Computer algebra independent integration tests

6-Hyperbolic-functions/6.1-Hyperbolic-sine/6.1.5-Hyperbolic-sine-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [369]. This is test number [163].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (369)	% 0. (0)
Mathematica	% 100. (369)	% 0. (0)
Maple	% 86.18 (318)	% 13.82 (51)
Maxima	% 56.91 (210)	% 43.09 (159)
Fricas	% 82.38 (304)	% 17.62 (65)
Sympy	% 29.27 (108)	% 70.73 (261)
Giac	% 72.36 (267)	% 27.64 (102)

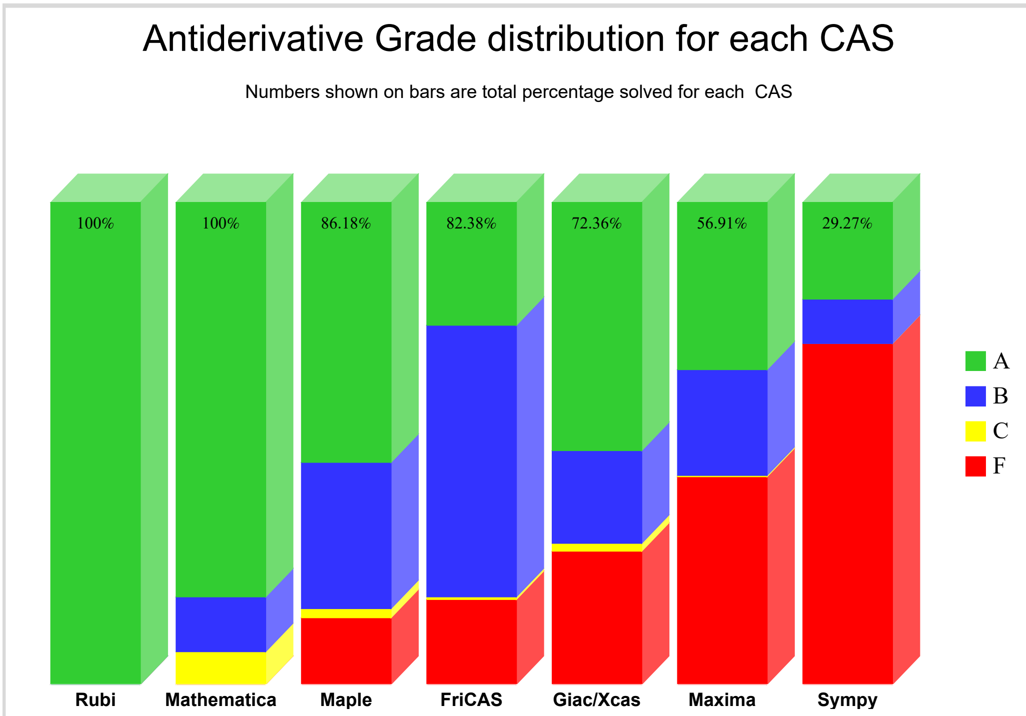
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

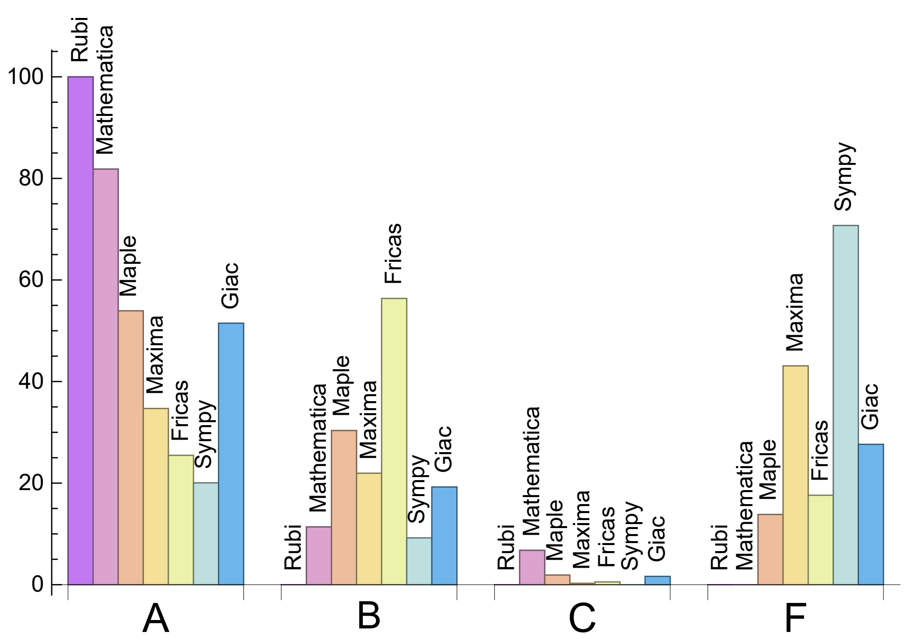
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	81.84	11.38	6.78	0.
Maple	53.93	30.35	1.9	13.82
Maxima	34.69	21.95	0.27	43.09
Fricas	25.47	56.37	0.54	17.62
Sympy	20.05	9.21	0.	70.73
Giac	51.49	19.24	1.63	27.64

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	86.08	0.99	66.	1.
Mathematica	0.59	123.75	1.42	72.	1.
Maple	0.05	163.44	1.85	100.5	1.52
Maxima	1.25	144.32	2.32	104.	1.73
Fricas	2.22	1098.11	10.99	420.5	6.2
Sympy	10.88	107.72	2.2	58.	1.64
Giac	1.25	214.24	2.19	101.	1.87

1.4 list of integrals that has no closed form antiderivative

{264, 265}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {285, 286, 300, 356, 363, 364, 368, 369}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command AboluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is ValueError. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception ValueError then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

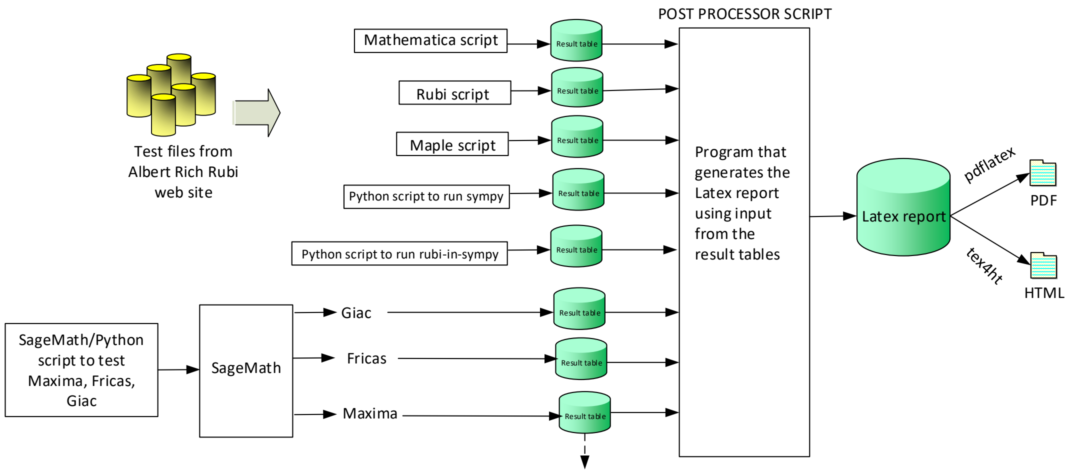
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 47, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 191, 193, 195, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 209, 211, 212, 214, 216, 219, 221, 222, 224, 226, 228, 230, 231, 232, 233, 234, 235, 236, 238, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 363, 364, 366, 367 }

B grade: { 1, 40, 42, 43, 48, 52, 53, 54, 55, 68, 92, 93, 94, 95, 115, 119, 158, 160, 162, 164, 171, 175, 194, 208, 210, 213, 215, 217, 218, 220, 223, 225, 227, 248, 274, 295, 297, 298, 300, 327, 362, 365 }

C grade: { 9, 13, 17, 21, 25, 29, 148, 188, 190, 192, 200, 202, 229, 237, 239, 259, 280, 284, 285, 316, 317, 320, 321, 368, 369 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 44, 45, 46, 47, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 79, 80, 81, 83, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98, 99, 100, 101, 102, 108, 110, 111, 116, 117, 118, 119, 120, 121, 122, 130, 133, 134, 136, 137, 140, 141, 142, 145, 152, 156, 157, 163, 165, 176, 183, 186, 193, 194, 195, 203, 205, 207, 212, 213, 222, 223, 224, 228, 230, 231, 232, 236, 238, 240, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 260, 264, 265, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 294, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 318, 319, 322, 323, 324, 329, 330, 331, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade: { 24, 40, 41, 42, 43, 48, 49, 50, 68, 72, 73, 82, 84, 93, 94, 95, 103, 104, 105, 106, 107, 109, 115, 126, 127, 128, 129, 131, 132, 135, 138, 139, 143, 144, 153, 154, 155, 158, 159, 160, 161, 162, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 187, 188, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 204, 206, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 225, 226, 227, 229, 233, 234, 235, 237, 239, 241, 242, 243, 253, 255, 256, 257, 258, 259, 295, 296, 297, 298, 299, 300 }

C grade: { 312, 313, 316, 317, 320, 321, 339 }

F grade: { 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 69, 70, 71, 112, 113, 114, 123, 124, 125, 146, 147, 148, 149, 150, 151, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 285, 286, 287, 288, 325, 326, 327, 328, 332, 333, 334, 335 }

2.1.4 Maxima

A grade: { 1, 2, 4, 6, 40, 41, 43, 44, 48, 49, 50, 56, 60, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 115, 119, 136, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 165, 171, 173, 175, 176, 181, 183, 184, 186, 193, 194, 196, 203, 204, 229, 231, 239, 247, 248, 260, 264, 265, 266, 267, 268, 269, 270, 272, 274, 275, 277, 301, 302, 303, 304, 305, 306, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367

B grade: { 3, 5, 42, 45, 46, 47, 51, 52, 53, 54, 55, 57, 58, 59, 61, 62, 63, 116, 117, 118, 120, 121, 122, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 172, 174, 177, 178, 179, 180, 182, 185, 187, 189, 191, 198, 201, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 232, 234, 237, 240, 242, 276, 278, 307, 309, 335 }

C grade: { 339 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 146, 147, 148, 149, 150, 151, 188, 190, 192, 195, 197, 199, 200, 202, 205, 207, 228, 230, 233, 235, 236, 238, 241, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 271, 273, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 336, 368, 369 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 40, 41, 43, 51, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 88, 89, 90, 92, 93, 96, 97, 98, 99, 100, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 134, 136, 163, 167, 171, 173, 175, 176, 178, 194, 212, 231, 232, 247, 248, 260, 264, 265, 266, 267, 268, 269, 270, 271, 274, 275, 276, 277, 285, 286, 287, 288, 289, 290, 295, 298, 301, 303, 305, 311, 316, 320, 329, 330, 331, 332, 336, 337, 338, 339, 340, 341, 349, 366, 367 }

B grade: { 5, 42, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 94, 95, 101, 102, 103, 104, 114, 123, 124, 125, 129, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 168, 169, 170, 172, 174, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 256, 259, 272, 273, 278, 291, 292, 293, 294, 296, 297, 299, 300, 302, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 324, 333, 334, 335, 342, 343, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

C grade: { 257, 258 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 105, 106, 107, 108, 109, 110, 111, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 261, 262, 263, 279, 280, 281, 282, 283, 284, 325, 326, 327, 328 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 40, 41, 42, 43, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 75, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 115, 119, 129, 133, 134, 142, 165, 171, 173, 175, 181, 184, 185, 192, 193, 201, 203, 211, 224, 246, 247, 248, 253, 274, 276, 301, 302, 303, 304, 311, 315, 319, 323, 324, 331, 336, 366, 367 }

B grade: { 95, 158, 159, 160, 161, 162, 163, 164, 172, 174, 176, 182, 183, 186, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 310, 314, 318 }

C grade: { }

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C grade: { 322, 323, 324, 339, 343, 346 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 68, 69, 70, 71, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 244, 245, 257, 258, 259, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 368, 369 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	14	23	12	35
normalized size	1	1.	2.1	1.1	1.4	2.3	1.2	3.5
time (sec)	N/A	0.006	0.011	0.002	1.097	1.958	0.166	1.437

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	43	59	46	65
normalized size	1	1.	0.92	1.08	1.72	2.36	1.84	2.6
time (sec)	N/A	0.009	0.024	0.006	1.08	2.06	0.262	1.326

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	73	105	36	65
normalized size	1	1.	1.07	0.85	2.7	3.89	1.33	2.41
time (sec)	N/A	0.014	0.012	0.009	1.098	1.956	0.564	1.462

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	39	81	134	95	92
normalized size	1	1.	0.72	0.85	1.76	2.91	2.07	2.
time (sec)	N/A	0.021	0.04	0.039	1.113	2.228	1.217	1.42

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	33	111	221	58	95
normalized size	1	1.	1.07	0.8	2.71	5.39	1.41	2.32
time (sec)	N/A	0.017	0.014	0.039	1.106	2.22	2.349	1.362

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	49	116	248	139	124
normalized size	1	1.	0.64	0.73	1.73	3.7	2.07	1.85
time (sec)	N/A	0.033	0.038	0.037	1.043	1.994	4.711	1.412

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	75	116	0	0	0	0
normalized size	1	1.	0.73	1.13	0.	0.	0.	0.
time (sec)	N/A	0.049	0.14	0.074	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	68	164	0	0	0	0
normalized size	1	1.	0.85	2.05	0.	0.	0.	0.
time (sec)	N/A	0.032	0.073	0.039	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	83	100	0	0	0	0
normalized size	1	1.	1.04	1.25	0.	0.	0.	0.
time (sec)	N/A	0.032	0.087	0.042	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	0	0	0
normalized size	1	1.	0.93	2.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.102	0.036	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	0	0	0
normalized size	1	1.	0.89	1.61	0.	0.	0.	0.
time (sec)	N/A	0.02	0.114	0.036	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	0	0	0
normalized size	1	1.	0.75	2.03	0.	0.	0.	0.
time (sec)	N/A	0.029	0.052	0.039	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	86	101	0	0	0	0
normalized size	1	1.	1.08	1.26	0.	0.	0.	0.
time (sec)	N/A	0.032	0.078	0.044	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	73	192	0	0	0	0
normalized size	1	1.	0.71	1.86	0.	0.	0.	0.
time (sec)	N/A	0.044	0.161	0.046	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	76	122	0	0	0	0
normalized size	1	1.	0.66	1.05	0.	0.	0.	0.
time (sec)	N/A	0.058	0.27	0.048	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	68	170	0	0	0	0
normalized size	1	1.	0.77	1.93	0.	0.	0.	0.
time (sec)	N/A	0.037	0.12	0.047	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	106	0	0	0	0
normalized size	1	1.	1.	1.2	0.	0.	0.	0.
time (sec)	N/A	0.037	0.125	0.04	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	52	111	0	0	0	0
normalized size	1	1.	0.93	1.98	0.	0.	0.	0.
time (sec)	N/A	0.022	0.045	0.043	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	89	0	0	0	0
normalized size	1	1.	0.96	1.59	0.	0.	0.	0.
time (sec)	N/A	0.022	0.034	0.034	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	62	159	0	0	0	0
normalized size	1	1.	0.72	1.85	0.	0.	0.	0.
time (sec)	N/A	0.038	0.062	0.046	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	84	114	0	0	0	0
normalized size	1	1.	0.93	1.27	0.	0.	0.	0.
time (sec)	N/A	0.038	0.095	0.043	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	79	205	0	0	0	0
normalized size	1	1.	0.67	1.74	0.	0.	0.	0.
time (sec)	N/A	0.056	0.163	0.049	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	65	122	0	0	0	0
normalized size	1	1.	0.71	1.34	0.	0.	0.	0.
time (sec)	N/A	0.034	0.164	0.048	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	169	0	0	0	0
normalized size	1	1.	0.89	2.73	0.	0.	0.	0.
time (sec)	N/A	0.02	0.059	0.05	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	94	104	0	0	0	0
normalized size	1	1.	1.52	1.68	0.	0.	0.	0.
time (sec)	N/A	0.021	0.135	0.048	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	91	0	0	0	0
normalized size	1	1.	0.93	3.03	0.	0.	0.	0.
time (sec)	N/A	0.009	0.02	0.045	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	68	0	0	0	0
normalized size	1	1.	0.93	2.27	0.	0.	0.	0.
time (sec)	N/A	0.009	0.023	0.038	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	50	159	0	0	0	0
normalized size	1	1.	0.86	2.74	0.	0.	0.	0.
time (sec)	N/A	0.019	0.094	0.052	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	83	113	0	0	0	0
normalized size	1	1.	1.34	1.82	0.	0.	0.	0.
time (sec)	N/A	0.02	0.059	0.048	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	80	204	0	0	0	0
normalized size	1	1.	0.88	2.24	0.	0.	0.	0.
time (sec)	N/A	0.033	0.13	0.054	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.053	0.024	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.036	0.073	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.034	0.059	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.04	0.046	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.038	0.04	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.037	0.023	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	65	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.045	0.18	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.04	0.143	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.04	0.143	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	134	138	80	192	58	68
normalized size	1	1.	2.91	3.	1.74	4.17	1.26	1.48
time (sec)	N/A	0.065	0.183	0.039	1.261	2.084	0.422	1.363

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	93	61	153	41	51
normalized size	1	1.	1.14	2.58	1.69	4.25	1.14	1.42
time (sec)	N/A	0.046	0.106	0.034	1.164	2.036	0.31	1.391

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	79	52	45	103	20	35
normalized size	1	1.	3.59	2.36	2.05	4.68	0.91	1.59
time (sec)	N/A	0.059	0.106	0.031	1.102	2.015	0.212	1.315

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	43	29	16	42	8	14
normalized size	1	1.	3.07	2.07	1.14	3.	0.57	1.
time (sec)	N/A	0.027	0.054	0.023	1.154	2.077	0.157	1.298

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	30	21	39	97	0	32
normalized size	1	1.	1.58	1.11	2.05	5.11	0.	1.68
time (sec)	N/A	0.039	0.019	0.028	1.19	2.12	0.	1.414

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	36	35	72	208	0	59
normalized size	1	1.	1.57	1.52	3.13	9.04	0.	2.57
time (sec)	N/A	0.058	0.039	0.028	1.235	2.102	0.	1.502

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	49	53	107	385	0	69
normalized size	1	1.	1.32	1.43	2.89	10.41	0.	1.86
time (sec)	N/A	0.073	0.177	0.031	1.13	2.168	0.	1.298

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	53	71	142	545	0	78
normalized size	1	1.	1.13	1.51	3.02	11.6	0.	1.66
time (sec)	N/A	0.073	0.216	0.035	1.183	2.059	0.	1.259

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	147	116	96	275	68	68
normalized size	1	1.	2.53	2.	1.66	4.74	1.17	1.17
time (sec)	N/A	0.102	0.185	0.055	1.235	2.035	0.531	1.383

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	45	75	80	217	53	51
normalized size	1	1.	1.02	1.7	1.82	4.93	1.2	1.16
time (sec)	N/A	0.129	0.12	0.05	1.314	2.06	0.401	1.384

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	55	52	54	150	39	30
normalized size	1	1.	1.72	1.62	1.69	4.69	1.22	0.94
time (sec)	N/A	0.061	0.141	0.045	1.218	2.074	0.304	1.323

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	25	109	95	36	27
normalized size	1	1.	0.71	0.81	3.52	3.06	1.16	0.87
time (sec)	N/A	0.03	0.009	0.033	1.216	1.985	0.262	1.308

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	91	44	74	240	0	46
normalized size	1	1.	2.68	1.29	2.18	7.06	0.	1.35
time (sec)	N/A	0.083	0.087	0.039	1.226	2.092	0.	1.365

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	88	58	109	404	0	62
normalized size	1	1.	2.1	1.38	2.6	9.62	0.	1.48
time (sec)	N/A	0.117	0.355	0.043	1.239	2.042	0.	1.321

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	131	76	142	578	0	80
normalized size	1	1.	2.26	1.31	2.45	9.97	0.	1.38
time (sec)	N/A	0.14	0.311	0.047	1.233	2.11	0.	1.366

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	143	92	174	730	0	113
normalized size	1	1.	2.23	1.44	2.72	11.41	0.	1.77
time (sec)	N/A	0.131	1.529	0.046	1.279	2.11	0.	1.361

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	42	20	27	36	19	20
normalized size	1	1.	1.56	0.74	1.	1.33	0.7	0.74
time (sec)	N/A	0.011	0.061	0.017	1.185	1.961	0.273	1.359

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	61	55	127	128	63	34
normalized size	1	1.	1.03	0.93	2.15	2.17	1.07	0.58
time (sec)	N/A	0.026	0.1	0.035	1.146	1.993	0.965	1.331

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	81	88	285	235	114	49
normalized size	1	1.	0.92	1.	3.24	2.67	1.3	0.56
time (sec)	N/A	0.042	0.12	0.046	1.062	1.981	2.867	1.371

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	87	121	502	338	156	63
normalized size	1	1.	0.74	1.03	4.29	2.89	1.33	0.54
time (sec)	N/A	0.06	0.164	0.049	1.186	2.072	6.287	1.434

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	42	20	27	38	20	20
normalized size	1	1.	1.56	0.74	1.	1.41	0.74	0.74
time (sec)	N/A	0.012	0.064	0.02	1.15	1.948	0.263	1.366

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	55	127	128	63	34
normalized size	1	1.	1.	0.93	2.15	2.17	1.07	0.58
time (sec)	N/A	0.027	0.071	0.033	1.103	1.939	0.935	1.367

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	81	88	285	234	114	49
normalized size	1	1.	0.92	1.	3.24	2.66	1.3	0.56
time (sec)	N/A	0.042	0.119	0.039	1.069	2.018	2.827	1.391

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	87	121	502	338	156	63
normalized size	1	1.	0.74	1.03	4.29	2.89	1.33	0.54
time (sec)	N/A	0.06	0.15	0.043	1.164	1.964	6.256	1.39

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	75	0	0	501	0	0
normalized size	1	1.	1.32	0.	0.	8.79	0.	0.
time (sec)	N/A	0.056	0.074	0.355	0.	2.13	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	76	0	0	506	0	0
normalized size	1	1.	1.33	0.	0.	8.88	0.	0.
time (sec)	N/A	0.056	0.077	0.304	0.	2.105	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	145	0	0	360	0	0
normalized size	1	1.	1.39	0.	0.	3.46	0.	0.
time (sec)	N/A	0.053	0.453	0.14	0.	1.984	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	113	0	0	259	0	0
normalized size	1	1.	1.64	0.	0.	3.75	0.	0.
time (sec)	N/A	0.031	0.206	0.119	0.	1.984	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	74	89	0	170	0	0
normalized size	1	1.	2.39	2.87	0.	5.48	0.	0.
time (sec)	N/A	0.014	0.041	0.125	0.	2.007	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	84	0	0	518	0	0
normalized size	1	1.	1.62	0.	0.	9.96	0.	0.
time (sec)	N/A	0.024	0.085	0.506	0.	2.113	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	156	0	0	1044	0	0
normalized size	1	1.	1.79	0.	0.	12.	0.	0.
time (sec)	N/A	0.045	0.226	0.118	0.	2.193	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	210	0	0	1395	0	0
normalized size	1	1.	1.72	0.	0.	11.43	0.	0.
time (sec)	N/A	0.068	0.196	0.118	0.	2.219	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	105	262	0	1947	0	211
normalized size	1	1.	0.97	2.43	0.	18.03	0.	1.95
time (sec)	N/A	0.315	0.431	0.027	0.	2.221	0.	1.347

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	174	0	1142	0	158
normalized size	1	1.	1.	2.12	0.	13.93	0.	1.93
time (sec)	N/A	0.185	0.136	0.026	0.	2.093	0.	1.4

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	92	0	640	0	116
normalized size	1	1.	1.07	1.61	0.	11.23	0.	2.04
time (sec)	N/A	0.116	0.093	0.022	0.	2.149	0.	1.484

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	63	0	367	296	90
normalized size	1	1.	1.11	1.34	0.	7.81	6.3	1.91
time (sec)	N/A	0.061	0.05	0.015	0.	2.03	96.174	1.37

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	58	49	0	456	0	111
normalized size	1	1.	1.16	0.98	0.	9.12	0.	2.22
time (sec)	N/A	0.076	0.049	0.022	0.	2.231	0.	1.383

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	73	0	926	0	132
normalized size	1	1.	1.37	1.24	0.	15.69	0.	2.24
time (sec)	N/A	0.121	0.402	0.027	0.	2.408	0.	1.388

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	118	108	0	2288	0	185
normalized size	1	1.	1.46	1.33	0.	28.25	0.	2.28
time (sec)	N/A	0.318	0.538	0.03	0.	2.96	0.	1.419

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	186	158	0	3970	0	231
normalized size	1	1.	1.71	1.45	0.	36.42	0.	2.12
time (sec)	N/A	0.491	0.934	0.031	0.	3.069	0.	1.448

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	118	296	0	4020	0	317
normalized size	1	1.	0.73	1.83	0.	24.81	0.	1.96
time (sec)	N/A	0.405	0.407	0.048	0.	2.785	0.	1.422

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	95	213	0	2426	0	248
normalized size	1	1.	0.83	1.85	0.	21.1	0.	2.16
time (sec)	N/A	0.238	0.352	0.04	0.	2.489	0.	1.354

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	86	175	0	1245	0	177
normalized size	1	1.	1.04	2.11	0.	15.	0.	2.13
time (sec)	N/A	0.134	0.178	0.033	0.	2.245	0.	1.452

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	97	0	855	0	134
normalized size	1	1.	1.13	1.62	0.	14.25	0.	2.23
time (sec)	N/A	0.074	0.101	0.02	0.	2.086	0.	1.392

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	91	166	0	1632	0	192
normalized size	1	1.	1.07	1.95	0.	19.2	0.	2.26
time (sec)	N/A	0.216	0.181	0.039	0.	3.46	0.	1.483

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	118	193	0	4055	0	277
normalized size	1	1.	1.03	1.68	0.	35.26	0.	2.41
time (sec)	N/A	0.372	0.649	0.047	0.	3.558	0.	1.402

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	156	227	0	8699	0	274
normalized size	1	1.	0.99	1.44	0.	55.06	0.	1.73
time (sec)	N/A	0.675	0.71	0.052	0.	5.241	0.	1.585

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	214	277	0	14660	0	319
normalized size	1	1.	1.08	1.4	0.	74.04	0.	1.61
time (sec)	N/A	0.884	0.956	0.052	0.	5.533	0.	1.859

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	81	42	26	104	34	45
normalized size	1	1.	1.11	0.58	0.36	1.42	0.47	0.62
time (sec)	N/A	0.028	0.034	0.023	1.685	2.563	0.614	1.682

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	142	82	107	323	87	96
normalized size	1	1.	1.39	0.8	1.05	3.17	0.85	0.94
time (sec)	N/A	0.051	0.246	0.043	1.6	2.183	1.174	1.607

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	204	124	167	625	156	126
normalized size	1	1.	1.56	0.95	1.27	4.77	1.19	0.96
time (sec)	N/A	0.085	0.485	0.051	1.677	2.193	2.557	1.307

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	265	164	225	972	223	155
normalized size	1	1.	1.66	1.02	1.41	6.08	1.39	0.97
time (sec)	N/A	0.126	0.58	0.053	1.689	2.212	5.194	1.292

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	171	44	49	80	31	42
normalized size	1	1.	4.62	1.19	1.32	2.16	0.84	1.14
time (sec)	N/A	0.013	0.034	0.023	1.761	2.026	0.599	1.245

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	183	134	86	298	88	93
normalized size	1	1.	2.77	2.03	1.3	4.52	1.33	1.41
time (sec)	N/A	0.035	0.228	0.043	1.782	2.086	1.117	1.246

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	277	224	146	594	158	123
normalized size	1	1.	2.92	2.36	1.54	6.25	1.66	1.29
time (sec)	N/A	0.067	0.648	0.05	1.601	2.103	2.603	1.272

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	308	314	205	946	219	153
normalized size	1	1.	2.48	2.53	1.65	7.63	1.77	1.23
time (sec)	N/A	0.102	1.768	0.052	1.713	2.173	5.23	1.293

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	138	155	367	548	314	366
normalized size	1	1.	0.75	0.85	2.01	2.99	1.72	2.
time (sec)	N/A	0.273	0.658	0.023	1.1	1.998	3.094	1.27

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	108	119	246	358	240	261
normalized size	1	1.	0.79	0.87	1.8	2.61	1.75	1.91
time (sec)	N/A	0.156	0.394	0.016	1.107	2.06	1.481	1.181

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	71	77	155	225	128	173
normalized size	1	1.	0.77	0.84	1.68	2.45	1.39	1.88
time (sec)	N/A	0.073	0.175	0.017	1.027	1.942	0.753	1.322

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	51	74	112	78	99
normalized size	1	1.	0.92	0.98	1.42	2.15	1.5	1.9
time (sec)	N/A	0.017	0.081	0.012	1.096	1.999	0.345	1.254

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	20	39	17	42
normalized size	1	1.	1.73	1.07	1.33	2.6	1.13	2.8
time (sec)	N/A	0.01	0.009	0.003	1.038	2.	0.167	1.242

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	52	43	0	424	189	90
normalized size	1	1.	1.18	0.98	0.	9.64	4.3	2.05
time (sec)	N/A	0.036	0.036	0.015	0.	1.995	20.61	1.192

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	85	118	0	1041	0	180
normalized size	1	1.	1.08	1.49	0.	13.18	0.	2.28
time (sec)	N/A	0.063	0.259	0.034	0.	2.106	0.	1.353

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	117	280	0	3005	0	319
normalized size	1	1.	0.92	2.2	0.	23.66	0.	2.51
time (sec)	N/A	0.122	0.272	0.049	0.	2.659	0.	1.311

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	159	494	0	6620	0	491
normalized size	1	1.	0.91	2.84	0.	38.05	0.	2.82
time (sec)	N/A	0.22	0.692	0.056	0.	2.751	0.	1.242

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	178	917	0	0	0	0
normalized size	1	1.	0.99	5.12	0.	0.	0.	0.
time (sec)	N/A	0.258	0.413	0.155	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	139	676	0	0	0	0
normalized size	1	1.	0.93	4.51	0.	0.	0.	0.
time (sec)	N/A	0.166	0.378	0.079	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	65	262	0	0	0	0
normalized size	1	1.	1.08	4.37	0.	0.	0.	0.
time (sec)	N/A	0.037	0.187	0.078	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	125	0	0	0	0
normalized size	1	1.	1.	2.08	0.	0.	0.	0.
time (sec)	N/A	0.037	0.186	0.075	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	81	456	0	0	0	0
normalized size	1	1.	0.86	4.85	0.	0.	0.	0.
time (sec)	N/A	0.06	0.15	0.095	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	166	438	0	0	0	0
normalized size	1	1.	0.84	2.22	0.	0.	0.	0.
time (sec)	N/A	0.212	0.618	0.147	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	101	218	0	0	0	0
normalized size	1	1.	0.79	1.7	0.	0.	0.	0.
time (sec)	N/A	0.114	0.352	0.079	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	100	0	0	414	0	0
normalized size	1	1.	0.89	0.	0.	3.7	0.	0.
time (sec)	N/A	0.1	0.349	0.121	0.	1.818	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	83	0	0	294	0	0
normalized size	1	1.	1.02	0.	0.	3.63	0.	0.
time (sec)	N/A	0.081	0.215	0.108	0.	1.765	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	66	0	0	189	0	0
normalized size	1	1.	1.38	0.	0.	3.94	0.	0.
time (sec)	N/A	0.054	0.074	0.108	0.	1.728	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	53	46	35	58	15	23
normalized size	1	1.	2.3	2.	1.52	2.52	0.65	1.
time (sec)	N/A	0.038	0.233	0.03	1.263	1.803	0.217	1.318

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	52	190	119	0	43
normalized size	1	1.	0.74	1.21	4.42	2.77	0.	1.
time (sec)	N/A	0.042	0.03	0.024	1.167	1.706	0.	1.263

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	91	378	209	0	62
normalized size	1	1.	0.74	1.34	5.56	3.07	0.	0.91
time (sec)	N/A	0.054	0.037	0.029	1.278	1.623	0.	1.208

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	67	128	632	305	0	81
normalized size	1	1.	0.74	1.41	6.95	3.35	0.	0.89
time (sec)	N/A	0.068	0.049	0.03	1.309	1.836	0.	1.141

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	59	46	36	59	15	24
normalized size	1	1.	2.19	1.7	1.33	2.19	0.56	0.89
time (sec)	N/A	0.043	0.075	0.03	1.197	1.72	0.21	1.291

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	32	52	190	119	0	43
normalized size	1	1.	0.65	1.06	3.88	2.43	0.	0.88
time (sec)	N/A	0.044	0.028	0.024	1.308	1.801	0.	1.292

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	92	91	379	208	0	62
normalized size	1	1.	1.21	1.2	4.99	2.74	0.	0.82
time (sec)	N/A	0.059	0.215	0.035	1.269	1.733	0.	1.338

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	63	128	632	304	0	81
normalized size	1	1.	0.62	1.27	6.26	3.01	0.	0.8
time (sec)	N/A	0.07	0.049	0.033	1.503	1.723	0.	1.286

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	85	0	0	738	0	0
normalized size	1	1.	1.29	0.	0.	11.18	0.	0.
time (sec)	N/A	0.066	0.119	0.11	0.	1.806	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	105	0	0	936	0	0
normalized size	1	1.	1.33	0.	0.	11.85	0.	0.
time (sec)	N/A	0.077	0.23	0.088	0.	1.931	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	184	0	0	1218	0	0
normalized size	1	1.	1.67	0.	0.	11.07	0.	0.
time (sec)	N/A	0.1	0.212	0.092	0.	1.947	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	241	1893	0	0	0	0
normalized size	1	1.	0.93	7.31	0.	0.	0.	0.
time (sec)	N/A	0.446	0.771	0.108	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	196	1037	0	0	0	0
normalized size	1	1.	0.95	5.01	0.	0.	0.	0.
time (sec)	N/A	0.311	0.55	0.157	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	151	897	0	0	0	0
normalized size	1	1.	0.92	5.47	0.	0.	0.	0.
time (sec)	N/A	0.212	0.401	0.093	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	61	101	0	387	469	101
normalized size	1	1.	1.11	1.84	0.	7.04	8.53	1.84
time (sec)	N/A	0.075	0.102	0.02	0.	1.879	97.36	1.229

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	82	113	0	1065	0	161
normalized size	1	1.	1.11	1.53	0.	14.39	0.	2.18
time (sec)	N/A	0.083	0.153	0.026	0.	1.886	0.	1.369

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	131	314	0	3617	0	377
normalized size	1	1.	1.02	2.45	0.	28.26	0.	2.95
time (sec)	N/A	0.175	0.27	0.036	0.	2.035	0.	1.275

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	189	633	0	8690	0	644
normalized size	1	1.	1.01	3.39	0.	46.47	0.	3.44
time (sec)	N/A	0.325	0.461	0.041	0.	2.334	0.	1.297

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	66	105	0	398	350	111
normalized size	1	1.	1.1	1.75	0.	6.63	5.83	1.85
time (sec)	N/A	0.084	0.069	0.02	0.	1.784	103.155	1.265

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	9	3	8
normalized size	1	1.	1.	1.17	0.	1.5	0.5	1.33
time (sec)	N/A	0.001	0.	0.002	0.	1.72	0.438	1.163

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	36	0	158	0	41
normalized size	1	1.	1.	3.	0.	13.17	0.	3.42
time (sec)	N/A	0.032	0.035	0.029	0.	1.633	0.	1.233

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	37	46	139	0	45
normalized size	1	1.	0.82	1.09	1.35	4.09	0.	1.32
time (sec)	N/A	0.037	0.086	0.016	1.567	1.753	0.	1.206

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	109	266	0	0	0	0
normalized size	1	1.	0.8	1.96	0.	0.	0.	0.
time (sec)	N/A	0.12	0.503	0.092	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	159	517	0	0	0	0
normalized size	1	1.	0.9	2.94	0.	0.	0.	0.
time (sec)	N/A	0.227	0.618	0.163	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	236	806	0	0	0	0
normalized size	1	1.	0.94	3.21	0.	0.	0.	0.
time (sec)	N/A	0.335	0.777	0.239	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	72	1462	0	162
normalized size	1	1.	0.68	0.6	1.36	27.58	0.	3.06
time (sec)	N/A	0.037	0.041	0.045	1.815	1.785	0.	1.313

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	47	672	0	95
normalized size	1	1.	0.76	0.71	1.38	19.76	0.	2.79
time (sec)	N/A	0.024	0.036	0.035	1.818	1.799	0.	1.186

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	23	216	19	46
normalized size	1	1.	1.	1.15	1.77	16.62	1.46	3.54
time (sec)	N/A	0.013	0.005	0.03	1.829	1.743	0.61	1.199

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	20	49	32	312	0	61
normalized size	1	1.	1.18	2.88	1.88	18.35	0.	3.59
time (sec)	N/A	0.013	0.006	0.053	1.809	1.714	0.	1.262

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	44	71	84	927	0	127
normalized size	1	1.	1.05	1.69	2.	22.07	0.	3.02
time (sec)	N/A	0.026	0.038	0.05	1.755	1.811	0.	1.338

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	67	89	130	2514	0	149
normalized size	1	1.	1.1	1.46	2.13	41.21	0.	2.44
time (sec)	N/A	0.037	0.1	0.053	1.827	1.963	0.	1.364

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	67	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.183	0.076	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	57	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.064	0.06	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.083	0.065	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.03	0.065	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	53	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.082	0.062	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	69	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.196	0.06	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	53	177	135	4963	0	154
normalized size	1	1.	0.4	1.34	1.02	37.6	0.	1.17
time (sec)	N/A	0.049	0.159	0.115	1.737	2.053	0.	1.273

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	38	131	85	2068	0	68
normalized size	1	1.	0.49	1.68	1.09	26.51	0.	0.87
time (sec)	N/A	0.032	0.091	0.091	1.776	1.845	0.	1.209

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	90	36	551	0	35
normalized size	1	1.	0.67	2.5	1.	15.31	0.	0.97
time (sec)	N/A	0.015	0.038	0.095	1.873	1.79	0.	1.226

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	56	24	338	0	18
normalized size	1	1.	1.	3.5	1.5	21.12	0.	1.12
time (sec)	N/A	0.014	0.006	0.083	1.785	1.68	0.	1.316

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	34	80	231	3163	0	47
normalized size	1	1.	0.5	1.18	3.4	46.51	0.	0.69
time (sec)	N/A	0.022	0.038	0.086	1.94	2.04	0.	1.272

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	47	96	630	9072	0	72
normalized size	1	1.	0.4	0.81	5.34	76.88	0.	0.61
time (sec)	N/A	0.033	0.058	0.086	2.016	2.564	0.	1.392

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	219	292	122	301	124	116
normalized size	1	1.	4.38	5.84	2.44	6.02	2.48	2.32
time (sec)	N/A	0.055	0.151	0.066	1.354	1.891	0.73	1.2

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	142	101	240	100	96
normalized size	1	1.	0.98	3.3	2.35	5.58	2.33	2.23
time (sec)	N/A	0.045	0.03	0.057	1.242	1.858	0.597	1.184

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	131	210	89	213	82	84
normalized size	1	1.	3.45	5.53	2.34	5.61	2.16	2.21
time (sec)	N/A	0.047	0.24	0.052	1.301	1.751	0.481	1.292

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	28	94	69	151	63	63
normalized size	1	1.	0.85	2.85	2.09	4.58	1.91	1.91
time (sec)	N/A	0.039	0.02	0.043	1.111	1.809	0.371	1.224

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	93	126	57	128	48	51
normalized size	1	1.	3.58	4.85	2.19	4.92	1.85	1.96
time (sec)	N/A	0.041	0.149	0.036	1.246	1.848	0.28	1.339

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	12	13	36	70	27	31
normalized size	1	1.	0.8	0.87	2.4	4.67	1.8	2.07
time (sec)	N/A	0.034	0.01	0.014	1.07	1.782	0.208	1.23

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	34	40	19	53	14	19
normalized size	1	1.	4.25	5.	2.38	6.62	1.75	2.38
time (sec)	N/A	0.031	0.046	0.028	1.256	1.698	0.145	1.233

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	7	28	8	15
normalized size	1	1.	1.	1.	1.	4.	1.14	2.14
time (sec)	N/A	0.018	0.005	0.008	1.185	1.823	0.147	1.325

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	18	43	55	158	0	69
normalized size	1	1.	0.75	1.79	2.29	6.58	0.	2.88
time (sec)	N/A	0.035	0.021	0.029	1.199	1.782	0.	1.292

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	49	72	76	0	39
normalized size	1	1.	0.88	1.96	2.88	3.04	0.	1.56
time (sec)	N/A	0.041	0.032	0.028	1.103	1.639	0.	1.272

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	61	91	124	451	0	124
normalized size	1	1.	1.17	1.75	2.38	8.67	0.	2.38
time (sec)	N/A	0.053	0.041	0.036	1.186	1.759	0.	1.328

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	93	277	197	0	72
normalized size	1	1.	0.95	2.51	7.49	5.32	0.	1.95
time (sec)	N/A	0.044	0.064	0.04	1.211	1.781	0.	1.27

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	94	137	189	801	0	159
normalized size	1	1.	1.18	1.71	2.36	10.01	0.	1.99
time (sec)	N/A	0.067	0.067	0.042	1.183	2.141	0.	1.267

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	121	166	73	177	65	68
normalized size	1	1.	3.02	4.15	1.82	4.42	1.62	1.7
time (sec)	N/A	0.074	0.179	0.054	1.128	2.08	0.442	1.263

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	70	53	107	44	47
normalized size	1	1.	1.29	5.	3.79	7.64	3.14	3.36
time (sec)	N/A	0.034	0.019	0.047	1.214	1.977	0.316	1.218

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	82	41	92	29	35
normalized size	1	1.	1.53	2.73	1.37	3.07	0.97	1.17
time (sec)	N/A	0.068	0.072	0.042	1.125	1.834	0.264	1.313

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	53	31	82	26	28
normalized size	1	1.	1.	3.79	2.21	5.86	1.86	2.
time (sec)	N/A	0.038	0.012	0.042	1.234	1.922	9.34	1.32

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	69	29	16	42	8	14
normalized size	1	1.	4.93	2.07	1.14	3.	0.57	1.
time (sec)	N/A	0.034	0.054	0.04	1.215	1.735	0.184	1.276

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	11	43	19	14
normalized size	1	1.	1.	1.	1.1	4.3	1.9	1.4
time (sec)	N/A	0.02	0.01	0.013	1.227	1.736	0.212	1.292

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	26	70	95	298	0	95
normalized size	1	1.	0.76	2.06	2.79	8.76	0.	2.79
time (sec)	N/A	0.039	0.043	0.04	1.128	1.849	0.	1.271

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	70	158	132	0	55
normalized size	1	1.	0.84	1.89	4.27	3.57	0.	1.49
time (sec)	N/A	0.071	0.022	0.039	1.317	1.736	0.	1.235

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	116	162	625	0	142
normalized size	1	1.	1.13	1.93	2.7	10.42	0.	2.37
time (sec)	N/A	0.055	0.044	0.05	1.303	1.857	0.	1.326

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	116	428	259	0	88
normalized size	1	1.	0.96	2.37	8.73	5.29	0.	1.8
time (sec)	N/A	0.078	0.036	0.052	1.162	1.763	0.	1.266

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	45	56	45	142	31	36
normalized size	1	1.	1.61	2.	1.61	5.07	1.11	1.29
time (sec)	N/A	0.044	0.086	0.053	1.198	1.746	27.575	1.245

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	40	36	72	84	32	22
normalized size	1	1.	2.	1.8	3.6	4.2	1.6	1.1
time (sec)	N/A	0.033	0.064	0.047	1.111	1.701	0.33	1.253

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	14	86	37	16
normalized size	1	1.	0.88	0.81	0.88	5.38	2.31	1.
time (sec)	N/A	0.021	0.026	0.015	1.182	1.683	0.388	1.253

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	45	56	45	142	31	36
normalized size	1	1.	1.73	2.15	1.73	5.46	1.19	1.38
time (sec)	N/A	0.04	0.086	0.055	1.137	1.891	27.517	1.22

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	38	36	72	82	32	22
normalized size	1	1.	1.9	1.8	3.6	4.1	1.6	1.1
time (sec)	N/A	0.033	0.062	0.043	1.194	1.746	0.332	1.263

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	14	85	36	16
normalized size	1	1.	0.88	0.81	0.88	5.31	2.25	1.
time (sec)	N/A	0.022	0.027	0.017	1.217	1.751	0.386	1.214

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	121	837	416	5296	0	343
normalized size	1	1.	0.88	6.07	3.01	38.38	0.	2.49
time (sec)	N/A	0.138	0.157	0.046	1.206	2.053	0.	1.258

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	857	674	0	3822	0	389
normalized size	1	1.	5.91	4.65	0.	26.36	0.	2.68
time (sec)	N/A	0.414	5.779	0.037	0.	1.926	0.	1.23

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	76	447	243	2228	0	188
normalized size	1	1.	0.94	5.52	3.	27.51	0.	2.32
time (sec)	N/A	0.091	0.088	0.033	1.251	1.921	0.	1.249

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	651	336	0	1523	0	227
normalized size	1	1.	6.71	3.46	0.	15.7	0.	2.34
time (sec)	N/A	0.241	3.499	0.032	0.	1.933	0.	1.242

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	185	109	629	0	82
normalized size	1	1.	1.	4.87	2.87	16.55	0.	2.16
time (sec)	N/A	0.061	0.035	0.028	1.294	1.808	0.	1.159

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	429	126	0	504	398	112
normalized size	1	1.	7.94	2.33	0.	9.33	7.37	2.07
time (sec)	N/A	0.109	1.03	0.024	0.	1.742	177.406	1.267

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	72	14	30
normalized size	1	1.	1.	1.09	1.36	6.55	1.27	2.73
time (sec)	N/A	0.025	0.005	0.01	1.067	2.026	0.425	1.263

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	99	71	89	177	0	120
normalized size	1	1.	2.06	1.48	1.85	3.69	0.	2.5
time (sec)	N/A	0.06	0.095	0.021	1.843	2.153	0.	1.241

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	67	71	0	693	0	117
normalized size	1	1.	1.14	1.2	0.	11.75	0.	1.98
time (sec)	N/A	0.079	0.167	0.026	0.	2.057	0.	1.324

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	353	215	1724	0	289
normalized size	1	1.	0.89	4.06	2.47	19.82	0.	3.32
time (sec)	N/A	0.122	0.124	0.033	1.842	2.311	0.	1.247

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	102	182	0	2807	0	243
normalized size	1	1.	1.02	1.82	0.	28.07	0.	2.43
time (sec)	N/A	0.213	0.333	0.034	0.	2.227	0.	1.361

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	135	1140	466	6700	0	498
normalized size	1	1.	1.	8.44	3.45	49.63	0.	3.69
time (sec)	N/A	0.197	0.226	0.043	1.865	2.854	0.	1.238

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	146	350	0	7626	0	436
normalized size	1	1.	1.	2.4	0.	52.23	0.	2.99
time (sec)	N/A	0.417	0.445	0.048	0.	2.393	0.	1.283

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	2901	290	0	2184	0	240
normalized size	1	1.	30.86	3.09	0.	23.23	0.	2.55
time (sec)	N/A	0.22	6.278	0.058	0.	2.148	0.	1.259

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	141	138	999	209	111
normalized size	1	1.	0.92	3.52	3.45	24.98	5.22	2.78
time (sec)	N/A	0.06	0.07	0.052	1.107	2.168	2.022	1.215

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	659	119	0	933	0	131
normalized size	1	1.	10.63	1.92	0.	15.05	0.	2.11
time (sec)	N/A	0.111	3.401	0.043	0.	2.101	0.	1.148

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	149	32	30
normalized size	1	1.	1.	1.08	1.38	11.46	2.46	2.31
time (sec)	N/A	0.025	0.013	0.019	1.209	1.959	1.015	1.122

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	121	201	201	963	0	251
normalized size	1	1.	1.53	2.54	2.54	12.19	0.	3.18
time (sec)	N/A	0.103	0.443	0.046	1.748	2.228	0.	1.132

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	94	138	0	1916	0	225
normalized size	1	1.	1.01	1.48	0.	20.6	0.	2.42
time (sec)	N/A	0.17	0.242	0.041	0.	2.158	0.	1.152

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	260	548	506	6197	0	398
normalized size	1	1.	1.91	4.03	3.72	45.57	0.	2.93
time (sec)	N/A	0.165	2.442	0.069	1.879	2.83	0.	1.176

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	137	266	0	7135	0	387
normalized size	1	1.	0.95	1.85	0.	49.55	0.	2.69
time (sec)	N/A	0.31	0.43	0.072	0.	2.457	0.	1.17

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	96	93	558	271	116	72
normalized size	1	1.	3.1	3.	18.	8.74	3.74	2.32
time (sec)	N/A	0.079	0.135	0.07	1.126	2.079	1.301	1.155

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	42	79	128	455	97	124
normalized size	1	1.	1.17	2.19	3.56	12.64	2.69	3.44
time (sec)	N/A	0.092	0.077	0.059	1.033	2.088	0.806	1.138

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	67	47	147	111	48	39
normalized size	1	1.	2.91	2.04	6.39	4.83	2.09	1.7
time (sec)	N/A	0.076	0.059	0.045	1.117	2.033	0.379	1.122

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	20	45	57	157	32	72
normalized size	1	1.	0.77	1.73	2.19	6.04	1.23	2.77
time (sec)	N/A	0.055	0.02	0.032	1.168	2.133	0.27	1.089

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	38	54	0	31
normalized size	1	1.	1.	0.89	2.	2.84	0.	1.63
time (sec)	N/A	0.032	0.008	0.022	1.037	2.05	0.	1.112

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	32	23	36	108	22	32
normalized size	1	1.	2.67	1.92	3.	9.	1.83	2.67
time (sec)	N/A	0.044	0.035	0.026	1.073	2.013	0.217	1.128

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	34	90	84	32	32
normalized size	1	1.	1.	2.27	6.	5.6	2.13	2.13
time (sec)	N/A	0.059	0.011	0.038	1.046	1.982	0.256	1.149

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	100	59	82	263	58	59
normalized size	1	1.	3.85	2.27	3.15	10.12	2.23	2.27
time (sec)	N/A	0.078	0.037	0.046	1.052	2.013	0.479	1.153

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	33	68	277	176	71	69
normalized size	1	1.	1.43	2.96	12.04	7.65	3.09	3.
time (sec)	N/A	0.077	0.01	0.066	1.069	1.973	0.754	1.134

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	164	93	123	437	104	84
normalized size	1	1.	4.56	2.58	3.42	12.14	2.89	2.33
time (sec)	N/A	0.097	0.04	0.07	1.089	2.138	1.377	1.126

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	112	116	774	351	139	88
normalized size	1	1.	2.38	2.47	16.47	7.47	2.96	1.87
time (sec)	N/A	0.123	0.146	0.093	1.113	2.003	2.533	1.164

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	52	114	155	621	129	138
normalized size	1	1.	0.79	1.73	2.35	9.41	1.95	2.09
time (sec)	N/A	0.065	0.1	0.078	1.119	2.028	1.588	1.184

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	84	70	266	171	71	55
normalized size	1	1.	2.27	1.89	7.19	4.62	1.92	1.49
time (sec)	N/A	0.118	0.092	0.059	1.088	2.013	0.823	1.133

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	29	66	82	284	60	89
normalized size	1	1.	0.81	1.83	2.28	7.89	1.67	2.47
time (sec)	N/A	0.037	0.03	0.051	1.071	2.06	0.486	1.131

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	65	162	36	45
normalized size	1	1.	1.	0.92	2.6	6.48	1.44	1.8
time (sec)	N/A	0.04	0.027	0.033	1.118	2.052	0.513	1.121

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	66	35	73	232	49	63
normalized size	1	1.	2.54	1.35	2.81	8.92	1.88	2.42
time (sec)	N/A	0.068	0.138	0.045	1.13	2.088	0.471	1.153

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	51	86	207	53	72
normalized size	1	1.	1.	1.76	2.97	7.14	1.83	2.48
time (sec)	N/A	0.048	0.016	0.06	1.142	2.062	0.688	1.121

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	107	58	90	302	66	68
normalized size	1	1.	3.82	2.07	3.21	10.79	2.36	2.43
time (sec)	N/A	0.094	0.054	0.051	1.03	2.033	0.55	1.141

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	68	231	166	66	51
normalized size	1	1.	1.	2.52	8.56	6.15	2.44	1.89
time (sec)	N/A	0.043	0.012	0.067	1.022	1.958	0.748	1.103

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	175	74	139	527	117	100
normalized size	1	1.	3.65	1.54	2.9	10.98	2.44	2.08
time (sec)	N/A	0.091	0.058	0.073	1.069	2.113	1.521	1.145

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	108	163	0	2944	0	266
normalized size	1	1.	0.87	1.31	0.	23.74	0.	2.15
time (sec)	N/A	0.184	0.352	0.039	0.	2.246	0.	1.195

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	153	357	216	1725	0	285
normalized size	1	1.	1.74	4.06	2.45	19.6	0.	3.24
time (sec)	N/A	0.175	0.175	0.036	1.573	2.268	0.	1.137

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	84	0	691	0	117
normalized size	1	1.	1.	1.22	0.	10.01	0.	1.7
time (sec)	N/A	0.091	0.188	0.029	0.	2.054	0.	1.198

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	36	84	89	177	0	120
normalized size	1	1.	0.75	1.75	1.85	3.69	0.	2.5
time (sec)	N/A	0.069	0.058	0.024	1.533	2.12	0.	1.132

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	62	116	0	53
normalized size	1	1.	1.	1.05	3.1	5.8	0.	2.65
time (sec)	N/A	0.041	0.01	0.02	1.06	2.027	0.	1.128

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	82	107	0	705	0	128
normalized size	1	1.	1.46	1.91	0.	12.59	0.	2.29
time (sec)	N/A	0.235	0.176	0.026	0.	2.23	0.	1.143

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	120	157	1165	0	169
normalized size	1	1.	0.87	2.31	3.02	22.4	0.	3.25
time (sec)	N/A	0.095	0.061	0.034	1.024	2.174	0.	1.142

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	176	232	0	3380	0	262
normalized size	1	1.	1.63	2.15	0.	31.3	0.	2.43
time (sec)	N/A	0.413	0.423	0.036	0.	3.47	0.	1.133

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	144	262	0	8182	0	394
normalized size	1	1.	0.64	1.17	0.	36.53	0.	1.76
time (sec)	N/A	0.435	0.39	0.062	0.	2.728	0.	1.224

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	150	491	506	6589	0	414
normalized size	1	1.	1.11	3.64	3.75	48.81	0.	3.07
time (sec)	N/A	0.356	0.662	0.062	1.594	2.792	0.	1.175

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	100	142	0	2144	0	244
normalized size	1	1.	0.69	0.99	0.	14.89	0.	1.69
time (sec)	N/A	0.249	0.265	0.05	0.	2.148	0.	1.184

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	146	248	209	1029	0	269
normalized size	1	1.	1.72	2.92	2.46	12.11	0.	3.16
time (sec)	N/A	0.103	0.225	0.042	1.571	2.199	0.	1.171

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	27	33	101	477	0	101
normalized size	1	1.	0.84	1.03	3.16	14.91	0.	3.16
time (sec)	N/A	0.053	0.047	0.033	1.026	2.127	0.	1.134

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	102	170	0	3102	0	200
normalized size	1	1.	1.27	2.12	0.	38.78	0.	2.5
time (sec)	N/A	0.402	0.478	0.041	0.	2.748	0.	1.186

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	184	273	3726	0	257
normalized size	1	1.	0.96	2.42	3.59	49.03	0.	3.38
time (sec)	N/A	0.11	0.197	0.056	1.051	2.292	0.	1.153

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	214	357	0	9129	0	327
normalized size	1	1.	1.35	2.25	0.	57.42	0.	2.06
time (sec)	N/A	0.671	0.818	0.056	0.	3.028	0.	1.205

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	0	1071	0	0
normalized size	1	1.	1.	0.81	0.	28.95	0.	0.
time (sec)	N/A	0.062	0.022	0.014	0.	4.919	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	1099	0	0
normalized size	1	1.	1.	0.79	0.	45.79	0.	0.
time (sec)	N/A	0.059	0.013	0.016	0.	2.583	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	59	88	0	458	695	117
normalized size	1	1.	1.16	1.73	0.	8.98	13.63	2.29
time (sec)	N/A	0.135	0.076	0.025	0.	2.093	99.557	1.134

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	48	46	26	90	20	30
normalized size	1	1.	1.92	1.84	1.04	3.6	0.8	1.2
time (sec)	N/A	0.083	0.066	0.032	1.035	2.047	1.311	1.136

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	81	44	27	89	20	28
normalized size	1	1.	3.	1.63	1.	3.3	0.74	1.04
time (sec)	N/A	0.087	0.088	0.037	1.055	2.082	0.366	1.141

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	132	150	0	510	0	166
normalized size	1	1.	1.48	1.69	0.	5.73	0.	1.87
time (sec)	N/A	0.177	0.355	0.032	0.	15.237	0.	1.178

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	65	73	0	502	0	138
normalized size	1	1.	1.08	1.22	0.	8.37	0.	2.3
time (sec)	N/A	0.15	0.159	0.033	0.	2.236	0.	1.173

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	93	150	0	510	0	166
normalized size	1	1.	1.04	1.69	0.	5.73	0.	1.87
time (sec)	N/A	0.252	0.249	0.032	0.	26.196	0.	1.168

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	86	0	482	0	122
normalized size	1	1.	1.16	1.48	0.	8.31	0.	2.1
time (sec)	N/A	0.157	0.116	0.03	0.	5.065	0.	1.172

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	85	213	0	616	1358	181
normalized size	1	1.	1.05	2.63	0.	7.6	16.77	2.23
time (sec)	N/A	0.169	0.284	0.056	0.	2.131	130.497	1.193

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	151	0	1339	0	262
normalized size	1	1.	1.	1.34	0.	11.85	0.	2.32
time (sec)	N/A	0.169	0.525	0.092	0.	2.165	0.	1.182

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	170	416	0	4178	0	586
normalized size	1	1.	0.94	2.31	0.	23.21	0.	3.26
time (sec)	N/A	0.27	0.679	0.116	0.	2.31	0.	1.174

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	235	844	0	9709	0	988
normalized size	1	1.	0.94	3.38	0.	38.84	0.	3.95
time (sec)	N/A	0.44	1.539	0.134	0.	2.649	0.	1.205

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	319	919	0	3956	0	0
normalized size	1	1.	0.73	2.09	0.	9.01	0.	0.
time (sec)	N/A	0.73	0.838	0.059	0.	2.466	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	240	710	0	2967	0	0
normalized size	1	1.	0.73	2.17	0.	9.07	0.	0.
time (sec)	N/A	0.538	0.694	0.049	0.	2.372	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	576	505	0	1968	0	0
normalized size	1	1.	2.68	2.35	0.	9.15	0.	0.
time (sec)	N/A	0.329	0.772	0.047	0.	2.253	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	57	204	0	57
normalized size	1	1.	0.87	1.	1.21	4.34	0.	1.21
time (sec)	N/A	0.456	0.104	0.103	1.308	2.009	0.	1.125

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.079	0.109	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.035	0.079	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.031	0.024	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	11.438	0.042	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	36.221	0.039	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	41	0	70	123	0	63
normalized size	1	1.	0.76	0.	1.3	2.28	0.	1.17
time (sec)	N/A	0.011	0.059	0.03	1.119	2.043	0.	1.132

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	55	0	90	258	0	228
normalized size	1	1.	0.62	0.	1.02	2.93	0.	2.59
time (sec)	N/A	0.021	0.101	0.095	1.161	2.026	0.	1.207

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	120	0	154	532	0	898
normalized size	1	1.	0.81	0.	1.03	3.57	0.	6.03
time (sec)	N/A	0.041	0.502	0.108	1.213	2.147	0.	1.275

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	167	0	174	801	0	1049
normalized size	1	1.	0.87	0.	0.91	4.19	0.	5.49
time (sec)	N/A	0.051	0.418	0.131	1.258	2.134	0.	1.28

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	0	86	304	0	317
normalized size	1	1.	0.74	0.	1.18	4.16	0.	4.34
time (sec)	N/A	0.024	0.133	0.033	1.118	2.087	0.	1.193

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	89	0	0	726	0	1023
normalized size	1	1.	0.74	0.	0.	6.05	0.	8.52
time (sec)	N/A	0.048	0.28	0.109	0.	2.187	0.	1.276

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	197	292	0	186	1542	0	4354
normalized size	1	0.97	1.44	0.	0.92	7.6	0.	21.45
time (sec)	N/A	0.086	1.327	0.132	1.23	2.168	0.	1.441

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	260	311	0	0	2880	0	9293
normalized size	1	0.98	1.17	0.	0.	10.83	0.	34.94
time (sec)	N/A	0.128	3.442	0.147	0.	2.313	0.	1.742

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	24	53	37	54
normalized size	1	1.	2.06	1.06	1.33	2.94	2.06	3.
time (sec)	N/A	0.017	0.014	0.006	1.024	2.041	1.722	1.109

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	66	123	0	109
normalized size	1	1.	0.92	1.33	1.69	3.15	0.	2.79
time (sec)	N/A	0.029	0.027	0.01	1.048	2.066	0.	1.214

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	45	36	116	208	82	109
normalized size	1	1.	1.05	0.84	2.7	4.84	1.91	2.53
time (sec)	N/A	0.035	0.014	0.007	1.036	2.097	37.068	1.188

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	84	126	270	0	154
normalized size	1	1.	0.7	1.15	1.73	3.7	0.	2.11
time (sec)	N/A	0.047	0.047	0.012	1.142	2.083	0.	1.184

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	68	51	176	421	0	155
normalized size	1	1.	1.05	0.78	2.71	6.48	0.	2.38
time (sec)	N/A	0.042	0.019	0.012	1.102	2.118	0.	1.211

