) + 1))
```

3.54 $\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=112

$$-\frac{ax(2a^2+b^2)}{2b^4}+\frac{(3a^2+2b^2)\sinh(x)}{3b^3}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4\sqrt{a-b}\sqrt{a+b}}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) + (2*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[
a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]) + ((3*a^2 + 2*b^2)*Sinh[x])/(3*b^3)
- (a*Cosh[x]*Sinh[x])/(2*b^2) + (Cosh[x]^2*Sinh[x])/(3*b)
```

Rubi [A] time = 0.308976, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2793, 3049, 3023, 2735, 2659, 208}

$$-\frac{ax(2a^2+b^2)}{2b^4}+\frac{(3a^2+2b^2)\sinh(x)}{3b^3}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4\sqrt{a-b}\sqrt{a+b}}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) + (2*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[
a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]) + ((3*a^2 + 2*b^2)*Sinh[x])/(3*b^3)
- (a*Cosh[x]*Sinh[x])/(2*b^2) + (Cosh[x]^2*Sinh[x])/(3*b)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\int \frac{\cosh(x)\left(2a+2b\cosh(x)-3a\cosh^2(x)\right)}{a+b\cosh(x)}\,dx}{3b}$$

$$= -\frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\int \frac{-3a^2+ab\cosh(x)+2\left(3a^2+2b^2\right)\cosh^2(x)}{a+b\cosh(x)}\,dx}{6b^2}$$

$$= \frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\int \frac{-3a^2b-3a\left(2a^2+b^2\right)\cosh(x)}{a+b\cosh(x)}\,dx}{6b^3}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{a^4\int \frac{1}{a+b\cosh(x)}\,\cdots}{b^4}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\left(2a^4\right)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}} + \frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(\cdots}{\cdots}$$

Mathematica [A] time = 0.186283, size = 99, normalized size = 0.88

$$\frac{-6ax\left(2a^2+b^2\right)+3b\left(4a^2+3b^2\right)\sinh(x)-\frac{24a^4\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-3ab^2\sinh(2x)+b^3\sinh(3x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (-6*a*(2*a^2 + b^2)*x - (24*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]
])/Sqrt[-a^2 + b^2] + 3*b*(4*a^2 + 3*b^2)*Sinh[x] - 3*a*b^2*Sinh[2*x] + b^3
*Sinh[3*x])/(12*b^4)
```

Maple [B] time = 0.032, size = 264, normalized size = 2.4

$$2\,\frac{a^4}{b^4\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right) - \frac{1}{3b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} + \frac{a}{2b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \frac{1}{2b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*cosh(x)),x)
```

```
[Out] 2*a^4/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)-1/3/b/(tanh(1/2*x)+1)^3+1/2/b^2/(tanh(1/2*x)+1)^2*a+1/2/b/(tanh(1/2*x)+1)
^2-1/b^3/(tanh(1/2*x)+1)*a^2-1/2/b^2/(tanh(1/2*x)+1)*a-1/b/(tanh(1/2*x)+1)-
a^3/b^4*ln(tanh(1/2*x)+1)-1/2*a/b^2*ln(tanh(1/2*x)+1)-1/3/b/(tanh(1/2*x)-1)
^3-1/2/b^2/(tanh(1/2*x)-1)^2*a-1/2/b/(tanh(1/2*x)-1)^2-1/b^3/(tanh(1/2*x)-1
)*a^2-1/2/b^2/(tanh(1/2*x)-1)*a-1/b/(tanh(1/2*x)-1)+a^3/b^4*ln(tanh(1/2*x)-
1)+1/2*a/b^2*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11282, size = 3691, normalized size = 32.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^2*b^3 - b^5)*cosh(x)^6 + (a^2*b^3 - b^5)*sinh(x)^6 - 3*(a^3*b^2 -
 a*b^4)*cosh(x)^5 - 3*(a^3*b^2 - a*b^4 - 2*(a^2*b^3 - b^5)*cosh(x))*sinh(x)
^5 - a^2*b^3 + b^5 - 12*(2*a^5 - a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4*b
- a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b - a^2*b^3 - 3*b^5 + 5*(a^2*b^3 -
b^5)*cosh(x)^2 - 5*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b^3 -
b^5)*cosh(x)^3 - 15*(a^3*b^2 - a*b^4)*cosh(x)^2 - 6*(2*a^5 - a^3*b^2 - a*b^
4)*x + 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - 3*(4*a^4*b - a^2*
b^3 - 3*b^5)*cosh(x)^2 - 3*(4*a^4*b - a^2*b^3 - 3*b^5 - 5*(a^2*b^3 - b^5)*c
osh(x)^4 + 10*(a^3*b^2 - a*b^4)*cosh(x)^3 + 12*(2*a^5 - a^3*b^2 - a*b^4)*x*
cosh(x) - 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^4*cosh
(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)*
```

```
sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) + b)) + 3*(a^3*b^2 - a*b^4)*cosh(x) + 3*(2*(a^2*b^3 - b^5)*cosh(x)^5
+ a^3*b^2 - a*b^4 - 5*(a^3*b^2 - a*b^4)*cosh(x)^4 - 12*(2*a^5 - a^3*b^2 - a
*b^4)*x*cosh(x)^2 + 4*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^3 - 2*(4*a^4*b -
a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^2*b^4
- b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^2*b^4 -
 b^6)*sinh(x)^3), 1/24*((a^2*b^3 - b^5)*cosh(x)^6 + (a^2*b^3 - b^5)*sinh(x)
^6 - 3*(a^3*b^2 - a*b^4)*cosh(x)^5 - 3*(a^3*b^2 - a*b^4 - 2*(a^2*b^3 - b^5)
*cosh(x))*sinh(x)^5 - a^2*b^3 + b^5 - 12*(2*a^5 - a^3*b^2 - a*b^4)*x*cosh(x
)^3 + 3*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b - a^2*b^3 - 3*b^
5 + 5*(a^2*b^3 - b^5)*cosh(x)^2 - 5*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 +
2*(10*(a^2*b^3 - b^5)*cosh(x)^3 - 15*(a^3*b^2 - a*b^4)*cosh(x)^2 - 6*(2*a^5
 - a^3*b^2 - a*b^4)*x + 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 -
3*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2 - 3*(4*a^4*b - a^2*b^3 - 3*b^5 - 5*
(a^2*b^3 - b^5)*cosh(x)^4 + 10*(a^3*b^2 - a*b^4)*cosh(x)^3 + 12*(2*a^5 - a^
3*b^2 - a*b^4)*x*cosh(x) - 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)
^2 - 48*(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2
+ a^4*sinh(x)^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a)/(a^2 - b^2)) + 3*(a^3*b^2 - a*b^4)*cosh(x) + 3*(2*(a^2*b^3 - b^
5)*cosh(x)^5 + a^3*b^2 - a*b^4 - 5*(a^3*b^2 - a*b^4)*cosh(x)^4 - 12*(2*a^5
- a^3*b^2 - a*b^4)*x*cosh(x)^2 + 4*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^3 -
2*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 - b^6)*cosh(x)^3
+ 3*(a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^2
 + (a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19972, size = 180, normalized size = 1.61

$$\frac{2\,a^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^4}+\frac{b^2e^{(3\,x)}-3\,abe^{(2\,x)}+12\,a^2e^x+9\,b^2e^x}{24\,b^3}-\frac{\left(2\,a^3+ab^2\right)x}{2\,b^4}+\frac{\left(3\,ab^2e^x-b^3-3\left(4\,a^2b+3\,b^3\right)e^{(2\,x)}\right)e^{(-3}}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^4) + 1/24*(b
^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 9*b^2*e^x)/b^3 - 1/2*(2*a^3 + a*b
^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b + 3*b^3)*e^(2*x))*e^(-3*x)
/b^4
```

3.55 $\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=85

$$\frac{x(2a^2+b^2)}{2b^3}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\sqrt{a+b}}-\frac{a\sinh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
 b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]) - (a*Sinh[x])/b^2 + (Cosh[x]*Sinh[x])/(
2*b)
```

Rubi [A] time = 0.168612, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2793, 3023, 2735, 2659, 208}

$$\frac{x(2a^2+b^2)}{2b^3}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\sqrt{a+b}}-\frac{a\sinh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
 b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]) - (a*Sinh[x])/b^2 + (Cosh[x]*Sinh[x])/(
2*b)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx &= \frac{\cosh(x)\sinh(x)}{2b}+\frac{\int \frac{a+b\cosh(x)-2a\cosh^2(x)}{a+b\cosh(x)}\,dx}{2b}\\
&= -\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}+\frac{\int \frac{ab+\left(2a^2+b^2\right)\cosh(x)}{a+b\cosh(x)}\,dx}{2b^2}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}-\frac{a^3\int \frac{1}{a+b\cosh(x)}\,dx}{b^3}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}-\frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.126834, size = 78, normalized size = 0.92

$$\frac{\frac{8a^3\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+4a^2x-4ab\sinh(x)+2b^2x+b^2\sinh(2x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (4*a^2*x + 2*b^2*x + (8*a^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2] - 4*a*b*Sinh[x] + b^2*Sinh[2*x])/(4*b^3)
```

Maple [B] time = 0.023, size = 174, normalized size = 2.1

$$-2\,\frac{a^3}{b^3\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)-\frac{1}{2\,b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{a}{b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{1}{2\,b}\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*cosh(x)),x)
```

```
[Out] -2*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-1/2/b/(tanh(1/2*x)+1)^2+1/b^2/(tanh(1/2*x)+1)*a+1/2/b/(tanh(1/2*x)+1)+1/
b^3*ln(tanh(1/2*x)+1)*a^2+1/2/b*ln(tanh(1/2*x)+1)+1/2/b/(tanh(1/2*x)-1)^2+1
/b^2/(tanh(1/2*x)-1)*a+1/2/b/(tanh(1/2*x)-1)-1/b^3*ln(tanh(1/2*x)-1)*a^2-1/
2/b*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39567, size = 2083, normalized size = 24.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2 - b^4)*sinh(x)^4 - a^2*b^2 + b^4
 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*
(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 - b^4)*
cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x
)^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a^2 -
b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(
b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))
/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))
 + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3
 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)*sinh(
x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5)*cosh(x)*sinh(x) + (a^2*b
^3 - b^5)*sinh(x)^2), 1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2 - b^4)*sinh
(x)^4 - a^2*b^2 + b^4 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b -
a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x))*sinh(x)^3 +
2*(3*(a^2*b^2 - b^4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x - 6*(a^3*b - a
*b^3)*cosh(x))*sinh(x)^2 + 16*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*
sinh(x)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a)/(a^2 - b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^2*b^
2 - b^4)*cosh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x) - 3*(a^3*b - a*b^3
)*cosh(x)^2)*sinh(x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5)*cosh(x
)*sinh(x) + (a^2*b^3 - b^5)*sinh(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23676, size = 124, normalized size = 1.46

$$-\frac{2\,a^3\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^3}+\frac{be^{(2\,x)}-4\,ae^x}{8\,b^2}+\frac{\left(2\,a^2+b^2\right)x}{2\,b^3}+\frac{\left(4\,abe^x-b^2\right)e^{(-2\,x)}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^3) + 1/8*(b
*e^(2*x) - 4*a*e^x)/b^2 + 1/2*(2*a^2 + b^2)*x/b^3 + 1/8*(4*a*b*e^x - b^2)*e
^(-2*x)/b^3
```

3.56 $\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{ax}{b^2}+\frac{\sinh(x)}{b}$$

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a
 - b]*b^2*Sqrt[a + b]) + Sinh[x]/b
```

Rubi [A] time = 0.106229, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2746, 12, 2735, 2659, 208}

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{ax}{b^2}+\frac{\sinh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a
 - b]*b^2*Sqrt[a + b]) + Sinh[x]/b
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx &= \frac{\sinh(x)}{b}-\frac{\int \frac{a\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\
&= \frac{\sinh(x)}{b}-\frac{a\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}+\frac{a^2\int \frac{1}{a+b\cosh(x)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}}+\frac{\sinh(x)}{b}
\end{aligned}$$

Mathematica [A] time = 0.105418, size = 57, normalized size = 0.92

$$\frac{a\left(-\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-x\right)+b\sinh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (a*(-x - (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2
]) + b*Sinh[x])/b^2
```

Maple [A] time = 0.02, size = 94, normalized size = 1.5

$$2\,{\frac{{a}^{2}}{{b}^{2}\sqrt{(a+b)(a-b)}}}{\rm Artanh}\left({\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}}\right)-{\frac{1}{b}}\left(\tanh\left({\frac{x}{2}}\right)+1\right)^{-1}-{\frac{a}{{b}^{2}}}\ln\left(\tanh\left({\frac{x}{2}}\right)+1\right)-{\frac{1}{b}}\left(\tanh\left({\frac{x}{2}}\right)-1\right)^{-1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*cosh(x)),x)
```

```
[Out] 2*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)-1/b/(tanh(1/2*x)+1)-a/b^2*ln(tanh(1/2*x)+1)-1/b/(tanh(1/2*x)-1)+a/b^2*ln(
tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25264, size = 1080, normalized size = 17.42

$$\left[-\frac{a^2b-b^3+2\left(a^3-ab^2\right)x\cosh(x)-\left(a^2b-b^3\right)\cosh(x)^2-\left(a^2b-b^3\right)\sinh(x)^2-2\left(a^2\cosh(x)+a^2\sinh(x)\right)\sqrt{a^2-b^2}}{2\left(\left(a^2b^2-b^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh(x) - (a^2*b - b^3)*cosh(x)^2 -
(a^2*b - b^3)*sinh(x)^2 - 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 - b^2)*log
((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh
(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh
(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 2*((a
^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(x))/((a^2*b^2 - b^4)*cosh(x) +
(a^2*b^2 - b^4)*sinh(x)), -1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh(x) - (
a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 + 4*(a^2*cosh(x) + a^2*sin
h(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)
/(a^2 - b^2)) + 2*((a^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(x))/((a^2*
b^2 - b^4)*cosh(x) + (a^2*b^2 - b^4)*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20146, size = 84, normalized size = 1.35

$$\frac{2\,a^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2}-\frac{ax}{b^2}-\frac{e^{(-x)}}{2\,b}+\frac{e^x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - a*x/b^2
 - 1/2*e^(-x)/b + 1/2*e^x/b
```

3.57 $\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{b}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqr
t[a + b])
```

Rubi [A] time = 0.0525138, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2735, 2659, 208}

$$\frac{x}{b}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Cosh[x]),x]
```

```
[Out] x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqr
t[a + b])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\cosh(x)}\,dx}{b}\\ &= \frac{x}{b}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{x}{b}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.0448116, size = 48, normalized size = 0.92

$$\frac{\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Cosh[x]),x]
```

```
[Out] (x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])/b
```

Maple [A] time = 0.014, size = 64, normalized size = 1.2

$$-2\,\frac{a}{b\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)+\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*cosh(x)),x)
```

```
[Out] -2*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1
/b*ln(tanh(1/2*x)+1)-1/b*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.30895, size = 535, normalized size = 10.29

$$\left[\frac{\sqrt{a^2-b^2}a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2(b\cosh(x)+a)\sinh(x)+b}\right)+\left(a^2-b^2\right)x}{a^2b-b^3}, \ 2\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) +
b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) + b)) + (a^2 - b^2)*x)/(a^2*b - b^3), (2*sqrt(-a^2 + b^2)*a*arcta
n(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2 - b^2)*
x)/(a^2*b - b^3)]
```

Sympy [A] time = 135.171, size = 241, normalized size = 4.63

$$\begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b= \\ \frac{x}{b}-\frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for } a=b \\ \frac{x}{b}-\frac{1}{b\tanh\left(\frac{x}{2}\right)} & \text{for } a=-b \\ \frac{\sinh(x)}{a} & \text{for } b=0 \\ \frac{ax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{a\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{a\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{bx\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b - tanh(x/2)/b, Eq(a, b)), (x/b
 - 1/(b*tanh(x/2)), Eq(a, -b)), (sinh(x)/a, Eq(b, 0)), (a*x*sqrt(a/(a - b)
+ b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a
- b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b
) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - a*log(sqrt(a/(a - b) +
 b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a
- b) + b/(a - b))) - b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))), True))
```

Giac [A] time = 1.19825, size = 57, normalized size = 1.1

$$-\frac{2\,a\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b) + x/b
```

3.58 $\int \frac{\textbf{sech}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] ArcTan[Sinh[x]]/a - (2*b*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*S
qrt[a - b]*Sqrt[a + b])
```

Rubi [A] time = 0.067154, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2747, 3770, 2659, 208}

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - (2*b*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*S
qrt[a - b]*Sqrt[a + b])
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{a}-\frac{b\int \frac{1}{a+b\cosh(x)}\,dx}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.0507831, size = 54, normalized size = 1.

$$\frac{2\left(\frac{b\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Cosh[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] + (b*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2]))/a
```

Maple [A] time = 0.02, size = 51, normalized size = 0.9

$$-2\,{\frac{b}{a\sqrt{(a+b)(a-b)}}}\operatorname{Artanh}\left({\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}}\right)+2\,{\frac{\arctan(\tanh(x/2))}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*cosh(x)),x)
```

```
[Out] -2*b/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2
/a*arctan(tanh(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.51079, size = 602, normalized size = 11.15

$$\left[\frac{\sqrt{a^2-b^2}b\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+2\left(a^2-b^2\right)\mathrm{arct}}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) +
b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) + b)) + 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), 2
*(sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(
a^2 - b^2)) + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*cosh(x)), x)
```

Giac [A] time = 1.18787, size = 61, normalized size = 1.13

$$-\frac{2\,b\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}a}+\frac{2\,\arctan\left(e^{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arctan(e
^x)/a
```

3.59 $\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=64

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{\tanh(x)}{a}$$

```
[Out] -((b*ArcTan[Sinh[x]])/a^2) + (2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
+ b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]) + Tanh[x]/a
```

Rubi [A] time = 0.116694, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2802, 12, 2747, 3770, 2659, 208}

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] -((b*ArcTan[Sinh[x]])/a^2) + (2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
+ b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]) + Tanh[x]/a
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = \frac{\tanh(x)}{a} - \frac{\int \frac{b\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{a}$$
$$= \frac{\tanh(x)}{a} - \frac{b\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{a}$$
$$= \frac{\tanh(x)}{a} - \frac{b\int \operatorname{sech}(x)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\cosh(x)}\,dx}{a^2}$$
$$= -\frac{b\tan^{-1}(\sinh(x))}{a^2} + \frac{\tanh(x)}{a} + \frac{(2b^2)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= -\frac{b\tan^{-1}(\sinh(x))}{a^2} + \frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}} + \frac{\tanh(x)}{a}$$

Mathematica [A] time = 0.102917, size = 63, normalized size = 0.98

$$\frac{-\frac{2b^2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + a\tanh(x) - 2b\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (-2*b*ArcTan[Tanh[x/2]] - (2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] + a*Tanh[x])/a^2
```

Maple [A] time = 0.03, size = 73, normalized size = 1.1

$$2\,\frac{b^2}{a^2\sqrt{(a+b)(a-b)}}\operatorname{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right) + 2\,\frac{\tanh(x/2)}{a\left((\tanh(x/2))^2+1\right)} - 2\,\frac{b\arctan(\tanh(x/2))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*cosh(x)),x)
```

```
[Out] 2*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)+2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2/a^2*b*arctan(tanh(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56925, size = 1301, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-(2*a^3 - 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
 + b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
+ 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x
) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
 + a)*sinh(x) + b)) + 2*(a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b -
 b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*arctan(cosh(x) + sinh(x)))
/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*cosh(x)*sin
h(x) + (a^4 - a^2*b^2)*sinh(x)^2), -2*(a^3 - a*b^2 + (b^2*cosh(x)^2 + 2*b^2
*cosh(x)*sinh(x) + b^2*sinh(x)^2 + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3 + (a^2*b - b
^3)*cosh(x)^2 + 2*(a^2*b - b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*
arctan(cosh(x) + sinh(x)))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(
a^4 - a^2*b^2)*cosh(x)*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{a + b\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*cosh(x)), x)
```

Giac [A] time = 1.16106, size = 82, normalized size = 1.28

$$\frac{2\,b^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^2}-\frac{2\,b\arctan\left(e^x\right)}{a^2}-\frac{2}{a\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^2) - 2*b*arc
tan(e^x)/a^2 - 2/(a*(e^(2*x) + 1))
```

3.60 $\int \frac{\mathbf{sech}^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=87

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^3}-\frac{b\tanh(x)}{a^2}+\frac{\tanh(x)\mathbf{sech}(x)}{2a}$$

```
[Out] ((a^2 + 2*b^2)*ArcTan[Sinh[x]])/(2*a^3) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tanh[
x/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]) - (b*Tanh[x])/a^2 + (Sech
[x]*Tanh[x])/(2*a)
```

Rubi [A] time = 0.299632, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2802, 3055, 3001, 3770, 2659, 208}

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^3}-\frac{b\tanh(x)}{a^2}+\frac{\tanh(x)\mathbf{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[Sinh[x]])/(2*a^3) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tanh[
x/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]) - (b*Tanh[x])/a^2 + (Sech
[x]*Tanh[x])/(2*a)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx = \frac{\operatorname{sech}(x)\tanh(x)}{2a} + \frac{\int \frac{\left(-2b+a\cosh(x)+b\cosh^2(x)\right)\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx}{2a}$$

$$= -\frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a} + \frac{\int \frac{\left(a^2+2b^2+ab\cosh(x)\right)\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{2a^2}$$

$$= -\frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a} - \frac{b^3\int \frac{1}{a+b\cosh(x)}\,dx}{a^3} + \frac{\left(a^2+2b^2\right)\int \operatorname{sech}(x)\,dx}{2a^3}$$

$$= \frac{\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^3} - \frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a} - \frac{\left(2b^3\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tan$$

$$= \frac{\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^3} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}} - \frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a}$$

Mathematica [A] time = 0.198691, size = 82, normalized size = 0.94

$$\frac{\frac{4b^3\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + 2\left(a^2+2b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + a\tanh(x)(a\operatorname{sech}(x)-2b)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (2*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] + (4*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt
[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(-2*b + a*Sech[x])*Tanh[x])/(2*a^3)
```

Maple [A] time = 0.027, size = 146, normalized size = 1.7

$$-2\frac{b^3}{a^3\sqrt{(a+b)(a-b)}}\operatorname{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right) - \frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2} - 2\frac{(\tanh(x/2))^3 b}{a^2\left((\tanh(x/2))^2+1\right)^2} + \frac{1}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*cosh(x)),x)
```

```
[Out] -2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-2/a^2/(tanh(1/2*x)^2+1)^2*tanh(1/2
*x)^3*b+1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)-2/a^2/(tanh(1/2*x)^2+1)^2*tanh(
1/2*x)*b+1/a*arctan(tanh(1/2*x))+2/a^3*arctan(tanh(1/2*x))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.53065, size = 3337, normalized size = 38.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3*b - 2*a*b^3 + (a^4 - a^2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^3
 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*cos
h(x))*sinh(x)^2 + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4
+ 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh
(x)^3 + b^3*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh
(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqr
t(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*
cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + ((a^4 + a^2*b^2 - 2*b^4)*cosh(x
)^4 + 4*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)*sinh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)
*sinh(x)^4 + a^4 + a^2*b^2 - 2*b^4 + 2*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)^2 +
2*(a^4 + a^2*b^2 - 2*b^4 + 3*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 +
 4*((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)*cosh(x))*si
nh(x))*arctan(cosh(x) + sinh(x)) - (a^4 - a^2*b^2)*cosh(x) - (a^4 - a^2*b^2
 - 3*(a^4 - a^2*b^2)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))*sinh(x))/(a^5 -
 a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3
+ (a^5 - a^3*b^2)*sinh(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^
2 + 3*(a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 +
```

```
 (a^5 - a^3*b^2)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3 + (a^4 - a^2*b^2)*co
sh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*
b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^2 + 2*(b^3*cosh(x)^4 + 4*b
^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cos
h(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
+ ((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)*si
nh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 + a^2*b^2 - 2*b^4 + 2*(a^
4 + a^2*b^2 - 2*b^4)*cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4 + 3*(a^4 + a^2*b^
2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^3 + (a
^4 + a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^4 -
a^2*b^2)*cosh(x) - (a^4 - a^2*b^2 - 3*(a^4 - a^2*b^2)*cosh(x)^2 - 4*(a^3*b
- a*b^3)*cosh(x))*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^4 + 4*(
a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sinh(x)^4 + 2*(a^5 - a^3
*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^2
 + 4*((a^5 - a^3*b^2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*cosh(x)), x)
```

Giac [A] time = 1.19855, size = 120, normalized size = 1.38

$$-\frac{2\,b^3\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^3}+\frac{\left(a^2+2\,b^2\right)\arctan\left(e^x\right)}{a^3}+\frac{ae^{(3\,x)}+2\,be^{(2\,x)}-ae^x+2\,b}{a^2\left(e^{(2\,x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a^2 +
 2*b^2)*arctan(e^x)/a^3 + (a*e^(3*x) + 2*b*e^(2*x) - a*e^x + 2*b)/(a^2*(e^(
```

```
2*x) + 1)^2)
```

3.61 $\int \frac{\text{sech}^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=114

$$\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}+\frac{(2a^2+3b^2)\tanh(x)}{3a^3}-\frac{b(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^4}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

```
[Out] -(b*(a^2 + 2*b^2)*ArcTan[Sinh[x]])/(2*a^4) + (2*b^4*ArcTanh[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]) + ((2*a^2 + 3*b^2)*Tan
h[x])/(3*a^3) - (b*Sech[x]*Tanh[x])/(2*a^2) + (Sech[x]^2*Tanh[x])/(3*a)
```

Rubi [A] time = 0.471966, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2802, 3055, 3001, 3770, 2659, 208}

$$\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}+\frac{(2a^2+3b^2)\tanh(x)}{3a^3}-\frac{b(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^4}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Cosh[x]),x]
```


```
[Out] -(b*(a^2 + 2*b^2)*ArcTan[Sinh[x]])/(2*a^4) + (2*b^4*ArcTanh[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]) + ((2*a^2 + 3*b^2)*Tan
h[x])/(3*a^3) - (b*Sech[x]*Tanh[x])/(2*a^2) + (Sech[x]^2*Tanh[x])/(3*a)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{\int \frac{(-3b+2a\cosh(x)+2b\cosh^2(x))\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx}{3a}$$

$$= -\frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{\int \frac{(2(2a^2+3b^2)+ab\cosh(x)-3b^2\cosh^2(x))\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx}{6a^2}$$

$$= \frac{(2a^2+3b^2)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{\int \frac{(-3b(a^2+2b^2)-3ab^2\cosh(x))\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{6a^3}$$

$$= \frac{(2a^2+3b^2)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{b^4\int \frac{1}{a+b\cosh(x)}\,dx}{a^4} - \frac{(b(a^2+2b^2)}{2}$$

$$= -\frac{b(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^4} + \frac{(2a^2+3b^2)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} +$$

$$= -\frac{b(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^4} + \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}} + \frac{(2a^2+3b^2)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tan}{2a^2}$$

Mathematica [A] time = 0.381951, size = 101, normalized size = 0.89

$$\frac{-\frac{12b^4\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - 6b(a^2+2b^2)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + a\tanh(x)\left(2a^2\operatorname{sech}^2(x)+4a^2-3ab\operatorname{sech}(x)+6b^2\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (-6*b*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] - (12*b^4*ArcTan[((a - b)*Tanh[x/2])/
Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(4*a^2 + 6*b^2 - 3*a*b*Sech[x] + 2*
a^2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

Maple [B] time = 0.029, size = 239, normalized size = 2.1

$$2\frac{b^4}{a^4\sqrt{(a+b)(a-b)}}\operatorname{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right) + 2\frac{(\tanh(x/2))^5}{a\left((\tanh(x/2))^2+1\right)^3} + \frac{b}{a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^5\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-3} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*cosh(x)),x)
```

```
[Out] 2*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)+2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5+1/a^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*
x)^5*b+2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*b^2+4/3/a/(tanh(1/2*x)^2+1)^
3*tanh(1/2*x)^3+4/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^2+2/a/(tanh(1/2*x
)^2+1)^3*tanh(1/2*x)+2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*b^2-1/a^2/(tanh(
1/2*x)^2+1)^3*tanh(1/2*x)*b-1/a^2*b*arctan(tanh(1/2*x))-2/a^4*arctan(tanh(1
/2*x))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.90383, size = 5894, normalized size = 51.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^5 + 4*a^
5 + 2*a^3*b^2 - 6*a*b^4 + 6*(a^3*b^2 - a*b^4)*cosh(x)^4 + 3*(2*a^3*b^2 - 2*
a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 6*(5*(a^4*b - a^2*b^3)*cos
h(x)^2 + 4*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a*b^4)*cosh(x)^
2 + 6*(2*a^5 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2 - a*b^4
)*cosh(x)^2)*sinh(x)^2 - 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*s
inh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*si
nh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*co
sh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(
x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(
x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt
```

```
(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*c
osh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 3*((a^4*b + a^2*b^3 - 2*b^5)*cos
h(x)^6 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)*sinh(x)^5 + (a^4*b + a^2*b^3 -
 2*b^5)*sinh(x)^6 + a^4*b + a^2*b^3 - 2*b^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c
osh(x)^4 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)
^2)*sinh(x)^4 + 4*(5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + 3*(a^4*b + a^2*b
^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*
(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^4 + 6*(a^4*b
 + a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b + a^2*b^3 - 2*b^5)*cos
h(x)^5 + 2*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + (a^4*b + a^2*b^3 - 2*b^5)*
cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - 3*(a^4*b - a^2*b^3)*cosh(x) -
 3*(a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 8*(a^3*b^2 - a*b^4)*c
osh(x)^3 - 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^6 + 6
*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4*b^2)*sinh(x)^6 + a^6 - a^4*
b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a^4*b^2)*cos
h(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 + 3*(a^6 - a^4*b^2 + 5*(a^6 -
 a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - a^
4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3 + (a^6 - a^4*b^2)*cosh(x))*s
inh(x)), -1/3*(3*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^
5 + 4*a^5 + 2*a^3*b^2 - 6*a*b^4 + 6*(a^3*b^2 - a*b^4)*cosh(x)^4 + 3*(2*a^3*
b^2 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 6*(5*(a^4*b - a^2*
b^3)*cosh(x)^2 + 4*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a*b^4)*
cosh(x)^2 + 6*(2*a^5 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2
 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5
 + b^4*sinh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 +
 b^4)*sinh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(
5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b
^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 3*((a^4*b + a^2*b^3 - 2*b^5)
*cosh(x)^6 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)*sinh(x)^5 + (a^4*b + a^2*b
^3 - 2*b^5)*sinh(x)^6 + a^4*b + a^2*b^3 - 2*b^5 + 3*(a^4*b + a^2*b^3 - 2*b^
5)*cosh(x)^4 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + 3*(a^4*b + a
^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2
 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^4 + 6*(a
^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b + a^2*b^3 - 2*b^5)
*cosh(x)^5 + 2*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + (a^4*b + a^2*b^3 - 2*b
^5)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - 3*(a^4*b - a^2*b^3)*cosh(
x) - 3*(a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 8*(a^3*b^2 - a*b^
4)*cosh(x)^3 - 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^6
 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4*b^2)*sinh(x)^6 + a^6 -
a^4*b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2
)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a^4*b^2)
*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 + 3*(a^6 - a^4*b^2 + 5*(a
```

```
^6 - a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^6
- a^4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3 + (a^6 - a^4*b^2)*cosh(x
))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*cosh(x)), x)
```

Giac [A] time = 1.27504, size = 166, normalized size = 1.46

$$\frac{2\,b^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^4} - \frac{\left(a^2b+2\,b^3\right)\arctan\left(e^x\right)}{a^4} - \frac{3\,abe^{(5\,x)}+6\,b^2e^{(4\,x)}+12\,a^2e^{(2\,x)}+12\,b^2e^{(2\,x)}-3\,abe^x+4\,a^2+6\,b^2}{3\,a^3\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - (a^2*b
+ 2*b^3)*arctan(e^x)/a^4 - 1/3*(3*a*b*e^(5*x) + 6*b^2*e^(4*x) + 12*a^2*e^(2
*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

3.62 $\int (a+b\cosh(c+dx))^5\,dx$

Optimal. Leaf size=183

$$\frac{b\left(192a^2b^2+107a^4+16b^4\right)\sinh(c+dx)}{30d}+\frac{b\left(47a^2+16b^2\right)\sinh(c+dx)(a+b\cosh(c+dx))^2}{60d}+\frac{7ab^2\left(22a^2+23b^2\right)\sinh}{120}$$

```
[Out] (a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*x)/8 + (b*(107*a^4 + 192*a^2*b^2 + 16*b^4)
*Sinh[c + d*x])/(30*d) + (7*a*b^2*(22*a^2 + 23*b^2)*Cosh[c + d*x]*Sinh[c +
d*x])/(120*d) + (b*(47*a^2 + 16*b^2)*(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])
/(60*d) + (9*a*b*(a + b*Cosh[c + d*x])^3*Sinh[c + d*x])/(20*d) + (b*(a + b*
Cosh[c + d*x])^4*Sinh[c + d*x])/(5*d)
```

Rubi [A] time = 0.260091, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2656, 2753, 2734}

$$\frac{b\left(192a^2b^2+107a^4+16b^4\right)\sinh(c+dx)}{30d}+\frac{b\left(47a^2+16b^2\right)\sinh(c+dx)(a+b\cosh(c+dx))^2}{60d}+\frac{7ab^2\left(22a^2+23b^2\right)\sinh}{120}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^5,x]
```

```
[Out] (a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*x)/8 + (b*(107*a^4 + 192*a^2*b^2 + 16*b^4)
*Sinh[c + d*x])/(30*d) + (7*a*b^2*(22*a^2 + 23*b^2)*Cosh[c + d*x]*Sinh[c +
d*x])/(120*d) + (b*(47*a^2 + 16*b^2)*(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])
/(60*d) + (9*a*b*(a + b*Cosh[c + d*x])^3*Sinh[c + d*x])/(20*d) + (b*(a + b*
Cosh[c + d*x])^4*Sinh[c + d*x])/(5*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^5\,dx = \frac{b(a+b\cosh(c+dx))^4\sinh(c+dx)}{5d} + \frac{1}{5}\int (a+b\cosh(c+dx))^3\left(5a^2+4b^2+9ab\cosh(c\right.$$

$$= \frac{9ab(a+b\cosh(c+dx))^3\sinh(c+dx)}{20d} + \frac{b(a+b\cosh(c+dx))^4\sinh(c+dx)}{5d} + \frac{1}{20}\int (a+$$

$$= \frac{b\left(47a^2+16b^2\right)(a+b\cosh(c+dx))^2\sinh(c+dx)}{60d} + \frac{9ab(a+b\cosh(c+dx))^3\sinh(c+d}{20d}$$

$$= \frac{1}{8}a\left(8a^4+40a^2b^2+15b^4\right)x + \frac{b\left(107a^4+192a^2b^2+16b^4\right)\sinh(c+dx)}{30d} + \frac{7ab^2\left(22a^2+23\right.}{}$$

Mathematica [A] time = 0.349365, size = 133, normalized size = 0.73

$$\frac{60a\left(40a^2b^2+8a^4+15b^4\right)(c+dx)+50b^3\left(8a^2+b^2\right)\sinh(3(c+dx))+600ab^2\left(2a^2+b^2\right)\sinh(2(c+dx))+300b\left(12a^2\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^5,x]
```

```
[Out] (60*a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*(c + d*x) + 300*b*(8*a^4 + 12*a^2*b^2 +
 b^4)*Sinh[c + d*x] + 600*a*b^2*(2*a^2 + b^2)*Sinh[2*(c + d*x)] + 50*b^3*(8
*a^2 + b^2)*Sinh[3*(c + d*x)] + 75*a*b^4*Sinh[4*(c + d*x)] + 6*b^5*Sinh[5*(
c + d*x)])/(480*d)
```

Maple [A] time = 0.014, size = 155, normalized size = 0.9

$$\frac{1}{d}\left(b^5\left(\frac{8}{15}+\frac{(\cosh(dx+c))^4}{5}+\frac{4\,(\cosh(dx+c))^2}{15}\right)\sinh(dx+c)+5\,ab^4\left(\left(1/4\,(\cosh(dx+c))^3+3/8\,\cosh(dx+c)\right)\sinh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(8/15+1/5*cosh(d*x+c)^4+4/15*cosh(d*x+c)^2)*sinh(d*x+c)+5*a*b^4*((
1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+10*a^2*b^3*(2
/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+10*a^3*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1
/2*d*x+1/2*c)+5*a^4*b*sinh(d*x+c)+a^5*(d*x+c))
```

Maxima [A] time = 1.12051, size = 369, normalized size = 2.02

$$\frac{5}{64}\,ab^4\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{5}{4}\,a^3b^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^5x+\frac{1}{480}\,b^5\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) + 5/4*a^3*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2
*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d + 25*e^(3*d*x + 3*c
)/d + 150*e^(d*x + c)/d - 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d - 3*e^
(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 5*a^4*b*sinh(d*x + c)/d
```

Fricas [A] time = 2.18402, size = 463, normalized size = 2.53

$$\frac{3\,b^5\sinh(dx+c)^5+5\left(6\,b^5\cosh(dx+c)^2+30\,ab^4\cosh(dx+c)+40\,a^2b^3+5\,b^5\right)\sinh(dx+c)^3+30\left(8\,a^5+40\,a^3b^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^5*sinh(d*x + c)^5 + 5*(6*b^5*cosh(d*x + c)^2 + 30*a*b^4*cosh(d*x
 + c) + 40*a^2*b^3 + 5*b^5)*sinh(d*x + c)^3 + 30*(8*a^5 + 40*a^3*b^2 + 15*a
*b^4)*d*x + 15*(b^5*cosh(d*x + c)^4 + 10*a*b^4*cosh(d*x + c)^3 + 80*a^4*b +
 120*a^2*b^3 + 10*b^5 + 5*(8*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 40*(2*a^3*b^2
 + a*b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 2.66193, size = 314, normalized size = 1.72

$$\begin{cases} a^5x + \frac{5a^4b\sinh(c+dx)}{d} - 5a^3b^2x\sinh^2(c+dx) + 5a^3b^2x\cosh^2(c+dx) + \frac{5a^3b^2\sinh(c+dx)\cosh(c+dx)}{d} - \frac{20a^2b^3\sinh^3(c+dx)}{3d} + \frac{10a \\ x(a+b\cosh(c))^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*sinh(c + d*x)/d - 5*a**3*b**2*x*sinh(c + d*x)*
*2 + 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c + d*
x)/d - 20*a**2*b**3*sinh(c + d*x)**3/(3*d) + 10*a**2*b**3*sinh(c + d*x)*cos
h(c + d*x)**2/d + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 - 15*a*b**4*sinh(c
 + d*x)**3*cosh(c + d*x)/(8*d) + 25*a*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(
8*d) + 8*b**5*sinh(c + d*x)**5/(15*d) - 4*b**5*sinh(c + d*x)**3*cosh(c + d*
x)**2/(3*d) + b**5*sinh(c + d*x)*cosh(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*c
osh(c))**5, True))
```

Giac [A] time = 1.18354, size = 362, normalized size = 1.98

$$\frac{6\,b^5e^{(5\,dx+5\,c)} + 75\,ab^4e^{(4\,dx+4\,c)} + 400\,a^2b^3e^{(3\,dx+3\,c)} + 50\,b^5e^{(3\,dx+3\,c)} + 1200\,a^3b^2e^{(2\,dx+2\,c)} + 600\,ab^4e^{(2\,dx+2\,c)} + 2400\,a^4be^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/960*(6*b^5*e^(5*d*x + 5*c) + 75*a*b^4*e^(4*d*x + 4*c) + 400*a^2*b^3*e^(3*
d*x + 3*c) + 50*b^5*e^(3*d*x + 3*c) + 1200*a^3*b^2*e^(2*d*x + 2*c) + 600*a*
b^4*e^(2*d*x + 2*c) + 2400*a^4*b*e^(d*x + c) + 3600*a^2*b^3*e^(d*x + c) + 3
00*b^5*e^(d*x + c) + 120*(8*a^5 + 40*a^3*b^2 + 15*a*b^4)*(d*x + c) - (75*a*
b^4*e^(d*x + c) + 6*b^5 + 300*(8*a^4*b + 12*a^2*b^3 + b^5)*e^(4*d*x + 4*c)
```

```
+ 600*(2*a^3*b^2 + a*b^4)*e^(3*d*x + 3*c) + 50*(8*a^2*b^3 + b^5)*e^(2*d*x +
 2*c))*e^(-5*d*x - 5*c))/d
```

3.63 $\int (a+b\cosh(c+dx))^4\,dx$

Optimal. Leaf size=137

$$\frac{ab\left(19a^2+16b^2\right)\sinh(c+dx)}{6d}+\frac{b^2\left(26a^2+9b^2\right)\sinh(c+dx)\cosh(c+dx)}{24d}+\frac{1}{8}x\left(24a^2b^2+8a^4+3b^4\right)+\frac{b\sinh(c+dx}{}$$

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 + 16*b^2)*Sinh[c + d*x])/
(6*d) + (b^2*(26*a^2 + 9*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(24*d) + (7*a*b*
(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])/(12*d) + (b*(a + b*Cosh[c + d*x])^3*
Sinh[c + d*x])/(4*d)
```

Rubi [A] time = 0.150763, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2656, 2753, 2734}

$$\frac{ab\left(19a^2+16b^2\right)\sinh(c+dx)}{6d}+\frac{b^2\left(26a^2+9b^2\right)\sinh(c+dx)\cosh(c+dx)}{24d}+\frac{1}{8}x\left(24a^2b^2+8a^4+3b^4\right)+\frac{b\sinh(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 + 16*b^2)*Sinh[c + d*x])/
(6*d) + (b^2*(26*a^2 + 9*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(24*d) + (7*a*b*
(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])/(12*d) + (b*(a + b*Cosh[c + d*x])^3*
Sinh[c + d*x])/(4*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^4\,dx = \frac{b(a+b\cosh(c+dx))^3\sinh(c+dx)}{4d} + \frac{1}{4}\int (a+b\cosh(c+dx))^2\left(4a^2+3b^2+7ab\cosh(c+\right.$$
$$= \frac{7ab(a+b\cosh(c+dx))^2\sinh(c+dx)}{12d} + \frac{b(a+b\cosh(c+dx))^3\sinh(c+dx)}{4d} + \frac{1}{12}\int (a+b$$
$$= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x + \frac{ab\left(19a^2+16b^2\right)\sinh(c+dx)}{6d} + \frac{b^2\left(26a^2+9b^2\right)\cosh(c+dx)\,\mathrm{s}}{24d}$$

Mathematica [A] time = 0.211921, size = 104, normalized size = 0.76

$$\frac{12\left(24a^2b^2+8a^4+3b^4\right)(c+dx)+24b^2\left(6a^2+b^2\right)\sinh(2(c+dx))+96ab\left(4a^2+3b^2\right)\sinh(c+dx)+32ab^3\sinh(3(c+d}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^4,x]
```

```
[Out] (12*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(c + d*x) + 96*a*b*(4*a^2 + 3*b^2)*Sinh[c
+ d*x] + 24*b^2*(6*a^2 + b^2)*Sinh[2*(c + d*x)] + 32*a*b^3*Sinh[3*(c + d*x)
] + 3*b^4*Sinh[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.013, size = 119, normalized size = 0.9

$$\frac{1}{d}\left(b^4\left(\left(\frac{(\cosh(dx+c))^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3\,dx}{8}+\frac{3\,c}{8}\right)+4\,ab^3\left(2/3+1/3\,(\cosh(dx+c))^2\right)\sinh(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+4*
a*b^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+6*a^2*b^2*(1/2*cosh(d*x+c)*sinh(d
*x+c)+1/2*d*x+1/2*c)+4*a^3*b*sinh(d*x+c)+a^4*(d*x+c))
```

Maxima [A] time = 1.04603, size = 247, normalized size = 1.8

$$\frac{1}{64}b^4\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{3}{4}a^2b^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+a^4x+\frac{1}{6}ab^3\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/64*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/4*a^2*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d
*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 4*a^3*b*sinh(d*x + c)/d
```

Fricas [A] time = 2.12324, size = 296, normalized size = 2.16

$$\frac{\left(3b^4\cosh(dx+c)+8ab^3\right)\sinh(dx+c)^3+3\left(8a^4+24a^2b^2+3b^4\right)dx+3\left(b^4\cosh(dx+c)^3+8ab^3\cosh(dx+c)^2+\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*((3*b^4*cosh(d*x + c) + 8*a*b^3)*sinh(d*x + c)^3 + 3*(8*a^4 + 24*a^2*b
^2 + 3*b^4)*d*x + 3*(b^4*cosh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^2 + 32*a^3
*b + 24*a*b^3 + 4*(6*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 1.40657, size = 240, normalized size = 1.75

$$\begin{cases} a^4x+\frac{4a^3b\sinh(c+dx)}{d}-3a^2b^2x\sinh^2(c+dx)+3a^2b^2x\cosh^2(c+dx)+\frac{3a^2b^2\sinh(c+dx)\cosh(c+dx)}{d}-\frac{8ab^3\sinh^3(c+dx)}{3d}+\frac{4ab^3\mathrm{s}}{} \\ x(a+b\cosh(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*sinh(c + d*x)/d - 3*a**2*b**2*x*sinh(c + d*x)*
*2 + 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c + d*
x)/d - 8*a*b**3*sinh(c + d*x)**3/(3*d) + 4*a*b**3*sinh(c + d*x)*cosh(c + d*
x)**2/d + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(c +
d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 - 3*b**4*sinh(c + d*x)**3*cosh(c +
d*x)/(8*d) + 5*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a
+ b*cosh(c))**4, True))
```

Giac [A] time = 1.12988, size = 259, normalized size = 1.89

$$\frac{3\,b^4e^{(4\,dx+4\,c)} + 32\,ab^3e^{(3\,dx+3\,c)} + 144\,a^2b^2e^{(2\,dx+2\,c)} + 24\,b^4e^{(2\,dx+2\,c)} + 384\,a^3be^{(dx+c)} + 288\,ab^3e^{(dx+c)} + 24\left(8\,a^4 + 24\,a^2b^2 +\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/192*(3*b^4*e^(4*d*x + 4*c) + 32*a*b^3*e^(3*d*x + 3*c) + 144*a^2*b^2*e^(2*
d*x + 2*c) + 24*b^4*e^(2*d*x + 2*c) + 384*a^3*b*e^(d*x + c) + 288*a*b^3*e^(
d*x + c) + 24*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(d*x + c) - (32*a*b^3*e^(d*x + c
) + 3*b^4 + 96*(4*a^3*b + 3*a*b^3)*e^(3*d*x + 3*c) + 24*(6*a^2*b^2 + b^4)*e
^(2*d*x + 2*c))*e^(-4*d*x - 4*c))/d
```

3.64 $\int (a+b\cosh(c+dx))^3\,dx$

Optimal. Leaf size=90

$$\frac{2b\left(4a^2+b^2\right)\sinh(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2+3b^2\right)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}$$

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 + (2*b*(4*a^2 + b^2)*Sinh[c + d*x])/(3*d) + (5*a*b^
2*Cosh[c + d*x]*Sinh[c + d*x])/(6*d) + (b*(a + b*Cosh[c + d*x])^2*Sinh[c +
d*x])/(3*d)
```

Rubi [A] time = 0.0682165, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$\frac{2b\left(4a^2+b^2\right)\sinh(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2+3b^2\right)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 + (2*b*(4*a^2 + b^2)*Sinh[c + d*x])/(3*d) + (5*a*b^
2*Cosh[c + d*x]*Sinh[c + d*x])/(6*d) + (b*(a + b*Cosh[c + d*x])^2*Sinh[c +
d*x])/(3*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{b(a+b\cosh(c+dx))^2\sinh(c+dx)}{3d}+\frac{1}{3}\int (a+b\cosh(c+dx))\left(3a^2+2b^2+5ab\cosh(c+\right.$$
$$=\frac{1}{2}a\left(2a^2+3b^2\right)x+\frac{2b\left(4a^2+b^2\right)\sinh(c+dx)}{3d}+\frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}+\frac{b(a+b}{}$$

Mathematica [A] time = 0.124111, size = 80, normalized size = 0.89

$$\frac{9b\left(4a^2+b^2\right)\sinh(c+dx)+12a^3c+12a^3dx+9ab^2\sinh(2(c+dx))+18ab^2c+18ab^2dx+b^3\sinh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^3,x]
```

```
[Out] (12*a^3*c + 18*a*b^2*c + 12*a^3*d*x + 18*a*b^2*d*x + 9*b*(4*a^2 + b^2)*Sinh
[c + d*x] + 9*a*b^2*Sinh[2*(c + d*x)] + b^3*Sinh[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.013, size = 77, normalized size = 0.9

$$\frac{1}{d}\left(b^3\left(\frac{2}{3}+\frac{(\cosh(dx+c))^2}{3}\right)\sinh(dx+c)+3\,ab^2\left(1/2\,\cosh(dx+c)\sinh(dx+c)+1/2\,dx+c/2\right)+3\,a^2b\sinh(dx+c)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh(
d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*sinh(d*x+c)+a^3*(d*x+c))
```

Maxima [A] time = 1.05084, size = 157, normalized size = 1.74

$$\frac{3}{8}ab^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^3x+\frac{1}{24}b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3\,a^2b\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 3/8*a*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b^3
*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)
/d) + 3*a^2*b*sinh(d*x + c)/d
```

Fricas [A] time = 2.21433, size = 188, normalized size = 2.09

$$\frac{b^3\sinh(dx+c)^3+6\left(2a^3+3ab^2\right)dx+3\left(b^3\cosh(dx+c)^2+6ab^2\cosh(dx+c)+12a^2b+3b^3\right)\sinh(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(b^3*sinh(d*x + c)^3 + 6*(2*a^3 + 3*a*b^2)*d*x + 3*(b^3*cosh(d*x + c)^
2 + 6*a*b^2*cosh(d*x + c) + 12*a^2*b + 3*b^3)*sinh(d*x + c))/d
```

Sympy [A] time = 0.665417, size = 128, normalized size = 1.42

$$\begin{cases} a^3x+\frac{3a^2b\sinh(c+dx)}{d}-\frac{3ab^2x\sinh^2(c+dx)}{2}+\frac{3ab^2x\cosh^2(c+dx)}{2}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{2b^3\sinh^3(c+dx)}{3d}+\frac{b^3\sinh(c+dx)\cosh^2(c+ \\ x\left(a+b\cosh(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)/d - 3*a*b**2*x*sinh(c + d*x)**2/
2 + 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/(2
*d) - 2*b**3*sinh(c + d*x)**3/(3*d) + b**3*sinh(c + d*x)*cosh(c + d*x)**2/d
, Ne(d, 0)), (x*(a + b*cosh(c))**3, True))
```

Giac [A] time = 1.1551, size = 167, normalized size = 1.86

$$\frac{b^3e^{(3dx+3c)}+9ab^2e^{(2dx+2c)}+36a^2be^{(dx+c)}+9b^3e^{(dx+c)}+12\left(2a^3+3ab^2\right)(dx+c)-\left(9ab^2e^{(dx+c)}+b^3+9\left(4a^2b+b^3\right)e^{}\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(b^3*e^(3*d*x + 3*c) + 9*a*b^2*e^(2*d*x + 2*c) + 36*a^2*b*e^(d*x + c)
+ 9*b^3*e^(d*x + c) + 12*(2*a^3 + 3*a*b^2)*(d*x + c) - (9*a*b^2*e^(d*x + c)
 + b^3 + 9*(4*a^2*b + b^3)*e^(2*d*x + 2*c))*e^(-3*d*x - 3*c))/d
```

3.65 $\int (a + b\cosh(c + dx))^2\, dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(2a^2 + b^2\right) + \frac{2ab\sinh(c + dx)}{d} + \frac{b^2\sinh(c + dx)\cosh(c + dx)}{2d}$$

```
[Out] ((2*a^2 + b^2)*x)/2 + (2*a*b*Sinh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rubi [A] time = 0.0172046, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2 + b^2\right) + \frac{2ab\sinh(c + dx)}{d} + \frac{b^2\sinh(c + dx)\cosh(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + (2*a*b*Sinh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a + b\cosh(c + dx))^2\, dx = \frac{1}{2}\left(2a^2 + b^2\right)x + \frac{2ab\sinh(c + dx)}{d} + \frac{b^2\cosh(c + dx)\sinh(c + dx)}{2d}$$

Mathematica [A] time = 0.0728621, size = 46, normalized size = 0.92

$$\frac{2\left(2a^2 + b^2\right)(c + dx) + 8ab\sinh(c + dx) + b^2\sinh(2(c + dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2)*(c + d*x) + 8*a*b*Sinh[c + d*x] + b^2*Sinh[2*(c + d*x)])/(
4*d)
```

Maple [A] time = 0.011, size = 51, normalized size = 1.

$$\frac{1}{d}\left(b^2\left(\frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,ab\sinh\left(dx+c\right)+a^2\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*sinh(d*x+c)+a^2*
(d*x+c))
```

Maxima [A] time = 1.04553, size = 74, normalized size = 1.48

$$\frac{1}{8}\,b^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^2x+\frac{2\,ab\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*sinh
(d*x + c)/d
```

Fricas [A] time = 2.07314, size = 96, normalized size = 1.92

$$\frac{\left(2\,a^2+b^2\right)dx+\left(b^2\cosh\left(dx+c\right)+4\,ab\right)\sinh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a^2 + b^2)*d*x + (b^2*cosh(d*x + c) + 4*a*b)*sinh(d*x + c))/d
```

Sympy [A] time = 0.307524, size = 78, normalized size = 1.56

$$\begin{cases} a^2x + \frac{2ab\sinh(c+dx)}{d} - \frac{b^2x\sinh^2(c+dx)}{2} + \frac{b^2x\cosh^2(c+dx)}{2} + \frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a + b\cosh(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sinh(c + d*x)/d - b**2*x*sinh(c + d*x)**2/2 + b**
2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)),
 (x*(a + b*cosh(c))**2, True))
```

Giac [A] time = 1.1479, size = 95, normalized size = 1.9

$$\frac{b^2e^{(2\,dx+2\,c)} + 8\,abe^{(dx+c)} + 4\left(2\,a^2 + b^2\right)(dx + c) - \left(8\,abe^{(dx+c)} + b^2\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2*e^(2*d*x + 2*c) + 8*a*b*e^(d*x + c) + 4*(2*a^2 + b^2)*(d*x + c) -
(8*a*b*e^(d*x + c) + b^2)*e^(-2*d*x - 2*c))/d
```

3.66 $\int (a+b\cosh (c+dx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\sinh (c+dx)}{d}$$

```
[Out] a*x + (b*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0084691, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2637}

$$ax+\frac{b\sinh (c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cosh[c + d*x],x]
```

```
[Out] a*x + (b*Sinh[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cosh (c+dx))\,dx &= ax+b\int \cosh (c+dx)\,dx\\ &= ax+\frac{b\sinh (c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0082455, size = 26, normalized size = 1.73

$$ax+\frac{b\sinh (c)\cosh (dx)}{d}+\frac{b\cosh (c)\sinh (dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cosh[c + d*x],x]
```

```
[Out] a*x + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.004, size = 16, normalized size = 1.1

$$ax + \frac{b\sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cosh(d*x+c),x)
```

```
[Out] a*x+b*sinh(d*x+c)/d
```

Maxima [A] time = 1.0172, size = 20, normalized size = 1.33

$$ax + \frac{b\sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*sinh(d*x + c)/d
```

Fricas [A] time = 2.09114, size = 39, normalized size = 2.6

$$\frac{adx + b\sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + b*sinh(d*x + c))/d
```

Sympy [A] time = 0.142993, size = 17, normalized size = 1.13

$$ax + b\left(\begin{cases} \frac{\sinh(c+dx)}{d} & \text{for } d \neq 0 \\ x\cosh(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True))
```

Giac [B] time = 1.17885, size = 43, normalized size = 2.87

$$ax + \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*b*(e^(d*x + c)/d - e^(-d*x - c)/d)
```

3.67 $\int \frac{1}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```

Rubi [A] time = 0.035172, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2659, 205}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-1),x]
```


```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```


Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}$$

Mathematica [A] time = 0.0482518, size = 48, normalized size = 0.98

$$-\frac{2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{d\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2]
*d)
```

Maple [A] time = 0.01, size = 44, normalized size = 0.9

$$2\,\frac{1}{d\sqrt{(a+b)\,(a-b)}}\text{Artanh}\left(\frac{(a-b)\tanh\,(1/2\,dx+c/2)}{\sqrt{(a+b)\,(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)),x)
```

```
[Out] 2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.25557, size = 585, normalized size = 11.94

$$\left[\frac{\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2-b^2+2\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)-2\,\sqrt{a^2-b^2}(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)+b}\right)}{\sqrt{a^2-b^2}d},-\frac{2\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] [log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a
^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b
*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b))/(sqrt(
a^2 - b^2)*d), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c
) + b*sinh(d*x + c) + a)/(a^2 - b^2))/((a^2 - b^2)*d)]
```

Sympy [A] time = 15.5237, size = 163, normalized size = 3.33

$$\begin{cases} \frac{\tilde{\infty} x}{\cosh(c)} & \text{for}\: a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} & \text{for}\: a = b \\ -\frac{1}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for}\: a = -b \\ \frac{x}{a+b\cosh(c)} & \text{for}\: d = 0 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/cosh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (tanh(c/2 + d*x/
2)/(b*d), Eq(a, b)), (-1/(b*d*tanh(c/2 + d*x/2)), Eq(a, -b)), (x/(a + b*cos
h(c)), Eq(d, 0)), (-log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(
a*d*sqrt(a/(a - b) + b/(a - b)) - b*d*sqrt(a/(a - b) + b/(a - b))) + log(sq
rt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(a*d*sqrt(a/(a - b) + b/(a -
 b)) - b*d*sqrt(a/(a - b) + b/(a - b))), True))
```

Giac [A] time = 1.22184, size = 53, normalized size = 1.08

$$\frac{2 \arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*d)
```

3.68 $\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

```
[Out] (2*a*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(
a + b)^(3/2)*d) - (b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c + d*x]))
```

Rubi [A] time = 0.0840867, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 12, 2659, 205}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-2),x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(
a + b)^(3/2)*d) - (b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx &= -\frac{b\sinh(c+dx)}{\left(a^2-b^2\right)d(a+b\cosh(c+dx))}-\frac{\int \frac{a}{a+b\cosh(c+dx)}\,dx}{-a^2+b^2}\\ &= -\frac{b\sinh(c+dx)}{\left(a^2-b^2\right)d(a+b\cosh(c+dx))}+\frac{a\int \frac{1}{a+b\cosh(c+dx)}\,dx}{a^2-b^2}\\ &= -\frac{b\sinh(c+dx)}{\left(a^2-b^2\right)d(a+b\cosh(c+dx))}-\frac{(2ia)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{\left(a^2-b^2\right)d}\\ &= \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\sinh(c+dx)}{\left(a^2-b^2\right)d(a+b\cosh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.206597, size = 84, normalized size = 0.98

$$\frac{\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{b\sinh(c+dx)}{(a-b)(a+b)(a+b\cosh(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-2),x]
```

```
[Out] ((2*a*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3
/2) - (b*Sinh[c + d*x])/((a - b)*(a + b)*(a + b*Cosh[c + d*x])))/d
```

Maple [A] time = 0.02, size = 118, normalized size = 1.4

$$\frac{1}{d}\left(2\,\frac{b\tanh\left(1/2\,dx+c/2\right)}{\left(a^2-b^2\right)\left(a\left(\tanh\left(1/2\,dx+c/2\right)\right)^2-\left(\tanh\left(1/2\,dx+c/2\right)\right)^2b-a-b\right)}+2\,\frac{a}{\left(a+b\right)\left(a-b\right)\sqrt{\left(a+b\right)\left(a-b\right)}}\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c))^2,x)
```

```
[Out] 1/d*(2*b/(a^2-b^2)*tanh(1/2*d*x+1/2*c)/(a*tanh(1/2*d*x+1/2*c)^2-tanh(1/2*d*
x+1/2*c)^2*b-a-b)+2*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.41934, size = 1787, normalized size = 20.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b - 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cosh
(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 - b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 - b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(
```

```
a^3 - a*b^2)*cosh(d*x + c) + 2*(a^3 - a*b^2)*sinh(d*x + c))/((a^4*b - 2*a^2
*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2
 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*
d + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4
)*d)*sinh(d*x + c)), 2*(a^2*b - b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x +
 c)^2 + 2*a^2*cosh(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x
 + c) + a)/(a^2 - b^2)) + (a^3 - a*b^2)*cosh(d*x + c) + (a^3 - a*b^2)*sinh(
d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b - 2*a^2*b^3
 + b^5)*d*sinh(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) + (
a^4*b - 2*a^2*b^3 + b^5)*d + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c) +
 (a^5 - 2*a^3*b^2 + a*b^4)*d)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23238, size = 138, normalized size = 1.6

$$\frac{2\,a\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^2d-b^2d\right)\sqrt{-a^2+b^2}}+\frac{2\left(ae^{(dx+c)}+b\right)}{\left(a^2d-b^2d\right)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^2*d - b^2*d)*sqrt(-a^2
 + b^2)) + 2*(a*e^(d*x + c) + b)/((a^2*d - b^2*d)*(b*e^(2*d*x + 2*c) + 2*a*
e^(d*x + c) + b))
```

3.69 $\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\sinh(c+dx)}{2d(a^2-b^2)^2(a+b\cosh(c+dx))}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

```
[Out] ((2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((a -
b)^(5/2)*(a + b)^(5/2)*d) - (b*Sinh[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cosh[
c + d*x])^2) - (3*a*b*Sinh[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cosh[c + d*x
]))
```

Rubi [A] time = 0.146591, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 205}

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\sinh(c+dx)}{2d(a^2-b^2)^2(a+b\cosh(c+dx))}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((a -
b)^(5/2)*(a + b)^(5/2)*d) - (b*Sinh[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cosh[
c + d*x])^2) - (3*a*b*Sinh[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cosh[c + d*x
]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{\int \frac{-2a+b\cosh(c+dx)}{(a+b\cosh(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))} + \frac{\int \frac{2a^2+b^2}{a+b\cosh(c+dx}}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))} + \frac{\left(2a^2+b^2\right)\int}{2\left(a^2\right.}$$

$$= -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))} - \frac{\left(i\left(2a^2+b^2\right)\right)}{}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab}{2\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 0.386033, size = 113, normalized size = 0.85

$$\frac{\frac{b\sinh(c+dx)\left(-4a^2-3ab\cosh(c+dx)+b^2\right)}{(a-b)^2(a+b)^2(a+b\cosh(c+dx))^2} - \frac{2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-3),x]
```

```
[Out] ((-2*(2*a^2 + b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-
a^2 + b^2)^(5/2) + (b*(-4*a^2 + b^2 - 3*a*b*Cosh[c + d*x])*Sinh[c + d*x])/(
(a - b)^2*(a + b)^2*(a + b*Cosh[c + d*x])^2))/(2*d)
```

Maple [A] time = 0.023, size = 186, normalized size = 1.4

$$\frac{1}{d}\left(-2\frac{1}{\left(a\left(\tanh\left(1/2\,dx+c/2\right)\right)^2-\left(\tanh\left(1/2\,dx+c/2\right)\right)^2b-a-b\right)^2}\left(-1/2\,\frac{(4\,a+b)\,b\left(\tanh\left(1/2\,dx+c/2\right)\right)^3}{(a-b)\left(a^2+2\,ab+b^2\right)}+1/2\,\frac{(4\,a-b}{(a+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/2*(4*
a-b)*b/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^2-
tanh(1/2*d*x+1/2*c)^2*b-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73872, size = 5711, normalized size = 42.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + 2*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^3 + 2
*(2*a^4*b - a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 - a^3*b^2 - a*b^4)*co
sh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*cosh
(d*x + c))*sinh(d*x + c)^2 + ((2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + (2*a^2*b^
2 + b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 + b^4 + 4*(2*a^3*b + a*b^3)*cosh(d*x +
 c)^3 + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)
^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 + 4*a^2*b^2 + b
^4 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c
))*sinh(d*x + c)^2 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c) + 4*(2*a^3*b + a*b^3
 + (2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)^2
+ (4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*l
og((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b*c
osh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
```

+ 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(10*a^4*b - 11*a^2*b^3 + b^5)*cosh(d*x + c) + 2*(10*a^4*b - 11*a^2*b^3 + b^5 + 3*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*sinh(d*x + c)^4 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d)*sinh(d*x + c)^3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + 2*(3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 6*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d + 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^3 + 3*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d)*sinh(d*x + c)), (3*a^3*b^2 - 3*a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^3 + (2*a^4*b - a^2*b^3 - b^5)*sinh(d*x + c)^3 + 3*(2*a^5 - a^3*b^2 - a*b^4)*cosh(d*x + c)^2 + 3*(2*a^5 - a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 - ((2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + (2*a^2*b^2 + b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 + b^4 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c) + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)^2 + (4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a)/(a^2 - b^2)) + (10*a^4*b - 11*a^2*b^3 + b^5)*cosh(d*x + c) + (10*a^4*b - 11*a^2*b^3 + b^5 + 3*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*sinh(d*x + c)^4 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d)*sinh(d*x + c)^3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + 2*(3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 6*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d + 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^3 + 3*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d)*sinh(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22912, size = 271, normalized size = 2.04

$$\frac{\left(2\,a^2+b^2\right)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^4d-2\,a^2b^2d+b^4d\right)\sqrt{-a^2+b^2}}+\frac{2\,a^2be^{(3\,dx+3\,c)}+b^3e^{(3\,dx+3\,c)}+6\,a^3e^{(2\,dx+2\,c)}+3\,ab^2e^{(2\,dx+2\,c)}+10\,a^2be^{(dx+c)}-b^3e^{(dx+c)}}{\left(a^4d-2\,a^2b^2d+b^4d\right)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (2*a^2 + b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^4*d - 2*a^2*
b^2*d + b^4*d)*sqrt(-a^2 + b^2)) + (2*a^2*b*e^(3*d*x + 3*c) + b^3*e^(3*d*x
+ 3*c) + 6*a^3*e^(2*d*x + 2*c) + 3*a*b^2*e^(2*d*x + 2*c) + 10*a^2*b*e^(d*x
+ c) - b^3*e^(d*x + c) + 3*a*b^2)/((a^4*d - 2*a^2*b^2*d + b^4*d)*(b*e^(2*d*
x + 2*c) + 2*a*e^(d*x + c) + b)^2)
```

3.70 $\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=184

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cosh(c+dx))}-\frac{5ab\sinh(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cosh(c+dx))^2}-\ldots 3$$

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(7/2)*(a + b)^(7/2)*d) - (b*Sinh[c + d*x])/(3*(a^2 - b^2)*d*(a + b*C
osh[c + d*x])^3) - (5*a*b*Sinh[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cosh[c +
 d*x])^2) - (b*(11*a^2 + 4*b^2)*Sinh[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Co
sh[c + d*x]))
```

Rubi [A] time = 0.253111, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 205}

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cosh(c+dx))}-\frac{5ab\sinh(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cosh(c+dx))^2}-\ldots 3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(7/2)*(a + b)^(7/2)*d) - (b*Sinh[c + d*x])/(3*(a^2 - b^2)*d*(a + b*C
osh[c + d*x])^3) - (5*a*b*Sinh[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cosh[c +
 d*x])^2) - (b*(11*a^2 + 4*b^2)*Sinh[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Co
sh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{\int \frac{-3a+2b\cosh(c+dx)}{(a+b\cosh(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))^2} + \frac{\int \frac{2\left(3a^2+2b^2\right)-5 \cdots}{(a+b\cos \cdots} \cdots}{6\left(a^2 \cdots\right.}$$

$$= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))^2} - \frac{b\left(11a^2+\cdots\right.}{6\left(a^2-b^2\right)^3 \cdots}$$

$$= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))^2} - \frac{b\left(11a^2+\cdots\right.}{6\left(a^2-b^2\right)^3 \cdots}$$

$$= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))^2} - \frac{b\left(11a^2+\cdots\right.}{6\left(a^2-b^2\right)^3 \cdots}$$

$$= \frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{\cdots}{6\left(a^2-b \cdots\right.}$$

Mathematica [A] time = 0.998127, size = 160, normalized size = 0.87

$$\frac{\frac{6a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} - \frac{b\sinh(c+dx)\left(6ab\left(9a^2+b^2\right)\cosh(c+dx)+\left(11a^2b^2+4b^4\right)\cosh(2(c+dx))+a^2b^2+36a^4+8b^4\right)}{2(a-b)^3(a+b)^3(a+b\cosh(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 + 3*b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])
/(-a^2 + b^2)^(7/2) - (b*(36*a^4 + a^2*b^2 + 8*b^4 + 6*a*b*(9*a^2 + b^2)*Co
sh[c + d*x] + (11*a^2*b^2 + 4*b^4)*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(2*(a
- b)^3*(a + b)^3*(a + b*Cosh[c + d*x])^3))/(6*d)
```

Maple [A] time = 0.024, size = 284, normalized size = 1.5

$$\frac{1}{d}\left(-2\,\frac{1}{\left(a\left(\tanh\left(1/2\,dx+c/2\right)\right)^{2}-\left(\tanh\left(1/2\,dx+c/2\right)\right)^{2}b-a-b\right)^{3}}\left(-1/2\,\frac{\left(6\,a^{2}+3\,ab+2\,b^{2}\right)b\left(\tanh\left(1/2\,dx+c/2\right)\right)^{5}}{\left(a-b\right)\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/
2*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tanh(
1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^2-tanh(1/2*d*x+1/2*c)^2*b-a-b)^3+a*(
2*a^2+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.95478, size = 12951, normalized size = 70.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/6*(22*a^4*b^3 - 14*a^2*b^5 - 8*b^7 + 6*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*c
osh(d*x + c)^5 + 6*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*sinh(d*x + c)^5 + 30*(2*
a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 + 30*(2*a^6*b + a^4*b^3 - 3*a^
2*b^5 + (2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*
```

(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c)^3 + 4*(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6 + 15*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^2 + 30*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 12*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7)*cosh(d*x + c)^2 + 12*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7 + 5*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^3 + 15*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^2 + (22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*((2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^6 + (2*a^3*b^3 + 3*a*b^5)*sinh(d*x + c)^6 + 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^5 + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 + 10*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^2 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 20*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^3 + 6*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c) + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^5 + 5*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^4 + 2*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 2*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c)^2 + (8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 30*(4*a^5*b^2 - 3*a^3*b^4 - a*b^6)*cosh(d*x + c) + 6*(20*a^5*b^2 - 15*a^3*b^4 - 5*a*b^6 + 5*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^4 + 20*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^3 + 2*(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c)^2 + 4*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7)*cosh(d*x + c))*sinh(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^6 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*sinh(d*x + c)^6 + 6*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^5 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^4 + 6*((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c) + (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c)^5 + 4*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d*cosh(d*x + c)^3 + 3*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^2 + 10*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d)*sinh(d*x + c)^4 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*

x + c)^3 + 15*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh
(d*x + c)^2 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*
cosh(d*x + c) + (2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d
)*sinh(d*x + c)^3 + 6*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10
)*d*cosh(d*x + c) + 3*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^1
1)*d*cosh(d*x + c)^4 + 20*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*
b^10)*d*cosh(d*x + c)^3 + 6*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^
7 + b^11)*d*cosh(d*x + c)^2 + 4*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b
^8 + 3*a*b^10)*d*cosh(d*x + c) + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a
^4*b^7 + b^11)*d)*sinh(d*x + c)^2 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^
2*b^9 + b^11)*d + 6*((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d
*cosh(d*x + c)^5 + 5*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)
*d*cosh(d*x + c)^4 + 2*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b
^11)*d*cosh(d*x + c)^3 + 2*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 +
3*a*b^10)*d*cosh(d*x + c)^2 + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*
b^7 + b^11)*d*cosh(d*x + c) + (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8
+ a*b^10)*d)*sinh(d*x + c)), 1/3*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 3*(2*a^5
*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^5 + 3*(2*a^5*b^2 + a^3*b^4 - 3*a*b^
6)*sinh(d*x + c)^5 + 15*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 + 1
5*(2*a^6*b + a^4*b^3 - 3*a^2*b^5 + (2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x
 + c))*sinh(d*x + c)^4 + 2*(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*co
sh(d*x + c)^3 + 2*(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6 + 15*(2*a^5*
b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^2 + 30*(2*a^6*b + a^4*b^3 - 3*a^2*b^
5)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 -
2*b^7)*cosh(d*x + c)^2 + 6*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7 + 5*(
2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^3 + 15*(2*a^6*b + a^4*b^3 - 3*
a^2*b^5)*cosh(d*x + c)^2 + (22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*co
sh(d*x + c))*sinh(d*x + c)^2 - 3*((2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^6 + (
2*a^3*b^3 + 3*a*b^5)*sinh(d*x + c)^6 + 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 +
 3*a^2*b^4)*cosh(d*x + c)^5 + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b
^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cos
h(d*x + c)^4 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*
cosh(d*x + c)^2 + 10*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4
 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 + 12*a^4*b
^2 + 9*a^2*b^4 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 +
3*a^2*b^4)*cosh(d*x + c)^2 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 +
3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4
 + 20*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^3 + 6*(8*a^5*b + 14*a^3*b^3 + 3
*a*b^5)*cosh(d*x + c)^2 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c))
*sinh(d*x + c)^2 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c) + 6*(2*a^4*b^2 +
 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^5 + 5*(2*a^4*b^2 + 3*a^2*b
^4)*cosh(d*x + c)^4 + 2*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 +
2*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c)^2 + (8*a^5*b + 14*a^3*b^3
+ 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2

```
 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a)/(a^2 - b^2)) + 15*(4*a^5*b^
2 - 3*a^3*b^4 - a*b^6)*cosh(d*x + c) + 3*(20*a^5*b^2 - 15*a^3*b^4 - 5*a*b^6
 + 5*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^4 + 20*(2*a^6*b + a^4*b^
3 - 3*a^2*b^5)*cosh(d*x + c)^3 + 2*(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a
*b^6)*cosh(d*x + c)^2 + 4*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7)*cosh(
d*x + c))*sinh(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*d*cosh(d*x + c)^6 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11
)*d*sinh(d*x + c)^6 + 6*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^
10)*d*cosh(d*x + c)^5 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7
+ b^11)*d*cosh(d*x + c)^4 + 6*((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9
 + b^11)*d*cosh(d*x + c) + (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a
*b^10)*d)*sinh(d*x + c)^5 + 4*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8
 + 3*a*b^10)*d*cosh(d*x + c)^3 + 3*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*
a^2*b^9 + b^11)*d*cosh(d*x + c)^2 + 10*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4
*a^3*b^8 + a*b^10)*d*cosh(d*x + c) + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 -
10*a^4*b^7 + b^11)*d)*sinh(d*x + c)^4 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b
^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a
^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^3 + 15*(a^9*b^2 - 4*a^7*b^4 + 6*
a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^2 + 3*(4*a^10*b - 15*a^8*b^3
+ 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c) + (2*a^11 - 5*a^9*b^2 + 1
0*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d)*sinh(d*x + c)^3 + 6*(a^9*b^2 - 4*a^7*
b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) + 3*(5*(a^8*b^3 - 4*a
^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^4 + 20*(a^9*b^2 - 4*
a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^3 + 6*(4*a^10*b -
 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^2 + 4*(2*a^11
 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d*cosh(d*x + c) + (4*a^1
0*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d)*sinh(d*x + c)^2 + (a^
8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d + 6*((a^8*b^3 - 4*a^6*b
^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^5 + 5*(a^9*b^2 - 4*a^7*b
^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^4 + 2*(4*a^10*b - 15*a
^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^3 + 2*(2*a^11 - 5*
a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d*cosh(d*x + c)^2 + (4*a^10*b
 - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c) + (a^9*b^2
- 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2372, size = 451, normalized size = 2.45

$$\frac{\left(2\,a^{3}+3\,ab^{2}\right)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\left(a^{6}d-3\,a^{4}b^{2}d+3\,a^{2}b^{4}d-b^{6}d\right)\sqrt{-a^{2}+b^{2}}}+\frac{6\,a^{3}b^{2}e^{(5\,dx+5\,c)}+9\,ab^{4}e^{(5\,dx+5\,c)}+30\,a^{4}be^{(4\,dx+4\,c)}+45\,a^{2}b^{3}e^{(4\,dx+4\,c)}+44\,a^{5}e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (2*a^3 + 3*a*b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^6*d - 3*
a^4*b^2*d + 3*a^2*b^4*d - b^6*d)*sqrt(-a^2 + b^2)) + 1/3*(6*a^3*b^2*e^(5*d*
x + 5*c) + 9*a*b^4*e^(5*d*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) + 45*a^2*b^3*
e^(4*d*x + 4*c) + 44*a^5*e^(3*d*x + 3*c) + 82*a^3*b^2*e^(3*d*x + 3*c) + 24*
a*b^4*e^(3*d*x + 3*c) + 102*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2
*c) + 12*b^5*e^(2*d*x + 2*c) + 60*a^3*b^2*e^(d*x + c) + 15*a*b^4*e^(d*x + c
) + 11*a^2*b^3 + 4*b^5)/((a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)*(b*e^(
2*d*x + 2*c) + 2*a*e^(d*x + c) + b)^3)
```

3.71 $\int \frac{1}{3+5\cosh(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

[Out] ArcTan[Tanh[(c + d*x)/2]/2]/(2*d)

Rubi [A] time = 0.0147783, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2659, 206}

$$\frac{\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(3 + 5*Cosh[c + d*x])^(-1),x]

[Out] ArcTan[Tanh[(c + d*x)/2]/2]/(2*d)

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{8-2x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Mathematica [A] time = 0.0344725, size = 23, normalized size = 1.05

$$-\frac{\tan^{-1}\left(2\coth\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-1),x]
```

```
[Out] -ArcTan[2*Coth[c/2 + (d*x)/2]]/(2*d)
```

Maple [A] time = 0.01, size = 18, normalized size = 0.8

$$\frac{1}{2d}\arctan\left(\frac{1}{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c)),x)
```

```
[Out] 1/2*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

Maxima [A] time = 1.56405, size = 26, normalized size = 1.18

$$-\frac{\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(5/4*e^(-d*x - c) + 3/4)/d
```

Fricas [A] time = 2.28335, size = 80, normalized size = 3.64

$$\frac{\arctan\left(\frac{5}{4}\cosh\left(dx+c\right)+\frac{5}{4}\sinh\left(dx+c\right)+\frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c) + 3/4)/d
```

Sympy [A] time = 0.863382, size = 24, normalized size = 1.09

$$\begin{cases} \frac{\operatorname{atan}{\left(\frac{\tanh{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{2}\right)}}{2d} & \text{for}\: d \neq 0 \\ \frac{x}{5\cosh{\left(c\right)}+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x)
```

```
[Out] Piecewise((atan(tanh(c/2 + d*x/2)/2)/(2*d), Ne(d, 0)), (x/(5*cosh(c) + 3),
True))
```

Giac [A] time = 1.19863, size = 22, normalized size = 1.

$$\frac{\arctan\left(\frac{5}{4}\,e^{\left(dx+c\right)}+\frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(5/4*e^(d*x + c) + 3/4)/d
```

3.72 $\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}$$

```
[Out] (-3*ArcTan[Tanh[(c + d*x)/2]/2])/(32*d) + (5*Sinh[c + d*x])/(16*d*(3 + 5*Co
sh[c + d*x]))
```

Rubi [A] time = 0.0316992, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 12, 2659, 206}

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-2),x]
```

```
[Out] (-3*ArcTan[Tanh[(c + d*x)/2]/2])/(32*d) + (5*Sinh[c + d*x])/(16*d*(3 + 5*Co
sh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx &= \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}+\frac{1}{16}\int -\frac{3}{3+5\cosh(c+dx)}\,dx\\ &= \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}-\frac{3}{16}\int \frac{1}{3+5\cosh(c+dx)}\,dx\\ &= \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}+\frac{(3i)\,\text{Subst}\left(\int \frac{1}{8-2x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{8d}\\ &= -\frac{3\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}+\frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.100258, size = 45, normalized size = 0.94

$$\frac{\frac{10\sinh(c+dx)}{5\cosh(c+dx)+3}-3\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-2),x]
```

```
[Out] (-3*ArcTan[Tanh[(c + d*x)/2]/2] + (10*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x]))
/(32*d)
```

Maple [A] time = 0.015, size = 48, normalized size = 1.

$$\frac{5}{16\,d}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+4\right)^{-1}-\frac{3}{32\,d}\arctan\left(\frac{1}{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c))^2,x)
```

```
[Out] 5/16/d*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+4)-3/32*arctan(1/2*tanh(1
/2*d*x+1/2*c))/d
```

Maxima [A] time = 1.53041, size = 86, normalized size = 1.79

$$\frac{3\arctan\left(\frac{5}{4}e^{(-dx-c)}+\frac{3}{4}\right)}{32\,d}+\frac{3\,e^{(-dx-c)}+5}{8\,d\left(6\,e^{(-dx-c)}+5\,e^{(-2\,dx-2\,c)}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/32*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/8*(3*e^(-d*x - c) + 5)/(d*(6*e^(-
d*x - c) + 5*e^(-2*d*x - 2*c) + 5))
```

Fricas [B] time = 2.09781, size = 435, normalized size = 9.06

$$-\frac{3\left(5\,\cosh(dx+c)^2+2\,(5\,\cosh(dx+c)+3)\sinh(dx+c)+5\,\sinh(dx+c)^2+6\,\cosh(dx+c)+5\right)\arctan\left(\frac{5}{4}\,\cosh(\right.}{32\left(5\,d\cosh(dx+c)^2+5\,d\sinh(dx+c)^2+6\,d\cosh(dx+c)+2\,(5\,d\cosh(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/32*(3*(5*cosh(d*x + c)^2 + 2*(5*cosh(d*x + c) + 3)*sinh(d*x + c) + 5*sin
h(d*x + c)^2 + 6*cosh(d*x + c) + 5)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x
 + c) + 3/4) + 12*cosh(d*x + c) + 12*sinh(d*x + c) + 20)/(5*d*cosh(d*x + c)
^2 + 5*d*sinh(d*x + c)^2 + 6*d*cosh(d*x + c) + 2*(5*d*cosh(d*x + c) + 3*d)*
sinh(d*x + c) + 5*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.15567, size = 74, normalized size = 1.54

$$-\frac{3\,\arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right)}{32\,d}-\frac{3\,e^{(dx+c)}+5}{8\,d\left(5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -3/32*arctan(5/4*e^(d*x + c) + 3/4)/d - 1/8*(3*e^(d*x + c) + 5)/(d*(5*e^(2*
d*x + 2*c) + 6*e^(d*x + c) + 5))
```

3.73 $\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=73

$$\frac{43\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d}-\frac{45\sinh(c+dx)}{512d(5\cosh(c+dx)+3)}+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

```
[Out] (43*ArcTan[Tanh[(c + d*x)/2]/2])/(1024*d) + (5*Sinh[c + d*x])/(32*d*(3 + 5*
Cosh[c + d*x])^2) - (45*Sinh[c + d*x])/(512*d*(3 + 5*Cosh[c + d*x]))
```

Rubi [A] time = 0.0646426, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 206}

$$\frac{43\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d}-\frac{45\sinh(c+dx)}{512d(5\cosh(c+dx)+3)}+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-3),x]
```

```
[Out] (43*ArcTan[Tanh[(c + d*x)/2]/2])/(1024*d) + (5*Sinh[c + d*x])/(32*d*(3 + 5*
Cosh[c + d*x])^2) - (45*Sinh[c + d*x])/(512*d*(3 + 5*Cosh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
```

```
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx &= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}+\frac{1}{32}\int \frac{-6+5\cosh(c+dx)}{(3+5\cosh(c+dx))^2}\,dx\\
&= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}+\frac{1}{512}\int \frac{43}{3+5\cosh(c+dx)}\,dx\\
&= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}+\frac{43}{512}\int \frac{1}{3+5\cosh(c+dx)}\,dx\\
&= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}-\frac{(43i)\,\mathrm{Subst}\left(\int \frac{1}{8-2x^2}\,dx,x,\tan\left(\frac{1}{2}\right.\right.}{256d}\\
&= \frac{43\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d}+\frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.152716, size = 55, normalized size = 0.75

$$\frac{43\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{10\sinh(c+dx)(45\cosh(c+dx)+11)}{(5\cosh(c+dx)+3)^2}}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-3),x]
```

```
[Out] (43*ArcTan[Tanh[(c + d*x)/2]/2] - (10*(11 + 45*Cosh[c + d*x])*Sinh[c + d*x]
)/(3 + 5*Cosh[c + d*x])^2)/(1024*d)
```

Maple [A] time = 0.016, size = 79, normalized size = 1.1

$$-\frac{85}{512\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}+4\right)^{-2}-\frac{35}{128\,d}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}+4\right)^{-2}+\frac{43}{1024\,d}\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c))^3,x)
```

```
[Out] -85/512/d/(tanh(1/2*d*x+1/2*c)^2+4)^2*tanh(1/2*d*x+1/2*c)^3-35/128/d/(tanh(
1/2*d*x+1/2*c)^2+4)^2*tanh(1/2*d*x+1/2*c)+43/1024*arctan(1/2*tanh(1/2*d*x+1
/2*c))/d
```

Maxima [A] time = 1.55, size = 146, normalized size = 2.

$$-\frac{43\,\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{1024\,d}-\frac{325\,e^{(-dx-c)}+387\,e^{(-2\,dx-2\,c)}+215\,e^{(-3\,dx-3\,c)}+225}{256\,d\left(60\,e^{(-dx-c)}+86\,e^{(-2\,dx-2\,c)}+60\,e^{(-3\,dx-3\,c)}+25\,e^{(-4\,dx-4\,c)}+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -43/1024*arctan(5/4*e^(-d*x - c) + 3/4)/d - 1/256*(325*e^(-d*x - c) + 387*e
^(-2*d*x - 2*c) + 215*e^(-3*d*x - 3*c) + 225)/(d*(60*e^(-d*x - c) + 86*e^(-
2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 25*e^(-4*d*x - 4*c) + 25))
```

Fricas [B] time = 2.21371, size = 1214, normalized size = 16.63

$$\frac{860\,\cosh\left(dx+c\right)^{3}+516\left(5\,\cosh\left(dx+c\right)+3\right)\sinh\left(dx+c\right)^{2}+860\,\sinh\left(dx+c\right)^{3}+43\left(25\,\cosh\left(dx+c\right)^{4}+20\left(5\,\cos\right.\right.}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(860*cosh(d*x + c)^3 + 516*(5*cosh(d*x + c) + 3)*sinh(d*x + c)^2 + 8
60*sinh(d*x + c)^3 + 43*(25*cosh(d*x + c)^4 + 20*(5*cosh(d*x + c) + 3)*sinh
(d*x + c)^3 + 25*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(75*cosh(d*x + c)
^2 + 90*cosh(d*x + c) + 43)*sinh(d*x + c)^2 + 86*cosh(d*x + c)^2 + 4*(25*co
sh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 43*cosh(d*x + c) + 15)*sinh(d*x + c) +
 60*cosh(d*x + c) + 25)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c) + 3/4)
 + 1548*cosh(d*x + c)^2 + 4*(645*cosh(d*x + c)^2 + 774*cosh(d*x + c) + 325)
*sinh(d*x + c) + 1300*cosh(d*x + c) + 900)/(25*d*cosh(d*x + c)^4 + 25*d*sin
h(d*x + c)^4 + 60*d*cosh(d*x + c)^3 + 20*(5*d*cosh(d*x + c) + 3*d)*sinh(d*x
 + c)^3 + 86*d*cosh(d*x + c)^2 + 2*(75*d*cosh(d*x + c)^2 + 90*d*cosh(d*x +
c) + 43*d)*sinh(d*x + c)^2 + 60*d*cosh(d*x + c) + 4*(25*d*cosh(d*x + c)^3 +
 45*d*cosh(d*x + c)^2 + 43*d*cosh(d*x + c) + 15*d)*sinh(d*x + c) + 25*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5\cosh(c+dx)+3)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))**3,x)
```

```
[Out] Integral((5*cosh(c + d*x) + 3)**(-3), x)
```

Giac [A] time = 1.13102, size = 104, normalized size = 1.42

$$\frac{43\arctan\left(\frac{5}{4}e^{(dx+c)}+\frac{3}{4}\right)}{1024\,d}+\frac{215\,e^{(3\,dx+3\,c)}+387\,e^{(2\,dx+2\,c)}+325\,e^{(dx+c)}+225}{256\,d\left(5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 43/1024*arctan(5/4*e^(d*x + c) + 3/4)/d + 1/256*(215*e^(3*d*x + 3*c) + 387*
e^(2*d*x + 2*c) + 325*e^(d*x + c) + 225)/(d*(5*e^(2*d*x + 2*c) + 6*e^(d*x +
```

```
c) + 5)^2)
```

3.74 $\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=98

$$-\frac{279\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d}+\frac{995\sinh(c+dx)}{24576d(5\cosh(c+dx)+3)}-\frac{25\sinh(c+dx)}{512d(5\cosh(c+dx)+3)^2}+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)}$$

```
[Out] (-279*ArcTan[Tanh[(c + d*x)/2]/2])/(16384*d) + (5*Sinh[c + d*x])/(48*d*(3 +
 5*Cosh[c + d*x])^3) - (25*Sinh[c + d*x])/(512*d*(3 + 5*Cosh[c + d*x])^2) +
 (995*Sinh[c + d*x])/(24576*d*(3 + 5*Cosh[c + d*x]))
```

Rubi [A] time = 0.097106, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 206}

$$-\frac{279\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d}+\frac{995\sinh(c+dx)}{24576d(5\cosh(c+dx)+3)}-\frac{25\sinh(c+dx)}{512d(5\cosh(c+dx)+3)^2}+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-4),x]
```



```
[Out] (-279*ArcTan[Tanh[(c + d*x)/2]/2])/(16384*d) + (5*Sinh[c + d*x])/(48*d*(3 +
 5*Cosh[c + d*x])^3) - (25*Sinh[c + d*x])/(512*d*(3 + 5*Cosh[c + d*x])^2) +
 (995*Sinh[c + d*x])/(24576*d*(3 + 5*Cosh[c + d*x]))
```


Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
```

```
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx &= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}+\frac{1}{48}\int \frac{-9+10\cosh(c+dx)}{(3+5\cosh(c+dx))^3}\,dx\\
&= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2}+\frac{\int \frac{154-75\cosh(c+dx)}{(3+5\cosh(c+dx))^2}\,dx}{1536}\\
&= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2}+\frac{995\sinh(c+dx)}{24576d(3+5\cosh(c+dx))}+\\
&= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2}+\frac{995\sinh(c+dx)}{24576d(3+5\cosh(c+dx))}-\\
&= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2}+\frac{995\sinh(c+dx)}{24576d(3+5\cosh(c+dx))}+\\
&= -\frac{279\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d}+\frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx}
\end{aligned}$$

Mathematica [A] time = 0.245403, size = 65, normalized size = 0.66

$$\frac{\frac{5\sinh(c+dx)(9540\cosh(c+dx)+4975\cosh(2(c+dx))+8141)}{(5\cosh(c+dx)+3)^3}-837\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{49152d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-4),x]
```

```
[Out] (-837*ArcTan[Tanh[(c + d*x)/2]/2] + (5*(8141 + 9540*Cosh[c + d*x] + 4975*Co
sh[2*(c + d*x)])*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x])^3)/(49152*d)
```

Maple [A] time = 0.016, size = 110, normalized size = 1.1

$$\frac{745}{8192\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}+4\right)^{-3}+\frac{265}{768\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}+4\right)^{-3}+\frac{295}{512\,d}\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c))^4,x)
```

```
[Out] 745/8192/d/(tanh(1/2*d*x+1/2*c)^2+4)^3*tanh(1/2*d*x+1/2*c)^5+265/768/d/(tan
h(1/2*d*x+1/2*c)^2+4)^3*tanh(1/2*d*x+1/2*c)^3+295/512/d/(tanh(1/2*d*x+1/2*c
)^2+4)^3*tanh(1/2*d*x+1/2*c)-279/16384*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

Maxima [A] time = 1.57465, size = 205, normalized size = 2.09

$$\frac{279\,\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{16384\,d}+\frac{68625\,e^{(-dx-c)}+119310\,e^{(-2\,dx-2\,c)}+111042\,e^{(-3\,dx-3\,c)}+62775\,e^{(-4\,dx-4\,c)}+20925\,e^{(-5}}{12288\,d\left(450\,e^{(-dx-c)}+915\,e^{(-2\,dx-2\,c)}+1116\,e^{(-3\,dx-3\,c)}+915\,e^{(-4\,dx-4\,c)}+450\,e^{(-5\,dx-5\,c)}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 279/16384*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/12288*(68625*e^(-d*x - c) +
119310*e^(-2*d*x - 2*c) + 111042*e^(-3*d*x - 3*c) + 62775*e^(-4*d*x - 4*c)
+ 20925*e^(-5*d*x - 5*c) + 24875)/(d*(450*e^(-d*x - c) + 915*e^(-2*d*x - 2*
```

```
c) + 1116*e^(-3*d*x - 3*c) + 915*e^(-4*d*x - 4*c) + 450*e^(-5*d*x - 5*c) +
125*e^(-6*d*x - 6*c) + 125))
```

Fricas [B] time = 2.48881, size = 2473, normalized size = 25.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/49152*(83700*cosh(d*x + c)^5 + 83700*(5*cosh(d*x + c) + 3)*sinh(d*x + c)
^4 + 83700*sinh(d*x + c)^5 + 251100*cosh(d*x + c)^4 + 2232*(375*cosh(d*x +
c)^2 + 450*cosh(d*x + c) + 199)*sinh(d*x + c)^3 + 444168*cosh(d*x + c)^3 +
24*(34875*cosh(d*x + c)^3 + 62775*cosh(d*x + c)^2 + 55521*cosh(d*x + c) + 1
9885)*sinh(d*x + c)^2 + 837*(125*cosh(d*x + c)^6 + 150*(5*cosh(d*x + c) + 3
)*sinh(d*x + c)^5 + 125*sinh(d*x + c)^6 + 450*cosh(d*x + c)^5 + 15*(125*cos
h(d*x + c)^2 + 150*cosh(d*x + c) + 61)*sinh(d*x + c)^4 + 915*cosh(d*x + c)^
4 + 4*(625*cosh(d*x + c)^3 + 1125*cosh(d*x + c)^2 + 915*cosh(d*x + c) + 279
)*sinh(d*x + c)^3 + 1116*cosh(d*x + c)^3 + 3*(625*cosh(d*x + c)^4 + 1500*co
sh(d*x + c)^3 + 1830*cosh(d*x + c)^2 + 1116*cosh(d*x + c) + 305)*sinh(d*x +
 c)^2 + 915*cosh(d*x + c)^2 + 6*(125*cosh(d*x + c)^5 + 375*cosh(d*x + c)^4
+ 610*cosh(d*x + c)^3 + 558*cosh(d*x + c)^2 + 305*cosh(d*x + c) + 75)*sinh(
d*x + c) + 450*cosh(d*x + c) + 125)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x
 + c) + 3/4) + 477240*cosh(d*x + c)^2 + 12*(34875*cosh(d*x + c)^4 + 83700*c
osh(d*x + c)^3 + 111042*cosh(d*x + c)^2 + 79540*cosh(d*x + c) + 22875)*sinh
(d*x + c) + 274500*cosh(d*x + c) + 99500)/(125*d*cosh(d*x + c)^6 + 125*d*si
nh(d*x + c)^6 + 450*d*cosh(d*x + c)^5 + 150*(5*d*cosh(d*x + c) + 3*d)*sinh(
d*x + c)^5 + 915*d*cosh(d*x + c)^4 + 15*(125*d*cosh(d*x + c)^2 + 150*d*cosh
(d*x + c) + 61*d)*sinh(d*x + c)^4 + 1116*d*cosh(d*x + c)^3 + 4*(625*d*cosh(
d*x + c)^3 + 1125*d*cosh(d*x + c)^2 + 915*d*cosh(d*x + c) + 279*d)*sinh(d*x
 + c)^3 + 915*d*cosh(d*x + c)^2 + 3*(625*d*cosh(d*x + c)^4 + 1500*d*cosh(d*
x + c)^3 + 1830*d*cosh(d*x + c)^2 + 1116*d*cosh(d*x + c) + 305*d)*sinh(d*x
+ c)^2 + 450*d*cosh(d*x + c) + 6*(125*d*cosh(d*x + c)^5 + 375*d*cosh(d*x +
c)^4 + 610*d*cosh(d*x + c)^3 + 558*d*cosh(d*x + c)^2 + 305*d*cosh(d*x + c)
+ 75*d)*sinh(d*x + c) + 125*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\cosh\left(c + dx\right) + 3\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))**4,x)
```

```
[Out] Integral((5*cosh(c + d*x) + 3)**(-4), x)
```

Giac [A] time = 1.13231, size = 134, normalized size = 1.37

$$-\frac{279 \arctan\left(\frac{5}{4} e^{(dx+c)} + \frac{3}{4}\right)}{16384\, d} - \frac{20925\, e^{(5\, dx+5\, c)} + 62775\, e^{(4\, dx+4\, c)} + 111042\, e^{(3\, dx+3\, c)} + 119310\, e^{(2\, dx+2\, c)} + 68625\, e^{(dx+c)} + 2}{12288\, d\left(5\, e^{(2\, dx+2\, c)} + 6\, e^{(dx+c)} + 5\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -279/16384*arctan(5/4*e^(d*x + c) + 3/4)/d - 1/12288*(20925*e^(5*d*x + 5*c)
 + 62775*e^(4*d*x + 4*c) + 111042*e^(3*d*x + 3*c) + 119310*e^(2*d*x + 2*c)
+ 68625*e^(d*x + c) + 24875)/(d*(5*e^(2*d*x + 2*c) + 6*e^(d*x + c) + 5)^3)
```

3.75 $\int \frac{1}{5+3\cosh(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{4}-\frac{\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}$$

```
[Out] x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)
```

Rubi [A] time = 0.0128323, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2657}

$$\frac{x}{4}-\frac{\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-1),x]
```

```
[Out] x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx=\frac{x}{4}-\frac{\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{2d}$$

Mathematica [B] time = 0.0291241, size = 77, normalized size = 2.48

$$\frac{\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}-\frac{\log\left(2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)-\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-1),x]
```

```
[Out] -Log[2*Cosh[c/2 + (d*x)/2] - Sinh[c/2 + (d*x)/2]]/(4*d) + Log[2*Cosh[c/2 +
(d*x)/2] + Sinh[c/2 + (d*x)/2]]/(4*d)
```

Maple [A] time = 0.014, size = 36, normalized size = 1.2

$$\frac{1}{4d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)-\frac{1}{4d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c)),x)
```

```
[Out] 1/4/d*ln(tanh(1/2*d*x+1/2*c)+2)-1/4/d*ln(tanh(1/2*d*x+1/2*c)-2)
```

Maxima [A] time = 1.03287, size = 50, normalized size = 1.61

$$-\frac{\log\left(3\,e^{(-dx-c)}+1\right)}{4\,d}+\frac{\log\left(e^{(-dx-c)}+3\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*log(3*e^(-d*x - c) + 1)/d + 1/4*log(e^(-d*x - c) + 3)/d
```

Fricas [A] time = 2.32233, size = 126, normalized size = 4.06

$$\frac{\log\left(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)+1\right)-\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+3\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh(
d*x + c) + 3))/d
```

Sympy [A] time = 0.726658, size = 41, normalized size = 1.32

$$\begin{cases} -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{4d}+\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{4d} & \text{for } d\neq 0 \\ \frac{x}{3\cosh(c)+5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x)
```

```
[Out] Piecewise((-log(tanh(c/2 + d*x/2) - 2)/(4*d) + log(tanh(c/2 + d*x/2) + 2)/(
4*d), Ne(d, 0)), (x/(3*cosh(c) + 5), True))
```

Giac [A] time = 1.16046, size = 42, normalized size = 1.35

$$\frac{\log\left(3\,e^{(dx+c)}+1\right)}{4\,d}-\frac{\log\left(e^{(dx+c)}+3\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*log(3*e^(d*x + c) + 1)/d - 1/4*log(e^(d*x + c) + 3)/d
```

3.76 $\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}-\frac{5\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{32d}+\frac{5x}{64}$$

```
[Out] (5*x)/64 - (5*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(32*d) - (3*Sinh[
c + d*x])/(16*d*(5 + 3*Cosh[c + d*x]))
```

Rubi [A] time = 0.0358887, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2664, 12, 2657}

$$-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}-\frac{5\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{32d}+\frac{5x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 - (5*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(32*d) - (3*Sinh[
c + d*x])/(16*d*(5 + 3*Cosh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
```

```
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx &= -\frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))} - \frac{1}{16}\int -\frac{5}{5+3\cosh(c+dx)}\,dx\\ &= -\frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))} + \frac{5}{16}\int \frac{1}{5+3\cosh(c+dx)}\,dx\\ &= \frac{5x}{64} - \frac{5\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{32d} - \frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.112503, size = 45, normalized size = 0.8

$$\frac{5\tanh^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \frac{6\sinh(c+dx)}{3\cosh(c+dx)+5}}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-2),x]
```

```
[Out] (5*ArcTanh[Tanh[(c + d*x)/2]/2] - (6*Sinh[c + d*x])/(5 + 3*Cosh[c + d*x]))/
(32*d)
```

Maple [A] time = 0.016, size = 72, normalized size = 1.3

$$\frac{3}{32\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-1} + \frac{5}{64\,d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right) + \frac{3}{32\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{-1} - \frac{5}{64\,d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c))^2,x)
```

```
[Out] 3/32/d/(tanh(1/2*d*x+1/2*c)+2)+5/64/d*ln(tanh(1/2*d*x+1/2*c)+2)+3/32/d/(tan
h(1/2*d*x+1/2*c)-2)-5/64/d*ln(tanh(1/2*d*x+1/2*c)-2)
```

Maxima [A] time = 1.04265, size = 109, normalized size = 1.95

$$-\frac{5\,\log\left(3\,e^{(-dx-c)}+1\right)}{64\,d}+\frac{5\,\log\left(e^{(-dx-c)}+3\right)}{64\,d}-\frac{5\,e^{(-dx-c)}+3}{8\,d\left(10\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -5/64*log(3*e^(-d*x - c) + 1)/d + 5/64*log(e^(-d*x - c) + 3)/d - 1/8*(5*e^(
-d*x - c) + 3)/(d*(10*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + 3))
```

Fricas [B] time = 2.57343, size = 621, normalized size = 11.09

$$\frac{5\left(3\,\cosh\left(dx+c\right)^{2}+2\left(3\,\cosh\left(dx+c\right)+5\right)\sinh\left(dx+c\right)+3\,\sinh\left(dx+c\right)^{2}+10\,\cosh\left(dx+c\right)+3\right)\log\left(3\,\cosh\left(dx+c\right.\right.}{64\left(3\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/64*(5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*sinh
(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(3*cosh(d*x + c) + 3*sinh(d*x + c) +
 1) - 5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*sinh
(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(cosh(d*x + c) + sinh(d*x + c) + 3)
+ 40*cosh(d*x + c) + 40*sinh(d*x + c) + 24)/(3*d*cosh(d*x + c)^2 + 3*d*sinh
(d*x + c)^2 + 10*d*cosh(d*x + c) + 2*(3*d*cosh(d*x + c) + 5*d)*sinh(d*x + c
) + 3*d)
```

Sympy [A] time = 2.42525, size = 199, normalized size = 3.55

$$\begin{cases}-\frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-256d}+\frac{20\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{64d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-256d}+\frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-256d}-\frac{20\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{64d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-256d}+\frac{12\,\mathrm{ta}}{64d\,\mathrm{tanh}} \\ \frac{x}{(3\cosh(c)+5)^{2}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((-5*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(64*d*tanh(c/
2 + d*x/2)**2 - 256*d) + 20*log(tanh(c/2 + d*x/2) - 2)/(64*d*tanh(c/2 + d*x
/2)**2 - 256*d) + 5*log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(64*d*t
anh(c/2 + d*x/2)**2 - 256*d) - 20*log(tanh(c/2 + d*x/2) + 2)/(64*d*tanh(c/2
 + d*x/2)**2 - 256*d) + 12*tanh(c/2 + d*x/2)/(64*d*tanh(c/2 + d*x/2)**2 - 2
56*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**2, True))
```

Giac [A] time = 1.15713, size = 93, normalized size = 1.66

$$\frac{5\,\log\left(3\,e^{(dx+c)}+1\right)}{64\,d}-\frac{5\,\log\left(e^{(dx+c)}+3\right)}{64\,d}+\frac{5\,e^{(dx+c)}+3}{8\,d\left(3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/64*log(3*e^(d*x + c) + 1)/d - 5/64*log(e^(d*x + c) + 3)/d + 1/8*(5*e^(d*x
 + c) + 3)/(d*(3*e^(2*d*x + 2*c) + 10*e^(d*x + c) + 3))
```

3.77 $\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=81

$$-\frac{45\sinh(c+dx)}{512d(3\cosh(c+dx)+5)}-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}-\frac{59\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{1024d}+\frac{59x}{2048}$$

```
[Out] (59*x)/2048 - (59*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(1024*d) - (3
*Sinh[c + d*x])/(32*d*(5 + 3*Cosh[c + d*x])^2) - (45*Sinh[c + d*x])/(512*d*
(5 + 3*Cosh[c + d*x]))
```

Rubi [A] time = 0.0631422, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 2754, 12, 2657}

$$-\frac{45\sinh(c+dx)}{512d(3\cosh(c+dx)+5)}-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}-\frac{59\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{1024d}+\frac{59x}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-3),x]
```

```
[Out] (59*x)/2048 - (59*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(1024*d) - (3
*Sinh[c + d*x])/(32*d*(5 + 3*Cosh[c + d*x])^2) - (45*Sinh[c + d*x])/(512*d*
(5 + 3*Cosh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
```

```
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx &= -\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2}-\frac{1}{32}\int \frac{-10+3\cosh(c+dx)}{(5+3\cosh(c+dx))^2}\,dx\\
&= -\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))}+\frac{1}{512}\int \frac{59}{5+3\cosh(c+dx)}\,dx\\
&= -\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))}+\frac{59}{512}\int \frac{1}{5+3\cosh(c+dx)}\,dx\\
&= \frac{59x}{2048}-\frac{59\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{1024d}-\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.187694, size = 58, normalized size = 0.72

$$\frac{59\tanh^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{3(182\sinh(c+dx)+45\sinh(2(c+dx)))}{(3\cosh(c+dx)+5)^2}}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-3),x]
```

```
[Out] (59*ArcTanh[Tanh[(c + d*x)/2]/2] - (3*(182*Sinh[c + d*x] + 45*Sinh[2*(c + d
*x)]))/(5 + 3*Cosh[c + d*x])^2)/(1024*d)
```

Maple [A] time = 0.017, size = 108, normalized size = 1.3

$$-\frac{9}{512\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-2}+\frac{69}{1024\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-1}+\frac{59}{2048\,d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)+\frac{9}{512\,d}\left(\tanh\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c))^3,x)
```

```
[Out] -9/512/d/(tanh(1/2*d*x+1/2*c)+2)^2+69/1024/d/(tanh(1/2*d*x+1/2*c)+2)+59/204
8/d*ln(tanh(1/2*d*x+1/2*c)+2)+9/512/d/(tanh(1/2*d*x+1/2*c)-2)^2+69/1024/d/(
tanh(1/2*d*x+1/2*c)-2)-59/2048/d*ln(tanh(1/2*d*x+1/2*c)-2)
```

Maxima [A] time = 1.03263, size = 169, normalized size = 2.09

$$-\frac{59\,\log\left(3\,e^{(-dx-c)}+1\right)}{2048\,d}+\frac{59\,\log\left(e^{(-dx-c)}+3\right)}{2048\,d}-\frac{3\left(241\,e^{(-dx-c)}+295\,e^{(-2\,dx-2\,c)}+59\,e^{(-3\,dx-3\,c)}+45\right)}{256\,d\left(60\,e^{(-dx-c)}+118\,e^{(-2\,dx-2\,c)}+60\,e^{(-3\,dx-3\,c)}+9\,e^{(-4\,dx-4\,c)}+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -59/2048*log(3*e^(-d*x - c) + 1)/d + 59/2048*log(e^(-d*x - c) + 3)/d - 3/25
6*(241*e^(-d*x - c) + 295*e^(-2*d*x - 2*c) + 59*e^(-3*d*x - 3*c) + 45)/(d*(
60*e^(-d*x - c) + 118*e^(-2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 9*e^(-4*d*x
- 4*c) + 9))
```

Fricas [B] time = 2.17252, size = 1661, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2048*(1416*cosh(d*x + c)^3 + 1416*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^2 +
 1416*sinh(d*x + c)^3 + 7080*cosh(d*x + c)^2 + 59*(9*cosh(d*x + c)^4 + 12*(
3*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x + c)
```

```
^3 + 2*(27*cosh(d*x + c)^2 + 90*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 118*c
osh(d*x + c)^2 + 4*(9*cosh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d*x +
c) + 15)*sinh(d*x + c) + 60*cosh(d*x + c) + 9)*log(3*cosh(d*x + c) + 3*sinh
(d*x + c) + 1) - 59*(9*cosh(d*x + c)^4 + 12*(3*cosh(d*x + c) + 5)*sinh(d*x
+ c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(27*cosh(d*x + c)^2 + 9
0*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 118*cosh(d*x + c)^2 + 4*(9*cosh(d*x
 + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d*x + c) + 15)*sinh(d*x + c) + 60*co
sh(d*x + c) + 9)*log(cosh(d*x + c) + sinh(d*x + c) + 3) + 24*(177*cosh(d*x
+ c)^2 + 590*cosh(d*x + c) + 241)*sinh(d*x + c) + 5784*cosh(d*x + c) + 1080
)/(9*d*cosh(d*x + c)^4 + 9*d*sinh(d*x + c)^4 + 60*d*cosh(d*x + c)^3 + 12*(3
*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^3 + 118*d*cosh(d*x + c)^2 + 2*(27*d*c
osh(d*x + c)^2 + 90*d*cosh(d*x + c) + 59*d)*sinh(d*x + c)^2 + 60*d*cosh(d*x
 + c) + 4*(9*d*cosh(d*x + c)^3 + 45*d*cosh(d*x + c)^2 + 59*d*cosh(d*x + c)
+ 15*d)*sinh(d*x + c) + 9*d)
```

Sympy [A] time = 5.73531, size = 445, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((-59*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**4/(2048*d*tanh
(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 472*log(tanh(c
/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*
d*tanh(c/2 + d*x/2)**2 + 32768*d) - 944*log(tanh(c/2 + d*x/2) - 2)/(2048*d*
tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 59*log(tan
h(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(2048*d*tanh(c/2 + d*x/2)**4 - 163
84*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 472*log(tanh(c/2 + d*x/2) + 2)*tanh(
c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2
 + 32768*d) + 944*log(tanh(c/2 + d*x/2) + 2)/(2048*d*tanh(c/2 + d*x/2)**4 -
 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 276*tanh(c/2 + d*x/2)**3/(2048*d
*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 816*tanh(
c/2 + d*x/2)/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 +
32768*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**3, True))
```

Giac [A] time = 1.15086, size = 123, normalized size = 1.52

$$\frac{59 \log\left(3 e^{(dx+c)} + 1\right)}{2048 d} - \frac{59 \log\left(e^{(dx+c)} + 3\right)}{2048 d} + \frac{3 \left(59 e^{(3 dx+3 c)} + 295 e^{(2 dx+2 c)} + 241 e^{(dx+c)} + 45\right)}{256 d\left(3 e^{(2 dx+2 c)} + 10 e^{(dx+c)} + 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 59/2048*log(3*e^(d*x + c) + 1)/d - 59/2048*log(e^(d*x + c) + 3)/d + 3/256*(
59*e^(3*d*x + 3*c) + 295*e^(2*d*x + 2*c) + 241*e^(d*x + c) + 45)/(d*(3*e^(2
*d*x + 2*c) + 10*e^(d*x + c) + 3)^2)
```

3.78 $\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=106

$$-\frac{311\sinh(c+dx)}{8192d(3\cosh(c+dx)+5)}-\frac{25\sinh(c+dx)}{512d(3\cosh(c+dx)+5)^2}-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}-\frac{385\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{16384d}+\frac{3}{32}$$

```
[Out] (385*x)/32768 - (385*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(16384*d)
- Sinh[c + d*x]/(16*d*(5 + 3*Cosh[c + d*x])^3) - (25*Sinh[c + d*x])/(512*d*
(5 + 3*Cosh[c + d*x])^2) - (311*Sinh[c + d*x])/(8192*d*(5 + 3*Cosh[c + d*x]
))
```

Rubi [A] time = 0.0965579, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 2754, 12, 2657}

$$-\frac{311\sinh(c+dx)}{8192d(3\cosh(c+dx)+5)}-\frac{25\sinh(c+dx)}{512d(3\cosh(c+dx)+5)^2}-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}-\frac{385\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{16384d}+\frac{3}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 - (385*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(16384*d)
- Sinh[c + d*x]/(16*d*(5 + 3*Cosh[c + d*x])^3) - (25*Sinh[c + d*x])/(512*d*
(5 + 3*Cosh[c + d*x])^2) - (311*Sinh[c + d*x])/(8192*d*(5 + 3*Cosh[c + d*x]
))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx &= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{1}{48}\int \frac{-15+6\cosh(c+dx)}{(5+3\cosh(c+dx))^3}\,dx\\
&= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2}+\frac{\int \frac{186-75\cosh(c+dx)}{(5+3\cosh(c+dx))^2}\,dx}{1536}\\
&= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2}-\frac{311\sinh(c+dx)}{8192d(5+3\cosh(c+dx))}-\frac{\int}{}\\
&= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2}-\frac{311\sinh(c+dx)}{8192d(5+3\cosh(c+dx))}+\frac{38}{}\\
&= \frac{385x}{32768}-\frac{385\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{16384d}-\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx)}
\end{aligned}
$$

Mathematica [A] time = 0.247145, size = 68, normalized size = 0.64

$$
\frac{770\tanh^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{9(4883\sinh(c+dx)+2340\sinh(2(c+dx))+311\sinh(3(c+dx)))}{(3\cosh(c+dx)+5)^3}}{32768d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-4),x]
```

```
[Out] (770*ArcTanh[Tanh[(c + d*x)/2]/2] - (9*(4883*Sinh[c + d*x] + 2340*Sinh[2*(c
 + d*x)] + 311*Sinh[3*(c + d*x)]))/(5 + 3*Cosh[c + d*x])^3)/(32768*d)
```

Maple [A] time = 0.019, size = 144, normalized size = 1.4

$$\frac{9}{2048\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-3}-\frac{81}{4096\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-2}+\frac{639}{16384\,d}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-1}+\frac{385}{32768\,d}\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c))^4,x)
```

```
[Out] 9/2048/d/(tanh(1/2*d*x+1/2*c)+2)^3-81/4096/d/(tanh(1/2*d*x+1/2*c)+2)^2+639/
16384/d/(tanh(1/2*d*x+1/2*c)+2)+385/32768/d*ln(tanh(1/2*d*x+1/2*c)+2)+9/204
8/d/(tanh(1/2*d*x+1/2*c)-2)^3+81/4096/d/(tanh(1/2*d*x+1/2*c)-2)^2+639/16384
/d/(tanh(1/2*d*x+1/2*c)-2)-385/32768/d*ln(tanh(1/2*d*x+1/2*c)-2)
```

Maxima [A] time = 1.0722, size = 228, normalized size = 2.15

$$-\frac{385\,\log\left(3\,e^{(-dx-c)}+1\right)}{32768\,d}+\frac{385\,\log\left(e^{(-dx-c)}+3\right)}{32768\,d}-\frac{73575\,e^{(-dx-c)}+218466\,e^{(-2\,dx-2\,c)}+239470\,e^{(-3\,dx-3\,c)}+86625}{12288\,d\left(270\,e^{(-dx-c)}+981\,e^{(-2\,dx-2\,c)}+1540\,e^{(-3\,dx-3\,c)}+981\,e^{(-4\,dx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -385/32768*log(3*e^(-d*x - c) + 1)/d + 385/32768*log(e^(-d*x - c) + 3)/d -
1/12288*(73575*e^(-d*x - c) + 218466*e^(-2*d*x - 2*c) + 239470*e^(-3*d*x -
3*c) + 86625*e^(-4*d*x - 4*c) + 10395*e^(-5*d*x - 5*c) + 8397)/(d*(270*e^(-
d*x - c) + 981*e^(-2*d*x - 2*c) + 1540*e^(-3*d*x - 3*c) + 981*e^(-4*d*x - 4
*c) + 270*e^(-5*d*x - 5*c) + 27*e^(-6*d*x - 6*c) + 27))
```

Fricas [B] time = 2.24239, size = 3320, normalized size = 31.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/98304*(83160*cosh(d*x + c)^5 + 138600*(3*cosh(d*x + c) + 5)*sinh(d*x + c)
^4 + 83160*sinh(d*x + c)^5 + 693000*cosh(d*x + c)^4 + 6160*(135*cosh(d*x +
c)^2 + 450*cosh(d*x + c) + 311)*sinh(d*x + c)^3 + 1915760*cosh(d*x + c)^3 +
 48*(17325*cosh(d*x + c)^3 + 86625*cosh(d*x + c)^2 + 119735*cosh(d*x + c) +
 36411)*sinh(d*x + c)^2 + 1747728*cosh(d*x + c)^2 + 1155*(27*cosh(d*x + c)^
6 + 54*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 270*cos
h(d*x + c)^5 + 9*(45*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh(d*x +
c)^4 + 981*cosh(d*x + c)^4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x + c)^2 +
 981*cosh(d*x + c) + 385)*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 + 3*(135*c
osh(d*x + c)^4 + 900*cosh(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 1540*cosh(d*x
 + c) + 327)*sinh(d*x + c)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(d*x + c)^5
+ 225*cosh(d*x + c)^4 + 654*cosh(d*x + c)^3 + 770*cosh(d*x + c)^2 + 327*cos
h(d*x + c) + 45)*sinh(d*x + c) + 270*cosh(d*x + c) + 27)*log(3*cosh(d*x + c
) + 3*sinh(d*x + c) + 1) - 1155*(27*cosh(d*x + c)^6 + 54*(3*cosh(d*x + c) +
 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 270*cosh(d*x + c)^5 + 9*(45*cosh
(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh(d*x + c)^4 + 981*cosh(d*x + c)^
4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x + c)^2 + 981*cosh(d*x + c) + 385)
*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 + 3*(135*cosh(d*x + c)^4 + 900*cosh
(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 1540*cosh(d*x + c) + 327)*sinh(d*x + c
)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(d*x + c)^5 + 225*cosh(d*x + c)^4 + 6
54*cosh(d*x + c)^3 + 770*cosh(d*x + c)^2 + 327*cosh(d*x + c) + 45)*sinh(d*x
 + c) + 270*cosh(d*x + c) + 27)*log(cosh(d*x + c) + sinh(d*x + c) + 3) + 24
*(17325*cosh(d*x + c)^4 + 115500*cosh(d*x + c)^3 + 239470*cosh(d*x + c)^2 +
 145644*cosh(d*x + c) + 24525)*sinh(d*x + c) + 588600*cosh(d*x + c) + 67176
)/(27*d*cosh(d*x + c)^6 + 27*d*sinh(d*x + c)^6 + 270*d*cosh(d*x + c)^5 + 54
*(3*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^5 + 981*d*cosh(d*x + c)^4 + 9*(45*
d*cosh(d*x + c)^2 + 150*d*cosh(d*x + c) + 109*d)*sinh(d*x + c)^4 + 1540*d*c
osh(d*x + c)^3 + 4*(135*d*cosh(d*x + c)^3 + 675*d*cosh(d*x + c)^2 + 981*d*c
osh(d*x + c) + 385*d)*sinh(d*x + c)^3 + 981*d*cosh(d*x + c)^2 + 3*(135*d*co
sh(d*x + c)^4 + 900*d*cosh(d*x + c)^3 + 1962*d*cosh(d*x + c)^2 + 1540*d*cos
h(d*x + c) + 327*d)*sinh(d*x + c)^2 + 270*d*cosh(d*x + c) + 6*(27*d*cosh(d*
x + c)^5 + 225*d*cosh(d*x + c)^4 + 654*d*cosh(d*x + c)^3 + 770*d*cosh(d*x +
 c)^2 + 327*d*cosh(d*x + c) + 45*d)*sinh(d*x + c) + 27*d)
```

Sympy [A] time = 12.9858, size = 784, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((-385*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**6/(32768*d*ta
nh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d
*x/2)**2 - 2097152*d) + 4620*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**
4/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d
*tanh(c/2 + d*x/2)**2 - 2097152*d) - 18480*log(tanh(c/2 + d*x/2) - 2)*tanh(
c/2 + d*x/2)**2/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)*
*4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 24640*log(tanh(c/2 + d*x
/2) - 2)/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 15
72864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 385*log(tanh(c/2 + d*x/2) + 2)*
tanh(c/2 + d*x/2)**6/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*
x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 4620*log(tanh(c/2 +
 d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*
tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 18480*
log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(32768*d*tanh(c/2 + d*x/2)*
*6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 20971
52*d) - 24640*log(tanh(c/2 + d*x/2) + 2)/(32768*d*tanh(c/2 + d*x/2)**6 - 39
3216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) +
 2556*tanh(c/2 + d*x/2)**5/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/
2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 14976*tanh(c/
2 + d*x/2)**3/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4
 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 23616*tanh(c/2 + d*x/2)/(3
2768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tan
h(c/2 + d*x/2)**2 - 2097152*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**4, True))
```

Giac [A] time = 1.13681, size = 153, normalized size = 1.44

$$\frac{385 \log\left(3 e^{(dx+c)} + 1\right)}{32768 d} - \frac{385 \log\left(e^{(dx+c)} + 3\right)}{32768 d} + \frac{10395 e^{(5 dx+5 c)} + 86625 e^{(4 dx+4 c)} + 239470 e^{(3 dx+3 c)} + 218466 e^{(2 dx+2 c}}{12288 d\left(3 e^{(2 dx+2 c)} + 10 e^{(dx+c)} + 3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 385/32768*log(3*e^(d*x + c) + 1)/d - 385/32768*log(e^(d*x + c) + 3)/d + 1/1
2288*(10395*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) + 239470*e^(3*d*x + 3*c
) + 218466*e^(2*d*x + 2*c) + 73575*e^(d*x + c) + 8397)/(d*(3*e^(2*d*x + 2*c
) + 10*e^(d*x + c) + 3)^3)
```

3.79 $\int (a+b\cosh(x))^{5/2}\,dx$

Optimal. Leaf size=153

$$\frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

```
[Out] (((-2*I)/15)*(23*a^2 + 9*b^2)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/Sqrt[(a + b*Cosh[x])/(a + b)] + (((16*I)/15)*a*(a^2 - b^2)*Sqrt[(
a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[
x]] + (16*a*b*Sqrt[a + b*Cosh[x]]*Sinh[x])/15 + (2*b*(a + b*Cosh[x])^(3/2)*
Sinh[x])/5
```

Rubi [A] time = 0.24402, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{16}{15}ab\,\text{s}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(5/2),x]
```

```
[Out] (((-2*I)/15)*(23*a^2 + 9*b^2)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/Sqrt[(a + b*Cosh[x])/(a + b)] + (((16*I)/15)*a*(a^2 - b^2)*Sqrt[(
a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[
x]] + (16*a*b*Sqrt[a + b*Cosh[x]]*Sinh[x])/15 + (2*b*(a + b*Cosh[x])^(3/2)*
Sinh[x])/5
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cosh(x))^{5/2}\,dx = \frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x) + \frac{2}{5}\int \sqrt{a+b\cosh(x)}\left(\frac{1}{2}\left(5a^2+3b^2\right)+4ab\cosh(x)\right)dx$$

$$= \frac{16}{15}ab\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x) + \frac{4}{15}\int \frac{\frac{1}{4}a\left(15a^2+17b^2\right)+\frac{1}{4}b\left(23\right.}{\sqrt{a+b\cosh}}$$

$$= \frac{16}{15}ab\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x) - \frac{1}{15}\left(8a\left(a^2-b^2\right)\right)\int \frac{1}{\sqrt{a+b\cosh}}$$

$$= \frac{16}{15}ab\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x) + \frac{\left(\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\right)\int}{15\sqrt{\frac{a+b\cosh(x}{a+b}}}$$

$$= -\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}} + \frac{16}{15}ab\sqrt{a+}$$

Mathematica [A] time = 0.501181, size = 150, normalized size = 0.98

$$\frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+b\sinh(x)\left(22a^2+28ab\cosh(x)+3b^2\cosh(2x)+3b^2\right)-2i\left(23a^2b+23a^3\right.}{15\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(5/2),x]
```

```
[Out] ((-2*I)*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cosh[x])/(a + b)]
*EllipticE[(I/2)*x, (2*b)/(a + b)] + (16*I)*a*(a^2 - b^2)*Sqrt[(a + b*Cosh[
x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2 + 28*a*b
*Cosh[x] + 3*b^2*Cosh[2*x])*Sinh[x])/(15*Sqrt[a + b*Cosh[x]])
```

Maple [B] time = 0.102, size = 685, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(5/2),x)
```

```
[Out] 2/15*(24*(-2*b/(a-b))^(1/2)*b^3*cosh(1/2*x)*sinh(1/2*x)^6+(56*(-2*b/(a-b))^
(1/2)*a*b^2+24*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(22*(-2*b/
(a-b))^(1/2)*a^2*b+28*(-2*b/(a-b))^(1/2)*a*b^2+6*(-2*b/(a-b))^(1/2)*b^3)*si
nh(1/2*x)^2*cosh(1/2*x)+15*a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*
(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-
b)/b)^(1/2))+23*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/
2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/
2))+17*a*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+9*b^3
*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ellipti
cF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-46*(2*b/(a-b)*sin
h(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*
(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2*b-18*(2*b/(a-b)*sinh(1/2*x)^
2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-
b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b^3)*((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)
^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/
sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cosh(x)^2 + 2\,ab\cosh(x) + a^2\right)\sqrt{b\cosh(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cosh(x)^2 + 2*a*b*cosh(x) + a^2)*sqrt(b*cosh(x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2),x, algorithm="giac")

[Out] integrate((b*cosh(x) + a)^(5/2), x)
```

3.80 $\int (a + b\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] (((-8*I)/3)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(
a + b*Cosh[x])/(a + b)] + (((2*I)/3)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a +
b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]] + (2*b*Sqrt[a +
b*Cosh[x]]*Sinh[x])/3
```

Rubi [A] time = 0.156512, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2656, 2752, 2663, 2661, 2655, 2653}

$$\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(3/2),x]
```


```
[Out] (((-8*I)/3)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(
a + b*Cosh[x])/(a + b)] + (((2*I)/3)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a +
b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]] + (2*b*Sqrt[a +
b*Cosh[x]]*Sinh[x])/3
```


Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cosh(x))^{3/2}\,dx = \frac{2}{3}b\sqrt{a+b\cosh(x)}\sinh(x)+\frac{2}{3}\int \frac{\frac{1}{2}\left(3a^2+b^2\right)+2ab\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$
$$= \frac{2}{3}b\sqrt{a+b\cosh(x)}\sinh(x)+\frac{1}{3}(4a)\int \sqrt{a+b\cosh(x)}\,dx+\frac{1}{3}\left(-a^2+b^2\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$$
$$= \frac{2}{3}b\sqrt{a+b\cosh(x)}\sinh(x)+\frac{\left(4a\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{\left(\left(-a^2+b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\right.}{3\sqrt{a+}}$$
$$= -\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}+\frac{2}{3}b\sqrt{a+b\cosh(x)}\sinh$$

Mathematica [A] time = 0.227168, size = 111, normalized size = 0.9

$$\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+2b\sinh(x)(a+b\cosh(x))-8ia(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(3/2),x]
```

```
[Out] ((-8*I)*a*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(a
 + b)] + (2*I)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x,
 (2*b)/(a + b)] + 2*b*(a + b*Cosh[x])*Sinh[x])/(3*Sqrt[a + b*Cosh[x]])
```

Maple [B] time = 0.063, size = 458, normalized size = 3.7

$$\frac{2}{3}\left(4\sqrt{-2\frac{b}{a-b}}b^2\cosh(x/2)(\sinh(x/2))^4+\left(2\sqrt{-2\frac{b}{a-b}}ab+2\sqrt{-2\frac{b}{a-b}}b^2\right)\left(\sinh\left(\frac{x}{2}\right)\right)^2\cosh\left(\frac{x}{2}\right)+3a^2\sqrt{2\frac{(\sinh(}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(3/2),x)
```

```
[Out] 2/3*(4*(-2*b/(a-b))^(1/2)*b^2*cosh(1/2*x)*sinh(1/2*x)^4+(2*(-2*b/(a-b))^(1/
2)*a*b+2*(-2*b/(a-b))^(1/2)*b^2)*sinh(1/2*x)^2*cosh(1/2*x)+3*a^2*(2*b/(a-b)
*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2
*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+4*a*b*(2*b/(a-b)*sinh(1/2*x)
^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a
-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b)
)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2
*(-2*(a-b)/b)^(1/2))-8*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1
/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2))*a*b)*((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/
(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*
b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(x)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cosh(x)+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(x) + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(3/2),x)
```

```
[Out] Integral((a + b*cosh(x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cosh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(3/2), x)
```

3.81 $\int \sqrt{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

Rubi [A] time = 0.0394434, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2655, 2653}

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = \frac{\sqrt{a+b\cosh(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$
$$= -\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.0850281, size = 61, normalized size = 1.

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

Maple [B] time = 0.062, size = 276, normalized size = 4.5

$$2\,\frac{\sqrt{-(\sinh(1/2\,dx+c/2))^2}\sqrt{\left(2\,(\cosh(1/2\,dx+c/2))^2\,b+a-b\right)(\sinh(1/2\,dx+c/2))^2}}{\sqrt{2\,b\,(\sinh(1/2\,dx+c/2))^4+(a+b)\,(\sinh(1/2\,dx+c/2))^2}\,\sinh(1/2\,dx+c/2)\,\sqrt{2\,(\sinh(1/2\,dx+c/2))^2\,b+a+b}d}\Bigg(a\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^(1/2),x)
```

```
[Out] 2*(a*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2
))+b*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2
))-2*b*EllipticE(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2)))*(-sinh(1/2*d*x+1/2*c)^2)^(1/2)*((2*cosh(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)*((2*cosh(1/2*d*x+1/2*c)^2*b+a-b)*sinh(1/2*d*x+1/2*c)^2)^(1/2)/(-2*b
/(a-b))^(1/2)/(2*b*sinh(1/2*d*x+1/2*c)^4+(a+b)*sinh(1/2*d*x+1/2*c)^2)^(1/2)
```

```
/sinh(1/2*d*x+1/2*c)/(2*sinh(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cosh(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cosh(c + d*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cosh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(d*x + c) + a), x)
```

3.82 $\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=46

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sq
rt[a + b*Cosh[x]]
```

Rubi [A] time = 0.0341934, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2663, 2661}

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sq
rt[a + b*Cosh[x]]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \frac{\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}\,dx}{\sqrt{a+b\cosh(x)}}$$
$$= -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

Mathematica [A] time = 0.036856, size = 46, normalized size = 1.