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	96	227	0	0	0	0
normalized size	1	1.	0.86	2.05	0.	0.	0.	0.
time (sec)	N/A	0.065	0.088	0.054	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	114	143	0	0	0	0
normalized size	1	1.	1.03	1.29	0.	0.	0.	0.
time (sec)	N/A	0.06	0.13	0.055	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	68	146	0	0	0	0
normalized size	1	1.	0.94	2.03	0.	0.	0.	0.
time (sec)	N/A	0.044	0.042	0.046	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	120	0	0	0	0
normalized size	1	1.	0.92	1.67	0.	0.	0.	0.
time (sec)	N/A	0.044	0.044	0.042	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	80	212	0	0	0	0
normalized size	1	1.	0.75	1.98	0.	0.	0.	0.
time (sec)	N/A	0.057	0.062	0.013	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	122	144	0	0	0	0
normalized size	1	1.	1.1	1.3	0.	0.	0.	0.
time (sec)	N/A	0.061	0.127	0.055	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	86	0	0	420	0	0
normalized size	1	1.	0.41	0.	0.	2.01	0.	0.
time (sec)	N/A	0.16	0.418	0.362	0.	2.18	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	74	0	0	306	0	0
normalized size	1	1.	0.72	0.	0.	2.97	0.	0.
time (sec)	N/A	0.087	0.269	0.155	0.	2.158	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	61	0	0	167	0	0
normalized size	1	1.	1.42	0.	0.	3.88	0.	0.
time (sec)	N/A	0.053	0.147	0.16	0.	2.432	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	121	0	0	312	0	0
normalized size	1	1.	1.17	0.	0.	3.03	0.	0.
time (sec)	N/A	0.079	0.26	0.163	0.	2.301	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	0	109	0	0
normalized size	1	1.	1.	1.06	0.	3.03	0.	0.
time (sec)	N/A	0.047	0.02	0.009	0.	2.187	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	50	0	165	0	0
normalized size	1	1.	0.95	1.28	0.	4.23	0.	0.
time (sec)	N/A	0.064	0.04	0.016	0.	2.044	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	74	0	269	0	0
normalized size	1	1.	0.92	1.25	0.	4.56	0.	0.
time (sec)	N/A	0.085	0.057	0.013	0.	2.133	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	70	113	0	532	0	0
normalized size	1	1.	0.95	1.53	0.	7.19	0.	0.
time (sec)	N/A	0.13	0.283	0.032	0.	1.98	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	85	120	0	589	0	0
normalized size	1	1.	1.06	1.5	0.	7.36	0.	0.
time (sec)	N/A	0.146	0.359	0.087	0.	2.117	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	172	228	0	1524	0	0
normalized size	1	1.	1.2	1.59	0.	10.66	0.	0.
time (sec)	N/A	0.246	0.459	0.088	0.	2.18	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	373	347	0	344	0	0
normalized size	1	1.	3.69	3.44	0.	3.41	0.	0.
time (sec)	N/A	0.179	0.706	0.028	0.	2.042	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	112	358	0	775	0	0
normalized size	1	1.	1.05	3.35	0.	7.24	0.	0.
time (sec)	N/A	0.182	0.849	0.086	0.	2.066	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	599	700	0	1536	0	0
normalized size	1	1.	3.09	3.61	0.	7.92	0.	0.
time (sec)	N/A	0.328	1.356	0.084	0.	2.145	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	449	459	0	417	0	0
normalized size	1	1.	3.71	3.79	0.	3.45	0.	0.
time (sec)	N/A	0.259	1.432	0.043	0.	2.043	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	136	468	0	1018	0	0
normalized size	1	1.	1.05	3.63	0.	7.89	0.	0.
time (sec)	N/A	0.279	2.	0.097	0.	2.317	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	671	922	0	2047	0	0
normalized size	1	1.	2.97	4.08	0.	9.06	0.	0.
time (sec)	N/A	0.476	6.181	0.105	0.	2.55	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	62	77	92	325	139	81
normalized size	1	1.	0.75	0.93	1.11	3.92	1.67	0.98
time (sec)	N/A	0.04	0.041	0.008	1.18	2.213	104.606	1.147

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	45	67	72	259	187	77
normalized size	1	1.	0.79	1.18	1.26	4.54	3.28	1.35
time (sec)	N/A	0.038	0.034	0.006	1.114	2.053	27.031	1.138

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	39	49	54	154	78	46
normalized size	1	1.	0.8	1.	1.1	3.14	1.59	0.94
time (sec)	N/A	0.031	0.021	0.008	1.058	1.995	7.023	1.176

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	32	132	63	32
normalized size	1	1.	1.	1.61	1.39	5.74	2.74	1.39
time (sec)	N/A	0.016	0.012	0.006	1.125	1.93	1.639	1.138

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	36	76	0	23
normalized size	1	1.	1.	1.	1.89	4.	0.	1.21
time (sec)	N/A	0.019	0.011	0.006	1.111	2.003	0.	1.13

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	39	70	462	0	72
normalized size	1	1.	0.9	0.93	1.67	11.	0.	1.71
time (sec)	N/A	0.031	0.069	0.009	1.116	1.954	0.	1.129

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	32	92	235	0	42
normalized size	1	1.	0.94	1.03	2.97	7.58	0.	1.35
time (sec)	N/A	0.028	0.017	0.045	1.113	1.929	0.	1.154

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	71	135	1963	0	107
normalized size	1	1.	0.74	0.7	1.34	19.44	0.	1.06
time (sec)	N/A	0.054	0.073	0.045	1.169	2.008	0.	1.127

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	44	61	232	635	0	57
normalized size	1	1.	0.67	0.92	3.52	9.62	0.	0.86
time (sec)	N/A	0.056	0.035	0.048	1.013	1.872	0.	1.112

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	23	170	42	23
normalized size	1	1.	1.	1.31	0.88	6.54	1.62	0.88
time (sec)	N/A	0.02	0.013	0.019	1.104	2.02	1.053	1.138

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	22	18	90	20	18
normalized size	1	1.	0.84	1.16	0.95	4.74	1.05	0.95
time (sec)	N/A	0.011	0.008	0.006	1.058	1.939	0.362	1.126

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	34	24	126	0	26
normalized size	1	1.	1.	3.09	2.18	11.45	0.	2.36
time (sec)	N/A	0.013	0.009	0.045	1.583	2.066	0.	1.146

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	46	43	678	0	45
normalized size	1	1.	1.	1.44	1.34	21.19	0.	1.41
time (sec)	N/A	0.024	0.036	0.046	1.547	2.041	0.	1.122

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	23	240	42	23
normalized size	1	1.	1.	1.31	0.88	9.23	1.62	0.88
time (sec)	N/A	0.02	0.016	0.013	1.082	1.962	0.954	1.143

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	26	18	128	20	18
normalized size	1	1.	0.84	1.37	0.95	6.74	1.05	0.95
time (sec)	N/A	0.012	0.009	0.016	1.108	1.966	0.329	1.117

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	22	79	99	288	0	58
normalized size	1	1.	0.41	1.46	1.83	5.33	0.	1.07
time (sec)	N/A	0.057	0.011	0.045	1.652	2.108	0.	1.133

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	34	148	115	1985	0	116
normalized size	1	1.	0.32	1.41	1.1	18.9	0.	1.1
time (sec)	N/A	0.139	0.026	0.055	1.685	2.146	0.	1.109

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	23	311	42	23
normalized size	1	1.	1.	1.31	0.88	11.96	1.62	0.88
time (sec)	N/A	0.021	0.016	0.013	1.053	2.042	0.955	1.106

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	26	18	154	20	18
normalized size	1	1.	1.	1.37	0.95	8.11	1.05	0.95
time (sec)	N/A	0.012	0.009	0.006	1.1	2.039	0.35	1.094

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	22	56	128	440	0	130
normalized size	1	1.	0.19	0.5	1.13	3.89	0.	1.15
time (sec)	N/A	0.077	0.011	0.055	1.599	2.318	0.	1.087

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	34	68	144	620	0	146
normalized size	1	1.	0.26	0.52	1.1	4.73	0.	1.11
time (sec)	N/A	0.088	0.025	0.058	1.595	2.798	0.	1.119

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	157	326	181	5434	0	1673
normalized size	1	1.	0.78	1.61	0.9	26.9	0.	8.28
time (sec)	N/A	0.082	0.666	0.061	1.191	2.56	0.	1.299

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	86	143	127	1783	604	1220
normalized size	1	1.	0.65	1.08	0.96	13.51	4.58	9.24
time (sec)	N/A	0.053	0.202	0.039	1.152	2.12	70.271	1.2

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	50	77	85	643	316	826
normalized size	1	1.	0.67	1.03	1.13	8.57	4.21	11.01
time (sec)	N/A	0.019	0.105	0.015	1.073	1.836	13.854	1.203

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	93	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	5.313	0.019	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	87	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	2.742	0.022	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	299	0	0	0	0	0
normalized size	1	1.	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	19.668	0.031	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	202	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	6.999	0.035	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	106	184	122	559	0	363
normalized size	1	1.	0.42	0.74	0.49	2.24	0.	1.45
time (sec)	N/A	0.248	0.11	0.163	1.596	1.855	0.	1.237

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	76	152	84	319	0	263
normalized size	1	1.	0.47	0.94	0.52	1.97	0.	1.62
time (sec)	N/A	0.139	0.058	0.109	1.665	1.882	0.	1.16

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	48	100	49	165	206	96
normalized size	1	1.	0.65	1.35	0.66	2.23	2.78	1.3
time (sec)	N/A	0.112	0.038	0.112	1.653	1.847	31.182	1.163

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	0	53	97	0	115
normalized size	1	1.	0.96	0.	1.15	2.11	0.	2.5
time (sec)	N/A	0.129	0.051	180.	1.602	1.821	0.	1.21

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	46	0	113	300	0	117
normalized size	1	1.	0.79	0.	1.95	5.17	0.	2.02
time (sec)	N/A	0.143	0.069	180.	1.567	1.76	0.	1.231

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	72	0	282	797	0	165
normalized size	1	1.	0.49	0.	1.92	5.42	0.	1.12
time (sec)	N/A	0.214	0.07	180.	1.626	1.874	0.	1.291

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	84	0	521	1512	0	217
normalized size	1	1.	0.42	0.	2.62	7.6	0.	1.09
time (sec)	N/A	0.279	0.078	180.	1.751	1.986	0.	1.332

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	28	62	0	134	99	43
normalized size	1	1.	0.68	1.51	0.	3.27	2.41	1.05
time (sec)	N/A	0.013	0.048	0.016	0.	1.746	0.843	1.164

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	72	88	286	0	99
normalized size	1	1.	0.94	0.85	1.04	3.36	0.	1.16
time (sec)	N/A	0.081	0.091	0.109	1.121	1.784	0.	1.159

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	92	97	109	367	0	123
normalized size	1	1.	0.91	0.96	1.08	3.63	0.	1.22
time (sec)	N/A	0.147	0.167	0.099	1.155	1.781	0.	1.125

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	61	130	0	61
normalized size	1	1.	0.78	0.8	0.94	2.	0.	0.94
time (sec)	N/A	0.065	0.066	0.073	1.061	1.889	0.	1.131

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	72	48	63	235	0	66
normalized size	1	1.	1.11	0.74	0.97	3.62	0.	1.02
time (sec)	N/A	0.083	0.107	0.073	1.088	1.868	0.	1.144

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	123	105	120	467	0	136
normalized size	1	1.	1.07	0.91	1.04	4.06	0.	1.18
time (sec)	N/A	0.168	0.396	0.145	1.064	1.824	0.	1.245

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	103	100	122	591	0	143
normalized size	1	1.	0.94	0.91	1.11	5.37	0.	1.3
time (sec)	N/A	0.154	0.138	0.134	1.06	1.942	0.	1.219

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	171	826	0	481
normalized size	1	1.	1.01	0.85	1.16	5.58	0.	3.25
time (sec)	N/A	0.194	0.752	0.173	1.626	1.836	0.	1.318

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	287	207	270	1287	0	301
normalized size	1	1.	1.2	0.87	1.13	5.38	0.	1.26
time (sec)	N/A	0.286	0.417	0.207	1.614	1.916	0.	1.34

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	124	126	138	699	0	181
normalized size	1	1.	1.08	1.1	1.2	6.08	0.	1.57
time (sec)	N/A	0.225	0.31	0.109	1.075	1.845	0.	1.282

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	193	956	0	527
normalized size	1	1.	1.37	0.98	1.2	5.94	0.	3.27
time (sec)	N/A	0.284	0.625	0.142	1.533	1.88	0.	1.353

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	354	265	308	1519	0	385
normalized size	1	1.	1.38	1.03	1.2	5.91	0.	1.5
time (sec)	N/A	0.477	0.767	0.179	1.583	1.941	0.	1.303

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	142	598	0	178
normalized size	1	1.	0.78	0.88	1.07	4.5	0.	1.34
time (sec)	N/A	0.198	0.158	0.106	1.033	1.764	0.	1.237

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	177	676	0	203
normalized size	1	1.	0.81	0.86	1.1	4.2	0.	1.26
time (sec)	N/A	0.226	0.264	0.122	1.071	1.808	0.	1.298

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	285	1202	0	356
normalized size	1	1.	0.79	0.86	1.05	4.44	0.	1.31
time (sec)	N/A	0.345	0.451	0.146	1.082	1.839	0.	1.218

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	76	70	93	410	0	101
normalized size	1	1.	0.94	0.86	1.15	5.06	0.	1.25
time (sec)	N/A	0.17	0.341	0.084	1.084	1.904	0.	1.288

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	135	720	0	144
normalized size	1	1.	1.4	0.79	1.05	5.62	0.	1.12
time (sec)	N/A	0.204	0.534	0.098	1.069	1.913	0.	1.276

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	272	144	193	1300	0	209
normalized size	1	1.	1.59	0.84	1.13	7.6	0.	1.22
time (sec)	N/A	0.3	1.207	0.125	1.076	2.083	0.	1.31

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	166	147	171	853	0	232
normalized size	1	1.	1.19	1.05	1.22	6.09	0.	1.66
time (sec)	N/A	0.321	0.73	0.175	1.049	2.037	0.	1.304

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	217	1118	0	267
normalized size	1	1.	1.41	0.97	1.19	6.11	0.	1.46
time (sec)	N/A	0.334	1.427	0.185	1.071	1.935	0.	1.274

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	300	300	480	302	355	2218	0	475
normalized size	1	1.	1.6	1.01	1.18	7.39	0.	1.58
time (sec)	N/A	0.587	5.804	0.241	1.086	2.165	0.	1.375

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	135	156	174	717	0	228
normalized size	1	1.	0.88	1.02	1.14	4.69	0.	1.49
time (sec)	N/A	0.301	0.335	0.116	1.057	1.952	0.	1.232

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	183	211	250	933	0	304
normalized size	1	1.	0.84	0.96	1.14	4.26	0.	1.39
time (sec)	N/A	0.364	0.615	0.184	1.061	1.975	0.	1.28

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	263	316	355	1434	0	463
normalized size	1	1.	0.83	1.	1.13	4.55	0.	1.47
time (sec)	N/A	0.462	0.984	0.163	1.094	2.044	0.	1.252

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	179	160	188	876	0	244
normalized size	1	1.	1.16	1.04	1.22	5.69	0.	1.58
time (sec)	N/A	0.332	0.786	0.184	1.057	1.872	0.	1.277

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	269	1269	0	323
normalized size	1	1.	1.14	0.96	1.2	5.64	0.	1.44
time (sec)	N/A	0.395	2.192	0.208	1.104	1.894	0.	1.295

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	2511	326	387	2248	0	498
normalized size	1	1.	7.77	1.01	1.2	6.96	0.	1.54
time (sec)	N/A	0.53	6.499	0.248	1.121	2.02	0.	1.246

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	252	186	204	973	0	282
normalized size	1	1.	1.57	1.16	1.27	6.04	0.	1.75
time (sec)	N/A	0.449	1.525	0.136	1.067	1.894	0.	1.235

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	290	1382	0	369
normalized size	1	1.	1.42	1.04	1.21	5.78	0.	1.54
time (sec)	N/A	0.546	6.061	0.167	1.081	2.156	0.	1.248

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	425	2453	0	582
normalized size	1	1.	8.69	1.12	1.24	7.13	0.	1.69
time (sec)	N/A	0.779	6.647	0.209	1.124	2.146	0.	1.323

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	47	80	41	47
normalized size	1	1.	1.	0.83	1.57	2.67	1.37	1.57
time (sec)	N/A	0.038	0.056	0.01	1.071	1.807	0.202	1.294

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	48	52	109	192	85	101
normalized size	1	1.	0.86	0.93	1.95	3.43	1.52	1.8
time (sec)	N/A	0.077	0.084	0.008	1.035	2.07	0.455	1.247

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	624	0	0
normalized size	1	1.	0.85	1.	0.	2.93	0.	0.
time (sec)	N/A	0.556	0.314	0.039	0.	2.183	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	248	370	0	1445	0	0
normalized size	1	1.	0.92	1.37	0.	5.33	0.	0.
time (sec)	N/A	0.809	0.515	0.039	0.	2.227	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [320] had the largest ratio of [1.5]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167
2	A	2	2	1.	8	0.25
3	A	2	1	1.	8	0.125
4	A	3	2	1.	8	0.25
5	A	2	1	1.	8	0.125
6	A	4	2	1.	8	0.25
7	A	4	3	1.	10	0.3
8	A	3	3	1.	10	0.3
9	A	3	3	1.	10	0.3
10	A	2	2	1.	10	0.2
11	A	2	2	1.	10	0.2
12	A	3	3	1.	10	0.3
13	A	3	3	1.	10	0.3
14	A	4	3	1.	10	0.3
15	A	4	3	1.	12	0.25
16	A	3	3	1.	12	0.25
17	A	3	3	1.	12	0.25
18	A	2	2	1.	12	0.167
19	A	2	2	1.	12	0.167
20	A	3	3	1.	12	0.25
21	A	3	3	1.	12	0.25
22	A	4	3	1.	12	0.25
23	A	3	2	1.	14	0.143
24	A	2	2	1.	14	0.143
25	A	2	2	1.	14	0.143
26	A	1	1	1.	14	0.071
27	A	1	1	1.	14	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	2	2	1.	14	0.143
29	A	2	2	1.	14	0.143
30	A	3	2	1.	14	0.143
31	A	1	1	1.	12	0.083
32	A	1	1	1.	12	0.083
33	A	1	1	1.	12	0.083
34	A	1	1	1.	12	0.083
35	A	1	1	1.	12	0.083
36	A	1	1	1.	12	0.083
37	A	1	1	1.	10	0.1
38	A	1	1	1.	12	0.083
39	A	1	1	1.	12	0.083
40	A	6	5	1.	13	0.385
41	A	2	2	1.	13	0.154
42	A	3	3	1.	13	0.231
43	A	2	2	1.	11	0.182
44	A	3	3	1.	11	0.273
45	A	5	5	1.	13	0.385
46	A	6	6	1.	13	0.462
47	A	6	5	1.	13	0.385
48	A	3	3	1.	13	0.231
49	A	6	6	1.	13	0.462
50	A	3	3	1.	13	0.231
51	A	2	2	1.	11	0.182
52	A	4	4	1.	11	0.364
53	A	6	6	1.	13	0.462
54	A	7	7	1.	13	0.538
55	A	7	6	1.	13	0.462
56	A	1	1	1.	14	0.071
57	A	2	2	1.	14	0.143
58	A	3	2	1.	14	0.143
59	A	4	2	1.	14	0.143
60	A	1	1	1.	14	0.071
61	A	2	2	1.	14	0.143
62	A	3	2	1.	14	0.143
63	A	4	2	1.	14	0.143
64	A	3	3	1.	16	0.188
65	A	3	3	1.	16	0.188
66	A	3	2	1.	17	0.118
67	A	2	2	1.	17	0.118
68	A	1	1	1.	17	0.059
69	A	2	2	1.	17	0.118
70	A	3	3	1.	17	0.176
71	A	4	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	7	7	1.	13	0.538
73	A	6	6	1.	13	0.462
74	A	6	6	1.	13	0.462
75	A	4	4	1.	11	0.364
76	A	5	5	1.	11	0.454
77	A	7	7	1.	13	0.538
78	A	7	7	1.	13	0.538
79	A	8	7	1.	13	0.538
80	A	7	7	1.	13	0.538
81	A	6	6	1.	13	0.462
82	A	5	5	1.	13	0.385
83	A	5	5	1.	11	0.454
84	A	6	6	1.	11	0.546
85	A	7	7	1.	13	0.538
86	A	8	7	1.	13	0.538
87	A	9	7	1.	13	0.538
88	A	4	3	1.	14	0.214
89	A	6	5	1.	14	0.357
90	A	7	6	1.	14	0.429
91	A	8	6	1.	14	0.429
92	A	1	1	1.	14	0.071
93	A	3	3	1.	14	0.214
94	A	4	4	1.	14	0.286
95	A	5	4	1.	14	0.286
96	A	4	3	1.	12	0.25
97	A	3	3	1.	12	0.25
98	A	2	2	1.	12	0.167
99	A	1	1	1.	12	0.083
100	A	2	1	1.	10	0.1
101	A	3	3	1.	12	0.25
102	A	5	5	1.	12	0.417
103	A	6	6	1.	12	0.5
104	A	7	6	1.	12	0.5
105	A	7	7	1.	10	0.7
106	A	6	6	1.	10	0.6
107	A	2	2	1.	10	0.2
108	A	2	2	1.	10	0.2
109	A	4	4	1.	10	0.4
110	A	7	7	1.	10	0.7
111	A	5	5	1.	13	0.385
112	A	4	3	1.	20	0.15
113	A	3	3	1.	20	0.15
114	A	2	2	1.	20	0.1
115	A	2	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	2	2	1.	15	0.133
117	A	3	3	1.	15	0.2
118	A	4	3	1.	15	0.2
119	A	2	2	1.	17	0.118
120	A	2	2	1.	17	0.118
121	A	3	3	1.	17	0.176
122	A	4	3	1.	17	0.176
123	A	3	3	1.	20	0.15
124	A	3	3	1.	20	0.15
125	A	4	4	1.	20	0.2
126	A	8	6	1.	17	0.353
127	A	7	6	1.	17	0.353
128	A	6	6	1.	17	0.353
129	A	4	4	1.	15	0.267
130	A	5	5	1.	15	0.333
131	A	6	5	1.	15	0.333
132	A	7	5	1.	15	0.333
133	A	4	4	1.	20	0.2
134	A	2	2	1.	20	0.1
135	A	2	2	1.	16	0.125
136	A	2	2	1.	13	0.154
137	A	5	5	1.	17	0.294
138	A	6	6	1.	17	0.353
139	A	7	6	1.	17	0.353
140	A	4	3	1.	10	0.3
141	A	3	3	1.	10	0.3
142	A	2	2	1.	10	0.2
143	A	2	2	1.	10	0.2
144	A	3	3	1.	10	0.3
145	A	4	3	1.	10	0.3
146	A	7	4	1.	10	0.4
147	A	5	4	1.	10	0.4
148	A	4	4	1.	10	0.4
149	A	4	4	1.	10	0.4
150	A	5	4	1.	10	0.4
151	A	7	4	1.	10	0.4
152	A	7	3	1.	10	0.3
153	A	5	3	1.	10	0.3
154	A	3	3	1.	10	0.3
155	A	3	3	1.	10	0.3
156	A	3	2	1.	10	0.2
157	A	3	2	1.	10	0.2
158	A	5	3	1.	13	0.231
159	A	3	2	1.	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	4	3	1.	13	0.231
161	A	3	2	1.	13	0.154
162	A	3	3	1.	13	0.231
163	A	2	1	1.	13	0.077
164	A	2	2	1.	13	0.154
165	A	2	2	1.	11	0.182
166	A	4	3	1.	11	0.273
167	A	3	3	1.	13	0.231
168	A	4	3	1.	13	0.231
169	A	3	2	1.	13	0.154
170	A	4	3	1.	13	0.231
171	A	4	4	1.	13	0.308
172	A	2	2	1.	13	0.154
173	A	3	3	1.	13	0.231
174	A	3	2	1.	13	0.154
175	A	2	2	1.	13	0.154
176	A	2	2	1.	11	0.182
177	A	4	3	1.	11	0.273
178	A	4	3	1.	13	0.231
179	A	4	3	1.	13	0.231
180	A	4	2	1.	13	0.154
181	A	3	2	1.	15	0.133
182	A	1	1	1.	15	0.067
183	A	2	2	1.	13	0.154
184	A	3	2	1.	15	0.133
185	A	1	1	1.	15	0.067
186	A	2	2	1.	13	0.154
187	A	3	2	1.	13	0.154
188	A	7	6	1.	13	0.462
189	A	3	2	1.	13	0.154
190	A	6	6	1.	13	0.462
191	A	3	2	1.	13	0.154
192	A	5	5	1.	13	0.385
193	A	2	2	1.	11	0.182
194	A	6	6	1.	11	0.546
195	A	5	5	1.	13	0.385
196	A	7	6	1.	13	0.462
197	A	6	6	1.	13	0.462
198	A	8	7	1.	13	0.538
199	A	7	6	1.	13	0.462
200	A	6	6	1.	13	0.462
201	A	3	2	1.	13	0.154
202	A	5	5	1.	13	0.385
203	A	2	2	1.	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
204	A	7	6	1.	11	0.546
205	A	6	6	1.	13	0.462
206	A	7	6	1.	13	0.462
207	A	7	6	1.	13	0.462
208	A	6	5	1.	13	0.385
209	A	6	5	1.	13	0.385
210	A	5	4	1.	13	0.308
211	A	5	5	1.	11	0.454
212	A	4	4	1.	11	0.364
213	A	4	4	1.	13	0.308
214	A	5	4	1.	13	0.308
215	A	5	5	1.	13	0.385
216	A	5	4	1.	13	0.308
217	A	6	5	1.	13	0.385
218	A	10	6	1.	13	0.462
219	A	4	3	1.	13	0.231
220	A	10	5	1.	13	0.385
221	A	4	3	1.	11	0.273
222	A	3	2	1.	11	0.182
223	A	7	5	1.	13	0.385
224	A	3	2	1.	13	0.154
225	A	9	6	1.	13	0.462
226	A	3	2	1.	13	0.154
227	A	11	5	1.	13	0.385
228	A	13	9	1.	13	0.692
229	A	7	6	1.	13	0.462
230	A	8	7	1.	13	0.538
231	A	6	5	1.	11	0.454
232	A	4	4	1.	11	0.364
233	A	7	7	1.	13	0.538
234	A	3	2	1.	13	0.154
235	A	7	7	1.	13	0.538
236	A	16	9	1.	13	0.692
237	A	7	6	1.	13	0.462
238	A	13	9	1.	13	0.692
239	A	6	5	1.	11	0.454
240	A	3	2	1.	11	0.182
241	A	8	7	1.	13	0.538
242	A	3	2	1.	13	0.154
243	A	8	7	1.	13	0.538
244	A	4	4	1.	13	0.308
245	A	3	3	1.	13	0.231
246	A	7	6	1.	15	0.4
247	A	5	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	5	4	1.	17	0.235
249	A	11	9	1.	15	0.6
250	A	9	8	1.	15	0.533
251	A	12	11	1.	15	0.733
252	A	6	6	1.	15	0.4
253	A	7	7	1.	31	0.226
254	A	8	8	1.	31	0.258
255	A	9	8	1.	31	0.258
256	A	10	8	1.	31	0.258
257	A	13	8	1.	14	0.571
258	A	11	7	1.	14	0.5
259	A	9	6	1.	12	0.5
260	A	13	5	1.	20	0.25
261	A	5	3	1.	36	0.083
262	A	4	3	1.	36	0.083
263	A	2	2	1.	34	0.059
264	A	0	0	0.	0	0.
265	A	0	0	0.	0	0.
266	A	1	1	1.	11	0.091
267	A	2	2	1.	13	0.154
268	A	2	2	1.	13	0.154
269	A	3	2	1.	13	0.154
270	A	1	1	1.	15	0.067
271	A	2	2	1.	17	0.118
272	A	2	2	0.97	17	0.118
273	A	3	2	0.98	17	0.118
274	A	2	1	1.	15	0.067
275	A	3	2	1.	17	0.118
276	A	3	1	1.	17	0.059
277	A	4	2	1.	17	0.118
278	A	3	1	1.	17	0.059
279	A	4	3	1.	19	0.158
280	A	4	3	1.	19	0.158
281	A	3	2	1.	19	0.105
282	A	3	2	1.	19	0.105
283	A	4	3	1.	19	0.158
284	A	4	3	1.	19	0.158
285	A	8	8	1.	18	0.444
286	A	6	6	1.	18	0.333
287	A	3	3	1.	18	0.167
288	A	4	4	1.	18	0.222
289	A	4	4	1.	10	0.4
290	A	5	5	1.	12	0.417
291	A	6	4	1.	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
292	A	5	5	1.	11	0.454
293	A	6	6	1.	13	0.462
294	A	9	5	1.	13	0.385
295	A	5	5	1.	14	0.357
296	A	6	6	1.	16	0.375
297	A	9	5	1.	16	0.312
298	A	6	6	1.	17	0.353
299	A	7	7	1.	19	0.368
300	A	10	6	1.	19	0.316
301	A	4	3	1.	16	0.188
302	A	5	4	1.	16	0.25
303	A	4	3	1.	16	0.188
304	A	4	3	1.	14	0.214
305	A	3	3	1.	14	0.214
306	A	4	4	1.	16	0.25
307	A	3	3	1.	16	0.188
308	A	6	5	1.	16	0.312
309	A	5	4	1.	16	0.25
310	A	4	3	1.	10	0.3
311	A	4	3	1.	8	0.375
312	A	5	5	1.	8	0.625
313	A	6	6	1.	10	0.6
314	A	4	3	1.	10	0.3
315	A	4	3	1.	8	0.375
316	A	9	9	1.	8	1.125
317	A	13	9	1.	10	0.9
318	A	4	3	1.	10	0.3
319	A	4	3	1.	8	0.375
320	A	15	12	1.	8	1.5
321	A	16	13	1.	10	1.3
322	A	2	2	1.	18	0.111
323	A	2	2	1.	18	0.111
324	A	1	1	1.	16	0.062
325	A	1	1	1.	16	0.062
326	A	1	1	1.	18	0.056
327	A	2	2	1.	18	0.111
328	A	2	2	1.	18	0.111
329	A	6	5	1.	25	0.2
330	A	6	5	1.	25	0.2
331	A	5	4	1.	25	0.16
332	A	4	4	1.	25	0.16
333	A	4	4	1.	25	0.16
334	A	6	5	1.	25	0.2
335	A	6	5	1.	25	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	1	1	1.	10	0.1
337	A	6	4	1.	12	0.333
338	A	6	4	1.	15	0.267
339	A	6	3	1.	12	0.25
340	A	4	2	1.	14	0.143
341	A	6	3	1.	17	0.176
342	A	8	5	1.	16	0.312
343	A	9	6	1.	18	0.333
344	A	14	5	1.	18	0.278
345	A	8	5	1.	19	0.263
346	A	9	6	1.	21	0.286
347	A	14	5	1.	21	0.238
348	A	8	4	1.	16	0.25
349	A	9	4	1.	18	0.222
350	A	14	4	1.	18	0.222
351	A	6	4	1.	18	0.222
352	A	7	4	1.	20	0.2
353	A	10	4	1.	20	0.2
354	A	8	5	1.	21	0.238
355	A	9	5	1.	23	0.217
356	A	14	5	1.	23	0.217
357	A	8	4	1.	19	0.21
358	A	10	4	1.	21	0.19
359	A	14	4	1.	21	0.19
360	A	8	5	1.	21	0.238
361	A	10	5	1.	23	0.217
362	A	14	5	1.	23	0.217
363	A	8	5	1.	24	0.208
364	A	10	5	1.	26	0.192
365	A	14	5	1.	26	0.192
366	A	6	5	1.	6	0.833
367	A	9	6	1.	6	1.
368	A	8	4	1.	16	0.25
369	A	8	4	1.	19	0.21

Chapter 3

Listing of integrals

3.1 $\int \sinh(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\cosh(a+bx)}{b}$$

```
[Out] Cosh[a + b*x]/b
```

Rubi [A] time = 0.0056658, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2638}

$$\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh(a+bx)\,dx = \frac{\cosh(a+bx)}{b}$$

Mathematica [B] time = 0.0109847, size = 21, normalized size = 2.1

$$\frac{\sinh(a)\sinh(bx)}{b} + \frac{\cosh(a)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x],x]
```

```
[Out] (Cosh[a]*Cosh[b*x])/b + (Sinh[a]*Sinh[b*x])/b
```

Maple [A] time = 0.002, size = 11, normalized size = 1.1

$$\frac{\cosh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a),x)
```

```
[Out] cosh(b*x+a)/b
```

Maxima [A] time = 1.09713, size = 14, normalized size = 1.4

$$\frac{\cosh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] cosh(b*x + a)/b
```

Fricas [A] time = 1.9584, size = 23, normalized size = 2.3

$$\frac{\cosh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] cosh(b*x + a)/b
```

Sympy [A] time = 0.166357, size = 12, normalized size = 1.2

$$\begin{cases} \frac{\cosh(a+bx)}{b} & \text{for}\ b \neq 0 \\ x\sinh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x)
```

```
[Out] Piecewise((cosh(a + b*x)/b, Ne(b, 0)), (x*sinh(a), True))
```

Giac [B] time = 1.43722, size = 35, normalized size = 3.5

$$\frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b
```

3.2 $\int \sinh^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}$$

```
[Out] -x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rubi [A] time = 0.0091, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2,x]
```

```
[Out] -x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^2(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{\int 1\,dx}{2}\\ &= -\frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.024184, size = 23, normalized size = 0.92

$$\frac{\sinh(2(a+bx))-2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2,x]
```

```
[Out] (-2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.006, size = 27, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)
```

Maxima [A] time = 1.07968, size = 43, normalized size = 1.72

$$-\frac{1}{2}x+\frac{e^{(2bx+2a)}}{8b}-\frac{e^{(-2bx-2a)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Fricas [A] time = 2.05974, size = 59, normalized size = 2.36

$$-\frac{bx-\cosh(bx+a)\sinh(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - cosh(b*x + a)*sinh(b*x + a))/b
```

Sympy [A] time = 0.26181, size = 46, normalized size = 1.84

$$\begin{cases}\frac{x\sinh^2(a+bx)}{2}-\frac{x\cosh^2(a+bx)}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b} & \text{for } b\neq 0\\ x\sinh^2(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh
(a + b*x)/(2*b), Ne(b, 0)), (x*sinh(a)**2, True))
```

Giac [B] time = 1.32558, size = 65, normalized size = 2.6

$$-\frac{4bx-\left(2e^{(2bx+2a)}-1\right)e^{(-2bx-2a)}+4a-e^{(2bx+2a)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b*x - (2*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 4*a - e^(2*b*x + 2
*a))/b
```

3.3 $\int \sinh^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cosh^3(a+bx)}{3b}-\frac{\cosh(a+bx)}{b}$$

```
[Out] -(Cosh[a + b*x]/b) + Cosh[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0139676, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cosh^3(a+bx)}{3b}-\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3,x]
```

```
[Out] -(Cosh[a + b*x]/b) + Cosh[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0117888, size = 29, normalized size = 1.07

$$\frac{\cosh(3(a+bx))}{12b}-\frac{3\cosh(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b)
```

Maple [A] time = 0.009, size = 23, normalized size = 0.9

$$\frac{\cosh(bx+a)}{b}\left(-\frac{2}{3}+\frac{(\sinh(bx+a))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)
```

Maxima [B] time = 1.09844, size = 73, normalized size = 2.7

$$\frac{e^{(3\,bx+3\,a)}}{24\,b}-\frac{3\,e^{(bx+a)}}{8\,b}-\frac{3\,e^{(-bx-a)}}{8\,b}+\frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b - 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b + 1/24*e^(-
3*b*x - 3*a)/b
```

Fricas [A] time = 1.95594, size = 105, normalized size = 3.89

$$\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-9\,\cosh\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 9*cosh(b*x + a))/
b
```

Sympy [A] time = 0.56425, size = 36, normalized size = 1.33

$$\begin{cases}\frac{\sinh^{2}(a+bx)\cosh(a+bx)}{b}-\frac{2\cosh^{3}(a+bx)}{3b} & \text{for } b\neq 0\\ x\sinh^{3}(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**2*cosh(a + b*x)/b - 2*cosh(a + b*x)**3/(3*b), Ne(
b, 0)), (x*sinh(a)**3, True))
```

Giac [A] time = 1.46189, size = 65, normalized size = 2.41

$$-\frac{\left(9\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}-e^{(3\,bx+3\,a)}+9\,e^{(bx+a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) + 9*e^(b*
x + a))/b
```

3.4 $\int \sinh^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

```
[Out] (3*x)/8 - (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]*Sinh[a + b
*x]^3)/(4*b)
```

Rubi [A] time = 0.0207817, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]*Sinh[a + b
*x]^3)/(4*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^4(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}-\frac{3}{4}\int \sinh^2(a+bx)\,dx\\ &= -\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0401589, size = 33, normalized size = 0.72

$$\frac{12(a+bx)-8\sinh(2(a+bx))+\sinh(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) - 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

Maple [A] time = 0.039, size = 39, normalized size = 0.9

$$\frac{1}{b}\left(\left(\frac{(\sinh(bx+a))^3}{4}-\frac{3\sinh(bx+a)}{8}\right)\cosh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^4,x)
```

```
[Out] 1/b*((1/4*sinh(b*x+a)^3-3/8*sinh(b*x+a))*cosh(b*x+a)+3/8*b*x+3/8*a)
```

Maxima [A] time = 1.11297, size = 81, normalized size = 1.76

$$\frac{3}{8}x+\frac{e^{(4bx+4a)}}{64b}-\frac{e^{(2bx+2a)}}{8b}+\frac{e^{(-2bx-2a)}}{8b}-\frac{e^{(-4bx-4a)}}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Fricas [A] time = 2.22838, size = 134, normalized size = 2.91

$$\frac{\cosh(bx+a)\sinh(bx+a)^3+3bx+\left(\cosh(bx+a)^3-4\cosh(bx+a)\right)\sinh(bx+a)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 - 4*cosh(b*x
+ a))*sinh(b*x + a))/b
```

Sympy [A] time = 1.21722, size = 95, normalized size = 2.07

$$\begin{cases}\frac{3x\sinh^4(a+bx)}{8}-\frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{4}+\frac{3x\cosh^4(a+bx)}{8}+\frac{5\sinh^3(a+bx)\cosh(a+bx)}{8b}-\frac{3\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b\neq 0\\ x\sinh^4(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/4
 + 3*x*cosh(a + b*x)**4/8 + 5*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) - 3*sinh
```

```
(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**4, True))
```

Giac [A] time = 1.41966, size = 92, normalized size = 2.

$$\frac{24\,bx - \left(18\,e^{(4\,bx+4\,a)} - 8\,e^{(2\,bx+2\,a)} + 1\right)e^{(-4\,bx-4\,a)} + 24\,a + e^{(4\,bx+4\,a)} - 8\,e^{(2\,bx+2\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/64*(24*b*x - (18*e^(4*b*x + 4*a) - 8*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4*a
) + 24*a + e^(4*b*x + 4*a) - 8*e^(2*b*x + 2*a))/b
```

3.5 $\int \sinh^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\cosh^5(a+bx)}{5b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}$$

```
[Out] Cosh[a + b*x]/b - (2*Cosh[a + b*x]^3)/(3*b) + Cosh[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0169643, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cosh^5(a+bx)}{5b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^5,x]
```

```
[Out] Cosh[a + b*x]/b - (2*Cosh[a + b*x]^3)/(3*b) + Cosh[a + b*x]^5/(5*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cosh(a+bx)\right)}{b}\\ &= \frac{\cosh(a+bx)}{b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0140689, size = 44, normalized size = 1.07

$$\frac{5\cosh(a+bx)}{8b}-\frac{5\cosh(3(a+bx))}{48b}+\frac{\cosh(5(a+bx))}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^5,x]
```

```
[Out] (5*Cosh[a + b*x])/(8*b) - (5*Cosh[3*(a + b*x)])/(48*b) + Cosh[5*(a + b*x)]/
(80*b)
```

Maple [A] time = 0.039, size = 33, normalized size = 0.8

$$\frac{\cosh(bx+a)}{b}\left(\frac{8}{15}+\frac{(\sinh(bx+a))^4}{5}-\frac{4(\sinh(bx+a))^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^5,x)
```

```
[Out] 1/b*(8/15+1/5*sinh(b*x+a)^4-4/15*sinh(b*x+a)^2)*cosh(b*x+a)
```

Maxima [B] time = 1.10583, size = 111, normalized size = 2.71

$$\frac{e^{(5\,bx+5\,a)}}{160\,b}-\frac{5\,e^{(3\,bx+3\,a)}}{96\,b}+\frac{5\,e^{(bx+a)}}{16\,b}+\frac{5\,e^{(-bx-a)}}{16\,b}-\frac{5\,e^{(-3\,bx-3\,a)}}{96\,b}+\frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b - 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b + 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

Fricas [B] time = 2.22009, size = 221, normalized size = 5.39

$$\frac{3\,\cosh\left(bx+a\right)^{5}+15\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}-25\,\cosh\left(bx+a\right)^{3}+15\left(2\,\cosh\left(bx+a\right)^{3}-5\,\cosh\left(bx+a\right)\right)\sinh\left(b\right.}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 - 25*cosh(b*x +
 a)^3 + 15*(2*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^2 + 150*cosh
(b*x + a))/b
```

Sympy [A] time = 2.34914, size = 58, normalized size = 1.41

$$\begin{cases}\frac{\sinh^{4}\left(a+bx\right)\cosh\left(a+bx\right)}{b}-\frac{4\sinh^{2}\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{3b}+\frac{8\cosh^{5}\left(a+bx\right)}{15b} & \text{for } b\neq 0\\ x\sinh^{5}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**5,x)
```

```
[Out] Piecewise((sinh(a + b*x)**4*cosh(a + b*x)/b - 4*sinh(a + b*x)**2*cosh(a + b
*x)**3/(3*b) + 8*cosh(a + b*x)**5/(15*b), Ne(b, 0)), (x*sinh(a)**5, True))
```

Giac [A] time = 1.36248, size = 95, normalized size = 2.32

$$\frac{\left(150\,e^{(4\,bx+4\,a)}-25\,e^{(2\,bx+2\,a)}+3\right)e^{(-5\,bx-5\,a)}+3\,e^{(5\,bx+5\,a)}-25\,e^{(3\,bx+3\,a)}+150\,e^{(bx+a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/480*((150*e^(4*b*x + 4*a) - 25*e^(2*b*x + 2*a) + 3)*e^(-5*b*x - 5*a) + 3*
e^(5*b*x + 5*a) - 25*e^(3*b*x + 3*a) + 150*e^(b*x + a))/b
```

3.6 $\int \sinh^6(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5\sinh^3(a+bx)\cosh(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{5x}{16}$$

```
[Out] (-5*x)/16 + (5*Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (5*Cosh[a + b*x]*Sinh[
a + b*x]^3)/(24*b) + (Cosh[a + b*x]*Sinh[a + b*x]^5)/(6*b)
```

Rubi [A] time = 0.0325209, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5\sinh^3(a+bx)\cosh(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^6,x]
```

```
[Out] (-5*x)/16 + (5*Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (5*Cosh[a + b*x]*Sinh[
a + b*x]^3)/(24*b) + (Cosh[a + b*x]*Sinh[a + b*x]^5)/(6*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^6(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}-\frac{5}{6}\int \sinh^4(a+bx)\,dx\\ &= -\frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}+\frac{5}{8}\int \sinh^2(a+bx)\,dx\\ &= \frac{5\cosh(a+bx)\sinh(a+bx)}{16b}-\frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}-\frac{5\int 1}{16}\\ &= -\frac{5x}{16}+\frac{5\cosh(a+bx)\sinh(a+bx)}{16b}-\frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.0384124, size = 43, normalized size = 0.64

$$\frac{45\sinh(2(a+bx))-9\sinh(4(a+bx))+\sinh(6(a+bx))-60a-60bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^6,x]
```

```
[Out] (-60*a - 60*b*x + 45*Sinh[2*(a + b*x)] - 9*Sinh[4*(a + b*x)] + Sinh[6*(a +
b*x)])/(192*b)
```

Maple [A] time = 0.037, size = 49, normalized size = 0.7

$$\frac{1}{b}\left(\left(\frac{(\sinh(bx+a))^5}{6}-\frac{5(\sinh(bx+a))^3}{24}+\frac{5\sinh(bx+a)}{16}\right)\cosh(bx+a)-\frac{5bx}{16}-\frac{5a}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^6,x)
```

```
[Out] 1/b*((1/6*sinh(b*x+a)^5-5/24*sinh(b*x+a)^3+5/16*sinh(b*x+a))*cosh(b*x+a)-5/
16*b*x-5/16*a)
```

Maxima [A] time = 1.04283, size = 116, normalized size = 1.73

$$-\frac{\left(9e^{(-2bx-2a)}-45e^{(-4bx-4a)}-1\right)e^{(6bx+6a)}}{384b}-\frac{5(bx+a)}{16b}-\frac{45e^{(-2bx-2a)}-9e^{(-4bx-4a)}+e^{(-6bx-6a)}}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-2*b*x - 2*a) - 45*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 5
/16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) - 9*e^(-4*b*x - 4*a) + e^(-6*b
*x - 6*a))/b
```

Fricas [A] time = 1.99368, size = 248, normalized size = 3.7

$$\frac{3\cosh(bx+a)\sinh(bx+a)^5+2\left(5\cosh(bx+a)^3-9\cosh(bx+a)\right)\sinh(bx+a)^3-30bx+3\left(\cosh(bx+a)^5-6\right.}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 9*cosh(b*x +
 a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh(b*x + a)^5 - 6*cosh(b*x + a)^3 + 15
*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [A] time = 4.71063, size = 139, normalized size = 2.07

$$\begin{cases}\frac{5x\sinh^6(a+bx)}{16}-\frac{15x\sinh^4(a+bx)\cosh^2(a+bx)}{16}+\frac{15x\sinh^2(a+bx)\cosh^4(a+bx)}{16}-\frac{5x\cosh^6(a+bx)}{16}+\frac{11\sinh^5(a+bx)\cosh(a+bx)}{16b}-\frac{5\sinh^3(a+\\ x\sinh^6(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((5*x*sinh(a + b*x)**6/16 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**2
/16 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - 5*x*cosh(a + b*x)**6/16 +
 11*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a + b*x
)**3/(6*b) + 5*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)
**6, True))
```

Giac [A] time = 1.41203, size = 124, normalized size = 1.85

$$-\frac{120\,bx-\left(110\,e^{(6\,bx+6\,a)}-45\,e^{(4\,bx+4\,a)}+9\,e^{(2\,bx+2\,a)}-1\right)e^{(-6\,bx-6\,a)}+120\,a-e^{(6\,bx+6\,a)}+9\,e^{(4\,bx+4\,a)}-45\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/384*(120*b*x - (110*e^(6*b*x + 6*a) - 45*e^(4*b*x + 4*a) + 9*e^(2*b*x +
2*a) - 1)*e^(-6*b*x - 6*a) + 120*a - e^(6*b*x + 6*a) + 9*e^(4*b*x + 4*a) -
45*e^(2*b*x + 2*a))/b
```

3.7 $\int \sinh^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=103

$$-\frac{10i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{21b\sqrt{\sinh(a+bx)}}+\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{10\sqrt{\sinh(a+bx)}\cosh(a+bx)}{21b}$$

```
[Out] (((-10*I)/21)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(
b*Sqrt[Sinh[a + b*x]]) - (10*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(21*b) + (2
*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(7*b)
```

Rubi [A] time = 0.048612, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2635, 2642, 2641}

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{10\sqrt{\sinh(a+bx)}\cosh(a+bx)}{21b}-\frac{10i\sqrt{i\sinh(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{21b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(
b*Sqrt[Sinh[a + b*x]]) - (10*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(21*b) + (2
*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} - \frac{5}{7}\int \sinh^{\frac{3}{2}}(a+bx)\,dx$$
$$= -\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} + \frac{5}{21}\int \frac{1}{\sqrt{\sinh(a+bx)}}dx$$
$$= -\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} + \frac{\left(5\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+}}}{21\sqrt{\sinh(a+bx)}}$$
$$= -\frac{10iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{21b\sqrt{\sinh(a+bx)}} - \frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sin}{7b}$$

Mathematica [A] time = 0.140299, size = 75, normalized size = 0.73

$$\frac{40i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)-26\sinh(2(a+bx))+3\sinh(4(a+bx))}{84b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(7/2),x]
```

```
[Out] ((40*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] -
 26*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)])/(84*b*Sqrt[Sinh[a + b*x]])
```

Maple [A] time = 0.074, size = 116, normalized size = 1.1

$$\frac{1}{b\cosh(bx+a)}\left(\frac{5i}{21}\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)+\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(7/2),x)
```

```
[Out] (5/21*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/7*sinh(b*x+a)*c
osh(b*x+a)^4-16/21*sinh(b*x+a)*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)
/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sinh\left(bx+a\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(bx+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(7/2), x)
```

3.8 $\int \sinh^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}+\frac{6i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqr
t[I*Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(5*b)
```

Rubi [A] time = 0.0320144, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2635, 2640, 2639}

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}+\frac{6i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(5/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqr
t[I*Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^{\frac{5}{2}}(a+bx)\,dx &= \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b}-\frac{3}{5}\int \sqrt{\sinh(a+bx)}\,dx\\ &= \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b}-\frac{\left(3\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{i\sinh(a+bx)}}\\ &= \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}}+\frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.0733094, size = 68, normalized size = 0.85

$$\frac{\sinh(a+bx)\sinh(2(a+bx))-6\sqrt{i\sinh(a+bx)}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)}{5b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + Sin
h[a + b*x]*Sinh[2*(a + b*x)])/(5*b*Sqrt[Sinh[a + b*x]])
```

Maple [A] time = 0.039, size = 164, normalized size = 2.1

$$\frac{1}{b\cosh(bx+a)}\left(-\frac{6\sqrt{2}}{5}\sqrt{1-i\sinh(bx+a)}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(5/2),x)
```

```
[Out] (-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a
))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x+a
))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1
-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)/cos
h(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sinh(bx+a)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(5/2), x)
```

3.9 $\int \sinh^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}$$

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*S
qrt[Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(3*b)
```

Rubi [A] time = 0.0318454, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2635, 2642, 2641}

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(3/2),x]
```

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*S
qrt[Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^{\frac{3}{2}}(a+bx)\,dx&=\frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b}-\frac{1}{3}\int\frac{1}{\sqrt{\sinh(a+bx)}}\,dx\\&=\frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b}-\frac{\sqrt{i\sinh(a+bx)}\int\frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{3\sqrt{\sinh(a+bx)}}\\&=\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}}+\frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b}\end{aligned}$$

Mathematica [C] time = 0.0872205, size = 83, normalized size = 1.04

$$\frac{\sinh(2(a+bx))-2\sqrt{-\sinh(2a+2bx)-\cosh(2a+2bx)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+bx))+\sinh(2(a+bx))\right)}{3b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(3/2),x]
```

```
[Out] (Sinh[2*(a + b*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)] +
 Sinh[2*(a + b*x)]]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2*b*x]])/(3*b*S
qrt[Sinh[a + b*x]])
```

Maple [A] time = 0.042, size = 100, normalized size = 1.3

$$\frac{1}{b\cosh(bx+a)}\left(-\frac{i}{3}\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\mathrm{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)+\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(3/2),x)
```

```
[Out] (-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*sinh(b*x+a)*c
osh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sinh(bx+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh^{\frac{3}{2}}(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(bx + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(3/2), x)
```

3.10 $\int \sqrt{\sinh(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I
*Sinh[a + b*x]])
```

Rubi [A] time = 0.0187614, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2640, 2639}

$$-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I
*Sinh[a + b*x]])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sinh(a+bx)}\,dx &= \frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{i\sinh(a+bx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.102381, size = 50, normalized size = 0.93

$$\frac{2\sqrt{i\sinh(a+bx)}E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|2\right)}{b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh
[a + b*x]])
```

Maple [A] time = 0.036, size = 108, normalized size = 2.

$$\frac{\sqrt{2}}{b\cosh(bx+a)}\sqrt{-i(\sinh(bx+a)+i)}\sqrt{-i(-\sinh(bx+a)+i)}\sqrt{i\sinh(bx+a)}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(1/2),x)
```

```
[Out] (-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x+
a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-I*
sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sinh(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\sinh(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sinh(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sinh(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sinh(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(b*x + a)), x)
```

3.11 $\int \frac{1}{\sqrt{\sinh(a+bx)}} \, dx$

Optimal. Leaf size=54

$$-\frac{2i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b\sqrt{\sinh(a+bx)}}$$

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt
[Sinh[a + b*x]])
```

Rubi [A] time = 0.019737, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2642, 2641}

$$-\frac{2i\sqrt{i\sinh(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt
[Sinh[a + b*x]])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sinh(a+bx)}} \, dx &= \frac{\sqrt{i\sinh(a+bx)} \int \frac{1}{\sqrt{i\sinh(a+bx)}} \, dx}{\sqrt{\sinh(a+bx)}} \\ &= -\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.114061, size = 48, normalized size = 0.89

$$-\frac{2\sqrt{\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2ia-2ibx+\pi),2\right)}{b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqr
t[I*Sinh[a + b*x]])
```

Maple [A] time = 0.036, size = 87, normalized size = 1.6

$$\frac{i\sqrt{2}}{b\cosh(bx+a)}\sqrt{-i(\sinh(bx+a)+i)}\sqrt{-i(-\sinh(bx+a)+i)}\sqrt{i\sinh(bx+a)}\text{EllipticF}\left(\sqrt{-i(\sinh(bx+a)+i)},\frac{\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(1/2),x)
```

```
[Out] I*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((-I*(sinh(b*x+a)+I))^(1/2),1/2*2^(1/2))/cosh(b*x+a)/s
inh(b*x+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sinh(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{\sinh(bx+a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(sinh(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sinh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sinh(b*x + a)), x)
```

3.12 $\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=76

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

```
[Out] (-2*Cosh[a + b*x])/(b*Sqrt[Sinh[a + b*x]]) - ((2*I)*EllipticE[(I*a - Pi/2 +
 I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rubi [A] time = 0.028671, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2636, 2640, 2639}

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(-3/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(b*Sqrt[Sinh[a + b*x]]) - ((2*I)*EllipticE[(I*a - Pi/2 +
 I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\int \sqrt{\sinh(a+bx)}\,dx\\ &= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{i\sinh(a+bx)}}\\ &= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0516042, size = 57, normalized size = 0.75

$$-\frac{2\left(\cosh(a+bx)-\sqrt{i\sinh(a+bx)}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(-3/2),x]
```

```
[Out] (-2*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sin
h[a + b*x]]))/(b*Sqrt[Sinh[a + b*x]])
```

Maple [A] time = 0.039, size = 154, normalized size = 2.

$$\frac{1}{b\cosh(bx+a)}\left(2\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},1/2\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(3/2),x)
```

```
[Out] (2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^
(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1/2
)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh
(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/
b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sinh(bx+a)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(-3/2), x)
```

3.13 $\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2i\sqrt{i\sinh(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}$$

```
[Out] (-2*Cosh[a + b*x])/(3*b*Sinh[a + b*x]^(3/2)) + (((2*I)/3)*EllipticF[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Rubi [A] time = 0.0317031, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2636, 2642, 2641}

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2i\sqrt{i\sinh(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(-5/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(3*b*Sinh[a + b*x]^(3/2)) + (((2*I)/3)*EllipticF[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$$
$$= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{3\sqrt{\sinh(a+bx)}}$$
$$= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} + \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}}$$

Mathematica [C] time = 0.0784565, size = 86, normalized size = 1.08

$$-\frac{2\left(\sinh(a+bx)\sqrt{-\sinh(2a+2bx)-\cosh(2a+2bx)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+bx))+\sinh(2(a+bx))\right)+\cosh(a+bx\right.}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(-5/2),x]
```

```
[Out] (-2*(Cosh[a + b*x] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)] + S
inh[2*(a + b*x)]]*Sinh[a + b*x]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2*b
*x]]))/(3*b*Sinh[a + b*x]^(3/2))
```

Maple [A] time = 0.044, size = 101, normalized size = 1.3

$$-\frac{1}{3\,b\cosh(bx+a)}\left(i\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\mathrm{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(5/2),x)
```

```
[Out] -1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))
^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))
*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sinh\left(bx+a\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)^(-5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(-5/2), x)
```

3.14 $\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=103

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}}+\frac{6i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

```
[Out] (-2*Cosh[a + b*x])/(5*b*Sinh[a + b*x]^(5/2)) + (6*Cosh[a + b*x])/(5*b*Sqrt[
Sinh[a + b*x]]) + (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh
[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rubi [A] time = 0.0442112, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2636, 2640, 2639}

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}}+\frac{6i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(-7/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(5*b*Sinh[a + b*x]^(5/2)) + (6*Cosh[a + b*x])/(5*b*Sqrt[
Sinh[a + b*x]]) + (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh
[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{3}{5}\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx \\ &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} - \frac{3}{5}\int \sqrt{\sinh(a+bx)}\,dx \\ &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} - \frac{\left(3\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{i\sinh(a+bx)}} \\ &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.16145, size = 73, normalized size = 0.71

$$\frac{3\sinh(2(a+bx)) - 2\coth(a+bx) + 6i(i\sinh(a+bx))^{3/2}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)}{5b\sinh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(-7/2),x]
```

```
[Out] (-2*Coth[a + b*x] + (6*I)*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Si
nh[a + b*x])^(3/2) + 3*Sinh[2*(a + b*x)])/(5*b*Sinh[a + b*x]^(3/2))
```

Maple [A] time = 0.046, size = 192, normalized size = 1.9

$$-\frac{1}{5\,b\cosh(bx+a)}\left(6\sqrt{-i(\sinh(bx+a)+i)}\sqrt{2}\sqrt{-i(-\sinh(bx+a)+i)}\sqrt{i\sinh(bx+a)}(\sinh(bx+a))^2\,\mathrm{EllipticE}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(7/2),x)
```

```
[Out] -1/5/sinh(b*x+a)^(5/2)*(6*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x
+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticE((-I*(sinh(b*x+a
)+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*
x+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticF((-I*(sinh(b*x+
a)+I))^(1/2),1/2*2^(1/2))-6*sinh(b*x+a)^4-4*sinh(b*x+a)^2+2)/cosh(b*x+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sinh\left(bx+a\right)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)^(-7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(-7/2), x)
```

3.15 $\int (b\sinh(c+dx))^{7/2}\,dx$

Optimal. Leaf size=116

$$-\frac{10ib^4\sqrt{i\sinh(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{21d\sqrt{b\sinh(c+dx)}}-\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d}+\frac{2b\cosh(c+dx)(b\sinh(c}{7d}$$

```
[Out] (((-10*I)/21)*b^4*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]
])/(d*Sqrt[b*Sinh[c + d*x]]) - (10*b^3*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])
/(21*d) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(5/2))/(7*d)
```

Rubi [A] time = 0.0578067, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2642, 2641}

$$-\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d}-\frac{10ib^4\sqrt{i\sinh(c+dx)}F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{21d\sqrt{b\sinh(c+dx)}}+\frac{2b\cosh(c+dx)(b\sinh(c+dx))^5}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*b^4*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]
])/(d*Sqrt[b*Sinh[c + d*x]]) - (10*b^3*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])
/(21*d) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(5/2))/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

Rubi steps

$$\int (b\sinh(c+dx))^{7/2}\,dx = \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} - \frac{1}{7}\left(5b^2\right)\int (b\sinh(c+dx))^{3/2}\,dx$$
$$= -\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} + \frac{1}{21}\left(5b^4\right)\int \frac{}{\sqrt{b\sin}}$$
$$= -\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} + \frac{\left(5b^4\sqrt{i\sinh(c+d}\right.}{21\sqrt{b\sin}}$$
$$= -\frac{10ib^4F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{21d\sqrt{b\sinh(c+dx)}} - \frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(}{}$$

Mathematica [A] time = 0.270165, size = 76, normalized size = 0.66

$$\frac{b^3\sqrt{b\sinh(c+dx)}\left(-\frac{20\text{EllipticF}\left(\frac{1}{4}(-2ic-2idx+\pi),2\right)}{\sqrt{i\sinh(c+dx)}} - 23\cosh(c+dx) + 3\cosh(3(c+dx))\right)}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (b^3*(-23*Cosh[c + d*x] + 3*Cosh[3*(c + d*x)] - (20*EllipticF[((-2*I)*c + P
i - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]])*Sqrt[b*Sinh[c + d*x]])/(42*d)
```

Maple [A] time = 0.048, size = 122, normalized size = 1.1

$$\frac{b^4}{21\,d\cosh(dx+c)}\left(5\,i\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\text{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(7/2),x)
```

```
[Out] 1/21*b^4*(5*I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*si
nh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+6*sinh(d*x+
c)*cosh(d*x+c)^4-16*cosh(d*x+c)^2*sinh(d*x+c))/cosh(d*x+c)/(b*sinh(d*x+c))^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh (dx + c)}b^3 \sinh (dx + c)^3 , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))*b^3*sinh(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh (dx + c))^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(7/2), x)
```

3.16 $\int (b\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=88

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}+\frac{6ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}}$$

```
[Out] (((6*I)/5)*b^2*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/
(d*Sqrt[I*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(3/2))/(5*
d)
```

Rubi [A] time = 0.036614, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2640, 2639}

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}+\frac{6ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((6*I)/5)*b^2*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/
(d*Sqrt[I*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(3/2))/(5*
d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\sinh(c+dx))^{5/2}\,dx &= \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}-\frac{1}{5}\left(3b^2\right)\int \sqrt{b\sinh(c+dx)}\,dx\\ &= \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}-\frac{\left(3b^2\sqrt{b\sinh(c+dx)}\right)\int \sqrt{i\sinh(c+dx)}\,dx}{5\sqrt{i\sinh(c+dx)}}\\ &= \frac{6ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}}+\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}\end{aligned}$$

Mathematica [A] time = 0.119512, size = 68, normalized size = 0.77

$$\frac{b^2\sqrt{b\sinh(c+dx)}\left(\sinh(2(c+dx))-\frac{6iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)}{\sqrt{i\sinh(c+dx)}}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Sinh[c + d*x]]*(((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4
, 2])/Sqrt[I*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(5*d)
```

Maple [A] time = 0.047, size = 170, normalized size = 1.9

$$-\frac{b^3}{5\,d\cosh(dx+c)}\left(6\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\text{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},1/2\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(5/2),x)
```

```
[Out] -1/5*b^3*(6*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh
(d*x+c))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-3*(1-I*sinh(d
*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*Elliptic
F((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^4+2*cosh(d*x+c)^2)/cos
h(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sinh(dx+c)}b^2\sinh(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))*b^2*sinh(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh (dx + c))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(5/2), x)
```

3.17 $\int (b\sinh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=88

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{b\sinh(c+dx)}}$$

```
[Out] (((2*I)/3)*b^2*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/
(d*Sqrt[b*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])/(3*d)
```

Rubi [A] time = 0.0368896, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 2642, 2641}

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*b^2*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/
(d*Sqrt[b*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b\sinh(c+dx))^{3/2}\,dx &= \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sinh(c+dx)}}\,dx\\
&= \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{\left(b^2\sqrt{i\sinh(c+dx)}\right)\int\frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3\sqrt{b\sinh(c+dx)}}\\
&= \frac{2ib^2F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3d\sqrt{b\sinh(c+dx)}}+\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}
\end{aligned}$$

Mathematica [C] time = 0.124739, size = 88, normalized size = 1.

$$\frac{b^2\left(\sinh(2(c+dx))-2\sqrt{-\sinh(2c+2dx)-\cosh(2c+2dx)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)\right)}{3d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (b^2*(Sinh[2*(c + d*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - Cosh[2*c + 2*d*x] - Sinh[2*c + 2*d*x]]))/
(3*d*Sqrt[b*Sinh[c + d*x]])
```

Maple [A] time = 0.04, size = 106, normalized size = 1.2

$$-\frac{b^2}{3\,d\cosh(dx+c)}\left(i\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\text{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(3/2),x)
```

```
[Out] -1/3*b^2*(I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh
(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)
^2*sinh(d*x+c))/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(b\sinh(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sinh(dx+c)}b\sinh(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))*b*sinh(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(3/2), x)
```

3.18 $\int \sqrt{b\sinh(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(d*Sqrt
[I*Sinh[c + d*x]])
```