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sq
rt[a + b*Cosh[x]]
```

Maple [B] time = 0.051, size = 146, normalized size = 3.2

$$2\,\frac{\sqrt{\left(2\,(\cosh(x/2))^2\,b+a-b\right)(\sinh(x/2))^2}\sqrt{-(\sinh(x/2))^2}}{\sqrt{2\,b\,(\sinh(x/2))^4+(a+b)\,(\sinh(x/2))^2}\,\sinh(x/2)\,\sqrt{2\,(\sinh(x/2))^2\,b+a+b}}\sqrt{\frac{2\,(\cosh(x/2))^2\,b+a-b}{a-b}}\text{EllipticF}\Bigg(\mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(1/2),x)
```

```
[Out] 2*((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)*((2*cosh
(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b*sinh(1/2*x)^4+(a+
b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a
-b)/b)^(1/2))/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\cosh(x)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cosh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + a), x)
```

3.83 $\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)
*Sqrt[(a + b*Cosh[x])/(a + b)]) - (2*b*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cos
h[x]])
```

Rubi [A] time = 0.0570477, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2664, 21, 2655, 2653}

$$-\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-3/2),x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)
*Sqrt[(a + b*Cosh[x])/(a + b)]) - (2*b*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cos
h[x]])
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx &= -\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx}{a^2-b^2}\\
&= -\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\int \sqrt{a+b\cosh(x)}\,dx}{a^2-b^2}\\
&= -\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}\\
&= -\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.120197, size = 68, normalized size = 0.81

$$-\frac{2\left(b\sinh(x)+i(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)\right)}{(a-b)(a+b)\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-3/2),x]
```

```
[Out] (-2*(I*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(a +
b)] + b*Sinh[x]))/((a - b)*(a + b)*Sqrt[a + b*Cosh[x]])
```

Maple [B] time = 0.079, size = 296, normalized size = 3.5

$$2\,\frac{1}{(a+b)\,(a-b)\sinh(x/2)\sqrt{2\,(\sinh(x/2))^2\,b+a+b}}\left(-2\sqrt{-2\,\frac{b}{a-b}}b\cosh(x/2)\,(\sinh(x/2))^2+\sqrt{-(\sinh(x/2))^2}\sqrt{2\,\frac{(\sin}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(3/2),x)
```

```
[Out] 2*(-2*(-2*b/(a-b))^(1/2)*b*cosh(1/2*x)*sinh(1/2*x)^2+(-sinh(1/2*x)^2)^(1/2)
*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a
-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a+(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh
(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(
-2*(a-b)/b)^(1/2))*b-2*(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2))*b)/(-2*b/(a-b))^(1/2)/(a-b)/(a+b)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cosh(x)+a}}{b^2\cosh(x)^2+2\,ab\cosh(x)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + a)/(b^2*cosh(x)^2 + 2*a*b*cosh(x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\cosh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(3/2),x)
```

```
[Out] Integral((a + b*cosh(x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(-3/2), x)
```

3.84 $\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=177

$$\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] (((-8*I)/3)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2
- b^2)^2*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/3)*Sqrt[(a + b*Cosh[x])/(
a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]]
) - (2*b*Sinh[x])/(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) - (8*a*b*Sinh[x])/(
3*(a^2 - b^2)^2*Sqrt[a + b*Cosh[x]])
```

Rubi [A] time = 0.206961, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-5/2),x]
```

```
[Out] (((-8*I)/3)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2
- b^2)^2*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/3)*Sqrt[(a + b*Cosh[x])/(
a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]]
) - (2*b*Sinh[x])/(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) - (8*a*b*Sinh[x])/(
3*(a^2 - b^2)^2*Sqrt[a + b*Cosh[x]])
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = -\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2+b^2\right)+ab\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx}{3\left(a^2-b^2\right)^2}$$

$$= -\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{(4a)\int \sqrt{a+b\cosh(x)}\,dx}{3\left(a^2-b^2\right)^2} - \int$$

$$= -\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{\left(4a\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+}}{3\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh}{a+b}}}$$

$$= -\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}} - \frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} -$$

Mathematica [A] time = 0.526546, size = 135, normalized size = 0.76

$$\frac{2i(a-b)(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+2b\sinh(x)\left(-5a^2-4ab\cosh(x)+b^2\right)-8ia(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}E}{3(a-b)^2(a+b)^2(a+b\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-5/2),x]
```

```
[Out] ((-8*I)*a*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*EllipticE[(I/2)*x, (2*b
)/(a + b)] + (2*I)*(a - b)*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*Ellipt
icF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cosh[x])*Sinh[x])/(
3*(a - b)^2*(a + b)^2*(a + b*Cosh[x])^(3/2))
```

Maple [B] time = 0.147, size = 459, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(5/2),x)
```

```
[Out] ((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/3/b/(a-b)/(a+b)*cosh(1/2*
x)*(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)/b
)^2-16/3*b*sinh(1/2*x)^2/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*cosh(1/2*x)^2*b+
a-b)*sinh(1/2*x)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)/(-2*b/(a-
b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b
*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b)
)^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-32/3*a*b/(a+b)^2/(a-b)^2*(-a+b)/(-2*b/(a-
b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b
*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(cosh(1/2*x)*
(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2*x)*(-2*b/(a
-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))))/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cosh(x)+a}}{b^3\cosh(x)^3+3\,ab^2\cosh(x)^2+3\,a^2b\cosh(x)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + a)/(b^3*cosh(x)^3 + 3*a*b^2*cosh(x)^2 + 3*a^2*b*c
osh(x) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(-5/2), x)
```

3.85 $\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$

Optimal. Leaf size=227

$$\frac{16ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{15\left(a^2-b^2\right)^3\sqrt{a+b\cosh(x)}}-\frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+$$

```
[Out] (((-2*I)/15)*(23*a^2 + 9*b^2)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/((a^2 - b^2)^3*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((16*I)/15)*a*Sq
rt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)
^2*Sqrt[a + b*Cosh[x]]) - (2*b*Sinh[x])/(5*(a^2 - b^2)*(a + b*Cosh[x])^(5/2
)) - (16*a*b*Sinh[x])/(15*(a^2 - b^2)^2*(a + b*Cosh[x])^(3/2)) - (2*b*(23*a
^2 + 9*b^2)*Sinh[x])/(15*(a^2 - b^2)^3*Sqrt[a + b*Cosh[x]])
```

Rubi [A] time = 0.312871, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{15\left(a^2-b^2\right)^3\sqrt{a+b\cosh(x)}}-\frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{16ia\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\right.}{15\left(a^2-b^2\right)^2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-7/2),x]
```



```
[Out] (((-2*I)/15)*(23*a^2 + 9*b^2)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/((a^2 - b^2)^3*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((16*I)/15)*a*Sq
rt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)
^2*Sqrt[a + b*Cosh[x]]) - (2*b*Sinh[x])/(5*(a^2 - b^2)*(a + b*Cosh[x])^(5/2
)) - (16*a*b*Sinh[x])/(15*(a^2 - b^2)^2*(a + b*Cosh[x])^(3/2)) - (2*b*(23*a
^2 + 9*b^2)*Sinh[x])/(15*(a^2 - b^2)^3*Sqrt[a + b*Cosh[x]])
```


Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}b\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx}{5\left(a^2-b^2\right)}$$

$$= -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}} + \frac{4\int \frac{\frac{3}{4}\left(5a^2+3b^2\right)-2ab\cosh(x)}{(a+b\cosh(x))^{3/2}}\,d}{15\left(a^2-b^2\right)^2}$$

$$= -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}} - \frac{2b\left(23a^2+9b^2\right)\sinh(x}{15\left(a^2-b^2\right)^3\sqrt{a+b\cosh}}$$

$$= -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}} - \frac{2b\left(23a^2+9b^2\right)\sinh(x}{15\left(a^2-b^2\right)^3\sqrt{a+b\cosh}}$$

$$= -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}} - \frac{2b\left(23a^2+9b^2\right)\sinh(x}{15\left(a^2-b^2\right)^3\sqrt{a+b\cosh}}$$

$$= -\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^3\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{16ia\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} - \frac{2b\sin}{5\left(a^2-b^2\right)(a+}$$

Mathematica [A] time = 0.65554, size = 165, normalized size = 0.73

$$\frac{2\left(\frac{b\sinh(x)\left(b^2\left(23a^2+9b^2\right)\cosh^2(x)+2ab\left(27a^2+5b^2\right)\cosh(x)-5a^2b^2+34a^4+3b^4\right)}{\left(b^2-a^2\right)^3} - \frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{5/2}\left(8a(b-a)\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+\left(23a^2+9b^2\right)E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)\right)}{(a-b)^3}\right)}{15(a+b\cosh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-7/2),x]
```

```
[Out] (2*(((-I)*((a + b*Cosh[x])/(a + b))^(5/2)*((23*a^2 + 9*b^2)*EllipticE[(I/2)
*x, (2*b)/(a + b)] + 8*a*(-a + b)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(a -
b)^3 + (b*(34*a^4 - 5*a^2*b^2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*Cosh[x] + b^
2*(23*a^2 + 9*b^2)*Cosh[x]^2)*Sinh[x])/(-a^2 + b^2)^3))/(15*(a + b*Cosh[x])
^(5/2))
```

Maple [B] time = 0.198, size = 566, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(7/2),x)
```

```
[Out] ((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/10/b^2/(a-b)/(a+b)*cosh(1
/2*x)*(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b
)/b)^3-8/15*a/b/(a+b)^2/(a-b)^2*cosh(1/2*x)*(2*b*sinh(1/2*x)^4+(a+b)*sinh(1
/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)/b)^2-4/15*b*sinh(1/2*x)^2/(a-b)^3/(
a+b)^3*cosh(1/2*x)*(23*a^2+9*b^2)/((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(
1/2)+2*(15*a^2-8*a*b+9*b^2)/(15*a^5+15*a^4*b-30*a^3*b^2-30*a^2*b^3+15*a*b^4
+15*b^5)/(-2*b/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/
2*x)^2)^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(
1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-8/15*b*(23*a^2+9*b^2)/(
a+b)^3/(a-b)^3*(-a+b)/(-2*b/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1
/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(2
*a-2*b)*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))
-EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))))/sinh(
1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cosh(x)+a}}{b^4\cosh(x)^4+4\,ab^3\cosh(x)^3+6\,a^2b^2\cosh(x)^2+4\,a^3b\cosh(x)+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + a)/(b^4*cosh(x)^4 + 4*a*b^3*cosh(x)^3 + 6*a^2*b^2
*cosh(x)^2 + 4*a^3*b*cosh(x) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cosh(x)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(-7/2), x)
```

3.86 $\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=100

$$\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a +
 b*Cosh[x])/(a + b)]) + ((2*I)*a*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I
/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

Rubi [A] time = 0.106668, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```



```
[Out] ((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a +
 b*Cosh[x])/(a + b)]) + ((2*I)*a*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I
/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```


Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx &= \frac{\int \sqrt{a+b\cosh(x)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{b}\\ &= \frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{\left(a\sqrt{\frac{a+b\cosh(x)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}\,dx}{b\sqrt{a+b\cosh(x)}}\\ &= -\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.346129, size = 73, normalized size = 0.73

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((a+b)E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)-a\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*EllipticE[(I/2)*x, (2*b)/(a
+ b)] - a*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*Cosh[x]])
```

Maple [A] time = 0.068, size = 181, normalized size = 1.8

$$2\,\frac{\sqrt{-\left(\sinh\left(x/2\right)\right)^{2}}\sqrt{\left(2\,\left(\cosh\left(x/2\right)\right)^{2}b+a-b\right)\left(\sinh\left(x/2\right)\right)^{2}}}{\sqrt{2\,b\left(\sinh\left(x/2\right)\right)^{4}+\left(a+b\right)\left(\sinh\left(x/2\right)\right)^{2}}\sinh\left(x/2\right)\sqrt{2\,\left(\sinh\left(x/2\right)\right)^{2}b+a+b}}\left(\mathrm{EllipticF}\left(\cosh\left(x/2\right)\sqrt{-2\,\frac{b}{a-b}},1/2\,\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*cosh(x))^(1/2),x)
```

```
[Out] 2*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*Ellip
ticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2)))*(-sinh(1/2*x)^
2)^(1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)*sin
h(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2
)^(1/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(x\right)}{\sqrt{b\cosh\left(x\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/sqrt(b*cosh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cosh\left(x\right)}{\sqrt{b\cosh\left(x\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(x)/sqrt(b*cosh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\sqrt{a + b\cosh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(a + b*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\sqrt{b\cosh(x) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)/sqrt(b*cosh(x) + a), x)
```

3.87 $\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=94

$$\frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a\cosh(x)+a}}+\frac{16}{105}a^2(7A+5B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{35}a(7A+5B)\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{2}{7}B\sinh($$

```
[Out] (64*a^3*(7*A + 5*B)*Sinh[x])/(105*Sqrt[a + a*Cosh[x]]) + (16*a^2*(7*A + 5*B
)*Sqrt[a + a*Cosh[x]]*Sinh[x])/105 + (2*a*(7*A + 5*B)*(a + a*Cosh[x])^(3/2)
*Sinh[x])/35 + (2*B*(a + a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rubi [A] time = 0.0922257, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2751, 2647, 2646}

$$\frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a\cosh(x)+a}}+\frac{16}{105}a^2(7A+5B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{35}a(7A+5B)\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{2}{7}B\sinh($$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (64*a^3*(7*A + 5*B)*Sinh[x])/(105*Sqrt[a + a*Cosh[x]]) + (16*a^2*(7*A + 5*B
)*Sqrt[a + a*Cosh[x]]*Sinh[x])/105 + (2*a*(7*A + 5*B)*(a + a*Cosh[x])^(3/2)
*Sinh[x])/35 + (2*B*(a + a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\cosh(x))^{5/2}(A + B\cosh(x))\,dx = \frac{2}{7}B(a + a\cosh(x))^{5/2}\sinh(x) + \frac{1}{7}(7A + 5B)\int (a + a\cosh(x))^{5/2}\,dx$$
$$= \frac{2}{35}a(7A + 5B)(a + a\cosh(x))^{3/2}\sinh(x) + \frac{2}{7}B(a + a\cosh(x))^{5/2}\sinh(x) + \frac{1}{35}$$
$$= \frac{16}{105}a^2(7A + 5B)\sqrt{a + a\cosh(x)}\sinh(x) + \frac{2}{35}a(7A + 5B)(a + a\cosh(x))^{3/2}\text{si}$$
$$= \frac{64a^3(7A + 5B)\sinh(x)}{105\sqrt{a + a\cosh(x)}} + \frac{16}{105}a^2(7A + 5B)\sqrt{a + a\cosh(x)}\sinh(x) + \frac{2}{35}a(7A$$

Mathematica [A] time = 0.119413, size = 60, normalized size = 0.64

$$\frac{1}{210}a^2\tanh\left(\frac{x}{2}\right)\sqrt{a(\cosh(x) + 1)}((392A + 505B)\cosh(x) + 6(7A + 20B)\cosh(2x) + 1246A + 15B\cosh(3x) + 1040B)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cosh[x])]*(1246*A + 1040*B + (392*A + 505*B)*Cosh[x] + 6*(
7*A + 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Tanh[x/2])/210
```

Maple [A] time = 0.046, size = 71, normalized size = 0.8

$$\frac{8\,a^3\sqrt{2}}{105}\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)\left(30\,B\,(\sinh(x/2))^6 + (21\,A + 105\,B)\left(\sinh\left(\frac{x}{2}\right)\right)^4 + (70\,A + 140\,B)\left(\sinh\left(\frac{x}{2}\right)\right)^2 + 105\,A + 10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x)
```

```
[Out] 8/105*cosh(1/2*x)*a^3*sinh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A+105*B)*sinh(1/2
*x)^4+(70*A+140*B)*sinh(1/2*x)^2+105*A+105*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/
2)
```

Maxima [B] time = 1.6449, size = 320, normalized size = 3.4

$$\frac{1}{60}\left(3\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{5}{2}x\right)} + 25\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{3}{2}x\right)} + 150\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{1}{2}x\right)} - 150\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{1}{2}x\right)} - 25\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{3}{2}x\right)} - 3\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{5}{2}x\right)}\right)A + \frac{1}{168}\left(\left(3\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/60*(3*sqrt(2)*a^(5/2)*e^(5/2*x) + 25*sqrt(2)*a^(5/2)*e^(3/2*x) + 150*sqrt
(2)*a^(5/2)*e^(1/2*x) - 150*sqrt(2)*a^(5/2)*e^(-1/2*x) - 25*sqrt(2)*a^(5/2)
*e^(-3/2*x) - 3*sqrt(2)*a^(5/2)*e^(-5/2*x))*A + 1/168*((3*sqrt(2)*a^(5/2)*e
^(-x) + 21*sqrt(2)*a^(5/2)*e^(-2*x) + 70*sqrt(2)*a^(5/2)*e^(-3*x) + 210*sqr
t(2)*a^(5/2)*e^(-4*x) - 105*sqrt(2)*a^(5/2)*e^(-5*x) - 7*sqrt(2)*a^(5/2)*e^
(-6*x))*e^(9/2*x) + (7*sqrt(2)*a^(5/2)*e^(-x) + 105*sqrt(2)*a^(5/2)*e^(-2*x
) - 210*sqrt(2)*a^(5/2)*e^(-3*x) - 70*sqrt(2)*a^(5/2)*e^(-4*x) - 21*sqrt(2)
*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a^(5/2)*e^(-6*x))*e^(5/2*x))*B
```

Fricas [B] time = 2.26143, size = 1553, normalized size = 16.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A + 5*B)*a
^2*cosh(x)^6 + 35*(10*A + 11*B)*a^2*cosh(x)^5 + 525*(4*A + 3*B)*a^2*cosh(x)
^4 + 21*(5*B*a^2*cosh(x) + (2*A + 5*B)*a^2)*sinh(x)^6 - 525*(4*A + 3*B)*a^2
*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A + 5*B)*a^2*cosh(x) + 5*(10*A +
 11*B)*a^2)*sinh(x)^5 - 35*(10*A + 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*cosh(
x)^3 + 9*(2*A + 5*B)*a^2*cosh(x)^2 + 5*(10*A + 11*B)*a^2*cosh(x) + 15*(4*A
+ 3*B)*a^2)*sinh(x)^4 - 21*(2*A + 5*B)*a^2*cosh(x) + 35*(15*B*a^2*cosh(x)^4
 + 12*(2*A + 5*B)*a^2*cosh(x)^3 + 10*(10*A + 11*B)*a^2*cosh(x)^2 + 60*(4*A
+ 3*B)*a^2*cosh(x) - 15*(4*A + 3*B)*a^2)*sinh(x)^3 - 15*B*a^2 + 35*(9*B*a^2
*cosh(x)^5 + 9*(2*A + 5*B)*a^2*cosh(x)^4 + 10*(10*A + 11*B)*a^2*cosh(x)^3 +
```

```
 90*(4*A + 3*B)*a^2*cosh(x)^2 - 45*(4*A + 3*B)*a^2*cosh(x) - (10*A + 11*B)*
a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A + 5*B)*a^2*cosh(x)^5 + 25*
(10*A + 11*B)*a^2*cosh(x)^4 + 300*(4*A + 3*B)*a^2*cosh(x)^3 - 225*(4*A + 3*
B)*a^2*cosh(x)^2 - 10*(10*A + 11*B)*a^2*cosh(x) - 3*(2*A + 5*B)*a^2)*sinh(x
))*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)
*sinh(x)^2 + sinh(x)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24837, size = 207, normalized size = 2.2

$$-\frac{1}{840}\sqrt{2}\left(\frac{\left(2100\,Aa^6e^{(3\,x)} + 1575\,Ba^6e^{(3\,x)} + 350\,Aa^6e^{(2\,x)} + 385\,Ba^6e^{(2\,x)} + 42\,Aa^6e^{x} + 105\,Ba^6e^{x} + 15\,Ba^6\right)e^{\left(-\frac{7}{2}\,x\right)}}{a^{\frac{7}{2}}} - \frac{15\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/840*sqrt(2)*((2100*A*a^6*e^(3*x) + 1575*B*a^6*e^(3*x) + 350*A*a^6*e^(2*x
) + 385*B*a^6*e^(2*x) + 42*A*a^6*e^x + 105*B*a^6*e^x + 15*B*a^6)*e^(-7/2*x)
/a^(7/2) - (15*B*a^(19/2)*e^(7/2*x) + 42*A*a^(19/2)*e^(5/2*x) + 105*B*a^(19
/2)*e^(5/2*x) + 350*A*a^(19/2)*e^(3/2*x) + 385*B*a^(19/2)*e^(3/2*x) + 2100*
A*a^(19/2)*e^(1/2*x) + 1575*B*a^(19/2)*e^(1/2*x))/a^7)
```

3.88 $\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=68

$$\frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a\cosh(x)+a}}+\frac{2}{15}a(5A+3B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

```
[Out] (8*a^2*(5*A + 3*B)*Sinh[x])/(15*Sqrt[a + a*Cosh[x]]) + (2*a*(5*A + 3*B)*Sqr
t[a + a*Cosh[x]]*Sinh[x])/15 + (2*B*(a + a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rubi [A] time = 0.0738993, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2751, 2647, 2646}

$$\frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a\cosh(x)+a}}+\frac{2}{15}a(5A+3B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (8*a^2*(5*A + 3*B)*Sinh[x])/(15*Sqrt[a + a*Cosh[x]]) + (2*a*(5*A + 3*B)*Sqr
t[a + a*Cosh[x]]*Sinh[x])/15 + (2*B*(a + a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx &= \frac{2}{5}B(a+a\cosh(x))^{3/2}\sinh(x)+\frac{1}{5}(5A+3B)\int (a+a\cosh(x))^{3/2}\,dx\\ &= \frac{2}{15}a(5A+3B)\sqrt{a+a\cosh(x)}\sinh(x)+\frac{2}{5}B(a+a\cosh(x))^{3/2}\sinh(x)+\frac{1}{15}(4\\ &= \frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a+a\cosh(x)}}+\frac{2}{15}a(5A+3B)\sqrt{a+a\cosh(x)}\sinh(x)+\frac{2}{5}B(a+a\,\text{co}\end{aligned}$$

Mathematica [A] time = 0.0872422, size = 46, normalized size = 0.68

$$\frac{1}{15}a\tanh\left(\frac{x}{2}\right)\sqrt{a(\cosh(x)+1)}(2(5A+9B)\cosh(x)+50A+3B\cosh(2x)+39B)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[x])]*(50*A + 39*B + 2*(5*A + 9*B)*Cosh[x] + 3*B*Cosh[2*
x])*Tanh[x/2])/15
```

Maple [A] time = 0.04, size = 57, normalized size = 0.8

$$\frac{4a^2\sqrt{2}}{15}\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)\left(6B(\sinh(x/2))^4+(5A+15B)\left(\sinh\left(\frac{x}{2}\right)\right)^2+15A+15B\right)\frac{1}{\sqrt{a\left(\cosh\left(\frac{x}{2}\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x)
```

```
[Out] 4/15*cosh(1/2*x)*a^2*sinh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A+15*B)*sinh(1/2*x)^
2+15*A+15*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [B] time = 1.61723, size = 220, normalized size = 3.24

$$\frac{1}{6}\left(\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{3}{2}x\right)} + 9\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)} - 9\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{1}{2}x\right)} - \sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}\right)A + \frac{1}{20}\left(\left(\sqrt{2}a^{\frac{3}{2}}e^{(-x)} + 5\sqrt{2}a^{\frac{3}{2}}e^{(-2x)} + 15\sqrt{2}a^{\frac{3}{2}}e^{(-3x)} - 5\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*a^(3/2)*e^(3/2*x) + 9*sqrt(2)*a^(3/2)*e^(1/2*x) - 9*sqrt(2)*a^
(3/2)*e^(-1/2*x) - sqrt(2)*a^(3/2)*e^(-3/2*x))*A + 1/20*((sqrt(2)*a^(3/2)*e
^(-x) + 5*sqrt(2)*a^(3/2)*e^(-2*x) + 15*sqrt(2)*a^(3/2)*e^(-3*x) - 5*sqrt(2
)*a^(3/2)*e^(-4*x))*e^(7/2*x) + (5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*a^(3
/2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2)*e^(-4*x))*e^(3/
2*x))*B
```

Fricas [B] time = 2.17396, size = 805, normalized size = 11.84

$$\frac{\sqrt{\frac{1}{2}}\left(3\,Ba\cosh\left(x\right)^{5} + 3\,Ba\sinh\left(x\right)^{5} + 5\,(2\,A + 3\,B)a\cosh\left(x\right)^{4} + 30\,(3\,A + 2\,B)a\cosh\left(x\right)^{3} + 5\,(3\,Ba\cosh\left(x\right) + (2\,A + 3\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A + 3*B)*a*cosh(x)
^4 + 30*(3*A + 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A + 3*B)*a)*sinh(x)
^4 - 30*(3*A + 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A + 3*B)*a*cos
h(x) + 3*(3*A + 2*B)*a)*sinh(x)^3 - 5*(2*A + 3*B)*a*cosh(x) + 30*(B*a*cosh(
x)^3 + (2*A + 3*B)*a*cosh(x)^2 + 3*(3*A + 2*B)*a*cosh(x) - (3*A + 2*B)*a)*s
inh(x)^2 - 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A + 3*B)*a*cosh(x)^3 + 18*(3*A
 + 2*B)*a*cosh(x)^2 - 12*(3*A + 2*B)*a*cosh(x) - (2*A + 3*B)*a)*sinh(x))*sq
rt(a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15364, size = 153, normalized size = 2.25

$$-\frac{1}{60}\sqrt{2}\left(\frac{\left(90\,Aa^{4}e^{(2\,x)}+60\,Ba^{4}e^{(2\,x)}+10\,Aa^{4}e^{x}+15\,Ba^{4}e^{x}+3\,Ba^{4}\right)e^{\left(-\frac{5}{2}\,x\right)}}{a^{\frac{5}{2}}}-\frac{3\,Ba^{\frac{13}{2}}e^{\left(\frac{5}{2}\,x\right)}+10\,Aa^{\frac{13}{2}}e^{\left(\frac{3}{2}\,x\right)}+15\,Ba^{\frac{13}{2}}e^{\left(\frac{3}{2}\,x\right)}\cdots}{a^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*((90*A*a^4*e^(2*x) + 60*B*a^4*e^(2*x) + 10*A*a^4*e^x + 15*B*a
^4*e^x + 3*B*a^4)*e^(-5/2*x)/a^(5/2) - (3*B*a^(13/2)*e^(5/2*x) + 10*A*a^(13
/2)*e^(3/2*x) + 15*B*a^(13/2)*e^(3/2*x) + 90*A*a^(13/2)*e^(1/2*x) + 60*B*a^
(13/2)*e^(1/2*x))/a^5)
```

3.89 $\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx$

Optimal. Leaf size=40

$$\frac{2a(3A+B)\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}$$

```
[Out] (2*a*(3*A + B)*Sinh[x])/(3*Sqrt[a + a*Cosh[x]]) + (2*B*Sqrt[a + a*Cosh[x]]*
Sinh[x])/3
```

Rubi [A] time = 0.0524759, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2751, 2646}

$$\frac{2a(3A+B)\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (2*a*(3*A + B)*Sinh[x])/(3*Sqrt[a + a*Cosh[x]]) + (2*B*Sqrt[a + a*Cosh[x]]*
Sinh[x])/3
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx = \frac{2}{3}B\sqrt{a+a\cosh(x)}\sinh(x)+\frac{1}{3}(3A+B)\int \sqrt{a+a\cosh(x)}\,dx$$
$$= \frac{2a(3A+B)\sinh(x)}{3\sqrt{a+a\cosh(x)}}+\frac{2}{3}B\sqrt{a+a\cosh(x)}\sinh(x)$$

Mathematica [A] time = 0.038382, size = 31, normalized size = 0.78

$$\frac{2}{3}\tanh\left(\frac{x}{2}\right)\sqrt{a(\cosh(x)+1)}(3A+B\cosh(x)+2B)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[x])]*(3*A + 2*B + B*Cosh[x])*Tanh[x/2])/3
```

Maple [A] time = 0.042, size = 39, normalized size = 1.

$$\frac{2\,a\sqrt{2}}{3}\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)\left(2\,B\left(\cosh\left(x/2\right)\right)^{2}+3\,A+B\right)\frac{1}{\sqrt{a\left(\cosh\left(\frac{x}{2}\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x)
```

```
[Out] 2/3*cosh(1/2*x)*a*sinh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A+B)*2^(1/2)/(a*cosh(1/2
*x)^2)^(1/2)
```

Maxima [B] time = 1.6292, size = 122, normalized size = 3.05

$$\left(\sqrt{2}\sqrt{a}e^{\left(\frac{1}{2}\,x\right)}-\sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}\,x\right)}\right)A+\frac{1}{6}\left(\left(\sqrt{2}\sqrt{a}e^{(-x)}+3\,\sqrt{2}\sqrt{a}e^{(-2\,x)}\right)e^{\left(\frac{5}{2}\,x\right)}-\left(3\,\sqrt{2}\sqrt{a}e^{(-x)}+\sqrt{2}\sqrt{a}e^{(-2\,x)}\right)e^{\left(\frac{1}{2}\,x\right)}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] (sqrt(2)*sqrt(a)*e^(1/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))*A + 1/6*((sqrt(2)*
sqrt(a)*e^(-x) + 3*sqrt(2)*sqrt(a)*e^(-2*x))*e^(5/2*x) - (3*sqrt(2)*sqrt(a)
*e^(-x) + sqrt(2)*sqrt(a)*e^(-2*x))*e^(1/2*x))*B
```

Fricas [B] time = 2.17391, size = 317, normalized size = 7.92

$$\frac{\sqrt{\frac{1}{2}}\left(B\cosh\left(x\right)^{3} + B\sinh\left(x\right)^{3} + 3\left(2\,A + B\right)\cosh\left(x\right)^{2} + 3\left(B\cosh\left(x\right) + 2\,A + B\right)\sinh\left(x\right)^{2} - 3\left(2\,A + B\right)\cosh\left(x\right) + 3\left(B\,c\right.\right.}{3\left(\cosh\left(x\right) + \sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A + B)*cosh(x)^2 + 3*(B*cos
h(x) + 2*A + B)*sinh(x)^2 - 3*(2*A + B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A +
 B)*cosh(x) - 2*A - B)*sinh(x) - B)*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x) +
sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\cosh\left(x\right) + 1\right)}\left(A + B\cosh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral(sqrt(a*(cosh(x) + 1))*(A + B*cosh(x)), x)
```

Giac [B] time = 1.11449, size = 93, normalized size = 2.32

$$\frac{\sqrt{2}\left(Ba^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)} + 6\,Aa^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)} + 3\,Ba^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)} - \frac{\left(6\,Aa^{3}e^{x} + 3\,Ba^{3}e^{x} + Ba^{3}\right)e^{\left(-\frac{3}{2}\,x\right)}}{a^{\frac{3}{2}}}\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(B*a^(3/2)*e^(3/2*x) + 6*A*a^(3/2)*e^(1/2*x) + 3*B*a^(3/2)*e^(1
/2*x) - (6*A*a^3*e^x + 3*B*a^3*e^x + B*a^3)*e^(-3/2*x)/a^(3/2))/a
```

3.90 $\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=98

$$-\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}}-\frac{16}{105}a^2(7A-5B)\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2}{35}a(7A-5B)\sinh(x)(a-a\cosh(x))^{3/2}+\frac{2}{7}B\sinh$$

```
[Out] (-64*a^3*(7*A - 5*B)*Sinh[x])/(105*Sqrt[a - a*Cosh[x]]) - (16*a^2*(7*A - 5*
B)*Sqrt[a - a*Cosh[x]]*Sinh[x])/105 - (2*a*(7*A - 5*B)*(a - a*Cosh[x])^(3/2
)*Sinh[x])/35 + (2*B*(a - a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rubi [A] time = 0.103554, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2751, 2647, 2646}

$$-\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}}-\frac{16}{105}a^2(7A-5B)\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2}{35}a(7A-5B)\sinh(x)(a-a\cosh(x))^{3/2}+\frac{2}{7}B\sinh$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (-64*a^3*(7*A - 5*B)*Sinh[x])/(105*Sqrt[a - a*Cosh[x]]) - (16*a^2*(7*A - 5*
B)*Sqrt[a - a*Cosh[x]]*Sinh[x])/105 - (2*a*(7*A - 5*B)*(a - a*Cosh[x])^(3/2
)*Sinh[x])/35 + (2*B*(a - a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{2}{7}B(a-a\cosh(x))^{5/2}\sinh(x)-\frac{1}{7}(-7A+5B)\int (a-a\cosh(x))^{5/2}\,dx$$
$$= -\frac{2}{35}a(7A-5B)(a-a\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a-a\cosh(x))^{5/2}\sinh(x)+\frac{1}{35}$$
$$= -\frac{16}{105}a^2(7A-5B)\sqrt{a-a\cosh(x)}\sinh(x)-\frac{2}{35}a(7A-5B)(a-a\cosh(x))^{3/2}\text{si}$$
$$= -\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}}-\frac{16}{105}a^2(7A-5B)\sqrt{a-a\cosh(x)}\sinh(x)-\frac{2}{35}a(7A$$

Mathematica [A] time = 0.134729, size = 61, normalized size = 0.62

$$\frac{1}{210}a^2\coth\left(\frac{x}{2}\right)\sqrt{a-a\cosh(x)}((505B-392A)\cosh(x)+6(7A-20B)\cosh(2x)+1246A+15B\cosh(3x)-1040B)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (a^2*Sqrt[a - a*Cosh[x]]*(1246*A - 1040*B + (-392*A + 505*B)*Cosh[x] + 6*(7
*A - 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Coth[x/2])/210
```

Maple [A] time = 0.05, size = 69, normalized size = 0.7

$$-\frac{16a^3}{105}\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\left(30B(\sinh(x/2))^6+(21A-15B)\left(\sinh\left(\frac{x}{2}\right)\right)^4+(-28A+20B)\left(\sinh\left(\frac{x}{2}\right)\right)^2+56A-40B\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x)
```

```
[Out] -16/105*sinh(1/2*x)*a^3*cosh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A-15*B)*sinh(1/
2*x)^4+(-28*A+20*B)*sinh(1/2*x)^2+56*A-40*B)/(-2*sinh(1/2*x)^2*a)^(1/2)
```

Maxima [B] time = 1.67311, size = 389, normalized size = 3.97

$$\frac{1}{60}\left(\frac{25\sqrt{2}a^{\frac{5}{2}}e^{(-x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}-\frac{150\sqrt{2}a^{\frac{5}{2}}e^{(-2x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}-\frac{150\sqrt{2}a^{\frac{5}{2}}e^{(-3x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}+\frac{25\sqrt{2}a^{\frac{5}{2}}e^{(-4x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}-\frac{3\sqrt{2}a^{\frac{5}{2}}e^{(-5x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}-\frac{3\sqrt{2}a^{\frac{5}{2}}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}\right)A+\frac{1}{168}B\left(\frac{\left(21\sqrt{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/60*(25*sqrt(2)*a^(5/2)*e^(-x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-2
*x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-3*x)/(-e^(-x))^(5/2) + 25*sqr
t(2)*a^(5/2)*e^(-4*x)/(-e^(-x))^(5/2) - 3*sqrt(2)*a^(5/2)*e^(-5*x)/(-e^(-x)
)^(5/2) - 3*sqrt(2)*a^(5/2)/(-e^(-x))^(5/2))*A + 1/168*B*((21*sqrt(2)*a^(5/
2)*e^(-x) - 70*sqrt(2)*a^(5/2)*e^(-2*x) + 210*sqrt(2)*a^(5/2)*e^(-3*x) + 10
5*sqrt(2)*a^(5/2)*e^(-4*x) - 7*sqrt(2)*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a^(5/2)
)*e^x/(-e^(-x))^(5/2) - (7*sqrt(2)*a^(5/2)*e^(-x) - 105*sqrt(2)*a^(5/2)*e^(
-2*x) - 210*sqrt(2)*a^(5/2)*e^(-3*x) + 70*sqrt(2)*a^(5/2)*e^(-4*x) - 21*sqr
t(2)*a^(5/2)*e^(-5*x) + 3*sqrt(2)*a^(5/2)*e^(-6*x))/(-e^(-x))^(5/2))
```

Fricas [B] time = 2.25447, size = 1554, normalized size = 15.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A - 5*B)*a
^2*cosh(x)^6 - 35*(10*A - 11*B)*a^2*cosh(x)^5 + 525*(4*A - 3*B)*a^2*cosh(x)
^4 + 21*(5*B*a^2*cosh(x) + (2*A - 5*B)*a^2)*sinh(x)^6 + 525*(4*A - 3*B)*a^2
*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A - 5*B)*a^2*cosh(x) - 5*(10*A -
 11*B)*a^2)*sinh(x)^5 - 35*(10*A - 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*cosh(
x)^3 + 9*(2*A - 5*B)*a^2*cosh(x)^2 - 5*(10*A - 11*B)*a^2*cosh(x) + 15*(4*A
- 3*B)*a^2)*sinh(x)^4 + 21*(2*A - 5*B)*a^2*cosh(x) + 35*(15*B*a^2*cosh(x)^4
 + 12*(2*A - 5*B)*a^2*cosh(x)^3 - 10*(10*A - 11*B)*a^2*cosh(x)^2 + 60*(4*A
```

```
- 3*B)*a^2*cosh(x) + 15*(4*A - 3*B)*a^2)*sinh(x)^3 + 15*B*a^2 + 35*(9*B*a^2
*cosh(x)^5 + 9*(2*A - 5*B)*a^2*cosh(x)^4 - 10*(10*A - 11*B)*a^2*cosh(x)^3 +
 90*(4*A - 3*B)*a^2*cosh(x)^2 + 45*(4*A - 3*B)*a^2*cosh(x) - (10*A - 11*B)*
a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A - 5*B)*a^2*cosh(x)^5 - 25*
(10*A - 11*B)*a^2*cosh(x)^4 + 300*(4*A - 3*B)*a^2*cosh(x)^3 + 225*(4*A - 3*
B)*a^2*cosh(x)^2 - 10*(10*A - 11*B)*a^2*cosh(x) + 3*(2*A - 5*B)*a^2)*sinh(x
))*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x
)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33842, size = 398, normalized size = 4.06

$$\frac{1}{840}\sqrt{2}\left(\frac{\left(2100\,Aa^6e^{(3\,x)}\mathrm{sgn}\left(-e^x+1\right)-1575\,Ba^6e^{(3\,x)}\mathrm{sgn}\left(-e^x+1\right)-350\,Aa^6e^{(2\,x)}\mathrm{sgn}\left(-e^x+1\right)+385\,Ba^6e^{(2\,x)}\mathrm{sgn}\left(-e^x\right.\right.}{\sqrt{-ae^x}a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/840*sqrt(2)*((2100*A*a^6*e^(3*x)*sgn(-e^x + 1) - 1575*B*a^6*e^(3*x)*sgn(-
e^x + 1) - 350*A*a^6*e^(2*x)*sgn(-e^x + 1) + 385*B*a^6*e^(2*x)*sgn(-e^x + 1
) + 42*A*a^6*e^x*sgn(-e^x + 1) - 105*B*a^6*e^x*sgn(-e^x + 1) + 15*B*a^6*sgn
(-e^x + 1))*e^(-3*x)/(sqrt(-a*e^x)*a^3) - (15*sqrt(-a*e^x)*B*a^9*e^(3*x)*sg
n(-e^x + 1) + 42*sqrt(-a*e^x)*A*a^9*e^(2*x)*sgn(-e^x + 1) - 105*sqrt(-a*e^x
)*B*a^9*e^(2*x)*sgn(-e^x + 1) - 350*sqrt(-a*e^x)*A*a^9*e^x*sgn(-e^x + 1) +
385*sqrt(-a*e^x)*B*a^9*e^x*sgn(-e^x + 1) + 2100*sqrt(-a*e^x)*A*a^9*sgn(-e^x
 + 1) - 1575*sqrt(-a*e^x)*B*a^9*sgn(-e^x + 1))/a^7)
```

3.91 $\int (a - a\cosh(x))^{3/2}(A + B\cosh(x))\,dx$

Optimal. Leaf size=71

$$-\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}}-\frac{2}{15}a(5A-3B)\sinh(x)\sqrt{a-a\cosh(x)}+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

```
[Out] (-8*a^2*(5*A - 3*B)*Sinh[x])/(15*Sqrt[a - a*Cosh[x]]) - (2*a*(5*A - 3*B)*Sq
rt[a - a*Cosh[x]]*Sinh[x])/15 + (2*B*(a - a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rubi [A] time = 0.0802921, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2751, 2647, 2646}

$$-\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}}-\frac{2}{15}a(5A-3B)\sinh(x)\sqrt{a-a\cosh(x)}+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (-8*a^2*(5*A - 3*B)*Sinh[x])/(15*Sqrt[a - a*Cosh[x]]) - (2*a*(5*A - 3*B)*Sq
rt[a - a*Cosh[x]]*Sinh[x])/15 + (2*B*(a - a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{5}B(a-a\cosh(x))^{3/2}\sinh(x)-\frac{1}{5}(-5A+3B)\int (a-a\cosh(x))^{3/2}\,dx$$
$$= -\frac{2}{15}a(5A-3B)\sqrt{a-a\cosh(x)}\sinh(x)+\frac{2}{5}B(a-a\cosh(x))^{3/2}\sinh(x)+\frac{1}{15}($$
$$= -\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}}-\frac{2}{15}a(5A-3B)\sqrt{a-a\cosh(x)}\sinh(x)+\frac{2}{5}B(a-a\,c$$

Mathematica [A] time = 0.0971045, size = 47, normalized size = 0.66

$$-\frac{1}{15}a\coth\left(\frac{x}{2}\right)\sqrt{a-a\cosh(x)}(2(5A-9B)\cosh(x)-50A+3B\cosh(2x)+39B)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] -(a*Sqrt[a - a*Cosh[x]]*(-50*A + 39*B + 2*(5*A - 9*B)*Cosh[x] + 3*B*Cosh[2*
x])*Coth[x/2])/15
```

Maple [A] time = 0.049, size = 55, normalized size = 0.8

$$\frac{8a^2}{15}\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\left(6B(\sinh(x/2))^4+(5A-3B)\left(\sinh\left(\frac{x}{2}\right)\right)^2-10A+6B\right)\frac{1}{\sqrt{-2(\sinh(x/2))^2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x)
```

```
[Out] 8/15*sinh(1/2*x)*a^2*cosh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A-3*B)*sinh(1/2*x)^2
-10*A+6*B)/(-2*sinh(1/2*x)^2*a)^(1/2)
```

Maxima [B] time = 1.6047, size = 269, normalized size = 3.79

$$\frac{1}{6}\left(\frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-x)}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}+\frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}\right)A+\frac{1}{20}B\left(\frac{\left(5\sqrt{2}a^{\frac{3}{2}}e^{(-x)}-15\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}-5\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}-\sqrt{2}\right.}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/6*(9*sqrt(2)*a^(3/2)*e^(-x)/(-e^(-x))^(3/2) + 9*sqrt(2)*a^(3/2)*e^(-2*x)/
(-e^(-x))^(3/2) - sqrt(2)*a^(3/2)*e^(-3*x)/(-e^(-x))^(3/2) - sqrt(2)*a^(3/2
)/(-e^(-x))^(3/2))*A + 1/20*B*((5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*a^(3/
2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2))*e^x/(-e^(-x))^(
3/2) - (5*sqrt(2)*a^(3/2)*e^(-x) + 15*sqrt(2)*a^(3/2)*e^(-2*x) - 5*sqrt(2)*
a^(3/2)*e^(-3*x) + sqrt(2)*a^(3/2)*e^(-4*x))/(-e^(-x))^(3/2))
```

Fricas [B] time = 2.15376, size = 807, normalized size = 11.37

$$-\frac{\sqrt{\frac{1}{2}}\left(3\,Ba\cosh\left(x\right)^{5}+3\,Ba\sinh\left(x\right)^{5}+5\,(2\,A-3\,B)a\cosh\left(x\right)^{4}-30\,(3\,A-2\,B)a\cosh\left(x\right)^{3}+5\,(3\,Ba\cosh\left(x\right)+(2\,A-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A - 3*B)*a*cosh(x
)^4 - 30*(3*A - 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A - 3*B)*a)*sinh(x
)^4 - 30*(3*A - 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A - 3*B)*a*co
sh(x) - 3*(3*A - 2*B)*a)*sinh(x)^3 + 5*(2*A - 3*B)*a*cosh(x) + 30*(B*a*cosh
(x)^3 + (2*A - 3*B)*a*cosh(x)^2 - 3*(3*A - 2*B)*a*cosh(x) - (3*A - 2*B)*a)*
sinh(x)^2 + 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A - 3*B)*a*cosh(x)^3 - 18*(3*
A - 2*B)*a*cosh(x)^2 - 12*(3*A - 2*B)*a*cosh(x) + (2*A - 3*B)*a)*sinh(x))*s
qrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25391, size = 286, normalized size = 4.03

$$\frac{1}{60}\sqrt{2}\left(\frac{\left(90\,Aa^4e^{(2\,x)}\operatorname{sgn}\left(-e^x+1\right) - 60\,Ba^4e^{(2\,x)}\operatorname{sgn}\left(-e^x+1\right) - 10\,Aa^4e^x\operatorname{sgn}\left(-e^x+1\right) + 15\,Ba^4e^x\operatorname{sgn}\left(-e^x+1\right) - 3\,Ba^4\mathrm{sg}\right.}{\sqrt{-ae^x}a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(2)*((90*A*a^4*e^(2*x)*sgn(-e^x + 1) - 60*B*a^4*e^(2*x)*sgn(-e^x +
 1) - 10*A*a^4*e^x*sgn(-e^x + 1) + 15*B*a^4*e^x*sgn(-e^x + 1) - 3*B*a^4*sgn
(-e^x + 1))*e^(-2*x)/(sqrt(-a*e^x)*a^2) + (3*sqrt(-a*e^x)*B*a^6*e^(2*x)*sgn
(-e^x + 1) + 10*sqrt(-a*e^x)*A*a^6*e^x*sgn(-e^x + 1) - 15*sqrt(-a*e^x)*B*a^
6*e^x*sgn(-e^x + 1) - 90*sqrt(-a*e^x)*A*a^6*sgn(-e^x + 1) + 60*sqrt(-a*e^x)
*B*a^6*sgn(-e^x + 1))/a^5)
```

3.92 $\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx$

Optimal. Leaf size=44

$$\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}$$

```
[Out] (-2*a*(3*A - B)*Sinh[x])/(3*Sqrt[a - a*Cosh[x]]) + (2*B*Sqrt[a - a*Cosh[x]]
*Sinh[x])/3
```

Rubi [A] time = 0.0576204, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2751, 2646}

$$\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (-2*a*(3*A - B)*Sinh[x])/(3*Sqrt[a - a*Cosh[x]]) + (2*B*Sqrt[a - a*Cosh[x]]
*Sinh[x])/3
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx = \frac{2}{3}B\sqrt{a-a\cosh(x)}\sinh(x) - \frac{1}{3}(-3A+B)\int \sqrt{a-a\cosh(x)}\,dx$$
$$= -\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}} + \frac{2}{3}B\sqrt{a-a\cosh(x)}\sinh(x)$$

Mathematica [A] time = 0.0506901, size = 32, normalized size = 0.73

$$\frac{2}{3}\coth\left(\frac{x}{2}\right)\sqrt{a-a\cosh(x)}(3A+B\cosh(x)-2B)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (2*Sqrt[a - a*Cosh[x]]*(3*A - 2*B + B*Cosh[x])*Coth[x/2])/3
```

Maple [A] time = 0.052, size = 39, normalized size = 0.9

$$-\frac{4\,a}{3}\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\left(2\,B\left(\cosh\left(x/2\right)\right)^{2}+3\,A-3\,B\right)\frac{1}{\sqrt{-2\,\left(\sinh\left(x/2\right)\right)^{2}a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x)
```

```
[Out] -4/3*sinh(1/2*x)*a*cosh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A-3*B)/(-2*sinh(1/2*x)^
2*a)^(1/2)
```

Maxima [B] time = 1.6375, size = 147, normalized size = 3.34

$$-\left(\frac{\sqrt{2}\sqrt{a}e^{(-x)}}{\sqrt{-e^{(-x)}}} + \frac{\sqrt{2}\sqrt{a}}{\sqrt{-e^{(-x)}}}\right)A + \frac{1}{6}\left(\frac{\left(3\sqrt{2}\sqrt{a}e^{(-x)} - \sqrt{2}\sqrt{a}\right)e^{x}}{\sqrt{-e^{(-x)}}} + \frac{3\sqrt{2}\sqrt{a}e^{(-x)} - \sqrt{2}\sqrt{a}e^{(-2\,x)}}{\sqrt{-e^{(-x)}}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*e^(-x)/sqrt(-e^(-x)) + sqrt(2)*sqrt(a)/sqrt(-e^(-x)))*A +
 1/6*((3*sqrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a))*e^x/sqrt(-e^(-x)) + (3*s
qrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a)*e^(-2*x))/sqrt(-e^(-x)))*B
```

Fricas [B] time = 2.14688, size = 319, normalized size = 7.25

$$\frac{\sqrt{\frac{1}{2}}\Big(B\cosh(x)^3 + B\sinh(x)^3 + 3(2A - B)\cosh(x)^2 + 3(B\cosh(x) + 2A - B)\sinh(x)^2 + 3(2A - B)\cosh(x) + 3\big(B\,\mathrm{c}}{3(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A - B)*cosh(x)^2 + 3*(B*cos
h(x) + 2*A - B)*sinh(x)^2 + 3*(2*A - B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A -
 B)*cosh(x) + 2*A - B)*sinh(x) + B)*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x) +
 sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a(\cosh(x) - 1)}\,(A + B\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral(sqrt(-a*(cosh(x) - 1))*(A + B*cosh(x)), x)
```

Giac [B] time = 1.23306, size = 166, normalized size = 3.77

$$-\frac{\sqrt{2}\left(\sqrt{-ae^x}Bae^x\mathrm{sgn}(-e^x+1) + 6\sqrt{-ae^x}Aa\mathrm{sgn}(-e^x+1) - 3\sqrt{-ae^x}Ba\mathrm{sgn}(-e^x+1) - \frac{\left(6Aa^3e^x\mathrm{sgn}(-e^x+1) - 3Ba^3e^x\mathrm{sgn}(-e^x+1) + B\right.}{\sqrt{-ae^x}a}\right.}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")

[Out] -1/6*sqrt(2)*(sqrt(-a*e^x)*B*a*e^x*sgn(-e^x + 1) + 6*sqrt(-a*e^x)*A*a*sgn(-e^x + 1) - 3*sqrt(-a*e^x)*B*a*sgn(-e^x + 1) - (6*A*a^3*e^x*sgn(-e^x + 1) - 3*B*a^3*e^x*sgn(-e^x + 1) + B*a^3*sgn(-e^x + 1))*e^(-x)/(sqrt(-a*e^x)*a))/a

3.93 $\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{(A-B)\sinh(x)}{\cosh(x)+1}+Bx$$

```
[Out] B*x + ((A - B)*Sinh[x])/(1 + Cosh[x])
```

Rubi [A] time = 0.0359926, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2735, 2648}

$$\frac{(A-B)\sinh(x)}{\cosh(x)+1}+Bx$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x]),x]
```

```
[Out] B*x + ((A - B)*Sinh[x])/(1 + Cosh[x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx &= Bx-(-A+B)\int \frac{1}{1+\cosh(x)}\,dx\\ &= Bx+\frac{(A-B)\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.0578666, size = 23, normalized size = 1.28

$$\frac{\sinh(x)\left(A+Bx\coth\left(\frac{x}{2}\right)-B\right)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x]),x]
```

```
[Out] ((A - B + B*x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])
```

Maple [A] time = 0.01, size = 34, normalized size = 1.9

$$A\tanh\left(\frac{x}{2}\right)-B\tanh\left(\frac{x}{2}\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1+cosh(x)),x)
```

```
[Out] A*tanh(1/2*x)-B*tanh(1/2*x)-B*ln(tanh(1/2*x)-1)+B*ln(tanh(1/2*x)+1)
```

Maxima [A] time = 1.04029, size = 35, normalized size = 1.94

$$B\left(x-\frac{2}{e^{(-x)}+1}\right)+\frac{2A}{e^{(-x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="maxima")
```

```
[Out] B*(x - 2/(e^(-x) + 1)) + 2*A/(e^(-x) + 1)
```

Fricas [A] time = 2.14689, size = 96, normalized size = 5.33

$$\frac{Bx\cosh(x)+Bx\sinh(x)+Bx-2A+2B}{\cosh(x)+\sinh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*cosh(x) + B*x*sinh(x) + B*x - 2*A + 2*B)/(cosh(x) + sinh(x) + 1)
```

Sympy [A] time = 0.411026, size = 15, normalized size = 0.83

$$A\tanh\left(\frac{x}{2}\right) + Bx - B\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x)
```

```
[Out] A*tanh(x/2) + B*x - B*tanh(x/2)
```

Giac [A] time = 1.16183, size = 23, normalized size = 1.28

$$Bx - \frac{2(A - B)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="giac")
```

```
[Out] B*x - 2*(A - B)/(e^x + 1)
```

3.94 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx$

Optimal. Leaf size=35

$$\frac{(A+2B)\sinh(x)}{3(\cosh(x)+1)}+\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}$$

```
[Out] ((A - B)*Sinh[x])/(3*(1 + Cosh[x])^2) + ((A + 2*B)*Sinh[x])/(3*(1 + Cosh[x]
))
```

Rubi [A] time = 0.0375198, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2750, 2648}

$$\frac{(A+2B)\sinh(x)}{3(\cosh(x)+1)}+\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

```
[Out] ((A - B)*Sinh[x])/(3*(1 + Cosh[x])^2) + ((A + 2*B)*Sinh[x])/(3*(1 + Cosh[x]
))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = \frac{(A-B)\sinh(x)}{3(1+\cosh(x))^2} + \frac{1}{3}(A+2B)\int \frac{1}{1+\cosh(x)}\,dx$$
$$= \frac{(A-B)\sinh(x)}{3(1+\cosh(x))^2} + \frac{(A+2B)\sinh(x)}{3(1+\cosh(x))}$$

Mathematica [A] time = 0.0504652, size = 25, normalized size = 0.71

$$\frac{\sinh(x)((A+2B)\cosh(x)+2A+B)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

```
[Out] ((2*A + B + (A + 2*B)*Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

Maple [A] time = 0.007, size = 34, normalized size = 1.

$$-\frac{A}{6}\left(\tanh\left(\frac{x}{2}\right)\right)^3 + \frac{B}{6}\left(\tanh\left(\frac{x}{2}\right)\right)^3 + \frac{A}{2}\tanh\left(\frac{x}{2}\right) + \frac{B}{2}\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1+cosh(x))^2,x)
```

```
[Out] -1/6*A*tanh(1/2*x)^3+1/6*B*tanh(1/2*x)^3+1/2*A*tanh(1/2*x)+1/2*B*tanh(1/2*x
)
```

Maxima [B] time = 1.03681, size = 174, normalized size = 4.97

$$\frac{2}{3}B\left(\frac{3e^{(-x)}}{3e^{(-x)}+3e^{(-2x)}+e^{(-3x)}+1} + \frac{3e^{(-2x)}}{3e^{(-x)}+3e^{(-2x)}+e^{(-3x)}+1} + \frac{2}{3e^{(-x)}+3e^{(-2x)}+e^{(-3x)}+1}\right) + \frac{2}{3}A\left(\frac{3e^{(-}}{3e^{(-x)}+3e^{(-2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*B*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 3*e^(-2*x)/(3*e^(-
x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1))
 + 2/3*A*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 1/(3*e^(-x) + 3
*e^(-2*x) + e^(-3*x) + 1))
```

Fricas [A] time = 2.07598, size = 165, normalized size = 4.71

$$-\frac{2\left((A+5B)\cosh(x)-(A-B)\sinh(x)+3A+3B\right)}{3\left(\cosh(x)^2+2\left(\cosh(x)+1\right)\sinh(x)+\sinh(x)^2+4\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*((A + 5*B)*cosh(x) - (A - B)*sinh(x) + 3*A + 3*B)/(cosh(x)^2 + 2*(cosh
(x) + 1)*sinh(x) + sinh(x)^2 + 4*cosh(x) + 3)
```

Sympy [A] time = 0.856958, size = 36, normalized size = 1.03

$$-\frac{A\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{2}+\frac{B\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{B\tanh\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))**2,x)
```

```
[Out] -A*tanh(x/2)**3/6 + A*tanh(x/2)/2 + B*tanh(x/2)**3/6 + B*tanh(x/2)/2
```

Giac [A] time = 1.21306, size = 41, normalized size = 1.17

$$-\frac{2\left(3Be^{(2x)}+3Ae^x+3Be^x+A+2B\right)}{3\left(e^x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x + 3*B*e^x + A + 2*B)/(e^x + 1)^3
```

3.95 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx$

Optimal. Leaf size=56

$$\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)}+\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)^2}+\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}$$

```
[Out] ((A - B)*Sinh[x])/(5*(1 + Cosh[x])^3) + ((2*A + 3*B)*Sinh[x])/(15*(1 + Cosh
[x])^2) + ((2*A + 3*B)*Sinh[x])/(15*(1 + Cosh[x]))
```

Rubi [A] time = 0.0482171, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2750, 2650, 2648}

$$\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)}+\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)^2}+\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

```
[Out] ((A - B)*Sinh[x])/(5*(1 + Cosh[x])^3) + ((2*A + 3*B)*Sinh[x])/(15*(1 + Cosh
[x])^2) + ((2*A + 3*B)*Sinh[x])/(15*(1 + Cosh[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx &= \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{1}{5}(2A+3B)\int \frac{1}{(1+\cosh(x))^2}\,dx\\ &= \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))^2}+\frac{1}{15}(2A+3B)\int \frac{1}{1+\cosh(x)}\,dx\\ &= \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))^2}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.0758902, size = 42, normalized size = 0.75

$$\frac{\sinh(x)(6(2A+3B)\cosh(x)+(2A+3B)\cosh(2x)+16A+9B)}{30(\cosh(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

```
[Out] ((16*A + 9*B + 6*(2*A + 3*B)*Cosh[x] + (2*A + 3*B)*Cosh[2*x])*Sinh[x])/(30*
(1 + Cosh[x])^3)
```

Maple [A] time = 0.009, size = 38, normalized size = 0.7

$$\frac{A-B}{20}\left(\tanh\left(\frac{x}{2}\right)\right)^5-\frac{A}{6}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{A}{4}\tanh\left(\frac{x}{2}\right)+\frac{B}{4}\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1+cosh(x))^3,x)
```

```
[Out] 1/20*(A-B)*tanh(1/2*x)^5-1/6*A*tanh(1/2*x)^3+1/4*A*tanh(1/2*x)+1/4*B*tanh(1
/2*x)
```

Maxima [B] time = 1.04365, size = 355, normalized size = 6.34

$$\frac{4}{15}A\left(\frac{5e^{(-x)}}{5e^{(-x)}+10e^{(-2x)}+10e^{(-3x)}+5e^{(-4x)}+e^{(-5x)}+1}+\frac{10e^{(-2x)}}{5e^{(-x)}+10e^{(-2x)}+10e^{(-3x)}+5e^{(-4x)}+e^{(-5x)}+1}+\frac{}{5e^{(-x)}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 4/15*A*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) + e^(-5
*x) + 1) + 10*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) +
 e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) + e^(
-5*x) + 1)) + 2/5*B*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(
-4*x) + e^(-5*x) + 1) + 5*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) +
5*e^(-4*x) + e^(-5*x) + 1) + 5*e^(-3*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*
x) + 5*e^(-4*x) + e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) +
 5*e^(-4*x) + e^(-5*x) + 1))
```

Fricas [B] time = 2.10774, size = 427, normalized size = 7.62

$$-\frac{2\left(15\,B\cosh\left(x\right)^{2}+15\,B\sinh\left(x\right)^{2}+2\,(11\,A+9\,B)\cosh\left(x\right)+6\,(5\,B\cosh\right.}{15\left(\cosh\left(x\right)^{4}+(4\,\cosh\left(x\right)+5)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+5\,\cosh\left(x\right)^{3}+\left(6\,\cosh\left(x\right)^{2}+15\,\cosh\left(x\right)+10\right)\sinh\left(x\right)^{2}+10\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A + 9*B)*cosh(x) + 6*(5*B*co
sh(x) + 3*A + 2*B)*sinh(x) + 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) + 5)*sinh
(x)^3 + sinh(x)^4 + 5*cosh(x)^3 + (6*cosh(x)^2 + 15*cosh(x) + 10)*sinh(x)^2
 + 10*cosh(x)^2 + (4*cosh(x)^3 + 15*cosh(x)^2 + 20*cosh(x) + 9)*sinh(x) + 1
1*cosh(x) + 5)
```

Sympy [A] time = 1.81946, size = 46, normalized size = 0.82

$$\frac{A\tanh^{5}\left(\frac{x}{2}\right)}{20}-\frac{A\tanh^{3}\left(\frac{x}{2}\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{4}-\frac{B\tanh^{5}\left(\frac{x}{2}\right)}{20}+\frac{B\tanh\left(\frac{x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))**3,x)
```

```
[Out] A*tanh(x/2)**5/20 - A*tanh(x/2)**3/6 + A*tanh(x/2)/4 - B*tanh(x/2)**5/20 +
B*tanh(x/2)/4
```

Giac [A] time = 1.17739, size = 62, normalized size = 1.11

$$-\frac{2\left(15\,Be^{(3\,x)}+20\,Ae^{(2\,x)}+15\,Be^{(2\,x)}+10\,Ae^{x}+15\,Be^{x}+2\,A+3\,B\right)}{15\,(e^{x}+1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*B*e^(3*x) + 20*A*e^(2*x) + 15*B*e^(2*x) + 10*A*e^x + 15*B*e^x + 2
*A + 3*B)/(e^x + 1)^5
```

3.96 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx$

Optimal. Leaf size=75

$$\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)}+\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)^2}+\frac{(3A+4B)\sinh(x)}{35(\cosh(x)+1)^3}+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

```
[Out] ((A - B)*Sinh[x])/(7*(1 + Cosh[x])^4) + ((3*A + 4*B)*Sinh[x])/(35*(1 + Cosh
[x])^3) + (2*(3*A + 4*B)*Sinh[x])/(105*(1 + Cosh[x])^2) + (2*(3*A + 4*B)*Si
nh[x])/(105*(1 + Cosh[x]))
```

Rubi [A] time = 0.0594869, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2750, 2650, 2648}

$$\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)}+\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)^2}+\frac{(3A+4B)\sinh(x)}{35(\cosh(x)+1)^3}+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

```
[Out] ((A - B)*Sinh[x])/(7*(1 + Cosh[x])^4) + ((3*A + 4*B)*Sinh[x])/(35*(1 + Cosh
[x])^3) + (2*(3*A + 4*B)*Sinh[x])/(105*(1 + Cosh[x])^2) + (2*(3*A + 4*B)*Si
nh[x])/(105*(1 + Cosh[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx &= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{1}{7}(3A+4B)\int \frac{1}{(1+\cosh(x))^3}\,dx\\ &= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3}+\frac{1}{35}(2(3A+4B))\int \frac{1}{(1+\cosh(x))^2}\,dx\\ &= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))^2}+\frac{1}{105}(2(3A+4B))\int \frac{1}{1+\cosh(x)}\\ &= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))^2}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.0918554, size = 57, normalized size = 0.76

$$\frac{\sinh(x)(29(3A+4B)\cosh(x)+8(3A+4B)\cosh(2x)+3A\cosh(3x)+96A+4B\cosh(3x)+58B)}{210(\cosh(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

```
[Out] ((96*A + 58*B + 29*(3*A + 4*B)*Cosh[x] + 8*(3*A + 4*B)*Cosh[2*x] + 3*A*Cosh
[3*x] + 4*B*Cosh[3*x])*Sinh[x])/(210*(1 + Cosh[x])^4)
```

Maple [A] time = 0.009, size = 55, normalized size = 0.7

$$-\frac{A-B}{56}\left(\tanh\left(\frac{x}{2}\right)\right)^7-\frac{-3\,A+B}{40}\left(\tanh\left(\frac{x}{2}\right)\right)^5-\frac{3\,A+B}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{A}{8}\tanh\left(\frac{x}{2}\right)+\frac{B}{8}\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1+cosh(x))^4,x)
```

```
[Out] -1/56*(A-B)*tanh(1/2*x)^7-1/40*(-3*A+B)*tanh(1/2*x)^5-1/24*(3*A+B)*tanh(1/2
*x)^3+1/8*A*tanh(1/2*x)+1/8*B*tanh(1/2*x)
```

Maxima [B] time = 1.0755, size = 606, normalized size = 8.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="maxima")
```

```
[Out] 8/105*B*(14*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21
*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 42*e^(-2*x)/(7*e^(-x) + 21*e^(-2*x
) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) +
35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*
x) + 7*e^(-6*x) + e^(-7*x) + 1) + 35*e^(-4*x)/(7*e^(-x) + 21*e^(-2*x) + 35*
e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 2/(7*e^
(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) +
 e^(-7*x) + 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) +
35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 21*e^(-2*x)/(7*e^(
-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) +
e^(-7*x) + 1) + 35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-
4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 1/(7*e^(-x) + 21*e^(-2*x)
 + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1))
```

Fricas [B] time = 2.09743, size = 586, normalized size = 7.81

$$-\frac{4\left((3A+74B)\cosh(x)^2+(3A+74\right.}{105\left(\cosh(x)^5+(5\cosh(x)+7)\sinh(x)^4+\sinh(x)^5+7\cosh(x)^4+\left(10\cosh(x)^2+28\cosh(x)+21\right)\sinh(x)^3+21\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="fricas")
```

```
[Out] -4/105*((3*A + 74*B)*cosh(x)^2 + (3*A + 74*B)*sinh(x)^2 + 14*(9*A + 7*B)*co
sh(x) - 6*((A - 22*B)*cosh(x) - 14*A - 7*B)*sinh(x) + 63*A + 84*B)/(cosh(x)
^5 + (5*cosh(x) + 7)*sinh(x)^4 + sinh(x)^5 + 7*cosh(x)^4 + (10*cosh(x)^2 +
28*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 + 42*cosh(x)^2 +
63*cosh(x) + 36)*sinh(x)^2 + 36*cosh(x)^2 + (5*cosh(x)^4 + 28*cosh(x)^3 + 6
```

```
3*cosh(x)^2 + 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) + 21)
```

Sympy [A] time = 4.36389, size = 78, normalized size = 1.04

$$-\frac{A\tanh^7\left(\frac{x}{2}\right)}{56}+\frac{3A\tanh^5\left(\frac{x}{2}\right)}{40}-\frac{A\tanh^3\left(\frac{x}{2}\right)}{8}+\frac{A\tanh\left(\frac{x}{2}\right)}{8}+\frac{B\tanh^7\left(\frac{x}{2}\right)}{56}-\frac{B\tanh^5\left(\frac{x}{2}\right)}{40}-\frac{B\tanh^3\left(\frac{x}{2}\right)}{24}+\frac{B\tanh\left(\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))**4,x)
```

```
[Out] -A*tanh(x/2)**7/56 + 3*A*tanh(x/2)**5/40 - A*tanh(x/2)**3/8 + A*tanh(x/2)/8
 + B*tanh(x/2)**7/56 - B*tanh(x/2)**5/40 - B*tanh(x/2)**3/24 + B*tanh(x/2)/
8
```

Giac [A] time = 1.19734, size = 81, normalized size = 1.08

$$-\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}+70\,Be^{(3\,x)}+63\,Ae^{(2\,x)}+84\,Be^{(2\,x)}+21\,Ae^{x}+28\,Be^{x}+3\,A+4\,B\right)}{105\,(e^{x}+1)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) + 70*B*e^(3*x) + 63*A*e^(2*x) + 84*B*e
^(2*x) + 21*A*e^x + 28*B*e^x + 3*A + 4*B)/(e^x + 1)^7
```

3.97 $\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx$

Optimal. Leaf size=20

$$-\frac{(A+B)\sinh(x)}{1-\cosh(x)}-Bx$$

```
[Out] -(B*x) - ((A + B)*Sinh[x])/(1 - Cosh[x])
```

Rubi [A] time = 0.0395878, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2735, 2648}

$$-\frac{(A+B)\sinh(x)}{1-\cosh(x)}-Bx$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x]),x]
```

```
[Out] -(B*x) - ((A + B)*Sinh[x])/(1 - Cosh[x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx &= -Bx-(-A-B)\int \frac{1}{1-\cosh(x)}\,dx\\ &= -Bx-\frac{(A+B)\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.0540247, size = 35, normalized size = 1.75

$$\frac{2\sinh\left(\frac{x}{2}\right)\left((A+B)\cosh\left(\frac{x}{2}\right)-Bx\sinh\left(\frac{x}{2}\right)\right)}{\cosh(x)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x]),x]
```

```
[Out] (2*Sinh[x/2]*((A + B)*Cosh[x/2] - B*x*Sinh[x/2]))/(-1 + Cosh[x])
```

Maple [A] time = 0.016, size = 37, normalized size = 1.9

$$-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+A\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+B\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x)),x)
```

```
[Out] -B*ln(tanh(1/2*x)+1)+1/tanh(1/2*x)*A+1/tanh(1/2*x)*B+B*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.03264, size = 36, normalized size = 1.8

$$-B\left(x+\frac{2}{e^{(-x)}-1}\right)-\frac{2A}{e^{(-x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="maxima")
```

```
[Out] -B*(x + 2/(e^(-x) - 1)) - 2*A/(e^(-x) - 1)
```

Fricas [A] time = 2.17154, size = 97, normalized size = 4.85

$$-\frac{Bx\cosh(x)+Bx\sinh(x)-Bx-2A-2B}{\cosh(x)+\sinh(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*cosh(x) + B*x*sinh(x) - B*x - 2*A - 2*B)/(cosh(x) + sinh(x) - 1)
```

Sympy [A] time = 0.66039, size = 15, normalized size = 0.75

$$\frac{A}{\tanh\left(\frac{x}{2}\right)} - Bx + \frac{B}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x)
```

```
[Out] A/tanh(x/2) - B*x + B/tanh(x/2)
```

Giac [A] time = 1.1627, size = 22, normalized size = 1.1

$$-Bx + \frac{2(A + B)}{e^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*(A + B)/(e^x - 1)
```

3.98 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx$

Optimal. Leaf size=37

$$-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}$$

```
[Out] -((A + B)*Sinh[x])/(3*(1 - Cosh[x])^2) - ((A - 2*B)*Sinh[x])/(3*(1 - Cosh[x
]))
```

Rubi [A] time = 0.0408369, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2750, 2648}

$$-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

```
[Out] -((A + B)*Sinh[x])/(3*(1 - Cosh[x])^2) - ((A - 2*B)*Sinh[x])/(3*(1 - Cosh[x
]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = -\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2} + \frac{1}{3}(A-2B)\int \frac{1}{1-\cosh(x)}\,dx$$
$$= -\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2} - \frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}$$

Mathematica [A] time = 0.0485829, size = 25, normalized size = 0.68

$$\frac{\sinh(x)((A-2B)\cosh(x)-2A+B)}{3(\cosh(x)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

```
[Out] ((-2*A + B + (A - 2*B)*Cosh[x])*Sinh[x])/(3*(-1 + Cosh[x])^2)
```

Maple [A] time = 0.013, size = 26, normalized size = 0.7

$$-\frac{-A+B}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - \frac{A+B}{6}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x))^2,x)
```

```
[Out] -1/2*(-A+B)/tanh(1/2*x)-1/6*(A+B)/tanh(1/2*x)^3
```

Maxima [B] time = 1.04776, size = 177, normalized size = 4.78

$$-\frac{2}{3}B\left(\frac{3e^{(-x)}}{3e^{(-x)}-3e^{(-2x)}+e^{(-3x)}-1} - \frac{3e^{(-2x)}}{3e^{(-x)}-3e^{(-2x)}+e^{(-3x)}-1} - \frac{2}{3e^{(-x)}-3e^{(-2x)}+e^{(-3x)}-1}\right) + \frac{2}{3}A\left(\frac{3e^{(-}}{3e^{(-x)}-3e^{(-2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*B*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 3*e^(-2*x)/(3*e^(
-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1)
) + 2/3*A*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 1/(3*e^(-x) -
3*e^(-2*x) + e^(-3*x) - 1))
```

Fricas [A] time = 2.06134, size = 163, normalized size = 4.41

$$\frac{2\left((A-5B)\cosh(x)-(A+B)\sinh(x)-3A+3B\right)}{3\left(\cosh(x)^2+2(\cosh(x)-1)\sinh(x)+\sinh(x)^2-4\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] 2/3*((A - 5*B)*cosh(x) - (A + B)*sinh(x) - 3*A + 3*B)/(cosh(x)^2 + 2*(cosh(
x) - 1)*sinh(x) + sinh(x)^2 - 4*cosh(x) + 3)
```

Sympy [A] time = 1.20393, size = 36, normalized size = 0.97

$$\frac{A}{2\tanh\left(\frac{x}{2}\right)}-\frac{A}{6\tanh^3\left(\frac{x}{2}\right)}-\frac{B}{2\tanh\left(\frac{x}{2}\right)}-\frac{B}{6\tanh^3\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))**2,x)
```

```
[Out] A/(2*tanh(x/2)) - A/(6*tanh(x/2)**3) - B/(2*tanh(x/2)) - B/(6*tanh(x/2)**3)
```

Giac [A] time = 1.16434, size = 43, normalized size = 1.16

$$-\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^x-3\,Be^x-A+2\,B\right)}{3\,(e^x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x - 3*B*e^x - A + 2*B)/(e^x - 1)^3
```

3.99 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx$

Optimal. Leaf size=60

$$-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

```
[Out] -((A + B)*Sinh[x])/(5*(1 - Cosh[x])^3) - ((2*A - 3*B)*Sinh[x])/(15*(1 - Cos
h[x])^2) - ((2*A - 3*B)*Sinh[x])/(15*(1 - Cosh[x]))
```

Rubi [A] time = 0.0550135, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2750, 2650, 2648}

$$-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

```
[Out] -((A + B)*Sinh[x])/(5*(1 - Cosh[x])^3) - ((2*A - 3*B)*Sinh[x])/(15*(1 - Cos
h[x])^2) - ((2*A - 3*B)*Sinh[x])/(15*(1 - Cosh[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx &= -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}+\frac{1}{5}(2A-3B)\int \frac{1}{(1-\cosh(x))^2}\,dx\\ &= -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}+\frac{1}{15}(2A-3B)\int \frac{1}{1-\cosh(x)}\,dx\\ &= -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.0727499, size = 42, normalized size = 0.7

$$\frac{\sinh(x)(-6(2A-3B)\cosh(x)+(2A-3B)\cosh(2x)+16A-9B)}{30(\cosh(x)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

```
[Out] ((16*A - 9*B - 6*(2*A - 3*B)*Cosh[x] + (2*A - 3*B)*Cosh[2*x])*Sinh[x])/(30*
(-1 + Cosh[x])^3)
```

Maple [A] time = 0.013, size = 39, normalized size = 0.7

$$-\frac{-A+B}{4}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{A}{6}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}-\frac{-A-B}{20}\left(\tanh\left(\frac{x}{2}\right)\right)^{-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x))^3,x)
```

```
[Out] -1/4*(-A+B)/tanh(1/2*x)-1/6*A/tanh(1/2*x)^3-1/20*(-A-B)/tanh(1/2*x)^5
```

Maxima [B] time = 1.0524, size = 360, normalized size = 6.

$$-\frac{2}{5}B\left(\frac{5e^{(-x)}}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}-\frac{5e^{(-2x)}}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}+\frac{}{5e^{(-x)}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*B*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) + e^(-5
*x) - 1) - 5*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) +
e^(-5*x) - 1) + 5*e^(-3*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*
x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) +
 e^(-5*x) - 1)) + 4/15*A*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) -
5*e^(-4*x) + e^(-5*x) - 1) - 10*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3
*x) - 5*e^(-4*x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x)
- 5*e^(-4*x) + e^(-5*x) - 1))
```

Fricas [B] time = 2.03179, size = 425, normalized size = 7.08

$$\frac{2\left(15B\cosh(x)^{2}+15B\sinh(x)^{2}+2(11A-9B)\cosh(x)+6(5B\cosh(x\right.}{15\left(\cosh(x)^{4}+(4\cosh(x)-5)\sinh(x)^{3}+\sinh(x)^{4}-5\cosh(x)^{3}+\left(6\cosh(x)^{2}-15\cosh(x)+10\right)\sinh(x)^{2}+10\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A - 9*B)*cosh(x) + 6*(5*B*cos
h(x) + 3*A - 2*B)*sinh(x) - 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) - 5)*sinh(
x)^3 + sinh(x)^4 - 5*cosh(x)^3 + (6*cosh(x)^2 - 15*cosh(x) + 10)*sinh(x)^2
+ 10*cosh(x)^2 + (4*cosh(x)^3 - 15*cosh(x)^2 + 20*cosh(x) - 9)*sinh(x) - 11
*cosh(x) + 5)
```

Sympy [A] time = 2.33447, size = 46, normalized size = 0.77

$$\frac{A}{4\tanh\left(\frac{x}{2}\right)}-\frac{A}{6\tanh^{3}\left(\frac{x}{2}\right)}+\frac{A}{20\tanh^{5}\left(\frac{x}{2}\right)}-\frac{B}{4\tanh\left(\frac{x}{2}\right)}+\frac{B}{20\tanh^{5}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))**3,x)
```

```
[Out] A/(4*tanh(x/2)) - A/(6*tanh(x/2)**3) + A/(20*tanh(x/2)**5) - B/(4*tanh(x/2)
) + B/(20*tanh(x/2)**5)
```

Giac [A] time = 1.179, size = 62, normalized size = 1.03

$$\frac{2\left(15\,Be^{(3\,x)} + 20\,Ae^{(2\,x)} - 15\,Be^{(2\,x)} - 10\,Ae^{x} + 15\,Be^{x} + 2\,A - 3\,B\right)}{15\,(e^{x} - 1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*B*e^(3*x) + 20*A*e^(2*x) - 15*B*e^(2*x) - 10*A*e^x + 15*B*e^x + 2*
A - 3*B)/(e^x - 1)^5
```

3.100 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx$

Optimal. Leaf size=81

$$-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

```
[Out] -((A + B)*Sinh[x])/(7*(1 - Cosh[x])^4) - ((3*A - 4*B)*Sinh[x])/(35*(1 - Cos
h[x])^3) - (2*(3*A - 4*B)*Sinh[x])/(105*(1 - Cosh[x])^2) - (2*(3*A - 4*B)*S
inh[x])/(105*(1 - Cosh[x]))
```

Rubi [A] time = 0.0664045, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2750, 2650, 2648}

$$-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```


```
[Out] -((A + B)*Sinh[x])/(7*(1 - Cosh[x])^4) - ((3*A - 4*B)*Sinh[x])/(35*(1 - Cos
h[x])^3) - (2*(3*A - 4*B)*Sinh[x])/(105*(1 - Cosh[x])^2) - (2*(3*A - 4*B)*S
inh[x])/(105*(1 - Cosh[x]))
```


Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx &= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}+\frac{1}{7}(3A-4B)\int \frac{1}{(1-\cosh(x))^3}\,dx\\
&= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}+\frac{1}{35}(2(3A-4B))\int \frac{1}{(1-\cosh(x))^2}\,dx\\
&= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}+\frac{1}{105}(2(3A-4B))\int \frac{1}{1-\cosh(x)}\\
&= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}
\end{aligned}$$

Mathematica [A] time = 0.0878756, size = 57, normalized size = 0.7

$$\frac{\sinh(x)(29(3A-4B)\cosh(x)-8(3A-4B)\cosh(2x)+3A\cosh(3x)-96A-4B\cosh(3x)+58B)}{210(\cosh(x)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```

```
[Out] ((-96*A + 58*B + 29*(3*A - 4*B)*Cosh[x] - 8*(3*A - 4*B)*Cosh[2*x] + 3*A*Cos
h[3*x] - 4*B*Cosh[3*x])*Sinh[x])/(210*(-1 + Cosh[x])^4)
```

Maple [A] time = 0.013, size = 56, normalized size = 0.7

$$-\frac{-A+B}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{3\,A-B}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}-\frac{A+B}{56}\left(\tanh\left(\frac{x}{2}\right)\right)^{-7}-\frac{-3\,A-B}{40}\left(\tanh\left(\frac{x}{2}\right)\right)^{-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x))^4,x)
```

```
[Out] -1/8*(-A+B)/tanh(1/2*x)-1/24*(3*A-B)/tanh(1/2*x)^3-1/56*(A+B)/tanh(1/2*x)^7
-1/40*(-3*A-B)/tanh(1/2*x)^5
```

Maxima [B] time = 1.066, size = 609, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="maxima")
```

```
[Out] -8/105*B*(14*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 2
1*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 42*e^(-2*x)/(7*e^(-x) - 21*e^(-2*
x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) +
 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5
*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 35*e^(-4*x)/(7*e^(-x) - 21*e^(-2*x) + 35
*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 2/(7*e
^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x)
+ e^(-7*x) - 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) -
 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 21*e^(-2*x)/(7*e^
(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) +
 e^(-7*x) - 1) + 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(
-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 1/(7*e^(-x) - 21*e^(-2*x
) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1))
```

Fricas [B] time = 2.08389, size = 585, normalized size = 7.22

$$\frac{4\left((3\,A-74\,B)\cosh\left(x\right)^{2}+(3\,A-74\,B\right.}{105\left(\cosh\left(x\right)^{5}+(5\,\cosh\left(x\right)-7)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}-7\,\cosh\left(x\right)^{4}+\left(10\,\cosh\left(x\right)^{2}-28\,\cosh\left(x\right)+21\right)\sinh\left(x\right)^{3}+21\,\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="fricas")
```

```
[Out] 4/105*((3*A - 74*B)*cosh(x)^2 + (3*A - 74*B)*sinh(x)^2 - 14*(9*A - 7*B)*cos
h(x) - 6*((A + 22*B)*cosh(x) + 14*A - 7*B)*sinh(x) + 63*A - 84*B)/(cosh(x)^
5 + (5*cosh(x) - 7)*sinh(x)^4 + sinh(x)^5 - 7*cosh(x)^4 + (10*cosh(x)^2 - 2
8*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 - 42*cosh(x)^2 + 6
3*cosh(x) - 36)*sinh(x)^2 - 36*cosh(x)^2 + (5*cosh(x)^4 - 28*cosh(x)^3 + 63
```

```
*cosh(x)^2 - 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) - 21)
```

Sympy [A] time = 5.14603, size = 78, normalized size = 0.96

$$\frac{A}{8\tanh\left(\frac{x}{2}\right)} - \frac{A}{8\tanh^3\left(\frac{x}{2}\right)} + \frac{3A}{40\tanh^5\left(\frac{x}{2}\right)} - \frac{A}{56\tanh^7\left(\frac{x}{2}\right)} - \frac{B}{8\tanh\left(\frac{x}{2}\right)} + \frac{B}{24\tanh^3\left(\frac{x}{2}\right)} + \frac{B}{40\tanh^5\left(\frac{x}{2}\right)} - \frac{B}{56\tanh^7\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))**4,x)
```

```
[Out] A/(8*tanh(x/2)) - A/(8*tanh(x/2)**3) + 3*A/(40*tanh(x/2)**5) - A/(56*tanh(x
/2)**7) - B/(8*tanh(x/2)) + B/(24*tanh(x/2)**3) + B/(40*tanh(x/2)**5) - B/(
56*tanh(x/2)**7)
```

Giac [A] time = 1.18975, size = 81, normalized size = 1.

$$-\frac{4\left(70\,Be^{(4\,x)} + 105\,Ae^{(3\,x)} - 70\,Be^{(3\,x)} - 63\,Ae^{(2\,x)} + 84\,Be^{(2\,x)} + 21\,Ae^{x} - 28\,Be^{x} - 3\,A + 4\,B\right)}{105\,(e^{x} - 1)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) - 70*B*e^(3*x) - 63*A*e^(2*x) + 84*B*e
^(2*x) + 21*A*e^x - 28*B*e^x - 3*A + 4*B)/(e^x - 1)^7
```

3.101 $\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}$$

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/S
qrt[a] + (2*B*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Rubi [A] time = 0.0655941, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2751, 2649, 206}

$$\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/S
qrt[a] + (2*B*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx &= \frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}+(A-B)\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx\\ &= \frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}+(2i(A-B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right)\\ &= \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.0341246, size = 41, normalized size = 0.73

$$\frac{2\cosh\left(\frac{x}{2}\right)\left((A-B)\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)+2B\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(\cosh(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] (2*Cosh[x/2]*((A - B)*ArcTan[Sinh[x/2]] + 2*B*Sinh[x/2]))/Sqrt[a*(1 + Cosh[
x])]
```

Maple [B] time = 0.053, size = 128, normalized size = 2.3

$$-\frac{\sqrt{2}}{a}\cosh\left(\frac{x}{2}\right)\sqrt{\left(\sinh\left(\frac{x}{2}\right)\right)^2 a}\left(\ln\left(2\frac{\sqrt{(\sinh(x/2))^2 a}\sqrt{-a}-a}{\cosh(x/2)}\right)aA-2B\sqrt{(\sinh(x/2))^2 a}\sqrt{-a}-\ln\left(2\frac{\sqrt{(\sinh(x/2))^2 a}}{\cosh(x/2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x)
```

```
[Out] -cosh(1/2*x)*(sinh(1/2*x)^2*a)^(1/2)*(ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(
1/2)*(-a)^(1/2)-a))*a*A-2*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)-ln(2/cosh(1/
```

```
2*x)*((sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)-a))*a*B)/a/(-a)^(1/2)/sinh(1/2*x)*
2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [B] time = 1.94619, size = 235, normalized size = 4.2

$$2\left(\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}+\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)-\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)A-\frac{1}{3}\left(3\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)-\sqrt{2}\left(\frac{3\arctan\left(e^{\left(-\frac{1}{2}x\right)}\right.}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) + e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))
- sqrt(2)*e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))*A - 1/3*(3*sqrt(2)*(arctan(e^(
1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) - sqrt(2)*(3*arctan(e^
(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/2*x)/(sqrt(a)*e^(-x) + sqr
t(a))) - (3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))/(a*e^x
+ a))*B
```

Fricas [A] time = 2.16759, size = 244, normalized size = 4.36

$$\frac{2\left(\sqrt{2}(A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(x)+\sinh(x))}{\sqrt{a}}\right)+\sqrt{\frac{1}{2}}(B\cosh(x)+B\sinh(x)-B)\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(
x)))*(cosh(x) + sinh(x))/sqrt(a)) + sqrt(1/2)*(B*cosh(x) + B*sinh(x) - B)*s
qrt(a/(cosh(x) + sinh(x))))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\cosh(x)}{\sqrt{a(\cosh(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral((A + B*cosh(x))/sqrt(a*(cosh(x) + 1)), x)
```

Giac [C] time = 1.23595, size = 100, normalized size = 1.79

$$\frac{1}{4}\sqrt{2}\left(\frac{8\,(A-B)\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{\sqrt{a}}+\frac{4\,Be^{\left(\frac{1}{2}\,x\right)}}{\sqrt{a}}-\frac{4\,Be^{\left(-\frac{1}{2}\,x\right)}}{\sqrt{a}}+\frac{8i\,A\sqrt{-a}\arctan\left(-i\right)-8i\,B\sqrt{-a}\arctan\left(-i\right)+8\,B\sqrt{-a}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(8*(A - B)*arctan(e^(1/2*x))/sqrt(a) + 4*B*e^(1/2*x)/sqrt(a) -
4*B*e^(-1/2*x)/sqrt(a) + (8*I*A*sqrt(-a)*arctan(-I) - 8*I*B*sqrt(-a)*arctan
(-I) + 8*B*sqrt(-a))/a)
```

3.102 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}$$

```
[Out] ((A + 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(2*Sqrt
[2]*a^(3/2)) + ((A - B)*Sinh[x])/(2*(a + a*Cosh[x])^(3/2))
```

Rubi [A] time = 0.0681411, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2750, 2649, 206}

$$\frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(2*Sqrt
[2]*a^(3/2)) + ((A - B)*Sinh[x])/(2*(a + a*Cosh[x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx &= \frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}}+\frac{(A+3B)\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx}{4a}\\ &= \frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}}+\frac{(i(A+3B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right)}{2a}\\ &= \frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0772987, size = 44, normalized size = 0.68

$$\frac{\frac{1}{2}(A-B)\sinh(x)+(A+3B)\cosh^3\left(\frac{x}{2}\right)\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)}{(a(\cosh(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^3 + ((A - B)*Sinh[x])/2)/(a*(1 + Cos
h[x]))^(3/2)
```

Maple [B] time = 0.062, size = 159, normalized size = 2.5

$$-\frac{\sqrt{2}}{4a^2}\sqrt{\left(\sinh\left(\frac{x}{2}\right)\right)^2 a}\left(A\ln\left(2\,\frac{\sqrt{(\sinh(x/2))^2 a}\sqrt{-a}-a}{\cosh(x/2)}\right)\left(\cosh\left(\frac{x}{2}\right)\right)^2 a+3\,B\ln\left(2\,\frac{\sqrt{(\sinh(x/2))^2 a}\sqrt{-a}-a}{\cosh(x/2)}\right)a\,(\cosh($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x)
```

```
[Out] -1/4*(sinh(1/2*x)^2*a)^(1/2)*(A*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/2)*(
-a)^(1/2)-a))*cosh(1/2*x)^2*a+3*B*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/2)
*(-a)^(1/2)-a))*a*cosh(1/2*x)^2-A*(-a)^(1/2)*(sinh(1/2*x)^2*a)^(1/2)+B*(sin
h(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/cosh(1/2*x)/a^2/(-a)^(1/2)/sinh(1/2*x)*2^(1
/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [B] time = 2.03622, size = 405, normalized size = 6.23

$$\frac{1}{6}\left(\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,x\right)}+8\,e^{\left(\frac{3}{2}\,x\right)}-3\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{3\,\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{a^{\frac{3}{2}}}\right)-\frac{8\,\sqrt{2}e^{\left(\frac{3}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}\right)A+\frac{1}{20}\left(\sqrt{2}\left(\frac{15\,e^{\left(\frac{5}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2))
 - 8*sqrt(2)*e^(3/2*x)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x
 + a^(3/2)))*A + 1/20*(sqrt(2)*((15*e^(5/2*x) + 40*e^(3/2*x) + 33*e^(1/2*x)
)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 15*arct
an(e^(1/2*x))/a^(3/2)) + 5*sqrt(2)*((3*e^(5/2*x) - 8*e^(3/2*x) - 3*e^(1/2*x
))/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arct
an(e^(1/2*x))/a^(3/2)) - 8*(5*sqrt(2)*sqrt(a)*e^(5/2*x) + sqrt(2)*sqrt(a)*e
^(1/2*x))/(a^2*e^(3*x) + 3*a^2*e^(2*x) + 3*a^2*e^x + a^2))*B
```

Fricas [B] time = 2.18368, size = 581, normalized size = 8.94

$$-\frac{\sqrt{2}\left((A+3\,B)\cosh\left(x\right)^{2}+(A+3\,B)\sinh\left(x\right)^{2}+2\,(A+3\,B)\cosh\left(x\right)+2\,((A+3\,B)\cosh\left(x\right)+A+3\,B)\sinh\left(x\right)+A+3\ldots\right.}{2\left(a^{2}\cosh\left(x\right)^{2}+a^{2}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*((A + 3*B)*cosh(x)^2 + (A + 3*B)*sinh(x)^2 + 2*(A + 3*B)*cosh
(x) + 2*((A + 3*B)*cosh(x) + A + 3*B)*sinh(x) + A + 3*B)*sqrt(a)*arctan(sqr
```

```
t(2)*sqrt(1/2)*sqrt(a)*sqrt(a/(cosh(x) + sinh(x)))/a) - 2*sqrt(1/2)*((A - B
)*cosh(x)^2 + (A - B)*sinh(x)^2 - (A - B)*cosh(x) + (2*(A - B)*cosh(x) - A
+ B)*sinh(x))*sqrt(a/(cosh(x) + sinh(x))))/(a^2*cosh(x)^2 + a^2*sinh(x)^2 +
 2*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) + a^2)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33583, size = 105, normalized size = 1.62

$$\frac{\left(\sqrt{2}A + 3\sqrt{2}B\right)\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{2a^{\frac{3}{2}}} + \frac{\sqrt{2}\left(Aa^{\frac{3}{2}}e^{\left(\frac{3}{2}x\right)} - Ba^{\frac{3}{2}}e^{\left(\frac{3}{2}x\right)} - Aa^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)} + Ba^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)}\right)}{2\left(ae^{x} + a\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*A + 3*sqrt(2)*B)*arctan(e^(1/2*x))/a^(3/2) + 1/2*sqrt(2)*(A*a^
(3/2)*e^(3/2*x) - B*a^(3/2)*e^(3/2*x) - A*a^(3/2)*e^(1/2*x) + B*a^(3/2)*e^(
1/2*x))/((a*e^x + a)^2*a)
```

3.103 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=93

$$\frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{16\sqrt{2}a^{5/2}}+\frac{(3A+5B)\sinh(x)}{16a(a\cosh(x)+a)^{3/2}}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

```
[Out] ((3*A + 5*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(16*S
qrt[2]*a^(5/2)) + ((A - B)*Sinh[x])/(4*(a + a*Cosh[x])^(5/2)) + ((3*A + 5*B
)*Sinh[x])/(16*a*(a + a*Cosh[x])^(3/2))
```

Rubi [A] time = 0.0885073, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2750, 2650, 2649, 206}

$$\frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{16\sqrt{2}a^{5/2}}+\frac{(3A+5B)\sinh(x)}{16a(a\cosh(x)+a)^{3/2}}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

```
[Out] ((3*A + 5*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(16*S
qrt[2]*a^(5/2)) + ((A - B)*Sinh[x])/(4*(a + a*Cosh[x])^(5/2)) + ((3*A + 5*B
)*Sinh[x])/(16*a*(a + a*Cosh[x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx &= \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\int \frac{1}{(a+a\cosh(x))^{3/2}}\,dx}{8a} \\
&= \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}} + \frac{(3A+5B)\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx}{32a^2} \\
&= \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}} + \frac{(i(3A+5B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right)}{16a^2} \\
&= \frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}} + \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.153856, size = 57, normalized size = 0.61

$$
\frac{\sinh(x)((3A+5B)\cosh(x)+7A+B)+4(3A+5B)\cosh^5\left(\frac{x}{2}\right)\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)}{16(a(\cosh(x)+1))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

```
[Out] (4*(3*A + 5*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^5 + (7*A + B + (3*A + 5*B)*Cosh[
x])*Sinh[x])/(16*(a*(1 + Cosh[x]))^(5/2))
```

Maple [B] time = 0.065, size = 209, normalized size = 2.3

$$-\frac{\sqrt{2}}{32\,a^{3}}\sqrt{\left(\sinh\left(\frac{x}{2}\right)\right)^{2} a}\left(3\,A\ln\left(2\,\frac{\sqrt{(\sinh(x/2))^{2}\,a}\sqrt{-a}-a}{\cosh(x/2)}\right)(\cosh(x/2))^{4}\,a+5\,B\ln\left(2\,\frac{\sqrt{(\sinh(x/2))^{2}\,a}\sqrt{-a}-a}{\cosh(x/2)}\right)(\cosh($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x)
```

```
[Out] -1/32*(sinh(1/2*x)^2*a)^(1/2)*(3*A*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/2
)*(-a)^(1/2)-a))*cosh(1/2*x)^4*a+5*B*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1
/2)*(-a)^(1/2)-a))*cosh(1/2*x)^4*a-3*A*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)*c
osh(1/2*x)^2-5*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)*cosh(1/2*x)^2-2*A*(-a)^
(1/2)*(sinh(1/2*x)^2*a)^(1/2)+2*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/cosh(
1/2*x)^3/a^3/(-a)^(1/2)/sinh(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [B] time = 2.0715, size = 576, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/80*(sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) + 128*e^(5/2*x) - 70*e^(3/2*x)
- 15*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x) +
 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x))/a^(5/
2)) - 128*sqrt(2)*e^(5/2*x)/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/
2)*e^(3*x) + 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)))*A + 1/672*(sqrt
(2)*((105*e^(9/2*x) + 490*e^(7/2*x) + 896*e^(5/2*x) + 790*e^(3/2*x) - 105*e
^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x) + 10*a^
(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 105*arctan(e^(1/2*x))/a^(5/2)) +
 7*sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) - 128*e^(5/2*x) - 70*e^(3/2*x) - 1
5*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x) + 10
*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x))/a^(5/2))
 - 128*(7*sqrt(2)*sqrt(a)*e^(7/2*x) + 3*sqrt(2)*sqrt(a)*e^(3/2*x))/(a^3*e^(
5*x) + 5*a^3*e^(4*x) + 10*a^3*e^(3*x) + 10*a^3*e^(2*x) + 5*a^3*e^x + a^3))*
B
```

Fricas [B] time = 2.24973, size = 1453, normalized size = 15.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)^4 + 4*(3*A + 5*
B)*cosh(x)^3 + 4*((3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x)^3 + 6*(3*A + 5*B
)*cosh(x)^2 + 6*((3*A + 5*B)*cosh(x)^2 + 2*(3*A + 5*B)*cosh(x) + 3*A + 5*B)
*sinh(x)^2 + 4*(3*A + 5*B)*cosh(x) + 4*((3*A + 5*B)*cosh(x)^3 + 3*(3*A + 5*
B)*cosh(x)^2 + 3*(3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x) + 3*A + 5*B)*sqrt
(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a)*sqrt(a/(cosh(x) + sinh(x)))/a) - 2*sqr
t(1/2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)^4 + (11*A - 3*B)*cosh(x
)^3 + (4*(3*A + 5*B)*cosh(x) + 11*A - 3*B)*sinh(x)^3 - (11*A - 3*B)*cosh(x)
^2 + (6*(3*A + 5*B)*cosh(x)^2 + 3*(11*A - 3*B)*cosh(x) - 11*A + 3*B)*sinh(x
)^2 - (3*A + 5*B)*cosh(x) + (4*(3*A + 5*B)*cosh(x)^3 + 3*(11*A - 3*B)*cosh(
x)^2 - 2*(11*A - 3*B)*cosh(x) - 3*A - 5*B)*sinh(x))*sqrt(a/(cosh(x) + sinh(
x))))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 + 4*a^3*cosh(x)^3 + 6*a^3*cosh(x)^2 +
4*a^3*cosh(x) + 4*(a^3*cosh(x) + a^3)*sinh(x)^3 + a^3 + 6*(a^3*cosh(x)^2 +
2*a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + 3*a^3*cosh(x)^2 + 3*a^3
*cosh(x) + a^3)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41516, size = 159, normalized size = 1.71

$$\frac{\sqrt{2}(3A+5B)\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{16a^{\frac{5}{2}}} + \frac{\sqrt{2}\left(3Aa^{\frac{7}{2}}e^{\left(\frac{7}{2}x\right)} + 5Ba^{\frac{7}{2}}e^{\left(\frac{7}{2}x\right)} + 11Aa^{\frac{7}{2}}e^{\left(\frac{5}{2}x\right)} - 3Ba^{\frac{7}{2}}e^{\left(\frac{5}{2}x\right)} - 11Aa^{\frac{7}{2}}e^{\left(\frac{3}{2}x\right)} + 3Ba^{\frac{7}{2}}e^{\left(\frac{3}{2}x\right)}}{16(ae^{x}+a)^{4}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*A + 5*B)*arctan(e^(1/2*x))/a^(5/2) + 1/16*sqrt(2)*(3*A*a^(7
/2)*e^(7/2*x) + 5*B*a^(7/2)*e^(7/2*x) + 11*A*a^(7/2)*e^(5/2*x) - 3*B*a^(7/2
)*e^(5/2*x) - 11*A*a^(7/2)*e^(3/2*x) + 3*B*a^(7/2)*e^(3/2*x) - 3*A*a^(7/2)*
e^(1/2*x) - 5*B*a^(7/2)*e^(1/2*x))/((a*e^x + a)^4*a^2)
```

3.104 $\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

Optimal. Leaf size=57

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}(A+B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

```
[Out] -((Sqrt[2]*(A + B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])
/Sqrt[a]) + (2*B*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Rubi [A] time = 0.0653115, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2751, 2649, 206}

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}(A+B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] -((Sqrt[2]*(A + B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])
/Sqrt[a]) + (2*B*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx &= \frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}} + (A+B)\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx \\
&= \frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}} + (2i(A+B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right) \\
&= -\frac{\sqrt{2}(A+B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}} + \frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.0541786, size = 40, normalized size = 0.7

$$\frac{2\sinh\left(\frac{x}{2}\right)\left((A+B)\log\left(\tanh\left(\frac{x}{4}\right)\right)+2B\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] (2*(2*B*Cosh[x/2] + (A + B)*Log[Tanh[x/4]])*Sinh[x/2])/Sqrt[a - a*Cosh[x]]
```

Maple [A] time = 0.066, size = 63, normalized size = 1.1

$$\sinh\left(\frac{x}{2}\right)\left(\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)A-\ln\left(1+\cosh\left(\frac{x}{2}\right)\right)A+B\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)-B\ln\left(1+\cosh\left(\frac{x}{2}\right)\right)+4B\cosh(x/2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x)
```

```
[Out] sinh(1/2*x)*(ln(-1+cosh(1/2*x))*A-ln(1+cosh(1/2*x))*A+B*ln(-1+cosh(1/2*x))-
B*ln(1+cosh(1/2*x))+4*B*cosh(1/2*x))/(-2*sinh(1/2*x)^2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cosh(x) + A}{\sqrt{-a\cosh(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/sqrt(-a*cosh(x) + a), x)
```

Fricas [B] time = 2.11703, size = 324, normalized size = 5.68

$$\frac{\sqrt{2}(A+B)a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right)-2\sqrt{\frac{1}{2}}(B\cosh(x)+B\sinh(x)+B)\sqrt{-}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(A + B)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) + s
inh(x)))*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x) +
 sinh(x) - 1)) - 2*sqrt(1/2)*(B*cosh(x) + B*sinh(x) + B)*sqrt(-a/(cosh(x) +
 sinh(x))))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cosh(x)}{\sqrt{-a(\cosh(x) - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))**(1/2),x)
```

```
[Out] Integral((A + B*cosh(x))/sqrt(-a*(cosh(x) - 1)), x)
```

Giac [C] time = 1.19808, size = 157, normalized size = 2.75

$$\frac{1}{4}\sqrt{2}\left(\frac{\left(8i\,A\sqrt{-a}\arctan\left(-i\right)+8i\,B\sqrt{-a}\arctan\left(-i\right)-8\,B\sqrt{-a}\right)\mathrm{sgn}\left(-e^{x}+1\right)}{a}-\frac{8\,(A+B)\arctan\left(\frac{\sqrt{-ae^{x}}}{\sqrt{a}}\right)}{\sqrt{a}\mathrm{sgn}\left(-e^{x}+1\right)}-\frac{4\,B}{\sqrt{-ae^{x}}\mathrm{sgn}\left(-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((8*I*A*sqrt(-a)*arctan(-I) + 8*I*B*sqrt(-a)*arctan(-I) - 8*B*s
qrt(-a))*sgn(-e^x + 1)/a - 8*(A + B)*arctan(sqrt(-a*e^x)/sqrt(a))/(sqrt(a)*
sgn(-e^x + 1)) - 4*B/(sqrt(-a*e^x)*sgn(-e^x + 1)) + 4*sqrt(-a*e^x)*B/(a*sgn
(-e^x + 1)))
```

3.105 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

```
[Out] -((A - 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/(2*Sqr
t[2]*a^(3/2)) - ((A + B)*Sinh[x])/(2*(a - a*Cosh[x])^(3/2))
```

Rubi [A] time = 0.0728487, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2750, 2649, 206}

$$-\frac{(A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

```
[Out] -((A - 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/(2*Sqr
t[2]*a^(3/2)) - ((A + B)*Sinh[x])/(2*(a - a*Cosh[x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx &= -\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{(A-3B)\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx}{4a}\\ &= -\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{(i(A-3B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right)}{2a}\\ &= -\frac{(A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.149487, size = 71, normalized size = 1.09

$$\frac{\sinh^3\left(\frac{x}{2}\right)\left((A+B)\mathbf{csch}^2\left(\frac{x}{4}\right)+(A+B)\mathbf{sech}^2\left(\frac{x}{4}\right)+4(A-3B)\log\left(\tanh\left(\frac{x}{4}\right)\right)\right)}{4a(\cosh(x)-1)\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

```
[Out] (((A + B)*Csch[x/4]^2 + 4*(A - 3*B)*Log[Tanh[x/4]] + (A + B)*Sech[x/4]^2)*S
inh[x/2]^3)/(4*a*(-1 + Cosh[x])*Sqrt[a - a*Cosh[x]])
```

Maple [A] time = 0.07, size = 83, normalized size = 1.3

$$-\frac{1}{4\,a}\left(\cosh\left(\frac{x}{2}\right)(-2\,A-2\,B)+\left(\ln\left(1+\cosh\left(\frac{x}{2}\right)\right)A-\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)A-3\,B\ln\left(1+\cosh\left(x/2\right)\right)+3\,B\ln\left(-1+\cosh\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x)
```

```
[Out] -1/4/a*(cosh(1/2*x)*(-2*A-2*B)+(ln(1+cosh(1/2*x))*A-ln(-1+cosh(1/2*x))*A-3*
B*ln(1+cosh(1/2*x))+3*B*ln(-1+cosh(1/2*x)))*sinh(1/2*x)^2)/sinh(1/2*x)/(-2*
```

```
sinh(1/2*x)^2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cosh(x) + A}{(-a\cosh(x) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(3/2), x)
```

Fricas [B] time = 2.19713, size = 680, normalized size = 10.46

$$\frac{\sqrt{2}\left((A-3B)\cosh(x)^2 + (A-3B)\sinh(x)^2 - 2(A-3B)\cosh(x) + 2((A-3B)\cosh(x) - A + 3B)\sinh(x) + A - 3}{4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((A - 3*B)*cosh(x)^2 + (A - 3*B)*sinh(x)^2 - 2*(A - 3*B)*cosh(
x) + 2*((A - 3*B)*cosh(x) - A + 3*B)*sinh(x) + A - 3*B)*sqrt(-a)*log((2*sqr
t(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x)) -
a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) - 4*sqrt(1/2)*((A + B)*
cosh(x)^2 + (A + B)*sinh(x)^2 + (A + B)*cosh(x) + (2*(A + B)*cosh(x) + A +
B)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x))))/(a^2*cosh(x)^2 + a^2*sinh(x)^2 -
2*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) - a^2)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22786, size = 150, normalized size = 2.31

$$-\frac{\left(\sqrt{2}A-3\sqrt{2}B\right)\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}\operatorname{sgn}\left(-e^x+1\right)}+\frac{\sqrt{2}\left(\sqrt{-ae^x}Aae^x+\sqrt{-ae^x}Bae^x+\sqrt{-ae^x}Aa+\sqrt{-ae^x}Ba\right)}{2\left(ae^x-a\right)^2 a\operatorname{sgn}\left(-e^x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*A - 3*sqrt(2)*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(3/2)*sgn(-e
^x + 1)) + 1/2*sqrt(2)*(sqrt(-a*e^x)*A*a*e^x + sqrt(-a*e^x)*B*a*e^x + sqrt(
-a*e^x)*A*a + sqrt(-a*e^x)*B*a)/((a*e^x - a)^2*a*sgn(-e^x + 1))
```

3.106 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{(3A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

```
[Out] -((3*A - 5*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/(16*
Sqrt[2]*a^(5/2)) - ((A + B)*Sinh[x])/(4*(a - a*Cosh[x])^(5/2)) - ((3*A - 5*
B)*Sinh[x])/(16*a*(a - a*Cosh[x])^(3/2))
```

Rubi [A] time = 0.095537, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2750, 2650, 2649, 206}

$$-\frac{(3A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
```

```
[Out] -((3*A - 5*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/(16*
Sqrt[2]*a^(5/2)) - ((A + B)*Sinh[x])/(4*(a - a*Cosh[x])^(5/2)) - ((3*A - 5*
B)*Sinh[x])/(16*a*(a - a*Cosh[x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx &= -\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}+\frac{(3A-5B)\int\frac{1}{(a-a\cosh(x))^{3/2}}\,dx}{8a}\\
&= -\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}+\frac{(3A-5B)\int\frac{1}{\sqrt{a-a\cosh(x)}}\,dx}{32a^2}\\
&= -\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}+\frac{(i(3A-5B))\,\mathrm{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right)}{16a^2}\\
&= -\frac{(3A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.360171, size = 108, normalized size = 1.15

$$\frac{\sinh^5\left(\frac{x}{2}\right)\left(-(A+B)\text{csch}^4\left(\frac{x}{4}\right)+2(3A-5B)\text{csch}^2\left(\frac{x}{4}\right)+(A+B)\text{sech}^4\left(\frac{x}{4}\right)+2(3A-5B)\text{sech}^2\left(\frac{x}{4}\right)+8(3A-5B)\log\left(\tan\right.\right.}{32a^2(\cosh(x)-1)^2\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
```

```
[Out] ((2*(3*A - 5*B)*Csch[x/4]^2 - (A + B)*Csch[x/4]^4 + 8*(3*A - 5*B)*Log[Tanh[
x/4]] + 2*(3*A - 5*B)*Sech[x/4]^2 + (A + B)*Sech[x/4]^4)*Sinh[x/2]^5)/(32*a
^2*(-1 + Cosh[x])^2*Sqrt[a - a*Cosh[x]])
```

Maple [A] time = 0.07, size = 118, normalized size = 1.3

$$-\frac{1}{32\,a^{2}}\left((-6\,A+10\,B)\cosh\left(\frac{x}{2}\right)\left(\sinh\left(\frac{x}{2}\right)\right)^{2}+(4\,A+4\,B)\cosh\left(\frac{x}{2}\right)+(3\,\ln\left(1+\cosh\left(x/2\right)\right)A-3\,\ln\left(-1+\cosh\left(x/2\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x)
```

```
[Out] -1/32/a^2*((-6*A+10*B)*cosh(1/2*x)*sinh(1/2*x)^2+(4*A+4*B)*cosh(1/2*x)+(3*l
n(1+cosh(1/2*x))*A-3*ln(-1+cosh(1/2*x))*A-5*B*ln(1+cosh(1/2*x))+5*B*ln(-1+c
osh(1/2*x)))*sinh(1/2*x)^4)/(1+cosh(1/2*x))/(-1+cosh(1/2*x))/sinh(1/2*x)/(-
2*sinh(1/2*x)^2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\cosh\left(x\right)+A}{\left(-a\cosh\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(5/2), x)
```

Fricas [B] time = 2.24865, size = 1553, normalized size = 16.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - 4*(3*A - 5*B
)*cosh(x)^3 + 4*((3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x)^3 + 6*(3*A - 5*B)
*cosh(x)^2 + 6*((3*A - 5*B)*cosh(x)^2 - 2*(3*A - 5*B)*cosh(x) + 3*A - 5*B)*
sinh(x)^2 - 4*(3*A - 5*B)*cosh(x) + 4*((3*A - 5*B)*cosh(x)^3 - 3*(3*A - 5*B
```

```
)*cosh(x)^2 + 3*(3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x) + 3*A - 5*B)*sqrt(
-a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x)
 + sinh(x)) - a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) - 4*sqrt(
1/2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - (11*A + 3*B)*cosh(x)^
3 + (4*(3*A - 5*B)*cosh(x) - 11*A - 3*B)*sinh(x)^3 - (11*A + 3*B)*cosh(x)^2
 + (6*(3*A - 5*B)*cosh(x)^2 - 3*(11*A + 3*B)*cosh(x) - 11*A - 3*B)*sinh(x)^
2 + (3*A - 5*B)*cosh(x) + (4*(3*A - 5*B)*cosh(x)^3 - 3*(11*A + 3*B)*cosh(x)
^2 - 2*(11*A + 3*B)*cosh(x) + 3*A - 5*B)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x
))))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 - 4*a^3*cosh(x)^3 + 6*a^3*cosh(x)^2 - 4
*a^3*cosh(x) + 4*(a^3*cosh(x) - a^3)*sinh(x)^3 + a^3 + 6*(a^3*cosh(x)^2 - 2
*a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 - 3*a^3*cosh(x)^2 + 3*a^3*
cosh(x) - a^3)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33991, size = 255, normalized size = 2.71

$$-\frac{\sqrt{2}(3A-5B)\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\operatorname{sgn}(-e^x+1)}+\frac{\sqrt{2}\left(3\sqrt{-ae^x}Aa^3e^{(3x)}-5\sqrt{-ae^x}Ba^3e^{(3x)}-11\sqrt{-ae^x}Aa^3e^{(2x)}-3\sqrt{-ae^x}Ba^3e^{(2x)}-11\sqrt{}\right.}{16\,(ae^x-a)^4a^2\operatorname{sgn}(-e^x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*A - 5*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(5/2)*sgn(-e^x +
1)) + 1/16*sqrt(2)*(3*sqrt(-a*e^x)*A*a^3*e^(3*x) - 5*sqrt(-a*e^x)*B*a^3*e^(
3*x) - 11*sqrt(-a*e^x)*A*a^3*e^(2*x) - 3*sqrt(-a*e^x)*B*a^3*e^(2*x) - 11*sq
rt(-a*e^x)*A*a^3*e^x - 3*sqrt(-a*e^x)*B*a^3*e^x + 3*sqrt(-a*e^x)*A*a^3 - 5*
sqrt(-a*e^x)*B*a^3)/((a*e^x - a)^4*a^2*sgn(-e^x + 1))
```

3.107 $\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=233

$$\frac{2i\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{105b\sqrt{a+b\cosh(x)}}+\frac{2}{105}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b}$$

```
[Out] (((-2*I)/105)*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[a + b*
Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)
]) + (((2*I)/105)*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[(a + b*
Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]
) + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cosh[x]]*Sinh[x])/105 +
(2*(7*A*b + 5*a*B)*(a + b*Cosh[x])^(3/2)*Sinh[x])/35 + (2*B*(a + b*Cosh[x])
^(5/2)*Sinh[x])/7
```

Rubi [A] time = 0.454098, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2}{105}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{2i\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{105b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```


```
[Out] (((-2*I)/105)*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[a + b*
Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)
]) + (((2*I)/105)*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[(a + b*
Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]
) + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cosh[x]]*Sinh[x])/105 +
(2*(7*A*b + 5*a*B)*(a + b*Cosh[x])^(3/2)*Sinh[x])/35 + (2*B*(a + b*Cosh[x])
^(5/2)*Sinh[x])/7
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{2}{7}B(a+b\cosh(x))^{5/2}\sinh(x)+\frac{2}{7}\int (a+b\cosh(x))^{3/2}\left(\frac{1}{2}(7aA+5bB)+\frac{1}{2}(7A\right.$$

$$= \frac{2}{35}(7Ab+5aB)(a+b\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a+b\cosh(x))^{5/2}\sinh(x)+\frac{4}{3}$$

$$= \frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\sinh(x)+\frac{2}{35}(7Ab+5aB)(a+$$

$$= \frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\sinh(x)+\frac{2}{35}(7Ab+5aB)(a+$$

$$= \frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\sinh(x)+\frac{2}{35}(7Ab+5aB)(a+$$

$$= -\frac{2i\left(161a^2Ab+63Ab^3+15a^3B+145ab^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{105b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i\left(a\right.}{}$$

Mathematica [A] time = 0.562536, size = 203, normalized size = 0.87

$$\frac{\sinh(x)(a+b\cosh(x))\left(90a^2B+6b\cosh(x)(15aB+7Ab)+154aAb+15b^2B\cosh(2x)+65b^2B\right)-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left(b\left(105a^3A\right.\right.}{}}{105\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*(b*(105*a^3*A + 119*a*A*b^2 + 135*a^
2*b*B + 25*b^3*B)*EllipticF[(I/2)*x, (2*b)/(a + b)] + (161*a^2*A*b + 63*A*b
^3 + 15*a^3*B + 145*a*b^2*B)*((a + b)*EllipticE[(I/2)*x, (2*b)/(a + b)] - a
*EllipticF[(I/2)*x, (2*b)/(a + b)])))/b + (a + b*Cosh[x])*(154*a*A*b + 90*a
^2*B + 65*b^2*B + 6*b*(7*A*b + 15*a*B)*Cosh[x] + 15*b^2*B*Cosh[2*x])*Sinh[x
])/(105*Sqrt[a + b*Cosh[x]])
```

Maple [B] time = 0.168, size = 1365, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x)
```

```
[Out] 2/105*(240*B*(-2*b/(a-b))^(1/2)*b^3*cosh(1/2*x)*sinh(1/2*x)^8+(168*A*(-2*b/
(a-b))^(1/2)*b^3+480*B*(-2*b/(a-b))^(1/2)*a*b^2+360*B*(-2*b/(a-b))^(1/2)*b^
3)*sinh(1/2*x)^6*cosh(1/2*x)+(392*A*(-2*b/(a-b))^(1/2)*a*b^2+168*A*(-2*b/(a
-b))^(1/2)*b^3+360*B*(-2*b/(a-b))^(1/2)*a^2*b+480*B*(-2*b/(a-b))^(1/2)*a*b^
2+280*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(154*A*(-2*b/(a-b
))^(1/2)*a^2*b+196*A*(-2*b/(a-b))^(1/2)*a*b^2+42*A*(-2*b/(a-b))^(1/2)*b^3+9
0*B*(-2*b/(a-b))^(1/2)*a^3+180*B*(-2*b/(a-b))^(1/2)*a^2*b+170*B*(-2*b/(a-b)
)^(1/2)*a*b^2+80*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^2*cosh(1/2*x)+105*A*
a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Elli
pticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+161*A*a^2*b*(2
*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(
cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+119*A*a*b^2*(2*b/(a-
b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1
/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+63*A*b^3*(2*b/(a-b)*sinh(1
/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2
*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-322*A*(-sinh(1/2*x)^2)^(1/2)*Ellipt
icE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*(2*b/(a-b)*sinh(
1/2*x)^2+(a+b)/(a-b))^(1/2)*a^2*b-126*A*(-sinh(1/2*x)^2)^(1/2)*EllipticE(co
sh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*(2*b/(a-b)*sinh(1/2*x)
^2+(a+b)/(a-b))^(1/2)*b^3+15*a^3*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1
/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2
*(a-b)/b)^(1/2))+135*B*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-
sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)
/b)^(1/2))+145*B*a*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1
/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2))+25*B*b^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^
(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-30*B
*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a
-b)/b)^(1/2))*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*a^3-290*B*(-sinh(
1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(
1/2))*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*a*b^2)*((2*cosh(1/2*x)^2*
b+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sin
h(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cosh(x) + A)(b\cosh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cosh(x)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cosh(x)^2 + \left(Ba^2 + 2\,Aab\right)\cosh(x)\right)\sqrt{b\cosh(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*cosh(x)^3 + A*a^2 + (2*B*a*b + A*b^2)*cosh(x)^2 + (B*a^2 +
2*A*a*b)*cosh(x))*sqrt(b*cosh(x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cosh(x) + A)(b\cosh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)
```

3.108 $\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=181

$$\frac{2i\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{15b\sqrt{a+b\cosh(x)}}-\frac{2i\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{15}\,\text{sinh}$$

```
[Out] (((-2*I)/15)*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(
I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/15)*(a^
2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (
2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]) + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Cosh
[x]]*Sinh[x])/15 + (2*B*(a + b*Cosh[x])^(3/2)*Sinh[x])/5
```

Rubi [A] time = 0.322156, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2i\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15b\sqrt{a+b\cosh(x)}}-\frac{2i\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{15}\sinh(x)(3aB$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)/15)*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(
I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/15)*(a^
2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (
2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]) + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Cosh
[x]]*Sinh[x])/15 + (2*B*(a + b*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh(x) + \frac{2}{5}\int \sqrt{a+b\cosh(x)}\left(\frac{1}{2}(5aA+3bB)+\frac{1}{2}(5Ab\right.$$

$$= \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh(x) + \frac{4}{15}\int$$

$$= \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh(x) - \frac{\left(\left(a^2\right.\right.}{}$$

$$= \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh(x) + \frac{\left(\left(20a\right.\right.}{}$$

$$= -\frac{2i\left(20aAb+3a^2B+9b^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i\left(a^2-b^2\right)(5Ab+3a}{15b\sqrt{a}}$$

Mathematica [A] time = 0.63913, size = 124, normalized size = 0.69

$$\frac{2}{15}\sqrt{a+b\cosh(x)}\left(\sinh(x)(6aB+5Ab+3bB\cosh(x)) - \frac{i\left(\left(3a^2B+20aAb+9b^2B\right)E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)-(a-b)(3aB+5Ab)\text{Ellip}\right.}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (2*Sqrt[a + b*Cosh[x]]*(((-I)*((20*a*A*b + 3*a^2*B + 9*b^2*B)*EllipticE[(I/
2)*x, (2*b)/(a + b)] - (a - b)*(5*A*b + 3*a*B)*EllipticF[(I/2)*x, (2*b)/(a
+ b)]))/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (5*A*b + 6*a*B + 3*b*B*Cosh[x])
*Sinh[x]))/15
```

Maple [B] time = 0.105, size = 973, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x)
```

```
[Out] 2/15*(24*B*(-2*b/(a-b))^(1/2)*b^2*cosh(1/2*x)*sinh(1/2*x)^6+(20*A*(-2*b/(a-
b))^(1/2)*b^2+36*B*(-2*b/(a-b))^(1/2)*a*b+24*B*(-2*b/(a-b))^(1/2)*b^2)*sinh
(1/2*x)^4*cosh(1/2*x)+(10*A*(-2*b/(a-b))^(1/2)*a*b+10*A*(-2*b/(a-b))^(1/2)*
b^2+12*B*(-2*b/(a-b))^(1/2)*a^2+18*B*(-2*b/(a-b))^(1/2)*a*b+6*B*(-2*b/(a-b)
)^(1/2)*b^2)*sinh(1/2*x)^2*cosh(1/2*x)+15*A*a^2*(2*b/(a-b)*sinh(1/2*x)^2+(a
+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^
(1/2),1/2*(-2*(a-b)/b)^(1/2))+20*A*a*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b)
)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2
*(-2*(a-b)/b)^(1/2))+5*A*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-
sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)
/b)^(1/2))-40*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2
)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a*
b+3*a^2*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2
)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+12*b*B*a
*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ellipti
cF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+9*B*b^2*(2*b/(a-b
)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/
2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-6*B*(2*b/(a-b)*sinh(1/2*x)^
2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-
b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2-18*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(
a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2)
,1/2*(-2*(a-b)/b)^(1/2))*b^2)*((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)
/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*
x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cosh(x) + A)(b\cosh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cosh(x)^2 + Aa + (Ba + Ab)\cosh(x)\right)\sqrt{b\cosh(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cosh(x)^2 + A*a + (B*a + A*b)*cosh(x))*sqrt(b*cosh(x) + a), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cosh(x) + A)(b\cosh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)
```

3.109 $\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx$

Optimal. Leaf size=138

$$\frac{2iB\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

```
[Out] (((-2*I)/3)*(3*A*b + a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a +
 b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/3)*(a^2 - b^2)*B*Sqrt[(a
+ b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh
[x]]) + (2*B*Sqrt[a + b*Cosh[x]]*Sinh[x])/3
```

Rubi [A] time = 0.206277, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2iB\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```


```
[Out] (((-2*I)/3)*(3*A*b + a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a +
 b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/3)*(a^2 - b^2)*B*Sqrt[(a
+ b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh
[x]]) + (2*B*Sqrt[a + b*Cosh[x]]*Sinh[x])/3
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \frac{2}{3}B\sqrt{a+b\cosh(x)}\sinh(x)+\frac{2}{3}\int \frac{\frac{1}{2}(3aA+bB)+\frac{1}{2}(3Ab+aB)\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$
$$= \frac{2}{3}B\sqrt{a+b\cosh(x)}\sinh(x)-\frac{\left(\left(a^2-b^2\right)B\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{3b}+\frac{(3Ab+aB)\int \sqrt{a}}{3b}$$
$$= \frac{2}{3}B\sqrt{a+b\cosh(x)}\sinh(x)+\frac{\left((3Ab+aB)\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$
$$= -\frac{2i(3Ab+aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i\left(a^2-b^2\right)B\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{a+b\cosh(x)}}$$

Mathematica [A] time = 0.333301, size = 123, normalized size = 0.89

$$\frac{2iB\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)-2i(a+b)(aB+3Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)+2bB\sinh(x)(a+b\cosh(x))}{3b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] ((-2*I)*(a + b)*(3*A*b + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)
*x, (2*b)/(a + b)] + (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*Elli
pticF[(I/2)*x, (2*b)/(a + b)] + 2*b*B*(a + b*Cosh[x])*Sinh[x])/(3*b*Sqrt[a
+ b*Cosh[x]])
```

Maple [B] time = 0.094, size = 605, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x)
```

```
[Out] 2/3*(4*B*(-2*b/(a-b))^(1/2)*b*cosh(1/2*x)*sinh(1/2*x)^4+(2*B*(-2*b/(a-b))^(
1/2)*a+2*B*(-2*b/(a-b))^(1/2)*b)*sinh(1/2*x)^2*cosh(1/2*x)+3*A*a*(2*b/(a-b)
```

```
*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2
*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+3*A*b*(2*b/(a-b)*sinh(1/2*x)
^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a
-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-6*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b)
)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2
*(-2*(a-b)/b)^(1/2))*b+a*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-si
nh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b
)^(1/2))+b*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*B*(
2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE
(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a)*((2*cosh(1/2*x)^
2*b+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*s
inh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cosh(x) + A)\sqrt{b\cosh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cosh(x) + A)\sqrt{b\cosh(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\cosh(x))\sqrt{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral((A + B*cosh(x))*sqrt(a + b*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cosh{(x)} + A)\sqrt{b\cosh{(x)} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

3.110 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}+\frac{Bx}{b}$$

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqr
t[a - b]*b*Sqrt[a + b])
```

Rubi [A] time = 0.0676142, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2735, 2659, 208}

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqr
t[a - b]*b*Sqrt[a + b])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx &= \frac{Bx}{b}-\frac{(-Ab+aB)\int \frac{1}{a+b\cosh(x)}\,dx}{b}\\ &= \frac{Bx}{b}-\frac{(2(-Ab+aB))\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b}+\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.0912761, size = 59, normalized size = 0.98

$$\frac{2(aB-Ab)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b\sqrt{b^2-a^2}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(
b*Sqrt[-a^2 + b^2])
```

Maple [B] time = 0.016, size = 103, normalized size = 1.7

$$2\frac{A}{\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)-2\frac{aB}{b\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)+\frac{B}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A-2/b/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*a*B+B/b*
```

```
ln(tanh(1/2*x)+1)-B/b*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27611, size = 574, normalized size = 9.57

$$\left[-\frac{(Ba-Ab)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-\left(Ba^2-\right.}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-((B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x)/(a^2*b - b^3), (2*(B*a - A*
b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a
^2 - b^2)) + (B*a^2 - B*b^2)*x)/(a^2*b - b^3)]
```

Sympy [A] time = 154.892, size = 403, normalized size = 6.72

$$\begin{cases} \tilde{\infty}\left(2A\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)+Bx\right) \\ \frac{A\tanh\left(\frac{x}{2}\right)}{b}+\frac{Bx}{b}-\frac{B\tanh\left(\frac{x}{2}\right)}{b} \\ -\frac{A}{b\tanh\left(\frac{x}{2}\right)}+\frac{Bx}{b}-\frac{B}{b\tanh\left(\frac{x}{2}\right)} \\ \frac{Ax+B\sinh(x)}{a} \\ -\frac{Ab\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ab\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Bax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ba\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{Ba\log\left(\sqrt{\cdots}\right.}{ab\sqrt{\frac{a}{a-b}+\cdots}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), (A*tanh(x
/2)/b + B*x/b - B*tanh(x/2)/b, Eq(a, b)), (-A/(b*tanh(x/2)) + B*x/b - B/(b*
tanh(x/2)), Eq(a, -b)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (-A*b*log(-sqrt(a/
(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*s
qrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/
2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) +
B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*s
qrt(a/(a - b) + b/(a - b))) + B*a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x
/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) -
 B*a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(
a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*x*sqrt(a/(a - b) + b/(a -
 b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))),
True))
```

Giac [A] time = 1.17967, size = 68, normalized size = 1.13

$$\frac{Bx}{b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - 2*(B*a - A*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2
)*b)
```

3.111 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=82

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sinh(x)(Ab-aB)}{\left(a^2-b^2\right)(a+b\cosh(x))}$$

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)
*(a + b)^(3/2)) - ((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x]))
```

Rubi [A] time = 0.0782561, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2754, 12, 2659, 208}

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sinh(x)(Ab-aB)}{\left(a^2-b^2\right)(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)
*(a + b)^(3/2)) - ((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx &= -\frac{(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{\int \frac{-aA+bB}{a+b\cosh(x)}\,dx}{-a^2+b^2}\\ &= -\frac{(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}\,dx}{a^2-b^2}\\ &= -\frac{(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{(2(aA-bB))\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\ &= \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)(a+b\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.167068, size = 81, normalized size = 0.99

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{\sinh(x)(aB-Ab)}{(a-b)(a+b)(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(
3/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b*Cosh[x]))
```

Maple [A] time = 0.018, size = 108, normalized size = 1.3

$$2\,\frac{(Ab-aB)\tanh(x/2)}{(a^2-b^2)\left(a\,(\tanh(x/2))^2-(\tanh(x/2))^2\,b-a-b\right)}+2\,\frac{Aa-Bb}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)\,(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^2,x)
```

```
[Out] 2*(A*b-B*a)/(a^2-b^2)*tanh(1/2*x)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+2*(
A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(
a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30036, size = 1914, normalized size = 23.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a*
b^2 - B*b^3)*cosh(x)^2 + (A*a*b^2 - B*b^3)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2
)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*cosh(x))*sinh(x))*sqrt
(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^
2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x
) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x
```

```
) + b)) + 2*(B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*cosh(x) + 2*(B*a^4 - A*
a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 + (a^4*b^2
 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(x)^2 + 2*(
a^5*b - 2*a^3*b^3 + a*b^5)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (a^4*b^
2 - 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 +
 A*b^4 + (A*a*b^2 - B*b^3 + (A*a*b^2 - B*b^3)*cosh(x)^2 + (A*a*b^2 - B*b^3)
*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^
2 - B*b^3)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
osh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b
^3)*cosh(x) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(x))/(a^4*b^2 - 2
*a^2*b^4 + b^6 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b
^4 + b^6)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cosh(x) + 2*(a^5*b - 2*
a^3*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1565, size = 144, normalized size = 1.76

$$\frac{2\,(Aa - Bb)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\left(a^{2} - b^{2}\right)\sqrt{-a^{2} + b^{2}}} - \frac{2\left(Ba^{2}e^{x} - Aabe^{x} + Bab - Ab^{2}\right)}{\left(a^{2}b - b^{3}\right)\left(be^{(2\,x)} + 2\,ae^{x} + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 +
 b^2)) - 2*(B*a^2*e^x - A*a*b*e^x + B*a*b - A*b^2)/((a^2*b - b^3)*(b*e^(2*x
) + 2*a*e^x + b))
```

3.112 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx$

Optimal. Leaf size=135

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/
((a - b)^(5/2)*(a + b)^(5/2)) - ((A*b - a*B)*Sinh[x])/(2*(a^2 - b^2)*(a + b
*Cosh[x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/(2*(a^2 - b^2)^2*(a +
b*Cosh[x]))
```

Rubi [A] time = 0.16936, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2754, 12, 2659, 208}

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/
((a - b)^(5/2)*(a + b)^(5/2)) - ((A*b - a*B)*Sinh[x])/(2*(a^2 - b^2)*(a + b
*Cosh[x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/(2*(a^2 - b^2)^2*(a +
b*Cosh[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = -\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\int \frac{-2(aA-bB)+(Ab-aB)\cosh(x)}{(a+b\cosh(x))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\left(3aAb-a^2B-2b^2B\right)\sinh(x)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}+\frac{\int \frac{2a^2A+Ab^2-3abB}{a+b\cosh(x)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\left(3aAb-a^2B-2b^2B\right)\sinh(x)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}+\frac{\left(2a^2A+Ab^2-3abB\right)\int \frac{1}{a+b\cosh(x)}}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\left(3aAb-a^2B-2b^2B\right)\sinh(x)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}+\frac{\left(2a^2A+Ab^2-3abB\right)\text{Subst}}{\left(a^2-\right.}$$

$$= \frac{\left(2a^2A+Ab^2-3abB\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\left(3aAb-a^2B-2\right.}{2\left(a^2-b^2\right)^2(a\ }$$

Mathematica [A] time = 0.395397, size = 134, normalized size = 0.99

$$\frac{1}{2}\left(-\frac{2\left(2a^2A-3abB+Ab^2\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}+\frac{\sinh(x)\left(a^2B-3aAb+2b^2B\right)}{(a-b)^2(a+b)^2(a+b\cosh(x))}+\frac{\sinh(x)(aB-Ab)}{(a-b)(a+b)(a+b\cosh(x))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2
]])/(-a^2 + b^2)^(5/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b*C
osh[x])^2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[x])/((a - b)^2*(a + b)^2*(a
 + b*Cosh[x])))/2
```

Maple [A] time = 0.023, size = 207, normalized size = 1.5

$$-2\,\frac{1}{\left(a\,(\tanh(x/2))^2-(\tanh(x/2))^2\,b-a-b\right)^2}\left(-1/2\,\frac{\left(4\,Aab+Ab^2-2\,a^2B-Bab-2\,Bb^2\right)(\tanh(x/2))^3}{(a-b)\left(a^2+2\,ab+b^2\right)}+1/2\,\frac{\left(4\,Aab-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*tanh(1
/2*x)^3+1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan
h(1/2*x))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)^2+(2*A*a^2+A*b^2-3*B*a*b)/(
a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-
b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.67215, size = 7004, normalized size = 51.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - 2*(2*
A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + 2*(2*B
*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*
B*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B
*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 +
 3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 +
 (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A
*b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^3 + 4*(2*
A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(
x))*sinh(x)^3 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^
5)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2
*b^3 + A*a*b^4)*cosh(x))*sinh(x)^2 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4
)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b
^4 + A*b^5)*cosh(x)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^2 +
 (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(
x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) +
b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) + b)) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 -
5*B*a*b^5 - A*b^6)*cosh(x) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A
*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3
*B*a*b^5 - A*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^
3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x)^
4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*sinh(x)^4 + 4*(a^7*b^2 - 3*a^5*
b^4 + 3*a^3*b^6 - a*b^8)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a
*b^8 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x))*sinh(x)^3 + 2*(2*a^
8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*cosh(x)^2 + 2*(2*a^8*b - 5*a^6
*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
)*cosh(x)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x))*sinh(x)^
2 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x) + 4*(a^7*b^2 - 3*a^
5*b^4 + 3*a^3*b^6 - a*b^8 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x)
^3 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x)^2 + (2*a^8*b - 5*a
^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*cosh(x))*sinh(x)), -(B*a^4*b^2 - 3*A*a^
3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^
2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b
^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + (2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*
A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*cosh(x)^2 + (2*B*a^6 - 6*A*a
^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - 3*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^
```

```
2 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*co
sh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 - 3*
B*a^2*b^3 + A*a*b^4)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (
2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x)^3 + 2*(4*A*a^4*b - 6*B*a^
3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3
*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
)*cosh(x)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x))*sinh(x)^2 +
4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*
b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^3 + 3*(2*A*a^3*b^
2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b
^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (4*B*a^5*b - 10*A*a^4*b
^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6)*cosh(x) + (4*B*a^5*b - 1
0*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4*b^2
 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*
A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*co
sh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + (a^6*b^3 - 3*a^4*b
^5 + 3*a^2*b^7 - b^9)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*s
inh(x)^4 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x)^3 + 4*(a^7*b
^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
)*cosh(x))*sinh(x)^3 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*
cosh(x)^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 + 3*(a^6*b^3
 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*
b^6 - a*b^8)*cosh(x))*sinh(x)^2 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^
8)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + (a^6*b^3 - 3*a^4*
b^5 + 3*a^2*b^7 - b^9)*cosh(x)^3 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b
^8)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*cosh(x))*
sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20303, size = 336, normalized size = 2.49

$$\frac{\left(2\,Aa^{2}-3\,Bab+Ab^{2}\right)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{-a^{2}+b^{2}}}+\frac{2\,Aa^{2}b^{2}e^{(3\,x)}-3\,Bab^{3}e^{(3\,x)}+Ab^{4}e^{(3\,x)}-2\,Ba^{4}e^{(2\,x)}+6\,Aa^{3}be^{(2\,x)}-5\,Ba^{2}b^{2}e^{(}}{\left(a^{4}b-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="giac")
```

```
[Out] (2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*
a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x)
 + A*b^4*e^(3*x) - 2*B*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x
) + 3*A*a*b^3*e^(2*x) - 2*B*b^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x
- 5*B*a*b^3*e^x - A*b^4*e^x - B*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b - 2*
a^2*b^3 + b^5)*(b*e^(2*x) + 2*a*e^x + b)^2)
```

3.113 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$

Optimal. Leaf size=197

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}-\frac{\sinh(x)\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)}{6\left(a^2-b^2\right)^3(a+b\cosh(x))}-\frac{\sinh(x)\left(-2a^2B+\right.}{6\left(a^2-b^2\right)^2(a+}$$

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/
Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)) - ((A*b - a*B)*Sinh[x])/(3*(a^2
 - b^2)*(a + b*Cosh[x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^2*B)*Sinh[x])/(6*(a^
2 - b^2)^2*(a + b*Cosh[x])^2) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2
*B)*Sinh[x])/(6*(a^2 - b^2)^3*(a + b*Cosh[x]))
```

Rubi [A] time = 0.347737, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2754, 12, 2659, 208}

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}-\frac{\sinh(x)\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)}{6\left(a^2-b^2\right)^3(a+b\cosh(x))}-\frac{\sinh(x)\left(-2a^2B+\right.}{6\left(a^2-b^2\right)^2(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/
Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)) - ((A*b - a*B)*Sinh[x])/(3*(a^2
 - b^2)*(a + b*Cosh[x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^2*B)*Sinh[x])/(6*(a^
2 - b^2)^2*(a + b*Cosh[x])^2) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2
*B)*Sinh[x])/(6*(a^2 - b^2)^3*(a + b*Cosh[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}-\frac{\int \frac{-3(aA-bB)+2(Ab-aB)\cosh(x)}{(a+b\cosh(x))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sinh(x)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2}+\frac{\int \frac{2\left(3a^2A+2Ab^2-5abB\right)-\left(5aAb-2a^2\right.}{(a+b\cosh(x))^2}}{6\left(a^2-b^2\right)^2}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sinh(x)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2}-\frac{\left(11a^2Ab+4Ab^3-2a^3B-1\right.}{6\left(a^2-b^2\right)^3(a+b\,\mathrm{co}}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sinh(x)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2}-\frac{\left(11a^2Ab+4Ab^3-2a^3B-1\right.}{6\left(a^2-b^2\right)^3(a+b\,\mathrm{co}}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}-\frac{\left(5aAb-2a^2B-3b^2B\right)\sinh(x)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2}-\frac{\left(11a^2Ab+4Ab^3-2a^3B-1\right.}{6\left(a^2-b^2\right)^3(a+b\,\mathrm{co}}$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}-\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}-\frac{\left(5aAb\right.}{6\left(a^2\right.}$$

Mathematica [A] time = 0.753572, size = 196, normalized size = 0.99

$$\frac{1}{6}\left(\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}+\frac{\sinh(x)\left(-11a^2Ab+2a^3B+13ab^2B-4Ab^3\right)}{(a-b)^3(a+b)^3(a+b\cosh(x))}+\frac{\sinh(x)\left(2a^2\right.}{(a-b)^2(a+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[x/2])/Sq
rt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (2*(-(A*b) + a*B)*Sinh[x])/((a - b)*(
a + b)*(a + b*Cosh[x])^3) + ((-5*a*A*b + 2*a^2*B + 3*b^2*B)*Sinh[x])/((a -
b)^2*(a + b)^2*(a + b*Cosh[x])^2) + ((-11*a^2*A*b - 4*A*b^3 + 2*a^3*B + 13*
a*b^2*B)*Sinh[x])/((a - b)^3*(a + b)^3*(a + b*Cosh[x])))/6
```

Maple [A] time = 0.025, size = 342, normalized size = 1.7

$$-2\frac{1}{\left(a\left(\tanh(x/2)\right)^2-\left(\tanh(x/2)\right)^2b-a-b\right)^3}\left(-1/2\frac{\left(6Aa^2b+3Aab^2+2Ab^3-2a^3B-2Ba^2b-6Bab^2-Bb^3\right)(\tanh(}{(a-b)\left(a^3+3a^2b+3ab^2+b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^4,x)
```

```
[Out] -2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^3)/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*x)^5+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-7*
B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2*x)^3-1/2*(6*A*a^2*b-3*A*a
*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tanh(1/2*x))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)^3+(2*A*a^3+3*A*a*b^
2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.39518, size = 17173, normalized size = 87.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 + 22*B*a^3*b^5 + 14*A*a^2*b^6 - 26*B*a*b^
7 + 8*A*b^8 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*
a*b^7 + B*b^8)*cosh(x)^5 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a
^2*b^6 - 3*A*a*b^7 + B*b^8)*sinh(x)^5 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a
^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^4 - 30*(2*A*a^6*b^2 -
 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7 + (2*A*a^5*b
^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x))*si
nh(x)^4 + 4*(4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b
^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x)^3 + 4*(4*B*a^8 - 22*
A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a
^2*b^6 + 12*A*a*b^7 - 15*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b
^6 - 3*A*a*b^7 + B*b^8)*cosh(x)^2 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b
^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x))*sinh(x)^3 + 12*(4*B*a^7*
b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6
 - 4*B*a*b^7 + 2*A*b^8)*cosh(x)^2 + 12*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5
*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 - 5*
(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*c
osh(x)^3 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^
2*b^6 + B*a*b^7)*cosh(x)^2 + (4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^
5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x))*si
nh(x)^2 - 3*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + (2*A*a^3*b^4 -
 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 +
3*A*a*b^6 - B*b^7)*sinh(x)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 -
 B*a*b^6)*cosh(x)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6
+ (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^5 + 3*(8
*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)
*cosh(x)^4 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3
*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x
)^2 + 10*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x))*sinh(
x)^4 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b
```

```
^5 - 3*B*a*b^6)*cosh(x)^3 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*
B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*
b^6 - B*b^7)*cosh(x)^3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*
b^6)*cosh(x)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5
 + 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 +
14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 3*(8*A*a^5*b^2
- 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^
3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^4 + 20*(2*A*a^4*b^3 - 4*B*
a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^3 + 6*(8*A*a^5*b^2 - 16*B*a^4*b^3
+ 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 4*(4*A*a^6*b
- 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh
(x))*sinh(x)^2 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh
(x) + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 -
 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^5 + 5*(2*A*a^4*b^3 - 4*B*a^3*b^4
+ 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a
^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^3 + 2*(4*A*a^6*b - 8*B*a^
5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x)^2 +
(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^
7)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2
*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b
^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) +
 2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4
*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8)*cosh(x) + 6*(4*B*a^
6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b
^7 + B*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a
*b^7 + B*b^8)*cosh(x)^4 - 20*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a
^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b + 28*B*
a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a
*b^7)*cosh(x)^2 + 4*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4
 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(x))*sinh(x))/(a^8
*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12 + (a^8*b^4 - 4*a^6*b^6 + 6
*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^6 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8
- 4*a^2*b^10 + b^12)*sinh(x)^6 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3
*b^9 + a*b^11)*cosh(x)^5 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 +
 a*b^11 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x))*si
nh(x)^5 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh
(x)^4 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12 + 5*(a^
8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^2 + 10*(a^9*b^3
- 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x))*sinh(x)^4 + 4*(2*a^1
1*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x)^3 + 4*(2*a^11
*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11 + 5*(a^8*b^4 - 4*a^6*b^
6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^3 + 15*(a^9*b^3 - 4*a^7*b^5 + 6*
a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a
^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x))*sinh(x)^3 + 3*(4*a^10*b^2 - 15*a^8*b^4
 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 +
```

```
 20*a^6*b^6 - 10*a^4*b^8 + b^12 + 5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^
2*b^10 + b^12)*cosh(x)^4 + 20*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9
+ a*b^11)*cosh(x)^3 + 6*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8
+ b^12)*cosh(x)^2 + 4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a
*b^11)*cosh(x))*sinh(x)^2 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9
+ a*b^11)*cosh(x) + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11
 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^5 + 5*(a^9
*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x)^4 + 2*(4*a^10*b^
2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^3 + 2*(2*a^11*b -
5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x)^2 + (4*a^10*b^2 - 1
5*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x))*sinh(x)), -1/3*(2*B*a^
5*b^3 - 11*A*a^4*b^4 + 11*B*a^3*b^5 + 7*A*a^2*b^6 - 13*B*a*b^7 + 4*A*b^8 -
3*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)
*cosh(x)^5 - 3*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a
*b^7 + B*b^8)*sinh(x)^5 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a
^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^4 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 +
 A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7 + (2*A*a^5*b^3 - 4*B*a^4*b
^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x))*sinh(x)^4 + 2*(4
*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*
b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b + 28*B
*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*
a*b^7 - 15*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7
 + B*b^8)*cosh(x)^2 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b
^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x))*sinh(x)^3 + 6*(4*B*a^7*b - 17*A*a^6*b^
2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 +
2*A*b^8)*cosh(x)^2 + 6*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*
b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 - 5*(2*A*a^5*b^3 - 4
*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x)^3 - 15*(2
*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)
*cosh(x)^2 + (4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*
b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x))*sinh(x)^2 + 3*(2*A
*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3
*A*a*b^6 - B*b^7)*cosh(x)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^
7)*sinh(x)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x
)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 -
 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^5 + 3*(8*A*a^5*b^2 - 16*
B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^4 + 3*(
8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7
 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 10*(2*A*a^
4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x))*sinh(x)^4 + 4*(4*A*a^
6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*
cosh(x)^3 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*
a^2*b^5 - 3*B*a*b^6 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cos
h(x)^3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^2 +
 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B
```

```
*b^7)*cosh(x))*sinh(x)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8
*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 +
 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*
b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^4 + 20*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^
2*b^5 - B*a*b^6)*cosh(x)^3 + 6*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 -
 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 4*(4*A*a^6*b - 8*B*a^5*b^2 +
12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x))*sinh(x)^2 +
 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x) + 6*(2*A*a^4
*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3
*A*a*b^6 - B*b^7)*cosh(x)^5 + 5*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 -
B*a*b^6)*cosh(x)^4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2
*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^3 + 2*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4
*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x)^2 + (8*A*a^5*b^2 - 1
6*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x))*sinh
(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/
(a^2 - b^2)) + 3*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5
- 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8)*cosh(x) + 3*(4*B*a^6*b^2 - 20*A*a^5*b^3
 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8 - 5*(2*A*
a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x
)^4 - 20*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6
 + B*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b
^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x)^2 + 4*
(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*
A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(x))*sinh(x))/(a^8*b^4 - 4*a^6*b^6 + 6
*a^4*b^8 - 4*a^2*b^10 + b^12 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^1
0 + b^12)*cosh(x)^6 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)
*sinh(x)^6 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(
x)^5 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11 + (a^8*b^4 -
 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x))*sinh(x)^5 + 3*(4*a^10*
b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^4 + 3*(4*a^10*b^
2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12 + 5*(a^8*b^4 - 4*a^6*b^6 +
6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^2 + 10*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*
b^7 - 4*a^3*b^9 + a*b^11)*cosh(x))*sinh(x)^4 + 4*(2*a^11*b - 5*a^9*b^3 + 10
*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x)^3 + 4*(2*a^11*b - 5*a^9*b^3 + 10*
a^5*b^7 - 10*a^3*b^9 + 3*a*b^11 + 5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^
2*b^10 + b^12)*cosh(x)^3 + 15*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9
+ a*b^11)*cosh(x)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8
+ b^12)*cosh(x))*sinh(x)^3 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a
^4*b^8 + b^12)*cosh(x)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4
*b^8 + b^12 + 5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(
x)^4 + 20*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x)^3
+ 6*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^2 +
4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x))*sinh
(x)^2 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x) +
6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11 + (a^8*b^4 - 4*a^6*
```

```
b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^5 + 5*(a^9*b^3 - 4*a^7*b^5 + 6
*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x)^4 + 2*(4*a^10*b^2 - 15*a^8*b^4 + 20*
a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^3 + 2*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b
^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x)^2 + (4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b
^6 - 10*a^4*b^8 + b^12)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19912, size = 612, normalized size = 3.11

$$\frac{\left(2\,Aa^3 - 4\,Ba^2b + 3\,Aab^2 - Bb^3\right)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6\right)\sqrt{-a^2+b^2}} + \frac{6\,Aa^3b^3e^{(5\,x)} - 12\,Ba^2b^4e^{(5\,x)} + 9\,Aab^5e^{(5\,x)} - 3\,Bb^6e^{(5\,x)} + 30\,Aa^4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="giac")
```

```
[Out] (2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^x + a)/sqrt(-a^2 + b^
2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + 1/3*(6*A*a^3*b
^3*e^(5*x) - 12*B*a^2*b^4*e^(5*x) + 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x) + 3
0*A*a^4*b^2*e^(4*x) - 60*B*a^3*b^3*e^(4*x) + 45*A*a^2*b^4*e^(4*x) - 15*B*a*
b^5*e^(4*x) - 8*B*a^6*e^(3*x) + 44*A*a^5*b*e^(3*x) - 64*B*a^4*b^2*e^(3*x) +
 82*A*a^3*b^3*e^(3*x) - 78*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) - 24*B*a^
5*b*e^(2*x) + 102*A*a^4*b^2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a^2*b^4*
e^(2*x) - 24*B*a*b^5*e^(2*x) + 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x + 60*A*a
^3*b^3*e^x - 66*B*a^2*b^4*e^x + 15*A*a*b^5*e^x + 3*B*b^6*e^x - 2*B*a^3*b^3
+ 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^6)/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*(b*e^(2*x) + 2*a*e^x + b)^3)
```

3.114 $\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=56

$$\frac{Bx}{b}-\frac{2B\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

```
[Out] (B*x)/b - (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt
[a + b]])/(a*b)
```

Rubi [A] time = 0.0784398, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2735, 2659, 208}

$$\frac{Bx}{b}-\frac{2B\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b - (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt
[a + b]])/(a*b)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{Bx}{b}-\frac{\left(aB-\frac{b^2B}{a}\right)\int\frac{1}{a+b\cosh(x)}\,dx}{b}$$
$$=\frac{Bx}{b}-\frac{\left(2\left(aB-\frac{b^2B}{a}\right)\right)\mathrm{Subst}\left(\int\frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$=\frac{Bx}{b}-\frac{2\sqrt{a-b}\sqrt{a+b}B\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

Mathematica [A] time = 0.0689199, size = 56, normalized size = 1.

$$\frac{B\left(\frac{2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b}+\frac{ax}{b}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*((a*x)/b + (2*Sqrt[-a^2 + b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[-a^2 + b
^2]])/b))/a
```

Maple [B] time = 0.021, size = 107, normalized size = 1.9

$$-2\,\frac{aB}{b\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)+2\,\frac{Bb}{a\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)+\frac{B}{b}\ln\left(\tanh\left(\frac{x}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] -2/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*a*B
+2*B/a*b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
+B/b*ln(tanh(1/2*x)+1)-B/b*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28327, size = 491, normalized size = 8.77

$$\left[\frac{Bax + \sqrt{a^2-b^2}B\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{ab}, \frac{Bax+2\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(B*a*x + sqrt(a^2 - b^2)*B*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)))/(a*b), (B*a*x + 2*sqrt(-a^2 + b^2)*B*arctan(-sqrt(
-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a*b)]
```

Sympy [A] time = 154.659, size = 170, normalized size = 3.04

$$\begin{cases} \text{NaN} & \text{for } a = \\ \frac{Bx}{b} & \text{for } a = \\ \frac{B\sinh(x)}{a} & \text{for } b = \\ \frac{Bx}{b} & \text{for } a = \\ \frac{Bx}{b} + \frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherv} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*x/b, Eq(a, b)), (B*sinh(x)/a, Eq(b
, 0)), (B*x/b, Eq(a, -b)), (B*x/b + B*log(-sqrt(a/(a - b) + b/(a - b)) + ta
nh(x/2))/(b*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b) + b/(a - b)
) + tanh(x/2))/(b*sqrt(a/(a - b) + b/(a - b))) + B*log(-sqrt(a/(a - b) + b/
(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b
) + b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))), True))
```

Giac [A] time = 1.20697, size = 77, normalized size = 1.38

$$\frac{Bx}{b} - \frac{2\left(Ba^2 - Bb^2\right)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - 2*(B*a^2 - B*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 +
 b^2)*a*b)
```

3.115 $\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

[Out] (B*x)/b

Rubi [A] time = 0.0013082, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

Mathematica [A] time = 0.0004243, size = 6, normalized size = 1.

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b
```

Maple [A] time = 0., size = 7, normalized size = 1.2

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] B*x/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01807, size = 9, normalized size = 1.5

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

Sympy [A] time = 0.388836, size = 3, normalized size = 0.5

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] B*x/b
```

Giac [A] time = 1.16252, size = 8, normalized size = 1.33

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*x/b
```

3.116 $\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx$

Optimal. Leaf size=11

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Rubi [A] time = 0.029401, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2754, 8}

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx = \frac{\sinh(x)}{b+a\cosh(x)} + \frac{\int 0\,dx}{a^2-b^2}$$
$$= \frac{\sinh(x)}{b+a\cosh(x)}$$

Mathematica [A] time = 0.0520125, size = 11, normalized size = 1.