Rubi [A] time = 0.0215649, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2640, 2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(d*Sqrt
[I*Sinh[c + d*x]])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sinh(c+dx)}\,dx &= \frac{\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{i\sinh(c+dx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0450161, size = 52, normalized size = 0.93

$$\frac{2iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[b*Sinh[c + d*x]])/(
d*Sqrt[I*Sinh[c + d*x]])
```

Maple [A] time = 0.043, size = 111, normalized size = 2.

$$\frac{b\sqrt{2}}{d\cosh(dx+c)}\sqrt{-i(\sinh(dx+c)+i)}\sqrt{-i(i-\sinh(dx+c))}\sqrt{i\sinh(dx+c)}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},1/2\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(1/2),x)
```

```
[Out] b*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(I*sinh(d*x
+c))^(1/2)*(2*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-I
*sinh(d*x+c))^(1/2),1/2*2^(1/2)))/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sinh(c + d*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(d*x + c)), x)
```

3.19 $\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b\sinh(c+dx)}}$$

```
[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt
[b*Sinh[c + d*x]])
```

Rubi [A] time = 0.0218033, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2642, 2641}

$$-\frac{2i\sqrt{i\sinh(c+dx)}F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt
[b*Sinh[c + d*x]])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx &= \frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{\sqrt{b\sinh(c+dx)}}\\ &= -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0340537, size = 54, normalized size = 0.96

$$\frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right),2\right)}{d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt[
b*Sinh[c + d*x]])
```

Maple [A] time = 0.034, size = 89, normalized size = 1.6

$$\frac{i\sqrt{2}}{d\cosh\left(dx+c\right)}\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\sqrt{-i\left(i-\sinh\left(dx+c\right)\right)}\sqrt{i\sinh\left(dx+c\right)}\text{EllipticF}\left(\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)},\frac{\sqrt{2}}{2}\right)\frac{}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(1/2),x)
```

```
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(I*sinh(d*x
+c))^(1/2)*EllipticF((-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/(b
*sinh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(dx+c\right)}}{b\sinh\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))/(b*sinh(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sinh{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(d*x + c)), x)
```

3.20 $\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

```
[Out] (-2*Cosh[c + d*x])/(b*d*Sqrt[b*Sinh[c + d*x]]) - ((2*I)*EllipticE[(I*c - Pi
/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(b^2*d*Sqrt[I*Sinh[c + d*x]])
```

Rubi [A] time = 0.0379691, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2640, 2639}

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(b*d*Sqrt[b*Sinh[c + d*x]]) - ((2*I)*EllipticE[(I*c - Pi
/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(b^2*d*Sqrt[I*Sinh[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx &= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}+\frac{\int \sqrt{b\sinh(c+dx)}\,dx}{b^2}\\ &= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}+\frac{\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}\,dx}{b^2\sqrt{i\sinh(c+dx)}}\\ &= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}-\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0615495, size = 62, normalized size = 0.72

$$-\frac{2\left(\cosh(c+dx)-\sqrt{i\sinh(c+dx)}E\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{bd\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sin
h[c + d*x]]))/(b*d*Sqrt[b*Sinh[c + d*x]])
```

Maple [A] time = 0.046, size = 159, normalized size = 1.9

$$\frac{1}{bd\cosh(dx+c)}\left(2\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\mathrm{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},1/2\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(3/2),x)
```

```
[Out] (2*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^
(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-(1-I*sinh(d*x+c))^(1/2
)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh
(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/b/cosh(d*x+c)/(b*sinh(d*x+c))^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(b\sinh(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh(dx+c)}}{b^2\sinh(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))/(b^2*sinh(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sinh(c + dx))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sinh(dx + c))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-3/2), x)
```

3.21 $\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}+\frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

```
[Out] (-2*Cosh[c + d*x])/(3*b*d*(b*Sinh[c + d*x])^(3/2)) + (((2*I)/3)*EllipticF[(
I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d*Sqrt[b*Sinh[c + d*x
]])
```

Rubi [A] time = 0.0379677, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2642, 2641}

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}+\frac{2i\sqrt{i\sinh(c+dx)}F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(3*b*d*(b*Sinh[c + d*x])^(3/2)) + (((2*I)/3)*EllipticF[(
I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d*Sqrt[b*Sinh[c + d*x
]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} - \frac{\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx}{3b^2}$$
$$= -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} - \frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3b^2\sqrt{b\sinh(c+dx)}}$$
$$= -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} + \frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3b^2d\sqrt{b\sinh(c+dx)}}$$

Mathematica [C] time = 0.0947008, size = 84, normalized size = 0.93

$$-\frac{2\left(\sqrt{2}\sqrt{\sinh^2(c+dx)(-(\coth(c+dx)+1))}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)+\coth(c+dx)\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d
*x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/(3
*b^2*d*Sqrt[b*Sinh[c + d*x]])
```

Maple [A] time = 0.043, size = 114, normalized size = 1.3

$$-\frac{1}{3\,b^2\sinh(dx+c)\cosh(dx+c)\,d}\left(i\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\mathrm{EllipticF}\left(\sqrt{1-i\sinh(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(5/2),x)
```

```
[Out] -1/3/b^2/sinh(d*x+c)*(I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(
1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))*s
inh(d*x+c)+2*cosh(d*x+c)^2)/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh(dx+c)}}{b^3\sinh(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))/(b^3*sinh(d*x + c)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-5/2), x)
```

3.22 $\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}}+\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}}-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}$$

```
[Out] (-2*Cosh[c + d*x])/(5*b*d*(b*Sinh[c + d*x])^(5/2)) + (6*Cosh[c + d*x])/(5*b
^3*d*Sqrt[b*Sinh[c + d*x]]) + (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2,
2]*Sqrt[b*Sinh[c + d*x]])/(b^4*d*Sqrt[I*Sinh[c + d*x]])
```

Rubi [A] time = 0.0560453, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2636, 2640, 2639}

$$\frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}}+\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}}-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(5*b*d*(b*Sinh[c + d*x])^(5/2)) + (6*Cosh[c + d*x])/(5*b
^3*d*Sqrt[b*Sinh[c + d*x]]) + (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2,
2]*Sqrt[b*Sinh[c + d*x]])/(b^4*d*Sqrt[I*Sinh[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} - \frac{3\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx}{5b^2}$$
$$= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} - \frac{3\int \sqrt{b\sinh(c+dx)}\,dx}{5b^4}$$
$$= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} - \frac{\left(3\sqrt{b\sinh(c+dx)}\right)\int \sqrt{i\sinh(c+dx)}\,d}{5b^4\sqrt{i\sinh(c+dx)}}$$
$$= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+d}}{5b^4d\sqrt{i\sinh(c+dx)}}$$

Mathematica [A] time = 0.163265, size = 79, normalized size = 0.67

$$-\frac{2\left(-3\cosh(c+dx)+\coth(c+dx)\text{csch}(c+dx)+3\sqrt{i\sinh(c+dx)}E\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{5b^3d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[((-2*I)*c
 + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(5*b^3*d*Sqrt[b*Sinh[c + d
*x]])
```

Maple [A] time = 0.049, size = 205, normalized size = 1.7

$$-\frac{1}{5\,b^3\left(\sinh\left(dx+c\right)\right)^2\cosh\left(dx+c\right)d}\left(6\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\sqrt{2}\sqrt{-i\left(i-\sinh\left(dx+c\right)\right)}\sqrt{i\sinh\left(dx+c\right)}\left(\sinh\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(7/2),x)
```

```
[Out] -1/5/b^3/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*
x+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+c)
+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*
x+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*x+c)
+I))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)/(b*s
inh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(dx+c\right)}}{b^4\sinh\left(dx+c\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))/(b^4*sinh(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-7/2), x)
```

3.23 $\int (i\sinh(c+dx))^{7/2}\,dx$

Optimal. Leaf size=91

$$-\frac{10i\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{21d}+\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}+\frac{10i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{21d}$$

```
[Out] (((-10*I)/21)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((10*I)/21)*Cosh[c
 + d*x]*Sqrt[I*Sinh[c + d*x]])/d + (((2*I)/7)*Cosh[c + d*x]*(I*Sinh[c + d*x
])^(5/2))/d
```

Rubi [A] time = 0.0343493, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2635, 2641}

$$-\frac{10iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{21d}+\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}+\frac{10i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((10*I)/21)*Cosh[c
 + d*x]*Sqrt[I*Sinh[c + d*x]])/d + (((2*I)/7)*Cosh[c + d*x]*(I*Sinh[c + d*x
])^(5/2))/d
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (i\sinh(c+dx))^{7/2}\,dx &= \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d}+\frac{5}{7}\int (i\sinh(c+dx))^{3/2}\,dx\\ &= \frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d}+\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d}+\frac{5}{21}\int \frac{1}{\sqrt{i\sinh(c+a}}\\ &= -\frac{10iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{21d}+\frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d}+\frac{2i\cosh(c+dx)(i\sinh(c+}{7d}\end{aligned}$$

Mathematica [A] time = 0.164432, size = 65, normalized size = 0.71

$$\frac{i\left(20\text{EllipticF}\left(\frac{1}{4}(-2ic-2idx+\pi),2\right)+\sqrt{i\sinh(c+dx)}(23\cosh(c+dx)-3\cosh(3(c+dx)))\right)}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(7/2),x]
```

```
[Out] ((I/42)*(20*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + (23*Cosh[c + d*x]
 - 3*Cosh[3*(c + d*x)])*Sqrt[I*Sinh[c + d*x]]))/d
```

Maple [A] time = 0.048, size = 122, normalized size = 1.3

$$\frac{-\frac{i}{21}}{d\cosh(dx+c)}\left(6i\sinh(dx+c)(\cosh(dx+c))^4-5\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\text{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(7/2),x)
```

```
[Out] -1/21*I*(6*I*sinh(d*x+c)*cosh(d*x+c)^4-5*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1
+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/
2),1/2*2^(1/2))-16*I*cosh(d*x+c)^2*sinh(d*x+c))/cosh(d*x+c)/(I*sinh(d*x+c))
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\sinh(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\sqrt{\frac{1}{2}}\left(-3i e^{(6dx+6c)}+23i e^{(4dx+4c)}+23i e^{(2dx+2c)}-3i\right)\sqrt{i e^{(2dx+2c)}-i}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}+84de^{(3dx+3c)}\text{integral}\left(-\frac{10i\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}}}{21\left(de^{(2dx+}\right.}\right.\right.}{84d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(sqrt(1/2)*(-3*I*e^(6*d*x + 6*c) + 23*I*e^(4*d*x + 4*c) + 23*I*e^(2*d*
x + 2*c) - 3*I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 84*d*e^(
3*d*x + 3*c)*integral(-10/21*I*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/
2*d*x - 1/2*c)/(d*e^(2*d*x + 2*c) - d), x))*e^(-3*d*x - 3*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\sinh(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(7/2), x)
```

3.24 $\int (i\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=62

$$\frac{2i(i\sinh(c+dx))^{3/2}\cosh(c+dx)}{5d}-\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}$$

```
[Out] (((-6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d
*x]*(I*Sinh[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.0198261, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2635, 2639}

$$\frac{2i(i\sinh(c+dx))^{3/2}\cosh(c+dx)}{5d}-\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d
*x]*(I*Sinh[c + d*x])^(3/2))/d
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (i\sinh(c+dx))^{5/2}\,dx &= \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}+\frac{3}{5}\int\sqrt{i\sinh(c+dx)}\,dx\\ &= -\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d}+\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}\end{aligned}$$

Mathematica [A] time = 0.0588187, size = 55, normalized size = 0.89

$$\frac{6iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)-\sqrt{i\sinh(c+dx)}\sinh(2(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(5/2),x]
```

```
[Out] ((6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] - Sqrt[I*Sinh[c + d*x]]*
Sinh[2*(c + d*x)])/(5*d)
```

Maple [B] time = 0.05, size = 169, normalized size = 2.7

$$\frac{\frac{i}{5}}{d\cosh(dx+c)}\left(6\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\,\text{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},1/2\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(5/2),x)
```

```
[Out] 1/5*I*(6*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*
x+c))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-3*(1-I*sinh(d*x+
c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((
1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^4+2*cosh(d*x+c)^2)/cosh(d
*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\sinh(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{\frac{1}{2}}\left(e^{(5\,dx+5\,c)}-2\,e^{(4\,dx+4\,c)}-12\,e^{(3\,dx+3\,c)}-24\,e^{(2\,dx+2\,c)}-e^{(dx+c)}+2\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-10\left(de^{(3\,dx+3\,c)}-2\,de^{\cdots}\right.}{10\left(de^{(3\,dx+3\,c)}-2\,de^{(2\,dx+2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(1/2)*(e^(5*d*x + 5*c) - 2*e^(4*d*x + 4*c) - 12*e^(3*d*x + 3*c)
- 24*e^(2*d*x + 2*c) - e^(d*x + c) + 2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2
*d*x - 1/2*c) - 10*(d*e^(3*d*x + 3*c) - 2*d*e^(2*d*x + 2*c))*integral(6/5*s
qrt(1/2)*(2*e^(2*d*x + 2*c) + 3*e^(d*x + c) - 2)*sqrt(I*e^(2*d*x + 2*c) - I
)*e^(-1/2*d*x - 1/2*c)/(d*e^(4*d*x + 4*c) - 4*d*e^(3*d*x + 3*c) + 3*d*e^(2*
d*x + 2*c) + 4*d*e^(d*x + c) - 4*d), x))/(d*e^(3*d*x + 3*c) - 2*d*e^(2*d*x
+ 2*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i \sinh (dx + c))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(5/2), x)
```

3.25 $\int (i \sinh(c+dx))^{3/2} \, dx$

Optimal. Leaf size=62

$$\frac{2i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{3d} - \frac{2i\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d}$$

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x]*Sqrt[I*Sinh[c + d*x]])/d
```

Rubi [A] time = 0.0205847, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2635, 2641}

$$\frac{2i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{3d} - \frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x]*Sqrt[I*Sinh[c + d*x]])/d
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (i\sinh(c+dx))^{3/2}\,dx &= \frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d} + \frac{1}{3}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx \\ &= -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{3d} + \frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d} \end{aligned}$$

Mathematica [C] time = 0.134912, size = 94, normalized size = 1.52

$$-\frac{2i\sqrt{i\sinh(c+dx)}\left(\text{csch}(c+dx)\sqrt{-\sinh(2c+2dx)-\cosh(2c+2dx)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[I*Sinh[c + d*x]]*(-Cosh[c + d*x] + Csch[c + d*x]*Hypergeom
etric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - Cos
h[2*c + 2*d*x] - Sinh[2*c + 2*d*x]]))/d
```

Maple [A] time = 0.048, size = 104, normalized size = 1.7

$$\frac{\frac{i}{3}}{d\cosh(dx+c)}\left(\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\mathrm{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)+2\,i\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(3/2),x)
```

```
[Out] 1/3*I*((1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+
c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+2*I*sinh(d*x+c)*co
sh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(i\,\sinh(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\sqrt{\frac{1}{2}}\left(ie^{(2dx+2c)}+i\right)\sqrt{ie^{(2dx+2c)}-i}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}+3de^{(dx+c)}\mathrm{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}-i}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{3\left(de^{(2dx+2c)}-d\right)},x\right)\right)e^{(-dx-c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(1/2)*(I*e^(2*d*x + 2*c) + I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*
d*x - 1/2*c) + 3*d*e^(d*x + c)*integral(-2/3*I*sqrt(1/2)*sqrt(I*e^(2*d*x +
2*c) - I)*e^(-1/2*d*x - 1/2*c)/(d*e^(2*d*x + 2*c) - d), x))*e^(-d*x - c)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(i\sinh(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i \sinh(dx + c))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(3/2), x)
```

3.26 $\int \sqrt{i \sinh(c+dx)}\,dx$

Optimal. Leaf size=30

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Rubi [A] time = 0.0091243, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{i \sinh(c+dx)}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

Mathematica [A] time = 0.0204305, size = 28, normalized size = 0.93

$$\frac{2iE\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[(Pi/2 - I*(c + d*x))/2, 2])/d
```

Maple [A] time = 0.045, size = 91, normalized size = 3.

$$\frac{i\sqrt{2}}{d\cosh(dx+c)}\sqrt{-i(\sinh(dx+c)+i)}\sqrt{-i(i-\sinh(dx+c))}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},1/2\sqrt{2}\right)-\mathrm{EllipticF}\left(\sqrt{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(1/2),x)
```

```
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(2*Elliptic
E((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(d*x+c))^(1/2),1/
2*2^(1/2)))/cosh(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i \sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*sinh(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}-i}\left(e^{(dx+c)}+2\right)e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}+\left(de^{(dx+c)}-2d\right)\mathrm{integral}\left(\frac{2\sqrt{\frac{1}{2}}\left(2e^{(2dx+2c)}+3e^{(dx+c)}-2\right)\sqrt{ie^{(2dx+2c)}-i}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{de^{(4dx+4c)}-4de^{(3dx+3c)}+3de^{(2dx+2c)}+4de^{(dx+c)}-4d},x\right)}{de^{(dx+c)}-2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (2*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*(e^(d*x + c) + 2)*e^(-1/2*d*x - 1/
2*c) + (d*e^(d*x + c) - 2*d)*integral(2*sqrt(1/2)*(2*e^(2*d*x + 2*c) + 3*e^
(d*x + c) - 2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c)/(d*e^(4*d*x
 + 4*c) - 4*d*e^(3*d*x + 3*c) + 3*d*e^(2*d*x + 2*c) + 4*d*e^(d*x + c) - 4*d
), x))/(d*e^(d*x + c) - 2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i \sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i \sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*sinh(d*x + c)), x)
```

3.27 $\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{2i\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{d}$$

[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d

Rubi [A] time = 0.0086927, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2641}

$$-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[I*Sinh[c + d*x]],x]

[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

Mathematica [A] time = 0.0226246, size = 28, normalized size = 0.93

$$\frac{2i\text{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right),2\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[I*Sinh[c + d*x]],x]

[Out] ((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2])/d

Maple [A] time = 0.038, size = 68, normalized size = 2.3

$$\frac{i\sqrt{2}}{d\cosh(dx+c)}\sqrt{-i(\sinh(dx+c)+i)}\sqrt{-i(i-\sinh(dx+c))}\text{EllipticF}\left(\sqrt{-i(\sinh(dx+c)+i)},\frac{\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(1/2),x)
```

```
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*EllipticF((
-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i \sinh(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(I*sinh(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{de^{(2\,dx+2\,c)}-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c)/(d
*e^(2*d*x + 2*c) - d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i \sinh(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i \sinh(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(I*sinh(d*x + c)), x)
```

3.28 $\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

```
[Out] ((2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + ((2*I)*Cosh[c + d*x])/(d*S
qrt[I*Sinh[c + d*x]])
```

Rubi [A] time = 0.0186412, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2636, 2639}

$$\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + ((2*I)*Cosh[c + d*x])/(d*S
qrt[I*Sinh[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx &= \frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}-\int \sqrt{i\sinh(c+dx)}\,dx\\ &= \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0938688, size = 50, normalized size = 0.86

$$\frac{2\left(\sqrt{i\sinh(c+dx)}\coth(c+dx)-iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (2*((-I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + Coth[c + d*x]*Sqrt[I
*Sinh[c + d*x]]))/d
```

Maple [A] time = 0.052, size = 159, normalized size = 2.7

$$\frac{-i}{d\cosh(dx+c)}\left(2\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\mathrm{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},1/2\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(3/2),x)
```

```
[Out] -I*(2*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c
))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-(1-I*sinh(d*x+c))^(
1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*s
inh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\sinh(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}-i}e^{\left(\frac{3}{2}dx+\frac{3}{2}c\right)}+\left(de^{(2dx+2c)}-d\right)\mathrm{integral}\left(-\frac{2\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}-i}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{de^{(2dx+2c)}-d},x\right)}{de^{(2dx+2c)}-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (4*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(3/2*d*x + 3/2*c) + (d*e^(2*d*x
+ 2*c) - d)*integral(-2*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(1/2*d*x +
1/2*c)/(d*e^(2*d*x + 2*c) - d), x))/(d*e^(2*d*x + 2*c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\sinh(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\sinh{(dx+c)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(-3/2), x)
```

3.29 $\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=62

$$\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}-\frac{2i\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d}$$

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x])/(d*(I*Sinh[c + d*x])^(3/2))
```

Rubi [A] time = 0.0201901, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2636, 2641}

$$\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x])/(d*(I*Sinh[c + d*x])^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx &= \frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx\\ &= -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{3d}+\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0586841, size = 83, normalized size = 1.34

$$\frac{2\left(\sqrt{2}\sqrt{\sinh^2(c+dx)(-(\coth(c+dx)+1))}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)+\coth(c+dx)\right)}{3d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/(3*
d*Sqrt[I*Sinh[c + d*x]])
```

Maple [A] time = 0.048, size = 113, normalized size = 1.8

$$\frac{-\frac{i}{3}}{\sinh(dx+c)\cosh(dx+c)\,d}\left(-\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(5/2),x)
```

```
[Out] -1/3*I/sinh(d*x+c)*(-(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2
)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))*sinh
(d*x+c)+2*I*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(-4i\,e^{(3\,dx+3\,c)}-4i\,e^{(dx+c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+3\left(de^{(4\,dx+4\,c)}-2\,de^{(2\,dx+2\,c)}+d\right)\mathrm{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,d\right.}}{3\left(de^{(2\,dx+2\,c)}-d\right)}\right.}{3\left(de^{(4\,dx+4\,c)}-2\,de^{(2\,dx+2\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(1/2)*(-4*I*e^(3*d*x + 3*c) - 4*I*e^(d*x + c))*sqrt(I*e^(2*d*x + 2
*c) - I)*e^(-1/2*d*x - 1/2*c) + 3*(d*e^(4*d*x + 4*c) - 2*d*e^(2*d*x + 2*c)
+ d)*integral(-2/3*I*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/
2*c)/(d*e^(2*d*x + 2*c) - d), x))/(d*e^(4*d*x + 4*c) - 2*d*e^(2*d*x + 2*c)
+ d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i \sinh (c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i \sinh (dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(-5/2), x)
```

3.30 $\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=91

$$\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

```
[Out] (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d*
x])/(d*(I*Sinh[c + d*x])^(5/2)) + (((6*I)/5)*Cosh[c + d*x])/(d*Sqrt[I*Sinh[
c + d*x]])
```

Rubi [A] time = 0.0333186, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2636, 2639}

$$\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d*
x])/(d*(I*Sinh[c + d*x])^(5/2)) + (((6*I)/5)*Cosh[c + d*x])/(d*Sqrt[I*Sinh[
c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx &= \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}+\frac{3}{5}\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx\\ &= \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}-\frac{3}{5}\int \sqrt{i\sinh(c+dx)}\,dx\\ &= \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d}+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.130367, size = 80, normalized size = 0.88

$$-\frac{2i\left(-3\cosh(c+dx)+\coth(c+dx)\operatorname{csch}(c+dx)+3\sqrt{i\sinh(c+dx)}E\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{5d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (((-2*I)/5)*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[(
(-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(d*Sqrt[I*Sinh[c +
 d*x]])
```

Maple [A] time = 0.054, size = 204, normalized size = 2.2

$$\frac{-\frac{i}{5}}{\left(\sinh\left(dx+c\right)\right)^{2}\cosh\left(dx+c\right)d}\left(6\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\sqrt{2}\sqrt{-i\left(i-\sinh\left(dx+c\right)\right)}\sqrt{i\sinh\left(dx+c\right)}\left(\sinh\left(dx+c\right)\right)^{2}\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(7/2),x)
```

```
[Out] -1/5*I/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+
c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+c)+I
))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+
c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*x+c)+I
))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)/(I*sin
h(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(i\sinh\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\sqrt{\frac{1}{2}}\left(3\,e^{(6\,dx+6\,c)}-8\,e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+5\left(de^{(6\,dx+6\,c)}-3\,de^{(4\,dx+4\,c)}+3\,de^{(2\,dx+2\,c)}-d\right)\mathrm{in}}{5\left(de^{(6\,dx+6\,c)}-3\,de^{(4\,dx+4\,c)}+3\,de^{(2\,dx+2\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(4*sqrt(1/2)*(3*e^(6*d*x + 6*c) - 8*e^(4*d*x + 4*c) + e^(2*d*x + 2*c))*
sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 5*(d*e^(6*d*x + 6*c) - 3
*d*e^(4*d*x + 4*c) + 3*d*e^(2*d*x + 2*c) - d)*integral(-6/5*sqrt(1/2)*sqrt(
I*e^(2*d*x + 2*c) - I)*e^(1/2*d*x + 1/2*c)/(d*e^(2*d*x + 2*c) - d), x))/(d*
```

```
e^(6*d*x + 6*c) - 3*d*e^(4*d*x + 4*c) + 3*d*e^(2*d*x + 2*c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i \sinh(dx + c))^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(-7/2), x)
```

3.31 $\int (b\sinh(c+dx))^{4/3}\,dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)}{7bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(7/3))/(7*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rubi [A] time = 0.0154059, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)}{7bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(4/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(7/3))/(7*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^{4/3}\,dx=\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{7/3}}{7bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.0525996, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\tanh(c+dx)(b\sinh(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(4/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2
]*(b*Sinh[c + d*x])^(4/3)*Tanh[c + d*x])/(7*d)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(4/3),x)
```

```
[Out] int((b*sinh(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sinh (dx+c))^{\frac{1}{3}}\, b\sinh (dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(1/3)*b*sinh(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(4/3), x)
```

3.32 $\int (b\sinh(c+dx))^{2/3}\,dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)}{5bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(5/3))/(5*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rubi [A] time = 0.015458, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)}{5bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(2/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(5/3))/(5*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^{2/3}\,dx=\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{5/3}}{5bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.0357521, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\tanh(c+dx)(b\sinh(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(2/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2
]*(b*Sinh[c + d*x])^(2/3)*Tanh[c + d*x])/(5*d)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(2/3),x)
```

```
[Out] int((b*sinh(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sinh (dx+c))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh (c+dx))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(2/3), x)
```

3.33 $\int \sqrt[3]{b \sinh (c+d x)} \, dx$

Optimal. Leaf size=60

$$\frac{3 \cosh (c+d x) (b \sinh (c+d x))^{4/3} {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh ^2(c+d x)\right)}{4 b d \sqrt{\cosh ^2(c+d x)}}$$

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(4/3))/(4*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rubi [A] time = 0.0163367, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3 \cosh (c+d x) (b \sinh (c+d x))^{4/3} {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh ^2(c+d x)\right)}{4 b d \sqrt{\cosh ^2(c+d x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(1/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(4/3))/(4*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b \sinh (c+d x)} \, dx = \frac{3 \cosh (c+d x) {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh ^2(c+d x)\right) (b \sinh (c+d x))^{4/3}}{4 b d \sqrt{\cosh ^2(c+d x)}}$$

Mathematica [A] time = 0.0342486, size = 57, normalized size = 0.95

$$\frac{3 \sqrt{\cosh ^2(c+d x)} \tanh (c+d x) \sqrt[3]{b \sinh (c+d x)} {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh ^2(c+d x)\right)}{4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]
*(b*Sinh[c + d*x])^(1/3)*Tanh[c + d*x])/(4*d)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sinh (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(1/3),x)
```

```
[Out] int((b*sinh(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh (dx + c))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b \sinh (dx + c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sinh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh (dx + c))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(1/3), x)
```

3.34 $\int \frac{1}{\sqrt[3]{b \sinh(c+dx)}} \, dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)}{2bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(2/3))/(2*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rubi [A] time = 0.0149039, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)}{2bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-1/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(2/3))/(2*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}}{2bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.0397989, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\tanh(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)}{2d\sqrt[3]{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]
*Tanh[c + d*x])/(2*d*(b*Sinh[c + d*x])^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\sinh\left(dx+c\right)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(1/3),x)
```

```
[Out] int(1/(b*sinh(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sinh\left(dx+c\right)\right)^{\frac{2}{3}}}{b\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(2/3)/(b*sinh(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\sinh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-1/3), x)
```

3.35 $\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3\cosh(c+dx)\sqrt[3]{b\sinh(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(1/3))/(b*d*Sqrt[Cosh[c + d*x]^2])
```

Rubi [A] time = 0.015846, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cosh(c+dx)\sqrt[3]{b\sinh(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-2/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(1/3))/(b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}}{bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.0381893, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\tanh(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)}{d(b\sinh(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-2/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]
*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(2/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^{-\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(2/3),x)
```

```
[Out] int(1/(b*sinh(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh (dx+c))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sinh (dx+c))^{\frac{1}{3}}}{b\sinh (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(1/3)/(b*sinh(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh (c+dx))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh (dx+c))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-2/3), x)
```

3.36 $\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\cosh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

```
[Out] (-3*Cosh[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^2])/(b*d
*Sqrt[Cosh[c + d*x]^2]*(b*Sinh[c + d*x])^(1/3))
```

Rubi [A] time = 0.0154207, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\cosh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Cosh[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^2])/(b*d
*Sqrt[Cosh[c + d*x]^2]*(b*Sinh[c + d*x])^(1/3))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = -\frac{3\cosh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

Mathematica [A] time = 0.0372699, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cosh^2(c+dx)}\tanh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{d(b\sinh(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^
2]*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(4/3))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (b \sinh (dx+c))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(4/3),x)
```

```
[Out] int(1/(b*sinh(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sinh (dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b \sinh (dx+c))^{\frac{2}{3}}}{b^2 \sinh (dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(2/3)/(b^2*sinh(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sinh (c+dx))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sinh (dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-4/3), x)
```

3.37 $\int (b\sinh(c+dx))^n\,dx$

Optimal. Leaf size=70

$$\frac{\cosh(c+dx)(b\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{bd(n+1)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] (Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x]^
2]*(b*Sinh[c + d*x])^(1 + n))/(b*d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rubi [A] time = 0.0193254, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2643}

$$\frac{\cosh(c+dx)(b\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{bd(n+1)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^n,x]
```

```
[Out] (Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x]^
2]*(b*Sinh[c + d*x])^(1 + n))/(b*d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^n\,dx=\frac{\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{1+n}}{bd(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.0450741, size = 65, normalized size = 0.93

$$\frac{\sqrt{\cosh^2(c+dx)}\tanh(c+dx)(b\sinh(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^n,x]
```

```
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*(b*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Maple [F] time = 0.18, size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^n,x)
```

```
[Out] int((b*sinh(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sinh (dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh (c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**n,x)
```

```
[Out] Integral((b*sinh(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^n, x)
```

3.38 $\int (i\sinh(c+dx))^n\,dx$

Optimal. Leaf size=72

$$-\frac{i\cosh(c+dx)(i\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] ((-I)*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c +
d*x]^2]*(I*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rubi [A] time = 0.0154995, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{i\cosh(c+dx)(i\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^n,x]
```

```
[Out] ((-I)*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c +
d*x]^2]*(I*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (i\sinh(c+dx))^n\,dx = -\frac{i\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.0397979, size = 67, normalized size = 0.93

$$\frac{\sqrt{\cosh^2(c+dx)}\tanh(c+dx)(i\sinh(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^n,x]
```

```
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*(I*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int (i \sinh (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^n,x)
```

```
[Out] int((I*sinh(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i \sinh (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{1}{2}\left(i e^{(2 dx+2 c)} - i\right)e^{(-dx-c)}\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((1/2*(I*e^(2*d*x + 2*c) - I)*e^(-d*x - c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (i \sinh (c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**n,x)
```

```
[Out] Integral((I*sinh(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i \sinh (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^n, x)
```

3.39 $\int(-i\sinh(c+dx))^n\,dx$

Optimal. Leaf size=72

$$\frac{i\cosh(c+dx)(-i\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] (I*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x
]^2]*((-I)*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rubi [A] time = 0.0157407, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{i\cosh(c+dx)(-i\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((-I)*Sinh[c + d*x])^n,x]
```

```
[Out] (I*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x
]^2]*((-I)*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int(-i\sinh(c+dx))^n\,dx=\frac{i\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.0400559, size = 67, normalized size = 0.93

$$\frac{\sqrt{\cosh^2(c+dx)}\tanh(c+dx)(-i\sinh(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-I)*Sinh[c + d*x])^n,x]
```

```
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*((-I)*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int (-i\sinh(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*sinh(d*x+c))^n,x)
```

```
[Out] int((-I*sinh(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-i\sinh(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((-I*sinh(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\frac{1}{2}\left(-i\,e^{(2\,dx+2\,c)}+i\right)e^{(-dx-c)}\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((1/2*(-I*e^(2*d*x + 2*c) + I)*e^(-d*x - c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-i\sinh(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))**n,x)
```

```
[Out] Integral((-I*sinh(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-i\sinh(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="giac")

[Out] integrate((-I*sinh(d*x + c))^n, x)
```

3.40 $\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=46

$$\frac{3ix}{2}+\frac{4\cosh^3(x)}{3}-4\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}-\frac{3}{2}i\sinh(x)\cosh(x)$$

```
[Out] ((3*I)/2)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 - ((3*I)/2)*Cosh[x]*Sinh[x] - (Co
sh[x]*Sinh[x]^3)/(I + Sinh[x])
```

Rubi [A] time = 0.065288, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2767, 2748, 2635, 8, 2633}

$$\frac{3ix}{2}+\frac{4\cosh^3(x)}{3}-4\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}-\frac{3}{2}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(I + Sinh[x]),x]
```

```
[Out] ((3*I)/2)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 - ((3*I)/2)*Cosh[x]*Sinh[x] - (Co
sh[x]*Sinh[x]^3)/(I + Sinh[x])
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx &= -\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)} + \int \sinh^2(x)(-3i+4\sinh(x))\,dx \\
&= -\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)} - 3i\int \sinh^2(x)\,dx + 4\int \sinh^3(x)\,dx \\
&= -\frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)} + \frac{3}{2}i\int 1\,dx - 4\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(x)\right) \\
&= \frac{3ix}{2} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}
\end{aligned}
$$

Mathematica [B] time = 0.182955, size = 134, normalized size = 2.91

$$
\frac{\cosh(x)\left(i\sinh^{-1}(\sinh(x))(\sinh(x)+i) + 2\sinh^3(x)\sqrt{\cosh^2(x)} - i\sinh^2(x)\sqrt{\cosh^2(x)} - \sinh(x)\left(7\sqrt{\cosh^2(x)} + 16\,\text{s}\right.\right.}{6(\sinh(x)+i)\sqrt{\cosh^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*((-16*I)*(ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + Sqrt[Cosh[x]^2]) -
 (16*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + 7*Sqrt[Cosh[x]^2])*Sinh[x] - I*S
qrt[Cosh[x]^2]*Sinh[x]^2 + 2*Sqrt[Cosh[x]^2]*Sinh[x]^3 + I*ArcSinh[Sinh[x]]
*(I + Sinh[x])))/(6*Sqrt[Cosh[x]^2]*(I + Sinh[x]))
```

Maple [B] time = 0.039, size = 138, normalized size = 3.

$$
\frac{3i}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \frac{3}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{1}{3}\Big(
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(I+sinh(x)),x)
```

```
[Out] 3/2*I*ln(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2+1/2*I/(tanh(1/2*x)+1)^2-3/2/(
tanh(1/2*x)+1)-1/2*I/(tanh(1/2*x)+1)+1/3/(tanh(1/2*x)+1)^3-3/2*I*ln(tanh(1/
2*x)-1)+3/2/(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)-1)-1/2/(tanh(1/2*x)-1)^2-1/2
*I/(tanh(1/2*x)-1)^2-1/3/(tanh(1/2*x)-1)^3-2*I/(tanh(1/2*x)+I)
```

Maxima [A] time = 1.26086, size = 80, normalized size = 1.74

$$
\frac{3}{2}i\,x - \frac{4\,e^{(-x)} - 36i\,e^{(-2\,x)} + 138\,e^{(-3\,x)} + 2i}{16\left(-3i\,e^{(-3\,x)} + 3\,e^{(-4\,x)}\right)} - \frac{7}{8}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)} + \frac{1}{24}\,e^{(-3\,x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 3/2*I*x - 1/16*(4*e^(-x) - 36*I*e^(-2*x) + 138*e^(-3*x) + 2*I)/(-3*I*e^(-3*
x) + 3*e^(-4*x)) - 7/8*e^(-x) + 1/8*I*e^(-2*x) + 1/24*e^(-3*x)
```

Fricas [A] time = 2.08395, size = 192, normalized size = 4.17

$$\frac{(36i\,x - 21i)e^{(4\,x)} - 3\,(12\,x + 23)e^{(3\,x)} + e^{(7\,x)} - 2i\,e^{(6\,x)} - 18\,e^{(5\,x)} - 18i\,e^{(2\,x)} - 2\,e^{x} + i}{24\left(e^{(4\,x)} + i\,e^{(3\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((36*I*x - 21*I)*e^(4*x) - 3*(12*x + 23)*e^(3*x) + e^(7*x) - 2*I*e^(6*
x) - 18*e^(5*x) - 18*I*e^(2*x) - 2*e^x + I)/(e^(4*x) + I*e^(3*x))
```

Sympy [A] time = 0.422437, size = 58, normalized size = 1.26

$$\frac{3ix}{2} + \frac{e^{3x}}{24} - \frac{ie^{2x}}{8} - \frac{7e^{x}}{8} - \frac{7e^{-x}}{8} + \frac{ie^{-2x}}{8} + \frac{e^{-3x}}{24} - \frac{2}{e^{x} + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(I+sinh(x)),x)
```

```
[Out] 3*I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 - 7*exp(x)/8 - 7*exp(-x)/8 + I*exp(-2*
x)/8 + exp(-3*x)/24 - 2/(exp(x) + I)
```

Giac [A] time = 1.36275, size = 68, normalized size = 1.48

$$\frac{3}{2}i\,x - \frac{\left(69\,e^{(3\,x)} + 18i\,e^{(2\,x)} + 2\,e^{x} - i\right)e^{(-3\,x)}}{24\,(e^{x} + i)} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} - \frac{7}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 3/2*I*x - 1/24*(69*e^(3*x) + 18*I*e^(2*x) + 2*e^x - I)*e^(-3*x)/(e^x + I) +
 1/24*e^(3*x) - 1/8*I*e^(2*x) - 7/8*e^x
```

3.41 $\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{3x}{2}-2i\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\sinh(x)+i}+\frac{3}{2}\sinh(x)\cosh(x)$$

```
[Out] (-3*x)/2 - (2*I)*Cosh[x] + (3*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^2)/(I +
 Sinh[x])
```

Rubi [A] time = 0.0463383, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2767, 2734}

$$-\frac{3x}{2}-2i\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\sinh(x)+i}+\frac{3}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (-3*x)/2 - (2*I)*Cosh[x] + (3*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^2)/(I +
 Sinh[x])
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx &= -\frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)}+\int \sinh(x)(-2i+3\sinh(x))\,dx\\ &= -\frac{3x}{2}-2i\cosh(x)+\frac{3}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.10594, size = 41, normalized size = 1.14

$$\frac{1}{2}\cosh(x)\left(-\frac{3\sinh^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}+\frac{\sinh^2(x)-i\sinh(x)+4}{\sinh(x)+i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*((-3*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (4 - I*Sinh[x] + Sinh[x]^
2)/(I + Sinh[x])))/2
```

Maple [B] time = 0.034, size = 93, normalized size = 2.6

$$\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-i\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{3}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+i\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(I+sinh(x)),x)
```

```
[Out] 1/2/(tanh(1/2*x)+1)-I/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2-3/2*ln(tanh(1/2
*x)+1)+1/2/(tanh(1/2*x)-1)+I/(tanh(1/2*x)-1)+1/2/(tanh(1/2*x)-1)^2+3/2*ln(t
anh(1/2*x)-1)+2/(tanh(1/2*x)+I)
```

Maxima [A] time = 1.16406, size = 61, normalized size = 1.69

$$-\frac{3}{2}x-\frac{3e^{(-x)}+20ie^{(-2x)}+i}{8\left(-ie^{(-2x)}+e^{(-3x)}\right)}-\frac{1}{2}ie^{(-x)}-\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x - 1/8*(3*e^(-x) + 20*I*e^(-2*x) + I)/(-I*e^(-2*x) + e^(-3*x)) - 1/2*
I*e^(-x) - 1/8*e^(-2*x)
```

Fricas [A] time = 2.03575, size = 153, normalized size = 4.25

$$-\frac{4(3x-1)e^{(3x)}-(-12ix-20i)e^{(2x)}-e^{(5x)}+3ie^{(4x)}-3e^{x}+i}{8\left(e^{(3x)}+ie^{(2x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(3*x - 1)*e^(3*x) - (-12*I*x - 20*I)*e^(2*x) - e^(5*x) + 3*I*e^(4*x
) - 3*e^x + I)/(e^(3*x) + I*e^(2*x))
```

Sympy [A] time = 0.310202, size = 41, normalized size = 1.14

$$-\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{ie^{x}}{2}-\frac{ie^{-x}}{2}-\frac{e^{-2x}}{8}-\frac{2i}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(I+sinh(x)),x)
```

```
[Out] -3*x/2 + exp(2*x)/8 - I*exp(x)/2 - I*exp(-x)/2 - exp(-2*x)/8 - 2*I/(exp(x)
+ I)
```

Giac [A] time = 1.39074, size = 51, normalized size = 1.42

$$-\frac{3}{2}\,x-\frac{\left(20i\,e^{(2\,x)}-3\,e^{x}+i\right)e^{(-2\,x)}}{8\,(e^{x}+i)}+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -3/2*x - 1/8*(20*I*e^(2*x) - 3*e^x + I)*e^(-2*x)/(e^x + I) + 1/8*e^(2*x) -
1/2*I*e^x
```

3.42 $\int \frac{\sinh^2(x)}{i+\sinh(x)} dx$

Optimal. Leaf size=22

$$-ix+\cosh(x)+\frac{i\cosh(x)}{\sinh(x)+i}$$

```
[Out] (-I)*x + Cosh[x] + (I*Cosh[x])/(I + Sinh[x])
```

Rubi [A] time = 0.0589099, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2746, 2735, 2648}

$$-ix+\cosh(x)+\frac{i\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(I + Sinh[x]),x]

[Out] (-I)*x + Cosh[x] + (I*Cosh[x])/(I + Sinh[x])
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx &= \cosh(x)-i\int \frac{\sinh(x)}{i+\sinh(x)}\,dx\\
&= -ix+\cosh(x)-\int \frac{1}{i+\sinh(x)}\,dx\\
&= -ix+\cosh(x)+\frac{i\cosh(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [B] time = 0.105787, size = 79, normalized size = 3.59

$$\frac{\cosh(x)\left(\sinh(x)+\frac{2\sinh(x)\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}}+\frac{2i\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}}+2i\right)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*(2*I + ((2*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2]
 + Sinh[x] + (2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sinh[x])/Sqrt[Cosh[x]^2
]))/(I + Sinh[x])
```

Maple [B] time = 0.031, size = 52, normalized size = 2.4

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+2i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(I+sinh(x)),x)
```

```
[Out] -I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)+I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)-1
)+2*I/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.10202, size = 45, normalized size = 2.05

$$-i\,x+\frac{10\,e^{(-x)}-2i}{4\left(-i\,e^{(-x)}+e^{(-2\,x)}\right)}+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 1/4*(10*e^(-x) - 2*I)/(-I*e^(-x) + e^(-2*x)) + 1/2*e^(-x)
```

Fricas [B] time = 2.01473, size = 103, normalized size = 4.68

$$\frac{(-2i\,x+i)e^{(2\,x)}+(2\,x+5)e^{x}+e^{(3\,x)}+i}{2\left(e^{(2\,x)}+i\,e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((-2*I*x + I)*e^(2*x) + (2*x + 5)*e^x + e^(3*x) + I)/(e^(2*x) + I*e^x)
```

Sympy [A] time = 0.212152, size = 20, normalized size = 0.91

$$-ix+\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{2}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(I+sinh(x)),x)
```

```
[Out] -I*x + exp(x)/2 + exp(-x)/2 + 2/(exp(x) + I)
```

Giac [A] time = 1.31539, size = 35, normalized size = 1.59

$$-i\,x + \frac{(5\,e^x + i)e^{(-x)}}{2\,(e^x + i)} + \frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -I*x + 1/2*(5*e^x + I)*e^(-x)/(e^x + I) + 1/2*e^x
```

3.43 $\int \frac{\sinh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=14

$$x-\frac{\cosh(x)}{\sinh(x)+i}$$

```
[Out] x - Cosh[x]/(I + Sinh[x])
```

Rubi [A] time = 0.0271906, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2735, 2648}

$$x-\frac{\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(I + Sinh[x]),x]
```

```
[Out] x - Cosh[x]/(I + Sinh[x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{i+\sinh(x)}\,dx &= x-i\int \frac{1}{i+\sinh(x)}\,dx\\ &= x-\frac{\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] time = 0.0538288, size = 43, normalized size = 3.07

$$i\operatorname{sech}(x)\left(i\sinh(x)+2\sqrt{\cosh^2(x)}\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(I + Sinh[x]),x]
```

```
[Out] I*Sech[x]*(1 + 2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2] + I*Si
nh[x])
```

Maple [B] time = 0.023, size = 29, normalized size = 2.1

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\,(\tanh(x/2)+i)^{-1}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(I+sinh(x)),x)
```

```
[Out] ln(tanh(1/2*x)+1)-2/(tanh(1/2*x)+I)-ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.15381, size = 16, normalized size = 1.14

$$x+\frac{2i}{e^{(-x)}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] x + 2*I/(e^(-x) - I)
```

Fricas [A] time = 2.07709, size = 42, normalized size = 3.

$$\frac{xe^{x}+ix+2i}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] (x*e^x + I*x + 2*I)/(e^x + I)
```

Sympy [A] time = 0.157277, size = 8, normalized size = 0.57

$$x+\frac{2i}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x)
```

```
[Out] x + 2*I/(exp(x) + I)
```

Giac [A] time = 1.29781, size = 14, normalized size = 1.

$$x+\frac{2i}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="giac")

[Out] x + 2*I/(e^x + I)
```

3.44 $\int \frac{\mathbf{csch}(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\cosh(x)}{\sinh(x)+i}+i\tanh^{-1}(\cosh(x))$$

```
[Out] I*ArcTanh[Cosh[x]] + Cosh[x]/(I + Sinh[x])
```

Rubi [A] time = 0.0388776, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2747, 2648, 3770}

$$\frac{\cosh(x)}{\sinh(x)+i}+i\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(I + Sinh[x]),x]
```

```
[Out] I*ArcTanh[Cosh[x]] + Cosh[x]/(I + Sinh[x])
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{csch}(x)\,dx\right)+i\int \frac{1}{i+\sinh(x)}\,dx\\ &= i\tanh^{-1}(\cosh(x))+\frac{\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0193429, size = 30, normalized size = 1.58

$$\operatorname{sech}(x)\left(\sinh(x)+i\sqrt{\cosh^2(x)}\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right)-i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(I + Sinh[x]),x]
```

```
[Out] Sech[x]*(-I + I*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + Sinh[x])
```

Maple [A] time = 0.028, size = 21, normalized size = 1.1

$$2\left(\tanh(x/2)+i\right)^{-1}-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+sinh(x)),x)
```

```
[Out] 2/(tanh(1/2*x)+I)-I*ln(tanh(1/2*x))
```

Maxima [A] time = 1.18967, size = 39, normalized size = 2.05

$$-\frac{2i}{e^{(-x)}-i}+i\log\left(e^{(-x)}+1\right)-i\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*I/(e^(-x) - I) + I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Fricas [B] time = 2.11965, size = 97, normalized size = 5.11

$$\frac{(ie^x-1)\log(e^x+1)+(-ie^x+1)\log(e^x-1)-2i}{e^x+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((I*e^x - 1)*log(e^x + 1) + (-I*e^x + 1)*log(e^x - 1) - 2*I)/(e^x + I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x)
```

```
[Out] Integral(csch(x)/(sinh(x) + I), x)
```

Giac [A] time = 1.41403, size = 32, normalized size = 1.68

$$-\frac{2i}{e^x+i}+i\log(e^x+1)-i\log(|e^x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -2*I/(e^x + I) + I*log(e^x + 1) - I*log(abs(e^x - 1))
```

3.45 $\int \frac{\mathbf{csch}^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$2i\coth(x)-\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{\sinh(x)+i}$$

```
[Out] -ArcTanh[Cosh[x]] + (2*I)*Coth[x] + Coth[x]/(I + Sinh[x])
```

Rubi [A] time = 0.0584309, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 8, 3770}

$$2i\coth(x)-\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(I + Sinh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + (2*I)*Coth[x] + Coth[x]/(I + Sinh[x])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \frac{\coth(x)}{i+\sinh(x)} + \int \operatorname{csch}^2(x)(-2i+\sinh(x))\,dx$$
$$= \frac{\coth(x)}{i+\sinh(x)} - 2i\int \operatorname{csch}^2(x)\,dx + \int \operatorname{csch}(x)\,dx$$
$$= -\tanh^{-1}(\cosh(x)) + \frac{\coth(x)}{i+\sinh(x)} - 2\,\mathrm{Subst}(\int 1\,dx, x, -i\coth(x))$$
$$= -\tanh^{-1}(\cosh(x)) + 2i\coth(x) + \frac{\coth(x)}{i+\sinh(x)}$$

Mathematica [A] time = 0.039471, size = 36, normalized size = 1.57

$$\operatorname{sech}(x)\left(2i\sinh(x) + i\operatorname{csch}(x) - \sqrt{\cosh^2(x)}\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right) + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(I + Sinh[x]),x]
```

```
[Out] Sech[x]*(1 - ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + I*Csch[x] + (2*I)*S
inh[x])
```

Maple [A] time = 0.028, size = 35, normalized size = 1.5

$$\frac{i}{2}\tanh\left(\frac{x}{2}\right) + \frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} + \ln\left(\tanh\left(\frac{x}{2}\right)\right) + 2i\left(\tanh\left(\frac{x}{2}\right) + i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(I+sinh(x)),x)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/2*I/tanh(1/2*x)+ln(tanh(1/2*x))+2*I/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.23491, size = 72, normalized size = 3.13

$$-\frac{4\left(-i e^{(-x)} + e^{(-2x)} - 2\right)}{2e^{(-x)} + 2i e^{(-2x)} - 2e^{(-3x)} - 2i} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -4*(-I*e^(-x) + e^(-2*x) - 2)/(2*e^(-x) + 2*I*e^(-2*x) - 2*e^(-3*x) - 2*I)
 - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [B] time = 2.10151, size = 208, normalized size = 9.04

$$-\frac{\left(e^{(3x)} + i e^{(2x)} - e^{x} - i\right)\log\left(e^{x}+1\right) - \left(e^{(3x)} + i e^{(2x)} - e^{x} - i\right)\log\left(e^{x}-1\right) - 2e^{(2x)} - 2i e^{x} + 4}{e^{(3x)} + i e^{(2x)} - e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -((e^(3*x) + I*e^(2*x) - e^x - I)*log(e^x + 1) - (e^(3*x) + I*e^(2*x) - e^x
 - I)*log(e^x - 1) - 2*e^(2*x) - 2*I*e^x + 4)/(e^(3*x) + I*e^(2*x) - e^x -
I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(I+sinh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50181, size = 59, normalized size = 2.57

$$\frac{2\left(e^{(2x)} + ie^{x} - 2\right)}{e^{(3x)} + ie^{(2x)} - e^{x} - i} - \log\left(e^{x} + 1\right) + \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 2*(e^(2*x) + I*e^x - 2)/(e^(3*x) + I*e^(2*x) - e^x - I) - log(e^x + 1) + lo
g(abs(e^x - 1))
```

3.46 $\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=37

$$-2\coth(x)-\frac{3}{2}i\tanh^{-1}(\cosh(x))+\frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{\sinh(x)+i}$$

```
[Out] ((-3*I)/2)*ArcTanh[Cosh[x]] - 2*Coth[x] + ((3*I)/2)*Coth[x]*Csch[x] + (Coth
[x]*Csch[x])/(I + Sinh[x])
```

Rubi [A] time = 0.073395, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$-2\coth(x)-\frac{3}{2}i\tanh^{-1}(\cosh(x))+\frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/2)*ArcTanh[Cosh[x]] - 2*Coth[x] + ((3*I)/2)*Coth[x]*Csch[x] + (Coth
[x]*Csch[x])/(I + Sinh[x])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\mathrm{csch}^3(x)}{i+\sinh(x)}\,dx &= \frac{\coth(x)\mathrm{csch}(x)}{i+\sinh(x)} + \int \mathrm{csch}^3(x)(-3i+2\sinh(x))\,dx \\
&= \frac{\coth(x)\mathrm{csch}(x)}{i+\sinh(x)} - 3i\int \mathrm{csch}^3(x)\,dx + 2\int \mathrm{csch}^2(x)\,dx \\
&= \frac{3}{2}i\coth(x)\mathrm{csch}(x) + \frac{\coth(x)\mathrm{csch}(x)}{i+\sinh(x)} + \frac{3}{2}i\int \mathrm{csch}(x)\,dx - 2i\,\mathrm{Subst}\Big(\int 1\,dx, x, -i\coth(x)\Big) \\
&= -\frac{3}{2}i\tanh^{-1}(\cosh(x)) - 2\coth(x) + \frac{3}{2}i\coth(x)\mathrm{csch}(x) + \frac{\coth(x)\mathrm{csch}(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [A] time = 0.177373, size = 49, normalized size = 1.32

$$\frac{1}{2}i\tanh(x)\left(\mathrm{csch}^3(x) + 2i\mathrm{csch}^2(x) + 3\mathrm{csch}(x) - 3\sqrt{\cosh^2(x)}\mathrm{csch}(x)\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right) + 4i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(I + Sinh[x]),x]
```

```
[Out] (I/2)*(4*I + 3*Csch[x] - 3*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2]*Csch[x]
 + (2*I)*Csch[x]^2 + Csch[x]^3)*Tanh[x]
```

Maple [A] time = 0.031, size = 53, normalized size = 1.4

$$-\frac{1}{2}\tanh\left(\frac{x}{2}\right) - \frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} + \frac{3i}{2}\ln\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - 2\left(\tanh(x/2)+i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(I+sinh(x)),x)
```

```
[Out] -1/2*tanh(1/2*x)-1/8*I*tanh(1/2*x)^2+1/8*I/tanh(1/2*x)^2+3/2*I*ln(tanh(1/2*
x))-1/2/tanh(1/2*x)-2/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.12972, size = 107, normalized size = 2.89

$$-\frac{8\left(e^{(-x)} + 5i\,e^{(-2x)} - 3\,e^{(-3x)} - 3i\,e^{(-4x)} - 4i\right)}{8\,e^{(-x)} + 16i\,e^{(-2x)} - 16\,e^{(-3x)} - 8i\,e^{(-4x)} + 8\,e^{(-5x)} - 8i} - \frac{3}{2}i\log\left(e^{(-x)}+1\right) + \frac{3}{2}i\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -8*(e^(-x) + 5*I*e^(-2*x) - 3*e^(-3*x) - 3*I*e^(-4*x) - 4*I)/(8*e^(-x) + 16
*I*e^(-2*x) - 16*e^(-3*x) - 8*I*e^(-4*x) + 8*e^(-5*x) - 8*I) - 3/2*I*log(e^
(-x) + 1) + 3/2*I*log(e^(-x) - 1)
```

Fricas [B] time = 2.16763, size = 385, normalized size = 10.41

$$\frac{\left(-3i\,e^{(5\,x)}+3\,e^{(4\,x)}+6i\,e^{(3\,x)}-6\,e^{(2\,x)}-3i\,e^{x}+3\right)\log\left(e^{x}+1\right)+\left(3i\,e^{(5\,x)}-3\,e^{(4\,x)}-6i\,e^{(3\,x)}+6\,e^{(2\,x)}+3i\,e^{x}-3\right)\log\left(e^{x}-1\right)}{2\,e^{(5\,x)}+2i\,e^{(4\,x)}-4\,e^{(3\,x)}-4i\,e^{(2\,x)}+2\,e^{x}+2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((-3*I*e^(5*x) + 3*e^(4*x) + 6*I*e^(3*x) - 6*e^(2*x) - 3*I*e^x + 3)*log(e^x
 + 1) + (3*I*e^(5*x) - 3*e^(4*x) - 6*I*e^(3*x) + 6*e^(2*x) + 3*I*e^x - 3)*l
og(e^x - 1) + 6*I*e^(4*x) - 6*e^(3*x) - 10*I*e^(2*x) + 2*e^x + 8*I)/(2*e^(5
*x) + 2*I*e^(4*x) - 4*e^(3*x) - 4*I*e^(2*x) + 2*e^x + 2*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(I+sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29808, size = 69, normalized size = 1.86

$$\frac{i\,e^{(3\,x)}-2\,e^{(2\,x)}+i\,e^{x}+2}{\left(e^{(2\,x)}-1\right)^{2}}+\frac{2i}{e^{x}+i}-\frac{3}{2}i\,\log\left(e^{x}+1\right)+\frac{3}{2}i\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] (I*e^(3*x) - 2*e^(2*x) + I*e^x + 2)/(e^(2*x) - 1)^2 + 2*I/(e^x + I) - 3/2*I
*log(e^x + 1) + 3/2*I*log(abs(e^x - 1))
```

3.47 $\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=47

$$\frac{4}{3}i\coth^3(x)-4i\coth(x)+\frac{3}{2}\tanh^{-1}(\cosh(x))-\frac{3}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{\sinh(x)+i}$$

```
[Out] (3*ArcTanh[Cosh[x]])/2 - (4*I)*Coth[x] + ((4*I)/3)*Coth[x]^3 - (3*Coth[x]*C
sch[x])/2 + (Coth[x]*Csch[x]^2)/(I + Sinh[x])
```

Rubi [A] time = 0.0726019, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 3768, 3770}

$$\frac{4}{3}i\coth^3(x)-4i\coth(x)+\frac{3}{2}\tanh^{-1}(\cosh(x))-\frac{3}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/2 - (4*I)*Coth[x] + ((4*I)/3)*Coth[x]^3 - (3*Coth[x]*C
sch[x])/2 + (Coth[x]*Csch[x]^2)/(I + Sinh[x])
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}+\int \operatorname{csch}^4(x)(-4i+3\sinh(x))\,dx\\ &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}-4i\int \operatorname{csch}^4(x)\,dx+3\int \operatorname{csch}^3(x)\,dx\\ &= -\frac{3}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}-\frac{3}{2}\int \operatorname{csch}(x)\,dx+4\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\coth(x)\right)\\ &= \frac{3}{2}\tanh^{-1}(\cosh(x))-4i\coth(x)+\frac{4}{3}i\coth^3(x)-\frac{3}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.21572, size = 53, normalized size = 1.13

$$\frac{1}{6}\operatorname{sech}(x)\left(-16i\sinh(x)+2i\operatorname{csch}^3(x)-3\operatorname{csch}^2(x)-8i\operatorname{csch}(x)+9\sqrt{\cosh^2(x)}\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right)-9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Sech[x]*(-9 + 9*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] - (8*I)*Csch[x] -
 3*Csch[x]^2 + (2*I)*Csch[x]^3 - (16*I)*Sinh[x]))/6
```

Maple [A] time = 0.035, size = 71, normalized size = 1.5

$$-\frac{7i}{8}\tanh\left(\frac{x}{2}\right)+\frac{i}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{i}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}-\frac{7i}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}-\frac{3}{2}\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(I+sinh(x)),x)
```

```
[Out] -7/8*I*tanh(1/2*x)+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2+1/24*I/tanh(1/2*x
)^3-7/8*I/tanh(1/2*x)-1/8/tanh(1/2*x)^2-3/2*ln(tanh(1/2*x))-2*I/(tanh(1/2*x
)+I)
```

Maxima [B] time = 1.18324, size = 142, normalized size = 3.02

$$\frac{16\left(-7i\,e^{(-x)}+39\,e^{(-2x)}+24i\,e^{(-3x)}-24\,e^{(-4x)}-9i\,e^{(-5x)}+9\,e^{(-6x)}-16\right)}{48\,e^{(-x)}+144i\,e^{(-2x)}-144\,e^{(-3x)}-144i\,e^{(-4x)}+144\,e^{(-5x)}+48i\,e^{(-6x)}-48\,e^{(-7x)}-48i}+\frac{3}{2}\log\left(e^{(-x)}+1\right)-\frac{3}{2}\log\left(e^{(-x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 16*(-7*I*e^(-x) + 39*e^(-2*x) + 24*I*e^(-3*x) - 24*e^(-4*x) - 9*I*e^(-5*x)
+ 9*e^(-6*x) - 16)/(48*e^(-x) + 144*I*e^(-2*x) - 144*e^(-3*x) - 144*I*e^(-4
*x) + 144*e^(-5*x) + 48*I*e^(-6*x) - 48*e^(-7*x) - 48*I) + 3/2*log(e^(-x) +
 1) - 3/2*log(e^(-x) - 1)
```

Fricas [B] time = 2.05862, size = 545, normalized size = 11.6

$$\frac{\left(9\,e^{(7\,x)} + 9i\,e^{(6\,x)} - 27\,e^{(5\,x)} - 27i\,e^{(4\,x)} + 27\,e^{(3\,x)} + 27i\,e^{(2\,x)} - 9\,e^{x} - 9i\right)\log\left(e^{x} + 1\right) - \left(9\,e^{(7\,x)} + 9i\,e^{(6\,x)} - 27\,e^{(5\,x)} - 27i\,e\right.}{6\,e^{(7\,x)} + 6i\,e^{(6\,x)} - 18\,e^{(5\,x)} - 18i\,e^{(4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((9*e^(7*x) + 9*I*e^(6*x) - 27*e^(5*x) - 27*I*e^(4*x) + 27*e^(3*x) + 27*I*e
^(2*x) - 9*e^x - 9*I)*log(e^x + 1) - (9*e^(7*x) + 9*I*e^(6*x) - 27*e^(5*x)
- 27*I*e^(4*x) + 27*e^(3*x) + 27*I*e^(2*x) - 9*e^x - 9*I)*log(e^x - 1) - 18
*e^(6*x) - 18*I*e^(5*x) + 48*e^(4*x) + 48*I*e^(3*x) - 78*e^(2*x) - 14*I*e^x
 + 32)/(6*e^(7*x) + 6*I*e^(6*x) - 18*e^(5*x) - 18*I*e^(4*x) + 18*e^(3*x) +
18*I*e^(2*x) - 6*e^x - 6*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(I+sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25883, size = 78, normalized size = 1.66

$$-\frac{2}{e^{x} + i} - \frac{3\,e^{(5\,x)} + 6i\,e^{(4\,x)} - 24i\,e^{(2\,x)} - 3\,e^{x} + 10i}{3\,\left(e^{(2\,x)} - 1\right)^{3}} + \frac{3}{2}\,\log\left(e^{x} + 1\right) - \frac{3}{2}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -2/(e^x + I) - 1/3*(3*e^(5*x) + 6*I*e^(4*x) - 24*I*e^(2*x) - 3*e^x + 10*I)/
(e^(2*x) - 1)^3 + 3/2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

3.48 $\int \frac{\sinh^4(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=58

$$-\frac{7x}{2}-\frac{16}{3}i\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}-\frac{8\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)}+\frac{7}{2}\sinh(x)\cosh(x)$$

```
[Out] (-7*x)/2 - ((16*I)/3)*Cosh[x] + (7*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^3)
/(3*(I + Sinh[x])^2) - (8*Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x]))
```

Rubi [A] time = 0.101936, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2765, 2977, 2734}

$$-\frac{7x}{2}-\frac{16}{3}i\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}-\frac{8\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)}+\frac{7}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-7*x)/2 - ((16*I)/3)*Cosh[x] + (7*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^3)
/(3*(I + Sinh[x])^2) - (8*Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2} + \frac{1}{3}\int \frac{\sinh^2(x)(-3i+5\sinh(x))}{i+\sinh(x)}\,dx$$
$$= -\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2} - \frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))} - \frac{1}{3}i\int (16+21i\sinh(x))\sinh(x)\,dx$$
$$= -\frac{7x}{2} - \frac{16}{3}i\cosh(x) + \frac{7}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2} - \frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))}$$

Mathematica [B] time = 0.185145, size = 147, normalized size = 2.53

$$-\frac{\sinh^3(x)\cosh(x)}{2(1-i\sinh(x))^2} - \frac{i\sqrt{2}\sqrt{1+\frac{1}{2}(-1+i\sinh(x))}\cosh(x)}{\sqrt{1+i\sinh(x)}} - \frac{31i\cosh(x)}{6(1-i\sinh(x))} + \frac{5i\cosh(x)}{6(1-i\sinh(x))^2} - \frac{7i\cosh(x)\sin^{-1}\left(\sqrt{\ldots}\right.}{\sqrt{1-i\sinh(x)}\sqrt{1+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (((5*I)/6)*Cosh[x])/(1 - I*Sinh[x])^2 - (((31*I)/6)*Cosh[x])/(1 - I*Sinh[x]
) - (I*Sqrt[2]*Cosh[x]*Sqrt[1 + (-1 + I*Sinh[x])/2])/Sqrt[1 + I*Sinh[x]] -
((7*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Cosh[x])/(Sqrt[1 - I*Sinh[x]]*Sq
rt[1 + I*Sinh[x]]) - (Cosh[x]*Sinh[x]^3)/(2*(1 - I*Sinh[x])^2)
```

Maple [B] time = 0.055, size = 116, normalized size = 2.

$$\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - 2i\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \frac{7}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1} + 2i\left(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(I+sinh(x))^2,x)
```

```
[Out] 1/2/(tanh(1/2*x)+1)-2*I/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2-7/2*ln(tanh(1
/2*x)+1)+1/2/(tanh(1/2*x)-1)+2*I/(tanh(1/2*x)-1)+1/2/(tanh(1/2*x)-1)^2+7/2*
ln(tanh(1/2*x)-1)+2*I/(tanh(1/2*x)+I)^2+4/3/(tanh(1/2*x)+I)^3+6/(tanh(1/2*x
)+I)
```

Maxima [A] time = 1.23467, size = 96, normalized size = 1.66

$$-\frac{7}{2}x + \frac{30e^{(-x)} + 478ie^{(-2x)} - 810e^{(-3x)} - 432ie^{(-4x)} + 6i}{16\left(3ie^{(-2x)} - 9e^{(-3x)} - 9ie^{(-4x)} + 3e^{(-5x)}\right)} - ie^{(-x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -7/2*x + 1/16*(30*e^(-x) + 478*I*e^(-2*x) - 810*e^(-3*x) - 432*I*e^(-4*x) +
 6*I)/(3*I*e^(-2*x) - 9*e^(-3*x) - 9*I*e^(-4*x) + 3*e^(-5*x)) - I*e^(-x) -
1/8*e^(-2*x)
```

Fricas [B] time = 2.03498, size = 275, normalized size = 4.74

$$-\frac{21\,(4\,x-3)e^{(5\,x)}-(-252i\,x-147i)e^{(4\,x)}-3\,(84\,x+127)e^{(3\,x)}-(84i\,x+239i)e^{(2\,x)}-3\,e^{(7\,x)}+15i\,e^{(6\,x)}+15\,e^{x}-3i}{24\,e^{(5\,x)}+72i\,e^{(4\,x)}-72\,e^{(3\,x)}-24i\,e^{(2\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(21*(4*x - 3)*e^(5*x) - (-252*I*x - 147*I)*e^(4*x) - 3*(84*x + 127)*e^(3*x
) - (84*I*x + 239*I)*e^(2*x) - 3*e^(7*x) + 15*I*e^(6*x) + 15*e^x - 3*I)/(24
*e^(5*x) + 72*I*e^(4*x) - 72*e^(3*x) - 24*I*e^(2*x))
```

Sympy [A] time = 0.531348, size = 68, normalized size = 1.17

$$-\frac{7x}{2}+\frac{-8ie^{2x}+14e^{x}+\frac{22i}{3}}{e^{3x}+3ie^{2x}-3e^{x}-i}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] -7*x/2 + (-8*I*exp(2*x) + 14*exp(x) + 22*I/3)/(exp(3*x) + 3*I*exp(2*x) - 3*
exp(x) - I) + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

Giac [A] time = 1.38319, size = 68, normalized size = 1.17

$$-\frac{7}{2}\,x-\frac{\left(216i\,e^{(4\,x)}-405\,e^{(3\,x)}-239i\,e^{(2\,x)}+15\,e^{x}-3i\right)e^{(-2\,x)}}{24\,(e^{x}+i)^{3}}+\frac{1}{8}\,e^{(2\,x)}-i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -7/2*x - 1/24*(216*I*e^(4*x) - 405*e^(3*x) - 239*I*e^(2*x) + 15*e^x - 3*I)*
e^(-2*x)/(e^x + I)^3 + 1/8*e^(2*x) - I*e^x
```

3.49 $\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=44

$$-2ix + \frac{4\cosh(x)}{3} - \frac{\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)^2} + \frac{2i\cosh(x)}{\sinh(x)+i}$$

```
[Out] (-2*I)*x + (4*Cosh[x])/3 - (Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x])^2) + ((2*I)
*Cosh[x])/(I + Sinh[x])
```

Rubi [A] time = 0.129489, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2765, 2968, 3023, 12, 2735, 2648}

$$-2ix + \frac{4\cosh(x)}{3} - \frac{\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)^2} + \frac{2i\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-2*I)*x + (4*Cosh[x])/3 - (Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x])^2) + ((2*I)
*Cosh[x])/(I + Sinh[x])
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx &= -\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int \frac{\sinh(x)(-2i+4\sinh(x))}{i+\sinh(x)}\,dx \\
&= -\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-\frac{1}{3}i\int \frac{2\sinh(x)+4i\sinh^2(x)}{i+\sinh(x)}\,dx \\
&= \frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int -\frac{6i\sinh(x)}{i+\sinh(x)}\,dx \\
&= \frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-2i\int \frac{\sinh(x)}{i+\sinh(x)}\,dx \\
&= -2ix+\frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-2\int \frac{1}{i+\sinh(x)}\,dx \\
&= -2ix+\frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{2i\cosh(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [A] time = 0.120159, size = 45, normalized size = 1.02

$$\frac{1}{3}\cosh(x)\left(\frac{3\sinh^2(x)+14i\sinh(x)-10}{(\sinh(x)+i)^2}-\frac{6i\sinh^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*(((-6*I)*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (-10 + (14*I)*Sinh[x]
 + 3*Sinh[x]^2)/(I + Sinh[x])^2))/3
```

Maple [B] time = 0.05, size = 75, normalized size = 1.7

$$-2\,i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+2\,i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+\frac{4\,i}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}+4\,i\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(I+sinh(x))^2,x)
```

```
[Out] -2*I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)+2*I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*
x)-1)+4/3*I/(tanh(1/2*x)+I)^3+4*I/(tanh(1/2*x)+I)-2/(tanh(1/2*x)+I)^2
```

Maxima [A] time = 1.31374, size = 80, normalized size = 1.82

$$-2ix-\frac{164e^{(-x)}+276ie^{(-2x)}-156e^{(-3x)}-12i}{8\left(3ie^{(-x)}-9e^{(-2x)}-9ie^{(-3x)}+3e^{(-4x)}\right)}+\frac{1}{2}e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*I*x - 1/8*(164*e^(-x) + 276*I*e^(-2*x) - 156*e^(-3*x) - 12*I)/(3*I*e^(-x
) - 9*e^(-2*x) - 9*I*e^(-3*x) + 3*e^(-4*x)) + 1/2*e^(-x)
```

Fricas [B] time = 2.06003, size = 217, normalized size = 4.93

$$\frac{(-12ix+9i)e^{(4x)}+6(6x+5)e^{(3x)}+(36ix+66i)e^{(2x)}-(12x+41)e^{x}+3e^{(5x)}-3i}{6e^{(4x)}+18ie^{(3x)}-18e^{(2x)}-6ie^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((-12*I*x + 9*I)*e^(4*x) + 6*(6*x + 5)*e^(3*x) + (36*I*x + 66*I)*e^(2*x) -
(12*x + 41)*e^x + 3*e^(5*x) - 3*I)/(6*e^(4*x) + 18*I*e^(3*x) - 18*e^(2*x) -
 6*I*e^x)
```

Sympy [A] time = 0.401296, size = 53, normalized size = 1.2

$$-2ix+\frac{6e^{2x}+10ie^{x}-\frac{16}{3}}{e^{3x}+3ie^{2x}-3e^{x}-i}+\frac{e^{x}}{2}+\frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] -2*I*x + (6*exp(2*x) + 10*I*exp(x) - 16/3)/(exp(3*x) + 3*I*exp(2*x) - 3*exp
(x) - I) + exp(x)/2 + exp(-x)/2
```

Giac [A] time = 1.38429, size = 51, normalized size = 1.16

$$-2ix+\frac{\left(39e^{(3x)}+69ie^{(2x)}-41e^{x}-3i\right)e^{(-x)}}{6(e^{x}+i)^{3}}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*I*x + 1/6*(39*e^(3*x) + 69*I*e^(2*x) - 41*e^x - 3*I)*e^(-x)/(e^x + I)^3
+ 1/2*e^x
```

3.50 $\int \frac{\sinh^2(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=32

$$x - \frac{5\cosh(x)}{3(\sinh(x)+i)} + \frac{i\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[Out] x + ((I/3)*Cosh[x])/(I + Sinh[x])^2 - (5*Cosh[x])/(3*(I + Sinh[x]))
```

Rubi [A] time = 0.0609489, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2758, 2735, 2648}

$$x - \frac{5\cosh(x)}{3(\sinh(x)+i)} + \frac{i\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] x + ((I/3)*Cosh[x])/(I + Sinh[x])^2 - (5*Cosh[x])/(3*(I + Sinh[x]))
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{(i+\sinh(x))^2} dx &= \frac{i\cosh(x)}{3(i+\sinh(x))^2} + \frac{1}{3}\int \frac{-2i+3\sinh(x)}{i+\sinh(x)} dx \\
&= x + \frac{i\cosh(x)}{3(i+\sinh(x))^2} - \frac{5}{3}i\int \frac{1}{i+\sinh(x)} dx \\
&= x + \frac{i\cosh(x)}{3(i+\sinh(x))^2} - \frac{5\cosh(x)}{3(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.140612, size = 55, normalized size = 1.72

$$-\frac{1}{3}i\cosh(x)\left(\frac{4-5i\sinh(x)}{(\sinh(x)+i)^2}-\frac{6\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (-I/3)*Cosh[x]*((-6*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2] +
(4 - (5*I)*Sinh[x])/(I + Sinh[x])^2)
```

Maple [B] time = 0.045, size = 52, normalized size = 1.6

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-2i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}-\frac{4}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}-2\left(\tanh(x/2)+i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(I+sinh(x))^2,x)
```

```
[Out] ln(tanh(1/2*x)+1)-ln(tanh(1/2*x)-1)-2*I/(tanh(1/2*x)+I)^2-4/3/(tanh(1/2*x)+
I)^3-2/(tanh(1/2*x)+I)
```

Maxima [A] time = 1.21758, size = 54, normalized size = 1.69

$$x-\frac{72\,e^{(-x)}+48i\,e^{(-2\,x)}-40i}{4\left(9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] x - 1/4*(72*e^(-x) + 48*I*e^(-2*x) - 40*I)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(
-3*x) - 3*I)
```

Fricas [B] time = 2.074, size = 150, normalized size = 4.69

$$\frac{3\,xe^{(3\,x)}+(9i\,x+12i)e^{(2\,x)}-9\,(x+2)e^{x}-3i\,x-10i}{3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (9*I*x + 12*I)*e^(2*x) - 9*(x + 2)*e^x - 3*I*x - 10*I)/(3*e^
(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

Sympy [A] time = 0.303963, size = 39, normalized size = 1.22

$$x + \frac{4ie^{2x} - 6e^{x} - \frac{10i}{3}}{e^{3x} + 3ie^{2x} - 3e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(I+sinh(x))**2,x)
```

```
[Out] x + (4*I*exp(2*x) - 6*exp(x) - 10*I/3)/(exp(3*x) + 3*I*exp(2*x) - 3*exp(x)
- I)
```

Giac [A] time = 1.32251, size = 30, normalized size = 0.94

$$x - \frac{-12i\,e^{(2\,x)} + 18\,e^{x} + 10i}{3\,(e^{x} + i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] x - 1/3*(-12*I*e^(2*x) + 18*e^x + 10*I)/(e^x + I)^3
```

3.51 $\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=31

$$-\frac{2i\cosh(x)}{3(\sinh(x)+i)}-\frac{\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[Out] -Cosh[x]/(3*(I + Sinh[x])^2) - (((2*I)/3)*Cosh[x])/(I + Sinh[x])
```

Rubi [A] time = 0.0295506, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2750, 2648}

$$-\frac{2i\cosh(x)}{3(\sinh(x)+i)}-\frac{\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(I + Sinh[x])^2,x]
```

```
[Out] -Cosh[x]/(3*(I + Sinh[x])^2) - (((2*I)/3)*Cosh[x])/(I + Sinh[x])
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx &= -\frac{\cosh(x)}{3(i+\sinh(x))^2}+\frac{2}{3}\int \frac{1}{i+\sinh(x)}\,dx\\ &= -\frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{2i\cosh(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.0087043, size = 22, normalized size = 0.71

$$\frac{(1-2i\sinh(x))\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(I + Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*(1 - (2*I)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

Maple [A] time = 0.033, size = 25, normalized size = 0.8

$$2\left(\tanh\left(x/2\right)+i\right)^{-2}-\frac{4\,i}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(I+sinh(x))^2,x)
```

```
[Out] 2/(tanh(1/2*x)+I)^2-4/3*I/(tanh(1/2*x)+I)^3
```

Maxima [B] time = 1.21631, size = 109, normalized size = 3.52

$$-\frac{6i\,e^{(-x)}}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}+\frac{6\,e^{(-2\,x)}}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}-\frac{4}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -6*I*e^(-x)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I) + 6*e^(-2*x)/(9*e^
(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I) - 4/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^
(-3*x) - 3*I)
```

Fricas [A] time = 1.98499, size = 95, normalized size = 3.06

$$-\frac{2\left(3\,e^{(2\,x)}+3i\,e^{x}-2\right)}{3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(3*e^(2*x) + 3*I*e^x - 2)/(3*e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

Sympy [A] time = 0.261966, size = 36, normalized size = 1.16

$$\frac{-2e^{2x}-2ie^{x}+\frac{4}{3}}{e^{3x}+3ie^{2x}-3e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))**2,x)
```

```
[Out] (-2*exp(2*x) - 2*I*exp(x) + 4/3)/(exp(3*x) + 3*I*exp(2*x) - 3*exp(x) - I)
```

Giac [A] time = 1.30797, size = 27, normalized size = 0.87

$$-\frac{2\left(3\,e^{(2\,x)} + 3i\,e^{x} - 2\right)}{3\,(e^{x} + i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) + 3*I*e^x - 2)/(e^x + I)^3
```

3.52 $\int \frac{\text{csch}(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=34

$$-\frac{4i\cosh(x)}{3(\sinh(x)+i)}+\frac{\cosh(x)}{3(\sinh(x)+i)^2}+\tanh^{-1}(\cosh(x))$$

```
[Out] ArcTanh[Cosh[x]] + Cosh[x]/(3*(I + Sinh[x])^2) - (((4*I)/3)*Cosh[x])/(I + S
inh[x])
```

Rubi [A] time = 0.0826279, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2766, 2978, 12, 3770}

$$-\frac{4i\cosh(x)}{3(\sinh(x)+i)}+\frac{\cosh(x)}{3(\sinh(x)+i)^2}+\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(I + Sinh[x])^2,x]
```

```
[Out] ArcTanh[Cosh[x]] + Cosh[x]/(3*(I + Sinh[x])^2) - (((4*I)/3)*Cosh[x])/(I + S
inh[x])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\text{csch}(x)}{(i+\sinh(x))^2}\,dx = \frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{1}{3}\int \frac{\text{csch}(x)(3i-\sinh(x))}{i+\sinh(x)}\,dx$$
$$= \frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{4i\cosh(x)}{3(i+\sinh(x))} + \frac{1}{3}i\int 3i\text{csch}(x)\,dx$$
$$= \frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{4i\cosh(x)}{3(i+\sinh(x))} - \int \text{csch}(x)\,dx$$
$$= \tanh^{-1}(\cosh(x)) + \frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{4i\cosh(x)}{3(i+\sinh(x))}$$

Mathematica [B] time = 0.0867691, size = 91, normalized size = 2.68

$$\frac{\cosh\left(\frac{x}{2}\right)\left(6-9\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\cosh\left(\frac{3x}{2}\right)\left(3\log\left(\tanh\left(\frac{x}{2}\right)\right)-8\right)+6i\sinh\left(\frac{x}{2}\right)\left(2\log\left(\tanh\left(\frac{x}{2}\right)\right)+\cosh(x)\log\left(\tan\right.\right.}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(I + Sinh[x])^2,x]
```

```
[Out] (Cosh[x/2]*(6 - 9*Log[Tanh[x/2]]) + Cosh[(3*x)/2]*(-8 + 3*Log[Tanh[x/2]]) +
 (6*I)*(-3 + 2*Log[Tanh[x/2]] + Cosh[x]*Log[Tanh[x/2]])*Sinh[x/2])/(6*(Cosh
[x/2] - I*Sinh[x/2])^3)
```

Maple [A] time = 0.039, size = 44, normalized size = 1.3

$$-\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{4i}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}-4i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}-2\left(\tanh(x/2)+i\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+sinh(x))^2,x)
```

```
[Out] -ln(tanh(1/2*x))+4/3*I/(tanh(1/2*x)+I)^3-4*I/(tanh(1/2*x)+I)-2/(tanh(1/2*x)
+I)^2
```

Maxima [B] time = 1.226, size = 74, normalized size = 2.18

$$\frac{2\left(-9i\,e^{(-x)}+3\,e^{(-2\,x)}-4\right)}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}+\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(-9*I*e^(-x) + 3*e^(-2*x) - 4)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*
I) + log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] time = 2.09156, size = 240, normalized size = 7.06

$$\frac{\left(3\,e^{(3\,x)} + 9i\,e^{(2\,x)} - 9\,e^{x} - 3i\right)\log\left(e^{x} + 1\right) - \left(3\,e^{(3\,x)} + 9i\,e^{(2\,x)} - 9\,e^{x} - 3i\right)\log\left(e^{x} - 1\right) - 6\,e^{(2\,x)} - 18i\,e^{x} + 8}{3\,e^{(3\,x)} + 9i\,e^{(2\,x)} - 9\,e^{x} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((3*e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)*log(e^x + 1) - (3*e^(3*x) + 9*I*e^
(2*x) - 9*e^x - 3*I)*log(e^x - 1) - 6*e^(2*x) - 18*I*e^x + 8)/(3*e^(3*x) +
9*I*e^(2*x) - 9*e^x - 3*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3646, size = 46, normalized size = 1.35

$$-\frac{2\left(3\,e^{(2\,x)} + 9i\,e^{x} - 4\right)}{3\left(e^{x} + i\right)^{3}} + \log\left(e^{x} + 1\right) - \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) + 9*I*e^x - 4)/(e^x + I)^3 + log(e^x + 1) - log(abs(e^x - 1
))
```

3.53 $\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\, dx$

Optimal. Leaf size=42

$$\frac{10\coth(x)}{3}+2i\tanh^{-1}(\cosh(x))-\frac{2i\coth(x)}{\sinh(x)+i}+\frac{\coth(x)}{3(\sinh(x)+i)^2}$$

```
[Out] (2*I)*ArcTanh[Cosh[x]] + (10*Coth[x])/3 + Coth[x]/(3*(I + Sinh[x])^2) - ((2
*I)*Coth[x])/(I + Sinh[x])
```

Rubi [A] time = 0.117322, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{10\coth(x)}{3}+2i\tanh^{-1}(\cosh(x))-\frac{2i\coth(x)}{\sinh(x)+i}+\frac{\coth(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*ArcTanh[Cosh[x]] + (10*Coth[x])/3 + Coth[x]/(3*(I + Sinh[x])^2) - ((2
*I)*Coth[x])/(I + Sinh[x])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx &= \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{1}{3}\int \frac{\operatorname{csch}^2(x)(4i-2\sinh(x))}{i+\sinh(x)}\,dx \\
&= \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)} + \frac{1}{3}\int \operatorname{csch}^2(x)(-10-6i\sinh(x))\,dx \\
&= \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)} - 2i\int \operatorname{csch}(x)\,dx - \frac{10}{3}\int \operatorname{csch}^2(x)\,dx \\
&= 2i\tanh^{-1}(\cosh(x)) + \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)} + \frac{10}{3}i\,\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right) \\
&= 2i\tanh^{-1}(\cosh(x)) + \frac{10\coth(x)}{3} + \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)}
\end{aligned}
$$

Mathematica [B] time = 0.35467, size = 88, normalized size = 2.1

$$
\frac{1}{6}\left(\frac{2}{\sinh(x)+i} + 3\tanh\left(\frac{x}{2}\right) + 3\coth\left(\frac{x}{2}\right) - 12i\log\left(\sinh\left(\frac{x}{2}\right)\right) + 12i\log\left(\cosh\left(\frac{x}{2}\right)\right) - \frac{4\sinh\left(\frac{x}{2}\right)(7\sinh(x)+8i)}{\left(\sinh\left(\frac{x}{2}\right)+i\cosh\left(\frac{x}{2}\right)\right)^3}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (3*Coth[x/2] + (12*I)*Log[Cosh[x/2]] - (12*I)*Log[Sinh[x/2]] + 2/(I + Sinh[
x]) - (4*Sinh[x/2]*(8*I + 7*Sinh[x]))/(I*Cosh[x/2] + Sinh[x/2])^3 + 3*Tanh[
x/2])/6
```

Maple [A] time = 0.043, size = 58, normalized size = 1.4

$$
\frac{1}{2}\tanh\left(\frac{x}{2}\right) - 2i\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - 2i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2} - \frac{4}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3} + 6\left(\tanh(x/2)+i\right)^{-1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(I+sinh(x))^2,x)
```

```
[Out] 1/2*tanh(1/2*x)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)-2*I/(tanh(1/2*x)+I)^2-4
/3/(tanh(1/2*x)+I)^3+6/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.23921, size = 109, normalized size = 2.6

$$\frac{4\left(12\,e^{(-x)}+11i\,e^{(-2\,x)}-9\,e^{(-3\,x)}-3i\,e^{(-4\,x)}-5i\right)}{9\,e^{(-x)}+12i\,e^{(-2\,x)}-12\,e^{(-3\,x)}-9i\,e^{(-4\,x)}+3\,e^{(-5\,x)}-3i}+2i\,\log\left(e^{(-x)}+1\right)-2i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 4*(12*e^(-x) + 11*I*e^(-2*x) - 9*e^(-3*x) - 3*I*e^(-4*x) - 5*I)/(9*e^(-x) +
 12*I*e^(-2*x) - 12*e^(-3*x) - 9*I*e^(-4*x) + 3*e^(-5*x) - 3*I) + 2*I*log(e
^(-x) + 1) - 2*I*log(e^(-x) - 1)
```

Fricas [B] time = 2.04185, size = 404, normalized size = 9.62

$$\frac{\left(6i\,e^{(5\,x)}-18\,e^{(4\,x)}-24i\,e^{(3\,x)}+24\,e^{(2\,x)}+18i\,e^{x}-6\right)\log\left(e^{x}+1\right)+\left(-6i\,e^{(5\,x)}+18\,e^{(4\,x)}+24i\,e^{(3\,x)}-24\,e^{(2\,x)}-18i\,e^{x}+6\right.}{3\,e^{(5\,x)}+9i\,e^{(4\,x)}-12\,e^{(3\,x)}-12i\,e^{(2\,x)}+9\,e^{x}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((6*I*e^(5*x) - 18*e^(4*x) - 24*I*e^(3*x) + 24*e^(2*x) + 18*I*e^x - 6)*log(
e^x + 1) + (-6*I*e^(5*x) + 18*e^(4*x) + 24*I*e^(3*x) - 24*e^(2*x) - 18*I*e^
x + 6)*log(e^x - 1) - 12*I*e^(4*x) + 36*e^(3*x) + 44*I*e^(2*x) - 48*e^x - 2
0*I)/(3*e^(5*x) + 9*I*e^(4*x) - 12*e^(3*x) - 12*I*e^(2*x) + 9*e^x + 3*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32053, size = 62, normalized size = 1.48

$$\frac{2}{e^{(2\,x)}-1}-\frac{2\left(6i\,e^{(2\,x)}-15\,e^{x}-7i\right)}{3\left(e^{x}+i\right)^{3}}+2i\,\log\left(e^{x}+1\right)-2i\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2/(e^(2*x) - 1) - 2/3*(6*I*e^(2*x) - 15*e^x - 7*I)/(e^x + I)^3 + 2*I*log(e^
x + 1) - 2*I*log(abs(e^x - 1))
```

3.54 $\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=58

$$\frac{16}{3}i\coth(x)-\frac{7}{2}\tanh^{-1}(\cosh(x))+\frac{7}{2}\coth(x)\operatorname{csch}(x)-\frac{8i\coth(x)\operatorname{csch}(x)}{3(\sinh(x)+i)}+\frac{\coth(x)\operatorname{csch}(x)}{3(\sinh(x)+i)^2}$$

```
[Out] (-7*ArcTanh[Cosh[x]])/2 + ((16*I)/3)*Coth[x] + (7*Coth[x]*Csch[x])/2 + (Cot
h[x]*Csch[x])/(3*(I + Sinh[x])^2) - (((8*I)/3)*Coth[x]*Csch[x])/(I + Sinh[x
])
```

Rubi [A] time = 0.139687, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$\frac{16}{3}i\coth(x)-\frac{7}{2}\tanh^{-1}(\cosh(x))+\frac{7}{2}\coth(x)\operatorname{csch}(x)-\frac{8i\coth(x)\operatorname{csch}(x)}{3(\sinh(x)+i)}+\frac{\coth(x)\operatorname{csch}(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(I + Sinh[x])^2,x]
```


```
[Out] (-7*ArcTanh[Cosh[x]])/2 + ((16*I)/3)*Coth[x] + (7*Coth[x]*Csch[x])/2 + (Cot
h[x]*Csch[x])/(3*(I + Sinh[x])^2) - (((8*I)/3)*Coth[x]*Csch[x])/(I + Sinh[x
])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx &= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{1}{3}\int \frac{\operatorname{csch}^3(x)(5i-3\sinh(x))}{i+\sinh(x)}\,dx\\
&= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))} + \frac{1}{3}\int \operatorname{csch}^3(x)(-21-16i\sinh(x))\,dx\\
&= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))} - \frac{16}{3}i\int \operatorname{csch}^2(x)\,dx - 7\int \operatorname{csch}^3(x)\,dx\\
&= \frac{7}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))} + \frac{7}{2}\int \operatorname{csch}(x)\,dx - \frac{16}{3}\operatorname{Subst}(\int 1
\\
&= -\frac{7}{2}\tanh^{-1}(\cosh(x)) + \frac{16}{3}i\coth(x) + \frac{7}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [B] time = 0.311149, size = 131, normalized size = 2.26

$$\frac{1}{24}\left(24i\tanh\left(\frac{x}{2}\right) + 24i\coth\left(\frac{x}{2}\right) + 3\operatorname{csch}^2\left(\frac{x}{2}\right) + 3\operatorname{sech}^2\left(\frac{x}{2}\right) + 84\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{160i\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)} + \frac{}{(\cosh(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] ((24*I)*Coth[x/2] + 3*Csch[x/2]^2 + 84*Log[Tanh[x/2]] + 3*Sech[x/2]^2 + 8/(
Cosh[x/2] - I*Sinh[x/2])^2 + ((160*I)*Sinh[x/2])/(Cosh[x/2] - I*Sinh[x/2])
+ (16*Sinh[x/2])/(I*Cosh[x/2] + Sinh[x/2])^3 + (24*I)*Tanh[x/2])/24
```

Maple [A] time = 0.047, size = 76, normalized size = 1.3

$$i\tanh\left(\frac{x}{2}\right) - \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + i\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} + \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} + \frac{7}{2}\ln\left(\tanh\left(\frac{x}{2}\right)\right) + 8i\left(\tanh\left(\frac{x}{2}\right) + i\right)^{-1} - \frac{4i}{3}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(I+sinh(x))^2,x)
```

```
[Out] I*tanh(1/2*x)-1/8*tanh(1/2*x)^2+I/tanh(1/2*x)+1/8/tanh(1/2*x)^2+7/2*ln(tanh
(1/2*x))+8*I/(tanh(1/2*x)+I)-4/3*I/(tanh(1/2*x)+I)^3+2/(tanh(1/2*x)+I)^2
```

Maxima [B] time = 1.23323, size = 142, normalized size = 2.45

$$-\frac{8\left(-75i\,e^{(-x)}+97\,e^{(-2\,x)}+126i\,e^{(-3\,x)}-98\,e^{(-4\,x)}-63i\,e^{(-5\,x)}+21\,e^{(-6\,x)}-32\right)}{72\,e^{(-x)}+120i\,e^{(-2\,x)}-168\,e^{(-3\,x)}-168i\,e^{(-4\,x)}+120\,e^{(-5\,x)}+72i\,e^{(-6\,x)}-24\,e^{(-7\,x)}-24i}-\frac{7}{2}\,\log\left(e^{(-x)}+1\right)+\frac{7}{2}\,\log\left(e^{(-x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -8*(-75*I*e^(-x) + 97*e^(-2*x) + 126*I*e^(-3*x) - 98*e^(-4*x) - 63*I*e^(-5*
x) + 21*e^(-6*x) - 32)/(72*e^(-x) + 120*I*e^(-2*x) - 168*e^(-3*x) - 168*I*e
^(-4*x) + 120*e^(-5*x) + 72*I*e^(-6*x) - 24*e^(-7*x) - 24*I) - 7/2*log(e^(-
x) + 1) + 7/2*log(e^(-x) - 1)
```

Fricas [B] time = 2.10982, size = 578, normalized size = 9.97

$$-\frac{\left(21\,e^{(7\,x)}+63i\,e^{(6\,x)}-105\,e^{(5\,x)}-147i\,e^{(4\,x)}+147\,e^{(3\,x)}+105i\,e^{(2\,x)}-63\,e^{x}-21i\right)\log\left(e^{x}+1\right)-\left(21\,e^{(7\,x)}+63i\,e^{(6\,x)}-105\,\right.}{6\,e^{(7\,x)}+18i\,e^{(6\,x)}-30\,e^{(5\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((21*e^(7*x) + 63*I*e^(6*x) - 105*e^(5*x) - 147*I*e^(4*x) + 147*e^(3*x) +
105*I*e^(2*x) - 63*e^x - 21*I)*log(e^x + 1) - (21*e^(7*x) + 63*I*e^(6*x) -
105*e^(5*x) - 147*I*e^(4*x) + 147*e^(3*x) + 105*I*e^(2*x) - 63*e^x - 21*I)*
log(e^x - 1) - 42*e^(6*x) - 126*I*e^(5*x) + 196*e^(4*x) + 252*I*e^(3*x) - 1
94*e^(2*x) - 150*I*e^x + 64)/(6*e^(7*x) + 18*I*e^(6*x) - 30*e^(5*x) - 42*I*
e^(4*x) + 42*e^(3*x) + 30*I*e^(2*x) - 18*e^x - 6*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36617, size = 80, normalized size = 1.38

$$\frac{e^{(3\,x)}+4i\,e^{(2\,x)}+e^{x}-4i}{\left(e^{(2\,x)}-1\right)^{2}}+\frac{2\left(9\,e^{(2\,x)}+21i\,e^{x}-10\right)}{3\left(e^{x}+i\right)^{3}}-\frac{7}{2}\,\log\left(e^{x}+1\right)+\frac{7}{2}\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] (e^(3*x) + 4*I*e^(2*x) + e^x - 4*I)/(e^(2*x) - 1)^2 + 2/3*(9*e^(2*x) + 21*I
*e^x - 10)/(e^x + I)^3 - 7/2*log(e^x + 1) + 7/2*log(abs(e^x - 1))
```

3.55 $\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=64

$$4\coth^3(x) - 12\coth(x) - 5i\tanh^{-1}(\cosh(x)) + 5i\coth(x)\operatorname{csch}(x) - \frac{10i\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)} + \frac{\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)^2}$$

```
[Out] (-5*I)*ArcTanh[Cosh[x]] - 12*Coth[x] + 4*Coth[x]^3 + (5*I)*Coth[x]*Csch[x]
+ (Coth[x]*Csch[x]^2)/(3*(I + Sinh[x])^2) - (((10*I)/3)*Coth[x]*Csch[x]^2)/
(I + Sinh[x])
```

Rubi [A] time = 0.130932, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 3768, 3770}

$$4\coth^3(x) - 12\coth(x) - 5i\tanh^{-1}(\cosh(x)) + 5i\coth(x)\operatorname{csch}(x) - \frac{10i\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)} + \frac{\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-5*I)*ArcTanh[Cosh[x]] - 12*Coth[x] + 4*Coth[x]^3 + (5*I)*Coth[x]*Csch[x]
+ (Coth[x]*Csch[x]^2)/(3*(I + Sinh[x])^2) - (((10*I)/3)*Coth[x]*Csch[x]^2)/
(I + Sinh[x])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx &= \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{1}{3}\int \frac{\operatorname{csch}^4(x)(6i-4\sinh(x))}{i+\sinh(x)}\,dx\\ &= \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))} + \frac{1}{3}\int \operatorname{csch}^4(x)(-36-30i\sinh(x))\,dx\\ &= \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))} - 10i\int \operatorname{csch}^3(x)\,dx - 12\int \operatorname{csch}^4(x)\,dx\\ &= 5i\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))} + 5i\int \operatorname{csch}(x)\,dx - 12i\,\mathrm{Subst}\Big(\\ &= -5i\tanh^{-1}(\cosh(x)) - 12\coth(x) + 4\coth^3(x) + 5i\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i}{3}\end{aligned}$$

Mathematica [B] time = 1.5293, size = 143, normalized size = 2.23

$$\frac{1}{24}\left(-44\coth\left(\frac{x}{2}\right) + 6i\operatorname{csch}^2\left(\frac{x}{2}\right) + \frac{1}{2}\sinh(x)\operatorname{csch}^4\left(\frac{x}{2}\right) + 2\left(-\frac{4}{\sinh(x)+i} - 22\tanh\left(\frac{x}{2}\right) + 3i\operatorname{sech}^2\left(\frac{x}{2}\right) + 60i\log\left(\mathrm{si}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-44*Coth[x/2] + (6*I)*Csch[x/2]^2 + (Csch[x/2]^4*Sinh[x])/2 + 2*((-60*I)*L
og[Cosh[x/2]] + (60*I)*Log[Sinh[x/2]] + (3*I)*Sech[x/2]^2 - 4*Csch[x]^3*Sin
h[x/2]^4 - 4/(I + Sinh[x]) + (8*Sinh[x/2]*(14*I + 13*Sinh[x]))/(I*Cosh[x/2]
 + Sinh[x/2])^3 - 22*Tanh[x/2]))/24
```

Maple [A] time = 0.046, size = 92, normalized size = 1.4

$$-\frac{15}{8}\tanh\left(\frac{x}{2}\right) + \frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{i}{4}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{i}{4}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} + 5i\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3} - \frac{15}{8}\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(I+sinh(x))^2,x)
```

```
[Out] -15/8*tanh(1/2*x)+1/24*tanh(1/2*x)^3-1/4*I*tanh(1/2*x)^2+1/4*I/tanh(1/2*x)^
2+5*I*ln(tanh(1/2*x))+1/24/tanh(1/2*x)^3-15/8/tanh(1/2*x)+2*I/(tanh(1/2*x)+
I)^2+4/3/(tanh(1/2*x)+I)^3-10/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.27862, size = 174, normalized size = 2.72

$$-\frac{16\left(57\,e^{(-x)}+99i\,e^{(-2\,x)}-155\,e^{(-3\,x)}-153i\,e^{(-4\,x)}+135\,e^{(-5\,x)}+85i\,e^{(-6\,x)}-45\,e^{(-7\,x)}-15i\,e^{(-8\,x)}-24i\right)}{72\,e^{(-x)}+144i\,e^{(-2\,x)}-240\,e^{(-3\,x)}-288i\,e^{(-4\,x)}+288\,e^{(-5\,x)}+240i\,e^{(-6\,x)}-144\,e^{(-7\,x)}-72i\,e^{(-8\,x)}+24\,e^{(-9\,x)}-24i}-5i\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -16*(57*e^(-x) + 99*I*e^(-2*x) - 155*e^(-3*x) - 153*I*e^(-4*x) + 135*e^(-5*
x) + 85*I*e^(-6*x) - 45*e^(-7*x) - 15*I*e^(-8*x) - 24*I)/(72*e^(-x) + 144*I
*e^(-2*x) - 240*e^(-3*x) - 288*I*e^(-4*x) + 288*e^(-5*x) + 240*I*e^(-6*x) -
 144*e^(-7*x) - 72*I*e^(-8*x) + 24*e^(-9*x) - 24*I) - 5*I*log(e^(-x) + 1) +
 5*I*log(e^(-x) - 1)
```

Fricas [B] time = 2.1101, size = 730, normalized size = 11.41

$$\frac{\left(-15i\,e^{(9\,x)}+45\,e^{(8\,x)}+90i\,e^{(7\,x)}-150\,e^{(6\,x)}-180i\,e^{(5\,x)}+180\,e^{(4\,x)}+150i\,e^{(3\,x)}-90\,e^{(2\,x)}-45i\,e^{x}+15\right)\log\left(e^{x}+1\right)+\left(15i\,\right.}{3\,e^{(9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((-15*I*e^(9*x) + 45*e^(8*x) + 90*I*e^(7*x) - 150*e^(6*x) - 180*I*e^(5*x) +
 180*e^(4*x) + 150*I*e^(3*x) - 90*e^(2*x) - 45*I*e^x + 15)*log(e^x + 1) + (
15*I*e^(9*x) - 45*e^(8*x) - 90*I*e^(7*x) + 150*e^(6*x) + 180*I*e^(5*x) - 18
0*e^(4*x) - 150*I*e^(3*x) + 90*e^(2*x) + 45*I*e^x - 15)*log(e^x - 1) + 30*I
*e^(8*x) - 90*e^(7*x) - 170*I*e^(6*x) + 270*e^(5*x) + 306*I*e^(4*x) - 310*e
^(3*x) - 198*I*e^(2*x) + 114*e^x + 48*I)/(3*e^(9*x) + 9*I*e^(8*x) - 18*e^(7
*x) - 30*I*e^(6*x) + 36*e^(5*x) + 36*I*e^(4*x) - 30*e^(3*x) - 18*I*e^(2*x)
+ 9*e^x + 3*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36133, size = 113, normalized size = 1.77

$$-\frac{2\left(-15i\,e^{(8\,x)} + 45\,e^{(7\,x)} + 85i\,e^{(6\,x)} - 135\,e^{(5\,x)} - 153i\,e^{(4\,x)} + 155\,e^{(3\,x)} + 99i\,e^{(2\,x)} - 57\,e^{x} - 24i\right)}{3\left(e^{(3\,x)} + i\,e^{(2\,x)} - e^{x} - i\right)^{3}} - 5i\,\log\left(e^{x} + 1\right) + 5i\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(-15*I*e^(8*x) + 45*e^(7*x) + 85*I*e^(6*x) - 135*e^(5*x) - 153*I*e^(4*
x) + 155*e^(3*x) + 99*I*e^(2*x) - 57*e^x - 24*I)/(e^(3*x) + I*e^(2*x) - e^x
 - I)^3 - 5*I*log(e^x + 1) + 5*I*log(abs(e^x - 1))
```

3.56 $\int \frac{1}{1+i\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

```
[Out] (I*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rubi [A] time = 0.0113079, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2648}

$$\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (I*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx=\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

Mathematica [A] time = 0.0608458, size = 42, normalized size = 1.56

$$\frac{2\sinh\left(\frac{1}{2}(c+dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Maple [A] time = 0.017, size = 20, normalized size = 0.7

$$2\,\frac{1}{d\,(-i+\tanh(1/2\,dx+c/2))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c)),x)
```

```
[Out] 2/d/(-I+tanh(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.18467, size = 27, normalized size = 1.

$$-\frac{2}{d\left(i\,e^{(-dx-c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/(d*(I*e^(-d*x - c) - 1))
```

Fricas [A] time = 1.96137, size = 36, normalized size = 1.33

$$\frac{2i}{de^{(dx+c)}-i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2*I/(d*e^(d*x + c) - I*d)
```

Sympy [A] time = 0.273093, size = 19, normalized size = 0.7

$$\frac{2ie^{c}}{d\left(ie^{c}+e^{-dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x)
```

```
[Out] 2*I*exp(c)/(d*(I*exp(c) + exp(-d*x)))
```

Giac [A] time = 1.3587, size = 20, normalized size = 0.74

$$\frac{2i}{d\left(e^{(dx+c)}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*I/(d*(e^(d*x + c) - I))
```

3.57 $\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=59

$$\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}$$

```
[Out] ((I/3)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^2) + ((I/3)*Cosh[c + d*x])/(
d*(1 + I*Sinh[c + d*x]))
```

Rubi [A] time = 0.0263944, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-2),x]
```

```
[Out] ((I/3)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^2) + ((I/3)*Cosh[c + d*x])/(
d*(1 + I*Sinh[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx &= \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1+i\sinh(c+dx)}\,dx\\ &= \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0996662, size = 61, normalized size = 1.03

$$\frac{-4\sinh(c+dx)+\sinh(2(c+dx))-4i\cosh(c+dx)-i\cosh(2(c+dx))+3i}{6d(\sinh(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-2),x]
```

```
[Out] (3*I - (4*I)*Cosh[c + d*x] - I*Cosh[2*(c + d*x)] - 4*Sinh[c + d*x] + Sinh[2
*(c + d*x)])/(6*d*(-I + Sinh[c + d*x])^2)
```

Maple [A] time = 0.035, size = 55, normalized size = 0.9

$$\frac{1}{d}\left(2\,\left(-i+\tanh\left(1/2\,dx+c/2\right)\right)^{-1}+2\,i\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}-\frac{4}{3}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(2/(-I+tanh(1/2*d*x+1/2*c))+2*I/(-I+tanh(1/2*d*x+1/2*c))^2-4/3/(-I+tanh
(1/2*d*x+1/2*c))^3)
```

Maxima [B] time = 1.14607, size = 127, normalized size = 2.15

$$\frac{6\,e^{(-dx-c)}}{d\left(9\,e^{(-dx-c)}-9i\,e^{(-2\,dx-2\,c)}-3\,e^{(-3\,dx-3\,c)}+3i\right)}+\frac{2i}{d\left(9\,e^{(-dx-c)}-9i\,e^{(-2\,dx-2\,c)}-3\,e^{(-3\,dx-3\,c)}+3i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 6*e^(-d*x - c)/(d*(9*e^(-d*x - c) - 9*I*e^(-2*d*x - 2*c) - 3*e^(-3*d*x - 3*
c) + 3*I)) + 2*I/(d*(9*e^(-d*x - c) - 9*I*e^(-2*d*x - 2*c) - 3*e^(-3*d*x -
3*c) + 3*I))
```

Fricas [A] time = 1.99318, size = 128, normalized size = 2.17

$$\frac{6\,e^{(dx+c)}-2i}{3\,de^{(3\,dx+3\,c)}-9i\,de^{(2\,dx+2\,c)}-9\,de^{(dx+c)}+3i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (6*e^(d*x + c) - 2*I)/(3*d*e^(3*d*x + 3*c) - 9*I*d*e^(2*d*x + 2*c) - 9*d*e^
(d*x + c) + 3*I*d)
```

Sympy [A] time = 0.964546, size = 63, normalized size = 1.07

$$\frac{\frac{2e^{-2c}e^{dx}}{d}-\frac{2ie^{-3c}}{3d}}{e^{3dx}-3ie^{-c}e^{2dx}-3e^{-2c}e^{dx}+ie^{-3c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))**2,x)
```

```
[Out] (2*exp(-2*c)*exp(d*x)/d - 2*I*exp(-3*c)/(3*d))/(exp(3*d*x) - 3*I*exp(-c)*ex
p(2*d*x) - 3*exp(-2*c)*exp(d*x) + I*exp(-3*c))
```

Giac [A] time = 1.33114, size = 34, normalized size = 0.58

$$\frac{6\,e^{(dx+c)} - 2i}{3\,d\left(e^{(dx+c)} - i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*e^(d*x + c) - 2*I)/(d*(e^(d*x + c) - I)^3)
```

3.58 $\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

```
[Out] ((I/5)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/15)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^2) + (((2*I)/15)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x]))
```

Rubi [A] time = 0.0416327, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((I/5)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/15)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^2) + (((2*I)/15)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx &= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2}{5}\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx\\ &= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{2}{15}\int \frac{1}{1+i\sinh(c+dx)}\,dx\\ &= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.119885, size = 81, normalized size = 0.92

$$\frac{15i\sinh(c+dx)-6i\sinh(2(c+dx))-i\sinh(3(c+dx))-15\cosh(c+dx)-6\cosh(2(c+dx))+\cosh(3(c+dx))+10}{30d(\sinh(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-3),x]
```

```
[Out] (10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] + (15*I)*S
inh[c + d*x] - (6*I)*Sinh[2*(c + d*x)] - I*Sinh[3*(c + d*x)])/(30*d*(-I + S
inh[c + d*x])^3)
```

Maple [A] time = 0.046, size = 88, normalized size = 1.

$$\frac{1}{d}\left(2\,(-i+\tanh(1/2\,dx+c/2))^{-1}+4\,i\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}+\frac{8}{5}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-5}-\frac{16}{3}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(2/(-I+tanh(1/2*d*x+1/2*c))+4*I/(-I+tanh(1/2*d*x+1/2*c))^2+8/5/(-I+tanh
(1/2*d*x+1/2*c))^5-16/3/(-I+tanh(1/2*d*x+1/2*c))^3-4*I/(-I+tanh(1/2*d*x+1/2
*c))^4)
```

Maxima [B] time = 1.06219, size = 285, normalized size = 3.24

$$\frac{20i\,e^{(-dx-c)}}{d\left(75i\,e^{(-dx-c)}+150\,e^{(-2\,dx-2\,c)}-150i\,e^{(-3\,dx-3\,c)}-75\,e^{(-4\,dx-4\,c)}+15i\,e^{(-5\,dx-5\,c)}-15\right)}+\frac{}{d\left(75i\,e^{(-dx-c)}+150\,e^{(-2\,dx-2\,c)}-150\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-
3*d*x - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) + 40*e^(-
2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x
 - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) - 4/(d*(75*I*e
^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) - 75*e^(-4*d*x
- 4*c) + 15*I*e^(-5*d*x - 5*c) - 15))
```

Fricas [A] time = 1.98127, size = 235, normalized size = 2.67

$$\frac{-40i\,e^{(2\,dx+2\,c)}-20\,e^{(dx+c)}+4i}{15\,de^{(5\,dx+5\,c)}-75i\,de^{(4\,dx+4\,c)}-150\,de^{(3\,dx+3\,c)}+150i\,de^{(2\,dx+2\,c)}+75\,de^{(dx+c)}-15i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (-40*I*e^(2*d*x + 2*c) - 20*e^(d*x + c) + 4*I)/(15*d*e^(5*d*x + 5*c) - 75*I
*d*e^(4*d*x + 4*c) - 150*d*e^(3*d*x + 3*c) + 150*I*d*e^(2*d*x + 2*c) + 75*d
*e^(d*x + c) - 15*I*d)
```

Sympy [A] time = 2.8668, size = 114, normalized size = 1.3

$$\frac{\frac{4ie^{5c}}{15d} + \frac{4e^{4c}e^{-dx}}{3d} - \frac{8ie^{3c}e^{-2dx}}{3d}}{ie^{5c} + 5e^{4c}e^{-dx} - 10ie^{3c}e^{-2dx} - 10e^{2c}e^{-3dx} + 5ie^{c}e^{-4dx} + e^{-5dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))**3,x)
```

```
[Out] (4*I*exp(5*c)/(15*d) + 4*exp(4*c)*exp(-d*x)/(3*d) - 8*I*exp(3*c)*exp(-2*d*x
)/(3*d))/(I*exp(5*c) + 5*exp(4*c)*exp(-d*x) - 10*I*exp(3*c)*exp(-2*d*x) - 1
0*exp(2*c)*exp(-3*d*x) + 5*I*exp(c)*exp(-4*d*x) + exp(-5*d*x))
```

Giac [A] time = 1.37125, size = 49, normalized size = 0.56

$$-\frac{40i\,e^{(2\,dx+2\,c)} + 20\,e^{(dx+c)} - 4i}{15\,d\left(e^{(dx+c)} - i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15*(40*I*e^(2*d*x + 2*c) + 20*e^(d*x + c) - 4*I)/(d*(e^(d*x + c) - I)^5)
```

3.59 $\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=117

$$\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

```
[Out] ((I/7)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^4) + (((3*I)/35)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x])^2) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rubi [A] time = 0.0596243, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((I/7)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^4) + (((3*I)/35)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x])^2) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int\frac{1}{(1+i\sinh(c+dx))^4}\,dx &= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3}{7}\int\frac{1}{(1+i\sinh(c+dx))^3}\,dx\\ &= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{6}{35}\int\frac{1}{(1+i\sinh(c+dx))^2}\,dx\\ &= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{2}{35}\int\frac{}{1+}\\ &= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{2i\cosh}{35d(1+i\sinh}\end{aligned}$$

Mathematica [A] time = 0.164024, size = 87, normalized size = 0.74

$$\frac{35\sinh\left(\frac{1}{2}(c+dx)\right)-7\sinh\left(\frac{5}{2}(c+dx)\right)+21i\cosh\left(\frac{3}{2}(c+dx)\right)-i\cosh\left(\frac{7}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((21*I)*Cosh[(3*(c + d*x))/2] - I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*x)
/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2])^7)
```

Maple [A] time = 0.049, size = 121, normalized size = 1.

$$\frac{1}{d}\left(2\,(-i+\tanh(1/2\,dx+c/2))^{-1}+6\,i\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}-12\,(-i+\tanh(1/2\,dx+c/2))^{-3}-16\,i\left(-i+\tanh\left(\frac{d}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c))^4,x)
```

```
[Out] 1/d*(2/(-I+tanh(1/2*d*x+1/2*c))+6*I/(-I+tanh(1/2*d*x+1/2*c))^2-12/(-I+tanh(
1/2*d*x+1/2*c))^3-16*I/(-I+tanh(1/2*d*x+1/2*c))^4+8*I/(-I+tanh(1/2*d*x+1/2*
c))^6-16/7/(-I+tanh(1/2*d*x+1/2*c))^7+72/5/(-I+tanh(1/2*d*x+1/2*c))^5)
```

Maxima [B] time = 1.18628, size = 502, normalized size = 4.29

$$\frac{28\,e^{(-dx-c)}}{d\left(245\,e^{(-dx-c)}-735i\,e^{(-2\,dx-2\,c)}-1225\,e^{(-3\,dx-3\,c)}+1225i\,e^{(-4\,dx-4\,c)}+735\,e^{(-5\,dx-5\,c)}-245i\,e^{(-6\,dx-6\,c)}-35\,e^{(-7\,dx-7\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 28*e^(-d*x - c)/(d*(245*e^(-d*x - c) - 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*
d*x - 3*c) + 1225*I*e^(-4*d*x - 4*c) + 735*e^(-5*d*x - 5*c) - 245*I*e^(-6*d
*x - 6*c) - 35*e^(-7*d*x - 7*c) + 35*I)) - 84*I*e^(-2*d*x - 2*c)/(d*(245*e^
(-d*x - c) - 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*d*x - 3*c) + 1225*I*e^(-4*
d*x - 4*c) + 735*e^(-5*d*x - 5*c) - 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d*x -
 7*c) + 35*I)) - 140*e^(-3*d*x - 3*c)/(d*(245*e^(-d*x - c) - 735*I*e^(-2*d*
x - 2*c) - 1225*e^(-3*d*x - 3*c) + 1225*I*e^(-4*d*x - 4*c) + 735*e^(-5*d*x
- 5*c) - 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d*x - 7*c) + 35*I)) + 4*I/(d*(24
5*e^(-d*x - c) - 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*d*x - 3*c) + 1225*I*e^
(-4*d*x - 4*c) + 735*e^(-5*d*x - 5*c) - 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d
*x - 7*c) + 35*I))
```

Fricas [A] time = 2.07162, size = 338, normalized size = 2.89

$$-\frac{140\,e^{(3\,dx+3\,c)}-84i\,e^{(2\,dx+2\,c)}-28\,e^{(dx+c)}+4i}{35\,de^{(7\,dx+7\,c)}-245i\,de^{(6\,dx+6\,c)}-735\,de^{(5\,dx+5\,c)}+1225i\,de^{(4\,dx+4\,c)}+1225\,de^{(3\,dx+3\,c)}-735i\,de^{(2\,dx+2\,c)}-245\,de^{(dx+c)}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -(140*e^(3*d*x + 3*c) - 84*I*e^(2*d*x + 2*c) - 28*e^(d*x + c) + 4*I)/(35*d*
e^(7*d*x + 7*c) - 245*I*d*e^(6*d*x + 6*c) - 735*d*e^(5*d*x + 5*c) + 1225*I*
d*e^(4*d*x + 4*c) + 1225*d*e^(3*d*x + 3*c) - 735*I*d*e^(2*d*x + 2*c) - 245*
d*e^(d*x + c) + 35*I*d)
```

Sympy [A] time = 6.2866, size = 156, normalized size = 1.33

$$\frac{-\frac{4e^{-4c}e^{3dx}}{d}+\frac{12ie^{-5c}e^{2dx}}{5d}+\frac{4e^{-6c}e^{dx}}{5d}-\frac{4ie^{-7c}}{35d}}{e^{7dx}-7ie^{-c}e^{6dx}-21e^{-2c}e^{5dx}+35ie^{-3c}e^{4dx}+35e^{-4c}e^{3dx}-21ie^{-5c}e^{2dx}-7e^{-6c}e^{dx}+ie^{-7c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))**4,x)
```

```
[Out] (-4*exp(-4*c)*exp(3*d*x)/d + 12*I*exp(-5*c)*exp(2*d*x)/(5*d) + 4*exp(-6*c)*
exp(d*x)/(5*d) - 4*I*exp(-7*c)/(35*d))/(exp(7*d*x) - 7*I*exp(-c)*exp(6*d*x)
 - 21*exp(-2*c)*exp(5*d*x) + 35*I*exp(-3*c)*exp(4*d*x) + 35*exp(-4*c)*exp(3
*d*x) - 21*I*exp(-5*c)*exp(2*d*x) - 7*exp(-6*c)*exp(d*x) + I*exp(-7*c))
```

Giac [A] time = 1.43353, size = 63, normalized size = 0.54

$$-\frac{140\,e^{(3\,dx+3\,c)}-84i\,e^{(2\,dx+2\,c)}-28\,e^{(dx+c)}+4i}{35\,d\left(e^{(dx+c)}-i\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/35*(140*e^(3*d*x + 3*c) - 84*I*e^(2*d*x + 2*c) - 28*e^(d*x + c) + 4*I)/(
d*(e^(d*x + c) - I)^7)
```

3.60 $\int \frac{1}{1-i\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

```
[Out] ((-I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rubi [A] time = 0.0120213, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2648}

$$-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((-I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx = -\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

Mathematica [A] time = 0.0643767, size = 42, normalized size = 1.56

$$\frac{2\sinh\left(\frac{1}{2}(c+dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]))
```

Maple [A] time = 0.02, size = 20, normalized size = 0.7

$$2\,\frac{1}{d\left(\tanh\left(1/2\,dx+c/2\right)+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c)),x)
```

```
[Out] 2/d/(tanh(1/2*d*x+1/2*c)+I)
```

Maxima [A] time = 1.15037, size = 27, normalized size = 1.

$$\frac{2}{d\left(ie^{(-dx-c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/(d*(I*e^(-d*x - c) + 1))
```

Fricas [A] time = 1.94836, size = 38, normalized size = 1.41

$$-\frac{2i}{de^{(dx+c)}+id}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2*I/(d*e^(d*x + c) + I*d)
```

Sympy [A] time = 0.262806, size = 20, normalized size = 0.74

$$-\frac{2ie^{c}}{d\left(-ie^{c}+e^{-dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x)
```

```
[Out] -2*I*exp(c)/(d*(-I*exp(c) + exp(-d*x)))
```

Giac [A] time = 1.36607, size = 20, normalized size = 0.74

$$-\frac{2i}{d\left(e^{(dx+c)}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*I/(d*(e^(d*x + c) + I))
```

3.61 $\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=59

$$-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}$$

```
[Out] ((-I/3)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^2) - ((I/3)*Cosh[c + d*x])/
(d*(1 - I*Sinh[c + d*x]))
```

Rubi [A] time = 0.0272801, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-2),x]
```

```
[Out] ((-I/3)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^2) - ((I/3)*Cosh[c + d*x])/
(d*(1 - I*Sinh[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx &= -\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-i\sinh(c+dx)}\,dx\\ &= -\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0710138, size = 59, normalized size = 1.

$$-\frac{\cosh\left(\frac{3}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)}{3d\left(\sinh\left(\frac{1}{2}(c+dx)\right)+i\cosh\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-2),x]
```

```
[Out] -(Cosh[(3*(c + d*x))/2] + (3*I)*Sinh[(c + d*x)/2])/(3*d*(I*Cosh[(c + d*x)/2
] + Sinh[(c + d*x)/2])^3)
```

Maple [A] time = 0.033, size = 55, normalized size = 0.9

$$\frac{1}{d}\left(-2\,i\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-2}-\frac{4}{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-3}+2\,(\tanh(1/2\,dx+c/2)+i)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*I/(tanh(1/2*d*x+1/2*c)+I)^2-4/3/(tanh(1/2*d*x+1/2*c)+I)^3+2/(tanh(1
/2*d*x+1/2*c)+I))
```

Maxima [B] time = 1.10345, size = 127, normalized size = 2.15

$$\frac{6\,e^{(-dx-c)}}{d\left(9\,e^{(-dx-c)}+9i\,e^{(-2\,dx-2\,c)}-3\,e^{(-3\,dx-3\,c)}-3i\right)}-\frac{2i}{d\left(9\,e^{(-dx-c)}+9i\,e^{(-2\,dx-2\,c)}-3\,e^{(-3\,dx-3\,c)}-3i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 6*e^(-d*x - c)/(d*(9*e^(-d*x - c) + 9*I*e^(-2*d*x - 2*c) - 3*e^(-3*d*x - 3*
c) - 3*I)) - 2*I/(d*(9*e^(-d*x - c) + 9*I*e^(-2*d*x - 2*c) - 3*e^(-3*d*x -
3*c) - 3*I))
```

Fricas [A] time = 1.93858, size = 128, normalized size = 2.17

$$\frac{6\,e^{(dx+c)}+2i}{3\,de^{(3\,dx+3\,c)}+9i\,de^{(2\,dx+2\,c)}-9\,de^{(dx+c)}-3i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (6*e^(d*x + c) + 2*I)/(3*d*e^(3*d*x + 3*c) + 9*I*d*e^(2*d*x + 2*c) - 9*d*e^
(d*x + c) - 3*I*d)
```

Sympy [A] time = 0.935036, size = 63, normalized size = 1.07

$$\frac{\frac{2e^{-2c}e^{dx}}{d}+\frac{2ie^{-3c}}{3d}}{e^{3dx}+3ie^{-c}e^{2dx}-3e^{-2c}e^{dx}-ie^{-3c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))**2,x)
```

```
[Out] (2*exp(-2*c)*exp(d*x)/d + 2*I*exp(-3*c)/(3*d))/(exp(3*d*x) + 3*I*exp(-c)*ex
p(2*d*x) - 3*exp(-2*c)*exp(d*x) - I*exp(-3*c))
```

Giac [A] time = 1.36704, size = 34, normalized size = 0.58

$$\frac{6\,e^{(dx+c)}+2i}{3\,d\left(e^{(dx+c)}+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*e^(d*x + c) + 2*I)/(d*(e^(d*x + c) + I)^3)
```

3.62 $\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

```
[Out] ((-I/5)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/15)*Cosh[c + d
*x])/(d*(1 - I*Sinh[c + d*x])^2) - (((2*I)/15)*Cosh[c + d*x])/(d*(1 - I*Sin
h[c + d*x]))
```

Rubi [A] time = 0.0418598, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((-I/5)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/15)*Cosh[c + d
*x])/(d*(1 - I*Sinh[c + d*x])^2) - (((2*I)/15)*Cosh[c + d*x])/(d*(1 - I*Sin
h[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx &= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}+\frac{2}{5}\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx\\ &= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}+\frac{2}{15}\int \frac{1}{1-i\sinh(c+dx)}\,dx\\ &= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.119002, size = 81, normalized size = 0.92

$$\frac{-15i\sinh(c+dx)+6i\sinh(2(c+dx))+i\sinh(3(c+dx))-15\cosh(c+dx)-6\cosh(2(c+dx))+\cosh(3(c+dx))+10}{30d(\sinh(c+dx)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-3),x]
```

```
[Out] (10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] - (15*I)*S
inh[c + d*x] + (6*I)*Sinh[2*(c + d*x)] + I*Sinh[3*(c + d*x)])/(30*d*(I + Si
nh[c + d*x])^3)
```

Maple [A] time = 0.039, size = 88, normalized size = 1.

$$\frac{1}{d}\left(2\,(\tanh(1/2\,dx+c/2)+i)^{-1}+\frac{8}{5}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-5}-4\,i\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-2}-\frac{16}{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(2/(tanh(1/2*d*x+1/2*c)+I)+8/5/(tanh(1/2*d*x+1/2*c)+I)^5-4*I/(tanh(1/2*
d*x+1/2*c)+I)^2-16/3/(tanh(1/2*d*x+1/2*c)+I)^3+4*I/(tanh(1/2*d*x+1/2*c)+I)^
4)
```