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Maple [B] time = 0.017, size = 29, normalized size = 2.6

$$2\,\frac{\tanh(x/2)}{a\,(\tanh(x/2))^2-(\tanh(x/2))^2\,b+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))/(b+a*cosh(x))^2,x)
```

```
[Out] 2*tanh(1/2*x)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08941, size = 159, normalized size = 14.45

$$-\frac{2\,(b\cosh(x)+b\sinh(x)+a)}{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) + a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18353, size = 35, normalized size = 3.18

$$-\frac{2\,(be^x+a)}{\left(ae^{(2\,x)}+2\,be^x+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a)
```

3.117 $\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx$

Optimal. Leaf size=36

$$\frac{5x}{\sqrt{3}}-x+\frac{10\tanh^{-1}\left(\frac{\sinh(x)}{-\cosh(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

```
[Out] -x + (5*x)/Sqrt[3] + (10*ArcTanh[Sinh[x]/(2 + Sqrt[3] - Cosh[x])])/Sqrt[3]
```

Rubi [A] time = 0.049653, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2735, 2657}

$$\frac{5x}{\sqrt{3}}-x+\frac{10\tanh^{-1}\left(\frac{\sinh(x)}{-\cosh(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Cosh[x])/(2 - Cosh[x]),x]
```

```
[Out] -x + (5*x)/Sqrt[3] + (10*ArcTanh[Sinh[x]/(2 + Sqrt[3] - Cosh[x])])/Sqrt[3]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -x + 5\int \frac{1}{2-\cosh(x)}\,dx$$
$$= -x + \frac{5x}{\sqrt{3}} + \frac{10\tanh^{-1}\left(\frac{\sinh(x)}{2+\sqrt{3}-\cosh(x)}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0715123, size = 24, normalized size = 0.67

$$\frac{10\tanh^{-1}\left(\sqrt{3}\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{3}} - x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + Cosh[x])/(2 - Cosh[x]),x]
```

```
[Out] -x + (10*ArcTanh[Sqrt[3]*Tanh[x/2]])/Sqrt[3]
```

Maple [A] time = 0.016, size = 32, normalized size = 0.9

$$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{10\sqrt{3}}{3}\mathrm{Artanh}\left(\tanh\left(\frac{x}{2}\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+cosh(x))/(2-cosh(x)),x)
```

```
[Out] -ln(tanh(1/2*x)+1)+ln(tanh(1/2*x)-1)+10/3*3^(1/2)*arctanh(tanh(1/2*x)*3^(1/
2))
```

Maxima [A] time = 1.53155, size = 46, normalized size = 1.28

$$\frac{5}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}+2}{\sqrt{3}+e^{(-x)}-2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="maxima")
```

```
[Out] 5/3*sqrt(3)*log(-(sqrt(3) - e^(-x) + 2)/(sqrt(3) + e^(-x) - 2)) - x
```

Fricas [A] time = 2.24681, size = 139, normalized size = 3.86

$$\frac{5}{3}\sqrt{3}\log\left(-\frac{2\left(\sqrt{3}-2\right)\cosh\left(x\right)-\left(2\sqrt{3}-3\right)\sinh\left(x\right)-\sqrt{3}+2}{\cosh\left(x\right)-2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="fricas")
```

```
[Out] 5/3*sqrt(3)*log(-(2*(sqrt(3) - 2)*cosh(x) - (2*sqrt(3) - 3)*sinh(x) - sqrt(
3) + 2)/(cosh(x) - 2)) - x
```

Sympy [A] time = 1.07708, size = 44, normalized size = 1.22

$$-x-\frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{\sqrt{3}}{3}\right)}{3}+\frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x)
```

```
[Out] -x - 5*sqrt(3)*log(tanh(x/2) - sqrt(3)/3)/3 + 5*sqrt(3)*log(tanh(x/2) + sqr
t(3)/3)/3
```

Giac [A] time = 1.19526, size = 50, normalized size = 1.39

$$-\frac{5}{3}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2e^{x}-4\right|}{\left|2\sqrt{3}+2e^{x}-4\right|}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="giac")
```

```
[Out] -5/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*e^x - 4)/abs(2*sqrt(3) + 2*e^x - 4)) -
x
```

3.118 $\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=108

$$-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] ((-2*I)*B*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a
 + b*Cosh[x])/(a + b)]) - ((2*I)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*
EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

Rubi [A] time = 0.111749, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2752, 2663, 2661, 2655, 2653}

$$-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]
```



```
[Out] ((-2*I)*B*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a
 + b*Cosh[x])/(a + b)]) - ((2*I)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*
EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```


Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx &= \frac{B\int \sqrt{a+b\cosh(x)}\,dx}{b}+\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{b}\\
&= \frac{\left(B\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{\left((Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}\,dx}{b\sqrt{a+b\cosh(x)}}\\
&= -\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.443158, size = 80, normalized size = 0.74

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((Ab-aB)\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+B(a+b)E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*B*EllipticE[(I/2)*x, (2*b)/(
a + b)] + (A*b - a*B)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*Cos
h[x]])
```

Maple [A] time = 0.096, size = 218, normalized size = 2.

$$2\,{\frac{\sqrt{-\left(\sinh\left(x/2\right)\right)^{2}}\sqrt{\left(2\,\left(\cosh\left(x/2\right)\right)^{2}b+a-b\right)\left(\sinh\left(x/2\right)\right)^{2}}}{\sqrt{2\,b\left(\sinh\left(x/2\right)\right)^{4}+\left(a+b\right)\left(\sinh\left(x/2\right)\right)^{2}}\sinh\left(x/2\right)\sqrt{2\,\left(\sinh\left(x/2\right)\right)^{2}b+a+b}}}\left(A{\rm EllipticF}\left(\cosh\left(x/2\right)\sqrt{-2\,{\frac{b}{a-b}}},1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x)
```

```
[Out] 2*(A*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+B*Ell
ipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*B*EllipticE
(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2)))*(-sinh(1/2*x)^2)^(
1/2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*((2*cosh(1/2*x)^2*b+a-b)*sinh(1/
2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1
/2)/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cosh\left(x\right) + A}{\sqrt{b\cosh\left(x\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{B\cosh\left(x\right) + A}{\sqrt{b\cosh\left(x\right) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cosh{(x)}}{\sqrt{a + b\cosh{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral((A + B*cosh(x))/sqrt(a + b*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cosh{(x)} + A}{\sqrt{b\cosh{(x)} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

3.119 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2\sinh(x)(Ab-aB)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] ((-2*I)*(A*b - a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/
(b*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]) - ((2*I)*B*Sqrt[(a + b*Cosh[x
])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]) - (2
*(A*b - a*B)*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]])
```

Rubi [A] time = 0.204889, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sinh(x)(Ab-aB)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```


```
[Out] ((-2*I)*(A*b - a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/
(b*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]) - ((2*I)*B*Sqrt[(a + b*Cosh[x
])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]) - (2
*(A*b - a*B)*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]])
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx = -\frac{2(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}} - \frac{2\int \frac{\frac{1}{2}(-aA+bB)-\frac{1}{2}(Ab-aB)\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}} + \frac{B\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{b} + \frac{(Ab-aB)\int \sqrt{a+b\cosh(x)}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}} + \frac{\left((Ab-aB)\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{\left(B\sqrt{\frac{a+b\cosh}{a+b}}\right.}{b\sqrt{}}$$

$$= -\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}} - \frac{2(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}$$

Mathematica [A] time = 0.356721, size = 133, normalized size = 0.88

$$\frac{-2iB\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+2b\sinh(x)(aB-Ab)+2i(a+b)(aB-Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b(a-b)(a+b)\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```

```
[Out] ((2*I)*(a + b)*(-(A*b) + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)
*x, (2*b)/(a + b)] - (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*Elli
pticF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-(A*b) + a*B)*Sinh[x])/((a - b)*b*(a +
 b)*Sqrt[a + b*Cosh[x]])
```

Maple [B] time = 0.197, size = 483, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x)
```

```
[Out] ((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)*(2*B/b/(-2*b/(a-b))^(1/2)*((2
*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b*sinh(1/2*x)^
4+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(
(-2*a+2*b)/b)^(1/2))+2*(A*b-B*a)/b/sinh(1/2*x)^2/(2*sinh(1/2*x)^2*b+a+b)/(-
2*b/(a-b))^(1/2)/(a^2-b^2)*(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)*(-
2*(-2*b/(a-b))^(1/2)*b*cosh(1/2*x)*sinh(1/2*x)^2+(2*b/(a-b)*sinh(1/2*x)^2+(
a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/
b)^(1/2))*(-sinh(1/2*x)^2)^(1/2)*a+(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1
/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*(-si
nh(1/2*x)^2)^(1/2)*b-2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*(-sinh(1/2*x)^2)
^(1/2)*b))/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cosh(x) + A)\sqrt{b\cosh(x) + a}}{b^2\cosh(x)^2 + 2\,ab\cosh(x) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cosh(x) + A)*sqrt(b*cosh(x) + a)/(b^2*cosh(x)^2 + 2*a*b*cosh(x)
 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

3.120 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=231

$$\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}-\frac{2i\left(a\right.}{}$$

```
[Out] (((-2*I)/3)*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)
*x, (2*b)/(a + b)])/(b*(a^2 - b^2)^2*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*
I)/3)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a
 + b)])/(b*(a^2 - b^2)*Sqrt[a + b*Cosh[x]]) - (2*(A*b - a*B)*Sinh[x])/(3*(a
^2 - b^2)*(a + b*Cosh[x])^(3/2)) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sinh[x])/
(3*(a^2 - b^2)^2*Sqrt[a + b*Cosh[x]])
```

Rubi [A] time = 0.349833, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(-B)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```


```
[Out] (((-2*I)/3)*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)
*x, (2*b)/(a + b)])/(b*(a^2 - b^2)^2*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*
I)/3)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a
 + b)])/(b*(a^2 - b^2)*Sqrt[a + b*Cosh[x]]) - (2*(A*b - a*B)*Sinh[x])/(3*(a
^2 - b^2)*(a + b*Cosh[x])^(3/2)) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sinh[x])/
(3*(a^2 - b^2)^2*Sqrt[a + b*Cosh[x]])
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
```

```
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aA-bB)+\frac{1}{2}(Ab-aB)\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2A+Ab^2-4abB\right)+\frac{1}{4}(4a \cdots}{\sqrt{a+b\,\mathrm{co}\cdots}}}{3\left(a^2-\cdots\right.}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} - \frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\cosh(x)}}\,\cdots}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{\left(\left(4aAb-a^2B-3b^2B\right)\sqrt{\cdots}\right.}{3b\left(a^2\cdots\right.}$$

$$= -\frac{2i\left(4aAb-a^2B-3b^2B\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}} - \cdots$$

Mathematica [A] time = 0.841302, size = 172, normalized size = 0.74

$$\frac{2\left(\frac{\sinh(x)\left(b\cosh(x)\left(a^2B-4aAb+3b^2B\right)-5a^2Ab+2a^3B+2ab^2B+Ab^3\right)}{\left(a^2-b^2\right)^2}+\frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}\left(\left(a^2B-4aAb+3b^2B\right)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-(a-b)(aB-Ab)\,\mathrm{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{b(a-b)^2}\right)}{3(a+b\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```

```
[Out] (2*((I*((a + b*Cosh[x])/(a + b))^(3/2)*((-4*a*A*b + a^2*B + 3*b^2*B)*Ellipt
icE[(I/2)*x, (2*b)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(I/2)*x, (2*
b)/(a + b)]))/((a - b)^2*b) + ((-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b^2*B +
b*(-4*a*A*b + a^2*B + 3*b^2*B)*Cosh[x])*Sinh[x])/(a^2 - b^2)^2))/(3*(a + b*
Cosh[x])^(3/2))
```

Maple [B] time = 0.346, size = 797, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x)
```

```
[Out] ((2*cosh(1/2*x)^2*b+a-b)*sinh(1/2*x)^2)^(1/2)*(-2*B/b/sinh(1/2*x)^2/(2*sinh
(1/2*x)^2*b+a+b)/(-2*b/(a-b))^(1/2)/(a^2-b^2)*(2*b*sinh(1/2*x)^4+(a+b)*sinh
(1/2*x)^2)^(1/2)*(2*(-2*b/(a-b))^(1/2)*b*cosh(1/2*x)*sinh(1/2*x)^2-(2*b/(a-
b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2
),1/2*((-2*a+2*b)/b)^(1/2))*(-sinh(1/2*x)^2)^(1/2)*a-(2*b/(a-b)*sinh(1/2*x)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2
*b)/b)^(1/2))*(-sinh(1/2*x)^2)^(1/2)*b+2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2)
)*(-sinh(1/2*x)^2)^(1/2)*b)+2*(A*b-B*a)/b*(-1/6/b/(a-b)/(a+b)*cosh(1/2*x)*(
2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)/b)^2-
8/3*b*sinh(1/2*x)^2/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*cosh(1/2*x)^2*b+a-b)*
sinh(1/2*x)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)/(-2*b/(a-b))^(1/
2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b*sinh(1
/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2)
,1/2*((-2*a+2*b)/b)^(1/2))-16/3*a*b/(a+b)^2/(a-b)^2*(-a+b)/(-2*b/(a-b))^(1/
2)*((2*cosh(1/2*x)^2*b+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b*sinh(1
/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(cosh(1/2*x)*(-2*b/(
a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1
/2),1/2*((-2*a+2*b)/b)^(1/2)))))/sinh(1/2*x)/(2*sinh(1/2*x)^2*b+a+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cosh(x) + A)\sqrt{b\cosh(x) + a}}{b^3\cosh(x)^3 + 3\,ab^2\cosh(x)^2 + 3\,a^2b\cosh(x) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cosh(x) + A)*sqrt(b*cosh(x) + a)/(b^3*cosh(x)^3 + 3*a*b^2*cosh(
x)^2 + 3*a^2*b*cosh(x) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

3.121 $\int \left(a \cosh^2(x)\right)^{7/2} dx$

Optimal. Leaf size=72

$$\frac{8}{35}a^2 \tanh(x) \left(a \cosh^2(x)\right)^{3/2} + \frac{16}{35}a^3 \tanh(x)\sqrt{a \cosh^2(x)} + \frac{1}{7} \tanh(x) \left(a \cosh^2(x)\right)^{7/2} + \frac{6}{35}a \tanh(x) \left(a \cosh^2(x)\right)^{5/2}$$

```
[Out] (16*a^3*Sqrt[a*Cosh[x]^2]*Tanh[x])/35 + (8*a^2*(a*Cosh[x]^2)^(3/2)*Tanh[x])
/35 + (6*a*(a*Cosh[x]^2)^(5/2)*Tanh[x])/35 + ((a*Cosh[x]^2)^(7/2)*Tanh[x])/
7
```

Rubi [A] time = 0.0549499, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3203, 3207, 2637}

$$\frac{8}{35}a^2 \tanh(x) \left(a \cosh^2(x)\right)^{3/2} + \frac{16}{35}a^3 \tanh(x)\sqrt{a \cosh^2(x)} + \frac{1}{7} \tanh(x) \left(a \cosh^2(x)\right)^{7/2} + \frac{6}{35}a \tanh(x) \left(a \cosh^2(x)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(7/2),x]
```


```
[Out] (16*a^3*Sqrt[a*Cosh[x]^2]*Tanh[x])/35 + (8*a^2*(a*Cosh[x]^2)^(3/2)*Tanh[x])
/35 + (6*a*(a*Cosh[x]^2)^(5/2)*Tanh[x])/35 + ((a*Cosh[x]^2)^(7/2)*Tanh[x])/
7
```


Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx = \frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x) + \frac{1}{7}(6a)\int \left(a\cosh^2(x)\right)^{5/2}\,dx$$
$$= \frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x) + \frac{1}{35}\left(24a^2\right)\int \left(a\cosh^2(x)\right)^{3/2}\,dx$$
$$= \frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x) + \frac{1}{35}\left(\right.$$
$$= \frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x) + \frac{1}{35}\left(\right.$$
$$= \frac{16}{35}a^3\sqrt{a\cosh^2(x)}\tanh(x) + \frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{7}\left(\right.$$

Mathematica [A] time = 0.025657, size = 42, normalized size = 0.58

$$\frac{a^3(1225\sinh(x) + 245\sinh(3x) + 49\sinh(5x) + 5\sinh(7x))\mathbf{sech}(x)\sqrt{a\cosh^2(x)}}{2240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(7/2),x]
```

```
[Out] (a^3*Sqrt[a*Cosh[x]^2]*Sech[x]*(1225*Sinh[x] + 245*Sinh[3*x] + 49*Sinh[5*x]
 + 5*Sinh[7*x]))/2240
```

Maple [A] time = 0.04, size = 38, normalized size = 0.5

$$\frac{a^4\cosh(x)\sinh(x)\left(5\,(\cosh(x))^6 + 6\,(\cosh(x))^4 + 8\,(\cosh(x))^2 + 16\right)}{35}\frac{1}{\sqrt{a\,(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(7/2),x)
```

```
[Out] 1/35*a^4*cosh(x)*sinh(x)*(5*cosh(x)^6+6*cosh(x)^4+8*cosh(x)^2+16)/(a*cosh(x
)^2)^(1/2)
```

Maxima [A] time = 1.52064, size = 96, normalized size = 1.33

$$\frac{1}{896}a^{\frac{7}{2}}e^{(7x)}+\frac{7}{640}a^{\frac{7}{2}}e^{(5x)}+\frac{7}{128}a^{\frac{7}{2}}e^{(3x)}-\frac{35}{128}a^{\frac{7}{2}}e^{(-x)}-\frac{7}{128}a^{\frac{7}{2}}e^{(-3x)}-\frac{7}{640}a^{\frac{7}{2}}e^{(-5x)}-\frac{1}{896}a^{\frac{7}{2}}e^{(-7x)}+\frac{35}{128}a^{\frac{7}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/896*a^(7/2)*e^(7*x) + 7/640*a^(7/2)*e^(5*x) + 7/128*a^(7/2)*e^(3*x) - 35/
128*a^(7/2)*e^(-x) - 7/128*a^(7/2)*e^(-3*x) - 7/640*a^(7/2)*e^(-5*x) - 1/89
6*a^(7/2)*e^(-7*x) + 35/128*a^(7/2)*e^x
```

Fricas [B] time = 2.28626, size = 2461, normalized size = 34.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(70*a^3*cosh(x)*e^x*sinh(x)^13 + 5*a^3*e^x*sinh(x)^14 + 7*(65*a^3*co
sh(x)^2 + 7*a^3)*e^x*sinh(x)^12 + 28*(65*a^3*cosh(x)^3 + 21*a^3*cosh(x))*e^
x*sinh(x)^11 + 7*(715*a^3*cosh(x)^4 + 462*a^3*cosh(x)^2 + 35*a^3)*e^x*sinh(
x)^10 + 70*(143*a^3*cosh(x)^5 + 154*a^3*cosh(x)^3 + 35*a^3*cosh(x))*e^x*sin
h(x)^9 + 35*(429*a^3*cosh(x)^6 + 693*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 + 35
*a^3)*e^x*sinh(x)^8 + 8*(2145*a^3*cosh(x)^7 + 4851*a^3*cosh(x)^5 + 3675*a^3
*cosh(x)^3 + 1225*a^3*cosh(x))*e^x*sinh(x)^7 + 7*(2145*a^3*cosh(x)^8 + 6468
*a^3*cosh(x)^6 + 7350*a^3*cosh(x)^4 + 4900*a^3*cosh(x)^2 - 175*a^3)*e^x*sin
h(x)^6 + 14*(715*a^3*cosh(x)^9 + 2772*a^3*cosh(x)^7 + 4410*a^3*cosh(x)^5 +
4900*a^3*cosh(x)^3 - 525*a^3*cosh(x))*e^x*sinh(x)^5 + 35*(143*a^3*cosh(x)^1
0 + 693*a^3*cosh(x)^8 + 1470*a^3*cosh(x)^6 + 2450*a^3*cosh(x)^4 - 525*a^3*c
osh(x)^2 - 7*a^3)*e^x*sinh(x)^4 + 140*(13*a^3*cosh(x)^11 + 77*a^3*cosh(x)^9
 + 210*a^3*cosh(x)^7 + 490*a^3*cosh(x)^5 - 175*a^3*cosh(x)^3 - 7*a^3*cosh(x
))*e^x*sinh(x)^3 + 7*(65*a^3*cosh(x)^12 + 462*a^3*cosh(x)^10 + 1575*a^3*cos
h(x)^8 + 4900*a^3*cosh(x)^6 - 2625*a^3*cosh(x)^4 - 210*a^3*cosh(x)^2 - 7*a^
3)*e^x*sinh(x)^2 + 14*(5*a^3*cosh(x)^13 + 42*a^3*cosh(x)^11 + 175*a^3*cosh(
```

```
x)^9 + 700*a^3*cosh(x)^7 - 525*a^3*cosh(x)^5 - 70*a^3*cosh(x)^3 - 7*a^3*cos
h(x))*e^x*sinh(x) + (5*a^3*cosh(x)^14 + 49*a^3*cosh(x)^12 + 245*a^3*cosh(x)
^10 + 1225*a^3*cosh(x)^8 - 1225*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 - 49*a^3*
cosh(x)^2 - 5*a^3)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^7
*e^(2*x) + (e^(2*x) + 1)*sinh(x)^7 + cosh(x)^7 + 7*(cosh(x)*e^(2*x) + cosh(
x))*sinh(x)^6 + 21*(cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^5 + 35*(cosh(x)^
3*e^(2*x) + cosh(x)^3)*sinh(x)^4 + 35*(cosh(x)^4*e^(2*x) + cosh(x)^4)*sinh(
x)^3 + 21*(cosh(x)^5*e^(2*x) + cosh(x)^5)*sinh(x)^2 + 7*(cosh(x)^6*e^(2*x)
+ cosh(x)^6)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25014, size = 107, normalized size = 1.49

$$\frac{1}{4480}\left(5\,a^3e^{(7\,x)} + 49\,a^3e^{(5\,x)} + 245\,a^3e^{(3\,x)} + 1225\,a^3e^x - \left(1225\,a^3e^{(6\,x)} + 245\,a^3e^{(4\,x)} + 49\,a^3e^{(2\,x)} + 5\,a^3\right)e^{(-7\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*e^(7*x) + 49*a^3*e^(5*x) + 245*a^3*e^(3*x) + 1225*a^3*e^x - (
1225*a^3*e^(6*x) + 245*a^3*e^(4*x) + 49*a^3*e^(2*x) + 5*a^3)*e^(-7*x))*sqrt
(a)
```

3.122 $\int \left(a\cosh^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\tanh(x)\sqrt{a\cosh^2(x)}+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{15}a\tanh(x)\left(a\cosh^2(x)\right)^{3/2}$$

```
[Out] (8*a^2*Sqrt[a*Cosh[x]^2]*Tanh[x])/15 + (4*a*(a*Cosh[x]^2)^(3/2)*Tanh[x])/15
 + ((a*Cosh[x]^2)^(5/2)*Tanh[x])/5
```

Rubi [A] time = 0.0356787, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3203, 3207, 2637}

$$\frac{8}{15}a^2\tanh(x)\sqrt{a\cosh^2(x)}+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{15}a\tanh(x)\left(a\cosh^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[a*Cosh[x]^2]*Tanh[x])/15 + (4*a*(a*Cosh[x]^2)^(3/2)*Tanh[x])/15
 + ((a*Cosh[x]^2)^(5/2)*Tanh[x])/5
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a\cosh^2(x)\right)^{5/2}\,dx &= \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{5}(4a)\int \left(a\cosh^2(x)\right)^{3/2}\,dx\\ &= \frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{15}\left(8a^2\right)\int \sqrt{a\cosh^2(x)}\,dx\\ &= \frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{15}\left(8a^2\sqrt{a\cosh^2(x)}\mathbf{sech}(x)\right)\int \cos\\ &= \frac{8}{15}a^2\sqrt{a\cosh^2(x)}\tanh(x)+\frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0162901, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2(150\sinh(x)+25\sinh(3x)+3\sinh(5x))\mathbf{sech}(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*Cosh[x]^2]*Sech[x]*(150*Sinh[x] + 25*Sinh[3*x] + 3*Sinh[5*x]))/
240
```

Maple [A] time = 0.037, size = 32, normalized size = 0.6

$$\frac{a^3\cosh(x)\sinh(x)\left(3\,(\cosh(x))^4+4\,(\cosh(x))^2+8\right)}{15}\frac{1}{\sqrt{a\,(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(5/2),x)
```

```
[Out] 1/15*a^3*cosh(x)*sinh(x)*(3*cosh(x)^4+4*cosh(x)^2+8)/(a*cosh(x)^2)^(1/2)
```

Maxima [A] time = 1.56399, size = 72, normalized size = 1.36

$$\frac{1}{160} a^{\frac{5}{2}} e^{(5x)} + \frac{5}{96} a^{\frac{5}{2}} e^{(3x)} - \frac{5}{16} a^{\frac{5}{2}} e^{(-x)} - \frac{5}{96} a^{\frac{5}{2}} e^{(-3x)} - \frac{1}{160} a^{\frac{5}{2}} e^{(-5x)} + \frac{5}{16} a^{\frac{5}{2}} e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) - 5/96*a
^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) + 5/16*a^(5/2)*e^x
```

Fricas [B] time = 2.20837, size = 1462, normalized size = 27.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cosh
(x)^2 + 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 + 5*a^2*cosh(x))*e^x*sin
h(x)^7 + 10*(63*a^2*cosh(x)^4 + 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6 +
4*(189*a^2*cosh(x)^5 + 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)^5 +
 10*(63*a^2*cosh(x)^6 + 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 - 15*a^2)*e^x
*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 + 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^3 - 15
*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 + 140*a^2*cosh(x)^6 + 450
*a^2*cosh(x)^4 - 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a^2*cosh(
x)^9 + 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 - 60*a^2*cosh(x)^3 - 5*a^2*cosh(
x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 + 25*a^2*cosh(x)^8 + 150*a^2*cosh(x)^6
- 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 - 3*a^2)*e^x)*sqrt(a*e^(4*x) + 2*a*e
^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) + 1)*sinh(x)^5 + cosh(x)^5
 + 5*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(2*x) + cosh(x
)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^2 + 5*(cosh(x)^
4*e^(2*x) + cosh(x)^4)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1672, size = 82, normalized size = 1.55

$$\frac{1}{480}\left(3\,a^2e^{(5\,x)}+25\,a^2e^{(3\,x)}+150\,a^2e^x-\left(150\,a^2e^{(4\,x)}+25\,a^2e^{(2\,x)}+3\,a^2\right)e^{(-5\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*e^(5*x) + 25*a^2*e^(3*x) + 150*a^2*e^x - (150*a^2*e^(4*x) + 25
*a^2*e^(2*x) + 3*a^2)*e^(-5*x))*sqrt(a)
```

3.123 $\int \left(a\cosh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=34

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}$$

```
[Out] (2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3 + ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rubi [A] time = 0.0225666, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3203, 3207, 2637}

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3 + ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a\cosh^2(x)\right)^{3/2} dx &= \frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{3}(2a)\int \sqrt{a\cosh^2(x)}\,dx\\ &= \frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{3}\left(2a\sqrt{a\cosh^2(x)}\,\mathbf{sech}(x)\right)\int \cosh(x)\,dx\\ &= \frac{2}{3}a\sqrt{a\cosh^2(x)}\tanh(x) + \frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0087829, size = 26, normalized size = 0.76

$$\frac{1}{12}a(9\sinh(x)+\sinh(3x))\mathbf{sech}(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))/12
```

Maple [A] time = 0.037, size = 24, normalized size = 0.7

$$\frac{a^2\cosh(x)\sinh(x)\left((\cosh(x))^2+2\right)}{3}\frac{1}{\sqrt{a(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(3/2),x)
```

```
[Out] 1/3*a^2*cosh(x)*sinh(x)*(cosh(x)^2+2)/(a*cosh(x)^2)^(1/2)
```

Maxima [A] time = 1.59739, size = 47, normalized size = 1.38

$$\frac{1}{24}a^{\frac{3}{2}}e^{(3x)} - \frac{3}{8}a^{\frac{3}{2}}e^{(-x)} - \frac{1}{24}a^{\frac{3}{2}}e^{(-3x)} + \frac{3}{8}a^{\frac{3}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*a^(3/2)*e^(3*x) - 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a^(
3/2)*e^x
```

Fricas [B] time = 2.42448, size = 672, normalized size = 19.76

$$\frac{\left(6\,a\cosh\left(x\right)e^{x}\sinh\left(x\right)^{5}+ae^{x}\sinh\left(x\right)^{6}+3\left(5\,a\cosh\left(x\right)^{2}+3\,a\right)e^{x}\sinh\left(x\right)^{4}+4\left(5\,a\cosh\left(x\right)^{3}+9\,a\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)^{3}}{24\left(\cosh\left(x\right)^{3}e^{(2\,x)}+\left(e^{(2\,x)}+1\right)\mathrm{si}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 + 3*a)
*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*cos
h(x)^4 + 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 + 6*a*cosh(x)
^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 + 9*a*cosh(x)^4 - 9*a*cosh(x)^
2 - a)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x) + (
e^(2*x) + 1)*sinh(x)^3 + cosh(x)^3 + 3*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^
2 + 3*(cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14336, size = 39, normalized size = 1.15

$$-\frac{1}{24}\left(\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-e^{(3\,x)}-9\,e^{x}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)*a^(3/2)
```

3.124 $\int \sqrt{a \cosh^2(x)}\, dx$

Optimal. Leaf size=13

$$\tanh(x)\sqrt{a \cosh^2(x)}$$

```
[Out] Sqrt[a*Cosh[x]^2]*Tanh[x]
```

Rubi [A] time = 0.0122538, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 2637}

$$\tanh(x)\sqrt{a \cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]^2],x]

[Out] Sqrt[a*Cosh[x]^2]*Tanh[x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\cosh^2(x)}\,dx = \left(\sqrt{a\cosh^2(x)}\mathbf{sech}(x)\right)\int \cosh(x)\,dx$$
$$= \sqrt{a\cosh^2(x)}\tanh(x)$$

Mathematica [A] time = 0.0043494, size = 13, normalized size = 1.

$$\tanh(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]^2],x]
```

```
[Out] Sqrt[a*Cosh[x]^2]*Tanh[x]
```

Maple [A] time = 0.028, size = 15, normalized size = 1.2

$$a\cosh(x)\sinh(x)\frac{1}{\sqrt{a(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(1/2),x)
```

```
[Out] 1/(a*cosh(x)^2)^(1/2)*a*cosh(x)*sinh(x)
```

Maxima [A] time = 1.59789, size = 23, normalized size = 1.77

$$-\frac{1}{2}\sqrt{a}e^{(-x)} + \frac{1}{2}\sqrt{a}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*e^(-x) + 1/2*sqrt(a)*e^x
```

Fricas [B] time = 2.39978, size = 216, normalized size = 16.62

$$\frac{\left(2\cosh\left(x\right)e^{x}\sinh\left(x\right)+e^{x}\sinh\left(x\right)^{2}+\left(\cosh\left(x\right)^{2}-1\right)e^{x}\right)\sqrt{ae^{(4x)}+2\,ae^{(2x)}+a}e^{(-x)}}{2\left(\cosh\left(x\right)e^{(2x)}+\left(e^{(2x)}+1\right)\sinh\left(x\right)+\cosh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 - 1)*e^x)*sqrt(a*e^
(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) + 1)*sinh(x) +
cosh(x))
```

Sympy [A] time = 0.617942, size = 19, normalized size = 1.46

$$\frac{\sqrt{a}\sqrt{\cosh^{2}\left(x\right)}\sinh\left(x\right)}{\cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(cosh(x)**2)*sinh(x)/cosh(x)
```

Giac [A] time = 1.15647, size = 19, normalized size = 1.46

$$-\frac{1}{2}\sqrt{a}\left(e^{(-x)}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(e^(-x) - e^x)
```

3.125 $\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{a\cosh^2(x)}}$$

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

Rubi [A] time = 0.0149485, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3770}

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]^2],x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \frac{\cosh(x)\int \mathbf{sech}(x)\,dx}{\sqrt{a\cosh^2(x)}} = \frac{\tan^{-1}(\sinh(x))\cosh(x)}{\sqrt{a\cosh^2(x)}}$$

Mathematica [A] time = 0.0065387, size = 21, normalized size = 1.31

$$\frac{2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]^2],x]
```

```
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

Maple [B] time = 0.044, size = 55, normalized size = 3.4

$$-\frac{\cosh(x)}{\sinh(x)}\sqrt{a(\sinh(x))^2}\ln\left(2\,\frac{\sqrt{-a}\sqrt{a(\sinh(x))^2}-a}{\cosh(x)}\right)\frac{1}{\sqrt{-a}}\frac{1}{\sqrt{a(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(1/2),x)
```

```
[Out] -cosh(x)*(a*sinh(x)^2)^(1/2)/(-a)^(1/2)*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/2
)-a)/cosh(x))/sinh(x)/(a*cosh(x)^2)^(1/2)
```

Maxima [A] time = 1.69339, size = 11, normalized size = 0.69

$$\frac{2\arctan(e^x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^x)/sqrt(a)
```

Fricas [B] time = 2.18351, size = 552, normalized size = 34.5

$$\left[-\frac{\sqrt{-a}\log\left(\frac{a\cosh(x)^2-2\sqrt{ae^{(4x)}+2ae^{(2x)}+a}(\cosh(x)e^x+e^x\sinh(x))\sqrt{-a}e^{(-x)}+\left(ae^{(2x)}+a\right)\sinh(x)^2+\left(a\cosh(x)^2-a\right)e^{(2x)}+2\left(a\cosh(x)e^{(2x)}+a\cosh(x)\right)\sin}{\left(e^{(2x)}+1\right)\sinh(x)^2+\cosh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}+2\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)+1}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log((a*cosh(x)^2 - 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*(cosh(x)*
e^x + e^x*sinh(x))*sqrt(-a)*e^(-x) + (a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)
^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x) - a)/((e^(2*x)
+ 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) +
 cosh(x))*sinh(x) + 1))/a, 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*arctan(cosh(
x) + sinh(x))/(a*e^(2*x) + a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.126 $\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\cosh(x)\tan^{-1}(\sinh(x))}{2a\sqrt{a\cosh^2(x)}}$$

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/(2*a*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(2*a*Sqrt[a*Cos
h[x]^2])
```

Rubi [A] time = 0.0248997, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3204, 3207, 3770}

$$\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\cosh(x)\tan^{-1}(\sinh(x))}{2a\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(-3/2),x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/(2*a*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(2*a*Sqrt[a*Cos
h[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx &= \frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx}{2a}\\ &= \frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\cosh(x)\int \operatorname{sech}(x)\,dx}{2a\sqrt{a\cosh^2(x)}}\\ &= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0144693, size = 31, normalized size = 0.74

$$\frac{\tanh(x)+2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(-3/2),x]
```

```
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x] + Tanh[x])/(2*a*Sqrt[a*Cosh[x]^2])
```

Maple [B] time = 0.059, size = 82, normalized size = 2.

$$\frac{1}{2\,a^2\cosh(x)\sinh(x)}\sqrt{a(\sinh(x))^2}\left(-\ln\left(2\,\frac{\sqrt{-a}\sqrt{a(\sinh(x))^2}-a}{\cosh(x)}\right)a(\cosh(x))^2+\sqrt{-a}\sqrt{a(\sinh(x))^2}\right)\frac{1}{\sqrt{-a}}\frac{1}{\sqrt{a(\cosh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(3/2),x)
```

```
[Out] 1/2/a^2/cosh(x)*(a*sinh(x)^2)^(1/2)*(-ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/2)-
a)/cosh(x))*a*cosh(x)^2+(-a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x)/
(a*cosh(x)^2)^(1/2)
```

Maxima [A] time = 1.77283, size = 55, normalized size = 1.31

$$\frac{e^{(3x)} - e^{x}}{a^{\frac{3}{2}}e^{(4x)} + 2\,a^{\frac{3}{2}}e^{(2x)} + a^{\frac{3}{2}}} + \frac{\arctan\left(e^{x}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (e^(3*x) - e^x)/(a^(3/2)*e^(4*x) + 2*a^(3/2)*e^(2*x) + a^(3/2)) + arctan(e^
x)/a^(3/2)
```

Fricas [B] time = 2.18795, size = 878, normalized size = 20.9

$$\frac{\left(3\cosh\left(x\right)e^{x}\sinh\left(x\right)^{2} + e^{x}\sinh\left(x\right)^{3} + \left(3\cosh\left(x\right)^{2} - 1\right)e^{x}\sinh\left(x\right) + \left(4\cosh\left(x\right)e^{x}\sinh\left(x\right)^{3} + e^{x}\sinh\left(x\right)^{4} + 2\left(3\cos\right.\right.}{a^{2}\cosh\left(x\right)^{4} + \left(a^{2}e^{(2x)} + a^{2}\right)\sinh\left(x\right)^{4} + 2\,a^{2}\cosh\left(x\right)^{2} + 4\left(a^{2}\cosh\left(x\right)e^{(2x)} + a^{2}\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + 2\left(3\,a^{2}\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (3*cosh(x)*e^x*sinh(x)^2 + e^x*sinh(x)^3 + (3*cosh(x)^2 - 1)*e^x*sinh(x) +
(4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x)^
2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^x
)*arctan(cosh(x) + sinh(x)) + (cosh(x)^3 - cosh(x))*e^x)*sqrt(a*e^(4*x) + 2
*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(x)^4 + (a^2*e^(2*x) + a^2)*sinh(x)^4 + 2*a
^2*cosh(x)^2 + 4*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^3 + 2*(3*a^2*c
osh(x)^2 + a^2 + (3*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*co
sh(x)^4 + 2*a^2*cosh(x)^2 + a^2)*e^(2*x) + 4*(a^2*cosh(x)^3 + a^2*cosh(x) +
 (a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15458, size = 76, normalized size = 1.81

$$\frac{\frac{\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)}{\sqrt{a}}-\frac{4\left(e^{(-x)}-e^{x}\right)}{\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)\sqrt{a}}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/sqrt(a) - 4*(e^(-x) - e^x)/(
((e^(-x) - e^x)^2 + 4)*sqrt(a)))/a
```

3.127 $\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{3\cosh(x)\tan^{-1}(\sinh(x))}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

```
[Out] (3*ArcTan[Sinh[x]]*Cosh[x])/(8*a^2*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(4*a*(a*Cos
h[x]^2)^(3/2)) + (3*Tanh[x])/(8*a^2*Sqrt[a*Cosh[x]^2])
```

Rubi [A] time = 0.0385913, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3204, 3207, 3770}

$$\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{3\cosh(x)\tan^{-1}(\sinh(x))}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(-5/2),x]
```

```
[Out] (3*ArcTan[Sinh[x]]*Cosh[x])/(8*a^2*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(4*a*(a*Cos
h[x]^2)^(3/2)) + (3*Tanh[x])/(8*a^2*Sqrt[a*Cosh[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx &= \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx}{4a}\\
&= \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{3\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx}{8a^2}\\
&= \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{(3\cosh(x))\int \operatorname{sech}(x)\,dx}{8a^2\sqrt{a\cosh^2(x)}}\\
&= \frac{3\tan^{-1}(\sinh(x))\cosh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.0350681, size = 40, normalized size = 0.66

$$
\frac{\tanh(x)\left(2\operatorname{sech}^2(x)+3\right)+6\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{8a^2\sqrt{a\cosh^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(-5/2),x]
```

```
[Out] (6*ArcTan[Tanh[x/2]]*Cosh[x] + (3 + 2*Sech[x]^2)*Tanh[x])/(8*a^2*Sqrt[a*Cos
h[x]^2])
```

Maple [B] time = 0.062, size = 102, normalized size = 1.7

$$\frac{1}{8 a^3 (\cosh(x))^3 \sinh(x)} \sqrt{a (\sinh(x))^2} \left(-3 \ln\left(2 \frac{\sqrt{-a}\sqrt{a (\sinh(x))^2} - a}{\cosh(x)}\right) a (\cosh(x))^4 + 3 \sqrt{a (\sinh(x))^2} (\cosh(x))^2 \sqrt{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(5/2),x)
```

```
[Out] 1/8/a^3/cosh(x)^3*(a*sinh(x)^2)^(1/2)*(-3*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1
/2)-a)/cosh(x))*a*cosh(x)^4+3*(a*sinh(x)^2)^(1/2)*cosh(x)^2*(-a)^(1/2)+2*(-
a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x)/(a*cosh(x)^2)^(1/2)
```

Maxima [A] time = 1.80192, size = 101, normalized size = 1.66

$$\frac{3 e^{(7x)} + 11 e^{(5x)} - 11 e^{(3x)} - 3 e^{x}}{4 \left(a^{\frac{5}{2}} e^{(8x)} + 4 a^{\frac{5}{2}} e^{(6x)} + 6 a^{\frac{5}{2}} e^{(4x)} + 4 a^{\frac{5}{2}} e^{(2x)} + a^{\frac{5}{2}}\right)} + \frac{3 \arctan\left(e^{x}\right)}{4 a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(7*x) + 11*e^(5*x) - 11*e^(3*x) - 3*e^x)/(a^(5/2)*e^(8*x) + 4*a^(5
/2)*e^(6*x) + 6*a^(5/2)*e^(4*x) + 4*a^(5/2)*e^(2*x) + a^(5/2)) + 3/4*arctan
(e^x)/a^(5/2)
```

Fricas [B] time = 2.33545, size = 2461, normalized size = 40.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(21*cosh(x)*e^x*sinh(x)^6 + 3*e^x*sinh(x)^7 + (63*cosh(x)^2 + 11)*e^x*s
inh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*e^x*sinh(x)^4 + (105*cosh(x)^4 + 1
10*cosh(x)^2 - 11)*e^x*sinh(x)^3 + (63*cosh(x)^5 + 110*cosh(x)^3 - 33*cosh(
```

```
x))*e^x*sinh(x)^2 + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*e^x*si
nh(x) + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*e^
x*sinh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 +
 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x
))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^x*sin
h(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*e^x*sinh(x) +
(cosh(x)^8 + 4*cosh(x)^6 + 6*cosh(x)^4 + 4*cosh(x)^2 + 1)*e^x)*arctan(cosh(
x) + sinh(x)) + (3*cosh(x)^7 + 11*cosh(x)^5 - 11*cosh(x)^3 - 3*cosh(x))*e^x
)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(a^3*cosh(x)^8 + 4*a^3*cosh(x)^6
 + (a^3*e^(2*x) + a^3)*sinh(x)^8 + 8*(a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*si
nh(x)^7 + 6*a^3*cosh(x)^4 + 4*(7*a^3*cosh(x)^2 + a^3 + (7*a^3*cosh(x)^2 + a
^3)*e^(2*x))*sinh(x)^6 + 8*(7*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (7*a^3*cosh(x
)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 4*a^3*cosh(x)^2 + 2*(35*a^3*cosh(
x)^4 + 30*a^3*cosh(x)^2 + 3*a^3 + (35*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + 3*
a^3)*e^(2*x))*sinh(x)^4 + 8*(7*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + 3*a^3*cos
h(x) + (7*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x
)^3 + a^3 + 4*(7*a^3*cosh(x)^6 + 15*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 + a^3 +
 (7*a^3*cosh(x)^6 + 15*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh
(x)^2 + (a^3*cosh(x)^8 + 4*a^3*cosh(x)^6 + 6*a^3*cosh(x)^4 + 4*a^3*cosh(x)^
2 + a^3)*e^(2*x) + 8*(a^3*cosh(x)^7 + 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 + a
^3*cosh(x) + (a^3*cosh(x)^7 + 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 + a^3*cosh(
x))*e^(2*x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1452, size = 100, normalized size = 1.64

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)}{16\,a^{\frac{5}{2}}}-\frac{3\,\sqrt{a}\left(e^{(-x)}-e^{x}\right)^{3}+20\,\sqrt{a}\left(e^{(-x)}-e^{x}\right)}{4\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a^(5/2) - 1/4*(3*sqrt(a)*(e^
(-x) - e^x)^3 + 20*sqrt(a)*(e^(-x) - e^x))/(((e^(-x) - e^x)^2 + 4)^2*a^3)
```

3.128 $\int \left(a\cosh^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=121

$$-\frac{26ia^2\text{EllipticF}\left(\frac{ix}{2},2\right)\sqrt{a\cosh^3(x)}}{77\cosh^{\frac{3}{2}}(x)}+\frac{2}{15}a^2\sinh(x)\cosh^5(x)\sqrt{a\cosh^3(x)}+\frac{26}{165}a^2\sinh(x)\cosh^3(x)\sqrt{a\cosh^3(x)}+\frac{78}{385}$$

```
[Out] (((-26*I)/77)*a^2*Sqrt[a*Cosh[x]^3]*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) +
(78*a^2*Cosh[x]*Sqrt[a*Cosh[x]^3]*Sinh[x])/385 + (26*a^2*Cosh[x]^3*Sqrt[a*C
osh[x]^3]*Sinh[x])/165 + (2*a^2*Cosh[x]^5*Sqrt[a*Cosh[x]^3]*Sinh[x])/15 + (
26*a^2*Sqrt[a*Cosh[x]^3]*Tanh[x])/77
```

Rubi [A] time = 0.0538256, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 2641}

$$\frac{2}{15}a^2\sinh(x)\cosh^5(x)\sqrt{a\cosh^3(x)}+\frac{26}{165}a^2\sinh(x)\cosh^3(x)\sqrt{a\cosh^3(x)}+\frac{78}{385}a^2\sinh(x)\cosh(x)\sqrt{a\cosh^3(x)}+\frac{26}{77}a^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(5/2),x]
```


```
[Out] (((-26*I)/77)*a^2*Sqrt[a*Cosh[x]^3]*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) +
(78*a^2*Cosh[x]*Sqrt[a*Cosh[x]^3]*Sinh[x])/385 + (26*a^2*Cosh[x]^3*Sqrt[a*C
osh[x]^3]*Sinh[x])/165 + (2*a^2*Cosh[x]^5*Sqrt[a*Cosh[x]^3]*Sinh[x])/15 + (
26*a^2*Sqrt[a*Cosh[x]^3]*Tanh[x])/77
```


Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\cosh^3(x)\right)^{5/2} dx = \frac{\left(a^2\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{15}{2}}(x)\,dx}{\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{15}a^2\cosh^5(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{\left(13a^2\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{11}{2}}(x)\,dx}{15\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{2}{15}a^2\cosh^5(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{\left(39a^2\sqrt{a\cosh^3(x)\right.}}{55\cos}$$

$$= \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{2}{15}a^2\cosh^5(x)\sqrt{a}$$

$$= \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{2}{15}a^2\cosh^5(x)\sqrt{a}$$

$$= -\frac{26ia^2\sqrt{a\cosh^3(x)}F\left(\frac{ix}{2}\middle|2\right)}{77\cosh^{\frac{3}{2}}(x)} + \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3}$$

Mathematica [A] time = 0.114683, size = 65, normalized size = 0.54

$$\frac{a\left(a\cosh^3(x)\right)^{3/2}\left((15465\sinh(x)+3657\sinh(3x)+749\sinh(5x)+77\sinh(7x))\sqrt{\cosh(x)}-12480i\text{EllipticF}\left(\frac{ix}{2},2\right)\right)}{36960\cosh^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(5/2),x]
```

```
[Out] (a*(a*Cosh[x]^3)^(3/2)*((-12480*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(1
5465*Sinh[x] + 3657*Sinh[3*x] + 749*Sinh[5*x] + 77*Sinh[7*x])))/(36960*Cosh
[x]^(9/2))
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \left(a \left(\cosh \left(x\right)\right)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(5/2),x)
```

```
[Out] int((a*cosh(x)^3)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \cosh \left(x\right)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\sqrt{a \cosh \left(x\right)^{3}} a^{2} \cosh \left(x\right)^{6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3)*a^2*cosh(x)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \cosh\left(x\right)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x)^3)^(5/2), x)
```

3.129 $\int \left(a\cosh^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=71

$$\frac{2}{9}a\sinh(x)\cosh^2(x)\sqrt{a\cosh^3(x)}+\frac{14}{45}a\sinh(x)\sqrt{a\cosh^3(x)}-\frac{14iaE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{15\cosh^{\frac{3}{2}}(x)}$$

```
[Out] (((-14*I)/15)*a*Sqrt[a*Cosh[x]^3]*EllipticE[(I/2)*x, 2])/Cosh[x]^(3/2) + (1
4*a*Sqrt[a*Cosh[x]^3]*Sinh[x])/45 + (2*a*Cosh[x]^2*Sqrt[a*Cosh[x]^3]*Sinh[x
])/9
```

Rubi [A] time = 0.0338387, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 2639}

$$\frac{2}{9}a\sinh(x)\cosh^2(x)\sqrt{a\cosh^3(x)}+\frac{14}{45}a\sinh(x)\sqrt{a\cosh^3(x)}-\frac{14iaE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{15\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(3/2),x]
```

```
[Out] (((-14*I)/15)*a*Sqrt[a*Cosh[x]^3]*EllipticE[(I/2)*x, 2])/Cosh[x]^(3/2) + (1
4*a*Sqrt[a*Cosh[x]^3]*Sinh[x])/45 + (2*a*Cosh[x]^2*Sqrt[a*Cosh[x]^3]*Sinh[x
])/9
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{9}{2}}(x)\,dx}{\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{\left(7a\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{5}{2}}(x)\,dx}{9\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{14}{45}a\sqrt{a\cosh^3(x)}\sinh(x) + \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{\left(7a\sqrt{a\cosh^3(x)}\right)\int \sqrt{\cosh(x)}\,dx}{15\cosh^{\frac{3}{2}}(x)}$$

$$= -\frac{14ia\sqrt{a\cosh^3(x)}E\left(\frac{ix}{2}\middle|2\right)}{15\cosh^{\frac{3}{2}}(x)} + \frac{14}{45}a\sqrt{a\cosh^3(x)}\sinh(x) + \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\sinh(x)$$

Mathematica [A] time = 0.0714354, size = 54, normalized size = 0.76

$$\frac{\left(a\cosh^3(x)\right)^{3/2}\left((38\sinh(2x)+5\sinh(4x))\sqrt{\cosh(x)}-168iE\left(\frac{ix}{2}\middle|2\right)\right)}{180\cosh^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(3/2),x]
```

```
[Out] ((a*Cosh[x]^3)^(3/2)*((-168*I)*EllipticE[(I/2)*x, 2] + Sqrt[Cosh[x]]*(38*Si
nh[2*x] + 5*Sinh[4*x])))/(180*Cosh[x]^(9/2))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \left(a \left(\cosh \left(x\right)\right)^{3}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(3/2),x)
```

```
[Out] int((a*cosh(x)^3)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \cosh \left(x\right)^{3}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a \cosh \left(x\right)^{3}} a \cosh \left(x\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3)*a*cosh(x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cosh\left(x\right)^{3}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x)^3)^(3/2), x)
```

3.130 $\int \sqrt{a\cosh^3(x)}\,dx$

Optimal. Leaf size=48

$$\frac{2}{3}\tanh(x)\sqrt{a\cosh^3(x)}-\frac{2i\text{EllipticF}\left(\frac{ix}{2},2\right)\sqrt{a\cosh^3(x)}}{3\cosh^{\frac{3}{2}}(x)}$$

```
[Out] (((-2*I)/3)*Sqrt[a*Cosh[x]^3]*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) + (2*Sqr
t[a*Cosh[x]^3]*Tanh[x])/3
```

Rubi [A] time = 0.0240773, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 2641}

$$\frac{2}{3}\tanh(x)\sqrt{a\cosh^3(x)}-\frac{2iF\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{3\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]^3],x]
```

```
[Out] (((-2*I)/3)*Sqrt[a*Cosh[x]^3]*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) + (2*Sqr
t[a*Cosh[x]^3]*Tanh[x])/3
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\cosh^3(x)}\,dx &= \frac{\sqrt{a\cosh^3(x)}\int \cosh^{\frac{3}{2}}(x)\,dx}{\cosh^{\frac{3}{2}}(x)}\\
&= \frac{2}{3}\sqrt{a\cosh^3(x)}\tanh(x)+\frac{\sqrt{a\cosh^3(x)}\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3\cosh^{\frac{3}{2}}(x)}\\
&= -\frac{2i\sqrt{a\cosh^3(x)}F\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)}+\frac{2}{3}\sqrt{a\cosh^3(x)}\tanh(x)
\end{aligned}$$

Mathematica [C] time = 0.0513766, size = 59, normalized size = 1.23

$$\frac{2}{3}\sqrt{a\cosh^3(x)}\left(\operatorname{sech}^2(x)\sqrt{\sinh(2x)+\cosh(2x)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2x)-\sinh(2x)\right)+\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]^3],x]
```

```
[Out] (2*Sqrt[a*Cosh[x]^3]*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*
x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \sqrt{a\left(\cosh(x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(1/2),x)
```

```
[Out] int((a*cosh(x)^3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \cosh(x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a \cosh(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**3)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \cosh (x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x)^3), x)
```

3.131 $\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh(x)\cosh(x)}{\sqrt{a\cosh^3(x)}}+\frac{2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh^3(x)}}$$

```
[Out] ((2*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2])/Sqrt[a*Cosh[x]^3] + (2*Cosh[x]*
Sinh[x])/Sqrt[a*Cosh[x]^3]
```

Rubi [A] time = 0.0238207, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2636, 2639}

$$\frac{2\sinh(x)\cosh(x)}{\sqrt{a\cosh^3(x)}}+\frac{2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]^3],x]
```

```
[Out] ((2*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2])/Sqrt[a*Cosh[x]^3] + (2*Cosh[x]*
Sinh[x])/Sqrt[a*Cosh[x]^3]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx &= \frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\cosh^3(x)}}\\ &= \frac{2\cosh(x)\sinh(x)}{\sqrt{a\cosh^3(x)}} - \frac{\cosh^{\frac{3}{2}}(x)\int \sqrt{\cosh(x)}\,dx}{\sqrt{a\cosh^3(x)}}\\ &= \frac{2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh^3(x)}} + \frac{2\cosh(x)\sinh(x)}{\sqrt{a\cosh^3(x)}}\end{aligned}$$

Mathematica [A] time = 0.021119, size = 36, normalized size = 0.78

$$\frac{2\cosh(x)\left(\sinh(x) + i\sqrt{\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)\right)}{\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]^3],x]
```

```
[Out] (2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/Sqrt[a*Cosh[x
]^3]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\cosh(x)\right)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \cosh(x)^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*cosh(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a \cosh(x)^3}}{a \cosh(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3)/(a*cosh(x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**3)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cosh(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cosh(x)^3), x)
```

3.132 $\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{10i\cosh^{\frac{3}{2}}(x)\text{EllipticF}\left(\frac{ix}{2},2\right)}{21a\sqrt{a\cosh^3(x)}}+\frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\text{sech}(x)}{7a\sqrt{a\cosh^3(x)}}$$

```
[Out] (((-10*I)/21)*Cosh[x]^(3/2)*EllipticF[(I/2)*x, 2])/(a*Sqrt[a*Cosh[x]^3]) +
(10*Sinh[x])/(21*a*Sqrt[a*Cosh[x]^3]) + (2*Sech[x]*Tanh[x])/(7*a*Sqrt[a*Cos
h[x]^3])
```

Rubi [A] time = 0.0342725, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2636, 2641}

$$\frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}}-\frac{10i\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\text{sech}(x)}{7a\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(-3/2),x]
```


```
[Out] (((-10*I)/21)*Cosh[x]^(3/2)*EllipticF[(I/2)*x, 2])/(a*Sqrt[a*Cosh[x]^3]) +
(10*Sinh[x])/(21*a*Sqrt[a*Cosh[x]^3]) + (2*Sech[x]*Tanh[x])/(7*a*Sqrt[a*Cos
h[x]^3])
```


Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx &= \frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\cosh^3(x)}} \\
&= \frac{2\mathbf{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}} + \frac{\left(5\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\cosh^3(x)}} \\
&= \frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}} + \frac{2\mathbf{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}} + \frac{\left(5\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{21a\sqrt{a\cosh^3(x)}} \\
&= -\frac{10i\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\cosh^3(x)}} + \frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}} + \frac{2\mathbf{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.0588545, size = 48, normalized size = 0.64

$$
\frac{2\cosh^2(x)\left(-5i\cosh^{\frac{5}{2}}(x)\mathbf{EllipticF}\left(\frac{ix}{2},2\right)+3\tanh(x)+5\sinh(x)\cosh(x)\right)}{21\left(a\cosh^3(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(-3/2),x]
```

```
[Out] (2*Cosh[x]^2*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2] + 5*Cosh[x]*Sinh[x
] + 3*Tanh[x]))/(21*(a*Cosh[x]^3)^(3/2))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \left(a \left(\cosh \left(x \right)\right)^{3}\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \cosh \left(x\right)^{3}\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{a \cosh \left(x\right)^{3}}}{a^{2} \cosh \left(x\right)^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3)/(a^2*cosh(x)^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \cosh\left(x\right)^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x)^3)^(-3/2), x)
```

3.133 $\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\mathbf{sech}^4(x)}{13a^2\sqrt{a\cosh^3(x)}}+\frac{22\tanh(x)\mathbf{sech}^2(x)}{117a^2\sqrt{a\cosh^3(x)}}$$

```
[Out] (((154*I)/195)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]^3])
 + (154*Cosh[x]*Sinh[x])/(195*a^2*Sqrt[a*Cosh[x]^3]) + (154*Tanh[x])/(585*a
^2*Sqrt[a*Cosh[x]^3]) + (22*Sech[x]^2*Tanh[x])/(117*a^2*Sqrt[a*Cosh[x]^3])
+ (2*Sech[x]^4*Tanh[x])/(13*a^2*Sqrt[a*Cosh[x]^3])
```

Rubi [A] time = 0.051845, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2636, 2639}

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\mathbf{sech}^4(x)}{13a^2\sqrt{a\cosh^3(x)}}+\frac{22\tanh(x)\mathbf{sech}^2(x)}{117a^2\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(-5/2),x]
```

```
[Out] (((154*I)/195)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]^3])
 + (154*Cosh[x]*Sinh[x])/(195*a^2*Sqrt[a*Cosh[x]^3]) + (154*Tanh[x])/(585*a
^2*Sqrt[a*Cosh[x]^3]) + (22*Sech[x]^2*Tanh[x])/(117*a^2*Sqrt[a*Cosh[x]^3])
+ (2*Sech[x]^4*Tanh[x])/(13*a^2*Sqrt[a*Cosh[x]^3])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\cosh^3(x)}}$$

$$= \frac{2\mathrm{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}} + \frac{\left(11\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\cosh^3(x)}}$$

$$= \frac{22\mathrm{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}} + \frac{2\mathrm{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}} + \frac{\left(77\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\cosh^3(x)}}$$

$$= \frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}} + \frac{22\mathrm{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}} + \frac{2\mathrm{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}} + \frac{\left(77\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{3}{2}}(x)}\,dx}{195a^2\sqrt{a\cosh^3(x)}}$$

$$= \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\cosh^3(x)}} + \frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}} + \frac{22\mathrm{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}} + \frac{2\mathrm{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}} - \frac{(77\,\mathrm{c}}{}$$

$$= \frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\cosh^3(x)}} + \frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}} + \frac{22\mathrm{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}} + \frac{2}{}$$

Mathematica [A] time = 0.100225, size = 61, normalized size = 0.5

$$\frac{462i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)+462\sinh(x)\cosh(x)+2\tanh(x)\left(45\mathrm{sech}^4(x)+55\mathrm{sech}^2(x)+77\right)}{585a^2\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(-5/2),x]
```

```
[Out] ((462*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 462*Cosh[x]*Sinh[x] + 2*(77
+ 55*Sech[x]^2 + 45*Sech[x]^4)*Tanh[x])/(585*a^2*Sqrt[a*Cosh[x]^3])
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \left(a\left(\cosh\left(x\right)\right)^{3}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cosh\left(x\right)^{3}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\cosh\left(x\right)^{3}}}{a^{3}\cosh\left(x\right)^{9}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3)/(a^3*cosh(x)^9), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cosh\left(x\right)^{3}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x)^3)^(-5/2), x)
```

3.134 $\int \left(a \cosh^4(x)\right)^{5/2} dx$

Optimal. Leaf size=132

$$\frac{1}{10}a^2 \sinh(x)\cosh^7(x)\sqrt{a\cosh^4(x)} + \frac{9}{80}a^2 \sinh(x)\cosh^5(x)\sqrt{a\cosh^4(x)} + \frac{21}{160}a^2 \sinh(x)\cosh^3(x)\sqrt{a\cosh^4(x)} + \frac{21}{128}a^2$$

```
[Out] (63*a^2*x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/256 + (21*a^2*Cosh[x]*Sqrt[a*Cosh[x]
^4]*Sinh[x])/128 + (21*a^2*Cosh[x]^3*Sqrt[a*Cosh[x]^4]*Sinh[x])/160 + (9*a^
2*Cosh[x]^5*Sqrt[a*Cosh[x]^4]*Sinh[x])/80 + (a^2*Cosh[x]^7*Sqrt[a*Cosh[x]^4
]*Sinh[x])/10 + (63*a^2*Sqrt[a*Cosh[x]^4]*Tanh[x])/256
```

Rubi [A] time = 0.0529856, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{10}a^2 \sinh(x)\cosh^7(x)\sqrt{a\cosh^4(x)} + \frac{9}{80}a^2 \sinh(x)\cosh^5(x)\sqrt{a\cosh^4(x)} + \frac{21}{160}a^2 \sinh(x)\cosh^3(x)\sqrt{a\cosh^4(x)} + \frac{21}{128}a^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(5/2),x]
```


```
[Out] (63*a^2*x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/256 + (21*a^2*Cosh[x]*Sqrt[a*Cosh[x]
^4]*Sinh[x])/128 + (21*a^2*Cosh[x]^3*Sqrt[a*Cosh[x]^4]*Sinh[x])/160 + (9*a^
2*Cosh[x]^5*Sqrt[a*Cosh[x]^4]*Sinh[x])/80 + (a^2*Cosh[x]^7*Sqrt[a*Cosh[x]^4
]*Sinh[x])/10 + (63*a^2*Sqrt[a*Cosh[x]^4]*Tanh[x])/256
```


Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx = \left(a^2\sqrt{a\cosh^4(x)}\,\mathbf{sech}^2(x)\right)\int \cosh^{10}(x)\,dx$$
$$= \frac{1}{10}a^2\cosh^7(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{10}\left(9a^2\sqrt{a\cosh^4(x)}\,\mathbf{sech}^2(x)\right)\int \cosh^8(x)\,dx$$
$$= \frac{9}{80}a^2\cosh^5(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{10}a^2\cosh^7(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{80}\Big(63a^2\sqrt{a\cosh^4(}$$
$$= \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{9}{80}a^2\cosh^5(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{10}a^2\cosh^7(x)\sqrt{a}$$
$$= \frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{9}{80}a^2\cosh^5(x)\sqrt{a}$$
$$= \frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{9}{80}a^2\cosh^5(x)\sqrt{a}$$
$$= \frac{63}{256}a^2x\sqrt{a\cosh^4(x)}\,\mathbf{sech}^2(x) + \frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4}$$

Mathematica [A] time = 0.116271, size = 53, normalized size = 0.4

$$\frac{a(2520x + 2100\sinh(2x) + 600\sinh(4x) + 150\sinh(6x) + 25\sinh(8x) + 2\sinh(10x))\,\mathbf{sech}^6(x)\left(a\cosh^4(x)\right)^{3/2}}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(5/2),x]
```

```
[Out] (a*(a*Cosh[x]^4)^(3/2)*Sech[x]^6*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x] +
 150*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

Maple [A] time = 0.138, size = 177, normalized size = 1.3

$$\frac{\sqrt{8}\,(\cosh(2x)+1)\sqrt{2}}{10240\,\sinh(2x)}\sqrt{a(-1+\cosh(2x))(\cosh(2x)+1)}\,a^{\frac{3}{2}}\Big(8\sqrt{a(\sinh(2x))^2}\sqrt{a}\,(\sinh(2x))^4 + 50\sqrt{a(\sinh(2x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(5/2),x)
```

```
[Out] 1/10240*8^(1/2)*(cosh(2*x)+1)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*2^(1/2
)*a^(3/2)*(8*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^4+50*(a*sinh(2*x)^2)^(
1/2)*a^(1/2)*cosh(2*x)*sinh(2*x)^2+160*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2
*x)^2+325*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(1/2)
*a^(1/2)+315*ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/(a*(c
osh(2*x)+1)^2)^(1/2)
```

Maxima [A] time = 1.72198, size = 135, normalized size = 1.02

$$\frac{63}{256}\,a^{\frac{5}{2}}x+\frac{1}{20480}\left(25\,a^{\frac{5}{2}}e^{(-2\,x)}+150\,a^{\frac{5}{2}}e^{(-4\,x)}+600\,a^{\frac{5}{2}}e^{(-6\,x)}+2100\,a^{\frac{5}{2}}e^{(-8\,x)}-2100\,a^{\frac{5}{2}}e^{(-12\,x)}-600\,a^{\frac{5}{2}}e^{(-14\,x)}-150\,a^{\frac{5}{2}}e^{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 63/256*a^(5/2)*x + 1/20480*(25*a^(5/2)*e^(-2*x) + 150*a^(5/2)*e^(-4*x) + 60
0*a^(5/2)*e^(-6*x) + 2100*a^(5/2)*e^(-8*x) - 2100*a^(5/2)*e^(-12*x) - 600*a
^(5/2)*e^(-14*x) - 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) - 2*a^(5/2)
*e^(-20*x) + 2*a^(5/2))*e^(10*x)
```

Fricas [B] time = 2.13916, size = 4963, normalized size = 37.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*(
76*a^2*cosh(x)^2 + 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 + 15*a^
2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 + 255*a^2*cosh(x)^2 +
 10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 + 425*a^2*cosh(x)^3 + 5
0*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 + 1275*a^2*cosh(
x)^4 + 300*a^2*cosh(x)^2 + 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2*cosh(
x)^7 + 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 + 70*a^2*cosh(x))*e^(2*x)*sin
```

```
h(x)^13 + 60*(4199*a^2*cosh(x)^8 + 7735*a^2*cosh(x)^6 + 4550*a^2*cosh(x)^4
+ 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*cosh(x)^9 +
 9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 + 2730*a^2*cosh(x)^3 + 315*a^2*cos
h(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 + 546975*a^2*cosh(x)^8
+ 600600*a^2*cosh(x)^6 + 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(x)^2 + 2520*
a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 + 60775*a^2*cosh(x)^9
+ 85800*a^2*cosh(x)^7 + 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(x)^3 + 2520*a^
2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 + 36465*a^2*cosh(x
)^10 + 64350*a^2*cosh(x)^8 + 60060*a^2*cosh(x)^6 + 34650*a^2*cosh(x)^4 + 75
60*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*a^2*cosh(x)^13 +
3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 + 8580*a^2*cosh(x)^7 + 6930*a^2*co
sh(x)^5 + 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sinh(x)^7 + 60*(12
92*a^2*cosh(x)^14 + 7735*a^2*cosh(x)^12 + 20020*a^2*cosh(x)^10 + 30030*a^2*
cosh(x)^8 + 32340*a^2*cosh(x)^6 + 17640*a^2*x*cosh(x)^4 - 980*a^2*cosh(x)^2
 - 10*a^2)*e^(2*x)*sinh(x)^6 + 24*(1292*a^2*cosh(x)^15 + 8925*a^2*cosh(x)^1
3 + 27300*a^2*cosh(x)^11 + 50050*a^2*cosh(x)^9 + 69300*a^2*cosh(x)^7 + 5292
0*a^2*x*cosh(x)^5 - 4900*a^2*cosh(x)^3 - 150*a^2*cosh(x))*e^(2*x)*sinh(x)^5
 + 30*(323*a^2*cosh(x)^16 + 2550*a^2*cosh(x)^14 + 9100*a^2*cosh(x)^12 + 200
20*a^2*cosh(x)^10 + 34650*a^2*cosh(x)^8 + 35280*a^2*x*cosh(x)^6 - 4900*a^2*
cosh(x)^4 - 300*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^4 + 120*(19*a^2*cosh
(x)^17 + 170*a^2*cosh(x)^15 + 700*a^2*cosh(x)^13 + 1820*a^2*cosh(x)^11 + 38
50*a^2*cosh(x)^9 + 5040*a^2*x*cosh(x)^7 - 980*a^2*cosh(x)^5 - 100*a^2*cosh(
x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x)^3 + 5*(76*a^2*cosh(x)^18 + 765*a^2*co
sh(x)^16 + 3600*a^2*cosh(x)^14 + 10920*a^2*cosh(x)^12 + 27720*a^2*cosh(x)^1
0 + 45360*a^2*x*cosh(x)^8 - 11760*a^2*cosh(x)^6 - 1800*a^2*cosh(x)^4 - 180*
a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^2 + 10*(4*a^2*cosh(x)^19 + 45*a^2*co
sh(x)^17 + 240*a^2*cosh(x)^15 + 840*a^2*cosh(x)^13 + 2520*a^2*cosh(x)^11 +
5040*a^2*x*cosh(x)^9 - 1680*a^2*cosh(x)^7 - 360*a^2*cosh(x)^5 - 60*a^2*cosh
(x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x) + (2*a^2*cosh(x)^20 + 25*a^2*cosh(x)
^18 + 150*a^2*cosh(x)^16 + 600*a^2*cosh(x)^14 + 2100*a^2*cosh(x)^12 + 5040*
a^2*x*cosh(x)^10 - 2100*a^2*cosh(x)^8 - 600*a^2*cosh(x)^6 - 150*a^2*cosh(x)
^4 - 25*a^2*cosh(x)^2 - 2*a^2)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*
e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^10*e^(4*x) + 2*cosh(x)^10*e^(2
*x) + (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^10 + cosh(x)^10 + 10*(cosh(x)*e^(4*
x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^9 + 45*(cosh(x)^2*e^(4*x) + 2*cos
h(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^8 + 120*(cosh(x)^3*e^(4*x) + 2*cosh(x)^
3*e^(2*x) + cosh(x)^3)*sinh(x)^7 + 210*(cosh(x)^4*e^(4*x) + 2*cosh(x)^4*e^(
2*x) + cosh(x)^4)*sinh(x)^6 + 252*(cosh(x)^5*e^(4*x) + 2*cosh(x)^5*e^(2*x)
+ cosh(x)^5)*sinh(x)^5 + 210*(cosh(x)^6*e^(4*x) + 2*cosh(x)^6*e^(2*x) + cos
h(x)^6)*sinh(x)^4 + 120*(cosh(x)^7*e^(4*x) + 2*cosh(x)^7*e^(2*x) + cosh(x)^
7)*sinh(x)^3 + 45*(cosh(x)^8*e^(4*x) + 2*cosh(x)^8*e^(2*x) + cosh(x)^8)*sin
h(x)^2 + 10*(cosh(x)^9*e^(4*x) + 2*cosh(x)^9*e^(2*x) + cosh(x)^9)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16287, size = 154, normalized size = 1.17

$$\frac{1}{20480}\left(5040\,a^2x + 2\,a^2e^{(10\,x)} + 25\,a^2e^{(8\,x)} + 150\,a^2e^{(6\,x)} + 600\,a^2e^{(4\,x)} + 2100\,a^2e^{(2\,x)} - \left(5754\,a^2e^{(10\,x)} + 2100\,a^2e^{(8\,x)} + 600\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20480*(5040*a^2*x + 2*a^2*e^(10*x) + 25*a^2*e^(8*x) + 150*a^2*e^(6*x) + 6
00*a^2*e^(4*x) + 2100*a^2*e^(2*x) - (5754*a^2*e^(10*x) + 2100*a^2*e^(8*x) +
 600*a^2*e^(6*x) + 150*a^2*e^(4*x) + 25*a^2*e^(2*x) + 2*a^2)*e^(-10*x))*sqr
t(a)
```

3.135 $\int \left(a \cosh^4(x)\right)^{3/2} \, dx$

Optimal. Leaf size=78

$$\frac{1}{6} a \sinh(x) \cosh^3(x) \sqrt{a \cosh^4(x)} + \frac{5}{24} a \sinh(x) \cosh(x) \sqrt{a \cosh^4(x)} + \frac{5}{16} a \tanh(x) \sqrt{a \cosh^4(x)} + \frac{5}{16} a x \mathbf{sech}^2(x) \sqrt{a \, \mathrm{co}}$$

```
[Out] (5*a*x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/16 + (5*a*Cosh[x]*Sqrt[a*Cosh[x]^4]*Sin
h[x])/24 + (a*Cosh[x]^3*Sqrt[a*Cosh[x]^4]*Sinh[x])/6 + (5*a*Sqrt[a*Cosh[x]^
4]*Tanh[x])/16
```

Rubi [A] time = 0.0352139, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{6} a \sinh(x) \cosh^3(x) \sqrt{a \cosh^4(x)} + \frac{5}{24} a \sinh(x) \cosh(x) \sqrt{a \cosh^4(x)} + \frac{5}{16} a \tanh(x) \sqrt{a \cosh^4(x)} + \frac{5}{16} a x \mathbf{sech}^2(x) \sqrt{a \, \mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(3/2),x]
```

```
[Out] (5*a*x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/16 + (5*a*Cosh[x]*Sqrt[a*Cosh[x]^4]*Sin
h[x])/24 + (a*Cosh[x]^3*Sqrt[a*Cosh[x]^4]*Sinh[x])/6 + (5*a*Sqrt[a*Cosh[x]^
4]*Tanh[x])/16
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a\cosh^4(x)\right)^{3/2} dx = \left(a\sqrt{a\cosh^4(x)}\mathbf{sech}^2(x)\right)\int \cosh^6(x)\,dx$$
$$= \frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{6}\left(5a\sqrt{a\cosh^4(x)}\mathbf{sech}^2(x)\right)\int \cosh^4(x)\,dx$$
$$= \frac{5}{24}a\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{8}\left(5a\sqrt{a\cosh^4(x)}\mathbf{sech}^2(\right.$$
$$= \frac{5}{24}a\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{5}{16}a\sqrt{a\cosh^4(x)}\tanh(x)$$
$$= \frac{5}{16}ax\sqrt{a\cosh^4(x)}\mathbf{sech}^2(x) + \frac{5}{24}a\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh($$

Mathematica [A] time = 0.0636691, size = 38, normalized size = 0.49

$$\frac{1}{192}(60x + 45\sinh(2x) + 9\sinh(4x) + \sinh(6x))\mathbf{sech}^6(x)\left(a\cosh^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(3/2),x]
```

```
[Out] ((a*Cosh[x]^4)^(3/2)*Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6*
x]))/192
```

Maple [B] time = 0.111, size = 131, normalized size = 1.7

$$\frac{\sqrt{8}\left(\cosh(2x)+1\right)\sqrt{2}}{384\sinh(2x)}\sqrt{a\left(-1+\cosh(2x)\right)\left(\cosh(2x)+1\right)}\sqrt{a}\left(2\sqrt{a\left(\sinh(2x)\right)^2}\sqrt{a}\left(\sinh(2x)\right)^2 + 9\cosh(2x)\sqrt{a\left(\sin\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(3/2),x)
```

```
[Out] 1/384*8^(1/2)*(cosh(2*x)+1)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*2^(1/2)*
a^(1/2)*(2*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2+9*cosh(2*x)*(a*sinh(2*
x)^2)^(1/2)*a^(1/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+15*ln(a^(1/2)*cosh(2*x
)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/(a*(cosh(2*x)+1)^2)^(1/2)
```

Maxima [A] time = 1.65225, size = 84, normalized size = 1.08

$$\frac{5}{16}\,a^{\frac{3}{2}}x + \frac{1}{384}\left(9\,a^{\frac{3}{2}}e^{(-2\,x)} + 45\,a^{\frac{3}{2}}e^{(-4\,x)} - 45\,a^{\frac{3}{2}}e^{(-8\,x)} - 9\,a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}e^{(-12\,x)} + a^{\frac{3}{2}}\right)e^{(6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^(3/2)*x + 1/384*(9*a^(3/2)*e^(-2*x) + 45*a^(3/2)*e^(-4*x) - 45*a^(3/
2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) - a^(3/2)*e^(-12*x) + a^(3/2))*e^(6*x)
```

Fricas [B] time = 2.1175, size = 2068, normalized size = 26.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*cos
h(x)^2 + 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 + 9*a*cosh(x))*e^(2*x
)*sinh(x)^9 + 45*(11*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8 + 7
2*(11*a*cosh(x)^5 + 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6*(15
4*a*cosh(x)^6 + 315*a*cosh(x)^4 + 210*a*cosh(x)^2 + 20*a*x)*e^(2*x)*sinh(x)
^6 + 36*(22*a*cosh(x)^7 + 63*a*cosh(x)^5 + 70*a*cosh(x)^3 + 20*a*x*cosh(x))
*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 + 42*a*cosh(x)^6 + 70*a*cosh(x)^4 +
 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 + 54*a*cosh(x
)^7 + 126*a*cosh(x)^5 + 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*sinh(x)^3
+ 3*(22*a*cosh(x)^10 + 135*a*cosh(x)^8 + 420*a*cosh(x)^6 + 600*a*x*cosh(x)^
4 - 90*a*cosh(x)^2 - 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11 + 15*a*cosh
(x)^9 + 60*a*cosh(x)^7 + 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 - 3*a*cosh(x))*
e^(2*x)*sinh(x) + (a*cosh(x)^12 + 9*a*cosh(x)^10 + 45*a*cosh(x)^8 + 120*a*x
*cosh(x)^6 - 45*a*cosh(x)^4 - 9*a*cosh(x)^2 - a)*e^(2*x))*sqrt(a*e^(8*x) +
4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^6*e^(4*x) +
```

```
2*cosh(x)^6*e^(2*x) + (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(
cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 + 15*(cosh(x)^2*e^
(4*x) + 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(cosh(x)^3*e^(4*x)
+ 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(x)^4*e^(4*x) + 2*co
sh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^5*e^(4*x) + 2*cosh(x)^5
*e^(2*x) + cosh(x)^5)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**4)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1516, size = 70, normalized size = 0.9

$$-\frac{1}{384}\left(\left(110\,e^{(6\,x)}+45\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)e^{(-6\,x)}-120\,x-e^{(6\,x)}-9\,e^{(4\,x)}-45\,e^{(2\,x)}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6*
x) - 9*e^(4*x) - 45*e^(2*x))*a^(3/2)
```

3.136 $\int \sqrt{a \cosh^4(x)} \, dx$

Optimal. Leaf size=36

$$\frac{1}{2}\tanh(x)\sqrt{a\cosh^4(x)}+\frac{1}{2}x\text{sech}^2(x)\sqrt{a\cosh^4(x)}$$

```
[Out] (x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/2 + (Sqrt[a*Cosh[x]^4]*Tanh[x])/2
```

Rubi [A] time = 0.0159334, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{2}\tanh(x)\sqrt{a\cosh^4(x)}+\frac{1}{2}x\text{sech}^2(x)\sqrt{a\cosh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]^4],x]
```

```
[Out] (x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/2 + (Sqrt[a*Cosh[x]^4]*Tanh[x])/2
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\cosh^4(x)}\,dx &= \left(\sqrt{a\cosh^4(x)}\,\mathrm{sech}^2(x)\right)\int \cosh^2(x)\,dx\\ &= \frac{1}{2}\sqrt{a\cosh^4(x)}\tanh(x) + \frac{1}{2}\left(\sqrt{a\cosh^4(x)}\,\mathrm{sech}^2(x)\right)\int 1\,dx\\ &= \frac{1}{2}x\sqrt{a\cosh^4(x)}\,\mathrm{sech}^2(x) + \frac{1}{2}\sqrt{a\cosh^4(x)}\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0135221, size = 25, normalized size = 0.69

$$\frac{1}{2}\mathrm{sech}^2(x)\sqrt{a\cosh^4(x)}(x+\sinh(x)\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]^4],x]
```

```
[Out] (Sqrt[a*Cosh[x]^4]*Sech[x]^2*(x + Cosh[x]*Sinh[x]))/2
```

Maple [B] time = 0.107, size = 89, normalized size = 2.5

$$\frac{\sqrt{8}\,(\cosh(2x)+1)\sqrt{2}}{16\,\sinh(2x)}\sqrt{a\,(-1+\cosh(2x))\,(\cosh(2x)+1)}\left(\sqrt{a\,(\sinh(2x))^2}\sqrt{a}+\ln\left(\sqrt{a}\cosh(2x)+\sqrt{a\,(\sinh(2x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(1/2),x)
```

```
[Out] 1/16*8^(1/2)*(cosh(2*x)+1)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*2^(1/2)*(
(a*sinh(2*x)^2)^(1/2)*a^(1/2)+ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a
)/a^(1/2)/sinh(2*x)/(a*(cosh(2*x)+1)^2)^(1/2)
```

Maxima [A] time = 1.63269, size = 36, normalized size = 1.

$$-\frac{1}{8}\left(\sqrt{a}e^{(-4x)}-\sqrt{a}\right)e^{(2x)}+\frac{1}{2}\sqrt{a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) + 1/2*sqrt(a)*x
```

Fricas [B] time = 1.97317, size = 551, normalized size = 15.31

$$\frac{\left(4\cosh\left(x\right)e^{\left(2x\right)}\sinh\left(x\right)^{3}+e^{\left(2x\right)}\sinh\left(x\right)^{4}+2\left(3\cosh\left(x\right)^{2}+2x\right)e^{\left(2x\right)}\sinh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}+2x\cosh\left(x\right)\right)e^{\left(2x\right)}\sinh}{8\left(\cosh\left(x\right)^{2}e^{\left(4x\right)}+2\cosh\left(x\right)^{2}e^{\left(2x\right)}+\left(e^{\left(4x\right)}+2e^{\left(2x\right)}+1\right)\sinh\left(x\right)^{2}+\cosh\left(x\right)^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 + 2*x
)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh(x
)^4 + 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x
) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) + 2*cosh(x)^2*e^(2*x) + (e
^(4*x) + 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) + 2*cosh
(x)*e^(2*x) + cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16474, size = 38, normalized size = 1.06

$$-\frac{1}{8}\left(\left(2e^{\left(2x\right)}+1\right)e^{\left(-2x\right)}-4x-e^{\left(2x\right)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))*sqrt(a)
```

3.137 $\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh(x)\cosh(x)}{\sqrt{a\cosh^4(x)}}$$

```
[Out] (Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

Rubi [A] time = 0.0166324, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 3767, 8}

$$\frac{\sinh(x)\cosh(x)}{\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]^4],x]
```

```
[Out] (Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \frac{\cosh^2(x)\int \mathrm{sech}^2(x)\,dx}{\sqrt{a\cosh^4(x)}}$$
$$= \frac{\left(i\cosh^2(x)\right)\mathrm{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)}{\sqrt{a\cosh^4(x)}}$$
$$= \frac{\cosh(x)\sinh(x)}{\sqrt{a\cosh^4(x)}}$$

Mathematica [A] time = 0.0048388, size = 15, normalized size = 1.

$$\frac{\sinh(x)\cosh(x)}{\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]^4],x]
```

```
[Out] (Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

Maple [B] time = 0.085, size = 56, normalized size = 3.7

$$\frac{\sqrt{8}\sqrt{2}}{4\,a\sinh(2\,x)}\sqrt{a\,(-1+\cosh(2\,x))\,(\cosh(2\,x)+1)}\sqrt{a\,(\sinh(2\,x))^2}\frac{1}{\sqrt{a\,(\cosh(2\,x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(1/2),x)
```

```
[Out] 1/4*8^(1/2)*2^(1/2)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/a*(a*sinh(2*x)^2
)^(1/2)/sinh(2*x)/(a*(cosh(2*x)+1)^2)^(1/2)
```

Maxima [A] time = 1.72424, size = 22, normalized size = 1.47

$$\frac{2}{\sqrt{a}e^{(-2\,x)} + \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/(sqrt(a)*e^(-2*x) + sqrt(a))
```

Fricas [B] time = 1.88053, size = 338, normalized size = 22.53

$$-\frac{2\,\sqrt{ae^{(8\,x)} + 4\,ae^{(6\,x)} + 6\,ae^{(4\,x)} + 4\,ae^{(2\,x)} + a}}{a\cosh\left(x\right)^{2} + \left(ae^{(4\,x)} + 2\,ae^{(2\,x)} + a\right)\sinh\left(x\right)^{2} + \left(a\cosh\left(x\right)^{2} + a\right)e^{(4\,x)} + 2\,\left(a\cosh\left(x\right)^{2} + a\right)e^{(2\,x)} + 2\,\left(a\cosh\left(x\right)e^{(4\,x)} + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)/(a*cosh(x)
^2 + (a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 + a)*e^(4*x) +
2*(a*cosh(x)^2 + a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x) +
a*cosh(x))*sinh(x) + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16102, size = 18, normalized size = 1.2

$$-\frac{2}{\sqrt{a}\left(e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(a)*(e^(2*x) + 1))
```

3.138 $\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\sinh(x)\cosh(x)}{a\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}-\frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}}$$

```
[Out] (Cosh[x]*Sinh[x])/(a*Sqrt[a*Cosh[x]^4]) - (2*Sinh[x]^2*Tanh[x])/(3*a*Sqrt[a
*Cosh[x]^4]) + (Sinh[x]^2*Tanh[x]^3)/(5*a*Sqrt[a*Cosh[x]^4])
```

Rubi [A] time = 0.025165, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3767}

$$\frac{\sinh(x)\cosh(x)}{a\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}-\frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(-3/2),x]
```

```
[Out] (Cosh[x]*Sinh[x])/(a*Sqrt[a*Cosh[x]^4]) - (2*Sinh[x]^2*Tanh[x])/(3*a*Sqrt[a
*Cosh[x]^4]) + (Sinh[x]^2*Tanh[x]^3)/(5*a*Sqrt[a*Cosh[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \frac{\cosh^2(x)\int \mathbf{sech}^6(x)\,dx}{a\sqrt{a\cosh^4(x)}}$$
$$= \frac{\left(i\cosh^2(x)\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(x)\right)}{a\sqrt{a\cosh^4(x)}}$$
$$= \frac{\cosh(x)\sinh(x)}{a\sqrt{a\cosh^4(x)}} - \frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}} + \frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}$$

Mathematica [A] time = 0.0267642, size = 30, normalized size = 0.45

$$\frac{\sinh(x)\cosh(x)(6\cosh(2x)+\cosh(4x)+8)}{15\left(a\cosh^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(-3/2),x]
```

```
[Out] (Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*Sinh[x])/(15*(a*Cosh[x]^4)^(3/2))
```

Maple [A] time = 0.096, size = 80, normalized size = 1.2

$$\frac{\sqrt{8}\sqrt{2}\left(2\,(\cosh(2\,x))^2+6\,\cosh(2\,x)+7\right)}{15\,a^2\,(\cosh(2\,x)+1)^2\sinh(2\,x)}\sqrt{a\,(\sinh(2\,x))^2}\sqrt{a\,(-1+\cosh(2\,x))\,(\cosh(2\,x)+1)}\frac{1}{\sqrt{a\,(\cosh(2\,x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(3/2),x)
```

```
[Out] 1/15*8^(1/2)/a^2*2^(1/2)*(2*cosh(2*x)^2+6*cosh(2*x)+7)*(a*sinh(2*x)^2)^(1/2
)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/(cosh(2*x)+1)^2/sinh(2*x)/(a*(cosh
(2*x)+1)^2)^(1/2)
```

Maxima [B] time = 1.58217, size = 223, normalized size = 3.33

$$\frac{16\,e^{(-2\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}+5\,a^{\frac{3}{2}}e^{(-8\,x)}+a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}\right)}+\frac{32\,e^{(-4\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}+5\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*
x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 32/3*e^(-4*x)/(5*a
^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*e^(
-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) + 10*a^(3/
2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x)
+ a^(3/2))
```

Fricas [B] time = 1.97198, size = 3163, normalized size = 47.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x)
^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (1
0*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*
e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) + 2*a^2*
e^(2*x) + a^2)*sinh(x)^10 + 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x) + 2*a
^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 + a^2 + (9
*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(9*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^
8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*cosh(x)^3
 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x
)^7 + 10*(21*a^2*cosh(x)^4 + 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cosh(x)^4 + 1
4*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(21*a^2*cosh(x)^4 + 14*a^2*cosh(x)^2 + a
^2)*e^(2*x))*sinh(x)^6 + 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5 + 70*a^2*co
sh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^3 + 15*a^2*co
sh(x))*e^(4*x) + 2*(63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^3 + 15*a^2*cosh(x))*e
^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*cosh(x
)^2 + a^2 + (21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 + a^2)*
e^(4*x) + 2*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 + a^2)*
```

```
e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^7 + 7*a^2*cosh(x)^
5 + 5*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*cosh(x)^7 + 7*a^2*cosh(x)^5 + 5*
a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh(x)^7 + 7*a^2*cosh(x)^5
 + 5*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 5*(9*a^2*cosh(x)^8 +
 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 + 12*a^2*cosh(x)^2 + a^2 + (9*a^2*cosh
(x)^8 + 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 + 12*a^2*cosh(x)^2 + a^2)*e^(4*
x) + 2*(9*a^2*cosh(x)^8 + 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 + 12*a^2*cosh
(x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*cosh(x)^10 + 5*a^2*cosh(x)^8 +
 10*a^2*cosh(x)^6 + 10*a^2*cosh(x)^4 + 5*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(
a^2*cosh(x)^10 + 5*a^2*cosh(x)^8 + 10*a^2*cosh(x)^6 + 10*a^2*cosh(x)^4 + 5*
a^2*cosh(x)^2 + a^2)*e^(2*x) + 10*(a^2*cosh(x)^9 + 4*a^2*cosh(x)^7 + 6*a^2*
cosh(x)^5 + 4*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^9 + 4*a^2*cosh(x)^
7 + 6*a^2*cosh(x)^5 + 4*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(a^2*cosh(
x)^9 + 4*a^2*cosh(x)^7 + 6*a^2*cosh(x)^5 + 4*a^2*cosh(x)^3 + a^2*cosh(x))*e
^(2*x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**4)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18343, size = 47, normalized size = 0.7

$$-\frac{16\left(10\sqrt{a}e^{(4x)}+5\sqrt{a}e^{(2x)}+\sqrt{a}\right)}{15\,a^2\left(e^{(2x)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*(10*sqrt(a)*e^(4*x) + 5*sqrt(a)*e^(2*x) + sqrt(a))/(a^2*(e^(2*x) + 1
)^5)
```

3.139 $\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^7(x)}{9a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}}+\frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}}$$

```
[Out] (Cosh[x]*Sinh[x])/(a^2*Sqrt[a*Cosh[x]^4]) - (4*Sinh[x]^2*Tanh[x])/(3*a^2*Sq
rt[a*Cosh[x]^4]) + (6*Sinh[x]^2*Tanh[x]^3)/(5*a^2*Sqrt[a*Cosh[x]^4]) - (4*S
inh[x]^2*Tanh[x]^5)/(7*a^2*Sqrt[a*Cosh[x]^4]) + (Sinh[x]^2*Tanh[x]^7)/(9*a^
2*Sqrt[a*Cosh[x]^4])
```

Rubi [A] time = 0.0351632, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3767}

$$\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^7(x)}{9a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}}+\frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sinh[x])/(a^2*Sqrt[a*Cosh[x]^4]) - (4*Sinh[x]^2*Tanh[x])/(3*a^2*Sq
rt[a*Cosh[x]^4]) + (6*Sinh[x]^2*Tanh[x]^3)/(5*a^2*Sqrt[a*Cosh[x]^4]) - (4*S
inh[x]^2*Tanh[x]^5)/(7*a^2*Sqrt[a*Cosh[x]^4]) + (Sinh[x]^2*Tanh[x]^7)/(9*a^
2*Sqrt[a*Cosh[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx &= \frac{\cosh^2(x)\int \operatorname{sech}^{10}(x)\,dx}{a^2\sqrt{a\cosh^4(x)}}\\ &= \frac{\left(i\cosh^2(x)\right)\operatorname{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -i\tanh(x)\right)}{a^2\sqrt{a\cosh^4(x)}}\\ &= \frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}} + \frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}} + \frac{\sinh^2(x)}{9a^2\sqrt{a\cosh^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.0481742, size = 47, normalized size = 0.4

$$\frac{(130\cosh(2x)+46\cosh(4x)+10\cosh(6x)+\cosh(8x)+128)\tanh(x)\operatorname{sech}^6(x)}{315a^2\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(-5/2),x]
```

```
[Out] ((128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*Sech[x]^6*
Tanh[x])/(315*a^2*Sqrt[a*Cosh[x]^4])
```

Maple [A] time = 0.095, size = 96, normalized size = 0.8

$$\frac{4\sqrt{8}\sqrt{2}\left(8\left(\cosh(2x)\right)^4+40\left(\cosh(2x)\right)^3+84\left(\cosh(2x)\right)^2+100\cosh(2x)+83\right)}{315a^3\left(\cosh(2x)+1\right)^4\sinh(2x)}\sqrt{a\left(\sinh(2x)\right)^2}\sqrt{a\left(-1+\cosh(2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(5/2),x)
```

```
[Out] 4/315*8^(1/2)*2^(1/2)/a^3*(8*cosh(2*x)^4+40*cosh(2*x)^3+84*cosh(2*x)^2+100*
cosh(2*x)+83)*(a*sinh(2*x)^2)^(1/2)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/
(cosh(2*x)+1)^4/sinh(2*x)/(a*(cosh(2*x)+1)^2)^(1/2)
```

Maxima [B] time = 1.65212, size = 617, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(-
6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-12*x)
+ 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(5/2))
 + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*
e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-12
*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(5
/2)) + 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5
/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^
(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) +
a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^
(5/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*
e^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x)
+ a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)
*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-1
2*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(
5/2))
```

Fricas [B] time = 2.49368, size = 9072, normalized size = 77.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(1008*cosh(x)*e^(2*x)*sinh(x)^7 + 126*e^(2*x)*sinh(x)^8 + 84*(42*c
osh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 504*(14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh
(x)^5 + 36*(245*cosh(x)^4 + 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(147*c
```

osh(x)^5 + 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(392*cosh(x)^6 +
140*cosh(x)^4 + 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(56*cosh(x)^7 + 2
8*cosh(x)^5 + 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (126*cosh(x)^8 + 84*
cosh(x)^6 + 36*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e
^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(a^3*cosh(x)^18 + 9*a^3*co
sh(x)^16 + (a^3*e^(4*x) + 2*a^3*e^(2*x) + a^3)*sinh(x)^18 + 18*(a^3*cosh(x)
*e^(4*x) + 2*a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^17 + 36*a^3*cosh(x)
^14 + 9*(17*a^3*cosh(x)^2 + a^3 + (17*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(17*
a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^16 + 48*(17*a^3*cosh(x)^3 + 3*a^3*cos
h(x) + (17*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x)^3 + 3
*a^3*cosh(x))*e^(2*x))*sinh(x)^15 + 84*a^3*cosh(x)^12 + 36*(85*a^3*cosh(x)^
4 + 30*a^3*cosh(x)^2 + a^3 + (85*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + a^3)*e^
(4*x) + 2*(85*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^14 +
504*(17*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^5
+ 10*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x)^5 + 10*a^3*c
osh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^13 + 126*a^3*cosh(x)^10 + 84*(221*
a^3*cosh(x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 + a^3 + (221*a^3*cosh(
x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(221*a^3*cos
h(x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^12 +
144*(221*a^3*cosh(x)^7 + 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 + 7*a^3*cosh(
x) + (221*a^3*cosh(x)^7 + 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 + 7*a^3*cosh
(x))*e^(4*x) + 2*(221*a^3*cosh(x)^7 + 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3
+ 7*a^3*cosh(x))*e^(2*x))*sinh(x)^11 + 126*a^3*cosh(x)^8 + 18*(2431*a^3*cos
h(x)^8 + 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 + 308*a^3*cosh(x)^2 + 7*a^
3 + (2431*a^3*cosh(x)^8 + 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 + 308*a^3
*cosh(x)^2 + 7*a^3)*e^(4*x) + 2*(2431*a^3*cosh(x)^8 + 4004*a^3*cosh(x)^6 +
2002*a^3*cosh(x)^4 + 308*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^10 + 4*(12
155*a^3*cosh(x)^9 + 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 + 4620*a^3*co
sh(x)^3 + 315*a^3*cosh(x) + (12155*a^3*cosh(x)^9 + 25740*a^3*cosh(x)^7 + 18
018*a^3*cosh(x)^5 + 4620*a^3*cosh(x)^3 + 315*a^3*cosh(x))*e^(4*x) + 2*(1215
5*a^3*cosh(x)^9 + 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 + 4620*a^3*cosh
(x)^3 + 315*a^3*cosh(x))*e^(2*x))*sinh(x)^9 + 84*a^3*cosh(x)^6 + 18*(2431*a
^3*cosh(x)^10 + 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 + 2310*a^3*cosh(x)^
4 + 315*a^3*cosh(x)^2 + 7*a^3 + (2431*a^3*cosh(x)^10 + 6435*a^3*cosh(x)^8 +
6006*a^3*cosh(x)^6 + 2310*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 + 7*a^3)*e^(4*
x) + 2*(2431*a^3*cosh(x)^10 + 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 + 231
0*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^8 + 144*(221*
a^3*cosh(x)^11 + 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^7 + 462*a^3*cosh(x)^5
+ 105*a^3*cosh(x)^3 + 7*a^3*cosh(x) + (221*a^3*cosh(x)^11 + 715*a^3*cosh(x)
^9 + 858*a^3*cosh(x)^7 + 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 + 7*a^3*cosh
(x))*e^(4*x) + 2*(221*a^3*cosh(x)^11 + 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^
7 + 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 + 7*a^3*cosh(x))*e^(2*x))*sinh(x)
^7 + 36*a^3*cosh(x)^4 + 84*(221*a^3*cosh(x)^12 + 858*a^3*cosh(x)^10 + 1287*
a^3*cosh(x)^8 + 924*a^3*cosh(x)^6 + 315*a^3*cosh(x)^4 + 42*a^3*cosh(x)^2 +
a^3 + (221*a^3*cosh(x)^12 + 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 + 924*a

```
^3*cosh(x)^6 + 315*a^3*cosh(x)^4 + 42*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(221
*a^3*cosh(x)^12 + 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 + 924*a^3*cosh(x)
^6 + 315*a^3*cosh(x)^4 + 42*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^6 + 504*(
17*a^3*cosh(x)^13 + 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 + 132*a^3*cosh(x)
^7 + 63*a^3*cosh(x)^5 + 14*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^13
 + 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 + 132*a^3*cosh(x)^7 + 63*a^3*cosh(
x)^5 + 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x)^13 + 78*
a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 + 132*a^3*cosh(x)^7 + 63*a^3*cosh(x)^5 +
 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 9*a^3*cosh(x)^2 + 36*
(85*a^3*cosh(x)^14 + 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 + 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 + 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 + a^3 +
(85*a^3*cosh(x)^14 + 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 + 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 + 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 + a^3)*e
^(4*x) + 2*(85*a^3*cosh(x)^14 + 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 +
1155*a^3*cosh(x)^8 + 735*a^3*cosh(x)^6 + 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)
^2 + a^3)*e^(2*x))*sinh(x)^4 + 48*(17*a^3*cosh(x)^15 + 105*a^3*cosh(x)^13 +
 273*a^3*cosh(x)^11 + 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 + 147*a^3*cosh(
x)^5 + 35*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (17*a^3*cosh(x)^15 + 105*a^3*cosh
(x)^13 + 273*a^3*cosh(x)^11 + 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 + 147*a
^3*cosh(x)^5 + 35*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x
)^15 + 105*a^3*cosh(x)^13 + 273*a^3*cosh(x)^11 + 385*a^3*cosh(x)^9 + 315*a^
3*cosh(x)^7 + 147*a^3*cosh(x)^5 + 35*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x)
)*sinh(x)^3 + a^3 + 9*(17*a^3*cosh(x)^16 + 120*a^3*cosh(x)^14 + 364*a^3*cos
h(x)^12 + 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 + 392*a^3*cosh(x)^6 + 140*
a^3*cosh(x)^4 + 24*a^3*cosh(x)^2 + a^3 + (17*a^3*cosh(x)^16 + 120*a^3*cosh(
x)^14 + 364*a^3*cosh(x)^12 + 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 + 392*a
^3*cosh(x)^6 + 140*a^3*cosh(x)^4 + 24*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(17*
a^3*cosh(x)^16 + 120*a^3*cosh(x)^14 + 364*a^3*cosh(x)^12 + 616*a^3*cosh(x)^
10 + 630*a^3*cosh(x)^8 + 392*a^3*cosh(x)^6 + 140*a^3*cosh(x)^4 + 24*a^3*cos
h(x)^2 + a^3)*e^(2*x))*sinh(x)^2 + (a^3*cosh(x)^18 + 9*a^3*cosh(x)^16 + 36*
a^3*cosh(x)^14 + 84*a^3*cosh(x)^12 + 126*a^3*cosh(x)^10 + 126*a^3*cosh(x)^8
 + 84*a^3*cosh(x)^6 + 36*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2
*(a^3*cosh(x)^18 + 9*a^3*cosh(x)^16 + 36*a^3*cosh(x)^14 + 84*a^3*cosh(x)^12
 + 126*a^3*cosh(x)^10 + 126*a^3*cosh(x)^8 + 84*a^3*cosh(x)^6 + 36*a^3*cosh(
x)^4 + 9*a^3*cosh(x)^2 + a^3)*e^(2*x) + 18*(a^3*cosh(x)^17 + 8*a^3*cosh(x)^
15 + 28*a^3*cosh(x)^13 + 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)^9 + 56*a^3*cosh
(x)^7 + 28*a^3*cosh(x)^5 + 8*a^3*cosh(x)^3 + a^3*cosh(x) + (a^3*cosh(x)^17
+ 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 + 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)
^9 + 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 + 8*a^3*cosh(x)^3 + a^3*cosh(x))*e
^(4*x) + 2*(a^3*cosh(x)^17 + 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 + 56*a^3*
cosh(x)^11 + 70*a^3*cosh(x)^9 + 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 + 8*a^3
*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15315, size = 72, normalized size = 0.62

$$-\frac{256 \left(126 \sqrt{a} e^{(8 x)} + 84 \sqrt{a} e^{(6 x)} + 36 \sqrt{a} e^{(4 x)} + 9 \sqrt{a} e^{(2 x)} + \sqrt{a}\right)}{315 a^{3} \left(e^{(2 x)} + 1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*(126*sqrt(a)*e^(8*x) + 84*sqrt(a)*e^(6*x) + 36*sqrt(a)*e^(4*x) + 9
*sqrt(a)*e^(2*x) + sqrt(a))/(a^3*(e^(2*x) + 1)^9)
```

3.140 $\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx$

Optimal. Leaf size=8

$$-\frac{1}{\cosh(x)+1}$$

```
[Out] -(1 + Cosh[x])^(-1)
```

Rubi [A] time = 0.0209781, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2667, 32}

$$-\frac{1}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Cosh[x])^2,x]
```

```
[Out] -(1 + Cosh[x])^(-1)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = \text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, \cosh(x)\right)$$
$$= -\frac{1}{1+\cosh(x)}$$

Mathematica [A] time = 0.0105182, size = 12, normalized size = 1.5

$$-\frac{1}{2}\text{sech}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Cosh[x])^2,x]
```

```
[Out] -Sech[x/2]^2/2
```

Maple [A] time = 0.004, size = 9, normalized size = 1.1

$$-(1+\cosh(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+cosh(x))^2,x)
```

```
[Out] -1/(1+cosh(x))
```

Maxima [A] time = 1.06784, size = 11, normalized size = 1.38

$$-\frac{1}{\cosh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(cosh(x) + 1)
```

Fricas [B] time = 2.10478, size = 122, normalized size = 15.25

$$-\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2
*cosh(x) + 1)
```

Sympy [A] time = 0.431506, size = 7, normalized size = 0.88

$$-\frac{1}{\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))**2,x)
```

```
[Out] -1/(cosh(x) + 1)
```

Giac [A] time = 1.15624, size = 14, normalized size = 1.75

$$-\frac{2\,e^{x}}{\left(e^{x}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*e^x/(e^x + 1)^2
```

3.141 $\int \frac{\sinh(x)}{(1-\cosh(x))^2} dx$

Optimal. Leaf size=8

$$\frac{1}{1-\cosh(x)}$$

```
[Out] (1 - Cosh[x])^(-1)
```

Rubi [A] time = 0.0216567, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 32}

$$\frac{1}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 - Cosh[x])^2,x]
```

```
[Out] (1 - Cosh[x])^(-1)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\operatorname{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, -\cosh(x)\right)$$
$$= \frac{1}{1-\cosh(x)}$$

Mathematica [A] time = 0.0100208, size = 12, normalized size = 1.5

$$-\frac{1}{2}\operatorname{csch}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 - Cosh[x])^2,x]
```

```
[Out] -Csch[x/2]^2/2
```

Maple [A] time = 0.005, size = 9, normalized size = 1.1

$$(1-\cosh(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1-cosh(x))^2,x)
```

```
[Out] 1/(1-cosh(x))
```

Maxima [A] time = 1.03423, size = 11, normalized size = 1.38

$$-\frac{1}{\cosh(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(cosh(x) - 1)
```

Fricas [B] time = 2.06581, size = 122, normalized size = 15.25

$$-\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-2\,\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2
*cosh(x) + 1)
```

Sympy [A] time = 0.439663, size = 7, normalized size = 0.88

$$-\frac{1}{\cosh\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))**2,x)
```

```
[Out] -1/(cosh(x) - 1)
```

Giac [A] time = 1.1428, size = 14, normalized size = 1.75

$$-\frac{2\,e^{x}}{\left(e^{x}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*e^x/(e^x - 1)^2
```

3.142 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx$

Optimal. Leaf size=12

$$x-\frac{2\sinh(x)}{\cosh(x)+1}$$

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rubi [A] time = 0.0314202, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2680, 8}

$$x-\frac{2\sinh(x)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 + Cosh[x])^2,x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\sinh(x)}{1+\cosh(x)} + \int 1\,dx$$
$$= x - \frac{2\sinh(x)}{1+\cosh(x)}$$

Mathematica [A] time = 0.0057045, size = 18, normalized size = 1.5

$$2\tanh^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - 2\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 + Cosh[x])^2,x]
```

```
[Out] 2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2]
```

Maple [A] time = 0.014, size = 24, normalized size = 2.

$$-2\,\tanh(x/2) - \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1+cosh(x))^2,x)
```

```
[Out] -2*tanh(1/2*x)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)
```

Maxima [A] time = 1.08282, size = 16, normalized size = 1.33

$$x - \frac{4}{e^{(-x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] x - 4/(e^(-x) + 1)
```

Fricas [A] time = 2.19453, size = 77, normalized size = 6.42

$$\frac{x\cosh(x) + x\sinh(x) + x + 4}{\cosh(x) + \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

Sympy [A] time = 0.733914, size = 7, normalized size = 0.58

$$x - 2\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+cosh(x))**2,x)
```

```
[Out] x - 2*tanh(x/2)
```

Giac [A] time = 1.22489, size = 14, normalized size = 1.17

$$x + \frac{4}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] x + 4/(e^x + 1)
```

3.143 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} dx$

Optimal. Leaf size=14

$$x + \frac{2\sinh(x)}{1 - \cosh(x)}$$

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rubi [A] time = 0.0311489, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2680, 8}

$$x + \frac{2\sinh(x)}{1 - \cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 - Cosh[x])^2,x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx = \frac{2\sinh(x)}{1-\cosh(x)} + \int 1\,dx$$
$$= x + \frac{2\sinh(x)}{1-\cosh(x)}$$

Mathematica [C] time = 0.0099743, size = 24, normalized size = 1.71

$$-2\coth\left(\frac{x}{2}\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 - Cosh[x])^2,x]
```

```
[Out] -2*Coth[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x/2]^2]
```

Maple [A] time = 0.016, size = 26, normalized size = 1.9

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - 2\,(\tanh(x/2))^{-1} - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1-cosh(x))^2,x)
```

```
[Out] ln(tanh(1/2*x)+1)-2/tanh(1/2*x)-ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.04929, size = 16, normalized size = 1.14

$$x + \frac{4}{e^{(-x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] x + 4/(e^(-x) - 1)
```

Fricas [A] time = 2.15343, size = 77, normalized size = 5.5

$$\frac{x\cosh(x) + x\sinh(x) - x - 4}{\cosh(x) + \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

Sympy [A] time = 1.42619, size = 7, normalized size = 0.5

$$x - \frac{2}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1-cosh(x))**2,x)
```

```
[Out] x - 2/tanh(x/2)
```

Giac [A] time = 1.13597, size = 14, normalized size = 1.

$$x - \frac{4}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] x - 4/(e^x - 1)
```

3.144 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^2} \, dx$

Optimal. Leaf size=10

$$\cosh(x) - 2\log(\cosh(x) + 1)$$

```
[Out] Cosh[x] - 2*Log[1 + Cosh[x]]
```

Rubi [A] time = 0.0383399, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 43}

$$\cosh(x) - 2\log(\cosh(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 + Cosh[x])^2,x]
```

```
[Out] Cosh[x] - 2*Log[1 + Cosh[x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = -\operatorname{Subst}\left(\int \frac{1-x}{1+x}\,dx, x, \cosh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-1+\frac{2}{1+x}\right)dx, x, \cosh(x)\right)$$
$$= \cosh(x) - 2\log(1+\cosh(x))$$

Mathematica [A] time = 0.0179428, size = 13, normalized size = 1.3

$$\cosh(x) - 4\log\left(\cosh\left(\frac{x}{2}\right)\right) - 1$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Cosh[x])^2,x]
```

```
[Out] -1 + Cosh[x] - 4*Log[Cosh[x/2]]
```

Maple [A] time = 0.012, size = 11, normalized size = 1.1

$$\cosh(x) - 2\ln(1+\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1+cosh(x))^2,x)
```

```
[Out] cosh(x)-2*ln(1+cosh(x))
```

Maxima [B] time = 1.13026, size = 31, normalized size = 3.1

$$-2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4\,\log\left(e^{(-x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^(-x) + 1)
```

Fricas [B] time = 2.2207, size = 197, normalized size = 19.7

$$\frac{4\,x\cosh\left(x\right)+\cosh\left(x\right)^{2}-8\,\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+2\,\left(2\,x+\cosh\left(x\right)\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+}{2\,\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*x*cosh(x) + cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
+ 1) + 2*(2*x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.658029, size = 70, normalized size = 7.

$$-\frac{2\log\left(\cosh\left(x\right)+1\right)\cosh\left(x\right)}{\cosh\left(x\right)+1}-\frac{2\log\left(\cosh\left(x\right)+1\right)}{\cosh\left(x\right)+1}+\frac{\sinh^{2}\left(x\right)\cosh\left(x\right)}{\cosh\left(x\right)+1}-\frac{\cosh^{3}\left(x\right)}{\cosh\left(x\right)+1}+\frac{\cosh^{2}\left(x\right)}{\cosh\left(x\right)+1}-\frac{2}{\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+cosh(x))**2,x)
```

```
[Out] -2*log(cosh(x) + 1)*cosh(x)/(cosh(x) + 1) - 2*log(cosh(x) + 1)/(cosh(x) + 1
) + sinh(x)**2*cosh(x)/(cosh(x) + 1) - cosh(x)**3/(cosh(x) + 1) + cosh(x)**
2/(cosh(x) + 1) - 2/(cosh(x) + 1)
```

Giac [A] time = 1.16163, size = 28, normalized size = 2.8

$$2\,x+\frac{1}{2}\,e^{\left(-x\right)}+\frac{1}{2}\,e^{x}-4\,\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] 2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^x + 1)
```

3.145 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} \, dx$

Optimal. Leaf size=12

$$\cosh(x) + 2\log(1 - \cosh(x))$$

```
[Out] Cosh[x] + 2*Log[1 - Cosh[x]]
```

Rubi [A] time = 0.0372116, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\cosh(x) + 2\log(1 - \cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 - Cosh[x])^2,x]
```

```
[Out] Cosh[x] + 2*Log[1 - Cosh[x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} dx = \text{Subst}\left(\int \frac{1-x}{1+x} dx, x, -\cosh(x)\right)$$
$$= \text{Subst}\left(\int \left(-1+\frac{2}{1+x}\right) dx, x, -\cosh(x)\right)$$
$$= \cosh(x) + 2\log(1-\cosh(x))$$

Mathematica [A] time = 0.0169673, size = 13, normalized size = 1.08

$$\cosh(x) + 4\log\left(\sinh\left(\frac{x}{2}\right)\right) - 1$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 - Cosh[x])^2,x]
```

```
[Out] -1 + Cosh[x] + 4*Log[Sinh[x/2]]
```

Maple [A] time = 0.015, size = 11, normalized size = 0.9

$$\cosh(x) + 2\ln(-1+\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1-cosh(x))^2,x)
```

```
[Out] cosh(x)+2*ln(-1+cosh(x))
```

Maxima [A] time = 1.02934, size = 31, normalized size = 2.58

$$2x + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x} + 4\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(e^(-x) - 1)
```

Fricas [B] time = 2.0958, size = 198, normalized size = 16.5

$$-\frac{4\,x\cosh\left(x\right)-\cosh\left(x\right)^{2}-8\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+2\left(2\,x-\cosh\left(x\right)\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}-}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*x*cosh(x) - cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
 - 1) + 2*(2*x - cosh(x))*sinh(x) - sinh(x)^2 - 1)/(cosh(x) + sinh(x))
```

Sympy [B] time = 0.643182, size = 70, normalized size = 5.83

$$\frac{2\log\left(\cosh\left(x\right)-1\right)\cosh\left(x\right)}{\cosh\left(x\right)-1}-\frac{2\log\left(\cosh\left(x\right)-1\right)}{\cosh\left(x\right)-1}-\frac{\sinh^{2}\left(x\right)\cosh\left(x\right)}{\cosh\left(x\right)-1}+\frac{\cosh^{3}\left(x\right)}{\cosh\left(x\right)-1}+\frac{\cosh^{2}\left(x\right)}{\cosh\left(x\right)-1}-\frac{2}{\cosh\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1-cosh(x))**2,x)
```

```
[Out] 2*log(cosh(x) - 1)*cosh(x)/(cosh(x) - 1) - 2*log(cosh(x) - 1)/(cosh(x) - 1)
 - sinh(x)**2*cosh(x)/(cosh(x) - 1) + cosh(x)**3/(cosh(x) - 1) + cosh(x)**2
/(cosh(x) - 1) - 2/(cosh(x) - 1)
```

Giac [A] time = 1.12046, size = 30, normalized size = 2.5

$$-2\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}+4\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(abs(e^x - 1))
```

3.146 $\int \frac{\sinh(x)}{(1+\cosh(x))^3} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{2(\cosh(x)+1)^2}$$

```
[Out] -1/(2*(1 + Cosh[x])^2)
```

Rubi [A] time = 0.0205692, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2667, 32}

$$-\frac{1}{2(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Cosh[x])^3,x]
```

```
[Out] -1/(2*(1 + Cosh[x])^2)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = \text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, \cosh(x)\right)$$
$$= -\frac{1}{2(1+\cosh(x))^2}$$

Mathematica [A] time = 0.0088983, size = 12, normalized size = 1.2

$$-\frac{1}{8}\text{sech}^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Cosh[x])^3,x]
```

```
[Out] -Sech[x/2]^4/8
```

Maple [A] time = 0.004, size = 9, normalized size = 0.9

$$-\frac{1}{2\,(1+\cosh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+cosh(x))^3,x)
```

```
[Out] -1/2/(1+cosh(x))^2
```

Maxima [A] time = 1.06343, size = 11, normalized size = 1.1

$$-\frac{1}{2\,(\cosh(x)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2/(cosh(x) + 1)^2
```

Fricas [B] time = 2.04528, size = 197, normalized size = 19.7

$$-\frac{2\,(\cosh(x)+\sinh(x))}{\cosh(x)^3+(3\,\cosh(x)+4)\sinh(x)^2+\sinh(x)^3+4\,\cosh(x)^2+\left(3\,\cosh(x)^2+8\,\cosh(x)+5\right)\sinh(x)+7\,\cosh(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) + 4)*sinh(x)^2 + sinh(x)^3 +
 4*cosh(x)^2 + (3*cosh(x)^2 + 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) + 4)
```

Sympy [A] time = 0.81847, size = 15, normalized size = 1.5

$$-\frac{1}{2\cosh^2(x)+4\cosh(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))**3,x)
```

```
[Out] -1/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

Giac [A] time = 1.18451, size = 16, normalized size = 1.6

$$-\frac{2\,e^{(2\,x)}}{(e^x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -2*e^(2*x)/(e^x + 1)^4
```

3.147 $\int \frac{\sinh(x)}{(1-\cosh(x))^3} dx$

Optimal. Leaf size=12

$$\frac{1}{2(1-\cosh(x))^2}$$

```
[Out] 1/(2*(1 - Cosh[x])^2)
```

Rubi [A] time = 0.0211197, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 32}

$$\frac{1}{2(1-\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 - Cosh[x])^3,x]
```

```
[Out] 1/(2*(1 - Cosh[x])^2)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = -\operatorname{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, -\cosh(x)\right)$$
$$= \frac{1}{2(1-\cosh(x))^2}$$

Mathematica [A] time = 0.0103202, size = 12, normalized size = 1.

$$\frac{1}{8}\operatorname{csch}^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 - Cosh[x])^3,x]
```

```
[Out] Csch[x/2]^4/8
```

Maple [A] time = 0.005, size = 11, normalized size = 0.9

$$\frac{1}{2\,(1-\cosh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1-cosh(x))^3,x)
```

```
[Out] 1/2/(1-cosh(x))^2
```

Maxima [A] time = 1.06197, size = 11, normalized size = 0.92

$$\frac{1}{2\,(\cosh(x)-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2/(cosh(x) - 1)^2
```

Fricas [B] time = 2.13166, size = 196, normalized size = 16.33

$$\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)-4\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}-4\,\cosh\left(x\right)^{2}+\left(3\,\cosh\left(x\right)^{2}-8\,\cosh\left(x\right)+5\right)\sinh\left(x\right)+7\,\cosh\left(x\right)-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 -
4*cosh(x)^2 + (3*cosh(x)^2 - 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) - 4)
```

Sympy [A] time = 0.842427, size = 14, normalized size = 1.17

$$\frac{1}{2\cosh^{2}\left(x\right)-4\cosh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))**3,x)
```

```
[Out] 1/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

Giac [A] time = 1.14395, size = 16, normalized size = 1.33

$$\frac{2\,e^{\left(2\,x\right)}}{\left(e^{x}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] 2*e^(2*x)/(e^x - 1)^4
```

3.148 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^3} \, dx$

Optimal. Leaf size=14

$$\frac{\sinh^3(x)}{3(\cosh(x)+1)^3}$$

```
[Out] Sinh[x]^3/(3*(1 + Cosh[x])^3)
```

Rubi [A] time = 0.0322497, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2671}

$$\frac{\sinh^3(x)}{3(\cosh(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 + Cosh[x])^3,x]
```

```
[Out] Sinh[x]^3/(3*(1 + Cosh[x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3} \, dx = \frac{\sinh^3(x)}{3(1+\cosh(x))^3}$$

Mathematica [A] time = 0.0264682, size = 12, normalized size = 0.86

$$\frac{1}{3}\tanh^3\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 + Cosh[x])^3,x]
```

```
[Out] Tanh[x/2]^3/3
```

Maple [A] time = 0.011, size = 9, normalized size = 0.6

$$\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1+cosh(x))^3,x)
```

```
[Out] 1/3*tanh(1/2*x)^3
```

Maxima [B] time = 1.06615, size = 66, normalized size = 4.71

$$\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{2}{3\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-2
*x) + e^(-3*x) + 1)
```

Fricas [B] time = 2.11234, size = 127, normalized size = 9.07

$$-\frac{4\,(2\,\cosh(x)+\sinh(x))}{3\left(\cosh(x)^{2}+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^{2}+4\,\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
 + 4*cosh(x) + 3)
```

Sympy [A] time = 1.50567, size = 7, normalized size = 0.5

$$\frac{\tanh^3\left(\frac{x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+cosh(x))**3,x)
```

```
[Out] tanh(x/2)**3/3
```

Giac [A] time = 1.16736, size = 22, normalized size = 1.57

$$-\frac{2\left(3\,e^{(2\,x)}+1\right)}{3\,(e^x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) + 1)/(e^x + 1)^3
```

3.149 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx$

Optimal. Leaf size=16

$$-\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

```
[Out] -Sinh[x]^3/(3*(1 - Cosh[x])^3)
```

Rubi [A] time = 0.0328942, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2671}

$$-\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

```
[Out] -Sinh[x]^3/(3*(1 - Cosh[x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = -\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

Mathematica [A] time = 0.0285295, size = 12, normalized size = 0.75

$$\frac{1}{3}\coth^3\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

```
[Out] Coth[x/2]^3/3
```

Maple [A] time = 0.013, size = 9, normalized size = 0.6

$$\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1-cosh(x))^3,x)
```

```
[Out] 1/3/tanh(1/2*x)^3
```

Maxima [B] time = 1.10497, size = 66, normalized size = 4.12

$$-\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}-\frac{2}{3\left(3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -2*e^(-2*x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/3/(3*e^(-x) - 3*e^(-
2*x) + e^(-3*x) - 1)
```

Fricas [B] time = 2.05595, size = 126, normalized size = 7.88

$$\frac{4\,(2\,\cosh\left(x\right)+\sinh\left(x\right))}{3\left(\cosh\left(x\right)^{2}+2\,(\cosh\left(x\right)-1)\sinh\left(x\right)+\sinh\left(x\right)^{2}-4\,\cosh\left(x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2
- 4*cosh(x) + 3)
```

Sympy [A] time = 2.57552, size = 8, normalized size = 0.5

$$\frac{1}{3\tanh^3\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1-cosh(x))**3,x)
```

```
[Out] 1/(3*tanh(x/2)**3)
```

Giac [A] time = 1.15602, size = 22, normalized size = 1.38

$$\frac{2\left(3\,e^{(2\,x)}+1\right)}{3\,(e^x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*e^(2*x) + 1)/(e^x - 1)^3
```

3.150 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} \, dx$

Optimal. Leaf size=14

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

```
[Out] 2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rubi [A] time = 0.0402647, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 43}

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

```
[Out] 2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = -\operatorname{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, \cosh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, \cosh(x)\right)$$
$$= \frac{2}{1+\cosh(x)} + \log(1+\cosh(x))$$

Mathematica [A] time = 0.0092576, size = 20, normalized size = 1.43

$$2\log\left(\cosh\left(\frac{x}{2}\right)\right) - \tanh^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

```
[Out] 2*Log[Cosh[x/2]] - Tanh[x/2]^2
```

Maple [A] time = 0.014, size = 15, normalized size = 1.1

$$2\,\left(1+\cosh\left(x\right)\right)^{-1}+\ln\left(1+\cosh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1+cosh(x))^3,x)
```

```
[Out] 2/(1+cosh(x))+ln(1+cosh(x))
```

Maxima [B] time = 1.15641, size = 42, normalized size = 3.

$$x + \frac{4\,e^{(-x)}}{2\,e^{(-x)} + e^{(-2\,x)} + 1} + 2\,\log\left(e^{(-x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] x + 4*e^(-x)/(2*e^(-x) + e^(-2*x) + 1) + 2*log(e^(-x) + 1)
```

Fricas [B] time = 1.86647, size = 336, normalized size = 24.

$$-\frac{x\cosh(x)^2 + x\sinh(x)^2 + 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\cosh(x) + 1\right)\mathrm{lo}}{\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\cosh(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*(
x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh
(x)^2 + 2*cosh(x) + 1)
```

Sympy [B] time = 0.914517, size = 194, normalized size = 13.86

$$\frac{2\log(\cosh(x)+1)\cosh^2(x)}{2\cosh^2(x)+4\cosh(x)+2} + \frac{4\log(\cosh(x)+1)\cosh(x)}{2\cosh^2(x)+4\cosh(x)+2} + \frac{2\log(\cosh(x)+1)}{2\cosh^2(x)+4\cosh(x)+2} + \frac{\sinh^2(x)\cosh^2(x)}{2\cosh^2(x)+4\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+cosh(x))**3,x)
```

```
[Out] 2*log(cosh(x) + 1)*cosh(x)**2/(2*cosh(x)**2 + 4*cosh(x) + 2) + 4*log(cosh(x
) + 1)*cosh(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2*log(cosh(x) + 1)/(2*cosh(
x)**2 + 4*cosh(x) + 2) + sinh(x)**2*cosh(x)**2/(2*cosh(x)**2 + 4*cosh(x) +
2) + 2*sinh(x)**2*cosh(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) - cosh(x)**4/(2*co
sh(x)**2 + 4*cosh(x) + 2) - 2*cosh(x)**3/(2*cosh(x)**2 + 4*cosh(x) + 2) + 4
*cosh(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 3/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

Giac [A] time = 1.14942, size = 28, normalized size = 2.

$$-x + \frac{4e^x}{(e^x+1)^2} + 2\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -x + 4*e^x/(e^x + 1)^2 + 2*log(e^x + 1)
```

3.151 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} dx$

Optimal. Leaf size=20

$$-\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

```
[Out] -2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rubi [A] time = 0.0404257, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$-\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

```
[Out] -2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = \text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, -\cosh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, -\cosh(x)\right)$$
$$= -\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

Mathematica [A] time = 0.0124578, size = 27, normalized size = 1.35

$$\coth^2\left(\frac{x}{2}\right) - 2\log\left(\tanh\left(\frac{x}{2}\right)\right) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

```
[Out] Coth[x/2]^2 - 2*Log[Cosh[x/2]] - 2*Log[Tanh[x/2]]
```

Maple [A] time = 0.018, size = 17, normalized size = 0.9

$$2\,(-1+\cosh(x))^{-1} - \ln(-1+\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1-cosh(x))^3,x)
```

```
[Out] 2/(-1+cosh(x))-ln(-1+cosh(x))
```

Maxima [A] time = 1.15768, size = 47, normalized size = 2.35

$$-x - \frac{4\,e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - 2\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -x - 4*e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 2*log(e^(-x) - 1)
```

Fricas [B] time = 1.88948, size = 335, normalized size = 16.75

$$\frac{x\cosh(x)^2 + x\sinh(x)^2 - 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\cosh(x) + 1\right)\log}{\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
- 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(x
*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 2*cosh(x) + 1)
```

Sympy [B] time = 0.949815, size = 194, normalized size = 9.7

$$-\frac{2\log(\cosh(x)-1)\cosh^2(x)}{2\cosh^2(x) - 4\cosh(x) + 2} + \frac{4\log(\cosh(x)-1)\cosh(x)}{2\cosh^2(x) - 4\cosh(x) + 2} - \frac{2\log(\cosh(x)-1)}{2\cosh^2(x) - 4\cosh(x) + 2} - \frac{\sinh^2(x)\cosh^2(x)}{2\cosh^2(x) - 4\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1-cosh(x))**3,x)
```

```
[Out] -2*log(cosh(x) - 1)*cosh(x)**2/(2*cosh(x)**2 - 4*cosh(x) + 2) + 4*log(cosh(
x) - 1)*cosh(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 2*log(cosh(x) - 1)/(2*cosh
(x)**2 - 4*cosh(x) + 2) - sinh(x)**2*cosh(x)**2/(2*cosh(x)**2 - 4*cosh(x) +
 2) + 2*sinh(x)**2*cosh(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) + cosh(x)**4/(2*c
osh(x)**2 - 4*cosh(x) + 2) - 2*cosh(x)**3/(2*cosh(x)**2 - 4*cosh(x) + 2) +
4*cosh(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 3/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

Giac [A] time = 1.13683, size = 27, normalized size = 1.35

$$x + \frac{4\,e^x}{(e^x - 1)^2} - 2\,\log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] x + 4*e^x/(e^x - 1)^2 - 2*log(abs(e^x - 1))
```

3.152 $\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=57

$$\frac{5x}{16a}+\frac{\sinh^7(x)}{7a}-\frac{\sinh^5(x)\cosh(x)}{6a}+\frac{5\sinh^3(x)\cosh(x)}{24a}-\frac{5\sinh(x)\cosh(x)}{16a}$$

```
[Out] (5*x)/(16*a) - (5*Cosh[x]*Sinh[x])/(16*a) + (5*Cosh[x]*Sinh[x]^3)/(24*a) -
(Cosh[x]*Sinh[x]^5)/(6*a) + Sinh[x]^7/(7*a)
```

Rubi [A] time = 0.059228, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$\frac{5x}{16a}+\frac{\sinh^7(x)}{7a}-\frac{\sinh^5(x)\cosh(x)}{6a}+\frac{5\sinh^3(x)\cosh(x)}{24a}-\frac{5\sinh(x)\cosh(x)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^8/(a + a*Cosh[x]),x]
```

```
[Out] (5*x)/(16*a) - (5*Cosh[x]*Sinh[x])/(16*a) + (5*Cosh[x]*Sinh[x]^3)/(24*a) -
(Cosh[x]*Sinh[x]^5)/(6*a) + Sinh[x]^7/(7*a)
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx = \frac{\sinh^7(x)}{7a} - \frac{\int \sinh^6(x)\,dx}{a}$$
$$= -\frac{\cosh(x)\sinh^5(x)}{6a} + \frac{\sinh^7(x)}{7a} + \frac{5\int \sinh^4(x)\,dx}{6a}$$
$$= \frac{5\cosh(x)\sinh^3(x)}{24a} - \frac{\cosh(x)\sinh^5(x)}{6a} + \frac{\sinh^7(x)}{7a} - \frac{5\int \sinh^2(x)\,dx}{8a}$$
$$= -\frac{5\cosh(x)\sinh(x)}{16a} + \frac{5\cosh(x)\sinh^3(x)}{24a} - \frac{\cosh(x)\sinh^5(x)}{6a} + \frac{\sinh^7(x)}{7a} + \frac{5\int 1\,dx}{16a}$$
$$= \frac{5x}{16a} - \frac{5\cosh(x)\sinh(x)}{16a} + \frac{5\cosh(x)\sinh^3(x)}{24a} - \frac{\cosh(x)\sinh^5(x)}{6a} + \frac{\sinh^7(x)}{7a}$$

Mathematica [A] time = 0.0671383, size = 51, normalized size = 0.89

$$\frac{420x-105\sinh(x)-315\sinh(2x)+63\sinh(3x)+63\sinh(4x)-21\sinh(5x)-7\sinh(6x)+3\sinh(7x)}{1344a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^8/(a + a*Cosh[x]),x]
```

```
[Out] (420*x - 105*Sinh[x] - 315*Sinh[2*x] + 63*Sinh[3*x] + 63*Sinh[4*x] - 21*Sin
h[5*x] - 7*Sinh[6*x] + 3*Sinh[7*x])/(1344*a)
```

Maple [B] time = 0.04, size = 208, normalized size = 3.7

$$-\frac{1}{7a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-7} + \frac{2}{3a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-6} - \frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5} + \frac{1}{4a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4} + \frac{11}{24a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^8/(a+a*cosh(x)),x)
```

```
[Out] -1/7/a/(tanh(1/2*x)+1)^7+2/3/a/(tanh(1/2*x)+1)^6-1/a/(tanh(1/2*x)+1)^5+1/4/
a/(tanh(1/2*x)+1)^4+11/24/a/(tanh(1/2*x)+1)^3+1/8/a/(tanh(1/2*x)+1)^2-5/16/
a/(tanh(1/2*x)+1)+5/16/a*ln(tanh(1/2*x)+1)-1/7/a/(tanh(1/2*x)-1)^7-2/3/a/(t
anh(1/2*x)-1)^6-1/a/(tanh(1/2*x)-1)^5-1/4/a/(tanh(1/2*x)-1)^4+11/24/a/(tanh
```

```
(1/2*x)-1)^3-1/8/a/(tanh(1/2*x)-1)^2-5/16/a/(tanh(1/2*x)-1)-5/16/a*ln(tanh(
1/2*x)-1)
```