Maxima [B] time = 1.06897, size = 285, normalized size = 3.24

$$\frac{20i\,e^{(-dx-c)}}{d\left(75i\,e^{(-dx-c)}-150\,e^{(-2\,dx-2\,c)}-150i\,e^{(-3\,dx-3\,c)}+75\,e^{(-4\,dx-4\,c)}+15i\,e^{(-5\,dx-5\,c)}+15\right)}-\frac{}{d\left(75i\,e^{(-dx-c)}-150\,e^{(-2\,dx-2\,c)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-
3*d*x - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) - 40*e^(-
2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x
 - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) + 4/(d*(75*I*e
^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) + 75*e^(-4*d*x
- 4*c) + 15*I*e^(-5*d*x - 5*c) + 15))
```

Fricas [A] time = 2.01792, size = 234, normalized size = 2.66

$$\frac{40i\,e^{(2\,dx+2\,c)}-20\,e^{(dx+c)}-4i}{15\,de^{(5\,dx+5\,c)}+75i\,de^{(4\,dx+4\,c)}-150\,de^{(3\,dx+3\,c)}-150i\,de^{(2\,dx+2\,c)}+75\,de^{(dx+c)}+15i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (40*I*e^(2*d*x + 2*c) - 20*e^(d*x + c) - 4*I)/(15*d*e^(5*d*x + 5*c) + 75*I*
d*e^(4*d*x + 4*c) - 150*d*e^(3*d*x + 3*c) - 150*I*d*e^(2*d*x + 2*c) + 75*d*
e^(d*x + c) + 15*I*d)
```

Sympy [A] time = 2.8268, size = 114, normalized size = 1.3

$$\frac{-\frac{4ie^{5c}}{15d} + \frac{4e^{4c}e^{-dx}}{3d} + \frac{8ie^{3c}e^{-2dx}}{3d}}{-ie^{5c} + 5e^{4c}e^{-dx} + 10ie^{3c}e^{-2dx} - 10e^{2c}e^{-3dx} - 5ie^{c}e^{-4dx} + e^{-5dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))**3,x)
```

```
[Out] (-4*I*exp(5*c)/(15*d) + 4*exp(4*c)*exp(-d*x)/(3*d) + 8*I*exp(3*c)*exp(-2*d*
x)/(3*d))/(-I*exp(5*c) + 5*exp(4*c)*exp(-d*x) + 10*I*exp(3*c)*exp(-2*d*x) -
 10*exp(2*c)*exp(-3*d*x) - 5*I*exp(c)*exp(-4*d*x) + exp(-5*d*x))
```

Giac [A] time = 1.3913, size = 49, normalized size = 0.56

$$-\frac{-40i\,e^{(2\,dx+2\,c)} + 20\,e^{(dx+c)} + 4i}{15\,d\left(e^{(dx+c)} + i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15*(-40*I*e^(2*d*x + 2*c) + 20*e^(d*x + c) + 4*I)/(d*(e^(d*x + c) + I)^5
)
```

3.63 $\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=117

$$-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

```
[Out] ((-I/7)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^4) - (((3*I)/35)*Cosh[c + d
*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*Sin
h[c + d*x])^2) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rubi [A] time = 0.0599475, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-I/7)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^4) - (((3*I)/35)*Cosh[c + d
*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*Sin
h[c + d*x])^2) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx &= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}+\frac{3}{7}\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx\\ &= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}+\frac{6}{35}\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx\\ &= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}+\frac{2}{35}\int\\ &= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}-\frac{2i\,c}{35d(1}\end{aligned}$$

Mathematica [A] time = 0.150469, size = 87, normalized size = 0.74

$$\frac{35\sinh\left(\frac{1}{2}(c+dx)\right)-7\sinh\left(\frac{5}{2}(c+dx)\right)-21i\cosh\left(\frac{3}{2}(c+dx)\right)+i\cosh\left(\frac{7}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-21*I)*Cosh[(3*(c + d*x))/2] + I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*x
)/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)
/2])^7)
```

Maple [A] time = 0.043, size = 121, normalized size = 1.

$$\frac{1}{d}\left(2\left(\tanh\left(1/2\,dx+c/2\right)+i\right)^{-1}+\frac{72}{5}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-5}-6\,i\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-2}-\frac{16}{7}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-7}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c))^4,x)
```

```
[Out] 1/d*(2/(tanh(1/2*d*x+1/2*c)+I)+72/5/(tanh(1/2*d*x+1/2*c)+I)^5-6*I/(tanh(1/2
*d*x+1/2*c)+I)^2-16/7/(tanh(1/2*d*x+1/2*c)+I)^7-12/(tanh(1/2*d*x+1/2*c)+I)^
3+16*I/(tanh(1/2*d*x+1/2*c)+I)^4-8*I/(tanh(1/2*d*x+1/2*c)+I)^6)
```

Maxima [B] time = 1.16414, size = 502, normalized size = 4.29

$$\frac{28\,e^{(-dx-c)}}{d\left(245\,e^{(-dx-c)}+735i\,e^{(-2\,dx-2\,c)}-1225\,e^{(-3\,dx-3\,c)}-1225i\,e^{(-4\,dx-4\,c)}+735\,e^{(-5\,dx-5\,c)}+245i\,e^{(-6\,dx-6\,c)}-35\,e^{(-7\,dx-7\,c)}-35i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 28*e^(-d*x - c)/(d*(245*e^(-d*x - c) + 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*
d*x - 3*c) - 1225*I*e^(-4*d*x - 4*c) + 735*e^(-5*d*x - 5*c) + 245*I*e^(-6*d
*x - 6*c) - 35*e^(-7*d*x - 7*c) - 35*I)) + 84*I*e^(-2*d*x - 2*c)/(d*(245*e^
(-d*x - c) + 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*d*x - 3*c) - 1225*I*e^(-4*
d*x - 4*c) + 735*e^(-5*d*x - 5*c) + 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d*x -
 7*c) - 35*I)) - 140*e^(-3*d*x - 3*c)/(d*(245*e^(-d*x - c) + 735*I*e^(-2*d*
x - 2*c) - 1225*e^(-3*d*x - 3*c) - 1225*I*e^(-4*d*x - 4*c) + 735*e^(-5*d*x
- 5*c) + 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d*x - 7*c) - 35*I)) - 4*I/(d*(24
5*e^(-d*x - c) + 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*d*x - 3*c) - 1225*I*e^
(-4*d*x - 4*c) + 735*e^(-5*d*x - 5*c) + 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d
*x - 7*c) - 35*I))
```

Fricas [A] time = 1.9639, size = 338, normalized size = 2.89

$$-\frac{140\,e^{(3\,dx+3\,c)}+84i\,e^{(2\,dx+2\,c)}-28\,e^{(dx+c)}-4i}{35\,de^{(7\,dx+7\,c)}+245i\,de^{(6\,dx+6\,c)}-735\,de^{(5\,dx+5\,c)}-1225i\,de^{(4\,dx+4\,c)}+1225\,de^{(3\,dx+3\,c)}+735i\,de^{(2\,dx+2\,c)}-245\,de^{(dx+c)}-35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -(140*e^(3*d*x + 3*c) + 84*I*e^(2*d*x + 2*c) - 28*e^(d*x + c) - 4*I)/(35*d*
e^(7*d*x + 7*c) + 245*I*d*e^(6*d*x + 6*c) - 735*d*e^(5*d*x + 5*c) - 1225*I*
d*e^(4*d*x + 4*c) + 1225*d*e^(3*d*x + 3*c) + 735*I*d*e^(2*d*x + 2*c) - 245*
d*e^(d*x + c) - 35*I*d)
```

Sympy [A] time = 6.2558, size = 156, normalized size = 1.33

$$\frac{-\frac{4e^{-4c}e^{3dx}}{d} - \frac{12ie^{-5c}e^{2dx}}{5d} + \frac{4e^{-6c}e^{dx}}{5d} + \frac{4ie^{-7c}}{35d}}{e^{7dx} + 7ie^{-c}e^{6dx} - 21e^{-2c}e^{5dx} - 35ie^{-3c}e^{4dx} + 35e^{-4c}e^{3dx} + 21ie^{-5c}e^{2dx} - 7e^{-6c}e^{dx} - ie^{-7c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))**4,x)
```

```
[Out] (-4*exp(-4*c)*exp(3*d*x)/d - 12*I*exp(-5*c)*exp(2*d*x)/(5*d) + 4*exp(-6*c)*
exp(d*x)/(5*d) + 4*I*exp(-7*c)/(35*d))/(exp(7*d*x) + 7*I*exp(-c)*exp(6*d*x)
 - 21*exp(-2*c)*exp(5*d*x) - 35*I*exp(-3*c)*exp(4*d*x) + 35*exp(-4*c)*exp(3
*d*x) + 21*I*exp(-5*c)*exp(2*d*x) - 7*exp(-6*c)*exp(d*x) - I*exp(-7*c))
```

Giac [A] time = 1.38993, size = 63, normalized size = 0.54

$$-\frac{140\,e^{(3\,dx+3\,c)} + 84i\,e^{(2\,dx+2\,c)} - 28\,e^{(dx+c)} - 4i}{35\,d\left(e^{(dx+c)} + i\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/35*(140*e^(3*d*x + 3*c) + 84*I*e^(2*d*x + 2*c) - 28*e^(d*x + c) - 4*I)/(
d*(e^(d*x + c) + I)^7)
```

3.64 $\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal. Leaf size=57

$$\frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rubi [A] time = 0.0559671, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2751, 2649, 206}

$$\frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx &= \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}+i\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx\\
&= \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}-2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)\\
&= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.073706, size = 75, normalized size = 1.32

$$\frac{2\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(-i\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)+(1+i)\sqrt[4]{-1}\tan^{-1}\left(\frac{\tanh\left(\frac{x}{4}\right)+i}{\sqrt{2}}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] (2*((1 + I)*(-1)^(1/4)*ArcTan[(I + Tanh[x/4])/Sqrt[2]] + Cosh[x/2] - I*Sinh
[x/2])*(Cosh[x/2] + I*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int \sinh(x)\frac{1}{\sqrt{a+ia\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+I*a*sinh(x))^(1/2),x)
```

```
[Out] int(sinh(x)/(a+I*a*sinh(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{\sqrt{i\,a\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

Fricas [B] time = 2.12969, size = 501, normalized size = 8.79

$$-\frac{2\sqrt{\frac{1}{2}}\sqrt{i\,ae^{(2x)}+2\,ae^{x}-i\,a}(i\,e^{x}-1)e^{\left(-\frac{1}{2}x\right)}+\frac{\sqrt{2}(ae^{x}-i\,a)\log\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{i\,ae^{(2x)}+2\,ae^{x}-i\,a}e^{\left(-\frac{1}{2}x\right)}+\frac{\sqrt{2}(ae^{x}-i\,a)}{\sqrt{a}}}{2\,e^{x}-2i}\right)}{\sqrt{a}}-\frac{\sqrt{2}(ae^{x}-i\,a)\log\left(2\sqrt{\frac{1}{2}}\sqrt{i\,ae^{(2x)}+2\,a\cdots}\right)}{\sqrt{a}}}{ae^{x}-i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(1/2)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*(I*e^x - 1)*e^(-1/2*x) + sq
rt(2)*(a*e^x - I*a)*log((2*sqrt(1/2)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*e^(-
1/2*x) + sqrt(2)*(a*e^x - I*a)/sqrt(a))/(2*e^x - 2*I))/sqrt(a) - sqrt(2)*(a
*e^x - I*a)*log((2*sqrt(1/2)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*e^(-1/2*x) -
```

```
sqrt(2)*(a*e^x - I*a)/sqrt(a))/(2*e^x - 2*I))/sqrt(a))/(a*e^x - I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{\sqrt{a\left(i\sinh(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(a*(I*sinh(x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{\sqrt{i\,a\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

3.65 $\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}} \, dx$

Optimal. Leaf size=57

$$\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a - I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a - I*a*Sinh[x]]
```

Rubi [A] time = 0.0557816, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2751, 2649, 206}

$$\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a - I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a - I*a*Sinh[x]]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}} \, dx &= \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}-i\int \frac{1}{\sqrt{a-ia\sinh(x)}} \, dx \\
&= \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}+2\,\text{Subst}\left(\int \frac{1}{2a-x^2} \, dx, x, -\frac{a\cosh(x)}{\sqrt{a-ia\sinh(x)}}\right) \\
&= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0768128, size = 76, normalized size = 1.33

$$\frac{2\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\left(\sinh\left(\frac{x}{2}\right)+(1+i)(-1)^{3/4}\tan^{-1}\left(\frac{\tanh\left(\frac{x}{4}\right)-i}{\sqrt{2}}\right)\right)\right)}{\sqrt{a-ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]
```

```
[Out] (2*(Cosh[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*((1 + I)*(-1)^(3/4)*ArcTan[(-I
+ Tanh[x/4])/Sqrt[2]] + Sinh[x/2])))/Sqrt[a - I*a*Sinh[x]]
```

Maple [F] time = 0.304, size = 0, normalized size = 0.

$$\int \sinh(x)\frac{1}{\sqrt{a-ia\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a-I*a*sinh(x))^(1/2),x)
```

```
[Out] int(sinh(x)/(a-I*a*sinh(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{\sqrt{-i\,a\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

Fricas [B] time = 2.10534, size = 506, normalized size = 8.88

$$-\frac{2\sqrt{\frac{1}{2}}\sqrt{-i\,ae^{(2x)}+2\,ae^{x}+i\,a}(-i\,e^{x}-1)e^{\left(-\frac{1}{2}x\right)}+\frac{\sqrt{2}(ae^{x}+i\,a)\log\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{-i\,ae^{(2x)}+2\,ae^{x}+i\,a}e^{\left(-\frac{1}{2}x\right)}+\frac{\sqrt{2}(ae^{x}+i\,a)}{\sqrt{a}}}{2\,e^{x}+2i}\right)}{\sqrt{a}}-\frac{\sqrt{2}(ae^{x}+i\,a)\log\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{-i\,ae^{(2x)}\ldots}}{\ldots}\right)}{\sqrt{a}}}{ae^{x}+i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(1/2)*sqrt(-I*a*e^(2*x) + 2*a*e^x + I*a)*(-I*e^x - 1)*e^(-1/2*x) +
sqrt(2)*(a*e^x + I*a)*log((2*sqrt(1/2)*sqrt(-I*a*e^(2*x) + 2*a*e^x + I*a)*e
^(-1/2*x) + sqrt(2)*(a*e^x + I*a)/sqrt(a))/(2*e^x + 2*I))/sqrt(a) - sqrt(2)
*(a*e^x + I*a)*log((2*sqrt(1/2)*sqrt(-I*a*e^(2*x) + 2*a*e^x + I*a)*e^(-1/2*
```

```
x) - sqrt(2)*(a*e^x + I*a)/sqrt(a))/(2*e^x + 2*I))/sqrt(a))/(a*e^x + I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{\sqrt{-a\,(i\sinh(x)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(-a*(I*sinh(x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{\sqrt{-i\,a\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

3.66 $\int (a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=104

$$\frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}}+\frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

```
[Out] (((64*I)/15)*a^3*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((16*I)/
15)*a^2*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d + (((2*I)/5)*a*Cosh[c
+ d*x]*(a + I*a*Sinh[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.0531309, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2647, 2646}

$$\frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}}+\frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((64*I)/15)*a^3*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((16*I)/
15)*a^2*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d + (((2*I)/5)*a*Cosh[c
+ d*x]*(a + I*a*Sinh[c + d*x])^(3/2))/d
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\sinh(c+dx))^{5/2}\,dx &= \frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}+\frac{1}{5}(8a)\int (a+ia\sinh(c+dx))^{3/2}\,dx\\ &= \frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}+\frac{1}{15}\Big(\cdots\\ &= \frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}}+\frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2ia\cosh(c+dx)(a\cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.452801, size = 145, normalized size = 1.39

$$\frac{a^2(\sinh(c+dx)-i)^2\sqrt{a+ia\sinh(c+dx)}\left(-150\sinh\left(\frac{1}{2}(c+dx)\right)+25\sinh\left(\frac{3}{2}(c+dx)\right)+3\sinh\left(\frac{5}{2}(c+dx)\right)-150i\cosh\cdots\right.}{30d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*((-150*I)*Cosh[(c +
 d*x)/2] - (25*I)*Cosh[(3*(c + d*x))/2] + (3*I)*Cosh[(5*(c + d*x))/2] - 150
*Sinh[(c + d*x)/2] + 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2]))/(
30*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int (a + ia \sinh(dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a \sinh(dx + c) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 1.98379, size = 360, normalized size = 3.46

$$-\frac{\sqrt{\frac{1}{2}}\left(3\,a^{2}e^{(5\,dx+5\,c)} - 25i\,a^{2}e^{(4\,dx+4\,c)} - 150\,a^{2}e^{(3\,dx+3\,c)} - 150i\,a^{2}e^{(2\,dx+2\,c)} - 25\,a^{2}e^{(dx+c)} + 3i\,a^{2}\right)\sqrt{i\,ae^{(2\,dx+2\,c)} + 2\,ae^{(dx+c)}}}{30\,de^{(3\,dx+3\,c)} - 30i\,de^{(2\,dx+2\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -sqrt(1/2)*(3*a^2*e^(5*d*x + 5*c) - 25*I*a^2*e^(4*d*x + 4*c) - 150*a^2*e^(3
*d*x + 3*c) - 150*I*a^2*e^(2*d*x + 2*c) - 25*a^2*e^(d*x + c) + 3*I*a^2)*sqr
t(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*c)/(30*d*e
^(3*d*x + 3*c) - 30*I*d*e^(2*d*x + 2*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.67 $\int (a+ia\sinh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}}+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}$$

```
[Out] (((8*I)/3)*a^2*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((2*I)/3)*
a*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d
```

Rubi [A] time = 0.031358, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2647, 2646}

$$\frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}}+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/3)*a^2*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((2*I)/3)*
a*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\sinh(c+dx))^{3/2}\,dx &= \frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}+\frac{1}{3}(4a)\int \sqrt{a+ia\sinh(c+dx)}\,dx\\ &= \frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}}+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] time = 0.205685, size = 113, normalized size = 1.64

$$-\frac{a(\sinh(c+dx)-i)\sqrt{a+ia\sinh(c+dx)}\left(-9i\sinh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{3}{2}(c+dx)\right)+9\cosh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{3}{2}(c\right.\right.}{3d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

```
[Out] -(a*(-I + Sinh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]]*(9*Cosh[(c + d*x)/2] +
 Cosh[(3*(c + d*x))/2] - (9*I)*Sinh[(c + d*x)/2] + I*Sinh[(3*(c + d*x))/2])
)/(3*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^3)
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (a + ia\sinh(dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\sinh(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 1.98399, size = 259, normalized size = 3.75

$$\frac{\sqrt{\frac{1}{2}}\left(i\,ae^{(3\,dx+3\,c)} + 9\,ae^{(2\,dx+2\,c)} + 9i\,ae^{(dx+c)} + a\right)\sqrt{i\,ae^{(2\,dx+2\,c)} + 2\,ae^{(dx+c)} - i\,a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{3\left(de^{(2\,dx+2\,c)} - i\,de^{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(I*a*e^(3*d*x + 3*c) + 9*a*e^(2*d*x + 2*c) + 9*I*a*e^(d*x + c
) + a)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*
c)/(d*e^(2*d*x + 2*c) - I*d*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh (dx + c) + a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(3/2), x)
```

3.68 $\int \sqrt{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

```
[Out] ((2*I)*a*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]])
```

Rubi [A] time = 0.0139036, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2646}

$$\frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*a*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Mathematica [B] time = 0.041178, size = 74, normalized size = 2.39

$$\frac{2\sqrt{a+ia\sinh(c+dx)}\left(\sinh\left(\frac{1}{2}(c+dx)\right)+i\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] (2*(I*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])*Sqrt[a + I*a*Sinh[c + d*x]])/(
d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Maple [B] time = 0.125, size = 89, normalized size = 2.9

$$\frac{i\sqrt{2}\left(e^{dx+c}+i\right)\left(e^{dx+c}-i\right)}{\left(ie^{2\,dx+2\,c}-i+2\,e^{dx+c}\right)d}\sqrt{a\left(ie^{2\,dx+2\,c}-i+2\,e^{dx+c}\right)e^{-dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(1/2),x)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*d*x+2*c)-I+2*exp(d*x+c))*exp(-d*x-c))^(1/2)/(I*exp(2*
d*x+2*c)-I+2*exp(d*x+c))*(exp(d*x+c)+I)*(exp(d*x+c)-I)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

Fricas [B] time = 2.00736, size = 170, normalized size = 5.48

$$\frac{\sqrt{\frac{1}{2}}\sqrt{i\,ae^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-i\,a}\left(2\,e^{(dx+c)}+2i\right)e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{de^{(dx+c)}-i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(1/2)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*(2*e^(d*x + c)
+ 2*I)*e^(-1/2*d*x - 1/2*c)/(d*e^(d*x + c) - I*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia\sinh\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*sinh(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

3.69 $\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx$

Optimal. Leaf size=52

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] (I*Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d
*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0243704, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2649, 206}

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d
*x]])])/(Sqrt[a]*d)
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx &= \frac{(2i)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{d}\\ &= \frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] time = 0.0852399, size = 84, normalized size = 1.62

$$\frac{(2+2i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1-i\tanh\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\sinh\left(\frac{1}{2}(c+dx)\right)-i\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] ((2 + 2*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4
])]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]))/(d*Sqrt[a + I*a*Sinh[c +
d*x]])
```

Maple [F] time = 0.506, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + ia\sinh(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*sinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+I*a*sinh(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i\,a\sinh(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

Fricas [B] time = 2.11263, size = 518, normalized size = 9.96

$$i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{\sqrt{2}\left(ade^{(dx+c)} - i\,ad\right)\sqrt{\frac{1}{ad^2}} + 2\sqrt{\frac{1}{2}}\sqrt{i\,ae^{(2\,dx+2\,c)} + 2\,ae^{(dx+c)} - i\,a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{2\,e^{(dx+c)} - 2i}\right) - i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{\sqrt{2}(ad}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] I*sqrt(2)*sqrt(1/(a*d^2))*log((sqrt(2)*(a*d*e^(d*x + c) - I*a*d)*sqrt(1/(a*
d^2)) + 2*sqrt(1/2)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(-1
/2*d*x - 1/2*c))/(2*e^(d*x + c) - 2*I)) - I*sqrt(2)*sqrt(1/(a*d^2))*log(-(s
qrt(2)*(a*d*e^(d*x + c) - I*a*d)*sqrt(1/(a*d^2)) - 2*sqrt(1/2)*sqrt(I*a*e^(
2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*c))/(2*e^(d*x + c)
- 2*I))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ia\sinh(c + dx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*a*sinh(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i a \sinh (dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

3.70 $\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

```
[Out] ((I/2)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d*x]]
)])/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x])^
(3/2))
```

Rubi [A] time = 0.044912, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2650, 2649, 206}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((I/2)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d*x]]
)])/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x])^
(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx}{4a}$$
$$= \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}} + \frac{i\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{2ad}$$
$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

Mathematica [A] time = 0.225912, size = 156, normalized size = 1.79

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\left(\sinh\left(\frac{1}{2}(c+dx)\right)+(1-i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1-i\,\text{t}\right.\right.\right.\right.}{2ad(\sinh(c+dx)-i)\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2] - I*((1 - I)*
(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + Sinh[(c + d*x)/2])))/(2*a*d*(-I + Si
nh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]])
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int (a+ia\sinh(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*sinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+I*a*sinh(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,a\sinh(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

Fricas [B] time = 2.19344, size = 1044, normalized size = 12.

$$\sqrt{\frac{1}{2}}\sqrt{i\,ae^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-i\,a}\left(-2i\,e^{(2\,dx+2\,c)}+2\,e^{(dx+c)}\right)e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+\sqrt{\frac{1}{2}}\left(i\,a^{2}de^{(3\,dx+3\,c)}+3\,a^{2}de^{(2\,dx+2\,c)}-3i\,a^{2}de^{(dx-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*(-2*I*e^(2*d*x
 + 2*c) + 2*e^(d*x + c))*e^(-1/2*d*x - 1/2*c) + sqrt(1/2)*(I*a^2*d*e^(3*d*x
 + 3*c) + 3*a^2*d*e^(2*d*x + 2*c) - 3*I*a^2*d*e^(d*x + c) - a^2*d)*sqrt(1/(
a^3*d^2))*log((sqrt(1/2)*(a^2*d*e^(d*x + c) - I*a^2*d)*sqrt(1/(a^3*d^2)) +
sqrt(1/2)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1
/2*c))/(e^(d*x + c) - I)) + sqrt(1/2)*(-I*a^2*d*e^(3*d*x + 3*c) - 3*a^2*d*e
^(2*d*x + 2*c) + 3*I*a^2*d*e^(d*x + c) + a^2*d)*sqrt(1/(a^3*d^2))*log(-(sqr
t(1/2)*(a^2*d*e^(d*x + c) - I*a^2*d)*sqrt(1/(a^3*d^2)) - sqrt(1/2)*sqrt(I*a
*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*c))/(e^(d*x + c
) - I)))/(2*a^2*d*e^(3*d*x + 3*c) - 6*I*a^2*d*e^(2*d*x + 2*c) - 6*a^2*d*e^(
d*x + c) + 2*I*a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ia\sinh{(c+dx)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*sinh(c + d*x) + a)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

3.71 $\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

```
[Out] (((3*I)/16)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c +
d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/4)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d
*x])^(5/2)) + (((3*I)/16)*Cosh[c + d*x])/(a*d*(a + I*a*Sinh[c + d*x])^(3/2)
)
```

Rubi [A] time = 0.0681726, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2650, 2649, 206}

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (((3*I)/16)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c +
d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/4)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d
*x])^(5/2)) + (((3*I)/16)*Cosh[c + d*x])/(a*d*(a + I*a*Sinh[c + d*x])^(3/2)
)
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx}{8a}$$
$$= \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx}{32a^2}$$
$$= \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \right.}{16a^2d}$$
$$= \frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx)}$$

Mathematica [A] time = 0.195971, size = 210, normalized size = 1.72

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4\sinh\left(\frac{1}{2}(c+dx)\right) + 4i\cosh\left(\frac{1}{2}(c+dx)\right) + 6\sinh\left(\frac{1}{2}(c+dx)\right)\left(\cosh\left(\frac{1}{2}(c+d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((4*I)*Cosh[(c + d*x)/2] + (3 -
3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + 4*Sinh[(c + d*x)/2] + 6*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2] + 3*((-I)*Cosh[(c +
d*x)/2] + Sinh[(c + d*x)/2])^3))/(16*d*(a + I*a*Sinh[c + d*x])^(5/2))
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int (a+ia\sinh(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

Fricas [B] time = 2.21884, size = 1395, normalized size = 11.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*sqrt(1/2)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*(-3*I*e^
(4*d*x + 4*c) - 11*e^(3*d*x + 3*c) - 11*I*e^(2*d*x + 2*c) - 3*e^(d*x + c))*
e^(-1/2*d*x - 1/2*c) + sqrt(1/2)*(12*I*a^3*d*e^(5*d*x + 5*c) + 60*a^3*d*e^(
4*d*x + 4*c) - 120*I*a^3*d*e^(3*d*x + 3*c) - 120*a^3*d*e^(2*d*x + 2*c) + 60
*I*a^3*d*e^(d*x + c) + 12*a^3*d)*sqrt(1/(a^5*d^2))*log(1/8*(sqrt(1/2)*(8*a^
3*d*e^(d*x + c) - 8*I*a^3*d)*sqrt(1/(a^5*d^2)) + 8*sqrt(1/2)*sqrt(I*a*e^(2*
d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*c))/(e^(d*x + c) - I)
) + sqrt(1/2)*(-12*I*a^3*d*e^(5*d*x + 5*c) - 60*a^3*d*e^(4*d*x + 4*c) + 120
*I*a^3*d*e^(3*d*x + 3*c) + 120*a^3*d*e^(2*d*x + 2*c) - 60*I*a^3*d*e^(d*x +
c) - 12*a^3*d)*sqrt(1/(a^5*d^2))*log(-1/8*(sqrt(1/2)*(8*a^3*d*e^(d*x + c) -
 8*I*a^3*d)*sqrt(1/(a^5*d^2)) - 8*sqrt(1/2)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*
e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*c))/(e^(d*x + c) - I)))/(8*a^3*d*e^(5*
d*x + 5*c) - 40*I*a^3*d*e^(4*d*x + 4*c) - 80*a^3*d*e^(3*d*x + 3*c) + 80*I*a
^3*d*e^(2*d*x + 2*c) + 40*a^3*d*e^(d*x + c) - 8*I*a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

3.72 $\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=108

$$-\frac{ax\left(2a^2-b^2\right)}{2b^4}-\frac{\left(2-\frac{3a^2}{b^2}\right)\cosh(x)}{3b}-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh^2(x)\cosh(x)}{3b}$$

```
[Out] -(a*(2*a^2 - b^2)*x)/(2*b^4) - (2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^4*Sqrt[a^2 + b^2]) - ((2 - (3*a^2)/b^2)*Cosh[x])/(3*b) - (a*Cosh[
x]*Sinh[x])/(2*b^2) + (Cosh[x]*Sinh[x]^2)/(3*b)
```

Rubi [A] time = 0.31549, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2793, 3049, 3023, 2735, 2660, 618, 206}

$$-\frac{ax\left(2a^2-b^2\right)}{2b^4}-\frac{\left(2-\frac{3a^2}{b^2}\right)\cosh(x)}{3b}-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh^2(x)\cosh(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -(a*(2*a^2 - b^2)*x)/(2*b^4) - (2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^4*Sqrt[a^2 + b^2]) - ((2 - (3*a^2)/b^2)*Cosh[x])/(3*b) - (a*Cosh[
x]*Sinh[x])/(2*b^2) + (Cosh[x]*Sinh[x]^2)/(3*b)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{\int \frac{\sinh(x)\left(2a+2b\sinh(x)+3a\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{3b}$$

$$= -\frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{\int \frac{-3a^2+ab\sinh(x)-2\left(3a^2-2b^2\right)\sinh^2(x)}{a+b\sinh(x)}\,dx}{6b^2}$$

$$= \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{i\int \frac{3ia^2b-3ia\left(2a^2-b^2\right)\sinh(x)}{a+b\sinh(x)}\,dx}{6b^3}$$

$$= -\frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} + \frac{a^4\int \frac{1}{a+b\sinh(x)}\,d}{b^4}$$

$$= -\frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} + \left(2a^4\right)\text{Subst}\left(\int\right.$$

$$= -\frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \left(4a^4\right)\text{Subst}\left(\int\right.$$

$$= -\frac{a\left(2a^2-b^2\right)x}{2b^4} - \frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}} + \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\,\text{s}}{3}$$

Mathematica [A] time = 0.431195, size = 105, normalized size = 0.97

$$\frac{3b\left(4a^2-3b^2\right)\cosh(x)+3a\left(\frac{8a^3\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-4a^2x+2b^2x-b^2\sinh(2x)\right)+b^3\cosh(3x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (3*b*(4*a^2 - 3*b^2)*Cosh[x] + b^3*Cosh[3*x] + 3*a*(-4*a^2*x + 2*b^2*x + (8
*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - b^2*Sin
h[2*x]))/(12*b^4)
```

Maple [B] time = 0.027, size = 262, normalized size = 2.4

$$\frac{1}{3\,b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{a}{2\,b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{2\,b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{a^2}{b^3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{a}{2\,b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*sinh(x)),x)
```

```
[Out] 1/3/b/(tanh(1/2*x)+1)^3+1/2/b^2/(tanh(1/2*x)+1)^2*a-1/2/b/(tanh(1/2*x)+1)^2
+1/b^3/(tanh(1/2*x)+1)*a^2-1/2/b^2/(tanh(1/2*x)+1)*a-1/2/b/(tanh(1/2*x)+1)-
a^3/b^4*ln(tanh(1/2*x)+1)+1/2*a/b^2*ln(tanh(1/2*x)+1)-1/3/b/(tanh(1/2*x)-1)
^3-1/2/b^2/(tanh(1/2*x)-1)^2*a-1/2/b/(tanh(1/2*x)-1)^2-1/b^3/(tanh(1/2*x)-1
)*a^2-1/2/b^2/(tanh(1/2*x)-1)*a+1/2/b/(tanh(1/2*x)-1)+a^3/b^4*ln(tanh(1/2*x
)-1)-1/2*a/b^2*ln(tanh(1/2*x)-1)+2*a^4/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a
*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22059, size = 1947, normalized size = 18.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((a^2*b^3 + b^5)*cosh(x)^6 + (a^2*b^3 + b^5)*sinh(x)^6 - 3*(a^3*b^2 +
a*b^4)*cosh(x)^5 - 3*(a^3*b^2 + a*b^4 - 2*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^
5 + a^2*b^3 + b^5 - 12*(2*a^5 + a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4*b +
 a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3 + b
^5)*cosh(x)^2 - 5*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b^3 + b
^5)*cosh(x)^3 - 15*(a^3*b^2 + a*b^4)*cosh(x)^2 - 6*(2*a^5 + a^3*b^2 - a*b^4
)*x + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 + 3*(4*a^4*b + a^2*b
^3 - 3*b^5)*cosh(x)^2 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3 + b^5)*co
sh(x)^4 - 10*(a^3*b^2 + a*b^4)*cosh(x)^3 - 12*(2*a^5 + a^3*b^2 - a*b^4)*x*c
osh(x) + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^4*cosh(
x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)*s
qrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 +
 b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sin
h(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sin
h(x) - b)) + 3*(a^3*b^2 + a*b^4)*cosh(x) + 3*(2*(a^2*b^3 + b^5)*cosh(x)^5 +
 a^3*b^2 + a*b^4 - 5*(a^3*b^2 + a*b^4)*cosh(x)^4 - 12*(2*a^5 + a^3*b^2 - a*
b^4)*x*cosh(x)^2 + 4*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^3 + 2*(4*a^4*b + a
^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^2*b^4 +
 b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 + b^6)*cosh(x)*sinh(x)^2 + (a^2*b^4 +
b^6)*sinh(x)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34664, size = 211, normalized size = 1.95

$$\frac{a^4 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4} + \frac{b^2e^{(3\,x)} - 3\,abe^{(2\,x)} + 12\,a^2e^x - 9\,b^2e^x}{24\,b^3} - \frac{\left(2\,a^3 - ab^2\right)x}{2\,b^4} + \frac{\left(3\,ab^2e^x + b^3 + 3\left(4\,a^2b - 3\,b^3\right)e^{(2\,x)}\right)e}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*
a^2*e^x - 9*b^2*e^x)/b^3 - 1/2*(2*a^3 - a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x +
b^3 + 3*(4*a^2*b - 3*b^3)*e^(2*x))*e^(-3*x)/b^4
```

3.73 $\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=82

$$\frac{x(2a^2-b^2)}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{a\cosh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

```
[Out] ((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2
]])/(b^3*Sqrt[a^2 + b^2]) - (a*Cosh[x])/b^2 + (Cosh[x]*Sinh[x])/(2*b)
```

Rubi [A] time = 0.184749, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2793, 3023, 2735, 2660, 618, 206}

$$\frac{x(2a^2-b^2)}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{a\cosh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2
]])/(b^3*Sqrt[a^2 + b^2]) - (a*Cosh[x])/b^2 + (Cosh[x]*Sinh[x])/(2*b)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx &= \frac{\cosh(x)\sinh(x)}{2b} - \frac{\int \frac{a+b\sinh(x)+2a\sinh^2(x)}{a+b\sinh(x)}\,dx}{2b}\\
&= -\frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{i\int \frac{-iab+i(2a^2-b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^2}\\
&= \frac{(2a^2-b^2)x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{a^3\int \frac{1}{a+b\sinh(x)}\,dx}{b^3}\\
&= \frac{(2a^2-b^2)x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{(2a^2-b^2)x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} + \frac{(4a^3)\,\mathrm{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{(2a^2-b^2)x}{2b^3} + \frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.136388, size = 82, normalized size = 1.

$$\frac{-\frac{8a^3\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 4a^2x - 4ab\cosh(x) - 2b^2x + b^2\sinh(2x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (4*a^2*x - 2*b^2*x - (8*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqr
t[-a^2 - b^2] - 4*a*b*Cosh[x] + b^2*Sinh[2*x])/(4*b^3)
```

Maple [B] time = 0.026, size = 174, normalized size = 2.1

$$-\frac{1}{2b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + \frac{1}{2b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{a}{b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{a^2}{b^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{2b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sinh(x)),x)
```

```
[Out] -1/2/b/(tanh(1/2*x)+1)^2+1/2/b/(tanh(1/2*x)+1)-1/b^2/(tanh(1/2*x)+1)*a+1/b^
3*ln(tanh(1/2*x)+1)*a^2-1/2/b*ln(tanh(1/2*x)+1)+1/2/b/(tanh(1/2*x)-1)^2+1/2
/b/(tanh(1/2*x)-1)+1/b^2/(tanh(1/2*x)-1)*a-1/b^3*ln(tanh(1/2*x)-1)*a^2+1/2/
b*ln(tanh(1/2*x)-1)-2*a^3/b^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-
2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.09283, size = 1142, normalized size = 13.93

$$\frac{\left(a^2b^2+b^4\right)\cosh(x)^4+\left(a^2b^2+b^4\right)\sinh(x)^4-a^2b^2-b^4+4\left(2\,a^4+a^2b^2-b^4\right)x\cosh(x)^2-4\left(a^3b+ab^3\right)\cosh(x)^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2*b^2 + b^4)*sinh(x)^4 - a^2*b^2 - b^4
+ 4*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4*(
a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 + b^4)*c
osh(x)^2 + 2*(2*a^4 + a^2*b^2 - b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sinh(x)
^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a^2 + b
^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b
^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/
(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b))
- 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(x)^3
- 2*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x
))/((a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^2*b^3 + b^5)*cosh(x)*sinh(x) + (a^2*b^
3 + b^5)*sinh(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4004, size = 158, normalized size = 1.93

$$-\frac{a^3 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3}+\frac{be^{(2\,x)}-4\,ae^x}{8\,b^2}+\frac{\left(2\,a^2-b^2\right)x}{2\,b^3}-\frac{\left(4\,abe^x+b^2\right)e^{(-2\,x)}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) + 1/8*(b*e^(2*x) - 4*a*e^x)/b^2 + 1/2*(2
*a^2 - b^2)*x/b^3 - 1/8*(4*a*b*e^x + b^2)*e^(-2*x)/b^3
```

3.74 $\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

```
[Out] -((a*x)/b^2) - (2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt
[a^2 + b^2]) + Cosh[x]/b
```

Rubi [A] time = 0.116113, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2746, 12, 2735, 2660, 618, 206}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -((a*x)/b^2) - (2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt
[a^2 + b^2]) + Cosh[x]/b
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx &= \frac{\cosh(x)}{b}-\frac{\int \frac{a\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{\cosh(x)}{b}-\frac{a\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{a^2\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}-\frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{\cosh(x)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0930584, size = 61, normalized size = 1.07

$$
\frac{a\left(\frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-x\right)+b\cosh(x)}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (a*(-x + (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])
 + b*Cosh[x])/b^2
```

Maple [A] time = 0.022, size = 92, normalized size = 1.6

$$
\frac{1}{b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{a}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{b}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+\frac{a}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\,\frac{a^2}{b^2\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sinh(x)),x)
```

```
[Out] 1/b/(tanh(1/2*x)+1)-a/b^2*ln(tanh(1/2*x)+1)-1/b/(tanh(1/2*x)-1)+a/b^2*ln(ta
nh(1/2*x)-1)+2*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a
```

```
^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14927, size = 640, normalized size = 11.23

$$\frac{a^2b + b^3 - 2\left(a^3 + ab^2\right)x\cosh\left(x\right) + \left(a^2b + b^3\right)\cosh\left(x\right)^2 + \left(a^2b + b^3\right)\sinh\left(x\right)^2 + 2\left(a^2\cosh\left(x\right) + a^2\sinh\left(x\right)\right)\sqrt{a^2 + b^2}}{2\left(\left(a^2b^2 + b^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b + b^3 - 2*(a^3 + a*b^2)*x*cosh(x) + (a^2*b + b^3)*cosh(x)^2 + (a
^2*b + b^3)*sinh(x)^2 + 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 + b^2)*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x
)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*((a^3
 + a*b^2)*x - (a^2*b + b^3)*cosh(x))*sinh(x))/((a^2*b^2 + b^4)*cosh(x) + (a
^2*b^2 + b^4)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48418, size = 116, normalized size = 2.04

$$\frac{a^2\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2 + b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2 + b^2}\right|}\right)}{\sqrt{a^2 + b^2}b^2} - \frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b
```

3.75 $\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=47

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{x}{b}$$

```
[Out] x/b + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0608682, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2735, 2660, 618, 206}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sinh[x]),x]
```

```
[Out] x/b + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = \frac{x}{b} - \frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b}$$
$$= \frac{x}{b} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{x}{b} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{x}{b} + \frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

Mathematica [A] time = 0.050002, size = 52, normalized size = 1.11

$$\frac{x - \frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x]),x]
```

```
[Out] (x - (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])/b
```

Maple [A] time = 0.015, size = 63, normalized size = 1.3

$$\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - 2\,\frac{a}{b\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tanh(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x)),x)
```

```
[Out] 1/b*ln(tanh(1/2*x)+1)-1/b*ln(tanh(1/2*x)-1)-2*a/b/(a^2+b^2)^(1/2)*arctanh(1
/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02995, size = 367, normalized size = 7.81

$$\frac{\sqrt{a^2+b^2}a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)+\left(a^2+b^2\right)x}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) + (a^2 + b^2)*x)/(a^2*b + b^3)
```

Sympy [A] time = 96.1744, size = 296, normalized size = 6.3

$$\begin{cases} \tilde{\infty}x & \text{for}\ a=0\wedge b=0\\ \frac{\cosh(x)}{a} & \text{for}\ b=0\\ \frac{x}{b} & \text{for}\ a=0\\ -\frac{b^2x\tanh\left(\frac{x}{2}\right)}{-b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}}+\frac{ibx\sqrt{b^2}}{-b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}}-\frac{2ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}{-b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} & \text{for}\ a=-\sqrt{-b^2}\\ \frac{b^2x\tanh\left(\frac{x}{2}\right)}{b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}}+\frac{ibx\sqrt{b^2}}{b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}}-\frac{2ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}{b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} & \text{for}\ a=\sqrt{-b^2}\\ \frac{a\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}}-\frac{a\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}}+\frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (cosh(x)/a, Eq(b, 0)), (x/b, Eq(a,
0)), (-b**2*x*tanh(x/2)/(-b**3*tanh(x/2) + I*b**2*sqrt(b**2)) + I*b*x*sqrt(
b**2)/(-b**3*tanh(x/2) + I*b**2*sqrt(b**2)) - 2*I*b*sqrt(b**2)*tanh(x/2)/(-
b**3*tanh(x/2) + I*b**2*sqrt(b**2)), Eq(a, -sqrt(-b**2))), (b**2*x*tanh(x/2
)/(b**3*tanh(x/2) + I*b**2*sqrt(b**2)) + I*b*x*sqrt(b**2)/(b**3*tanh(x/2) +
 I*b**2*sqrt(b**2)) - 2*I*b*sqrt(b**2)*tanh(x/2)/(b**3*tanh(x/2) + I*b**2*s
qrt(b**2)), Eq(a, sqrt(-b**2))), (a*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)
/a)/(b*sqrt(a**2 + b**2)) - a*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b
*sqrt(a**2 + b**2)) + x/b, True))
```

Giac [A] time = 1.36999, size = 90, normalized size = 1.91

$$-\frac{a\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b) + x/b
```

3.76 $\int \frac{\mathbf{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=50

$$\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{a}$$

```
[Out] -(ArcTanh[Cosh[x]]/a) + (2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a
*Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0756402, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2747, 3770, 2660, 618, 206}

$$\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Sinh[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/a) + (2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a
*Sqrt[a^2 + b^2])
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx &= \frac{\int \operatorname{csch}(x)\,dx}{a}-\frac{b\int \frac{1}{a+b\sinh(x)}\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a}-\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a}+\frac{(4b)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a}+\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.0494246, size = 58, normalized size = 1.16

$$\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2b\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Sinh[x]),x]
```

```
[Out] ((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[T
anh[x/2]])/a
```

Maple [A] time = 0.022, size = 49, normalized size = 1.

$$\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-2\,\frac{b}{a\sqrt{a^2+b^2}}\operatorname{Artanh}\left(1/2\,\frac{2\,a\tanh(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*sinh(x)),x)
```

```
[Out] 1/a*ln(tanh(1/2*x))-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)
/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.23058, size = 456, normalized size = 9.12

$$\frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-\left(a^2+b^2\right)\log(\cos}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*log(c
osh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*sinh(x)), x)
```

Giac [A] time = 1.38303, size = 111, normalized size = 2.22

$$-\frac{b\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a}-\frac{\log\left(e^x+1\right)}{a}+\frac{\log\left(\left|e^x-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a) - log(e^x + 1)/a + log(abs(e^x - 1))/a
```

3.77 $\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(a^2*Sqrt[a^2 + b^2]) - Coth[x]/a
```

Rubi [A] time = 0.121137, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 12, 2747, 3770, 2660, 618, 206}

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(a^2*Sqrt[a^2 + b^2]) - Coth[x]/a
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx &= -\frac{\coth(x)}{a}-\frac{\int \frac{b\operatorname{csch}(x)}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{\coth(x)}{a}-\frac{b\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{\coth(x)}{a}-\frac{b\int \operatorname{csch}(x)\,dx}{a^2}+\frac{b^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2}\\
&= \frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}-\frac{\left(4b^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\coth(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.401641, size = 81, normalized size = 1.37

$$
-\frac{2b\left(\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2b\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+a\tanh\left(\frac{x}{2}\right)+a\coth\left(\frac{x}{2}\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -(a*Coth[x/2] + 2*b*((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt
[-a^2 - b^2] + Log[Tanh[x/2]]) + a*Tanh[x/2])/(2*a^2)
```

Maple [A] time = 0.027, size = 73, normalized size = 1.2

$$-\frac{1}{2a}\tanh\left(\frac{x}{2}\right)-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{b}{a^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\frac{b^2}{a^2\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\frac{2a\tanh(x/2)-2b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*sinh(x)),x)
```

```
[Out] -1/2/a*tanh(1/2*x)-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))+2*b^2/a^2/(a^2
+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.40844, size = 926, normalized size = 15.69

$$\frac{2a^3+2ab^2-\left(b^2\cosh(x)^2+2b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2-b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a\cdots}{b\cosh(x)^2+b\sinh\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 -
 b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) + (a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b + b^3
)*cosh(x)*sinh(x) - (a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (
a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b + b^3)*cosh(x)*sinh(x) - (
a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^4 + a^2*b^2 - (a^4 +
 a^2*b^2)*cosh(x)^2 - 2*(a^4 + a^2*b^2)*cosh(x)*sinh(x) - (a^4 + a^2*b^2)*s
inh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*sinh(x)), x)
```

Giac [A] time = 1.38792, size = 132, normalized size = 2.24

$$\frac{b^2 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2}+\frac{b\log\left(e^x+1\right)}{a^2}-\frac{b\log\left(\left|e^x-1\right|\right)}{a^2}-\frac{2}{a\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + b*log(e^x + 1)/a^2 - b*log(abs(e^x - 1)
)/a^2 - 2/(a*(e^(2*x) - 1))
```

3.78 $\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=81

$$\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^3}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

```
[Out] ((a^2 - 2*b^2)*ArcTanh[Cosh[x]])/(2*a^3) + (2*b^3*ArcTanh[(b - a*Tanh[x/2])
/Sqrt[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) + (b*Coth[x])/a^2 - (Coth[x]*Csch[
x])/(2*a)
```

Rubi [A] time = 0.31795, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^3}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 - 2*b^2)*ArcTanh[Cosh[x]])/(2*a^3) + (2*b^3*ArcTanh[(b - a*Tanh[x/2])
/Sqrt[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) + (b*Coth[x])/a^2 - (Coth[x]*Csch[
x])/(2*a)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx &= -\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{i\int \frac{\operatorname{csch}^2(x)\left(2ib+ia\sinh(x)+ib\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{2a}\\
&= \frac{b\coth(x)}{a^2}-\frac{\coth(x)\operatorname{csch}(x)}{2a}-\frac{\int \frac{\operatorname{csch}(x)\left(a^2-2b^2+ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{2a^2}\\
&= \frac{b\coth(x)}{a^2}-\frac{\coth(x)\operatorname{csch}(x)}{2a}-\frac{b^3\int \frac{1}{a+b\sinh(x)}\,dx}{a^3}-\frac{\left(a^2-2b^2\right)\int \operatorname{csch}(x)\,dx}{2a^3}\\
&= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^3}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\operatorname{csch}(x)}{2a}-\frac{\left(2b^3\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^3}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{\left(4b^3\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b
\end{aligned}
$$

$$
= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\operatorname{csch}(x)}{2a}
$$

Mathematica [A] time = 0.537556, size = 118, normalized size = 1.46

$$-\frac{4\left(a^2-2b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{16b^3\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+a^2\text{csch}^2\left(\frac{x}{2}\right)+a^2\text{sech}^2\left(\frac{x}{2}\right)-4ab\tanh\left(\frac{x}{2}\right)-4ab\coth\left(\frac{x}{2}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -((16*b^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*
a*b*Coth[x/2] + a^2*Csch[x/2]^2 + 4*(a^2 - 2*b^2)*Log[Tanh[x/2]] + a^2*Sech
[x/2]^2 - 4*a*b*Tanh[x/2])/(8*a^3)
```

Maple [A] time = 0.03, size = 108, normalized size = 1.3

$$\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{b}{2a^2}\tanh\left(\frac{x}{2}\right)-\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}-\frac{1}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{b^2}{a^3}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{b}{2a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*sinh(x)),x)
```

```
[Out] 1/8/a*tanh(1/2*x)^2+1/2/a^2*tanh(1/2*x)*b-1/8/a/tanh(1/2*x)^2-1/2/a*ln(tanh
(1/2*x))+1/a^3*ln(tanh(1/2*x))*b^2+1/2*b/a^2/tanh(1/2*x)-2/a^3*b^3/(a^2+b^2
)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.9604, size = 2288, normalized size = 28.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*s
inh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 + a^
2*b^2)*cosh(x))*sinh(x)^2 - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^
3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)^2 +
 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^
2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sin
```

```
h(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh
(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(a^4 + a^2*b^2)*c
osh(x) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cos
h(x)*sinh(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^2 - 2*b^4
- 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 -
 a^2*b^2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(x)
^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) +
 ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)*sin
h(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4
 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2
 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^3 - (a^
4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^4
+ a^2*b^2 + 3*(a^4 + a^2*b^2)*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x))*sinh(x
))/(a^5 + a^3*b^2 + (a^5 + a^3*b^2)*cosh(x)^4 + 4*(a^5 + a^3*b^2)*cosh(x)*s
inh(x)^3 + (a^5 + a^3*b^2)*sinh(x)^4 - 2*(a^5 + a^3*b^2)*cosh(x)^2 - 2*(a^5
 + a^3*b^2 - 3*(a^5 + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + a^3*b^2)*co
sh(x)^3 - (a^5 + a^3*b^2)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*sinh(x)), x)
```

Giac [A] time = 1.41891, size = 185, normalized size = 2.28

$$-\frac{b^3\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3}+\frac{\left(a^2-2\,b^2\right)\log\left(e^x+1\right)}{2\,a^3}-\frac{\left(a^2-2\,b^2\right)\log\left(\left|e^x-1\right|\right)}{2\,a^3}-\frac{ae^{(3\,x)}-2\,be^{(2\,x)}+ae^x+2\,b}{a^2\left(e^{(2\,x)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/2*(a^2 - 2*b^2)*log(e^x + 1)/a^3 - 1
/2*(a^2 - 2*b^2)*log(abs(e^x - 1))/a^3 - (a*e^(3*x) - 2*b*e^(2*x) + a*e^x +
 2*b)/(a^2*(e^(2*x) - 1)^2)
```

3.79 $\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=109

$$-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3}-\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}+\frac{b\coth(x)\operatorname{csch}(x)}{2a^2}-\frac{\coth(x)\operatorname{csch}}{3a}$$

```
[Out] -(b*(a^2 - 2*b^2)*ArcTanh[Cosh[x]])/(2*a^4) - (2*b^4*ArcTanh[(b - a*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a^4*Sqrt[a^2 + b^2]) + ((2*a^2 - 3*b^2)*Coth[x])/(3*
a^3) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)
```

Rubi [A] time = 0.491286, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3}-\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}+\frac{b\coth(x)\operatorname{csch}(x)}{2a^2}-\frac{\coth(x)\operatorname{csch}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -(b*(a^2 - 2*b^2)*ArcTanh[Cosh[x]])/(2*a^4) - (2*b^4*ArcTanh[(b - a*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a^4*Sqrt[a^2 + b^2]) + ((2*a^2 - 3*b^2)*Coth[x])/(3*
a^3) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = -\frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{i\int \frac{\operatorname{csch}^3(x)\left(3ib+2ia\sinh(x)+2ib\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{3a}$$

$$= \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} - \frac{\int \frac{\operatorname{csch}^2(x)\left(2\left(2a^2-3b^2\right)+ab\sinh(x)-3b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{6a^2}$$

$$= \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} - \frac{i\int \frac{\operatorname{csch}(x)\left(3ib\left(a^2-2b^2\right)+3iab^2\sinh(x)\right)}{a+b\sinh(x)}\,dx}{6a^3}$$

$$= \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{b^4\int \frac{1}{a+b\sinh(x)}\,dx}{a^4} + \frac{\left(b\left(a^2-2b^2\right)\right)}{2a}$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} +$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} -$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} + \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}}{2a^2}$$

Mathematica [A] time = 0.934462, size = 186, normalized size = 1.71

$$\frac{4a\left(2a^2-3b^2\right)\coth\left(\frac{x}{2}\right)+\frac{48b^4\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+3a^2b\text{csch}^2\left(\frac{x}{2}\right)+3a^2b\text{sech}^2\left(\frac{x}{2}\right)+12a^2b\log\left(\tanh\left(\frac{x}{2}\right)\right)+8a^3\tanh\left(\right.}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((48*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 4*a
*(2*a^2 - 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 + 12*a^2*b*Log[Tanh[x/2]]
- 24*b^3*Log[Tanh[x/2]] + 3*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4
 - (a^3*Csch[x/2]^4*Sinh[x])/2 + 8*a^3*Tanh[x/2] - 12*a*b^2*Tanh[x/2])/(24*
a^4)
```

Maple [A] time = 0.031, size = 158, normalized size = 1.5

$$-\frac{1}{24\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}-\frac{b}{8\,a^{2}}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+\frac{3}{8\,a}\tanh\left(\frac{x}{2}\right)-\frac{b^{2}}{2\,a^{3}}\tanh\left(\frac{x}{2}\right)-\frac{1}{24\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}+\frac{3}{8\,a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*sinh(x)),x)
```

```
[Out] -1/24/a*tanh(1/2*x)^3-1/8/a^2*b*tanh(1/2*x)^2+3/8/a*tanh(1/2*x)-1/2/a^3*b^2
*tanh(1/2*x)-1/24/a/tanh(1/2*x)^3+3/8/a/tanh(1/2*x)-1/2/a^3/tanh(1/2*x)*b^2
+1/8/a^2*b/tanh(1/2*x)^2+1/2/a^2*b*ln(tanh(1/2*x))-1/a^4*b^3*ln(tanh(1/2*x)
)+2/a^4*b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.06914, size = 3970, normalized size = 36.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 8*a^5
- 4*a^3*b^2 - 12*a*b^4 - 12*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(2*a^3*b^2 + 2*
```

```
a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 12*(5*(a^4*b + a^2*b^3)*co
sh(x)^2 - 4*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 24*(a^5 - a*b^4)*cosh(x)
^2 - 12*(2*a^5 - 2*a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2 + a*b
^4)*cosh(x)^2)*sinh(x)^2 + 6*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4
*sinh(x)^6 - 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 - b^4)*
sinh(x)^4 - b^4 + 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*
cosh(x)^4 - 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 - 2*b^4*cos
h(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(a^4*b + a^2*b^3)*cosh(x) -
3*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^6 + 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x
)*sinh(x)^5 + (a^4*b - a^2*b^3 - 2*b^5)*sinh(x)^6 - a^4*b + a^2*b^3 + 2*b^5
 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 3*(a^4*b - a^2*b^3 - 2*b^5 - 5*(
a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - a^2*b^3 - 2*b
^5)*cosh(x)^3 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b -
 a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*(a^4*b - a^2*b^3 - 2*b^5 + 5*(a^4*b - a^2*b
^3 - 2*b^5)*cosh(x)^4 - 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 +
6*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^5 - 2*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x
)^3 + (a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1
) + 3*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^6 + 6*(a^4*b - a^2*b^3 - 2*b^5)*co
sh(x)*sinh(x)^5 + (a^4*b - a^2*b^3 - 2*b^5)*sinh(x)^6 - a^4*b + a^2*b^3 + 2
*b^5 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 3*(a^4*b - a^2*b^3 - 2*b^5 -
 5*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - a^2*b^3 -
 2*b^5)*cosh(x)^3 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4
*b - a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*(a^4*b - a^2*b^3 - 2*b^5 + 5*(a^4*b - a
^2*b^3 - 2*b^5)*cosh(x)^4 - 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^
2 + 6*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^5 - 2*(a^4*b - a^2*b^3 - 2*b^5)*co
sh(x)^3 + (a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
 - 1) - 6*(a^4*b + a^2*b^3 - 5*(a^4*b + a^2*b^3)*cosh(x)^4 + 8*(a^3*b^2 + a
*b^4)*cosh(x)^3 + 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 + a^4*b^2)*cosh(x
)^6 + 6*(a^6 + a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 + a^4*b^2)*sinh(x)^6 - a^6
 - a^4*b^2 - 3*(a^6 + a^4*b^2)*cosh(x)^4 - 3*(a^6 + a^4*b^2 - 5*(a^6 + a^4*
b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + a^4*b^2)*cosh(x)^3 - 3*(a^6 + a^4*b
^2)*cosh(x))*sinh(x)^3 + 3*(a^6 + a^4*b^2)*cosh(x)^2 + 3*(a^6 + a^4*b^2 + 5
*(a^6 + a^4*b^2)*cosh(x)^4 - 6*(a^6 + a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a
^6 + a^4*b^2)*cosh(x)^5 - 2*(a^6 + a^4*b^2)*cosh(x)^3 + (a^6 + a^4*b^2)*cos
h(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44821, size = 231, normalized size = 2.12

$$\frac{b^4 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^4} - \frac{\left(a^2b-2\,b^3\right)\log\left(e^x+1\right)}{2\,a^4} + \frac{\left(a^2b-2\,b^3\right)\log\left(\left|e^x-1\right|\right)}{2\,a^4} + \frac{3\,abe^{(5\,x)}-6\,b^2e^{(4\,x)}-12\,a^2e^{(2\,x)}+12\,b}{3\,a^3\left(e^{(2\,x)}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) - 1/2*(a^2*b - 2*b^3)*log(e^x + 1)/a^4 +
1/2*(a^2*b - 2*b^3)*log(abs(e^x - 1))/a^4 + 1/3*(3*a*b*e^(5*x) - 6*b^2*e^(4
*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 - 6*b^2)/(a^3*(e^
(2*x) - 1)^3)
```

3.80 $\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=162

$$\frac{x\left(6a^2-b^2\right)}{2b^4}-\frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)}+\frac{2a^3\left(3a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}}-\frac{a^2\sinh^2(x)\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(3a^2+b^2\right)\mathrm{s}\cdots}{2b^2\left(\cdots\right.}$$

```
[Out] ((6*a^2 - b^2)*x)/(2*b^4) + (2*a^3*(3*a^2 + 4*b^2)*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^4*(a^2 + b^2)^(3/2)) - (a*(3*a^2 + 2*b^2)*Cosh[x])/(
b^3*(a^2 + b^2)) + ((3*a^2 + b^2)*Cosh[x]*Sinh[x])/(2*b^2*(a^2 + b^2)) - (a
^2*Cosh[x]*Sinh[x]^2)/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.405337, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2792, 3049, 3023, 2735, 2660, 618, 206}

$$\frac{x\left(6a^2-b^2\right)}{2b^4}-\frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)}+\frac{2a^3\left(3a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}}-\frac{a^2\sinh^2(x)\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(3a^2+b^2\right)\mathrm{s}\cdots}{2b^2\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a^2 - b^2)*x)/(2*b^4) + (2*a^3*(3*a^2 + 4*b^2)*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^4*(a^2 + b^2)^(3/2)) - (a*(3*a^2 + 2*b^2)*Cosh[x])/(
b^3*(a^2 + b^2)) + ((3*a^2 + b^2)*Cosh[x]*Sinh[x])/(2*b^2*(a^2 + b^2)) - (a
^2*Cosh[x]*Sinh[x]^2)/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{a^2\cosh(x)\sinh^2(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{\sinh(x)\left(2a^2-ab\sinh(x)+\left(3a^2+b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}dx}{b\left(a^2+b^2\right)}$$

$$= \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{-a\left(3a^2+b^2\right)+b\left(a^2-b^2\right)\sinh(x)-2a\left(3a^2+2b^2\right)}{a+b\sinh(x)}}{2b^2\left(a^2+b^2\right)}$$

$$= -\frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{i\int \frac{iab\left(3a^2+\right.}{}}{}$$

$$= \frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}$$

$$= \frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}$$

$$= \frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}$$

$$= \frac{\left(6a^2-b^2\right)x}{2b^4} + \frac{2a^3\left(3a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}} - \frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b^2\right)\cosh}{2b^2\left(a^2+\right.}$$

Mathematica [A] time = 0.407256, size = 118, normalized size = 0.73

$$\frac{-2x\left(b^2-6a^2\right)+\frac{8a^3\left(3a^2+4b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}-\frac{4a^4b\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-8ab\cosh(x)+b^2\sinh(2x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*(-6*a^2 + b^2)*x + (8*a^3*(3*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt
[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) - 8*a*b*Cosh[x] - (4*a^4*b*Cosh[x])/((a^2
 + b^2)*(a + b*Sinh[x])) + b^2*Sinh[2*x])/(4*b^4)
```

Maple [A] time = 0.048, size = 296, normalized size = 1.8

$$-\frac{1}{2\,b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2\,b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-2\,\frac{a}{b^3\left(\tanh\left(x/2\right)+1\right)}+3\,\frac{\ln\left(\tanh\left(x/2\right)+1\right)a^2}{b^4}-\frac{1}{2\,b^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] -1/2/b^2/(tanh(1/2*x)+1)^2+1/2/b^2/(tanh(1/2*x)+1)-2/b^3/(tanh(1/2*x)+1)*a+
3/b^4*ln(tanh(1/2*x)+1)*a^2-1/2/b^2*ln(tanh(1/2*x)+1)+1/2/b^2/(tanh(1/2*x)-
1)^2+1/2/b^2/(tanh(1/2*x)-1)+2/b^3/(tanh(1/2*x)-1)*a-3/b^4*ln(tanh(1/2*x)-1
)*a^2+1/2/b^2*ln(tanh(1/2*x)-1)+2/b^2*a^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-
```

```
a)/(a^2+b^2)*tanh(1/2*x)+2/b^3*a^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2
+b^2)-6/b^4*a^5/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)
^(1/2))-8/b^2*a^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^
2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78534, size = 4020, normalized size = 24.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^4*b^3 + 2*a^2*b^5 + b^7 + (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^6 + (a
^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^6 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(
x)^5 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6 - (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)
)*sinh(x)^5 - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a
^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x)^4 - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^
5 + b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^2 - 4*(6*a^6*b + 11*a^4*b^
3 + 4*a^2*b^5 - b^7)*x + 30*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^
4 + 8*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)*x)*cosh
(x)^3 + 4*(4*a^7 + 4*a^5*b^2 + 5*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^3 - 15
*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^2 + 2*(6*a^7 + 11*a^5*b^2 + 4*a^3*b^
4 - a*b^6)*x - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*
a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x)^3 - (32*a^6*b + 49*a^4*b^3 +
 18*a^2*b^5 + b^7 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x)^2
 - (32*a^6*b + 49*a^4*b^3 + 18*a^2*b^5 + b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^
7)*cosh(x)^4 + 60*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 6*(16*a^6*b + 3
3*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x
)*cosh(x)^2 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x - 24*(2*a^7 + 2*
a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)*x)*cosh(x))*sinh(x)^2 +
8*((3*a^5*b + 4*a^3*b^3)*cosh(x)^4 + (3*a^5*b + 4*a^3*b^3)*sinh(x)^4 + 2*(3
*a^6 + 4*a^4*b^2)*cosh(x)^3 + 2*(3*a^6 + 4*a^4*b^2 + 2*(3*a^5*b + 4*a^3*b^3
)*cosh(x))*sinh(x)^3 - (3*a^5*b + 4*a^3*b^3)*cosh(x)^2 - (3*a^5*b + 4*a^3*b
^3 - 6*(3*a^5*b + 4*a^3*b^3)*cosh(x)^2 - 6*(3*a^6 + 4*a^4*b^2)*cosh(x))*sin
h(x)^2 + 2*(2*(3*a^5*b + 4*a^3*b^3)*cosh(x)^3 + 3*(3*a^6 + 4*a^4*b^2)*cosh(
x)^2 - (3*a^5*b + 4*a^3*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cos
h(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b
)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b
*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 6*(a^5*b^2 + 2
*a^3*b^4 + a*b^6)*cosh(x) + 2*(3*a^5*b^2 + 6*a^3*b^4 + 3*a*b^6 + 3*(a^4*b^3
 + 2*a^2*b^5 + b^7)*cosh(x)^5 - 15*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^4
 - 2*(16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a^4*b^3 + 4
*a^2*b^5 - b^7)*x)*cosh(x)^3 + 12*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2
+ 4*a^3*b^4 - a*b^6)*x)*cosh(x)^2 - (32*a^6*b + 49*a^4*b^3 + 18*a^2*b^5 + b
```

```
^7 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x))/((a^4*
b^5 + 2*a^2*b^7 + b^9)*cosh(x)^4 + (a^4*b^5 + 2*a^2*b^7 + b^9)*sinh(x)^4 +
2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x)^3 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8
+ 2*(a^4*b^5 + 2*a^2*b^7 + b^9)*cosh(x))*sinh(x)^3 - (a^4*b^5 + 2*a^2*b^7 +
 b^9)*cosh(x)^2 - (a^4*b^5 + 2*a^2*b^7 + b^9 - 6*(a^4*b^5 + 2*a^2*b^7 + b^9
)*cosh(x)^2 - 6*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2 + 2*(2*(a^
4*b^5 + 2*a^2*b^7 + b^9)*cosh(x)^3 + 3*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x
)^2 - (a^4*b^5 + 2*a^2*b^7 + b^9)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42231, size = 317, normalized size = 1.96

$$-\frac{\left(3\,a^5+4\,a^3b^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{\left(6\,a^2-b^2\right)x}{2\,b^4}+\frac{b^2e^{(2\,x)}-8\,abe^x}{8\,b^4}+\frac{\left(a^2b^3+b^5+8\left(2\,a^5-a^3b^2-ab^4\right)e^{(3\,x)}-(32}{8\left(a^2+b^2\right)(be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(3*a^5 + 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/2*(6*a^2
 - b^2)*x/b^4 + 1/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(a^2*b^3 + b^5 + 8*
(2*a^5 - a^3*b^2 - a*b^4)*e^(3*x) - (32*a^4*b + 17*a^2*b^3 + b^5)*e^(2*x) +
 6*(a^3*b^2 + a*b^4)*e^x)*e^(-2*x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x - b)*b
^4)
```

3.81 $\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=115

$$\frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)}-\frac{2a^2(2a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3(a^2+b^2)^{3/2}}-\frac{a^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{2ax}{b^3}$$

```
[Out] (-2*a*x)/b^3 - (2*a^2*(2*a^2 + 3*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^3*(a^2 + b^2)^(3/2)) + ((2*a^2 + b^2)*Cosh[x])/(b^2*(a^2 + b^2))
- (a^2*Cosh[x]*Sinh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.238455, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2792, 3023, 2735, 2660, 618, 206}

$$\frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)}-\frac{2a^2(2a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3(a^2+b^2)^{3/2}}-\frac{a^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*x)/b^3 - (2*a^2*(2*a^2 + 3*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^3*(a^2 + b^2)^(3/2)) + ((2*a^2 + b^2)*Cosh[x])/(b^2*(a^2 + b^2))
- (a^2*Cosh[x]*Sinh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\int \frac{a^2-ab\sinh(x)+\left(2a^2+b^2\right)\sinh^2(x)}{a+b\sinh(x)}\,dx}{b\left(a^2+b^2\right)}\\
&= \frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)}-\frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{i\int \frac{-ia^2b+2ia\left(a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}\,dx}{b^2\left(a^2+b^2\right)}\\
&= -\frac{2ax}{b^3}+\frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)}-\frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(a^2\left(2a^2+3b^2\right)\right)\int \frac{1}{a+b\sinh(x)}\,dx}{b^3\left(a^2+b^2\right)}\\
&= -\frac{2ax}{b^3}+\frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)}-\frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(2a^2\left(2a^2+3b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax}\right.}{b^3\left(a^2+b^2\right)}\\
&= -\frac{2ax}{b^3}+\frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)}-\frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\left(4a^2\left(2a^2+3b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-}\right.}{b^3\left(a^2+b^2\right.}\\
&= -\frac{2ax}{b^3}-\frac{2a^2\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\left(a^2+b^2\right)^{3/2}}+\frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)}-\frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.35159, size = 95, normalized size = 0.83

$$
\frac{-\frac{2a^2\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+\cosh(x)\left(\frac{a^3b}{\left(a^2+b^2\right)(a+b\sinh(x))}+b\right)-2ax}{b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*x - (2*a^2*(2*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]]
)/(-a^2 - b^2)^(3/2) + Cosh[x]*(b + (a^3*b)/((a^2 + b^2)*(a + b*Sinh[x]))))
```

```
/b^3
```

Maple [A] time = 0.04, size = 213, normalized size = 1.9

$$\frac{1}{b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-2\,\frac{a\ln\left(\tanh\left(x/2\right)+1\right)}{b^3}-\frac{1}{b^2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+2\,\frac{a\ln\left(\tanh\left(x/2\right)-1\right)}{b^3}-2\,\frac{a}{b\left(a\left(\tanh\left(x/2\right)\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] 1/b^2/(tanh(1/2*x)+1)-2*a/b^3*ln(tanh(1/2*x)+1)-1/b^2/(tanh(1/2*x)-1)+2*a/b
^3*ln(tanh(1/2*x)-1)-2/b*a^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)*
tanh(1/2*x)-2/b^2*a^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)+4/b^3*a
^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+6/b*a
^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.48933, size = 2426, normalized size = 21.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^4 - (
a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^
5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^3 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a
^5*b + 2*a^3*b^3 + a*b^5)*x + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x
)^3 + 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x)*cosh(x)^2 + 2*(2*
a^6 + 2*a^4*b^2 - 3*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 + 4*(a^6 + 2*a^4*
b^2 + a^2*b^4)*x - 3*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^5*b + 2*a^3*b^3 + a*
b^5)*x)*cosh(x))*sinh(x)^2 - 2*((2*a^4*b + 3*a^2*b^3)*cosh(x)^3 + (2*a^4*b
+ 3*a^2*b^3)*sinh(x)^3 + 2*(2*a^5 + 3*a^3*b^2)*cosh(x)^2 + (4*a^5 + 6*a^3*b
^2 + 3*(2*a^4*b + 3*a^2*b^3)*cosh(x))*sinh(x)^2 - (2*a^4*b + 3*a^2*b^3)*cos
h(x) - (2*a^4*b + 3*a^2*b^3 - 3*(2*a^4*b + 3*a^2*b^3)*cosh(x)^2 - 4*(2*a^5
+ 3*a^3*b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(3*a^5*b + 4*a^3*b^3 + a*b^5
 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x) - 2*(3*a^5*b + 4*a^3*b^3 + a*b^
```

```
5 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^5*b + 2*a^3*b^3 + a*b^5
- 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5
)*x - 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x)*cosh(x))*sinh(x))
/((a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x)^3 + (a^4*b^4 + 2*a^2*b^6 + b^8)*sinh(
x)^3 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x)^2 + (2*a^5*b^3 + 4*a^3*b^5 +
 2*a*b^7 + 3*(a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x))*sinh(x)^2 - (a^4*b^4 + 2*
a^2*b^6 + b^8)*cosh(x) - (a^4*b^4 + 2*a^2*b^6 + b^8 - 3*(a^4*b^4 + 2*a^2*b^
6 + b^8)*cosh(x)^2 - 4*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35389, size = 248, normalized size = 2.16

$$\frac{\left(2\,a^4+3\,a^2b^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2b^3+b^5\right)\sqrt{a^2+b^2}}-\frac{2\,ax}{b^3}+\frac{e^x}{2\,b^2}-\frac{\left(a^2b^2+b^4+\left(4\,a^4-a^2b^2-b^4\right)e^{(2\,x)}-2\left(3\,a^3b+ab^3\right)e^x\right)e^{(-x)}}{2\left(a^2+b^2\right)\left(be^{(2\,x)}+2\,ae^x-b\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^4 + 3*a^2*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^3 + b^5)*sqrt(a^2 + b^2)) - 2*a*x/b^3 +
 1/2*e^x/b^2 - 1/2*(a^2*b^2 + b^4 + (4*a^4 - a^2*b^2 - b^4)*e^(2*x) - 2*(3*
a^3*b + a*b^3)*e^x)*e^(-x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x - b)*b^3)
```

3.82 $\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=83

$$\frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2 + (2*a*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2
*(a^2 + b^2)^(3/2)) - (a^2*Cosh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.133505, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2790, 2735, 2660, 618, 206}

$$\frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] x/b^2 + (2*a*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2
*(a^2 + b^2)^(3/2)) - (a^2*Cosh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{i\int \frac{-iab+i\left(a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}\,dx}{b\left(a^2+b^2\right)}\\
&= \frac{x}{b^2}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\left(a\left(a^2+2b^2\right)\right)\int \frac{1}{a+b\sinh(x)}\,dx}{b^2\left(a^2+b^2\right)}\\
&= \frac{x}{b^2}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\left(2a\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2\left(a^2+b^2\right)}\\
&= \frac{x}{b^2}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(4a\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2\left(a^2+b^2\right)}\\
&= \frac{x}{b^2}+\frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.178002, size = 86, normalized size = 1.04

$$\frac{\frac{2a\left(a^2+2b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (x + (2*a*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 -
 b^2)^(3/2) - (a^2*b*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))/b^2
```

Maple [B] time = 0.033, size = 175, normalized size = 2.1

$$\frac{1}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\frac{a\tanh(x/2)}{\left(a(\tanh(x/2))^2-2\tanh(x/2)b-a\right)\left(a^2+b^2\right)}+2\frac{}{b\left(a(\tanh(x/2))^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] 1/b^2*ln(tanh(1/2*x)+1)-1/b^2*ln(tanh(1/2*x)-1)+2*a/(a*tanh(1/2*x)^2-2*tanh
(1/2*x)*b-a)/(a^2+b^2)*tanh(1/2*x)+2*a^2/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b
-a)/(a^2+b^2)-2/b^2*a^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(
a^2+b^2)^(1/2))-4*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+
```

```
b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.24456, size = 1245, normalized size = 15.

$$\frac{2\,a^4b + 2\,a^2b^3 - \left(a^4b + 2\,a^2b^3 + b^5\right)x\cosh\left(x\right)^2 - \left(a^4b + 2\,a^2b^3 + b^5\right)x\sinh\left(x\right)^2 + \left(a^3b + 2\,ab^3 - \left(a^3b + 2\,ab^3\right)\cosh\left(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a^4*b + 2*a^2*b^3 - (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x)^2 - (a^4*b + 2*a
^2*b^3 + b^5)*x*sinh(x)^2 + (a^3*b + 2*a*b^3 - (a^3*b + 2*a*b^3)*cosh(x)^2
- (a^3*b + 2*a*b^3)*sinh(x)^2 - 2*(a^4 + 2*a^2*b^2)*cosh(x) - 2*(a^4 + 2*a^
2*b^2 + (a^3*b + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x
)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*s
inh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*si
nh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (a^4*b + 2*a^2*b^
3 + b^5)*x - 2*(a^5 + a^3*b^2 + (a^5 + 2*a^3*b^2 + a*b^4)*x)*cosh(x) - 2*(a
^5 + a^3*b^2 + (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x) + (a^5 + 2*a^3*b^2 + a*b
^4)*x)*sinh(x))/(a^4*b^3 + 2*a^2*b^5 + b^7 - (a^4*b^3 + 2*a^2*b^5 + b^7)*co
sh(x)^2 - (a^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^2 - 2*(a^5*b^2 + 2*a^3*b^4 +
a*b^6)*cosh(x) - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6 + (a^4*b^3 + 2*a^2*b^5 + b^
7)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45168, size = 177, normalized size = 2.13

$$-\frac{\left(a^3 + 2\,ab^2\right)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2 + b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2 + b^2}\right|}\right)}{\left(a^2b^2 + b^4\right)\sqrt{a^2 + b^2}} + \frac{2\left(a^3e^x - a^2b\right)}{\left(a^2b^2 + b^4\right)\left(be^{(2\,x)} + 2\,ae^x - b\right)} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3 + 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2
*a + 2*sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^3*e^x - a
^2*b)/((a^2*b^2 + b^4)*(b*e^(2*x) + 2*a*e^x - b)) + x/b^2
```

3.83 $\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=60

$$\frac{a\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}-\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

```
[Out] (-2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (a*Co
sh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.0737309, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2754, 12, 2660, 618, 206}

$$\frac{a\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}-\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (a*Co
sh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx &= \frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\int \frac{b}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{b\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{(4b)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.101181, size = 68, normalized size = 1.13

$$\frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{2b\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (a*C
osh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Maple [A] time = 0.02, size = 97, normalized size = 1.6

$$4\,\frac{2\,\tanh(x/2)\,b+2\,a}{\left(-4\,a^2-4\,b^2\right)\left(a\left(\tanh(x/2)\right)^2-2\,\tanh(x/2)\,b-a\right)}-8\,\frac{b}{\left(-4\,a^2-4\,b^2\right)\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tanh(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x))^2,x)
```

```
[Out] 4*(2*tanh(1/2*x)*b+2*a)/(-4*a^2-4*b^2)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-
8*b/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b
^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0855, size = 855, normalized size = 14.25

$$-\frac{2\,a^3b+2\,ab^3+\left(b^3\cosh\left(x\right)^2+b^3\sinh\left(x\right)^2+2\,ab^2\cosh\left(x\right)-b^3+2\left(b^3\cosh\left(x\right)+ab^2\right)\sinh\left(x\right)\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cos}{}\right.}{a^4b^2+2\,a^2b^4+b^6-\left(a^4b^2+2\,a^2b^4+b^6\right)\cosh\left(x\right)^2-\left(a^4b^2+2\,a^2b^4+b^6\right)\sinh\left(x\right)^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b + 2*a*b^3 + (b^3*cosh(x)^2 + b^3*sinh(x)^2 + 2*a*b^2*cosh(x) - b^
3 + 2*(b^3*cosh(x) + a*b^2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b
^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x)
- 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4 + a^2*b^2)*cosh(x
) - 2*(a^4 + a^2*b^2)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2 + 2*a^
2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^2 - 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 + (a^4*b^2 + 2*a
^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39155, size = 134, normalized size = 2.23

$$\frac{b\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a^2e^x-ab\right)}{\left(a^2b+b^3\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^2*e^x - a*b)/((a^2*b + b^3)*(b*e^(2*x) +
 2*a*e^x - b))
```

3.84 $\int \frac{\mathrm{csch}(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=85

$$\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\tanh^{-1}(\cosh(x))}{a^2}$$

```
[Out] -(ArcTanh[Cosh[x]]/a^2) + (2*b*(2*a^2 + b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)) + (b^2*Cosh[x])/(a*(a^2 + b^2)*(a + b
*Sinh[x]))
```

Rubi [A] time = 0.215688, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2802, 3001, 3770, 2660, 618, 206}

$$\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\tanh^{-1}(\cosh(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -(ArcTanh[Cosh[x]]/a^2) + (2*b*(2*a^2 + b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)) + (b^2*Cosh[x])/(a*(a^2 + b^2)*(a + b
*Sinh[x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx &= \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\int \frac{\operatorname{csch}(x)\left(a^2+b^2-ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}\\ &= \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\int \operatorname{csch}(x)\,dx}{a^2}-\frac{\left(b\left(2a^2+b^2\right)\right)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a^2}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\left(2b\left(2a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tan\right.}{a^2\left(a^2+b^2\right)}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a^2}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(4b\left(2a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b\right.}{a^2\left(a^2+b^2\right)}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a^2}+\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.18121, size = 91, normalized size = 1.07

$$\frac{\frac{2b\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+\frac{ab^2\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*b*(2*a^2 + b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2
)^(3/2) + Log[Tanh[x/2]] + (a*b^2*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))/a
^2
```

Maple [B] time = 0.039, size = 166, normalized size = 2.

$$\frac{1}{a^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-2\,\frac{b^3\tanh(x/2)}{a^2\left(a(\tanh(x/2))^2-2\,\tanh(x/2)\,b-a\right)\left(a^2+b^2\right)}-2\,\frac{b^2}{a\left(a(\tanh(x/2))^2-2\,\tanh(x/2)\,b-a\right)\left(a^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*sinh(x))^2,x)
```

```
[Out] 1/a^2*ln(tanh(1/2*x))-2/a^2*b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^
2)*tanh(1/2*x)-2/a*b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)-4*b/(a
^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-2/a^2*b^3/
(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.45987, size = 1632, normalized size = 19.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b^2 + 2*a*b^4 - (2*a^2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cosh(x)^2 - (2
*a^2*b^2 + b^4)*sinh(x)^2 - 2*(2*a^3*b + a*b^3)*cosh(x) - 2*(2*a^3*b + a*b^
3 + (2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
+ b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x
)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4*b + a^2*b^3)*c
osh(x) + (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (
a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) -
2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))*lo
g(cosh(x) + sinh(x) + 1) - (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 +
b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 +
 a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*co
sh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(a^4*b + a^2*b^3)*sinh(x))/(
a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b + 2*a^4*b^3 + a^2*b^5)*cosh(x)^2 - (a^
6*b + 2*a^4*b^3 + a^2*b^5)*sinh(x)^2 - 2*(a^7 + 2*a^5*b^2 + a^3*b^4)*cosh(x
) - 2*(a^7 + 2*a^5*b^2 + a^3*b^4 + (a^6*b + 2*a^4*b^3 + a^2*b^5)*cosh(x))*s
inh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)/(a + b*sinh(x))**2, x)
```

Giac [A] time = 1.48278, size = 192, normalized size = 2.26

$$-\frac{(2\,a^2b+b^3)\log\left(\frac{|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}|}{|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}|}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}}-\frac{2\,(abe^x-b^2)}{(a^3+ab^2)(be^{(2\,x)}+2\,ae^x-b)}-\frac{\log(e^x+1)}{a^2}+\frac{\log(|e^x-1|)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*b + b^3)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2
*a + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) - 2*(a*b*e^x - b
^2)/((a^3 + a*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - log(e^x + 1)/a^2 + log(abs(
e^x - 1))/a^2
```

3.85 $\int \frac{\mathrm{csch}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{2b^2\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{3/2}}-\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{2b\tanh^{-1}(\cosh(x))}{a^3}$$

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*b^2*(3*a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2
])/Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(3/2)) - ((a^2 + 2*b^2)*Coth[x])/(a^2
*(a^2 + b^2)) + (b^2*Coth[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.371979, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2b^2\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{3/2}}-\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{2b\tanh^{-1}(\cosh(x))}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*b^2*(3*a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2
])/Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(3/2)) - ((a^2 + 2*b^2)*Coth[x])/(a^2
*(a^2 + b^2)) + (b^2*Coth[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx &= \frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\int \frac{\operatorname{csch}^2(x)\left(a^2+2b^2-ab\sinh(x)+b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}\\
&= -\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{i\int \frac{\operatorname{csch}(x)\left(2ib\left(a^2+b^2\right)-iab^2\sinh(x)\right)}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}\\
&= -\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{(2b)\int \operatorname{csch}(x)\,dx}{a^3}+\frac{\left(b^2\left(3a^2+2b^2\right)\right)\int \frac{}{a+b}}{a^3\left(a^2+b^2\right)}\\
&= \frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(2b^2\left(3a^2+2b^2\right)\right)\mathrm{Sub}}{a}\\
&= \frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\left(4b^2\left(3a^2+2b^2\right)\right)\mathrm{Sub}}{}\\
&= \frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2b^2\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{3/2}}-\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2 c}{a\left(a^2+b^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.648951, size = 118, normalized size = 1.03

$$-\frac{\frac{4b^2(3a^2+2b^2)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+\frac{2ab^3\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}+a\tanh\left(\frac{x}{2}\right)+a\coth\left(\frac{x}{2}\right)+4b\log\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] -((4*b^2*(3*a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
- b^2)^(3/2) + a*Coth[x/2] + 4*b*Log[Tanh[x/2]] + (2*a*b^3*Cosh[x])/((a^2 +
 b^2)*(a + b*Sinh[x])) + a*Tanh[x/2])/(2*a^3)
```

Maple [A] time = 0.047, size = 193, normalized size = 1.7

$$-\frac{1}{2a^2}\tanh\left(\frac{x}{2}\right)-\frac{1}{2a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-2\frac{b\ln(\tanh(x/2))}{a^3}+2\frac{b^4\tanh(x/2)}{a^3\left(a(\tanh(x/2))^2-2\tanh(x/2)b-a\right)(a^2+b^2)}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] -1/2/a^2*tanh(1/2*x)-1/2/a^2/tanh(1/2*x)-2/a^3*b*ln(tanh(1/2*x))+2/a^3*b^4/
(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)*tanh(1/2*x)+2/a^2*b^3/(a*tanh
(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)+6/a*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*
(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+4/a^3*b^4/(a^2+b^2)^(3/2)*arctanh(1/
2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.55821, size = 4055, normalized size = 35.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a^5*b + 6*a^3*b^3 + 4*a*b^5 + 2*(a^4*b^2 + a^2*b^4)*cosh(x)^3 + 2*(a^4*b
^2 + a^2*b^4)*sinh(x)^3 - 2*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 - 2*(a^
5*b + 3*a^3*b^3 + 2*a*b^5 - 3*(a^4*b^2 + a^2*b^4)*cosh(x))*sinh(x)^2 + (3*a
```

```
^2*b^3 + 2*b^5 + (3*a^2*b^3 + 2*b^5)*cosh(x)^4 + (3*a^2*b^3 + 2*b^5)*sinh(x
)^4 + 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x)^3 + 2*(3*a^3*b^2 + 2*a*b^4 + 2*(3*a^2
*b^3 + 2*b^5)*cosh(x))*sinh(x)^3 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 2*(3*a
^2*b^3 + 2*b^5 - 3*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3*(3*a^3*b^2 + 2*a*b^4)*
cosh(x))*sinh(x)^2 - 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x) - 2*(3*a^3*b^2 + 2*a*b
^4 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^3 - 3*(3*a^3*b^2 + 2*a*b^4)*cosh(x)^2 +
2*(3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
+ b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x
)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(2*a^6 + 5*a^4*b^2
+ 3*a^2*b^4)*cosh(x) + 2*(a^4*b^2 + 2*a^2*b^4 + b^6 + (a^4*b^2 + 2*a^2*b^4
+ b^6)*cosh(x)^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 + 2*(a^5*b + 2*a^3
*b^3 + a*b^5)*cosh(x)^3 + 2*(a^5*b + 2*a^3*b^3 + a*b^5 + 2*(a^4*b^2 + 2*a^2
*b^4 + b^6)*cosh(x))*sinh(x)^3 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 -
2*(a^4*b^2 + 2*a^2*b^4 + b^6 - 3*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 - 3*
(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x))*sinh(x)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b
^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*
cosh(x)^3 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x)^2 + 2*(a^4*b^2 + 2*a^2*b^
4 + b^6)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*(a^4*b^2 + 2*a^2*
b^4 + b^6 + (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^4 + (a^4*b^2 + 2*a^2*b^4 +
b^6)*sinh(x)^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x)^3 + 2*(a^5*b + 2*a^3
*b^3 + a*b^5 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 - 2*(a^4*b^
2 + 2*a^2*b^4 + b^6)*cosh(x)^2 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6 - 3*(a^4*b^2
+ 2*a^2*b^4 + b^6)*cosh(x)^2 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x))*sinh(
x)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5
 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^3 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*
cosh(x)^2 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) - 1) - 2*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 3*(a^4*b^2 + a^2*b^4)*cosh
(x)^2 + 2*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x))*sinh(x))/(a^7*b + 2*a^5*b^
3 + a^3*b^5 + (a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^4 + (a^7*b + 2*a^5*b^3
+ a^3*b^5)*sinh(x)^4 + 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*cosh(x)^3 + 2*(a^8 + 2
*a^6*b^2 + a^4*b^4 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x))*sinh(x)^3 - 2
*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^2 - 2*(a^7*b + 2*a^5*b^3 + a^3*b^5 -
 3*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^2 - 3*(a^8 + 2*a^6*b^2 + a^4*b^4)*
cosh(x))*sinh(x)^2 - 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*cosh(x) - 2*(a^8 + 2*a^6
*b^2 + a^4*b^4 - 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^3 - 3*(a^8 + 2*a^6
*b^2 + a^4*b^4)*cosh(x)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x))*sinh(x
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**2/(a + b*sinh(x))**2, x)
```

Giac [A] time = 1.4016, size = 277, normalized size = 2.41

$$\frac{\left(3\,a^2b^2 + 2\,b^4\right)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^5 + a^3b^2\right)\sqrt{a^2+b^2}} + \frac{2\left(ab^2e^{(3\,x)} - a^2be^{(2\,x)} - 2\,b^3e^{(2\,x)} - 2\,a^3e^x - 3\,ab^2e^x + a^2b + 2\,b^3\right)}{\left(a^4 + a^2b^2\right)\left(be^{(4\,x)} + 2\,ae^{(3\,x)} - 2\,be^{(2\,x)} - 2\,ae^x + b\right)} + \frac{2\,b\log(e^x + }{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^2*b^2 + 2*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^2*e^
(3*x) - a^2*b*e^(2*x) - 2*b^3*e^(2*x) - 2*a^3*e^x - 3*a*b^2*e^x + a^2*b + 2
*b^3)/((a^4 + a^2*b^2)*(b*e^(4*x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x + b
)) + 2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3
```

3.86 $\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=158

$$\frac{2b^3\left(4a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\left(a^2+b^2\right)^{3/2}}+\frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)}+\frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}-\frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x}{2a^2\left(a^2+b^2\right)}$$

```
[Out] ((a^2 - 6*b^2)*ArcTanh[Cosh[x]])/(2*a^4) + (2*b^3*(4*a^2 + 3*b^2)*ArcTanh[(
b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^4*(a^2 + b^2)^(3/2)) + (b*(2*a^2 + 3*
b^2)*Coth[x])/(a^3*(a^2 + b^2)) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*a^2*(a
^2 + b^2)) + (b^2*Coth[x]*Csch[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.675246, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$\frac{2b^3\left(4a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\left(a^2+b^2\right)^{3/2}}+\frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)}+\frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}-\frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x}{2a^2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Sinh[x])^2,x]
```


```
[Out] ((a^2 - 6*b^2)*ArcTanh[Cosh[x]])/(2*a^4) + (2*b^3*(4*a^2 + 3*b^2)*ArcTanh[(
b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^4*(a^2 + b^2)^(3/2)) + (b*(2*a^2 + 3*
b^2)*Coth[x])/(a^3*(a^2 + b^2)) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*a^2*(a
^2 + b^2)) + (b^2*Coth[x]*Csch[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{b^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{\text{csch}^3(x)\left(a^2+3b^2-ab\sinh(x)+2b^2\sinh^2(x)\right)}{a+b\sinh(x)}dx}{a\left(a^2+b^2\right)}$$

$$= -\frac{\left(a^2+3b^2\right)\coth(x)\text{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{i\int \frac{\text{csch}^2(x)\left(2ib\left(2a^2+3b^2\right)+ia\left(a^2-b^2\right)\sinh(x\right.}{a+b\sinh(x)}}{2a^2\left(a^2+b^2\right)}$$

$$= \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+3b^2\right)\coth(x)\text{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\int \frac{\text{csch}(x)\left(a^4-\right.}{}}{}$$

$$= \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+3b^2\right)\coth(x)\text{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\left(a^2-6b^2\right)\int}{2a}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+3b^2\right)\coth(x)\text{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{b^2\coth}{a\left(a^2+b^2\right)}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+3b^2\right)\coth(x)\text{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{b^2\coth}{a\left(a^2+b^2\right)}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{2b^3\left(4a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\left(a^2+b^2\right)^{3/2}} + \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)} -$$

Mathematica [A] time = 0.70981, size = 156, normalized size = 0.99

$$\frac{-4\left(a^2-6b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{16b^3\left(4a^2+3b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} + \frac{8ab^4\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} - a^2\text{csch}^2\left(\frac{x}{2}\right) - a^2\text{sech}^2\left(\frac{x}{2}\right) + 8ab\tanh\left(\right.}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((16*b^3*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
- b^2)^(3/2) + 8*a*b*Coth[x/2] - a^2*Csch[x/2]^2 - 4*(a^2 - 6*b^2)*Log[Tanh
[x/2]] - a^2*Sech[x/2]^2 + (8*a*b^4*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
+ 8*a*b*Tanh[x/2])/(8*a^4)
```

Maple [A] time = 0.052, size = 227, normalized size = 1.4

$$\frac{1}{8a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{b}{a^3}\tanh\left(\frac{x}{2}\right) - \frac{1}{8a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} - \frac{1}{2a^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right) + 3\frac{\ln\left(\tanh\left(x/2\right)\right)b^2}{a^4} + \frac{b}{a^3}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] 1/8/a^2*tanh(1/2*x)^2+1/a^3*tanh(1/2*x)*b-1/8/a^2/tanh(1/2*x)^2-1/2/a^2*ln(
tanh(1/2*x))+3/a^4*ln(tanh(1/2*x))*b^2+b/a^3/tanh(1/2*x)-2/a^4*b^5/(a*tanh(
1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)*tanh(1/2*x)-2/a^3*b^4/(a*tanh(1/2*x)^
```

```
2-2*tanh(1/2*x)*b-a)/(a^2+b^2)-8/a^2*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*t
anh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-6/a^4*b^5/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a
*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.24119, size = 8699, normalized size = 55.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a^5*b^2 + 20*a^3*b^4 + 12*a*b^6 - 2*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)
*cosh(x)^5 - 2*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*sinh(x)^5 - 4*(a^7 - 4*a^3*b
^4 - 3*a*b^6)*cosh(x)^4 - 2*(2*a^7 - 8*a^3*b^4 - 6*a*b^6 + 5*(a^6*b + 4*a^4
*b^3 + 3*a^2*b^5)*cosh(x))*sinh(x)^4 + 8*(2*a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*
cosh(x)^3 + 4*(4*a^6*b + 10*a^4*b^3 + 6*a^2*b^5 - 5*(a^6*b + 4*a^4*b^3 + 3*
a^2*b^5)*cosh(x)^2 - 4*(a^7 - 4*a^3*b^4 - 3*a*b^6)*cosh(x))*sinh(x)^3 - 4*(
a^7 + 6*a^5*b^2 + 11*a^3*b^4 + 6*a*b^6)*cosh(x)^2 - 4*(a^7 + 6*a^5*b^2 + 11
*a^3*b^4 + 6*a*b^6 + 5*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*cosh(x)^3 + 6*(a^7 -
 4*a^3*b^4 - 3*a*b^6)*cosh(x)^2 - 6*(2*a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*cosh(
x))*sinh(x)^2 + 2*((4*a^2*b^4 + 3*b^6)*cosh(x)^6 + (4*a^2*b^4 + 3*b^6)*sinh
(x)^6 - 4*a^2*b^4 - 3*b^6 + 2*(4*a^3*b^3 + 3*a*b^5)*cosh(x)^5 + 2*(4*a^3*b^
3 + 3*a*b^5 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x)^5 - 3*(4*a^2*b^4 + 3*b
^6)*cosh(x)^4 - (12*a^2*b^4 + 9*b^6 - 15*(4*a^2*b^4 + 3*b^6)*cosh(x)^2 - 10
*(4*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^4 - 4*(4*a^3*b^3 + 3*a*b^5)*cosh(x)
^3 - 4*(4*a^3*b^3 + 3*a*b^5 - 5*(4*a^2*b^4 + 3*b^6)*cosh(x)^3 - 5*(4*a^3*b^
3 + 3*a*b^5)*cosh(x)^2 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x)^3 + 3*(4*a^
2*b^4 + 3*b^6)*cosh(x)^2 + (12*a^2*b^4 + 9*b^6 + 15*(4*a^2*b^4 + 3*b^6)*cos
h(x)^4 + 20*(4*a^3*b^3 + 3*a*b^5)*cosh(x)^3 - 18*(4*a^2*b^4 + 3*b^6)*cosh(x
)^2 - 12*(4*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^2 + 2*(4*a^3*b^3 + 3*a*b^5)
*cosh(x) + 2*(4*a^3*b^3 + 3*a*b^5 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x)^5 + 5*(4*
a^3*b^3 + 3*a*b^5)*cosh(x)^4 - 6*(4*a^2*b^4 + 3*b^6)*cosh(x)^3 - 6*(4*a^3*b
^3 + 3*a*b^5)*cosh(x)^2 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x))*sqrt(a^2
+ b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2
*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a
))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b
)) - 2*(7*a^6*b + 16*a^4*b^3 + 9*a^2*b^5)*cosh(x) - (a^6*b - 4*a^4*b^3 - 11
*a^2*b^5 - 6*b^7 - (a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^6 - (a^
6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*sinh(x)^6 - 2*(a^7 - 4*a^5*b^2 - 11*a
^3*b^4 - 6*a*b^6)*cosh(x)^5 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3
*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x)^5 + 3*(a^6*b - 4
*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^4 + (3*a^6*b - 12*a^4*b^3 - 33*a^2*b
^5 - 18*b^7 - 15*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 10*(a
^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^4 + 4*(a^7 - 4*a^5*
```

```
b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^3 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6
*a*b^6 - 5*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 - 5*(a^7 - 4*
a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b
^5 - 6*b^7)*cosh(x))*sinh(x)^3 - 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)
*cosh(x)^2 - (3*a^6*b - 12*a^4*b^3 - 33*a^2*b^5 - 18*b^7 + 15*(a^6*b - 4*a^
4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^4 + 20*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 -
6*a*b^6)*cosh(x)^3 - 18*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2
- 12*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 -
 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x) - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^
4 - 6*a*b^6 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^5 + 5*(a^7
 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^4 - 6*(a^6*b - 4*a^4*b^3 - 11*
a^2*b^5 - 6*b^7)*cosh(x)^3 - 6*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cos
h(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x))*log(c
osh(x) + sinh(x) + 1) + (a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7 - (a^6*b -
4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^6 - (a^6*b - 4*a^4*b^3 - 11*a^2*b^5
 - 6*b^7)*sinh(x)^6 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^5
- 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2
*b^5 - 6*b^7)*cosh(x))*sinh(x)^5 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^
7)*cosh(x)^4 + (3*a^6*b - 12*a^4*b^3 - 33*a^2*b^5 - 18*b^7 - 15*(a^6*b - 4*
a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 10*(a^7 - 4*a^5*b^2 - 11*a^3*b^4
- 6*a*b^6)*cosh(x))*sinh(x)^4 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*
cosh(x)^3 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 - 5*(a^6*b - 4*a^4*b^
3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 - 5*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b
^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x)
^3 - 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - (3*a^6*b - 12*a
^4*b^3 - 33*a^2*b^5 - 18*b^7 + 15*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*
cosh(x)^4 + 20*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^3 - 18*(a^6
*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 12*(a^7 - 4*a^5*b^2 - 11*a
^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*
a*b^6)*cosh(x) - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3*(a^6*b - 4*a
^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^5 + 5*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 -
6*a*b^6)*cosh(x)^4 - 6*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 -
 6*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^
3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(7
*a^6*b + 16*a^4*b^3 + 9*a^2*b^5 + 5*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*cosh(x)
^4 + 8*(a^7 - 4*a^3*b^4 - 3*a*b^6)*cosh(x)^3 - 12*(2*a^6*b + 5*a^4*b^3 + 3*
a^2*b^5)*cosh(x)^2 + 4*(a^7 + 6*a^5*b^2 + 11*a^3*b^4 + 6*a*b^6)*cosh(x))*si
nh(x))/(a^8*b + 2*a^6*b^3 + a^4*b^5 - (a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)
^6 - (a^8*b + 2*a^6*b^3 + a^4*b^5)*sinh(x)^6 - 2*(a^9 + 2*a^7*b^2 + a^5*b^4
)*cosh(x)^5 - 2*(a^9 + 2*a^7*b^2 + a^5*b^4 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5
)*cosh(x))*sinh(x)^5 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^4 + (3*a^8*b
 + 6*a^6*b^3 + 3*a^4*b^5 - 15*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^2 - 10*
(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x))*sinh(x)^4 + 4*(a^9 + 2*a^7*b^2 + a^5*b
^4)*cosh(x)^3 + 4*(a^9 + 2*a^7*b^2 + a^5*b^4 - 5*(a^8*b + 2*a^6*b^3 + a^4*b
^5)*cosh(x)^3 - 5*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x)^2 + 3*(a^8*b + 2*a^6*
b^3 + a^4*b^5)*cosh(x))*sinh(x)^3 - 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)
^2 - (3*a^8*b + 6*a^6*b^3 + 3*a^4*b^5 + 15*(a^8*b + 2*a^6*b^3 + a^4*b^5)*co
sh(x)^4 + 20*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x)^3 - 18*(a^8*b + 2*a^6*b^3
+ a^4*b^5)*cosh(x)^2 - 12*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x))*sinh(x)^2 -
2*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x) - 2*(a^9 + 2*a^7*b^2 + a^5*b^4 + 3*(a
^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^5 + 5*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(
x)^4 - 6*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^3 - 6*(a^9 + 2*a^7*b^2 + a^5
*b^4)*cosh(x)^2 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.5849, size = 274, normalized size = 1.73

$$-\frac{\left(4\,a^{2}b^{3}+3\,b^{5}\right)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{6}+a^{4}b^{2}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(ab^{3}e^{x}-b^{4}\right)}{\left(a^{5}+a^{3}b^{2}\right)\left(be^{(2\,x)}+2\,ae^{x}-b\right)}+\frac{\left(a^{2}-6\,b^{2}\right)\log\left(e^{x}+1\right)}{2\,a^{4}}-\frac{\left(a^{2}-6\,b^{2}\right)\log\left(|\right.}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(4*a^2*b^3 + 3*b^5)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + a^4*b^2)*sqrt(a^2 + b^2)) - 2*(a*b^3*e
^x - b^4)/((a^5 + a^3*b^2)*(b*e^(2*x) + 2*a*e^x - b)) + 1/2*(a^2 - 6*b^2)*l
og(e^x + 1)/a^4 - 1/2*(a^2 - 6*b^2)*log(abs(e^x - 1))/a^4 - (a*e^(3*x) - 4*
b*e^(2*x) + a*e^x + 4*b)/(a^3*(e^(2*x) - 1)^2)
```

3.87 $\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2} dx$

Optimal. Leaf size=198

$$-\frac{2b^4(5a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5(a^2+b^2)^{3/2}}+\frac{(-7a^2b^2+2a^4-12b^4)\coth(x)}{3a^4(a^2+b^2)}-\frac{b(a^2-4b^2)\tanh^{-1}(\cosh(x))}{a^5}-\frac{(a^2+4b^2)\operatorname{co}}{3a^2(a}$$

```
[Out] -((b*(a^2 - 4*b^2)*ArcTanh[Cosh[x]])/a^5) - (2*b^4*(5*a^2 + 4*b^2)*ArcTanh[
(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^5*(a^2 + b^2)^(3/2)) + ((2*a^4 - 7*a
^2*b^2 - 12*b^4)*Coth[x])/(3*a^4*(a^2 + b^2)) + (b*(a^2 + 2*b^2)*Coth[x]*Cs
ch[x])/(a^3*(a^2 + b^2)) - ((a^2 + 4*b^2)*Coth[x]*Csch[x]^2)/(3*a^2*(a^2 +
b^2)) + (b^2*Coth[x]*Csch[x]^2)/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.883786, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2b^4(5a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5(a^2+b^2)^{3/2}}+\frac{(-7a^2b^2+2a^4-12b^4)\coth(x)}{3a^4(a^2+b^2)}-\frac{b(a^2-4b^2)\tanh^{-1}(\cosh(x))}{a^5}-\frac{(a^2+4b^2)\operatorname{co}}{3a^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Sinh[x])^2,x]
```


```
[Out] -((b*(a^2 - 4*b^2)*ArcTanh[Cosh[x]])/a^5) - (2*b^4*(5*a^2 + 4*b^2)*ArcTanh[
(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^5*(a^2 + b^2)^(3/2)) + ((2*a^4 - 7*a
^2*b^2 - 12*b^4)*Coth[x])/(3*a^4*(a^2 + b^2)) + (b*(a^2 + 2*b^2)*Coth[x]*Cs
ch[x])/(a^3*(a^2 + b^2)) - ((a^2 + 4*b^2)*Coth[x]*Csch[x]^2)/(3*a^2*(a^2 +
b^2)) + (b^2*Coth[x]*Csch[x]^2)/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{\operatorname{csch}^4(x)\left(a^2+4b^2-ab\sinh(x)+3b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= -\frac{\left(a^2+4b^2\right)\coth(x)\operatorname{csch}^2(x)}{3a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{i\int \frac{\operatorname{csch}^3(x)\left(6ib\left(a^2+2b^2\right)+ia\left(2a^2-b^2\right)\sinh(\right.}{a+b\sinh(x)}}{3a^2\left(a^2+b^2\right.}$$

$$= \frac{b\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+4b^2\right)\coth(x)\operatorname{csch}^2(x)}{3a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} - \int$$

$$= \frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{3a^4\left(a^2+b^2\right)} + \frac{b\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+4b^2\right)\coth(x)\operatorname{csch}^2(x)}{3a^2\left(a^2+b^2\right)} +$$

$$= \frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{3a^4\left(a^2+b^2\right)} + \frac{b\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+4b^2\right)\coth(x)\operatorname{csch}^2(x)}{3a^2\left(a^2+b^2\right)} +$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} + \frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{3a^4\left(a^2+b^2\right)} + \frac{b\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3\left(a^2+b^2\right)}$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} + \frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{3a^4\left(a^2+b^2\right)} + \frac{b\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3\left(a^2+b^2\right)}$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} - \frac{2b^4\left(5a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5\left(a^2+b^2\right)^{3/2}} + \frac{\left(2a^4-7a^2b^2-12b^4\right)\mathrm{c}}{3a^4\left(a^2+b^2\right)}$$

Mathematica [A] time = 0.955536, size = 214, normalized size = 1.08

$$\frac{4a\left(2a^2-9b^2\right)\tanh\left(\frac{x}{2}\right) + 4a\left(2a^2-9b^2\right)\coth\left(\frac{x}{2}\right) - \frac{48b^4\left(5a^2+4b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} - \frac{24ab^5\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + 6a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) + 6}{24a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((-48*b^4*(5*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
 - b^2)^(3/2) + 4*a*(2*a^2 - 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 + 24*(a
 - 2*b)*b*(a + 2*b)*Log[Tanh[x/2]] + 6*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*
Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2 - (24*a*b^5*Cosh[x])/((a^2 + b^2)
*(a + b*Sinh[x])) + 4*a*(2*a^2 - 9*b^2)*Tanh[x/2])/(24*a^5)
```

Maple [A] time = 0.052, size = 277, normalized size = 1.4

$$-\frac{1}{24a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{b}{4a^3}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{3}{8a^2}\tanh\left(\frac{x}{2}\right) - \frac{3b^2}{2a^4}\tanh\left(\frac{x}{2}\right) - \frac{1}{24a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3} + \frac{3}{8a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] -1/24/a^2*tanh(1/2*x)^3-1/4/a^3*b*tanh(1/2*x)^2+3/8/a^2*tanh(1/2*x)-3/2/a^4
*b^2*tanh(1/2*x)-1/24/a^2/tanh(1/2*x)^3+3/8/a^2/tanh(1/2*x)-3/2/a^4/tanh(1/
2*x)*b^2+1/4/a^3*b/tanh(1/2*x)^2+1/a^3*b*ln(tanh(1/2*x))-4/a^5*b^3*ln(tanh(
1/2*x))+2/a^5*b^6/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)*tanh(1/2*x)
+2/a^4*b^5/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)+10/a^3*b^4/(a^2+b^
2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+8/a^5*b^6/(a^2+
b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.53278, size = 14660, normalized size = 74.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*a^7*b - 10*a^5*b^3 - 38*a^3*b^5 - 24*a*b^7 - 6*(a^6*b^2 + 3*a^4*b^4
 + 2*a^2*b^6)*cosh(x)^7 - 6*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*sinh(x)^7 - 6
*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^6 - 6*(2*a^7*b + a^5*b^3
 - 5*a^3*b^5 - 4*a*b^7 + 7*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x))*sinh(
x)^6 + 6*(7*a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh(x)^5 + 6*(7*a^6*b^2 + 1
7*a^4*b^4 + 10*a^2*b^6 - 21*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^2 - 6
*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^5 + 6*(2*a^7*b
- 5*a^5*b^3 - 19*a^3*b^5 - 12*a*b^7)*cosh(x)^4 + 6*(2*a^7*b - 5*a^5*b^3 - 1
9*a^3*b^5 - 12*a*b^7 - 35*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^3 - 15*
(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 5*(7*a^6*b^2 + 17*a^4
*b^4 + 10*a^2*b^6)*cosh(x))*sinh(x)^4 + 6*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 -
 14*a^2*b^6)*cosh(x)^3 + 6*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 - 14*a^2*b^6 - 3
5*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^4 - 20*(2*a^7*b + a^5*b^3 - 5*a
^3*b^5 - 4*a*b^7)*cosh(x)^3 + 10*(7*a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh
(x)^2 + 4*(2*a^7*b - 5*a^5*b^3 - 19*a^3*b^5 - 12*a*b^7)*cosh(x))*sinh(x)^3
- 2*(2*a^7*b - 23*a^5*b^3 - 61*a^3*b^5 - 36*a*b^7)*cosh(x)^2 - 2*(2*a^7*b -
 23*a^5*b^3 - 61*a^3*b^5 - 36*a*b^7 + 63*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*
cosh(x)^5 + 45*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 30*(7*
a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh(x)^3 - 18*(2*a^7*b - 5*a^5*b^3 - 19
*a^3*b^5 - 12*a*b^7)*cosh(x)^2 - 9*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 - 14*a^2
*b^6)*cosh(x))*sinh(x)^2 - 3*((5*a^2*b^5 + 4*b^7)*cosh(x)^8 + (5*a^2*b^5 +
4*b^7)*sinh(x)^8 + 2*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^7 + 2*(5*a^3*b^4 + 4*a*b
^6 + 4*(5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^7 + 5*a^2*b^5 + 4*b^7 - 4*(5*a^
2*b^5 + 4*b^7)*cosh(x)^6 - 2*(10*a^2*b^5 + 8*b^7 - 14*(5*a^2*b^5 + 4*b^7)*c
osh(x)^2 - 7*(5*a^3*b^4 + 4*a*b^6)*cosh(x))*sinh(x)^6 - 6*(5*a^3*b^4 + 4*a*
b^6)*cosh(x)^5 - 2*(15*a^3*b^4 + 12*a*b^6 - 28*(5*a^2*b^5 + 4*b^7)*cosh(x)^
3 - 21*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^2 + 12*(5*a^2*b^5 + 4*b^7)*cosh(x))*si
nh(x)^5 + 6*(5*a^2*b^5 + 4*b^7)*cosh(x)^4 + 2*(15*a^2*b^5 + 12*b^7 + 35*(5*
a^2*b^5 + 4*b^7)*cosh(x)^4 + 35*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^3 - 30*(5*a^2
```

```
*b^5 + 4*b^7)*cosh(x)^2 - 15*(5*a^3*b^4 + 4*a*b^6)*cosh(x))*sinh(x)^4 + 6*(
5*a^3*b^4 + 4*a*b^6)*cosh(x)^3 + 2*(15*a^3*b^4 + 12*a*b^6 + 28*(5*a^2*b^5 +
 4*b^7)*cosh(x)^5 + 35*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^4 - 40*(5*a^2*b^5 + 4*
b^7)*cosh(x)^3 - 30*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^2 + 12*(5*a^2*b^5 + 4*b^7
)*cosh(x))*sinh(x)^3 - 4*(5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 2*(10*a^2*b^5 + 8*
b^7 - 14*(5*a^2*b^5 + 4*b^7)*cosh(x)^6 - 21*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^5
 + 30*(5*a^2*b^5 + 4*b^7)*cosh(x)^4 + 30*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^3 -
18*(5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 9*(5*a^3*b^4 + 4*a*b^6)*cosh(x))*sinh(x)
^2 - 2*(5*a^3*b^4 + 4*a*b^6)*cosh(x) + 2*(4*(5*a^2*b^5 + 4*b^7)*cosh(x)^7 -
 5*a^3*b^4 - 4*a*b^6 + 7*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^6 - 12*(5*a^2*b^5 +
4*b^7)*cosh(x)^5 - 15*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^4 + 12*(5*a^2*b^5 + 4*b
^7)*cosh(x)^3 + 9*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^2 - 4*(5*a^2*b^5 + 4*b^7)*c
osh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b
*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*
(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(
b*cosh(x) + a)*sinh(x) - b)) - 2*(4*a^8 - 7*a^6*b^2 - 29*a^4*b^4 - 18*a^2*b
^6)*cosh(x) + 3*((a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^8 + (a^6
*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*sinh(x)^8 + a^6*b^2 - 2*a^4*b^4 - 7*a
^2*b^6 - 4*b^8 + 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^7 + 2*
(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7 + 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b
^6 - 4*b^8)*cosh(x))*sinh(x)^7 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8
)*cosh(x)^6 - 2*(2*a^6*b^2 - 4*a^4*b^4 - 14*a^2*b^6 - 8*b^8 - 14*(a^6*b^2 -
 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 7*(a^7*b - 2*a^5*b^3 - 7*a^3*b^
5 - 4*a*b^7)*cosh(x))*sinh(x)^6 - 6*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^
7)*cosh(x)^5 - 2*(3*a^7*b - 6*a^5*b^3 - 21*a^3*b^5 - 12*a*b^7 - 28*(a^6*b^2
 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 21*(a^7*b - 2*a^5*b^3 - 7*a^3
*b^5 - 4*a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*co
sh(x))*sinh(x)^5 + 6*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 +
2*(3*a^6*b^2 - 6*a^4*b^4 - 21*a^2*b^6 - 12*b^8 + 35*(a^6*b^2 - 2*a^4*b^4 -
7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 35*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)
*cosh(x)^3 - 30*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 15*(a
^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^4 + 6*(a^7*b - 2*a
^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 + 2*(3*a^7*b - 6*a^5*b^3 - 21*a^3*b
^5 - 12*a*b^7 + 28*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^5 + 35
*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 40*(a^6*b^2 - 2*a^4*
b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 30*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*
a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*si
nh(x)^3 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 2*(2*a^6*
b^2 - 4*a^4*b^4 - 14*a^2*b^6 - 8*b^8 - 14*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6
- 4*b^8)*cosh(x)^6 - 21*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^5
 + 30*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 30*(a^7*b - 2*a
^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 - 18*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b
^6 - 4*b^8)*cosh(x)^2 - 9*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)
)*sinh(x)^2 - 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x) - 2*(a^7*
b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 -
4*b^8)*cosh(x)^7 - 7*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^6 +
12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^5 + 15*(a^7*b - 2*a^5*
b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6
- 4*b^8)*cosh(x)^3 - 9*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^2
+ 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x))*log(cosh(x)
 + sinh(x) + 1) - 3*((a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^8 +
(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*sinh(x)^8 + a^6*b^2 - 2*a^4*b^4 -
 7*a^2*b^6 - 4*b^8 + 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^7
+ 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7 + 4*(a^6*b^2 - 2*a^4*b^4 - 7*a
^2*b^6 - 4*b^8)*cosh(x))*sinh(x)^7 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4
*b^8)*cosh(x)^6 - 2*(2*a^6*b^2 - 4*a^4*b^4 - 14*a^2*b^6 - 8*b^8 - 14*(a^6*b
^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 7*(a^7*b - 2*a^5*b^3 - 7*a^
3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^6 - 6*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*
a*b^7)*cosh(x)^5 - 2*(3*a^7*b - 6*a^5*b^3 - 21*a^3*b^5 - 12*a*b^7 - 28*(a^6
```

```
*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 21*(a^7*b - 2*a^5*b^3 - 7
*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8
)*cosh(x))*sinh(x)^5 + 6*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^
4 + 2*(3*a^6*b^2 - 6*a^4*b^4 - 21*a^2*b^6 - 12*b^8 + 35*(a^6*b^2 - 2*a^4*b^
4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 35*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*
b^7)*cosh(x)^3 - 30*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 1
5*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^4 + 6*(a^7*b -
 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 + 2*(3*a^7*b - 6*a^5*b^3 - 21*a
^3*b^5 - 12*a*b^7 + 28*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^5
+ 35*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 40*(a^6*b^2 - 2*
a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 30*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5
- 4*a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)
)*sinh(x)^3 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 2*(2*
a^6*b^2 - 4*a^4*b^4 - 14*a^2*b^6 - 8*b^8 - 14*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*
b^6 - 4*b^8)*cosh(x)^6 - 21*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(
x)^5 + 30*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 30*(a^7*b -
 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 - 18*(a^6*b^2 - 2*a^4*b^4 - 7*a
^2*b^6 - 4*b^8)*cosh(x)^2 - 9*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cos
h(x))*sinh(x)^2 - 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x) - 2*(
a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^
6 - 4*b^8)*cosh(x)^7 - 7*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^
6 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^5 + 15*(a^7*b - 2*
a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*
b^6 - 4*b^8)*cosh(x)^3 - 9*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x
)^2 + 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x))*log(cos
h(x) + sinh(x) - 1) - 2*(4*a^8 - 7*a^6*b^2 - 29*a^4*b^4 - 18*a^2*b^6 + 21*(
a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^6 + 18*(2*a^7*b + a^5*b^3 - 5*a^3*
b^5 - 4*a*b^7)*cosh(x)^5 - 15*(7*a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh(x)
^4 - 12*(2*a^7*b - 5*a^5*b^3 - 19*a^3*b^5 - 12*a*b^7)*cosh(x)^3 - 9*(4*a^8
- 3*a^6*b^2 - 21*a^4*b^4 - 14*a^2*b^6)*cosh(x)^2 + 2*(2*a^7*b - 23*a^5*b^3
- 61*a^3*b^5 - 36*a*b^7)*cosh(x))*sinh(x))/(a^9*b + 2*a^7*b^3 + a^5*b^5 + (
a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^8 + (a^9*b + 2*a^7*b^3 + a^5*b^5)*sinh
(x)^8 + 2*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^7 + 2*(a^10 + 2*a^8*b^2 + a^
6*b^4 + 4*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x))*sinh(x)^7 - 4*(a^9*b + 2*a
^7*b^3 + a^5*b^5)*cosh(x)^6 - 2*(2*a^9*b + 4*a^7*b^3 + 2*a^5*b^5 - 14*(a^9*
b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^2 - 7*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)
)*sinh(x)^6 - 6*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^5 - 2*(3*a^10 + 6*a^8*
b^2 + 3*a^6*b^4 - 28*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^3 - 21*(a^10 + 2
*a^8*b^2 + a^6*b^4)*cosh(x)^2 + 12*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x))*s
inh(x)^5 + 6*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^4 + 2*(3*a^9*b + 6*a^7*b
^3 + 3*a^5*b^5 + 35*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^4 + 35*(a^10 + 2*
a^8*b^2 + a^6*b^4)*cosh(x)^3 - 30*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^2 -
 15*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x))*sinh(x)^4 + 6*(a^10 + 2*a^8*b^2 +
 a^6*b^4)*cosh(x)^3 + 2*(3*a^10 + 6*a^8*b^2 + 3*a^6*b^4 + 28*(a^9*b + 2*a^7
*b^3 + a^5*b^5)*cosh(x)^5 + 35*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^4 - 40*
(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^3 - 30*(a^10 + 2*a^8*b^2 + a^6*b^4)*c
osh(x)^2 + 12*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x))*sinh(x)^3 - 4*(a^9*b +
 2*a^7*b^3 + a^5*b^5)*cosh(x)^2 - 2*(2*a^9*b + 4*a^7*b^3 + 2*a^5*b^5 - 14*(
a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^6 - 21*(a^10 + 2*a^8*b^2 + a^6*b^4)*co
sh(x)^5 + 30*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^4 + 30*(a^10 + 2*a^8*b^2
 + a^6*b^4)*cosh(x)^3 - 18*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^2 - 9*(a^1
0 + 2*a^8*b^2 + a^6*b^4)*cosh(x))*sinh(x)^2 - 2*(a^10 + 2*a^8*b^2 + a^6*b^4
)*cosh(x) - 2*(a^10 + 2*a^8*b^2 + a^6*b^4 - 4*(a^9*b + 2*a^7*b^3 + a^5*b^5)
*cosh(x)^7 - 7*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^6 + 12*(a^9*b + 2*a^7*b
^3 + a^5*b^5)*cosh(x)^5 + 15*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^4 - 12*(a
^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^3 - 9*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh
(x)^2 + 4*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.85886, size = 319, normalized size = 1.61

$$\frac{\left(5\,a^2b^4+4\,b^6\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^7+a^5b^2\right)\sqrt{a^2+b^2}}+\frac{2\left(ab^4e^x-b^5\right)}{\left(a^6+a^4b^2\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}-\frac{\left(a^2b-4\,b^3\right)\log\left(e^x+1\right)}{a^5}+\frac{\left(a^2b-4\,b^3\right)\log\left(|e^x\right.}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (5*a^2*b^4 + 4*b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^7 + a^5*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^4*e^
x - b^5)/((a^6 + a^4*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - (a^2*b - 4*b^3)*log(
e^x + 1)/a^5 + (a^2*b - 4*b^3)*log(abs(e^x - 1))/a^5 + 2/3*(3*a*b*e^(5*x) -
 9*b^2*e^(4*x) - 6*a^2*e^(2*x) + 18*b^2*e^(2*x) - 3*a*b*e^x + 2*a^2 - 9*b^2
)/(a^4*(e^(2*x) - 1)^3)
```

3.88 $\int \frac{1}{3+5i\sinh(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[Out] ((I/4)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - ((I/4)*Log[Cosh[
(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d
```

Rubi [A] time = 0.0281812, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2660, 616, 31}

$$\frac{i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((I/4)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - ((I/4)*Log[Cosh[
(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+5i\sinh(c+dx)}\,dx &= -\frac{(2i)\,\mathrm{Subst}\left(\int \frac{1}{3+10x+3x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}\\ &= -\frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{1+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{4d}+\frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{9+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx\right.\right.}{4d}\\ &= \frac{i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}\end{aligned}$$

Mathematica [A] time = 0.0341815, size = 81, normalized size = 1.11

$$\frac{\tan^{-1}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{i\log(4-5\cosh(c+dx))}{8d}+\frac{i\log(5\cosh(c+dx)+4)}{8d}+\frac{\tan^{-1}\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ArcTan[3*Coth[(c + d*x)/2]]/(4*d) + ArcTan[3*Tanh[(c + d*x)/2]]/(4*d) - ((I
/8)*Log[4 - 5*Cosh[c + d*x]])/d + ((I/8)*Log[4 + 5*Cosh[c + d*x]])/d
```

Maple [A] time = 0.023, size = 42, normalized size = 0.6

$$\frac{-\frac{i}{4}}{d}\ln\left(3\tanh\left(1/2\,dx+c/2\right)-i\right)+\frac{\frac{i}{4}}{d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c)),x)
```

```
[Out] -1/4*I/d*ln(3*tanh(1/2*d*x+1/2*c)-I)+1/4*I/d*ln(tanh(1/2*d*x+1/2*c)-3*I)
```

Maxima [A] time = 1.68477, size = 26, normalized size = 0.36

$$\frac{\arctan\left(\frac{5}{4}i\,e^{(-dx-c)}-\frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(5/4*I*e^(-d*x - c) - 3/4)/d
```

Fricas [A] time = 2.56265, size = 104, normalized size = 1.42

$$\frac{i\,\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)-i\,\log\left(e^{(dx+c)}-\frac{3}{5}i-\frac{4}{5}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*log(e^(d*x + c) - 3/5*I + 4/5) - I*log(e^(d*x + c) - 3/5*I - 4/5))/d
```

Sympy [A] time = 0.613609, size = 34, normalized size = 0.47

$$\frac{\operatorname{RootSum}\left(16z^{2}+1,\left(i\mapsto i\log\left(-\frac{16iie^{-c}}{5}+e^{dx}-\frac{3ie^{-c}}{5}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x)
```

```
[Out] RootSum(16*_z**2 + 1, Lambda(_i, _i*log(-16*_i*I*exp(-c)/5 + exp(d*x) - 3*I
*exp(-c)/5)))/d
```

Giac [A] time = 1.68185, size = 45, normalized size = 0.62

$$\frac{i \log\left(-(i-2)\, e^{(dx+c)} - 2i + 1\right)}{4\, d} - \frac{i \log\left(-(2i-1)\, e^{(dx+c)} + i - 2\right)}{4\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*I*log(-(I - 2)*e^(d*x + c) - 2*I + 1)/d - 1/4*I*log(-(2*I - 1)*e^(d*x +
 c) + I - 2)/d
```

3.89 $\int \frac{1}{(3+5i\sinh(c+dx))^2} dx$

Optimal. Leaf size=102

$$\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{3i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{3i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+d\right.\right.}{64d}$$

```
[Out] (((-3*I)/64)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((3*I)/64
)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/16)*Cosh[c
+ d*x])/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rubi [A] time = 0.0508041, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2664, 12, 2660, 616, 31}

$$\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{3i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{3i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+d\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (((-3*I)/64)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((3*I)/64
)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/16)*Cosh[c
+ d*x])/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx &= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} + \frac{1}{16}\int -\frac{3}{3+5i\sinh(c+dx)}\,dx\\ &= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} - \frac{3}{16}\int \frac{1}{3+5i\sinh(c+dx)}\,dx\\ &= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} + \frac{(3i)\operatorname{Subst}\left(\int \frac{1}{3+10x+3x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{8d}\\ &= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} + \frac{(9i)\operatorname{Subst}\left(\int \frac{1}{1+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{64d} - \frac{(9i)\operatorname{Subst}\left(\int\right.}{}\\ &= -\frac{3i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{3i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{5i\cosh(c+}{16d(3+5i\sinh(}\end{aligned}$$

Mathematica [A] time = 0.245723, size = 142, normalized size = 1.39

$$\frac{40\sinh\left(\frac{1}{2}(c+dx)\right)\left(\frac{3}{\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)} + \frac{1}{3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)}\right) - 9\left(2\tan^{-1}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right) - i\,\mathrm{lo}\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-9*(2*ArcTan[3*Coth[(c + d*x)/2]] + 2*ArcTan[3*Tanh[(c + d*x)/2]] - I*Log[
4 - 5*Cosh[c + d*x]] + I*Log[4 + 5*Cosh[c + d*x]]) + 40*((3*Cosh[(c + d*x)/
2] + I*Sinh[(c + d*x)/2])^(-1) + 3/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x
)/2]))*Sinh[(c + d*x)/2])/(384*d)
```