Maxima [B] time = 1.11207, size = 138, normalized size = 2.42

$$-\frac{\left(7\,e^{(-x)}+21\,e^{(-2\,x)}-63\,e^{(-3\,x)}-63\,e^{(-4\,x)}+315\,e^{(-5\,x)}+105\,e^{(-6\,x)}-3\right)e^{(7\,x)}}{2688\,a}+\frac{5\,x}{16\,a}+\frac{105\,e^{(-x)}+315\,e^{(-2\,x)}-63\,e^{(-3\,x)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/2688*(7*e^(-x) + 21*e^(-2*x) - 63*e^(-3*x) - 63*e^(-4*x) + 315*e^(-5*x)
+ 105*e^(-6*x) - 3)*e^(7*x)/a + 5/16*x/a + 1/2688*(105*e^(-x) + 315*e^(-2*x
) - 63*e^(-3*x) - 63*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) - 3*e^(-7*x))/a
```

Fricas [B] time = 1.99905, size = 340, normalized size = 5.96

$$\frac{3\,\sinh\left(x\right)^{7}+21\,\left(3\,\cosh\left(x\right)^{2}-2\,\cosh\left(x\right)-1\right)\sinh\left(x\right)^{5}+7\,\left(15\,\cosh\left(x\right)^{4}-20\,\cosh\left(x\right)^{3}-30\,\cosh\left(x\right)^{2}+36\,\cosh\left(x\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/1344*(3*sinh(x)^7 + 21*(3*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^5 + 7*(15*co
sh(x)^4 - 20*cosh(x)^3 - 30*cosh(x)^2 + 36*cosh(x) + 9)*sinh(x)^3 + 21*(cos
h(x)^6 - 2*cosh(x)^5 - 5*cosh(x)^4 + 12*cosh(x)^3 + 9*cosh(x)^2 - 30*cosh(x
) - 5)*sinh(x) + 420*x)/a
```

Sympy [B] time = 22.9578, size = 1253, normalized size = 21.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**8/(a+a*cosh(x)),x)
```

```
[Out] 105*x*tanh(x/2)**14/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*ta
nh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2
)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 735*x*tanh(x/2)**12/(336*a*tanh(x/2)*
*14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 +
11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) +
 2205*x*tanh(x/2)**10/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*
tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x
/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 3675*x*tanh(x/2)**8/(336*a*tanh(x/2
)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8
+ 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a)
 + 3675*x*tanh(x/2)**6/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a
*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(
x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 2205*x*tanh(x/2)**4/(336*a*tanh(x/
2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8
 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a
) + 735*x*tanh(x/2)**2/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a
*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(
x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 105*x/(336*a*tanh(x/2)**14 - 2352*
a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tan
h(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 210*tanh(x
/2)**13/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10
- 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*
a*tanh(x/2)**2 - 336*a) + 1400*tanh(x/2)**11/(336*a*tanh(x/2)**14 - 2352*a*
tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(
x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 3962*tanh(x/
2)**9/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 -
11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*
tanh(x/2)**2 - 336*a) - 6144*tanh(x/2)**7/(336*a*tanh(x/2)**14 - 2352*a*tan
h(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2
)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) + 3962*tanh(x/2)*
*5/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 117
60*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tan
h(x/2)**2 - 336*a) - 1400*tanh(x/2)**3/(336*a*tanh(x/2)**14 - 2352*a*tanh(x
/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**
6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) + 210*tanh(x/2)/(336
*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*ta
nh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)*
*2 - 336*a)
```

Giac [A] time = 1.17844, size = 122, normalized size = 2.14

$$\frac{\left(105\,e^{(6\,x)} + 315\,e^{(5\,x)} - 63\,e^{(4\,x)} - 63\,e^{(3\,x)} + 21\,e^{(2\,x)} + 7\,e^{x} - 3\right)e^{(-7\,x)} + 840\,x + 3\,e^{(7\,x)} - 7\,e^{(6\,x)} - 21\,e^{(5\,x)} + 63\,e^{(4\,x)} + 63\,e^{(3}}{2688\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/2688*((105*e^(6*x) + 315*e^(5*x) - 63*e^(4*x) - 63*e^(3*x) + 21*e^(2*x) +
 7*e^x - 3)*e^(-7*x) + 840*x + 3*e^(7*x) - 7*e^(6*x) - 21*e^(5*x) + 63*e^(4
*x) + 63*e^(3*x) - 315*e^(2*x) - 105*e^x)/a
```

3.153 $\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=46

$$\frac{(a-a\cosh(x))^6}{6a^7}-\frac{4(a-a\cosh(x))^5}{5a^6}+\frac{(a-a\cosh(x))^4}{a^5}$$

```
[Out] (a - a*Cosh[x])^4/a^5 - (4*(a - a*Cosh[x])^5)/(5*a^6) + (a - a*Cosh[x])^6/(
6*a^7)
```

Rubi [A] time = 0.0649232, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\frac{(a-a\cosh(x))^6}{6a^7}-\frac{4(a-a\cosh(x))^5}{5a^6}+\frac{(a-a\cosh(x))^4}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^7/(a + a*Cosh[x]),x]
```

```
[Out] (a - a*Cosh[x])^4/a^5 - (4*(a - a*Cosh[x])^5)/(5*a^6) + (a - a*Cosh[x])^6/(
6*a^7)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx = -\frac{\text{Subst}\left(\int (a-x)^3(a+x)^2\,dx, x, a\cosh(x)\right)}{a^7}$$
$$= -\frac{\text{Subst}\left(\int \left(4a^2(a-x)^3-4a(a-x)^4+(a-x)^5\right)dx, x, a\cosh(x)\right)}{a^7}$$
$$= \frac{(a-a\cosh(x))^4}{a^5} - \frac{4(a-a\cosh(x))^5}{5a^6} + \frac{(a-a\cosh(x))^6}{6a^7}$$

Mathematica [A] time = 0.0302479, size = 27, normalized size = 0.59

$$\frac{4\sinh^8\left(\frac{x}{2}\right)(28\cosh(x)+5\cosh(2x)+27)}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^7/(a + a*Cosh[x]),x]
```

```
[Out] (4*(27 + 28*Cosh[x] + 5*Cosh[2*x])*Sinh[x/2]^8)/(15*a)
```

Maple [B] time = 0.034, size = 107, normalized size = 2.3

$$128\,\frac{1}{a}\left(\frac{1}{768\,(\tanh(x/2)+1)^6} - \frac{7}{1280\,(\tanh(x/2)+1)^5} + \frac{7}{1024\,(\tanh(x/2)+1)^4} - \frac{7}{2048\,(\tanh(x/2)+1)^2} - \frac{}{2048\,\tan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a+a*cosh(x)),x)
```

```
[Out] 128/a*(1/768/(tanh(1/2*x)+1)^6-7/1280/(tanh(1/2*x)+1)^5+7/1024/(tanh(1/2*x)
+1)^4-7/2048/(tanh(1/2*x)+1)^2-7/2048/(tanh(1/2*x)+1)+1/768/(tanh(1/2*x)-1)
^6+7/1280/(tanh(1/2*x)-1)^5+7/1024/(tanh(1/2*x)-1)^4-7/2048/(tanh(1/2*x)-1)
^2+7/2048/(tanh(1/2*x)-1))
```

Maxima [A] time = 1.09906, size = 113, normalized size = 2.46

$$-\frac{\left(12\,e^{(-x)}+30\,e^{(-2\,x)}-100\,e^{(-3\,x)}-75\,e^{(-4\,x)}+600\,e^{(-5\,x)}-5\right)e^{(6\,x)}}{1920\,a} - \frac{600\,e^{(-x)}-75\,e^{(-2\,x)}-100\,e^{(-3\,x)}+30\,e^{(-4\,x)}+12\,e^{(}}{1920\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*e^(-x) + 30*e^(-2*x) - 100*e^(-3*x) - 75*e^(-4*x) + 600*e^(-5*x
) - 5)*e^(6*x)/a - 1/1920*(600*e^(-x) - 75*e^(-2*x) - 100*e^(-3*x) + 30*e^(
-4*x) + 12*e^(-5*x) - 5*e^(-6*x))/a
```

Fricas [B] time = 1.81454, size = 313, normalized size = 6.8

$$\frac{5\cosh(x)^6 + 5\sinh(x)^6 - 12\cosh(x)^5 + 15\left(5\cosh(x)^2 - 4\cosh(x) - 2\right)\sinh(x)^4 - 30\cosh(x)^4 + 100\cosh(x)^3 + 1}{960\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(5*cosh(x)^6 + 5*sinh(x)^6 - 12*cosh(x)^5 + 15*(5*cosh(x)^2 - 4*cosh(
x) - 2)*sinh(x)^4 - 30*cosh(x)^4 + 100*cosh(x)^3 + 15*(5*cosh(x)^4 - 8*cosh
(x)^3 - 12*cosh(x)^2 + 20*cosh(x) + 5)*sinh(x)^2 + 75*cosh(x)^2 - 600*cosh(
x))/a
```

Sympy [B] time = 12.7959, size = 218, normalized size = 4.74

$$\frac{16\tanh^{12}\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right) - 90a\tanh^{10}\left(\frac{x}{2}\right) + 225a\tanh^{8}\left(\frac{x}{2}\right) - 300a\tanh^{6}\left(\frac{x}{2}\right) + 225a\tanh^{4}\left(\frac{x}{2}\right) - 90a\tanh^{2}\left(\frac{x}{2}\right) + 15a} - \frac{}{15a\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**7/(a+a*cosh(x)),x)
```

```
[Out] 16*tanh(x/2)**12/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2)
**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a) -
 96*tanh(x/2)**10/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2
)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
+ 240*tanh(x/2)**8/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/
2)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
```

Giac [A] time = 1.20752, size = 101, normalized size = 2.2

$$-\frac{\left(600\,e^{(5\,x)}-75\,e^{(4\,x)}-100\,e^{(3\,x)}+30\,e^{(2\,x)}+12\,e^{x}-5\right)e^{(-6\,x)}-5\,e^{(6\,x)}+12\,e^{(5\,x)}+30\,e^{(4\,x)}-100\,e^{(3\,x)}-75\,e^{(2\,x)}+600\,e^{x}}{1920\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/1920*((600*e^(5*x) - 75*e^(4*x) - 100*e^(3*x) + 30*e^(2*x) + 12*e^x - 5)
*e^(-6*x) - 5*e^(6*x) + 12*e^(5*x) + 30*e^(4*x) - 100*e^(3*x) - 75*e^(2*x)
+ 600*e^x)/a
```

3.154 $\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=44

$$-\frac{3x}{8a}+\frac{\sinh^5(x)}{5a}-\frac{\sinh^3(x)\cosh(x)}{4a}+\frac{3\sinh(x)\cosh(x)}{8a}$$

```
[Out] (-3*x)/(8*a) + (3*Cosh[x]*Sinh[x])/(8*a) - (Cosh[x]*Sinh[x]^3)/(4*a) + Sinh
[x]^5/(5*a)
```

Rubi [A] time = 0.0544319, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$-\frac{3x}{8a}+\frac{\sinh^5(x)}{5a}-\frac{\sinh^3(x)\cosh(x)}{4a}+\frac{3\sinh(x)\cosh(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^6/(a + a*Cosh[x]),x]
```


```
[Out] (-3*x)/(8*a) + (3*Cosh[x]*Sinh[x])/(8*a) - (Cosh[x]*Sinh[x]^3)/(4*a) + Sinh
[x]^5/(5*a)
```


Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = \frac{\sinh^5(x)}{5a} - \frac{\int \sinh^4(x)\,dx}{a}$$
$$= -\frac{\cosh(x)\sinh^3(x)}{4a} + \frac{\sinh^5(x)}{5a} + \frac{3\int \sinh^2(x)\,dx}{4a}$$
$$= \frac{3\cosh(x)\sinh(x)}{8a} - \frac{\cosh(x)\sinh^3(x)}{4a} + \frac{\sinh^5(x)}{5a} - \frac{3\int 1\,dx}{8a}$$
$$= -\frac{3x}{8a} + \frac{3\cosh(x)\sinh(x)}{8a} - \frac{\cosh(x)\sinh^3(x)}{4a} + \frac{\sinh^5(x)}{5a}$$

Mathematica [A] time = 0.051559, size = 39, normalized size = 0.89

$$\frac{-60x + 20\sinh(x) + 40\sinh(2x) - 10\sinh(3x) - 5\sinh(4x) + 2\sinh(5x)}{160a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (-60*x + 20*Sinh[x] + 40*Sinh[2*x] - 10*Sinh[3*x] - 5*Sinh[4*x] + 2*Sinh[5*
x])/(160*a)
```

Maple [B] time = 0.033, size = 156, normalized size = 3.6

$$-\frac{1}{5\,a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5} + \frac{3}{4\,a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4} - \frac{3}{4\,a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} - \frac{1}{4\,a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \frac{3}{8\,a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a+a*cosh(x)),x)
```

```
[Out] -1/5/a/(tanh(1/2*x)+1)^5+3/4/a/(tanh(1/2*x)+1)^4-3/4/a/(tanh(1/2*x)+1)^3-1/
4/a/(tanh(1/2*x)+1)^2+3/8/a/(tanh(1/2*x)+1)-3/8/a*ln(tanh(1/2*x)+1)-1/5/a/(
tanh(1/2*x)-1)^5-3/4/a/(tanh(1/2*x)-1)^4-3/4/a/(tanh(1/2*x)-1)^3+1/4/a/(tan
h(1/2*x)-1)^2+3/8/a/(tanh(1/2*x)-1)+3/8/a*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.10902, size = 105, normalized size = 2.39

$$-\frac{\left(5\,e^{(-x)}+10\,e^{(-2\,x)}-40\,e^{(-3\,x)}-20\,e^{(-4\,x)}-2\right)e^{(5\,x)}}{320\,a}-\frac{3\,x}{8\,a}-\frac{20\,e^{(-x)}+40\,e^{(-2\,x)}-10\,e^{(-3\,x)}-5\,e^{(-4\,x)}+2\,e^{(-5\,x)}}{320\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/320*(5*e^(-x) + 10*e^(-2*x) - 40*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x)/a -
 3/8*x/a - 1/320*(20*e^(-x) + 40*e^(-2*x) - 10*e^(-3*x) - 5*e^(-4*x) + 2*e^
(-5*x))/a
```

Fricas [A] time = 1.89219, size = 188, normalized size = 4.27

$$\frac{\sinh(x)^5+5\left(2\,\cosh(x)^2-2\,\cosh(x)-1\right)\sinh(x)^3+5\left(\cosh(x)^4-2\,\cosh(x)^3-3\,\cosh(x)^2+8\,\cosh(x)+2\right)\sinh(x}{80\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/80*(sinh(x)^5 + 5*(2*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^3 + 5*(cosh(x)^4
- 2*cosh(x)^3 - 3*cosh(x)^2 + 8*cosh(x) + 2)*sinh(x) - 30*x)/a
```

Sympy [B] time = 7.6424, size = 692, normalized size = 15.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**6/(a+a*cosh(x)),x)
```

```
[Out] -15*x*tanh(x/2)**10/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x
/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 75*x*tanh(x/2)**
8/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tan
h(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 150*x*tanh(x/2)**6/(40*a*tanh(x/2)
**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a
*tanh(x/2)**2 - 40*a) + 150*x*tanh(x/2)**4/(40*a*tanh(x/2)**10 - 200*a*tanh
```

```
(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 4
0*a) - 75*x*tanh(x/2)**2/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*t
anh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 15*x/(40*a*
tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**
4 + 200*a*tanh(x/2)**2 - 40*a) + 30*tanh(x/2)**9/(40*a*tanh(x/2)**10 - 200*
a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)*
*2 - 40*a) - 140*tanh(x/2)**7/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 40
0*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 256*ta
nh(x/2)**5/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 -
400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 140*tanh(x/2)**3/(40*a*ta
nh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4
+ 200*a*tanh(x/2)**2 - 40*a) - 30*tanh(x/2)/(40*a*tanh(x/2)**10 - 200*a*tan
h(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 -
40*a)
```

Giac [A] time = 1.25476, size = 89, normalized size = 2.02

$$-\frac{\left(20\,e^{(4\,x)} + 40\,e^{(3\,x)} - 10\,e^{(2\,x)} - 5\,e^{x} + 2\right)e^{(-5\,x)} + 120\,x - 2\,e^{(5\,x)} + 5\,e^{(4\,x)} + 10\,e^{(3\,x)} - 40\,e^{(2\,x)} - 20\,e^{x}}{320\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/320*((20*e^(4*x) + 40*e^(3*x) - 10*e^(2*x) - 5*e^x + 2)*e^(-5*x) + 120*x
 - 2*e^(5*x) + 5*e^(4*x) + 10*e^(3*x) - 40*e^(2*x) - 20*e^x)/a
```

3.155 $\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=33

$$\frac{(a-a\cosh(x))^4}{4a^5}-\frac{2(a-a\cosh(x))^3}{3a^4}$$

```
[Out] (-2*(a - a*Cosh[x])^3)/(3*a^4) + (a - a*Cosh[x])^4/(4*a^5)
```

Rubi [A] time = 0.0556596, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\frac{(a-a\cosh(x))^4}{4a^5}-\frac{2(a-a\cosh(x))^3}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (-2*(a - a*Cosh[x])^3)/(3*a^4) + (a - a*Cosh[x])^4/(4*a^5)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)\,dx, x, a\cosh(x)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a-x)^2-(a-x)^3\right)\,dx, x, a\cosh(x)\right)}{a^5}$$
$$= -\frac{2(a-a\cosh(x))^3}{3a^4}+\frac{(a-a\cosh(x))^4}{4a^5}$$

Mathematica [A] time = 0.0203131, size = 21, normalized size = 0.64

$$\frac{2\sinh^6\left(\frac{x}{2}\right)(3\cosh(x)+5)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (2*(5 + 3*Cosh[x])*Sinh[x/2]^6)/(3*a)
```

Maple [B] time = 0.027, size = 87, normalized size = 2.6

$$32\,\frac{1}{a}\left(\frac{1}{128\,(\tanh(x/2)+1)^4}-\frac{5}{192\,(\tanh(x/2)+1)^3}+\frac{5}{256\,(\tanh(x/2)+1)^2}+\frac{5}{256\,\tanh(x/2)+256}+\frac{1}{128\,(\tanh(x/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a+a*cosh(x)),x)
```

```
[Out] 32/a*(1/128/(tanh(1/2*x)+1)^4-5/192/(tanh(1/2*x)+1)^3+5/256/(tanh(1/2*x)+1)
^2+5/256/(tanh(1/2*x)+1)+1/128/(tanh(1/2*x)-1)^4+5/192/(tanh(1/2*x)-1)^3+5/
256/(tanh(1/2*x)-1)^2-5/256/(tanh(1/2*x)-1))
```

Maxima [A] time = 1.14559, size = 81, normalized size = 2.45

$$-\frac{\left(8\,e^{(-x)}+12\,e^{(-2\,x)}-72\,e^{(-3\,x)}-3\right)e^{(4\,x)}}{192\,a}+\frac{72\,e^{(-x)}-12\,e^{(-2\,x)}-8\,e^{(-3\,x)}+3\,e^{(-4\,x)}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*e^(-x) + 12*e^(-2*x) - 72*e^(-3*x) - 3)*e^(4*x)/a + 1/192*(72*e^(
-x) - 12*e^(-2*x) - 8*e^(-3*x) + 3*e^(-4*x))/a
```

Fricas [A] time = 1.79853, size = 165, normalized size = 5.

$$\frac{3\cosh(x)^4 + 3\sinh(x)^4 - 8\cosh(x)^3 + 6\left(3\cosh(x)^2 - 4\cosh(x) - 2\right)\sinh(x)^2 - 12\cosh(x)^2 + 72\cosh(x)}{96a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(x)^4 + 3*sinh(x)^4 - 8*cosh(x)^3 + 6*(3*cosh(x)^2 - 4*cosh(x)
- 2)*sinh(x)^2 - 12*cosh(x)^2 + 72*cosh(x))/a
```

Sympy [B] time = 4.05777, size = 104, normalized size = 3.15

$$-\frac{4\tanh^8\left(\frac{x}{2}\right)}{3a\tanh^8\left(\frac{x}{2}\right) - 12a\tanh^6\left(\frac{x}{2}\right) + 18a\tanh^4\left(\frac{x}{2}\right) - 12a\tanh^2\left(\frac{x}{2}\right) + 3a} + \frac{16\tanh^6\left(\frac{x}{2}\right)}{3a\tanh^8\left(\frac{x}{2}\right) - 12a\tanh^6\left(\frac{x}{2}\right) + 18a\tanh^4\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**5/(a+a*cosh(x)),x)
```

```
[Out] -4*tanh(x/2)**8/(3*a*tanh(x/2)**8 - 12*a*tanh(x/2)**6 + 18*a*tanh(x/2)**4 -
 12*a*tanh(x/2)**2 + 3*a) + 16*tanh(x/2)**6/(3*a*tanh(x/2)**8 - 12*a*tanh(x
/2)**6 + 18*a*tanh(x/2)**4 - 12*a*tanh(x/2)**2 + 3*a)
```

Giac [A] time = 1.20462, size = 69, normalized size = 2.09

$$\frac{\left(72e^{(3x)} - 12e^{(2x)} - 8e^{x} + 3\right)e^{(-4x)} + 3e^{(4x)} - 8e^{(3x)} - 12e^{(2x)} + 72e^{x}}{192a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/192*((72*e^(3*x) - 12*e^(2*x) - 8*e^x + 3)*e^(-4*x) + 3*e^(4*x) - 8*e^(3*
x) - 12*e^(2*x) + 72*e^x)/a
```

3.156 $\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{2a}+\frac{\sinh^3(x)}{3a}-\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] x/(2*a) - (Cosh[x]*Sinh[x])/(2*a) + Sinh[x]^3/(3*a)
```

Rubi [A] time = 0.0459738, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$\frac{x}{2a}+\frac{\sinh^3(x)}{3a}-\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] x/(2*a) - (Cosh[x]*Sinh[x])/(2*a) + Sinh[x]^3/(3*a)
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a + a\cosh(x)}\,dx = \frac{\sinh^3(x)}{3a} - \frac{\int \sinh^2(x)\,dx}{a}$$
$$= -\frac{\cosh(x)\sinh(x)}{2a} + \frac{\sinh^3(x)}{3a} + \frac{\int 1\,dx}{2a}$$
$$= \frac{x}{2a} - \frac{\cosh(x)\sinh(x)}{2a} + \frac{\sinh^3(x)}{3a}$$

Mathematica [A] time = 0.0345503, size = 25, normalized size = 0.81

$$\frac{6x - 3\sinh(x) - 3\sinh(2x) + \sinh(3x)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (6*x - 3*Sinh[x] - 3*Sinh[2*x] + Sinh[3*x])/(12*a)
```

Maple [B] time = 0.023, size = 103, normalized size = 3.3

$$-\frac{1}{3\,a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} + \frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \frac{1}{2\,a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{1}{2\,a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{3\,a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+a*cosh(x)),x)
```

```
[Out] -1/3/a/(tanh(1/2*x)+1)^3+1/a/(tanh(1/2*x)+1)^2-1/2/a/(tanh(1/2*x)+1)+1/2/a*
ln(tanh(1/2*x)+1)-1/3/a/(tanh(1/2*x)-1)^3-1/a/(tanh(1/2*x)-1)^2-1/2/a/(tanh
(1/2*x)-1)-1/2/a*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.1133, size = 73, normalized size = 2.35

$$-\frac{\left(3\,e^{(-x)} + 3\,e^{(-2\,x)} - 1\right)e^{(3\,x)}}{24\,a} + \frac{x}{2\,a} + \frac{3\,e^{(-x)} + 3\,e^{(-2\,x)} - e^{(-3\,x)}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*e^(-x) + 3*e^(-2*x) - 1)*e^(3*x)/a + 1/2*x/a + 1/24*(3*e^(-x) + 3*
e^(-2*x) - e^(-3*x))/a
```

Fricas [A] time = 1.8617, size = 89, normalized size = 2.87

$$\frac{\sinh(x)^3 + 3\left(\cosh(x)^2 - 2\cosh(x) - 1\right)\sinh(x) + 6x}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(x)^3 + 3*(cosh(x)^2 - 2*cosh(x) - 1)*sinh(x) + 6*x)/a
```

Sympy [B] time = 2.14946, size = 294, normalized size = 9.48

$$\frac{3x\tanh^6\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} - \frac{9x\tanh^4\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right) - 18a\tanh^4\left(\frac{x}{2}\right) + 18a\tanh^2\left(\frac{x}{2}\right) - 6a} + \frac{}{6a\tanh^6\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+a*cosh(x)),x)
```

```
[Out] 3*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2
- 6*a) - 9*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh
(x/2)**2 - 6*a) + 9*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 +
18*a*tanh(x/2)**2 - 6*a) - 3*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a
*tanh(x/2)**2 - 6*a) - 6*tanh(x/2)**5/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) - 16*tanh(x/2)**3/(6*a*tanh(x/2)**6 - 18*a*tanh
(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)/(6*a*tanh(x/2)**6 - 18*a*
tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

Giac [A] time = 1.2793, size = 54, normalized size = 1.74

$$\frac{\left(3e^{(2x)} + 3e^{x} - 1\right)e^{(-3x)} + 12x + e^{(3x)} - 3e^{(2x)} - 3e^{x}}{24a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/24*((3*e^(2*x) + 3*e^x - 1)*e^(-3*x) + 12*x + e^(3*x) - 3*e^(2*x) - 3*e^x
)/a
```

3.157 $\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\cosh^2(x)}{2a}-\frac{\cosh(x)}{a}$$

```
[Out] -(Cosh[x]/a) + Cosh[x]^2/(2*a)
```

Rubi [A] time = 0.0432605, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2667}

$$\frac{\cosh^2(x)}{2a}-\frac{\cosh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] -(Cosh[x]/a) + Cosh[x]^2/(2*a)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx &= -\frac{\text{Subst}\left(\int (a-x)\,dx, x, a\cosh(x)\right)}{a^3}\\ &= -\frac{\cosh(x)}{a}+\frac{\cosh^2(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0116991, size = 13, normalized size = 0.68

$$\frac{2\sinh^4\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (2*Sinh[x/2]^4)/a
```

Maple [B] time = 0.02, size = 47, normalized size = 2.5

$$8\frac{1}{a}\left(1/16\left(\tanh(x/2)+1\right)^{-2}-3/16\left(\tanh(x/2)+1\right)^{-1}+1/16\left(\tanh(x/2)-1\right)^{-2}+3/16\left(\tanh(x/2)-1\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+a*cosh(x)),x)
```

```
[Out] 8/a*(1/16/(tanh(1/2*x)+1)^2-3/16/(tanh(1/2*x)+1)+1/16/(tanh(1/2*x)-1)^2+3/1
6/(tanh(1/2*x)-1))
```

Maxima [B] time = 1.11178, size = 49, normalized size = 2.58

$$-\frac{\left(4\,e^{(-x)}-1\right)e^{(2\,x)}}{8\,a}-\frac{4\,e^{(-x)}-e^{(-2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*e^(-x) - 1)*e^(2*x)/a - 1/8*(4*e^(-x) - e^(-2*x))/a
```

Fricas [A] time = 1.88021, size = 58, normalized size = 3.05

$$\frac{\cosh(x)^2+\sinh(x)^2-4\,\cosh(x)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(x)^2 + sinh(x)^2 - 4*cosh(x))/a
```

Sympy [B] time = 1.106, size = 27, normalized size = 1.42

$$\frac{2\tanh^4\left(\frac{x}{2}\right)}{a\tanh^4\left(\frac{x}{2}\right) - 2a\tanh^2\left(\frac{x}{2}\right) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+a*cosh(x)),x)
```

```
[Out] 2*tanh(x/2)**4/(a*tanh(x/2)**4 - 2*a*tanh(x/2)**2 + a)
```

Giac [A] time = 1.24729, size = 36, normalized size = 1.89

$$-\frac{(4e^x - 1)e^{(-2x)} - e^{(2x)} + 4e^x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/8*((4*e^x - 1)*e^(-2*x) - e^(2*x) + 4*e^x)/a
```

3.158 $\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh(x)}{a}-\frac{x}{a}$$

```
[Out] -(x/a) + Sinh[x]/a
```

Rubi [A] time = 0.0392258, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2682, 8}

$$\frac{\sinh(x)}{a}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -(x/a) + Sinh[x]/a
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx &= \frac{\sinh(x)}{a}-\frac{\int 1\,dx}{a}\\ &= -\frac{x}{a}+\frac{\sinh(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0082304, size = 17, normalized size = 1.31

$$\frac{2\left(\frac{\sinh(x)}{2}-\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (2*(-x/2 + Sinh[x]/2))/a
```

Maple [B] time = 0.017, size = 51, normalized size = 3.9

$$-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+a*cosh(x)),x)
```

```
[Out] -1/a/(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)+1)-1/a/(tanh(1/2*x)-1)+1/a*ln(tanh(
1/2*x)-1)
```

Maxima [A] time = 1.10447, size = 31, normalized size = 2.38

$$-\frac{x}{a}-\frac{e^{(-x)}}{2\,a}+\frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -x/a - 1/2*e^(-x)/a + 1/2*e^x/a
```

Fricas [A] time = 1.82705, size = 24, normalized size = 1.85

$$-\frac{x-\sinh\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(x - sinh(x))/a
```

Sympy [B] time = 0.554878, size = 46, normalized size = 3.54

$$-\frac{x\tanh^2\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}+\frac{x}{a\tanh^2\left(\frac{x}{2}\right)-a}-\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+a*cosh(x)),x)
```

```
[Out] -x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) - 2*tanh(x/2)
/(a*tanh(x/2)**2 - a)
```

Giac [A] time = 1.18697, size = 23, normalized size = 1.77

$$-\frac{2\,x+e^{(-x)}-e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/2*(2*x + e^(-x) - e^x)/a
```

3.159 $\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(\cosh(x)+1)}{a}$$

```
[Out] Log[1 + Cosh[x]]/a
```

Rubi [A] time = 0.0236929, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 31}

$$\frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + a*Cosh[x]),x]

[Out] Log[1 + Cosh[x]]/a
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\cosh(x)\right)}{a}$$
$$= \frac{\log(1+\cosh(x))}{a}$$

Mathematica [A] time = 0.0054675, size = 12, normalized size = 1.33

$$\frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + a*Cosh[x]),x]
```

```
[Out] (2*Log[Cosh[x/2]])/a
```

Maple [A] time = 0.006, size = 12, normalized size = 1.3

$$\frac{\ln\left(a+a\cosh\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+a*cosh(x)),x)
```

```
[Out] ln(a+a*cosh(x))/a
```

Maxima [A] time = 1.06024, size = 15, normalized size = 1.67

$$\frac{\log\left(a\cosh\left(x\right)+a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] log(a*cosh(x) + a)/a
```

Fricas [A] time = 1.96665, size = 53, normalized size = 5.89

$$-\frac{x - 2\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [A] time = 0.156929, size = 7, normalized size = 0.78

$$\frac{\log\left(\cosh\left(x\right) + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x)
```

```
[Out] log(cosh(x) + 1)/a
```

Giac [A] time = 1.13179, size = 23, normalized size = 2.56

$$-\frac{x}{a} + \frac{2\log\left(e^{x} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -x/a + 2*log(e^x + 1)/a
```

3.160 $\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2(a\cosh(x)+a)}-\frac{\tanh^{-1}(\cosh(x))}{2a}$$

```
[Out] -ArcTanh[Cosh[x]]/(2*a) + 1/(2*(a + a*Cosh[x]))
```

Rubi [A] time = 0.052373, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2667, 44, 206}

$$\frac{1}{2(a\cosh(x)+a)}-\frac{\tanh^{-1}(\cosh(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + a*Cosh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/(2*a) + 1/(2*(a + a*Cosh[x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{csch}(x)}{a+a\cosh(x)}\,dx &= -\left(a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx,x,a\cosh(x)\right)\right)\\ &= -\left(a\,\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a\left(a^2-x^2\right)}\right)dx,x,a\cosh(x)\right)\right)\\ &= \frac{1}{2(a+a\cosh(x))}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx,x,a\cosh(x)\right)\\ &= -\frac{\tanh^{-1}(\cosh(x))}{2a}+\frac{1}{2(a+a\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.0274464, size = 42, normalized size = 1.83

$$\frac{1-2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{2a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + a*Cosh[x]),x]
```

```
[Out] (1 - 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(2*a*(1 + Cosh[x]))
```

Maple [A] time = 0.016, size = 23, normalized size = 1.

$$-\frac{1}{4\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{1}{2\,a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+a*cosh(x)),x)
```

```
[Out] -1/4/a*tanh(1/2*x)^2+1/2/a*ln(tanh(1/2*x))
```

Maxima [B] time = 1.01682, size = 63, normalized size = 2.74

$$\frac{e^{(-x)}}{2\,ae^{(-x)} + ae^{(-2\,x)} + a} - \frac{\log\left(e^{(-x)} + 1\right)}{2\,a} + \frac{\log\left(e^{(-x)} - 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(-
x) - 1)/a
```

Fricas [B] time = 1.83715, size = 397, normalized size = 17.26

$$-\frac{\left(\cosh\left(x\right)^{2} + 2\left(\cosh\left(x\right) + 1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 2\,\cosh\left(x\right) + 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - \left(\cosh\left(x\right)^{2} + 2\,(\cos\right.}{2\left(a\cosh\left(x\right)^{2} + a\sinh\left(x\right)^{2} + 2\,a\cosh\left(x\right) + 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) - 2*cosh(x) - 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{csch}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)/(cosh(x) + 1), x)/a
```

Giac [B] time = 1.18903, size = 70, normalized size = 3.04

$$-\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{4\,a}+\frac{e^{(-x)}+e^{x}+6}{4\,a\left(e^{(-x)}+e^{x}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x + 6)/(a*(e^(-x) + e^x + 2))
```

3.161 $\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=24

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\coth(x)}{3a}$$

```
[Out] (-2*Coth[x])/(3*a) + Csch[x]/(3*(a + a*Cosh[x]))
```

Rubi [A] time = 0.0483311, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 3767, 8}

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\coth(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (-2*Coth[x])/(3*a) + Csch[x]/(3*(a + a*Cosh[x]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx &= \frac{\operatorname{csch}(x)}{3(a+a\cosh(x))}+\frac{2\int \operatorname{csch}^2(x)\,dx}{3a}\\ &= \frac{\operatorname{csch}(x)}{3(a+a\cosh(x))}-\frac{(2i)\,\text{Subst}\left(\int 1\,dx,x,-i\coth(x)\right)}{3a}\\ &= -\frac{2\coth(x)}{3a}+\frac{\operatorname{csch}(x)}{3(a+a\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.0460004, size = 30, normalized size = 1.25

$$-\frac{(2\cosh(x)+\cosh(2x))\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}^3\left(\frac{x}{2}\right)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -((2*Cosh[x] + Cosh[2*x])*Csch[x/2]*Sech[x/2]^3)/(12*a)
```

Maple [A] time = 0.016, size = 29, normalized size = 1.2

$$\frac{1}{4a}\left(\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3-2\,\tanh(x/2)-\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+a*cosh(x)),x)
```

```
[Out] 1/4/a*(1/3*tanh(1/2*x)^3-2*tanh(1/2*x)-1/tanh(1/2*x))
```

Maxima [B] time = 1.00997, size = 80, normalized size = 3.33

$$-\frac{8\,e^{(-x)}}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}-\frac{4}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 4/3/(2*a*e^(-x)
- 2*a*e^(-3*x) - a*e^(-4*x) + a)
```

Fricas [B] time = 1.85497, size = 292, normalized size = 12.17

$$-\frac{4\,(2\,\cosh(x)+2\,\sinh(x)+1)}{3\left(a\cosh(x)^{4}+a\sinh(x)^{4}+2\,a\cosh(x)^{3}+2\,(2\,a\cosh(x)+a)\sinh(x)^{3}+6\left(a\cosh(x)^{2}+a\cosh(x)\right)\sinh(x)^{2}-2\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + 2*sinh(x) + 1)/(a*cosh(x)^4 + a*sinh(x)^4 + 2*a*cosh(x)^3
 + 2*(2*a*cosh(x) + a)*sinh(x)^3 + 6*(a*cosh(x)^2 + a*cosh(x))*sinh(x)^2 -
2*a*cosh(x) + 2*(2*a*cosh(x)^3 + 3*a*cosh(x)^2 - a)*sinh(x) - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{csch}^{2}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**2/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.17293, size = 47, normalized size = 1.96

$$-\frac{1}{2\,a(e^{x}-1)}+\frac{3\,e^{(2\,x)}+12\,e^{x}+5}{6\,a(e^{x}+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="giac")

[Out] -1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 12*e^x + 5)/(a*(e^x + 1)^3)
```

3.162 $\int \frac{\text{csch}^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=49

$$-\frac{a}{8(a\cosh(x)+a)^2}+\frac{1}{8(a-a\cosh(x))}-\frac{1}{4(a\cosh(x)+a)}+\frac{3\tanh^{-1}(\cosh(x))}{8a}$$

```
[Out] (3*ArcTanh[Cosh[x]])/(8*a) + 1/(8*(a - a*Cosh[x])) - a/(8*(a + a*Cosh[x])^2
) - 1/(4*(a + a*Cosh[x]))
```

Rubi [A] time = 0.0811718, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 206}

$$-\frac{a}{8(a\cosh(x)+a)^2}+\frac{1}{8(a-a\cosh(x))}-\frac{1}{4(a\cosh(x)+a)}+\frac{3\tanh^{-1}(\cosh(x))}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/(8*a) + 1/(8*(a - a*Cosh[x])) - a/(8*(a + a*Cosh[x])^2
) - 1/(4*(a + a*Cosh[x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx &= a^3\operatorname{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, a\cosh(x)\right)\\
&= a^3\operatorname{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3\left(a^2-x^2\right)}\right)dx, x, a\cosh(x)\right)\\
&= \frac{1}{8(a-a\cosh(x))}-\frac{a}{8(a+a\cosh(x))^2}-\frac{1}{4(a+a\cosh(x))}+\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\cosh(x)\right)\\
&= \frac{3\tanh^{-1}(\cosh(x))}{8a}+\frac{1}{8(a-a\cosh(x))}-\frac{a}{8(a+a\cosh(x))^2}-\frac{1}{4(a+a\cosh(x))}
\end{aligned}$$

Mathematica [A] time = 0.129217, size = 60, normalized size = 1.22

$$-\frac{2\coth^2\left(\frac{x}{2}\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+4}{16a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] -(4 + 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]) + Se
ch[x/2]^2)/(16*a*(1 + Cosh[x]))
```

Maple [A] time = 0.02, size = 45, normalized size = 0.9

$$-\frac{1}{32\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^4+\frac{3}{16\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{3}{8\,a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{16\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+a*cosh(x)),x)
```

```
[Out] -1/32/a*tanh(1/2*x)^4+3/16/a*tanh(1/2*x)^2-3/8/a*ln(tanh(1/2*x))-1/16/a/tan
h(1/2*x)^2
```

Maxima [B] time = 1.06089, size = 139, normalized size = 2.84

$$-\frac{3\,e^{(-x)}+6\,e^{(-2\,x)}-2\,e^{(-3\,x)}+6\,e^{(-4\,x)}+3\,e^{(-5\,x)}}{4\left(2\,ae^{(-x)}-ae^{(-2\,x)}-4\,ae^{(-3\,x)}-ae^{(-4\,x)}+2\,ae^{(-5\,x)}+ae^{(-6\,x)}+a\right)}+\frac{3\,\log\left(e^{(-x)}+1\right)}{8\,a}-\frac{3\,\log\left(e^{(-x)}-1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(3*e^(-x) + 6*e^(-2*x) - 2*e^(-3*x) + 6*e^(-4*x) + 3*e^(-5*x))/(2*a*e^
(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) +
 a) + 3/8*log(e^(-x) + 1)/a - 3/8*log(e^(-x) - 1)/a
```

Fricas [B] time = 1.95053, size = 2066, normalized size = 42.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(6*cosh(x)^5 + 6*(5*cosh(x) + 2)*sinh(x)^4 + 6*sinh(x)^5 + 12*cosh(x)^
4 + 4*(15*cosh(x)^2 + 12*cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + 12*(5*cosh(
x)^3 + 6*cosh(x)^2 - cosh(x) + 1)*sinh(x)^2 + 12*cosh(x)^2 - 3*(cosh(x)^6 +
 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10
*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x
) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2
 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2
*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x
) + sinh(x) + 1) + 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 +
 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5
*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(
x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 +
 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*si
nh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(5*cosh(x)^4 + 8*cosh
(x)^3 - 2*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 6*cosh(x))/(a*cosh(x)^6 + a*
```

```
sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4 + (
15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*cosh(
x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*cosh(
x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*a*c
osh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^2 -
 a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{csch}^3(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**3/(cosh(x) + 1), x)/a
```

Giac [B] time = 1.18777, size = 127, normalized size = 2.59

$$\frac{3\,\log\left(e^{(-x)}+e^x+2\right)}{16\,a}-\frac{3\,\log\left(e^{(-x)}+e^x-2\right)}{16\,a}+\frac{3\,e^{(-x)}+3\,e^x-10}{16\,a\left(e^{(-x)}+e^x-2\right)}-\frac{9\left(e^{(-x)}+e^x\right)^2+52\,e^{(-x)}+52\,e^x+84}{32\,a\left(e^{(-x)}+e^x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3/16*log(e^(-x) + e^x + 2)/a - 3/16*log(e^(-x) + e^x - 2)/a + 1/16*(3*e^(-x
) + 3*e^x - 10)/(a*(e^(-x) + e^x - 2)) - 1/32*(9*(e^(-x) + e^x)^2 + 52*e^(-
x) + 52*e^x + 84)/(a*(e^(-x) + e^x + 2)^2)
```

3.163 $\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{4\coth^3(x)}{15a}+\frac{4\coth(x)}{5a}+\frac{\operatorname{csch}^3(x)}{5(a\cosh(x)+a)}$$

```
[Out] (4*Coth[x])/(5*a) - (4*Coth[x]^3)/(15*a) + Csch[x]^3/(5*(a + a*Cosh[x]))
```

Rubi [A] time = 0.0507936, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2672, 3767}

$$-\frac{4\coth^3(x)}{15a}+\frac{4\coth(x)}{5a}+\frac{\operatorname{csch}^3(x)}{5(a\cosh(x)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (4*Coth[x])/(5*a) - (4*Coth[x]^3)/(15*a) + Csch[x]^3/(5*(a + a*Cosh[x]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx = \frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}+\frac{4\int \operatorname{csch}^4(x)\,dx}{5a}$$
$$= \frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}+\frac{(4i)\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\coth(x)\right)}{5a}$$
$$= \frac{4\coth(x)}{5a}-\frac{4\coth^3(x)}{15a}+\frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}$$

Mathematica [A] time = 0.0533196, size = 38, normalized size = 1.03

$$\frac{(-6\cosh(x)-2\cosh(2x)+2\cosh(3x)+\cosh(4x))\operatorname{csch}^3(x)}{15a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] ((-6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(15*a*(1 +
 Cosh[x]))
```

Maple [A] time = 0.02, size = 45, normalized size = 1.2

$$\frac{1}{16\,a}\left(\frac{1}{5}\left(\tanh\left(\frac{x}{2}\right)\right)^5-\frac{4}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3+6\,\tanh(x/2)+4\,(\tanh(x/2))^{-1}-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/16/a*(1/5*tanh(1/2*x)^5-4/3*tanh(1/2*x)^3+6*tanh(1/2*x)+4/tanh(1/2*x)-1/3
/tanh(1/2*x)^3)
```

Maxima [B] time = 1.03411, size = 315, normalized size = 8.51

$$\frac{32\,e^{(-x)}}{15\left(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}+6\,ae^{(-5\,x)}+2\,ae^{(-6\,x)}-2\,ae^{(-7\,x)}-ae^{(-8\,x)}+a\right)}-\frac{3}{15\left(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 32/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a
*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 32/15*e^(-2*x)/(2*a*e^(-x) - 2
*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a
*e^(-8*x) + a) - 32/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) +
6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 16/15/(2*a*e
^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(
-7*x) - a*e^(-8*x) + a)
```

Fricas [B] time = 1.75972, size = 809, normalized size = 21.86

$$-\frac{}{15\left(a\cosh\left(x\right)^{7}+a\sinh\left(x\right)^{7}+2\,a\cosh\left(x\right)^{6}+\left(7\,a\cosh\left(x\right)+2\,a\right)\sinh\left(x\right)^{6}-2\,a\cosh\left(x\right)^{5}+\left(21\,a\cosh\left(x\right)^{2}+12\,a\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -16/15*(6*cosh(x)^2 + 3*(4*cosh(x) + 1)*sinh(x) + 6*sinh(x)^2 + cosh(x) - 2
)/(a*cosh(x)^7 + a*sinh(x)^7 + 2*a*cosh(x)^6 + (7*a*cosh(x) + 2*a)*sinh(x)^
6 - 2*a*cosh(x)^5 + (21*a*cosh(x)^2 + 12*a*cosh(x) - 2*a)*sinh(x)^5 - 6*a*c
osh(x)^4 + (35*a*cosh(x)^3 + 30*a*cosh(x)^2 - 10*a*cosh(x) - 6*a)*sinh(x)^4
 + (35*a*cosh(x)^4 + 40*a*cosh(x)^3 - 20*a*cosh(x)^2 - 24*a*cosh(x))*sinh(x
)^3 + 6*a*cosh(x)^2 + (21*a*cosh(x)^5 + 30*a*cosh(x)^4 - 20*a*cosh(x)^3 - 3
6*a*cosh(x)^2 + 6*a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 12*a*cosh(x)^
5 - 10*a*cosh(x)^4 - 24*a*cosh(x)^3 + 12*a*cosh(x) + 3*a)*sinh(x) - 2*a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1316, size = 80, normalized size = 2.16

$$\frac{9\,e^{(2\,x)} - 24\,e^{x} + 11}{24\,a{\left(e^{x} - 1\right)}^{3}} - \frac{45\,e^{(4\,x)} + 240\,e^{(3\,x)} + 490\,e^{(2\,x)} + 320\,e^{x} + 73}{120\,a{\left(e^{x} + 1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(9*e^(2*x) - 24*e^x + 11)/(a*(e^x - 1)^3) - 1/120*(45*e^(4*x) + 240*e^
(3*x) + 490*e^(2*x) + 320*e^x + 73)/(a*(e^x + 1)^5)
```

3.164 $\int \frac{\text{csch}^5(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=78

$$\frac{a^2}{24(a\cosh(x)+a)^3}-\frac{a}{32(a-a\cosh(x))^2}+\frac{3a}{32(a\cosh(x)+a)^2}-\frac{1}{8(a-a\cosh(x))}+\frac{3}{16(a\cosh(x)+a)}-\frac{5\tanh^{-1}(\cosh(x))}{16a}$$

```
[Out] (-5*ArcTanh[Cosh[x]])/(16*a) - a/(32*(a - a*Cosh[x])^2) - 1/(8*(a - a*Cosh[
x])) + a^2/(24*(a + a*Cosh[x])^3) + (3*a)/(32*(a + a*Cosh[x])^2) + 3/(16*(a
 + a*Cosh[x]))
```

Rubi [A] time = 0.106355, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 206}

$$\frac{a^2}{24(a\cosh(x)+a)^3}-\frac{a}{32(a-a\cosh(x))^2}+\frac{3a}{32(a\cosh(x)+a)^2}-\frac{1}{8(a-a\cosh(x))}+\frac{3}{16(a\cosh(x)+a)}-\frac{5\tanh^{-1}(\cosh(x))}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^5/(a + a*Cosh[x]),x]
```


```
[Out] (-5*ArcTanh[Cosh[x]])/(16*a) - a/(32*(a - a*Cosh[x])^2) - 1/(8*(a - a*Cosh[
x])) + a^2/(24*(a + a*Cosh[x])^3) + (3*a)/(32*(a + a*Cosh[x])^2) + 3/(16*(a
 + a*Cosh[x]))
```


Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = -\left(a^5\operatorname{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^4}\,dx, x, a\cosh(x)\right)\right)$$
$$= -\left(a^5\operatorname{Subst}\left(\int\left(\frac{1}{16a^4(a-x)^3}+\frac{1}{8a^5(a-x)^2}+\frac{1}{8a^3(a+x)^4}+\frac{3}{16a^4(a+x)^3}+\frac{3}{16a^5(a+x)^2}+\frac{}{16a^5\ldots}\right.\right.\right.$$
$$= -\frac{a}{32(a-a\cosh(x))^2}-\frac{1}{8(a-a\cosh(x))}+\frac{a^2}{24(a+a\cosh(x))^3}+\frac{3a}{32(a+a\cosh(x))^2}+\frac{3}{16(a+a\operatorname{co}\ldots}$$
$$= -\frac{5\tanh^{-1}(\cosh(x))}{16a}-\frac{a}{32(a-a\cosh(x))^2}-\frac{1}{8(a-a\cosh(x))}+\frac{a^2}{24(a+a\cosh(x))^3}+\frac{3a}{32(a+a\operatorname{co}\ldots}$$

Mathematica [A] time = 0.256084, size = 89, normalized size = 1.14

$$\frac{\cosh^2\left(\frac{x}{2}\right)\left(-3\operatorname{csch}^4\left(\frac{x}{2}\right)+24\operatorname{csch}^2\left(\frac{x}{2}\right)+2\operatorname{sech}^6\left(\frac{x}{2}\right)+9\operatorname{sech}^4\left(\frac{x}{2}\right)+36\operatorname{sech}^2\left(\frac{x}{2}\right)+120\log\left(\sinh\left(\frac{x}{2}\right)\right)-120\log\left(\cosh\left(\ldots\right.\right.\right.}{192(a\cosh(x)+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]^2*(24*Csch[x/2]^2 - 3*Csch[x/2]^4 - 120*Log[Cosh[x/2]] + 120*Log
[Sinh[x/2]] + 36*Sech[x/2]^2 + 9*Sech[x/2]^4 + 2*Sech[x/2]^6))/(192*(a + a*
Cosh[x]))
```

Maple [A] time = 0.022, size = 67, normalized size = 0.9

$$-\frac{1}{192\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^6+\frac{5}{128\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^4-\frac{5}{32\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{128\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-4}+\frac{5}{16\,a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{5}{64\,a}\left(\tanh\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^5/(a+a*cosh(x)),x)
```

```
[Out] -1/192/a*tanh(1/2*x)^6+5/128/a*tanh(1/2*x)^4-5/32/a*tanh(1/2*x)^2-1/128/a/t
anh(1/2*x)^4+5/16/a*ln(tanh(1/2*x))+5/64/a/tanh(1/2*x)^2
```

Maxima [B] time = 1.02713, size = 209, normalized size = 2.68

$$\frac{15\,e^{(-x)} + 30\,e^{(-2\,x)} - 40\,e^{(-3\,x)} - 110\,e^{(-4\,x)} + 18\,e^{(-5\,x)} - 110\,e^{(-6\,x)} - 40\,e^{(-7\,x)} + 30\,e^{(-8\,x)} + 15\,e^{(-9\,x)}}{24\left(2\,ae^{(-x)} - 3\,ae^{(-2\,x)} - 8\,ae^{(-3\,x)} + 2\,ae^{(-4\,x)} + 12\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 8\,ae^{(-7\,x)} - 3\,ae^{(-8\,x)} + 2\,ae^{(-9\,x)} + ae^{(-10\,x)} + a\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(15*e^(-x) + 30*e^(-2*x) - 40*e^(-3*x) - 110*e^(-4*x) + 18*e^(-5*x) -
110*e^(-6*x) - 40*e^(-7*x) + 30*e^(-8*x) + 15*e^(-9*x))/(2*a*e^(-x) - 3*a*e
^(-2*x) - 8*a*e^(-3*x) + 2*a*e^(-4*x) + 12*a*e^(-5*x) + 2*a*e^(-6*x) - 8*a*
e^(-7*x) - 3*a*e^(-8*x) + 2*a*e^(-9*x) + a*e^(-10*x) + a) - 5/16*log(e^(-x)
 + 1)/a + 5/16*log(e^(-x) - 1)/a
```

Fricas [B] time = 2.08501, size = 5257, normalized size = 67.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/48*(30*cosh(x)^9 + 30*(9*cosh(x) + 2)*sinh(x)^8 + 30*sinh(x)^9 + 60*cosh(
x)^8 + 40*(27*cosh(x)^2 + 12*cosh(x) - 2)*sinh(x)^7 - 80*cosh(x)^7 + 20*(12
6*cosh(x)^3 + 84*cosh(x)^2 - 28*cosh(x) - 11)*sinh(x)^6 - 220*cosh(x)^6 + 1
2*(315*cosh(x)^4 + 280*cosh(x)^3 - 140*cosh(x)^2 - 110*cosh(x) + 3)*sinh(x)
^5 + 36*cosh(x)^5 + 20*(189*cosh(x)^5 + 210*cosh(x)^4 - 140*cosh(x)^3 - 165
*cosh(x)^2 + 9*cosh(x) - 11)*sinh(x)^4 - 220*cosh(x)^4 + 40*(63*cosh(x)^6 +
 84*cosh(x)^5 - 70*cosh(x)^4 - 110*cosh(x)^3 + 9*cosh(x)^2 - 22*cosh(x) - 2
)*sinh(x)^3 - 80*cosh(x)^3 + 60*(18*cosh(x)^7 + 28*cosh(x)^6 - 28*cosh(x)^5
 - 55*cosh(x)^4 + 6*cosh(x)^3 - 22*cosh(x)^2 - 4*cosh(x) + 1)*sinh(x)^2 + 6
0*cosh(x)^2 - 15*(cosh(x)^10 + 2*(5*cosh(x) + 1)*sinh(x)^9 + sinh(x)^10 + 2
*cosh(x)^9 + 3*(15*cosh(x)^2 + 6*cosh(x) - 1)*sinh(x)^8 - 3*cosh(x)^8 + 8*(
15*cosh(x)^3 + 9*cosh(x)^2 - 3*cosh(x) - 1)*sinh(x)^7 - 8*cosh(x)^7 + 2*(10
5*cosh(x)^4 + 84*cosh(x)^3 - 42*cosh(x)^2 - 28*cosh(x) + 1)*sinh(x)^6 + 2*c
```

```
osh(x)^6 + 12*(21*cosh(x)^5 + 21*cosh(x)^4 - 14*cosh(x)^3 - 14*cosh(x)^2 +
cosh(x) + 1)*sinh(x)^5 + 12*cosh(x)^5 + 2*(105*cosh(x)^6 + 126*cosh(x)^5 -
105*cosh(x)^4 - 140*cosh(x)^3 + 15*cosh(x)^2 + 30*cosh(x) + 1)*sinh(x)^4 +
2*cosh(x)^4 + 8*(15*cosh(x)^7 + 21*cosh(x)^6 - 21*cosh(x)^5 - 35*cosh(x)^4
+ 5*cosh(x)^3 + 15*cosh(x)^2 + cosh(x) - 1)*sinh(x)^3 - 8*cosh(x)^3 + 3*(15
*cosh(x)^8 + 24*cosh(x)^7 - 28*cosh(x)^6 - 56*cosh(x)^5 + 10*cosh(x)^4 + 40
*cosh(x)^3 + 4*cosh(x)^2 - 8*cosh(x) - 1)*sinh(x)^2 - 3*cosh(x)^2 + 2*(5*co
sh(x)^9 + 9*cosh(x)^8 - 12*cosh(x)^7 - 28*cosh(x)^6 + 6*cosh(x)^5 + 30*cosh
(x)^4 + 4*cosh(x)^3 - 12*cosh(x)^2 - 3*cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1
)*log(cosh(x) + sinh(x) + 1) + 15*(cosh(x)^10 + 2*(5*cosh(x) + 1)*sinh(x)^9
 + sinh(x)^10 + 2*cosh(x)^9 + 3*(15*cosh(x)^2 + 6*cosh(x) - 1)*sinh(x)^8 -
3*cosh(x)^8 + 8*(15*cosh(x)^3 + 9*cosh(x)^2 - 3*cosh(x) - 1)*sinh(x)^7 - 8*
cosh(x)^7 + 2*(105*cosh(x)^4 + 84*cosh(x)^3 - 42*cosh(x)^2 - 28*cosh(x) + 1
)*sinh(x)^6 + 2*cosh(x)^6 + 12*(21*cosh(x)^5 + 21*cosh(x)^4 - 14*cosh(x)^3
- 14*cosh(x)^2 + cosh(x) + 1)*sinh(x)^5 + 12*cosh(x)^5 + 2*(105*cosh(x)^6 +
 126*cosh(x)^5 - 105*cosh(x)^4 - 140*cosh(x)^3 + 15*cosh(x)^2 + 30*cosh(x)
+ 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(15*cosh(x)^7 + 21*cosh(x)^6 - 21*cosh(x)^
5 - 35*cosh(x)^4 + 5*cosh(x)^3 + 15*cosh(x)^2 + cosh(x) - 1)*sinh(x)^3 - 8*
cosh(x)^3 + 3*(15*cosh(x)^8 + 24*cosh(x)^7 - 28*cosh(x)^6 - 56*cosh(x)^5 +
10*cosh(x)^4 + 40*cosh(x)^3 + 4*cosh(x)^2 - 8*cosh(x) - 1)*sinh(x)^2 - 3*co
sh(x)^2 + 2*(5*cosh(x)^9 + 9*cosh(x)^8 - 12*cosh(x)^7 - 28*cosh(x)^6 + 6*co
sh(x)^5 + 30*cosh(x)^4 + 4*cosh(x)^3 - 12*cosh(x)^2 - 3*cosh(x) + 1)*sinh(x
) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 10*(27*cosh(x)^8 + 48*cosh(
x)^7 - 56*cosh(x)^6 - 132*cosh(x)^5 + 18*cosh(x)^4 - 88*cosh(x)^3 - 24*cosh
(x)^2 + 12*cosh(x) + 3)*sinh(x) + 30*cosh(x))/(a*cosh(x)^10 + a*sinh(x)^10
+ 2*a*cosh(x)^9 + 2*(5*a*cosh(x) + a)*sinh(x)^9 - 3*a*cosh(x)^8 + 3*(15*a*c
osh(x)^2 + 6*a*cosh(x) - a)*sinh(x)^8 - 8*a*cosh(x)^7 + 8*(15*a*cosh(x)^3 +
 9*a*cosh(x)^2 - 3*a*cosh(x) - a)*sinh(x)^7 + 2*a*cosh(x)^6 + 2*(105*a*cosh
(x)^4 + 84*a*cosh(x)^3 - 42*a*cosh(x)^2 - 28*a*cosh(x) + a)*sinh(x)^6 + 12*
a*cosh(x)^5 + 12*(21*a*cosh(x)^5 + 21*a*cosh(x)^4 - 14*a*cosh(x)^3 - 14*a*c
osh(x)^2 + a*cosh(x) + a)*sinh(x)^5 + 2*a*cosh(x)^4 + 2*(105*a*cosh(x)^6 +
126*a*cosh(x)^5 - 105*a*cosh(x)^4 - 140*a*cosh(x)^3 + 15*a*cosh(x)^2 + 30*a
*cosh(x) + a)*sinh(x)^4 - 8*a*cosh(x)^3 + 8*(15*a*cosh(x)^7 + 21*a*cosh(x)^
6 - 21*a*cosh(x)^5 - 35*a*cosh(x)^4 + 5*a*cosh(x)^3 + 15*a*cosh(x)^2 + a*co
sh(x) - a)*sinh(x)^3 - 3*a*cosh(x)^2 + 3*(15*a*cosh(x)^8 + 24*a*cosh(x)^7 -
 28*a*cosh(x)^6 - 56*a*cosh(x)^5 + 10*a*cosh(x)^4 + 40*a*cosh(x)^3 + 4*a*co
sh(x)^2 - 8*a*cosh(x) - a)*sinh(x)^2 + 2*a*cosh(x) + 2*(5*a*cosh(x)^9 + 9*a
*cosh(x)^8 - 12*a*cosh(x)^7 - 28*a*cosh(x)^6 + 6*a*cosh(x)^5 + 30*a*cosh(x)
^4 + 4*a*cosh(x)^3 - 12*a*cosh(x)^2 - 3*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**5/(a+a*cosh(x)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.16065, size = 157, normalized size = 2.01

$$-\frac{5\,\log\left(e^{(-x)}+e^{x}+2\right)}{32\,a}+\frac{5\,\log\left(e^{(-x)}+e^{x}-2\right)}{32\,a}-\frac{15\left(e^{(-x)}+e^{x}\right)^{2}-76\,e^{(-x)}-76\,e^{x}+100}{64\,a\left(e^{(-x)}+e^{x}-2\right)^{2}}+\frac{55\left(e^{(-x)}+e^{x}\right)^{3}+402\left(e^{(-x)}\right.}{192\,a\left(e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -5/32*log(e^(-x) + e^x + 2)/a + 5/32*log(e^(-x) + e^x - 2)/a - 1/64*(15*(e^
(-x) + e^x)^2 - 76*e^(-x) - 76*e^x + 100)/(a*(e^(-x) + e^x - 2)^2) + 1/192*
(55*(e^(-x) + e^x)^3 + 402*(e^(-x) + e^x)^2 + 1020*e^(-x) + 1020*e^x + 936)
/(a*(e^(-x) + e^x + 2)^3)
```

3.165 $\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=140

$$\frac{(a^2-3b^2)\cosh^4(x)}{4b^3}-\frac{a(a^2-3b^2)\cosh^3(x)}{3b^4}+\frac{(-3a^2b^2+a^4+3b^4)\cosh^2(x)}{2b^5}-\frac{a(-3a^2b^2+a^4+3b^4)\cosh(x)}{b^6}+\frac{(a^2-b^2}{}$$

```
[Out] -((a*(a^4 - 3*a^2*b^2 + 3*b^4)*Cosh[x])/b^6) + ((a^4 - 3*a^2*b^2 + 3*b^4)*C
osh[x]^2)/(2*b^5) - (a*(a^2 - 3*b^2)*Cosh[x]^3)/(3*b^4) + ((a^2 - 3*b^2)*Co
sh[x]^4)/(4*b^3) - (a*Cosh[x]^5)/(5*b^2) + Cosh[x]^6/(6*b) + ((a^2 - b^2)^3
*Log[a + b*Cosh[x]])/b^7
```

Rubi [A] time = 0.167012, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{(a^2-3b^2)\cosh^4(x)}{4b^3}-\frac{a(a^2-3b^2)\cosh^3(x)}{3b^4}+\frac{(-3a^2b^2+a^4+3b^4)\cosh^2(x)}{2b^5}-\frac{a(-3a^2b^2+a^4+3b^4)\cosh(x)}{b^6}+\frac{(a^2-b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^7/(a + b*Cosh[x]),x]
```

```
[Out] -((a*(a^4 - 3*a^2*b^2 + 3*b^4)*Cosh[x])/b^6) + ((a^4 - 3*a^2*b^2 + 3*b^4)*C
osh[x]^2)/(2*b^5) - (a*(a^2 - 3*b^2)*Cosh[x]^3)/(3*b^4) + ((a^2 - 3*b^2)*Co
sh[x]^4)/(4*b^3) - (a*Cosh[x]^5)/(5*b^2) + Cosh[x]^6/(6*b) + ((a^2 - b^2)^3
*Log[a + b*Cosh[x]])/b^7
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{a+x}\,dx, x, b\cosh(x)\right)}{b^7}$$

$$= -\frac{\text{Subst}\left(\int \left(a^5\left(1+\frac{3b^2\left(-a^2+b^2\right)}{a^4}\right) - \left(a^4-3a^2b^2+3b^4\right)x + a\left(a^2-3b^2\right)x^2 - \left(a^2-3b^2\right)x^3 + ax^4 - \right.\right.}{b^7}$$

$$= -\frac{a\left(a^4-3a^2b^2+3b^4\right)\cosh(x)}{b^6} + \frac{\left(a^4-3a^2b^2+3b^4\right)\cosh^2(x)}{2b^5} - \frac{a\left(a^2-3b^2\right)\cosh^3(x)}{3b^4} + \frac{\left(a^2-3\right.}{}$$

Mathematica [A] time = 0.17459, size = 144, normalized size = 1.03

$$\frac{-30b^4\left(2b^2-a^2\right)\cosh(4x)+15b^2\left(-40a^2b^2+16a^4+29b^4\right)\cosh(2x)-120ab\left(-22a^2b^2+8a^4+19b^4\right)\cosh(x)+960\left(a\right.}{960b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^7/(a + b*Cosh[x]),x]
```

```
[Out] (-120*a*b*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Cosh[x] + 15*b^2*(16*a^4 - 40*a^2*b
^2 + 29*b^4)*Cosh[2*x] - 20*a*(2*a - 3*b)*b^3*(2*a + 3*b)*Cosh[3*x] - 30*b^
4*(-a^2 + 2*b^2)*Cosh[4*x] - 12*a*b^5*Cosh[5*x] + 5*b^6*Cosh[6*x] + 960*(a^
2 - b^2)^3*Log[a + b*Cosh[x]])/(960*b^7)
```

Maple [B] time = 0.034, size = 1039, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a+b*cosh(x)),x)
```

```
[Out] 7/12/b/(tanh(1/2*x)+1)^3+5/16/b/(tanh(1/2*x)+1)^2-11/16/b/(tanh(1/2*x)+1)-7
/12/b/(tanh(1/2*x)-1)^3+5/16/b/(tanh(1/2*x)-1)^2+11/16/b/(tanh(1/2*x)-1)+1/
b*ln(tanh(1/2*x)+1)+1/b*ln(tanh(1/2*x)-1)-3/b^2/(a-b)*ln(a*tanh(1/2*x)^2-ta
nh(1/2*x)^2*b-a-b)*a^2-1/b/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a+
1/b^7/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^7-1/b^6/(a-b)*ln(a*ta
```

```
nh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^6-3/b^5/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/
2*x)^2*b-a-b)*a^5+3/b^4/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^4+3
/b^3/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^3+1/2/b/(tanh(1/2*x)-1
)^5+1/8/b/(tanh(1/2*x)-1)^4+1/6/b/(tanh(1/2*x)+1)^6+1/6/b/(tanh(1/2*x)-1)^6
+1/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)-1/2/b/(tanh(1/2*x)+1)^5+1/
8/b/(tanh(1/2*x)+1)^4-3/b^3*ln(tanh(1/2*x)+1)*a^2-3/b^3*ln(tanh(1/2*x)-1)*a
^2-7/8/b^2/(tanh(1/2*x)+1)^2*a+9/8/b^3/(tanh(1/2*x)+1)*a^2-15/8/b^2/(tanh(1
/2*x)+1)*a-7/8/b^2/(tanh(1/2*x)-1)^2*a-9/8/b^3/(tanh(1/2*x)-1)*a^2+15/8/b^2
/(tanh(1/2*x)-1)*a-1/5/b^2/(tanh(1/2*x)+1)^5*a+1/4/b^3/(tanh(1/2*x)+1)^4*a^
2+1/2/b^2/(tanh(1/2*x)+1)^4*a-1/3/b^4/(tanh(1/2*x)+1)^3*a^3-1/2/b^3/(tanh(1
/2*x)+1)^3*a^2+1/4/b^2/(tanh(1/2*x)+1)^3*a+3/b^5*ln(tanh(1/2*x)+1)*a^4+1/2/
b^5/(tanh(1/2*x)+1)^2*a^4+1/2/b^4/(tanh(1/2*x)+1)^2*a^3-7/8/b^3/(tanh(1/2*x
)+1)^2*a^2-1/b^6/(tanh(1/2*x)+1)*a^5-1/2/b^5/(tanh(1/2*x)+1)*a^4+5/2/b^4/(t
anh(1/2*x)+1)*a^3+1/5/b^2/(tanh(1/2*x)-1)^5*a+1/4/b^3/(tanh(1/2*x)-1)^4*a^2
+1/2/b^2/(tanh(1/2*x)-1)^4*a+1/3/b^4/(tanh(1/2*x)-1)^3*a^3+1/2/b^3/(tanh(1/
2*x)-1)^3*a^2-1/4/b^2/(tanh(1/2*x)-1)^3*a-1/b^7*ln(tanh(1/2*x)-1)*a^6+3/b^5
*ln(tanh(1/2*x)-1)*a^4+1/2/b^5/(tanh(1/2*x)-1)^2*a^4+1/2/b^4/(tanh(1/2*x)-1
)^2*a^3-7/8/b^3/(tanh(1/2*x)-1)^2*a^2+1/b^6/(tanh(1/2*x)-1)*a^5+1/2/b^5/(ta
nh(1/2*x)-1)*a^4-5/2/b^4/(tanh(1/2*x)-1)*a^3-1/b^7*ln(tanh(1/2*x)+1)*a^6
```

Maxima [B] time = 1.06457, size = 419, normalized size = 2.99

$$-\frac{\left(12\,ab^{4}e^{(-x)}-5\,b^{5}-30\,\left(a^{2}b^{3}-2\,b^{5}\right)e^{(-2\,x)}+20\,\left(4\,a^{3}b^{2}-9\,ab^{4}\right)e^{(-3\,x)}-15\,\left(16\,a^{4}b-40\,a^{2}b^{3}+29\,b^{5}\right)e^{(-4\,x)}+120\,\left(8\,a^{5}-\right.\right.}{1920\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 - 2*b^5)*e^(-2*x) + 20*(4*a^
3*b^2 - 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-4*x) +
120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 - 1/1920*(12*a*b^
4*e^(-5*x) - 5*b^5*e^(-6*x) + 120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-x) -
15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 - 9*a*b^4)*e^(
-3*x) - 30*(a^2*b^3 - 2*b^5)*e^(-4*x))/b^6 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*x/b^7 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(2*a*e^(-x) + b*e^(-2*
x) + b)/b^7
```

Fricas [B] time = 2.15659, size = 5296, normalized size = 37.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*b
^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^10 + 6*(55*b^
6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 - 10*b^6)*sinh(x)^10 - 20*(4*a^3
*b^3 - 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 - 4*a
^3*b^3 + 9*a*b^5 + 15*(a^2*b^4 - 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^4*b^2
 - 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5*cosh(
x)^3 + 16*a^4*b^2 - 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 - 2*b^6)*cosh(x)^2 -
12*(4*a^3*b^3 - 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*x*cosh(x)^6 - 120*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^7 +
 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b + 22*a^3*b^3 - 19*a*b
^5 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^2 + (
16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 - 12*a*b^5*cosh(x) + 1
2*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 - 2*b^6)*cosh(x)^
4 - 140*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 - 40*a^2*b^4 + 29*
b^6)*cosh(x)^2 - 160*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 70*(8*a^5*b -
22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 - 120*(8*a^5*b - 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b^5*cosh(x)^6 - 40
*a^5*b + 110*a^3*b^3 - 95*a*b^5 + 315*(a^2*b^4 - 2*b^6)*cosh(x)^5 - 105*(4*
a^3*b^3 - 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x
)^3 - 480*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x) - 105*(8*a^5*b - 22
*a^3*b^3 + 19*a*b^5)*cosh(x)^2)*sinh(x)^5 + 15*(16*a^4*b^2 - 40*a^2*b^4 + 2
9*b^6)*cosh(x)^4 + 15*(165*b^6*cosh(x)^8 - 264*a*b^5*cosh(x)^7 + 420*(a^2*b
^4 - 2*b^6)*cosh(x)^6 + 16*a^4*b^2 - 40*a^2*b^4 + 29*b^6 - 168*(4*a^3*b^3 -
 9*a*b^5)*cosh(x)^5 + 70*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^4 - 192
0*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 - 280*(8*a^5*b - 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^3 - 40*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x))*si
nh(x)^4 - 20*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^3 + 20*(55*b^6*cosh(x)^9 - 99*a*
b^5*cosh(x)^8 + 180*(a^2*b^4 - 2*b^6)*cosh(x)^7 - 84*(4*a^3*b^3 - 9*a*b^5)*
cosh(x)^6 - 4*a^3*b^3 + 9*a*b^5 + 42*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cos
h(x)^5 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 - 210*(8*a^5*
b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^4 - 60*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)
*cosh(x)^2 + 3*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^3 + 30*(
a^2*b^4 - 2*b^6)*cosh(x)^2 + 30*(11*b^6*cosh(x)^10 - 22*a*b^5*cosh(x)^9 + 4
5*(a^2*b^4 - 2*b^6)*cosh(x)^8 - 24*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^7 + 14*(16
*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^6 + a^2*b^4 - 2*b^6 - 960*(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^4 - 84*(8*a^5*b - 22*a^3*b^3 + 19*a*b^
5)*cosh(x)^5 - 40*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^3 + 3*(16*a^4*b
^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^2 - 2*(4*a^3*b^3 - 9*a*b^5)*cosh(x))*sinh
(x)^2 + 1920*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5*sinh(x) + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^
```

```
4 - b^6)*cosh(x)^4*sinh(x)^2 + 20*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(
x)^3*sinh(x)^3 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^4
 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*sinh(x)^6)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x)))
 + 12*(5*b^6*cosh(x)^11 - 11*a*b^5*cosh(x)^10 + 25*(a^2*b^4 - 2*b^6)*cosh(x
)^9 - 15*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^8 + 10*(16*a^4*b^2 - 40*a^2*b^4 + 29
*b^6)*cosh(x)^7 - 960*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 - 70*
(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^6 - a*b^5 - 50*(8*a^5*b - 22*a^3*
b^3 + 19*a*b^5)*cosh(x)^4 + 5*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^3
- 5*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^2 + 5*(a^2*b^4 - 2*b^6)*cosh(x))*sinh(x))
/(b^7*cosh(x)^6 + 6*b^7*cosh(x)^5*sinh(x) + 15*b^7*cosh(x)^4*sinh(x)^2 + 20
*b^7*cosh(x)^3*sinh(x)^3 + 15*b^7*cosh(x)^2*sinh(x)^4 + 6*b^7*cosh(x)*sinh(
x)^5 + b^7*sinh(x)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**7/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13178, size = 309, normalized size = 2.21

$$5\,b^5\left(e^{(-x)}+e^x\right)^6-12\,ab^4\left(e^{(-x)}+e^x\right)^5+30\,a^2b^3\left(e^{(-x)}+e^x\right)^4-90\,b^5\left(e^{(-x)}+e^x\right)^4-80\,a^3b^2\left(e^{(-x)}+e^x\right)^3+240\,ab^4\left(e^{(-x)}+e^x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*b^5*(e^(-x) + e^x)^6 - 12*a*b^4*(e^(-x) + e^x)^5 + 30*a^2*b^3*(e^
(-x) + e^x)^4 - 90*b^5*(e^(-x) + e^x)^4 - 80*a^3*b^2*(e^(-x) + e^x)^3 + 240
*a*b^4*(e^(-x) + e^x)^3 + 240*a^4*b*(e^(-x) + e^x)^2 - 720*a^2*b^3*(e^(-x)
+ e^x)^2 + 720*b^5*(e^(-x) + e^x)^2 - 960*a^5*(e^(-x) + e^x) + 2880*a^3*b^2
*(e^(-x) + e^x) - 2880*a*b^4*(e^(-x) + e^x))/b^6 + (a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*log(abs(b*(e^(-x) + e^x) + 2*a))/b^7
```

3.166 $\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=154

$$-\frac{ax\left(-20a^2b^2+8a^4+15b^4\right)}{8b^6}+\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^3}+\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{8b^5}$$

```
[Out] -(a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) + (2*(a - b)^(5/2)*(a + b)^(5/
2)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/b^6 + ((8*(a^2 - b^2)^2 -
a*b*(4*a^2 - 7*b^2)*Cosh[x])*Sinh[x])/(8*b^5) + ((4*(a^2 - b^2) - 3*a*b*Cos
h[x])*Sinh[x]^3)/(12*b^3) + Sinh[x]^5/(5*b)
```

Rubi [A] time = 0.425108, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2695, 2865, 2735, 2659, 208}

$$-\frac{ax\left(-20a^2b^2+8a^4+15b^4\right)}{8b^6}+\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^3}+\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{8b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^6/(a + b*Cosh[x]),x]
```


```
[Out] -(a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) + (2*(a - b)^(5/2)*(a + b)^(5/
2)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/b^6 + ((8*(a^2 - b^2)^2 -
a*b*(4*a^2 - 7*b^2)*Cosh[x])*Sinh[x])/(8*b^5) + ((4*(a^2 - b^2) - 3*a*b*Cos
h[x])*Sinh[x]^3)/(12*b^3) + Sinh[x]^5/(5*b)
```


Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx = \frac{\sinh^5(x)}{5b} + \frac{\int \frac{(-b-a\cosh(x))\sinh^4(x)}{a+b\cosh(x)}\,dx}{b}$$

$$= \frac{\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)\sinh^3(x)}{12b^3} + \frac{\sinh^5(x)}{5b} - \frac{\int \frac{\left(b\left(a^2-4b^2\right)+a\left(4a^2-7b^2\right)\cosh(x)\right)\sinh^2(x)}{a+b\cosh(x)}\,dx}{4b^3}$$

$$= \frac{\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)\sinh^3(x)}{12b^3} + \frac{\sinh^5(}{5b}$$

$$= -\frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{\left(4\left(a^2-b^2\right)-3\right.}{}$$

$$= -\frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{\left(4\left(a^2-b^2\right)-3\right.}{}$$

$$= -\frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{2(a-b)^{5/2}(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^6} + \frac{\left(8\left(a^2-b^2\right)^2-ab\left(4a\right.\right.}{}$$

Mathematica [A] time = 0.225557, size = 154, normalized size = 1.

$$\frac{-60ax\left(-20a^2b^2+8a^4+15b^4\right)-10b^3\left(7b^2-4a^2\right)\sinh(3x)-120ab^2\left(a^2-2b^2\right)\sinh(2x)+60b\left(-18a^2b^2+8a^4+11b^4\right)}{480b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] (-60*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x + 960*(-a^2 + b^2)^(5/2)*ArcTan[((a
- b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + 60*b*(8*a^4 - 18*a^2*b^2 + 11*b^4)*Sinh
[x] - 120*a*b^2*(a^2 - 2*b^2)*Sinh[2*x] - 10*b^3*(-4*a^2 + 7*b^2)*Sinh[3*x]
 - 15*a*b^4*Sinh[4*x] + 6*b^5*Sinh[5*x])/(480*b^6)
```

Maple [B] time = 0.028, size = 679, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a+b*cosh(x)),x)
```

```
[Out] -2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/12/
b/(tanh(1/2*x)+1)^3-5/8/b/(tanh(1/2*x)+1)^2-1/b/(tanh(1/2*x)+1)+1/12/b/(tan
h(1/2*x)-1)^3+5/8/b/(tanh(1/2*x)-1)^2-1/b/(tanh(1/2*x)-1)+2/b^6/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*a^6-1/5/b/(tanh(1/2
*x)-1)^5-1/2/b/(tanh(1/2*x)-1)^4-1/5/b/(tanh(1/2*x)+1)^5+1/2/b/(tanh(1/2*x)
+1)^4-6*a^4/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))
^(1/2))+6*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b
))^(1/2))-a^5/b^6*ln(tanh(1/2*x)+1)+a^5/b^6*ln(tanh(1/2*x)-1)-5/8/b^2/(tanh
(1/2*x)+1)^2*a+2/b^3/(tanh(1/2*x)+1)*a^2+7/8/b^2/(tanh(1/2*x)+1)*a+5/2*a^3/
b^4*ln(tanh(1/2*x)+1)-15/8*a/b^2*ln(tanh(1/2*x)+1)+5/8/b^2/(tanh(1/2*x)-1)^
2*a+2/b^3/(tanh(1/2*x)-1)*a^2+7/8/b^2/(tanh(1/2*x)-1)*a-5/2*a^3/b^4*ln(tanh
(1/2*x)-1)+15/8*a/b^2*ln(tanh(1/2*x)-1)+1/4/b^2/(tanh(1/2*x)+1)^4*a-1/3/b^3
/(tanh(1/2*x)+1)^3*a^2-1/2/b^2/(tanh(1/2*x)+1)^3*a+1/2/b^4/(tanh(1/2*x)+1)^
2*a^3+1/2/b^3/(tanh(1/2*x)+1)^2*a^2-1/b^5/(tanh(1/2*x)+1)*a^4-1/2/b^4/(tanh
(1/2*x)+1)*a^3-1/4/b^2/(tanh(1/2*x)-1)^4*a-1/3/b^3/(tanh(1/2*x)-1)^3*a^2-1/
2/b^2/(tanh(1/2*x)-1)^3*a-1/2/b^4/(tanh(1/2*x)-1)^2*a^3-1/2/b^3/(tanh(1/2*x
)-1)^2*a^2-1/b^5/(tanh(1/2*x)-1)*a^4-1/2/b^4/(tanh(1/2*x)-1)*a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34401, size = 7443, normalized size = 48.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^
5*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^8 + 5*(54*b^5
```

```
*cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 - 14*b^5)*sinh(x)^8 - 120*(a^3*b^
2 - 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*
b^2 + 12*a*b^4 + 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 - 20
*a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(
x)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b - 54*a^2*b^3 +
33*b^5 + 14*(4*a^2*b^3 - 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 - 2*a*b^4)*cosh(x))
*sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4
+ 280*(4*a^2*b^3 - 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 - 2*a*b^4)*cosh(x)^2 -
60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*
cosh(x))*sinh(x)^5 - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^4 +
 10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 - 48*a^4*b + 108*a^2*b^3 - 66*
b^5 + 70*(4*a^2*b^3 - 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 - 2*a*b^4)*cosh(x)^3
- 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b - 18*a^2*b^3 +
 11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 - 2*a*b^4)*cosh(x)^3 + 20*(36*
b^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 - 7*b^5)*cosh(x)^5 + 6*a
^3*b^2 - 12*a*b^4 - 210*(a^3*b^2 - 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 - 20*a^3*
b^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3
- 12*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 - 10*(4*a^2*b^3 - 7
*b^5)*cosh(x)^2 + 10*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3
 - 7*b^5)*cosh(x)^6 - 252*(a^3*b^2 - 2*a*b^4)*cosh(x)^5 - 4*a^2*b^3 + 7*b^5
 - 120*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b - 18*a^2*b
^3 + 11*b^5)*cosh(x)^4 - 36*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*
(a^3*b^2 - 2*a*b^4)*cosh(x))*sinh(x)^2 + 960*((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^5 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^3*sinh(x)^2 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 +
 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 - 2*a^2*b^2 + b^4)*sinh
(x)^5)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) + 5*(12*b^5*cosh(x)^9 - 27*a*b^4*cosh(x)^8 + 16*(4*a^2*b
^3 - 7*b^5)*cosh(x)^7 - 168*(a^3*b^2 - 2*a*b^4)*cosh(x)^6 - 120*(8*a^5 - 20
*a^3*b^2 + 15*a*b^4)*x*cosh(x)^4 + 72*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(
x)^5 + 3*a*b^4 - 48*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3 + 72*(a^3*b^2
 - 2*a*b^4)*cosh(x)^2 - 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x))/(b^6*cosh(x
)^5 + 5*b^6*cosh(x)^4*sinh(x) + 10*b^6*cosh(x)^3*sinh(x)^2 + 10*b^6*cosh(x)
^2*sinh(x)^3 + 5*b^6*cosh(x)*sinh(x)^4 + b^6*sinh(x)^5), 1/960*(6*b^5*cosh(
x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^5*cosh(x) - a*b^4)*
sinh(x)^9 + 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^8 + 5*(54*b^5*cosh(x)^2 - 27*a*b
^4*cosh(x) + 8*a^2*b^3 - 14*b^5)*sinh(x)^8 - 120*(a^3*b^2 - 2*a*b^4)*cosh(x
)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*b^2 + 12*a*b^4 + 4*
(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*x*cosh(x)^5 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^6 + 20*(63*b^5*c
osh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b - 54*a^2*b^3 + 33*b^5 + 14*(4*a^2*
b^3 - 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 - 2*a*b^4)*cosh(x))*sinh(x)^6 + 15*a*b
^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4 + 280*(4*a^2*b^3 -
```

```
7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 - 2*a*b^4)*cosh(x)^2 - 60*(8*a^5 - 20*a^3*
b^2 + 15*a*b^4)*x + 180*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^5
- 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^4 + 10*(126*b^5*cosh(x
)^6 - 189*a*b^4*cosh(x)^5 - 48*a^4*b + 108*a^2*b^3 - 66*b^5 + 70*(4*a^2*b^3
 - 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 - 2*a*b^4)*cosh(x)^3 - 60*(8*a^5 - 20*a^
3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^2)
*sinh(x)^4 + 120*(a^3*b^2 - 2*a*b^4)*cosh(x)^3 + 20*(36*b^5*cosh(x)^7 - 63*
a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 - 7*b^5)*cosh(x)^5 + 6*a^3*b^2 - 12*a*b^4 -
 210*(a^3*b^2 - 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*c
osh(x)^2 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3 - 12*(8*a^4*b - 18*
a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 - 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^2 + 1
0*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3 - 7*b^5)*cosh(x)^6
 - 252*(a^3*b^2 - 2*a*b^4)*cosh(x)^5 - 4*a^2*b^3 + 7*b^5 - 120*(8*a^5 - 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x
)^4 - 36*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*(a^3*b^2 - 2*a*b^4)
*cosh(x))*sinh(x)^2 - 1920*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 5*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3*sin
h(x)^2 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 + 5*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^5)*sqrt(-a^2 +
 b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 5
*(12*b^5*cosh(x)^9 - 27*a*b^4*cosh(x)^8 + 16*(4*a^2*b^3 - 7*b^5)*cosh(x)^7
- 168*(a^3*b^2 - 2*a*b^4)*cosh(x)^6 - 120*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x
*cosh(x)^4 + 72*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^5 + 3*a*b^4 - 48*(8
*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3 + 72*(a^3*b^2 - 2*a*b^4)*cosh(x)^2
- 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x))/(b^6*cosh(x)^5 + 5*b^6*cosh(x)^4*
sinh(x) + 10*b^6*cosh(x)^3*sinh(x)^2 + 10*b^6*cosh(x)^2*sinh(x)^3 + 5*b^6*c
osh(x)*sinh(x)^4 + b^6*sinh(x)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**6/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18367, size = 359, normalized size = 2.33

$$\frac{6\,b^4e^{(5\,x)} - 15\,ab^3e^{(4\,x)} + 40\,a^2b^2e^{(3\,x)} - 70\,b^4e^{(3\,x)} - 120\,a^3be^{(2\,x)} + 240\,ab^3e^{(2\,x)} + 480\,a^4e^x - 1080\,a^2b^2e^x + 660\,b^4e^x}{960\,b^5} - \frac{(8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) - 70*b^4*e^(3*
x) - 120*a^3*b*e^(2*x) + 240*a*b^3*e^(2*x) + 480*a^4*e^x - 1080*a^2*b^2*e^x
 + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*(15
*a*b^4*e^x - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(a^3*
b^2 - 2*a*b^4)*e^(3*x) - 10*(4*a^2*b^3 - 7*b^5)*e^(2*x))*e^(-5*x)/b^6 + 2*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sq
rt(-a^2 + b^2)*b^6)
```

3.167 $\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=83

$$\frac{\left(a^2-2b^2\right)\cosh^2(x)}{2b^3}-\frac{a\left(a^2-2b^2\right)\cosh(x)}{b^4}+\frac{\left(a^2-b^2\right)^2\log(a+b\cosh(x))}{b^5}-\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}$$

```
[Out] -((a*(a^2 - 2*b^2)*Cosh[x])/b^4) + ((a^2 - 2*b^2)*Cosh[x]^2)/(2*b^3) - (a*C
osh[x]^3)/(3*b^2) + Cosh[x]^4/(4*b) + ((a^2 - b^2)^2*Log[a + b*Cosh[x]])/b^
5
```

Rubi [A] time = 0.107238, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{\left(a^2-2b^2\right)\cosh^2(x)}{2b^3}-\frac{a\left(a^2-2b^2\right)\cosh(x)}{b^4}+\frac{\left(a^2-b^2\right)^2\log(a+b\cosh(x))}{b^5}-\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^5/(a + b*Cosh[x]),x]
```


```
[Out] -((a*(a^2 - 2*b^2)*Cosh[x])/b^4) + ((a^2 - 2*b^2)*Cosh[x]^2)/(2*b^3) - (a*C
osh[x]^3)/(3*b^2) + Cosh[x]^4/(4*b) + ((a^2 - b^2)^2*Log[a + b*Cosh[x]])/b^
5
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{a+x}\,dx, x, b\cosh(x)\right)}{b^5}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3\left(1-\frac{2b^2}{a^2}\right)+\left(a^2-2b^2\right)x-ax^2+x^3+\frac{(a^2-b^2)^2}{a+x}\right)dx, x, b\cosh(x)\right)}{b^5}$$
$$= -\frac{a\left(a^2-2b^2\right)\cosh(x)}{b^4}+\frac{\left(a^2-2b^2\right)\cosh^2(x)}{2b^3}-\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}+\frac{\left(a^2-b^2\right)^2\log(a+b\cosh(x))}{b^5}$$

Mathematica [A] time = 0.107378, size = 84, normalized size = 1.01

$$\frac{-12b^2\left(3b^2-2a^2\right)\cosh(2x)-24ab\left(4a^2-7b^2\right)\cosh(x)+96\left(a^2-b^2\right)^2\log(a+b\cosh(x))-8ab^3\cosh(3x)+3b^4\cosh(4x)}{96b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] (-24*a*b*(4*a^2 - 7*b^2)*Cosh[x] - 12*b^2*(-2*a^2 + 3*b^2)*Cosh[2*x] - 8*a*
b^3*Cosh[3*x] + 3*b^4*Cosh[4*x] + 96*(a^2 - b^2)^2*Log[a + b*Cosh[x]])/(96*
b^5)
```

Maple [B] time = 0.026, size = 599, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a+b*cosh(x)),x)
```

```
[Out] -1/2/b/(tanh(1/2*x)+1)^3-3/8/b/(tanh(1/2*x)+1)^2+5/8/b/(tanh(1/2*x)+1)+1/2/
b/(tanh(1/2*x)-1)^3-3/8/b/(tanh(1/2*x)-1)^2-5/8/b/(tanh(1/2*x)-1)-1/b*ln(ta
nh(1/2*x)+1)-1/b*ln(tanh(1/2*x)-1)+2/b^2/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*
x)^2*b-a-b)*a^2+1/b/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a+1/b^5/(
a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^5-1/b^4/(a-b)*ln(a*tanh(1/2*
x)^2-tanh(1/2*x)^2*b-a-b)*a^4-2/b^3/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*
b-a-b)*a^3+1/4/b/(tanh(1/2*x)-1)^4-1/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2
```

```
*b-a-b)+1/4/b/(tanh(1/2*x)+1)^4+2/b^3*ln(tanh(1/2*x)+1)*a^2+2/b^3*ln(tanh(1
/2*x)-1)*a^2+1/2/b^2/(tanh(1/2*x)+1)^2*a-1/2/b^3/(tanh(1/2*x)+1)*a^2+3/2/b^
2/(tanh(1/2*x)+1)*a+1/2/b^2/(tanh(1/2*x)-1)^2*a+1/2/b^3/(tanh(1/2*x)-1)*a^2
-3/2/b^2/(tanh(1/2*x)-1)*a-1/3/b^2/(tanh(1/2*x)+1)^3*a-1/b^5*ln(tanh(1/2*x)
+1)*a^4+1/2/b^3/(tanh(1/2*x)+1)^2*a^2-1/b^4/(tanh(1/2*x)+1)*a^3+1/3/b^2/(ta
nh(1/2*x)-1)^3*a-1/b^5*ln(tanh(1/2*x)-1)*a^4+1/2/b^3/(tanh(1/2*x)-1)^2*a^2+
1/b^4/(tanh(1/2*x)-1)*a^3
```

Maxima [B] time = 1.0499, size = 240, normalized size = 2.89

$$-\frac{\left(8\,ab^{2}e^{(-x)}-3\,b^{3}-12\left(2\,a^{2}b-3\,b^{3}\right)e^{(-2\,x)}+24\left(4\,a^{3}-7\,ab^{2}\right)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,b^{4}}-\frac{8\,ab^{2}e^{(-3\,x)}-3\,b^{3}e^{(-4\,x)}+24\left(4\,a^{3}-7\,ab^{2}\right)e^{(-x}}{192\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b - 3*b^3)*e^(-2*x) + 24*(4*a^3
- 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 - 1/192*(8*a*b^2*e^(-3*x) - 3*b^3*e^(-4*x)
 + 24*(4*a^3 - 7*a*b^2)*e^(-x) - 12*(2*a^2*b - 3*b^3)*e^(-2*x))/b^4 + (a^4
- 2*a^2*b^2 + b^4)*x/b^5 + (a^4 - 2*a^2*b^2 + b^4)*log(2*a*e^(-x) + b*e^(-2
*x) + b)/b^5
```

Fricas [B] time = 1.93527, size = 2228, normalized size = 26.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*cos
h(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 - 3*b^4)*cosh(x)^6 + 4*(21*b^4*cosh
(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 - 9*b^4)*sinh(x)^6 - 192*(a^4 - 2*a^2*
b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b - 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*cosh(
x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b + 7*a*b^3 + 3*(2*a^2*b^2 - 3*b^4)*cosh(x
))*sinh(x)^5 - 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cosh(x)^3
 + 90*(2*a^2*b^2 - 3*b^4)*cosh(x)^2 - 96*(a^4 - 2*a^2*b^2 + b^4)*x - 60*(4*
a^3*b - 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 - 24*(4*a^3*b - 7*a*b^3)*cosh(x
)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 - 12*a^3*b + 21*a*b^3 + 30*(
```

```
2*a^2*b^2 - 3*b^4)*cosh(x)^3 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x) - 30*(4
*a^3*b - 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 - 3*b^4)*cosh(x)^2 +
 12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 - 3*b^4)*cosh(x)^
4 + 2*a^2*b^2 - 3*b^4 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 20*(4*a^3*
b - 7*a*b^3)*cosh(x)^3 - 6*(4*a^3*b - 7*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3*sinh(
x) + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4)*log(2*(b*cosh(x)
 + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3*cosh(x)^6 + 9*(2*
a^2*b^2 - 3*b^4)*cosh(x)^5 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^3 - 15*(4
*a^3*b - 7*a*b^3)*cosh(x)^4 - a*b^3 - 9*(4*a^3*b - 7*a*b^3)*cosh(x)^2 + 3*(
2*a^2*b^2 - 3*b^4)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(
x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**5/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22426, size = 167, normalized size = 2.01

$$\frac{3\,b^3\left(e^{(-x)}+e^x\right)^4-8\,ab^2\left(e^{(-x)}+e^x\right)^3+24\,a^2b\left(e^{(-x)}+e^x\right)^2-48\,b^3\left(e^{(-x)}+e^x\right)^2-96\,a^3\left(e^{(-x)}+e^x\right)+192\,ab^2\left(e^{(-x)}+e^x\right)}{192\,b^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b^3*(e^(-x) + e^x)^4 - 8*a*b^2*(e^(-x) + e^x)^3 + 24*a^2*b*(e^(-x)
 + e^x)^2 - 48*b^3*(e^(-x) + e^x)^2 - 96*a^3*(e^(-x) + e^x) + 192*a*b^2*(e^
(-x) + e^x))/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(b*(e^(-x) + e^x) + 2*a))
/b^5
```

3.168 $\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=104

$$-\frac{ax\left(2a^2-3b^2\right)}{2b^4}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2b^3}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{\sinh^3(x)}{3b}$$

```
[Out] -(a*(2*a^2 - 3*b^2)*x)/(2*b^4) + (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTanh[(Sq
rt[a - b]*Tanh[x/2])/Sqrt[a + b]])/b^4 + ((2*(a^2 - b^2) - a*b*Cosh[x])*Sin
h[x])/(2*b^3) + Sinh[x]^3/(3*b)
```

Rubi [A] time = 0.240763, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2695, 2865, 2735, 2659, 208}

$$-\frac{ax\left(2a^2-3b^2\right)}{2b^4}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2b^3}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{\sinh^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] -(a*(2*a^2 - 3*b^2)*x)/(2*b^4) + (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTanh[(Sq
rt[a - b]*Tanh[x/2])/Sqrt[a + b]])/b^4 + ((2*(a^2 - b^2) - a*b*Cosh[x])*Sin
h[x])/(2*b^3) + Sinh[x]^3/(3*b)
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
```

```
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \frac{\sinh^3(x)}{3b} + \frac{\int \frac{(-b-a\cosh(x))\sinh^2(x)}{a+b\cosh(x)}\,dx}{b}$$

$$= \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2b^3} + \frac{\sinh^3(x)}{3b} - \frac{\int \frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\cosh(x)}{a+b\cosh(x)}\,dx}{2b^3}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2b^3} + \frac{\sinh^3(x)}{3b} + \frac{\left(a^2-b^2\right)^2\int \frac{1}{a+b\cosh(x)}\,dx}{b^4}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2b^3} + \frac{\sinh^3(x)}{3b} + \frac{\left(2\left(a^2-b^2\right)^2\right)\text{Subst}\left(\int \frac{}{a+b}\right.}{b^4}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4} + \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(}{2b^3}$$

Mathematica [A] time = 0.179862, size = 95, normalized size = 0.91

$$\frac{-24\left(b^2-a^2\right)^{3/2}\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)+12a^2b\sinh(x)-12a^3x+18ab^2x-3ab^2\sinh(2x)-15b^3\sinh(x)+b^3\sinh(3x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (-12*a^3*x + 18*a*b^2*x - 24*(-a^2 + b^2)^(3/2)*ArcTan[((a - b)*Tanh[x/2])/
Sqrt[-a^2 + b^2]] + 12*a^2*b*Sinh[x] - 15*b^3*Sinh[x] - 3*a*b^2*Sinh[2*x] +
 b^3*Sinh[3*x])/(12*b^4)
```

Maple [B] time = 0.021, size = 338, normalized size = 3.3

$$2\,\frac{a^4}{b^4\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)\,(a-b)}}\right)-4\,\frac{a^2}{b^2\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)\,(a-b)}}\right)+2\,\frac{1}{\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*cosh(x)),x)
```

```
[Out] 2*a^4/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)-4*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/
2))+2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/
3/b/(tanh(1/2*x)+1)^3+1/2/b^2/(tanh(1/2*x)+1)^2*a+1/2/b/(tanh(1/2*x)+1)^2-1
/b^3/(tanh(1/2*x)+1)*a^2-1/2/b^2/(tanh(1/2*x)+1)*a+1/b/(tanh(1/2*x)+1)-a^3/
b^4*ln(tanh(1/2*x)+1)+3/2*a/b^2*ln(tanh(1/2*x)+1)-1/3/b/(tanh(1/2*x)-1)^3-1
/2/b^2/(tanh(1/2*x)-1)^2*a-1/2/b/(tanh(1/2*x)-1)^2-1/b^3/(tanh(1/2*x)-1)*a^
2-1/2/b^2/(tanh(1/2*x)-1)*a+1/b/(tanh(1/2*x)-1)+a^3/b^4*ln(tanh(1/2*x)-1)-3
/2*a/b^2*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08383, size = 2843, normalized size = 27.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x)
 - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b - 5*b^3
)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b - 5*b^3)*sinh(
x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2*a^
3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x)^3 - b^3 - 3*(4*a^2*b
- 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 - 4*a^2*b + 5*
b^3 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b - 5*b^3)*cosh(x)^2)*sinh(
x)^2 - 24*((a^2 - b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2*sinh(x) + 3*(a^2
 - b^2)*cosh(x)*sinh(x)^2 + (a^2 - b^2)*sinh(x)^3)*sqrt(a^2 - b^2)*log((b^2
*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) +
 a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 3*(2*b^3*c
osh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*
b - 5*b^3)*cosh(x)^3 + a*b^2 - 2*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x))/(b^4*c
osh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^
3), 1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh
(x) - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b - 5*
b^3)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b - 5*b^3)*si
nh(x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2
*a^3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x)^3 - b^3 - 3*(4*a^2
*b - 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 - 4*a^2*b +
 5*b^3 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b - 5*b^3)*cosh(x)^2)*si
nh(x)^2 - 48*((a^2 - b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2*sinh(x) + 3*(
a^2 - b^2)*cosh(x)*sinh(x)^2 + (a^2 - b^2)*sinh(x)^3)*sqrt(-a^2 + b^2)*arct
an(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 3*(2*b^3*co
sh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*b
 - 5*b^3)*cosh(x)^3 + a*b^2 - 2*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x))/(b^4*co
sh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1654, size = 197, normalized size = 1.89

$$\frac{b^2e^{(3x)} - 3\,abe^{(2x)} + 12\,a^2e^x - 15\,b^2e^x}{24\,b^3} - \frac{\left(2\,a^3 - 3\,ab^2\right)x}{2\,b^4} + \frac{\left(3\,ab^2e^x - b^3 - 3\left(4\,a^2b - 5\,b^3\right)e^{(2x)}\right)e^{(-3x)}}{24\,b^4} + \frac{2\left(a^4 - 2\,a^2b^2 + b^4\right.}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x - 15*b^2*e^x)/b^3 - 1/2*(2*a
^3 - 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b - 5*b^3)*e^(2*x)
)*e^(-3*x)/b^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^x + a)/sqrt(-a^2 + b
^2))/(sqrt(-a^2 + b^2)*b^4)
```

3.169 $\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=40

$$\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}-\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}$$

```
[Out] -((a*Cosh[x])/b^2) + Cosh[x]^2/(2*b) + ((a^2 - b^2)*Log[a + b*Cosh[x]])/b^3
```

Rubi [A] time = 0.0689581, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}-\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -((a*Cosh[x])/b^2) + Cosh[x]^2/(2*b) + ((a^2 - b^2)*Log[a + b*Cosh[x]])/b^3
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx, x, b\cosh(x)\right)}{b^3}$$
$$= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right) dx, x, b\cosh(x)\right)}{b^3}$$
$$= -\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}$$

Mathematica [A] time = 0.0528166, size = 40, normalized size = 1.

$$\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}-\frac{a\cosh(x)}{b^2}+\frac{\cosh(2x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -((a*Cosh[x])/b^2) + Cosh[2*x]/(4*b) + ((a^2 - b^2)*Log[a + b*Cosh[x]])/b^3
```

Maple [B] time = 0.02, size = 283, normalized size = 7.1

$$\frac{a^3}{b^3(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2 b-a-b\right)-\frac{a^2}{b^2(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2 b-a-b\right)-\frac{a}{(a-b)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*cosh(x)),x)
```

```
[Out] 1/b^3/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^3-1/b^2/(a-b)*ln(a*ta
nh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^2-1/b/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*
x)^2*b-a-b)*a+1/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+1/2/b/(tanh(1
/2*x)+1)^2-1/b^2/(tanh(1/2*x)+1)*a-1/2/b/(tanh(1/2*x)+1)-1/b^3*ln(tanh(1/2*
x)+1)*a^2+1/b*ln(tanh(1/2*x)+1)+1/2/b/(tanh(1/2*x)-1)^2+1/b^2/(tanh(1/2*x)-
1)*a+1/2/b/(tanh(1/2*x)-1)-1/b^3*ln(tanh(1/2*x)-1)*a^2+1/b*ln(tanh(1/2*x)-1
)
```

Maxima [B] time = 1.02369, size = 113, normalized size = 2.82

$$-\frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2}-\frac{4\,ae^{(-x)}-be^{(-2\,x)}}{8\,b^2}+\frac{\left(a^2-b^2\right)x}{b^3}+\frac{\left(a^2-b^2\right)\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 - 1/8*(4*a*e^(-x) - b*e^(-2*x))/b^2 + (a^
2 - b^2)*x/b^3 + (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/b^3
```

Fricas [B] time = 1.94193, size = 629, normalized size = 15.72

$$\frac{b^2\cosh(x)^4+b^2\sinh(x)^4-4\,ab\cosh(x)^3-8\left(a^2-b^2\right)x\cosh(x)^2+4\left(b^2\cosh(x)-ab\right)\sinh(x)^3-4\,ab\cosh(x)+2\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 - b^2)*x*cosh
(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*b^2*cosh(x)^
2 - 6*a*b*cosh(x) - 4*(a^2 - b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 - b^2)*cosh(
x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2)*log(2*(b*cosh
(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4*(a^2
 - b^2)*x*cosh(x) - a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) +
b^3*sinh(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18362, size = 76, normalized size = 1.9

$$\frac{b\left(e^{(-x)}+e^{x}\right)^{2}-4\,a\left(e^{(-x)}+e^{x}\right)}{8\,b^{2}}+\frac{\left(a^{2}-b^{2}\right)\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2\,a\right|\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*(e^(-x) + e^x)^2 - 4*a*(e^(-x) + e^x))/b^2 + (a^2 - b^2)*log(abs(b*(
e^(-x) + e^x) + 2*a))/b^3
```

3.170 $\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{ax}{b^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}$$

```
[Out] -((a*x)/b^2) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/S
qrt[a + b]])/b^2 + Sinh[x]/b
```

Rubi [A] time = 0.112383, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2695, 2735, 2659, 208}

$$-\frac{ax}{b^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] -((a*x)/b^2) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/S
qrt[a + b]])/b^2 + Sinh[x]/b
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx &= \frac{\sinh(x)}{b}+\frac{\int \frac{-b-a\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\ &= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}-\left(1-\frac{a^2}{b^2}\right)\int \frac{1}{a+b\cosh(x)}\,dx\\ &= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}-\left(2\left(1-\frac{a^2}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\ &= -\frac{ax}{b^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}\end{aligned}$$

Mathematica [A] time = 0.0780804, size = 54, normalized size = 0.92

$$\frac{2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)-ax+b\sinh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (-(a*x) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] +
 b*Sinh[x])/b^2
```

Maple [B] time = 0.019, size = 129, normalized size = 2.2

$$2\,\frac{a^2}{b^2\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)-2\,\frac{1}{\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)-\frac{1}{b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*cosh(x)),x)
```

```
[Out] 2*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)-2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/b/
(tanh(1/2*x)+1)-a/b^2*ln(tanh(1/2*x)+1)-1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1
/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.06663, size = 805, normalized size = 13.64

$$\left[-\frac{2\,ax\cosh\left(x\right) - b\cosh\left(x\right)^{2} - b\sinh\left(x\right)^{2} - 2\,\sqrt{a^{2}-b^{2}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh(x)+2\,a^{2}-b^{2}+2}{b\cosh(x)^{2}+b\sinh(x)^{2}+2\,a}\right.}{2\left(b^{2}\cosh\left(x\right)+b^{2}\sinh\left(x\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(
x) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 -
b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh
(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh
(x) + b)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x)), -
1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 + 4*sqrt(-a^2 + b^2)*(cosh(x
) + sinh(x))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^
2)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12556, size = 92, normalized size = 1.56

$$-\frac{ax}{b^2} - \frac{e^{(-x)}}{2b} + \frac{e^x}{2b} + \frac{2\left(a^2 - b^2\right)\arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -a*x/b^2 - 1/2*e^(-x)/b + 1/2*e^x/b + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqrt
(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2)
```

3.171 $\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\cosh(x))}{b}$$

```
[Out] Log[a + b*Cosh[x]]/b
```

Rubi [A] time = 0.0265945, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 31}

$$\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[a + b*Cosh[x]]/b
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{b}\\ &= \frac{\log(a+b\cosh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0149837, size = 11, normalized size = 1.

$$\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[a + b*Cosh[x]]/b
```

Maple [A] time = 0.006, size = 12, normalized size = 1.1

$$\frac{\ln\left(a+b\cosh\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*cosh(x)),x)
```

```
[Out] ln(a+b*cosh(x))/b
```

Maxima [A] time = 0.999493, size = 15, normalized size = 1.36

$$\frac{\log\left(b\cosh\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] log(b*cosh(x) + a)/b
```

Fricas [B] time = 1.91312, size = 72, normalized size = 6.55

$$-\frac{x-\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(x - log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/b
```

Sympy [A] time = 0.414641, size = 14, normalized size = 1.27

$$\begin{cases} \frac{\log\left(\frac{a}{b}+\cosh(x)\right)}{b} & \text{for } b \neq 0 \\ \frac{\cosh(x)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((log(a/b + cosh(x))/b, Ne(b, 0)), (cosh(x)/a, True))
```

Giac [A] time = 1.16356, size = 26, normalized size = 2.36

$$\frac{\log\left(\left|b\left(e^{(-x)} + e^{x}\right) + 2\,a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*(e^(-x) + e^x) + 2*a))/b
```

3.172 $\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=53

$$\frac{b\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) - Log[1 + Cosh[x]]/(2*(a - b)) + (b*Log[a + b*
Cosh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0770954, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2668, 706, 31, 633}

$$\frac{b\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) - Log[1 + Cosh[x]]/(2*(a - b)) + (b*Log[a + b*
Cosh[x]])/(a^2 - b^2)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx &= -\left(b\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x)\right)\right)\\ &= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{a^2-b^2}+\frac{b\,\mathrm{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx, x, b\cosh(x)\right)}{a^2-b^2}\\ &= \frac{b\log(a+b\cosh(x))}{a^2-b^2}+\frac{\mathrm{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\cosh(x)\right)}{2(a-b)}-\frac{\mathrm{Subst}\left(\int \frac{1}{b-x}\,dx, x, b\cosh(x)\right)}{2(a+b)}\\ &= \frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(a+b\cosh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.0646628, size = 37, normalized size = 0.7

$$\frac{b\log(a+b\cosh(x))+a\log\left(\tanh\left(\frac{x}{2}\right)\right)-b\log(\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Cosh[x]),x]
```

```
[Out] (b*Log[a + b*Cosh[x]] - b*Log[Sinh[x]] + a*Log[Tanh[x/2]])/(a^2 - b^2)
```

Maple [A] time = 0.016, size = 52, normalized size = 1.

$$\frac{b}{(a+b)(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a-b\right)+\frac{1}{a+b}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*cosh(x)),x)
```

```
[Out] b/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+1/(a+b)*ln(tanh(1/2*x
))
```

Maxima [A] time = 1.03591, size = 80, normalized size = 1.51

$$\frac{b\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^2-b^2}-\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] b*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
log(e^(-x) - 1)/(a + b)
```

Fricas [A] time = 2.01879, size = 181, normalized size = 3.42

$$\frac{b\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] (b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh(
x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*cosh(x)), x)
```

Giac [A] time = 1.20039, size = 90, normalized size = 1.7

$$\frac{b^2 \log \left(\left|b\left(e^{(-x)}+e^x\right)+2 a\right|\right)}{a^2 b-b^3}-\frac{\log \left(e^{(-x)}+e^x+2\right)}{2 (a-b)}+\frac{\log \left(e^{(-x)}+e^x-2\right)}{2 (a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) - 1/2*log(e^(-x) + e^x +
 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

3.173 $\int \frac{\text{csch}^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=67

$$\frac{\text{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] (2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)
^(3/2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Rubi [A] time = 0.0898146, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2696, 12, 2659, 208}

$$\frac{\text{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Cosh[x]),x]
```


```
[Out] (2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)
^(3/2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```


Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx &= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}-\frac{\int \frac{b^2}{a+b\cosh(x)}\,dx}{-a^2+b^2}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}+\frac{b^2\int \frac{1}{a+b\cosh(x)}\,dx}{a^2-b^2}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.198267, size = 77, normalized size = 1.15

$$\frac{2b^2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - C
oth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

Maple [A] time = 0.019, size = 78, normalized size = 1.2

$$-\frac{1}{2a-2b}\tanh\left(\frac{x}{2}\right)+2\frac{b^2}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)-\frac{1}{2a+2b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*cosh(x)),x)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)+2/(a+b)/(a-b)*b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/2/(a+b)/tanh(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.06479, size = 1172, normalized size = 17.49

$$\left[\frac{2a^3-2ab^2+\left(b^2\cosh(x)^2+2b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2-b^2\right)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2-}{b\cosh(x)^2+b\sinh(x)}\right.}{a^4-2a^2b^2+b^4-\left(a^4-2a^2b^2+b^4\right)\cosh(x)^2-2\left(a^4-2a^2b^2+b^4\right)c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
- b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(a^
4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^
```

```
2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a*b
^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
- (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x
) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*cosh(x)), x)
```

Giac [A] time = 1.16409, size = 103, normalized size = 1.54

$$\frac{2\,b^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}+\frac{2\,(be^x-a)}{\left(a^2-b^2\right)\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) +
 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

3.174 $\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=91

$$\frac{b^3\log(a+b\cosh(x))}{(a^2-b^2)^2}+\frac{\operatorname{csch}^2(x)(b-a\cosh(x))}{2(a^2-b^2)}-\frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(a-2b)\log(\cosh(x)+1)}{4(a-b)^2}$$

```
[Out] ((b - a*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) - ((a + 2*b)*Log[1 - Cosh[x]])/
(4*(a + b)^2) + ((a - 2*b)*Log[1 + Cosh[x]])/(4*(a - b)^2) + (b^3*Log[a + b
*Cosh[x]])/(a^2 - b^2)^2
```

Rubi [A] time = 0.15923, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2668, 741, 801}

$$\frac{b^3\log(a+b\cosh(x))}{(a^2-b^2)^2}+\frac{\operatorname{csch}^2(x)(b-a\cosh(x))}{2(a^2-b^2)}-\frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(a-2b)\log(\cosh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((b - a*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) - ((a + 2*b)*Log[1 - Cosh[x]])/
(4*(a + b)^2) + ((a - 2*b)*Log[1 + Cosh[x]])/(4*(a - b)^2) + (b^3*Log[a + b
*Cosh[x]])/(a^2 - b^2)^2
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = b^3\operatorname{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cosh(x)\right)$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \frac{a^2-2b^2+ax}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x)\right)}{2\left(a^2-b^2\right)}$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right) dx, x, b\cosh(x)\right)}{2\left(a^2-b^2\right)}$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2} + \frac{(a-2b)\log(1+\cosh(x))}{4(a-b)^2} + \frac{b^3\log(a+b}{\left(a^2-b\right.}$$

Mathematica [A] time = 0.269375, size = 100, normalized size = 1.1

$$-\frac{4a^3\log\left(\tanh\left(\frac{x}{2}\right)\right) - 12ab^2\log\left(\tanh\left(\frac{x}{2}\right)\right) - 8b^3\log(a+b\cosh(x)) + (a-b)^2(a+b)\operatorname{csch}^2\left(\frac{x}{2}\right) + (a-b)(a+b)^2\operatorname{sech}^2}{8(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -((a - b)^2*(a + b)*Csch[x/2]^2 - 8*b^3*Log[a + b*Cosh[x]] + 8*b^3*Log[Sinh
[x]] + 4*a^3*Log[Tanh[x/2]] - 12*a*b^2*Log[Tanh[x/2]] + (a - b)*(a + b)^2*S
ech[x/2]^2)/(8*(a - b)^2*(a + b)^2)
```

Maple [A] time = 0.023, size = 97, normalized size = 1.1

$$\frac{1}{8a-8b}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{b^3}{(a+b)^2(a-b)^2}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2 - \left(\tanh\left(\frac{x}{2}\right)\right)^2 b - a - b\right) - \frac{1}{8a+8b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} - \frac{a}{2(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*cosh(x)),x)
```

```
[Out] 1/8*tanh(1/2*x)^2/(a-b)+b^3/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^
2*b-a-b)-1/8/(a+b)/tanh(1/2*x)^2-1/2/(a+b)^2*ln(tanh(1/2*x))*a-1/(a+b)^2*ln
(tanh(1/2*x))*b
```

Maxima [A] time = 1.10484, size = 208, normalized size = 2.29

$$\frac{b^{3}\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^{4}-2\,a^{2}b^{2}+b^{4}}+\frac{(a-2\,b)\log\left(e^{(-x)}+1\right)}{2\left(a^{2}-2\,ab+b^{2}\right)}-\frac{(a+2\,b)\log\left(e^{(-x)}-1\right)}{2\left(a^{2}+2\,ab+b^{2}\right)}-\frac{ae^{(-x)}-2\,be^{(-2\,x)}+ae^{(-3\,x)}}{a^{2}-b^{2}-2\left(a^{2}-b^{2}\right)e^{(-2\,x)}+\left(a^{2}-b^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a - 2*b
)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*(a + 2*b)*log(e^(-x) - 1)/(a^2
+ 2*a*b + b^2) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(a^2
 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

Fricas [B] time = 2.25945, size = 2072, normalized size = 22.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b^
3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 + 2*
(a^3 - a*b^2)*cosh(x) - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*si
nh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)^2 + 4*(
b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh
(x))) - ((a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 - 2*b^3)*cosh
(x)*sinh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 -
 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3 - 3*(a^3 -
3*a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^
```

```
3 - (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) +
((a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 + 2*b^3)*cosh(x)*sinh
(x)^3 + (a^3 - 3*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 + 2*b^3 - 2*(a^3
- 3*a*b^2 + 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 + 2*b^3 - 3*(a^3 - 3*a*b^2
+ 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^3 - (a^3
 - 3*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^3 -
 a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a^2*b - b^3)*cosh(x))*sinh(x))/((a^
4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^
3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 - 2
*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*cosh(x)), x)
```

Giac [B] time = 1.15935, size = 242, normalized size = 2.66

$$\frac{b^4\log\left(\left|b\left(e^{(-x)}+e^x\right)+2a\right|\right)}{a^4b-2a^2b^3+b^5}+\frac{(a-2b)\log\left(e^{(-x)}+e^x+2\right)}{4\left(a^2-2ab+b^2\right)}-\frac{(a+2b)\log\left(e^{(-x)}+e^x-2\right)}{4\left(a^2+2ab+b^2\right)}+\frac{b^3\left(e^{(-x)}+e^x\right)^2-2a^3\left(e^{(-x)}\right.}{2\left(a^4-2a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(a - 2
*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*(a + 2*b)*log(e^(-x) +
e^x - 2)/(a^2 + 2*a*b + b^2) + 1/2*(b^3*(e^(-x) + e^x)^2 - 2*a^3*(e^(-x) +
e^x) + 2*a*b^2*(e^(-x) + e^x) + 4*a^2*b - 8*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(
(e^(-x) + e^x)^2 - 4))
```

3.175 $\int \frac{\text{csch}^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=110

$$\frac{\text{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}+\frac{\text{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{3\left(a^2-b^2\right)^2}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] (2*b^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)
^(5/2)) + ((3*b^3 + a*(2*a^2 - 5*b^2)*Cosh[x])*Csch[x])/(3*(a^2 - b^2)^2) +
 ((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2))
```

Rubi [A] time = 0.245871, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2696, 2866, 12, 2659, 208}

$$\frac{\text{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}+\frac{\text{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{3\left(a^2-b^2\right)^2}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)
^(5/2)) + ((3*b^3 + a*(2*a^2 - 5*b^2)*Cosh[x])*Csch[x])/(3*(a^2 - b^2)^2) +
 ((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\int \frac{\left(-2a^2+3b^2-2ab\cosh(x)\right)\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{\left(3b^3+a\left(2a^2-5b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\int \frac{3b^4}{a+b\cosh(x)}\,dx}{3\left(a^2-b^2\right)^2}$$

$$= \frac{\left(3b^3+a\left(2a^2-5b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{b^4\int \frac{1}{a+b\cosh(x)}\,dx}{\left(a^2-b^2\right)^2}$$

$$= \frac{\left(3b^3+a\left(2a^2-5b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\left(2b^4\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,\right.}{\left(a^2-b^2\right)^2}$$

$$= \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{\left(3b^3+a\left(2a^2-5b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.534108, size = 141, normalized size = 1.28

$$\frac{1}{24}\left(-\frac{48b^4\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} - \frac{14b\tanh\left(\frac{x}{2}\right)}{(a-b)^2} + \frac{8a\tanh\left(\frac{x}{2}\right)}{(a-b)^2} + \frac{2(4a+7b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\sinh^4\left(\frac{x}{2}\right)\operatorname{csch}^3(x)}{a-b} - \frac{\sinh(x)\mathrm{c}}{2(a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] ((-48*b^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
+ (2*(4*a + 7*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) -
 (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/2])/(a - b)^2 - (14*b*Tanh
[x/2])/(a - b)^2)/24
```

Maple [A] time = 0.024, size = 127, normalized size = 1.2

$$-\frac{1}{8\,(a-b)^2}\left(\frac{a}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{b}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - 3\,a\tanh(x/2) + 5\,\tanh(x/2)\,b\right) + 2\,\frac{b^4}{(a-b)^2\,(a+b)^2\,\sqrt{(a+b)\,(a-b)}}\mathrm{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*cosh(x)),x)
```

```
[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*tanh(1/2*x)^3*b-3*a*tanh(1/2*x)+5*tan
h(1/2*x)*b)+2/(a-b)^2/(a+b)^2*b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/
2*x)/((a+b)*(a-b))^(1/2))-1/24/(a+b)/tanh(1/2*x)^3-1/8/(a+b)^2*(-3*a-5*b)/t
anh(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15178, size = 5405, normalized size = 49.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/3*(6*(a^2*b^3 - b^5)*cosh(x)^5 + 6*(a^2*b^3 - b^5)*sinh(x)^5 + 4*a^5 - 1
4*a^3*b^2 + 10*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*b^4 -
 5*(a^2*b^3 - b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*cos
h(x)^3 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 - b^5)*cosh(x)^2 - 6*
(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(
x)^2 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 - b^5)*cosh(x)^3 + 3*(a^3
*b^2 - a*b^4)*cosh(x)^2 - (2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2
+ 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 - 3*b^4*cosh(x
)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 - b^4)*sinh(x)^4 - b^4 + 4*(5*b^
4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 - 6*b^4*cosh(x)
^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 - 2*b^4*cosh(x)^3 + b^4*cosh(x))*sin
h(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
```

```
 a)*sinh(x) + b)) + 6*(a^2*b^3 - b^5)*cosh(x) + 6*(a^2*b^3 - b^5 + 5*(a^2*b
^3 - b^5)*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^3 + 2*(2*a^4*b - 7*a^2*b^
3 + 5*b^5)*cosh(x)^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 -
 a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3
*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 +
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)), 2/3*(3*(a^2*b^3 - b^
5)*cosh(x)^5 + 3*(a^2*b^3 - b^5)*sinh(x)^5 + 2*a^5 - 7*a^3*b^2 + 5*a*b^4 -
3*(a^3*b^2 - a*b^4)*cosh(x)^4 - 3*(a^3*b^2 - a*b^4 - 5*(a^2*b^3 - b^5)*cosh
(x))*sinh(x)^4 + 2*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x)^3 + 2*(2*a^4*b - 7
*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 - b^5)*cosh(x)^2 - 6*(a^3*b^2 - a*b^4)*cosh(
x))*sinh(x)^3 - 6*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(x)^2 - 6*(a^5 - 3*a^3*b^
2 + 2*a*b^4 - 5*(a^2*b^3 - b^5)*cosh(x)^3 + 3*(a^3*b^2 - a*b^4)*cosh(x)^2 -
 (2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2 - 3*(b^4*cosh(x)^6 + 6*b^
4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 - 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3
*(5*b^4*cosh(x)^2 - b^4)*sinh(x)^4 - b^4 + 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(
x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 - 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(
b^4*cosh(x)^5 - 2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 3*(a^2*b^
3 - b^5)*cosh(x) + 3*(a^2*b^3 - b^5 + 5*(a^2*b^3 - b^5)*cosh(x)^4 - 4*(a^3*
b^2 - a*b^4)*cosh(x)^3 + 2*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x)^2 - 4*(a^5
 - 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4
+ b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh
(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
h(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19373, size = 211, normalized size = 1.92

$$\frac{2\,b^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{2\left(3\,b^3e^{(5\,x)}-3\,ab^2e^{(4\,x)}+4\,a^2be^{(3\,x)}-10\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}+12\,ab^2e^{(2\,x)}+3\,b^3e^x+2\,a^3\right.}{3\left(a^4-2\,a^2b^2+b^4\right)\left(e^{(2\,x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a
^2 + b^2)) + 2/3*(3*b^3*e^(5*x) - 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) - 10*b^
3*e^(3*x) - 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*b^3*e^x + 2*a^3 - 5*a*b^2)
/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

3.176 $\int \frac{\text{csch}^5(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=151

$$\frac{b^5\log(a+b\cosh(x))}{\left(a^2-b^2\right)^3}+\frac{\left(3a^2+9ab+8b^2\right)\log(1-\cosh(x))}{16(a+b)^3}-\frac{\left(3a^2-9ab+8b^2\right)\log(\cosh(x)+1)}{16(a-b)^3}+\frac{\text{csch}^4(x)(b-a\cosh}{4\left(a^2-b^2\right)}$$

```
[Out] ((4*b^3 + a*(3*a^2 - 7*b^2)*Cosh[x])*Csch[x]^2)/(8*(a^2 - b^2)^2) + ((b - a
*Cosh[x])*Csch[x]^4)/(4*(a^2 - b^2)) + ((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Cos
h[x]])/(16*(a + b)^3) - ((3*a^2 - 9*a*b + 8*b^2)*Log[1 + Cosh[x]])/(16*(a -
 b)^3) + (b^5*Log[a + b*Cosh[x]])/(a^2 - b^2)^3
```

Rubi [A] time = 0.253579, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2668, 741, 823, 801}

$$\frac{b^5\log(a+b\cosh(x))}{\left(a^2-b^2\right)^3}+\frac{\left(3a^2+9ab+8b^2\right)\log(1-\cosh(x))}{16(a+b)^3}-\frac{\left(3a^2-9ab+8b^2\right)\log(\cosh(x)+1)}{16(a-b)^3}+\frac{\text{csch}^4(x)(b-a\cosh}{4\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] ((4*b^3 + a*(3*a^2 - 7*b^2)*Cosh[x])*Csch[x]^2)/(8*(a^2 - b^2)^2) + ((b - a
*Cosh[x])*Csch[x]^4)/(4*(a^2 - b^2)) + ((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Cos
h[x]])/(16*(a + b)^3) - ((3*a^2 - 9*a*b + 8*b^2)*Log[1 + Cosh[x]])/(16*(a -
 b)^3) + (b^5*Log[a + b*Cosh[x]])/(a^2 - b^2)^3
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
```

```
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = -\left(b^5\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\cosh(x)\right)\right)$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} - \frac{b^3\,\mathrm{Subst}\left(\int \frac{3a^2-4b^2+3ax}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cosh(x)\right)}{4\left(a^2-b^2\right)}$$

$$= \frac{\left(4b^3+a\left(3a^2-7b^2\right)\cosh(x)\right)\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} + \frac{b\,\mathrm{Subst}\left(\int \frac{-3a^4+7a^2b^2-8b^4-a(}{(a+x)\left(b^2-x\right.}\right.}{8\left(a^2-\right.}$$

$$= \frac{\left(4b^3+a\left(3a^2-7b^2\right)\cosh(x)\right)\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} + \frac{b\,\mathrm{Subst}\left(\int\left(-\frac{(a-b)^2\left(3a^2+9ab-\right.}{2b(a+b)(b-x}\right.\right.}{}$$

$$= \frac{\left(4b^3+a\left(3a^2-7b^2\right)\cosh(x)\right)\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} + \frac{\left(3a^2+9ab+8b^2\right)\log(1-c}{16(a+b)^3}$$

Mathematica [A] time = 0.861324, size = 148, normalized size = 0.98

$$\frac{1}{64}\left(\frac{2\left(3a^2-8ab+5b^2\right)\operatorname{sech}^2\left(\frac{x}{2}\right)+\frac{8\left(a\left(-10a^2b^2+3a^4+15b^4\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)+8b^5\log(a+b\cosh(x))-8b^5\log(\sinh(x))\right)}{(a+b)^3}+(a-b)^2\operatorname{sech}^4\left(\frac{x}{2}\right)}{(a-b)^3}-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] ((2*(3*a + 5*b)*Csch[x/2]^2)/(a + b)^2 - Csch[x/2]^4/(a + b) + ((8*(8*b^5*L
og[a + b*Cosh[x]] - 8*b^5*Log[Sinh[x]] + a*(3*a^4 - 10*a^2*b^2 + 15*b^4)*Lo
g[Tanh[x/2]]))/(a + b)^3 + 2*(3*a^2 - 8*a*b + 5*b^2)*Sech[x/2]^2 + (a - b)^
2*Sech[x/2]^4)/(a - b)^3)/64
```

Maple [A] time = 0.026, size = 191, normalized size = 1.3

$$\frac{a}{64\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^4-\frac{b}{64\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^4-\frac{a}{8\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{3\,b}{16\,(a-b)^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{b^5}{(a-b)^3\,(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^5/(a+b*cosh(x)),x)
```

```
[Out] 1/64/(a-b)^2*tanh(1/2*x)^4*a-1/64/(a-b)^2*tanh(1/2*x)^4*b-1/8/(a-b)^2*tanh(
1/2*x)^2*a+3/16/(a-b)^2*tanh(1/2*x)^2*b+1/(a-b)^3*b^5/(a+b)^3*ln(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-a-b)-1/64/(a+b)/tanh(1/2*x)^4+3/16/(a+b)^2/tanh(1/2*x
)^2*b+1/8/(a+b)^2/tanh(1/2*x)^2*a+3/8/(a+b)^3*ln(tanh(1/2*x))*a^2+9/8/(a+b)
^3*ln(tanh(1/2*x))*a*b+1/(a+b)^3*ln(tanh(1/2*x))*b^2
```

Maxima [B] time = 1.14881, size = 470, normalized size = 3.11

$$\frac{b^5\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(3\,a^2-9\,ab+8\,b^2\right)\log\left(e^{(-x)}+1\right)}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{\left(3\,a^2+9\,ab+8\,b^2\right)\log\left(e^{(-x)}-1\right)}{8\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\frac{8\,b^3e^{(-2\,x)}+8\,b}{4\left(a^4-2\,a^2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] b^5*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
1/8*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3)
 + 1/8*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b
^3) + 1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 - 7*a*b^2)*e^(-x) - (11
*a^3 - 15*a*b^2)*e^(-3*x) + 16*(a^2*b - 2*b^3)*e^(-4*x) - (11*a^3 - 15*a*b^
2)*e^(-5*x) + (3*a^3 - 7*a*b^2)*e^(-7*x))/(a^4 - 2*a^2*b^2 + b^4 - 4*(a^4 -
 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a^2*b^2 + b^4)*e^(-4*x) - 4*(a^4 -
2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2*b^2 + b^4)*e^(-8*x))
```

Fricas [B] time = 2.76161, size = 8195, normalized size = 54.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^7 + 2*(3*a^5 - 10*a^3*b^2 + 7
*a*b^4)*sinh(x)^7 + 16*(a^2*b^3 - b^5)*cosh(x)^6 + 2*(8*a^2*b^3 - 8*b^5 + 7
*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^6 - 2*(11*a^5 - 26*a^3*b^2
 + 15*a*b^4)*cosh(x)^5 - 2*(11*a^5 - 26*a^3*b^2 + 15*a*b^4 - 21*(3*a^5 - 10
*a^3*b^2 + 7*a*b^4)*cosh(x)^2 - 48*(a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 + 32*
(a^4*b - 3*a^2*b^3 + 2*b^5)*cosh(x)^4 + 2*(16*a^4*b - 48*a^2*b^3 + 32*b^5 +
 35*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^3 + 120*(a^2*b^3 - b^5)*cosh(x)^
2 - 5*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 - 2*(11*a^5 - 26*
a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 2*(11*a^5 - 26*a^3*b^2 + 15*a*b^4 - 35*(3*a
^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^4 - 160*(a^2*b^3 - b^5)*cosh(x)^3 + 10*(
11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x)^2 - 64*(a^4*b - 3*a^2*b^3 + 2*b^5)*
cosh(x))*sinh(x)^3 + 16*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(21*(3*a^5 - 10*a^3*b
^2 + 7*a*b^4)*cosh(x)^5 + 8*a^2*b^3 - 8*b^5 + 120*(a^2*b^3 - b^5)*cosh(x)^4
 - 10*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 96*(a^4*b - 3*a^2*b^3 +
2*b^5)*cosh(x)^2 - 3*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 +
2*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x) + 8*(b^5*cosh(x)^8 + 8*b^5*cosh(x)
*sinh(x)^7 + b^5*sinh(x)^8 - 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 - 4*b^5*cosh
(x)^2 + 4*(7*b^5*cosh(x)^2 - b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 - 3*b^5*co
sh(x))*sinh(x)^5 + b^5 + 2*(35*b^5*cosh(x)^4 - 30*b^5*cosh(x)^2 + 3*b^5)*si
nh(x)^4 + 8*(7*b^5*cosh(x)^5 - 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3
+ 4*(7*b^5*cosh(x)^6 - 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 - b^5)*sinh(x)^2
+ 8*(b^5*cosh(x)^7 - 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 - b^5*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - ((3*a^5 - 10*a^3*b^2 + 15*
a*b^4 + 8*b^5)*cosh(x)^8 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x
)*sinh(x)^7 + (3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*sinh(x)^8 - 4*(3*a^5
```

```
- 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^6 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b
^4 + 8*b^5 - 7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^6
 + 8*(7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^3 - 3*(3*a^5 - 10*a
^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 + 3*a^5 - 10*a^3*b^2 + 15*a*b
^4 + 8*b^5 + 6*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(9*a^5
 - 30*a^3*b^2 + 45*a*b^4 + 24*b^5 + 35*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b
^5)*cosh(x)^4 - 30*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(
x)^4 + 8*(7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^5 - 10*(3*a^5 -
 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^3 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^
4 + 8*b^5)*cosh(x))*sinh(x)^3 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*c
osh(x)^2 + 4*(7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^6 - 3*a^5 +
 10*a^3*b^2 - 15*a*b^4 - 8*b^5 - 15*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)
*cosh(x)^4 + 9*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^2
 + 8*((3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^7 - 3*(3*a^5 - 10*a^3
*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^5 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b
^5)*cosh(x)^3 - (3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x))*sinh(x))*l
og(cosh(x) + sinh(x) + 1) + ((3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x
)^8 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)*sinh(x)^7 + (3*a^5
- 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*sinh(x)^8 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b
^4 - 8*b^5)*cosh(x)^6 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5 - 7*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 - 10*a
^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^3 - 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8
*b^5)*cosh(x))*sinh(x)^5 + 3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5 + 6*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^4 + 2*(9*a^5 - 30*a^3*b^2 + 45*a*
b^4 - 24*b^5 + 35*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^4 - 30*(3
*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 -
10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^5 - 10*(3*a^5 - 10*a^3*b^2 + 15*a*b^
4 - 8*b^5)*cosh(x)^3 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x))*s
inh(x)^3 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2 + 4*(7*(3*a^
5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^6 - 3*a^5 + 10*a^3*b^2 - 15*a*b^
4 + 8*b^5 - 15*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^4 + 9*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 - 10*a^3
*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^7 - 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b
^5)*cosh(x)^5 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^3 - (3*a^
5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
- 1) + 2*(7*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^6 + 48*(a^2*b^3 - b^5)*c
osh(x)^5 + 3*a^5 - 10*a^3*b^2 + 7*a*b^4 - 5*(11*a^5 - 26*a^3*b^2 + 15*a*b^4
)*cosh(x)^4 + 64*(a^4*b - 3*a^2*b^3 + 2*b^5)*cosh(x)^3 - 3*(11*a^5 - 26*a^3
*b^2 + 15*a*b^4)*cosh(x)^2 + 16*(a^2*b^3 - b^5)*cosh(x))*sinh(x))/((a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 8*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)*sinh(x)^7 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^8 - 4*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 - 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6 - 7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^6 + a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 8*(7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^5 + 6*(a^6
```

```
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 2*(3*a^6 - 9*a^4*b^2 + 9*a^2*b^
4 - 3*b^6 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 30*(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^5 - 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^
3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 - 4*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 4*(7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 15*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^4 + 9*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2
)*sinh(x)^2 + 8*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 - 3*(a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)^3 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**5/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16045, size = 456, normalized size = 3.02

$$\frac{b^6 \log\left(\left|b\left(e^{(-x)} + e^x\right) + 2\,a\right|\right)}{a^6 b - 3\,a^4 b^3 + 3\,a^2 b^5 - b^7} - \frac{\left(3\,a^2 - 9\,ab + 8\,b^2\right)\log\left(e^{(-x)} + e^x + 2\right)}{16\left(a^3 - 3\,a^2 b + 3\,ab^2 - b^3\right)} + \frac{\left(3\,a^2 + 9\,ab + 8\,b^2\right)\log\left(e^{(-x)} + e^x - 2\right)}{16\left(a^3 + 3\,a^2 b + 3\,ab^2 + b^3\right)} + \frac{3\,b^5(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
- 1/16*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) + 1/16*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) + e^x - 2)/(a^3 + 3*a^2
*b + 3*a*b^2 + b^3) + 1/4*(3*b^5*(e^(-x) + e^x)^4 + 3*a^5*(e^(-x) + e^x)^3
- 10*a^3*b^2*(e^(-x) + e^x)^3 + 7*a*b^4*(e^(-x) + e^x)^3 + 8*a^2*b^3*(e^(-x
) + e^x)^2 - 32*b^5*(e^(-x) + e^x)^2 - 20*a^5*(e^(-x) + e^x) + 56*a^3*b^2*(
e^(-x) + e^x) - 36*a*b^4*(e^(-x) + e^x) + 16*a^4*b - 64*a^2*b^3 + 96*b^5)/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

3.177 $\int \frac{\mathbf{csch}^6(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=159

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}+\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{15\left(a^2-b^2\right)^2}+\frac{\operatorname{csch}(x)\left(15b^5-a\left(-26a^2b^2+8a^4+33b^4\right)\cosh(x)\right)}{15\left(a^2-b^2\right)^3}$$

```
[Out] (2*b^6*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)
^(7/2)) + ((15*b^5 - a*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Cosh[x])*Csch[x])/(15*
(a^2 - b^2)^3) + ((5*b^3 + a*(4*a^2 - 9*b^2)*Cosh[x])*Csch[x]^3)/(15*(a^2 -
 b^2)^2) + ((b - a*Cosh[x])*Csch[x]^5)/(5*(a^2 - b^2))
```

Rubi [A] time = 0.476846, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2696, 2866, 12, 2659, 208}

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}+\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{15\left(a^2-b^2\right)^2}+\frac{\operatorname{csch}(x)\left(15b^5-a\left(-26a^2b^2+8a^4+33b^4\right)\cosh(x)\right)}{15\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^6*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)
^(7/2)) + ((15*b^5 - a*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Cosh[x])*Csch[x])/(15*
(a^2 - b^2)^3) + ((5*b^3 + a*(4*a^2 - 9*b^2)*Cosh[x])*Csch[x]^3)/(15*(a^2 -
 b^2)^2) + ((b - a*Cosh[x])*Csch[x]^5)/(5*(a^2 - b^2))
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\text{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{(b-a\cosh(x))\text{csch}^5(x)}{5\left(a^2-b^2\right)} + \frac{\int \frac{\left(-4a^2+5b^2-4ab\cosh(x)\right)\text{csch}^4(x)}{a+b\cosh(x)}\,dx}{5\left(a^2-b^2\right)}$$

$$= \frac{\left(5b^3+a\left(4a^2-9b^2\right)\cosh(x)\right)\text{csch}^3(x)}{15\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\text{csch}^5(x)}{5\left(a^2-b^2\right)} + \frac{\int \frac{\left(8a^4-18a^2b^2+15b^4+2ab\left(4a^2-9b^2\right)\right.}{a+b\cosh(x)}}{15\left(a^2-b^2\right)^2}$$

$$= \frac{\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)\text{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a\left(4a^2-9b^2\right)\cosh(x)\right)\text{csch}^3(x)}{15\left(a^2-b^2\right)^2} + \frac{(b}{}$$

$$= \frac{\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)\text{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a\left(4a^2-9b^2\right)\cosh(x)\right)\text{csch}^3(x)}{15\left(a^2-b^2\right)^2} + \frac{(b}{}$$

$$= \frac{\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)\text{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a\left(4a^2-9b^2\right)\cosh(x)\right)\text{csch}^3(x)}{15\left(a^2-b^2\right)^2} + \frac{(b}{}$$

$$= \frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} + \frac{\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)\text{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a\left(4a^2-\right.\right.}{15}$$

Mathematica [A] time = 1.71403, size = 201, normalized size = 1.26

$$\frac{1}{480}\left(-\frac{2\left(64a^2-183ab+149b^2\right)\tanh\left(\frac{x}{2}\right)}{(a-b)^3} - \frac{2\left(64a^2+183ab+149b^2\right)\coth\left(\frac{x}{2}\right)}{(a+b)^3} + \frac{960b^6\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} - \frac{96\sinh}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] ((960*b^6*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2)
- (2*(64*a^2 + 183*a*b + 149*b^2)*Coth[x/2])/(a + b)^3 - (8*(19*a - 29*b)*C
sch[x]^3*Sinh[x/2]^4)/(a - b)^2 - (96*Csch[x]^5*Sinh[x/2]^6)/(a - b) + ((19
*a + 29*b)*Csch[x/2]^4*Sinh[x])/(2*(a + b)^2) - (3*Csch[x/2]^6*Sinh[x])/(2*
(a + b)) - (2*(64*a^2 - 183*a*b + 149*b^2)*Tanh[x/2])/(a - b)^3)/480
```

Maple [A] time = 0.026, size = 213, normalized size = 1.3

$$-\frac{1}{32\,(a-b)^3}\left(\frac{a^2}{5}\left(\tanh\left(\frac{x}{2}\right)\right)^5-\frac{2\,ab}{5}\left(\tanh\left(\frac{x}{2}\right)\right)^5+\frac{b^2}{5}\left(\tanh\left(\frac{x}{2}\right)\right)^5-\frac{5\,a^2}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3+4\,(\tanh(x/2))^3\,ab-\frac{7\,b^2}{3}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^6/(a+b*cosh(x)),x)
```

```
[Out] -1/32/(a-b)^3*(1/5*tanh(1/2*x)^5*a^2-2/5*tanh(1/2*x)^5*a*b+1/5*b^2*tanh(1/2
*x)^5-5/3*tanh(1/2*x)^3*a^2+4*tanh(1/2*x)^3*a*b-7/3*tanh(1/2*x)^3*b^2+10*a^
2*tanh(1/2*x)-28*a*b*tanh(1/2*x)+22*b^2*tanh(1/2*x))+2/(a-b)^3/(a+b)^3*b^6/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/160/(a
+b)/tanh(1/2*x)^5-1/96*(-5*a-7*b)/(a+b)^2/tanh(1/2*x)^3-1/32/(a+b)^3*(10*a^
2+28*a*b+22*b^2)/tanh(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78489, size = 15046, normalized size = 94.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/15*(30*(a^2*b^5 - b^7)*cosh(x)^9 + 30*(a^2*b^5 - b^7)*sinh(x)^9 - 30*(a^
3*b^4 - a*b^6)*cosh(x)^8 - 30*(a^3*b^4 - a*b^6 - 9*(a^2*b^5 - b^7)*cosh(x))
*sinh(x)^8 + 40*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^7 + 40*(a^4*b^3 - 5*a
^2*b^5 + 4*b^7 + 27*(a^2*b^5 - b^7)*cosh(x)^2 - 6*(a^3*b^4 - a*b^6)*cosh(x)
)*sinh(x)^7 - 16*a^7 + 68*a^5*b^2 - 118*a^3*b^4 + 66*a*b^6 - 60*(a^5*b^2 -
```

4*a^3*b^4 + 3*a*b^6)*cosh(x)^6 - 20*(3*a^5*b^2 - 12*a^3*b^4 + 9*a*b^6 - 126
*(a^2*b^5 - b^7)*cosh(x)^3 + 42*(a^3*b^4 - a*b^6)*cosh(x)^2 - 14*(a^4*b^3 -
 5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^6 + 4*(24*a^6*b - 92*a^4*b^3 + 157*a^2
*b^5 - 89*b^7)*cosh(x)^5 + 4*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7
+ 945*(a^2*b^5 - b^7)*cosh(x)^4 - 420*(a^3*b^4 - a*b^6)*cosh(x)^3 + 210*(a^
4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 90*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*c
osh(x))*sinh(x)^5 - 20*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)
^4 - 20*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6 - 189*(a^2*b^5 - b^7)*c
osh(x)^5 + 105*(a^3*b^4 - a*b^6)*cosh(x)^4 - 70*(a^4*b^3 - 5*a^2*b^5 + 4*b^
7)*cosh(x)^3 + 45*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^2 - (24*a^6*b - 9
2*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x))*sinh(x)^4 + 40*(a^4*b^3 - 5*a^2*
b^5 + 4*b^7)*cosh(x)^3 + 40*(a^4*b^3 - 5*a^2*b^5 + 4*b^7 + 63*(a^2*b^5 - b^
7)*cosh(x)^6 - 42*(a^3*b^4 - a*b^6)*cosh(x)^5 + 35*(a^4*b^3 - 5*a^2*b^5 + 4
*b^7)*cosh(x)^4 - 30*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^3 + (24*a^6*b
- 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^2 - 2*(8*a^7 - 31*a^5*b^2 + 47
*a^3*b^4 - 24*a*b^6)*cosh(x))*sinh(x)^3 + 20*(4*a^7 - 17*a^5*b^2 + 28*a^3*b
^4 - 15*a*b^6)*cosh(x)^2 + 20*(54*(a^2*b^5 - b^7)*cosh(x)^7 + 4*a^7 - 17*a^
5*b^2 + 28*a^3*b^4 - 15*a*b^6 - 42*(a^3*b^4 - a*b^6)*cosh(x)^6 + 42*(a^4*b^
3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 45*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(
x)^4 + 2*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^3 - 6*(8*a^
7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)^2 + 6*(a^4*b^3 - 5*a^2*b^5
+ 4*b^7)*cosh(x))*sinh(x)^2 - 15*(b^6*cosh(x)^10 + 10*b^6*cosh(x)*sinh(x)^9
 + b^6*sinh(x)^10 - 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^6 - 10*b^6*cosh(x)^4 +
 5*(9*b^6*cosh(x)^2 - b^6)*sinh(x)^8 + 5*b^6*cosh(x)^2 + 40*(3*b^6*cosh(x)^
3 - b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 - 14*b^6*cosh(x)^2 + b^6)
sinh(x)^6 - b^6 + 4(63*b^6*cosh(x)^5 - 70*b^6*cosh(x)^3 + 15*b^6*cosh(x))
sinh(x)^5 + 10(21*b^6*cosh(x)^6 - 35*b^6*cosh(x)^4 + 15*b^6*cosh(x)^2 - b
^6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 - 7*b^6*cosh(x)^5 + 5*b^6*cosh(x)^3 - b
^6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8 - 28*b^6*cosh(x)^6 + 30*b^6*cosh
(x)^4 - 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2 + 10*(b^6*cosh(x)^9 - 4*b^6*cosh(
x)^7 + 6*b^6*cosh(x)^5 - 4*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x))*sqrt(a^2 -
 b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*
(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a)
)/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)
) + 30*(a^2*b^5 - b^7)*cosh(x) + 10*(27*(a^2*b^5 - b^7)*cosh(x)^8 - 24*(a^3
*b^4 - a*b^6)*cosh(x)^7 + 3*a^2*b^5 - 3*b^7 + 28*(a^4*b^3 - 5*a^2*b^5 + 4*b
^7)*cosh(x)^6 - 36*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^5 + 2*(24*a^6*b
- 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^4 - 8*(8*a^7 - 31*a^5*b^2 + 47
*a^3*b^4 - 24*a*b^6)*cosh(x)^3 + 12*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^2
 + 4*(4*a^7 - 17*a^5*b^2 + 28*a^3*b^4 - 15*a*b^6)*cosh(x))*sinh(x))/((a^8 -
 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^10 + 10*(a^8 - 4*a^6*b^2
+ 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^9 + (a^8 - 4*a^6*b^2 + 6*a^4
*b^4 - 4*a^2*b^6 + b^8)*sinh(x)^10 - 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8)*cosh(x)^8 - 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 -
9*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^8 - a^

```
8 + 4*a^6*b^2 - 6*a^4*b^4 + 4*a^2*b^6 - b^8 + 40*(3*(a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b
^6 + b^8)*cosh(x))*sinh(x)^7 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6
+ b^8)*cosh(x)^6 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 + 21*(
a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 14*(a^8 - 4*a^6*
b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 - 4*a^
6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 - 70*(a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 + 15*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^
2*b^6 + b^8)*cosh(x))*sinh(x)^5 - 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b
^6 + b^8)*cosh(x)^4 - 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 - 2
1*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^6 + 35*(a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 15*(a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^8 - 4*a^6*b^2 + 6
*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^7 - 7*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*
a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8
)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x))*sinh
(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2 + 5*(9*
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^8 + a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 - 28*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*cosh(x)^6 + 30*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cos
h(x)^4 - 12*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh
(x)^2 + 10*((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^9 - 4*(
a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^7 + 6*(a^8 - 4*a^6*b
^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 - 4*(a^8 - 4*a^6*b^2 + 6*a^4*b^
4 - 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
 b^8)*cosh(x))*sinh(x)), 2/15*(15*(a^2*b^5 - b^7)*cosh(x)^9 + 15*(a^2*b^5 -
 b^7)*sinh(x)^9 - 15*(a^3*b^4 - a*b^6)*cosh(x)^8 - 15*(a^3*b^4 - a*b^6 - 9*
(a^2*b^5 - b^7)*cosh(x))*sinh(x)^8 + 20*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(
x)^7 + 20*(a^4*b^3 - 5*a^2*b^5 + 4*b^7 + 27*(a^2*b^5 - b^7)*cosh(x)^2 - 6*(
a^3*b^4 - a*b^6)*cosh(x))*sinh(x)^7 - 8*a^7 + 34*a^5*b^2 - 59*a^3*b^4 + 33*
a*b^6 - 30*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^6 - 10*(3*a^5*b^2 - 12*a
^3*b^4 + 9*a*b^6 - 126*(a^2*b^5 - b^7)*cosh(x)^3 + 42*(a^3*b^4 - a*b^6)*cos
h(x)^2 - 14*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^6 + 2*(24*a^6*b
- 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^5 + 2*(24*a^6*b - 92*a^4*b^3 +
 157*a^2*b^5 - 89*b^7 + 945*(a^2*b^5 - b^7)*cosh(x)^4 - 420*(a^3*b^4 - a*b^
6)*cosh(x)^3 + 210*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 90*(a^5*b^2 -
4*a^3*b^4 + 3*a*b^6)*cosh(x))*sinh(x)^5 - 10*(8*a^7 - 31*a^5*b^2 + 47*a^3*b
^4 - 24*a*b^6)*cosh(x)^4 - 10*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6 -
 189*(a^2*b^5 - b^7)*cosh(x)^5 + 105*(a^3*b^4 - a*b^6)*cosh(x)^4 - 70*(a^4*
b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^3 + 45*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cos
h(x)^2 - (24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x))*sinh(x)^4
+ 20*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^3 + 20*(a^4*b^3 - 5*a^2*b^5 + 4*
b^7 + 63*(a^2*b^5 - b^7)*cosh(x)^6 - 42*(a^3*b^4 - a*b^6)*cosh(x)^5 + 35*(a
^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^4 - 30*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*
cosh(x)^3 + (24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^2 - 2*(8
```

```
*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x))*sinh(x)^3 + 10*(4*a^7 -
 17*a^5*b^2 + 28*a^3*b^4 - 15*a*b^6)*cosh(x)^2 + 10*(54*(a^2*b^5 - b^7)*cos
h(x)^7 + 4*a^7 - 17*a^5*b^2 + 28*a^3*b^4 - 15*a*b^6 - 42*(a^3*b^4 - a*b^6)*
cosh(x)^6 + 42*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 45*(a^5*b^2 - 4*a^
3*b^4 + 3*a*b^6)*cosh(x)^4 + 2*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^
7)*cosh(x)^3 - 6*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)^2 + 6
*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^2 - 15*(b^6*cosh(x)^10 + 10
*b^6*cosh(x)*sinh(x)^9 + b^6*sinh(x)^10 - 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^
6 - 10*b^6*cosh(x)^4 + 5*(9*b^6*cosh(x)^2 - b^6)*sinh(x)^8 + 5*b^6*cosh(x)^
2 + 40*(3*b^6*cosh(x)^3 - b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 - 1
4*b^6*cosh(x)^2 + b^6)*sinh(x)^6 - b^6 + 4*(63*b^6*cosh(x)^5 - 70*b^6*cosh(
x)^3 + 15*b^6*cosh(x))*sinh(x)^5 + 10*(21*b^6*cosh(x)^6 - 35*b^6*cosh(x)^4
+ 15*b^6*cosh(x)^2 - b^6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 - 7*b^6*cosh(x)^5
 + 5*b^6*cosh(x)^3 - b^6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8 - 28*b^6*c
osh(x)^6 + 30*b^6*cosh(x)^4 - 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2 + 10*(b^6*c
osh(x)^9 - 4*b^6*cosh(x)^7 + 6*b^6*cosh(x)^5 - 4*b^6*cosh(x)^3 + b^6*cosh(x
))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x
) + a)/(a^2 - b^2)) + 15*(a^2*b^5 - b^7)*cosh(x) + 5*(27*(a^2*b^5 - b^7)*co
sh(x)^8 - 24*(a^3*b^4 - a*b^6)*cosh(x)^7 + 3*a^2*b^5 - 3*b^7 + 28*(a^4*b^3
- 5*a^2*b^5 + 4*b^7)*cosh(x)^6 - 36*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)
^5 + 2*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^4 - 8*(8*a^7
- 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)^3 + 12*(a^4*b^3 - 5*a^2*b^5 +
 4*b^7)*cosh(x)^2 + 4*(4*a^7 - 17*a^5*b^2 + 28*a^3*b^4 - 15*a*b^6)*cosh(x))
*sinh(x))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^10 + 10*
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^9 + (a^8 -
4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sinh(x)^10 - 5*(a^8 - 4*a^6*b^2 +
6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^8 - 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4
*a^2*b^6 + b^8 - 9*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^
2)*sinh(x)^8 - a^8 + 4*a^6*b^2 - 6*a^4*b^4 + 4*a^2*b^6 - b^8 + 40*(3*(a^8 -
 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x))*sinh(x)^7 + 10*(a^8 - 4*a^6*b^2 + 6*a^4
*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^6 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8 + 21*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 -
 14*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^6 +
4*(63*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 - 70*(a^8 -
 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 + 15*(a^8 - 4*a^6*b^2 +
 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x))*sinh(x)^5 - 10*(a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*
a^2*b^6 + b^8 - 21*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^
6 + 35*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 15*(a^8
- 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^
8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^7 - 7*(a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4
- 4*a^2*b^6 + b^8)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b
^8)*cosh(x))*sinh(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*
```

```
cosh(x)^2 + 5*(9*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^8
+ a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 - 28*(a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^6 + 30*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a
^2*b^6 + b^8)*cosh(x)^4 - 12*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8
)*cosh(x)^2)*sinh(x)^2 + 10*((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8
)*cosh(x)^9 - 4*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^7 +
 6*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 - 4*(a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*
b^4 - 4*a^2*b^6 + b^8)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**6/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19861, size = 409, normalized size = 2.57

$$\frac{2\,b^6\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{2\left(15\,b^5e^{(9\,x)}-15\,ab^4e^{(8\,x)}+20\,a^2b^3e^{(7\,x)}-80\,b^5e^{(7\,x)}-30\,a^3b^2e^{(6\,x)}+90\,ab^4e^{(6\,x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^6*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*sqrt(-a^2 + b^2)) + 2/15*(15*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2*b
^3*e^(7*x) - 80*b^5*e^(7*x) - 30*a^3*b^2*e^(6*x) + 90*a*b^4*e^(6*x) + 48*a^
4*b*e^(5*x) - 136*a^2*b^3*e^(5*x) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) + 230*
a^3*b^2*e^(4*x) - 240*a*b^4*e^(4*x) + 20*a^2*b^3*e^(3*x) - 80*b^5*e^(3*x) +
 40*a^5*e^(2*x) - 130*a^3*b^2*e^(2*x) + 150*a*b^4*e^(2*x) + 15*b^5*e^x - 8*
a^5 + 26*a^3*b^2 - 33*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(e^(2*x)
- 1)^5)
```

3.178 $\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=67

$$-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2 - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2
*Sqrt[a + b]) - Sinh[x]/(b*(a + b*Cosh[x]))
```

Rubi [A] time = 0.104158, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2693, 2735, 2659, 208}

$$-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

```
[Out] x/b^2 - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2
*Sqrt[a + b]) - Sinh[x]/(b*(a + b*Cosh[x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx &= -\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\ &= \frac{x}{b^2}-\frac{\sinh(x)}{b(a+b\cosh(x))}-\frac{a\int \frac{1}{a+b\cosh(x)}\,dx}{b^2}\\ &= \frac{x}{b^2}-\frac{\sinh(x)}{b(a+b\cosh(x))}-\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\ &= \frac{x}{b^2}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}}-\frac{\sinh(x)}{b(a+b\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.0990332, size = 61, normalized size = 0.91

$$\frac{\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-\frac{b\sinh(x)}{a+b\cosh(x)}+x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

```
[Out] (x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] -
(b*Sinh[x])/(a + b*Cosh[x]))/b^2
```

Maple [A] time = 0.023, size = 99, normalized size = 1.5

$$2\frac{\tanh(x/2)}{b\left(a(\tanh(x/2))^2-(\tanh(x/2))^2b-a-b\right)}-2\frac{a}{b^2\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)+\frac{1}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*cosh(x))^2,x)
```

```
[Out] 2/b*tanh(1/2*x)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)-2/b^2*a/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/b^2*ln(tanh(1/2*x)+
1)-1/b^2*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1027, size = 1663, normalized size = 24.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] [((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)) + (a^2*b - b^3)*x + 2*(a^3 - a*b^2 + (a^3 - a*b^2)*
x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*cosh(x) + (a^3 - a*b^2)*x)*si
nh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 - b^5)*sinh(x)
```

```
^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*cos
h(x))*sinh(x)), ((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh(x)^2 + 2*
a^2*b - 2*b^3 + 2*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*
(a*b*cosh(x) + a^2)*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
osh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3)*x + 2*(a^3 - a*b^2 + (
a^3 - a*b^2)*x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*cosh(x) + (a^3 -
 a*b^2)*x)*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 -
 b^5)*sinh(x)^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b
^3 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16578, size = 92, normalized size = 1.37

$$-\frac{2\,a\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}b^{2}}+\frac{x}{b^{2}}+\frac{2\,(ae^{x}+b)}{\left(be^{(2\,x)}+2\,ae^{x}+b\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) + x/b^2 +
2*(a*e^x + b)/((b*e^(2*x) + 2*a*e^x + b)*b^2)
```

3.179 $\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=113

$$-\frac{(4a^2-3b^2)\tanh(x)}{3a^3}+\frac{b(3a^2-2b^2)\tan^{-1}(\sinh(x))}{2a^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{b\tanh(x)\mathrm{sech}(x)}{2a^2}$$

```
[Out] (b*(3*a^2 - 2*b^2)*ArcTan[Sinh[x]])/(2*a^4) + (2*(a - b)^(3/2)*(a + b)^(3/2
)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/a^4 - ((4*a^2 - 3*b^2)*Tanh
[x])/(3*a^3) - (b*Sech[x]*Tanh[x])/(2*a^2) + (Sech[x]^2*Tanh[x])/(3*a)
```

Rubi [A] time = 0.405609, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2725, 3055, 3001, 3770, 2659, 208}

$$-\frac{(4a^2-3b^2)\tanh(x)}{3a^3}+\frac{b(3a^2-2b^2)\tan^{-1}(\sinh(x))}{2a^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{b\tanh(x)\mathrm{sech}(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Cosh[x]),x]
```


```
[Out] (b*(3*a^2 - 2*b^2)*ArcTan[Sinh[x]])/(2*a^4) + (2*(a - b)^(3/2)*(a + b)^(3/2
)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/a^4 - ((4*a^2 - 3*b^2)*Tanh
[x])/(3*a^3) - (b*Sech[x]*Tanh[x])/(2*a^2) + (Sech[x]^2*Tanh[x])/(3*a)
```


Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = -\frac{b\,\mathrm{sech}(x)\tanh(x)}{2a^2} + \frac{\mathrm{sech}^2(x)\tanh(x)}{3a} - \frac{\int \frac{\left(2\left(4a^2-3b^2\right)-ab\cosh(x)-3\left(2a^2-b^2\right)\cosh^2(x)\right)\mathrm{sech}^2(x)}{a+b\cosh(x)}\,dx}{6a^2}$$

$$= -\frac{\left(4a^2-3b^2\right)\tanh(x)}{3a^3} - \frac{b\,\mathrm{sech}(x)\tanh(x)}{2a^2} + \frac{\mathrm{sech}^2(x)\tanh(x)}{3a} - \frac{\int \frac{\left(-3b\left(3a^2-2b^2\right)-3a\left(2a^2-b^2\right)\cosh(x)\right)\mathrm{se}}{a+b\cosh(x)}}{6a^3}$$

$$= -\frac{\left(4a^2-3b^2\right)\tanh(x)}{3a^3} - \frac{b\,\mathrm{sech}(x)\tanh(x)}{2a^2} + \frac{\mathrm{sech}^2(x)\tanh(x)}{3a} + \frac{\left(b\left(3a^2-2b^2\right)\right)\int \mathrm{sech}(x)\,dx}{2a^4} +$$

$$= \frac{b\left(3a^2-2b^2\right)\tan^{-1}(\sinh(x))}{2a^4} - \frac{\left(4a^2-3b^2\right)\tanh(x)}{3a^3} - \frac{b\,\mathrm{sech}(x)\tanh(x)}{2a^2} + \frac{\mathrm{sech}^2(x)\tanh(x)}{3a} +$$

$$= \frac{b\left(3a^2-2b^2\right)\tan^{-1}(\sinh(x))}{2a^4} + \frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4} - \frac{\left(4a^2-3b^2\right)\tanh(x)}{3a^3}$$

Mathematica [A] time = 0.411587, size = 100, normalized size = 0.88

$$\frac{-12\left(b^2-a^2\right)^{3/2}\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right) + 6b\left(3a^2-2b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + a\tanh(x)\left(2a^2\mathrm{sech}^2(x) - 8a^2 - 3ab\,\mathrm{sech}(x) + \right.}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (6*b*(3*a^2 - 2*b^2)*ArcTan[Tanh[x/2]] - 12*(-a^2 + b^2)^(3/2)*ArcTan[((a -
 b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + a*(-8*a^2 + 6*b^2 - 3*a*b*Sech[x] + 2*a^
2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

Maple [B] time = 0.032, size = 315, normalized size = 2.8

$$2\,\frac{1}{\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right) - 4\,\frac{b^2}{a^2\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right) + 2\,\frac{b^4}{a^4\sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*cosh(x)),x)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-4*b^2/
a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2*b^
4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-2/
a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5+1/a^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5
*b+2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*b^2-20/3/a/(tanh(1/2*x)^2+1)^3*t
anh(1/2*x)^3+4/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^2-2/a/(tanh(1/2*x)^2
+1)^3*tanh(1/2*x)+2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*b^2-1/a^2/(tanh(1/2
*x)^2+1)^3*tanh(1/2*x)*b+3/a^2*b*arctan(tanh(1/2*x))-2/a^4*arctan(tanh(1/2*
x))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73491, size = 5057, normalized size = 44.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*a^2*b*cosh(x)^5 + 3*a^2*b*sinh(x)^5 - 6*(2*a^3 - a*b^2)*cosh(x)^4
+ 3*(5*a^2*b*cosh(x) - 4*a^3 + 2*a*b^2)*sinh(x)^4 - 3*a^2*b*cosh(x) + 6*(5*
a^2*b*cosh(x)^2 - 4*(2*a^3 - a*b^2)*cosh(x))*sinh(x)^3 - 8*a^3 + 6*a*b^2 -
12*(a^3 - a*b^2)*cosh(x)^2 + 6*(5*a^2*b*cosh(x)^3 - 2*a^3 + 2*a*b^2 - 6*(2*
a^3 - a*b^2)*cosh(x)^2)*sinh(x)^2 + 3*((a^2 - b^2)*cosh(x)^6 + 6*(a^2 - b^2
)*cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4 + 3*(
5*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*cosh(x)^3
 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(5*(a^2 -
 b^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^
2 + 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x
))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*co
```

```
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)) - 3*((3*a^2*b - 2*b^3)*cosh(x)^6 + 6*(3*a^2*b - 2*b
^3)*cosh(x)*sinh(x)^5 + (3*a^2*b - 2*b^3)*sinh(x)^6 + 3*(3*a^2*b - 2*b^3)*c
osh(x)^4 + 3*(3*a^2*b - 2*b^3 + 5*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^4 +
4*(5*(3*a^2*b - 2*b^3)*cosh(x)^3 + 3*(3*a^2*b - 2*b^3)*cosh(x))*sinh(x)^3 +
 3*a^2*b - 2*b^3 + 3*(3*a^2*b - 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b - 2*b^3)*c
osh(x)^4 + 3*a^2*b - 2*b^3 + 6*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 6*(
(3*a^2*b - 2*b^3)*cosh(x)^5 + 2*(3*a^2*b - 2*b^3)*cosh(x)^3 + (3*a^2*b - 2*
b^3)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(5*a^2*b*cosh(x)^4 - 8
*(2*a^3 - a*b^2)*cosh(x)^3 - a^2*b - 8*(a^3 - a*b^2)*cosh(x))*sinh(x))/(a^4
*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*
a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x
)^3 + 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4
)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x)), -
1/3*(3*a^2*b*cosh(x)^5 + 3*a^2*b*sinh(x)^5 - 6*(2*a^3 - a*b^2)*cosh(x)^4 +
3*(5*a^2*b*cosh(x) - 4*a^3 + 2*a*b^2)*sinh(x)^4 - 3*a^2*b*cosh(x) + 6*(5*a^
2*b*cosh(x)^2 - 4*(2*a^3 - a*b^2)*cosh(x))*sinh(x)^3 - 8*a^3 + 6*a*b^2 - 12
*(a^3 - a*b^2)*cosh(x)^2 + 6*(5*a^2*b*cosh(x)^3 - 2*a^3 + 2*a*b^2 - 6*(2*a^
3 - a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^2 - b^2)*cosh(x)^6 + 6*(a^2 - b^2)*
cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4 + 3*(5*
(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*cosh(x)^3 +
 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(5*(a^2 - b
^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2
+ 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))
*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x)
+ a)/(a^2 - b^2)) - 3*((3*a^2*b - 2*b^3)*cosh(x)^6 + 6*(3*a^2*b - 2*b^3)*co
sh(x)*sinh(x)^5 + (3*a^2*b - 2*b^3)*sinh(x)^6 + 3*(3*a^2*b - 2*b^3)*cosh(x)
^4 + 3*(3*a^2*b - 2*b^3 + 5*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(
3*a^2*b - 2*b^3)*cosh(x)^3 + 3*(3*a^2*b - 2*b^3)*cosh(x))*sinh(x)^3 + 3*a^2
*b - 2*b^3 + 3*(3*a^2*b - 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b - 2*b^3)*cosh(x)
^4 + 3*a^2*b - 2*b^3 + 6*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^2
*b - 2*b^3)*cosh(x)^5 + 2*(3*a^2*b - 2*b^3)*cosh(x)^3 + (3*a^2*b - 2*b^3)*c
osh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(5*a^2*b*cosh(x)^4 - 8*(2*a^
3 - a*b^2)*cosh(x)^3 - a^2*b - 8*(a^3 - a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(
x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*co
sh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 +
3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh
(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*cosh(x)), x)
```

<u>Giac</u> [A] time = 1.20763, size = 194, normalized size = 1.72

$$\frac{(3a^2b - 2b^3)\arctan(e^x)}{a^4} + \frac{2(a^4 - 2a^2b^2 + b^4)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^4} - \frac{3abe^{(5x)} - 12a^2e^{(4x)} + 6b^2e^{(4x)} - 12a^2e^{(2x)} + 12b}{3a^3\left(e^{(2x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] (3*a^2*b - 2*b^3)*arctan(e^x)/a^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^x
 + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - 1/3*(3*a*b*e^(5*x) - 12*a^
2*e^(4*x) + 6*b^2*e^(4*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x - 8
*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

3.180 $\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=57

$$\frac{(a^2-b^2)\log(\cosh(x))}{a^3}-\frac{(a^2-b^2)\log(a+b\cosh(x))}{a^3}-\frac{b\text{sech}(x)}{a^2}+\frac{\text{sech}^2(x)}{2a}$$

```
[Out] ((a^2 - b^2)*Log[Cosh[x]])/a^3 - ((a^2 - b^2)*Log[a + b*Cosh[x]])/a^3 - (b*
Sech[x])/a^2 + Sech[x]^2/(2*a)
```

Rubi [A] time = 0.0988762, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2721, 894}

$$\frac{(a^2-b^2)\log(\cosh(x))}{a^3}-\frac{(a^2-b^2)\log(a+b\cosh(x))}{a^3}-\frac{b\text{sech}(x)}{a^2}+\frac{\text{sech}^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((a^2 - b^2)*Log[Cosh[x]])/a^3 - ((a^2 - b^2)*Log[a + b*Cosh[x]])/a^3 - (b*
Sech[x])/a^2 + Sech[x]^2/(2*a)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = -\operatorname{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx, x, b\cosh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{b^2}{ax^3}-\frac{b^2}{a^2x^2}+\frac{-a^2+b^2}{a^3x}+\frac{a^2-b^2}{a^3(a+x)}\right)dx, x, b\cosh(x)\right)$$
$$= \frac{(a^2-b^2)\log(\cosh(x))}{a^3}-\frac{(a^2-b^2)\log(a+b\cosh(x))}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\operatorname{sech}^2(x)}{2a}$$

Mathematica [A] time = 0.0925007, size = 46, normalized size = 0.81

$$\frac{2(a^2-b^2)(\log(\cosh(x))-\log(a+b\cosh(x)))+a^2\operatorname{sech}^2(x)-2ab\operatorname{sech}(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (2*(a^2 - b^2)*(Log[Cosh[x]] - Log[a + b*Cosh[x]]) - 2*a*b*Sech[x] + a^2*Se
ch[x]^2)/(2*a^3)
```

Maple [B] time = 0.03, size = 140, normalized size = 2.5

$$-\frac{1}{a}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a-b\right)+\frac{b^2}{a^3}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a-b\right)+\frac{1}{a}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*cosh(x)),x)
```

```
[Out] -1/a*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+1/a^3*ln(a*tanh(1/2*x)^2-tanh(
1/2*x)^2*b-a-b)*b^2+1/a*ln(tanh(1/2*x)^2+1)-1/a^3*ln(tanh(1/2*x)^2+1)*b^2-2
/a/(tanh(1/2*x)^2+1)-2/a^2/(tanh(1/2*x)^2+1)*b+2/a/(tanh(1/2*x)^2+1)^2
```

Maxima [A] time = 1.56926, size = 130, normalized size = 2.28

$$-\frac{2\left(be^{(-x)}-ae^{(-2x)}+be^{(-3x)}\right)}{2a^2e^{(-2x)}+a^2e^{(-4x)}+a^2}-\frac{(a^2-b^2)\log\left(2ae^{(-x)}+be^{(-2x)}+b\right)}{a^3}+\frac{(a^2-b^2)\log\left(e^{(-2x)}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*(b*e^(-x) - a*e^(-2*x) + b*e^(-3*x))/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a^
2) - (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/a^3 + (a^2 - b^2)*log(e^(
-2*x) + 1)/a^3
```

Fricas [B] time = 2.00553, size = 1166, normalized size = 20.46

$$-\frac{2\,ab\cosh\left(x\right)^{3}+2\,ab\sinh\left(x\right)^{3}-2\,a^{2}\cosh\left(x\right)^{2}+2\,ab\cosh\left(x\right)+2\left(3\,ab\cosh\left(x\right)-a^{2}\right)\sinh\left(x\right)^{2}+\left(\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{4}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 + 2*a*b*cosh(x) + 2*(
3*a*b*cosh(x) - a^2)*sinh(x)^2 + ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cos
h(x)*sinh(x)^3 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^
2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh
(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh
(x))) - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b
^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2
- b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(
x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)^2 - 2*a^
2*cosh(x) + a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*si
nh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(
a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^{3}\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*cosh(x)), x)
```

Giac [B] time = 1.15605, size = 155, normalized size = 2.72

$$\frac{\left(a^2-b^2\right)\log\left(e^{(-x)}+e^x\right)}{a^3}-\frac{\left(a^2b-b^3\right)\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^3b}-\frac{3\,a^2\left(e^{(-x)}+e^x\right)^2-3\,b^2\left(e^{(-x)}+e^x\right)^2+4\,ab\left(e^{(-x)}+e^x\right)}{2\,a^3\left(e^{(-x)}+e^x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] (a^2 - b^2)*log(e^(-x) + e^x)/a^3 - (a^2*b - b^3)*log(abs(b*(e^(-x) + e^x)
+ 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) + e^x)^2 - 3*b^2*(e^(-x) + e^x)^2 + 4*
a*b*(e^(-x) + e^x) - 4*a^2)/(a^3*(e^(-x) + e^x)^2)
```

3.181 $\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}+\frac{b\tan^{-1}(\sinh(x))}{a^2}-\frac{\tanh(x)}{a}$$

```
[Out] (b*ArcTan[Sinh[x]])/a^2 + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*T
anh[x/2])/Sqrt[a + b]])/a^2 - Tanh[x]/a
```

Rubi [A] time = 0.231897, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2723, 3056, 3001, 3770, 2659, 208}

$$\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}+\frac{b\tan^{-1}(\sinh(x))}{a^2}-\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (b*ArcTan[Sinh[x]])/a^2 + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*T
anh[x/2])/Sqrt[a + b]])/a^2 - Tanh[x]/a
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
```

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = -\int \frac{\left(1-\cosh^2(x)\right)\mathbf{sech}^2(x)}{a+b\cosh(x)}\,dx$$
$$= -\frac{\tanh(x)}{a} - \frac{\int \frac{(-b-a\cosh(x))\mathbf{sech}(x)}{a+b\cosh(x)}\,dx}{a}$$
$$= -\frac{\tanh(x)}{a} + \frac{b\int \mathbf{sech}(x)\,dx}{a^2} - \frac{\left(-a^2+b^2\right)\int \frac{1}{a+b\cosh(x)}\,dx}{a^2}$$
$$= \frac{b\tan^{-1}(\sinh(x))}{a^2} - \frac{\tanh(x)}{a} - \frac{\left(2\left(-a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{b\tan^{-1}(\sinh(x))}{a^2} + \frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2} - \frac{\tanh(x)}{a}$$

Mathematica [A] time = 0.104588, size = 61, normalized size = 1.

$$\frac{2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right) - a\tanh(x) + 2b\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*b*ArcTan[Tanh[x/2]] + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqrt
[-a^2 + b^2]] - a*Tanh[x])/a^2
```

Maple [B] time = 0.022, size = 108, normalized size = 1.8

$$2\frac{1}{\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right) - 2\frac{b^2}{a^2\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right) - 2\frac{\tanh(x/2)}{a\left((\tanh(x/2))^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*cosh(x)),x)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-2*b^2/
a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-2/a*
```

```
tanh(1/2*x)/(tanh(1/2*x)^2+1)+2/a^2*b*arctan(tanh(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.17282, size = 984, normalized size = 16.13

$$\left[\frac{\sqrt{a^2-b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+}\right.}{a^2\cosh(x)^2+2\,a^2\cosh(x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((b^2*
cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) +
a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
+ b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 2*(b*cosh(x
)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x)) + 2*
a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2), -2*(sqrt(
-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(-sqrt(-a
^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x)) - a)/(a^2*cosh(x)
^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*cosh(x)), x)
```

Giac [A] time = 1.14741, size = 90, normalized size = 1.48

$$\frac{2\,b\arctan\left(e^{x}\right)}{a^{2}}+\frac{2\left(a^{2}-b^{2}\right)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}a^{2}}+\frac{2}{a\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b*arctan(e^x)/a^2 + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(s
qrt(-a^2 + b^2)*a^2) + 2/(a*(e^(2*x) + 1))
```

3.182 $\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\log(\cosh(x))}{a}-\frac{\log(a+b\cosh(x))}{a}$$

```
[Out] Log[Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

Rubi [A] time = 0.0423172, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\cosh(x))}{a}-\frac{\log(a+b\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\cosh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\cosh(x)\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{a}$$
$$= \frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}$$

Mathematica [A] time = 0.009067, size = 20, normalized size = 1.

$$\frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

Maple [A] time = 0.014, size = 21, normalized size = 1.1

$$\frac{\ln(\cosh(x))}{a} - \frac{\ln(a+b\cosh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*cosh(x)),x)
```

```
[Out] ln(cosh(x))/a-ln(a+b*cosh(x))/a
```

Maxima [A] time = 1.5941, size = 45, normalized size = 2.25

$$-\frac{\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{a} + \frac{\log\left(e^{(-2\,x)} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -log(2*a*e^(-x) + b*e^(-2*x) + b)/a + log(e^(-2*x) + 1)/a
```

Fricas [A] time = 1.95711, size = 116, normalized size = 5.8

$$-\frac{\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x) - sin
h(x))))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*cosh(x)), x)
```

Giac [A] time = 1.23281, size = 45, normalized size = 2.25

$$\frac{\log\left(e^{(-x)}+e^{x}\right)}{a}-\frac{\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2\,a\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="giac")

[Out] log(e^(-x) + e^x)/a - log(abs(b*(e^(-x) + e^x) + 2*a))/a
```

3.183 $\int \frac{\coth(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{a\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) + Log[1 + Cosh[x]]/(2*(a - b)) - (a*Log[a + b*
Cosh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0704049, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2721, 801}

$$-\frac{a\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) + Log[1 + Cosh[x]]/(2*(a - b)) - (a*Log[a + b*
Cosh[x]])/(a^2 - b^2)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = -\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)} + \frac{a}{(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)(b+x)}\right) dx, x, b\cosh(x)\right)$$
$$= \frac{\log(1-\cosh(x))}{2(a+b)} + \frac{\log(1+\cosh(x))}{2(a-b)} - \frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

Mathematica [A] time = 0.0695403, size = 38, normalized size = 0.7

$$-\frac{a\log(a+b\cosh(x)) - a\log(\sinh(x)) + b\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Cosh[x]),x]
```

```
[Out] -((a*Log[a + b*Cosh[x]] - a*Log[Sinh[x]] + b*Log[Tanh[x/2]])/(a^2 - b^2))
```

Maple [A] time = 0.02, size = 53, normalized size = 1.

$$-\frac{a}{(a+b)(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2 - \left(\tanh\left(\frac{x}{2}\right)\right)^2 b - a - b\right) + \frac{1}{a+b}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*cosh(x)),x)
```

```
[Out] -a/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+1/(a+b)*ln(tanh(1/2*
x))
```

Maxima [A] time = 1.03312, size = 80, normalized size = 1.48

$$-\frac{a\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{a^2-b^2} + \frac{\log\left(e^{(-x)}+1\right)}{a-b} + \frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -a*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) + log(e^(-x) + 1)/(a - b) +
 log(e^(-x) - 1)/(a + b)
```

Fricas [A] time = 2.09963, size = 182, normalized size = 3.37

$$-\frac{a\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh
(x) + 1) - (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*cosh(x)), x)
```

Giac [A] time = 1.2286, size = 90, normalized size = 1.67

$$-\frac{ab\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^2b-b^3}+\frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) + 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

3.184 $\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=77

$$-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] (2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)
^(3/2)) - (a*Coth[x])/(a^2 - b^2) + (b*Csch[x])/(a^2 - b^2)
```

Rubi [A] time = 0.0935763, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2727, 3767, 8, 2606, 2659, 208}

$$-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)
^(3/2)) - (a*Coth[x])/(a^2 - b^2) + (b*Csch[x])/(a^2 - b^2)
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx &= \frac{a\int \operatorname{csch}^2(x)\,dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a+b\cosh(x)}\,dx}{a^2-b^2}-\frac{b\int \coth(x)\operatorname{csch}(x)\,dx}{a^2-b^2}\\
&= -\frac{(ia)\operatorname{Subst}\left(\int 1\,dx,x,-i\coth(x)\right)}{a^2-b^2}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}+\frac{(ib)\operatorname{Subst}(\int 1}{a^2}\\
&= \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.20125, size = 77, normalized size = 1.

$$\frac{2a^2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*a^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - C
oth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

Maple [A] time = 0.024, size = 78, normalized size = 1.

$$-\frac{1}{2\,a-2\,b}\tanh\left(\frac{x}{2}\right)+2\,\frac{a^2}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}\text{Artanh}\left(\frac{(a-b)\tanh\,(x/2)}{\sqrt{(a+b)\,(a-b)}}\right)-\frac{1}{2\,a+2\,b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*cosh(x)),x)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)+2/(a+b)/(a-b)*a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/2/(a+b)/tanh(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.96197, size = 1172, normalized size = 15.22

$$\left[\frac{2\,a^3-2\,ab^2+\left(a^2\cosh\,(x)^2+2\,a^2\cosh\,(x)\sinh\,(x)+a^2\sinh\,(x)^2-a^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2}{b\cosh(x)^2+b\sinh(}\right.}{a^4-2\,a^2b^2+b^4-\left(a^4-2\,a^2b^2+b^4\right)\cosh\,(x)^2-2\left(a^4-2\,a^2b^2+b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
- a^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(a^
4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a*b
^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
- (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x
) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*cosh(x)), x)
```

Giac [A] time = 1.19427, size = 103, normalized size = 1.34

$$\frac{2\,a^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}} + \frac{2\,(be^x-a)}{\left(a^2-b^2\right)\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) +
 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

3.185 $\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{a^3\log(a+b\cosh(x))}{\left(a^2-b^2\right)^2}-\frac{\operatorname{csch}^2(x)(a-b\cosh(x))}{2\left(a^2-b^2\right)}+\frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(2a-b)\log(\cosh(x)+1)}{4(a-b)^2}$$

```
[Out] -((a - b*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) + ((2*a + b)*Log[1 - Cosh[x]])
/(4*(a + b)^2) + ((2*a - b)*Log[1 + Cosh[x]])/(4*(a - b)^2) - (a^3*Log[a +
b*Cosh[x]])/(a^2 - b^2)^2
```

Rubi [A] time = 0.199426, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2721, 1647, 801}

$$-\frac{a^3\log(a+b\cosh(x))}{\left(a^2-b^2\right)^2}-\frac{\operatorname{csch}^2(x)(a-b\cosh(x))}{2\left(a^2-b^2\right)}+\frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(2a-b)\log(\cosh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -((a - b*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) + ((2*a + b)*Log[1 - Cosh[x]])
/(4*(a + b)^2) + ((2*a - b)*Log[1 + Cosh[x]])/(4*(a - b)^2) - (a^3*Log[a +
b*Cosh[x]])/(a^2 - b^2)^2
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = \text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cosh(x)\right)$$

$$= -\frac{(a-b\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(2a^2-b^2\right)x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x)\right)}{2b^2}$$

$$= -\frac{(a-b\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2a-b)b^2}{2(a-b)^2(b+x)}\right)dx, x, b\cosh(x\right.}{2b^2}$$

$$= -\frac{(a-b\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2} + \frac{(2a-b)\log(1+\cosh(x))}{4(a-b)^2} - \frac{a^3\log(a+b\,\text{c}}{\left(a^2-b\right.}$$

Mathematica [A] time = 0.210392, size = 101, normalized size = 1.07

$$\frac{-12a^2b\log\left(\tanh\left(\frac{x}{2}\right)\right) - 8a^3\log(a+b\cosh(x)) + 8a^3\log(\sinh(x)) - (a-b)^2(a+b)\text{csch}^2\left(\frac{x}{2}\right) + (a-b)(a+b)^2\text{sech}^2\left(\frac{x}{2}\right)}{8(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (-((a - b)^2*(a + b)*Csch[x/2]^2) - 8*a^3*Log[a + b*Cosh[x]] + 8*a^3*Log[Si
nh[x]] - 12*a^2*b*Log[Tanh[x/2]] + 4*b^3*Log[Tanh[x/2]] + (a - b)*(a + b)^2
*Sech[x/2]^2)/(8*(a - b)^2*(a + b)^2)
```

Maple [A] time = 0.029, size = 97, normalized size = 1.

$$-\frac{1}{8a-8b}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-\frac{a^{3}}{(a+b)^{2}(a-b)^{2}}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-\left(\tanh\left(\frac{x}{2}\right)\right)^{2}b-a-b\right)-\frac{1}{8a+8b}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{a}{(a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*cosh(x)),x)
```

```
[Out] -1/8*tanh(1/2*x)^2/(a-b)-a^3/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)
^2*b-a-b)-1/8/(a+b)/tanh(1/2*x)^2+1/(a+b)^2*ln(tanh(1/2*x))*a+1/2/(a+b)^2*l
n(tanh(1/2*x))*b
```

Maxima [A] time = 1.09367, size = 211, normalized size = 2.24

$$-\frac{a^{3}\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^{4}-2\,a^{2}b^{2}+b^{4}}+\frac{(2\,a-b)\log\left(e^{(-x)}+1\right)}{2\left(a^{2}-2\,ab+b^{2}\right)}+\frac{(2\,a+b)\log\left(e^{(-x)}-1\right)}{2\left(a^{2}+2\,ab+b^{2}\right)}+\frac{be^{(-x)}-2\,ae^{(-2\,x)}+be^{(-3\,x}}{a^{2}-b^{2}-2\left(a^{2}-b^{2}\right)e^{(-2\,x)}+\left(a^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(2*a -
b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + b)*log(e^(-x) - 1)/(a^2
 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2 - 2*(a^
2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

Fricas [B] time = 2.24211, size = 2071, normalized size = 22.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^2*b - b^3)*cosh(x)^3 + 2*(a^2*b - b^3)*sinh(x)^3 - 4*(a^3 - a*b^2
)*cosh(x)^2 - 2*(2*a^3 - 2*a*b^2 - 3*(a^2*b - b^3)*cosh(x))*sinh(x)^2 + 2*(
a^2*b - b^3)*cosh(x) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sin
```

```
h(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 + 4*(a
^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(
x))) + ((2*a^3 + 3*a^2*b - b^3)*cosh(x)^4 + 4*(2*a^3 + 3*a^2*b - b^3)*cosh(
x)*sinh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*sinh(x)^4 + 2*a^3 + 3*a^2*b - b^3 -
2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 - 2*(2*a^3 + 3*a^2*b - b^3 - 3*(2*a^3 +
 3*a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a^2*b - b^3)*cosh(x)^3
 - (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + (
(2*a^3 - 3*a^2*b + b^3)*cosh(x)^4 + 4*(2*a^3 - 3*a^2*b + b^3)*cosh(x)*sinh(
x)^3 + (2*a^3 - 3*a^2*b + b^3)*sinh(x)^4 + 2*a^3 - 3*a^2*b + b^3 - 2*(2*a^3
 - 3*a^2*b + b^3)*cosh(x)^2 - 2*(2*a^3 - 3*a^2*b + b^3 - 3*(2*a^3 - 3*a^2*b
 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a^2*b + b^3)*cosh(x)^3 - (2*a^
3 - 3*a^2*b + b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^2*b
- b^3 + 3*(a^2*b - b^3)*cosh(x)^2 - 4*(a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3
 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a
^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 - 2*
a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*cosh(x)), x)
```

Giac [A] time = 1.19589, size = 240, normalized size = 2.55

$$-\frac{a^3 b \log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^4 b - 2\,a^2 b^3 + b^5} + \frac{(2\,a-b)\log\left(e^{(-x)}+e^x+2\right)}{4\left(a^2-2\,ab+b^2\right)} + \frac{(2\,a+b)\log\left(e^{(-x)}+e^x-2\right)}{4\left(a^2+2\,ab+b^2\right)} - \frac{a^3\left(e^{(-x)}+e^x\right)^2 - 2\,a^2 b\left(e^{(-\cdots}{2\left(a^4-2\,a^2b^2+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(2*
a - b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + b)*log(e^(-x)
 + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a^2*b*(e^(-
x) + e^x) + 2*b^3*(e^(-x) + e^x) - 4*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*((e^(-
x) + e^x)^2 - 4))
```

3.186 $\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=137

$$-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{a^3\coth(x)}{\left(a^2-b^2\right)^2}+\frac{b\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\operatorname{csch}(x)}{\left(a^2-b^2\right)^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] (2*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)
^(5/2)) - (a^3*Coth[x])/(a^2 - b^2)^2 - (a*Coth[x]^3)/(3*(a^2 - b^2)) + (a^
2*b*Csch[x])/(a^2 - b^2)^2 + (b*Csch[x])/(a^2 - b^2) + (b*Csch[x]^3)/(3*(a^
2 - b^2))
```

Rubi [A] time = 0.19578, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2659, 208}

$$-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{a^3\coth(x)}{\left(a^2-b^2\right)^2}+\frac{b\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\operatorname{csch}(x)}{\left(a^2-b^2\right)^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (2*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)
^(5/2)) - (a^3*Coth[x])/(a^2 - b^2)^2 - (a*Coth[x]^3)/(3*(a^2 - b^2)) + (a^
2*b*Csch[x])/(a^2 - b^2)^2 + (b*Csch[x])/(a^2 - b^2) + (b*Csch[x]^3)/(3*(a^
2 - b^2))
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{a\int \coth^2(x)\text{csch}^2(x)\,dx}{a^2-b^2} + \frac{a^2\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx}{a^2-b^2} - \frac{b\int \coth^3(x)\text{csch}(x)\,dx}{a^2-b^2}$$

$$= \frac{a^3\int \text{csch}^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{a^4\int \frac{1}{a+b\cosh(x)}\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(a^2b\right)\int \coth(x)\text{csch}(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{(ia)\,\text{Subst}\left(\int x^2\,dx, x, i\coth\right.}{a^2-b^2}$$

$$= -\frac{a\coth^3(x)}{3\left(a^2-b^2\right)} + \frac{b\text{csch}(x)}{a^2-b^2} + \frac{b\text{csch}^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(ia^3\right)\text{Subst}(\int 1\,dx, x, -i\coth(x))}{\left(a^2-b^2\right)^2} + \frac{\left(2a^4\right)\text{Subst}\left(\int \frac{}{a+b\cdot}\right.}{\left(a^2\right.}$$

$$= \frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{a^3\coth(x)}{\left(a^2-b^2\right)^2} - \frac{a\coth^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\text{csch}(x)}{\left(a^2-b^2\right)^2} + \frac{b\text{csch}(x)}{a^2-b^2} + \frac{b\text{csch}^3(x)}{3\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.507083, size = 131, normalized size = 0.96

$$\frac{1}{24}\left(-\frac{48a^4\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{2(5b-8a)\tanh\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{2(8a+5b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\sinh^4\left(\frac{x}{2}\right)\text{csch}^3(x)}{a-b} - \frac{\sinh(x)\text{csch}^4\left(\frac{x}{2}\right.}{2(a+b)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] ((-48*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
- (2*(8*a + 5*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) -
 (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (2*(-8*a + 5*b)*Tanh[x/2])/(a - b)^2)/
24
```

Maple [A] time = 0.029, size = 127, normalized size = 0.9

$$-\frac{1}{8\,(a-b)^2}\left(\frac{a}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{b}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3 + 5\,a\tanh(x/2) - 3\,\tanh(x/2)\,b\right) + 2\,\frac{a^4}{(a-b)^2\,(a+b)^2\,\sqrt{(a+b)\,(a-b)}}\text{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*cosh(x)),x)
```

```
[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*tanh(1/2*x)^3*b+5*a*tanh(1/2*x)-3*tan
h(1/2*x)*b)+2/(a-b)^2/(a+b)^2*a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/
2*x)/((a+b)*(a-b))^(1/2))-1/24/(a+b)/tanh(1/2*x)^3-1/8*(5*a+3*b)/(a+b)^2/ta
nh(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30164, size = 5677, normalized size = 41.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/3*(6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b - 3*a^2*b^3 + b^
5)*sinh(x)^5 - 8*a^5 + 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^4)
*cosh(x)^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b - 3*a^2*b^3 + b^5)*c
osh(x))*sinh(x)^4 - 4*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^3 - 4*(4*a^4*b -
5*a^2*b^3 + b^5 - 15*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^2 + 6*(2*a^5 - 3*a
^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(x)^2 + 12*(a^5
 - a^3*b^2 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(2*a^5 - 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 + 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 - 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(5*a^4*cosh
(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 - 6*a^4*cosh(x)^2 + a
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) + b)) + 6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*a^4*b - 3*a^2*b^
3 + b^5 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 - 3*a^3*b^2 +
```

```
a*b^4)*cosh(x)^3 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^2 + 4*(a^5 - a^3*b
^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
- 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 +
3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 +
6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh
(x)), 2/3*(3*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^5 + 3*(2*a^4*b - 3*a^2*b^3
 + b^5)*sinh(x)^5 - 4*a^5 + 5*a^3*b^2 - a*b^4 - 3*(2*a^5 - 3*a^3*b^2 + a*b^
4)*cosh(x)^4 - 3*(2*a^5 - 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b - 3*a^2*b^3 + b^5)
*cosh(x))*sinh(x)^4 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^3 - 2*(4*a^4*b
- 5*a^2*b^3 + b^5 - 15*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^2 + 6*(2*a^5 - 3
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 6*(a^5 - a^3*b^2)*cosh(x)^2 + 6*(a^5
 - a^3*b^2 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(2*a^5 - 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 - 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 - 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(5*a^4*cosh
(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 - 6*a^4*cosh(x)^2 + a
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2
- b^2)) + 3*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x) + 3*(2*a^4*b - 3*a^2*b^3 +
b^5 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 - 3*a^3*b^2 + a*b^
4)*cosh(x)^3 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*
cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*cosh(x)), x)
```

Giac [A] time = 1.21027, size = 232, normalized size = 1.69

$$\frac{2\,a^4 \arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\left(a^4 - 2\,a^2b^2 + b^4\right)\sqrt{-a^2 + b^2}} + \frac{2\left(6\,a^2be^{(5\,x)} - 3\,b^3e^{(5\,x)} - 6\,a^3e^{(4\,x)} + 3\,ab^2e^{(4\,x)} - 8\,a^2be^{(3\,x)} + 2\,b^3e^{(3\,x)} + 6\,a^3e^{(2\,x)} + 6\,a^2b}{3\left(a^4 - 2\,a^2b^2 + b^4\right)\left(e^{(2\,x)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a
^2 + b^2)) + 2/3*(6*a^2*b*e^(5*x) - 3*b^3*e^(5*x) - 6*a^3*e^(4*x) + 3*a*b^2
*e^(4*x) - 8*a^2*b*e^(3*x) + 2*b^3*e^(3*x) + 6*a^3*e^(2*x) + 6*a^2*b*e^x -
3*b^3*e^x - 4*a^3 + a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

3.187 $\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^5(x)}{5a}+\frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{\tanh^3(x)\mathbf{sech}(x)}{4a}-\frac{3\tanh(x)\mathbf{sech}(x)}{8a}$$

```
[Out] (3*ArcTan[Sinh[x]])/(8*a) - (3*Sech[x]*Tanh[x])/(8*a) - (Sech[x]*Tanh[x]^3)
/(4*a) - Tanh[x]^5/(5*a)
```

Rubi [A] time = 0.0928183, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\tanh^5(x)}{5a}+\frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{\tanh^3(x)\mathbf{sech}(x)}{4a}-\frac{3\tanh(x)\mathbf{sech}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(8*a) - (3*Sech[x]*Tanh[x])/(8*a) - (Sech[x]*Tanh[x]^3)
/(4*a) - Tanh[x]^5/(5*a)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh^4(x)\,dx}{a}-\frac{\int \operatorname{sech}^2(x)\tanh^4(x)\,dx}{a}\\ &= -\frac{\operatorname{sech}(x)\tanh^3(x)}{4a}+\frac{i\,\text{Subst}\left(\int x^4\,dx,x,i\tanh(x)\right)}{a}+\frac{3\int \operatorname{sech}(x)\tanh^2(x)\,dx}{4a}\\ &= -\frac{3\operatorname{sech}(x)\tanh(x)}{8a}-\frac{\operatorname{sech}(x)\tanh^3(x)}{4a}-\frac{\tanh^5(x)}{5a}+\frac{3\int \operatorname{sech}(x)\,dx}{8a}\\ &= \frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{3\operatorname{sech}(x)\tanh(x)}{8a}-\frac{\operatorname{sech}(x)\tanh^3(x)}{4a}-\frac{\tanh^5(x)}{5a}\end{aligned}$$

Mathematica [A] time = 0.0845774, size = 58, normalized size = 1.26

$$\frac{\cosh^2\left(\frac{x}{2}\right)\left(30\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)\left(-8\operatorname{sech}^4(x)+10\operatorname{sech}^3(x)+16\operatorname{sech}^2(x)-25\operatorname{sech}(x)-8\right)\right)}{20a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]^2*(30*ArcTan[Tanh[x/2]] + (-8 - 25*Sech[x] + 16*Sech[x]^2 + 10*S
ech[x]^3 - 8*Sech[x]^4)*Tanh[x]))/(20*a*(1 + Cosh[x]))
```

Maple [B] time = 0.06, size = 115, normalized size = 2.5

$$\frac{3}{4a}\left(\tanh\left(\frac{x}{2}\right)\right)^{9}\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-5}+\frac{7}{2a}\left(\tanh\left(\frac{x}{2}\right)\right)^{7}\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-5}-\frac{32}{5a}\left(\tanh\left(\frac{x}{2}\right)\right)^{5}\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+a*cosh(x)),x)
```

```
[Out] 3/4/a/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^9+7/2/a/(tanh(1/2*x)^2+1)^5*tanh(1/2*
x)^7-32/5/a/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^5-7/2/a/(tanh(1/2*x)^2+1)^5*tan
h(1/2*x)^3-3/4/a/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)+3/4/a*arctan(tanh(1/2*x))
```

Maxima [B] time = 1.55973, size = 120, normalized size = 2.61

$$-\frac{25\,e^{(-x)}+10\,e^{(-3\,x)}+80\,e^{(-4\,x)}-10\,e^{(-7\,x)}+40\,e^{(-8\,x)}-25\,e^{(-9\,x)}+8}{20\left(5\,ae^{(-2\,x)}+10\,ae^{(-4\,x)}+10\,ae^{(-6\,x)}+5\,ae^{(-8\,x)}+ae^{(-10\,x)}+a\right)}-\frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/20*(25*e^(-x) + 10*e^(-3*x) + 80*e^(-4*x) - 10*e^(-7*x) + 40*e^(-8*x) -
25*e^(-9*x) + 8)/(5*a*e^(-2*x) + 10*a*e^(-4*x) + 10*a*e^(-6*x) + 5*a*e^(-8*
x) + a*e^(-10*x) + a) - 3/4*arctan(e^(-x))/a
```

Fricas [B] time = 1.88848, size = 2522, normalized size = 54.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/20*(25*cosh(x)^9 + 5*(45*cosh(x) - 8)*sinh(x)^8 + 25*sinh(x)^9 - 40*cosh
(x)^8 + 10*(90*cosh(x)^2 - 32*cosh(x) + 1)*sinh(x)^7 + 10*cosh(x)^7 + 70*(3
0*cosh(x)^3 - 16*cosh(x)^2 + cosh(x))*sinh(x)^6 + 70*(45*cosh(x)^4 - 32*cos
h(x)^3 + 3*cosh(x)^2)*sinh(x)^5 + 10*(315*cosh(x)^5 - 280*cosh(x)^4 + 35*co
```

```
sh(x)^3 - 8)*sinh(x)^4 - 80*cosh(x)^4 + 10*(210*cosh(x)^6 - 224*cosh(x)^5 +
 35*cosh(x)^4 - 32*cosh(x) - 1)*sinh(x)^3 - 10*cosh(x)^3 + 10*(90*cosh(x)^7
 - 112*cosh(x)^6 + 21*cosh(x)^5 - 48*cosh(x)^2 - 3*cosh(x))*sinh(x)^2 - 15*
(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 1)*sinh(
x)^8 + 5*cosh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*sinh(x)^7 + 10*(21*cosh(x)^
4 + 14*cosh(x)^2 + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*cosh(x)^5 + 70*cosh(
x)^3 + 15*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 + 15*cosh(x)
^2 + 1)*sinh(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^5 + 5*cosh(x
)^3 + cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 1
2*cosh(x)^2 + 1)*sinh(x)^2 + 5*cosh(x)^2 + 10*(cosh(x)^9 + 4*cosh(x)^7 + 6*
cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) +
 5*(45*cosh(x)^8 - 64*cosh(x)^7 + 14*cosh(x)^6 - 64*cosh(x)^3 - 6*cosh(x)^2
 - 5)*sinh(x) - 25*cosh(x) - 8)/(a*cosh(x)^10 + 10*a*cosh(x)*sinh(x)^9 + a*
sinh(x)^10 + 5*a*cosh(x)^8 + 5*(9*a*cosh(x)^2 + a)*sinh(x)^8 + 40*(3*a*cosh
(x)^3 + a*cosh(x))*sinh(x)^7 + 10*a*cosh(x)^6 + 10*(21*a*cosh(x)^4 + 14*a*c
osh(x)^2 + a)*sinh(x)^6 + 4*(63*a*cosh(x)^5 + 70*a*cosh(x)^3 + 15*a*cosh(x)
)*sinh(x)^5 + 10*a*cosh(x)^4 + 10*(21*a*cosh(x)^6 + 35*a*cosh(x)^4 + 15*a*c
osh(x)^2 + a)*sinh(x)^4 + 40*(3*a*cosh(x)^7 + 7*a*cosh(x)^5 + 5*a*cosh(x)^3
 + a*cosh(x))*sinh(x)^3 + 5*a*cosh(x)^2 + 5*(9*a*cosh(x)^8 + 28*a*cosh(x)^6
 + 30*a*cosh(x)^4 + 12*a*cosh(x)^2 + a)*sinh(x)^2 + 10*(a*cosh(x)^9 + 4*a*c
osh(x)^7 + 6*a*cosh(x)^5 + 4*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tanh^6(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**6/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.18083, size = 78, normalized size = 1.7

$$\frac{3\,\arctan\left(e^x\right)}{4\,a} - \frac{25\,e^{(9\,x)} - 40\,e^{(8\,x)} + 10\,e^{(7\,x)} - 80\,e^{(4\,x)} - 10\,e^{(3\,x)} - 25\,e^x - 8}{20\,a\left(e^{(2\,x)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3/4*arctan(e^x)/a - 1/20*(25*e^(9*x) - 40*e^(8*x) + 10*e^(7*x) - 80*e^(4*x)
 - 10*e^(3*x) - 25*e^x - 8)/(a*(e^(2*x) + 1)^5)
```

3.188 $\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^4(x)}{4a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\operatorname{sech}(x)}{a}$$

```
[Out] -(Sech[x]/a) + Sech[x]^3/(3*a) - Tanh[x]^4/(4*a)
```

Rubi [A] time = 0.0858699, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2607, 30, 2606}

$$-\frac{\tanh^4(x)}{4a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] -(Sech[x]/a) + Sech[x]^3/(3*a) - Tanh[x]^4/(4*a)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh^3(x)\,dx}{a}-\frac{\int \operatorname{sech}^2(x)\tanh^3(x)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^3\,dx,x,i\tanh(x)\right)}{a}+\frac{\operatorname{Subst}\left(\int\left(-1+x^2\right)\,dx,x,\operatorname{sech}(x)\right)}{a}\\ &= -\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\tanh^4(x)}{4a}\end{aligned}$$

Mathematica [A] time = 0.0273653, size = 25, normalized size = 0.83

$$\frac{2\sinh^6\left(\frac{x}{2}\right)(5\cosh(x)+3)\operatorname{sech}^4(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (2*(3 + 5*Cosh[x])*Sech[x]^4*Sinh[x/2]^6)/(3*a)
```

Maple [A] time = 0.051, size = 30, normalized size = 1.

$$\frac{1}{a}\left(-\left(\cosh(x)\right)^{-1}+\frac{1}{3\left(\cosh(x)\right)^3}-\frac{1}{4\left(\cosh(x)\right)^4}+\frac{1}{2\left(\cosh(x)\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+a*cosh(x)),x)
```

```
[Out] 1/a*(-1/cosh(x)+1/3/cosh(x)^3-1/4/cosh(x)^4+1/2/cosh(x)^2)
```

Maxima [B] time = 1.08009, size = 301, normalized size = 10.03

$$-\frac{2\,e^{(-x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}+\frac{2\,e^{(-2\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}-\frac{10\,e}{3\left(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) + 2
*e^(-2*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) - 1
0/3*e^(-3*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a)
- 10/3*e^(-5*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a) + 2*e^(-6*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a) - 2*e^(-7*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a)
```

Fricas [B] time = 1.8375, size = 562, normalized size = 18.73

$$-\frac{2\left(3\,\cosh\left(x\right)^{4}+3\left(4\,\cosh\left(x\right)-1\right)\sinh\left(x\right)^{3}+3\,\sinh\left(x\right)^{4}-3\,\cosh\left(x\right)^{3}+\left(18\,\cosh\left(x\right)^{2}-9\,\cosh\left(x\right)+8\right)\sinh\left(x\right)^{2}+\right.}{3\left(a\cosh\left(x\right)^{5}+5\,a\cosh\left(x\right)\sinh\left(x\right)^{4}+a\sinh\left(x\right)^{5}+5\,a\cosh\left(x\right)^{3}+\left(10\,a\cosh\left(x\right)^{2}+3\,a\right)\sinh\left(x\right)^{3}+5\left(2\,a\cosh\left(x\right)^{3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(x)^4 + 3*(4*cosh(x) - 1)*sinh(x)^3 + 3*sinh(x)^4 - 3*cosh(x)^3
 + (18*cosh(x)^2 - 9*cosh(x) + 8)*sinh(x)^2 + 8*cosh(x)^2 + (12*cosh(x)^3 -
 9*cosh(x)^2 + 4*cosh(x) + 3)*sinh(x) - 3*cosh(x) + 5)/(a*cosh(x)^5 + 5*a*c
osh(x)*sinh(x)^4 + a*sinh(x)^5 + 5*a*cosh(x)^3 + (10*a*cosh(x)^2 + 3*a)*sin
h(x)^3 + 5*(2*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^2 + 10*a*cosh(x) + (5*a*co
sh(x)^4 + 9*a*cosh(x)^2 + 2*a)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tanh^{5}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**5/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.19375, size = 65, normalized size = 2.17

$$-\frac{2\left(3\left(e^{(-x)}+e^{x}\right)^{3}-3\left(e^{(-x)}+e^{x}\right)^{2}-4\,e^{(-x)}-4\,e^{x}+6\right)}{3\,a\left(e^{(-x)}+e^{x}\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -2/3*(3*(e^(-x) + e^x)^3 - 3*(e^(-x) + e^x)^2 - 4*e^(-x) - 4*e^x + 6)/(a*(e
^(-x) + e^x)^4)
```

3.189 $\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^3(x)}{3a}+\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)\mathrm{sech}(x)}{2a}$$

```
[Out] ArcTan[Sinh[x]]/(2*a) - (Sech[x]*Tanh[x])/(2*a) - Tanh[x]^3/(3*a)
```

Rubi [A] time = 0.0797613, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\tanh^3(x)}{3a}+\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)\mathrm{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) - (Sech[x]*Tanh[x])/(2*a) - Tanh[x]^3/(3*a)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh^2(x)\,dx}{a}-\frac{\int \operatorname{sech}^2(x)\tanh^2(x)\,dx}{a}\\ &= -\frac{\operatorname{sech}(x)\tanh(x)}{2a}-\frac{i\,\operatorname{Subst}\left(\int x^2\,dx,x,i\tanh(x)\right)}{a}+\frac{\int \operatorname{sech}(x)\,dx}{2a}\\ &= \frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\tanh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.0593945, size = 46, normalized size = 1.39

$$\frac{\cosh^2\left(\frac{x}{2}\right)\left(6\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)\left(2\operatorname{sech}^2(x)-3\operatorname{sech}(x)-2\right)\right)}{3a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]^2*(6*ArcTan[Tanh[x/2]] + (-2 - 3*Sech[x] + 2*Sech[x]^2)*Tanh[x])
)/(3*a*(1 + Cosh[x]))
```

Maple [B] time = 0.036, size = 71, normalized size = 2.2

$$\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)\right)^5\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-3}-\frac{8}{3\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-3}-\frac{1}{a}\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-3}+\frac{1}{a}\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5-8/3/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)
^3-1/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)+1/a*arctan(tanh(1/2*x))
```

Maxima [B] time = 1.55904, size = 77, normalized size = 2.33

$$-\frac{3\,e^{(-x)}+6\,e^{(-4\,x)}-3\,e^{(-5\,x)}+2}{3\left(3\,ae^{(-2\,x)}+3\,ae^{(-4\,x)}+ae^{(-6\,x)}+a\right)}-\frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*e^(-x) + 6*e^(-4*x) - 3*e^(-5*x) + 2)/(3*a*e^(-2*x) + 3*a*e^(-4*x)
+ a*e^(-6*x) + a) - arctan(e^(-x))/a
```

Fricas [B] time = 1.73557, size = 1031, normalized size = 31.24

$$-\frac{3\,\cosh\left(x\right)^{5}+3\,(5\,\cosh\left(x\right)-2)\sinh\left(x\right)^{4}+3\,\sinh\left(x\right)^{5}-6\,\cosh\left(x\right)^{4}+6\left(5\,\cosh\left(x\right)^{2}-4\,\cosh\left(x\right)\right)\sinh\left(x\right)^{3}+6\left(5\,c\right.}{3\left(a\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*cosh(x)^5 + 3*(5*cosh(x) - 2)*sinh(x)^4 + 3*sinh(x)^5 - 6*cosh(x)^4
 + 6*(5*cosh(x)^2 - 4*cosh(x))*sinh(x)^3 + 6*(5*cosh(x)^3 - 6*cosh(x)^2)*si
nh(x)^2 - 3*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 +
 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*
cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cos
h(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(5*cosh(x)^4 -
 8*cosh(x)^3 - 1)*sinh(x) - 3*cosh(x) - 2)/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(
x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5
*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 +
6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*
```

```
sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tanh^4(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**4/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.18156, size = 53, normalized size = 1.61

$$\frac{\arctan(e^x)}{a} - \frac{3\,e^{(5\,x)} - 6\,e^{(4\,x)} - 3\,e^x - 2}{3\,a\left(e^{(2\,x)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] arctan(e^x)/a - 1/3*(3*e^(5*x) - 6*e^(4*x) - 3*e^x - 2)/(a*(e^(2*x) + 1)^3)
```

3.190 $\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\operatorname{sech}^2(x)}{2a}-\frac{\operatorname{sech}(x)}{a}$$

```
[Out] -(Sech[x]/a) + Sech[x]^2/(2*a)
```

Rubi [A] time = 0.0659567, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2606, 30, 8}

$$\frac{\operatorname{sech}^2(x)}{2a}-\frac{\operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] -(Sech[x]/a) + Sech[x]^2/(2*a)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh(x)\,dx}{a} - \frac{\int \operatorname{sech}^2(x)\tanh(x)\,dx}{a}\\ &= -\frac{\operatorname{Subst}(\int 1\,dx, x, \operatorname{sech}(x))}{a} + \frac{\operatorname{Subst}(\int x\,dx, x, \operatorname{sech}(x))}{a}\\ &= -\frac{\operatorname{sech}(x)}{a} + \frac{\operatorname{sech}^2(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0213752, size = 17, normalized size = 0.89

$$\frac{2\sinh^4\left(\frac{x}{2}\right)\operatorname{sech}^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (2*Sech[x]^2*Sinh[x/2]^4)/a
```

Maple [A] time = 0.026, size = 18, normalized size = 1.

$$\frac{1}{a}\left(-(\cosh(x))^{-1} + \frac{1}{2(\cosh(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+a*cosh(x)),x)
```

```
[Out] 1/a*(-1/cosh(x)+1/2/cosh(x)^2)
```

Maxima [B] time = 1.07107, size = 95, normalized size = 5.

$$-\frac{2\,e^{(-x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} + \frac{2\,e^{(-2\,x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} - \frac{2\,e^{(-3\,x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-x)/(2*a*e^(-2*x) + a*e^(-4*x) + a) + 2*e^(-2*x)/(2*a*e^(-2*x) + a*e^
(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
```

Fricas [B] time = 1.82161, size = 221, normalized size = 11.63

$$-\frac{2\left(\cosh\left(x\right)^{2} + \left(2\,\cosh\left(x\right) - 1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} - \cosh\left(x\right) + 1\right)}{a\cosh\left(x\right)^{3} + 3\,a\cosh\left(x\right)\sinh\left(x\right)^{2} + a\sinh\left(x\right)^{3} + 3\,a\cosh\left(x\right) + \left(3\,a\cosh\left(x\right)^{2} + a\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2*(cosh(x)^2 + (2*cosh(x) - 1)*sinh(x) + sinh(x)^2 - cosh(x) + 1)/(a*cosh(
x)^3 + 3*a*cosh(x)*sinh(x)^2 + a*sinh(x)^3 + 3*a*cosh(x) + (3*a*cosh(x)^2 +
 a)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tanh^{3}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**3/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.15578, size = 30, normalized size = 1.58

$$-\frac{2\left(e^{(-x)}+e^{x}-1\right)}{a\left(e^{(-x)}+e^{x}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*(e^(-x) + e^x - 1)/(a*(e^(-x) + e^x)^2)
```

3.191 $\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a}$$

```
[Out] ArcTan[Sinh[x]]/a - Tanh[x]/a
```

Rubi [A] time = 0.0488648, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 3767, 8, 3770}

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - Tanh[x]/a
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{a}-\frac{\int \operatorname{sech}^2(x)\,dx}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{i\operatorname{Subst}(\int 1\,dx,x,-i\tanh(x))}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.045499, size = 18, normalized size = 1.2

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + a*Cosh[x]),x]

[Out] (2*ArcTan[Tanh[x/2]] - Tanh[x])/a
```

Maple [A] time = 0.02, size = 31, normalized size = 2.1

$$-2\,\frac{\tanh(x/2)}{a\left((\tanh(x/2))^2+1\right)}+2\,\frac{\arctan(\tanh(x/2))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+a*cosh(x)),x)

[Out] -2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)+2/a*arctan(tanh(1/2*x))
```

Maxima [A] time = 1.54332, size = 31, normalized size = 2.07

$$-\frac{2\arctan\left(e^{(-x)}\right)}{a}-\frac{2}{ae^{(-2x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-2*x) + a)
```

Fricas [B] time = 1.92385, size = 185, normalized size = 12.33

$$\frac{2\left(\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\arctan(\cosh(x) + \sinh(x)) + 1\right)}{a\cosh(x)^2 + 2a\cosh(x)\sinh(x) + a\sinh(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(cosh(x) + sinh(x)
) + 1)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tanh^2(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**2/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.16883, size = 30, normalized size = 2.

$$\frac{2\arctan(e^x)}{a} + \frac{2}{a\left(e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)/a + 2/(a*(e^(2*x) + 1))
```

3.192 $\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(\cosh(x))}{a}-\frac{\log(\cosh(x)+1)}{a}$$

```
[Out] Log[Cosh[x]]/a - Log[1 + Cosh[x]]/a
```

Rubi [A] time = 0.0399798, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2707, 36, 29, 31}

$$\frac{\log(\cosh(x))}{a}-\frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + a*Cosh[x]),x]
```

```
[Out] Log[Cosh[x]]/a - Log[1 + Cosh[x]]/a
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\cosh(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a\cosh(x)\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\cosh(x)\right)}{a}\\ &= \frac{\log(\cosh(x))}{a} - \frac{\log(1+\cosh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.0167471, size = 12, normalized size = 0.67

$$-\frac{2\tanh^{-1}(2\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + a*Cosh[x]),x]
```

```
[Out] (-2*ArcTanh[1 + 2*Cosh[x]])/a
```

Maple [A] time = 0.016, size = 19, normalized size = 1.1

$$\frac{\ln(\cosh(x))}{a} - \frac{\ln(1+\cosh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+a*cosh(x)),x)
```

```
[Out] ln(cosh(x))/a-ln(1+cosh(x))/a
```

Maxima [A] time = 1.05412, size = 32, normalized size = 1.78

$$-\frac{2\log\left(e^{(-x)}+1\right)}{a} + \frac{\log\left(e^{(-2x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*log(e^(-x) + 1)/a + log(e^(-2*x) + 1)/a
```

Fricas [A] time = 1.90531, size = 96, normalized size = 5.33

$$\frac{\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)-2\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] (log(2*cosh(x)/(cosh(x) - sinh(x))) - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tanh\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.24794, size = 30, normalized size = 1.67

$$\frac{\log\left(e^{(2\,x)}+1\right)}{a}-\frac{2\,\log\left(e^{x}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] log(e^(2*x) + 1)/a - 2*log(e^x + 1)/a
```

3.193 $\int \frac{\coth(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\operatorname{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

```
[Out] -ArcTanh[Cosh[x]]/(2*a) - (Coth[x]*Csch[x])/(2*a) + Csch[x]^2/(2*a)
```

Rubi [A] time = 0.0651714, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2706, 2606, 30, 2611, 3770}

$$\frac{\operatorname{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + a*Cosh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/(2*a) - (Coth[x]*Csch[x])/(2*a) + Csch[x]^2/(2*a)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^2(x)\mathbf{csch}(x)\,dx}{a}-\frac{\int \coth(x)\mathbf{csch}^2(x)\,dx}{a}\\ &= -\frac{\coth(x)\mathbf{csch}(x)}{2a}+\frac{\int \mathbf{csch}(x)\,dx}{2a}-\frac{\mathrm{Subst}(\int x\,dx,x,-i\mathbf{csch}(x))}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\mathbf{csch}(x)}{2a}+\frac{\mathbf{csch}^2(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0371304, size = 42, normalized size = 1.27

$$-\frac{2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+1}{2a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + a*Cosh[x]),x]
```

```
[Out] -(1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(2*a*(1 + Cosh[x]))
```

Maple [A] time = 0.019, size = 23, normalized size = 0.7

$$\frac{1}{4\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{1}{2\,a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+a*cosh(x)),x)
```

```
[Out] 1/4/a*tanh(1/2*x)^2+1/2/a*ln(tanh(1/2*x))
```

Maxima [A] time = 1.07166, size = 65, normalized size = 1.97

$$-\frac{e^{(-x)}}{2\,ae^{(-x)} + ae^{(-2\,x)} + a} - \frac{\log\left(e^{(-x)} + 1\right)}{2\,a} + \frac{\log\left(e^{(-x)} - 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(
-x) - 1)/a
```

Fricas [B] time = 1.96863, size = 397, normalized size = 12.03

$$-\frac{\left(\cosh\left(x\right)^{2} + 2\left(\cosh\left(x\right) + 1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 2\,\cosh\left(x\right) + 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - \left(\cosh\left(x\right)^{2} + 2\,(\mathrm{cos}\right.}{2\left(a\cosh\left(x\right)^{2} + a\sinh\left(x\right)^{2} + 2\,a\cosh\left(x\right) + 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\coth\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.22914, size = 70, normalized size = 2.12

$$-\frac{\log\left(e^{(-x)} + e^{x} + 2\right)}{4\,a} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{4\,a} + \frac{e^{(-x)} + e^{x} - 2}{4\,a\left(e^{(-x)} + e^{x} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x - 2)/(a*(e^(-x) + e^x + 2))
```

3.194 $\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=30

$$\frac{\coth^3(x)}{3a}-\frac{\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

```
[Out] Coth[x]^3/(3*a) - Csch[x]/a - Csch[x]^3/(3*a)
```

Rubi [A] time = 0.0785009, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2607, 30, 2606}

$$\frac{\coth^3(x)}{3a}-\frac{\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] Coth[x]^3/(3*a) - Csch[x]/a - Csch[x]^3/(3*a)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^3(x)\operatorname{csch}(x)\,dx}{a}-\frac{\int \coth^2(x)\operatorname{csch}^2(x)\,dx}{a}\\ &= \frac{i\,\mathrm{Subst}\left(\int x^2\,dx,x,i\coth(x)\right)}{a}+\frac{i\,\mathrm{Subst}\left(\int \left(-1+x^2\right)\,dx,x,-i\operatorname{csch}(x)\right)}{a}\\ &= \frac{\coth^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}-\frac{\operatorname{csch}^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.0480718, size = 25, normalized size = 0.83

$$\frac{(-4\cosh(x)+\cosh(2x)-3)\operatorname{csch}(x)}{6a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] ((-3 - 4*Cosh[x] + Cosh[2*x])*Csch[x])/(6*a*(1 + Cosh[x]))
```

Maple [A] time = 0.021, size = 29, normalized size = 1.

$$\frac{1}{4\,a}\left(\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3+2\,\tanh(x/2)-\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+a*cosh(x)),x)
```

```
[Out] 1/4/a*(1/3*tanh(1/2*x)^3+2*tanh(1/2*x)-1/tanh(1/2*x))
```

Maxima [B] time = 1.01716, size = 163, normalized size = 5.43

$$-\frac{2\,e^{(-x)}}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}-\frac{2\,e^{(-2\,x)}}{2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a}-\frac{2\,e^{(-3\,x)}}{2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a}+\frac{}{3\left(2\,ae^{(-x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a*
e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-
3*x) - a*e^(-4*x) + a) + 2/3/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a)
```

Fricas [B] time = 1.76574, size = 292, normalized size = 9.73

$$-\frac{2\left(3\,\cosh\left(x\right)^{2}+2\left(3\,\cosh\left(x\right)+2\right)\sinh\left(x\right)+3\,\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)}{3\left(a\cosh\left(x\right)^{3}+a\sinh\left(x\right)^{3}+2\,a\cosh\left(x\right)^{2}+\left(3\,a\cosh\left(x\right)+2\,a\right)\sinh\left(x\right)^{2}-a\cosh\left(x\right)+\left(3\,a\cosh\left(x\right)^{2}+4\,a\cosh\left(x\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(x)^2 + 2*(3*cosh(x) + 2)*sinh(x) + 3*sinh(x)^2 + 2*cosh(x) + 1
)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2 + (3*a*cosh(x) + 2*a)*sinh(x)^
2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) - 2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\coth^{2}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)**2/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.18759, size = 47, normalized size = 1.57

$$-\frac{1}{2\,a(e^{x}-1)}-\frac{9\,e^{(2\,x)}+12\,e^{x}+7}{6\,a(e^{x}+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/2/(a*(e^x - 1)) - 1/6*(9*e^(2*x) + 12*e^x + 7)/(a*(e^x + 1)^3)
```

3.195 $\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=46

$$\frac{\coth^4(x)}{4a}-\frac{3\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth^3(x)\text{csch}(x)}{4a}-\frac{3\coth(x)\text{csch}(x)}{8a}$$

```
[Out] (-3*ArcTanh[Cosh[x]])/(8*a) + Coth[x]^4/(4*a) - (3*Coth[x]*Csch[x])/(8*a) -
 (Coth[x]^3*Csch[x])/(4*a)
```

Rubi [A] time = 0.106865, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\coth^4(x)}{4a}-\frac{3\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth^3(x)\text{csch}(x)}{4a}-\frac{3\coth(x)\text{csch}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (-3*ArcTanh[Cosh[x]])/(8*a) + Coth[x]^4/(4*a) - (3*Coth[x]*Csch[x])/(8*a) -
 (Coth[x]^3*Csch[x])/(4*a)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^4(x)\mathbf{csch}(x)\,dx}{a}-\frac{\int \coth^3(x)\mathbf{csch}^2(x)\,dx}{a}\\ &= -\frac{\coth^3(x)\mathbf{csch}(x)}{4a}+\frac{3\int \coth^2(x)\mathbf{csch}(x)\,dx}{4a}+\frac{\operatorname{Subst}\left(\int x^3\,dx,x,i\coth(x)\right)}{a}\\ &= \frac{\coth^4(x)}{4a}-\frac{3\coth(x)\mathbf{csch}(x)}{8a}-\frac{\coth^3(x)\mathbf{csch}(x)}{4a}+\frac{3\int \mathbf{csch}(x)\,dx}{8a}\\ &= -\frac{3\tanh^{-1}(\cosh(x))}{8a}+\frac{\coth^4(x)}{4a}-\frac{3\coth(x)\mathbf{csch}(x)}{8a}-\frac{\coth^3(x)\mathbf{csch}(x)}{4a}\end{aligned}$$

Mathematica [A] time = 0.0990674, size = 60, normalized size = 1.3

$$\frac{-2\coth^2\left(\frac{x}{2}\right)+\mathbf{sech}^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)-8}{16a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (-8 - 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]) + Se
ch[x/2]^2)/(16*a*(1 + Cosh[x]))
```

Maple [A] time = 0.027, size = 45, normalized size = 1.

$$\frac{1}{32a}\left(\tanh\left(\frac{x}{2}\right)\right)^4+\frac{3}{16a}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{3}{8a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{16a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+a*cosh(x)),x)
```

```
[Out] 1/32/a*tanh(1/2*x)^4+3/16/a*tanh(1/2*x)^2+3/8/a*ln(tanh(1/2*x))-1/16/a/tanh
(1/2*x)^2
```

Maxima [B] time = 1.08625, size = 139, normalized size = 3.02

$$-\frac{5e^{(-x)}+2e^{(-2x)}+2e^{(-3x)}+2e^{(-4x)}+5e^{(-5x)}}{4\left(2ae^{(-x)}-ae^{(-2x)}-4ae^{(-3x)}-ae^{(-4x)}+2ae^{(-5x)}+ae^{(-6x)}+a\right)}-\frac{3\log\left(e^{(-x)}+1\right)}{8a}+\frac{3\log\left(e^{(-x)}-1\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(5*e^(-x) + 2*e^(-2*x) + 2*e^(-3*x) + 2*e^(-4*x) + 5*e^(-5*x))/(2*a*e^
(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) +
 a) - 3/8*log(e^(-x) + 1)/a + 3/8*log(e^(-x) - 1)/a
```

Fricas [B] time = 2.01018, size = 2071, normalized size = 45.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(10*cosh(x)^5 + 2*(25*cosh(x) + 2)*sinh(x)^4 + 10*sinh(x)^5 + 4*cosh(x
)^4 + 4*(25*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x)^3 + 4*cosh(x)^3 + 4*(25*cosh
(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 + 3*(cosh(x)^6
 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 +
10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh
(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)
```

```
^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 -
 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh
(x) + sinh(x) + 1) - 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6
 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*
(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cos
h(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2
 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*
sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(25*cosh(x)^4 + 8*c
osh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x) + 10*cosh(x))/(a*cosh(x)^6
+ a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4
 + (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*c
osh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*c
osh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2
*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)
^2 - a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\coth^3(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)**3/(cosh(x) + 1), x)/a
```

Giac [B] time = 1.17454, size = 127, normalized size = 2.76

$$-\frac{3\,\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a}+\frac{3\,\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a}-\frac{3\,e^{(-x)}+3\,e^{x}-2}{16\,a\left(e^{(-x)}+e^{x}-2\right)}+\frac{9\left(e^{(-x)}+e^{x}\right)^{2}+4\,e^{(-x)}+4\,e^{x}-12}{32\,a\left(e^{(-x)}+e^{x}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -3/16*log(e^(-x) + e^x + 2)/a + 3/16*log(e^(-x) + e^x - 2)/a - 1/16*(3*e^(-
x) + 3*e^x - 2)/(a*(e^(-x) + e^x - 2)) + 1/32*(9*(e^(-x) + e^x)^2 + 4*e^(-x
) + 4*e^x - 12)/(a*(e^(-x) + e^x + 2)^2)
```

3.196 $\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=41

$$\frac{\coth^5(x)}{5a}-\frac{\operatorname{csch}^5(x)}{5a}-\frac{2\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

```
[Out] Coth[x]^5/(5*a) - Csch[x]/a - (2*Csch[x]^3)/(3*a) - Csch[x]^5/(5*a)
```

Rubi [A] time = 0.0807787, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\coth^5(x)}{5a}-\frac{\operatorname{csch}^5(x)}{5a}-\frac{2\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] Coth[x]^5/(5*a) - Csch[x]/a - (2*Csch[x]^3)/(3*a) - Csch[x]^5/(5*a)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^5(x)\text{csch}(x)\,dx}{a}-\frac{\int \coth^4(x)\text{csch}^2(x)\,dx}{a}\\ &= -\frac{i\,\text{Subst}\left(\int x^4\,dx,x,i\coth(x)\right)}{a}-\frac{i\,\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,-i\text{csch}(x)\right)}{a}\\ &= \frac{\coth^5(x)}{5a}-\frac{i\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-i\text{csch}(x)\right)}{a}\\ &= \frac{\coth^5(x)}{5a}-\frac{\text{csch}(x)}{a}-\frac{2\text{csch}^3(x)}{3a}-\frac{\text{csch}^5(x)}{5a}\end{aligned}$$

Mathematica [A] time = 0.0728887, size = 41, normalized size = 1.

$$-\frac{(8\cosh(x)+36\cosh(2x)+24\cosh(3x)-3\cosh(4x)-25)\text{csch}^3(x)}{120a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] -((-25 + 8*Cosh[x] + 36*Cosh[2*x] + 24*Cosh[3*x] - 3*Cosh[4*x])*Csch[x]^3)/
(120*a*(1 + Cosh[x]))
```

Maple [A] time = 0.028, size = 45, normalized size = 1.1

$$\frac{1}{16\,a}\left(\frac{1}{5}\left(\tanh\left(\frac{x}{2}\right)\right)^5+\frac{4}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^3+6\,\tanh\left(x/2\right)-4\,\left(\tanh\left(x/2\right)\right)^{-1}-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/16/a*(1/5*tanh(1/2*x)^5+4/3*tanh(1/2*x)^3+6*tanh(1/2*x)-4/tanh(1/2*x)-1/3
/tanh(1/2*x)^3)
```

Maxima [B] time = 1.0675, size = 633, normalized size = 15.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -6/5*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*
e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 14/5*e^(-2*x)/(2*a*e^(-x) - 2*a
*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e
^(-8*x) + a) - 26/15*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6
*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 10/3*e^(-4*x)
/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) -
2*a*e^(-7*x) - a*e^(-8*x) + a) + 2/3*e^(-5*x)/(2*a*e^(-x) - 2*a*e^(-2*x) -
6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
 - 2*e^(-6*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*
a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 2*e^(-7*x)/(2*a*e^(-x) - 2*a*
e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^
(-8*x) + a) + 2/5/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x)
+ 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Fricas [B] time = 1.85135, size = 728, normalized size = 17.76

$$-\frac{2\left(15\cosh(x)^4+6\,(10\cosh(x)+3)\sinh(x)^3+15\sinh(x)^4+12\cosh\right.}{15\left(a\cosh(x)^5+a\sinh(x)^5+2\,a\cosh(x)^4+(5\,a\cosh(x)+2\,a)\sinh(x)^4-3\,a\cosh(x)^3+\left(10\,a\cosh(x)^2+8\,a\cosh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2/15*(15*cosh(x)^4 + 6*(10*cosh(x) + 3)*sinh(x)^3 + 15*sinh(x)^4 + 12*cosh
(x)^3 + 2*(45*cosh(x)^2 + 18*cosh(x) + 2)*sinh(x)^2 + 4*cosh(x)^2 + 2*(30*c
osh(x)^3 + 27*cosh(x)^2 - 14*cosh(x) - 23)*sinh(x) - 4*cosh(x) + 13)/(a*cos
h(x)^5 + a*sinh(x)^5 + 2*a*cosh(x)^4 + (5*a*cosh(x) + 2*a)*sinh(x)^4 - 3*a*
cosh(x)^3 + (10*a*cosh(x)^2 + 8*a*cosh(x) - a)*sinh(x)^3 - 8*a*cosh(x)^2 +
(10*a*cosh(x)^3 + 12*a*cosh(x)^2 - 9*a*cosh(x) - 8*a)*sinh(x)^2 + 2*a*cosh(
x) + (5*a*cosh(x)^4 + 8*a*cosh(x)^3 - 3*a*cosh(x)^2 - 8*a*cosh(x) - 2*a)*si
nh(x) + 6*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\coth^4(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)**4/(cosh(x) + 1), x)/a
```

Giac [A] time = 1.31358, size = 80, normalized size = 1.95

$$-\frac{15\,e^{(2\,x)}-24\,e^{x}+13}{24\,a(e^{x}-1)^{3}}-\frac{165\,e^{(4\,x)}+480\,e^{(3\,x)}+650\,e^{(2\,x)}+400\,e^{x}+113}{120\,a(e^{x}+1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/24*(15*e^(2*x) - 24*e^x + 13)/(a*(e^x - 1)^3) - 1/120*(165*e^(4*x) + 480
*e^(3*x) + 650*e^(2*x) + 400*e^x + 113)/(a*(e^x + 1)^5)
```

3.197 $\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx$

Optimal. Leaf size=37

$$2\sqrt{a+b\cosh(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

Rubi [A] time = 0.0626052, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2721, 50, 63, 207}

$$2\sqrt{a+b\cosh(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
```

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\cosh(x)\right)\\ &= 2\sqrt{a+b\cosh(x)} + a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\cosh(x)\right)\\ &= 2\sqrt{a+b\cosh(x)} + (2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\cosh(x)}\right)\\ &= -2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right) + 2\sqrt{a+b\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.0198663, size = 37, normalized size = 1.

$$2\sqrt{a+b\cosh(x)} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
```

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

Maple [A] time = 0.013, size = 30, normalized size = 0.8

$$-2\,\text{Artanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)\sqrt{a} + 2\sqrt{a+b\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(1/2)*tanh(x),x)
```

```
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*cosh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cosh\left(x\right)+a}\tanh\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

Fricas [B] time = 4.11359, size = 1162, normalized size = 31.41

$$\left[\frac{1}{2}\sqrt{a}\log\left(-\frac{b^2\cosh\left(x\right)^4+b^2\sinh\left(x\right)^4+16\,ab\cosh\left(x\right)^3+4\left(b^2\cosh\left(x\right)+4\,ab\right)\sinh\left(x\right)^3+16\,ab\cosh\left(x\right)+2\left(16\,a^2+\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^
2*cosh(x) + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^2
+ 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*cosh
(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b*cos
h(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*sqrt(
a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^2)*cos
h(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*
sqrt(b*cosh(x) + a), sqrt(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*c
osh(x) + 2*(b*cosh(x) + 2*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqrt(-a)/(a*b
*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^2)*si
nh(x))) + 2*sqrt(b*cosh(x) + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\cosh(x)}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(1/2)*tanh(x),x)
```

```
[Out] Integral(sqrt(a + b*cosh(x))*tanh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cosh(x) + a}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

3.198 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0570442, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2721, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\cosh(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\cosh(x)}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.010613, size = 24, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.016, size = 19, normalized size = 0.8

$$-2\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*cosh(x))^(1/2),x)
```

```
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

Fricas [B] time = 2.04826, size = 1099, normalized size = 45.79

$$\left[\frac{\log\left(\frac{b^2\cosh(x)^4+b^2\sinh(x)^4+16\,ab\cosh(x)^3+4\left(b^2\cosh(x)+4\,ab\right)\sinh(x)^3+16\,ab\cosh(x)+2\left(16\,a^2+b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+24\,ab\cosh(x)+16\,a^2+\ldots\right.}{\cosh(x)^4+4\,\cosh(x)\sin\ldots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x)
 + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^2 + 2*(3*b^
2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*cosh(x)^3 + b
*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b*cosh(x) + (3
*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*sqrt(a) + b^2
+ 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^2)*cosh(x))*sin
h(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*si
nh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1))/sqrt(a), sqrt
(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2
*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^
2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(a + b*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

3.199 $\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=56

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[a + b*Cosh[x]])/b
```

Rubi [A] time = 0.127799, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4401, 2659, 208, 2668, 31}

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[a + b*Cosh[x]])/b
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\sinh(x)}{a+b\cosh(x)}\right)dx\\ &= A\int \frac{1}{a+b\cosh(x)}\,dx+B\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx\\ &= (2A)\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{B\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cosh(x)\right)}{b}\\ &= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0822838, size = 55, normalized size = 0.98

$$\frac{B\log(a+b\cosh(x))}{b}-\frac{2A\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*L
og[a + b*Cosh[x]])/b
```

Maple [B] time = 0.017, size = 137, normalized size = 2.5

$$\frac{aB}{b(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a-b\right)-\frac{B}{a-b}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a-b\right)+2\frac{A}{\sqrt{(a+b)(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] 1/b/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a*B-1/(a-b)*ln(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-a-b)*B+2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x
)/((a+b)*(a-b))^(1/2))*A-B/b*ln(tanh(1/2*x)+1)-B/b*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91991, size = 717, normalized size = 12.8

$$\left[\frac{\sqrt{a^2-b^2}Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-\left(Ba^2-Bb^2\right)x}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
```

```
 a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x + (B*a^2 - B*b^2)*log(2*(b*cosh(x) +
a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*A*b*arctan(-sq
rt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^2 - B*b^2)*x
 - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b - b^3
)]
```

Sympy [A] time = 138.828, size = 741, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(
tanh(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (A*tanh(x/2)/b + B*x/b - 2*B*log(
tanh(x/2) + 1)/b, Eq(a, b)), (-A/(b*tanh(x/2)) + B*x/b - 2*B*log(tanh(x/2)
+ 1)/b + 2*B*log(tanh(x/2))/b, Eq(a, -b)), ((A*x + B*cosh(x))/a, Eq(b, 0)),
 (-A*b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a - b) + b
/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a -
b) + b/(a - b))) + B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a - b) + b/(a
- b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b
/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a - b) + b/(a -
 b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(
a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - 2*B*a*sqrt(a/(a - b) + b/(a -
 b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a -
 b) + b/(a - b))) - B*b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) +
 b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt(a/(a - b) + b/(a
 - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt(a/(a - b) + b/(a
- b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/
(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + 2*B*b*sqrt(a/(a - b) + b/(a
- b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a
- b) + b/(a - b))), True))
```

Giac [A] time = 1.21904, size = 81, normalized size = 1.45

$$\frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}-\frac{Bx}{b}+\frac{B\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*x/b + B*log(b
*e^(2*x) + 2*a*e^x + b)/b
```

3.200 $\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{A\sinh(x)}{\cosh(x)+1}+B\log(\cosh(x)+1)$$

```
[Out] B*Log[1 + Cosh[x]] + (A*Sinh[x])/(1 + Cosh[x])
```

Rubi [A] time = 0.0765717, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4401, 2648, 2667, 31}

$$\frac{A\sinh(x)}{\cosh(x)+1}+B\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] B*Log[1 + Cosh[x]] + (A*Sinh[x])/(1 + Cosh[x])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx &= \int \left(\frac{A}{1+\cosh(x)}+\frac{B\sinh(x)}{1+\cosh(x)}\right)dx\\ &= A\int \frac{1}{1+\cosh(x)}\,dx + B\int \frac{\sinh(x)}{1+\cosh(x)}\,dx\\ &= \frac{A\sinh(x)}{1+\cosh(x)} + B\,\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cosh(x)\right)\\ &= B\log(1+\cosh(x)) + \frac{A\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.0319336, size = 19, normalized size = 1.06

$$A\tanh\left(\frac{x}{2}\right)+2B\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] 2*B*Log[Cosh[x/2]] + A*Tanh[x/2]
```

Maple [A] time = 0.011, size = 28, normalized size = 1.6

$$A\tanh\left(\frac{x}{2}\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(1+cosh(x)),x)
```

```
[Out] A*tanh(1/2*x)-B*ln(tanh(1/2*x)-1)-B*ln(tanh(1/2*x)+1)
```

Maxima [A] time = 1.05537, size = 26, normalized size = 1.44

$$B \log\left(\cosh\left(x\right) + 1\right) + \frac{2 \, A}{e^{\left(-x\right)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

```
[Out] B*log(cosh(x) + 1) + 2*A/(e^(-x) + 1)
```

Fricas [B] time = 2.05236, size = 169, normalized size = 9.39

$$-\frac{B x \cosh\left(x\right) + B x \sinh\left(x\right) + B x - 2 \, {\left(B \cosh\left(x\right) + B \sinh\left(x\right) + B\right)} \log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + 2 \, A}{\cosh\left(x\right) + \sinh\left(x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*cosh(x) + B*x*sinh(x) + B*x - 2*(B*cosh(x) + B*sinh(x) + B)*log(cosh(
x) + sinh(x) + 1) + 2*A)/(cosh(x) + sinh(x) + 1)
```

Sympy [A] time = 0.408678, size = 20, normalized size = 1.11

$$A \tanh{\left (\frac{x}{2} \right)} + B x - 2 B \log{\left (\tanh{\left (\frac{x}{2} \right)} + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x)
```

```
[Out] A*tanh(x/2) + B*x - 2*B*log(tanh(x/2) + 1)
```

Giac [A] time = 1.1553, size = 30, normalized size = 1.67

$$-B x + 2 \, B \log\left(e^{x} + 1\right) - \frac{2 \, A}{e^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*B*log(e^x + 1) - 2*A/(e^x + 1)
```

3.201 $\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{A\sinh(x)}{1-\cosh(x)}-B\log(1-\cosh(x))$$

```
[Out] -(B*Log[1 - Cosh[x]]) - (A*Sinh[x])/(1 - Cosh[x])
```

Rubi [A] time = 0.0859135, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4401, 2648, 2667, 31}

$$-\frac{A\sinh(x)}{1-\cosh(x)}-B\log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(1 - Cosh[x]),x]
```

```
[Out] -(B*Log[1 - Cosh[x]]) - (A*Sinh[x])/(1 - Cosh[x])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx &= \int \left(-\frac{A}{-1+\cosh(x)}-\frac{B\sinh(x)}{-1+\cosh(x)}\right)dx\\ &= -\left(A\int \frac{1}{-1+\cosh(x)}\,dx\right)-B\int \frac{\sinh(x)}{-1+\cosh(x)}\,dx\\ &= -\frac{A\sinh(x)}{1-\cosh(x)}-B\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\cosh(x)\right)\\ &= -B\log(1-\cosh(x))-\frac{A\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.0441071, size = 19, normalized size = 0.79

$$A\coth\left(\frac{x}{2}\right)-2B\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(1 - Cosh[x]),x]
```

```
[Out] A*Coth[x/2] - 2*B*Log[Sinh[x/2]]
```

Maple [A] time = 0.017, size = 36, normalized size = 1.5

$$B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+A\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-2\,B\ln\left(\tanh\left(x/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(1-cosh(x)),x)
```

```
[Out] B*ln(tanh(1/2*x)+1)+B*ln(tanh(1/2*x)-1)+1/tanh(1/2*x)*A-2*B*ln(tanh(1/2*x))
```

Maxima [A] time = 1.06148, size = 27, normalized size = 1.12

$$-B\log\left(\cosh\left(x\right)-1\right)-\frac{2\,A}{e^{(-x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="maxima")
```

```
[Out] -B*log(cosh(x) - 1) - 2*A/(e^(-x) - 1)
```

Fricas [B] time = 1.79008, size = 167, normalized size = 6.96

$$\frac{Bx\cosh\left(x\right)+Bx\sinh\left(x\right)-Bx-2\left(B\cosh\left(x\right)+B\sinh\left(x\right)-B\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+2\,A}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*cosh(x) + B*x*sinh(x) - B*x - 2*(B*cosh(x) + B*sinh(x) - B)*log(cosh(x
) + sinh(x) - 1) + 2*A)/(cosh(x) + sinh(x) - 1)
```

Sympy [A] time = 0.665453, size = 31, normalized size = 1.29

$$\frac{A}{\tanh\left(\frac{x}{2}\right)}-Bx+2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)-2B\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x)
```

```
[Out] A/tanh(x/2) - B*x + 2*B*log(tanh(x/2) + 1) - 2*B*log(tanh(x/2))
```

Giac [A] time = 1.15983, size = 30, normalized size = 1.25

$$Bx-2\,B\log\left(|e^{x}-1|\right)+\frac{2\,A}{e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="giac")
```

```
[Out] B*x - 2*B*log(abs(e^x - 1)) + 2*A/(e^x - 1)
```

3.202 $\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=65

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{B\log(a+b\cosh(x))}{a}+\frac{B\log(\cosh(x))}{a}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[Cosh[x]])/a - (B*Log[a + b*Cosh[x]])/a
```

Rubi [A] time = 0.147995, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4401, 2659, 208, 2721, 36, 29, 31}

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{B\log(a+b\cosh(x))}{a}+\frac{B\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tanh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[Cosh[x]])/a - (B*Log[a + b*Cosh[x]])/a
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\tanh(x)}{a+b\cosh(x)}\right)dx\\
&= A\int \frac{1}{a+b\cosh(x)}\,dx+B\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx\\
&= (2A)\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+B\,\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx,x,b\cosh(x)\right)\\
&= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,b\cosh(x)\right)}{a}-\frac{B\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cosh(x)\right)}{a}\\
&= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(\cosh(x))}{a}-\frac{B\log(a+b\cosh(x))}{a}
\end{aligned}$$

Mathematica [A] time = 0.142697, size = 61, normalized size = 0.94

$$\frac{B(\log(\cosh(x))-\log(a+b\cosh(x)))}{a}-\frac{2A\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tanh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*(
Log[Cosh[x]] - Log[a + b*Cosh[x]]))/a
```

Maple [B] time = 0.029, size = 125, normalized size = 1.9

$$-\frac{B}{a-b}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a-b\right)+\frac{Bb}{a(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a-b\right)+2\frac{A}{\sqrt{(a+b)(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tanh(x))/(a+b*cosh(x)),x)
```

```
[Out] -1/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*B+1/a/(a-b)*ln(a*tanh(1/2*
x)^2-tanh(1/2*x)^2*b-a-b)*B*b+2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*
x)/((a+b)*(a-b))^(1/2))*A+B/a*ln(tanh(1/2*x)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.09463, size = 806, normalized size = 12.4

$$\left[\frac{\sqrt{a^2-b^2}Aa\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-\left(Ba^2-Bb^2\right)\mathrm{lo}}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) + b)) - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x
))) + (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2), -(
2*sqrt(-a^2 + b^2)*A*a*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)
/(a^2 - b^2)) + (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x)))
- (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*tanh(x))/(a + b*cosh(x)), x)
```

Giac [A] time = 1.19342, size = 89, normalized size = 1.37

$$\frac{2\,A\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\frac{B\log\left(be^{(2\,x)}+2\,ae^x+b\right)}{a}+\frac{B\log\left(e^{(2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(b*e^(2*x)
 + 2*a*e^x + b)/a + B*log(e^(2*x) + 1)/a
```

3.203 $\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=100

$$-\frac{aB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}+\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[1 - Cosh[x]])/(2*(a + b)) + (B*Log[1 + Cosh[x]])/(2*(a - b)) - (
a*B*Log[a + b*Cosh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.166308, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4401, 2659, 208, 2721, 801}

$$-\frac{aB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}+\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[1 - Cosh[x]])/(2*(a + b)) + (B*Log[1 + Cosh[x]])/(2*(a - b)) - (
a*B*Log[a + b*Cosh[x]])/(a^2 - b^2)
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\coth(x)}{a+b\cosh(x)}\right) dx\\
&= A\int \frac{1}{a+b\cosh(x)}\,dx + B\int \frac{\coth(x)}{a+b\cosh(x)}\,dx\\
&= (2A)\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - B\,\mathrm{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x\right.\\
&= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} - B\,\mathrm{Subst}\left(\int\left(\frac{1}{2(a+b)(b-x)}+\frac{a}{(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)(b+\right.\right.\\
&= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{B\log(1-\cosh(x))}{2(a+b)} + \frac{B\log(1+\cosh(x))}{2(a-b)} - \frac{aB\log(a+b\cosh(x))}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.243717, size = 81, normalized size = 0.81

$$\frac{B\left(a\log(a+b\cosh(x))-a\log(\sinh(x))+b\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{b^2-a^2}-\frac{2A\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*(
a*Log[a + b*Cosh[x]] - a*Log[Sinh[x]] + b*Log[Tanh[x/2]]))/(-a^2 + b^2)
```

Maple [A] time = 0.026, size = 139, normalized size = 1.4

$$-\frac{aB}{(a+b)(a-b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-\left(\tanh\left(\frac{x}{2}\right)\right)^2b-a-b\right)+2\frac{Aa}{(a+b)\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*coth(x))/(a+b*cosh(x)),x)
```

```
[Out] -1/(a+b)*a*B/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+2/(a+b)/((a+b)*(
a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A*a+2/(a+b)/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A*b+B/(a+b)*
ln(tanh(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 14.0187, size = 842, normalized size = 8.42

$$\left[-\frac{Ba\log\left(\frac{2(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\sqrt{a^2-b^2}A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2-b^2}(b\cosh(x)+b\,\mathrm{s}}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2(b\cosh(x)+a)\sinh(x)+b}\right.}{a^2-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - sqrt(a^2 - b^2)*A*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x
)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*a +
B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x) + sinh(x) - 1))/(
a^2 - b^2), -(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + 2*sqrt(-a^2
+ b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
- (B*a + B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x) + sinh(x
) - 1))/(a^2 - b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\coth(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*coth(x))/(a + b*cosh(x)), x)
```

Giac [A] time = 1.1764, size = 122, normalized size = 1.22

$$-\frac{Ba\log\left(be^{(2x)} + 2ae^{x} + b\right)}{a^2 - b^2} + \frac{2A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2 + b^2}} + \frac{B\log\left(e^{x} + 1\right)}{a - b} + \frac{B\log\left(|e^{x} - 1|\right)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -B*a*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqrt
(-a^2 + b^2))/sqrt(-a^2 + b^2) + B*log(e^x + 1)/(a - b) + B*log(abs(e^x - 1
))/(a + b)
```

3.204 $\int \frac{A+B\textbf{sech}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{B\tan^{-1}(\sinh(x))}{a}$$

```
[Out] (B*ArcTan[Sinh[x]])/a + (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt
[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b])
```

Rubi [A] time = 0.133207, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2828, 3001, 3770, 2659, 208}

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{B\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sech[x])/(a + b*Cosh[x]),x]
```


```
[Out] (B*ArcTan[Sinh[x]])/a + (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt
[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b])
```


Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\mathbf{sech}(x)}{a+b\cosh(x)}\,dx &= \int \frac{(B+A\cosh(x))\mathbf{sech}(x)}{a+b\cosh(x)}\,dx\\
&= \frac{B\int \mathbf{sech}(x)\,dx}{a}+\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}\,dx}{a}\\
&= \frac{B\tan^{-1}(\sinh(x))}{a}+\frac{(2(aA-bB))\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\
&= \frac{B\tan^{-1}(\sinh(x))}{a}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}
\end{aligned}
$$

Mathematica [A] time = 0.113905, size = 63, normalized size = 1.02

$$
\frac{2\left(\frac{(bB-aA)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+B\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sech[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*(B*ArcTan[Tanh[x/2]] + ((-(a*A) + b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-
a^2 + b^2]])/Sqrt[-a^2 + b^2]))/a
```

Maple [A] time = 0.03, size = 89, normalized size = 1.4

$$2\,{\frac {A}{\sqrt {\left(a+b \right) \left(a-b \right) }}{\rm Artanh} \left({\frac { \left(a-b \right) \tanh \left(x/2 \right) }{\sqrt {\left(a+b \right) \left(a-b \right) }}}\right)}-2\,{\frac {Bb}{a\sqrt {\left(a+b \right) \left(a-b \right) }}{\rm Artanh} \left({\frac { \left(a-b \right) \tanh \left(x/2 \right) }{\sqrt {\left(a+b \right) \left(a-b \right) }}}\right)}+2\,{\frac {B\arctan \left(\tanh \left(\right.\right.}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sech(x))/(a+b*cosh(x)),x)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A-2*B/
a*b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2*B/
a*arctan(tanh(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.64704, size = 643, normalized size = 10.37

$$\left[-\frac{(Aa-Bb)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-2\left(Ba\right.}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-((A*a - B*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)) - 2*(B*a^2 - B*b^2)*arctan(cosh(x) + sinh(x)))/(
a^3 - a*b^2), -2*((A*a - B*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (B*a^2 - B*b^2)*arctan(cosh(x) + si
nh(x)))/(a^3 - a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B \operatorname{sech}(x)}{a + b \cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*sech(x))/(a + b*cosh(x)), x)
```

Giac [A] time = 1.21048, size = 72, normalized size = 1.16

$$\frac{2\, B \arctan\left(e^{x}\right)}{a} + \frac{2\, (Aa - Bb) \arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2} + b^{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*B*arctan(e^x)/a + 2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqr
t(-a^2 + b^2)*a)
```

3.205 $\int \frac{A+B\mathbf{csch}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=99

$$\frac{bB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}-\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[1 - Cosh[x]])/(2*(a + b)) - (B*Log[1 + Cosh[x]])/(2*(a - b)) + (
b*B*Log[a + b*Cosh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.305051, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {4225, 4401, 2659, 208, 2668, 706, 31, 633}

$$\frac{bB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}-\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[1 - Cosh[x]])/(2*(a + b)) - (B*Log[1 + Cosh[x]])/(2*(a - b)) + (
b*B*Log[a + b*Cosh[x]])/(a^2 - b^2)
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rubi steps

$$\int \frac{A + B\text{csch}(x)}{a + b\cosh(x)}\,dx = -\left(i \int \frac{\text{csch}(x)(iB + iA\sinh(x))}{a + b\cosh(x)}\,dx\right)$$

$$= \int \left(\frac{A}{a + b\cosh(x)} + \frac{B\text{csch}(x)}{a + b\cosh(x)}\right) dx$$

$$= A \int \frac{1}{a + b\cosh(x)}\,dx + B \int \frac{\text{csch}(x)}{a + b\cosh(x)}\,dx$$

$$= (2A)\,\text{Subst}\left(\int \frac{1}{a + b - (a - b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - (bB)\,\text{Subst}\left(\int \frac{1}{(a + x)\left(b^2 - x^2\right)}\,dx, x, b\cos\right.$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a - b}\sqrt{a + b}} + \frac{(bB)\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{a^2 - b^2} + \frac{(bB)\,\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx, x, b\,c\right.}{a^2 - b^2}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a - b}\sqrt{a + b}} + \frac{bB\log(a + b\cosh(x))}{a^2 - b^2} + \frac{B\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\cosh(x)\right)}{2(a - b)} - \frac{B\,\text{Sub}}{}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a - b}\sqrt{a + b}} + \frac{B\log(1 - \cosh(x))}{2(a + b)} - \frac{B\log(1 + \cosh(x))}{2(a - b)} + \frac{bB\log(a + b\cosh(x))}{a^2 - b^2}$$

Mathematica [A] time = 0.187558, size = 81, normalized size = 0.82

$$\frac{B\left(b\log(a + b\cosh(x)) + a\log\left(\tanh\left(\frac{x}{2}\right)\right) - b\log(\sinh(x))\right)}{a^2 - b^2} - \frac{2A\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2 - a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*(
b*Log[a + b*Cosh[x]] - b*Log[Sinh[x]] + a*Log[Tanh[x/2]]))/(a^2 - b^2)
```

Maple [A] time = 0.025, size = 138, normalized size = 1.4

$$\frac{Bb}{(a + b)(a - b)}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2 - \left(\tanh\left(\frac{x}{2}\right)\right)^2 b - a - b\right) + 2\frac{Aa}{(a + b)\sqrt{(a + b)(a - b)}}\text{Artanh}\left(\frac{(a - b)\tanh(x/2)}{\sqrt{(a + b)(a - b)}}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*csch(x))/(a+b*cosh(x)),x)
```

```
[Out] 1/(a+b)*B*b/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+2/(a+b)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A*a+2/(a+b)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A*b+B/(a+b)*l
n(tanh(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 22.6808, size = 840, normalized size = 8.48

$$\left[\frac{Bb\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2-b^2}A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right.}{a^2-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + sqrt(a^2 - b^2)*A*log((b
^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x)
 + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)
^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*a + B
*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) + sinh(x) - 1))/(a
^2 - b^2), (B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - 2*sqrt(-a^2 +
b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) -
(B*a + B*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) + sinh(x)
- 1))/(a^2 - b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\,\mathrm{csch}\,(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*csch(x))/(a + b*cosh(x)), x)
```

Giac [A] time = 1.23717, size = 122, normalized size = 1.23

$$\frac{Bb\log\left(be^{(2x)} + 2\,ae^{x} + b\right)}{a^2 - b^2} + \frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - \frac{B\log\left(e^{x}+1\right)}{a-b} + \frac{B\log\left(|e^{x}-1|\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*b*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqrt(
-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(e^x + 1)/(a - b) + B*log(abs(e^x - 1)
)/(a + b)
```

3.206 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx$

Optimal. Leaf size=86

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\sqrt{a+b}}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b
]])/(Sqrt[a - b]*b*Sqrt[a + b]*e) + (C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

Rubi [A] time = 0.153965, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4377, 2735, 2659, 205, 2668, 31}

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\sqrt{a+b}}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b
]])/(Sqrt[a - b]*b*Sqrt[a + b]*e) + (C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = C\int \frac{\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx + \int \frac{A+B\cosh(d+ex)}{a+b\cosh(d+ex)}\,dx$$

$$= \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+b\cosh(d+ex)}\,dx}{b} + \frac{C\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(d+e$$

$$= \frac{Bx}{b} + \frac{C\log(a+b\cosh(d+ex))}{be} - \frac{(2i(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,\right.}{be}$$

$$= \frac{Bx}{b} + \frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}e} + \frac{C\log(a+b\cosh(d+ex)}{be}$$

Mathematica [A] time = 0.234088, size = 81, normalized size = 0.94

$$\frac{\frac{2(aB-Ab)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+C\log(a+b\cosh(d+ex))+B(d+ex)}{be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

```
[Out] (B*(d + e*x) + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a
^2 + b^2]])/Sqrt[-a^2 + b^2] + C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

Maple [B] time = 0.036, size = 276, normalized size = 3.2

$$\frac{aC}{eb\,(a-b)}\ln\left(a\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2-\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2b-a-b\right)-\frac{C}{e\,(a-b)}\ln\left(a\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2-\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x)
```

```
[Out] 1/e/b/(a-b)*ln(a*tanh(1/2*e*x+1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)*a*C-1/e
/(a-b)*ln(a*tanh(1/2*e*x+1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)*C+2/e/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2))*A-2/e/
b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)
)*a*B+1/e/b*ln(tanh(1/2*e*x+1/2*d)+1)*B-1/e/b*ln(tanh(1/2*e*x+1/2*d)+1)*C-1
/e/b*ln(tanh(1/2*e*x+1/2*d)-1)*B-1/e/b*ln(tanh(1/2*e*x+1/2*d)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0403, size = 948, normalized size = 11.02

$$\left[\frac{\left((B-C)a^2-(B-C)b^2\right)ex-(Ba-Ab)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(ex+d)^2+b^2\sinh(ex+d)^2+2\,ab\cosh(ex+d)+2\,a^2-b^2+2\left(b^2\cosh(ex+d)+ab\right)\sinh}{b\cosh(ex+d)^2+b\sinh(ex+d)^2+2\,a\cosh(ex+d)+2\,(b\cosh(e}\right.}{\left(a^2b-b^3\right)e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="f
ricas")
```

```
[Out] [(((B - C)*a^2 - (B - C)*b^2)*e*x - (B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*co
sh(e*x + d)^2 + b^2*sinh(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2
*(b^2*cosh(e*x + d) + a*b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x +
d) + b*sinh(e*x + d) + a))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cos
h(e*x + d) + 2*(b*cosh(e*x + d) + a)*sinh(e*x + d) + b)) + (C*a^2 - C*b^2)*
log(2*(b*cosh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d))))/((a^2*b - b^3
)*e), (((B - C)*a^2 - (B - C)*b^2)*e*x + 2*(B*a - A*b)*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2))
+ (C*a^2 - C*b^2)*log(2*(b*cosh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d
))))/((a^2*b - b^3)*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22631, size = 135, normalized size = 1.57

$$\frac{(xe+d)(B-C)e^{(-1)}}{b}+\frac{Ce^{(-1)}\log\left(be^{(2\,xe+2\,d)}+2\,ae^{(xe+d)}+b\right)}{b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{be^{(xe+d)}+a}{\sqrt{-a^2+b^2}}\right)e^{(-1)}}{\sqrt{-a^2+b^2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="g
iac")
```

```
[Out] (x*e + d)*(B - C)*e^(-1)/b + C*e^(-1)*log(b*e^(2*x*e + 2*d) + 2*a*e^(x*e +
d) + b)/b - 2*(B*a - A*b)*arctan((b*e^(x*e + d) + a)/sqrt(-a^2 + b^2))*e^(-
1)/(sqrt(-a^2 + b^2)*b)
```

3.207 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{3/2}(a+b)^{3/2}}-\frac{C}{be(a+b\cosh(d+ex))}$$

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b]])/((a -
b)^(3/2)*(a + b)^(3/2)*e) - C/(b*e*(a + b*Cosh[d + e*x])) - ((A*b - a*B)*Si
nh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x]))
```

Rubi [A] time = 0.176173, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4377, 2754, 12, 2659, 205, 2668, 32}

$$-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{3/2}(a+b)^{3/2}}-\frac{C}{be(a+b\cosh(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b]])/((a -
b)^(3/2)*(a + b)^(3/2)*e) - C/(b*e*(a + b*Cosh[d + e*x])) - ((A*b - a*B)*Si
nh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x]))
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx + \int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$$

$$= -\frac{(Ab-aB)\sinh(d+ex)}{\left(a^2-b^2\right)e(a+b\cosh(d+ex))} + \frac{\int \frac{-aA+bB}{a+b\cosh(d+ex)}\,dx}{-a^2+b^2} + \frac{C\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\ldots}\right.}{}$$

$$= -\frac{C}{be(a+b\cosh(d+ex))} - \frac{(Ab-aB)\sinh(d+ex)}{\left(a^2-b^2\right)e(a+b\cosh(d+ex))} + \frac{(aA-bB)\int}{a^2\ldots}$$

$$= -\frac{C}{be(a+b\cosh(d+ex))} - \frac{(Ab-aB)\sinh(d+ex)}{\left(a^2-b^2\right)e(a+b\cosh(d+ex))} - \frac{(2i(aA-bB)\ldots}{}$$

$$= \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}e} - \frac{C}{be(a+b\cosh(d+ex))} - \frac{(A\ldots}{\left(a^2-\ldots\right.}$$

Mathematica [A] time = 0.408815, size = 115, normalized size = 0.95

$$\frac{\frac{C\left(b^2-a^2\right)-b(Ab-aB)\sinh(d+ex)}{b(a-b)(a+b)(a+b\cosh(d+ex))} + \frac{2(aA-bB)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,x
]
```

```
[Out] ((2*(a*A - b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2
 + b^2)^(3/2) + ((-a^2 + b^2)*C - b*(A*b - a*B)*Sinh[d + e*x])/((a - b)*b*(
a + b)*(a + b*Cosh[d + e*x])))/e
```

Maple [A] time = 0.036, size = 144, normalized size = 1.2

$$\frac{1}{e}\left(-2\,\frac{1}{a\left(\tanh\left(1/2\,ex+d/2\right)\right)^2-\left(\tanh\left(1/2\,ex+d/2\right)\right)^2b-a-b}\left(-\frac{(Ab-aB)\tanh\left(1/2\,ex+d/2\right)}{a^2-b^2}+\frac{C}{a-b}\right)+2\,\frac{\ldots}{(a+b)\,(\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x)
```

```
[Out] 1/e*(-2*(-(A*b-B*a)/(a^2-b^2)*tanh(1/2*e*x+1/2*d)+C/(a-b))/(a*tanh(1/2*e*x+
1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)+2*(A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22388, size = 2384, normalized size = 19.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm=
"fricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a*
b^2 - B*b^3)*cosh(e*x + d)^2 + (A*a*b^2 - B*b^3)*sinh(e*x + d)^2 + 2*(A*a^2
*b - B*a*b^2)*cosh(e*x + d) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*cosh
(e*x + d))*sinh(e*x + d))*sqrt(a^2 - b^2)*log((b^2*cosh(e*x + d)^2 + b^2*si
nh(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2*(b^2*cosh(e*x + d) +
a*b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) +
 a))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(b*cosh
(e*x + d) + a)*sinh(e*x + d) + b)) + 2*((B + C)*a^4 - A*a^3*b - (B + 2*C)*a
^2*b^2 + A*a*b^3 + C*b^4)*cosh(e*x + d) + 2*((B + C)*a^4 - A*a^3*b - (B + 2
*C)*a^2*b^2 + A*a*b^3 + C*b^4)*sinh(e*x + d))/((a^4*b^2 - 2*a^2*b^4 + b^6)*
e*cosh(e*x + d)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*e*sinh(e*x + d)^2 + 2*(a^5*
b - 2*a^3*b^3 + a*b^5)*e*cosh(e*x + d) + (a^4*b^2 - 2*a^2*b^4 + b^6)*e + 2*
((a^4*b^2 - 2*a^2*b^4 + b^6)*e*cosh(e*x + d) + (a^5*b - 2*a^3*b^3 + a*b^5)*
```

```
e)*sinh(e*x + d)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4 + (A*a*b^2 - B
*b^3 + (A*a*b^2 - B*b^3)*cosh(e*x + d)^2 + (A*a*b^2 - B*b^3)*sinh(e*x + d)^
2 + 2*(A*a^2*b - B*a*b^2)*cosh(e*x + d) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 -
 B*b^3)*cosh(e*x + d))*sinh(e*x + d))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2)) + ((B + C)*a^4 -
A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*cosh(e*x + d) + ((B + C)*a^4
 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*sinh(e*x + d))/((a^4*b^2
- 2*a^2*b^4 + b^6)*e*cosh(e*x + d)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*e*sinh(e
*x + d)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*cosh(e*x + d) + (a^4*b^2 - 2*a^
2*b^4 + b^6)*e + 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*e*cosh(e*x + d) + (a^5*b -
2*a^3*b^3 + a*b^5)*e)*sinh(e*x + d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21669, size = 225, normalized size = 1.86

$$\frac{2\,(Aa-Bb)\arctan\left(\frac{be^{(xe+d)}+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^2e-b^2e\right)\sqrt{-a^2+b^2}}-\frac{2\left(Ba^2e^{(xe+d)}+Ca^2e^{(xe+d)}-Aabe^{(xe+d)}-Cb^2e^{(xe+d)}+Bab-Ab^2\right)}{\left(a^2be-b^3e\right)\left(be^{(2\,xe+2\,d)}+2\,ae^{(xe+d)}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm=
"giac")
```

```
[Out] 2*(A*a - B*b)*arctan((b*e^(x*e + d) + a)/sqrt(-a^2 + b^2))/((a^2*e - b^2*e)
*sqrt(-a^2 + b^2)) - 2*(B*a^2*e^(x*e + d) + C*a^2*e^(x*e + d) - A*a*b*e^(x*
e + d) - C*b^2*e^(x*e + d) + B*a*b - A*b^2)/((a^2*b*e - b^3*e)*(b*e^(2*x*e
+ 2*d) + 2*a*e^(x*e + d) + b))
```

3.208 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$

Optimal. Leaf size=187

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{2e\left(a^2-b^2\right)^2(a+b\cosh(d+ex))}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex)}$$

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*e) - C/(2*b*e*(a + b*Cosh[d + e*x])^2)
 - ((A*b - a*B)*Sinh[d + e*x])/(2*(a^2 - b^2)*e*(a + b*Cosh[d + e*x])^2) -
((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[d + e*x])/(2*(a^2 - b^2)^2*e*(a + b*Cosh[
d + e*x]))
```

Rubi [A] time = 0.264141, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4377, 2754, 12, 2659, 205, 2668, 32}

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{2e\left(a^2-b^2\right)^2(a+b\cosh(d+ex))}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*e) - C/(2*b*e*(a + b*Cosh[d + e*x])^2)
 - ((A*b - a*B)*Sinh[d + e*x])/(2*(a^2 - b^2)*e*(a + b*Cosh[d + e*x])^2) -
((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[d + e*x])/(2*(a^2 - b^2)^2*e*(a + b*Cosh[
d + e*x]))
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx + \int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$$

$$= -\frac{(Ab-aB)\sinh(d+ex)}{2\left(a^2-b^2\right)e(a+b\cosh(d+ex))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx}{2\left(a^2-b^2\right)} + \cdots$$

$$= -\frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2\left(a^2-b^2\right)e(a+b\cosh(d+ex))^2} - \frac{\left(3aAb-\cdots\right.}{2\left(a^2-b\cdots\right.}$$

$$= -\frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2\left(a^2-b^2\right)e(a+b\cosh(d+ex))^2} - \frac{\left(3aAb-\cdots\right.}{2\left(a^2-b\cdots\right.}$$

$$= -\frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2\left(a^2-b^2\right)e(a+b\cosh(d+ex))^2} - \frac{\left(3aAb-\cdots\right.}{2\left(a^2-b\cdots\right.}$$

$$= \frac{\left(2a^2A+Ab^2-3abB\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}e} - \frac{C}{2be(a+b\cosh(d+ex\cdots}$$

Mathematica [A] time = 0.784789, size = 175, normalized size = 0.94

$$\frac{\frac{C\left(b^2-a^2\right)-b(Ab-aB)\sinh(d+ex)}{b(a-b)(a+b)(a+b\cosh(d+ex))^2} - \frac{2\left(2a^2A-3abB+Ab^2\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{\left(a^2B-3aAb+2b^2B\right)\sinh(d+ex)}{(a-b)^2(a+b)^2(a+b\cosh(d+ex))}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,x
]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a
^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[d + e*x
])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])) + ((-a^2 + b^2)*C - b*(A*b -
 a*B)*Sinh[d + e*x])/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^2))/(2*e)
```

Maple [A] time = 0.042, size = 273, normalized size = 1.5

$$\frac{1}{e}\left(-2\,\frac{1}{\left(a\left(\tanh\left(1/2\,ex+d/2\right)\right)^{2}-\left(\tanh\left(1/2\,ex+d/2\right)\right)^{2}b-a-b\right)^{2}}\left(-1/2\,\frac{\left(4\,Aab+Ab^{2}-2\,a^{2}B-Bab-2\,Bb^{2}\right)\left(\tanh\left(1\right.\right.}{\left(a-b\right)\left(a^{2}+2\,ab+b^{2}\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x)
```

```
[Out] 1/e*(-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*t
anh(1/2*e*x+1/2*d)^3+C/(a-b)*tanh(1/2*e*x+1/2*d)^2+1/2*(4*A*a*b-A*b^2-2*B*a
^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)-a*C/(a^2-2*a*b+
b^2))/(a*tanh(1/2*e*x+1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)^2+(2*A*a^2+A*b^
2-3*B*a*b)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e
*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.98059, size = 8046, normalized size = 43.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm=
"fricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d)^3 -
```

2*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(e*x + d
)^3 + 2*(2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*
(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6)*cosh(e*x + d)^2 + 2*(2*(B +
C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^
4 + 3*A*a*b^5 - 2*(B + C)*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 +
3*B*a*b^5 - A*b^6)*cosh(e*x + d))*sinh(e*x + d)^2 - (2*A*a^2*b^3 - 3*B*a*b^
4 + A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^4 + (2*A*a^2*b^
3 - 3*B*a*b^4 + A*b^5)*sinh(e*x + d)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a
*b^4)*cosh(e*x + d)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b
^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d))*sinh(e*x + d)^3 + 2*(4*A*a^4*b - 6*B
*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^2 + 2*(4*A*a^4*b
- 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + 3*(2*A*a^2*b^3 - 3*B*a*b^
4 + A*b^5)*cosh(e*x + d)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(e
*x + d))*sinh(e*x + d)^2 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(e*x
 + d) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 +
 A*b^5)*cosh(e*x + d)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(e*x
+ d)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e
*x + d))*sinh(e*x + d))*sqrt(a^2 - b^2)*log((b^2*cosh(e*x + d)^2 + b^2*sinh
(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2*(b^2*cosh(e*x + d) + a*
b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a
))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(b*cosh(e
*x + d) + a)*sinh(e*x + d) + b)) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3
+ 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6)*cosh(e*x + d) + 2*(4*B*a^5*b - 10*A*a^4
*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4*b^2 - 3*B*
a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d)^2 + 2*(2*(B + C)*a^6
 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*
A*a*b^5 - 2*(B + C)*b^6)*cosh(e*x + d))*sinh(e*x + d))/((a^6*b^3 - 3*a^4*b^
5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 -
 b^9)*e*sinh(e*x + d)^4 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e*cos
h(e*x + d)^3 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*e*cosh(e
x + d)^2 + 4((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d) + (a
^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e)*sinh(e*x + d)^3 + 4*(a^7*b^2 - 3
*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e*cosh(e*x + d) + 2*(3*(a^6*b^3 - 3*a^4*b^5 +
 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 -
a*b^8)*e*cosh(e*x + d) + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*
e)*sinh(e*x + d)^2 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e + 4*((a^6*b^
3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^3 + 3*(a^7*b^2 - 3*a^5*b^4
 + 3*a^3*b^6 - a*b^8)*e*cosh(e*x + d)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5
+ a^2*b^7 - b^9)*e*cosh(e*x + d) + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
)*e)*sinh(e*x + d)), -(B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*
B*b^6 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*
x + d)^3 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh
(e*x + d)^3 + (2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^
3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6)*cosh(e*x + d)^2 + (2*(
B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^

```
2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^
4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d))*sinh(e*x + d)^2 + (2*A*a^2*b^3 - 3*B*
a*b^4 + A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^4 + (2*A*a^
2*b^3 - 3*B*a*b^4 + A*b^5)*sinh(e*x + d)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 +
 A*a*b^4)*cosh(e*x + d)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a
^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d))*sinh(e*x + d)^3 + 2*(4*A*a^4*b -
 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^2 + 2*(4*A*a^
4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + 3*(2*A*a^2*b^3 - 3*B*
a*b^4 + A*b^5)*cosh(e*x + d)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*co
sh(e*x + d))*sinh(e*x + d)^2 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh
(e*x + d) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b
^4 + A*b^5)*cosh(e*x + d)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(
e*x + d)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*co
sh(e*x + d))*sinh(e*x + d))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
sh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2)) + (4*B*a^5*b - 10*A*a^4*b^2
 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6)*cosh(e*x + d) + (4*B*a^5*b
 - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4
*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d)^2 + 2*(2*
(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a
^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6)*cosh(e*x + d))*sinh(e*x + d))/((a^6*b^3
 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^4 + (a^6*b^3 - 3*a^4*b^5 +
3*a^2*b^7 - b^9)*e*sinh(e*x + d)^4 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a
*b^8)*e*cosh(e*x + d)^3 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^
9)*e*cosh(e*x + d)^2 + 4*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*
x + d) + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e)*sinh(e*x + d)^3 + 4*(
a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e*cosh(e*x + d) + 2*(3*(a^6*b^3 -
3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3
*a^3*b^6 - a*b^8)*e*cosh(e*x + d) + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*
b^7 - b^9)*e)*sinh(e*x + d)^2 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e +
 4*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^3 + 3*(a^7*b^2
- 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e*cosh(e*x + d)^2 + (2*a^8*b - 5*a^6*b^3 +
 3*a^4*b^5 + a^2*b^7 - b^9)*e*cosh(e*x + d) + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*
b^6 - a*b^8)*e)*sinh(e*x + d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17601, size = 539, normalized size = 2.88

$$\frac{\left(2\,Aa^2-3\,Bab+Ab^2\right)\arctan\left(\frac{be^{(xe+d)}+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^4e-2\,a^2b^2e+b^4e\right)\sqrt{-a^2+b^2}}+\frac{2\,Aa^2b^2e^{(3\,xe+3\,d)}-3\,Bab^3e^{(3\,xe+3\,d)}+Ab^4e^{(3\,xe+3\,d)}-2\,Ba^4e^{(2\,xe+2\,d)}-2\,Ca^4e^{(2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm=
"giac")
```

```
[Out] (2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^(x*e + d) + a)/sqrt(-a^2 + b^2))/((
a^4*e - 2*a^2*b^2*e + b^4*e)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*x*e + 3*
d) - 3*B*a*b^3*e^(3*x*e + 3*d) + A*b^4*e^(3*x*e + 3*d) - 2*B*a^4*e^(2*x*e +
 2*d) - 2*C*a^4*e^(2*x*e + 2*d) + 6*A*a^3*b*e^(2*x*e + 2*d) - 5*B*a^2*b^2*e
^(2*x*e + 2*d) + 4*C*a^2*b^2*e^(2*x*e + 2*d) + 3*A*a*b^3*e^(2*x*e + 2*d) -
2*B*b^4*e^(2*x*e + 2*d) - 2*C*b^4*e^(2*x*e + 2*d) - 4*B*a^3*b*e^(x*e + d) +
 10*A*a^2*b^2*e^(x*e + d) - 5*B*a*b^3*e^(x*e + d) - A*b^4*e^(x*e + d) - B*a
^2*b^2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b*e - 2*a^2*b^3*e + b^5*e)*(b*e^(2*x*e
+ 2*d) + 2*a*e^(x*e + d) + b)^2)
```

3.209 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$

Optimal. Leaf size=260

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)\sinh(d+ex)}{6e\left(a^2-b^2\right)^3(a+b\cosh(d+ex))}-\frac{\left(-2a^2\right.}{6e\left(a\right.}$$

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e
*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*e) - C/(3*b*e*(a + b*Cos
h[d + e*x])^3) - ((A*b - a*B)*Sinh[d + e*x])/(3*(a^2 - b^2)*e*(a + b*Cosh[d
 + e*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^2*B)*Sinh[d + e*x])/(6*(a^2 - b^2)^
2*e*(a + b*Cosh[d + e*x])^2) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*
B)*Sinh[d + e*x])/(6*(a^2 - b^2)^3*e*(a + b*Cosh[d + e*x]))
```

Rubi [A] time = 0.447967, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4377, 2754, 12, 2659, 205, 2668, 32}

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)\sinh(d+ex)}{6e\left(a^2-b^2\right)^3(a+b\cosh(d+ex))}-\frac{\left(-2a^2\right.}{6e\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,x]
```


```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e
*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*e) - C/(3*b*e*(a + b*Cos
h[d + e*x])^3) - ((A*b - a*B)*Sinh[d + e*x])/(3*(a^2 - b^2)*e*(a + b*Cosh[d
 + e*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^2*B)*Sinh[d + e*x])/(6*(a^2 - b^2)^
2*e*(a + b*Cosh[d + e*x])^2) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*
B)*Sinh[d + e*x])/(6*(a^2 - b^2)^3*e*(a + b*Cosh[d + e*x]))
```


Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
```

```
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx + \int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$$

$$= -\frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} - \frac{\int \frac{-3(aA-bB)+2(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx}{3\left(a^2-b^2\right)} +$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} - \frac{\left(5aAb\right.}{6\left(a^2-\right.}$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} - \frac{\left(5aAb\right.}{6\left(a^2-\right.}$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} - \frac{\left(5aAb\right.}{6\left(a^2-\right.}$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} - \frac{\left(5aAb\right.}{6\left(a^2-\right.}$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}e} - \frac{}{3be(a+b\,\mathrm{c}}$$

Mathematica [A] time = 2.40705, size = 245, normalized size = 0.94

$$\frac{\frac{2C\left(b^2-a^2\right)-2b(Ab-aB)\sinh(d+ex)}{b(a-b)(a+b)(a+b\cosh(d+ex))^3} + \frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} + \frac{\left(-11a^2Ab+2a^3B+13ab^2B-4Ab^3\right)\sinh(d+ex)}{(a-b)^3(a+b)^3(a+b\cosh(d+ex))} + \frac{\left(2a^2B-5\right.}{(a-b)^2(\ldots}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,x
]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[(d + e*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + ((-5*a*A*b + 2*a^2*B + 3*b^2*
B)*Sinh[d + e*x])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])^2) + ((-11*a^2
*A*b - 4*A*b^3 + 2*a^3*B + 13*a*b^2*B)*Sinh[d + e*x])/((a - b)^3*(a + b)^3*
(a + b*Cosh[d + e*x])) + (2*(-a^2 + b^2)*C - 2*b*(A*b - a*B)*Sinh[d + e*x])
```

```
/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^3))/(6*e)
```