Maple [A] time = 0.043, size = 82, normalized size = 0.8

$$\frac{\frac{3i}{64}}{d}\ln\left(3\tanh\left(1/2\,dx+c/2\right)-i\right)+\frac{5}{48\,d}\left(3\tanh\left(1/2\,dx+c/2\right)-i\right)^{-1}-\frac{\frac{3i}{64}}{d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,i\right)+\frac{5}{16\,d}\left(\tanh\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c))^2,x)
```

```
[Out] 3/64*I/d*ln(3*tanh(1/2*d*x+1/2*c)-I)+5/48/d/(3*tanh(1/2*d*x+1/2*c)-I)-3/64*
I/d*ln(tanh(1/2*d*x+1/2*c)-3*I)+5/16/d/(tanh(1/2*d*x+1/2*c)-3*I)
```

Maxima [A] time = 1.60047, size = 107, normalized size = 1.05

$$\frac{3i\,\log\left(\frac{10\,e^{(-dx-c)}+6i-8}{10\,e^{(-dx-c)}+6i+8}\right)}{64\,d} + \frac{3i\,e^{(-dx-c)}-5}{-8\,d\left(-6i\,e^{(-dx-c)}-5\,e^{(-2\,dx-2\,c)}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/64*I*log((10*e^(-d*x - c) + 6*I - 8)/(10*e^(-d*x - c) + 6*I + 8))/d + (3*
I*e^(-d*x - c) - 5)/(d*(48*I*e^(-d*x - c) + 40*e^(-2*d*x - 2*c) - 40))
```

Fricas [A] time = 2.1833, size = 323, normalized size = 3.17

$$\frac{\left(-15i\, e^{(2\,dx+2\,c)} - 18\, e^{(dx+c)} + 15i\right) \log\left(e^{(dx+c)} - \frac{3}{5}i + \frac{4}{5}\right) + \left(15i\, e^{(2\,dx+2\,c)} + 18\, e^{(dx+c)} - 15i\right) \log\left(e^{(dx+c)} - \frac{3}{5}i - \frac{4}{5}\right) + 24i\, e^{(dx+c}}{64\left(5\,de^{(2\,dx+2\,c)} - 6i\,de^{(dx+c)} - 5\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/64*((-15*I*e^(2*d*x + 2*c) - 18*e^(d*x + c) + 15*I)*log(e^(d*x + c) - 3/5
*I + 4/5) + (15*I*e^(2*d*x + 2*c) + 18*e^(d*x + c) - 15*I)*log(e^(d*x + c)
- 3/5*I - 4/5) + 24*I*e^(d*x + c) + 40)/(5*d*e^(2*d*x + 2*c) - 6*I*d*e^(d*x
 + c) - 5*d)
```

Sympy [A] time = 1.17428, size = 87, normalized size = 0.85

$$\frac{\frac{3ie^{-c}e^{dx}}{40d} + \frac{e^{-2c}}{8d}}{e^{2dx} - \frac{6ie^{-c}e^{dx}}{5} - e^{-2c}} + \frac{\operatorname{RootSum}\left(4096z^2 + 9, \left(i \mapsto i \log\left(\frac{256iie^{-c}}{15} + e^{dx} - \frac{3ie^{-c}}{5}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**2,x)
```

```
[Out] (3*I*exp(-c)*exp(d*x)/(40*d) + exp(-2*c)/(8*d))/(exp(2*d*x) - 6*I*exp(-c)*e
xp(d*x)/5 - exp(-2*c)) + RootSum(4096*_z**2 + 9, Lambda(_i, _i*log(256*_i*I
*exp(-c)/15 + exp(d*x) - 3*I*exp(-c)/5)))/d
```

Giac [A] time = 1.60727, size = 96, normalized size = 0.94

$$-\frac{3i\,\log\left(-(i-2)\,e^{(dx+c)} - 2i + 1\right)}{64\,d} + \frac{3i\,\log\left(-(2i-1)\,e^{(dx+c)} + i - 2\right)}{64\,d} + \frac{3i\,e^{(dx+c)} + 5}{8\,d\left(5\,e^{(2\,dx+2\,c)} - 6i\,e^{(dx+c)} - 5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -3/64*I*log(-(I - 2)*e^(d*x + c) - 2*I + 1)/d + 3/64*I*log(-(2*I - 1)*e^(d*
x + c) + I - 2)/d + 1/8*(3*I*e^(d*x + c) + 5)/(d*(5*e^(2*d*x + 2*c) - 6*I*e
^(d*x + c) - 5))
```

3.90 $\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}+\frac{43i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}-\frac{43i\log}{}$$

```
[Out] (((43*I)/2048)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - (((43*I)
/2048)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/32)*Co
sh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) - (((45*I)/512)*Cosh[c + d*x])
/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rubi [A] time = 0.0851339, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2664, 2754, 12, 2660, 616, 31}

$$-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}+\frac{43i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}-\frac{43i\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (((43*I)/2048)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - (((43*I)
/2048)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/32)*Co
sh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) - (((45*I)/512)*Cosh[c + d*x])
/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx &= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}+\frac{1}{32}\int \frac{-6+5i\sinh(c+dx)}{(3+5i\sinh(c+dx))^2}\,dx\\
&= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{1}{512}\int \frac{43}{3+5i\sinh(c+dx)}\,dx\\
&= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{43}{512}\int \frac{1}{3+5i\sinh(c+dx)}\,dx\\
&= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}-\frac{(43i)\,\mathrm{Subst}\left(\int \frac{1}{3+10x+3x^2}\,dx,x,\mathrm{ta}\right.}{256d}\\
&= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}-\frac{(129i)\,\mathrm{Subst}\left(\int \frac{1}{1+3x}\,dx,x,\tan\right.}{2048d}\\
&= \frac{43i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}-\frac{43i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}+\frac{5i\cosh(c+d}{32d(3+5i\sinh(c}
\end{aligned}$$

Mathematica [A] time = 0.484808, size = 204, normalized size = 1.56

$$\frac{86\tan^{-1}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)-43i\log(4-5\cosh(c+dx))+43i\log(5\cosh(c+dx)+4)+\sinh\left(\frac{1}{2}(c+dx)\right)\left(-\frac{}{\cosh\left(\frac{1}{2}(c+\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (86*ArcTan[3*Coth[(c + d*x)/2]] + 86*ArcTan[3*Tanh[(c + d*x)/2]] - (43*I)*L
og[4 - 5*Cosh[c + d*x]] + (43*I)*Log[4 + 5*Cosh[c + d*x]] - (80*I)/(3*Cosh[
(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + (80*I)/(Cosh[(c + d*x)/2] + (3*I)*S
inh[(c + d*x)/2])^2 + (-120/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) - 3
60/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(4096*
d)
```

Maple [A] time = 0.051, size = 124, normalized size = 1.

$$-\frac{\frac{43\,i}{2048}}{d}\ln\left(3\tanh\left(1/2\,dx+c/2\right)-i\right)-\frac{\frac{25\,i}{1152}}{d}\left(3\tanh\left(1/2\,dx+c/2\right)-i\right)^{-2}-\frac{155}{4608\,d}\left(3\tanh\left(1/2\,dx+c/2\right)-i\right)^{-1}+\frac{\frac{25\,i}{128}}{d}\left(\mathrm{tan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c))^3,x)
```

```
[Out] -43/2048*I/d*ln(3*tanh(1/2*d*x+1/2*c)-I)-25/1152*I/d/(3*tanh(1/2*d*x+1/2*c)
-I)^2-155/4608/d/(3*tanh(1/2*d*x+1/2*c)-I)+25/128*I/d/(tanh(1/2*d*x+1/2*c)-
3*I)^2+43/2048*I/d*ln(tanh(1/2*d*x+1/2*c)-3*I)+15/512/d/(tanh(1/2*d*x+1/2*c
)-3*I)
```

Maxima [A] time = 1.67748, size = 167, normalized size = 1.27

$$-\frac{43i\log\left(\frac{10e^{(-dx-c)}+6i-8}{10e^{(-dx-c)}+6i+8}\right)}{2048d}-\frac{-325ie^{(-dx-c)}-387e^{(-2dx-2c)}+215ie^{(-3dx-3c)}+225}{d\left(-15360ie^{(-dx-c)}-22016e^{(-2dx-2c)}+15360ie^{(-3dx-3c)}+6400e^{(-4dx-4c)}+6400\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -43/2048*I*log((10*e^(-d*x - c) + 6*I - 8)/(10*e^(-d*x - c) + 6*I + 8))/d -
 (-325*I*e^(-d*x - c) - 387*e^(-2*d*x - 2*c) + 215*I*e^(-3*d*x - 3*c) + 225
)/(d*(-15360*I*e^(-d*x - c) - 22016*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x -
3*c) + 6400*e^(-4*d*x - 4*c) + 6400))
```

Fricas [A] time = 2.19338, size = 625, normalized size = 4.77

$$\frac{\left(1075ie^{(4dx+4c)}+2580e^{(3dx+3c)}-3698ie^{(2dx+2c)}-2580e^{(dx+c)}+1075i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)+\left(-1075ie^{(4dx+4c)}-2\right.}{51200de^{(4dx+4c)}-122880ide^{(3dx+3c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((1075*I*e^(4*d*x + 4*c) + 2580*e^(3*d*x + 3*c) - 3698*I*e^(2*d*x + 2*c) -
2580*e^(d*x + c) + 1075*I)*log(e^(d*x + c) - 3/5*I + 4/5) + (-1075*I*e^(4*d
*x + 4*c) - 2580*e^(3*d*x + 3*c) + 3698*I*e^(2*d*x + 2*c) + 2580*e^(d*x + c
) - 1075*I)*log(e^(d*x + c) - 3/5*I - 4/5) - 1720*I*e^(3*d*x + 3*c) - 3096*
e^(2*d*x + 2*c) + 2600*I*e^(d*x + c) + 1800)/(51200*d*e^(4*d*x + 4*c) - 122
880*I*d*e^(3*d*x + 3*c) - 176128*d*e^(2*d*x + 2*c) + 122880*I*d*e^(d*x + c)
 + 51200*d)
```

Sympy [A] time = 2.55733, size = 156, normalized size = 1.19

$$\frac{-\frac{9e^{4c}}{256d}+\frac{13ie^{3c}e^{-dx}}{256d}+\frac{387e^{2c}e^{-2dx}}{6400d}-\frac{43ie^{c}e^{-3dx}}{1280d}}{e^{4c}-\frac{12ie^{3c}e^{-dx}}{5}-\frac{86e^{2c}e^{-2dx}}{25}+\frac{12ie^{c}e^{-3dx}}{5}+e^{-4dx}}+\frac{\operatorname{RootSum}\left(4194304z^{2}+1849,\left(i\mapsto i\log\left(-\frac{8192iie^{c}}{215}+\frac{3ie^{c}}{5}+e^{-dx}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**3,x)
```

[Out] (-9*exp(4*c)/(256*d) + 13*I*exp(3*c)*exp(-d*x)/(256*d) + 387*exp(2*c)*exp(-2*d*x)/(6400*d) - 43*I*exp(c)*exp(-3*d*x)/(1280*d))/(exp(4*c) - 12*I*exp(3*c)*exp(-d*x)/5 - 86*exp(2*c)*exp(-2*d*x)/25 + 12*I*exp(c)*exp(-3*d*x)/5 + exp(-4*d*x)) + RootSum(4194304*_z**2 + 1849, Lambda(_i, _i*log(-8192*_i*I*exp(c)/215 + 3*I*exp(c)/5 + exp(-d*x))))/d

---

Giac [A] time = 1.30706, size = 126, normalized size = 0.96

$$\frac{43i \log\left(-(i-2)\,e^{(dx+c)} - 2i + 1\right)}{2048\,d} - \frac{43i \log\left(-(2i-1)\,e^{(dx+c)} + i - 2\right)}{2048\,d} - \frac{-215i\,e^{(3\,dx+3\,c)} - 387\,e^{(2\,dx+2\,c)} + 325i\,e^{(dx+c)} + 2}{256\,d\left(-5i\,e^{(2\,dx+2\,c)} - 6\,e^{(dx+c)} + 5i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 43/2048*I*log(-(I - 2)*e^(d*x + c) - 2*I + 1)/d - 43/2048*I*log(-(2*I - 1)*
e^(d*x + c) + I - 2)/d - 1/256*(-215*I*e^(3*d*x + 3*c) - 387*e^(2*d*x + 2*c
) + 325*I*e^(d*x + c) + 225)/(d*(-5*I*e^(2*d*x + 2*c) - 6*e^(d*x + c) + 5*I
)^2)
```

3.91 $\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=160

$$\frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{279i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)\right.}{32768}$$

```
[Out] (((-279*I)/32768)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((27
9*I)/32768)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/4
8)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^3) - (((25*I)/512)*Cosh[c +
d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) + (((995*I)/24576)*Cosh[c + d*x])/(d*
(3 + (5*I)*Sinh[c + d*x]))
```

Rubi [A] time = 0.126119, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2664, 2754, 12, 2660, 616, 31}

$$\frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{279i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)\right.}{32768}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (((-279*I)/32768)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((27
9*I)/32768)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/4
8)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^3) - (((25*I)/512)*Cosh[c +
d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) + (((995*I)/24576)*Cosh[c + d*x])/(d*
(3 + (5*I)*Sinh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} + \frac{1}{48}\int \frac{-9+10i\sinh(c+dx)}{(3+5i\sinh(c+dx))^3}\,dx$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{\int \frac{154-75i\sinh(c+dx)}{(3+5i\sinh(c+dx))^2}\,dx}{1536}$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))} +$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))} -$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))} +$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))} +$$

$$= -\frac{279i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{279i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{5i\cosh(c}{48d(3+5i\sin}$$

Mathematica [A] time = 0.579572, size = 265, normalized size = 1.66

$$-5022\tan^{-1}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 2511i\log(4-5\cosh(c+dx)) - 2511i\log(5\cosh(c+dx)+4) + 40\sinh\left(\frac{1}{2}(c+dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (-5022*ArcTan[3*Coth[(c + d*x)/2]] - 5022*ArcTan[3*Tanh[(c + d*x)/2]] + (25
11*I)*Log[4 - 5*Cosh[c + d*x]] - (2511*I)*Log[4 + 5*Cosh[c + d*x]] + (4640*
I)/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 - (1440*I)/(Cosh[(c + d*x)
/2] + (3*I)*Sinh[(c + d*x)/2])^2 + 40*(80/(3*Cosh[(c + d*x)/2] + I*Sinh[(c
+ d*x)/2])^3 + 199/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) + 240/(Cosh[
(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2])^3 + 597/(Cosh[(c + d*x)/2] + (3*I)*
Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(589824*d)
```

Maple [A] time = 0.053, size = 164, normalized size = 1.

$$\frac{\frac{275 i}{27648}}{d}(3\tanh(1/2 dx + c/2) - i)^{-2} + \frac{\frac{279 i}{32768}}{d}\ln(3\tanh(1/2 dx + c/2) - i) - \frac{125}{20736 d}(3\tanh(1/2 dx + c/2) - i)^{-3} + \frac{3}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c))^4,x)
```

```
[Out] 275/27648*I/d/(3*tanh(1/2*d*x+1/2*c)-I)^2+279/32768*I/d*ln(3*tanh(1/2*d*x+1
/2*c)-I)-125/20736/d/(3*tanh(1/2*d*x+1/2*c)-I)^3+3505/221184/d/(3*tanh(1/2*
d*x+1/2*c)-I)-279/32768*I/d*ln(tanh(1/2*d*x+1/2*c)-3*I)+75/1024*I/d/(tanh(1
/2*d*x+1/2*c)-3*I)^2-125/768/d/(tanh(1/2*d*x+1/2*c)-3*I)^3+345/8192/d/(tanh
(1/2*d*x+1/2*c)-3*I)
```

Maxima [A] time = 1.68885, size = 225, normalized size = 1.41

$$\frac{279 i \log\left(\frac{10 e^{(-dx-c)}+6i-8}{10 e^{(-dx-c)}+6i+8}\right)}{32768 d} + \frac{68625 i e^{(-dx-c)} + 119310 e^{(-2 dx-2 c)} - 111042 i e^{(-3 dx-3 c)} - 62775 e^{(-4 dx}}{d\left(5529600 i e^{(-dx-c)} + 11243520 e^{(-2 dx-2 c)} - 13713408 i e^{(-3 dx-3 c)} - 11243520 e^{(-4 dx-4 c)} + 5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 279/32768*I*log((10*e^(-d*x - c) + 6*I - 8)/(10*e^(-d*x - c) + 6*I + 8))/d
+ (68625*I*e^(-d*x - c) + 119310*e^(-2*d*x - 2*c) - 111042*I*e^(-3*d*x - 3*
c) - 62775*e^(-4*d*x - 4*c) + 20925*I*e^(-5*d*x - 5*c) - 24875)/(d*(5529600
*I*e^(-d*x - c) + 11243520*e^(-2*d*x - 2*c) - 13713408*I*e^(-3*d*x - 3*c) -
 11243520*e^(-4*d*x - 4*c) + 5529600*I*e^(-5*d*x - 5*c) + 1536000*e^(-6*d*x
 - 6*c) - 1536000))
```

Fricas [B] time = 2.21196, size = 972, normalized size = 6.08

$$\frac{\left(-104625 i e^{(6 dx+6 c)} - 376650 e^{(5 dx+5 c)} + 765855 i e^{(4 dx+4 c)} + 934092 e^{(3 dx+3 c)} - 765855 i e^{(2 dx+2 c)} - 376650 e^{(dx+c)} + 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((-104625*I*e^(6*d*x + 6*c) - 376650*e^(5*d*x + 5*c) + 765855*I*e^(4*d*x +
4*c) + 934092*e^(3*d*x + 3*c) - 765855*I*e^(2*d*x + 2*c) - 376650*e^(d*x +
c) + 104625*I)*log(e^(d*x + c) - 3/5*I + 4/5) + (104625*I*e^(6*d*x + 6*c) +
 376650*e^(5*d*x + 5*c) - 765855*I*e^(4*d*x + 4*c) - 934092*e^(3*d*x + 3*c)
 + 765855*I*e^(2*d*x + 2*c) + 376650*e^(d*x + c) - 104625*I)*log(e^(d*x + c
) - 3/5*I - 4/5) + 167400*I*e^(5*d*x + 5*c) + 502200*e^(4*d*x + 4*c) - 8883
36*I*e^(3*d*x + 3*c) - 954480*e^(2*d*x + 2*c) + 549000*I*e^(d*x + c) + 1990
00)/(12288000*d*e^(6*d*x + 6*c) - 44236800*I*d*e^(5*d*x + 5*c) - 89948160*d
*e^(4*d*x + 4*c) + 109707264*I*d*e^(3*d*x + 3*c) + 89948160*d*e^(2*d*x + 2*
c) - 44236800*I*d*e^(d*x + c) - 12288000*d)
```

Sympy [A] time = 5.19372, size = 223, normalized size = 1.39

$$\frac{\frac{279ie^{-c}e^{5dx}}{20480d}+\frac{837e^{-2c}e^{4dx}}{20480d}-\frac{18507ie^{-3c}e^{3dx}}{256000d}-\frac{3977e^{-4c}e^{2dx}}{51200d}+\frac{183ie^{-5c}e^{dx}}{4096d}+\frac{199e^{-6c}}{12288d}}{e^{6dx}-\frac{18ie^{-c}e^{5dx}}{5}-\frac{183e^{-2c}e^{4dx}}{25}+\frac{1116ie^{-3c}e^{3dx}}{125}+\frac{183e^{-4c}e^{2dx}}{25}-\frac{18ie^{-5c}e^{dx}}{5}-e^{-6c}}+\frac{\operatorname{RootSum}\left(1073741824z^2+77841,\left(i\mapsto i\log\left(\right.\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**4,x)
```

```
[Out] (279*I*exp(-c)*exp(5*d*x)/(20480*d) + 837*exp(-2*c)*exp(4*d*x)/(20480*d) -
18507*I*exp(-3*c)*exp(3*d*x)/(256000*d) - 3977*exp(-4*c)*exp(2*d*x)/(51200*
d) + 183*I*exp(-5*c)*exp(d*x)/(4096*d) + 199*exp(-6*c)/(12288*d))/(exp(6*d*
x) - 18*I*exp(-c)*exp(5*d*x)/5 - 183*exp(-2*c)*exp(4*d*x)/25 + 1116*I*exp(-
3*c)*exp(3*d*x)/125 + 183*exp(-4*c)*exp(2*d*x)/25 - 18*I*exp(-5*c)*exp(d*x)
/5 - exp(-6*c)) + RootSum(1073741824*_z**2 + 77841, Lambda(_i, _i*log(13107
2*_i*I*exp(-c)/1395 + exp(d*x) - 3*I*exp(-c)/5)))/d
```

Giac [A] time = 1.29235, size = 155, normalized size = 0.97

$$-\frac{279i\log\left(-(i-2)e^{(dx+c)}-2i+1\right)}{32768d}+\frac{279i\log\left(-(2i-1)e^{(dx+c)}+i-2\right)}{32768d}+\frac{20925ie^{(5dx+5c)}+62775e^{(4dx+4c)}-111042}{12288d\left(5e^{(2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -279/32768*I*log(-(I - 2)*e^(d*x + c) - 2*I + 1)/d + 279/32768*I*log(-(2*I
- 1)*e^(d*x + c) + I - 2)/d + 1/12288*(20925*I*e^(5*d*x + 5*c) + 62775*e^(4
*d*x + 4*c) - 111042*I*e^(3*d*x + 3*c) - 119310*e^(2*d*x + 2*c) + 68625*I*e
^(d*x + c) + 24875)/(d*(5*e^(2*d*x + 2*c) - 6*I*e^(d*x + c) - 5)^3)
```

3.92 $\int \frac{1}{5+3i\sinh(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{x}{4}-\frac{i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

```
[Out] x/4 - ((I/2)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
```

Rubi [A] time = 0.0132772, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2657}

$$\frac{x}{4}-\frac{i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-1),x]

[Out] x/4 - ((I/2)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx=\frac{x}{4}-\frac{i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

Mathematica [B] time = 0.0335293, size = 171, normalized size = 4.62

$$-\frac{\log(5\cosh(c+dx)-4\sinh(c+dx))}{8d}+\frac{\log(4\sinh(c+dx)+5\cosh(c+dx))}{8d}-\frac{i\tan^{-1}\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-1),x]

[Out] ((-I/4)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/2]
 - 2*Sinh[(c + d*x)/2])])/d + ((I/4)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(c
+ d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])])/d - Log[5*Cosh[c + d
*x] - 4*Sinh[c + d*x]]/(8*d) + Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]]/(8*d)
```

Maple [A] time = 0.023, size = 44, normalized size = 1.2

$$\frac{1}{4d}\ln\left(5\,\tanh\left(1/2\,dx+c/2\right)+4-3\,i\right)-\frac{1}{4d}\ln\left(5\,\tanh\left(1/2\,dx+c/2\right)-4-3\,i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c)),x)
```

```
[Out] 1/4/d*ln(5*tanh(1/2*d*x+1/2*c)+4-3*I)-1/4/d*ln(5*tanh(1/2*d*x+1/2*c)-4-3*I)
```

Maxima [A] time = 1.76108, size = 49, normalized size = 1.32

$$\frac{\log\left(-\frac{6\left(-i\,e^{(-dx-c)}+3\right)}{6i\,e^{(-dx-c)}-2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*log(-6*(-I*e^(-d*x - c) + 3)/(6*I*e^(-d*x - c) - 2))/d
```

Fricas [A] time = 2.02601, size = 80, normalized size = 2.16

$$\frac{\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-\log\left(e^{(dx+c)}-3i\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(e^(d*x + c) - 1/3*I) - log(e^(d*x + c) - 3*I))/d
```

Sympy [A] time = 0.599206, size = 31, normalized size = 0.84

$$\frac{-\frac{\log\left(e^{dx}-3ie^{-c}\right)}{4}+\frac{\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x)
```

```
[Out] (-log(exp(d*x) - 3*I*exp(-c))/4 + log(exp(d*x) - I*exp(-c)/3)/4)/d
```

Giac [A] time = 1.24547, size = 42, normalized size = 1.14

$$\frac{\log\left(3\,e^{(dx+c)}-i\right)}{4\,d}-\frac{\log\left(e^{(dx+c)}-3i\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*log(3*e^(d*x + c) - I)/d - 1/4*log(e^(d*x + c) - 3*I)/d
```

3.93 $\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}-\frac{5i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d}+\frac{5x}{64}$$

```
[Out] (5*x)/64 - (((5*I)/32)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d - (((
3*I)/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rubi [A] time = 0.0346976, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2664, 12, 2657}

$$-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}-\frac{5i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d}+\frac{5x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 - (((5*I)/32)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d - (((
3*I)/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx &= -\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}-\frac{1}{16}\int -\frac{5}{5+3i\sinh(c+dx)}\,dx\\ &= -\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}+\frac{5}{16}\int \frac{1}{5+3i\sinh(c+dx)}\,dx\\ &= \frac{5x}{64}-\frac{5i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d}-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.228439, size = 183, normalized size = 2.77

$$\frac{-\frac{120\cosh(c+dx)}{3\sinh(c+dx)-5i}-25\log(5\cosh(c+dx)-4\sinh(c+dx))+25\log(4\sinh(c+dx)+5\cosh(c+dx))-50i\tan^{-1}\left(\frac{2\cosh}{\cosh(}\right.}{640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (24*I - (50*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c +
d*x)/2] - 2*Sinh[(c + d*x)/2])] + (50*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh
[(c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 25*Log[5*Cosh[c
 + d*x] - 4*Sinh[c + d*x]] + 25*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] - (1
20*Cosh[c + d*x])/(-5*I + 3*Sinh[c + d*x]))/(640*d)
```

Maple [B] time = 0.043, size = 134, normalized size = 2.

$$-\frac{9}{80d}(5\tanh(1/2dx+c/2)+4-3i)^{-1}-\frac{\frac{3i}{20}}{d}(5\tanh(1/2dx+c/2)+4-3i)^{-1}+\frac{5}{64d}\ln(5\tanh(1/2dx+c/2)+4\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c))^2,x)
```

```
[Out] -9/80/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)-3/20*I/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I
)+5/64/d*ln(5*tanh(1/2*d*x+1/2*c)+4-3*I)-9/80/d/(5*tanh(1/2*d*x+1/2*c)-4-3*
I)+3/20*I/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)-5/64/d*ln(5*tanh(1/2*d*x+1/2*c)-4
-3*I)
```

Maxima [A] time = 1.78205, size = 86, normalized size = 1.3

$$-\frac{5i\arctan\left(\frac{3}{4}e^{(-dx-c)}+\frac{5}{4}i\right)}{32d}-\frac{5ie^{(-dx-c)}-3}{-8d\left(-10ie^{(-dx-c)}-3e^{(-2dx-2c)}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -5/32*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (5*I*e^(-d*x - c) - 3)/(d*(80*
I*e^(-d*x - c) + 24*e^(-2*d*x - 2*c) - 24))
```

Fricas [A] time = 2.08619, size = 298, normalized size = 4.52

$$\frac{5\left(3e^{(2dx+2c)}-10ie^{(dx+c)}-3\right)\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-5\left(3e^{(2dx+2c)}-10ie^{(dx+c)}-3\right)\log\left(e^{(dx+c)}-3i\right)-40ie^{(dx+c)}-24}{64\left(3de^{(2dx+2c)}-10ide^{(dx+c)}-3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/64*(5*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 1/3*I)
 - 5*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 3*I) - 40
*I*e^(d*x + c) - 24)/(3*d*e^(2*d*x + 2*c) - 10*I*d*e^(d*x + c) - 3*d)
```

Sympy [A] time = 1.11671, size = 88, normalized size = 1.33

$$\frac{-\frac{5ie^{-c}e^{dx}}{24d}-\frac{e^{-2c}}{8d}}{e^{2dx}-\frac{10ie^{-c}e^{dx}}{3}-e^{-2c}}+\frac{-\frac{5\log\left(e^{dx}-3ie^{-c}\right)}{64}+\frac{5\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{64}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**2,x)
```

```
[Out] (-5*I*exp(-c)*exp(d*x)/(24*d) - exp(-2*c)/(8*d))/(exp(2*d*x) - 10*I*exp(-c)
*exp(d*x)/3 - exp(-2*c)) + (-5*log(exp(d*x) - 3*I*exp(-c))/64 + 5*log(exp(d
*x) - I*exp(-c)/3)/64)/d
```

Giac [A] time = 1.24642, size = 93, normalized size = 1.41

$$\frac{5\log\left(3e^{(dx+c)}-i\right)}{64d}-\frac{5\log\left(e^{(dx+c)}-3i\right)}{64d}+\frac{-5ie^{(dx+c)}-3}{8d\left(3e^{(2dx+2c)}-10ie^{(dx+c)}-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/64*log(3*e^(d*x + c) - I)/d - 5/64*log(e^(d*x + c) - 3*I)/d + 1/8*(-5*I*e
^(d*x + c) - 3)/(d*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3))
```

3.94 $\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=95

$$-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{59i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d}+\frac{59x}{2048}$$

```
[Out] (59*x)/2048 - (((59*I)/1024)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
 - (((3*I)/32)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((45*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rubi [A] time = 0.0674412, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2664, 2754, 12, 2657}

$$-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{59i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d}+\frac{59x}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (59*x)/2048 - (((59*I)/1024)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
 - (((3*I)/32)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((45*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{1}{32}\int \frac{-10+3i\sinh(c+dx)}{(5+3i\sinh(c+dx))^2}\,dx$$
$$= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))} + \frac{1}{512}\int \frac{59}{5+3i\sinh(c+dx)}\,dx$$
$$= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))} + \frac{59}{512}\int \frac{1}{5+3i\sinh(c+dx)}\,dx$$
$$= \frac{59x}{2048} - \frac{59i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d} - \frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}$$

Mathematica [B] time = 0.648357, size = 277, normalized size = 2.92

$$-\frac{144\sinh\left(\frac{1}{2}(c+dx)\right)\left(5\sinh\left(\frac{1}{2}(c+dx)\right)-3i\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{3\sinh(c+dx)-5i} + \frac{48}{\left((1+2i)\cosh\left(\frac{1}{2}(c+dx)\right)-(2+i)\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{48}{\left((1+2i)\sinh\left(\frac{1}{2}(c+dx)\right)+(2+i)\cosh\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((-118*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/
2] - 2*Sinh[(c + d*x)/2])] + (118*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(c
+ d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 59*Log[5*Cosh[c + d
*x] - 4*Sinh[c + d*x]] + 59*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] + 48/((1
 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2])^2 + 48/((2 + I)*Cosh
[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x)/2])^2 - (144*Sinh[(c + d*x)/2]*((-
3*I)*Cosh[(c + d*x)/2] + 5*Sinh[(c + d*x)/2]))/(-5*I + 3*Sinh[c + d*x]))/(4
096*d)
```

Maple [B] time = 0.05, size = 224, normalized size = 2.4

$$\frac{63}{3200\,d}\left(5\,\tanh\left(1/2\,dx+c/2\right)+4-3\,i\right)^{-2} - \frac{\frac{27\,i}{400}}{d}\left(5\,\tanh\left(1/2\,dx+c/2\right)+4-3\,i\right)^{-2} - \frac{963}{12800\,d}\left(5\,\tanh\left(1/2\,dx+c/2\right)+4-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c))^3,x)
```

```
[Out] 63/3200/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^2-27/400*I/d/(5*tanh(1/2*d*x+1/2*c)
+4-3*I)^2-963/12800/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)-123/1600*I/d/(5*tanh(1/
2*d*x+1/2*c)+4-3*I)+59/2048/d*ln(5*tanh(1/2*d*x+1/2*c)+4-3*I)-63/3200/d/(5*
tanh(1/2*d*x+1/2*c)-4-3*I)^2-27/400*I/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^2-963
/12800/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)+123/1600*I/d/(5*tanh(1/2*d*x+1/2*c)-
4-3*I)-59/2048/d*ln(5*tanh(1/2*d*x+1/2*c)-4-3*I)
```

Maxima [A] time = 1.60141, size = 146, normalized size = 1.54

$$-\frac{59i\,\arctan\left(\frac{3}{4}\,e^{(-dx-c)}+\frac{5}{4}i\right)}{1024\,d} - \frac{-723i\,e^{(-dx-c)}-885\,e^{(-2\,dx-2\,c)}+177i\,e^{(-3\,dx-3\,c)}+135}{d\left(-15360i\,e^{(-dx-c)}-30208\,e^{(-2\,dx-2\,c)}+15360i\,e^{(-3\,dx-3\,c)}+2304\,e^{(-4\,dx-4\,c)}+2304\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -59/1024*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (-723*I*e^(-d*x - c) - 885*
e^(-2*d*x - 2*c) + 177*I*e^(-3*d*x - 3*c) + 135)/(d*(-15360*I*e^(-d*x - c)
- 30208*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x - 3*c) + 2304*e^(-4*d*x - 4*c)
 + 2304))
```

Fricas [B] time = 2.10321, size = 594, normalized size = 6.25

$$\frac{\left(531\,e^{(4\,dx+4\,c)} - 3540i\,e^{(3\,dx+3\,c)} - 6962\,e^{(2\,dx+2\,c)} + 3540i\,e^{(dx+c)} + 531\right)\log\left(e^{(dx+c)} - \frac{1}{3}i\right) - \left(531\,e^{(4\,dx+4\,c)} - 3540i\,e^{(3\,dx+}\right.}{18432\,de^{(4\,dx+4\,c)} - 122880i\,de^{(3\,dx+3\,c)} - 2416}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((531*e^(4*d*x + 4*c) - 3540*I*e^(3*d*x + 3*c) - 6962*e^(2*d*x + 2*c) + 354
0*I*e^(d*x + c) + 531)*log(e^(d*x + c) - 1/3*I) - (531*e^(4*d*x + 4*c) - 35
40*I*e^(3*d*x + 3*c) - 6962*e^(2*d*x + 2*c) + 3540*I*e^(d*x + c) + 531)*log
(e^(d*x + c) - 3*I) - 1416*I*e^(3*d*x + 3*c) - 7080*e^(2*d*x + 2*c) + 5784*
I*e^(d*x + c) + 1080)/(18432*d*e^(4*d*x + 4*c) - 122880*I*d*e^(3*d*x + 3*c)
 - 241664*d*e^(2*d*x + 2*c) + 122880*I*d*e^(d*x + c) + 18432*d)
```

Sympy [A] time = 2.60271, size = 158, normalized size = 1.66

$$\frac{-\frac{15e^{4c}}{256d} + \frac{241ie^{3c}e^{-dx}}{768d} + \frac{295e^{2c}e^{-2dx}}{768d} - \frac{59ie^{c}e^{-3dx}}{768d}}{e^{4c} - \frac{20ie^{3c}e^{-dx}}{3} - \frac{118e^{2c}e^{-2dx}}{9} + \frac{20ie^{c}e^{-3dx}}{3} + e^{-4dx}} + \frac{-\frac{59\log\left(\frac{ie^{c}}{3}+e^{-dx}\right)}{2048} + \frac{59\log\left(3ie^{c}+e^{-dx}\right)}{2048}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**3,x)
```

```
[Out] (-15*exp(4*c)/(256*d) + 241*I*exp(3*c)*exp(-d*x)/(768*d) + 295*exp(2*c)*exp
(-2*d*x)/(768*d) - 59*I*exp(c)*exp(-3*d*x)/(768*d))/(exp(4*c) - 20*I*exp(3*
c)*exp(-d*x)/3 - 118*exp(2*c)*exp(-2*d*x)/9 + 20*I*exp(c)*exp(-3*d*x)/3 + e
xp(-4*d*x)) + (-59*log(I*exp(c)/3 + exp(-d*x))/2048 + 59*log(3*I*exp(c) + e
xp(-d*x))/2048)/d
```

Giac [A] time = 1.27225, size = 123, normalized size = 1.29

$$\frac{59\,\log\left(3\,e^{(dx+c)} - i\right)}{2048\,d} - \frac{59\,\log\left(e^{(dx+c)} - 3i\right)}{2048\,d} - \frac{-177i\,e^{(3\,dx+3\,c)} - 885\,e^{(2\,dx+2\,c)} + 723i\,e^{(dx+c)} + 135}{256\,d\left(-3i\,e^{(2\,dx+2\,c)} - 10\,e^{(dx+c)} + 3i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 59/2048*log(3*e^(d*x + c) - I)/d - 59/2048*log(e^(d*x + c) - 3*I)/d - 1/256
*(-177*I*e^(3*d*x + 3*c) - 885*e^(2*d*x + 2*c) + 723*I*e^(d*x + c) + 135)/(
d*(-3*I*e^(2*d*x + 2*c) - 10*e^(d*x + c) + 3*I)^2)
```

3.95 $\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=124

$$-\frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}-\frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}-\frac{385i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d}+$$

```
[Out] (385*x)/32768 - (((385*I)/16384)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])
])/d - ((I/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^3) - (((25*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((311*I)/8192)*Cosh[c
+ d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rubi [A] time = 0.101751, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2664, 2754, 12, 2657}

$$-\frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}-\frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}-\frac{385i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d}+$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```


```
[Out] (385*x)/32768 - (((385*I)/16384)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])
])/d - ((I/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^3) - (((25*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((311*I)/8192)*Cosh[c
+ d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```


Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
```

```
PosQ[a]
```

Rubi steps

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{1}{48}\int \frac{-15+6i\sinh(c+dx)}{(5+3i\sinh(c+dx))^3}\,dx$$

$$= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} + \frac{\int \frac{186-75i\sinh(c+dx)}{(5+3i\sinh(c+dx))^2}\,dx}{1536}$$

$$= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))} -$$

$$= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))} +$$

$$= \frac{385x}{32768} - \frac{385i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d} - \frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx)}$$

Mathematica [B] time = 1.76811, size = 308, normalized size = 2.48

$$\frac{\frac{2(-298563i\sinh(c+dx)+89364i\sinh(2(c+dx))+8397i\sinh(3(c+dx))+166615\cosh(c+dx)+82530\cosh(2(c+dx))-13995\cosh(3(c+dx))-235150)}{(3\sinh(c+dx)-5i)^3} + \frac{}{\left((1+2i)\cosh\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-3850*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)
/2] - 2*Sinh[(c + d*x)/2])] + (3850*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(
c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 1925*Log[5*Cosh[c
 + d*x] - 4*Sinh[c + d*x]] + 1925*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] +
(2656 - 192*I)/((1 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2])^2
+ (2656 + 192*I)/((2 + I)*Cosh[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x)/2])^
2 + (2*(-235150 + 166615*Cosh[c + d*x] + 82530*Cosh[2*(c + d*x)] - 13995*Co
sh[3*(c + d*x)] - (298563*I)*Sinh[c + d*x] + (89364*I)*Sinh[2*(c + d*x)] +
(8397*I)*Sinh[3*(c + d*x)]))/(-5*I + 3*Sinh[c + d*x])^3)/(327680*d)
```

Maple [B] time = 0.052, size = 314, normalized size = 2.5

$$\frac{1053}{32000\,d}(5\,\tanh(1/2\,dx+c/2)+4-3\,i)^{-3} - \frac{\frac{99\,i}{8000}}{d}(5\,\tanh(1/2\,dx+c/2)+4-3\,i)^{-3} + \frac{783}{128000\,d}(5\,\tanh(1/2\,dx+c/2)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c))^4,x)
```

```
[Out] 1053/32000/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^3-99/8000*I/d/(5*tanh(1/2*d*x+1/
2*c)+4-3*I)^3+783/128000/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^2-3753/64000*I/d/(
5*tanh(1/2*d*x+1/2*c)+4-3*I)^2-39933/1024000/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I
)-8361/256000*I/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)+385/32768/d*ln(5*tanh(1/2*d
*x+1/2*c)+4-3*I)+1053/32000/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^3+99/8000*I/d/(
5*tanh(1/2*d*x+1/2*c)-4-3*I)^3-783/128000/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^2
```

```
-3753/64000*I/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^2-39933/1024000/d/(5*tanh(1/2
*d*x+1/2*c)-4-3*I)+8361/256000*I/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)-385/32768/
d*ln(5*tanh(1/2*d*x+1/2*c)-4-3*I)
```

Maxima [A] time = 1.71295, size = 205, normalized size = 1.65

$$-\frac{385i\arctan\left(\frac{3}{4}e^{(-dx-c)}+\frac{5}{4}i\right)}{16384\,d}-\frac{73575i\,e^{(-dx-c)}+218466\,e^{(-2\,dx-2\,c)}-239470i\,e^{(-3\,dx-3\,c)}-86625}{d\left(3317760i\,e^{(-dx-c)}+12054528\,e^{(-2\,dx-2\,c)}-18923520i\,e^{(-3\,dx-3\,c)}-12054528\,e^{(-4\,dx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -385/16384*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (73575*I*e^(-d*x - c) + 2
18466*e^(-2*d*x - 2*c) - 239470*I*e^(-3*d*x - 3*c) - 86625*e^(-4*d*x - 4*c)
 + 10395*I*e^(-5*d*x - 5*c) - 8397)/(d*(3317760*I*e^(-d*x - c) + 12054528*e
^(-2*d*x - 2*c) - 18923520*I*e^(-3*d*x - 3*c) - 12054528*e^(-4*d*x - 4*c) +
 3317760*I*e^(-5*d*x - 5*c) + 331776*e^(-6*d*x - 6*c) - 331776))
```

Fricas [B] time = 2.17341, size = 946, normalized size = 7.63

$$\frac{\left(31185\,e^{(6\,dx+6\,c)}-311850i\,e^{(5\,dx+5\,c)}-1133055\,e^{(4\,dx+4\,c)}+1778700i\,e^{(3\,dx+3\,c)}+1133055\,e^{(2\,dx+2\,c)}-311850i\,e^{(dx+c)}-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((31185*e^(6*d*x + 6*c) - 311850*I*e^(5*d*x + 5*c) - 1133055*e^(4*d*x + 4*c
) + 1778700*I*e^(3*d*x + 3*c) + 1133055*e^(2*d*x + 2*c) - 311850*I*e^(d*x +
 c) - 31185)*log(e^(d*x + c) - 1/3*I) - (31185*e^(6*d*x + 6*c) - 311850*I*e
^(5*d*x + 5*c) - 1133055*e^(4*d*x + 4*c) + 1778700*I*e^(3*d*x + 3*c) + 1133
055*e^(2*d*x + 2*c) - 311850*I*e^(d*x + c) - 31185)*log(e^(d*x + c) - 3*I)
- 83160*I*e^(5*d*x + 5*c) - 693000*e^(4*d*x + 4*c) + 1915760*I*e^(3*d*x + 3
*c) + 1747728*e^(2*d*x + 2*c) - 588600*I*e^(d*x + c) - 67176)/(2654208*d*e^
(6*d*x + 6*c) - 26542080*I*d*e^(5*d*x + 5*c) - 96436224*d*e^(4*d*x + 4*c) +
 151388160*I*d*e^(3*d*x + 3*c) + 96436224*d*e^(2*d*x + 2*c) - 26542080*I*d*
e^(d*x + c) - 2654208*d)
```

Sympy [B] time = 5.23034, size = 219, normalized size = 1.77

$$\frac{-\frac{385ie^{-c}e^{5dx}}{12288d}-\frac{9625e^{-2c}e^{4dx}}{36864d}+\frac{119735ie^{-3c}e^{3dx}}{165888d}+\frac{12137e^{-4c}e^{2dx}}{18432d}-\frac{2725ie^{-5c}e^{dx}}{12288d}-\frac{311e^{-6c}}{12288d}}{e^{6dx}-10ie^{-c}e^{5dx}-\frac{109e^{-2c}e^{4dx}}{3}+\frac{1540ie^{-3c}e^{3dx}}{27}+\frac{109e^{-4c}e^{2dx}}{3}-10ie^{-5c}e^{dx}-e^{-6c}}+\frac{-\frac{385\log\left(e^{dx}-3ie^{-c}\right)}{32768}+\frac{385\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{32768}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**4,x)
```

```
[Out] (-385*I*exp(-c)*exp(5*d*x)/(12288*d) - 9625*exp(-2*c)*exp(4*d*x)/(36864*d)
+ 119735*I*exp(-3*c)*exp(3*d*x)/(165888*d) + 12137*exp(-4*c)*exp(2*d*x)/(18
```

```
432*d) - 2725*I*exp(-5*c)*exp(d*x)/(12288*d) - 311*exp(-6*c)/(12288*d))/(ex
p(6*d*x) - 10*I*exp(-c)*exp(5*d*x) - 109*exp(-2*c)*exp(4*d*x)/3 + 1540*I*ex
p(-3*c)*exp(3*d*x)/27 + 109*exp(-4*c)*exp(2*d*x)/3 - 10*I*exp(-5*c)*exp(d*x
) - exp(-6*c)) + (-385*log(exp(d*x) - 3*I*exp(-c))/32768 + 385*log(exp(d*x)
 - I*exp(-c)/3)/32768)/d
```

Giac [A] time = 1.29281, size = 153, normalized size = 1.23

$$\frac{385 \log\left(3 e^{(dx+c)} - i\right)}{32768 d} - \frac{385 \log\left(e^{(dx+c)} - 3i\right)}{32768 d} - \frac{10395i e^{(5dx+5c)} + 86625 e^{(4dx+4c)} - 239470i e^{(3dx+3c)} - 218466 e^{(2dx+2c)}}{12288 d\left(3 e^{(2dx+2c)} - 10i e^{(dx+c)} - 3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 385/32768*log(3*e^(d*x + c) - I)/d - 385/32768*log(e^(d*x + c) - 3*I)/d - 1
/12288*(10395*I*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) - 239470*I*e^(3*d*x
 + 3*c) - 218466*e^(2*d*x + 2*c) + 73575*I*e^(d*x + c) + 8397)/(d*(3*e^(2*d
*x + 2*c) - 10*I*e^(d*x + c) - 3)^3)
```

3.96 $\int (a+b\sinh(c+dx))^5\,dx$

Optimal. Leaf size=183

$$\frac{b\left(-192a^2b^2+107a^4+16b^4\right)\cosh(c+dx)}{30d}+\frac{b\left(47a^2-16b^2\right)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d}+\frac{7ab^2\left(22a^2-23b^2\right)}{}$$

```
[Out] (a*(8*a^4 - 40*a^2*b^2 + 15*b^4)*x)/8 + (b*(107*a^4 - 192*a^2*b^2 + 16*b^4)
*Cosh[c + d*x])/(30*d) + (7*a*b^2*(22*a^2 - 23*b^2)*Cosh[c + d*x]*Sinh[c +
d*x])/(120*d) + (b*(47*a^2 - 16*b^2)*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)
/(60*d) + (9*a*b*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^3)/(20*d) + (b*Cosh[c
+ d*x]*(a + b*Sinh[c + d*x])^4)/(5*d)
```

Rubi [A] time = 0.273472, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2656, 2753, 2734}

$$\frac{b\left(-192a^2b^2+107a^4+16b^4\right)\cosh(c+dx)}{30d}+\frac{b\left(47a^2-16b^2\right)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d}+\frac{7ab^2\left(22a^2-23b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^5,x]
```


```
[Out] (a*(8*a^4 - 40*a^2*b^2 + 15*b^4)*x)/8 + (b*(107*a^4 - 192*a^2*b^2 + 16*b^4)
*Cosh[c + d*x])/(30*d) + (7*a*b^2*(22*a^2 - 23*b^2)*Cosh[c + d*x]*Sinh[c +
d*x])/(120*d) + (b*(47*a^2 - 16*b^2)*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)
/(60*d) + (9*a*b*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^3)/(20*d) + (b*Cosh[c
+ d*x]*(a + b*Sinh[c + d*x])^4)/(5*d)
```


Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^5\,dx = \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d} + \frac{1}{5}\int (a+b\sinh(c+dx))^3\left(5a^2-4b^2+9ab\sinh(c+\right.$$
$$= \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d} + \frac{1}{20}\int (a+b$$
$$= \frac{b\left(47a^2-16b^2\right)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d} + \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d}$$
$$= \frac{1}{8}a\left(8a^4-40a^2b^2+15b^4\right)x + \frac{b\left(107a^4-192a^2b^2+16b^4\right)\cosh(c+dx)}{30d} + \frac{7ab^2\left(22a^2-23b^2\right.}{}$$

Mathematica [A] time = 0.657863, size = 138, normalized size = 0.75

$$\frac{15a\left(4\left(-40a^2b^2+8a^4+15b^4\right)(c+dx)+40\left(2a^2b^2-b^4\right)\sinh(2(c+dx))+5b^4\sinh(4(c+dx))\right)+300b\left(-12a^2b^2+8a^4\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^5,x]
```

```
[Out] (300*b*(8*a^4 - 12*a^2*b^2 + b^4)*Cosh[c + d*x] + 50*(8*a^2*b^3 - b^5)*Cosh
[3*(c + d*x)] + 6*b^5*Cosh[5*(c + d*x)] + 15*a*(4*(8*a^4 - 40*a^2*b^2 + 15*
b^4)*(c + d*x) + 40*(2*a^2*b^2 - b^4)*Sinh[2*(c + d*x)] + 5*b^4*Sinh[4*(c +
 d*x)]))/(480*d)
```

Maple [A] time = 0.023, size = 155, normalized size = 0.9

$$\frac{1}{d}\left(b^5\left(\frac{8}{15}+\frac{(\sinh(dx+c))^4}{5}-\frac{4\,(\sinh(dx+c))^2}{15}\right)\cosh(dx+c)+5\,ab^4\left(\left(1/4\,(\sinh(dx+c))^3-3/8\,\sinh(dx+c)\right)\cosh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+5*a*b^4*((
1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+10*a^2*b^3*(-
2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+10*a^3*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-
1/2*d*x-1/2*c)+5*a^4*b*cosh(d*x+c)+a^5*(d*x+c))
```

Maxima [A] time = 1.09957, size = 367, normalized size = 2.01

$$\frac{5}{64}ab^4\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{5}{4}a^3b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^5x+\frac{1}{480}b^5\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 5/4*a^3*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2
*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c
)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^
```

```
(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 5*a^4*b*cosh(d*x + c)/d
```

Fricas [A] time = 1.99772, size = 548, normalized size = 2.99

$$\frac{3 b^5 \cosh(dx+c)^5 + 15 b^5 \cosh(dx+c)\sinh(dx+c)^4 + 150 ab^4 \cosh(dx+c)\sinh(dx+c)^3 + 25\left(8 a^2b^3 - b^5\right)\cosh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^5*cosh(d*x + c)^5 + 15*b^5*cosh(d*x + c)*sinh(d*x + c)^4 + 150*a
*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 25*(8*a^2*b^3 - b^5)*cosh(d*x + c)^3 +
 30*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*d*x + 15*(2*b^5*cosh(d*x + c)^3 + 5*(8*
a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + 150*(8*a^4*b - 12*a^2*b^3 +
 b^5)*cosh(d*x + c) + 150*(a*b^4*cosh(d*x + c)^3 + 4*(2*a^3*b^2 - a*b^4)*co
sh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 3.09409, size = 314, normalized size = 1.72

$$\begin{cases} a^5x + \frac{5a^4b\cosh(c+dx)}{d} + 5a^3b^2x\sinh^2(c+dx) - 5a^3b^2x\cosh^2(c+dx) + \frac{5a^3b^2\sinh(c+dx)\cosh(c+dx)}{d} + \frac{10a^2b^3\sinh^2(c+dx)\cosh}{d} \\ x(a+b\sinh(c))^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*cosh(c + d*x)/d + 5*a**3*b**2*x*sinh(c + d*x)*
*2 - 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c + d*
x)/d + 10*a**2*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 20*a**2*b**3*cosh(c
+ d*x)**3/(3*d) + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 + 25*a*b**4*sinh(c
 + d*x)**3*cosh(c + d*x)/(8*d) - 15*a*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(
8*d) + b**5*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b**5*sinh(c + d*x)**2*cosh
(c + d*x)**3/(3*d) + 8*b**5*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*s
inh(c))**5, True))
```

Giac [A] time = 1.27042, size = 366, normalized size = 2.

$$\frac{6 b^5 e^{(5 dx+5 c)} + 75 ab^4 e^{(4 dx+4 c)} + 400 a^2b^3 e^{(3 dx+3 c)} - 50 b^5 e^{(3 dx+3 c)} + 1200 a^3b^2 e^{(2 dx+2 c)} - 600 ab^4 e^{(2 dx+2 c)} + 2400 a^4be}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/960*(6*b^5*e^(5*d*x + 5*c) + 75*a*b^4*e^(4*d*x + 4*c) + 400*a^2*b^3*e^(3*
d*x + 3*c) - 50*b^5*e^(3*d*x + 3*c) + 1200*a^3*b^2*e^(2*d*x + 2*c) - 600*a*
b^4*e^(2*d*x + 2*c) + 2400*a^4*b*e^(d*x + c) - 3600*a^2*b^3*e^(d*x + c) + 3
00*b^5*e^(d*x + c) + 120*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*(d*x + c) - (75*a*
```

```
b^4*e^(d*x + c) - 6*b^5 - 300*(8*a^4*b - 12*a^2*b^3 + b^5)*e^(4*d*x + 4*c)
+ 600*(2*a^3*b^2 - a*b^4)*e^(3*d*x + 3*c) - 50*(8*a^2*b^3 - b^5)*e^(2*d*x +
 2*c))*e^(-5*d*x - 5*c))/d
```

3.97 $\int (a+b\sinh(c+dx))^4\,dx$

Optimal. Leaf size=137

$$\frac{ab\left(19a^2-16b^2\right)\cosh(c+dx)}{6d}+\frac{b^2\left(26a^2-9b^2\right)\sinh(c+dx)\cosh(c+dx)}{24d}+\frac{1}{8}x\left(-24a^2b^2+8a^4+3b^4\right)+\frac{b\cosh(c+$$

```
[Out] ((8*a^4 - 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 - 16*b^2)*Cosh[c + d*x])/
(6*d) + (b^2*(26*a^2 - 9*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(24*d) + (7*a*b*
Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)/(12*d) + (b*Cosh[c + d*x]*(a + b*Sin
h[c + d*x])^3)/(4*d)
```

Rubi [A] time = 0.155736, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2656, 2753, 2734}

$$\frac{ab\left(19a^2-16b^2\right)\cosh(c+dx)}{6d}+\frac{b^2\left(26a^2-9b^2\right)\sinh(c+dx)\cosh(c+dx)}{24d}+\frac{1}{8}x\left(-24a^2b^2+8a^4+3b^4\right)+\frac{b\cosh(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^4,x]
```

```
[Out] ((8*a^4 - 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 - 16*b^2)*Cosh[c + d*x])/
(6*d) + (b^2*(26*a^2 - 9*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(24*d) + (7*a*b*
Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)/(12*d) + (b*Cosh[c + d*x]*(a + b*Sin
h[c + d*x])^3)/(4*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^4\,dx = \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d} + \frac{1}{4}\int (a+b\sinh(c+dx))^2\left(4a^2-3b^2+7ab\sinh(c+d\right.$$
$$= \frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{12d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d} + \frac{1}{12}\int (a+b$$
$$= \frac{1}{8}\left(8a^4-24a^2b^2+3b^4\right)x + \frac{ab\left(19a^2-16b^2\right)\cosh(c+dx)}{6d} + \frac{b^2\left(26a^2-9b^2\right)\cosh(c+dx)\,\mathrm{si}}{24d}$$

Mathematica [A] time = 0.393612, size = 108, normalized size = 0.79

$$\frac{3\left(4\left(-24a^2b^2+8a^4+3b^4\right)(c+dx)+8\left(6a^2b^2-b^4\right)\sinh(2(c+dx))+b^4\sinh(4(c+dx))\right)+96ab\left(4a^2-3b^2\right)\cosh(c+d}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^4,x]
```

```
[Out] (96*a*b*(4*a^2 - 3*b^2)*Cosh[c + d*x] + 32*a*b^3*Cosh[3*(c + d*x)] + 3*(4*(
8*a^4 - 24*a^2*b^2 + 3*b^4)*(c + d*x) + 8*(6*a^2*b^2 - b^4)*Sinh[2*(c + d*x
)] + b^4*Sinh[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.016, size = 119, normalized size = 0.9

$$\frac{1}{d}\left(b^4\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4ab^3\left(-2/3+1/3(\sinh(dx+c))^2\right)\cosh(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+4*
a*b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+6*a^2*b^2*(1/2*cosh(d*x+c)*sinh(
d*x+c)-1/2*d*x-1/2*c)+4*a^3*b*cosh(d*x+c)+a^4*(d*x+c))
```

Maxima [A] time = 1.10724, size = 246, normalized size = 1.8

$$\frac{1}{64}b^4\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{3}{4}a^2b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^4x+\frac{1}{6}ab^3\left(\frac{e^{(3}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/64*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 3/4*a^2*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d
*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 4*a^3*b*cosh(d*x + c)/d
```

Fricas [A] time = 2.05994, size = 358, normalized size = 2.61

$$\frac{3\,b^4\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+8\,ab^3\cosh\left(dx+c\right)^3+24\,ab^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\left(8\,a^4-24\,a^2b^2+3\,b^4\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^3 + 24*a*
b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*d*x + 24
*(4*a^3*b - 3*a*b^3)*cosh(d*x + c) + 3*(b^4*cosh(d*x + c)^3 + 4*(6*a^2*b^2
- b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 1.48116, size = 240, normalized size = 1.75

$$\begin{cases} a^4x+\frac{4a^3b\cosh\left(c+dx\right)}{d}+3a^2b^2x\sinh^2\left(c+dx\right)-3a^2b^2x\cosh^2\left(c+dx\right)+\frac{3a^2b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{d}+\frac{4ab^3\sinh^2\left(c+dx\right)\cosh\left(c\right.}{d} \\ x\left(a+b\sinh\left(c\right)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*cosh(c + d*x)/d + 3*a**2*b**2*x*sinh(c + d*x)*
*2 - 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c + d*
x)/d + 4*a*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 8*a*b**3*cosh(c + d*x)**
3/(3*d) + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(c +
d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 + 5*b**4*sinh(c + d*x)**3*cosh(c +
d*x)/(8*d) - 3*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a
+ b*sinh(c))**4, True))
```

Giac [A] time = 1.18111, size = 261, normalized size = 1.91

$$\frac{3\,b^4e^{(4\,dx+4\,c)}+32\,ab^3e^{(3\,dx+3\,c)}+144\,a^2b^2e^{(2\,dx+2\,c)}-24\,b^4e^{(2\,dx+2\,c)}+384\,a^3be^{(dx+c)}-288\,ab^3e^{(dx+c)}+24\left(8\,a^4-24\,a^2b\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/192*(3*b^4*e^(4*d*x + 4*c) + 32*a*b^3*e^(3*d*x + 3*c) + 144*a^2*b^2*e^(2*
d*x + 2*c) - 24*b^4*e^(2*d*x + 2*c) + 384*a^3*b*e^(d*x + c) - 288*a*b^3*e^(
d*x + c) + 24*(8*a^4 - 24*a^2*b^2 + 3*b^4)*(d*x + c) + (32*a*b^3*e^(d*x + c
) - 3*b^4 + 96*(4*a^3*b - 3*a*b^3)*e^(3*d*x + 3*c) - 24*(6*a^2*b^2 - b^4)*e
^(2*d*x + 2*c))*e^(-4*d*x - 4*c))/d
```

3.98 $\int (a+b\sinh(c+dx))^3\,dx$

Optimal. Leaf size=92

$$\frac{2b\left(4a^2-b^2\right)\cosh(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2-3b^2\right)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}$$

```
[Out] (a*(2*a^2 - 3*b^2)*x)/2 + (2*b*(4*a^2 - b^2)*Cosh[c + d*x])/(3*d) + (5*a*b^
2*Cosh[c + d*x]*Sinh[c + d*x])/(6*d) + (b*Cosh[c + d*x]*(a + b*Sinh[c + d*x
])^2)/(3*d)
```

Rubi [A] time = 0.0728995, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$\frac{2b\left(4a^2-b^2\right)\cosh(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2-3b^2\right)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 3*b^2)*x)/2 + (2*b*(4*a^2 - b^2)*Cosh[c + d*x])/(3*d) + (5*a*b^
2*Cosh[c + d*x]*Sinh[c + d*x])/(6*d) + (b*Cosh[c + d*x]*(a + b*Sinh[c + d*x
])^2)/(3*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^3\,dx=\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}+\frac{1}{3}\int (a+b\sinh(c+dx))\left(3a^2-2b^2+5ab\sinh(c+d$$
$$=\frac{1}{2}a\left(2a^2-3b^2\right)x+\frac{2b\left(4a^2-b^2\right)\cosh(c+dx)}{3d}+\frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}+\frac{b\cosh(c$$

Mathematica [A] time = 0.174541, size = 71, normalized size = 0.77

$$\frac{6a\left(2a^2-3b^2\right)(c+dx)-9b\left(b^2-4a^2\right)\cosh(c+dx)+9ab^2\sinh(2(c+dx))+b^3\cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (6*a*(2*a^2 - 3*b^2)*(c + d*x) - 9*b*(-4*a^2 + b^2)*Cosh[c + d*x] + b^3*Cos
h[3*(c + d*x)] + 9*a*b^2*Sinh[2*(c + d*x)])/(12*d)
```

Maple [A] time = 0.017, size = 77, normalized size = 0.8

$$\frac{1}{d}\left(b^3\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)+3\,ab^2\,(1/2\,\cosh(dx+c)\sinh(dx+c)-1/2\,dx-c/2)+3\,a^2b\cosh(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh
(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*cosh(d*x+c)+a^3*(d*x+c))
```

Maxima [A] time = 1.02683, size = 155, normalized size = 1.68

$$-\frac{3}{8}\,ab^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^3x+\frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3\,a^2b\cosh(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b^
3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c
)/d) + 3*a^2*b*cosh(d*x + c)/d
```

Fricas [A] time = 1.94243, size = 225, normalized size = 2.45

$$\frac{b^3\cosh(dx+c)^3+3\,b^3\cosh(dx+c)\sinh(dx+c)^2+18\,ab^2\cosh(dx+c)\sinh(dx+c)+6\left(2\,a^3-3\,ab^2\right)dx+9\left(4\,a^2\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(b^3*cosh(d*x + c)^3 + 3*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 18*a*b^2*
cosh(d*x + c)*sinh(d*x + c) + 6*(2*a^3 - 3*a*b^2)*d*x + 9*(4*a^2*b - b^3)*c
osh(d*x + c))/d
```

Sympy [A] time = 0.753248, size = 128, normalized size = 1.39

$$\begin{cases} a^3x+\frac{3a^2b\cosh(c+dx)}{d}+\frac{3ab^2x\sinh^2(c+dx)}{2}-\frac{3ab^2x\cosh^2(c+dx)}{2}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{b^3\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{2b^3\cosh^3(c}{3d} \\ x\,(a+b\sinh(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*cosh(c + d*x)/d + 3*a*b**2*x*sinh(c + d*x)**2/
2 - 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/(2
*d) + b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b**3*cosh(c + d*x)**3/(3*d)
, Ne(d, 0)), (x*(a + b*sinh(c))**3, True))
```