Maple [A] time = 0.046, size = 459, normalized size = 1.8

$$\frac{1}{e}\left(-2\,\frac{1}{\left(a\left(\tanh\left(1/2\,ex+d/2\right)\right)^{2}-\left(\tanh\left(1/2\,ex+d/2\right)\right)^{2}b-a-b\right)^{3}}\left(-1/2\,\frac{\left(6\,Aa^{2}b+3\,Aab^{2}+2\,Ab^{3}-2\,a^{3}B-2\,Ba^{2}b-6\right.}{(a-b)\left(a^{3}+3\,a^{2}b+3\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x)
```

```
[Out] 1/e*(-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^
3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*e*x+1/2*d)^5+C/(a-b)*tanh(1/2*e
*x+1/2*d)^4+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-7*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*
a*b+b^2)*tanh(1/2*e*x+1/2*d)^3-2*a*C/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)^2-
1/2*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tanh(1/2*e*x+1/2*d)+1/3*C*(3*a^2+b^2)/(a^3-3*a^2*b
+3*a*b^2-b^3))/(a*tanh(1/2*e*x+1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)^3+(2*A
*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.96464, size = 19134, normalized size = 73.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm=
"fricas")
```

```
[Out] [-1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 + 22*B*a^3*b^5 + 14*A*a^2*b^6 - 26*B*a*b^
7 + 8*A*b^8 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*
a*b^7 + B*b^8)*cosh(e*x + d)^5 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 +
 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*sinh(e*x + d)^5 - 30*(2*A*a^6*b^2 - 4*B*a
^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d)^4 -
 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*
a*b^7 + (2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 +
B*b^8)*cosh(e*x + d))*sinh(e*x + d)^4 + 4*(4*(B + C)*a^8 - 22*A*a^7*b + 4*(
7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (
39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d)^3 + 4*(4*(B + C)
*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4
*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8 - 15*(2*
A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh
(e*x + d)^2 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A
*a^2*b^6 + B*a*b^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 12*(4*B*a^7*b - 17*A*a
^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b
^7 + 2*A*b^8)*cosh(e*x + d)^2 + 12*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3
 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 - 5*(2*A
*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(
e*x + d)^3 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*
a^2*b^6 + B*a*b^7)*cosh(e*x + d)^2 + (4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B -
 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B
+ 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d))*sinh(e*x + d)^2 - 3*
(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + (2*A*a^3*b^4 - 4*B*a^2*b^5
 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*
b^6 - B*b^7)*sinh(e*x + d)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 -
 B*a*b^6)*cosh(e*x + d)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*
a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh
(e*x + d)^5 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 +
3*A*a*b^6 - B*b^7)*cosh(e*x + d)^4 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a
^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3
*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 10*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2
*b^5 - B*a*b^6)*cosh(e*x + d))*sinh(e*x + d)^4 + 4*(4*A*a^6*b - 8*B*a^5*b^2
 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d)^3 +
 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3
*B*a*b^6 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^
3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^2
+ 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 -
B*b^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*
A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 3*(8*A*a^5*b
^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A
```

*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^4 + 20*(2*A*a^4*b
^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^3 + 6*(8*A*a^5*b^2
- 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x +
d)^2 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*
b^5 - 3*B*a*b^6)*cosh(e*x + d))*sinh(e*x + d)^2 + 6*(2*A*a^4*b^3 - 4*B*a^3*
b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d) + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 +
3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*co
sh(e*x + d)^5 + 5*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(
e*x + d)^4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3
*A*a*b^6 - B*b^7)*cosh(e*x + d)^3 + 2*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b
^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d)^2 + (8*A*a^5*b^2
- 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x
+ d))*sinh(e*x + d))*sqrt(a^2 - b^2)*log((b^2*cosh(e*x + d)^2 + b^2*sinh(e*
x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2*(b^2*cosh(e*x + d) + a*b)*
sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a))/
(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(b*cosh(e*x
+ d) + a)*sinh(e*x + d) + b)) + 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^
4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8)*cosh(e*x + d) + 6*(4*B
*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*
a*b^7 + B*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*
A*a*b^7 + B*b^8)*cosh(e*x + d)^4 - 20*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^
4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d)^3 + 2*(4*(B + C)*a^8
- 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4
+ 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x +
d)^2 + 4*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3
*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(e*x + d))*sinh(e*x + d))/((a
^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^6 + (a^
8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*sinh(e*x + d)^6 + 6*(a
^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^5 + 3*
(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^
4 + 6*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d
) + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e)*sinh(e*x + d)
^5 + 4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e
x + d)^3 + 3(5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*co
sh(e*x + d)^2 + 10*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e
*cosh(e*x + d) + (4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)
*e)*sinh(e*x + d)^4 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8
+ b^12)*e*cosh(e*x + d)^2 + 4*(5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b
^10 + b^12)*e*cosh(e*x + d)^3 + 15*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3
*b^9 + a*b^11)*e*cosh(e*x + d)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6
- 10*a^4*b^8 + b^12)*e*cosh(e*x + d) + (2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 -
10*a^3*b^9 + 3*a*b^11)*e)*sinh(e*x + d)^3 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5
*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d) + 3*(5*(a^8*b^4 - 4*a^6*b^6 + 6*
a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^4 + 20*(a^9*b^3 - 4*a^7*b^5 +
6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^3 + 6*(4*a^10*b^2 - 15*a^8*

b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^2 + 4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d) + (4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e)*sinh(e*x + d)^2 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e + 6*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^5 + 5*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^4 + 2*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^3 + 2*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d)^2 + (4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d) + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e)*sinh(e*x + d)), -1/3*(2*B*a^5*b^3 - 11*A*a^4*b^4 + 11*B*a^3*b^5 + 7*A*a^2*b^6 - 13*B*a*b^7 + 4*A*b^8 - 3*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(e*x + d)^5 - 3*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*sinh(e*x + d)^5 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d)^4 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7 + (2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(e*x + d))*sinh(e*x + d)^4 + 2*(4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d)^3 + 2*(4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8 - 15*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(e*x + d)^2 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 6*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(e*x + d)^2 + 6*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(e*x + d)^3 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d)^2 + (4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d))*sinh(e*x + d)^2 + 3*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*sinh(e*x + d)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh(e*x + d)^5 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^4 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 10*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d))*sinh(e*x + d)^4 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d)^3 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6 +

5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^4 + 20*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^3 + 6*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d))*sinh(e*x + d)^2 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d) + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^5 + 5*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^3 + 2*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d)^2 + (8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh(e*x + d))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2)) + 3*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8)*cosh(e*x + d) + 3*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(e*x + d)^4 - 20*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d)^3 + 2*(4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d)^2 + 4*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(e*x + d))*sinh(e*x + d))/((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^6 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*sinh(e*x + d)^6 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^5 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^4 + 6*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d) + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e)*sinh(e*x + d)^5 + 4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d)^3 + 3*(5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^2 + 10*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d) + (4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e)*sinh(e*x + d)^4 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^2 + 4*(5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^3 + 15*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d) + (2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e)*sinh(e*x + d)^3 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^

```
5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d) + 3*(5*(a^8*b^4 - 4*a^6*b^6 + 6
*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^4 + 20*(a^9*b^3 - 4*a^7*b^5 +
 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^3 + 6*(4*a^10*b^2 - 15*a^8
*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^2 + 4*(2*a^11*b - 5*
a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d) + (4*a^10*b^2
 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e)*sinh(e*x + d)^2 + (a^8*b
^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e + 6*((a^8*b^4 - 4*a^6*b^6
 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^5 + 5*(a^9*b^3 - 4*a^7*b^
5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^4 + 2*(4*a^10*b^2 - 15*
a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^3 + 2*(2*a^11*b -
 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d)^2 + (4*a^1
0*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d) + (a^9
*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e)*sinh(e*x + d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29966, size = 946, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm=
"giac")
```

```
[Out] (2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^(x*e + d) + a)/sqrt(-
a^2 + b^2))/((a^6*e - 3*a^4*b^2*e + 3*a^2*b^4*e - b^6*e)*sqrt(-a^2 + b^2))
+ 1/3*(6*A*a^3*b^3*e^(5*x*e + 5*d) - 12*B*a^2*b^4*e^(5*x*e + 5*d) + 9*A*a*b
^5*e^(5*x*e + 5*d) - 3*B*b^6*e^(5*x*e + 5*d) + 30*A*a^4*b^2*e^(4*x*e + 4*d)
 - 60*B*a^3*b^3*e^(4*x*e + 4*d) + 45*A*a^2*b^4*e^(4*x*e + 4*d) - 15*B*a*b^5
*e^(4*x*e + 4*d) - 8*B*a^6*e^(3*x*e + 3*d) - 8*C*a^6*e^(3*x*e + 3*d) + 44*A
```

```
*a^5*b*e^(3*x*e + 3*d) - 64*B*a^4*b^2*e^(3*x*e + 3*d) + 24*C*a^4*b^2*e^(3*x
*e + 3*d) + 82*A*a^3*b^3*e^(3*x*e + 3*d) - 78*B*a^2*b^4*e^(3*x*e + 3*d) - 2
4*C*a^2*b^4*e^(3*x*e + 3*d) + 24*A*a*b^5*e^(3*x*e + 3*d) + 8*C*b^6*e^(3*x*e
 + 3*d) - 24*B*a^5*b*e^(2*x*e + 2*d) + 102*A*a^4*b^2*e^(2*x*e + 2*d) - 102*
B*a^3*b^3*e^(2*x*e + 2*d) + 36*A*a^2*b^4*e^(2*x*e + 2*d) - 24*B*a*b^5*e^(2*
x*e + 2*d) + 12*A*b^6*e^(2*x*e + 2*d) - 12*B*a^4*b^2*e^(x*e + d) + 60*A*a^3
*b^3*e^(x*e + d) - 66*B*a^2*b^4*e^(x*e + d) + 15*A*a*b^5*e^(x*e + d) + 3*B*
b^6*e^(x*e + d) - 2*B*a^3*b^3 + 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^6)/((a^6*
b*e - 3*a^4*b^3*e + 3*a^2*b^5*e - b^7*e)*(b*e^(2*x*e + 2*d) + 2*a*e^(x*e +
d) + b)^3)
```

3.210 $\int \frac{x}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=191

$$\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\sqrt{a+b}}$$

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqrt[
a + b]) - (x*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqr
t[a]*Sqrt[a + b]) + PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a +
b]))]/(4*Sqrt[a]*Sqrt[a + b]) - PolyLog[2, -((b*E^(2*x))/(2*a + b + 2*Sqrt[
a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b])
```

Rubi [A] time = 0.376604, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5630, 3320, 2264, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cosh[x]^2),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqrt[
a + b]) - (x*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqr
t[a]*Sqrt[a + b]) + PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a +
b]))]/(4*Sqrt[a]*Sqrt[a + b]) - PolyLog[2, -((b*E^(2*x))/(2*a + b + 2*Sqrt[
a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b])
```

Rule 5630

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = 2\int \frac{x}{2a+b+b\cosh(2x)}\,dx$$
$$= 4\int \frac{e^{2x}x}{b+2(2a+b)e^{2x}+be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}} - \frac{(2b)\int \frac{e^{2x}x}{4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{\int \log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)}\right)dx}{2\sqrt{a}\sqrt{a+b}} + \frac{\int \log}{}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)}\right)}{x}\,dx, x, e^{2x}\right)}{4\sqrt{a}\sqrt{a+b}}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{\text{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \frac{\text{Li}_2\left(-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{}}\right)}{4\sqrt{a}\sqrt{a+b}}$$

Mathematica [C] time = 0.562128, size = 536, normalized size = 2.81

$$-\frac{i\left(\text{PolyLog}\left(2,\frac{\left(-2i\sqrt{-a(a+b)}+2a+b\right)\left(-i\sqrt{-a(a+b)}\tanh(x)+a+b\right)}{b\left(i\sqrt{-a(a+b)}\tanh(x)+a+b\right)}\right)-\text{PolyLog}\left(2,\frac{\left(2i\sqrt{-a(a+b)}+2a+b\right)\left(-i\sqrt{-a(a+b)}\tanh(x)+a+b\right)}{b\left(i\sqrt{-a(a+b)}\tanh(x)+a+b\right)}\right)\right)+4x\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cosh[x]^2),x]
```

```
[Out] -(4*x*ArcTan[((a + b)*Coth[x])/Sqrt[-(a*(a + b))]] + (2*I)*ArcCos[-1 - (2*a
)/b]*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]] + (ArcCos[-1 - (2*a)/b] + 2*Arc
Tan[((a + b)*Coth[x])/Sqrt[-(a*(a + b))]] - 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(
a + b))]])*Log[(Sqrt[2]*Sqrt[-(a*(a + b))])/(Sqrt[b]*E^x*Sqrt[2*a + b + b*C
osh[2*x]])] + (ArcCos[-1 - (2*a)/b] - 2*ArcTan[((a + b)*Coth[x])/Sqrt[-(a*(
a + b))]] + 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]])*Log[(Sqrt[2]*Sqrt[-(a
*(a + b))]*E^x)/(Sqrt[b]*Sqrt[2*a + b + b*Cosh[2*x]])] - (ArcCos[-1 - (2*a)
/b] - 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]])*Log[(2*(a + b)*(a + I*Sqrt[
-(a*(a + b))])*(-1 + Tanh[x]))/(b*(a + b + I*Sqrt[-(a*(a + b))]*Tanh[x]))]
- (ArcCos[-1 - (2*a)/b] + 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]])*Log[((2
*I)*(a + b)*(I*a + Sqrt[-(a*(a + b))])*(1 + Tanh[x]))/(b*(a + b + I*Sqrt[-(
a*(a + b))]*Tanh[x]))] + I*(PolyLog[2, ((2*a + b - (2*I)*Sqrt[-(a*(a + b))]
```

```
)*(a + b - I*Sqrt[-(a*(a + b))]*Tanh[x]))/(b*(a + b + I*Sqrt[-(a*(a + b))]*
Tanh[x]))] - PolyLog[2, ((2*a + b + (2*I)*Sqrt[-(a*(a + b))])*(a + b - I*Sq
rt[-(a*(a + b))]*Tanh[x]))/(b*(a + b + I*Sqrt[-(a*(a + b))]*Tanh[x]))]))/(4
*Sqrt[-(a*(a + b))])
```

Maple [B] time = 0.046, size = 487, normalized size = 2.6

$$\frac{x}{2}\ln\left(1-be^{2x}\left(2\sqrt{a(a+b)}-2a-b\right)^{-1}\right)\frac{1}{\sqrt{a(a+b)}}-\frac{x^2}{2}\frac{1}{\sqrt{a(a+b)}}+\frac{1}{4}\mathrm{polylog}\left(2,be^{2x}\left(2\sqrt{a(a+b)}-2a-b\right)^{-1}\right)\frac{1}{\sqrt{a(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cosh(x)^2),x)
```

```
[Out] 1/2/(a*(a+b))^(1/2)*x*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-1/2/(a*(a+
b))^(1/2)*x^2+1/4/(a*(a+b))^(1/2)*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2
*a-b))+1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a
-b))*x+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*
(a+b))^(1/2)-2*a-b))*a*x+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(
1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b*x-1/(-2*(a*(a+b))^(1/2)-2*a-b)*x
^2-1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^2-1/2/(a*(a+b))^(1/2)/(
-2*(a*(a+b))^(1/2)-2*a-b)*b*x^2+1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*
exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2
)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+1/4/(a*(a+b))^(
1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*
a-b))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cosh(x)^2 + a), x)
```

Fricas [B] time = 1.95436, size = 1968, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b
)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x*sqrt((a^
2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b
)/b) + b)/b) - b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) + (2*a +
b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqr
t((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + b*sqrt((a^2 + a*b)/b^2)*dilog(-(
((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^
2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) + b)/b + 1) +
 b*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b) - b)/b + 1) - b*sqrt((a^2 + a*b)/b^2)*dilog(-(((2*a + b)
*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b
^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) + b)/b + 1) - b*sqrt((a^
2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x)
+ b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a -
 b)/b) - b)/b + 1))/(a^2 + a*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cosh(x)^2 + a), x)
```

3.211 $\int \frac{x^2}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=291

$$\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{\text{PolyLog}\left(3,-\frac{}{2\sqrt{a}\cdots}\right.}{4\sqrt{a}\sqrt{a+\cdots}}$$

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqr
t[a + b]) - (x^2*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2
*Sqrt[a]*Sqrt[a + b]) + (x*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sq
rt[a + b]))])/(2*Sqrt[a]*Sqrt[a + b]) - (x*PolyLog[2, -((b*E^(2*x))/(2*a +
b + 2*Sqrt[a]*Sqrt[a + b]))])/(2*Sqrt[a]*Sqrt[a + b]) - PolyLog[3, -((b*E^(
2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b]) + PolyLog
[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b
])
```

Rubi [A] time = 0.566805, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5630, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{\text{PolyLog}\left(3,-\frac{}{2\sqrt{a}\cdots}\right.}{4\sqrt{a}\sqrt{a+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cosh[x]^2),x]
```


```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqr
t[a + b]) - (x^2*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2
*Sqrt[a]*Sqrt[a + b]) + (x*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sq
rt[a + b]))])/(2*Sqrt[a]*Sqrt[a + b]) - (x*PolyLog[2, -((b*E^(2*x))/(2*a +
b + 2*Sqrt[a]*Sqrt[a + b]))])/(2*Sqrt[a]*Sqrt[a + b]) - PolyLog[3, -((b*E^(
2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b]) + PolyLog
[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b
])
```


Rule 5630

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
```

```
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cosh^2(x)}\,dx = 2\int \frac{x^2}{2a+b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x^2}{b+2(2a+b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^2}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}} - \frac{(2b)\int \frac{e^{2x}x^2}{4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{\int x\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)}\right)dx}{\sqrt{a}\sqrt{a+b}} + \frac{\int}{}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\text{Li}_2\left(-\frac{b}{2a+b+}\right.}{2\sqrt{a}\sqrt{a}}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\text{Li}_2\left(-\frac{b}{2a+b+}\right.}{2\sqrt{a}\sqrt{a}}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\text{Li}_2\left(-\frac{b}{2a+b+}\right.}{2\sqrt{a}\sqrt{a}}$$

Mathematica [A] time = 0.717135, size = 221, normalized size = 0.76

$$\frac{2x\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right) - 2x\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right) - \text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right) + \text{PolyLog}\left(3,-\frac{}{2\sqrt{a}}\right.}{4\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cosh[x]^2),x]
```

```
[Out] (2*x^2*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])] - 2*x^2*Log[1
 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])] + 2*x*PolyLog[2, -((b*E^(
2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))] - 2*x*PolyLog[2, -((b*E^(2*x))/(2
```

```
*a + b + 2*Sqrt[a]*Sqrt[a + b]))] - PolyLog[3, -((b*E^(2*x))/(2*a + b - 2*S
qrt[a]*Sqrt[a + b]))] + PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[
a + b]))])/(4*Sqrt[a]*Sqrt[a + b])
```

Maple [B] time = 0.038, size = 686, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cosh(x)^2),x)
```

```
[Out] -2/3/(-2*(a*(a+b))^(1/2)-2*a-b)*x^3+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x^2*ln(1-b
*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x*polylo
g(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*p
olylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-2/3/(a*(a+b))^(1/2)/(-2*(a*
(a+b))^(1/2)-2*a-b)*a*x^3+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^
2*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(a*(a+b))^(1/2)/(-2*(a*(a+b
))^(1/2)-2*a-b)*a*x*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/2/(a
*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*polylog(3,b*exp(2*x)/(-2*(a*(a+b
))^(1/2)-2*a-b))-1/3/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^3+1/2/(
a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^2*ln(1-b*exp(2*x)/(-2*(a*(a+b
))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x*polylog
(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*(a+b))^(1/2)/(-2*(a*(a+b))
^(1/2)-2*a-b)*b*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/3/(a*(a+
b))^(1/2)*x^3+1/2/(a*(a+b))^(1/2)*x^2*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*
a-b))+1/2/(a*(a+b))^(1/2)*x*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))
-1/4/(a*(a+b))^(1/2)*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cosh(x)^2 + a), x)
```

Fricas [C] time = 2.12077, size = 2967, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((2*a
 + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x^2*sq
rt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2
*a - b)/b) + b)/b) - b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) +
 (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 2*b*x*sqrt((a^2 + a*b)/b
^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(
x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) +
 b)/b + 1) + 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a +
 b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 2*b*x*sqrt((a^2 + a*b)/b^2
)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x)
)*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) + b)
/b + 1) - 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)
*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) - 2*b*sqrt((a^2 + a*b)/b^2)*poly
log(3, ((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*s
qrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)/b) - 2
*b*sqrt((a^2 + a*b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b)/b) + 2*b*sqrt((a^2 + a*b)/b^2)*polylog(3, ((2*a + b
)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/
b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)/b) + 2*b*sqrt((a^2 + a*
b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) +
 b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a -
b)/b)/b))/(a^2 + a*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cosh(x)^2 + a), x)
```

3.212 $\int \frac{x^3}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=391

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{3x\text{PolyLog}}{4}$$

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqr
t[a + b]) - (x^3*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2
*Sqrt[a]*Sqrt[a + b]) + (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a
]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a + b]) - (3*x^2*PolyLog[2, -((b*E^(2*x))
/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a + b]) - (3*x*PolyLo
g[3, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a +
 b]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))])/(
4*Sqrt[a]*Sqrt[a + b]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*S
qrt[a + b]))])/(8*Sqrt[a]*Sqrt[a + b]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a +
 b + 2*Sqrt[a]*Sqrt[a + b]))])/(8*Sqrt[a]*Sqrt[a + b])
```

Rubi [A] time = 0.599144, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5630, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{3x\text{PolyLog}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cosh[x]^2),x]
```


```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqr
t[a + b]) - (x^3*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2
*Sqrt[a]*Sqrt[a + b]) + (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a
]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a + b]) - (3*x^2*PolyLog[2, -((b*E^(2*x))
/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a + b]) - (3*x*PolyLo
g[3, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a +
 b]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))])/(
4*Sqrt[a]*Sqrt[a + b]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*S
qrt[a + b]))])/(8*Sqrt[a]*Sqrt[a + b]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a +
 b + 2*Sqrt[a]*Sqrt[a + b]))])/(8*Sqrt[a]*Sqrt[a + b])
```

Rule 5630

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx = 2\int \frac{x^3}{2a+b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x^3}{b+2(2a+b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^3}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}} - \frac{(2b)\int \frac{e^{2x}x^3}{4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{3\int x^2\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)}\right)dx}{2\sqrt{a}\sqrt{a+b}} + \cdots$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \frac{3x^2\text{Li}_2\left(-\cdots\right)}{4\sqrt{a}\cdots}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \frac{3x^2\text{Li}_2\left(-\cdots\right)}{4\sqrt{a}\cdots}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \frac{3x^2\text{Li}_2\left(-\cdots\right)}{4\sqrt{a}\cdots}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \frac{3x^2\text{Li}_2\left(-\cdots\right)}{4\sqrt{a}\cdots}$$

Mathematica [A] time = 0.610059, size = 295, normalized size = 0.75

$$\frac{6x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right) - 6x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right) - 6x\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right) + 6x\text{PolyLog}\left(3,\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cosh[x]^2),x]
```

```
[Out] (4*x^3*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])] - 4*x^3*Log[1
 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])] + 6*x^2*PolyLog[2, -((b*E
^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))] - 6*x^2*PolyLog[2, -((b*E^(2*x)
)/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))] - 6*x*PolyLog[3, -((b*E^(2*x))/(2*a +
 b - 2*Sqrt[a]*Sqrt[a + b]))] + 6*x*PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*S
qrt[a]*Sqrt[a + b]))] + 3*PolyLog[4, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqr
```

```
t[a + b]))] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))
])/(8*Sqrt[a]*Sqrt[a + b])
```

Maple [B] time = 0.039, size = 889, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cosh(x)^2),x)
```

```
[Out] 1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x^
3+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b)
)^(1/2)-2*a-b))*a*x^3+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b
*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b*x^3-1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*
x^4-1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^4-1/4/(a*(a+b))^(1/2
)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^4+3/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2
,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x^2+3/2/(a*(a+b))^(1/2)/(-2*(a*(a+b
))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x^2+3/4/
(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b
))^(1/2)-2*a-b))*b*x^2-3/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(3,b*exp(2*x)/
(-2*(a*(a+b))^(1/2)-2*a-b))*x-3/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b
)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x-3/4/(a*(a+b))^(1/2)/
(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))
*b*x+3/4/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(4,b*exp(2*x)/(-2*(a*(a+b))^(1/2
)-2*a-b))+3/4/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(4,b*exp(2*
x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+3/8/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*
a-b)*polylog(4,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b+1/2/(a*(a+b))^(1/2)
*x^3*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*(a+b))^(1/2)*x^4+3/4
/(a*(a+b))^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-3/4/(a
*(a+b))^(1/2)*x*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))+3/8/(a*(a+b
))^(1/2)*polylog(4,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*cosh(x)^2 + a), x)
```

Fricas [C] time = 2.36334, size = 3956, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((2*a
 + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x^3*sq
rt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2
*a - b)/b) + b)/b) - b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) +
 (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 3*b*x^2*sqrt((a^2 + a*b)
/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sin
h(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)
 + b)/b + 1) + 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2
*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 3*b*x^2*sqrt((a^2 + a*
b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*s
inh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b
) + b)/b + 1) - 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (
2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2
*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) - 6*b*x*sqrt((a^2 + a*b)
/b^2)*polylog(3, ((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*
sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)/b) - 6*b*x*sqrt((a^2 + a*b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a
+ b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)/b) + 6*b*x*sqrt((a^2 + a*b)/b^2)*polylo
g(3, ((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqr
t((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)/b) + 6*b*
x*sqrt((a^2 + a*b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x)
+ 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*
b)/b^2) - 2*a - b)/b)/b) + 6*b*sqrt((a^2 + a*b)/b^2)*polylog(4, ((2*a + b)*
```

```
cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^
2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)/b) + 6*b*sqrt((a^2 + a*b
)/b^2)*polylog(4, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)/b) - 6*b*sqrt((a^2 + a*b)/b^2)*polylog(4, ((2*a + b)*cosh(x) + (2*a +
 b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sq
rt((a^2 + a*b)/b^2) - 2*a - b)/b)/b) - 6*b*sqrt((a^2 + a*b)/b^2)*polylog(4,
 -((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((
a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)/b))/(a^2 + a
*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\cosh{(x)}^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cosh(x)^2 + a), x)
```

3.213 $\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

```
[Out] (-3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - CoshIntegral[(3*Sqrt
[1 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rubi [A] time = 0.118876, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3301}

$$-\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - CoshIntegral[(3*Sqrt
[1 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{3\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}\\
&= -\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0771611, size = 55, normalized size = 0.95

$$\frac{-3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - CoshIntegral[(3*Sqrt[1 - a*
x])/Sqrt[1 + a*x]])/(4*a)
```

Maple [F] time = 0.133, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\cosh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] -integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cosh^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.214 $\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] -CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]/(2*a)
```

Rubi [A] time = 0.0813098, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3301}

$$-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]/(2*a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x} + \frac{\cosh(2x)}{2x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} \\
&= -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.034599, size = 57, normalized size = 0.98

$$-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\log(1-ax)}{4a} + \frac{\log(ax+1)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[1 - a*x]/(4*a) +
 Log[1 + a*x]/(4*a)
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\cosh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

```
[Out] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(ax+1\right)}{4\,a}-\frac{\log\left(ax-1\right)}{4\,a}-\frac{1}{4}\int\frac{e^{\left(\frac{2\,\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}}{a^{2}x^{2}-1}\,dx-\frac{1}{4}\int\frac{e^{\left(-\frac{2\,\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] 1/4*log(a*x + 1)/a - 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x + 1)
/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/sqrt
(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}{a^{2}x^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\cosh^{2}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

3.215 $\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\mathrm{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] -(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi [A] time = 0.038764, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6681, 3301}

$$-\frac{\mathrm{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]

[Out] -(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 0.0325367, size = 26, normalized size = 1.

$$-\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\cosh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] -integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.216 $\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=39

$$\text{Unintegrable}\left(\frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/((1 - a*x)*(1 + a*x)), x]
```

Rubi [A] time = 0.0375866, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Sech[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\text{Subst}\left(\int \frac{\operatorname{sech}(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 5.70574, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Maple [A] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\cosh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(a^2x^2-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="m
axima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="f
ricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^2x^2\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1)) - cosh(sqrt(-a*x
+ 1)/sqrt(a*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="g
iac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.217 $\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=41

$$\text{Unintegrable}\left(\frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/((1 - a*x)*(1 + a*x)), x]
```

Rubi [A] time = 0.0798475, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Sech[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/
a)
```

Rubi steps

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\operatorname{sech}^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 23.4857, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Maple [A] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\cosh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{ax+1}}{\sqrt{-ax+1}ae^{\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}+\sqrt{-ax+1}a}+2\int\frac{\sqrt{ax+1}}{\left(a^2x^2-1\right)\sqrt{-ax+1}e^{\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}+\left(a^2x^2-1\right)\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"maxima")
```

```
[Out] 2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + sqrt
(-a*x + 1)*a) + 2*integrate(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(
2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"fricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

3.218 $\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=60

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}$$

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]
) - x/(b*(a + b*Cosh[x]))
```

Rubi [A] time = 0.0568916, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5465, 2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]
) - x/(b*(a + b*Cosh[x]))
```

Rule 5465

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*(a + b*Cosh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx &= -\frac{x}{b(a+b\cosh(x))}+\frac{\int \frac{1}{a+b\cosh(x)}\,dx}{b}\\ &= -\frac{x}{b(a+b\cosh(x))}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.124802, size = 59, normalized size = 0.98

$$-\frac{2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b\sqrt{b^2-a^2}}-\frac{x}{b(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) - x/
(b*(a + b*Cosh[x]))
```

Maple [B] time = 0.069, size = 138, normalized size = 2.3

$$-2\,\frac{x{e}^{x}}{b\left(b{e}^{2\,x}+2\,a{e}^{x}+b\right)}+\frac{1}{b}\ln\left({e}^{x}+\frac{1}{b}\left(a\sqrt{a^2-b^2}-a^2+b^2\right)\frac{1}{\sqrt{a^2-b^2}}\right)\frac{1}{\sqrt{a^2-b^2}}-\frac{1}{b}\ln\left({e}^{x}+\frac{1}{b}\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x)/(a+b*cosh(x))^2,x)
```

```
[Out] -2*x/b*exp(x)/(b*exp(2*x)+2*a*exp(x)+b)+1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(a*(a
^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)-1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(a*
(a^2-b^2)^(1/2)+a^2-b^2)/(a^2-b^2)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93979, size = 1202, normalized size = 20.03

$$\left[-\frac{2\left(a^2-b^2\right)x\cosh(x)+2\left(a^2-b^2\right)x\sinh(x)-\left(b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}{a^2b^2-b^4+\left(a^2b^2-b^4\right)\cosh(x)^2+\left(a^2b^2-b^4\right)\sinh(x)^2+2\left(a^3b-ab^3\right)c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*(a^2 - b^2)*x*cosh(x) + 2*(a^2 - b^2)*x*sinh(x) - (b*cosh(x)^2 + b*sin
h(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(a^2 - b^2)*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x
)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)))/(a^2*b^2
 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b -
 a*b^3)*cosh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x)), -2*
((a^2 - b^2)*x*cosh(x) + (a^2 - b^2)*x*sinh(x) + (b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(-a^2 + b^2)*arctan(-sq
rt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a^2*b^2 - b^4 + (
a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b - a*b^3)*co
sh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sinh(x)}{(b\cosh(x)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] integrate(x*sinh(x)/(b*cosh(x) + a)^2, x)
```

3.219 $\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx$

Optimal. Leaf size=87

$$-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}-\frac{x}{2b(a+b\cosh(x))^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] (a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*b*(a + b)^(
3/2)) - x/(2*b*(a + b*Cosh[x])^2) - Sinh[x]/(2*(a^2 - b^2)*(a + b*Cosh[x]))
```

Rubi [A] time = 0.0892023, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5465, 2664, 12, 2659, 208}

$$-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}-\frac{x}{2b(a+b\cosh(x))^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```



```
[Out] (a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*b*(a + b)^(
3/2)) - x/(2*b*(a + b*Cosh[x])^2) - Sinh[x]/(2*(a^2 - b^2)*(a + b*Cosh[x]))
```


Rule 5465

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*(a + b*Cosh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx &= -\frac{x}{2b(a+b\cosh(x))^2}+\frac{\int \frac{1}{(a+b\cosh(x))^2}\,dx}{2b}\\
&= -\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{\int \frac{a}{a+b\cosh(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= -\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{a\int \frac{1}{a+b\cosh(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= -\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b\left(a^2-b^2\right)}\\
&= \frac{a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}}-\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.243949, size = 87, normalized size = 1.

$$\frac{1}{2}\left(\frac{\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{x}{(a+b\cosh(x))^2}}{b}-\frac{\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] (((2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - x
/(a + b*Cosh[x])^2)/b - Sinh[x]/((a - b)*(a + b)*(a + b*Cosh[x])))/2
```

Maple [B] time = 0.083, size = 231, normalized size = 2.7

$$-\frac{2a^2xe^{2x}-abe^{3x}-2b^2xe^{2x}-2a^2e^{2x}-b^2e^{2x}-3ae^xb-b^2}{b\left(be^{2x}+2ae^x+b\right)^2\left(a^2-b^2\right)}+\frac{a}{(2a+2b)(a-b)b}\ln\left(e^x+\frac{1}{b}\left(a\sqrt{a^2-b^2}-a^2+b^2\right)\frac{1}{\sqrt{a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x)/(a+b*cosh(x))^3,x)
```

```
[Out] -1/b*(2*a^2*x*exp(2*x)-a*b*exp(3*x)-2*b^2*x*exp(2*x)-2*a^2*exp(2*x)-b^2*exp
(2*x)-3*a*exp(x)*b-b^2)/(b*exp(2*x)+2*a*exp(x)+b)^2/(a^2-b^2)+1/2/(a^2-b^2)
^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2
)/b)-1/2/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)+a^2-b
^2)/(a^2-b^2)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05765, size = 3969, normalized size = 45.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*b^2 - 2*b^4 + 2*(a^3*b - a*b^3)*cosh(x)^3 + 2*(a^3*b - a*b^3)*s
inh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x)^
2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x + 3*(a^3*b - a*b
^3)*cosh(x))*sinh(x)^2 - (a*b^2*cosh(x)^4 + a*b^2*sinh(x)^4 + 4*a^2*b*cosh(
x)^3 + 4*a^2*b*cosh(x) + 4*(a*b^2*cosh(x) + a^2*b)*sinh(x)^3 + a*b^2 + 2*(2
*a^3 + a*b^2)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + 6*a^2*b*cosh(x) + 2*a^3 +
a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + 3*a^2*b*cosh(x)^2 + a^2*b + (2*a^3
+ a*b^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)
^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a
^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cos
h(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(a^3*b - a*b^3)*cosh(x) + 2*(3*a
^3*b - 3*a*b^3 + 3*(a^3*b - a*b^3)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2
*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^4*b^3 - 2*a^2*b^5 + b^7 +
(a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^4*b^3 - 2*a^2*b^5 + b^7)*sinh(x)
^4 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 4*(a^5*b^2 - 2*a^3*b^4 + a
*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 + 2*(2*a^6*b - 3*a^4*
b^3 + b^7)*cosh(x)^2 + 2*(2*a^6*b - 3*a^4*b^3 + b^7 + 3*(a^4*b^3 - 2*a^2*b^
5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^2 + 4
*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x) + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (
a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh
(x)^2 + (2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x))*sinh(x)), (a^2*b^2 - b^4 + (a^
3*b - a*b^3)*cosh(x)^3 + (a^3*b - a*b^3)*sinh(x)^3 + (2*a^4 - a^2*b^2 - b^4
 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x)^2 + (2*a^4 - a^2*b^2 - b^4 - 2*(a^4
 - 2*a^2*b^2 + b^4)*x + 3*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 - (a*b^2*cosh(
x)^4 + a*b^2*sinh(x)^4 + 4*a^2*b*cosh(x)^3 + 4*a^2*b*cosh(x) + 4*(a*b^2*cos
h(x) + a^2*b)*sinh(x)^3 + a*b^2 + 2*(2*a^3 + a*b^2)*cosh(x)^2 + 2*(3*a*b^2*
cosh(x)^2 + 6*a^2*b*cosh(x) + 2*a^3 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3
 + 3*a^2*b*cosh(x)^2 + a^2*b + (2*a^3 + a*b^2)*cosh(x))*sinh(x))*sqrt(-a^2
+ b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) +
3*(a^3*b - a*b^3)*cosh(x) + (3*a^3*b - 3*a*b^3 + 3*(a^3*b - a*b^3)*cosh(x)^
2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x
))/(a^4*b^3 - 2*a^2*b^5 + b^7 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^
```

```
4*b^3 - 2*a^2*b^5 + b^7)*sinh(x)^4 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x
)^3 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x))
*sinh(x)^3 + 2*(2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x)^2 + 2*(2*a^6*b - 3*a^4*b
^3 + b^7 + 3*(a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 - 2*a^3*b^4
 + a*b^6)*cosh(x))*sinh(x)^2 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x) + 4*
(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^3 + 3*(a
^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x)^2 + (2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x)
)*sinh(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(x)}{(b \cosh(x) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="giac")
```

```
[Out] integrate(x*sinh(x)/(b*cosh(x) + a)^3, x)
```

3.220 $\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=47

$$\frac{9}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

```
[Out] (9*CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (9*Co
sh[a]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rubi [A] time = 0.462328, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6742, 3303, 3298, 3301, 5448}

$$\frac{9}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

```
[Out] (9*CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (9*Co
sh[a]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+\cosh^2(a+bx)\right)\sinh(a+bx)}{x}\,dx &= \int \left(\frac{2\sinh(a+bx)}{x}+\frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\right)dx \\
&= 2\int \frac{\sinh(a+bx)}{x}\,dx+\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx \\
&= (2\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+(2\sinh(a))\int \frac{\cosh(bx)}{x}\,dx+\int \left(\frac{\sinh(a+bx)}{4x}+\right. \\
&= 2\mathbf{Chi}(bx)\sinh(a)+2\cosh(a)\mathbf{Shi}(bx)+\frac{1}{4}\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{4}\int \frac{\sinh(3a}{x} \\
&= 2\mathbf{Chi}(bx)\sinh(a)+2\cosh(a)\mathbf{Shi}(bx)+\frac{1}{4}\cosh(a)\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{4}\cosh(3a \\
&= \frac{9}{4}\mathbf{Chi}(bx)\sinh(a)+\frac{1}{4}\mathbf{Chi}(3bx)\sinh(3a)+\frac{9}{4}\cosh(a)\mathbf{Shi}(bx)+\frac{1}{4}\cosh(3a)\mathbf{Sh}
\end{aligned}
$$

Mathematica [A] time = 0.106146, size = 41, normalized size = 0.87

$$\frac{1}{4}(9\sinh(a)\mathbf{Chi}(bx)+\sinh(3a)\mathbf{Chi}(3bx)+9\cosh(a)\mathbf{Shi}(bx)+\cosh(3a)\mathbf{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

```
[Out] (9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] + 9*Cosh[a]*Si
nhIntegral[b*x] + Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Maple [A] time = 0.075, size = 47, normalized size = 1.

$$\frac{e^{-3a}\mathrm{Ei}(1,3bx)}{8}+\frac{9\,e^{-a}\mathrm{Ei}(1,bx)}{8}-\frac{9\,e^{a}\mathrm{Ei}(1,-bx)}{8}-\frac{e^{3a}\mathrm{Ei}(1,-3bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x)
```

```
[Out] 1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)-9/8*exp(a)*Ei(1,-b*x)-1/8*e
xp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.31264, size = 57, normalized size = 1.21

$$\frac{1}{8}\,\mathrm{Ei}\,(3\,bx)\,e^{(3\,a)}-\frac{9}{8}\,\mathrm{Ei}\,(-bx)\,e^{(-a)}-\frac{1}{8}\,\mathrm{Ei}\,(-3\,bx)\,e^{(-3\,a)}+\frac{9}{8}\,\mathrm{Ei}\,(bx)\,e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 9/8
*Ei(b*x)*e^a
```

Fricas [A] time = 1.82911, size = 204, normalized size = 4.34

$$\frac{1}{8}\,(\mathrm{Ei}\,(3\,bx)-\mathrm{Ei}\,(-3\,bx))\cosh\,(3\,a)+\frac{9}{8}\,(\mathrm{Ei}\,(bx)-\mathrm{Ei}\,(-bx))\cosh\,(a)+\frac{1}{8}\,(\mathrm{Ei}\,(3\,bx)+\mathrm{Ei}\,(-3\,bx))\sinh\,(3\,a)+\frac{9}{8}\,(\mathrm{Ei}\,(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9/8*(Ei(b*x) - Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) + 9/8*(Ei(b*x) + Ei(-b*x))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\cosh^2\left(a+bx\right)+2\right)\sinh\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)**2)*sinh(b*x+a)/x,x)
```

```
[Out] Integral((cosh(a + b*x)**2 + 2)*sinh(a + b*x)/x, x)
```

Giac [A] time = 1.17129, size = 57, normalized size = 1.21

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{\left(3\,a\right)}-\frac{9}{8}\,\mathrm{Ei}\left(-bx\right)e^{\left(-a\right)}-\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{\left(-3\,a\right)}+\frac{9}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 9/8
*Ei(b*x)*e^a
```

3.221 $\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)},x\right)$$

```
[Out] Unintegrable[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]), x]
```

Rubi [A] time = 0.0381827, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Defer[Int][(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Mathematica [A] time = 5.09777, size = 0, normalized size = 0.

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]), x]
```

Maple [A] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{x^m \sinh(dx+c)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{(2\,dx+m\log(x)+2\,c)}}{b(m+1)e^{(2\,dx+2\,c)}+2\,a(m+1)e^{(dx+c)}+b(m+1)}-\frac{1}{2}\int\frac{2\left(2\,adxe^{(3\,dx+3\,c)}+2\,a(m+1)e^{(dx+c)}+b(m+1)+\right.}{b^2(m+1)e^{(4\,dx+4\,c)}+4\,ab(m+1)e^{(3\,dx+3\,c)}+4\,ab(m+1)e^{(dx+c)}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] x*e^(2*d*x + m*log(x) + 2*c)/(b*(m + 1)*e^(2*d*x + 2*c) + 2*a*(m + 1)*e^(d*
x + c) + b*(m + 1)) - 1/2*integrate(2*(2*a*d*x*e^(3*d*x + 3*c) + 2*a*(m + 1
)*e^(d*x + c) + b*(m + 1) + (2*b*d*x*e^(2*c) + b*(m + 1)*e^(2*c))*e^(2*d*x)
)*x^m/(b^2*(m + 1)*e^(4*d*x + 4*c) + 4*a*b*(m + 1)*e^(3*d*x + 3*c) + 4*a*b*
(m + 1)*e^(d*x + c) + b^2*(m + 1) + 2*(2*a^2*(m + 1)*e^(2*c) + b^2*(m + 1)*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \sinh(dx+c)}{b\cosh(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**m*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sinh(dx+c)}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

3.222 $\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=327

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}-\frac{6x\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{6x\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^3}+$$

```
[Out] -x^4/(4*b) + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (3*x^2*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (3*x^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^3) + (6*PolyLog[4, -((b*E^(c + d*x))/(a
- Sqrt[a^2 - b^2]))])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/(b*d^4)
```

Rubi [A] time = 0.479493, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5562, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}-\frac{6x\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{6x\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```


```
[Out] -x^4/(4*b) + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (3*x^2*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (3*x^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^3) + (6*PolyLog[4, -((b*E^(c + d*x))/(a
- Sqrt[a^2 - b^2]))])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/(b*d^4)
```


Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^4}{4b} + \int \frac{e^{c+dx}x^3}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx + \int \frac{e^{c+dx}x^3}{a+\sqrt{a^2-b^2}+be^{c+dx}}\,dx$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{3\int x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{bd} - \frac{3\int x^2}{}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+}}{a+\sqrt{a}}\right.}{bd^2}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+}}{a+\sqrt{a}}\right.}{bd^2}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+}}{a+\sqrt{a}}\right.}{bd^2}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+}}{a+\sqrt{a}}\right.}{bd^2}$$

Mathematica [A] time = 0.0326848, size = 326, normalized size = 1.

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2} - \frac{6x\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3} - \frac{6x\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^3} +$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -x^4/(4*b) + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (3*x^2*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (3*x^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^3) + (6*PolyLog[4, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 - b^2])])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2
 - b^2]))])/(b*d^4)
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{x^3 \sinh(dx+c)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{4\,b} - \frac{1}{2}\int \frac{4\left(ax^3e^{(dx+c)} + bx^3\right)}{b^2e^{(2\,dx+2\,c)} + 2\,abe^{(dx+c)} + b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/b - 1/2*integrate(4*(a*x^3*e^(d*x + c) + b*x^3)/(b^2*e^(2*d*x + 2*c
) + 2*a*b*e^(d*x + c) + b^2), x)
```

Fricas [C] time = 2.09096, size = 1585, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^4*x^4 - 12*d^2*x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 12*d^2*
x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 4*c^3*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*c^3*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 24*d*x*pol
ylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*d*x*polylog(3, -(a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/
b) - 4*(d^3*x^3 + c^3)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 4*(d^3*x^3 + c^3)*
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 - b^2)/b^2) + b)/b) - 24*polylog(4, -(a*cosh(d*x + c) + a*sinh(
```

```
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) -
24*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b))/(b*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sinh(dx + c)}{b \cosh(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^3*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

3.223 $\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}-\frac{2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^3}+\frac{x^2}{}$$

```
[Out] -x^3/(3*b) + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (2*x*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (2*x*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (2*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (2*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2]))])/(b*d^3)
```

Rubi [A] time = 0.390186, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5562, 2190, 2531, 2282, 6589}

$$\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}-\frac{2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^3}+\frac{x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -x^3/(3*b) + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (2*x*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (2*x*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (2*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (2*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2]))])/(b*d^3)
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^3}{3b} + \int \frac{e^{c+dx}x^2}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx + \int \frac{e^{c+dx}x^2}{a+\sqrt{a^2-b^2}+be^{c+dx}}\,dx$$
$$= -\frac{x^3}{3b} + \frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{2\int x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{bd} - \frac{2\int x\ }{}$$
$$= -\frac{x^3}{3b} + \frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{2x\text{Li}_2\left(-\frac{be^{c+}}{a+\sqrt{a^2}}\right.}{bd^2}$$
$$= -\frac{x^3}{3b} + \frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{2x\text{Li}_2\left(-\frac{be^{c+}}{a+\sqrt{a^2}}\right.}{bd^2}$$
$$= -\frac{x^3}{3b} + \frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{2x\text{Li}_2\left(-\frac{be^{c+}}{a+\sqrt{a^2}}\right.}{bd^2}$$

Mathematica [A] time = 0.0206282, size = 244, normalized size = 1.

$$\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2} - \frac{2\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right)}{bd^3} - \frac{2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^3} + \frac{x^2\ \text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -x^3/(3*b) + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (2*x*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (2*x*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (2*PolyLog[3, (b*E^(c + d*x
))/(-a + Sqrt[a^2 - b^2])])/(b*d^3) - (2*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 - b^2]))])/(b*d^3)
```

Maple [F] time = 0.172, size = 0, normalized size = 0.

$$\int \frac{x^2 \sinh(dx+c)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3\,b} - \frac{1}{2}\int \frac{4\left(ax^2e^{(dx+c)} + bx^2\right)}{b^2e^{(2\,dx+2\,c)} + 2\,abe^{(dx+c)} + b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/b - 1/2*integrate(4*(a*x^2*e^(d*x + c) + b*x^2)/(b^2*e^(2*d*x + 2*c
) + 2*a*b*e^(d*x + c) + b^2), x)
```

Fricas [C] time = 1.99546, size = 1245, normalized size = 5.08

$$-\frac{d^3x^3 - 6\,dx\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right) - 6\,dx\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(d^3*x^3 - 6*d*x*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 6*d*x*dilog(
-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*c^2*log(2*b*cosh(d*x + c) + 2*b*sinh(d
*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*c^2*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*(d^2*x^2 - c^2)*lo
g((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 - b^2)/b^2) + b)/b) - 3*(d^2*x^2 - c^2)*log((a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) +
 b)/b) + 6*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 6*polylog(3, -(a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2))/b))/(b*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sinh (c + dx)}{a + b \cosh (c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**2*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sinh (dx + c)}{b \cosh (dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

3.224 $\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

```
[Out] -x^2/(2*b) + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E^(
c + d*x))/(a - Sqrt[a^2 - b^2]))]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 - b^2]))]/(b*d^2)
```

Rubi [A] time = 0.24167, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5562, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -x^2/(2*b) + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E^(
c + d*x))/(a - Sqrt[a^2 - b^2]))]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 - b^2]))]/(b*d^2)
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx &= -\frac{x^2}{2b}+\int \frac{e^{c+dx}x}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx+\int \frac{e^{c+dx}x}{a+\sqrt{a^2-b^2}+be^{c+dx}}\,dx\\
&= -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{\int \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{\int \log\left(1+\right.}{}\\
&= -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a-\sqrt{a^2-b^2}}\right)}{x}\,dx,x,e^{c+dx}\right)}{bd^2}-\\
&= -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}\end{aligned}$$

Mathematica [A] time = 0.0121135, size = 160, normalized size = 0.99

$$\frac{\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -x^2/(2*b) + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, (b*E^(c
```

```
+ d*x))/(-a + Sqrt[a^2 - b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 - b^2]))]/(b*d^2)
```

Maple [B] time = 0.047, size = 368, normalized size = 2.3

$$-\frac{x^2}{2b} - 2\frac{cx}{bd} - \frac{c^2}{d^2b} + \frac{x}{bd}\ln\left(\left(-b e^{dx+c} + \sqrt{a^2-b^2} - a\right)\left(-a + \sqrt{a^2-b^2}\right)^{-1}\right) + \frac{c}{d^2b}\ln\left(\left(-b e^{dx+c} + \sqrt{a^2-b^2} - a\right)\left(-a + \sqrt{a^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] -1/2*x^2/b-2/d/b*c*x-1/d^2/b*c^2+1/d/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)
/(-a+(a^2-b^2)^(1/2)))*x+1/d^2/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(
a^2-b^2)^(1/2)))*c+1/d/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(
1/2)))*x+1/d^2/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c
+1/d^2/b*dilog((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+1/d^
2/b*dilog((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))+2/d^2/b*c*l
n(exp(d*x+c))-1/d^2/b*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{2b} - \frac{1}{2}\int \frac{4\left(axe^{(dx+c)} + bx\right)}{b^2e^{(2dx+2c)} + 2abe^{(dx+c)} + b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/2*integrate(4*(a*x*e^(d*x + c) + b*x)/(b^2*e^(2*d*x + 2*c) +
2*a*b*e^(d*x + c) + b^2), x)
```

Fricas [B] time = 1.97892, size = 902, normalized size = 5.6

$$-\frac{d^2x^2 + 2c\log\left(2b\cosh(dx+c) + 2b\sinh(dx+c) + 2b\sqrt{\frac{a^2-b^2}{b^2}} + 2a\right) + 2c\log\left(2b\cosh(dx+c) + 2b\sinh(dx+c) - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x^2 + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a
^2 - b^2)/b^2) + 2*a) + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b
*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b)
 - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 2*dilog(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^
2) + b)/b + 1) - 2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1))/(b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh (c + dx)}{a + b \cosh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh (dx + c)}{b \cosh (dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

3.225 $\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

```
[Out] Log[a + b*Cosh[c + d*x]]/(b*d)
```

Rubi [A] time = 0.0318788, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 31}

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Log[a + b*Cosh[c + d*x]]/(b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\cosh(c+dx))}{bd}\end{aligned}$$

Mathematica [A] time = 0.0370468, size = 18, normalized size = 1.

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Log[a + b*Cosh[c + d*x]]/(b*d)
```

Maple [A] time = 0.004, size = 19, normalized size = 1.1

$$\frac{\ln(a+b\cosh(dx+c))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] ln(a+b*cosh(d*x+c))/b/d
```

Maxima [A] time = 1.00467, size = 24, normalized size = 1.33

$$\frac{\log(b\cosh(dx+c)+a)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*cosh(d*x + c) + a)/(b*d)
```

Fricas [B] time = 1.86296, size = 104, normalized size = 5.78

$$-\frac{dx-\log\left(\frac{2\,(b\cosh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x - log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))))/(b*d)
```

Sympy [A] time = 1.03296, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x \sinh(c)}{a} & \text{for}\ b = 0 \wedge d = 0 \\ \frac{x \sinh(c)}{a + b\cosh(c)} & \text{for}\ d = 0 \\ \frac{\cosh(c + dx)}{ad} & \text{for}\ b = 0 \\ \frac{\log\left(\frac{a}{b} + \cosh(c + dx)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Piecewise((x*sinh(c)/a, Eq(b, 0) & Eq(d, 0)), (x*sinh(c)/(a + b*cosh(c)), E
q(d, 0)), (cosh(c + d*x)/(a*d), Eq(b, 0)), (log(a/b + cosh(c + d*x))/(b*d),
 True))
```

Giac [A] time = 1.32136, size = 42, normalized size = 2.33

$$\frac{\log\left(\left|b\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 2\,a\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/(b*d)
```

3.226 $\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi [A] time = 0.0364959, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx = \int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx$$

Mathematica [A] time = 13.0255, size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])), x]
```

Maple [A] time = 0.127, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{x(a+b\cosh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\log(x)}{b} - \frac{1}{2}\int \frac{4\left(ae^{(dx+c)}+b\right)}{b^2xe^{(2\,dx+2\,c)}+2\,abxe^{(dx+c)}+b^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(x)/b - 1/2*integrate(4*(a*e^(d*x + c) + b)/(b^2*x*e^(2*d*x + 2*c) + 2*a
*b*x*e^(d*x + c) + b^2*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh(dx+c)}{bx\cosh(dx+c)+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)/(b*x*cosh(d*x + c) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{(b\cosh(dx+c)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/((b*cosh(d*x + c) + a)*x), x)
```

3.227 $\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)},x\right)$$

```
[Out] Unintegrable[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]), x]
```

Rubi [A] time = 0.0608897, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]

[Out] Defer[Int][(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Mathematica [A] time = 23.6409, size = 0, normalized size = 0.

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]

[Out] Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]), x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{x^{m} \left(\sinh \left(dx + c\right)\right)^{2}}{a + b \cosh \left(dx + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \sinh \left(dx + c\right)^{2}}{b \cosh \left(dx + c\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{m} \sinh \left(dx + c\right)^{2}}{b \cosh \left(dx + c\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \sinh^{2}\left(c + dx\right)}{a + b \cosh \left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**m*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sinh (dx+c)^2}{b \cosh (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.228 $\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=495

$$\frac{3x^2\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{3x^2\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}-\frac{6x\sqrt{a^2-b^2}\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\ldots$$

```
[Out] -(a*x^4)/(4*b^2) - (6*Cosh[c + d*x])/(b*d^4) - (3*x^2*Cosh[c + d*x])/(b*d^2
) + (Sqrt[a^2 - b^2]*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b
^2*d) - (Sqrt[a^2 - b^2]*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])]
)/(b^2*d) + (3*Sqrt[a^2 - b^2]*x^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 - b^2]))])/(b^2*d^2) - (3*Sqrt[a^2 - b^2]*x^2*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^2) - (6*Sqrt[a^2 - b^2]*x*PolyLog[3, -((
b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 - b^2]*x*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^3) + (6*Sqrt[a^2
 - b^2]*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^4) - (
6*Sqrt[a^2 - b^2]*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^
2*d^4) + (6*x*Sinh[c + d*x])/(b*d^3) + (x^3*Sinh[c + d*x])/(b*d)
```

Rubi [A] time = 0.838589, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5566, 30, 3296, 2638, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{3x^2\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}-\frac{6x\sqrt{a^2-b^2}\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -(a*x^4)/(4*b^2) - (6*Cosh[c + d*x])/(b*d^4) - (3*x^2*Cosh[c + d*x])/(b*d^2
) + (Sqrt[a^2 - b^2]*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b
^2*d) - (Sqrt[a^2 - b^2]*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])]
)/(b^2*d) + (3*Sqrt[a^2 - b^2]*x^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 - b^2]))])/(b^2*d^2) - (3*Sqrt[a^2 - b^2]*x^2*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^2) - (6*Sqrt[a^2 - b^2]*x*PolyLog[3, -((
b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 - b^2]*x*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^3) + (6*Sqrt[a^2
 - b^2]*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^4) - (
6*Sqrt[a^2 - b^2]*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^
```

```
2*d^4) + (6*x*Sinh[c + d*x])/(b*d^3) + (x^3*Sinh[c + d*x])/(b*d)
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^3\,dx}{b^2} + \frac{\int x^3\cosh(c+dx)\,dx}{b} + \frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax^4}{4b^2} + \frac{x^3\sinh(c+dx)}{bd} + \frac{\left(2\left(a^2-b^2\right)\right)\int \frac{e^{c+dx}x^3}{b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2} - \frac{3\int x^2\sinh(c+dx)\,dx}{bd}$$

$$= -\frac{ax^4}{4b^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{x^3\sinh(c+dx)}{bd} + \frac{\left(2\sqrt{a^2-b^2}\right)\int \frac{e^{c+dx}x^3}{2a-2\sqrt{a^2-b^2}+2be^{c+dx}}\,dx}{b} - \frac{\left(2\sqrt{a^2-\right.}{}$$

$$= -\frac{ax^4}{4b^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d} +$$

$$= -\frac{ax^4}{4b^2} - \frac{6\cosh(c+dx)}{bd^4} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}x^3\log}{b}$$

$$= -\frac{ax^4}{4b^2} - \frac{6\cosh(c+dx)}{bd^4} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}x^3\log}{b}$$

$$= -\frac{ax^4}{4b^2} - \frac{6\cosh(c+dx)}{bd^4} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}x^3\log}{b}$$

$$= -\frac{ax^4}{4b^2} - \frac{6\cosh(c+dx)}{bd^4} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}x^3\log}{b}$$

Mathematica [A] time = 1.39152, size = 386, normalized size = 0.78

$$4\sqrt{a^2-b^2}\left(3d^2x^2\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right) - 3d^2x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right) - 6dx\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right) + 6dx\text{PolyLog}\left(3,\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-(a*d^4*x^4) + 4*Sqrt[a^2 - b^2]*(d^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 3
*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 3*d^2*x^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 6*d*x*PolyLog[3, (b*E^(
c + d*x))/(-a + Sqrt[a^2 - b^2])] + 6*d*x*PolyLog[3, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 - b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])]
 - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 4*b*Cosh[d*x]*(
-3*(2 + d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*Sinh[c]) + 4*b*(d*x*(6 + d^2*x
```

```
^2)*Cosh[c] - 3*(2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/(4*b^2*d^4)
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int \frac{x^3 (\sinh (dx + c))^2}{a + b \cosh (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.21682, size = 2889, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d^4*x^4*cosh(d*x + c) + 2*b*d^3*x^3 + 6*b*d^2*x^2 + 12*b*d*x - 2*(b
*d^3*x^3 - 3*b*d^2*x^2 + 6*b*d*x - 6*b)*cosh(d*x + c)^2 - 2*(b*d^3*x^3 - 3*
b*d^2*x^2 + 6*b*d*x - 6*b)*sinh(d*x + c)^2 - 12*(b*d^2*x^2*cosh(d*x + c) +
b*d^2*x^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) +
 b)/b + 1) + 12*(b*d^2*x^2*cosh(d*x + c) + b*d^2*x^2*sinh(d*x + c))*sqrt((a
```

```
^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 4*(b*c^3*cosh(d*x +
 c) + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*
b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*(b*c^3*cosh(d*x + c)
 + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 4*((b*d^3*x^3 + b*c^3)*co
sh(d*x + c) + (b*d^3*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2) + b)/b) + 4*((b*d^3*x^3 + b*c^3)*cosh(d*x + c) + (b*d^3
*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)
+ b)/b) - 24*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*poly
log(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 - b^2)/b^2))/b) + 24*(b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*(b*d*x*cosh(d*
x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2))/b) - 24*(b*d*x*cosh(d*x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + (a*d^4*x^4 - 4*(b*d^3*x^3 -
3*b*d^2*x^2 + 6*b*d*x - 6*b)*cosh(d*x + c))*sinh(d*x + c) + 12*b)/(b^2*d^4*
cosh(d*x + c) + b^2*d^4*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sinh(dx+c)^2}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")

[Out] integrate(x^3*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.229 $\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=370

$$\frac{2x\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{2x\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}-\frac{2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{2\sqrt{a^2}}{}$$

```
[Out] -(a*x^3)/(3*b^2) - (2*x*Cosh[c + d*x])/(b*d^2) + (Sqrt[a^2 - b^2]*x^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (Sqrt[a^2 - b^2]*x^2*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (2*Sqrt[a^2 - b^2]
*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^2) - (2*Sqr
t[a^2 - b^2]*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d
^2) - (2*Sqrt[a^2 - b^2]*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])
)])/(b^2*d^3) + (2*Sqrt[a^2 - b^2]*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/(b^2*d^3) + (2*Sinh[c + d*x])/(b*d^3) + (x^2*Sinh[c + d*x])/(
b*d)
```

Rubi [A] time = 0.704118, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5566, 30, 3296, 2637, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{2x\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{2x\sqrt{a^2-b^2}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}-\frac{2\sqrt{a^2-b^2}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{2\sqrt{a^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -(a*x^3)/(3*b^2) - (2*x*Cosh[c + d*x])/(b*d^2) + (Sqrt[a^2 - b^2]*x^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (Sqrt[a^2 - b^2]*x^2*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (2*Sqrt[a^2 - b^2]
*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^2) - (2*Sqr
t[a^2 - b^2]*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d
^2) - (2*Sqrt[a^2 - b^2]*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])
)])/(b^2*d^3) + (2*Sqrt[a^2 - b^2]*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/(b^2*d^3) + (2*Sinh[c + d*x])/(b*d^3) + (x^2*Sinh[c + d*x])/(
b*d)
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^2\,dx}{b^2} + \frac{\int x^2\cosh(c+dx)\,dx}{b} + \frac{(a^2-b^2)\int \frac{x^2}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax^3}{3b^2} + \frac{x^2\sinh(c+dx)}{bd} + \frac{\left(2\left(a^2-b^2\right)\right)\int \frac{e^{c+dx}x^2}{b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2} - \frac{2\int x\sinh(c+dx)\,dx}{bd}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{x^2\sinh(c+dx)}{bd} + \frac{\left(2\sqrt{a^2-b^2}\right)\int \frac{e^{c+dx}x^2}{2a-2\sqrt{a^2-b^2}+2be^{c+dx}}\,dx}{b} - \frac{\left(2\sqrt{a^2-\right.}{}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}$$

Mathematica [A] time = 1.19684, size = 293, normalized size = 0.79

$$3\sqrt{a^2-b^2}\left(2dx\,\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right) - 2dx\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right) - 2\,\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right) + 2\,\text{PolyLog}\left(3,-\frac{be^{c+d}}{\sqrt{a^2-b^2}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-(a*d^3*x^3) + 3*Sqrt[a^2 - b^2]*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 2
*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 2*d*x*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-
a + Sqrt[a^2 - b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]
))]) + 3*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]) + 3*b*((2 + d
^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/(3*b^2*d^3)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{x^2 (\sinh (dx + c))^2}{a + b \cosh (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.13667, size = 2303, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*d^3*x^3*cosh(d*x + c) + 3*b*d^2*x^2 + 6*b*d*x - 3*(b*d^2*x^2 - 2*
b*d*x + 2*b)*cosh(d*x + c)^2 - 3*(b*d^2*x^2 - 2*b*d*x + 2*b)*sinh(d*x + c)^
2 - 12*(b*d*x*cosh(d*x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*di
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 12*(b*d*x*cosh(d*x + c) + b*d*x*sinh
(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 6
*(b*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 6*(b
*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cos
```

```
h(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 6*((b*d
^2*x^2 - b*c^2)*cosh(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x + c))*sqrt((a^
2 - b^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 6*((b*d^2*x^2 - b*c^2)*cosh
(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 - b^2)/b^2) + b)/b) + 12*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 12*(b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) +
 2*(a*d^3*x^3 - 3*(b*d^2*x^2 - 2*b*d*x + 2*b)*cosh(d*x + c))*sinh(d*x + c)
+ 6*b)/(b^2*d^3*cosh(d*x + c) + b^2*d^3*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sinh (dx + c)^2}{b \cosh (dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.230 $\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=244

$$\frac{\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{x\sqrt{a^2-b^2}\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^2d}-\frac{x\sqrt{a^2-b^2}\log\left(\right.}{b^2d}$$

```
[Out] -(a*x^2)/(2*b^2) - Cosh[c + d*x]/(b*d^2) + (Sqrt[a^2 - b^2]*x*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (Sqrt[a^2 - b^2]*x*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (Sqrt[a^2 - b^2]*PolyLog[2,
-((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^2) - (Sqrt[a^2 - b^2]*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^2) + (x*Sinh[c +
d*x])/(b*d)
```

Rubi [A] time = 0.417587, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5566, 30, 3296, 2638, 3320, 2264, 2190, 2279, 2391}

$$\frac{\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{\sqrt{a^2-b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{x\sqrt{a^2-b^2}\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^2d}-\frac{x\sqrt{a^2-b^2}\log\left(\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```


```
[Out] -(a*x^2)/(2*b^2) - Cosh[c + d*x]/(b*d^2) + (Sqrt[a^2 - b^2]*x*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (Sqrt[a^2 - b^2]*x*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (Sqrt[a^2 - b^2]*PolyLog[2,
-((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^2) - (Sqrt[a^2 - b^2]*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^2) + (x*Sinh[c +
d*x])/(b*d)
```


Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x\,dx}{b^2}+\frac{\int x\cosh(c+dx)\,dx}{b}+\frac{\left(a^2-b^2\right)\int \frac{x}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax^2}{2b^2}+\frac{x\sinh(c+dx)}{bd}+\frac{\left(2\left(a^2-b^2\right)\right)\int \frac{e^{c+dx}x}{b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2}-\frac{\int \sinh(c+dx)\,dx}{bd}$$

$$= -\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}+\frac{\left(2\sqrt{a^2-b^2}\right)\int \frac{e^{c+dx}x}{2a-2\sqrt{a^2-b^2}+2be^{c+dx}}\,dx}{b}-\frac{\left(2\sqrt{a^2-b^2}\right)}{}$$

$$= -\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{x\sin}{}$$

$$= -\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{x\sin}{}$$

$$= -\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{\sqrt{a^2}}{}$$

Mathematica [A] time = 0.949367, size = 187, normalized size = 0.77

$$\frac{2\sqrt{a^2-b^2}\left(\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right)-\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)+dx\left(\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)-\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)\right)\right)+a(c-dx)}{2b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (a*(c - d*x)*(c + d*x) - 2*b*Cosh[c + d*x] + 2*Sqrt[a^2 - b^2]*(d*x*(Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 - b^2])]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - Poly
```

```
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 2*b*d*x*Sinh[c + d*x])/
(2*b^2*d^2)
```

Maple [B] time = 0.056, size = 862, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] -1/2*a*x^2/b^2+1/2*(d*x-1)/b/d^2*exp(d*x+c)-1/2*(d*x+1)/b/d^2*exp(-d*x-c)+1
/b^2/d/(a^2-b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(
1/2)))*x*a^2-1/d/(a^2-b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(
a^2-b^2)^(1/2)))*x-1/b^2/d/(a^2-b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)
+a)/(a+(a^2-b^2)^(1/2)))*x*a^2+1/d/(a^2-b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^
2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x+1/b^2/d^2/(a^2-b^2)^(1/2)*ln((-b*exp(d*x
+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c*a^2-1/d^2/(a^2-b^2)^(1/2)*ln
((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c-1/b^2/d^2/(a^2-b
^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c*a^2+1/
d^2/(a^2-b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2))
)*c+1/b^2/d^2/(a^2-b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(
a^2-b^2)^(1/2)))*a^2-1/d^2/(a^2-b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2-b^2)^(
1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/b^2/d^2/(a^2-b^2)^(1/2)*dilog((b*exp(d*x+c)
+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*a^2+1/d^2/(a^2-b^2)^(1/2)*dilog((b
*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/b^2/d^2*c/(-a^2+b^2)^
(1/2)*arctan(1/2*(2*b*exp(d*x+c)+2*a)/(-a^2+b^2)^(1/2))*a^2+2/d^2*c/(-a^2+b
^2)^(1/2)*arctan(1/2*(2*b*exp(d*x+c)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11274, size = 1666, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^2*x^2*cosh(d*x + c) + b*d*x - (b*d*x - b)*cosh(d*x + c)^2 - (b*d*
x - b)*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 -
b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 2*(b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1)
- 2*(b*c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 2*(b*
c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*((b*d*x +
b*c)*cosh(d*x + c) + (b*d*x + b*c)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log
((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 - b^2)/b^2) + b)/b) + 2*((b*d*x + b*c)*cosh(d*x + c) + (b*d*x + b*
c)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (
a*d^2*x^2 - 2*(b*d*x - b)*cosh(d*x + c))*sinh(d*x + c) + b)/(b^2*d^2*cosh(d
*x + c) + b^2*d^2*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(dx+c)^2}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

3.231 $\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}-\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] -((a*x)/b^2) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*
x)/2])/Sqrt[a + b]])/(b^2*d) + Sinh[c + d*x]/(b*d)
```

Rubi [A] time = 0.123418, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2695, 2735, 2659, 205}

$$\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}-\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```


```
[Out] -((a*x)/b^2) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*
x)/2])/Sqrt[a + b]])/(b^2*d) + Sinh[c + d*x]/(b*d)
```


Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{\sinh(c+dx)}{bd}+\frac{\int \frac{-b-a\cosh(c+dx)}{a+b\cosh(c+dx)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}-\left(1-\frac{a^2}{b^2}\right)\int \frac{1}{a+b\cosh(c+dx)}\,dx\\
&= -\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}+\frac{\left(2i\left(1-\frac{a^2}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}\\
&= -\frac{ax}{b^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{\sinh(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] time = 0.180409, size = 69, normalized size = 0.95

$$\frac{2\sqrt{b^2-a^2}\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)-a(c+dx)+b\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x)) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqr
t[-a^2 + b^2]] + b*Sinh[c + d*x])/(b^2*d)
```

Maple [B] time = 0.009, size = 177, normalized size = 2.4

$$2\,\frac{a^{2}}{db^{2}\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh\,(1/2\,dx+c/2)}{\sqrt{(a+b)\,(a-b)}}\right)-2\,\frac{1}{d\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh\,(1/2\,dx+c/2)}{\sqrt{(a+b)\,(a-b)}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] 2/d/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*a^2-2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))-1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2
*c)+1)-1/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03002, size = 1085, normalized size = 14.86

$$\left[-\frac{2\,adx\cosh\,(dx+c)-b\cosh\,(dx+c)^{2}-b\sinh\,(dx+c)^{2}-2\,\sqrt{a^{2}-b^{2}}(\cosh\,(dx+c)+\sinh\,(dx+c))\log\left(\frac{b^{2}\cosh(dx+c)^{2}+}{}\right.}{2\left(b^{2}d\cosh\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 - 2*sq
rt(a^2 - b^2)*(cosh(d*x + c) + sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b^
2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c
) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
```

```
c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*
cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(a*d*x - b*cosh(d*x + c))*sinh(d
*x + c) + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c)), -1/2*(2*a*d*x*cos
h(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 + 4*sqrt(-a^2 + b^2)*(co
sh(d*x + c) + sinh(d*x + c))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c) + b*
sinh(d*x + c) + a)/(a^2 - b^2)) + 2*(a*d*x - b*cosh(d*x + c))*sinh(d*x + c)
 + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24338, size = 131, normalized size = 1.79

$$-\frac{(dx+c)a}{b^2d}+\frac{e^{(dx+c)}}{2\,bd}-\frac{e^{(-dx-c)}}{2\,bd}+\frac{2\left(a^2-b^2\right)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c)/(b*d) + 2*(
a^2 - b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b
^2*d)
```

3.232 $\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi [A] time = 0.0569911, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx=\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Mathematica [A] time = 108.57, size = 0, normalized size = 0.

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])), x]
```

Maple [A] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(\sinh (dx+c))^2}{x (a+b\cosh (dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2 \left(a^2 e^c - b^2 e^c\right) \int \frac{e^{(dx)}}{b^3 x e^{(2\,dx+2\,c)} + 2\,ab^2 x e^{(dx+c)} + b^3 x}\,dx + \frac{\mathrm{Ei}\,(-dx)\,e^{(-c)}}{2\,b} + \frac{\mathrm{Ei}\,(dx)\,e^c}{2\,b} - \frac{a\log (x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(a^2*e^c - b^2*e^c)*integrate(e^(d*x)/(b^3*x*e^(2*d*x + 2*c) + 2*a*b^2*x*
e^(d*x + c) + b^3*x), x) + 1/2*Ei(-d*x)*e^(-c)/b + 1/2*Ei(d*x)*e^c/b - a*lo
g(x)/b^2
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sinh (dx+c)^2}{bx\cosh (dx+c)+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)^2/(b*x*cosh(d*x + c) + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (dx+c)^2}{(b\cosh (dx+c)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/((b*cosh(d*x + c) + a)*x), x)
```

3.233 $\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)},x\right)$$

```
[Out] Unintegrable[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]), x]
```

Rubi [A] time = 0.0637631, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]

[Out] Defer[Int][(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx=\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Mathematica [A] time = 31.0466, size = 0, normalized size = 0.

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]

[Out] Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]), x]
```

Maple [A] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{x^m (\sinh (dx + c))^3}{a + b \cosh (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sinh (dx + c)^3}{b \cosh (dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \sinh (dx + c)^3}{b \cosh (dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sinh^3 (c + dx)}{a + b \cosh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**m*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \sinh (dx+c)^3}{b \cosh (dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.234 $\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=586

$$\frac{3x^2\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{3x^2\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{6x\left(a^2-b^2\right)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-$$

```
[Out] (3*x)/(8*b*d^3) + x^3/(4*b*d) - ((a^2 - b^2)*x^4)/(4*b^3) - (6*a*x*Cosh[c +
 d*x])/(b^2*d^3) - (a*x^3*Cosh[c + d*x])/(b^2*d) + ((a^2 - b^2)*x^3*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b^2)*x^3*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + (3*(a^2 - b^2)*x^2*PolyL
og[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + (3*(a^2 - b^2)
*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) - (6*(
a^2 - b^2)*x*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^3
) - (6*(a^2 - b^2)*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/
(b^3*d^3) + (6*(a^2 - b^2)*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2
]))])/(b^3*d^4) + (6*(a^2 - b^2)*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2
 - b^2]))])/(b^3*d^4) + (6*a*Sinh[c + d*x])/(b^2*d^4) + (3*a*x^2*Sinh[c + d
*x])/(b^2*d^2) - (3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*x^2*Cosh[c
+ d*x]*Sinh[c + d*x])/(4*b*d^2) + (3*x*Sinh[c + d*x]^2)/(4*b*d^3) + (x^3*Si
nh[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.686392, antiderivative size = 586, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5566, 3296, 2637, 5372, 3311, 30, 2635, 8, 5562, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{3x^2\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{6x\left(a^2-b^2\right)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```


```
[Out] (3*x)/(8*b*d^3) + x^3/(4*b*d) - ((a^2 - b^2)*x^4)/(4*b^3) - (6*a*x*Cosh[c +
 d*x])/(b^2*d^3) - (a*x^3*Cosh[c + d*x])/(b^2*d) + ((a^2 - b^2)*x^3*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b^2)*x^3*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + (3*(a^2 - b^2)*x^2*PolyL
og[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + (3*(a^2 - b^2)
*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) - (6*(
```

```
a^2 - b^2)*x*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^3
) - (6*(a^2 - b^2)*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/
(b^3*d^3) + (6*(a^2 - b^2)*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2
]))])/(b^3*d^4) + (6*(a^2 - b^2)*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2
 - b^2]))])/(b^3*d^4) + (6*a*Sinh[c + d*x])/(b^2*d^4) + (3*a*x^2*Sinh[c + d
*x])/(b^2*d^2) - (3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*x^2*Cosh[c
+ d*x]*Sinh[c + d*x])/(4*b*d^2) + (3*x*Sinh[c + d*x]^2)/(4*b*d^3) + (x^3*Si
nh[c + d*x]^2)/(2*b*d)
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^3\sinh(c+dx)\,dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)\,dx}{b}+\frac{\left(a^2-b^2\right)\int\frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{x^3\sinh^2(c+dx)}{2bd}+\frac{\left(a^2-b^2\right)\int\frac{e^{c+dx}x^3}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx}{b^2}+\frac{\left(a^2\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{\left(a^2-b^2\right)x^3\log\left(1+\right.}{b^3d}$$

$$= \frac{x^3}{4bd}-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right.}{b^3d}$$

$$= \frac{3x}{8bd^3}+\frac{x^3}{4bd}-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\log\left(1+\right.}{b^3d}$$

$$= \frac{3x}{8bd^3}+\frac{x^3}{4bd}-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\log\left(1+\right.}{b^3d}$$

$$= \frac{3x}{8bd^3}+\frac{x^3}{4bd}-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\log\left(1+\right.}{b^3d}$$

Mathematica [A] time = 11.8514, size = 1082, normalized size = 1.85

$$-\frac{(b-a)(a+b)\cosh\left(\frac{c}{2}\right)\mathbf{sech}(c)\sinh\left(\frac{c}{2}\right)x^4}{2b^3}-\frac{a\cosh(dx)\left(d^3\cosh(c)x^3-3d^2\sinh(c)x^2+6d\cosh(c)x-6\sinh(c)\right)}{b^2d^4}+\frac{\text{co}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -((-a + b)*(a + b)*x^4*Cosh[c/2]*Sech[c]*Sinh[c/2])/(2*b^3) - (a*Cosh[d*x]*
(6*d*x*Cosh[c] + d^3*x^3*Cosh[c] - 6*Sinh[c] - 3*d^2*x^2*Sinh[c]))/(b^2*d^4
) + (Cosh[2*d*x]*(6*d*x*Cosh[2*c] + 4*d^3*x^3*Cosh[2*c] - 3*Sinh[2*c] - 6*d
^2*x^2*Sinh[2*c]))/(16*b*d^4) - (a*(-6*Cosh[c] - 3*d^2*x^2*Cosh[c] + 6*d*x*
Sinh[c] + d^3*x^3*Sinh[c])*Sinh[d*x])/(b^2*d^4) + ((-3*Cosh[2*c] - 6*d^2*x^
2*Cosh[2*c] + 6*d*x*Sinh[2*c] + 4*d^3*x^3*Sinh[2*c])*Sinh[2*d*x])/(16*b*d^4
) + ((-a^2 + b^2)*(-x^4 + (2*b^2*(d^3*x^3*Log[1 + ((a - Sqrt[a^2 - b^2])*(C
osh[c + d*x] - Sinh[c + d*x]))/b] - 3*d^2*x^2*PolyLog[2, ((-a + Sqrt[a^2 -
b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 6*d*x*PolyLog[3, ((-a + Sqrt[a^
2 - b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 6*PolyLog[4, ((-a + Sqrt[a^
2 - b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b])*(1 + Cosh[2*c] + Sinh[2*c]))
/(Sqrt[a^2 - b^2]*(-a + Sqrt[a^2 - b^2])*d^4) + (2*b^2*(d^3*x^3*Log[1 + ((a
 + Sqrt[a^2 - b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 3*d^2*x^2*PolyLog
[2, ((a + Sqrt[a^2 - b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 6*d*x*Pol
yLog[3, ((a + Sqrt[a^2 - b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 6*Pol
yLog[4, ((a + Sqrt[a^2 - b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b])*(1 + C
osh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2])*d^4) + (2*a*(
d^3*x^3*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 - b^2])]
+ 3*d^2*x^2*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 -
 b^2])] - 6*d*x*PolyLog[3, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a
^2 - b^2])] + 6*PolyLog[4, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a
^2 - b^2])])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]*d^4) - (2*a*(d^3
*x^3*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 - b^2])] + 3
*d^2*x^2*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 - b
^2]))] - 6*d*x*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a
^2 - b^2]))] + 6*PolyLog[4, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt
[a^2 - b^2]))])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]*d^4))*(1 - Ta
nh[c]))/(4*b^3)
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int \frac{x^3 (\sinh (dx+c))^3}{a+b\cosh (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(8\left(a^2d^4e^{(2c)}-b^2d^4e^{(2c)}\right)x^4+\left(4\,b^2d^3x^3e^{(4c)}-6\,b^2d^2x^2e^{(4c)}+6\,b^2dxe^{(4c)}-3\,b^2e^{(4c)}\right)e^{(2\,dx)}-16\left(abd^3x^3e^{(3c)}-3\,abd^2x^2e^{(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/32*(8*(a^2*d^4*e^(2*c) - b^2*d^4*e^(2*c))*x^4 + (4*b^2*d^3*x^3*e^(4*c) -
6*b^2*d^2*x^2*e^(4*c) + 6*b^2*d*x*e^(4*c) - 3*b^2*e^(4*c))*e^(2*d*x) - 16*(
a*b*d^3*x^3*e^(3*c) - 3*a*b*d^2*x^2*e^(3*c) + 6*a*b*d*x*e^(3*c) - 6*a*b*e^(
3*c))*e^(d*x) - 16*(a*b*d^3*x^3*e^c + 3*a*b*d^2*x^2*e^c + 6*a*b*d*x*e^c + 6
*a*b*e^c)*e^(-d*x) + (4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + 6*b^2*d*x + 3*b^2)*e^
(-2*d*x))*e^(-2*c)/(b^3*d^4) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x^3*
e^(d*x) + (a^2*b - b^3)*x^3)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b
^4), x)
```

Fricas [C] time = 2.47236, size = 4741, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^
2*d*x - 3*b^2)*cosh(d*x + c)^4 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x
```

- 3*b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 16*(a*b*d^3*x^3 - 3*a*b*d^2*x^2 + 6
*a*b*d*x - 6*a*b)*cosh(d*x + c)^3 - 4*(4*a*b*d^3*x^3 - 12*a*b*d^2*x^2 + 24*
a*b*d*x - 24*a*b - (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^4*x^4 - 2*(a^2 - b^2)*c^4)*co
sh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^4*x^4 - 8*(a^2 - b^2)*c^4 - 3*(4*b^2*d^3
*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)*cosh(d*x + c)^2 + 24*(a*b*d^3*x^3
 - 3*a*b*d^2*x^2 + 6*a*b*d*x - 6*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*b^
2 - 16*(a*b*d^3*x^3 + 3*a*b*d^2*x^2 + 6*a*b*d*x + 6*a*b)*cosh(d*x + c) + 96
*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d^2*x^2*cosh(d*x + c)
*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)
/b^2) + b)/b + 1) + 96*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)
*d^2*x^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*sinh(d*x + c)^2)
*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 32*((a^2 - b^2)*c^3*cosh(d*x + c)
^2 + 2*(a^2 - b^2)*c^3*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^3*sinh(d
*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)
/b^2) + 2*a) - 32*((a^2 - b^2)*c^3*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^3*cosh
(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^3*sinh(d*x + c)^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 32*(((a^2 -
b^2)*d^3*x^3 + (a^2 - b^2)*c^3)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^3*x^3 +
(a^2 - b^2)*c^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^3*x^3 + (a^2
- b^2)*c^3)*sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 32*(((a^2 -
b^2)*d^3*x^3 + (a^2 - b^2)*c^3)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^3*x^3 +
(a^2 - b^2)*c^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^3*x^3 + (a^2
- b^2)*c^3)*sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 192*((a^2 -
b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*sinh(d*x + c)^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 192*((a^2 - b^2)
*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*
sinh(d*x + c)^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 192*((a^2 - b^2)*d*x
*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*d*x*sinh(d*x + c)^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 192*((a^2 -
b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x + c)*sinh(d*x + c) +
(a^2 - b^2)*d*x*sinh(d*x + c)^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 4*(4
*a*b*d^3*x^3 + 12*a*b*d^2*x^2 + 24*a*b*d*x - (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2
 + 6*b^2*d*x - 3*b^2)*cosh(d*x + c)^3 + 12*(a*b*d^3*x^3 - 3*a*b*d^2*x^2 + 6
*a*b*d*x - 6*a*b)*cosh(d*x + c)^2 + 24*a*b + 4*((a^2 - b^2)*d^4*x^4 - 2*(a^
2 - b^2)*c^4)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^4*cosh(d*x + c)^2 + 2*b^
3*d^4*cosh(d*x + c)*sinh(d*x + c) + b^3*d^4*sinh(d*x + c)^2)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sinh (dx+c)^3}{b \cosh (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^3*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.235 $\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=432

$$\frac{2x\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{2x\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{2\left(a^2-b^2\right)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{2\left(a\right.}{}$$

```
[Out] x^2/(4*b*d) - ((a^2 - b^2)*x^3)/(3*b^3) - (2*a*Cosh[c + d*x])/(b^2*d^3) - (
a*x^2*Cosh[c + d*x])/(b^2*d) + ((a^2 - b^2)*x^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b^2)*x^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2])])/(b^3*d) + (2*(a^2 - b^2)*x*PolyLog[2, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + (2*(a^2 - b^2)*x*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) - (2*(a^2 - b^2)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^3) - (2*(a^2 - b^2)*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^3) + (2*a*x*Sinh[c +
d*x])/(b^2*d^2) - (x*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d^2) + Sinh[c + d*x]
^2/(4*b*d^3) + (x^2*Sinh[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.563287, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5566, 3296, 2638, 5372, 3310, 30, 5562, 2190, 2531, 2282, 6589}

$$\frac{2x\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{2x\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{2\left(a^2-b^2\right)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{2\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] x^2/(4*b*d) - ((a^2 - b^2)*x^3)/(3*b^3) - (2*a*Cosh[c + d*x])/(b^2*d^3) - (
a*x^2*Cosh[c + d*x])/(b^2*d) + ((a^2 - b^2)*x^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b^2)*x^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2])])/(b^3*d) + (2*(a^2 - b^2)*x*PolyLog[2, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + (2*(a^2 - b^2)*x*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) - (2*(a^2 - b^2)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^3) - (2*(a^2 - b^2)*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^3) + (2*a*x*Sinh[c +
d*x])/(b^2*d^2) - (x*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d^2) + Sinh[c + d*x]
^2/(4*b*d^3) + (x^2*Sinh[c + d*x]^2)/(2*b*d)
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
```

```
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^2\sinh(c+dx)\,dx}{b^2} + \frac{\int x^2\cosh(c+dx)\sinh(c+dx)\,dx}{b} + \frac{\left(a^2-b^2\right)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{x^2\sinh^2(c+dx)}{2bd} + \frac{\left(a^2-b^2\right)\int \frac{e^{c+dx}x^2}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx}{b^2} + \frac{\left(a^2 \cdots\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{\left(a^2-b^2\right)x^2\log\left(1+\cdots\right.}{b^3d}$$

$$= \frac{x^2}{4bd} - \frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{2a\cosh(c+dx)}{b^2d^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}\cdots$$

$$= \frac{x^2}{4bd} - \frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{2a\cosh(c+dx)}{b^2d^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}\cdots$$

$$= \frac{x^2}{4bd} - \frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{2a\cosh(c+dx)}{b^2d^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}\cdots$$

Mathematica [A] time = 8.91343, size = 831, normalized size = 1.92

$$8\left(a^2-b^2\right)\tanh(c)x^3 - \frac{24ab\cosh(dx)\left(\left(d^2x^2+2\right)\cosh(c)-2dx\sinh(c)\right)}{d^3} + \frac{3b^2\cosh(2dx)\left(\left(2d^2x^2+1\right)\cosh(2c)-2dx\sinh(2c)\right)}{d^3} - \frac{24ab\left(\left(d^2x^2+2\right)\sinh(\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] ((-24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^2*C
osh[2*d*x]*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (24*a*b*(-2
*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*Cosh[
2*c] + (1 + 2*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^3 + 4*(-a^2 + b^2)*(-2*x^3
 + (3*b^2*(d^2*x^2*Log[1 + ((a - Sqrt[a^2 - b^2])*(Cosh[c + d*x] - Sinh[c +
 d*x]))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 - b^2])*(Cosh[c + d*x] - Sinh
[c + d*x]))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 - b^2])*(Cosh[c + d*x] - Sinh
[c + d*x]))/b])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]*(-a + Sqrt[a^
2 - b^2])*d^3) + (3*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 - b^2])*(Cosh[c + d
*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((a + Sqrt[a^2 - b^2])*(-Cosh[c
 + d*x] + Sinh[c + d*x]))/b] - 2*PolyLog[3, ((a + Sqrt[a^2 - b^2])*(-Cosh[c
```

```
 + d*x] + Sinh[c + d*x]))/b])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]
*(a + Sqrt[a^2 - b^2])*d^3) + (3*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sin
h[c + d*x]))/(a - Sqrt[a^2 - b^2])] + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x] +
Sinh[c + d*x]))/(-a + Sqrt[a^2 - b^2])] - 2*PolyLog[3, (b*(Cosh[c + d*x] +
Sinh[c + d*x]))/(-a + Sqrt[a^2 - b^2])])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt
[a^2 - b^2]*d^3) - (3*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x])
)/(a + Sqrt[a^2 - b^2])] + 2*d*x*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c +
d*x]))/(a + Sqrt[a^2 - b^2]))] - 2*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c
+ d*x]))/(a + Sqrt[a^2 - b^2]))])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 -
b^2]*d^3))*(1 - Tanh[c]) + 8*(a^2 - b^2)*x^3*Tanh[c])/(24*b^3)
```

Maple [F] time = 0.17, size = 0, normalized size = 0.

$$\int \frac{x^2 (\sinh (dx + c))^3}{a + b \cosh (dx + c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(16 \left(a^2 d^3 e^{(2\,c)} - b^2 d^3 e^{(2\,c)}\right) x^3 + 3 \left(2\,b^2 d^2 x^2 e^{(4\,c)} - 2\,b^2 d x e^{(4\,c)} + b^2 e^{(4\,c)}\right) e^{(2\,dx)} - 24 \left(abd^2 x^2 e^{(3\,c)} - 2\,abdxe^{(3\,c)} + 2\,abe^{(3\,c)}\right) e^{(dx}}{48\,b^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(a^2*d^3*e^(2*c) - b^2*d^3*e^(2*c))*x^3 + 3*(2*b^2*d^2*x^2*e^(4*c)
 - 2*b^2*d*x*e^(4*c) + b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*x^2*e^(3*c) - 2
*a*b*d*x*e^(3*c) + 2*a*b*e^(3*c))*e^(d*x) - 24*(a*b*d^2*x^2*e^c + 2*a*b*d*x
*e^c + 2*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*x^2 + 2*b^2*d*x + b^2)*e^(-2*d*x)
)*e^(-2*c)/(b^3*d^3) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x^2*e^(d*x)
+ (a^2*b - b^3)*x^2)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4), x)
```

Fricas [C] time = 2.24674, size = 3811, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^2*d^2*x^2 + 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c)^4 +
 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 24*(a*b*
d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*x^2 - 4*a*b*d*
x + 4*a*b - (2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c))*sinh(d*x + c)^
3 - 16*((a^2 - b^2)*d^3*x^3 + 2*(a^2 - b^2)*c^3)*cosh(d*x + c)^2 - 2*(8*(a^
2 - b^2)*d^3*x^3 + 16*(a^2 - b^2)*c^3 - 9*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)
*cosh(d*x + c)^2 + 36*(a*b*d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c))*sinh
(d*x + c)^2 + 3*b^2 - 24*(a*b*d^2*x^2 + 2*a*b*d*x + 2*a*b)*cosh(d*x + c) +
96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x + c)*sinh(
d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)
/b + 1) + 96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x
+ c)*sinh(d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)
/b^2) + b)/b + 1) + 48*((a^2 - b^2)*c^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^2
*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^2*sinh(d*x + c)^2)*log(2*b*cos
h(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 48*((a^
2 - b^2)*c^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^2*cosh(d*x + c)*sinh(d*x + c
) + (a^2 - b^2)*c^2*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 48*(((a^2 - b^2)*d^2*x^2 - (a^2 -
b^2)*c^2)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*cosh(
d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*sinh(d*x +
 c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 48*(((a^2 - b^2)*d^2*x^2 - (a^2 -
b^2)*c^2)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*cosh(
d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*sinh(d*x +
 c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 96*((a^2 - b^2)*cosh(d*x + c)^2 +
2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2)*po
lylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 - b^2)/b^2))/b) - 96*((a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^
2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2)*polylog
(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 - b^2)/b^2))/b) - 4*(6*a*b*d^2*x^2 + 12*a*b*d*x - 3*(2*b^2*d^2
*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c)^3 + 18*(a*b*d^2*x^2 - 2*a*b*d*x + 2*a
*b)*cosh(d*x + c)^2 + 12*a*b + 8*((a^2 - b^2)*d^3*x^3 + 2*(a^2 - b^2)*c^3)*
```

```
cosh(d*x + c))*sinh(d*x + c))/(b^3*d^3*cosh(d*x + c)^2 + 2*b^3*d^3*cosh(d*x
 + c)*sinh(d*x + c) + b^3*d^3*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sinh (dx+c)^3}{b \cosh (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.236 $\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=288

$$\frac{(a^2-b^2)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{(a^2-b^2)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}+\frac{x(a^2-b^2)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^3d}+\frac{x(a^2-b^2)\,\text{lo}}{}$$

```
[Out] x/(4*b*d) - ((a^2 - b^2)*x^2)/(2*b^3) - (a*x*Cosh[c + d*x])/(b^2*d) + ((a^2
 - b^2)*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 -
 b^2)*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b
^2)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + ((a^2
 - b^2)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) + (
a*Sinh[c + d*x])/(b^2*d^2) - (Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2) + (x*S
inh[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.336127, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {5566, 3296, 2637, 5372, 2635, 8, 5562, 2190, 2279, 2391}

$$\frac{(a^2-b^2)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{(a^2-b^2)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}+\frac{x(a^2-b^2)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^3d}+\frac{x(a^2-b^2)\,\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```


```
[Out] x/(4*b*d) - ((a^2 - b^2)*x^2)/(2*b^3) - (a*x*Cosh[c + d*x])/(b^2*d) + ((a^2
 - b^2)*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 -
 b^2)*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b
^2)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + ((a^2
 - b^2)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) + (
a*Sinh[c + d*x])/(b^2*d^2) - (Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2) + (x*S
inh[c + d*x]^2)/(2*b*d)
```


Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
```

```
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x\sinh(c+dx)\,dx}{b^2} + \frac{\int x\cosh(c+dx)\sinh(c+dx)\,dx}{b} + \frac{\left(a^2-b^2\right)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{\left(a^2-b^2\right)x^2}{2b^3} - \frac{ax\cosh(c+dx)}{b^2d} + \frac{x\sinh^2(c+dx)}{2bd} + \frac{\left(a^2-b^2\right)\int \frac{e^{c+dx}x}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx}{b^2} + \frac{\left(a^2-\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)x^2}{2b^3} - \frac{ax\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{\left(a^2-b^2\right)x\log\left(1+\frac{be}{a+\sqrt{}}\right.}{b^3d}$$

$$= \frac{x}{4bd} - \frac{\left(a^2-b^2\right)x^2}{2b^3} - \frac{ax\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{\left(a^2-b^2\right)x\log\left(1\right.}{b^3d}$$

$$= \frac{x}{4bd} - \frac{\left(a^2-b^2\right)x^2}{2b^3} - \frac{ax\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{\left(a^2-b^2\right)x\log\left(1\right.}{b^3d}$$

Mathematica [A] time = 2.92643, size = 414, normalized size = 1.44

$$\frac{4\left(a^2-b^2\right)\left(2\text{PolyLog}\left(2,\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2-b^2}-a}\right)+2\text{PolyLog}\left(2,-\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2-b^2}+a}\right)+2(c+dx)\log\left(\frac{b(\sinh(c+dx)+\text{c}}{a-\sqrt{a^2-}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-8*a*b*d*x*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*(2*
c*(c + d*x) - (c + d*x)^2 + (4*a*Sqrt[-(a^2 - b^2)^2]*c*ArcTan[(a + b*Cosh[
c + d*x] + b*Sinh[c + d*x])/Sqrt[-a^2 + b^2]])/(a^2 - b^2)^(3/2) + (4*a*Sqr
t[-(a^2 - b^2)^2]*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^
2 - b^2]])/(-a^2 + b^2)^(3/2) - 2*c*Log[2*(a + b*Cosh[c + d*x])*(Cosh[c + d
*x] + Sinh[c + d*x])] + 2*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*
x]))/(a - Sqrt[a^2 - b^2])] + 2*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[
c + d*x]))/(a + Sqrt[a^2 - b^2])] + 2*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c
 + d*x]))/(-a + Sqrt[a^2 - b^2])] + 2*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh
[c + d*x]))/(a + Sqrt[a^2 - b^2]))]) + 8*a*b*Sinh[c + d*x] - b^2*Sinh[2*(c
+ d*x)])/(8*b^3*d^2)
```

Maple [B] time = 0.066, size = 860, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] -1/d^2/b*dilog((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/d^
2/b*dilog((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))+1/d^2/b*c^2
-1/d^2/b^3*a^2*c^2+1/d^2/b^3*a^2*dilog((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-
a+(a^2-b^2)^(1/2)))+1/d^2/b^3*a^2*dilog((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a
+(a^2-b^2)^(1/2)))+1/16*(2*d*x-1)/b/d^2*exp(2*d*x+2*c)+1/16*(2*d*x+1)/b/d^2
*exp(-2*d*x-2*c)+1/2*x^2/b+2/d/b*c*x-1/d/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2
)-a)/(-a+(a^2-b^2)^(1/2)))*x-1/d^2/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(
-a+(a^2-b^2)^(1/2)))*c-1/d/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^
2)^(1/2)))*x-1/d^2/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)
))*c-2/d^2/b*c*ln(exp(d*x+c))+1/d^2/b*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)+
b)-1/2*x^2/b^3*a^2-1/2*a*(d*x-1)/b^2/d^2*exp(d*x+c)-1/2*a*(d*x+1)/b^2/d^2*e
xp(-d*x-c)-2/d/b^3*a^2*c*x+1/d/b^3*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a
+(a^2-b^2)^(1/2)))*a^2*x+1/d^2/b^3*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a
+(a^2-b^2)^(1/2)))*a^2*c+1/d/b^3*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^
2-b^2)^(1/2)))*a^2*x+1/d^2/b^3*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-
b^2)^(1/2)))*a^2*c+2/d^2/b^3*c*a^2*ln(exp(d*x+c))-1/d^2/b^3*c*a^2*ln(b*exp(
2*d*x+2*c)+2*a*exp(d*x+c)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(8\left(a^2d^2e^{(2c)} - b^2d^2e^{(2c)}\right)x^2 + \left(2b^2dxe^{(4c)} - b^2e^{(4c)}\right)e^{(2dx)} - 8\left(abdxe^{(3c)} - abe^{(3c)}\right)e^{(dx)} - 8\left(abdxe^{c} + abe^{c}\right)e^{(-dx)} + \left(2b^2d\right.}{16b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(8*(a^2*d^2*e^(2*c) - b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*x*e^(4*c) - b^2*
e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) - 8*(a*b*d*x
*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2)
 - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x*e^(d*x) + (a^2*b - b^3)*x)/(b^
4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4), x)
```

Fricas [B] time = 2.11964, size = 2828, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*b^2*d*x - b^2)*cosh(d*x + c)^4 + (2*b^2*d*x - b^2)*sinh(d*x + c)^4
 + 2*b^2*d*x - 8*(a*b*d*x - a*b)*cosh(d*x + c)^3 - 4*(2*a*b*d*x - 2*a*b - (
2*b^2*d*x - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^2*x^2 -
2*(a^2 - b^2)*c^2)*cosh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^2*x^2 - 8*(a^2 - b^
2)*c^2 - 3*(2*b^2*d*x - b^2)*cosh(d*x + c)^2 + 12*(a*b*d*x - a*b)*cosh(d*x
+ c))*sinh(d*x + c)^2 + b^2 - 8*(a*b*d*x + a*b)*cosh(d*x + c) + 16*((a^2 -
b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 16*((a^2 - b^
2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2
)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 16*((a^2 - b^2)
*c*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*c*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt
((a^2 - b^2)/b^2) + 2*a) - 16*((a^2 - b^2)*c*cosh(d*x + c)^2 + 2*(a^2 - b^2
)*c*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c*sinh(d*x + c)^2)*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 16*(((
a^2 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d*x + (a^2
 - b^2)*c)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d*x + (a^2 - b^2)*c)*
```

```
sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 16*(((a^2 - b^2)*d*x + (
a^2 - b^2)*c)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*
x + c)*sinh(d*x + c) + ((a^2 - b^2)*d*x + (a^2 - b^2)*c)*sinh(d*x + c)^2)*l
og((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 - b^2)/b^2) + b)/b) - 4*(2*a*b*d*x - (2*b^2*d*x - b^2)*cosh(d*x
+ c)^3 + 6*(a*b*d*x - a*b)*cosh(d*x + c)^2 + 2*a*b + 4*((a^2 - b^2)*d^2*x^2
 - 2*(a^2 - b^2)*c^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^2*cosh(d*x + c)^
2 + 2*b^3*d^2*cosh(d*x + c)*sinh(d*x + c) + b^3*d^2*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

3.237 $\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{(a^2-b^2)\log(a+b\cosh(c+dx))}{b^3d}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}$$

```
[Out] -((a*Cosh[c + d*x])/(b^2*d)) + Cosh[c + d*x]^2/(2*b*d) + ((a^2 - b^2)*Log[a
 + b*Cosh[c + d*x]])/(b^3*d)
```

Rubi [A] time = 0.0727722, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{(a^2-b^2)\log(a+b\cosh(c+dx))}{b^3d}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -((a*Cosh[c + d*x])/(b^2*d)) + Cosh[c + d*x]^2/(2*b*d) + ((a^2 - b^2)*Log[a
 + b*Cosh[c + d*x]])/(b^3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx, x, b\cosh(c+dx)\right)}{b^3 d}$$
$$= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)dx, x, b\cosh(c+dx)\right)}{b^3 d}$$
$$= -\frac{a\cosh(c+dx)}{b^2 d} + \frac{\cosh^2(c+dx)}{2bd} + \frac{\left(a^2-b^2\right)\log(a+b\cosh(c+dx))}{b^3 d}$$

Mathematica [A] time = 0.1017, size = 55, normalized size = 0.9

$$\frac{4\left(a^2-b^2\right)\log(a+b\cosh(c+dx)) - 4ab\cosh(c+dx) + b^2\cosh(2(c+dx))}{4b^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-4*a*b*Cosh[c + d*x] + b^2*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*Log[a + b*Cos
h[c + d*x]])/(4*b^3*d)
```

Maple [B] time = 0.026, size = 415, normalized size = 6.8

$$\frac{a^3}{db^3(a-b)}\ln\left(a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 b-a-b\right)-\frac{a^2}{db^2(a-b)}\ln\left(a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\left(\tanh\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] 1/d/b^3/(a-b)*ln(a*tanh(1/2*d*x+1/2*c)^2-tanh(1/2*d*x+1/2*c)^2*b-a-b)*a^3-1
/d/b^2/(a-b)*ln(a*tanh(1/2*d*x+1/2*c)^2-tanh(1/2*d*x+1/2*c)^2*b-a-b)*a^2-1/
d/b/(a-b)*ln(a*tanh(1/2*d*x+1/2*c)^2-tanh(1/2*d*x+1/2*c)^2*b-a-b)*a+1/d/(a-
b)*ln(a*tanh(1/2*d*x+1/2*c)^2-tanh(1/2*d*x+1/2*c)^2*b-a-b)+1/2/d/b/(tanh(1/
2*d*x+1/2*c)+1)^2-1/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a-1/2/d/b/(tanh(1/2*d*x+1
/2*c)+1)-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2+1/d/b*ln(tanh(1/2*d*x+1/2*c)
+1)+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a+1/2
/d/b/(tanh(1/2*d*x+1/2*c)-1)-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2+1/d/b*ln
```

```
(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.05682, size = 176, normalized size = 2.89

$$-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d}+\frac{\left(a^2-b^2\right)(dx+c)}{b^3d}-\frac{4\,ae^{(-dx-c)}-be^{(-2\,dx-2\,c)}}{8\,b^2d}+\frac{\left(a^2-b^2\right)\log\left(2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}+b\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + (a^2 - b^2)*(d*x + c)
/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b*e^(-2*d*x - 2*c))/(b^2*d) + (a^2 - b^2
)*log(2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) + b)/(b^3*d)
```

Fricas [B] time = 1.93135, size = 848, normalized size = 13.9

$$\frac{b^2\cosh(dx+c)^4+b^2\sinh(dx+c)^4-8\left(a^2-b^2\right)dx\cosh(dx+c)^2-4\,ab\cosh(dx+c)^3+4\left(b^2\cosh(dx+c)-ab\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 - 8*(a^2 - b^2)*d*x*cosh(d*x
 + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)
^3 - 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 - 4*(a^2 - b^2)*d*x - 6
*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + b^2 + 8*((a^2 - b^2)*cosh(d*x + c)^2
+ 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2)*
log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b^2*cosh(
d*x + c)^3 - 4*(a^2 - b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2 - a*b)
*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh(d*x + c
) + b^3*d*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17586, size = 123, normalized size = 2.02

$$\frac{\left(a^2-b^2\right)\log\left(\left|b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3d}+\frac{bd\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\,ad\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{8\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^2 - b^2)*log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/(b^3*d) + 1/8*(b
*d*(e^(d*x + c) + e^(-d*x - c))^2 - 4*a*d*(e^(d*x + c) + e^(-d*x - c)))/(b^
2*d^2)
```

3.238 $\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi [A] time = 0.0550612, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Mathematica [F] time = 180., size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.148, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^3}{x(a+b\cosh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\mathrm{Ei}(2\,dx)\,e^{(2\,c)}}{4\,b}+\frac{a\mathrm{Ei}(-dx)\,e^{(-c)}}{2\,b^2}-\frac{\mathrm{Ei}(-2\,dx)\,e^{(-2\,c)}}{4\,b}-\frac{a\mathrm{Ei}(dx)\,e^{c}}{2\,b^2}+\frac{\left(a^2-b^2\right)\log(x)}{b^3}-\frac{1}{8}\int\frac{16\left(a^2b-b^3+\left(a^3e^c-ab^2e^c\right)e\right.}{b^4xe^{(2\,dx+2\,c)}+2\,ab^3xe^{(dx+c)}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*Ei(2*d*x)*e^(2*c)/b + 1/2*a*Ei(-d*x)*e^(-c)/b^2 - 1/4*Ei(-2*d*x)*e^(-2*
c)/b - 1/2*a*Ei(d*x)*e^c/b^2 + (a^2 - b^2)*log(x)/b^3 - 1/8*integrate(16*(a
^2*b - b^3 + (a^3*e^c - a*b^2*e^c)*e^(d*x))/(b^4*x*e^(2*d*x + 2*c) + 2*a*b^
3*x*e^(d*x + c) + b^4*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sinh(dx+c)^3}{bx\cosh(dx+c)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)^3/(b*x*cosh(d*x + c) + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (dx+c)^3}{(b\cosh (dx+c)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^3/((b*cosh(d*x + c) + a)*x), x)
```

3.239 $\int \cosh(a+b\log(cx^n))\,dx$

Optimal. Leaf size=54

$$\frac{x\cosh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

```
[Out] (x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2) - (b*n*x*Sinh[a + b*Log[c*x^n]])/(
1 - b^2*n^2)
```

Rubi [A] time = 0.0119098, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5518}

$$\frac{x\cosh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] (x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2) - (b*n*x*Sinh[a + b*Log[c*x^n]])/(
1 - b^2*n^2)
```

Rule 5518

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Simp[(
x*Cosh[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] + Simp[(b*d*n*x*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] /; FreeQ[{a, b, c, d, n}, x] &&
NeQ[b^2*d^2*n^2 - 1, 0]
```

Rubi steps

$$\int \cosh(a+b\log(cx^n))\,dx=\frac{x\cosh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

Mathematica [A] time = 0.0598323, size = 41, normalized size = 0.76

$$\frac{x(bn\sinh(a+b\log(cx^n))-\cosh(a+b\log(cx^n)))}{b^2n^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] (x*(-Cosh[a + b*Log[c*x^n]] + b*n*Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \cosh\left(a + b\ln\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n)),x)
```

```
[Out] int(cosh(a+b*ln(c*x^n)),x)
```

Maxima [A] time = 1.09895, size = 69, normalized size = 1.28

$$\frac{c^b x e^{(b\log(x^n)+a)}}{2\,(bn+1)} - \frac{x e^{(-a)}}{2\left(bc^b n - c^b\right)(x^n)^b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) - 1/2*x*e^(-a)/((b*c^b*n - c^b)*(x^n
)^b)
```

Fricas [A] time = 1.78747, size = 123, normalized size = 2.28

$$\frac{bnx\sinh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right) - x\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)}{b^2n^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*sinh(b*n*log(x) + b*log(c) + a) - x*cosh(b*n*log(x) + b*log(c) + a))
/(b^2*n^2 - 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.19559, size = 63, normalized size = 1.17

$$\frac{c^b x x^{bn} e^a}{2(bn+1)} - \frac{x e^{(-a)}}{2(bn-1)c^b x^{bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) - 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

3.240 $\int \cosh^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=88

$$\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2b^2n^2x}{1-4b^2n^2}$$

```
[Out] (-2*b^2*n^2*x)/(1 - 4*b^2*n^2) + (x*Cosh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^
2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 4*b^2*n^2
)
```

Rubi [A] time = 0.0200897, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5520, 8}

$$\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2b^2n^2x}{1-4b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (-2*b^2*n^2*x)/(1 - 4*b^2*n^2) + (x*Cosh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^
2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 4*b^2*n^2
)
```

Rule 5520

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Cosh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (Dist[(b^2
*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Cosh[d*(a + b*Log[c*x^n])]^(
p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n}, x
] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh^2(a+b\log(cx^n))\,dx = \frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2} - \frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2} - \frac{(2b^2n^2)\int}{1-4b^2n}$$
$$= -\frac{2b^2n^2x}{1-4b^2n^2} + \frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2} - \frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n)}{1-4b^2n^2}$$

Mathematica [A] time = 0.0878339, size = 56, normalized size = 0.64

$$\frac{x\left(2bn\sinh(2(a+b\log(cx^n))) - \cosh(2(a+b\log(cx^n))) + 4b^2n^2 - 1\right)}{8b^2n^2-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-1 + 4*b^2*n^2 - Cosh[2*(a + b*Log[c*x^n])] + 2*b*n*Sinh[2*(a + b*Log[c
*x^n])]))/(-2 + 8*b^2*n^2)
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int (\cosh(a+b\ln(cx^n)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^2,x)
```

Maxima [A] time = 1.11203, size = 90, normalized size = 1.02

$$\frac{c^{2\,b}xe^{(2\,b\log(x^n)+2\,a)}}{4\,(2\,bn+1)} + \frac{1}{2}\,x - \frac{xe^{(-2\,a)}}{4\left(2\,bc^{2\,b}n - c^{2\,b}\right)(x^n)^{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) + 1/2*x - 1/4*x*e^(-2*a)/(
(2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

Fricas [A] time = 1.9206, size = 258, normalized size = 2.93

$$\frac{4\,bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-x\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\left(4\,b^{2}n^{2}-1\right)x}{2\left(4\,b^{2}n^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) + a)
^2 + (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.27695, size = 228, normalized size = 2.59

$$\frac{bc^{2\,b}nxx^{2\,bn}e^{(2\,a)}}{2\left(4\,b^{2}n^{2}-1\right)}+\frac{2\,b^{2}n^{2}x}{4\,b^{2}n^{2}-1}-\frac{c^{2\,b}xx^{2\,bn}e^{(2\,a)}}{4\left(4\,b^{2}n^{2}-1\right)}-\frac{bnxe^{(-2\,a)}}{2\left(4\,b^{2}n^{2}-1\right)c^{2\,b}x^{2\,bn}}-\frac{x}{2\left(4\,b^{2}n^{2}-1\right)}-\frac{xe^{(-2\,a)}}{4\left(4\,b^{2}n^{2}-1\right)c^{2\,b}x^{2\,bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) + 2*b^2*n^2*x/(4*b^2*n^
2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-2*
```

```
a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

3.241 $\int \cosh^3(a + b\log(cx^n))\, dx$

Optimal. Leaf size=149

$$\frac{6b^3n^3x\sinh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}+\frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{6b^2n^2x\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}-\frac{3bnx\sinh(a+b\log(cx^n))\,c}{1-9b^2n}$$

```
[Out] (-6*b^2*n^2*x*Cosh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) + (x*Cos
h[a + b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2) + (6*b^3*n^3*x*Sinh[a + b*Log[c*x^n]
])/(1 - 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cosh[a + b*Log[c*x^n]]^2*Sinh[a
+ b*Log[c*x^n]])/(1 - 9*b^2*n^2)
```

Rubi [A] time = 0.0388181, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5520, 5518}

$$\frac{6b^3n^3x\sinh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}+\frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{6b^2n^2x\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}-\frac{3bnx\sinh(a+b\log(cx^n))\,c}{1-9b^2n}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^2*n^2*x*Cosh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) + (x*Cos
h[a + b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2) + (6*b^3*n^3*x*Sinh[a + b*Log[c*x^n]
])/(1 - 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cosh[a + b*Log[c*x^n]]^2*Sinh[a
+ b*Log[c*x^n]])/(1 - 9*b^2*n^2)
```

Rule 5520

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Cosh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (Dist[(b^2
*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Cosh[d*(a + b*Log[c*x^n])]^(
p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n}, x
] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rule 5518

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Simp[(
x*Cosh[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] + Simp[(b*d*n*x*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] /; FreeQ[{a, b, c, d, n}, x] &&
```

```
NeQ[b^2*d^2*n^2 - 1, 0]
```

Rubi steps

$$\int \cosh^3(a+b\log(cx^n))\,dx = \frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2} - \frac{3bnx\cosh^2(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-9b^2n^2} - \frac{(6b^2n^2)\int}{}$$
$$= -\frac{6b^2n^2x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} + \frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2} + \frac{6b^3n^3x\sinh(a+b\log(cx^n)}{1-10b^2n^2+9b^4n^4}$$

Mathematica [A] time = 0.520587, size = 117, normalized size = 0.79

$$\frac{x\left(\left(3-27b^2n^2\right)\cosh(a+b\log(cx^n)) + \left(1-b^2n^2\right)\cosh(3(a+b\log(cx^n))) + 6bn\sinh(a+b\log(cx^n))\left(\left(b^2n^2-1\right)\cosh\right.\right.}{36b^4n^4-40b^2n^2+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*((3 - 27*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (1 - b^2*n^2)*Cosh[3*(a + b*L
og[c*x^n])] + 6*b*n*(-1 + 5*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a + b*Log[c*x^
n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int (\cosh(a+b\ln(cx^n)))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^3,x)
```

Maxima [A] time = 1.19777, size = 155, normalized size = 1.04

$$\frac{c^{3\,b}xe^{(3\,b\log(x^n)+3\,a)}}{8\,(3\,bn+1)} + \frac{3\,c^b xe^{(b\log(x^n)+a)}}{8\,(bn+1)} - \frac{3\,xe^{(-b\log(x^n)-a)}}{8\left(bc^bn-c^b\right)} - \frac{xe^{(-3\,a)}}{8\left(3\,bc^{3\,b}n-c^{3\,b}\right)(x^n)^{3\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) + 3/8*c^b*x*e^(b*log(x^n)
+ a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) - 1/8*x*e^(-3*a)
/((3*b*c^(3*b)*n - c^(3*b))*(x^n)^(3*b))
```

Fricas [A] time = 1.87502, size = 536, normalized size = 3.6

$$-\frac{(b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^3+3(b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(b^2*n^2 - 1)*x
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - 3*(b^3
*n^3 - b*n)*x*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(9*b^2*n^2 - 1)*x*cosh(
b*n*log(x) + b*log(c) + a) - 3*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log
(c) + a)^2 + (9*b^3*n^3 - b*n)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*b^4*n
^4 - 10*b^2*n^2 + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.36022, size = 898, normalized size = 6.03

$$\frac{3\,b^3c^{3\,b}n^3xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)}+\frac{27\,b^3c^bn^3xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)}-\frac{b^2c^{3\,b}n^2xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)}-\frac{27\,b^2c^bn^2xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)}-\frac{3\,bc^{3\,b}nx}{8\,(9\,b^4n^4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/8
*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)*n
^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^2*c^b*n^2*x*x^
(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a
)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) - 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 +
1)*c^(3*b)*x^(3*b*n)) - 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 +
 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b
^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) - 1/8*b^2*n^2*x*
e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) + 3/8*c^b*x*x^(b*
n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) + 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c
^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n))
 + 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n))
```

3.242 $\int \cosh^4(a + b\log(cx^n))\,dx$

Optimal. Leaf size=191

$$-\frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{1-16b^2n^2}+\frac{24b}{}$$

```
[Out] (24*b^4*n^4*x)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (12*b^2*n^2*x*Cosh[a + b*Log
[c*x^n]]^2)/(1 - 20*b^2*n^2 + 64*b^4*n^4) + (x*Cosh[a + b*Log[c*x^n]]^4)/(1
 - 16*b^2*n^2) + (24*b^3*n^3*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]
])/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cosh[a + b*Log[c*x^n]]^3*Sinh[a
 + b*Log[c*x^n]])/(1 - 16*b^2*n^2)
```

Rubi [A] time = 0.0509666, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5520, 8}

$$-\frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{1-16b^2n^2}+\frac{24b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (12*b^2*n^2*x*Cosh[a + b*Log
[c*x^n]]^2)/(1 - 20*b^2*n^2 + 64*b^4*n^4) + (x*Cosh[a + b*Log[c*x^n]]^4)/(1
 - 16*b^2*n^2) + (24*b^3*n^3*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]
])/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cosh[a + b*Log[c*x^n]]^3*Sinh[a
 + b*Log[c*x^n]])/(1 - 16*b^2*n^2)
```

Rule 5520

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Cosh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (Dist[(b^2
*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Cosh[d*(a + b*Log[c*x^n])]^(
p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n}, x
] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cosh^4(a+b\log(cx^n))\,dx = \frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2} - \frac{4bnx\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-16b^2n^2} - \frac{(12b^2n^2)}{}$$
$$= -\frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} + \frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2} + \frac{24b^3n^3x\cosh(a+b\log(c}{1-20b^2n^2}$$
$$= \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4} - \frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} + \frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2} + 24b$$

Mathematica [A] time = 0.416361, size = 167, normalized size = 0.87

$$\frac{x\left(128b^3n^3\sinh(2(a+b\log(cx^n))) + 16b^3n^3\sinh(4(a+b\log(cx^n))) + \left(4-64b^2n^2\right)\cosh(2(a+b\log(cx^n))) + \left(1-4b\right.\right.}{8\left(64b^4n^4-2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (4 - 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x^n
])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] - 8*b*n*Sinh[2*(a + b*Log[
c*x^n])] + 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b*Log
[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2 + 64
*b^4*n^4))
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int (\cosh(a+b\ln(cx^n)))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^4,x)
```

Maxima [A] time = 1.22116, size = 174, normalized size = 0.91

$$\frac{c^{4\,b}xe^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}}{16\,\left(4\,bn+1\right)}+\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}}{4\,\left(2\,bn+1\right)}+\frac{3}{8}\,x-\frac{xe^{\left(-2\,b\log\left(x^{n}\right)-2\,a\right)}}{4\,\left(2\,bc^{2\,b}n-c^{2\,b}\right)}-\frac{xe^{\left(-4\,a\right)}}{16\,\left(4\,bc^{4\,b}n-c^{4\,b}\right)\left(x^{n}\right)^{4\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) + 1/4*c^(2*b)*x*e^(2*b*lo
g(x^n) + 2*a)/(2*b*n + 1) + 3/8*x - 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^(2
*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b))
```

Fricas [A] time = 1.80707, size = 801, normalized size = 4.19

$$-\frac{\left(4\,b^{2}n^{2}-1\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-16\left(4\,b^{3}n^{3}-bn\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] -1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3 -
 b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 +
 (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 + 4*(16*b^2*n^2 - 1)*x
*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*log(x)
 + b*log(c) + a)^2 + 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a)^
2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^3 + (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) +
a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29388, size = 1049, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 8*b^3*c
^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4*x
/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)/(6
4*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64*b^4
*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 -
20*b^2*n^2 + 1) - 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*
n^2 + 1) - 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2
*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n
)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x*x^(4*b*n
)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(
64*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(4*b)*x^(4*b*n)) + 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 +
 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*
c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/4*x*e^(-2*a)/(
(64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e^(-4*a)/((64*b^4
*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

3.243 $\int x^m \cosh(a+b\log(cx^n))\,dx$

Optimal. Leaf size=73

$$\frac{(m+1)x^{m+1}\cosh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}-\frac{bnx^{m+1}\sinh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}$$

```
[Out] ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2) - (b*n*x^(
1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)
```

Rubi [A] time = 0.0225428, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5528}

$$\frac{(m+1)x^{m+1}\cosh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}-\frac{bnx^{m+1}\sinh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2) - (b*n*x^(
1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)
```

Rule 5528

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])])/(b^2*d
^2*e*n^2 - e*(m + 1)^2), x] + Simp[(b*d*n*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c
*x^n])])/(b^2*d^2*e*n^2 - e*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n},
x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rubi steps

$$\int x^m \cosh(a+b\log(cx^n))\,dx=\frac{(1+m)x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}-\frac{bnx^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

Mathematica [A] time = 0.122304, size = 54, normalized size = 0.74

$$\frac{x^{m+1}((m+1)\cosh(a+b\log(cx^n))-bn\sinh(a+b\log(cx^n)))}{(-bn+m+1)(bn+m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] (x^(1 + m)*((1 + m)*Cosh[a + b*Log[c*x^n]] - b*n*Sinh[a + b*Log[c*x^n]]))/(
(1 + m - b*n)*(1 + m + b*n))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int x^m \cosh\left(a + b\ln\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n)),x)
```

Maxima [A] time = 1.11951, size = 86, normalized size = 1.18

$$\frac{c^b x e^{(b\log(x^n)+m\log(x)+a)}}{2\,(bn+m+1)} - \frac{x e^{(-b\log(x^n)+m\log(x)-a)}}{2\left(bc^b n - c^b(m+1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 1/2*x*e^(-b*log(x^n
) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

Fricas [A] time = 1.81374, size = 305, normalized size = 4.18

$$-\frac{(m+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\cosh\left(m\log\left(x\right)\right)+(m+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(m\log\left(x\right)\right)-}{b^2n^2-m^2-2\,m-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -((m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (m + 1)*x*cosh
(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)) - (b*n*x*cosh(m*log(x)) + b*n*x*
sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m - 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.18171, size = 317, normalized size = 4.34

$$\frac{bc^b nxx^{bn}x^m e^a}{2\left(b^2n^2 - m^2 - 2\,m - 1\right)} - \frac{c^b mxx^{bn}x^m e^a}{2\left(b^2n^2 - m^2 - 2\,m - 1\right)} - \frac{c^b xx^{bn}x^m e^a}{2\left(b^2n^2 - m^2 - 2\,m - 1\right)} - \frac{bnxx^m e^{(-a)}}{2\left(b^2n^2 - m^2 - 2\,m - 1\right)c^b x^{bn}} - \frac{}{2\left(b^2n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b*
n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2 -
 m^2 - 2*m - 1) - 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*
n)) - 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) - 1/2*x*x^
m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

3.244 $\int x^m \cosh^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=120

$$\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

```
[Out] (-2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 +
m)*Cosh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 4*b^2*n^2) - (2*b*n*x^(1 + m)*Cos
h[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2)
```

Rubi [A] time = 0.0481035, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5530, 30}

$$\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```


```
[Out] (-2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 +
m)*Cosh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 4*b^2*n^2) - (2*b*n*x^(1 + m)*Cos
h[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2)
```


Rule 5530

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (Dist[(b^2*d^2*n^2*p*(p - 1))/(b^2
*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2),
x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]*Cosh[d*(a +
 b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{a
, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2, 0
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx = \frac{(1+m)x^{1+m}\cosh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2} - \frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log}{(1+m)^2-4b^2n^2}$$
$$= -\frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)} + \frac{(1+m)x^{1+m}\cosh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2} - \frac{2bnx^{1+m}\cosh}{}$$

Mathematica [A] time = 0.271697, size = 87, normalized size = 0.72

$$\frac{x^{m+1}\left(-2b(m+1)n\sinh(2(a+b\log(cx^n)))+(m+1)^2\cosh(2(a+b\log(cx^n)))-4b^2n^2+m^2+2m+1\right)}{2(m+1)(-2bn+m+1)(2bn+m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^(1 + m)*(1 + 2*m + m^2 - 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n])
] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n)*(
1 + m + 2*b*n))
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int x^m (\cosh(a+b\ln(cx^n)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97615, size = 726, normalized size = 6.05

$$\frac{\left(m^2+2\,m+1\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2\cosh\left(m\log\left(x\right)\right)-\left(4\,b^2n^2-m^2-2\,m-1\right)x\cosh\left(m\log\left(x\right)\right)+\left(\left(m^2+2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) - (
4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*log
(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2
- 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*m +
b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x) + b*
log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 - (4*b^2
*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2 + 3*m^2
 + 3*m + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.2969, size = 1025, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="giac")

[Out] 1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 -
3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4
*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)
/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 2*b^2*n^2*x*x^m/(4*b^2
*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n)*x^m
*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*x*
x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1
/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*b*m*n*
x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x
^(2*b*n)) - m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4
*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2
*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3
*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m
^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^
3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)/((4*b^2*m*n^2
+ 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))
```

3.245 $\int x^m \cosh^3(a + b\log(cx^n))\,dx$

Optimal. Leaf size=203

$$\frac{6b^3n^3x^{m+1}\sinh(a+b\log(cx^n))}{\left((m+1)^2-9b^2n^2\right)\left((m+1)^2-b^2n^2\right)}+\frac{(m+1)x^{m+1}\cosh^3(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}-\frac{6b^2(m+1)n^2x^{m+1}\cosh(a+b\log(cx^n))}{\left((m+1)^2-9b^2n^2\right)\left((m+1)^2-b^2n^2\right)}-$$

```
[Out] (-6*b^2*(1 + m)*n^2*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/(((1 + m)^2 - 9*b^2*n
^2)*((1 + m)^2 - b^2*n^2)) + ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3)/(
(1 + m)^2 - 9*b^2*n^2) + (6*b^3*n^3*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/(((1
+ m)^2 - 9*b^2*n^2)*((1 + m)^2 - b^2*n^2)) - (3*b*n*x^(1 + m)*Cosh[a + b*Lo
g[c*x^n]]^2*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 9*b^2*n^2)
```

Rubi [A] time = 0.0834603, antiderivative size = 197, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5530, 5528}

$$\frac{6b^3n^3x^{m+1}\sinh(a+b\log(cx^n))}{-10b^2(m+1)^2n^2+9b^4n^4+(m+1)^4}+\frac{(m+1)x^{m+1}\cosh^3(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}-\frac{6b^2(m+1)n^2x^{m+1}\cosh(a+b\log(cx^n))}{-10b^2(m+1)^2n^2+9b^4n^4+(m+1)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^2*(1 + m)*n^2*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^4 - 10*b^2*(
1 + m)^2*n^2 + 9*b^4*n^4) + ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3)/((
1 + m)^2 - 9*b^2*n^2) + (6*b^3*n^3*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 +
m)^4 - 10*b^2*(1 + m)^2*n^2 + 9*b^4*n^4) - (3*b*n*x^(1 + m)*Cosh[a + b*Log[
c*x^n]]^2*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 9*b^2*n^2)
```

Rule 5530

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (Dist[(b^2*d^2*n^2*p*(p - 1))/(b^2
*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2),
x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]*Cosh[d*(a +
 b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{a
, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2, 0
]
```

Rule 5528

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])])/(b^2*d
^2*e*n^2 - e*(m + 1)^2), x] + Simp[(b*d*n*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c
*x^n])])/(b^2*d^2*e*n^2 - e*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n},
x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rubi steps

$$\int x^m \cosh^3(a+b\log(cx^n))\,dx = \frac{(1+m)x^{1+m}\cosh^3(a+b\log(cx^n))}{(1+m)^2-9b^2n^2} - \frac{3bnx^{1+m}\cosh^2(a+b\log(cx^n))\sinh(a+b\log}{(1+m)^2-9b^2n^2}$$

$$= -\frac{6b^2(1+m)n^2x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^4-10b^2(1+m)^2n^2+9b^4n^4} + \frac{(1+m)x^{1+m}\cosh^3(a+b\log(cx^n))}{(1+m)^2-9b^2n^2} + \frac{}{(}$$

Mathematica [A] time = 1.3393, size = 292, normalized size = 1.44

$$\frac{1}{4}x^{m+1}\left(\frac{3\sinh(bn\log(x))((m+1)\sinh(a+b\log(cx^n)-bn\log(x))-bn\cosh(a+b\log(cx^n)-bn\log(x)))}{(-bn+m+1)(bn+m+1)} + \frac{3\cosh(b}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^(1 + m)*((3*Sinh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]])
 + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*
n)) + (3*Cosh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]] - b*
n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n)) + (Si
nh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*
Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*b*n))
 + (Cosh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] - 3
*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*
b*n))))/4
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int x^m(\cosh(a+b\ln(cx^n)))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n))^3,x)
```

Maxima [A] time = 1.21915, size = 186, normalized size = 0.92

$$\frac{c^{3\,b}xe^{(3\,b\log(x^n)+m\log(x)+3\,a)}}{8\,(3\,bn+m+1)}+\frac{3\,c^{b}xe^{(b\log(x^n)+m\log(x)+a)}}{8\,(bn+m+1)}-\frac{3\,xe^{(-b\log(x^n)+m\log(x)-a)}}{8\left(bc^{b}n-c^{b}(m+1)\right)}-\frac{xe^{(-3\,b\log(x^n)+m\log(x)-3\,a)}}{8\left(3\,bc^{3\,b}n-c^{3\,b}(m+1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) + 3/8*c^b*x
*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*log
(x) - a)/(b*c^b*n - c^b*(m + 1)) - 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3*a)
/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

Fricas [B] time = 1.93076, size = 1547, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^3*cosh(m*log(x)) + 3*(m^3 - 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*
x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + 3*((b^3*n^3 - (b*m^2 + 2
*b*m + b)*n)*x*cosh(m*log(x)) + (b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*sinh(m*
log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*((m^3 - (b^2*m + b^2)*n^2 +
3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (m^3 -
(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh
(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 3*(3*(b^3*n^3 - (b*m^2 + 2*
b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) + (9*b^3*n^3
 - (b*m^2 + 2*b*m + b)*n)*x*cosh(m*log(x)) + (3*(b^3*n^3 - (b*m^2 + 2*b*m +
 b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + (9*b^3*n^3 - (b*m^2 + 2*b*m +
b)*n)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^3 - (b^2*m +
 b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(m^3 -
```

```
 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a))*
sinh(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b^2*m + b^2)*n^2
 + 6*m^2 + 4*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50957, size = 4354, normalized size = 21.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 27/8*b^3*c^b*n^
3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x^m*e
^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^
3 + 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10
*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) -
 3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 -
20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b
)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 +
m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^b*m^2*n*x*x^(b*n)*x^m
*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2
*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1/8
*c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m
*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^(3*b)*m*n*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
```

```
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^b*m^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^
4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m
+ 1) - 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2
*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m^2*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*x^m*e^(3*a)
/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*
m^2 + 4*m + 1) - 27/8*b^3*n^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/
8*b^3*n^3*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
- 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*c^b*m^2*x*
x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n
^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^b*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 -
10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)
 + 3/8*c^(3*b)*m*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b
^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*m*n^2*x*x
^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 +
4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 1/8*b^2*m*n^2*x*x^m*e^(-3*a)/((9*b^
4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 +
4*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*c^b*m*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b
^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1
/8*c^(3*b)*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n
^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*m^2*n*x*x^m*e^(-a)
/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6
*m^2 + 4*m + 1)*c^b*x^(b*n)) - 27/8*b^2*n^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b
^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b
*x^(b*n)) + 3/8*b*m^2*n*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^
2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) -
1/8*b^2*n^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^
4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*c^b*x*x^
(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2
 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*m^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2
*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*
n)) + 3/4*b*m*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 +
m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 1/8*m^3*x*x^m*e^
(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m
^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/4*b*m*n*x*x^m*e^(-3*a)/((9*b^4
*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4
*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*m^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*
n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n
)) + 3/8*b*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/8*m^2*x*x^m*e^(-3
*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*b*n*x*x^m*e^(-3*a)/((9*b^4*n^4
- 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m +
```

```
1)*c^(3*b)*x^(3*b*n)) + 9/8*m*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/
8*m*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*x^m*e^(-a)/((9
*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2
 + 4*m + 1)*c^b*x^(b*n)) + 1/8*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2
- 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b
*n))
```

3.246 $\int x^m \cosh^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=266

$$\frac{(m+1)x^{m+1}\cosh^4(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}-\frac{12b^2(m+1)n^2x^{m+1}\cosh^2(a+b\log(cx^n))}{\left((m+1)^2-16b^2n^2\right)\left((m+1)^2-4b^2n^2\right)}-\frac{4bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^3}{(m+1)^2-16b^2n^2}$$

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2
*n^2)) - (12*b^2*(1 + m)*n^2*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^2)/(((1 + m)^
2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 + m)*Cosh[a + b*Lo
g[c*x^n]]^4)/((1 + m)^2 - 16*b^2*n^2) + (24*b^3*n^3*x^(1 + m)*Cosh[a + b*Lo
g[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*
b^2*n^2)) - (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]
])/((1 + m)^2 - 16*b^2*n^2)
```

Rubi [A] time = 0.127537, antiderivative size = 260, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5530, 30}

$$\frac{(m+1)x^{m+1}\cosh^4(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}-\frac{12b^2(m+1)n^2x^{m+1}\cosh^2(a+b\log(cx^n))}{-20b^2(m+1)^2n^2+64b^4n^4+(m+1)^4}-\frac{4bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^3}{(m+1)^2-16b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2
*n^2)) - (12*b^2*(1 + m)*n^2*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^2)/((1 + m)^4
 - 20*b^2*(1 + m)^2*n^2 + 64*b^4*n^4) + ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c
*x^n]]^4)/((1 + m)^2 - 16*b^2*n^2) + (24*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c
*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^4 - 20*b^2*(1 + m)^2*n^2 + 64*b^4*n
^4) - (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/((1
 + m)^2 - 16*b^2*n^2)
```

Rule 5530

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (Dist[(b^2*d^2*n^2*p*(p - 1))/(b^2
*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2),
x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]*Cosh[d*(a +
```

```
 b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{a
, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2, 0
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \frac{(1+m)x^{1+m}\cosh^4(a+b\log(cx^n))}{(1+m)^2-16b^2n^2} - \frac{4bnx^{1+m}\cosh^3(a+b\log(cx^n))\sinh(a+b\log}{(1+m)^2-16b^2n^2}$$

$$= -\frac{12b^2(1+m)n^2x^{1+m}\cosh^2(a+b\log(cx^n))}{(1+m)^4-20b^2(1+m)^2n^2+64b^4n^4} + \frac{(1+m)x^{1+m}\cosh^4(a+b\log(cx^n))}{(1+m)^2-16b^2n^2} +$$

$$= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^4-20b^2(1+m)^2n^2+64b^4n^4\right)} - \frac{12b^2(1+m)n^2x^{1+m}\cosh^2(a+b\log}{(1+m)^4-20b^2(1+m)^2n^2+64}$$

Mathematica [A] time = 3.37795, size = 311, normalized size = 1.17

$$\frac{1}{8}x^{m+1}\left(\frac{4\sinh(2bn\log(x))\left((m+1)\sinh(2(a+b\log(cx^n)-bn\log(x)))-2bn\cosh(2(a+b\log(cx^n)-bn\log(x)))\right)}{(-2bn+m+1)(2bn+m+1)}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^(1 + m)*(3/(1 + m) + (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[x
] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1
+ m - 2*b*n)*(1 + m + 2*b*n)) + (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a -
b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]
))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cosh[4*(
a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*Log[c*
x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1 + m)*C
osh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[x] + b*L
og[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int x^{m} \left(\cosh \left(a + b \ln \left(c x^{n}\right)\right)\right)^{4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07424, size = 2880, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] 1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*co
sh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) + 4*(m^4 + 4*m^3 - 16*(b^2*m
^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a
)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^
2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*
n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^4
+ 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m
^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b
*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b^2*m^2 + 2
```

```
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^4 + 4*m^3 -
 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2*cosh(m*log(x)) + 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)
*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 - 4*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 +
2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sin
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b^3*m + b^3)*n^3 -
 (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3*cosh(
m*log(x)) + (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + (16*(b^3*m
 + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^4 + 4*m^3 - 4*
(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^4 + 4*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m
 + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 2
0*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sinh(m*log(x)))/(m^5
+ 64*(b^4*m + b^4)*n^4 + 5*m^4 + 10*m^3 - 20*(b^2*m^3 + 3*b^2*m^2 + 3*b^2*m
 + b^2)*n^2 + 10*m^2 + 5*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.72975, size = 9288, normalized size = 34.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b
^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*
```

```
m^3 + 10*m^2 + 5*m + 1) + 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*m^2*n
^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n
^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*
n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 8*b^3*c^(2
*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*
m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4
+ 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(4*b)*m*n^2*x*x^(4*b
*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^
2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) -
1/2*b*c^(2*b)*m^3*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20
*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) - 8*b^2*c^(2*b)*m*n^2*x*x^(2*b*n)*x^m*e^(2*a)/(64
*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*
n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/16*c^(4*b)*m^4*x*
x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) - 3/4*b*c^(4*b)*m^2*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4*c^(2*b)*m
^4*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) - 3/2*b*c^(2*b)*m^2*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b
^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*
b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 15/2*b^2*m
^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
 m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4
*c^(4*b)*m^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1) - 3/4*b*c^(4*b)*m*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^
4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m
^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + c^(2*b)*m^3*x*x^(2*b*n)*x^m*
```

```
e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
- 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*b*c^
(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n
^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) - 8*b^3*m*n^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - b^3*m*n^3*x*x^m*e^(-4*a)/(
(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2
*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n))
 - 15*b^2*m*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) + 3/8*c^(4*b)*m^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/2*c^(2*b)*m^2*x*x^(
2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2
*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
 - 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*
b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10
*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*m^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64
*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 2
0*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 8*b^3*n^3*x*x^m
*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m
^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)
*x^(2*b*n)) - 1/4*b^2*m^2*n^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - b^3*n^3*x*x^m*e^(-4*a)/((6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m
*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) +
 3/8*m^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2
 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1
5/2*b^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*
n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
+ 1/4*c^(4*b)*m*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2
*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^
3 + 10*m^2 + 5*m + 1) + c^(2*b)*m*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 6
4*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 -
20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/2*b*m^3*n*x*x^m*e^(-2*a)/((64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 8*
b^2*m*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*
b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5
*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*m^3*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64
*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 2
```

0*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b^2*m*n^2*x
*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2
 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(
4*b)*x^(4*b*n)) + 3/2*m^3*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 1/16*c^(4*b)*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4
*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^
2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4*c^(2*b)*x*x^(2*b*n)*x^m*e^(2*a)/(6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m
*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4*m^4*x*x^m*e^(-
2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2
*b*n)) + 3/2*b*m^2*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 4*b^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m
*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 +
5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*m
^4*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2
*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
*c^(4*b)*x^(4*b*n)) + 3/4*b*m^2*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/4*b^2*n^2*x*x^m*e^(-
4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4
*b*n)) + 9/4*m^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2
*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m
+ 1) + m^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1)*c^(2*b)*x^(2*b*n)) + 3/2*b*m*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*
b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20
*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*m^3*x*x^m*e^
(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
- 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^
(4*b*n)) + 3/4*b*m*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/2*m*x*x^m/(64*b^4*m*n^4 + 64*b^4*
n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2
*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/2*m^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 +
 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
- 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/2*b*n*x*x^
m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b
)*x^(2*b*n)) + 3/8*m^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*
m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3
 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 1/4*b*n*x*x^m*e^(-4*a)/((64*b^4*m

```
*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 +
5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/8*x*
x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60
*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + m*x*x^m*e^(-
2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2
*b*n)) + 1/4*m*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
- 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^
2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 1/4*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^
4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b
^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*x^m*e^(-4*a
)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*
n))
```

3.247 $\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=18

$$\frac{\sinh(a+b\log(cx^n))}{bn}$$

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n)
```

Rubi [A] time = 0.0160305, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2637}

$$\frac{\sinh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\sinh(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [B] time = 0.0105704, size = 37, normalized size = 2.06

$$\frac{\sinh(a)\cosh(b\log(cx^n))}{bn}+\frac{\cosh(a)\sinh(b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] (Cosh[b*Log[c*x^n]]*Sinh[a])/(b*n) + (Cosh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

Maple [A] time = 0.007, size = 19, normalized size = 1.1

$$\frac{\sinh\left(a+b\ln\left(cx^{n}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))/x,x)
```

```
[Out] sinh(a+b*ln(c*x^n))/b/n
```

Maxima [A] time = 1.0494, size = 24, normalized size = 1.33

$$\frac{\sinh\left(b\log\left(cx^{n}\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] sinh(b*log(c*x^n) + a)/(b*n)
```

Fricas [A] time = 1.91868, size = 53, normalized size = 2.94

$$\frac{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] sinh(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [A] time = 1.1883, size = 41, normalized size = 2.28

$$\begin{cases} \log(x)\cosh(a) & \text{for } b = 0 \wedge n = 0 \\ \log(x)\cosh(a + b\log(c)) & \text{for } n = 0 \\ \log(x)\cosh(a) & \text{for } b = 0 \\ \frac{\sinh(a+bn\log(x)+b\log(c))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((log(x)*cosh(a), Eq(b, 0) & Eq(n, 0)), (log(x)*cosh(a + b*log(c))
, Eq(n, 0)), (log(x)*cosh(a), Eq(b, 0)), (sinh(a + b*n*log(x) + b*log(c))/(
b*n), True))
```

Giac [B] time = 1.13579, size = 57, normalized size = 3.17

$$\frac{\left(c^{2b}x^{bn}e^{(2a)} - \frac{1}{x^{bn}}\right)e^{(-a)}}{2bc^{b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 1/2*(c^(2*b)*x^(b*n)*e^(2*a) - 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

3.248 $\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=39

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\log(x)}{2}$$

```
[Out] Log[x]/2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b*n)
```

Rubi [A] time = 0.031073, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2635, 8}

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Log[x]/2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \cosh^2(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int 1\,dx, x, \log(cx^n)\right)}{2n}$$
$$= \frac{\log(x)}{2} + \frac{\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{2bn}$$

Mathematica [A] time = 0.0267756, size = 36, normalized size = 0.92

$$\frac{2(a+b\log(cx^n))+\sinh(2(a+b\log(cx^n)))}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] (2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

Maple [A] time = 0.014, size = 52, normalized size = 1.3

$$\frac{\cosh(a+b\ln(cx^n))\sinh(a+b\ln(cx^n))}{2\,bn} + \frac{\ln(cx^n)}{2\,n} + \frac{a}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^2/x,x)
```

```
[Out] 1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/2*ln(c*x^n)/n+1/2/b/n*a
```

Maxima [A] time = 1.05581, size = 66, normalized size = 1.69

$$\frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} - \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) +
1/2*log(x)
```

Fricas [A] time = 1.98861, size = 122, normalized size = 3.13

$$\frac{bn\log(x) + \cosh(bn\log(x) + b\log(c) + a)\sinh(bn\log(x) + b\log(c) + a)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*n*log(x) + cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a))/(b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(a + b\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(cosh(a + b*log(c*x**n))**2/x, x)
```

Giac [B] time = 1.19596, size = 108, normalized size = 2.77

$$\frac{\left(4\,bc^{2\,b}ne^{(2\,a)}\log(x) + c^{4\,b}x^{2\,bn}e^{(4\,a)} - \frac{2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{2\,bn}}\right)e^{(-2\,a)}}{8\,bc^{2\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] 1/8*(4*b*c^(2*b)*n*e^(2*a)*log(x) + c^(4*b)*x^(2*b*n)*e^(4*a) - (2*c^(2*b)*
x^(2*b*n)*e^(2*a) + 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

3.249 $\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh(a+b\log(cx^n))}{bn}$$

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + Sinh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rubi [A] time = 0.0324898, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2633}

$$\frac{\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + Sinh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx&=\frac{\text{Subst}\left(\int \cosh^3(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\&=\frac{i\,\text{Subst}\left(\int \left(1-x^2\right)dx,x,-i\sinh(a+b\log(cx^n))\right)}{bn}\\&=\frac{\sinh(a+b\log(cx^n))}{bn}+\frac{\sinh^3(a+b\log(cx^n))}{3bn}\end{aligned}$$

Mathematica [A] time = 0.0094332, size = 42, normalized size = 1.

$$\frac{\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + Sinh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Maple [A] time = 0.014, size = 36, normalized size = 0.9

$$\frac{\sinh(a+b\ln(cx^n))}{nb}\left(\frac{2}{3}+\frac{(\cosh(a+b\ln(cx^n)))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] 1/n/b*(2/3+1/3*cosh(a+b*ln(c*x^n))^2)*sinh(a+b*ln(c*x^n))
```

Maxima [B] time = 1.02788, size = 116, normalized size = 2.76

$$\frac{e^{(3\,b\log(cx^n)+3\,a)}}{24\,bn}+\frac{3\,e^{(b\log(cx^n)+a)}}{8\,bn}-\frac{3\,e^{(-b\log(cx^n)-a)}}{8\,bn}-\frac{e^{(-3\,b\log(cx^n)-3\,a)}}{24\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) + 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8*
e^(-b*log(c*x^n) - a)/(b*n) - 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

Fricas [A] time = 1.91825, size = 167, normalized size = 3.98

$$\frac{\sinh(bn\log(x)+b\log(c)+a)^3+3\left(\cosh(bn\log(x)+b\log(c)+a)^2+3\right)\sinh(bn\log(x)+b\log(c)+a)}{12\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) + a
)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [A] time = 16.1491, size = 87, normalized size = 2.07

$$\begin{cases} \log(x)\cosh^3(a) & \text{for } b = 0 \wedge n = 0 \\ \log(x)\cosh^3(a + b\log(c)) & \text{for } n = 0 \\ \log(x)\cosh^3(a) & \text{for } b = 0 \\ -\frac{2\sinh^3(a+bn\log(x)+b\log(c))}{3bn} + \frac{\sinh(a+bn\log(x)+b\log(c))\cosh^2(a+bn\log(x)+b\log(c))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*cosh(a)**3, Eq(b, 0) & Eq(n, 0)), (log(x)*cosh(a + b*log(
c))**3, Eq(n, 0)), (log(x)*cosh(a)**3, Eq(b, 0)), (-2*sinh(a + b*n*log(x) +
 b*log(c))**3/(3*b*n) + sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n*log(x)
 + b*log(c))**2/(b*n), True))
```

Giac [B] time = 1.20838, size = 109, normalized size = 2.6

$$\frac{\left(c^{6\,b}x^{3\,bn}e^{(6\,a)} + 9\,c^{4\,b}x^{bn}e^{(4\,a)} - \frac{9\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{3\,bn}}\right)e^{(-3\,a)}}{24\,bc^{3\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) + 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x^
(2*b*n)*e^(2*a) + 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

3.250 $\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=73

$$\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4bn}+\frac{3\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{8bn}+\frac{3\log(x)}{8}$$

```
[Out] (3*Log[x])/8 + (3*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(8*b*n) +
(Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/(4*b*n)
```

Rubi [A] time = 0.0471974, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2635, 8}

$$\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4bn}+\frac{3\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{8bn}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (3*Log[x])/8 + (3*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(8*b*n) +
(Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/(4*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \cosh^4(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{4bn} + \frac{3\,\text{Subst}\left(\int \cosh^2(a+bx)\,dx, x, \log(cx^n)\right)}{4n}$$
$$= \frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn} + \frac{\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx$$
$$= \frac{3\log(x)}{8} + \frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn} + \frac{\cosh^3(a+b\log(cx^n))\sinh(a}{4bn}$$

Mathematica [A] time = 0.0458092, size = 51, normalized size = 0.7

$$\frac{12(a+b\log(cx^n))+8\sinh(2(a+b\log(cx^n)))+\sinh(4(a+b\log(cx^n)))}{32bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (12*(a + b*Log[c*x^n]) + 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[c
*x^n])])/(32*b*n)
```

Maple [A] time = 0.015, size = 84, normalized size = 1.2

$$\frac{(\cosh(a+b\ln(cx^n)))^3\sinh(a+b\ln(cx^n))}{4\,bn} + \frac{3\,\cosh(a+b\ln(cx^n))\sinh(a+b\ln(cx^n))}{8\,bn} + \frac{3\,\ln(cx^n)}{8\,n} + \frac{3\,a}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] 1/4*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/b/n+3/8*cosh(a+b*ln(c*x^n))*s
inh(a+b*ln(c*x^n))/b/n+3/8*ln(c*x^n)/n+3/8/b/n*a
```

Maxima [A] time = 1.04559, size = 126, normalized size = 1.73

$$\frac{e^{(4\,b\log(cx^n)+4\,a)}}{64\,bn} + \frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} - \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} - \frac{e^{(-4\,b\log(cx^n)-4\,a)}}{64\,bn} + \frac{3}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) + 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) -
1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n)
+ 3/8*log(x)
```

Fricas [A] time = 1.87579, size = 270, normalized size = 3.7

$$\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\,bn\log\left(x\right)+\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+4\right.}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 + 4*cosh(b*n*log(x) + b*log
(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^{4}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(cosh(a + b*log(c*x**n))**4/x, x)
```

Giac [A] time = 1.2682, size = 154, normalized size = 2.11

$$\frac{\left(24\,bc^{4\,b}ne^{(4\,a)}\log\left(x\right)+c^{8\,b}x^{4\,bn}e^{(8\,a)}+8\,c^{6\,b}x^{2\,bn}e^{(6\,a)}-\frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+8\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] 1/64*(24*b*c^(4*b)*n*e^(4*a)*log(x) + c^(8*b)*x^(4*b*n)*e^(8*a) + 8*c^(6*b)
*x^(2*b*n)*e^(6*a) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) + 8*c^(2*b)*x^(2*b*n)*e^
(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

3.251 $\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=65

$$\frac{\sinh^5(a+b\log(cx^n))}{5bn}+\frac{2\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh(a+b\log(cx^n))}{bn}$$

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + (2*Sinh[a + b*Log[c*x^n]]^3)/(3*b*n) + Sinh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Rubi [A] time = 0.0372635, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2633}

$$\frac{\sinh^5(a+b\log(cx^n))}{5bn}+\frac{2\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + (2*Sinh[a + b*Log[c*x^n]]^3)/(3*b*n) + Sinh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh^5(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{i\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -i\sinh(a+b\log(cx^n))\right)}{bn}\\ &= \frac{\sinh(a+b\log(cx^n))}{bn}+\frac{2\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh^5(a+b\log(cx^n))}{5bn}\end{aligned}$$

Mathematica [A] time = 0.0178927, size = 65, normalized size = 1.

$$\frac{\sinh^5(a+b\log(cx^n))}{5bn}+\frac{2\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + (2*Sinh[a + b*Log[c*x^n]]^3)/(3*b*n) + Sinh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Maple [A] time = 0.017, size = 51, normalized size = 0.8

$$\frac{\sinh(a+b\ln(cx^n))}{nb}\left(\frac{8}{15}+\frac{(\cosh(a+b\ln(cx^n)))^4}{5}+\frac{4(\cosh(a+b\ln(cx^n)))^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] 1/n/b*(8/15+1/5*cosh(a+b*ln(c*x^n))^4+4/15*cosh(a+b*ln(c*x^n))^2)*sinh(a+b*
ln(c*x^n))
```

Maxima [B] time = 1.03966, size = 176, normalized size = 2.71

$$\frac{e^{(5b\log(cx^n)+5a)}}{160bn}+\frac{5e^{(3b\log(cx^n)+3a)}}{96bn}+\frac{5e^{(b\log(cx^n)+a)}}{16bn}-\frac{5e^{(-b\log(cx^n)-a)}}{16bn}-\frac{5e^{(-3b\log(cx^n)-3a)}}{96bn}-\frac{e^{(-5b\log(cx^n)-5a)}}{160bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) + 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
+ 5/16*e^(b*log(c*x^n) + a)/(b*n) - 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/96
*e^(-3*b*log(c*x^n) - 3*a)/(b*n) - 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

Fricas [A] time = 1.94643, size = 333, normalized size = 5.12

$$\frac{3 \sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{5}+5\left(6 \cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+5\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+15\left(c\right.}{240\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*n*log(x) + b*log(c) + a)^5 + 5*(6*cosh(b*n*log(x) + b*log(c
) + a)^2 + 5)*sinh(b*n*log(x) + b*log(c) + a)^3 + 15*(cosh(b*n*log(x) + b*l
og(c) + a)^4 + 5*cosh(b*n*log(x) + b*log(c) + a)^2 + 10)*sinh(b*n*log(x) +
b*log(c) + a))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19242, size = 157, normalized size = 2.42

$$\frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)}+25\,c^{8\,b}x^{3\,bn}e^{(8\,a)}+150\,c^{6\,b}x^{bn}e^{(6\,a)}-\frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+25\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) + 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*c
^(6*b)*x^(b*n)*e^(6*a) - (150*c^(4*b)*x^(4*b*n)*e^(4*a) + 25*c^(2*b)*x^(2*b
*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

3.252 $\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x} dx$

Optimal. Leaf size=67

$$\frac{2\sinh(a+b\log(cx^n))\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{5bn}-\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn}$$

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Cosh[a + b*L
og[c*x^n]]^(3/2)*Sinh[a + b*Log[c*x^n]])/(5*b*n)
```

Rubi [A] time = 0.0444049, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2635, 2639}

$$\frac{2\sinh(a+b\log(cx^n))\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{5bn}-\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Cosh[a + b*L
og[c*x^n]]^(3/2)*Sinh[a + b*Log[c*x^n]])/(5*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \cosh^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\cosh^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{5bn} + \frac{3\,\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log(\right.}{5n}$$
$$= -\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn} + \frac{2\cosh^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{5bn}$$

Mathematica [A] time = 0.056488, size = 62, normalized size = 0.93

$$\frac{\sinh(2(a+b\log(cx^n)))\sqrt{\cosh(a+b\log(cx^n))} - 6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] ((-6*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sqrt[Cosh[a + b*Log[c*x^n]
]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n)
```

Maple [B] time = 0.085, size = 256, normalized size = 3.8

$$\frac{2}{5\,bn}\sqrt{\left(2\,(\cosh(a/2+1/2\,b\ln(cx^n)))^2-1\right)\left(\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)^2}\left(8\,(\cosh(a/2+1/2\,b\ln(cx^n)))^7-16\,(\cosh(a/2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^(5/2)/x,x)
```

```
[Out] 2/5/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+10*c
osh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*c
osh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n))
,2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+s
inh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2
```

```
*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (b \log (cx^n) + a)^{\frac{5}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh (b \log (cx^n) + a)^{\frac{5}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

3.253 $\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=67

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}{3bn}-\frac{2i\text{EllipticF}\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right),2\right)}{3bn}$$

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sqrt[Cosh[a
+ b*Log[c*x^n]]]*Sinh[a + b*Log[c*x^n]])/(3*b*n)
```

Rubi [A] time = 0.0443683, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2635, 2641}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}{3bn}-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sqrt[Cosh[a
+ b*Log[c*x^n]]]*Sinh[a + b*Log[c*x^n]])/(3*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \cosh^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\sqrt{\cosh(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n}$$
$$= -\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{3bn} + \frac{2\sqrt{\cosh(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{3bn}$$

Mathematica [C] time = 0.121836, size = 114, normalized size = 1.7

$$\frac{2\sqrt{\sinh(2(a+b\log(cx^n)))+\cosh(2(a+b\log(cx^n)))+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+b\log(cx^n)))-\sinh(2(a+b\log(\right.}{3bn\sqrt{\cosh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (Sinh[2*(a + b*Log[c*x^n])] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(a
 + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[1 + Cosh[2*(a + b*Log[
c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]])/(3*b*n*Sqrt[Cosh[a + b*Log[c*x^n]]]
)
```

Maple [B] time = 0.09, size = 237, normalized size = 3.5

$$\frac{2}{3\,nb}\sqrt{\left(2\,(\cosh(a/2+1/2\,b\ln(cx^n)))^2-1\right)\left(\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)^2}\left(4\,(\cosh(a/2+1/2\,b\ln(cx^n)))^5-6\,(\cosh(a/2+1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] 2/3/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-sin
h(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2
)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(c*x^
```

```
n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/
sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

3.254 $\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=28

$$-\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rubi [A] time = 0.0275534, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2639}

$$-\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0207976, size = 28, normalized size = 1.

$$-\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Maple [B] time = 0.07, size = 183, normalized size = 6.5

$$-2\frac{\sqrt{\left(2\left(\cosh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}-1\right)\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}}\sqrt{-\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}}\sqrt{-2\left(\cosh\left(a/2\right.\right.}}{n\sqrt{2\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{4}+\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}}\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] -2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^
2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a+1/2
*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*
x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cosh\left(b\log\left(cx^{n}\right)+a\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh\left(a + b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(cosh(a + b*log(c*x**n)))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cosh\left(b\log\left(cx^{n}\right) + a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

3.255 $\int \frac{1}{x\sqrt{\cosh(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=28

$$-\frac{2i\text{EllipticF}\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right),2\right)}{bn}$$

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rubi [A] time = 0.0277238, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2641}

$$-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0203363, size = 28, normalized size = 1.

$$-\frac{2i\text{EllipticF}\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right),2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Maple [B] time = 0.069, size = 183, normalized size = 6.5

$$2\frac{\sqrt{\left(2\left(\cosh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}-1\right)\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}}\sqrt{-\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}}\sqrt{-2\left(\cosh\left(a/2+1/\right.\right.}}{n\sqrt{2\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{4}+\left(\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\right)^{2}}\sinh\left(a/2+1/2\,b\ln\left(cx^{n}\right)\right)\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cosh(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] 2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/
2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2
+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*ln(c*x
^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cosh\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x\sqrt{\cosh\left(b\log\left(cx^{n}\right)+a\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(cosh(a + b*log(c*x**n)))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cosh\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

3.256 $\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=63

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}+\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sinh[a + b*Log[c*
x^n]])/(b*n*Sqrt[Cosh[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.0421568, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2636, 2639}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}+\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sinh[a + b*Log[c*
x^n]])/(b*n*Sqrt[Cosh[a + b*Log[c*x^n]]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\sinh(a+b\log(cx^n))}{bn\sqrt{\cosh(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn} + \frac{2\sinh(a+b\log(cx^n))}{bn\sqrt{\cosh(a+b\log(cx^n))}}$$

Mathematica [A] time = 0.0567587, size = 58, normalized size = 0.92

$$\frac{2\left(\frac{\sinh(a+b\log(cx^n))}{\sqrt{\cosh(a+b\log(cx^n))}} + iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*(I*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]/Sqrt[
Cosh[a + b*Log[c*x^n]]]))/(b*n)
```

Maple [A] time = 0.098, size = 141, normalized size = 2.2

$$2\,\frac{\sqrt{-2\,(\sinh(a/2+1/2\,b\ln(cx^n)))^2-1}\sqrt{-(\sinh(a/2+1/2\,b\ln(cx^n)))^2}\text{EllipticE}\left(\cosh(a/2+1/2\,b\ln(cx^n)),\sqrt{2}\right)+2}{n\sinh(a/2+1/2\,b\ln(cx^n))\sqrt{2\,(\cosh(a/2+1/2\,b\ln(cx^n)))^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cosh(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 2/n*((-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n
))^2)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*sinh(1/2*a+1/2
*b*ln(c*x^n))^2*cosh(1/2*a+1/2*b*ln(c*x^n)))/sinh(1/2*a+1/2*b*ln(c*x^n))/(2
*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

3.257 $\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=67

$$\frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{2i\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3bn}$$

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sinh[a + b*L
og[c*x^n]])/(3*b*n*Cosh[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] time = 0.0421404, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2636, 2641}

$$\frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sinh[a + b*L
og[c*x^n]])/(3*b*n*Cosh[a + b*Log[c*x^n]]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= -\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{3bn} + \frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}$$

Mathematica [C] time = 0.0851685, size = 122, normalized size = 1.82

$$\frac{2\left(\cosh(a+b\log(cx^n))\sqrt{\sinh(2(a+b\log(cx^n)))+\cosh(2(a+b\log(cx^n)))+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+b\log(cx^n\right.\right.}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*(Sinh[a + b*Log[c*x^n]] + Cosh[a + b*Log[c*x^n]]*Hypergeometric2F1[1/4,
1/2, 5/4, -Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[1
+ Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Cosh[a
+ b*Log[c*x^n]]^(3/2))
```

Maple [B] time = 0.089, size = 295, normalized size = 4.4

$$\frac{2}{3\,nb}\left(2\sqrt{-(\sinh(a/2+1/2\,b\ln(cx^n)))^2}\sqrt{-2\,(\sinh(a/2+1/2\,b\ln(cx^n)))^2-1}\text{EllipticF}\left(\cosh(a/2+1/2\,b\ln(cx^n)),\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cosh(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] 2/3/n*(2*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x
^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sinh(1/2*a+1
```

```
/2*b*ln(c*x^n))^2+(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2
*b*ln(c*x^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*s
inh(1/2*a+1/2*b*ln(c*x^n))^2*cosh(1/2*a+1/2*b*ln(c*x^n)))*((2*cosh(1/2*a+1/
2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*sinh(1/2*a+1/2*
b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cosh(1/2*a+1/2*b*ln(
c*x^n))^2-1)^(3/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\cosh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\cosh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*ln(c*x**n))**(5/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \cosh \left(b \log \left(c x^{n}\right)+a\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

3.258 $\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx$

Optimal. Leaf size=206

$$\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(e^{-2a}(cx^n)^{-4/n}+1\right)^2}-\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(e^{-2a}(cx^n)^{-4/n}+1\right)}-\frac{5e^{-3a}x(cx^n)^{-6/n}\operatorname{csch}^{-1}}{4\left(e^{-2a}\right.}$$

```
[Out] -(x*Cosh[a + (2*Log[c*x^n])/n]^(5/2))/4 + (5*x*Cosh[a + (2*Log[c*x^n])/n]^(
5/2))/(4*E^(2*a)*(c*x^n)^(4/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^2) + (5*x*Co
sh[a + (2*Log[c*x^n])/n]^(5/2))/(12*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))) - (5*x
*ArcCsch[E^a*(c*x^n)^(2/n)]*Cosh[a + (2*Log[c*x^n])/n]^(5/2))/(4*E^(3*a)*(c
*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))
```

Rubi [A] time = 0.153696, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5526, 5534, 353, 349, 345, 242, 277, 215}

$$\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(e^{-2a}(cx^n)^{-4/n}+1\right)^2}-\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(e^{-2a}(cx^n)^{-4/n}+1\right)}-\frac{5e^{-3a}x(cx^n)^{-6/n}\operatorname{csch}^{-1}}{4\left(e^{-2a}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] -(x*Cosh[a + (2*Log[c*x^n])/n]^(5/2))/4 + (5*x*Cosh[a + (2*Log[c*x^n])/n]^(
5/2))/(4*E^(2*a)*(c*x^n)^(4/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^2) + (5*x*Co
sh[a + (2*Log[c*x^n])/n]^(5/2))/(12*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))) - (5*x
*ArcCsch[E^a*(c*x^n)^(2/n)]*Cosh[a + (2*Log[c*x^n])/n]^(5/2))/(4*E^(3*a)*(c
*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))
```

Rule 5526

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5534

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 353

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &&
 IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 349

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*
(a + b*x^n)^p)/(m + 1), x] - Dist[(b*n*p)/(m + 1), Int[x^(m + n)*(a + b*x^n
)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && GtQ
[p, 0]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(x)}{n}\right)dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}\left(1+e^{-2a}x^{-4/n}\right)^{5/2}dx,x,cx^n\right)}{n\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{\left(5x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}\left(1+\right.\right.}{2n\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)}+\frac{\left(5e^{-2a}x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\right.\right.}{2}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)}+\frac{\left(5e^{-2a}x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\right.\right.}{4\left(\right.}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)}-\frac{\left(5e^{-2a}x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\right.\right.}{4\left(\right.}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2}+\frac{5x\cosh^{\frac{5}{2}}\left(a+\right.}{12\left(1+e^{-2a}(c\right.}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2}+\frac{5x\cosh^{\frac{5}{2}}\left(a+\right.}{12\left(1+e^{-2a}(c\right.}$$

Mathematica [C] time = 0.459269, size = 85, normalized size = 0.41

$$\frac{1}{14}e^{2a}x(cx^n)^{4/n}\left(e^{2a}(cx^n)^{4/n}+1\right){}_2F_1\left(2,\frac{7}{2};\frac{9}{2};e^{2a}(cx^n)^{4/n}+1\right)\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] (E^(2*a)*x*(c*x^n)^(4/n)*(1 + E^(2*a)*(c*x^n)^(4/n))*Cosh[a + (2*Log[c*x^n]
)/n]^(5/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + E^(2*a)*(c*x^n)^(4/n)])/14
```

Maple [F] time = 0.334, size = 0, normalized size = 0.

$$\int \left(\cosh\left(a+2\,\frac{\ln\left(cx^n\right)}{n}\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

```
[Out] int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(a+\frac{2\,\log\left(cx^n\right)}{n}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(5/2), x)
```

Fricas [A] time = 1.86262, size = 475, normalized size = 2.31

$$\frac{\left(15\,\sqrt{2}x^3e^{\left(\frac{3\,(an+2\,\log(c))}{2\,n}\right)}\log\left(\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\,\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^2}}+2}{x^4}\right)+4\,\sqrt{\frac{1}{2}}\left(2\,x^8e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+14\,x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}\cdots\right.\right.}{192\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(15*sqrt(2)*x^3*e^(3/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n + 2*log
(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2
) + 2)/x^4) + 4*sqrt(1/2)*(2*x^8*e^(4*(a*n + 2*log(c))/n) + 14*x^4*e^(2*(a*
```

```
n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-1/2*
(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*log(c))/n)/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*ln(c*x**n)/n)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(a + \frac{2\,\log\left(cx^n\right)}{n}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(5/2), x)
```

3.259 $\int \sqrt{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=102

$$\frac{1}{2}x\sqrt{\cosh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}-\frac{e^{-a}x\left(cx^n\right)^{-2/n}\operatorname{csch}^{-1}\left(e^a\left(cx^n\right)^{2/n}\right)\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

```
[Out] (x*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/2 - (x*ArcCsch[E^a*(c*x^n)^(2/n)]*Sqrt
[Cosh[a + (2*Log[c*x^n])/n]])/(2*E^a*(c*x^n)^(2/n)*Sqrt[1 + 1/(E^(2*a)*(c*x
^n)^(4/n))])
```

Rubi [A] time = 0.0823893, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5526, 5534, 345, 242, 277, 215}

$$\frac{1}{2}x\sqrt{\cosh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}-\frac{e^{-a}x\left(cx^n\right)^{-2/n}\operatorname{csch}^{-1}\left(e^a\left(cx^n\right)^{2/n}\right)\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/2 - (x*ArcCsch[E^a*(c*x^n)^(2/n)]*Sqrt
[Cosh[a + (2*Log[c*x^n])/n]])/(2*E^a*(c*x^n)^(2/n)*Sqrt[1 + 1/(E^(2*a)*(c*x
^n)^(4/n))])
```

Rule 5526

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5534

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
```

```
a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} \, dx = \frac{\left(x \left(c x^n\right)^{-1/n}\right) \text{Subst}\left(\int x^{-1+\frac{1}{n}} \sqrt{\cosh \left(a+\frac{2 \log (x)}{n}\right)} \, dx, x, c x^n\right)}{n}$$

$$= \frac{\left(x \left(c x^n\right)^{-2/n} \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)}\right) \text{Subst}\left(\int x^{-1+\frac{2}{n}} \sqrt{1+e^{-2 a} x^{-4/n}} \, dx, x, c x^n\right)}{n \sqrt{1+e^{-2 a} \left(c x^n\right)^{-4/n}}}$$

$$= \frac{\left(x \left(c x^n\right)^{-2/n} \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)}\right) \text{Subst}\left(\int \sqrt{1+\frac{e^{-2 a}}{x^2}} \, dx, x, \left(c x^n\right)^{2/n}\right)}{2 \sqrt{1+e^{-2 a} \left(c x^n\right)^{-4/n}}}$$

$$= -\frac{\left(x \left(c x^n\right)^{-2/n} \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)}\right) \text{Subst}\left(\int \frac{\sqrt{1+e^{-2 a} x^2}}{x^2} \, dx, x, \left(c x^n\right)^{-2/n}\right)}{2 \sqrt{1+e^{-2 a} \left(c x^n\right)^{-4/n}}}$$

$$= \frac{1}{2} x \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} - \frac{\left(e^{-2 a} x \left(c x^n\right)^{-2/n} \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1+e^{-}}}\right.}{2 \sqrt{1+e^{-2 a} \left(c x^n\right)^{-4/n}}}$$

$$= \frac{1}{2} x \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} - \frac{e^{-a} x \left(c x^n\right)^{-2/n} \sinh^{-1}\left(e^{-a} \left(c x^n\right)^{-2/n}\right) \sqrt{\cosh \left(a+\frac{2 \log (c x}{n}\right.}}{2 \sqrt{1+e^{-2 a} \left(c x^n\right)^{-4/n}}}$$

Mathematica [A] time = 0.31763, size = 74, normalized size = 0.73

$$\frac{1}{2} x \left(1-\frac{\tanh^{-1}\left(\sqrt{e^{2 a} \left(c x^n\right)^{4/n}+1}\right)}{\sqrt{e^{2 a} \left(c x^n\right)^{4/n}+1}}\right) \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*(1 - ArcTanh[Sqrt[1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[1 + E^(2*a)*(c*x^n)^(
4/n)])*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/2
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \sqrt{\cosh\left(a + 2\frac{\ln\left(cx^n\right)}{n}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
```

```
[Out] int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cosh\left(a + \frac{2\log\left(cx^n\right)}{n}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(a + 2*log(c*x^n)/n)), x)
```

Fricas [A] time = 1.79661, size = 360, normalized size = 3.53

$$\frac{1}{8}\left(4\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}+1}{x^2}}e^{\left(\frac{an+2\log(c)}{2n}\right)} + \sqrt{2}e^{\left(\frac{an+2\log(c)}{2n}\right)}\log\left(\frac{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)} - 2\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}+1}{x^2}} + 2}{x^4}\right)\right)e^{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(1/2*(a*n
 + 2*log(c))/n) + sqrt(2)*e^(1/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n + 2
*log(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)
```

```
/x^2) + 2)/x^4))*e^(-(a*n + 2*log(c))/n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cosh\left(a + \frac{2\log\left(cx^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*ln(c*x**n)/n)**(1/2),x)
```

```
[Out] Integral(sqrt(cosh(a + 2*log(c*x**n)/n)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.260 $\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=42

$$-\frac{x\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(2*Cosh[a + (2*Log[c*x^n])/n]^(3/2))
```

Rubi [A] time = 0.0498428, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5526, 5534, 264}

$$-\frac{x\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] -(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(2*Cosh[a + (2*Log[c*x^n])/n]^(3/2))
```

Rule 5526

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5534

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(x\,(cx^n)^{2/n}\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{\left(1+e^{-2a}x^{-4/n}\right)^{3/2}}\,dx,x,cx^n\right)}{n\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$
$$= -\frac{x\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A] time = 0.149444, size = 61, normalized size = 1.45

$$\frac{\sinh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)-\cosh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)}{x\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] (-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n]
)/n])/(x*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])
```

Maple [F] time = 0.149, size = 0, normalized size = 0.

$$\int \left(\cosh\left(a+2\,\frac{\ln(cx^n)}{n}\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

```
[Out] int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

Fricas [A] time = 1.80027, size = 167, normalized size = 3.98

$$-\frac{2\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}+1}{x^2}}e^{\left(-\frac{an+2\log(c)}{2n}\right)}}{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n +
2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a + \frac{2\log(cx^n)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*ln(c*x**n)/n)**(3/2),x)
```

```
[Out] Integral(cosh(a + 2*log(c*x**n)/n)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh\left(a + \frac{2 \log(cx^n)}{n}\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

3.261 $\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=101

$$-\frac{e^{-2a}x\,(cx^n)^{-4/n}\left(e^{-2a}\,(cx^n)^{-4/n}+1\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(e^{-2a}\,(cx^n)^{-4/n}+1\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(6*Cosh[a + (2*Log[c*x^n])/n]^(7/2)) -
 (x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(15*E^(2*a)*(c*x^n)^(4/n)*Cosh[a + (2*
Log[c*x^n])/n]^(7/2))
```

Rubi [A] time = 0.0752656, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5526, 5534, 271, 264}

$$-\frac{e^{-2a}x\,(cx^n)^{-4/n}\left(e^{-2a}\,(cx^n)^{-4/n}+1\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(e^{-2a}\,(cx^n)^{-4/n}+1\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] -(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(6*Cosh[a + (2*Log[c*x^n])/n]^(7/2)) -
 (x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(15*E^(2*a)*(c*x^n)^(4/n)*Cosh[a + (2*
Log[c*x^n])/n]^(7/2))
```

Rule 5526

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5534

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
```

```
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx &= \frac{\left(x\,(cx^n)^{-1/n}\right)\mathrm{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}\\
&= \frac{\left(x\,(cx^n)^{6/n}\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)^{7/2}\right)\mathrm{Subst}\left(\int \frac{x^{-1-\frac{6}{n}}}{\left(1+e^{-2a}x^{-4/n}\right)^{7/2}}\,dx, x, cx^n\right)}{n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\\
&= -\frac{x\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} + \frac{\left(2e^{-2a}x\,(cx^n)^{6/n}\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)^{7/2}\right)\mathrm{Subst}\left(\int \frac{x^{-1-\frac{10}{n}}}{\left(1+e^{-2a}x^{-4/n}\right)^{7/2}}\right.}{3n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\\
&= -\frac{x\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{e^{-2a}x\,(cx^n)^{-4/n}\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}
\end{aligned}$$

Mathematica [A] time = 0.252413, size = 121, normalized size = 1.2

$$\frac{\left(\left(5x^4-2\right)\sinh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)+\left(5x^4+2\right)\cosh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)\right)\left(\sinh\left(2a+\frac{4\log(cx^n)}{n}-4\log(x)\right)-\right.}{15x^5\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] (((2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (-2 + 5*x^4)*Sinh[a -
 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n] +
Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Cosh[a + (2*Log[c*x^n])/n
]^(5/2))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \left(\cosh \left(a + 2\,\frac{\ln \left(cx^{n}\right)}{n}\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
```

```
[Out] int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh \left(a + \frac{2\,\log\left(cx^{n}\right)}{n}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

Fricas [A] time = 1.87371, size = 312, normalized size = 3.09

$$-\frac{8\,\sqrt{\frac{1}{2}}\left(5\,x^{5}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+2\,x\right)\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^{2}}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{15\left(x^{12}e^{\left(\frac{6\,(an+2\,\log(c))}{n}\right)}+3\,x^{8}e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+3\,x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) + 2*x)*sqrt((x^4*e^(2*(a*n
+ 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*lo
g(c))/n) + 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/n)
+ 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*ln(c*x**n)/n)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

3.262 $\int \cosh\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=101

$$\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] ((c + d*x)*Cosh[(a + b*x)/(c + d*x)])/d + ((b*c - a*d)*CoshIntegral[(b*c -
a*d)/(d*(c + d*x))]*Sinh[b/d])/d^2 - ((b*c - a*d)*Cosh[b/d]*SinhIntegral[(b
*c - a*d)/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.176673, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5608, 3297, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((c + d*x)*Cosh[(a + b*x)/(c + d*x)])/d + ((b*c - a*d)*CoshIntegral[(b*c -
a*d)/(d*(c + d*x))]*Sinh[b/d])/d^2 - ((b*c - a*d)*Cosh[b/d]*SinhIntegral[(b
*c - a*d)/(d*(c + d*x))])/d^2
```

Rule 5608

```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Cosh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad)\sinh\right.}{}$$

$$= \frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right)}{d^2} - \frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [B] time = 0.351606, size = 373, normalized size = 3.69

$$\frac{(bc-ad)\left(\sinh\left(\frac{b}{d}\right)-\cosh\left(\frac{b}{d}\right)\right)\text{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)+(bc-ad)\left(\sinh\left(\frac{b}{d}\right)+\cosh\left(\frac{b}{d}\right)\right)\text{Chi}\left(\frac{ad-bc}{d(c+dx)}\right)-ad\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{xd^2+cd}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(a + b*x)/(c + d*x)],x]
```

```
[Out] (2*c*d*Cosh[(a + b*x)/(c + d*x)] + 2*d^2*x*Cosh[(a + b*x)/(c + d*x)] + (b*c
 - a*d)*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)]*(-Cosh[b/d] + Sinh[b/d]) +
(b*c - a*d)*CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cosh[b/d] + Sinh[b/
d]) + b*c*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Cosh[b
/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + b*c*Sinh[b/d]*SinhIntegral
[(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/
(d*(c + d*x))] - b*c*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + a*
d*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + b*c*Sinh[b/d]*SinhInt
egral[(b*c - a*d)/(c*d + d^2*x)] - a*d*Sinh[b/d]*SinhIntegral[(b*c - a*d)/(
c*d + d^2*x)])/(2*d^2)
```

Maple [B] time = 0.04, size = 347, normalized size = 3.4

$$\frac{a}{2}e^{-\frac{bx+a}{dx+c}}\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}\right)^{-1}-\frac{bc}{2d}e^{-\frac{bx+a}{dx+c}}\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}\right)^{-1}-\frac{a}{2d}e^{-\frac{b}{d}}\mathrm{Ei}\left(1,\frac{ad-bc}{d(dx+c)}\right)+\frac{bc}{2d^{2}}e^{-\frac{b}{d}}\mathrm{Ei}\left(1,\frac{ad-bc}{d(dx+c)}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((b*x+a)/(d*x+c)),x)
```

```
[Out] 1/2*exp(-(b*x+a)/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*b*c)*a-1/2/d*exp(-(b*x+a)/
(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*b*c)*b*c-1/2/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(
d*x+c))*a+1/2/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/2*d*exp((b*x+a)
/(d*x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*b*c+1/2*exp((b
*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+1/2
/d*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d
/(d*x+c))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(\frac{bx+a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cosh((b*x + a)/(d*x + c)), x)
```

Fricas [A] time = 1.9023, size = 344, normalized size = 3.41

$$\frac{2\left(d^2x+cd\right)\cosh\left(\frac{bx+a}{dx+c}\right)-\left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)-(bc-ad)\mathrm{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)+\left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(d^2*x + c*d)*cosh((b*x + a)/(d*x + c)) - ((b*c - a*d)*Ei((b*c - a*d
)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d) +
((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a*d)/(d
^2*x + c*d)))*sinh(b/d))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(\frac{bx+a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh((b*x + a)/(d*x + c)), x)
```

3.263 $\int \cosh^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] ((c + d*x)*Cosh[(a + b*x)/(c + d*x)]^2)/d + ((b*c - a*d)*CoshIntegral[(2*(b
*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2 - ((b*c - a*d)*Cosh[(2*b)/d]*S
inhIntegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.193896, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5608, 3313, 12, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Cosh[(a + b*x)/(c + d*x)]^2)/d + ((b*c - a*d)*CoshIntegral[(2*(b
*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2 - ((b*c - a*d)*Cosh[(2*b)/d]*S
inhIntegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 5608

```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Cosh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

```
LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\cosh^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2i(bc-ad))\,\text{Subst}\left(\int -\frac{i\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad)\sin\right.}{}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} - \frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

Mathematica [A] time = 0.28457, size = 111, normalized size = 1.04

$$\frac{2\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(ad-bc)}{d(c+dx)}\right) + 2\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(ad-bc)}{d(c+dx)}\right) + d\left((c+dx)\cosh\left(\frac{2(a+bx)}{c+dx}\right)+dx\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] (d*(d*x + (c + d*x)*Cosh[(2*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)*CoshInte
gral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sinh[(2*b)/d] + 2*(b*c - a*d)*Cosh[(
2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(2*d^2)
```

Maple [B] time = 0.091, size = 358, normalized size = 3.4

$$\frac{x}{2} + \frac{a}{4}e^{-2\frac{bx+a}{dx+c}}\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}\right)^{-1} - \frac{bc}{4d}e^{-2\frac{bx+a}{dx+c}}\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}\right)^{-1} - \frac{a}{2d}e^{-2\frac{b}{d}}\text{Ei}\left(1,2\frac{ad-bc}{d(dx+c)}\right) + \frac{bc}{2d^2}e^{-2\frac{b}{d}}\text{Ei}\left(1,2\frac{a}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((b*x+a)/(d*x+c))^2,x)
```

```
[Out] 1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*b*c)*a-1/4/d*exp(-
2*(b*x+a)/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*b*c)*b*c-1/2/d*exp(-2*b/d)*Ei(1,2
*(a*d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2*b/d)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*b*c
+1/4*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/(a*d
-b*c)*x*b*c+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+a)/(d
*x+c))/(a*d-b*c)*c^2*b+1/2/d*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*a-1/2/
d^2*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x + \frac{1}{4}\int e^{\left(\frac{2bc}{d^2x+cd} - \frac{2a}{dx+c} - \frac{2b}{d}\right)}\,dx + \frac{1}{4}\int e^{\left(-\frac{2bc}{d^2x+cd} + \frac{2a}{dx+c} + \frac{2b}{d}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x) +
 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

Fricas [B] time = 1.83696, size = 774, normalized size = 7.23

$$\frac{d^2x + \left(d^2x + cd\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2 + \left(d^2x - (bc - ad)\mathrm{Ei}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right) + cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2 + \left((bc - ad)\mathrm{Ei}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*x + (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 + (d^2*x - (b*c - a*
d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d*x
+ c))^2 + ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x
 + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) + ((b*c
 - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2 - (b*c
 - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2 + (b*c
```

```
 - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*x + a)/(
d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(\frac{bx+a}{dx+c}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cosh((b*x + a)/(d*x + c))^2, x)
```

3.264 $\int e^{a+bx}\cosh^4(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] -E^(-3*a - 3*b*x)/(48*b) - E^(-a - b*x)/(4*b) + (3*E^(a + b*x))/(8*b) + E^(
3*a + 3*b*x)/(12*b) + E^(5*a + 5*b*x)/(80*b)
```

Rubi [A] time = 0.0386232, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 270}

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

```
[Out] -E^(-3*a - 3*b*x)/(48*b) - E^(-a - b*x)/(4*b) + (3*E^(a + b*x))/(8*b) + E^(
3*a + 3*b*x)/(12*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\cosh^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{16x^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^4}\,dx, x, e^{a+bx}\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(6+\frac{1}{x^4}+\frac{4}{x^2}+4x^2+x^4\right)dx, x, e^{a+bx}\right)}{16b}\\
&= -\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}
\end{aligned}$$

Mathematica [A] time = 0.0416132, size = 62, normalized size = 0.75

$$\frac{e^{-3(a+bx)}\left(-60e^{2(a+bx)}+90e^{4(a+bx)}+20e^{6(a+bx)}+3e^{8(a+bx)}-5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

```
[Out] (-5 - 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) + 20*E^(6*(a + b*x)) + 3*E^(8
*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

Maple [A] time = 0.011, size = 77, normalized size = 0.9

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^2(\cosh(bx+a))^3}{5}+\frac{(\sinh(bx+a))^2\cosh(bx+a)}{5}+\frac{\cosh(bx+a)}{5}+\left(\frac{8}{15}+\frac{(\cosh(bx+a))^4}{5}+\frac{4\,(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^4,x)
```

```
[Out] 1/b*(1/5*sinh(b*x+a)^2*cosh(b*x+a)^3+1/5*sinh(b*x+a)^2*cosh(b*x+a)+1/5*cosh
(b*x+a)+(8/15+1/5*cosh(b*x+a)^4+4/15*cosh(b*x+a)^2)*sinh(b*x+a))
```

Maxima [A] time = 1.06531, size = 92, normalized size = 1.11

$$\frac{e^{(5\,bx+5\,a)}}{80\,b}+\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{3\,e^{(bx+a)}}{8\,b}-\frac{e^{(-bx-a)}}{4\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/80*e^(5*b*x + 5*a)/b + 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 1/4*e
^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

Fricas [A] time = 1.79349, size = 325, normalized size = 3.92

$$-\frac{\cosh\left(bx+a\right)^{4}-16\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+2\left(3\,\cosh\left(bx+a\right)^{2}+10\right)\sinh\left(bx+a\right)^{2}+20\,\cos}{120\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(b*x + a)^2 + 20*cosh(b*x + a)^2 - 16*(c
osh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) - b*
sinh(b*x + a))
```

Sympy [A] time = 82.5895, size = 139, normalized size = 1.67

$$\begin{cases}\frac{8e^{a}e^{bx}\sinh^{4}\left(a+bx\right)}{15b}-\frac{8e^{a}e^{bx}\sinh^{3}\left(a+bx\right)\cosh\left(a+bx\right)}{15b}-\frac{4e^{a}e^{bx}\sinh^{2}\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{5b}+\frac{4e^{a}e^{bx}\sinh\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{5b}+\frac{e^{a}e^{bx}\cosh^{4}\left(a+bx\right)}{5b}\\ xe^{a}\cosh^{4}\left(a\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((8*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) - 8*exp(a)*exp(b*x)*si
nh(a + b*x)**3*cosh(a + b*x)/(15*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*co
sh(a + b*x)**2/(5*b) + 4*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(5*
b) + exp(a)*exp(b*x)*cosh(a + b*x)**4/(5*b), Ne(b, 0)), (x*exp(a)*cosh(a)**
4, True))
```

Giac [A] time = 1.32643, size = 81, normalized size = 0.98

$$-\frac{5\left(12\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}-3\,e^{(5\,bx+5\,a)}-20\,e^{(3\,bx+3\,a)}-90\,e^{(bx+a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/240*(5*(12*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - 3*e^(5*b*x + 5*a) - 2
0*e^(3*b*x + 3*a) - 90*e^(b*x + a))/b
```

3.265 $\int e^{a+bx}\cosh^3(a+bx)\,dx$

Optimal. Leaf size=57

$$-\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

```
[Out] -E^(-2*a - 2*b*x)/(16*b) + (3*E^(2*a + 2*b*x))/(16*b) + E^(4*a + 4*b*x)/(32
*b) + (3*x)/8
```

Rubi [A] time = 0.0367967, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 266, 43}

$$-\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3,x]
```


```
[Out] -E^(-2*a - 2*b*x)/(16*b) + (3*E^(2*a + 2*b*x))/(16*b) + E^(4*a + 4*b*x)/(32
*b) + (3*x)/8
```


Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cosh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{2a+2bx}\right)}{16b}\\ &= -\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}\end{aligned}$$

Mathematica [A] time = 0.0333287, size = 47, normalized size = 0.82

$$\frac{-e^{-2(a+bx)}+3e^{2(a+bx)}+\frac{1}{2}e^{4(a+bx)}+6bx}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3,x]
```

```
[Out] (-E^(-2*(a + b*x)) + 3*E^(2*(a + b*x)) + E^(4*(a + b*x))/2 + 6*b*x)/(16*b)
```

Maple [A] time = 0.01, size = 67, normalized size = 1.2

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^2(\cosh(bx+a))^2}{4}+\frac{(\cosh(bx+a))^2}{4}+\left(\frac{(\cosh(bx+a))^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3\,bx}{8}+\frac{3}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/4*sinh(b*x+a)^2*cosh(b*x+a)^2+1/4*cosh(b*x+a)^2+(1/4*cosh(b*x+a)^3+3
/8*cosh(b*x+a))*sinh(b*x+a)+3/8*b*x+3/8*a)
```

Maxima [A] time = 1.01722, size = 72, normalized size = 1.26

$$\frac{3(bx+a)}{8b}+\frac{e^{(4bx+4a)}}{32b}+\frac{3e^{(2bx+2a)}}{16b}-\frac{e^{(-2bx-2a)}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b + 3/16*e^(2*b*x + 2*a)/b - 1/16*e^
(-2*b*x - 2*a)/b
```

Fricas [B] time = 1.68195, size = 263, normalized size = 4.61

$$-\frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2-3\sinh(bx+a)^3-6(2bx+1)\cosh(bx+a)+3\left(4bx-3\cosh(bx+\right.}{32(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 3*sinh(b*x + a)^
3 - 6*(2*b*x + 1)*cosh(b*x + a) + 3*(4*b*x - 3*cosh(b*x + a)^2 - 2)*sinh(b*
x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 21.1572, size = 182, normalized size = 3.19

$$\begin{cases}\frac{3xe^{a}e^{bx}\sinh^{3}(a+bx)}{8}-\frac{3xe^{a}e^{bx}\sinh^{2}(a+bx)\cosh(a+bx)}{8}-\frac{3xe^{a}e^{bx}\sinh(a+bx)\cosh^{2}(a+bx)}{8}+\frac{3xe^{a}e^{bx}\cosh^{3}(a+bx)}{8}-\frac{3e^{a}e^{bx}\sinh^{3}(a+bx)}{8b}+\frac{3e^{a}e^{bx}\text{s}\ldots \\ xe^{a}\cosh^{3}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*sin
h(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a +
b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - 3*exp(a)*exp(b*x)*sinh
(a + b*x)**3/(8*b) + 3*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**2/(4*b)
 - exp(a)*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*cosh(a)**3,
 True))
```

<u>Giac</u> [A] time = 1.28948, size = 77, normalized size = 1.35

$$\frac{12\,bx - 2\left(3\,e^{(2\,bx+2\,a)} + 1\right)e^{(-2\,bx-2\,a)} + 12\,a + e^{(4\,bx+4\,a)} + 6\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*x
 + 4*a) + 6*e^(2*b*x + 2*a))/b
```

3.266 $\int e^{a+bx} \cosh^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

```
[Out] -E^(-a - b*x)/(4*b) + E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(12*b)
```

Rubi [A] time = 0.029496, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 270}

$$-\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2,x]
```

```
[Out] -E^(-a - b*x)/(4*b) + E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(12*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{4x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, e^{a+bx}\right)}{4b}$$
$$= -\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

Mathematica [A] time = 0.0204152, size = 39, normalized size = 0.8

$$\frac{e^{-a-bx}\left(6e^{2(a+bx)}+e^{4(a+bx)}-3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (E^(-a - b*x)*(-3 + 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

Maple [A] time = 0.01, size = 49, normalized size = 1.

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^2\cosh(bx+a)}{3}+\frac{\cosh(bx+a)}{3}+\left(\frac{2}{3}+\frac{(\cosh(bx+a))^2}{3}\right)\sinh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/3*sinh(b*x+a)^2*cosh(b*x+a)+1/3*cosh(b*x+a)+(2/3+1/3*cosh(b*x+a)^2)*
sinh(b*x+a))
```

Maxima [A] time = 1.18352, size = 54, normalized size = 1.1

$$\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*e^(3*b*x + 3*a)/b + 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b
```

Fricas [A] time = 2.01223, size = 154, normalized size = 3.14

$$-\frac{\cosh\left(bx+a\right)^{2}-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-3}{6\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 3
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 5.69518, size = 78, normalized size = 1.59

$$\begin{cases} -\frac{2e^{a}e^{bx}\sinh^{2}\left(a+bx\right)}{3b}+\frac{2e^{a}e^{bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{3b}+\frac{e^{a}e^{bx}\cosh^{2}\left(a+bx\right)}{3b} & \text{for}\ b\neq 0\\ xe^{a}\cosh^{2}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*si
nh(a + b*x)*cosh(a + b*x)/(3*b) + exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), N
e(b, 0)), (x*exp(a)*cosh(a)**2, True))
```

Giac [A] time = 1.22713, size = 46, normalized size = 0.94

$$\frac{e^{(3\,bx+3\,a)} + 6\,e^{(bx+a)} - 3\,e^{(-bx-a)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*(e^(3*b*x + 3*a) + 6*e^(b*x + a) - 3*e^(-b*x - a))/b
```

3.267 $\int e^{a+bx}\cosh(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{2a+2bx}}{4b}+\frac{x}{2}$$

```
[Out] E^(2*a + 2*b*x)/(4*b) + x/2
```

Rubi [A] time = 0.0151755, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 12, 14}

$$\frac{e^{2a+2bx}}{4b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) + x/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{4b}+\frac{x}{2}$$

Mathematica [A] time = 0.0111907, size = 23, normalized size = 1.

$$\frac{e^{2a+2bx}}{4b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) + x/2
```

Maple [A] time = 0.007, size = 37, normalized size = 1.6

$$\frac{1}{b}\left(\frac{(\cosh(bx+a))^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a),x)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)^2+1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 1.11335, size = 32, normalized size = 1.39

$$\frac{1}{2}x+\frac{a}{2b}+\frac{e^{(2bx+2a)}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

Fricas [B] time = 1.79648, size = 131, normalized size = 5.7

$$\frac{(2\,bx+1)\cosh\left(bx+a\right)-(2\,bx-1)\sinh\left(bx+a\right)}{4\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*x + 1)*cosh(b*x + a) - (2*b*x - 1)*sinh(b*x + a))/(b*cosh(b*x + a
) - b*sinh(b*x + a))
```

Sympy [A] time = 1.25577, size = 80, normalized size = 3.48

$$\begin{cases} -\frac{xe^{a}e^{bx}\sinh\left(a+bx\right)}{2}+\frac{xe^{a}e^{bx}\cosh\left(a+bx\right)}{2}+\frac{e^{a}e^{bx}\sinh\left(a+bx\right)}{b}-\frac{e^{a}e^{bx}\cosh\left(a+bx\right)}{2b} & \text{for } b\neq 0\\ xe^{a}\cosh\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)/2 + x*exp(a)*exp(b*x)*cosh(a +
b*x)/2 + exp(a)*exp(b*x)*sinh(a + b*x)/b - exp(a)*exp(b*x)*cosh(a + b*x)/(2
*b), Ne(b, 0)), (x*exp(a)*cosh(a), True))
```

Giac [A] time = 1.19928, size = 30, normalized size = 1.3

$$\frac{2\,bx+2\,a+e^{(2\,bx+2\,a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x + 2*a + e^(2*b*x + 2*a))/b
```

3.268 $\int e^{a+bx}\textbf{sech}(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{\log\left(e^{2a+2bx}+1\right)}{b}$$

```
[Out] Log[1 + E^(2*a + 2*b*x)]/b
```

Rubi [A] time = 0.0173843, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 12, 260}

$$\frac{\log\left(e^{2a+2bx}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x],x]
```

```
[Out] Log[1 + E^(2*a + 2*b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int e^{a+bx}\operatorname{sech}(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{2\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\log\left(1+e^{2a+2bx}\right)}{b}$$

Mathematica [A] time = 0.0124252, size = 17, normalized size = 1.

$$\frac{\log\left(e^{2a+2bx}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x],x]
```

```
[Out] Log[1 + E^(2*a + 2*b*x)]/b
```

Maple [A] time = 0.006, size = 19, normalized size = 1.1

$$x+\frac{\ln\left(\cosh\left(bx+a\right)\right)}{b}+\frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a),x)
```

```
[Out] x+1/b*ln(cosh(b*x+a))+1/b*a
```

Maxima [A] time = 1.59275, size = 22, normalized size = 1.29

$$\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(2*b*x + 2*a) + 1)/b
```

Fricas [A] time = 1.78251, size = 76, normalized size = 4.47

$$\frac{\log\left(\frac{2\,\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\,\mathrm{sech}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x), x)
```

Giac [A] time = 1.16051, size = 22, normalized size = 1.29

$$\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] log(e^(2*b*x + 2*a) + 1)/b
```

3.269 $\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}$$

```
[Out] (-2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))) + (2*ArcTan[E^(a + b*x)])/b
```

Rubi [A] time = 0.0293319, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 288, 203}

$$\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^2,x]
```

```
[Out] (-2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))) + (2*ArcTan[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^2}{\left(1+x^2\right)^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{4\operatorname{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= -\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{2\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= -\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0610293, size = 36, normalized size = 0.9

$$\frac{2\left(\tan^{-1}\left(e^{a+bx}\right)-\frac{e^{a+bx}}{e^{2(a+bx)}+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^2,x]
```

```
[Out] (2*(-(E^(a + b*x)/(1 + E^(2*(a + b*x)))) + ArcTan[E^(a + b*x)]))/b
```

Maple [A] time = 0.011, size = 45, normalized size = 1.1

$$\frac{(\sinh(bx+a))^2}{b\cosh(bx+a)}-\frac{\cosh(bx+a)}{b}+2\,\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] 1/b*sinh(b*x+a)^2/cosh(b*x+a)-1/b*cosh(b*x+a)+2*arctan(exp(b*x+a))/b
```

Maxima [A] time = 1.65203, size = 50, normalized size = 1.25

$$\frac{2\arctan\left(e^{(bx+a)}\right)}{b} - \frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(b*x + a))/b - 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) + 1))
```

Fricas [B] time = 1.73908, size = 304, normalized size = 7.6

$$\frac{2\left(\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(b\right.}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*
arctan(cosh(b*x + a) + sinh(b*x + a)) - cosh(b*x + a) - sinh(b*x + a))/(b*c
osh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**2, x)
```

Giac [A] time = 1.30845, size = 50, normalized size = 1.25

$$\frac{2 \arctan\left(e^{(bx+a)}\right)}{b} - \frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*arctan(e^(b*x + a))/b - 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) + 1))
```

3.270 $\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}+1\right)^2}$$

```
[Out] (2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

Rubi [A] time = 0.0265666, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 264}

$$\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^3,x]
```

```
[Out] (2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{8x^3}{(1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{8\,\text{Subst}\left(\int \frac{x^3}{(1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{2e^{4a+4bx}}{b\left(1+e^{2a+2bx}\right)^2}$$

Mathematica [A] time = 0.0177166, size = 29, normalized size = 1.

$$\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^3,x]
```

```
[Out] (2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

Maple [A] time = 0.037, size = 30, normalized size = 1.

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^2}{2\,(\cosh(bx+a))^2}+\tanh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] 1/b*(1/2*sinh(b*x+a)^2/cosh(b*x+a)^2+tanh(b*x+a))
```

Maxima [B] time = 1.06647, size = 92, normalized size = 3.17

$$-\frac{4\,e^{(2\,bx+2\,a)}}{b\left(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1\right)}-\frac{2}{b\left(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1)) - 2/(b*(e^
(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1))
```

Fricas [B] time = 1.65953, size = 238, normalized size = 8.21

$$-\frac{2\,(3\,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))}{b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)+\left(3\,b\cosh\left(bx+a\right)^{2}+b\right)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**3, x)
```

Giac [A] time = 1.28117, size = 42, normalized size = 1.45

$$-\frac{2\left(2\,e^{(2\,bx+2\,a)}+1\right)}{b\left(e^{(2\,bx+2\,a)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

3.271 $\int e^{a+bx}\mathbf{sech}^4(a+bx)\,dx$

Optimal. Leaf size=95

$$\frac{e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)^2}-\frac{8e^{3a+3bx}}{3b\left(e^{2a+2bx}+1\right)^3}+\frac{\tan^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (-8*E^(3*a + 3*b*x))/(3*b*(1 + E^(2*a + 2*b*x))^3) - (2*E^(a + b*x))/(b*(1
+ E^(2*a + 2*b*x))^2) + E^(a + b*x)/(b*(1 + E^(2*a + 2*b*x))) + ArcTan[E^(a
 + b*x)]/b
```

Rubi [A] time = 0.0467283, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2282, 12, 288, 199, 203}

$$\frac{e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)^2}-\frac{8e^{3a+3bx}}{3b\left(e^{2a+2bx}+1\right)^3}+\frac{\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^4,x]
```

```
[Out] (-8*E^(3*a + 3*b*x))/(3*b*(1 + E^(2*a + 2*b*x))^3) - (2*E^(a + b*x))/(b*(1
+ E^(2*a + 2*b*x))^2) + E^(a + b*x)/(b*(1 + E^(2*a + 2*b*x))) + ArcTan[E^(a
 + b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{16x^4}{(1+x^2)^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{16\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3} + \frac{8\,\text{Subst}\left(\int \frac{x^2}{(1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{2\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} + \frac{\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [A] time = 0.0828558, size = 64, normalized size = 0.67

$$\frac{e^{a+bx}\left(-8e^{2(a+bx)}+3e^{4(a+bx)}-3\right)}{3b\left(e^{2(a+bx)}+1\right)^3} + \frac{\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^4,x]
```

```
[Out] (E^(a + b*x)*(-3 - 8*E^(2*(a + b*x)) + 3*E^(4*(a + b*x))))/(3*b*(1 + E^(2*(
a + b*x)))^3) + ArcTan[E^(a + b*x)]/b
```

Maple [A] time = 0.04, size = 83, normalized size = 0.9

$$\frac{(\sinh(bx+a))^2}{3\,b\,(\cosh(bx+a))^3} + \frac{(\sinh(bx+a))^2}{3\,b\cosh(bx+a)} - \frac{\cosh(bx+a)}{3\,b} + \frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2\,b} + \frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^4,x)
```

```
[Out] 1/3/b*sinh(b*x+a)^2/cosh(b*x+a)^3+1/3/b*sinh(b*x+a)^2/cosh(b*x+a)-1/3/b*cos
h(b*x+a)+1/2/b*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*x+a))/b
```

Maxima [A] time = 1.56712, size = 112, normalized size = 1.18

$$\frac{\arctan\left(e^{(bx+a)}\right)}{b}+\frac{3\,e^{(5\,bx+5\,a)}-8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{3\,b\left(e^{(6\,bx+6\,a)}+3\,e^{(4\,bx+4\,a)}+3\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] arctan(e^(b*x + a))/b + 1/3*(3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(b
*x + a))/(b*(e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) + 3*e^(2*b*x + 2*a) + 1))
```

Fricas [B] time = 1.8413, size = 1424, normalized size = 14.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)
^5 + 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^3 - 8*cosh(b*x + a)^3 + 6*(5*
cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6
*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x +
 a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b
*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + co
sh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(
5*cosh(b*x + a)^4 - 8*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(b*x + a))
/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6
 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5
*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)
```

```
^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*
(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) +
 b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{sech}^{4}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**4,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**4, x)
```

Giac [A] time = 1.25156, size = 82, normalized size = 0.86

$$\frac{\arctan\left(e^{(bx+a)}\right)}{b}+\frac{3\,e^{(5\,bx+5\,a)}-8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{3\,b\left(e^{(2\,bx+2\,a)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] arctan(e^(b*x + a))/b + 1/3*(3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(b
*x + a))/(b*(e^(2*b*x + 2*a) + 1)^3)
```

3.272 $\int e^{a+bx}\textbf{sech}^5(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{8}{b\left(e^{2a+2bx}+1\right)^2}+\frac{32}{3b\left(e^{2a+2bx}+1\right)^3}-\frac{4}{b\left(e^{2a+2bx}+1\right)^4}$$

```
[Out] -4/(b*(1 + E^(2*a + 2*b*x))^4) + 32/(3*b*(1 + E^(2*a + 2*b*x))^3) - 8/(b*(1
 + E^(2*a + 2*b*x))^2)
```

Rubi [A] time = 0.0477429, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 266, 43}

$$-\frac{8}{b\left(e^{2a+2bx}+1\right)^2}+\frac{32}{3b\left(e^{2a+2bx}+1\right)^3}-\frac{4}{b\left(e^{2a+2bx}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^5,x]
```

```
[Out] -4/(b*(1 + E^(2*a + 2*b*x))^4) + 32/(3*b*(1 + E^(2*a + 2*b*x))^3) - 8/(b*(1
 + E^(2*a + 2*b*x))^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{32x^5}{\left(1+x^2\right)^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{32\,\text{Subst}\left(\int \frac{x^5}{\left(1+x^2\right)^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{16\,\text{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{16\,\text{Subst}\left(\int \left(\frac{1}{(1+x)^5}-\frac{2}{(1+x)^4}+\frac{1}{(1+x)^3}\right)dx, x, e^{2a+2bx}\right)}{b}\\
&= -\frac{4}{b\left(1+e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{8}{b\left(1+e^{2a+2bx}\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0342389, size = 44, normalized size = 0.73

$$-\frac{4\left(4e^{2(a+bx)}+6e^{4(a+bx)}+1\right)}{3b\left(e^{2(a+bx)}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^5,x]
```

```
[Out] (-4*(1 + 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(1 + E^(2*(a + b*x)))
^4)
```

Maple [A] time = 0.035, size = 61, normalized size = 1.

$$\frac{1}{b}\left(\frac{(\sinh(bx+a))^2}{4\,(\cosh(bx+a))^4}+\frac{(\sinh(bx+a))^2}{4\,(\cosh(bx+a))^2}+\left(\frac{2}{3}+\frac{(\mathrm{sech}\,(bx+a))^2}{3}\right)\tanh(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^5,x)
```

```
[Out] 1/b*(1/4*sinh(b*x+a)^2/cosh(b*x+a)^4+1/4*sinh(b*x+a)^2/cosh(b*x+a)^2+(2/3+1
/3*sech(b*x+a)^2)*tanh(b*x+a))
```

Maxima [B] time = 1.04132, size = 232, normalized size = 3.87

$$-\frac{8\,e^{(4\,bx+4\,a)}}{b\left(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\right)}-\frac{16\,e^{(2\,bx+2\,a)}}{3\,b\left(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4
*a) + 4*e^(2*b*x + 2*a) + 1)) - 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a) +
4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b*(e
^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a)
+ 1))
```

Fricas [B] time = 1.78063, size = 635, normalized size = 10.58

$$-\frac{}{3\left(b\cosh(bx+a)^6+6\,b\cosh(bx+a)\sinh(bx+a)^5+b\sinh(bx+a)^6+4\,b\cosh(bx+a)^4+\left(15\,b\cosh(bx+a)^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)^
2 + 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x
```

```
+ a)^6 + 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + 4*b)*sinh(b*x + a)^4
 + 4*(5*b*cosh(b*x + a)^3 + 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*cosh(b
*x + a)^2 + (15*b*cosh(b*x + a)^4 + 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b*x +
a)^2 + 2*(3*b*cosh(b*x + a)^5 + 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x + a))*si
nh(b*x + a) + 4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21968, size = 57, normalized size = 0.95

$$-\frac{4\left(6e^{(4bx+4a)} + 4e^{(2bx+2a)} + 1\right)}{3b\left(e^{(2bx+2a)} + 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^4
)
```

3.273 $\int e^x \cosh^2(2x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}$$

```
[Out] -1/(12*E^(3*x)) + E^x/2 + E^(5*x)/20
```

Rubi [A] time = 0.0202289, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 12, 270}

$$-\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[2*x]^2,x]
```

```
[Out] -1/(12*E^(3*x)) + E^x/2 + E^(5*x)/20
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^x \cosh^2(2x)\,dx &= \text{Subst}\left(\int \frac{\left(1+x^4\right)^2}{4x^4}\,dx, x, e^x\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{\left(1+x^4\right)^2}{x^4}\,dx, x, e^x\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \left(2+\frac{1}{x^4}+x^4\right)dx, x, e^x\right)\\
&= -\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}
\end{aligned}
$$

Mathematica [A] time = 0.0133952, size = 26, normalized size = 1.

$$
-\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[2*x]^2,x]
```

```
[Out] -1/(12*E^(3*x)) + E^x/2 + E^(5*x)/20
```

Maple [A] time = 0.019, size = 34, normalized size = 1.3

$$
\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}+\frac{\cosh(x)}{2}-\frac{\cosh(3x)}{12}+\frac{\cosh(5x)}{20}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(2*x)^2,x)
```

```
[Out] 1/2*sinh(x)+1/12*sinh(3*x)+1/20*sinh(5*x)+1/2*cosh(x)-1/12*cosh(3*x)+1/20*c
osh(5*x)
```

Maxima [A] time = 1.06247, size = 23, normalized size = 0.88

$$
\frac{1}{20}\,e^{(5x)}-\frac{1}{12}\,e^{(-3x)}+\frac{1}{2}\,e^x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

Fricas [B] time = 1.82604, size = 170, normalized size = 6.54

$$-\frac{\cosh\left(x\right)^{4} - 16\,\cosh\left(x\right)^{3}\sinh\left(x\right) + 6\,\cosh\left(x\right)^{2}\sinh\left(x\right)^{2} - 16\,\cosh\left(x\right)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} - 15}{30\left(\cosh\left(x\right) - \sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4 - 15)/(cosh(x) - sinh(x))
```

Sympy [B] time = 0.814416, size = 42, normalized size = 1.62

$$-\frac{8e^{x}\sinh^{2}\left(2x\right)}{15} + \frac{4e^{x}\sinh\left(2x\right)\cosh\left(2x\right)}{15} + \frac{7e^{x}\cosh^{2}\left(2x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)**2,x)
```

```
[Out] -8*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 + 7*exp(x)*cosh
(2*x)**2/15
```

Giac [A] time = 1.23804, size = 23, normalized size = 0.88

$$\frac{1}{20}\,e^{\left(5\,x\right)} - \frac{1}{12}\,e^{\left(-3\,x\right)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

3.274 $\int e^x \cosh(2x) \, dx$

Optimal. Leaf size=19

$$\frac{e^{3x}}{6} - \frac{e^{-x}}{2}$$

```
[Out] -1/(2*E^x) + E^(3*x)/6
```

Rubi [A] time = 0.0113203, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{3x}}{6} - \frac{e^{-x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[2*x],x]
```

```
[Out] -1/(2*E^x) + E^(3*x)/6
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$
\begin{aligned}
\int e^x \cosh(2x)\,dx &= \text{Subst}\left(\int \frac{1+x^4}{2x^2}\,dx, x, e^x\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^4}{x^2}\,dx, x, e^x\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^2}+x^2\right)dx, x, e^x\right)\\
&= -\frac{e^{-x}}{2}+\frac{e^{3x}}{6}
\end{aligned}
$$

Mathematica [A] time = 0.0083832, size = 16, normalized size = 0.84

$$\frac{1}{6}e^{-x}\left(e^{4x}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[2*x],x]
```

```
[Out] (-3 + E^(4*x))/(6*E^x)
```

Maple [A] time = 0.007, size = 22, normalized size = 1.2

$$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}-\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(2*x),x)
```

```
[Out] 1/2*sinh(x)+1/6*sinh(3*x)-1/2*cosh(x)+1/6*cosh(3*x)
```

Maxima [A] time = 1.00421, size = 18, normalized size = 0.95

$$\frac{1}{6}\,e^{(3\,x)}-\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x, algorithm="maxima")
```

```
[Out] 1/6*e^(3*x) - 1/2*e^(-x)
```

Fricas [A] time = 1.74545, size = 95, normalized size = 5.

$$-\frac{\cosh(x)^2 - 4\cosh(x)\sinh(x) + \sinh(x)^2}{3(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x, algorithm="fricas")
```

```
[Out] -1/3*(cosh(x)^2 - 4*cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

Sympy [A] time = 0.302073, size = 20, normalized size = 1.05

$$\frac{2e^x\sinh(2x)}{3} - \frac{e^x\cosh(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x)
```

```
[Out] 2*exp(x)*sinh(2*x)/3 - exp(x)*cosh(2*x)/3
```

Giac [A] time = 1.25778, size = 18, normalized size = 0.95

$$\frac{1}{6}e^{(3x)} - \frac{1}{2}e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x, algorithm="giac")
```

```
[Out] 1/6*e^(3*x) - 1/2*e^(-x)
```

3.275 $\int e^x \mathbf{sech}(2x)\,dx$

Optimal. Leaf size=92

$$\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2] + Log[
1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*
Sqrt[2])
```

Rubi [A] time = 0.0635706, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2282, 12, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2] + Log[
1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*
Sqrt[2])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int e^x\mathrm{sech}(2x)\,dx &= \mathrm{Subst}\left(\int\frac{2x^2}{1+x^4}\,dx,x,e^x\right)\\ &= 2\,\mathrm{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,e^x\right)\\ &= -\,\mathrm{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,e^x\right)+\mathrm{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,e^x\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,e^x\right)+\frac{1}{2}\,\mathrm{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,e^x\right)+\frac{\mathrm{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,\right.}{2\sqrt{2}}\\ &= \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}+\frac{\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,\right.}{\sqrt{2}}\\ &= -\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.0102933, size = 24, normalized size = 0.26

$$\frac{2}{3}e^{3x}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{4x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x],x]
```

```
[Out] (2*E^(3*x)*Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)])/3
```

Maple [C] time = 0.041, size = 25, normalized size = 0.3

$$2\sum_{\_R=\mathrm{RootOf}\left(256\_Z^4+1\right)}\_R\ln\left(64\_R^3+e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x),x)
```

```
[Out] 2*sum(_R*ln(64*_R^3+exp(x)),_R=RootOf(256*_Z^4+1))
```

Maxima [A] time = 1.56204, size = 103, normalized size = 1.12

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

Fricas [A] time = 1.96356, size = 354, normalized size = 3.85

$$-\sqrt{2}\arctan\left(-\sqrt{2}e^x+\sqrt{2}\sqrt{\sqrt{2}e^x+e^{(2x)}+1}-1\right)-\sqrt{2}\arctan\left(-\sqrt{2}e^x+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^x+4e^{(2x)}+4}+1\right)-\frac{1}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1)
 - sqrt(2)*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x
) + 4) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1/4*sqrt(2)*
log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{sech}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x)
```

```
[Out] Integral(exp(x)*sech(2*x), x)
```

Giac [A] time = 1.16802, size = 103, normalized size = 1.12

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

3.276 $\int e^x \mathbf{sech}^2(2x)\,dx$

Optimal. Leaf size=111

$$-\frac{e^x}{e^{4x}+1}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}$$

```
[Out] -(E^x/(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 + Sqr
t[2]*E^x]/(2*Sqrt[2]) - Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) + Log[1
+ Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rubi [A] time = 0.0747904, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2282, 12, 288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{e^x}{e^{4x}+1}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]^2,x]
```

```
[Out] -(E^x/(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 + Sqr
t[2]*E^x]/(2*Sqrt[2]) - Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) + Log[1
+ Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^x \operatorname{sech}^2(2x)\,dx = \text{Subst}\left(\int \frac{4x^4}{\left(1+x^4\right)^2}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^4\right)^2}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1+e^{4x}} + \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1+e^{4x}} + \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1+e^{4x}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{e^x}{1+e^{4x}} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\text{Sub}}{}$$
$$= -\frac{e^x}{1+e^{4x}} - \frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.0851417, size = 106, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{8e^x}{e^{4x}+1} - \sqrt{2}\log\left(-\sqrt{2}e^x+e^{2x}+1\right) + \sqrt{2}\log\left(\sqrt{2}e^x+e^{2x}+1\right) - 2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}e^x\right) + 2\sqrt{2}\tan^{-1}\left(\sqrt{2}e^x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]^2,x]
```

```
[Out] ((-8*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^x] + 2*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]*E^x + E^(2*x)] + Sqrt[2]*Log
[1 + Sqrt[2]*E^x + E^(2*x)])/8
```

Maple [C] time = 0.043, size = 36, normalized size = 0.3

$$-\frac{e^{x}}{1+e^{4 x}}+4 \sum_{\_R=\mathrm{RootOf}\left(65536\_Z^{4}+1\right)} \_R \ln\left(e^{x}+16\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)^2,x)
```

```
[Out] -exp(x)/(1+exp(4*x))+4*sum(_R*ln(exp(x)+16*_R),_R=RootOf(65536*_Z^4+1))
```

Maxima [A] time = 1.56247, size = 119, normalized size = 1.07

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

Fricas [B] time = 1.93908, size = 494, normalized size = 4.45

$$-\frac{4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)+4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+\right.}{8\left(e^{(4x)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt
(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)
*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - (sqrt(2)*e^(
4*x) + sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*e^(4*x) + sqr
```

```
t(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 8*e^x)/(e^(4*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{sech}^2(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)**2,x)
```

```
[Out] Integral(exp(x)*sech(2*x)**2, x)
```

Giac [A] time = 1.22135, size = 119, normalized size = 1.07

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

3.277 $\int e^x \cosh^2(3x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

```
[Out] -1/(20*E^(5*x)) + E^x/2 + E^(7*x)/28
```

Rubi [A] time = 0.0191598, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 12, 270}

$$-\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[3*x]^2,x]
```

```
[Out] -1/(20*E^(5*x)) + E^x/2 + E^(7*x)/28
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int e^x \cosh^2(3x)\,dx = \text{Subst}\left(\int \frac{(1+x^6)^2}{4x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{(1+x^6)^2}{x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \left(2+\frac{1}{x^6}+x^6\right)dx, x, e^x\right)$$
$$= -\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Mathematica [A] time = 0.0148954, size = 26, normalized size = 1.

$$-\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[3*x]^2,x]
```

```
[Out] -1/(20*E^(5*x)) + E^x/2 + E^(7*x)/28
```

Maple [A] time = 0.016, size = 34, normalized size = 1.3

$$\frac{\sinh(x)}{2}+\frac{\sinh(5x)}{20}+\frac{\sinh(7x)}{28}+\frac{\cosh(x)}{2}-\frac{\cosh(5x)}{20}+\frac{\cosh(7x)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(3*x)^2,x)
```

```
[Out] 1/2*sinh(x)+1/20*sinh(5*x)+1/28*sinh(7*x)+1/2*cosh(x)-1/20*cosh(5*x)+1/28*c
osh(7*x)
```

Maxima [A] time = 1.04664, size = 23, normalized size = 0.88

$$\frac{1}{28}\,e^{(7\,x)}-\frac{1}{20}\,e^{(-5\,x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

Fricas [B] time = 1.78877, size = 240, normalized size = 9.23

$$-\frac{\cosh(x)^6 - 36\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 - 120\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 - 36\cosh}{70\left(\cosh(x) - \sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)^6
 - 35)/(cosh(x) - sinh(x))
```

Sympy [B] time = 0.802695, size = 42, normalized size = 1.62

$$-\frac{18e^x\sinh^2(3x)}{35} + \frac{6e^x\sinh(3x)\cosh(3x)}{35} + \frac{17e^x\cosh^2(3x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)**2,x)
```

```
[Out] -18*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 + 17*exp(x)*co
sh(3*x)**2/35
```

Giac [A] time = 1.20983, size = 23, normalized size = 0.88

$$\frac{1}{28}e^{(7x)} - \frac{1}{20}e^{(-5x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

3.278 $\int e^x \cosh(3x)\,dx$

Optimal. Leaf size=19

$$\frac{e^{4x}}{8}-\frac{1}{4}e^{-2x}$$

```
[Out] -1/(4*E^(2*x)) + E^(4*x)/8
```

Rubi [A] time = 0.0116903, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{4x}}{8}-\frac{1}{4}e^{-2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[3*x],x]
```

```
[Out] -1/(4*E^(2*x)) + E^(4*x)/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int e^x \cosh(3x)\,dx &= \text{Subst}\left(\int \frac{1+x^6}{2x^3}\,dx, x, e^x\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^6}{x^3}\,dx, x, e^x\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^3}+x^3\right)dx, x, e^x\right)\\ &= -\frac{1}{4}e^{-2x}+\frac{e^{4x}}{8}\end{aligned}$$

Mathematica [A] time = 0.0092524, size = 16, normalized size = 0.84

$$\frac{1}{8}e^{-2x}\left(e^{6x}-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[3*x],x]
```

```
[Out] (-2 + E^(6*x))/(8*E^(2*x))
```

Maple [A] time = 0.018, size = 26, normalized size = 1.4

$$\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}-\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(3*x),x)
```

```
[Out] 1/4*sinh(2*x)+1/8*sinh(4*x)-1/4*cosh(2*x)+1/8*cosh(4*x)
```

Maxima [A] time = 1.05605, size = 18, normalized size = 0.95

$$\frac{1}{8}\,e^{(4\,x)}-\frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x, algorithm="maxima")
```

```
[Out] 1/8*e^(4*x) - 1/4*e^(-2*x)
```

Fricas [B] time = 1.9006, size = 130, normalized size = 6.84

$$-\frac{\cosh\left(x\right)^{3}-9\,\cosh\left(x\right)^{2}\sinh\left(x\right)+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}-3\,\sinh\left(x\right)^{3}}{8\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(x)^3 - 9*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 - 3*sinh(x)^3)/
(cosh(x) - sinh(x))
```

Sympy [A] time = 0.301793, size = 20, normalized size = 1.05

$$\frac{3e^{x}\sinh\left(3x\right)}{8}-\frac{e^{x}\cosh\left(3x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x)
```

```
[Out] 3*exp(x)*sinh(3*x)/8 - exp(x)*cosh(3*x)/8
```

Giac [A] time = 1.22303, size = 18, normalized size = 0.95

$$\frac{1}{8}\,e^{\left(4\,x\right)}-\frac{1}{4}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x, algorithm="giac")
```

```
[Out] 1/8*e^(4*x) - 1/4*e^(-2*x)
```

3.279 $\int e^x \mathbf{sech}(3x)\,dx$

Optimal. Leaf size=55

$$-\frac{1}{3}\log\left(e^{2x}+1\right)+\frac{1}{6}\log\left(-e^{2x}+e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*E^(2*x))/Sqrt[3]]/Sqrt[3]) - Log[1 + E^(2*x)]/3 + Log[1 - E
^(2*x) + E^(4*x)]/6
```

Rubi [A] time = 0.0584103, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2282, 12, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{3}\log\left(e^{2x}+1\right)+\frac{1}{6}\log\left(-e^{2x}+e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[3*x],x]
```

```
[Out] -(ArcTan[(1 - 2*E^(2*x))/Sqrt[3]]/Sqrt[3]) - Log[1 + E^(2*x)]/3 + Log[1 - E
^(2*x) + E^(4*x)]/6
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^x \mathrm{sech}(3x)\,dx = \mathrm{Subst}\left(\int \frac{2x^3}{1+x^6}\,dx, x, e^x\right)$$
$$= 2\,\mathrm{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, e^x\right)$$
$$= \mathrm{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, e^{2x}\right)$$
$$= -\left(\frac{1}{3}\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^{2x}\right)\right) + \frac{1}{3}\mathrm{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, e^{2x}\right)$$
$$= -\frac{1}{3}\log\left(1+e^{2x}\right) + \frac{1}{6}\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, e^{2x}\right) + \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, e^{2x}\right)$$
$$= -\frac{1}{3}\log\left(1+e^{2x}\right) + \frac{1}{6}\log\left(1-e^{2x}+e^{4x}\right) - \mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2e^{2x}\right)$$
$$= \frac{\tan^{-1}\left(\frac{-1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log\left(1+e^{2x}\right) + \frac{1}{6}\log\left(1-e^{2x}+e^{4x}\right)$$

Mathematica [C] time = 0.0107056, size = 24, normalized size = 0.44

$$\frac{1}{2}e^{4x}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-e^{6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[3*x],x]
```

```
[Out] (E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, -E^(6*x)])/2
```

Maple [C] time = 0.04, size = 79, normalized size = 1.4

$$-\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{3} + \frac{1}{6}\ln\left(\mathrm{e}^{2x}-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right) + \frac{i}{6}\ln\left(\mathrm{e}^{2x}-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3} + \frac{1}{6}\ln\left(\mathrm{e}^{2x}-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) - \frac{i}{6}\ln\left(\mathrm{e}^{2x}-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(3*x),x)
```

```
[Out] -1/3*ln(1+exp(2*x))+1/6*ln(exp(2*x)-1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)-1/
2+1/2*I*3^(1/2))*3^(1/2)+1/6*ln(exp(2*x)-1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(2*
```

```
x)-1/2-1/2*I*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.56601, size = 96, normalized size = 1.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\sqrt{3}+2e^{x}\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\sqrt{3}+2e^{x}\right)+\frac{1}{6}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)+\frac{1}{6}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{1}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(sqrt(3) + 2*e^x) + 1/3*sqrt(3)*arctan(-sqrt(3) + 2*e^x)
 + 1/6*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*log(-sqrt(3)*e^x + e^(2*x) + 1)
 - 1/3*log(e^(2*x) + 1)
```

Fricas [A] time = 2.06412, size = 288, normalized size = 5.24

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)+3\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)+\frac{1}{6}\log\left(\frac{2\cosh(x)^{2}+2\sinh(x)^{2}-1}{\cosh(x)^{2}-2\cosh(x)\sinh(x)+\sinh(x)^{2}}\right)-\frac{1}{3}\log\left(\frac{2\cos}{\cosh(x)-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + 3*sqrt(3)*sinh(x))/(cosh(x) - s
inh(x))) + 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 - 1)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 1/3*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\operatorname{sech}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x)
```

```
[Out] Integral(exp(x)*sech(3*x), x)
```

Giac [A] time = 1.20861, size = 59, normalized size = 1.07

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}-1\right)\right)+\frac{1}{6}\log\left(e^{(4x)}-e^{(2x)}+1\right)-\frac{1}{3}\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) - 1)) + 1/6*log(e^(4*x) - e^(2*x)
 + 1) - 1/3*log(e^(2*x) + 1)
```

3.280 $\int e^x \mathbf{sech}^2(3x)\,dx$

Optimal. Leaf size=110

$$-\frac{2e^x}{3\left(e^{6x}+1\right)}-\frac{\log\left(-\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}}+\frac{\log\left(\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}}+\frac{2}{9}\tan^{-1}\left(e^x\right)-\frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right)+\frac{1}{9}\tan^{-1}\left(2e^x+\sqrt{3}\right)$$

```
[Out] (-2*E^x)/(3*(1 + E^(6*x))) + (2*ArcTan[E^x])/9 - ArcTan[Sqrt[3] - 2*E^x]/9
+ ArcTan[Sqrt[3] + 2*E^x]/9 - Log[1 - Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3]) +
Log[1 + Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3])
```

Rubi [A] time = 0.205163, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2282, 12, 288, 209, 634, 618, 204, 628, 203}

$$-\frac{2e^x}{3\left(e^{6x}+1\right)}-\frac{\log\left(-\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}}+\frac{\log\left(\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}}+\frac{2}{9}\tan^{-1}\left(e^x\right)-\frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right)+\frac{1}{9}\tan^{-1}\left(2e^x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[3*x]^2,x]
```


```
[Out] (-2*E^x)/(3*(1 + E^(6*x))) + (2*ArcTan[E^x])/9 - ArcTan[Sqrt[3] - 2*E^x]/9
+ ArcTan[Sqrt[3] + 2*E^x]/9 - Log[1 - Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3]) +
Log[1 + Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3])
```


Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int e^x \operatorname{sech}^2(3x)\,dx = \text{Subst}\left(\int \frac{4x^6}{\left(1+x^6\right)^2}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^6}{\left(1+x^6\right)^2}\,dx, x, e^x\right)$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^6}\,dx, x, e^x\right)$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)} + \frac{2}{9}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) + \frac{2}{9}\text{Subst}\left(\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right) + \frac{2}{9}\text{Subst}\left(\int -\right.$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)} + \frac{2}{9}\tan^{-1}\left(e^x\right) + \frac{1}{18}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right) + \frac{1}{18}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,d\right.$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)} + \frac{2}{9}\tan^{-1}\left(e^x\right) - \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} + \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{-1-x^2}\,d\right.$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)} + \frac{2}{9}\tan^{-1}\left(e^x\right) - \frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right) + \frac{1}{9}\tan^{-1}\left(\sqrt{3}+2e^x\right) - \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} + \cdots$$

Mathematica [C] time = 0.0200028, size = 34, normalized size = 0.31

$$\frac{2}{3}e^x\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-e^{6x}\right)-\frac{1}{e^{6x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[3*x]^2,x]
```

```
[Out] (2*E^x*(-(1 + E^(6*x))^(-1) + Hypergeometric2F1[1/6, 1, 7/6, -E^(6*x)]))/3
```

Maple [C] time = 0.053, size = 59, normalized size = 0.5

$$-\frac{2\,e^{x}}{3+3\,e^{6\,x}}+\frac{i}{9}\ln\left(e^{x}+i\right)-\frac{i}{9}\ln\left(e^{x}-i\right)+4\sum_{\_R=\text{RootOf}\left(1679616\,\_Z^4-1296\,\_Z^2+1\right)}\_R\,\ln\left(e^{x}+36\,\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(3*x)^2,x)
```

```
[Out] -2/3*exp(x)/(1+exp(6*x))+1/9*I*ln(exp(x)+I)-1/9*I*ln(exp(x)-I)+4*sum(_R*ln(
exp(x)+36*_R),_R=RootOf(1679616*_Z^4-1296*_Z^2+1))
```

Maxima [A] time = 1.59934, size = 107, normalized size = 0.97

$$\frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{2\,e^{x}}{3\left(e^{(6x)}+1\right)}+\frac{1}{9}\arctan\left(\sqrt{3}+2\,e^{x}\right)+\frac{1}{9}\arctan\left(-\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) - 1/18*sqrt(3)*log(-sqrt(3)*e^x
 + e^(2*x) + 1) - 2/3*e^x/(e^(6*x) + 1) + 1/9*arctan(sqrt(3) + 2*e^x) + 1/9
*arctan(-sqrt(3) + 2*e^x) + 2/9*arctan(e^x)
```

Fricas [A] time = 2.08446, size = 474, normalized size = 4.31

$$-\frac{4\left(e^{(6x)}+1\right)\arctan\left(\sqrt{3}+\sqrt{-4\sqrt{3}e^{x}+4\,e^{(2x)}+4}-2\,e^{x}\right)+4\left(e^{(6x)}+1\right)\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}e^{x}+e^{(2x)}+1}-2\,e^{x}\right)+\cdots}{18\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(4*(e^(6*x) + 1)*arctan(sqrt(3) + sqrt(-4*sqrt(3)*e^x + 4*e^(2*x) + 4
) - 2*e^x) + 4*(e^(6*x) + 1)*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*e^x + e^(2*x)
 + 1) - 2*e^x) - 4*(e^(6*x) + 1)*arctan(e^x) - (sqrt(3)*e^(6*x) + sqrt(3))*
log(4*sqrt(3)*e^x + 4*e^(2*x) + 4) + (sqrt(3)*e^(6*x) + sqrt(3))*log(-4*sqr
t(3)*e^x + 4*e^(2*x) + 4) + 12*e^x)/(e^(6*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\operatorname{sech}^{2}{\left(3x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)**2,x)
```

```
[Out] Integral(exp(x)*sech(3*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\operatorname{sech}\left(3\,x\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)^2,x, algorithm="giac")
```

```
[Out] integrate(e^x*sech(3*x)^2, x)
```

3.281 $\int e^x \cosh^2(4x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}$$

```
[Out] -1/(28*E^(7*x)) + E^x/2 + E^(9*x)/36
```

Rubi [A] time = 0.0190408, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 12, 270}

$$-\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[4*x]^2,x]
```

```
[Out] -1/(28*E^(7*x)) + E^x/2 + E^(9*x)/36
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \cosh^2(4x)\,dx &= \text{Subst}\left(\int \frac{(1+x^8)^2}{4x^8}\,dx, x, e^x\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(1+x^8)^2}{x^8}\,dx, x, e^x\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(2+\frac{1}{x^8}+x^8\right)dx, x, e^x\right)\\ &= -\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}\end{aligned}$$

Mathematica [A] time = 0.014904, size = 26, normalized size = 1.

$$-\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[4*x]^2,x]
```

```
[Out] -1/(28*E^(7*x)) + E^x/2 + E^(9*x)/36
```

Maple [A] time = 0.013, size = 34, normalized size = 1.3

$$\frac{\sinh(x)}{2}+\frac{\sinh(7x)}{28}+\frac{\sinh(9x)}{36}+\frac{\cosh(x)}{2}-\frac{\cosh(7x)}{28}+\frac{\cosh(9x)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(4*x)^2,x)
```

```
[Out] 1/2*sinh(x)+1/28*sinh(7*x)+1/36*sinh(9*x)+1/2*cosh(x)-1/28*cosh(7*x)+1/36*c
osh(9*x)
```

Maxima [A] time = 1.0343, size = 23, normalized size = 0.88

$$\frac{1}{36}\,e^{(9x)}-\frac{1}{28}\,e^{(-7x)}+\frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)^2,x, algorithm="maxima")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

Fricas [B] time = 1.9059, size = 311, normalized size = 11.96

$$-\frac{\cosh(x)^8 - 64\cosh(x)^7\sinh(x) + 28\cosh(x)^6\sinh(x)^2 - 448\cosh(x)^5\sinh(x)^3 + 70\cosh(x)^4\sinh(x)^4 - 448\ \mathrm{co}}{126(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*cos
h(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*co
sh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 - 63)/(cosh(x) - sinh(
x))
```

Sympy [B] time = 0.816193, size = 42, normalized size = 1.62

$$-\frac{32e^x\sinh^2(4x)}{63} + \frac{8e^x\sinh(4x)\cosh(4x)}{63} + \frac{31e^x\cosh^2(4x)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)**2,x)
```

```
[Out] -32*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 + 31*exp(x)*co
sh(4*x)**2/63
```

Giac [A] time = 1.3319, size = 23, normalized size = 0.88

$$\frac{1}{36}e^{(9x)} - \frac{1}{28}e^{(-7x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)^2,x, algorithm="giac")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

3.282 $\int e^x \cosh(4x) \, dx$

Optimal. Leaf size=19

$$\frac{e^{5x}}{10} - \frac{1}{6}e^{-3x}$$

```
[Out] -1/(6*E^(3*x)) + E^(5*x)/10
```

Rubi [A] time = 0.0113104, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{5x}}{10} - \frac{1}{6}e^{-3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[4*x],x]
```

```
[Out] -1/(6*E^(3*x)) + E^(5*x)/10
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int e^x\cosh(4x)\,dx &= \text{Subst}\left(\int\frac{1+x^8}{2x^4}\,dx,x,e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\frac{1+x^8}{x^4}\,dx,x,e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{1}{x^4}+x^4\right)\,dx,x,e^x\right)\\ &= -\frac{1}{6}e^{-3x}+\frac{e^{5x}}{10}\end{aligned}$$

Mathematica [A] time = 0.009291, size = 19, normalized size = 1.

$$\frac{e^{5x}}{10}-\frac{1}{6}e^{-3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[4*x],x]
```

```
[Out] -1/(6*E^(3*x)) + E^(5*x)/10
```

Maple [A] time = 0.008, size = 26, normalized size = 1.4

$$\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}-\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(4*x),x)
```

```
[Out] 1/6*sinh(3*x)+1/10*sinh(5*x)-1/6*cosh(3*x)+1/10*cosh(5*x)
```

Maxima [A] time = 1.04033, size = 18, normalized size = 0.95

$$\frac{1}{10}e^{(5x)}-\frac{1}{6}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x, algorithm="maxima")
```

```
[Out] 1/10*e^(5*x) - 1/6*e^(-3*x)
```

Fricas [B] time = 1.86518, size = 163, normalized size = 8.58

$$-\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4}{15(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x, algorithm="fricas")
```

```
[Out] -1/15*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

Sympy [A] time = 0.303538, size = 20, normalized size = 1.05

$$\frac{4e^x\sinh(4x)}{15} - \frac{e^x\cosh(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x)
```

```
[Out] 4*exp(x)*sinh(4*x)/15 - exp(x)*cosh(4*x)/15
```

Giac [A] time = 1.26228, size = 18, normalized size = 0.95

$$\frac{1}{10}e^{(5x)} - \frac{1}{6}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x, algorithm="giac")
```

```
[Out] 1/10*e^(5*x) - 1/6*e^(-3*x)
```

3.283 $\int e^x \mathbf{sech}(4x)\,dx$

Optimal. Leaf size=371

$$-\frac{\log\left(-\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(-\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2
])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 -
 Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt
[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/
(2*Sqrt[2*(2 - Sqrt[2])]) - Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(4*Sqr
t[2*(2 - Sqrt[2])]) + Log[1 + Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2
 - Sqrt[2])]) + Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2 + Sqr
t[2])]) - Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2 + Sqrt[2])]
)
```

Rubi [A] time = 0.321078, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2282, 12, 299, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{\log\left(-\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(-\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[4*x],x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2
])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 -
 Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt
[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/
(2*Sqrt[2*(2 - Sqrt[2])]) - Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(4*Sqr
t[2*(2 - Sqrt[2])]) + Log[1 + Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2
 - Sqrt[2])]) + Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2 + Sqr
t[2])]) - Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2 + Sqrt[2])]
)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^x \operatorname{sech}(4x)\,dx = \text{Subst}\left(\int \frac{2x^4}{1+x^8}\,dx, x, e^x\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^4}{1+x^8}\,dx, x, e^x\right)$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}}$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{2\sqrt{2}} - \text{S}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x\right.}{4\sqrt{2}}$$
$$= -\frac{\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\log\left(1+\right.}{4}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\log\left(1-\sqrt{2}\right.}{4\sqrt{2(}}$$

Mathematica [C] time = 0.0101792, size = 24, normalized size = 0.06

$$\frac{2}{5}e^{5x}\,{}_2F_1\left(\frac{5}{8},1;\frac{13}{8};-e^{8x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[4*x],x]
```

```
[Out] (2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, -E^(8*x)])/5
```

Maple [C] time = 0.036, size = 25, normalized size = 0.1

$$2\sum_{\_R=\mathrm{RootOf}\left(16777216\_Z^8+1\right)}\_R\ \ln\left(-32768\_R^5+e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(4*x),x)
```

```
[Out] 2*sum(_R*ln(-32768*_R^5+exp(x)),_R=RootOf(16777216*_Z^8+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{sech}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x, algorithm="maxima")
```

```
[Out] integrate(e^x*sech(4*x), x)
```

Fricas [B] time = 2.21957, size = 3322, normalized size = 8.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqr
t(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-s
qrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/
(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2)
- sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*e^x - sqrt(2)*sqrt(-2*sqrt
(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) +
4) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqr
t(2) + 2))) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*
arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sq
rt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) + sqrt(-s
qrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sqrt(
sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*
sqrt(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x +
4*e^(2*x) + 4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2)
 - sqrt(-sqrt(2) + 2))) - 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-s
```

```
qrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2)
+ 2)*e^x + 4*e^(2*x) + 4) + 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(
-sqrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2
) + 2)*e^x + 4*e^(2*x) + 4) - 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqr
t(-sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqr
t(2) + 2)*e^x + 4*e^(2*x) + 4) + 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*
sqrt(-sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-
sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - 1/4*sqrt(sqrt(2) + 2)*arctan((2*sqrt(sq
rt(sqrt(2) + 2)*e^x + e^(2*x) + 1) - sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(
2) + 2)) - 1/4*sqrt(sqrt(2) + 2)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*e^x + e^
(2*x) + 1) + sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(-sqr
t(2) + 2)*arctan((2*sqrt(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - sqrt(-sqrt
(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2)) + 1/4*sqrt(-sqrt(2) + 2)*arctan((2*sqr
t(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt
(sqrt(2) + 2)) - 1/16*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2*x
) + 1) + 1/16*sqrt(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1)
+ 1/16*sqrt(sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/16*s
qrt(sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{sech}(4x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x)
```

```
[Out] Integral(exp(x)*sech(4*x), x)
```

Giac [A] time = 1.35742, size = 336, normalized size = 0.91

$$\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^x}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^x}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2}{\sqrt{\sqrt{2}+2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) + 2)
) + 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2)
 + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt(sq
rt(2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^x)/s
qrt(sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2
*x) + 1) + 1/8*sqrt(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1)
 + 1/8*sqrt(sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/8*sq
rt(sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

3.284 $\int e^x \mathbf{sech}^2(4x)\,dx$

Optimal. Leaf size=379

$$-\frac{e^x}{2\left(e^{8x}+1\right)}-\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(-\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)+\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)-\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(-\right.$$

```
[Out] -E^x/(2*(1 + E^(8*x))) - ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2
]]]/(8*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 -
 Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/S
qrt[2 + Sqrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2
*E^x)/Sqrt[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log
[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt
[2 - Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt
[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*E^x
+ E^(2*x)])/32
```

Rubi [A] time = 0.319149, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2282, 12, 288, 213, 1169, 634, 618, 204, 628}

$$-\frac{e^x}{2\left(e^{8x}+1\right)}-\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(-\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)+\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)-\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(-\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[4*x]^2,x]
```


```
[Out] -E^x/(2*(1 + E^(8*x))) - ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2
]]]/(8*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 -
 Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/S
qrt[2 + Sqrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2
*E^x)/Sqrt[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log
[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt
[2 - Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt
[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*E^x
+ E^(2*x)])/32
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int e^x \text{sech}^2(4x)\,dx = \text{Subst}\left(\int \frac{4x^8}{\left(1+x^8\right)^2}\,dx, x, e^x\right)$$

$$= 4\,\text{Subst}\left(\int \frac{x^8}{\left(1+x^8\right)^2}\,dx, x, e^x\right)$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^8}\,dx, x, e^x\right)$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}}$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} + \text{Sul}$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} + \frac{1}{16}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right) + \frac{1}{16}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subs}$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} - \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) + \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) - $$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} - \frac{1}{16}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{16}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{16}\sqrt{2}$$

Mathematica [C] time = 0.0200419, size = 34, normalized size = 0.09

$$\frac{1}{2}e^x\left({}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-e^{8x}\right) - \frac{1}{e^{8x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[4*x]^2,x]
```

```
[Out] (E^x*(-(1 + E^(8*x))^(-1) + Hypergeometric2F1[1/8, 1, 9/8, -E^(8*x)]))/2
```

Maple [C] time = 0.046, size = 36, normalized size = 0.1

$$-\frac{e^{x}}{2+2\,e^{8 x}}+4 \sum_{\_R=\mathrm{RootOf}\left(281474976710656\,\_Z^{8}+1\right)} \_R \ln\left(e^{x}+64\,\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(4*x)^2,x)
```

```
[Out] -1/2*exp(x)/(1+exp(8*x))+4*sum(_R*ln(exp(x)+64*_R),_R=RootOf(28147497671065
6*_Z^8+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{x}}{2\left(e^{(8\,x)}+1\right)}+4\int\frac{e^{x}}{8\left(e^{(8\,x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^x/(e^(8*x) + 1) + 4*integrate(1/8*e^x/(e^(8*x) + 1), x)
```

Fricas [B] time = 2.1124, size = 4143, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(8*sqrt(-sqrt(2) + 2)*(e^(8*x) + 1)*arctan((2*sqrt(sqrt(sqrt(2) + 2)
*e^x + e^(2*x) + 1) - sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) + 8*sq
rt(-sqrt(2) + 2)*(e^(8*x) + 1)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*e^x + e^(2
*x) + 1) + sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) - 2*sqrt(-sqrt(2)
 + 2)*(e^(8*x) + 1)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 2*sqrt(-sqr
t(2) + 2)*(e^(8*x) + 1)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 8*(sqr
t(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*arctan((2*sqrt(sqrt(-sqrt(2) +
```

```
2)*e^x + e^(2*x) + 1) - sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2)) + 8*
(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*arctan((2*sqrt(-sqrt(-sqrt(
2) + 2)*e^x + e^(2*x) + 1) + sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2))
 + 4*(sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(
-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan(-(2*sqrt(2)*e^x - sqrt(2)
*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x +
4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2)
 + sqrt(-sqrt(2) + 2))) + 4*(sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e
^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan(-(
2*sqrt(2)*e^x - sqrt(2)*sqrt(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*s
qrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2)
+ 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 4*(sqrt(2)*sqrt(sqrt(2) +
 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt
(sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2)
+ 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2)
 + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 4*(
sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(
2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(-
2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2
*x) + 4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqr
t(-sqrt(2) + 2))) - (sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e^(8*x) +
 sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(2*sqrt(2)*sqr
t(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - (s
qrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2
) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2
*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*sqrt(sqrt(2) +
2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(
sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2
) + 2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)
*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(-2*
sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x
) + 4) - 2*(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*log(sqrt(sqrt(2)
 + 2)*e^x + e^(2*x) + 1) + 2*(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2)
)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 64*e^x)/(e^(8*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{sech}^2(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)**2,x)
```

```
[Out] Integral(exp(x)*sech(4*x)**2, x)
```

Giac [A] time = 1.25401, size = 352, normalized size = 0.93

$$\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+}}{\sqrt{\sqrt{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) +
2)) + 1/16*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqr
t(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqr
t(sqrt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^
x)/sqrt(sqrt(2) + 2)) + 1/32*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x +
e^(2*x) + 1) - 1/32*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x)
+ 1) + 1/32*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) -
1/32*sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/2*e^
x/(e^(8*x) + 1)
```

3.285 $\int F^{c(a+bx)} \cosh^3(d+ex)\,dx$

Optimal. Leaf size=202

$$\frac{6e^3\sinh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}-\frac{6bce^2\log(F)\cosh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}+3e\,s$$

```
[Out] -((b*c*F^(c*(a + b*x))*Cosh[d + e*x]^3*Log[F])/(9*e^2 - b^2*c^2*Log[F]^2))
- (6*b*c*e^2*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(9*e^4 - 10*b^2*c^2*e^2*
Log[F]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d
+ e*x])/(9*e^2 - b^2*c^2*Log[F]^2) + (6*e^3*F^(c*(a + b*x))*Sinh[d + e*x])/
(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4)
```

Rubi [A] time = 0.0781238, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5477, 5475}

$$\frac{6e^3\sinh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}-\frac{6bce^2\log(F)\cosh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}+3e\,s$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

```
[Out] -((b*c*F^(c*(a + b*x))*Cosh[d + e*x]^3*Log[F])/(9*e^2 - b^2*c^2*Log[F]^2))
- (6*b*c*e^2*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(9*e^4 - 10*b^2*c^2*e^2*
Log[F]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d
+ e*x])/(9*e^2 - b^2*c^2*Log[F]^2) + (6*e^3*F^(c*(a + b*x))*Sinh[d + e*x])/
(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4)
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\cosh^3(d+ex)\log(F)}{9e^2-b^2c^2\log^2(F)}+\frac{3eF^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)}{9e^2-b^2c^2\log^2(F)}+\frac{\left(6e^2\right)\int F^{c(a+b}}{9e^2-b}$$

$$= -\frac{bcF^{c(a+bx)}\cosh^3(d+ex)\log(F)}{9e^2-b^2c^2\log^2(F)}-\frac{6bce^2F^{c(a+bx)}\cosh(d+ex)\log(F)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}+\frac{3eF^{c(a+bx)}\cos}{9e^2}$$

Mathematica [A] time = 0.656973, size = 159, normalized size = 0.79

$$\frac{F^{c(a+bx)}\left(3\cosh(d+ex)\left(b^3c^3\log^3(F)-9bce^2\log(F)\right)+\cosh(3(d+ex))\left(b^3c^3\log^3(F)-bce^2\log(F)\right)+6e\sinh(d+ex)\left(\right.\right.}{4\left(-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(3*Cosh[d + e*x]*(-9*b*c*e^2*Log[F] + b^3*c^3*Log[F]^3) +
Cosh[3*(d + e*x)]*(-(b*c*e^2*Log[F]) + b^3*c^3*Log[F]^3) + 6*e*(5*e^2 - b^2
*c^2*Log[F]^2 + Cosh[2*(d + e*x)]*(e^2 - b^2*c^2*Log[F]^2))*Sinh[d + e*x]))
/(4*(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A] time = 0.065, size = 326, normalized size = 1.6

$$\left((\ln(F))^3b^3c^3e^{6ex+6d}+3\,(\ln(F))^3b^3c^3e^{4ex+4d}-3\,(\ln(F))^2b^2c^2ee^{6ex+6d}+3\,(\ln(F))^3b^3c^3e^{2ex+2d}-3\,(\ln(F))^2b^2c^2ee^{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d)^3,x)
```

```
[Out] 1/8*(ln(F)^3*b^3*c^3*exp(6*e*x+6*d)+3*ln(F)^3*b^3*c^3*exp(4*e*x+4*d)-3*ln(F
)^2*b^2*c^2*e*exp(6*e*x+6*d)+3*ln(F)^3*b^3*c^3*exp(2*e*x+2*d)-3*ln(F)^2*b^2
```

```
*c^2*e*exp(4*e*x+4*d)-ln(F)*b*c*e^2*exp(6*e*x+6*d)+ln(F)^3*b^3*c^3+3*ln(F)^
2*b^2*c^2*e*exp(2*e*x+2*d)-27*ln(F)*b*c*e^2*exp(4*e*x+4*d)+3*e^3*exp(6*e*x+
6*d)+3*ln(F)^2*b^2*c^2*e-27*ln(F)*b*c*e^2*exp(2*e*x+2*d)+27*e^3*exp(4*e*x+4
*d)-ln(F)*b*c*e^2-27*e^3*exp(2*e*x+2*d)-3*e^3)/(b*c*ln(F)-e)*exp(-3*e*x-3*d
)/(b*c*ln(F)-3*e)/(e+b*c*ln(F))/(b*c*ln(F)+3*e)*F^(c*(b*x+a))
```

Maxima [A] time = 1.08815, size = 181, normalized size = 0.9

$$\frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log{(F)}+3\,e)}+\frac{3\,F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log{(F)}+e)}+\frac{3\,F^{ac}e^{(bcx\log(F)-ex)}}{8\left(bce^{d}\log{(F)}-ee^{d}\right)}+\frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\left(bce^{(3\,d)}\log{(F)}-3\,ee^{(3\,d)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) + 3/8*F^(a*c)
*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F)
- e*x)/(b*c*e^d*log(F) - e*e^d) + 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(b*c
*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Fricas [B] time = 2.03196, size = 5434, normalized size = 26.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((3*e^3*cosh(e*x + d)^6 + 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c^3
*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*cosh
(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e*x +
 d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*l
og(F)^3 + 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^2 - (5*
b*c*e^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*c^3*cos
h(e*x + d)^6 + 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2 + b^3*
c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 + 27*e^3*cosh(e*x + d) + (5*b^3*c
^3*cosh(e*x + d)^3 + 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^2*e*cos
h(e*x + d)^3 + b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)
```

```
^3 + 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 - 3*e^3 - 3*(b^2*c^2
*e*cosh(e*x + d)^6 + b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x + d)^2
- b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 + 54*e^3*cosh(e*x + d)^2
+ (5*b^3*c^3*cosh(e*x + d)^4 + 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*log(F)^
3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 + 6*b^2*c^2*e*cosh(e*x + d)^2 - b
^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 + 54*b*c*e^2*cosh(e*x + d)^
2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 + 27*b*c*
e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 + b*c*e^2)*log(F) + 6*(3*e
^3*cosh(e*x + d)^5 + 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) + (b^3*c^
3*cosh(e*x + d)^5 + 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d))*log(
F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 + 2*b^2*c^2*e*cosh(e*x + d)^3 - b^2*c^2
*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 + 18*b*c*e^2*cosh(e*x
 + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*cosh((b*c*x + a*c
)*log(F)) + (3*e^3*cosh(e*x + d)^6 + 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(
F)^3 - 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(
b^3*c^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e
^2*cosh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cos
h(e*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 + b^3
*c^3)*log(F)^3 + 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^
2 - (5*b*c*e^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 + 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2
 + b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 + 27*e^3*cosh(e*x + d) + (
5*b^3*c^3*cosh(e*x + d)^3 + 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^
2*e*cosh(e*x + d)^3 + b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e
*x + d)^3 + 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 - 3*e^3 - 3*(
b^2*c^2*e*cosh(e*x + d)^6 + b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x
+ d)^2 - b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 + 54*e^3*cosh(e*x
+ d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 + 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*
log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 + 6*b^2*c^2*e*cosh(e*x + d
)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 + 54*b*c*e^2*cosh(e*
x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 +
27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 + b*c*e^2)*log(F) +
 6*(3*e^3*cosh(e*x + d)^5 + 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) +
(b^3*c^3*cosh(e*x + d)^5 + 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d
))*log(F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 + 2*b^2*c^2*e*cosh(e*x + d)^3 -
b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 + 18*b*c*e^2*c
osh(e*x + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*sinh((b*c*
x + a*c)*log(F)))/(b^4*c^4*cosh(e*x + d)^3*log(F)^4 - 10*b^2*c^2*e^2*cosh(e
*x + d)^3*log(F)^2 + 9*e^4*cosh(e*x + d)^3 + (b^4*c^4*log(F)^4 - 10*b^2*c^2
*e^2*log(F)^2 + 9*e^4)*sinh(e*x + d)^3 + 3*(b^4*c^4*cosh(e*x + d)*log(F)^4
- 10*b^2*c^2*e^2*cosh(e*x + d)*log(F)^2 + 9*e^4*cosh(e*x + d))*sinh(e*x + d
)^2 + 3*(b^4*c^4*cosh(e*x + d)^2*log(F)^4 - 10*b^2*c^2*e^2*cosh(e*x + d)^2*
log(F)^2 + 9*e^4*cosh(e*x + d)^2)*sinh(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.46632, size = 1673, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*
b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*
c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3
*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))
+ 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) + 3/4*(2*(b*
c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn
(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2)
- (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi
*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
+ e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/2*I*(-6*I*e^
(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*
c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) + 6*I*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)
/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*log(
abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e)*cos(-
1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b
*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*
```

```
c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c
)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(
F)) + (b*c*log(abs(F)) - e)*x - d) - 1/2*I*(-6*I*e^(1/2*I*pi*b*c*x*sgn(F) -
 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) -
8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*e) + 6*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1
/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8
*I*pi*b*c + 16*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F
)) - e)*x - d) + 1/4*(2*(b*c*log(abs(F)) - 3*e)*cos(-1/2*pi*b*c*x*sgn(F) +
1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2
+ 4*(b*c*log(abs(F)) - 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x
*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) -
pi*b*c)^2 + 4*(b*c*log(abs(F)) - 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs
(F)) - 3*e)*x - 3*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*
x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 1
6*b*c*log(abs(F)) - 48*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
- 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16
*b*c*log(abs(F)) - 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x -
3*d)
```

3.286 $\int F^{c(a+bx)}\cosh^2(d+ex)\,dx$

Optimal. Leaf size=132

$$-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(c
*(a + b*x))*Cosh[d + e*x]^2*Log[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*F^(c*
(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0528336, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5477, 2194}

$$-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```


```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(c
*(a + b*x))*Cosh[d + e*x]^2*Log[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*F^(c*
(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```


Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} + \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} + \frac{(2e^2)\int F^{c(a+bx}}{4e^2-b^2c^2\log}$$
$$= \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} - \frac{bcF^{c(a+bx)}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} + \frac{2eF^{c(a+bx)}\cosh(d+ex)}{4e^2-b^2c^2\log}$$

Mathematica [A] time = 0.225002, size = 85, normalized size = 0.64

$$\frac{F^{c(a+bx)}\left(b^2c^2\log^2(F)\cosh(2(d+ex))+b^2c^2\log^2(F)-2bce\log(F)\sinh(2(d+ex))-4e^2\right)}{2b^3c^3\log^3(F)-8bce^2\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(-4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Log
[F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*L
og[F]^3)
```

Maple [A] time = 0.037, size = 143, normalized size = 1.1

$$\frac{\left((\ln(F))^2b^2c^2e^{4ex+4d}+2(\ln(F))^2b^2c^2e^{2ex+2d}-2\ln(F)bcee^{4ex+4d}+b^2c^2(\ln(F))^2+2\ln(F)bce-8e^2e^{2ex+2d}\right)e^{-2ex-}}{4bc\ln(F)(bc\ln(F)-2e)(bc\ln(F)+2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d)^2,x)
```

```
[Out] 1/4*(ln(F)^2*b^2*c^2*exp(4*e*x+4*d)+2*ln(F)^2*b^2*c^2*exp(2*e*x+2*d)-2*ln(F
)*b*c*e*exp(4*e*x+4*d)+b^2*c^2*ln(F)^2+2*ln(F)*b*c*e-8*e^2*exp(2*e*x+2*d))/
ln(F)/b/c/(b*c*ln(F)-2*e)*exp(-2*e*x-2*d)/(b*c*ln(F)+2*e)*F^(c*(b*x+a))
```

Maxima [A] time = 1.08939, size = 127, normalized size = 0.96

$$\frac{F^{ac}e^{(bcx\log(F)+2ex+2d)}}{4(bc\log(F)+2e)} + \frac{F^{ac}e^{(bcx\log(F)-2ex)}}{4\left(bce^{(2d)}\log(F)-2ee^{(2d)}\right)} + \frac{F^{bcx+ac}}{2bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c)
*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 1/2*F^(b*c*x
 + a*c)/(b*c*log(F))
```

Fricas [B] time = 2.52552, size = 1783, normalized size = 13.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x
+ d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*
sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 + b^
2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x +
 d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)
^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) + 4*e^2*cosh(e*x + d
) - (b^2*c^2*cosh(e*x + d)^3 + b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x +
d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(
e*x + d)^4 - 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*
b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 + 2*
b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*lo
g(F) - (3*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*sinh(e*x + d
)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3
*log(F) + 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^2*c^2*cosh(e*x
 + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/(b^3*c^3*cosh(e
*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^3*c^3*log(F)^3 -
 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x + d)*log(F)^3 - 4*
b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

Sympy [A] time = 110.87, size = 604, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d)**2,x)
```

```
[Out] Piecewise((-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cos
h(d + e*x)/(2*e), Eq(F, 1)), (zoo*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c
))**(b*c*x)*sinh(d + e*x)**2 + zoo*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*
c))**(b*c*x)*sinh(d + e*x)*cosh(d + e*x) + zoo*e**2*exp(-2*e/(b*c))**(a*c)*
exp(-2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(-2*e/(b*c)))), (zoo*e*
*2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)**2 + zoo*e**
2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)*cosh(d + e*x)
 + zoo*e**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2,
 Eq(F, exp(2*e/(b*c)))), (F**(a*c)*(-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)
**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (-x*sinh(d + e*x)**2
/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (
F**(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*cosh(d + e*x)**2/(b**3*c**3*log(F)*
*3 - 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x)*b*c*e*log(F)*sinh(d + e*x)*
cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) + 2*F**(a*c)*F**(b*
c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F*
*(a*c)*F**(b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*l
og(F)), True))
```

Giac [C] time = 1.45306, size = 1219, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi
*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) -
(pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c
*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) +
 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) +
 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi
*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b
*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x +
```

```
2*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) +
 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*c
*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sg
n(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2
*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F
)) - 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) - 1/2*I
*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) + 2
*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I
*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a
*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d)
```

3.287 $\int F^{c(a+bx)} \cosh(d+ex)\,dx$

Optimal. Leaf size=75

$$\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

```
[Out] -((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2)) + (e
*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0182983, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5475}

$$\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

```
[Out] -((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2)) + (e
*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx=-\frac{bcF^{c(a+bx)}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

Mathematica [A] time = 0.103601, size = 50, normalized size = 0.67

$$\frac{F^{c(a+bx)}(e\sinh(d+ex)-bc\log(F)\cosh(d+ex))}{(e-bc\log(F))(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(-(b*c*Cosh[d + e*x]*Log[F]) + e*Sinh[d + e*x]))/((e - b*c
*Log[F])*(e + b*c*Log[F]))
```

Maple [A] time = 0.017, size = 74, normalized size = 1.

$$\frac{\left(\ln(F)\,bc\mathrm{e}^{2\,ex+2\,d}+bc\ln(F)-e\mathrm{e}^{2\,ex+2\,d}+e\right)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{(2\,bc\ln(F)-2\,e)\,(e+bc\ln(F))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d),x)
```

```
[Out] 1/2*(ln(F)*b*c*exp(2*e*x+2*d)+b*c*ln(F)-e*exp(2*e*x+2*d)+e)/(b*c*ln(F)-e)*e
xp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

Maxima [A] time = 1.04999, size = 85, normalized size = 1.13

$$\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{2\,(bc\log(F)+e)}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{2\left(bce^{d}\log(F)-ee^{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 1/2*F^(a*c)*e^(b*
c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

Fricas [B] time = 2.51874, size = 643, normalized size = 8.57

$$-\frac{\left(e\cosh(ex+d)^{2}-(bc\log(F)-e)\sinh(ex+d)^{2}-\left(bc\cosh(ex+d)^{2}+bc\right)\log(F)-2\,(bc\cosh(ex+d)\log(F)-e\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x
 + d)^2 + b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh
(e*x + d) - e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*log(F
) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 + b*c)*log(F) - 2*(b*c*cosh(e
*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) - e)*sinh((b*c*x + a*c)*log
(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*log(F)
^2 - e^2)*sinh(e*x + d))
```

Sympy [A] time = 12.0146, size = 316, normalized size = 4.21

$$\begin{cases} -\frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\sinh(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\cosh(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}\sinh(d+ex)}{2e} & \text{for } F=-1 \wedge b=-\frac{ie}{\pi c} \\ x\cosh(d) & \text{for } F=1 \wedge e=0 \\ \tilde{\infty}e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F=e^{-\frac{e}{bc}} \\ \tilde{\infty}e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F=e^{\frac{e}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log(F)\cosh(d+ex)}{b^2c^2\log(F)^2-e^2} - \frac{F^{ac}F^{bcx}e\sinh(d+ex)}{b^2c^2\log(F)^2-e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d),x)
```

```
[Out] Piecewise((-(-1)**(a*c)*(-1)**(-I*e*x/pi)*x*sinh(d + e*x)/2 + (-1)**(a*c)*(
-1)**(-I*e*x/pi)*x*cosh(d + e*x)/2 + (-1)**(a*c)*(-1)**(-I*e*x/pi)*sinh(d +
 e*x)/(2*e), Eq(F, -1) & Eq(b, -I*e/(pi*c))), (x*cosh(d), Eq(F, 1) & Eq(e,
0)), (zoo*e*exp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo
*e*exp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/
(b*c)))), (zoo*e*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) +
zoo*e*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(e/
(b*c)))), (F**(a*c)*F**(b*c*x)*b*c*log(F)*cosh(d + e*x)/(b**2*c**2*log(F)**
2 - e**2) - F**(a*c)*F**(b*c*x)*e*sinh(d + e*x)/(b**2*c**2*log(F)**2 - e**2
), True))
```

Giac [C] time = 1.3714, size = 825, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="giac")
```

```
[Out] (2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) +
e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/4*I*(-
2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I
*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) + 2*I*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)
/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(abs(F)
) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c
*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F)
- pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/
2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c
*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c
*log(abs(F)) - e)*x - d) - 1/4*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*
b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2
*b*c*log(abs(F)) - 2*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*lo
g(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
```

3.288 $\int F^{c(a+bx)}\mathbf{sech}(d+ex)\,dx$

Optimal. Leaf size=68

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e),
 (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rubi [A] time = 0.0217432, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5492}

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x],x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e),
 (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx=\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

Mathematica [A] time = 0.0186107, size = 70, normalized size = 1.03

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc\log(F)}{2e}+\frac{1}{2};\frac{bc\log(F)}{2e}+\frac{3}{2};-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x],x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 + (b*c*Log[F])/(2*e
), 3/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\operatorname{sech}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-4\,F^{ac}e\int\frac{e^{(bcx\log(F)+ex+d)}}{bc\log(F)+\left(bce^{(4\,d)}\log(F)-ee^{(4\,d)}\right)e^{(4\,ex)}+2\left(bce^{(2\,d)}\log(F)-ee^{(2\,d)}\right)e^{(2\,ex)}-e}\,dx+\frac{2\,F^{ac}e^{(bcx\log(F}}{bc\log(F)+\left(bce^{(2\,d)}\log(F\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="maxima")
```

```
[Out] -4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*d
)*log(F) - e*e^(4*d))*e^(4*e*x) + 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2*e
*x) - e), x) + 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(2
*d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\operatorname{sech}\left(ex+d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\operatorname{sech}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\operatorname{sech}\left(ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d), x)
```

3.289 $\int F^{c(a+bx)}\mathbf{sech}^2(d+ex)\,dx$

Optimal. Leaf size=70

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rubi [A] time = 0.0292432, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5492}

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx=\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

Mathematica [A] time = 0.016046, size = 70, normalized size = 1.

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\operatorname{sech}\left(ex+d\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16\,F^{ac}bce\int\frac{F^{bcx}}{b^{2}c^{2}\log\left(F\right)^{2}-6\,bce\log\left(F\right)+8\,e^{2}+\left(b^{2}c^{2}e^{(6\,d)}\log\left(F\right)^{2}-6\,bcee^{(6\,d)}\log\left(F\right)+8\,e^{2}e^{(6\,d)}\right)e^{(6\,ex)}+3\left(b^{2}c^{2}e^{(4\,d)}\log\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) + 8
*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d))*
e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2*e^
(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F) + 8
*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) - 4*(4*F^(a*c)*e - (F^(a*c)*b*c*e^(2*d)
*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b
```

```
*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) +
8*e^2*e^(4*d))*e^(4*e*x) + 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*lo
g(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\,\text{sech}\left(ex+d\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\,\text{sech}^{2}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\,\text{sech}\left(ex+d\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^2, x)
```

3.290 $\int F^{c(a+bx)}\mathbf{sech}^3(d+ex)\,dx$

Optimal. Leaf size=124

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{e^2}+\frac{bc\log(F)\mathbf{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)}{}$$

```
[Out] (E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (
3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]))/e^2 + (b*c*F^(c*
(a + b*x))*Log[F]*Sech[d + e*x])/(2*e^2) + (F^(c*(a + b*x))*Sech[d + e*x]*T
anh[d + e*x])/(2*e)
```

Rubi [A] time = 0.0524655, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5490, 5492}

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{e^2}+\frac{bc\log(F)\mathbf{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

```
[Out] (E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (
3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]))/e^2 + (b*c*F^(c*
(a + b*x))*Log[F]*Sech[d + e*x])/(2*e^2) + (F^(c*(a + b*x))*Sech[d + e*x]*T
anh[d + e*x])/(2*e)
```

Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{bcF^{c(a+bx)}\log(F)\text{sech}(d+ex)}{2e^2} + \frac{F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)}{2e} + \frac{1}{2}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)$$

$$= \frac{e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)(e-bc\log(F))}{e^2} + \frac{bcF^{c(a+bx)}\log(F)}{2e^2}$$

Mathematica [A] time = 0.216721, size = 96, normalized size = 0.77

$$\frac{F^{c(a+bx)}\left(2e^{d+ex}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)+\text{sech}(d+ex)(bc\log(F)+e\tanh(d+ex))\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(2*E^(d + e*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]) + Sech[d + e*x
]*(b*c*Log[F] + e*Tanh[d + e*x])))/(2*e^2)
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\text{sech}\left(ex+d\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$48\left(F^{ac}bcee^{d}\log\left(F\right)+F^{ac}e^{2}e^{d}\right)\int\frac{}{b^{2}c^{2}\log\left(F\right)^{2}-8\,bce\log\left(F\right)+15\,e^{2}+\left(b^{2}c^{2}e^{(8\,d)}\log\left(F\right)^{2}-8\,bcee^{(8\,d)}\log\left(F\right)+15\,e^{2}e^{(8\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F) +
 e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(F)
^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) + 4*(b^2*c^2*e^(6*d
)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^2*c^
2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x) + 4
*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*
e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) - (F^(a*c)*b*c*e^(3*d)*log(F) - 5*F^
(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) +
15*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^
2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F)
 + 15*e^2*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(F^{bcx+ac}\,\mathrm{sech}\left(ex+d\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c} \operatorname{sech}(ex+d)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^3, x)
```

3.291 $\int F^{c(a+bx)}\mathbf{sech}^4(d+ex)\,dx$

Optimal. Leaf size=133

$$\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{3e^2}+\frac{bc\log(F)\mathbf{sech}^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tanh(d+$$

```
[Out] (2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(2*e - b*c*Log[F]))/(3*e^2)
+ (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x]^2)/(6*e^2) + (F^(c*(a + b*x))*S
ech[d + e*x]^2*Tanh[d + e*x])/(3*e)
```

Rubi [A] time = 0.0585902, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5490, 5492}

$$\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{3e^2}+\frac{bc\log(F)\mathbf{sech}^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tanh(d+$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```



```
[Out] (2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(2*e - b*c*Log[F]))/(3*e^2)
+ (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x]^2)/(6*e^2) + (F^(c*(a + b*x))*S
ech[d + e*x]^2*Tanh[d + e*x])/(3*e)
```


Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{sech}^4(d+ex)\,dx = \frac{bcF^{c(a+bx)}\log(F)\text{sech}^2(d+ex)}{6e^2} + \frac{F^{c(a+bx)}\text{sech}^2(d+ex)\tanh(d+ex)}{3e} + \frac{1}{6}\left(4-\frac{b^2c^2\log^2(F)}{e^2}\right.$$
$$= \frac{2e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)(2e-bc\log(F))}{3e^2} + \frac{bcF^{c(a+bx)}\log(}{6}$$

Mathematica [A] time = 0.185042, size = 101, normalized size = 0.76

$$\frac{F^{c(a+bx)}\left(4e^{2(d+ex)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)+\text{sech}^2(d+ex)(bc\log(F)+2e\tanh(d+ex))\right)}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(4*E^(2*(d + e*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(2*e - b*c*Log[F]) + Sech[d
 + e*x]^2*(b*c*Log[F] + 2*e*Tanh[d + e*x])))/(6*e^2)
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\text{sech}\left(ex+d\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(F^(b
*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) - 192*
e^3 + (b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*log(F)^2 + 104*b*c
*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) + 5*(b^3*c^3*e^(8*d)*lo
g(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e
^3*e^(8*d))*e^(8*e*x) + 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)
*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x) + 10*(b
^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*
d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x) + 5*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b
^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e
^(2*e*x)), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F^(a*c)*b^2*
c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(a*c)*e^2*e^(
4*d))*e^(4*e*x) - 8*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*e^2*e^(2*d))*
e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^
2*log(F) - 192*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F
)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*x) + 4*(b^3*c^3*
e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(
F) - 192*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*
e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x
) + 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e
^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\operatorname{sech}\left(ex+d\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c} \operatorname{sech}(ex+d)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^4, x)
```

3.292 $\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=250

$$-\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\cosh^2(ac+bc}{32bc}$$

```
[Out] -(Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(128*b*c*E^(4*c*(a + b*x)))
- (5*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))
) + (5*E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(32*b
*c) + (5*E^(4*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(12
8*b*c) + (E^(6*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(1
92*b*c) + (5*x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/16
```

Rubi [A] time = 0.228918, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\cosh^2(ac+bc}{32bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] -(Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(128*b*c*E^(4*c*(a + b*x)))
- (5*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))
) + (5*E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(32*b
*c) + (5*E^(4*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(12
8*b*c) + (E^(6*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(1
92*b*c) + (5*x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/16
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx = \left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\int e^{c(a+bx)}\cosh^5(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int \frac{(1+x^2)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int \frac{(1+x^2)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc}$$

$$= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int \frac{(1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc}$$

$$= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int \left(10+\frac{1}{x^3}+\frac{5}{x^2}+\frac{10}{x}+5x+x^2\right)dx, x, e^{2}\right.}{64bc}$$

$$= -\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{128bc} - \frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac +}{64bc}$$

Mathematica [A] time = 0.0973858, size = 106, normalized size = 0.42

$$\frac{\left(-\frac{1}{2}e^{-4c(a+bx)} - 5e^{-2c(a+bx)} + 10e^{2c(a+bx)} + \frac{5}{2}e^{4c(a+bx)} + \frac{1}{3}e^{6c(a+bx)} + 20bcx\right)\cosh^2(c(a+bx))^{5/2}\mathrm{sech}^5(c(a+bx))}{64bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((-1/(2*E^(4*c*(a + b*x))) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) + (
5*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 + 20*b*c*x)*(Cosh[c*(a + b*x)]
^2)^(5/2)*Sech[c*(a + b*x)]^5)/(64*b*c)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left((\cosh(bcx+ac))^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x)
```

Maxima [A] time = 1.09616, size = 151, normalized size = 0.6

$$\frac{5\,(bcx+ac)}{16\,bc}+\frac{e^{(6\,bcx+6\,ac)}}{192\,bc}+\frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc}+\frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc}-\frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc}-\frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*c
*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2*a
*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

Fricas [A] time = 2.19512, size = 562, normalized size = 2.25

$$-\frac{\cosh\left(bcx+ac\right)^{5}+5\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{4}-5\,\sinh\left(bcx+ac\right)^{5}-5\left(10\,\cosh\left(bcx+ac\right)^{2}+9\right)\sinh\left(bcx+ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*s
inh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 + 1
5*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*sin
h(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x + a
*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*c*cos
h(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21151, size = 136, normalized size = 0.54

$$\frac{120\,bcx - 3\left(30\,e^{(4\,bcx+4\,ac)} + 10\,e^{(2\,bcx+2\,ac)} + 1\right)e^{(-4\,bcx-4\,ac)} + \left(2\,e^{(6\,bcx+18\,ac)} + 15\,e^{(4\,bcx+16\,ac)} + 60\,e^{(2\,bcx+14\,ac)}\right)e^{(-12\,ac)}}{384\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(120*b*c*x - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2*a*c) + 1)*
e^(-4*b*c*x - 4*a*c) + (2*e^(6*b*c*x + 18*a*c) + 15*e^(4*b*c*x + 16*a*c) +
60*e^(2*b*c*x + 14*a*c))*e^(-12*a*c))/(b*c)
```

3.293 $\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=162

$$-\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{se}}{32bc}$$

```
[Out] -(Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(16*b*c*E^(2*c*(a + b*x))) +
 (3*E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(16*b*c)
 + (E^(4*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(32*b*c)
 + (3*x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/8
```

Rubi [A] time = 0.123512, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{se}}{32bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(16*b*c*E^(2*c*(a + b*x))) +
 (3*E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(16*b*c)
 + (E^(4*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(32*b*c)
 + (3*x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/8
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx &= \left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\int e^{c(a+bx)}\cosh^3(ac+bcx)\,dx\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int\frac{\left(1+x^2\right)^3}{8x^3}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int\frac{\left(1+x^2\right)^3}{x^3}\,dx,x,e^{c(a+bx)}\right)}{8bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int\frac{(1+x)^3}{x^2}\,dx,x,e^{2c(a+bx)}\right)}{16bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int\left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx,x,e^{2c(a+bx)}\right)}{16bc}\\
&= -\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+}{16bc}
\end{aligned}
$$

Mathematica [A] time = 0.107306, size = 78, normalized size = 0.48

$$\frac{\left(-e^{-2c(a+bx)}+3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\cosh^2(c(a+bx))^{3/2}\mathrm{sech}^3(c(a+bx))}{16bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((-E^(-2*c*(a + b*x)) + 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x
)*(Cosh[c*(a + b*x)]^2)^(3/2)*Sech[c*(a + b*x)]^3)/(16*b*c)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left((\cosh(bcx+ac))^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x)
```

Maxima [A] time = 1.15058, size = 100, normalized size = 0.62

$$\frac{3(bcx+ac)}{8\,bc}+\frac{e^{(4\,bcx+4\,ac)}}{32\,bc}+\frac{3\,e^{(2\,bcx+2\,ac)}}{16\,bc}-\frac{e^{(-2\,bcx-2\,ac)}}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x
+ 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)
```

Fricas [A] time = 2.01751, size = 323, normalized size = 1.99

$$-\frac{\cosh(bcx+ac)^3+3\,\cosh(bcx+ac)\sinh(bcx+ac)^2-3\,\sinh(bcx+ac)^3-6\,(2\,bcx+1)\cosh(bcx+ac)+3\,\left(4\,bcx-3\right.}{32\,(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*si
nh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*cosh
(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b
*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2913, size = 99, normalized size = 0.61

$$\frac{12\,bcx-2\left(3\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-2\,bcx-2\,ac)}+\left(e^{(4\,bcx+8\,ac)}+6\,e^{(2\,bcx+6\,ac)}\right)e^{(-4\,ac)}}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x - 2*a*c) + (e^(4
*b*c*x + 8*a*c) + 6*e^(2*b*c*x + 6*a*c))*e^(-4*a*c))/(b*c)
```

3.294 $\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)$$

```
[Out] (E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(4*b*c) + (
x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/2
```

Rubi [A] time = 0.0993262, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 14}

$$\frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(4*b*c) + (
x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/2
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx &= \left(\sqrt{\cosh^2(ac+bcx)}\mathbf{sech}(ac+bcx)\right)\int e^{c(a+bx)}\cosh(ac+bcx)\,dx\\ &= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\mathbf{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int \frac{1+x^2}{2x}\,dx,x,e^{c(a+bx)}\right)}{bc}\\ &= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\mathbf{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int \frac{1+x^2}{x}\,dx,x,e^{c(a+bx)}\right)}{2bc}\\ &= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\mathbf{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int \left(\frac{1}{x}+x\right)dx,x,e^{c(a+bx)}\right)}{2bc}\\ &= \frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\mathbf{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\mathbf{sech}(ac+bcx)\end{aligned}$$

Mathematica [A] time = 0.0383602, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}+2bcx\right)\sqrt{\cosh^2(c(a+bx))}\mathbf{sech}(c(a+bx))}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) + 2*b*c*x)*Sqrt[Cosh[c*(a + b*x)]^2]*Sech[c*(a + b*x)])
/(4*b*c)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int e^{c(bx+a)} \sqrt{(\cosh(bcx+ac))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x)
```

Maxima [A] time = 1.15082, size = 39, normalized size = 0.53

$$\frac{1}{2}x + \frac{a}{2b} + \frac{e^{(2bcx+2ac)}}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Fricas [A] time = 1.72602, size = 163, normalized size = 2.2

$$\frac{(2bcx+1)\cosh(bcx+ac) - (2bcx-1)\sinh(bcx+ac)}{4(bc\cosh(bcx+ac) - bc\sinh(bcx+ac))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.2278, size = 31, normalized size = 0.42

$$\frac{1}{2}x + \frac{e^{(2bcx+2ac)}}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

3.295 $\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx$

Optimal. Leaf size=44

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] (Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[a*c + b*c*x]^
2])
```

Rubi [A] time = 0.113316, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 260}

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[a*c + b*c*x]^
2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx &= \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\mathbf{sech}(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}\\ &= \frac{\cosh(ac+bcx)\,\mathrm{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\ &= \frac{(2\cosh(ac+bcx))\,\mathrm{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\ &= \frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\end{aligned}$$

Mathematica [A] time = 0.0541603, size = 42, normalized size = 0.95

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(c(a+bx))}{bc\sqrt{\cosh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[c*(a + b*x)]^
2])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \mathrm{e}^{c(bx+a)} \frac{1}{\sqrt{\left(\cosh \left(bcx+ac\right)\right)^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x)
```

Maxima [A] time = 1.7139, size = 28, normalized size = 0.64

$$\frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Fricas [A] time = 1.82579, size = 97, normalized size = 2.2

$$\frac{\log\left(\frac{2\,\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac} \int \frac{e^{bcx}}{\sqrt{\cosh^{2}\left(ac+bcx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(cosh(a*c + b*c*x)**2), x)
```

Giac [A] time = 1.23292, size = 27, normalized size = 0.61

$$\frac{\log\left(e^{(2\,bcx)} + e^{(-2\,ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(e^(2*b*c*x) + e^(-2*a*c))/(b*c)
```

3.296 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] (2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt
[Cosh[a*c + b*c*x]^2])
```

Rubi [A] time = 0.128711, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 264}

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
```


```
[Out] (2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt
[Cosh[a*c + b*c*x]^2])
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx &= \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\operatorname{sech}^3(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}\\ &= \frac{\cosh(ac+bcx)\operatorname{Subst}\left(\int \frac{8x^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\ &= \frac{(8\cosh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\ &= \frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\cosh^2(ac+bcx)}}\end{aligned}$$

Mathematica [A] time = 0.0717623, size = 46, normalized size = 0.82

$$\frac{4e^{5c(a+bx)}\sqrt{\cosh^2(c(a+bx))}}{bc\left(e^{2c(a+bx)}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (4*E^(5*c*(a + b*x))*Sqrt[Cosh[c*(a + b*x)]^2])/(b*c*(1 + E^(2*c*(a + b*x))
)^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \mathrm{e}^{c(bx+a)} \left((\cosh(bcx+ac))^2 \right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x)
```

Maxima [A] time = 1.06753, size = 113, normalized size = 2.02

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc\left(e^{(4\,bcx+4\,ac)} + 2\,e^{(2\,bcx+2\,ac)} + 1\right)} - \frac{2}{bc\left(e^{(4\,bcx+4\,ac)} + 2\,e^{(2\,bcx+2\,ac)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 1.79339, size = 302, normalized size = 5.39

$$-\frac{2\,(3\,\cosh(bcx+ac) + \sinh(bcx+ac))}{bc\cosh(bcx+ac)^3 + 3\,bc\cosh(bcx+ac)\sinh(bcx+ac)^2 + bc\sinh(bcx+ac)^3 + 3\,bc\cosh(bcx+ac) + \left(3\,bc\cosh(bcx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*c*
cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25059, size = 51, normalized size = 0.91

$$-\frac{2\left(2\,e^{(2\,bcx+2\,ac)}+1\right)}{bc\left(e^{(2\,bcx+2\,ac)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)
```

3.297 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{8\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}+\frac{32\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}-\frac{4\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] (-4*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Cosh[a*c + b*c*x
]^2]) + (32*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3*Sqrt[Cosh[a
*c + b*c*x]^2]) - (8*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt
[Cosh[a*c + b*c*x]^2])
```

Rubi [A] time = 0.19712, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{8\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}+\frac{32\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}-\frac{4\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```


```
[Out] (-4*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Cosh[a*c + b*c*x
]^2]) + (32*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3*Sqrt[Cosh[a
*c + b*c*x]^2]) - (8*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt
[Cosh[a*c + b*c*x]^2])
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\mathrm{sech}^5(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{\cosh(ac+bcx)\,\mathrm{Subst}\left(\int \frac{32x^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(32\cosh(ac+bcx))\,\mathrm{Subst}\left(\int \frac{x^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(16\cosh(ac+bcx))\,\mathrm{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(16\cosh(ac+bcx))\,\mathrm{Subst}\left(\int \left(\frac{1}{(1+x)^5}-\frac{2}{(1+x)^4}+\frac{1}{(1+x)^3}\right)dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= -\frac{4\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(ac+bcx)}} + \frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\cosh^2(ac+bcx)}} - \frac{8\,c}{bc\left(1+e^{2c(a\cdot\right.}}$$

Mathematica [A] time = 0.0691754, size = 72, normalized size = 0.51

$$-\frac{4\left(4e^{2c(a+bx)}+6e^{4c(a+bx)}+1\right)\cosh(c(a+bx))}{3bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)])/(3*b
*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Cosh[c*(a + b*x)]^2])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left((\cosh(bcx+ac))^2\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x)
```

Maxima [A] time = 1.05603, size = 282, normalized size = 2.

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bc}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 1.9234, size = 797, normalized size = 5.65

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}+4\,bc\cosh\left(bcx+ac\right)^{4}+\left(15\,bc\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20836, size = 69, normalized size = 0.49

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) + 1)^4)
```

3.298 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{64\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}+\frac{48\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+bcx)}}-\frac{192\cosh(ac+bcx)}{5bc\left(e^{2c(a+bx)}+1\right)^5\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] (32*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^6*Sqrt[Cosh[a*c + b*c
*x]^2]) - (192*Cosh[a*c + b*c*x])/(5*b*c*(1 + E^(2*c*(a + b*x)))^5*Sqrt[Cos
h[a*c + b*c*x]^2]) + (48*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*
Sqrt[Cosh[a*c + b*c*x]^2]) - (64*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a +
 b*x)))^3*Sqrt[Cosh[a*c + b*c*x]^2])
```

Rubi [A] time = 0.258773, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{64\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}+\frac{48\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+bcx)}}-\frac{192\cosh(ac+bcx)}{5bc\left(e^{2c(a+bx)}+1\right)^5\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```


```
[Out] (32*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^6*Sqrt[Cosh[a*c + b*c
*x]^2]) - (192*Cosh[a*c + b*c*x])/(5*b*c*(1 + E^(2*c*(a + b*x)))^5*Sqrt[Cos
h[a*c + b*c*x]^2]) + (48*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*
Sqrt[Cosh[a*c + b*c*x]^2]) - (64*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a +
 b*x)))^3*Sqrt[Cosh[a*c + b*c*x]^2])
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\operatorname{sech}^7(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{\cosh(ac+bcx)\,\mathrm{Subst}\left(\int \frac{128x^7}{(1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(128\cosh(ac+bcx))\,\mathrm{Subst}\left(\int \frac{x^7}{(1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(64\cosh(ac+bcx))\,\mathrm{Subst}\left(\int \frac{x^3}{(1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(64\cosh(ac+bcx))\,\mathrm{Subst}\left(\int \left(-\frac{1}{(1+x)^7}+\frac{3}{(1+x)^6}-\frac{3}{(1+x)^5}+\frac{1}{(1+x)^4}\right)dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^6\sqrt{\cosh^2(ac+bcx)}} - \frac{192\cosh(ac+bcx)}{5bc\left(1+e^{2c(a+bx)}\right)^5\sqrt{\cosh^2(ac+bcx)}} + \frac{48}{bc\left(1+e^{2c(\right.}}$$

Mathematica [A] time = 0.0786676, size = 84, normalized size = 0.44

$$-\frac{16\left(6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}+1\right)\cosh(c(a+bx))}{15bc\left(e^{2c(a+bx)}+1\right)^6\sqrt{\cosh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x))
)*Cosh[c*(a + b*x)])/(15*b*c*(1 + E^(2*c*(a + b*x)))^6*Sqrt[Cosh[c*(a + b*x
)]^2])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left((\cosh(bcx+ac))^2\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x)
```

Maxima [B] time = 1.08167, size = 521, normalized size = 2.73

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc\left(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{}{bc\left(e^{(12\,bcx+12\,ac)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
+ 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 1.85763, size = 1516, normalized size = 7.94

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^{9}+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{8}+bc\sinh\left(bcx+ac\right)^{9}+6\,bc\cosh\left(bcx+ac\right)^{7}+6\left(6\,bc\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)
+ 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*
```

```
sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)^7 +
 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*
x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a*c)^2 +
 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b
*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh
(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*x + a*c)^4
 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 21*b*c*cosh(
b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*x + a*c)^5 +
 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2
 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(
b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26918, size = 86, normalized size = 0.45

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

3.299 $\int e^x \cosh(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{e^x\cosh(a+bx)}{1-b^2}-\frac{be^x\sinh(a+bx)}{1-b^2}$$

```
[Out] (E^x*Cosh[a + b*x])/(1 - b^2) - (b*E^x*Sinh[a + b*x])/(1 - b^2)
```

Rubi [A] time = 0.0133061, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5475}

$$\frac{e^x\cosh(a+bx)}{1-b^2}-\frac{be^x\sinh(a+bx)}{1-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[a + b*x],x]
```

```
[Out] (E^x*Cosh[a + b*x])/(1 - b^2) - (b*E^x*Sinh[a + b*x])/(1 - b^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\cosh(a+bx)\,dx=\frac{e^x\cosh(a+bx)}{1-b^2}-\frac{be^x\sinh(a+bx)}{1-b^2}$$

Mathematica [A] time = 0.0464297, size = 28, normalized size = 0.68

$$\frac{e^x(b\sinh(a+bx)-\cosh(a+bx))}{b^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[a + b*x],x]
```

```
[Out] (E^x*(-Cosh[a + b*x] + b*Sinh[a + b*x]))/(-1 + b^2)
```

Maple [A] time = 0.016, size = 62, normalized size = 1.5

$$\frac{\sinh\left(\left(b-1\right)x+a\right)}{2b-2}+\frac{\sinh\left(\left(1+b\right)x+a\right)}{2+2b}-\frac{\cosh\left(\left(b-1\right)x+a\right)}{2b-2}+\frac{\cosh\left(\left(1+b\right)x+a\right)}{2+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(b*x+a),x)
```

```
[Out] 1/2/(b-1)*sinh((b-1)*x+a)+1/2/(1+b)*sinh((1+b)*x+a)-1/2*cosh((b-1)*x+a)/(b-
1)+1/2*cosh((1+b)*x+a)/(1+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8884, size = 135, normalized size = 3.29

$$-\frac{\cosh\left(bx+a\right)\cosh\left(x\right)-\left(b\cosh\left(x\right)+b\sinh\left(x\right)\right)\sinh\left(bx+a\right)+\cosh\left(bx+a\right)\sinh\left(x\right)}{b^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)*cosh(x) - (b*cosh(x) + b*sinh(x))*sinh(b*x + a) + cosh(b*x
+ a)*sinh(x))/(b^2 - 1)
```

Sympy [A] time = 1.42752, size = 99, normalized size = 2.41

$$\begin{cases} \frac{xe^x \sinh(a-x)}{2} + \frac{xe^x \cosh(a-x)}{2} - \frac{e^x \sinh(a-x)}{2} & \text{for } b = -1 \\ -\frac{xe^x \sinh(a+x)}{2} + \frac{xe^x \cosh(a+x)}{2} + \frac{e^x \sinh(a+x)}{2} & \text{for } b = 1 \\ \frac{be^x \sinh(a+bx)}{b^2-1} - \frac{e^x \cosh(a+bx)}{b^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 - exp(x)*sinh(a
- x)/2, Eq(b, -1)), (-x*exp(x)*sinh(a + x)/2 + x*exp(x)*cosh(a + x)/2 + exp
(x)*sinh(a + x)/2, Eq(b, 1)), (b*exp(x)*sinh(a + b*x)/(b**2 - 1) - exp(x)*c
osh(a + b*x)/(b**2 - 1), True))
```

Giac [A] time = 1.23169, size = 43, normalized size = 1.05

$$\frac{e^{(bx+a+x)}}{2(b+1)} - \frac{e^{(-bx-a+x)}}{2(b-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a + x)/(b + 1) - 1/2*e^(-b*x - a + x)/(b - 1)
```

3.300 $\int e^x \cosh\left(a+cx^2\right) dx$

Optimal. Leaf size=85

$$\frac{\sqrt{\pi}e^{a-\frac{1}{4c}}\text{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}e^{\frac{1}{4c}-a}\text{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -(E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c]) + (E^
(a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Rubi [A] time = 0.0811729, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5513, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}e^{a-\frac{1}{4c}}\text{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}e^{\frac{1}{4c}-a}\text{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[a + c*x^2],x]
```

```
[Out] -(E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c]) + (E^
(a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^x \cosh\left(a+cx^2\right)dx &= \int\left(\frac{1}{2}e^{-a+x-cx^2}+\frac{1}{2}e^{a+x+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-a+x-cx^2}dx+\frac{1}{2}\int e^{a+x+cx^2}dx\\ &= \frac{1}{2}e^{a-\frac{1}{4c}}\int e^{\frac{(1+2cx)^2}{4c}}dx+\frac{1}{2}e^{-a+\frac{1}{4c}}\int e^{-\frac{(1-2cx)^2}{4c}}dx\\ &= -\frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\operatorname{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0807592, size = 79, normalized size = 0.93

$$\frac{\sqrt{\pi}e^{-\frac{1}{4}/c}\left(e^{\frac{1}{2}/c}(\cosh(a)-\sinh(a))\operatorname{Erf}\left(\frac{2cx-1}{2\sqrt{c}}\right)+(\sinh(a)+\cosh(a))\operatorname{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a]) +
Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4*c)))
```

Maple [A] time = 0.124, size = 72, normalized size = 0.9

$$\frac{\sqrt{\pi}}{4}e^{-\frac{4ac-1}{4c}}\operatorname{Erf}\left(\sqrt{c}x-\frac{1}{2}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}+\frac{\sqrt{\pi}}{4}e^{\frac{4ac-1}{4c}}\operatorname{Erf}\left(\sqrt{-c}x-\frac{1}{2}\frac{1}{\sqrt{-c}}\right)\frac{1}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(c*x^2+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4*P
i^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf((-c)^(1/2)*x-1/2/(-c)^(1/2))
```

Maxima [A] time = 1.10022, size = 88, normalized size = 1.04

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}x - \frac{1}{2\sqrt{-c}}\right)e^{\left(a-\frac{1}{4c}\right)}}{4\sqrt{-c}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x - \frac{1}{2\sqrt{c}}\right)e^{\left(-a+\frac{1}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) + 1/4*sq
rt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

Fricas [A] time = 1.85297, size = 286, normalized size = 3.36

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right) + \sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{(2\,cx+1)\sqrt{-c}}{2\,c}\right) - \sqrt{\pi}\sqrt{c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right) - \sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{2\,cx-1}{2\sqrt{c}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c))
*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) - sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)/c
) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\cosh\left(a + cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*cosh(a + c*x**2), x)
```

Giac [A] time = 1.31035, size = 99, normalized size = 1.16

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{1}{c}\right)\right)e^{\left(\frac{4ac-1}{4c}\right)}}{4\sqrt{-c}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{1}{c}\right)\right)e^{\left(-\frac{4ac-1}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c)
 - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt(c
)
```

3.301 $\int e^x \cosh\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=101

$$\frac{\sqrt{\pi}e^{a-\frac{(b+1)^2}{4c}}\operatorname{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}e^{\frac{(1-b)^2}{4c}-a}\operatorname{Erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -(E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/(4*Sq
rt[c]) + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])
])/(4*Sqrt[c])
```

Rubi [A] time = 0.125484, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5513, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}e^{a-\frac{(b+1)^2}{4c}}\operatorname{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}e^{\frac{(1-b)^2}{4c}-a}\operatorname{Erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[a + b*x + c*x^2],x]
```

```
[Out] -(E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/(4*Sq
rt[c]) + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])
])/(4*Sqrt[c])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^x \cosh\left(a+bx+cx^2\right)dx &= \int\left(\frac{1}{2}e^{-a+(1-b)x-cx^2}+\frac{1}{2}e^{a+(1+b)x+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-a+(1-b)x-cx^2}\,dx+\frac{1}{2}\int e^{a+(1+b)x+cx^2}\,dx\\ &= \frac{1}{2}e^{-a+\frac{(1-b)^2}{4c}}\int e^{-\frac{(1-b-2cx)^2}{4c}}\,dx+\frac{1}{2}e^{a-\frac{(1+b)^2}{4c}}\int e^{\frac{(1+b+2cx)^2}{4c}}\,dx\\ &= -\frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\text{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.146048, size = 91, normalized size = 0.9

$$\frac{\sqrt{\pi}e^{-\frac{(b+1)^2}{4c}}\left(e^{\frac{b^2+1}{2c}}(\cosh(a)-\sinh(a))\text{Erf}\left(\frac{b+2cx-1}{2\sqrt{c}}\right)+(\sinh(a)+\cosh(a))\text{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] -
 Sinh[a]) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt
[c]*E^((1 + b)^2/(4*c)))
```

Maple [A] time = 0.099, size = 97, normalized size = 1.

$$\frac{\sqrt{\pi}}{4}\mathrm{e}^{-\frac{4ac-b^2+2b-1}{4c}}\text{Erf}\left(\sqrt{c}x-\frac{1-b}{2}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}-\frac{\sqrt{\pi}}{4}\mathrm{e}^{\frac{4ac-b^2-2b-1}{4c}}\text{Erf}\left(-\sqrt{-c}x+\frac{1+b}{2}\frac{1}{\sqrt{-c}}\right)\frac{1}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(c*x^2+b*x+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b)/
c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-(-c)^(1/
2)*x+1/2*(1+b)/(-c)^(1/2))
```

Maxima [A] time = 1.09015, size = 109, normalized size = 1.08

$$\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{-c}x - \frac{b+1}{2\sqrt{-c}}\right)e^{\left(a - \frac{(b+1)^2}{4c}\right)}}{4\sqrt{-c}} + \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{c}x + \frac{b-1}{2\sqrt{c}}\right)e^{\left(-a + \frac{(b-1)^2}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c)
/sqrt(-c) + 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4*(
b - 1)^2/c)/sqrt(c)
```

Fricas [A] time = 1.75541, size = 367, normalized size = 3.63

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right) + \sinh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+b+1)\sqrt{-c}}{2\,c}\right) - \sqrt{\pi}\sqrt{c}\left(\cosh\left(-\frac{b^2-4\,ac-2\,b+1}{4\,c}\right) - \sinh\left(-\frac{b^2-4\,a}{4}\right.\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*(
b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) - sqrt(pi)*s
qrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c - 2*b
 + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \cosh\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*cosh(a + b*x + c*x**2), x)
```

Giac [A] time = 1.25795, size = 123, normalized size = 1.22

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c}\left(2\,x+\frac{b+1}{c}\right)\right)e^{\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)}}{4\,\sqrt{-c}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{c}\left(2\,x+\frac{b-1}{c}\right)\right)e^{\left(\frac{b^2-4\,ac-2\,b+1}{4\,c}\right)}}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c + 2
*b + 1)/c)/sqrt(-c) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^(1
/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

3.302 $\int e^{x^2} \cosh(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}\sqrt{\pi}e^{-a-\frac{b^2}{4}}\text{Erfi}\left(\frac{1}{2}(2x-b)\right)+\frac{1}{4}\sqrt{\pi}e^{a-\frac{b^2}{4}}\text{Erfi}\left(\frac{1}{2}(b+2x)\right)$$

```
[Out] (E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2])/4 + (E^(a - b^2/4)*Sqrt[Pi]*Er
fi[(b + 2*x)/2])/4
```

Rubi [A] time = 0.0644386, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5513, 2234, 2204}

$$\frac{1}{4}\sqrt{\pi}e^{-a-\frac{b^2}{4}}\text{Erfi}\left(\frac{1}{2}(2x-b)\right)+\frac{1}{4}\sqrt{\pi}e^{a-\frac{b^2}{4}}\text{Erfi}\left(\frac{1}{2}(b+2x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cosh[a + b*x],x]
```

```
[Out] (E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2])/4 + (E^(a - b^2/4)*Sqrt[Pi]*Er
fi[(b + 2*x)/2])/4
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\cosh(a+bx)\,dx &= \int\left(\frac{1}{2}e^{-a-bx+x^2}+\frac{1}{2}e^{a+bx+x^2}\right)dx\\ &= \frac{1}{2}\int e^{-a-bx+x^2}\,dx+\frac{1}{2}\int e^{a+bx+x^2}\,dx\\ &= \frac{1}{2}e^{-a-\frac{b^2}{4}}\int e^{\frac{1}{4}(-b+2x)^2}\,dx+\frac{1}{2}e^{a-\frac{b^2}{4}}\int e^{\frac{1}{4}(b+2x)^2}\,dx\\ &= \frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(b+2x)\right)\end{aligned}$$

Mathematica [A] time = 0.0671981, size = 51, normalized size = 0.78

$$\frac{1}{4}\sqrt{\pi}e^{-\frac{b^2}{4}}\left((\sinh(a)-\cosh(a))\mathbf{Erfi}\left(\frac{b}{2}-x\right)+(\sinh(a)+\cosh(a))\mathbf{Erfi}\left(\frac{b}{2}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cosh[a + b*x],x]
```

```
[Out] (Sqrt[Pi]*(Erfi[b/2 - x]*(-Cosh[a] + Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + Si
nh[a])))/(4*E^(b^2/4))
```

Maple [C] time = 0.069, size = 52, normalized size = 0.8

$$\frac{i}{4}\sqrt{\pi}\mathrm{e}^{-a-\frac{b^2}{4}}\mathrm{Erf}\left(-ix+\frac{i}{2}b\right)-\frac{i}{4}\sqrt{\pi}\mathrm{e}^{a-\frac{b^2}{4}}\mathrm{Erf}\left(ix+\frac{i}{2}b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(b*x+a),x)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4*b
^2)*erf(I*x+1/2*I*b)
```

Maxima [C] time = 1.06126, size = 61, normalized size = 0.94

$$-\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}-\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(-1
/2*I*b + I*x)*e^(-1/4*b^2 - a)
```

Fricas [A] time = 1.92477, size = 130, normalized size = 2.

$$\frac{1}{4}\sqrt{\pi}\left(\operatorname{erfi}\left(\frac{1}{2}b+x\right)e^{\left(\frac{1}{4}b^2+a\right)}+\operatorname{erfi}\left(-\frac{1}{2}b+x\right)e^{\left(\frac{1}{4}b^2-a\right)}\right)e^{\left(-\frac{1}{2}b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(erfi(1/2*b + x)*e^(1/4*b^2 + a) + erfi(-1/2*b + x)*e^(1/4*b^2
 - a))*e^(-1/2*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cosh(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cosh(a + b*x), x)
```

Giac [C] time = 1.204, size = 61, normalized size = 0.94

$$\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}ib-ix\right)e^{\left(-\frac{1}{4}b^2+a\right)}+\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}ib-ix\right)e^{\left(-\frac{1}{4}b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(1/
2*I*b - I*x)*e^(-1/4*b^2 - a)
```

3.303 $\int e^{x^2} \cosh\left(a + cx^2\right) dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}e^{-a}\text{Erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}}+\frac{\sqrt{\pi}e^{a}\text{Erfi}\left(\sqrt{c+1}x\right)}{4\sqrt{c+1}}$$

```
[Out] (Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(4*Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqr
t[1 + c]*x])/(4*Sqrt[1 + c])
```

Rubi [A] time = 0.0789304, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5513, 2204}

$$\frac{\sqrt{\pi}e^{-a}\text{Erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}}+\frac{\sqrt{\pi}e^{a}\text{Erfi}\left(\sqrt{c+1}x\right)}{4\sqrt{c+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cosh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(4*Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqr
t[1 + c]*x])/(4*Sqrt[1 + c])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2}\cosh\left(a+cx^2\right)dx=\int\left(\frac{1}{2}e^{-a+(1-c)x^2}+\frac{1}{2}e^{a+(1+c)x^2}\right)dx$$
$$=\frac{1}{2}\int e^{-a+(1-c)x^2}\,dx+\frac{1}{2}\int e^{a+(1+c)x^2}\,dx$$
$$=\frac{e^{-a}\sqrt{\pi}\text{erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}}+\frac{e^{a}\sqrt{\pi}\text{erfi}\left(\sqrt{1+c}x\right)}{4\sqrt{1+c}}$$

Mathematica [A] time = 0.0964044, size = 71, normalized size = 1.09

$$\frac{\sqrt{\pi}\left(\sqrt{c-1}(c+1)(\cosh(a)-\sinh(a))\text{Erf}\left(\sqrt{c-1}x\right)+(c-1)\sqrt{c+1}(\sinh(a)+\cosh(a))\text{Erfi}\left(\sqrt{c+1}x\right)\right)}{4\left(c^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cosh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]) + (
-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)
)
```

Maple [A] time = 0.072, size = 48, normalized size = 0.7

$$\frac{\sqrt{\pi}\mathrm{e}^{-a}}{4}\text{Erf}\left(\sqrt{c-1}x\right)\frac{1}{\sqrt{c-1}}+\frac{\sqrt{\pi}\mathrm{e}^{a}}{4}\text{Erf}\left(\sqrt{-1-c}x\right)\frac{1}{\sqrt{-1-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(c*x^2+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(-1
-c)^(1/2)*erf((-1-c)^(1/2)*x)
```

Maxima [A] time = 1.08295, size = 63, normalized size = 0.97

$$\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{c-1}x\right)e^{(-a)}}{4\,\sqrt{c-1}}+\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{-c-1}x\right)e^{a}}{4\,\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqrt(
-c - 1)*x)*e^a/sqrt(-c - 1)
```

Fricas [A] time = 1.88621, size = 234, normalized size = 3.6

$$\frac{\sqrt{\pi}((c+1)\cosh(a)-(c+1)\sinh(a))\sqrt{c-1}\operatorname{erf}\left(\sqrt{c-1}x\right)-\sqrt{\pi}((c-1)\cosh(a)+(c-1)\sinh(a))\sqrt{-c-1}\operatorname{erf}\left(\sqrt{-c-1}x\right)}{4\left(c^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c -
1)*x) - sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sqrt(
-c - 1)*x))/(c^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\cosh\left(a+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cosh(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*cosh(a + c*x**2), x)
```

Giac [A] time = 1.27693, size = 66, normalized size = 1.02

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{c-1}x\right)e^{(-a)}}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c-1}x\right)e^{a}}{4\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-sq
rt(-c - 1)*x)*e^a/sqrt(-c - 1)
```

3.304 $\int e^{x^2} \cosh\left(a+bx+cx^2\right) dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\pi}e^{a-\frac{b^2}{4(c+1)}}\operatorname{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}-\frac{\sqrt{\pi}e^{-a-\frac{b^2}{4(1-c)}}\operatorname{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}$$

```
[Out] -(E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])]
)/(4*Sqrt[1 - c]) + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*x
)/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Rubi [A] time = 0.158538, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5513, 2234, 2204}

$$\frac{\sqrt{\pi}e^{a-\frac{b^2}{4(c+1)}}\operatorname{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}-\frac{\sqrt{\pi}e^{-a-\frac{b^2}{4(1-c)}}\operatorname{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cosh[a + b*x + c*x^2],x]
```

```
[Out] -(E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])]
)/(4*Sqrt[1 - c]) + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*x
)/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{x^2}\cosh\left(a+bx+cx^2\right)dx &= \int\left(\frac{1}{2}e^{-a-bx+(1-c)x^2}+\frac{1}{2}e^{a+bx+(1+c)x^2}\right)dx\\
&= \frac{1}{2}\int e^{-a-bx+(1-c)x^2}\,dx+\frac{1}{2}\int e^{a+bx+(1+c)x^2}\,dx\\
&= \frac{1}{2}e^{-a-\frac{b^2}{4(1-c)}}\int e^{\frac{(-b+2(1-c)x)^2}{4(1-c)}}\,dx+\frac{1}{2}e^{a-\frac{b^2}{4(1+c)}}\int e^{\frac{(b+2(1+c)x)^2}{4(1+c)}}\,dx\\
&= -\frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}
\end{aligned}$$

Mathematica [A] time = 0.35318, size = 122, normalized size = 1.06

$$\frac{\sqrt{\pi}e^{-\frac{b^2}{4c+4}}\left(\sqrt{c-1}(c+1)e^{\frac{b^2c}{2(c^2-1)}}(\cosh(a)-\sinh(a))\mathrm{Erf}\left(\frac{b+2(c-1)x}{2\sqrt{c-1}}\right)+(c-1)\sqrt{c+1}(\sinh(a)+\cosh(a))\mathrm{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)\right)}{4\left(c^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cosh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-1 +
 c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a]) + (-1 + c)*Sqrt[1 + c]*Erfi[(b
 + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*E^(b^2
/(4 + 4*c)))
```

Maple [A] time = 0.139, size = 105, normalized size = 0.9

$$\frac{\sqrt{\pi}}{4}\mathrm{e}^{-\frac{4ac-b^2-4a}{4c-4}}\mathrm{Erf}\left(\sqrt{c-1}x+\frac{b}{2}\frac{1}{\sqrt{c-1}}\right)\frac{1}{\sqrt{c-1}}-\frac{\sqrt{\pi}}{4}\mathrm{e}^{\frac{4ac-b^2+4a}{4+4c}}\mathrm{Erf}\left(-\sqrt{-1-c}x+\frac{b}{2}\frac{1}{\sqrt{-1-c}}\right)\frac{1}{\sqrt{-1-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(c*x^2+b*x+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*x+
1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(1+c))/(-1-c)^(1/2)
*erf(-(-1-c)^(1/2)*x+1/2*b/(-1-c)^(1/2))
```

Maxima [A] time = 1.07839, size = 120, normalized size = 1.04

$$\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{-c-1}x-\frac{b}{2\sqrt{-c-1}}\right)e^{\left(a-\frac{b^2}{4(c+1)}\right)}}{4\sqrt{-c-1}}+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{c-1}x+\frac{b}{2\sqrt{c-1}}\right)e^{\left(-a+\frac{b^2}{4(c-1)}\right)}}{4\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c + 1
))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(-a
 + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

Fricas [A] time = 1.82221, size = 466, normalized size = 4.05

$$\frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\operatorname{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)-\sqrt{\pi}\left((c-1)\cosh\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)+(c-\right.}{4\left(c^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*sin
h(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x + b)/
sqrt(c - 1)) - sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)) + (
c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2*(c +
 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cosh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cosh(a + b*x + c*x**2), x)
```

Giac [A] time = 1.22805, size = 136, normalized size = 1.18

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c-1}\left(2x+\frac{b}{c+1}\right)\right)e^{\left(-\frac{b^2-4ac-4a}{4(c+1)}\right)}}{4\sqrt{-c-1}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c-1}\left(2x+\frac{b}{c-1}\right)\right)e^{\left(\frac{b^2-4ac+4a}{4(c-1)}\right)}}{4\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*c
 - 4*a)/(c + 1))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x + b/
(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

3.305 $\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx$

Optimal. Leaf size=110

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)$$

```
[Out] (E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f])
/(2*Sqrt[f])])/4 + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi
[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rubi [A] time = 0.148254, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + f*x^2],x]
```

```
[Out] (E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f])
/(2*Sqrt[f])])/4 + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi
[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-d-fx^2}f^{a+bx}+\frac{1}{2}e^{d+fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{2}\int e^{-d-fx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{d+fx^2}f^{a+bx}\,dx\\ &= \frac{1}{2}\int e^{-d-fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{2}\int e^{d+fx^2+a\log(f)+bx\log(f)}\,dx\\ &= \frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{-\frac{(-2fx+b\log(f))^2}{4f}}\,dx\\ &= \frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\end{aligned}$$

Mathematica [A] time = 0.122241, size = 102, normalized size = 0.93

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{b^2\log^2(f)}{4f}}\left(e^{\frac{b^2\log^2(f)}{2f}}(\cosh(d)-\sinh(d))\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+(\sinh(d)+\cosh(d))\mathrm{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + f*x^2],x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi]*(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(2*
Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(Cosh[
```

```
d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A] time = 0.122, size = 100, normalized size = 0.9

$$-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{\frac{(\ln(f))^2b^2-4df}{4f}}\mathrm{Erf}\left(-\sqrt{f}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{f}}-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2-4df}{4f}}\mathrm{Erf}\left(-\sqrt{-f}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-f}}\right)\frac{1}{\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-4*d*f)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2
*ln(f)*b/f^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*f)/f)/(-f)^(1/
2)*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
```

Maxima [A] time = 1.08289, size = 122, normalized size = 1.11

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\,\mathrm{erf}\left(\sqrt{f}x-\frac{b\log(f)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*l
og(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f))*e
^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Fricas [B] time = 1.87246, size = 591, normalized size = 5.37

$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)\mathrm{erf}\left(\frac{(2fx+b\log(f))\sqrt{-f}}{2f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)\mathrm{erf}\left(-\frac{2fx-b\log(f)}{2\sqrt{f}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*e
rf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) + sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(
f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + sqrt
(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 +
4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqr
t(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\cosh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + f*x**2), x)
```

Giac [A] time = 1.23386, size = 143, normalized size = 1.3

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)}}{4\sqrt{f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4
*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*l
og(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

3.306 $\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=148

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\text{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\text{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] (E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Log
[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)
*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + f^(a + b*x)/(
2*b*Log[f])
```

Rubi [A] time = 0.191135, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5513, 2194, 2287, 2234, 2205, 2204}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\text{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\text{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```


```
[Out] (E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Log
[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)
*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + f^(a + b*x)/(
2*b*Log[f])
```


Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh^2\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\ &= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int e^{-2d-2fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{4}\int e^{2d+2fx^2+a\log(f)+bx\log(f)}\,dx\\ &= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{2d-\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{\frac{(4fx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{(-4fx+b\log(f))^2}{8f}}\,d\\ &= \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{4fx+b\log(\right.}{2\sqrt{2}\sqrt{f}}\right.\end{aligned}$$

Mathematica [A] time = 0.676545, size = 149, normalized size = 1.01

$$\frac{1}{16}f^a\left(\frac{\sqrt{2\pi}e^{\frac{b^2\log^2(f)}{8f}}(\cosh(2d)-\sinh(2d))\text{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)}{\sqrt{f}}+\frac{\sqrt{2\pi}e^{-\frac{b^2\log^2(f)}{8f}}(\sinh(2d)+\cosh(2d))\text{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)}{\sqrt{f}}+\frac{}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*((8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(4*
f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] + (Sq
rt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh[2*d
]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16
```

Maple [A] time = 0.172, size = 126, normalized size = 0.9

$$-\frac{\sqrt{2}f^a\sqrt{\pi}}{16}\mathrm{e}^{\frac{(\ln(f))^2b^2-16df}{8f}}\mathrm{Erf}\left(-\sqrt{2}\sqrt{f}x+\frac{b\ln(f)\sqrt{2}}{4}\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{f}}-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2-16df}{8f}}\mathrm{Erf}\left(-\sqrt{-2f}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-2f}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(ln(f)^2*b^2-16*d*f)/f)*2^(1/2)/f^(1/2)*erf(-2^(
1/2)*f^(1/2)*x+1/4*ln(f)*b*2^(1/2)/f^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/8*(ln(f
)^2*b^2-16*d*f)/f)/(-2*f)^(1/2)*erf(-(-2*f)^(1/2)*x+1/2*ln(f)*b/(-2*f)^(1/2
))+1/2*f^a/ln(f)/b*f^(b*x)
```

Maxima [A] time = 1.57384, size = 171, normalized size = 1.16

$$\frac{\sqrt{2}\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{8f}-2d\right)}}{16\sqrt{f}}+\frac{\sqrt{2}\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{8f}+2d\right)}}{16\sqrt{-f}}+\frac{f^{bx+a}}{2b\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqrt
(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf(s
qrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/f +
 2*d)/sqrt(-f) + 1/2*f^(b*x + a)/(b*log(f))
```

Fricas [B] time = 1.95898, size = 826, normalized size = 5.58

$$-\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-8\,af\log(f)-16\,df}{8\,f}\right)\text{erf}\left(\frac{\sqrt{2}(4\,fx+b\log(f))\sqrt{-f}}{4\,f}\right)\log(f)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+8\,af\log(f)-16\,df}{8\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2)*
sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*erf(-
1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)
*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2
 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*
(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f*log(f)
 - 16*d*f)/f) - 8*f*cosh((b*x + a)*log(f)) - 8*f*sinh((b*x + a)*log(f)))/(b
*f*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + f*x**2)**2, x)
```

Giac [C] time = 1.29996, size = 479, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8*
(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)*e
rf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*
log(f) - 16*d*f)/f)/sqrt(-f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1
/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f)
- pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2
*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^
(b*x*log(abs(f)) + a*log(abs(f))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/
2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b +
4*b*log(abs(f))) + 2*I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*
sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*
x*log(abs(f)) + a*log(abs(f)))
```

3.307 $\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=239

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\mathrm{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\mathrm{Erfi}$$

```
[Out] (3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f
])/(2*Sqrt[f])])/16 + (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[P
i/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E^(d - (b^2*Log[f
]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/16
+ (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x + b*
Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rubi [A] time = 0.285771, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\mathrm{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\mathrm{Erfi}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f
])/(2*Sqrt[f])])/16 + (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[P
i/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E^(d - (b^2*Log[f
]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/16
+ (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x + b*
Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh^3\left(d+fx^2\right)dx &= \int\left(\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}+\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx}\,dx+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{d+fx^2}f^{a+bx}\\ &= \frac{1}{8}\int e^{-3d-3fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{8}\int e^{3d+3fx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}\int e^{-d-fx^2+a\log(f)}\\ &= \frac{1}{8}\left(3e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{8}\left(e^{3d-\frac{b^2\log^2(f)}{12f}}f^a\right)\int e^{\frac{(6fx+b\log(f))^2}{12f}}\,dx+\frac{1}{8}\left(e^{-3d+\frac{b^2}{}}\right.\\ &= \frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{6fx-b\log}{2\sqrt{3}\sqrt{}}\right.\end{aligned}$$

Mathematica [A] time = 0.41045, size = 286, normalized size = 1.2

$$\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{-\frac{b^2\log^2(f)}{4f}}\left(3\sqrt{3}e^{\frac{b^2\log^2(f)}{2f}}(\cosh(d)-\sinh(d))\operatorname{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{3f}}(\cosh(3d)-\sinh(3d))\operatorname{Erf}\left(\frac{6fx}{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi/3]*(3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt
[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sqrt[
3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(2
*Sqrt[f])]*(Cosh[d] - Sinh[d]) + 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[
f])]*Sinh[d] + E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*S
qrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*f*x + b
*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A] time = 0.208, size = 207, normalized size = 0.9

$$-\frac{\sqrt{3}f^{a}\sqrt{\pi}}{48}\mathrm{e}^{\frac{(\ln(f))^2b^2-36df}{12f}}\mathrm{Erf}\left(-\sqrt{3}\sqrt{f}x+\frac{b\ln(f)\sqrt{3}}{6}\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{f}}-\frac{f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^2b^2-36df}{12f}}\mathrm{Erf}\left(-\sqrt{-3f}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-3f}}\right)\frac{1}{\sqrt{\ }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+d)^3,x)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(1/12*(ln(f)^2*b^2-36*d*f)/f)*3^(1/2)/f^(1/2)*erf(-3^
(1/2)*f^(1/2)*x+1/6*ln(f)*b*3^(1/2)/f^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/12*(l
n(f)^2*b^2-36*d*f)/f)/(-3*f)^(1/2)*erf(-(-3*f)^(1/2)*x+1/2*ln(f)*b/(-3*f)^(
1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-4*d*f)/f)/f^(1/2)*erf(-f^(1/2)
*x+1/2*ln(f)*b/f^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*f)/f)/(
-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
```

Maxima [A] time = 1.66498, size = 270, normalized size = 1.13

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log(f)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}+\frac{\sqrt{3}\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}b\log(f)}{6\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{12f}-3d\right)}}{48\sqrt{f}}+\frac{\sqrt{3}\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{3}\sqrt{-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqr
```

```
t(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt(3)
*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e^(-1
/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2
*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Fricas [B] time = 2.09782, size = 1287, normalized size = 5.38

$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}\left(6\,fx+b\log(f)\right)\sqrt{-f}}{6\,f}\right)+\sqrt{3}\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)\operatorname{erf}\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) + sqrt(3)*sqrt(pi
)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*sqr
t(3)*(6*f*x - b*log(f))/sqrt(f)) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3
)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*
d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt
(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) + 9*sqrt(pi)*s
qrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(2*f*x +
b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + 4*a*f*l
og(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(pi)*sqrt(f)
*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4*a*f*log(f)
 - 4*d*f)/f) - 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f)*s
inh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30216, size = 301, normalized size = 1.26

$$-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)}}{48\sqrt{f}}-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-12\,af\log}{12\,f}\right.}}{48\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/12
*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(pi)
*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 - 12*
a*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*
log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 3/16*s
qrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f
*log(f) - 4*d*f)/f)/sqrt(-f)
```

3.308 $\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=115

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\operatorname{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\operatorname{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

```
[Out] (E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b*L
og[f])/(2*Sqrt[f])])/4 + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[
Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rubi [A] time = 0.21951, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\operatorname{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\operatorname{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
```


```
[Out] (E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b*L
og[f])/(2*Sqrt[f])])/4 + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[
Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```


Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx}\,dx\\ &= \frac{1}{2}\int e^{-d-fx^2+a\log(f)-x(e-b\log(f))}\,dx+\frac{1}{2}\int e^{d+fx^2+a\log(f)+x(e+b\log(f))}\,dx\\ &= \frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4f}}f^a\right)\int e^{\frac{(e+2fx+b\log(f))^2}{4f}}\,dx\\ &= \frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2\right.\end{aligned}$$

Mathematica [A] time = 0.266512, size = 123, normalized size = 1.07

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+e^2}{4f}}\left((\cosh(d)-\sinh(d))e^{\frac{b^2\log^2(f)+e^2}{2f}}\mathrm{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+(\sinh(d)+\cosh(d))\mathrm{Erfi}\left(\frac{b\log(f)+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e +
2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(e + 2*f*x + b*Lo
```

```
g[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*f)))
```

Maple [A] time = 0.098, size = 126, normalized size = 1.1

$$-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{\frac{(\ln(f))^2b^2-2\ln(f)be-4df+e^2}{4f}}\mathrm{Erf}\left(-\sqrt{f}x+\frac{b\ln(f)-e}{2}\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{f}}-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2+2\ln(f)be-4df+e^2}{4f}}\mathrm{Erf}\left(-\sqrt{-f}x+\frac{e+b\ln}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*er
f(-f^(1/2)*x+1/2*(b*ln(f)-e)/f^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^
2+2*ln(f)*b*e-4*d*f+e^2)/f)/(-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-
f)^(1/2))
```

Maxima [A] time = 1.0277, size = 138, normalized size = 1.2

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\,\mathrm{erf}\left(\sqrt{f}x-\frac{b\log(f)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d
+ 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f)
 + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

Fricas [B] time = 1.97204, size = 699, normalized size = 6.08

$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+e^2-4df+2(be-2af)\log(f)}{4f}\right)\mathrm{erf}\left(\frac{(2fx+b\log(f)+e)\sqrt{-f}}{2f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+e^2-4df-2(be-2af)\log(f)}{4f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a
*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + sqrt(pi)*sqrt(f
)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(-1/
2*(2*f*x - b*log(f) + e)/sqrt(f)) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*lo
g(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a*f
)*log(f))/f) + sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(f))*si
nh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + e*x + f*x**2), x)
```

Giac [A] time = 1.22867, size = 181, normalized size = 1.57

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4af\log(f)-2be\log(f)-4df+e^2}{4f}\right)}}{4\sqrt{f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4af\log(\right.}{}}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)
^2 + 4*a*f*log(f) - 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(f) - 1/4*sqrt(pi)*e
rf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*lo
g(f) + 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(-f)
```

3.309 $\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=161

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\text{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\text{Erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] (E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*f
*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*f)
)*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])
])/8 + f^(a + b*x)/(2*b*Log[f])
```

Rubi [A] time = 0.272695, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5513, 2194, 2287, 2234, 2205, 2204}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\text{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\text{Erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*f
*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*f)
)*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])
])/8 + f^(a + b*x)/(2*b*Log[f])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int\exp\left(-2d-2fx^2+a\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int\exp\left(2d\right.\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^a\right)\int e^{-\frac{(-2e-4fx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^a\right)\int e\\
&= \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{er}\end{aligned}$$

Mathematica [A] time = 0.626115, size = 220, normalized size = 1.37

$$\frac{f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+4e^2}{8f}}\left(\sqrt{\pi}b\log(f)(\cosh(2d)-\sinh(2d))e^{\frac{b^2\log^2(f)+4e^2}{4f}}\operatorname{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+4\sqrt{2}f^{b\left(\frac{e}{2f}+x\right)+\frac{1}{2}}e^{\frac{b^2\log^2(f)+4e^2}{8f}}+\right.}{8\sqrt{2}b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1/2
 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2*e
+ 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d]) + b
*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2
*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[f])
```

Maple [A] time = 0.127, size = 158, normalized size = 1.

$$-\frac{\sqrt{2}f^{a}\sqrt{\pi}}{16}\mathrm{e}^{\frac{(\ln(f))^{2}b^{2}-4\ln(f)be-16df+4e^{2}}{8f}}\mathrm{Erf}\left(-\sqrt{2}\sqrt{f}x+\frac{(b\ln(f)-2e)\sqrt{2}}{4}\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{f}}-\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}+4\ln(f)be-16df+4e^{2}}{8f}}\mathrm{Erf}\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(ln(f)^2*b^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)*2^(1/2
)/f^(1/2)*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))-1/8*Pi^
(1/2)*f^a*exp(-1/8*(ln(f)^2*b^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)/(-2*f)^(1/2)*e
rf(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))+1/2*f^a/ln(f)/b*f^(b*x)
```

Maxima [A] time = 1.53616, size = 193, normalized size = 1.2

$$\frac{\sqrt{2}\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}(b\log(f)+2e)}{4\sqrt{-f}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^{2}}{8f}\right)}}{16\sqrt{-f}}+\frac{\sqrt{2}\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}(b\log(f)-2e)}{4\sqrt{f}}\right)e^{\left(-2d+\frac{(b\log(f)-2e)^{2}}{8f}\right)}}{16\sqrt{f}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)*s
qrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f))*e
```

```
^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) + 1/2*f^(b*x + a)/(b*log(f))
```

Fricas [B] time = 1.92515, size = 956, normalized size = 5.94

$$-\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^2\log\left(f\right)^2+4e^2-16df+4\left(be-2af\right)\log\left(f\right)}{8f}\right)\operatorname{erf}\left(\frac{\sqrt{2}\left(4fx+b\log\left(f\right)+2e\right)\sqrt{-f}}{4f}\right)\log\left(f\right)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\cosh\left(\frac{b^2\log\left(f\right)^2+4\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*e)*sqrt(
-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 -
 16*d*f - 4*(b*e - 2*a*f)*log(f))/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f) + 2
*e)/sqrt(f))*log(f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*(4*f*x +
b*log(f) + 2*e)*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*sqrt(2)*
(4*f*x - b*log(f) + 2*e)/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 1
6*d*f - 4*(b*e - 2*a*f)*log(f))/f) - 8*f*cosh((b*x + a)*log(f)) - 8*f*sinh(
(b*x + a)*log(f)))/(b*f*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.29423, size = 525, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(f) - 2*e)/f))
*e^(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 4*b*e*log(f) - 16*d*f + 4*e^2)/f)/sq
rt(f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) +
2*e)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*log(f) + 4*b*e*log(f) - 16*d*f + 4*e
^2)/f)/sqrt(-f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f
) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) -
(pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f)
+ 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(
f)) + a*log(abs(f))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x +
1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f
))) + 2*I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*
I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f))
 + a*log(abs(f)))
```

3.310 $\int f^{a+bx} \cosh^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=257

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\text{Erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e$$

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b
*Log[f])/(2*Sqrt[f])])/16 + (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E
^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log
[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 + a)*S
qrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rubi [A] time = 0.46604, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\text{Erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e$$

Antiderivative was successfully verified.

[In] Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]


```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b
*Log[f])/(2*Sqrt[f])])/16 + (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E
^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log
[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 + a)*S
qrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```


Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a+bx}+\frac{3}{8}\mathrm{ex}\right.\\ &= \frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx}\,dx+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+fx^2\right)\right)f^{a+bx}\,d\\ &= \frac{1}{8}\int\exp\left(-3d-3fx^2+a\log(f)-x(3e-b\log(f))\right)dx+\frac{1}{8}\int\exp\left(3d+3fx^2+a\right.\\ &= \frac{1}{8}\left(3e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}dx+\frac{1}{8}\left(e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^a\right)\int e^{-\frac{(-3e-6fx+b\log}{12f}}\\ &= \frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\mathrm{er}\end{aligned}$$

Mathematica [A] time = 0.770232, size = 353, normalized size = 1.37

$$\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+3e^2}{4f}}\left(3\sqrt{3}(\cosh(d)-\sinh(d))e^{\frac{b^2\log^2(f)+2e^2}{2f}}\mathrm{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+(\cosh(3d)-\sinh(3d))e^{\frac{2b^2}{}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi[(
e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erf
i[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 + b^2
*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d
]) + 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]*Sinh[
d] + E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqr
t[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(3*e
 + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^2 + b^2*L
og[f]^2)/(4*f)))
```

Maple [A] time = 0.171, size = 265, normalized size = 1.

$$-\frac{\sqrt{3}f^{a}\sqrt{\pi}}{48}\mathrm{e}^{\frac{(\ln(f))^{2}b^{2}-6\ln(f)be-36df+9e^{2}}{12f}}\mathrm{Erf}\left(-\sqrt{3}\sqrt{f}x+\frac{\left(b\ln\left(f\right)-3e\right)\sqrt{3}}{6}\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{f}}-\frac{f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}+6\ln(f)be-36df+9e^{2}}{12f}}\mathrm{Erf}\left(-\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(1/12*(ln(f)^2*b^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)*3^(1/
2)/f^(1/2)*erf(-3^(1/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))-1/16*P
i^(1/2)*f^a*exp(-1/12*(ln(f)^2*b^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)/(-3*f)^(1/2
)*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp
(1/4*(ln(f)^2*b^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2*(b*l
n(f)-e)/f^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*f+
e^2)/f)/(-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))
```

Maxima [A] time = 1.56483, size = 308, normalized size = 1.2

$$\frac{\sqrt{3}\sqrt{\pi}f^{a}\,\mathrm{erf}\left(\sqrt{3}\sqrt{-f}x-\frac{\sqrt{3}\left(b\log\left(f\right)+3e\right)}{6\sqrt{-f}}\right)e^{\left(3d-\frac{\left(b\log\left(f\right)+3e\right)^{2}}{12f}\right)}}{48\sqrt{-f}}+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\,\mathrm{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{\left(b\log\left(f\right)-e\right)^{2}}{4f}\right)}+\frac{\sqrt{3}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(pi)
```

```
*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*log
(f) - e)^2/f) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqrt(
3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt(f) +
 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d - 1/4
*(b*log(f) + e)^2/f)/sqrt(-f)
```

Fricas [B] time = 2.10165, size = 1519, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f +
 6*(b*e - 2*a*f)*log(f))/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sqrt(-
f)/f) + sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f -
 6*(b*e - 2*a*f)*log(f))/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/sqrt(
f)) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sq
rt(-f)/f)*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f + 6*(b*e - 2*a*f)*log(f)
)/f) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/s
qrt(f))*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f - 6*(b*e - 2*a*f)*log(f))/
f) + 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*
a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + 9*sqrt(pi)*sqr
t(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(
-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) - 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x +
 b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e -
2*a*f)*log(f))/f) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt
(f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2826, size = 385, normalized size = 1.5

$$-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)-3e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12af\log(f)-6be\log(f)-36df+9e^2}{12f}\right)}}{48\sqrt{f}}-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)+3e}{f}\right)\right)}{48\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*log(f) - 3*e)/f))
*e^(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 6*b*e*log(f) - 36*d*f + 9*e^2)/f)/
sqrt(f) - 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f)
+ 3*e)/f))*e^(-1/12*(b^2*log(f)^2 - 12*a*f*log(f) + 6*b*e*log(f) - 36*d*f +
 9*e^2)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/
f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 2*b*e*log(f) - 4*d*f + e^2)/f)/sq
rt(f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(
b^2*log(f)^2 - 4*a*f*log(f) + 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(-f)
```

3.311 $\int f^{a+cx^2}\cosh(d+ex)\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -(E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[
Log[f]])
```

Rubi [A] time = 0.193849, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5513, 2287, 2234, 2204}

$$\frac{\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] -(E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[
Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh(d+ex)\,dx &= \int\left(\frac{1}{2}e^{-d-ex}f^{a+cx^2}+\frac{1}{2}e^{d+ex}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{d+ex}f^{a+cx^2}\,dx\\ &= \frac{1}{2}\int e^{-d-ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{2}\int e^{d+ex+a\log(f)+cx^2\log(f)}\,dx\\ &= \frac{1}{2}\left(e^{-d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{2}\left(e^{d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+2cx\log(f))^2}{4c\log(f)}}\,dx\\ &= -\frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.150874, size = 104, normalized size = 0.78

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}}\left((\cosh(d)-\sinh(d))\mathrm{Erfi}\left(\frac{2cx\log(f)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(d)+\cosh(d))\mathrm{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d]
- Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + S
inh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.097, size = 117, normalized size = 0.9

$$\frac{f^{a}\sqrt{\pi}}{4}\mathrm{e}^{-\frac{4d\ln(f)c+e^{2}}{4c\ln(f)}}\operatorname{Erf}\left(\sqrt{-c\ln\left(f\right)}x+\frac{e}{2}\frac{1}{\sqrt{-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{-c\ln\left(f\right)}}-\frac{f^{a}\sqrt{\pi}}{4}\mathrm{e}^{\frac{4d\ln(f)c-e^{2}}{4c\ln(f)}}\operatorname{Erf}\left(-\sqrt{-c\ln\left(f\right)}x+\frac{e}{2}\frac{1}{\sqrt{-c\ln\left(f\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(e*x+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((
-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(
f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f)
)^(1/2))
```

Maxima [A] time = 1.06411, size = 142, normalized size = 1.07

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^{2}}{4c\log(f)}\right)}}{4\sqrt{-c\log\left(f\right)}}+\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x+\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^{2}}{4c\log(f)}\right)}}{4\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*
e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] time = 1.93468, size = 598, normalized size = 4.5

$$-\frac{\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{4ac\log(f)^{2}+4cd\log(f)-e^{2}}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4ac\log(f)^{2}+4cd\log(f)-e^{2}}{4c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2cx\log(f)+e)\sqrt{-c\log(f)}}{2c\log(f)}\right)+\sqrt{-c}}{4c\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e
^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(
c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-
c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f
))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))*
erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\cosh(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x), x)
```

Giac [A] time = 1.28414, size = 178, normalized size = 1.34

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log}{4c\log(f)}\right.}}{4\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*
log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*e
rf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*
d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.312 $\int f^{a+cx^2} \cosh^2(d+ex)\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{c\log(f)}-2d}\text{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)}}\text{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[Log
[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*E
rfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.21993, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5513, 2204, 2287, 2234}

$$-\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{c\log(f)}-2d}\text{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)}}\text{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[Log
[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*E
rfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cosh^2(d+ex)\,dx &= \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2ex}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d-2ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{4}\int e^{2d+2ex+a\log(f)+cx^2\log(f)}\,d\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{4}\left(e^{2d-\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(2e+2}{4}}\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e+cx\log(}{\sqrt{c}\sqrt{\log(f}}\right.}{8\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A] time = 0.225014, size = 131, normalized size = 0.81

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{c\log(f)}}\left((\cosh(2d)-\sinh(2d))\text{Erfi}\left(\frac{cx\log(f)-e}{\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\text{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)+2e^{\frac{e^2}{c\log(f)}}\text{Erfi}\left(\sqrt{c}x\sqrt{\log(\right.\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(-e
 + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e +
c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c]*E^
(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.113, size = 139, normalized size = 0.9

$$\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{-\frac{2d\ln(f)c+e^{2}}{c\ln(f)}}\operatorname{Erf}\left(\sqrt{-c\ln\left(f\right)}x+e\frac{1}{\sqrt{-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{-c\ln\left(f\right)}}-\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{\frac{2d\ln(f)c-e^{2}}{c\ln(f)}}\operatorname{Erf}\left(-\sqrt{-c\ln\left(f\right)}x+e\frac{1}{\sqrt{-c\ln\left(f\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*l
n(f))^(1/2)*x+e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))+1/4*f^a
*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [A] time = 1.04816, size = 177, normalized size = 1.1

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x-\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(2d-\frac{e^{2}}{c\log(f)}\right)}}{8\sqrt{-c\log\left(f\right)}}+\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x+\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(-2d-\frac{e^{2}}{c\log(f)}\right)}}{8\sqrt{-c\log\left(f\right)}}+\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{\right.}}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(c
*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sqrt
(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*e
rf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Fricas [A] time = 2.01814, size = 678, normalized size = 4.21

$$-\frac{2\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(a\log\left(f\right)\right)+\sqrt{\pi}\sinh\left(a\log\left(f\right)\right)\right)\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)+\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{ac\log(f)^{2}+2cd\log}{c\log(f)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f)))
*erf(sqrt(-c*log(f))*x) + sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 + 2*
c*d*log(f) - e^2)/(c*log(f))) + sqrt(pi)*sinh((a*c*log(f)^2 + 2*c*d*log(f)
- e^2)/(c*log(f))))*erf((c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqrt
(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))
+ sqrt(pi)*sinh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))))*erf((c*x*l
og(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x)**2, x)
```

Giac [A] time = 1.36981, size = 203, normalized size = 1.26

$$-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2+2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2-2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) - e^
2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x - e/(
c*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(
f))
```

3.313 $\int f^{a+cx^2} \cosh^3(d+ex)\,dx$

Optimal. Leaf size=271

$$-\frac{3\sqrt{\pi} f^a e^{-\frac{e^2}{4c\log(f)}-d}\text{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi} f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\text{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi} f^a e^{d-\frac{e^2}{4c\log(f)}}\text{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\sqrt{\pi} f$$

```
[Out] (-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(-3*d - (9*e^2)/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*
Sqrt[c]*Sqrt[Log[f]]) + (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*
d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.337881, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5513, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi} f^a e^{-\frac{e^2}{4c\log(f)}-d}\text{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi} f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\text{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi} f^a e^{d-\frac{e^2}{4c\log(f)}}\text{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\sqrt{\pi} f$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(-3*d - (9*e^2)/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*
Sqrt[c]*Sqrt[Log[f]]) + (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*
d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh^3(d+ex)\,dx &= \int\left(\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}+\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)dx\\
&= \frac{1}{8}\int e^{-3d-3ex}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3d+3ex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{d+ex}f^{a+cx^2}\,dx\\
&= \frac{1}{8}\int e^{-3d-3ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{8}\int e^{3d+3ex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}\int e^{-d-ex+a\log(f)+cx^2\log\ldots}\\
&= \frac{1}{8}\left(e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-3e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\left(e^{3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(3e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\left(3e^{-d-\frac{e^2}{4c\log\ldots}}\right.\\
&= -\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\ldots}}{16\sqrt{c}\ldots}\end{aligned}$$

Mathematica [A] time = 0.44809, size = 214, normalized size = 0.79

$$\frac{\sqrt{\pi}f^a e^{-\frac{9e^2}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\left(3(\cosh(2d)-\sinh(2d))e^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{2cx\log(f)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\mathrm{Erfi}\left(\frac{2cx\log\ldots}{2\sqrt{c}\sqrt{\ldots}}\right.\right.\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*c*
x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e + 2
*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e
+ 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + Erfi[(
-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d])))/(1
6*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.14, size = 234, normalized size = 0.9

$$\frac{f^a\sqrt{\pi}}{16}e^{-\frac{12d\ln(f)c+9e^2}{4c\ln(f)}}\mathrm{Erf}\left(\sqrt{-c\ln(f)}x+\frac{3e}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{16}e^{\frac{12d\ln(f)c-9e^2}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{3e}{2}\frac{1}{\sqrt{-c\ln(f)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(e*x+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f((-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(3/4*(4*d
*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+3/2*e/(-c
*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln
(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a
*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*
x+1/2*e/(-c*ln(f))^(1/2))
```

Maxima [A] time = 1.06688, size = 285, normalized size = 1.05

$$\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{3e}{2\sqrt{-c\log(f)}}\right)e^{\left(3d-\frac{9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}f^a\,\mathrm{e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d - 9
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*
x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/1
```

```
6*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e
^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] time = 2.02335, size = 1202, normalized size = 4.44

$$-\frac{\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log\left(f\right)^{2}+12\,cd\log\left(f\right)-9\,e^{2}}{4\,c\log\left(f\right)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log\left(f\right)^{2}+12\,cd\log\left(f\right)-9\,e^{2}}{4\,c\log\left(f\right)}\right)\right)\operatorname{erf}\left(\frac{\left(2\,cx\log\left(f\right)+3\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) - 9
*e^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f)))
 + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^
2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(c
*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(
-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(
f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))
*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(
sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))) + sq
rt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))))*erf(1
/2*(2*c*x*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34824, size = 356, normalized size = 1.31

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{3\,e}{c\log\left(f\right)}\right)\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}+12\,cd\log\left(f\right)-9\,e^{2}}{4\,c\log\left(f\right)}\right)}}{16\,\sqrt{-c\log\left(f\right)}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{e}{c\log\left(f\right)}\right)\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}}{4\,c}\right.}}{16\,\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*a
*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqr
t(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2
 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f
) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f
))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/
(c*log(f)))/sqrt(-c*log(f))
```

3.314 $\int f^{a+cx^2} \cosh\left(d+fx^2\right) dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\pi}e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}e^d f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}$$

```
[Out] (f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(4*E^d*Sqrt[f - c*Log[f]]) + (E^d*
f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Rubi [A] time = 0.158194, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5513, 2287, 2205, 2204}

$$\frac{\sqrt{\pi}e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}e^d f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(4*E^d*Sqrt[f - c*Log[f]]) + (E^d*
f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-d-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+cx^2}\right)dx\\ &=\frac{1}{2}\int e^{-d-fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{d+fx^2}f^{a+cx^2}\,dx\\ &=\frac{1}{2}\int e^{-d+a\log(f)-x^2(f-c\log(f))}\,dx+\frac{1}{2}\int e^{d+a\log(f)+x^2(f+c\log(f))}\,dx\\ &=\frac{e^{-d}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.317373, size = 75, normalized size = 0.93

$$\frac{1}{4}\sqrt{\pi}f^a\left(\frac{(\cosh(d)-\sinh(d))\mathbf{Erf}\left(x\sqrt{f-c\log(f)}\right)}{\sqrt{f-c\log(f)}}+\frac{(\sinh(d)+\cosh(d))\mathbf{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{\sqrt{c\log(f)+f}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f - c*L
og[f]] + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*Log[f]
]))/4
```

Maple [A] time = 0.073, size = 70, normalized size = 0.9

$$\frac{f^a\sqrt{\pi}\mathrm{e}^{-d}}{4}\mathrm{Erf}\left(x\sqrt{f-c\ln\left(f\right)}\right)\frac{1}{\sqrt{f-c\ln\left(f\right)}}+\frac{f^a\sqrt{\pi}\mathrm{e}^{d}}{4}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)-f}x\right)\frac{1}{\sqrt{-c\ln\left(f\right)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+1/4*Pi^
(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Maxima [A] time = 1.0347, size = 93, normalized size = 1.15

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right)e^{(-d)}}{4\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x\right)e^{d}}{4\sqrt{-c\log\left(f\right)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) + 1/
4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

Fricas [B] time = 1.88687, size = 412, normalized size = 5.09

$$-\frac{\left(\sqrt{\pi}\left(c\log\left(f\right)+f\right)\cosh\left(a\log\left(f\right)-d\right)+\sqrt{\pi}\left(c\log\left(f\right)+f\right)\sinh\left(a\log\left(f\right)-d\right)\right)\sqrt{-c\log\left(f\right)+f}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}\right.}{4\left(c^{2}\mathrm{l}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f)
*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) + (sqrt
(pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*log
(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2 - f
^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^{2}}\cosh\left(d+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + f*x**2), x)
```

Giac [A] time = 1.3165, size = 101, normalized size = 1.25

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{\left(a\log\left(f\right)+d\right)}}{4\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+f}x\right)e^{\left(a\log\left(f\right)-d\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f) -
 f) - 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*log
(f) + f)
```

3.315 $\int f^{a+cx^2} \cosh^2(d+fx^2)\,dx$

Optimal. Leaf size=128

$$\frac{\sqrt{\pi}e^{-2d}f^a\text{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}e^{2d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^a
*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]) + (
E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f]])
```

Rubi [A] time = 0.203445, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5513, 2204, 2287, 2205}

$$\frac{\sqrt{\pi}e^{-2d}f^a\text{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}e^{2d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^a
*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]) + (
E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\ &= \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d+a\log(f)-x^2(2f-c\log(f))}\,dx+\frac{1}{4}\int e^{2d+a\log(f)+x^2(2f+c\log}\\ &= \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{2f+c}\right.}{8\sqrt{2f+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.542172, size = 179, normalized size = 1.4

$$\frac{\sqrt{\pi}f^a\left(\left(2c^2\log^2(f)-8f^2\right)\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)+\sqrt{c}\sqrt{\log(f)}\left(\sqrt{2f-c\log(f)}(c\log(f)+2f)(\sinh(2d)-\cosh(2d))\text{Erf}\left(\right.\right.\right.}{8\sqrt{c}\sqrt{\log(f)}\left(c^2\log^2(f)-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(-8*f^2 + 2*c^2*Log[f]^2) + Sqr
t[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f +
c*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f - c*
Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log
[f]]*(-4*f^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.091, size = 101, normalized size = 0.8

$$\frac{f^{a}\sqrt{\pi}\mathrm{e}^{-2d}}{8}\mathrm{Erf}\left(x\sqrt{2f-c\ln\left(f\right)}\right)\frac{1}{\sqrt{2f-c\ln\left(f\right)}}+\frac{f^{a}\sqrt{\pi}\mathrm{e}^{2d}}{8}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)-2f}x\right)\frac{1}{\sqrt{-c\ln\left(f\right)-2f}}+\frac{f^{a}\sqrt{\pi}}{4}\mathrm{Erf}\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-2*d)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+1
/8*Pi^(1/2)*f^a*exp(2*d)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)+1
/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [A] time = 1.10348, size = 135, normalized size = 1.05

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2f}x\right)e^{(2d)}}{8\sqrt{-c\log\left(f\right)-2f}}+\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2f}x\right)e^{(-2d)}}{8\sqrt{-c\log\left(f\right)+2f}}+\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)}{4\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f)
 + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f) +
2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Fricas [B] time = 2.01842, size = 720, normalized size = 5.62

$$-\frac{\left(\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}+2cf\log\left(f\right)\right)\cosh\left(a\log\left(f\right)-2d\right)+\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}+2cf\log\left(f\right)\right)\sinh\left(a\log\left(f\right)-2d\right)\right)\sqrt{-c\log\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(p
i)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2*f
)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(f) +
 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) + 2*(sqrt(pi)*(c^
2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(a
*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*c*f^2*l
og(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + f*x**2)**2, x)
```

Giac [A] time = 1.35794, size = 144, normalized size = 1.12

$$-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}x\right)}{4\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-2\,f}x\right)e^{\left(a\log\left(f\right)+2\,d\right)}}{8\sqrt{-c\log\left(f\right)-2\,f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+2\,f}x\right)e^{\left(a\log\left(f\right)-2\,d\right)}}{8\sqrt{-c\log\left(f\right)+2\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/8*
sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f) +
2*f)
```

3.316 $\int f^{a+cx^2} \cosh^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=171

$$\frac{3\sqrt{\pi}e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}e^{-3d}f^a\text{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}e^{d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}e^{3d}f^a\text{Erfi}\left(x\cdots\right.}{16\sqrt{c\log\cdots}}$$

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) + (f
^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]])
+ (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]) +
 (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*Log[f
]])
```

Rubi [A] time = 0.296568, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5513, 2287, 2205, 2204}

$$\frac{3\sqrt{\pi}e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}e^{-3d}f^a\text{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}e^{d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}e^{3d}f^a\text{Erfi}\left(x\cdots\right.}{16\sqrt{c\log\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) + (f
^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]])
+ (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]) +
 (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*Log[f
]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)dx = \int\left(\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}+\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right)dx$$
$$= \frac{1}{8}\int e^{-3d-3fx^2}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3d+3fx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{d+fx^2}f^{a}$$
$$= \frac{1}{8}\int e^{-3d+a\log(f)-x^2(3f-c\log(f))}\,dx+\frac{1}{8}\int e^{3d+a\log(f)+x^2(3f+c\log(f))}\,dx+\frac{3}{8}\int e^{-d+a\log(f)-x^2}$$
$$= \frac{3e^{-d}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e^{d}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{\right.}}{16\sqrt{f+c\,l}}$$

Mathematica [A] time = 1.19563, size = 270, normalized size = 1.58

$$\frac{\sqrt{\pi}f^a\left((f-c\log(f))\left(\sqrt{3f-c\log(f)}\left(c^2\log^2(f)+4cf\log(f)+3f^2\right)(\cosh(3d)-\sinh(3d))\text{Erf}\left(x\sqrt{3f-c\log(f)}\right)+(3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*
f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) + (f - c*Lo
g[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[
f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x*Sqr
t[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) +
Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d]
```

```
+ Sinh[3*d])))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] time = 0.127, size = 144, normalized size = 0.8

$$\frac{f^{a}\sqrt{\pi}e^{-3d}}{16}\mathrm{Erf}\left(x\sqrt{3f-c\ln\left(f\right)}\right)\frac{1}{\sqrt{3f-c\ln\left(f\right)}}+\frac{f^{a}\sqrt{\pi}e^{3d}}{16}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)-3f}x\right)\frac{1}{\sqrt{-c\ln\left(f\right)-3f}}+\frac{3f^{a}\sqrt{\pi}e^{-d}}{16}\mathrm{Er}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3*d)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))+
1/16*Pi^(1/2)*f^a*exp(3*d)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*x)
+3/16*Pi^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+3/16*
Pi^(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Maxima [A] time = 1.12808, size = 193, normalized size = 1.13

$$\frac{\sqrt{\pi}f^{a}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-3f}x\right)e^{(3d)}}{16\sqrt{-c\log\left(f\right)-3f}}+\frac{3\sqrt{\pi}f^{a}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right)e^{(-d)}}{16\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}f^{a}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+3f}x\right)e^{(-3d)}}{16\sqrt{-c\log\left(f\right)+3f}}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*f
) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f)
 + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f) +
 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f
)
```

Fricas [B] time = 2.04465, size = 1301, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log
(f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log(f)
 + 3*f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) -
9*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f
^2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(
f) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) +
9*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f
^2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(
f) - f)*x) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*
f^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f
^2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*erf(sqrt(-c*
log(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3398, size = 209, normalized size = 1.22

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-3\,f}x\right)e^{\left(a\log\left(f\right)+3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}-\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{\left(a\log\left(f\right)+d\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}-\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+ \cdots}\right.}{16\,\sqrt{-c\log\left(f\right)\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*log
(f) - 3*f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqr
t(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) -
```

```
d)/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^(a*l
og(f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

3.317 $\int f^{a+cx^2} \cosh\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=140

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d} \operatorname{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}} \operatorname{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

```
[Out] (E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]))/
(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - e^2/(4*(f + c*Log
[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/
(4*Sqrt[f + c*Log[f]])
```

Rubi [A] time = 0.310736, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d} \operatorname{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}} \operatorname{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]))/
(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - e^2/(4*(f + c*Log
[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/
(4*Sqrt[f + c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex-fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+cx^2}\,dx\\ &= \frac{1}{2}\int e^{-d-ex+a\log(f)-x^2(f-c\log(f))}\,dx+\frac{1}{2}\int e^{d+ex+a\log(f)+x^2(f+c\log(f))}\,dx\\ &= \frac{1}{2}\left(e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{2}\left(e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\right)\int e\\ &= \frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.651889, size = 165, normalized size = 1.18

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4(c\log(f)+f)}}\left(\sqrt{c\log(f)+f}(\cosh(d)-\sinh(d))e^{\frac{e^2 f}{2f^2-2c^2\log^2(f)}}\operatorname{Erf}\left(\frac{-2cx\log(f)+e+2fx}{2\sqrt{f-c\log(f)}}\right)+\sqrt{f-c\log(f)}(\sinh(d)+\cosh(d\right.}{4\sqrt{f-c\log(f)}\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c*x*
Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d]) + Er
fi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[f]]*(C
osh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]*Sqrt[f
 + c*Log[f]])
```

Maple [A] time = 0.157, size = 147, normalized size = 1.1

$$\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{4d\ln(f)c-4df+e^2}{4c\ln(f)-4f}}\operatorname{Erf}\left(x\sqrt{f-c\ln(f)}+\frac{e}{2}\frac{1}{\sqrt{f-c\ln(f)}}\right)\frac{1}{\sqrt{f-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{\frac{4d\ln(f)c+4df-e^2}{4c\ln(f)+4f}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)-\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))
^(1/2)*erf(x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*ex
p(1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f
)-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))
```

Maxima [A] time = 1.07335, size = 171, normalized size = 1.22

$$\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}x-\frac{e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4(c\log(f)+f)}\right)}}{4\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}x+\frac{e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e^2}{4(c\log(\ldots\right.}}}{4\sqrt{-c\log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(
d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*f^a*erf(sqrt
(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) -
f))/sqrt(-c*log(f) + f)
```

Fricas [B] time = 2.01145, size = 855, normalized size = 6.11

$$-\frac{\left(\sqrt{\pi}\left(c\log\left(f\right)+f\right)\cosh\left(\frac{4\,ac\log\left(f\right)^{2}-e^{2}+4\,df-4\,\left(cd+af\right)\log\left(f\right)}{4\,\left(c\log\left(f\right)-f\right)}\right)+\sqrt{\pi}\left(c\log\left(f\right)+f\right)\sinh\left(\frac{4\,ac\log\left(f\right)^{2}-e^{2}+4\,df-4\,\left(cd+af\right)\log\left(f\right)}{4\,\left(c\log\left(f\right)-f\right)}\right)\right)\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(
c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(1/4*(4*a*
c*log(f)^2 - e^2 + 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*l
og(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f)
 - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f +
4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(1/4*(4
*a*c*log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-
c*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log
(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^{2}}\cosh\left(d+ex+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2), x)
```

Giac [A] time = 1.30027, size = 232, normalized size = 1.66

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{e}{c\log\left(f\right)+f}\right)\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}+4\,cd\log\left(f\right)+4\,af\log\left(f\right)+4\,df-e^{2}}{4\,\left(c\log\left(f\right)+f\right)}\right)}}{4\,\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x-\cdots\right.\right.}{4\,\sqrt{-c\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4
*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + f
))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e
/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) + 4*
d*f - e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

3.318 $\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=183

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{2f-c\log(f)}-2d}\text{Erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)+2f}}\text{Erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/Sqrt
[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8*Sqr
t[2*f + c*Log[f]])
```

Rubi [A] time = 0.33219, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5513, 2204, 2287, 2234, 2205}

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{2f-c\log(f)}-2d}\text{Erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)+2f}}\text{Erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/Sqrt
[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8*Sqr
t[2*f + c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)dx = \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx$$

$$=\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int\exp\left(-2d-2ex+a\log(f)-x^2(2f-c\log(f))\right)d$$

$$=\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e+2x(-2f+c\log(f)}{4(-2f+c\log(f))}\right.$$

$$=\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{e^2}{2f+c\log(f)}}}{8}$$

Mathematica [A] time = 1.42698, size = 258, normalized size = 1.41

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{2f-c\log(f)}}\left(-\sqrt{c}\sqrt{\log(f)}\left((2f-c\log(f))\sqrt{c\log(f)+2f}(\sinh(2d)+\cosh(2d))e^{\frac{4e^2f}{c^2\log^2(f)-4f^2}}\text{Erfi}\left(\frac{cx\log(f)+e+2fx}{\sqrt{c\log(f)+2f}}\right)+\right.\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(-2*E^(e^2/(-2*f + c*Log[f]))*Erfi[S
qrt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf[(
e + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c
*Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2))*Er
fi[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqrt[2*f
 + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f^2 + c
^2*Log[f]^2))
```

Maple [A] time = 0.168, size = 177, normalized size = 1.

$$\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{2d\ln(f)c-4df+e^2}{-2f+c\ln(f)}}\mathrm{Erf}\left(x\sqrt{2f-c\ln(f)}+e\frac{1}{\sqrt{2f-c\ln(f)}}\right)\frac{1}{\sqrt{2f-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{\frac{2d\ln(f)c+4df-e^2}{2f+c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))
^(1/2)*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*ex
p((2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f
)-2*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*er
f((-c*ln(f))^(1/2)*x)
```

Maxima [A] time = 1.09979, size = 217, normalized size = 1.19

$$\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-2f}x-\frac{e}{\sqrt{-c\log(f)-2f}}\right)e^{\left(2d-\frac{e^2}{c\log(f)+2f}\right)}}{8\sqrt{-c\log(f)-2f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+2f}x+\frac{e}{\sqrt{-c\log(f)+2f}}\right)e^{\left(-2d-\frac{}{c\log}\right.}}{8\sqrt{-c\log(f)+2f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^(
2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f) -
2*f))/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(
-c*log(f))
```

Fricas [B] time = 1.91734, size = 1119, normalized size = 6.11

$$-\frac{2\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2-4f^2\right)\cosh\left(a\log\left(f\right)\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2-4f^2\right)\sinh\left(a\log\left(f\right)\right)\right)\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log
(f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) + (sq
rt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(
c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f
))*sinh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c*d + a*f)*log(f))/(c*log(f) - 2*f
)))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + 2*f
)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh((a*c*log
(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c
^2*log(f)^2 - 2*c*f*log(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f
)*log(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c*x*log(f) + 2*f*x
+ e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)))/(c^3*log(f)^3 - 4*c*f^2*log(f
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24007, size = 267, normalized size = 1.46

$$-\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}x\right)}{4\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-2f}\left(x+\frac{e}{c\log\left(f\right)+2f}\right)\right)e^{\left(\frac{ac\log\left(f\right)^{2}+2cd\log\left(f\right)+2af\log\left(f\right)+4df-e^{2}}{c\log\left(f\right)+2f}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c*d
*log(f) + 2*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*
f) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*e^((
a*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) + 4*d*f - e^2)/(c*log(f) - 2*f))
/sqrt(-c*log(f) + 2*f)
```

3.319 $\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=300

$$\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{Erfi}\left(\frac{2x(c\log(f}{2\sqrt{c\log(}}\right.}{16\sqrt{c\log(f)+f}}$$

```
[Out] (3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f])
)/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^(-3*d + (9*e^2)/(12
*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f
 - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) + (3*E^(d - e^2/(4*(f + c*Log[f])
))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*
Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f +
 c*Log[f]])
```

Rubi [A] time = 0.5812, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{Erfi}\left(\frac{2x(c\log(f}{2\sqrt{c\log(}}\right.}{16\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f])
)/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^(-3*d + (9*e^2)/(12
*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f
 - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) + (3*E^(d - e^2/(4*(f + c*Log[f])
))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*
Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f +
 c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a+cx^2}+\frac{3}{8}\text{ex}\right.\\
&= \frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}\,dx+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+fx^2\right)\right)f^{a+cx^2}\,d\\
&= \frac{1}{8}\int\exp\left(-3d-3ex+a\log(f)-x^2(3f-c\log(f))\right)dx+\frac{1}{8}\int\exp\left(3d+3ex+a\,\text{lo}\right.\\
&= \frac{1}{8}\left(3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{8}\left(e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\right.\\
&= \frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{3e+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right.}{16\sqrt{3f-c\log(f)}}
\end{aligned}
$$

Mathematica [A] time = 5.74456, size = 478, normalized size = 1.59

$$\sqrt{\pi}f^a\exp\left(-\frac{1}{4}e^2\left(\frac{9}{c\log(f)+3f}+\frac{1}{c\log(f)+f}\right)\right)\left((f-c\log(f))\left(\sqrt{3f-c\log(f)}\left(c^2\log^2(f)+4cf\log(f)+3f^2\right)(\cosh(3d)-\mathrm{s}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3*
f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*S
qrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*
(Cosh[d] - Sinh[d]) + (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f + c*
Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f])/(2*
Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log
[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(3*f + c
*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f +
 c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) + E^(e^2/(4*(f + c*Log[f]))
)*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f]
)*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*((f + c*Log[
f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f
]^4))
```

Maple [A] time = 0.236, size = 302, normalized size = 1.

$$\frac{f^a\sqrt{\pi}}{16}\mathrm{e}^{-\frac{12d\ln(f)c-36df+9e^2}{4c\ln(f)-12f}}\mathrm{Erf}\left(x\sqrt{3f-c\ln(f)}+\frac{3e}{2}\frac{1}{\sqrt{3f-c\ln(f)}}\right)\frac{1}{\sqrt{3f-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{16}\mathrm{e}^{\frac{12d\ln(f)c+36df-9e^2}{4c\ln(f)+12f}}\mathrm{Erf}\left(-\sqrt{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(-3*f+c*ln(f)))/(3*f-
c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))-1/16*Pi
^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))/(-c*ln(f)-3*f)
^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))+3/16*Pi^(1/2
)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(
x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(4*d
*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)
```

```
*x+1/2*e/(-c*ln(f)-f)^(1/2))
```

Maxima [A] time = 1.13557, size = 355, normalized size = 1.18

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x-\frac{3\,e}{2\,\sqrt{-c\log\left(f\right)-3\,f}}\right)e^{\left(3\,d-\frac{9\,e^{2}}{4\,\left(c\log\left(f\right)+3\,f\right)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}+\frac{3\,\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{e}{2\,\sqrt{-c\log\left(f\right)-f}}\right)e^{\left(d-\frac{e^{2}}{4\,\left(c\right.}\right.}}{16\,\sqrt{-c\log\left(f\right)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f)
)*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*
f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^2/(
c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) +
 f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqrt(-c*
log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/sqrt(-c
*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Fricas [B] time = 2.15398, size = 2219, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(f)
 - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3
)*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log
(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3*e)*sqr
t(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*lo
g(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4
*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f
)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(c
*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(1/2*(2*c*x*log(f
```

```
) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)*(c^3*log(f
)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e
^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3
 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2
+ 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/
2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) + (sqrt(pi
)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(1/4*(4*a*c*
log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)) + sqrt
(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(1/4*(4*a
*c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)))*sq
rt(-c*log(f) - 3*f)*erf(1/2*(2*c*x*log(f) + 6*f*x + 3*e)*sqrt(-c*log(f) - 3
*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38782, size = 475, normalized size = 1.58

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{3\,e}{c\log\left(f\right)+3\,f}\right)\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}+12\,cd\log\left(f\right)+12\,af\log\left(f\right)+36\,df-9\,e^{2}}{4\,\left(c\log\left(f\right)+3\,f\right)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f)))
*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) + 36*d*f - 9*e^2)/(
c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) +
4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16*sqrt
```

```
(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4*a*c*l
og(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) - f))/sqrt(-
c*log(f) + f) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x - 3*e/(c*
log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f) + 36
*d*f - 9*e^2)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

3.320 $\int f^{a+bx+cx^2} \cosh(d+ex)\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{\pi} f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}} \operatorname{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi} f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d} \operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -(E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f] -
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d -
 (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*Log
[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.291038, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5513, 2287, 2234, 2204}

$$\frac{\sqrt{\pi} f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}} \operatorname{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi} f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d} \operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] -(E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f] -
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d -
 (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*Log
[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh(d+ex)\,dx &= \int \left(\frac{1}{2}e^{-d-ex}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{d+ex}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\int \exp\left(-d+a\log(f)+cx^2\log(f)-x(e-b\log(f))\right)dx+\frac{1}{2}\int \exp\left(d+a\log(f)+cx^2\right.\\ &= \frac{1}{2}\left(e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\right)\int e\\ &= -\frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.309041, size = 134, normalized size = 0.88

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{e(2b\log(f)+e)}{4c\log(f)}}\left(e^{\frac{be}{c}}(\cosh(d)-\sinh(d))\mathbf{Erfi}\left(\frac{\log(f)(b+2cx)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(d)+\cosh(d))\mathbf{Erfi}\left(\frac{\log(f)(b+2cx)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*
Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d]) + Erfi[(e + (b + 2*c*x)*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e + 2*b*L
og[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.106, size = 156, normalized size = 1.

$$-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2-2\ln(f)be+4d\ln(f)c+e^2}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-e}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2+2\ln(f)be-}{4c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c+e^2)/ln(f)/
c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1/2
))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1
/2))
```

Maxima [A] time = 1.08879, size = 174, normalized size = 1.14

$$\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}\,x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}\,x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f))
)*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(
b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] time = 2.02907, size = 717, normalized size = 4.69

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)\right)\mathrm{erf}\left(\frac{((2cx+b)\log}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(p
i)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log
(f))))*erf(1/2*((2*c*x + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log
(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} \cosh(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x), x)
```

Giac [A] time = 1.30882, size = 228, normalized size = 1.49

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b\log\left(f\right)-e}{c\log\left(f\right)}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2+4cd\log\left(f\right)-2be\log\left(f\right)+e^2}{4c\log\left(f\right)}\right)}}{4\sqrt{-c\log\left(f\right)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b\log(f}{c\log(}\right.\right.}{4\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*e
^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/
(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x +
(b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*
```

```
log(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.321 $\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx$

Optimal. Leaf size=219

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*
Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*Sqr
t[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.368765, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5513, 2234, 2204, 2287}

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*
Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*Sqr
t[c]*Sqrt[Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx &= \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2ex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= \frac{1}{4}\int \exp\left(-2d+a\log(f)+cx^2\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int \exp\left(2d+a\log(f\right.\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-2e+b\log(f)+2cx\log}{4c\log(f)}\right.\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2d-\frac{(2e+b}{4c}}}{}
\end{aligned}$$

Mathematica [A] time = 0.522342, size = 183, normalized size = 0.84

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{e(b\log(f)+e)}{c\log(f)}}\left(e^{\frac{2be}{c}}(\cosh(2d)-\sinh(2d))\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)-2e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)+2e\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b +
 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c*x)
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e + (b
 + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sq
rt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.171, size = 211, normalized size = 1.

$$-\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}-4\ln(f)be+8d\ln(f)c+4e^{2}}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{b\ln(f)-2e}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}+4\ln(f)be}{4c\ln(f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*ln(f)*b*e+8*d*ln(f)*c+4*e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^
(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*ln(f)*b*e-8*d*ln(f)*c+4*e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*l
n(f))^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Maxima [A] time = 1.08145, size = 250, normalized size = 1.14

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+2e}{2\sqrt{-c\log(f)}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^{2}}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-2e}{2\sqrt{-c\log(f)}}\right)e^{\left(-2d-\frac{(b\log(f)-2e)^{2}}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(f
)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(
pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^(-2
*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*
```

```
erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1
/4*b^2/c))
```

Fricas [B] time = 1.978, size = 934, normalized size = 4.26

$$-\frac{2\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(-\frac{\left(b^2-4ac\right)\log\left(f\right)}{4c}\right)+\sqrt{\pi}\sinh\left(-\frac{\left(b^2-4ac\right)\log\left(f\right)}{4c}\right)\right)\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log\left(f\right)}}{2c}\right)+\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(
pi)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/
c) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 -
4*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log
(f)^2 + 4*e^2 - 4*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*l
og(f) + 2*e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-
1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f))/(c*log(f))) +
 sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f
))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*e)*sqrt(-c*log(f))/(c*log(f
))))/(c*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24701, size = 304, normalized size = 1.39

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^{2}\log\left(f\right)-4\,ac\log\left(f\right)}{4\,c}\right)}}{4\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log\left(f\right)-2\,e}{c\log\left(f\right)}\right)\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}-4\,ac\log\left(f\right)^{2}+}{4\,c\,l}\right.}}{8\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4
*a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*
x + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
8*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(-1/4*
(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)/(c*lo
g(f)))/sqrt(-c*log(f))
```

3.322 $\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx$

Optimal. Leaf size=315

$$-\frac{3\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}}{16\sqrt{}}$$

```
[Out] (-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(
-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f] -
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*d - (
3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.457259, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5513, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}}{16\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(
-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f] -
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*d - (
3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx &= \int\left(\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}+\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{8}\int e^{-3d-3ex}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3d+3ex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{d+e\ldots}\\ &= \frac{1}{8}\int\exp\left(-3d+a\log(f)+cx^2\log(f)-x(3e-b\log(f))\right)dx+\frac{1}{8}\int\exp\left(3d+a\log(f)\ldots\right.\\ &= \frac{1}{8}\left(3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx+\frac{1}{8}\left(e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\right)\ldots\\ &= -\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3e-b\log(f)-2cx\log\ldots}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}\ldots\end{aligned}$$

Mathematica [A] time = 0.957081, size = 262, normalized size = 0.83

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{3e(2b\log(f)+3e)}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\left(3(\cosh(2d)-\sinh(2d))e^{\frac{2e(b\log(f)+e)}{c\log(f)}}\text{Erfi}\left(\frac{\log(f)(b+2cx)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2\ldots\right.\right.}{16\sqrt{c}\sqrt{\log(f\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((e*(2*e + b*Log[f]))
/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^
((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + E^((3*b*e)/c)*Erfi[(-3*
e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d])))
/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.159, size = 316, normalized size = 1.

$$-\frac{f^a\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^2b^2-6\ln(f)be+12d\ln(f)c+9e^2}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{b\ln(f)-3e}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^2b^2+6\ln(f}{4c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*ln(f)*b*e+12*d*ln(f)*c+9*e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(-c*ln(f)
)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*ln(f)*b*e-12*d*ln(f)*c+9
*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln
(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e
)/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d
*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln
(f))/(-c*ln(f))^(1/2))
```

Maxima [A] time = 1.10904, size = 355, normalized size = 1.13

$$\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+3e}{2\sqrt{-c\log(f)}}\right)e^{\left(3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log(
f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^(d
- 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(
sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b*log(
f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f
))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f) - 3*e)
^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] time = 1.99052, size = 1434, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2
- 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*l
og(f)^2 + 9*e^2 - 6*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)
*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f)))
 + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f
))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + e)*sqrt(-c*log(f))/(c*log(f))
) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) - e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt
(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c
*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/
(c*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30674, size = 463, normalized size = 1.47

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b\log(f)-3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log\left(f\right)}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f)))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) + 9
*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*
(2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
 4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b
^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f)
))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f
) + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f
) + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.323 $\int f^{a+bx+cx^2} \cosh\left(d + fx^2\right) dx$

Optimal. Leaf size=154

$$\frac{\sqrt{\pi} f^a e^{d-\frac{b^2 \log^2(f)}{4(c \log(f)+f)}} \operatorname{Erfi}\left(\frac{b \log(f)+2x(c \log(f)+f)}{2\sqrt{c \log(f)+f}}\right)}{4\sqrt{c \log(f)+f}} - \frac{\sqrt{\pi} f^a e^{\frac{b^2 \log^2(f)}{4f-4c \log(f)}-d} \operatorname{Erf}\left(\frac{b \log(f)-2x(f-c \log(f))}{2\sqrt{f-c \log(f)}}\right)}{4\sqrt{f-c \log(f)}}$$

```
[Out] -(E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2
*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d
- (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f +
 c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rubi [A] time = 0.320591, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi} f^a e^{d-\frac{b^2 \log^2(f)}{4(c \log(f)+f)}} \operatorname{Erfi}\left(\frac{b \log(f)+2x(c \log(f)+f)}{2\sqrt{c \log(f)+f}}\right)}{4\sqrt{c \log(f)+f}} - \frac{\sqrt{\pi} f^a e^{\frac{b^2 \log^2(f)}{4f-4c \log(f)}-d} \operatorname{Erf}\left(\frac{b \log(f)-2x(f-c \log(f))}{2\sqrt{f-c \log(f)}}\right)}{4\sqrt{f-c \log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```


```
[Out] -(E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2
*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d
- (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f +
 c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```


Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{d+fx^2}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\int\exp\left(-d+a\log(f)+bx\log(f)-x^2(f-c\log(f))\right)dx+\frac{1}{2}\int\exp\left(d+a\log(f)\right.\\ &= \frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\right.\\ &= -\frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.658025, size = 185, normalized size = 1.2

$$\frac{\sqrt{\pi}f^ae^{-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\left(\sqrt{f-c\log(f)}(c\log(f)+f)(\cosh(d)-\sinh(d))e^{\frac{b^2f\log^2(f)}{2f^2-2c^2\log^2(f)}}\operatorname{Erf}\left(\frac{2fx-\log(f)(b+2cx)}{2\sqrt{f-c\log(f)}}\right)+(f-c\log(f))\sqrt{c\right.}{4\left(f^2-c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x - (
b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[f]
)*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log
[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b^2*L
og[f]^2)/(4*(f + c*Log[f])))*(f^2 - c^2*Log[f]^2))
```

Maple [A] time = 0.164, size = 160, normalized size = 1.

$$-\frac{f^{a}\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}+4d\ln(f)c-4df}{4c\ln(f)-4f}}\mathrm{Erf}\left(-x\sqrt{f-c\ln(f)}+\frac{b\ln(f)}{2}\frac{1}{\sqrt{f-c\ln(f)}}\right)\frac{1}{\sqrt{f-c\ln(f)}}-\frac{f^{a}\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}-4d\ln(f)c-4df}{4c\ln(f)+4f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*d*ln(f)*c-4*d*f)/(-f+c*ln(f)))/(f
-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))-1/4
*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))/(-c*ln(
f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))
```

Maxima [A] time = 1.07464, size = 188, normalized size = 1.22

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)-f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{b^{2}\log(f)^{2}}{4(c\log(f)+f)}+d\right)}}{4\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)+f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{b^{2}\log(f)^{2}}{4(c\log(f)-f)}\right.}}{4\sqrt{-c\log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 1/4*sqrt
(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e^(-
```

```
1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

Fricas [B] time = 1.91437, size = 878, normalized size = 5.7

$$-\frac{\left(\sqrt{\pi}\left(c\log\left(f\right)+f\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log\left(f\right)^2-4\,df+4\left(cd+af\right)\log\left(f\right)}{4\left(c\log\left(f\right)-f\right)}\right)+\sqrt{\pi}\left(c\log\left(f\right)+f\right)\sinh\left(-\frac{\left(b^2-4\,ac\right)\log\left(f\right)^2-4\,df+4\left(cd+af\right)\mathrm{lo}}{4\left(c\log\left(f\right)-f\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f +
4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqr
t(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/
(c*log(f) - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^
2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f
) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*l
og(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2), x)
```

Giac [A] time = 1.2679, size = 244, normalized size = 1.58

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2x+\frac{b\log\left(f\right)}{c\log\left(f\right)+f}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2-4cd\log\left(f\right)-4af\log\left(f\right)-4df}{4\left(c\log\left(f\right)+f\right)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d
*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log
(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
 + f)
```

3.324 $\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=225

$$-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\operatorname{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}\ldots}{2\ldots}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*f^a
*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])])/
(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*f^a
*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f]])])
/(8*Sqrt[2*f + c*Log[f]])
```

Rubi [A] time = 0.349837, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5513, 2234, 2204, 2287, 2205}

$$-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\operatorname{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}\ldots}{2\ldots}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*f^a
*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])])/
(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*f^a
*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f]])])
/(8*Sqrt[2*f + c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)dx=\int\left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx$$

$$=\frac{1}{4}\int\exp\left(-2d+a\log(f)+bx\log(f)-x^2(2f-c\log(f))\right)dx+\frac{1}{4}\int\exp\left(2d+a\log(\right.$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-2f+c\log}{4(-2f+c\log(f))}\right.$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{b^2}{8f-}}}{}$$

Mathematica [A] time = 2.20027, size = 257, normalized size = 1.14

$$\frac{1}{8}\sqrt{\pi}f^a\left(\frac{2f^{-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\left(\sqrt{2f-c\log(f)}(c\log(f)+2f)(\cosh(2d)-\sinh(2d))e^{\frac{b^2f\log^2(f)}{4f^2-c^2\log^2(f)}}\operatorname{Erf}\left(\right.\right.}{c^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f^
(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))*Er
f[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*Log[f
]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x)*Log
[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[
2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log
[f]^2))))/8
```

Maple [A] time = 0.213, size = 217, normalized size = 1.

$$-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2+8d\ln(f)c-16df}{4c\ln(f)-8f}}\operatorname{Erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{b\ln(f)}{2}\frac{1}{\sqrt{2f-c\ln(f)}}\right)\frac{1}{\sqrt{2f-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2-8d\ln}{8f+4c\ln}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*d*ln(f)*c-16*d*f)/(-2*f+c*ln(f)))
/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(
1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*d*ln(f)*c-16*d*f)/(2*f+c*ln(
f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)
-2*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Maxima [A] time = 1.07874, size = 269, normalized size = 1.2

$$\frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2\,f}x-\frac{b\log\left(f\right)}{2\,\sqrt{-c\log\left(f\right)-2\,f}}\right)e^{\left(-\frac{b^2\log\left(f\right)^2}{4\,(c\log\left(f\right)+2\,f)}+2\,d\right)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}}+\frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2\,f}x-\frac{b\log\left(f\right)}{2\,\sqrt{-c\log\left(f\right)+2\,f}}\right)e^{\left(-\frac{}{4}\right.}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
- 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f)
+ 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) + 2*f
) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/
(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [B] time = 2.07509, size = 1269, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log
(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*d +
 a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2
*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*
log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*
d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))/(c*log(
f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f))*sqrt
(-c*log(f) - 2*f)/(c*log(f) + 2*f)) + 2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*co
sh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(-1/4
*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f
))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29487, size = 323, normalized size = 1.44

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2f}\left(2x+\frac{b\log\left(f\right)}{c\log\left(f\right)+2f}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2-8cd\log\left(f\right)-8af\log\left(f\right)-16df}{4\left(c\log\left(f\right)+2f\right)}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2*
f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f) -
 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sq
rt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) - 2*f)
)/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c)
)*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

3.325 $\int f^{a+bx+cx^2} \cosh^3\left(d+fx^2\right) dx$

Optimal. Leaf size=323

$$-\frac{3\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\text{Erfi}}{16\sqrt{c\log}}$$

```
[Out] (-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^
(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) +
(3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] +
2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(
3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*
x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Rubi [A] time = 0.51246, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5513, 2287, 2234, 2205, 2204}

$$-\frac{3\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\text{Erfi}}{16\sqrt{c\log}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^
(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) +
(3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] +
2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(
3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*
x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)dx = \int\left(\frac{1}{8}e^{-3d-3fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{d+fx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx+cx^2}\cdots\right.$$

$$=\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\cdots$$

$$=\frac{1}{8}\int\exp\left(-3d+a\log(f)+bx\log(f)-x^2(3f-c\log(f))\right)dx+\frac{1}{8}\int\exp\left(3d+a\log\cdots\right.$$

$$=\frac{1}{8}\left(3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{8}\left(e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}\cdots\right.$$

$$=-\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(\cdots}{2\sqrt{3f-\cdots}}\right)}{16\sqrt{3f-c\log(f)}}\cdots$$

Mathematica [B] time = 6.49794, size = 2511, normalized size = 7.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(27*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f^3*Cosh[d]*Erf[(2*
f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]] +
 27*c*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f^2*Cosh[d]*Erf[(2*f*x - b*Log[
f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]] - 3*c^
2*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f*Cosh[d]*Erf[(2*f*x - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]] - 3*c^3*E^
((b^2*Log[f]^2)/(4*(f - c*Log[f])))*Cosh[d]*Erf[(2*f*x - b*Log[f] - 2*c*x*L
og[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]] + 3*E^((b^2*Log[
f]^2)/(4*(3*f - c*Log[f])))*f^3*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log
[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]] + c*E^((b^2*Log[f]^2)/(
4*(3*f - c*Log[f])))*f^2*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]] - 3*c^2*E^((b^2*Log[f]^
2)/(4*(3*f - c*Log[f])))*f*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/
(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]] - c^3*E^((b^2*Log[f
]^2)/(4*(3*f - c*Log[f])))*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/
(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]] + (27*f^3*Cosh[d]*E
rfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Lo
g[f]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (27*c*f^2*Cosh[d]*Erfi[(2*f*
x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[
f]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^2*f*Cosh[d]*Erfi[(2*f*x +
 b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f
]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^3*Cosh[d]*Erfi[(2*f*x + b*
Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]])
/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*f^3*Cosh[3*d]*Erfi[(6*f*x + b*L
og[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((b
^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(6*f*x + b*Log[f
] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^
((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cosh[3*d]*Erfi[(6*f*x + b*
Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[
f]])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(6*f*x +
 b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*L
og[f]])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - 27*E^((b^2*Log[f]^2)/(4*(
f - c*Log[f])))*f^3*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log
[f]])]*Sqrt[f - c*Log[f]]*Sinh[d] - 27*c*E^((b^2*Log[f]^2)/(4*(f - c*Log[f]
)))*f^2*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f
]*Sqrt[f - c*Log[f]]*Sinh[d] + 3*c^2*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*
```

```
f*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sq
rt[f - c*Log[f]]*Sinh[d] + 3*c^3*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*Erf[
(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f -
 c*Log[f]]*Sinh[d] + (27*f^3*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt
[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Lo
g[f]))) - (27*c*f^2*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Lo
g[f]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[
f]))) - (3*c^2*f*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f
]))) + (3*c^3*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])
]*Log[f]^3*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))
) - 3*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f^3*Erf[(6*f*x - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*Sinh[3*d] - c*E
^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f^2*Erf[(6*f*x - b*Log[f] - 2*c*x*Lo
g[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d] + 3*c
^2*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f*Erf[(6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]]*Sinh[3*d] +
 c^3*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*Erf[(6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d] +
 (3*f^3*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sq
rt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*
f^2*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]
*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) -
(3*c^2*f*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*L
og[f]^2*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f
]))) + (c^3*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])
]*Log[f]^3*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Lo
g[f])))))/(16*(f - c*Log[f])*(3*f - c*Log[f])*(f + c*Log[f])*(3*f + c*Log[f
]))
```

Maple [A] time = 0.233, size = 326, normalized size = 1.

$$-\frac{f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^2 b^2+12 d\ln(f)c-36 d f}{4 c\ln(f)-12 f}}\mathrm{Erf}\left(-x\sqrt{3 f-c\ln(f)}+\frac{b\ln(f)}{2}\frac{1}{\sqrt{3 f-c\ln(f)}}\right)\frac{1}{\sqrt{3 f-c\ln(f)}}-\frac{f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^2 b^2-12 d}{4 c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*d*ln(f)*c-36*d*f)/(-3*f+c*ln(f)
))/(3*f-c*ln(f))^(1/2)*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f-c*ln(f))
```

```
^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-12*d*ln(f)*c-36*d*f)/(3*f+c
*ln(f)))/(-c*ln(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*l
n(f)-3*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*d*ln(f)*c-4*d*f)
/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*
ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*ln(f)*c-4*d*f)/(f
+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(
f)-f)^(1/2))
```

Maxima [A] time = 1.1103, size = 387, normalized size = 1.2

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 3 f} x - \frac{b \log\left(f\right)}{2 \sqrt{-c \log\left(f\right) - 3 f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \left(c \log\left(f\right) + 3 f\right)} + 3 d\right)}}{16 \sqrt{-c \log\left(f\right) - 3 f}} + \frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - f} x - \frac{b \log\left(f\right)}{2 \sqrt{-c \log\left(f\right) - f}}\right) e^{\left(-\frac{b}{4(\cdots}\right.}}{16 \sqrt{-c \log\left(f\right) - f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*f)
 + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 3/16
*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))
*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) + 1/16*sqrt(p
i)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3*f))*e^
(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Fricas [B] time = 2.08794, size = 2249, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f
) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^
```

```
3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*l
og(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(f))*
sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f
*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d
*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f
*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d
*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2
*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi
)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2
- 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi
)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(-1/4*((b^2
- 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*
log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*log(f) - f)/(c*log
(f) + f)) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f
^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/(c*
log(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) +
 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))
/(c*log(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(
f))*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log
(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**3,x)

[Out] Timed out
```

Giac [A] time = 1.2674, size = 498, normalized size = 1.54

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3f}\left(2x+\frac{b\log\left(f\right)}{c\log\left(f\right)+3f}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2-12cd\log\left(f\right)-12af\log\left(f\right)-36df}{4\left(c\log\left(f\right)+3f\right)}\right)}}{16\sqrt{-c\log\left(f\right)-3f}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) + 3
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f
) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/
2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) + f))/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + b*lo
g(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f)
 + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) - 1/16*sqrt(pi
)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f)))*e^(-1/4
*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 36*d*f)/(
c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

3.326 $\int f^{a+bx+cx^2} \cosh\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=161

$$\frac{\sqrt{\pi} f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d} \operatorname{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}} \operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

```
[Out] (E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[f
] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (
E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f]
 + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rubi [A] time = 0.425727, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi} f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d} \operatorname{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}} \operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[f
] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (
E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f]
 + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\int \exp\left(-d+a\log(f)-x(e-b\log(f))-x^2(f-c\log(f))\right)dx+\frac{1}{2}\int \exp\left(d+a\right.\\ &= \frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{2}\left(e^{d-\frac{(e+b}{4(f}}\right.\\ &= \frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+2}{2\sqrt{f+}}\right.}{4\sqrt{f+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 1.49355, size = 251, normalized size = 1.56

$$\frac{\sqrt{\pi}e^{-\frac{b^2\log^2(f)+e^2}{4(c\log(f)+f)}}f^{a+\frac{bef}{c^2\log^2(f)-f^2}}\left(\sqrt{f-c\log(f)}(c\log(f)+f)(\cosh(d)-\sinh(d))f^{\frac{be}{2(c\log(f)+f)}}\exp\left(\frac{f\left(b^2\log^2(f)+e^2\right)}{2\left(f^2-c^2\log^2(f)\right)}\right)\mathrm{Erf}\left(\frac{-\log(f)}{2\sqrt{}}\right.\right.}{4\left(f^2-c^2\log^2(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(E^((f*(e^2 + b^2*Log[f]^2)
)/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x - (
b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[f]
)*(Cosh[d] - Sinh[d]) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x + (b +
 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]]*(
Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f])))*(f^2 -
c^2*Log[f]^2))
```

Maple [A] time = 0.124, size = 186, normalized size = 1.2

$$-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2-2\ln(f)be+4d\ln(f)c-4df+e^2}{4c\ln(f)-4f}}\mathrm{Erf}\left(-x\sqrt{f-c\ln(f)}+\frac{b\ln(f)-e}{2}\frac{1}{\sqrt{f-c\ln(f)}}\right)\frac{1}{\sqrt{f-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f)}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/
(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f
-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(
f)*c-4*d*f+e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1
/2*(e+b*ln(f))/(-c*ln(f)-f)^(1/2))
```

Maxima [A] time = 1.08126, size = 204, normalized size = 1.27

$$\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}+d\right)}}{4\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)+f}\,x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{(b\log(f)-}{4(c\log(f)}\right.}}{4\sqrt{-c\log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*log
(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f)
```

```
+ 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-c*l
og(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f
)
```

Fricas [B] time = 1.99659, size = 975, normalized size = 6.06

$$-\frac{\left(\sqrt{\pi}\left(c\log\left(f\right)+f\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log\left(f\right)^2+e^2-4\,df+2\left(2\,cd-be+2\,af\right)\log\left(f\right)}{4\left(c\log\left(f\right)-f\right)}\right)+\sqrt{\pi}\left(c\log\left(f\right)+f\right)\sinh\left(-\frac{\left(b^2-4\,ac\right)\log\left(f\right)^2+e^2-4\,df+}{4\left(c\log\left(f\right.\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*
d*f + 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f)
+ f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a
*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x +
 b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + (sqrt(pi)*(c*log(f) -
 f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*
f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(-1/4*((b^2 - 4*a*
c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + f))
)*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) + e)*sqrt(-c*log(
f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x + f*x**2), x)
```

Giac [A] time = 1.25372, size = 282, normalized size = 1.75

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2x+\frac{b\log\left(f\right)+e}{c\log\left(f\right)+f}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2-4cd\log\left(f\right)-4af\log\left(f\right)+2be\log\left(f\right)-4df+e^2}{4\left(c\log\left(f\right)+f\right)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
+ f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f)
 + 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sq
rt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f)))*
e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 2*b*
e*log(f) - 4*d*f + e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

3.327 $\int f^{a+bx+cx^2} \cosh^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=239

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right) \operatorname{Erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(2d-\frac{(b\log(f)+2e}{4c\log(f)+8}\right.}{8\sqrt{c}}$$

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f]))
*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*L
og[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f + 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqr
t[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Rubi [A] time = 0.505905, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5513, 2234, 2204, 2287, 2205}

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right) \operatorname{Erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(2d-\frac{(b\log(f)+2e}{4c\log(f)+8}\right.}{8\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f]))
*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*L
og[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f + 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqr
t[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,d\\
&= \frac{1}{4}\int\exp\left(-2d+a\log(f)-x(2e-b\log(f))-x^2(2f-c\log(f))\right)dx+\frac{1}{4}\int\exp\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-}{}\right.\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erf}\left(\frac{2e-b\log(f)+2x(}{2\sqrt{2f-c\log}}\right.}{8\sqrt{2f-c\log(f)}}\end{aligned}$$

Mathematica [A] time = 5.97796, size = 339, normalized size = 1.42

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}e^{-\frac{b^2\log^2(f)+4e^2}{4c\log(f)+8f}}f^{a+\frac{4bef}{c^2\log^2(f)-4f^2}}\left(\sqrt{2f-c\log(f)}(c\log(f)+2f)(\cosh(2d)-\sinh(2d))f^{\frac{}{c\log}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*Sqrt[P
i]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2*f + c
*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])
]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f^((b*e)/
(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[2*f + c
*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d])))/
(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.149, size = 249, normalized size = 1.

$$-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2-4\ln(f)be+8d\ln(f)c-16df+4e^2}{4c\ln(f)-8f}}\mathrm{Erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{b\ln(f)-2e}{2}\frac{1}{\sqrt{2f-c\ln(f)}}\right)\frac{1}{\sqrt{2f-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*ln(f)*b*e+8*d*ln(f)*c-16*d*f+4*e^
2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln
(f)-2*e)/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*ln(f
)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-
c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f)-2*f)^(1/2))-1/4*Pi^(1/2)*f
^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*
ln(f))^(1/2))
```

Maxima [A] time = 1.1485, size = 290, normalized size = 1.21

$$\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)-2f}\,x-\frac{b\log(f)+2e}{2\sqrt{-c\log(f)-2f}}\right)e^{\left(-\frac{(b\log(f)+2e)^2}{4(c\log(f)+2f)}+2d\right)}}{8\sqrt{-c\log(f)-2f}}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)+2f}\,x-\frac{b\log(f)-2e}{2\sqrt{-c\log(f)+2f}}\right)e^{\left(-\right.}}{8\sqrt{-c\log(f)+2f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-c
*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*
log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*log(f
) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) - 2*f)
 - 2*d)/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/
2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [B] time = 1.98213, size = 1382, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)) +
 sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-
c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2*c*x + b)*log(f) + 2*e)*sqrt(-c*log(f)
+ 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/
4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f)
)/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(-1/4*((b^
2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*l
og(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f) +
2*e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) + 2*(sqrt(pi)*(c^2*log(f)^2 -
4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*
sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt
(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33177, size = 369, normalized size = 1.54

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2f}\left(2x+\frac{b\log\left(f\right)+2e}{c\log\left(f\right)+2f}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2-8cd\log\left(f\right)-8af\log\left(f\right)+4be\log\left(f\right)-16df+4e^2}{4\left(c\log\left(f\right)+2f\right)}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*log
(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*
log(f) + 4*b*e*log(f) - 16*d*f + 4*e^2)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2*e)/
(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) +
8*a*f*log(f) - 4*b*e*log(f) - 16*d*f + 4*e^2)/(c*log(f) - 2*f))/sqrt(-c*log
(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^
2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

3.328 $\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=344

$$\frac{\sqrt{\pi}f^a\exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\text{Erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\text{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a}{}$$

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log
[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]])
+ (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e
- b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f
- c*Log[f]]) + (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[
f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])
])/(16*Sqrt[3*f + c*Log[f]])
```

Rubi [A] time = 0.734441, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}f^a\exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\text{Erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\text{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a}{}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log
[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]])
+ (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e
- b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f
- c*Log[f]]) + (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[
f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])
])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)dx = \int\left(\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a+bx+c}\right.$$

$$= \frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+fx^2\right)\right)f$$

$$= \frac{1}{8}\int\exp\left(-3d+a\log(f)-x(3e-b\log(f))-x^2(3f-c\log(f))\right)dx+\frac{1}{8}\int\exp\left(\right.$$

$$= \frac{1}{8}\left(\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-3e+b\log(f)+2x(-3f+c\log(f}{4(-3f+c\log(f))}\right.$$

$$= \frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^a\sqrt{\pi}}{16\sqrt{3f-c}}$$

Mathematica [B] time = 6.64304, size = 2991, normalized size = 8.69

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log
[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] -
2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e^2 +
 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*Erf[(e
 + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f
 - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) -
 (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log
[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)
/(4*(f - c*Log[f]))) + (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x
*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e
*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c*f^2*Cosh[3*d]*Erf[(3*e +
 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f
 - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f])
)) - (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt
[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f]
 - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c^3*Cosh[3*d]*Erf[(3*e + 6*f*x -
b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Lo
g[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (2
7*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[
f]])]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*
Log[f]))) - (27*c*f^2*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2
*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b
^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^2*f*Cosh[d]*Erfi[(e + 2*f*x + b*Log
[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]])/E^
((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^3*Cosh[d]*E
rfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*
Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]
))) + (3*f^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt
[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log
[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f
] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^
((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cos
h[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]]
)]*Log[f]^2*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/
(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x
```

```
*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*Log[f]])/E^((9*e^2
 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (27*f^3*Erf[(e + 2*
f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*S
inh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*c
*f^2*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[
f]*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(
f - c*Log[f]))) + (3*c^2*f*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqr
t[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log
[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (3*c^3*Erf[(e + 2*f*x - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]]*Sinh[d]
)/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (27*f^3*Erf
i[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*
Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f])))
 - (27*c*f^2*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]
^2)/(4*(f + c*Log[f]))) - (3*c^2*f*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f
])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2
*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^3*Erfi[(e + 2*f*x +
b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]
]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*
f^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*S
qrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4
*(3*f - c*Log[f]))) - (c*f^2*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 +
 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*c^2*f*Erf[(3*e + 6
*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f
 - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f -
 c*Log[f]))) + (c^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f
 - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*
Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*f^3*Erfi[(3*e + 6*f*x + b
*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]]*Sinh
[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*
f^2*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*
Log[f]*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f
]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*
Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/
E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Erfi[
(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*
Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4
*(3*f + c*Log[f])))))/(16*(f - c*Log[f])*(3*f - c*Log[f])*(f + c*Log[f])*(3
*f + c*Log[f]))
```

Maple [A] time = 0.197, size = 384, normalized size = 1.1

$$-\frac{f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}-6\ln(f)be+12d\ln(f)c-36df+9e^{2}}{4c\ln(f)-12f}}\mathrm{Erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{b\ln(f)-3e}{2}\frac{1}{\sqrt{3f-c\ln(f)}}\right)\frac{1}{\sqrt{3f-c\ln(f)}}-\frac{f^{a}\sqrt{}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*ln(f)*b*e+12*d*ln(f)*c-36*d*f+9*
e^2)/(-3*f+c*ln(f)))/(3*f-c*ln(f))^(1/2)*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b*
ln(f)-3*e)/(3*f-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*l
n(f)*b*e-12*d*ln(f)*c-36*d*f+9*e^2)/(3*f+c*ln(f)))/(-c*ln(f)-3*f)^(1/2)*erf
(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f)-3*f)^(1/2))-3/16*Pi^(1
/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f
)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^
(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*
f+e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*l
n(f))/(-c*ln(f)-f)^(1/2))
```

Maxima [A] time = 1.13338, size = 425, normalized size = 1.24

$$\frac{\sqrt{\pi}f^{a}\,\mathrm{erf}\left(\sqrt{-c\log(f)-3f}x-\frac{b\log(f)+3e}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(-\frac{(b\log(f)+3e)^{2}}{4(c\log(f)+3f)}+3d\right)}}{16\sqrt{-c\log(f)-3f}}+\frac{3\sqrt{\pi}f^{a}\,\mathrm{erf}\left(\sqrt{-c\log(f)-f}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\right.}}{16\sqrt{-c\log(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(-
c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c
*log(f) - 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f
) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sq
rt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*lo
g(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)
/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*
(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/(c*log(f
```

```
) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Fricas [B] time = 2.1873, size = 2454, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(2*c*d - b*e + 2*a*f)
*log(f))/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*
f^2*log(f) - 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*
(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(
-1/2*(6*f*x - (2*c*x + b)*log(f) + 3*e)*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3
*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)
*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a*f)*
log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2
*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d
 - b*e + 2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*
x - (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(p
i)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2
 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f
) + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*
log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*l
og(f) + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) + (sqrt(pi)*(c^3*log(f)^3 -
3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2
+ 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + 3*f)) + sqrt
(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((b
^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*
log(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f) +
 3*e)*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*l
og(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29481, size = 582, normalized size = 1.69

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3f}\left(2x+\frac{b\log(f)+3e}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)-12af\log(f)+6be\log(f)-36df+9e^2}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log\left(f\right)-3f}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*lo
g(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a
*f*log(f) + 6*b*e*log(f) - 36*d*f + 9*e^2)/(c*log(f) + 3*f))/sqrt(-c*log(f)
 - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/
(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*
a*f*log(f) + 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f
) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log
(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*lo
g(f) - 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f) - 1/
16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e)/(c*log(f
) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*
log(f) - 6*b*e*log(f) - 36*d*f + 9*e^2)/(c*log(f) - 3*f))/sqrt(-c*log(f) +
3*f)
```

3.329 $\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)dx$

Optimal. Leaf size=20

$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

```
[Out] -4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

Rubi [A] time = 0.0498668, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3315}

$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] -4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)dx &= \int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\cosh(x)}\,dx\\ &= -4\sqrt{\cosh(x)}+\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}\end{aligned}$$

Mathematica [B] time = 0.346142, size = 46, normalized size = 2.3

$$\frac{2\sinh(x)\left(x-\frac{2\sinh(x)\cosh(x)\sqrt{\tanh^2\left(\frac{x}{2}\right)}}{(\cosh(x)-1)^{3/2}\sqrt{\cosh(x)+1}}\right)}{\sqrt{\cosh(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] (2*Sinh[x]*(x - (2*Cosh[x]*Sinh[x]*Sqrt[Tanh[x/2]^2])/((-1 + Cosh[x])^(3/2)
*Sqrt[1 + Cosh[x]])))/Sqrt[Cosh[x]]
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int x\left(\cosh\left(x\right)\right)^{-\frac{3}{2}}+x\sqrt{\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\cosh^2(x)+1\right)}{\cosh^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(3/2)+x*cosh(x)**(1/2),x)
```

```
[Out] Integral(x*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

3.330 $\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx$

Optimal. Leaf size=24

$$\frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

```
[Out] 4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

Rubi [A] time = 0.0511449, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3315}

$$\frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] 4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned} \int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx &= -\left(\frac{1}{3} \int \frac{x}{\sqrt{\cosh(x)}}\, dx \right) + \int \frac{x}{\cosh^{\frac{5}{2}}(x)}\, dx \\ &= \frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)} \end{aligned}$$

Mathematica [A] time = 0.0750161, size = 16, normalized size = 0.67

$$\frac{2(x\tanh(x)+2)}{3\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] (2*(2 + x*Tanh[x]))/(3*Sqrt[Cosh[x]])
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int x\left(\cosh\left(x\right)\right)^{-\frac{5}{2}}-\frac{x}{3}\frac{1}{\sqrt{\cosh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x}{3\sqrt{\cosh\left(x\right)}}+\frac{x}{\cosh\left(x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Fricas [B] time = 1.79833, size = 374, normalized size = 15.58

$$\frac{4\left(\left(x+2\right)\cosh\left(x\right)^{3}+3\left(x+2\right)\cosh\left(x\right)\sinh\left(x\right)^{2}+\left(x+2\right)\sinh\left(x\right)^{3}-\left(x-2\right)\cosh\left(x\right)+\left(3\left(x+2\right)\cosh\left(x\right)^{2}-x+2\right)\sin\right.}{3\left(\cosh\left(x\right)^{4}+4\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{2}+2\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 -
(x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 - x + 2)*sinh(x))*sqrt(cosh(x))/(cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(5/2)-1/3*x/cosh(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

3.331 $\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx$

Optimal. Leaf size=47

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

```
[Out] 4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/2
)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

Rubi [A] time = 0.0708335, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3315}

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] 4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/2
)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx = \frac{3}{5}\int x\sqrt{\cosh(x)}\,dx+\int \frac{x}{\cosh^{\frac{7}{2}}(x)}\,dx$$
$$= \frac{4}{15\cosh^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx+\frac{3}{5}\int x\sqrt{\cosh(x)}\,dx$$
$$= \frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

Mathematica [A] time = 0.583576, size = 64, normalized size = 1.36

$$\frac{1}{5}\sqrt{\cosh(x)}\left(6x\tanh(x)+\left(2x\tanh(x)+\frac{4}{3}\right)\operatorname{sech}^2(x)-\frac{12\sinh^2(x)}{\sqrt{\cosh(x)-1}(\cosh(x)+1)^{3/2}\sqrt{\tanh^2\left(\frac{x}{2}\right)}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] (Sqrt[Cosh[x]]*((-12*Sinh[x]^2)/(Sqrt[-1 + Cosh[x]]*(1 + Cosh[x])^(3/2)*Sqr
t[Tanh[x/2]^2]) + 6*x*Tanh[x] + Sech[x]^2*(4/3 + 2*x*Tanh[x])))/5
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int x(\cosh(x))^{-\frac{7}{2}}+\frac{3x}{5}\sqrt{\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5}x\sqrt{\cosh(x)}+\frac{x}{\cosh(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(7/2)+3/5*x*cosh(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5} x \sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

3.332 $\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)}\right) dx$

Optimal. Leaf size=36

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}} - 8x\sqrt{\cosh(x)} - 16iE\left(\frac{ix}{2}\middle|2\right)$$

```
[Out] -8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[Co
sh[x]]
```

Rubi [A] time = 0.0941286, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3316, 2639}

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}} - 8x\sqrt{\cosh(x)} - 16iE\left(\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] -8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[Co
sh[x]]
```

Rule 3316

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[((c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[(d^2*m*(m - 1))/(b^2*f^2*(n + 1)*(n + 2)), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[(d*m*(c + d*x)^(m - 1)*(b*
Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)dx=\int \frac{x^2}{\cosh^{\frac{3}{2}}(x)}\,dx+\int x^2\sqrt{\cosh(x)}\,dx$$
$$=-8x\sqrt{\cosh(x)}+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}+8\int \sqrt{\cosh(x)}\,dx$$
$$=-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}$$

Mathematica [C] time = 0.179557, size = 76, normalized size = 2.11

$$\frac{4\sqrt{\cosh(x)}(\sinh(x)+\cosh(x))\left(8\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2x}\right)(\sinh(x)-\cosh(x))\sqrt{\sinh(2x)+\cosh(2x)+1}+x^2\sinh(x)-4(x-}{e^{2x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] (4*Sqrt[Cosh[x]]*(Cosh[x] + Sinh[x])*(-4*(-2 + x)*Cosh[x] + x^2*Sinh[x] + 8
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*x)]*(-Cosh[x] + Sinh[x])*Sqrt[1 +
Cosh[2*x] + Sinh[2*x]]))/(1 + E^(2*x))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int x^2\left(\cosh(x)\right)^{-\frac{3}{2}}+x^2\sqrt{\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

```
[Out] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\cosh(x)}+\frac{x^2}{\cosh(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/cosh(x)**(3/2)+x**2*cosh(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

3.333 $\int (x+\cosh(x))^2\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x}{2}+2x\sinh(x)-2\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] x/2 + x^3/3 - 2*Cosh[x] + 2*x*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Rubi [A] time = 0.038134, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2638, 2635, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2x\sinh(x)-2\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x])^2,x]
```

```
[Out] x/2 + x^3/3 - 2*Cosh[x] + 2*x*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (x+\cosh(x))^2\,dx &= \int \left(x^2+2x\cosh(x)+\cosh^2(x)\right)\,dx\\ &= \frac{x^3}{3}+2\int x\cosh(x)\,dx+\int \cosh^2(x)\,dx\\ &= \frac{x^3}{3}+2x\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)+\frac{\int 1\,dx}{2}-2\int \sinh(x)\,dx\\ &= \frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0641519, size = 26, normalized size = 0.87

$$\frac{1}{6}\left(x\left(2x^2+12\sinh(x)+3\right)+3(\sinh(x)-4)\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x])^2,x]
```

```
[Out] (3*Cosh[x]*(-4 + Sinh[x]) + x*(3 + 2*x^2 + 12*Sinh[x]))/6
```

Maple [A] time = 0.007, size = 25, normalized size = 0.8

$$\frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x))^2,x)
```

```
[Out] 1/2*x+1/3*x^3-2*cosh(x)+2*x*sinh(x)+1/2*cosh(x)*sinh(x)
```

Maxima [A] time = 1.04878, size = 49, normalized size = 1.63

$$\frac{1}{3}x^3 - (x+1)e^{(-x)} + (x-1)e^x + \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Fricas [A] time = 1.83525, size = 80, normalized size = 2.67

$$\frac{1}{3}x^3 + \frac{1}{2}(4x + \cosh(x))\sinh(x) + \frac{1}{2}x - 2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*(4*x + cosh(x))*sinh(x) + 1/2*x - 2*cosh(x)
```

Sympy [A] time = 0.2522, size = 41, normalized size = 1.37

$$\frac{x^3}{3} - \frac{x\sinh^2(x)}{2} + 2x\sinh(x) + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2} - 2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))**2,x)
```

```
[Out] x**3/3 - x*sinh(x)**2/2 + 2*x*sinh(x) + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
- 2*cosh(x)
```

Giac [A] time = 1.2375, size = 49, normalized size = 1.63

$$\frac{1}{3}x^3 - (x+1)e^{(-x)} + (x-1)e^x + \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

3.334 $\int (x+\cosh (x))^3 \, dx$

Optimal. Leaf size=56

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sinh(x)+\frac{\sinh^3(x)}{3}+7\sinh(x)-\frac{3\cosh^2(x)}{4}-6x\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

```
[Out] (3*x^2)/4 + x^4/4 - 6*x*Cosh[x] - (3*Cosh[x]^2)/4 + 7*Sinh[x] + 3*x^2*Sinh[
x] + (3*x*Cosh[x]*Sinh[x])/2 + Sinh[x]^3/3
```

Rubi [A] time = 0.0745582, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6742, 3296, 2637, 3310, 30, 2633}

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sinh(x)+\frac{\sinh^3(x)}{3}+7\sinh(x)-\frac{3\cosh^2(x)}{4}-6x\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x])^3,x]
```

```
[Out] (3*x^2)/4 + x^4/4 - 6*x*Cosh[x] - (3*Cosh[x]^2)/4 + 7*Sinh[x] + 3*x^2*Sinh[
x] + (3*x*Cosh[x]*Sinh[x])/2 + Sinh[x]^3/3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int (x+\cosh(x))^3\,dx &= \int \left(x^3+3x^2\cosh(x)+3x\cosh^2(x)+\cosh^3(x)\right)dx\\
&= \frac{x^4}{4}+3\int x^2\cosh(x)\,dx+3\int x\cosh^2(x)\,dx+\int \cosh^3(x)\,dx\\
&= \frac{x^4}{4}-\frac{3\cosh^2(x)}{4}+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+i\,\text{Subst}\left(\int \left(1-x^2\right)dx,x,-i\sinh(x)\right)+\\
&= \frac{3x^2}{4}+\frac{x^4}{4}-6x\cosh(x)-\frac{3\cosh^2(x)}{4}+\sinh(x)+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+\frac{\sinh^3(x)}{3}\\
&= \frac{3x^2}{4}+\frac{x^4}{4}-6x\cosh(x)-\frac{3\cosh^2(x)}{4}+7\sinh(x)+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+\frac{\sinh^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0875466, size = 51, normalized size = 0.91

$$\frac{1}{12}\left(3x^4+9x^2+9\left(4x^2+9\right)\sinh(x)+9x\sinh(2x)+\sinh(3x)\right)-6x\cosh(x)-\frac{3}{8}\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x])^3,x]
```

```
[Out] -6*x*Cosh[x] - (3*Cosh[2*x])/8 + (9*x^2 + 3*x^4 + 9*(9 + 4*x^2)*Sinh[x] + 9
*x*Sinh[2*x] + Sinh[3*x])/12
```

Maple [A] time = 0.007, size = 52, normalized size = 0.9

$$\left(\frac{2}{3}+\frac{(\cosh(x))^2}{3}\right)\sinh(x)+\frac{3x\cosh(x)\sinh(x)}{2}+\frac{3x^2}{4}-\frac{3(\cosh(x))^2}{4}+3x^2\sinh(x)-6x\cosh(x)+6\sinh(x)+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x))^3,x)
```

```
[Out] (2/3+1/3*cosh(x)^2)*sinh(x)+3/2*x*cosh(x)*sinh(x)+3/4*x^2-3/4*cosh(x)^2+3*x
^2*sinh(x)-6*x*cosh(x)+6*sinh(x)+1/4*x^4
```

Maxima [A] time = 1.04289, size = 109, normalized size = 1.95

$$\frac{1}{4}x^4+\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}-\frac{3}{2}\left(x^2+2x+2\right)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{2}\left(x^2-2x+2\right)e^x+\frac{1}{24}e^{(3x)}-\frac{3}{8}e^{(-x)}-\frac{1}{24}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/2*(x^2 + 2*x + 2)*e^(-x) - 3
/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(-x
) - 1/24*e^(-3*x) + 3/8*e^x
```

Fricas [A] time = 1.80534, size = 184, normalized size = 3.29

$$\frac{1}{4}x^4+\frac{1}{12}\sinh(x)^3+\frac{3}{4}x^2-6x\cosh(x)-\frac{3}{8}\cosh(x)^2+\frac{1}{4}\left(12x^2+6x\cosh(x)+\cosh(x)^2+27\right)\sinh(x)-\frac{3}{8}\sinh(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/12*sinh(x)^3 + 3/4*x^2 - 6*x*cosh(x) - 3/8*cosh(x)^2 + 1/4*(12*
x^2 + 6*x*cosh(x) + cosh(x)^2 + 27)*sinh(x) - 3/8*sinh(x)^2
```

Sympy [A] time = 0.89551, size = 85, normalized size = 1.52

$$\frac{x^4}{4} - \frac{3x^2\sinh^2(x)}{4} + 3x^2\sinh(x) + \frac{3x^2\cosh^2(x)}{4} + \frac{3x\sinh(x)\cosh(x)}{2} - 6x\cosh(x) - \frac{2\sinh^3(x)}{3} - \frac{3\sinh^2(x)}{4} + \mathrm{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))**3,x)
```

```
[Out] x**4/4 - 3*x**2*sinh(x)**2/4 + 3*x**2*sinh(x) + 3*x**2*cosh(x)**2/4 + 3*x*s
inh(x)*cosh(x)/2 - 6*x*cosh(x) - 2*sinh(x)**3/3 - 3*sinh(x)**2/4 + sinh(x)*
cosh(x)**2 + 6*sinh(x)
```

Giac [A] time = 1.2129, size = 101, normalized size = 1.8

$$\frac{1}{4}x^4 + \frac{3}{4}x^2 + \frac{3}{16}(2x-1)e^{(2x)} - \frac{3}{8}\left(4x^2+8x+9\right)e^{(-x)} - \frac{3}{16}(2x+1)e^{(-2x)} + \frac{3}{8}\left(4x^2-8x+9\right)e^{x} + \frac{1}{24}e^{(3x)} - \frac{1}{24}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/8*(4*x^2 + 8*x + 9)*e^(-x) -
 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 9)*e^x + 1/24*e^(3*x) - 1/24*
e^(-3*x)
```

3.335 $\int \frac{\cosh(a+bx)}{c+dx^2} dx$

Optimal. Leaf size=213

$$\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a-\frac{b\sqrt{-}}{\sqrt{d}}\right.}{2}$$

```
[Out] (Cosh[a + (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(
2*Sqrt[-c]*Sqrt[d]) - (Cosh[a - (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[
-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a + (b*Sqrt[-c])/Sqrt[d]]
*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a -
 (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c
]*Sqrt[d])
```

Rubi [A] time = 0.523346, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5281, 3303, 3298, 3301}

$$\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a-\frac{b\sqrt{-}}{\sqrt{d}}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x^2),x]
```

```
[Out] (Cosh[a + (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(
2*Sqrt[-c]*Sqrt[d]) - (Cosh[a - (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[
-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a + (b*Sqrt[-c])/Sqrt[d]]
*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a -
 (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c
]*Sqrt[d])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\cosh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)} + \frac{\sqrt{-c}\cosh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$= -\frac{\int \frac{\cosh(a+bx)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\int \frac{\cosh(a+bx)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cosh\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cosh\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \cdots}{2\sqrt{-c}}$$

$$= \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(\cdots\right)}{2\sqrt{-c}\sqrt{d}}$$

Mathematica [C] time = 0.280674, size = 180, normalized size = 0.85

$$\frac{i\left(\cosh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{CosIntegral}\left(-\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)-\cosh\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{CosIntegral}\left(\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)+i\left(\sinh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{Si}\left(\frac{b\sqrt{c}}{\sqrt{d}}-\cdots\right.\right.\right.}{2\sqrt{c}\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*(Cosh[a - (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[-((b*Sqrt[c])/Sqrt[d])
+ I*b*x] - Cosh[a + (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[(b*Sqrt[c])/Sqrt[d]
+ I*b*x] + I*(Sinh[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] - I*b*x] + Sinh[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] + I*b*x])))/(Sqrt[c]*Sqrt[d])
```

Maple [A] time = 0.039, size = 212, normalized size = 1.

$$\frac{1}{4}e^{\frac{1}{d}\left(b\sqrt{-cd}-da\right)}\mathrm{Ei}\left(1,\frac{1}{d}\left(b\sqrt{-cd}+(bx+a)\,d-da\right)\right)\frac{1}{\sqrt{-cd}}-\frac{1}{4}e^{-\frac{1}{d}\left(b\sqrt{-cd}+da\right)}\mathrm{Ei}\left(1,-\frac{1}{d}\left(b\sqrt{-cd}-(bx+a)\,d+da\right)\right)\frac{1}{\sqrt{-cd}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x^2+c),x)
```

```
[Out] 1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)-d*a)/d)*Ei(1,(b*(-c*d)^(1/2)+(b*x+a)*d
-d*a)/d)-1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+d*a)/d)*Ei(1,-(b*(-c*d)^(1/2
)-(b*x+a)*d+d*a)/d)-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-
c*d)^(1/2)-(b*x+a)*d+d*a)/d)+1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)-d*a)/d)*
Ei(1,-(b*(-c*d)^(1/2)+(b*x+a)*d-d*a)/d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84985, size = 624, normalized size = 2.93

$$-\frac{\left(\sqrt{-\frac{b^2c}{d}}\mathrm{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2c}{d}}\mathrm{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\mathrm{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2c}{d}}\mathrm{Ei}\left(-bx-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2*c/
d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-b^2*c/
d)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)/(d*x^2 + c), x)
```

3.336 $\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx$

Optimal. Leaf size=271

$$\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right.}{\sqrt{d}}$$

```
[Out] (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CoshIntegral[(b*(d - Sqrt[d^2
- 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d + Sqrt[d^2 - 4
*c*e]))/(2*e)]*CoshIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[
d^2 - 4*c*e] + (Sinh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b
*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Sinh[a - (b*(d
 + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e
) + b*x])/Sqrt[d^2 - 4*c*e]
```

Rubi [A] time = 0.750514, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6728, 3303, 3298, 3301}

$$\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right.}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```



```
[Out] (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CoshIntegral[(b*(d - Sqrt[d^2
- 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d + Sqrt[d^2 - 4
*c*e]))/(2*e)]*CoshIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[
d^2 - 4*c*e] + (Sinh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b
*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Sinh[a - (b*(d
 + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e
) + b*x])/Sqrt[d^2 - 4*c*e]
```


Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)dx$$

$$= \frac{(2e)\int \frac{\cosh(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\cosh(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cosh\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cosh\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

Mathematica [C] time = 0.467385, size = 248, normalized size = 0.92

$$\frac{\cosh\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right)\text{CosIntegral}\left(\frac{ib\left(-\sqrt{d^2-4ce}+d+2ex\right)}{2e}\right)-\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{ib\left(\sqrt{d^2-4ce}+d+2ex\right)}{2e}\right)-\sinh}{\sqrt{d^2-4ce}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (Cosh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[((I/2)*b*(d - Sqr
t[d^2 - 4*c*e] + 2*e*x))/e] - Cosh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*C
osIntegral[((I/2)*b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/e] - Sinh[a - (b*(d +
Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))
/(2*e)] + I*Sinh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[((I/2)
*b*(-d + Sqrt[d^2 - 4*c*e]))/e - I*b*x])/Sqrt[d^2 - 4*c*e]
```

Maple [A] time = 0.04, size = 370, normalized size = 1.4

$$\frac{b}{2}e^{\frac{1}{2e}\left(-2ea+bd+\sqrt{-4b^2ce+b^2d^2}\right)}\text{Ei}\left(1,\frac{1}{2e}\left(2(bx+a)e-2ea+bd+\sqrt{-4b^2ce+b^2d^2}\right)\right)\frac{1}{\sqrt{-4b^2ce+b^2d^2}}-\frac{b}{2}e^{-\frac{1}{2e}\left(2ea-bd+\sqrt{-4b^2c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(e*x^2+d*x+c),x)
```

```
[Out] 1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2*(-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(
1/2))/e)*Ei(1,1/2*(2*(b*x+a)*e-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2
*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1
/2)))*Ei(1,-1/2*(-2*(b*x+a)*e+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1/2*
b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2*(-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2
))/e)*Ei(1,-1/2*(2*(b*x+a)*e-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2*b
/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2)
))*Ei(1,1/2*(-2*(b*x+a)*e+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98171, size = 1445, normalized size = 5.33

$$-\frac{\left(e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(-\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)\right)\cosh\left(\frac{bd-2\,ae+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)-\left(e\sqrt{\frac{b^2d^2-}{e^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*((e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^
2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*
b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*cosh(1/2*(b*d - 2*a*e
+ e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2
)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b
^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c
*e)/e^2))/e))*cosh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e
) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^2*
d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*
e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*sinh(1/2*(b*d - 2*a*e +
e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*
Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2
*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e
)/e^2))/e))*sinh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))
/(b*d^2 - 4*b*c*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(bx+a)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)/(e*x^2 + d*x + c), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```