<u>Giac</u> [A] time = 1.32152, size = 173, normalized size = 1.88

$$\frac{b^{3} e^{(3 dx+3 c)} + 9 ab^{2} e^{(2 dx+2 c)} + 36 a^{2} b e^{(dx+c)} - 9 b^{3} e^{(dx+c)} + 12 \left(2 a^{3} - 3 ab^{2}\right)(dx + c) - \left(9 ab^{2} e^{(dx+c)} - b^{3} - 9 \left(4 a^{2} b - b^{3}\right) e^{(2 d}\right.}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(b^3*e^(3*d*x + 3*c) + 9*a*b^2*e^(2*d*x + 2*c) + 36*a^2*b*e^(d*x + c)
- 9*b^3*e^(d*x + c) + 12*(2*a^3 - 3*a*b^2)*(d*x + c) - (9*a*b^2*e^(d*x + c)
 - b^3 - 9*(4*a^2*b - b^3)*e^(2*d*x + 2*c))*e^(-3*d*x - 3*c))/d
```

3.99 $\int (a+b\sinh(c+dx))^2\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}x\left(2a^2-b^2\right)+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] ((2*a^2 - b^2)*x)/2 + (2*a*b*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rubi [A] time = 0.0165329, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2-b^2\right)+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*x)/2 + (2*a*b*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^2\,dx=\frac{1}{2}\left(2a^2-b^2\right)x+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A] time = 0.0809233, size = 48, normalized size = 0.92

$$\frac{2\left(2a^2-b^2\right)(c+dx)+8ab\cosh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*(2*a^2 - b^2)*(c + d*x) + 8*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)])/(
4*d)
```

Maple [A] time = 0.012, size = 51, normalized size = 1.

$$\frac{1}{d}\left(b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2\,ab\cosh(dx+c)+a^2\,(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*cosh(d*x+c)+a^2*
(d*x+c))
```

Maxima [A] time = 1.09643, size = 74, normalized size = 1.42

$$-\frac{1}{8}\,b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^2x+\frac{2\,ab\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*cos
h(d*x + c)/d
```

Fricas [A] time = 1.99934, size = 112, normalized size = 2.15

$$\frac{b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(2\,a^2-b^2\right)dx+4\,ab\cosh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(d*x + c)*sinh(d*x + c) + (2*a^2 - b^2)*d*x + 4*a*b*cosh(d*x +
 c))/d
```

Sympy [A] time = 0.345343, size = 78, normalized size = 1.5

$$\begin{cases} a^2x+\frac{2ab\cosh\left(c+dx\right)}{d}+\frac{b^2x\sinh^2\left(c+dx\right)}{2}-\frac{b^2x\cosh^2\left(c+dx\right)}{2}+\frac{b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x\left(a+b\sinh\left(c\right)\right)^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*cosh(c + d*x)/d + b**2*x*sinh(c + d*x)**2/2 - b**
2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)),
 (x*(a + b*sinh(c))**2, True))
```

Giac [A] time = 1.25374, size = 99, normalized size = 1.9

$$\frac{b^2e^{(2\,dx+2\,c)}+8\,abe^{(dx+c)}+4\left(2\,a^2-b^2\right)(dx+c)+\left(8\,abe^{(dx+c)}-b^2\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2*e^(2*d*x + 2*c) + 8*a*b*e^(d*x + c) + 4*(2*a^2 - b^2)*(d*x + c) +
(8*a*b*e^(d*x + c) - b^2)*e^(-2*d*x - 2*c))/d
```

3.100 $\int (a+b\sinh (c+dx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\cosh (c+dx)}{d}$$

```
[Out] a*x + (b*Cosh[c + d*x])/d
```

Rubi [A] time = 0.0097391, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2638}

$$ax+\frac{b\cosh (c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x],x]

[Out] a*x + (b*Cosh[c + d*x])/d
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sinh (c+dx))\,dx &= ax+b\int \sinh (c+dx)\,dx\\ &= ax+\frac{b\cosh (c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0094447, size = 26, normalized size = 1.73

$$ax+\frac{b\sinh (c)\sinh (dx)}{d}+\frac{b\cosh (c)\cosh (dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x],x]

[Out] a*x + (b*Cosh[c]*Cosh[d*x])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.003, size = 16, normalized size = 1.1

$$ax+\frac{b\cosh \left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c),x)
```

```
[Out] a*x+b*cosh(d*x+c)/d
```

Maxima [A] time = 1.03804, size = 20, normalized size = 1.33

$$ax + \frac{b\cosh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*cosh(d*x + c)/d
```

Fricas [A] time = 2.00046, size = 39, normalized size = 2.6

$$\frac{adx + b\cosh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + b*cosh(d*x + c))/d
```

Sympy [A] time = 0.167423, size = 17, normalized size = 1.13

$$ax + b\left(\begin{cases}\frac{\cosh(c+dx)}{d} & \text{for}\ d \neq 0\\ x\sinh(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

Giac [B] time = 1.24204, size = 42, normalized size = 2.8

$$ax + \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d)
```

3.101 $\int \frac{1}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

```
[Out] (-2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Rubi [A] time = 0.0355395, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2660, 618, 204}

$$-\frac{2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$
$$= \frac{(4i)\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$
$$= -\frac{2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Mathematica [A] time = 0.0360528, size = 52, normalized size = 1.18

$$\frac{2\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{d\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d)
```

Maple [A] time = 0.015, size = 43, normalized size = 1.

$$2\,\frac{1}{d\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tanh(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)),x)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99491, size = 424, normalized size = 9.64

$$\frac{\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)-2\,\sqrt{a^2+b^2}(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^
2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*
cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^
2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b))/(sqrt(a
^2 + b^2)*d)
```

Sympy [A] time = 20.6097, size = 189, normalized size = 4.3

$$\begin{cases} -\frac{2i\sqrt{b^2}}{-b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ibd\sqrt{b^2}} & \text{for } a=-\sqrt{-b^2} \\ -\frac{2i\sqrt{b^2}}{b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ibd\sqrt{b^2}} & \text{for } a=\sqrt{-b^2} \\ \frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd} & \text{for } a=0 \\ \frac{x}{a+b\sinh(c)} & \text{for } d=0 \\ -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}}+\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((-2*I*sqrt(b**2)/(-b**2*d*tanh(c/2 + d*x/2) + I*b*d*sqrt(b**2)),
Eq(a, -sqrt(-b**2))), (-2*I*sqrt(b**2)/(b**2*d*tanh(c/2 + d*x/2) + I*b*d*sq
rt(b**2)), Eq(a, sqrt(-b**2))), (log(tanh(c/2 + d*x/2))/(b*d), Eq(a, 0)), (
x/(a + b*sinh(c)), Eq(d, 0)), (-log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b
**2)/a)/(d*sqrt(a**2 + b**2)) + log(tanh(c/2 + d*x/2) - b/a + sqrt(a**2 + b
**2)/a)/(d*sqrt(a**2 + b**2)), True))
```

Giac [A] time = 1.19226, size = 90, normalized size = 2.05

$$\frac{\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.102 $\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}$$

```
[Out] (-2*a*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2
)*d) - (b*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 0.0628787, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 12, 2660, 618, 204}

$$-\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-2*a*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2
)*d) - (b*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = -\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}+\frac{\int \frac{a}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}+\frac{a\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}-\frac{(2ia)\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{\left(a^2+b^2\right)d}$$

$$= -\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}+\frac{(4ia)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(ic+\right.\right.}{\left(a^2+b^2\right)d}$$

$$= -\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}-\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}$$

Mathematica [A] time = 0.25853, size = 85, normalized size = 1.08

$$-\frac{\frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-2),x]
```

```
[Out] -(((2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3
/2) + (b*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])))/d)
```

Maple [A] time = 0.034, size = 118, normalized size = 1.5

$$\frac{1}{d}\left(-2\,\frac{1}{(\tanh(1/2\,dx+c/2))^2\,a-2\,\tanh(1/2\,dx+c/2)\,b-a}\left(-\frac{b^2\tanh(1/2\,dx+c/2)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)+2\,\frac{a}{\left(a^2+b^2\right)^{3/2}}\text{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*(-b^2/a/(a^2+b^2)*tanh(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*ta
nh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10616, size = 1041, normalized size = 13.18

$$-\frac{2\,a^2b + 2\,b^3 - \left(ab\cosh\left(dx+c\right)^2 + ab\sinh\left(dx+c\right)^2 + 2\,a^2\cosh\left(dx+c\right) - ab + 2\left(ab\cosh\left(dx+c\right) + a^2\right)\sinh\left(dx+\right.\right.}{\left(a^4b + 2\,a^2b^3 + b^5\right)d\cosh\left(dx+c\right)^2 + \left(a^4b + 2\,a^2b^3 + b^5\right)d\sinh\left(dx+c\right)^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b + 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cosh
(d*x + c) - a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 + b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(
a^3 + a*b^2)*cosh(d*x + c) - 2*(a^3 + a*b^2)*sinh(d*x + c))/((a^4*b + 2*a^2
*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2
 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) - (a^4*b + 2*a^2*b^3 + b^5)*
d + 2*((a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4
)*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35334, size = 180, normalized size = 2.28

$$-\frac{a\log\left(\frac{\left|-2\,be^{(dx+c)} - 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|-2\,be^{(dx+c)} - 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2d + b^2d\right)\sqrt{a^2+b^2}} + \frac{2\left(ae^{(dx+c)} - b\right)}{\left(a^2d + b^2d\right)\left(be^{(2\,dx+2\,c)} + 2\,ae^{(dx+c)} - b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -a*log(abs(-2*b*e^(d*x + c) - 2*a - 2*sqrt(a^2 + b^2))/abs(-2*b*e^(d*x + c)
 - 2*a + 2*sqrt(a^2 + b^2)))/((a^2*d + b^2*d)*sqrt(a^2 + b^2)) + 2*(a*e^(d*
x + c) - b)/((a^2*d + b^2*d)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b))
```

3.103 $\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$-\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}-\frac{3ab\cosh(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\sinh(c+dx))}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

```
[Out] -(((2*a^2 - b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(5/2)*d)) - (b*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x])
^2) - (3*a*b*Cosh[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 0.122238, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$-\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}-\frac{3ab\cosh(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\sinh(c+dx))}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-3),x]
```

```
[Out] -(((2*a^2 - b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(5/2)*d)) - (b*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x])
^2) - (3*a*b*Cosh[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{\int \frac{-2a+b\sinh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx}{2\left(a^2+b^2\right)}$$

$$= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} + \frac{\int \frac{2a^2-b^2}{a+b\sinh(c+dx)}\,d}{2\left(a^2+b^2\right)^2}$$

$$= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} + \frac{\left(2a^2-b^2\right)\int \frac{}{a+b}}{2\left(a^2+\right.}$$

$$= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} - \frac{\left(i\left(2a^2-b^2\right)\right)\mathrm{Su}}{}$$

$$= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} + \frac{\left(2i\left(2a^2-b^2\right)\right)\mathrm{S}}{}$$

$$= -\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} - \frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\,\mathrm{c}}{2\left(a^2+b^2\right)^2 d}$$

Mathematica [A] time = 0.272079, size = 117, normalized size = 0.92

$$\frac{\frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \frac{b\cosh(c+dx)\left(4a^2+3ab\sinh(c+dx)+b^2\right)}{(a+b\sinh(c+dx))^2}}{2d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((2*(2*a^2 - b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[
-a^2 - b^2] - (b*Cosh[c + d*x]*(4*a^2 + b^2 + 3*a*b*Sinh[c + d*x]))/(a + b*
Sinh[c + d*x])^2)/(2*(a^2 + b^2)^2*d)
```

Maple [B] time = 0.049, size = 280, normalized size = 2.2

$$\frac{1}{d}\left(-2\,\frac{1}{\left(\left(\tanh\left(1/2\,dx+c/2\right)\right)^2a-2\,\tanh\left(1/2\,dx+c/2\right)b-a\right)^2}\left(-1/2\,\frac{b^2\left(5\,a^2+2\,b^2\right)\left(\tanh\left(1/2\,dx+c/2\right)\right)^3}{a\left(a^4+2\,a^2b^2+b^4\right)}-1/2\,\frac{b\left(4\,a\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3
-1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tanh(1/2*d*x+1/2*c)^
2+1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tanh(1/2*d*x+1/2*c)+1/2*b*(4
*a^2+b^2)/(a^4+2*a^2*b^2+b^4))/(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)*b-a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.65862, size = 3005, normalized size = 23.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*b^2 + 6*a*b^4 + 2*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^3 + 2*
(2*a^4*b + a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 + a^3*b^2 - a*b^4)*cos
h(d*x + c)^2 + 6*(2*a^5 + a^3*b^2 - a*b^4 + (2*a^4*b + a^2*b^3 - b^5)*cosh(
d*x + c))*sinh(d*x + c)^2 - ((2*a^2*b^2 - b^4)*cosh(d*x + c)^4 + (2*a^2*b^2
 - b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 - b^4 + 4*(2*a^3*b - a*b^3)*cosh(d*x +
c)^3 + 4*(2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cosh(d*x + c))*sinh(d*x + c)^
3 + 2*(4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 - 4*a^2*b^2 + b^
4 + 3*(2*a^2*b^2 - b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^2 - 4*(2*a^3*b - a*b^3)*cosh(d*x + c) - 4*(2*a^3*b - a*b^3
- (2*a^2*b^2 - b^4)*cosh(d*x + c)^3 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^2 -
 (4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*lo
g((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
+ b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*co
sh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(10*a
^4*b + 11*a^2*b^3 + b^5)*cosh(d*x + c) - 2*(10*a^4*b + 11*a^2*b^3 + b^5 - 3
*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^2 - 6*(2*a^5 + a^3*b^2 - a*b^4)*co
sh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh
(d*x + c)^4 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*sinh(d*x + c)^4 + 4
```

```
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 + 5*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 + 3*a^4
*b^4 + 3*a^2*b^6 + b^8)*d*cosh(d*x + c) + (a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*d)*sinh(d*x + c)^3 - 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos
h(d*x + c) + 2*(3*(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh(d*x + c)^2
 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c) + (2*a^8 + 5*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 + 3*a^4*b
^4 + 3*a^2*b^6 + b^8)*d + 4*((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh
(d*x + c)^3 + 3*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 +
 (2*a^8 + 5*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d*cosh(d*x + c) - (a^7*b +
 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31144, size = 319, normalized size = 2.51

$$-\frac{\left(2\,a^2-b^2\right)\log\left(\frac{\left|-2\,be^{(dx+c)}-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|-2\,be^{(dx+c)}-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\left(a^4d+2\,a^2b^2d+b^4d\right)\sqrt{a^2+b^2}}+\frac{2\,a^2be^{(3\,dx+3\,c)}-b^3e^{(3\,dx+3\,c)}+6\,a^3e^{(2\,dx+2\,c)}-3\,ab^2e^{(2\,dx+2\,c)}-10\,a^2be^{(dx+c)}-b^3}{\left(a^4d+2\,a^2b^2d+b^4d\right)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2 - b^2)*log(abs(-2*b*e^(d*x + c) - 2*a - 2*sqrt(a^2 + b^2))/abs(
-2*b*e^(d*x + c) - 2*a + 2*sqrt(a^2 + b^2)))/((a^4*d + 2*a^2*b^2*d + b^4*d)
*sqrt(a^2 + b^2)) + (2*a^2*b*e^(3*d*x + 3*c) - b^3*e^(3*d*x + 3*c) + 6*a^3*
e^(2*d*x + 2*c) - 3*a*b^2*e^(2*d*x + 2*c) - 10*a^2*b*e^(d*x + c) - b^3*e^(d
*x + c) + 3*a*b^2)/((a^4*d + 2*a^2*b^2*d + b^4*d)*(b*e^(2*d*x + 2*c) + 2*a*
e^(d*x + c) - b)^2)
```

3.104 $\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=174

$$-\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{7/2}}-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{6d\left(a^2+b^2\right)^3(a+b\sinh(c+dx))}-\frac{5ab\cosh(c+dx)}{6d\left(a^2+b^2\right)^2(a+b\sinh(c+dx))^2}-$$

```
[Out] -((a*(2*a^2 - 3*b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((
a^2 + b^2)^(7/2)*d)) - (b*Cosh[c + d*x])/(3*(a^2 + b^2)*d*(a + b*Sinh[c + d
*x])^3) - (5*a*b*Cosh[c + d*x])/(6*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x])^2)
 - (b*(11*a^2 - 4*b^2)*Cosh[c + d*x])/(6*(a^2 + b^2)^3*d*(a + b*Sinh[c + d*
x]))
```

Rubi [A] time = 0.219641, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$-\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{7/2}}-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{6d\left(a^2+b^2\right)^3(a+b\sinh(c+dx))}-\frac{5ab\cosh(c+dx)}{6d\left(a^2+b^2\right)^2(a+b\sinh(c+dx))^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-4),x]
```

```
[Out] -((a*(2*a^2 - 3*b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((
a^2 + b^2)^(7/2)*d)) - (b*Cosh[c + d*x])/(3*(a^2 + b^2)*d*(a + b*Sinh[c + d
*x])^3) - (5*a*b*Cosh[c + d*x])/(6*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x])^2)
 - (b*(11*a^2 - 4*b^2)*Cosh[c + d*x])/(6*(a^2 + b^2)^3*d*(a + b*Sinh[c + d*
x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{\int \frac{-3a+2b\sinh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} + \frac{\int \frac{2\left(3a^2-2b^2\right)-5ab}{(a+b\sinh(c}}{6\left(a^2+\right.}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} - \frac{b\left(11a^2-4b\right.}{6\left(a^2+b^2\right)^3 d(a}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} - \frac{b\left(11a^2-4b\right.}{6\left(a^2+b^2\right)^3 d(a}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} - \frac{b\left(11a^2-4b\right.}{6\left(a^2+b^2\right)^3 d(a}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} - \frac{b\left(11a^2-4b\right.}{6\left(a^2+b^2\right)^3 d(a}$$

$$= -\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2} d} - \frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5a}{6\left(a^2+b^2\right)}$$

Mathematica [A] time = 0.692069, size = 159, normalized size = 0.91

$$\frac{\frac{6a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{b\cosh(c+dx)\left(\left(4b^4-11a^2b^2\right)\sinh^2(c+dx)+3ab\left(b^2-9a^2\right)\sinh(c+dx)-5a^2b^2-18a^4-2b^4\right)}{(a+b\sinh(c+dx))^3}}{6d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 - 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2] + (b*Cosh[c + d*x]*(-18*a^4 - 5*a^2*b^2 - 2*b^4 + 3*a*b*(-9
*a^2 + b^2)*Sinh[c + d*x] + (-11*a^2*b^2 + 4*b^4)*Sinh[c + d*x]^2))/(a + b*
Sinh[c + d*x])^3)/(6*(a^2 + b^2)^3*d)
```

Maple [B] time = 0.056, size = 494, normalized size = 2.8

$$\frac{1}{d}\left(-2\,\frac{1}{\left(\left(\tanh\left(1/2\,dx+c/2\right)\right)^{2}a-2\,\tanh\left(1/2\,dx+c/2\right)b-a\right)^{3}}\left(-1/2\,\frac{b^{2}\left(9\,a^{4}+6\,a^{2}b^{2}+2\,b^{4}\right)\left(\tanh\left(1/2\,dx+c/2\right)\right)^{5}}{a\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(9*a^4+6*a^2*b^2+2*b^4)/a/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*t
anh(1/2*d*x+1/2*c)^5-1/2*b*(6*a^6-27*a^4*b^2-12*a^2*b^4-4*b^6)/a^2/(a^6+3*a
^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6-21*a^4*b^2-
4*a^2*b^4-4*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^3+1/a^2*
b*(6*a^6-20*a^4*b^2-3*a^2*b^4-2*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2
*d*x+1/2*c)^2-1/2/a*b^2*(27*a^4+4*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b
^6)*tanh(1/2*d*x+1/2*c)-1/6*b*(18*a^4+5*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2
*b^4+b^6))/(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)^3+a*(2*a^2-3
*b^2)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75107, size = 6620, normalized size = 38.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(22*a^4*b^3 + 14*a^2*b^5 - 8*b^7 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*c
osh(d*x + c)^5 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*sinh(d*x + c)^5 - 30*(2*
a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 - 30*(2*a^6*b - a^4*b^3 - 3*a^
2*b^5 + (2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*
(22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c)^3 - 4*(22*a^7 -
 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6 + 15*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*co
sh(d*x + c)^2 + 30*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 12*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7)*cosh(d*x + c)^2 + 1
```

```
2*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7 - 5*(2*a^5*b^2 - a^3*b^4 - 3*a
*b^6)*cosh(d*x + c)^3 - 15*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^2
- (22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c))*sinh(d*x + c
)^2 + 3*((2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^6 + (2*a^3*b^3 - 3*a*b^5)*sinh
(d*x + c)^6 - 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)
^5 + 6*(2*a^4*b^2 - 3*a^2*b^4 + (2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c))*sinh(d
*x + c)^5 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 3*(8*a^5*b
 - 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^2 + 10*(2*a
^4*b^2 - 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^6 - 12*a^4*b^2
+ 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 - 12*a^4*b^2 + 9*a^2*b^4 + 5*(2*a^3
*b^3 - 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)^
2 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(
8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 - 3*(8*a^5*b - 14*a^3*b^3 +
 3*a*b^5 - 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^4 - 20*(2*a^4*b^2 - 3*a^2*
b^4)*cosh(d*x + c)^3 - 6*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 -
 4*(4*a^6 - 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*a
^4*b^2 - 3*a^2*b^4)*cosh(d*x + c) + 6*(2*a^4*b^2 - 3*a^2*b^4 + (2*a^3*b^3 -
 3*a*b^5)*cosh(d*x + c)^5 + 5*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)^4 + 2*(
8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 2*(4*a^6 - 12*a^4*b^2 + 9
*a^2*b^4)*cosh(d*x + c)^2 - (8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c
)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(
d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c)
+ a)*sinh(d*x + c) - b)) - 30*(4*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(d*x + c)
 - 6*(20*a^5*b^2 + 15*a^3*b^4 - 5*a*b^6 + 5*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)
*cosh(d*x + c)^4 + 20*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^3 + 2*(
22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c)^2 - 4*(17*a^6*b
+ 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7)*cosh(d*x + c))*sinh(d*x + c))/((a^8*b^3 +
 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^6 + (a^8*b^3 + 4
*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*sinh(d*x + c)^6 + 6*(a^9*b^2 + 4
*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^5 + 3*(4*a^10*b
+ 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*cosh(d*x + c)^4 + 6*((a^8*
b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c) + (a^9*b^2
+ 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c)^5 + 4*(2*a^1
1 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 - 3*a*b^10)*d*cosh(d*x + c)^3 + 3*(
5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^2 +
10*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) +
 (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d)*sinh(d*x + c)^
4 - 3*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*cosh(d*x +
 c)^2 + 4*(5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*
x + c)^3 + 15*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh
(d*x + c)^2 + 3*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*
cosh(d*x + c) + (2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 - 3*a*b^10)*d
)*sinh(d*x + c)^3 + 6*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10
)*d*cosh(d*x + c) + 3*(5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^1
1)*d*cosh(d*x + c)^4 + 20*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*
b^10)*d*cosh(d*x + c)^3 + 6*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^
7 - b^11)*d*cosh(d*x + c)^2 + 4*(2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b
^8 - 3*a*b^10)*d*cosh(d*x + c) - (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a
^4*b^7 - b^11)*d)*sinh(d*x + c)^2 - (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^
2*b^9 + b^11)*d + 6*((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d
*cosh(d*x + c)^5 + 5*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)
*d*cosh(d*x + c)^4 + 2*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b
^11)*d*cosh(d*x + c)^3 + 2*(2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 -
3*a*b^10)*d*cosh(d*x + c)^2 - (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*
b^7 - b^11)*d*cosh(d*x + c) + (a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8
+ a*b^10)*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24228, size = 491, normalized size = 2.82

$$\frac{\left(2\,a^3-3\,ab^2\right)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\left(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d\right)\sqrt{a^2+b^2}}+\frac{6\,a^3b^2e^{(5\,dx+5\,c)}-9\,ab^4e^{(5\,dx+5\,c)}+30\,a^4be^{(4\,dx+4\,c)}-45\,a^2b^3e^{(4\,dx+4\,c)}+44}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3 - 3*a*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/ab
s(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^6*d + 3*a^4*b^2*d + 3*a^2
*b^4*d + b^6*d)*sqrt(a^2 + b^2)) + 1/3*(6*a^3*b^2*e^(5*d*x + 5*c) - 9*a*b^4
*e^(5*d*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) - 45*a^2*b^3*e^(4*d*x + 4*c) +
44*a^5*e^(3*d*x + 3*c) - 82*a^3*b^2*e^(3*d*x + 3*c) + 24*a*b^4*e^(3*d*x + 3
*c) - 102*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2*c) - 12*b^5*e^(2*
d*x + 2*c) + 60*a^3*b^2*e^(d*x + c) - 15*a*b^4*e^(d*x + c) - 11*a^2*b^3 + 4
*b^5)/((a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d + b^6*d)*(b*e^(2*d*x + 2*c) + 2*a
*e^(d*x + c) - b)^3)
```

3.105 $\int (a+b\sinh(x))^{5/2}\,dx$

Optimal. Leaf size=179

$$-\frac{16ia\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{15\sqrt{a+b\sinh(x)}}+\frac{2i\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))$$

```
[Out] (16*a*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*b*Cosh[x]*(a + b*Sinh[x])^(3/2
))/5 + (((2*I)/15)*(23*a^2 - 9*b^2)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a +
b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (((16*I)/15)*a*(
a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/
(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rubi [A] time = 0.257808, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{16ia\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15\sqrt{a+b\sinh(x)}}+\frac{2i\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (16*a*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*b*Cosh[x]*(a + b*Sinh[x])^(3/2
))/5 + (((2*I)/15)*(23*a^2 - 9*b^2)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a +
b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (((16*I)/15)*a*(
a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/
(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sinh(x))^{5/2}\,dx = \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{5}\int \sqrt{a+b\sinh(x)}\left(\frac{1}{2}\left(5a^2-3b^2\right)+4ab\sinh(x)\right)dx$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{4}{15}\int \frac{\frac{1}{4}a\left(15a^2-17b^2\right)+\frac{1}{4}b\left(\right.}{\sqrt{a+b\,\mathrm{si}}}$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{1}{15}\left(23a^2-9b^2\right)\int \sqrt{a+b\,\mathrm{sin}}$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{\left(\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}\right.}{15\sqrt{\frac{a+b\,\mathrm{sin}}{a-i}}}$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2i\left(23a^2-9b^2\right)E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right.}{15\sqrt{\frac{a+b\sinh(x}{a-ib}}}$$

Mathematica [A] time = 0.413323, size = 178, normalized size = 0.99

$$\frac{-16ia\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+b\cosh(x)\left(22a^2+28ab\sinh(x)+3b^2\cosh(2x)-3b^2\right)+2}{15\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (2*((23*I)*a^3 + 23*a^2*b - (9*I)*a*b^2 - 9*b^3)*EllipticE[(Pi - (2*I)*x)/4
, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (16*I)*a*(a^2 + b
^2)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/
```

```
(a - I*b)] + b*Cosh[x]*(22*a^2 - 3*b^2 + 3*b^2*Cosh[2*x] + 28*a*b*Sinh[x]))
/(15*Sqrt[a + b*Sinh[x]])
```

Maple [B] time = 0.155, size = 917, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(5/2),x)
```

```
[Out] 2/15*(8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+
sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a
)/(I*b+a))^(1/2))*a^3*b+8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-
a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^3+15*(-(a+b*sinh(x))/(I*b-a))^(1/2)
*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a
+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4+6*(-(a+b*sinh(x))/
(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*
EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2-
9*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))
*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a
))^(1/2))*b^4-23*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/
2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(
-(I*b-a)/(I*b+a))^(1/2))*a^4-14*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))
*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(
I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2+9*(-(a+b*sinh(x))/(I*b-a))^
(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE
((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^4+3*b^4*sinh(x)
^4+14*a*b^3*sinh(x)^3+11*a^2*b^2*sinh(x)^2+3*b^4*sinh(x)^2+14*a*b^3*sinh(x)
+11*a^2*b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \sinh(x)^2 + 2\,ab\sinh(x) + a^2\right)\sqrt{b\sinh(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sinh(x)^2 + 2*a*b*sinh(x) + a^2)*sqrt(b*sinh(x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(x) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(5/2), x)
```

3.106 $\int (a+b\sinh(x))^{3/2}\,dx$

Optimal. Leaf size=150

$$-\frac{2i(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{3\sqrt{a+b\sinh(x)}}+\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8ia\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] (2*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x
, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (
((2*I)/3)*(a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
b*Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rubi [A] time = 0.165804, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2656, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2i(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3\sqrt{a+b\sinh(x)}}+\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8ia\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x
, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (
((2*I)/3)*(a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
b*Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sinh(x))^{3/2}\,dx &= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{3}\int \frac{\frac{1}{2}\left(3a^2-b^2\right)+2ab\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx\\ &= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}(4a)\int \sqrt{a+b\sinh(x)}\,dx+\frac{1}{3}\left(-a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sinh(x)}}\,d\\ &= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{\left(4a\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{\left(\left(-a^2-b^2\right)\sqrt{\frac{a+b\sinh}{a}}\right.}{3\sqrt{a}}\\ &= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2i\left(a^2+b^2\right)F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3\sqrt{a+b\sinh(}}\end{aligned}$$

Mathematica [A] time = 0.378487, size = 139, normalized size = 0.93

$$\frac{-2i\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+2b\cosh(x)(a+b\sinh(x))+8a(b+ia)\sqrt{\frac{a+b\sinh(x)}{a-ib}}E\left(\frac{1}{4}(\pi-2\right.}{3\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Cosh[x]*(a + b*Sinh[x]) + 8*a*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((
-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*Ell
ipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*
b)])/(3*Sqrt[a + b*Sinh[x]])
```

Maple [B] time = 0.079, size = 676, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(3/2),x)
```

```
[Out] 2/3*(I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sin
h(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a^2*b+I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a
))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(
1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3+3*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sin
h(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(
x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3+3*(-(a+b*sinh(x))/(I*b-a))
^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Elliptic
F((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^2-4*(-(a+b*s
inh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a)
)^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*
a^3-4*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh
(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I
*b+a))^(1/2))*a*b^2+b^3*sinh(x)^3+a*b^2*sinh(x)^2+b^3*sinh(x)+a*b^2)/b/cosh
(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh(x) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sinh(x) + a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(x) + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sinh(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a + b*sinh(x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sinh(x) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(3/2), x)
```

3.107 $\int \sqrt{a+b\sinh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt
[(a + b*Sinh[x])/(a - I*b)]
```

Rubi [A] time = 0.0371274, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2655, 2653}

$$\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt
[(a + b*Sinh[x])/(a - I*b)]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh(x)}\,dx &= \frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\\ &= \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\end{aligned}$$

Mathematica [A] time = 0.186684, size = 65, normalized size = 1.08

$$\frac{2(b+ia)\sqrt{\frac{a+b\sinh(x)}{a-ib}}E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)}{\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*
Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Maple [B] time = 0.078, size = 262, normalized size = 4.4

$$2\frac{ib-a}{b\cosh(x)\sqrt{a+b\sinh(x)}}\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\left(i\mathrm{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(1/2),x)
```

```
[Out] 2/b*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((
I+sinh(x))*b/(I*b-a))^(1/2)*(I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b-I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))*b+EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*
b+a))^(1/2))*a-a*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a)
)^(1/2)))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh(x)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh (x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x) + a), x)
```

3.108 $\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{\sqrt{a+b\sinh(x)}}$$

```
[Out] ((2*I)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a -
 I*b)])/Sqrt[a + b*Sinh[x]]
```

Rubi [A] time = 0.0368365, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2663, 2661}

$$\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a -
 I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx &= \frac{\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{\sqrt{a+b\sinh(x)}}\\ &= \frac{2iF\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}\end{aligned}$$

Mathematica [A] time = 0.185854, size = 60, normalized size = 1.

$$\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)}{\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Maple [A] time = 0.075, size = 125, normalized size = 2.1

$$-2\,\frac{ib-a}{b\cosh(x)\sqrt{a+b\sinh(x)}}\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))\,b}{ib+a}}\sqrt{\frac{(i+\sinh(x))\,b}{ib-a}}\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-}{ib+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x))^(1/2),x)
```

```
[Out] -2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b\sinh(x)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sinh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh\left(x\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(x) + a), x)
```

3.109 $\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] (-2*b*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)*Sqrt[(a + b*Sinh[x])/(a - I*b)])
```

Rubi [A] time = 0.0597764, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2664, 21, 2655, 2653}

$$-\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*b*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)*Sqrt[(a + b*Sinh[x])/(a - I*b)])
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x, a + b*x])
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = -\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} - \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{a^2+b^2}$$
$$= -\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{\int \sqrt{a+b\sinh(x)}\,dx}{a^2+b^2}$$
$$= -\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$
$$= -\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Mathematica [A] time = 0.15025, size = 81, normalized size = 0.86

$$\frac{-2b\cosh(x)+2(b+ia)\sqrt{\frac{a+b\sinh(x)}{a-ib}}E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*b*Cosh[x] + 2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b
)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Maple [B] time = 0.095, size = 456, normalized size = 4.9

$$2\frac{1}{\left(a^2+b^2\right)b\cosh(x)\sqrt{a+b\sinh(x)}}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh}{ib-a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x))^(3/2),x)
```

```
[Out] 2*((-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x)
)*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
a))^(1/2))*a^2+(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)
*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b^2-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I
*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a
))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-s
inh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sin
h(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^2-b^2*sinh(x)^2-b^2)/(a^2+
b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh(x)+a}}{b^2\sinh(x)^2+2\,ab\sinh(x)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(x) + a)/(b^2*sinh(x)^2 + 2*a*b*sinh(x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+b\sinh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a + b*sinh(x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(b\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(-3/2), x)
```

3.110 $\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{3\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}-\frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}+\frac{8ia\sqrt{a+b\sin}}{3\left(a^2+b\right.}$$

```
[Out] (-2*b*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (8*a*b*Cosh[x])/(3*(
a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x,
(2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(
a - I*b)]) - (((2*I)/3)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a
+ b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rubi [A] time = 0.211873, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}\right.}{3\left(a^2+b^2\right)^2\sqrt{\frac{a+b}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*b*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (8*a*b*Cosh[x])/(3*(
a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x,
(2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(
a - I*b)]) - (((2*I)/3)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a
+ b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2-b^2\right)+ab\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{3\left(a^2+b^2\right)^2}$$

$$= -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{(4a)\int \sqrt{a+b\sinh(x)}\,dx}{3\left(a^2+b^2\right)^2} - \frac{\int \cdots}{\cdots}$$

$$= -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{\left(4a\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\cdots}}{3\left(a^2+b^2\right)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\cdots$$

$$= -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh\cdots}}{3\left(a^2+b^2\right)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\cdots$$

Mathematica [A] time = 0.61847, size = 166, normalized size = 0.84

$$\frac{-2i\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}(a+b\sinh(x))\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)-2b\cosh(x)\left(5a^2+4ab\sinh(x)+b^2\right)+\frac{8ia(a+b\sinh\cdots}{\cdots}}{3\left(a^2+b^2\right)^2(a+b\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(-5/2),x]
```

```
[Out] (((8*I)*a*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x])
^2)/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*EllipticF[(Pi - (2*
I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a - I*
b)] - 2*b*Cosh[x]*(5*a^2 + b^2 + 4*a*b*Sinh[x]))/(3*(a^2 + b^2)^2*(a + b*Si
nh[x])^(3/2))
```

Maple [A] time = 0.147, size = 438, normalized size = 2.2

$$\frac{1}{\cosh(x)}\sqrt{(a+b\sinh(x))(\cosh(x))^2}\left(-\frac{2}{3b(a^2+b^2)}\sqrt{(a+b\sinh(x))(\cosh(x))^2}\left(\sinh(x)+\frac{a}{b}\right)^{-2}-\frac{8b(\cosh(x))^2 a}{3(a^2+b^2)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x))^(5/2),x)
```

```
[Out] ((a+b*sinh(x))*cosh(x)^2)^(1/2)*(-2/3/b/(a^2+b^2)*((a+b*sinh(x))*cosh(x)^2)
^(1/2)/(sinh(x)+a/b)^2-8/3*b*cosh(x)^2/(a^2+b^2)^2*a/((a+b*sinh(x))*cosh(x)
^2)^(1/2)+2*(3*a^2-b^2)/(3*a^4+6*a^2*b^2+3*b^4)*(a/b-I)*((-b*sinh(x)-a)/(I*
b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a
+b*sinh(x))*cosh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I
*b)/(I*b+a))^(1/2))+8/3*a*b/(a^2+b^2)^2*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1
/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(
x))*cosh(x)^2)^(1/2)*((-a/b-I)*EllipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a
-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(
I*b+a))^(1/2))))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh(x)+a}}{b^3\sinh(x)^3+3ab^2\sinh(x)^2+3a^2b\sinh(x)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(x) + a)/(b^3*sinh(x)^3 + 3*a*b^2*sinh(x)^2 + 3*a^2*b*s
inh(x) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\sinh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))**(5/2),x)
```

```
[Out] Integral((a + b*sinh(x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(-5/2), x)
```

3.111 $\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=128

$$\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2ia\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{b\sqrt{a+b\sinh(x)}}$$

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*S
qrt[(a + b*Sinh[x])/(a - I*b)]) - ((2*I)*a*EllipticF[Pi/4 - (I/2)*x, (2*b)/
(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rubi [A] time = 0.114254, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2ia\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```


```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*S
qrt[(a + b*Sinh[x])/(a - I*b)]) - ((2*I)*a*EllipticF[Pi/4 - (I/2)*x, (2*b)/
(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```


Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \frac{\int \sqrt{a+b\sinh(x)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b}$$
$$= \frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{\left(a\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{b\sqrt{a+b\sinh(x)}}$$
$$= \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2iaF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

Mathematica [A] time = 0.351524, size = 101, normalized size = 0.79

$$\frac{2\sqrt{\frac{a+b\sinh(x)}{a-ib}}\left((b+ia)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)-ia\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*((I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - I*a*Ellip
ticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sinh[x])/(a - I*b
)])/(b*Sqrt[a + b*Sinh[x]])
```

Maple [A] time = 0.079, size = 218, normalized size = 1.7

$$2\frac{ib-a}{b^2\cosh(x)\sqrt{a+b\sinh(x)}}\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))\,b}{ib+a}}\sqrt{\frac{(i+\sinh(x))\,b}{ib-a}}\left(i\text{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib}{ib}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x))^(1/2),x)
```

```
[Out] 2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+
sinh(x))*b/(I*b-a))^(1/2)*(I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*
b-a)/(I*b+a))^(1/2))*b-I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)
/(I*b+a))^(1/2))*b+EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(x\right)}{\sqrt{b\sinh\left(x\right) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(x)/sqrt(b*sinh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(x\right)}{\sqrt{a + b\sinh\left(x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(a + b*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(x\right)}{\sqrt{b\sinh\left(x\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

3.112 $\int (a + ia\sinh(x))^{5/2}(A + B\sinh(x))\,dx$

Optimal. Leaf size=112

$$\frac{64a^3(5B+7iA)\cosh(x)}{105\sqrt{a+ia\sinh(x)}} + \frac{16}{105}a^2(5B+7iA)\cosh(x)\sqrt{a+ia\sinh(x)} + \frac{2}{35}a(5B+7iA)\cosh(x)(a+ia\sinh(x))^{3/2} + \frac{2}{7}B\,\mathrm{c}$$

```
[Out] (64*a^3*((7*I)*A + 5*B)*Cosh[x])/(105*Sqrt[a + I*a*Sinh[x]]) + (16*a^2*((7*
I)*A + 5*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/105 + (2*a*((7*I)*A + 5*B)*Cosh[
x]*(a + I*a*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(5/2))/7
```

Rubi [A] time = 0.100143, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2751, 2647, 2646}

$$\frac{64a^3(5B+7iA)\cosh(x)}{105\sqrt{a+ia\sinh(x)}} + \frac{16}{105}a^2(5B+7iA)\cosh(x)\sqrt{a+ia\sinh(x)} + \frac{2}{35}a(5B+7iA)\cosh(x)(a+ia\sinh(x))^{3/2} + \frac{2}{7}B\,\mathrm{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (64*a^3*((7*I)*A + 5*B)*Cosh[x])/(105*Sqrt[a + I*a*Sinh[x]]) + (16*a^2*((7*
I)*A + 5*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/105 + (2*a*((7*I)*A + 5*B)*Cosh[
x]*(a + I*a*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(5/2))/7
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2} + \frac{1}{7}(7A-5iB)\int (a+ia\sinh(x))^{5/2}\,dx$$
$$= \frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\sinh(x))^{3/2} + \frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2} +$$
$$= \frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)} + \frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\,\mathrm{s}$$
$$= \frac{64a^3(7iA+5B)\cosh(x)}{105\sqrt{a+ia\sinh(x)}} + \frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)} + \frac{2}{35}a$$

Mathematica [A] time = 0.348767, size = 100, normalized size = 0.89

$$\frac{a^2\sqrt{a+ia\sinh(x)}\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)((-392A+505iB)\sinh(x)+(-120B-42iA)\cosh(2x)+1246iA-15iB\sin}{210\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (a^2*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((1246*I)*A + 1040*B +
 ((-42*I)*A - 120*B)*Cosh[2*x] + (-392*A + (505*I)*B)*Sinh[x] - (15*I)*B*Si
nh[3*x]))/(210*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (a+ia\sinh(x))^{\frac{5}{2}}(A+B\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

```
[Out] int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x)+A)(ia\sinh(x)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

Fricas [A] time = 1.81776, size = 414, normalized size = 3.7

$$-\frac{\sqrt{\frac{1}{2}}\left(15\,Ba^2e^{(7\,x)}+21\,(2\,A-5i\,B)a^2e^{(6\,x)}-(350i\,A+385\,B)a^2e^{(5\,x)}-525\,(4\,A-3i\,B)a^2e^{(4\,x)}-(2100i\,A+1575\,B)a^2e^{(}\right.}{420\left(e^{(4\,x)}-i\,e^{(3\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/420*sqrt(1/2)*(15*B*a^2*e^(7*x) + 21*(2*A - 5*I*B)*a^2*e^(6*x) - (350*I*
A + 385*B)*a^2*e^(5*x) - 525*(4*A - 3*I*B)*a^2*e^(4*x) - (2100*I*A + 1575*B
)*a^2*e^(3*x) - 35*(10*A - 11*I*B)*a^2*e^(2*x) - (-42*I*A - 105*B)*a^2*e^x
- 15*I*B*a^2)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*e^(-1/2*x)/(e^(4*x) - I*e^(
3*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))**(5/2)*(A+B*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x) + A)(i\,a\sinh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

3.113 $\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=81

$$\frac{8a^2(3B+5iA)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(3B+5iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

```
[Out] (8*a^2*((5*I)*A + 3*B)*Cosh[x])/(15*Sqrt[a + I*a*Sinh[x]]) + (2*a*((5*I)*A
+ 3*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/15 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(
3/2))/5
```

Rubi [A] time = 0.0808424, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2751, 2647, 2646}

$$\frac{8a^2(3B+5iA)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(3B+5iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (8*a^2*((5*I)*A + 3*B)*Cosh[x])/(15*Sqrt[a + I*a*Sinh[x]]) + (2*a*((5*I)*A
+ 3*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/15 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(
3/2))/5
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx &= \frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{1}{5}(5A-3iB)\int (a+ia\sinh(x))^{3/2}\,dx\\ &= \frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}+\\ &= \frac{8a^2(5iA+3B)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cos\end{aligned}$$

Mathematica [A] time = 0.214721, size = 83, normalized size = 1.02

$$-\frac{a\sqrt{a+ia\sinh(x)}\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)(2(5A-9iB)\sinh(x)-50iA+3B\cosh(2x)-39B)}{15\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] -(a*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((-50*I)*A - 39*B + 3*B
*Cosh[2*x] + 2*(5*A - (9*I)*B)*Sinh[x]))/(15*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int (a+ia\sinh(x))^{\frac{3}{2}}(A+B\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

```
[Out] int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x)+A)(i\,a\sinh(x)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)
```

Fricas [A] time = 1.76496, size = 294, normalized size = 3.63

$$\frac{\sqrt{\frac{1}{2}}\left(3i\,Bae^{(5\,x)}+(10i\,A+15\,B)ae^{(4\,x)}+30\,(3\,A-2i\,B)ae^{(3\,x)}+(90i\,A+60\,B)ae^{(2\,x)}+5\,(2\,A-3i\,B)ae^{x}-3\,Ba\right)\sqrt{i\,ae^{(2\,x)}+}}{30\left(e^{(3\,x)}-i\,e^{(2\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*I*B*a*e^(5*x) + (10*I*A + 15*B)*a*e^(4*x) + 30*(3*A - 2*I
*B)*a*e^(3*x) + (90*I*A + 60*B)*a*e^(2*x) + 5*(2*A - 3*I*B)*a*e^x - 3*B*a)*
sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*e^(-1/2*x)/(e^(3*x) - I*e^(2*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))**(3/2)*(A+B*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x) + A)(i\,a\sinh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)
```

3.114 $\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx$

Optimal. Leaf size=48

$$\frac{2a(B+3iA)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

```
[Out] (2*a*((3*I)*A + B)*Cosh[x])/(3*Sqrt[a + I*a*Sinh[x]]) + (2*B*Cosh[x]*Sqrt[a
 + I*a*Sinh[x]])/3
```

Rubi [A] time = 0.0538788, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2751, 2646}

$$\frac{2a(B+3iA)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*a*((3*I)*A + B)*Cosh[x])/(3*Sqrt[a + I*a*Sinh[x]]) + (2*B*Cosh[x]*Sqrt[a
 + I*a*Sinh[x]])/3
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx &= \frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{1}{3}(3A-iB)\int \sqrt{a+ia\sinh(x)}\,dx\\ &= \frac{2a(3iA+B)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0739076, size = 66, normalized size = 1.38

$$\frac{2\sqrt{a+ia\sinh(x)}\left(\sinh\left(\frac{x}{2}\right)+i\cosh\left(\frac{x}{2}\right)\right)(3A+B\sinh(x)-2iB)}{3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*(I*Cosh[x/2] + Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*(3*A - (2*I)*B + B*Sinh[
x]))/(3*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int \sqrt{a + ia\sinh(x)}\,(A + B\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

```
[Out] int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x) + A)\sqrt{i\,a\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

Fricas [B] time = 1.72814, size = 189, normalized size = 3.94

$$\frac{\sqrt{\frac{1}{2}}\left(Be^{(3\,x)} + 3\,(2\,A - i\,B)e^{(2\,x)} + (6i\,A + 3\,B)e^{x} - i\,B\right)\sqrt{i\,ae^{(2\,x)} + 2\,ae^{x} - i\,a}\,e^{\left(-\frac{1}{2}\,x\right)}}{3\left(e^{(2\,x)} - i\,e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(B*e^(3*x) + 3*(2*A - I*B)*e^(2*x) + (6*I*A + 3*B)*e^x - I*B)
*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*e^(-1/2*x)/(e^(2*x) - I*e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,(i\sinh(x) + 1)}\,(A + B\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))**(1/2)*(A+B*sinh(x)),x)
```

```
[Out] Integral(sqrt(a*(I*sinh(x) + 1))*(A + B*sinh(x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x)+A)\sqrt{i\,a\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

3.115 $\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$Bx-\frac{(B+iA)\cosh(x)}{\sinh(x)+i}$$

```
[Out] B*x - ((I*A + B)*Cosh[x])/(I + Sinh[x])
```

Rubi [A] time = 0.037834, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2735, 2648}

$$Bx-\frac{(B+iA)\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x]),x]
```

```
[Out] B*x - ((I*A + B)*Cosh[x])/(I + Sinh[x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx &= Bx-(-A+iB)\int \frac{1}{i+\sinh(x)}\,dx\\ &= Bx-\frac{(iA+B)\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] time = 0.23284, size = 53, normalized size = 2.3

$$\cosh(x)\left(\frac{2iB\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}}-\frac{B+iA}{\sinh(x)+i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x]),x]
```

```
[Out] Cosh[x]*(((2*I)*B*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2] - (I
*A + B)/(I + Sinh[x]))
```

Maple [B] time = 0.03, size = 46, normalized size = 2.

$$B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\frac{B}{\tanh\left(x/2\right)+i}-2iA\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x)),x)
```

```
[Out] B*ln(tanh(1/2*x)+1)-2/(tanh(1/2*x)+I)*B-2*I/(tanh(1/2*x)+I)*A-B*ln(tanh(1/2
*x)-1)
```

Maxima [A] time = 1.26283, size = 35, normalized size = 1.52

$$B\left(x+\frac{2i}{e^{(-x)}-i}\right)-\frac{2A}{e^{(-x)}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] B*(x + 2*I/(e^(-x) - I)) - 2*A/(e^(-x) - I)
```

Fricas [A] time = 1.8033, size = 58, normalized size = 2.52

$$\frac{Bxe^{x}+iBx-2A+2iB}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*e^x + I*B*x - 2*A + 2*I*B)/(e^x + I)
```

Sympy [A] time = 0.217398, size = 15, normalized size = 0.65

$$Bx+\frac{-2A+2iB}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x)
```

```
[Out] B*x + (-2*A + 2*I*B)/(exp(x) + I)
```

Giac [A] time = 1.3178, size = 23, normalized size = 1.

$$Bx-\frac{2(A-iB)}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] B*x - 2*(A - I*B)/(e^x + I)
```

3.116 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=43

$$-\frac{(A+2iB)\cosh(x)}{3(\sinh(x)+i)}-\frac{(B+iA)\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[Out] -((I*A + B)*Cosh[x])/(3*(I + Sinh[x])^2) - ((A + (2*I)*B)*Cosh[x])/(3*(I +
Sinh[x]))
```

Rubi [A] time = 0.0422746, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2750, 2648}

$$-\frac{(A+2iB)\cosh(x)}{3(\sinh(x)+i)}-\frac{(B+iA)\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
```

```
[Out] -((I*A + B)*Cosh[x])/(3*(I + Sinh[x])^2) - ((A + (2*I)*B)*Cosh[x])/(3*(I +
Sinh[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx &= -\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}+\frac{1}{3}(-iA+2B)\int \frac{1}{i+\sinh(x)}\,dx\\ &= -\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}-\frac{(A+2iB)\cosh(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.030382, size = 32, normalized size = 0.74

$$\frac{\cosh(x)(-(A+2iB)\sinh(x)-2iA+B)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*((-2*I)*A + B - (A + (2*I)*B)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

Maple [A] time = 0.024, size = 52, normalized size = 1.2

$$-2\,\frac{A}{\tanh(x/2)+i}-(-2\,iA-2\,B)\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}-\frac{4\,iB-4\,A}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x))^2,x)
```

```
[Out] -2*A/(tanh(1/2*x)+I)-(-2*I*A-2*B)/(tanh(1/2*x)+I)^2-2/3*(2*I*B-2*A)/(tanh(1
/2*x)+I)^3
```

Maxima [B] time = 1.16712, size = 190, normalized size = 4.42

$$-2\,A\left(\frac{3\,e^{(-x)}}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}-\frac{i}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}\right)+\frac{1}{2}\,B\left(-\frac{12i\,e^{(-x)}}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}+\frac{}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*A*(3*e^(-x)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I) - I/(9*e^(-x) +
 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I)) + 1/2*B*(-12*I*e^(-x)/(9*e^(-x) + 9*I*e^
(-2*x) - 3*e^(-3*x) - 3*I) + 12*e^(-2*x)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3
*x) - 3*I) - 8/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I))
```

Fricas [A] time = 1.70625, size = 119, normalized size = 2.77

$$-\frac{6\,Be^{(2\,x)}+6\,(A+i\,B)e^{x}+2i\,A-4\,B}{3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(6*B*e^(2*x) + 6*(A + I*B)*e^x + 2*I*A - 4*B)/(3*e^(3*x) + 9*I*e^(2*x) - 9
*e^x - 3*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26339, size = 43, normalized size = 1.

$$-\frac{6\,Be^{(2\,x)}+6\,Ae^{x}+6i\,Be^{x}+2i\,A-4\,B}{3\,(e^{x}+i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*B*e^(2*x) + 6*A*e^x + 6*I*B*e^x + 2*I*A - 4*B)/(e^x + I)^3
```

3.117 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3} \, dx$

Optimal. Leaf size=68

$$\frac{(-3B+2iA)\cosh(x)}{15(\sinh(x)+i)}-\frac{(2A+3iB)\cosh(x)}{15(\sinh(x)+i)^2}-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

```
[Out] -((I*A + B)*Cosh[x])/(5*(I + Sinh[x])^3) - ((2*A + (3*I)*B)*Cosh[x])/(15*(I
 + Sinh[x])^2) + (((2*I)*A - 3*B)*Cosh[x])/(15*(I + Sinh[x]))
```

Rubi [A] time = 0.0535386, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2750, 2650, 2648}

$$\frac{(-3B+2iA)\cosh(x)}{15(\sinh(x)+i)}-\frac{(2A+3iB)\cosh(x)}{15(\sinh(x)+i)^2}-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
```


```
[Out] -((I*A + B)*Cosh[x])/(5*(I + Sinh[x])^3) - ((2*A + (3*I)*B)*Cosh[x])/(15*(I
 + Sinh[x])^2) + (((2*I)*A - 3*B)*Cosh[x])/(15*(I + Sinh[x]))
```


Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx &= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}+\frac{1}{5}(-2iA+3B)\int \frac{1}{(i+\sinh(x))^2}\,dx\\
&= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}-\frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2}+\frac{1}{15}(-2A-3iB)\int \frac{1}{i+\sinh(x)}\,dx\\
&= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}-\frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2}+\frac{(2iA-3B)\cosh(x)}{15(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.0374888, size = 50, normalized size = 0.74

$$\frac{\cosh(x)\left((-3B+2iA)\sinh^2(x)-3(2A+3iB)\sinh(x)-7iA+3B\right)}{15(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
```

```
[Out] (Cosh[x]*((-7*I)*A + 3*B - 3*(2*A + (3*I)*B)*Sinh[x] + ((2*I)*A - 3*B)*Sinh
[x]^2))/(15*(I + Sinh[x])^3)
```

Maple [A] time = 0.029, size = 91, normalized size = 1.3

$$-\frac{8A-8iB}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4}+2iA\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}-\frac{-8iA-8B}{5}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-5}-\frac{16iA+12B}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x))^3,x)
```

```
[Out] -1/2*(8*A-8*I*B)/(tanh(1/2*x)+I)^4+2*I*A/(tanh(1/2*x)+I)-2/5*(-4*I*A-4*B)/(
tanh(1/2*x)+I)^5-2/3*(8*I*A+6*B)/(tanh(1/2*x)+I)^3-(-4*A+2*I*B)/(tanh(1/2*x
)+I)^2
```

Maxima [B] time = 1.27767, size = 378, normalized size = 5.56

$$A\left(\frac{20i\,e^{(-x)}}{75\,e^{(-x)}+150i\,e^{(-2x)}-150\,e^{(-3x)}-75i\,e^{(-4x)}+15\,e^{(-5x)}-15i}-\frac{40\,e^{(-2x)}}{75\,e^{(-x)}+150i\,e^{(-2x)}-150\,e^{(-3x)}-75i\,e^{(-4x)}+15\,e^{(-5x)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="maxima")
```

```
[Out] A*(20*I*e^(-x)/(75*e^(-x) + 150*I*e^(-2*x) - 150*e^(-3*x) - 75*I*e^(-4*x) +
 15*e^(-5*x) - 15*I) - 40*e^(-2*x)/(75*e^(-x) + 150*I*e^(-2*x) - 150*e^(-3*
x) - 75*I*e^(-4*x) + 15*e^(-5*x) - 15*I) + 4/(75*e^(-x) + 150*I*e^(-2*x) -
150*e^(-3*x) - 75*I*e^(-4*x) + 15*e^(-5*x) - 15*I)) - 1/2*B*(20*e^(-x)/(25*
e^(-x) + 50*I*e^(-2*x) - 50*e^(-3*x) - 25*I*e^(-4*x) + 5*e^(-5*x) - 5*I) +
20*I*e^(-2*x)/(25*e^(-x) + 50*I*e^(-2*x) - 50*e^(-3*x) - 25*I*e^(-4*x) + 5*
e^(-5*x) - 5*I) - 20*e^(-3*x)/(25*e^(-x) + 50*I*e^(-2*x) - 50*e^(-3*x) - 25
*I*e^(-4*x) + 5*e^(-5*x) - 5*I) - 4*I/(25*e^(-x) + 50*I*e^(-2*x) - 50*e^(-3
*x) - 25*I*e^(-4*x) + 5*e^(-5*x) - 5*I))
```

Fricas [A] time = 1.62266, size = 209, normalized size = 3.07

$$-\frac{30\,Be^{(3x)}+10\,(4\,A+3i\,B)e^{(2x)}-(-20i\,A+30\,B)e^{x}-4\,A-6i\,B}{15\,e^{(5x)}+75i\,e^{(4x)}-150\,e^{(3x)}-150i\,e^{(2x)}+75\,e^{x}+15i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -(30*B*e^(3*x) + 10*(4*A + 3*I*B)*e^(2*x) - (-20*I*A + 30*B)*e^x - 4*A - 6*
I*B)/(15*e^(5*x) + 75*I*e^(4*x) - 150*e^(3*x) - 150*I*e^(2*x) + 75*e^x + 15
*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20797, size = 62, normalized size = 0.91

$$-\frac{30\,Be^{(3\,x)} + 40\,Ae^{(2\,x)} + 30i\,Be^{(2\,x)} + 20i\,Ae^{x} - 30\,Be^{x} - 4\,A - 6i\,B}{15\,(e^{x} + i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="giac")
```

```
[Out] -1/15*(30*B*e^(3*x) + 40*A*e^(2*x) + 30*I*B*e^(2*x) + 20*I*A*e^x - 30*B*e^x
 - 4*A - 6*I*B)/(e^x + I)^5
```

3.118 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx$

Optimal. Leaf size=91

$$\frac{2(3A+4iB)\cosh(x)}{105(\sinh(x)+i)}+\frac{2(-4B+3iA)\cosh(x)}{105(\sinh(x)+i)^2}-\frac{(3A+4iB)\cosh(x)}{35(\sinh(x)+i)^3}-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

```
[Out] -((I*A + B)*Cosh[x])/(7*(I + Sinh[x])^4) - ((3*A + (4*I)*B)*Cosh[x])/(35*(I
 + Sinh[x])^3) + (2*((3*I)*A - 4*B)*Cosh[x])/(105*(I + Sinh[x])^2) + (2*(3*
A + (4*I)*B)*Cosh[x])/(105*(I + Sinh[x]))
```

Rubi [A] time = 0.0676414, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2750, 2650, 2648}

$$\frac{2(3A+4iB)\cosh(x)}{105(\sinh(x)+i)}+\frac{2(-4B+3iA)\cosh(x)}{105(\sinh(x)+i)^2}-\frac{(3A+4iB)\cosh(x)}{35(\sinh(x)+i)^3}-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
```

```
[Out] -((I*A + B)*Cosh[x])/(7*(I + Sinh[x])^4) - ((3*A + (4*I)*B)*Cosh[x])/(35*(I
 + Sinh[x])^3) + (2*((3*I)*A - 4*B)*Cosh[x])/(105*(I + Sinh[x])^2) + (2*(3*
A + (4*I)*B)*Cosh[x])/(105*(I + Sinh[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx &= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}+\frac{1}{7}(-3iA+4B)\int\frac{1}{(i+\sinh(x))^3}\,dx\\ &= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}-\frac{1}{35}(2(3A+4iB))\int\frac{1}{(i+\sinh(x))^2}\,dx\\ &= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}+\frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2}+\frac{1}{105}(2(3iA-4B))\int\frac{}{i+\text{si}}\\ &= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}+\frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2}+\frac{2(3A+4iB)\cosh(x)}{105(i+\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.0490861, size = 67, normalized size = 0.74

$$\frac{\cosh(x)\left((6A+8iB)\sinh^3(x)+8i(3A+4iB)\sinh^2(x)-13(3A+4iB)\sinh(x)-36iA+13B\right)}{105(\sinh(x)+i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
```

```
[Out] (Cosh[x]*((-36*I)*A + 13*B - 13*(3*A + (4*I)*B)*Sinh[x] + (8*I)*(3*A + (4*I
)*B)*Sinh[x]^2 + (6*A + (8*I)*B)*Sinh[x]^3))/(105*(I + Sinh[x])^4)
```

Maple [A] time = 0.03, size = 128, normalized size = 1.4

$$-(6\,iA+2\,B)\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}-\frac{16\,A-16\,iB}{7}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-7}+2\,\frac{A}{\tanh(x/2)+i}-\frac{-32\,iA-24\,B}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x))^4,x)
```

```
[Out] -(6*I*A+2*B)/(tanh(1/2*x)+I)^2-2/7*(8*A-8*I*B)/(tanh(1/2*x)+I)^7+2*A/(tanh(
1/2*x)+I)-1/2*(-32*I*A-24*B)/(tanh(1/2*x)+I)^4-2/5*(-36*A+32*I*B)/(tanh(1/2
*x)+I)^5-2/3*(18*A-10*I*B)/(tanh(1/2*x)+I)^3-1/3*(24*I*A+24*B)/(tanh(1/2*x)
+I)^6
```

Maxima [B] time = 1.30924, size = 632, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 1/2*B*(224*I*e^(-x)/(735*e^(-x) + 2205*I*e^(-2*x) - 3675*e^(-3*x) - 3675*I*
e^(-4*x) + 2205*e^(-5*x) + 735*I*e^(-6*x) - 105*e^(-7*x) - 105*I) - 672*e^(
-2*x)/(735*e^(-x) + 2205*I*e^(-2*x) - 3675*e^(-3*x) - 3675*I*e^(-4*x) + 220
5*e^(-5*x) + 735*I*e^(-6*x) - 105*e^(-7*x) - 105*I) - 560*I*e^(-3*x)/(735*e
^(-x) + 2205*I*e^(-2*x) - 3675*e^(-3*x) - 3675*I*e^(-4*x) + 2205*e^(-5*x) +
 735*I*e^(-6*x) - 105*e^(-7*x) - 105*I) + 560*e^(-4*x)/(735*e^(-x) + 2205*I
*e^(-2*x) - 3675*e^(-3*x) - 3675*I*e^(-4*x) + 2205*e^(-5*x) + 735*I*e^(-6*x
) - 105*e^(-7*x) - 105*I) + 32/(735*e^(-x) + 2205*I*e^(-2*x) - 3675*e^(-3*x
```

```
) - 3675*I*e^(-4*x) + 2205*e^(-5*x) + 735*I*e^(-6*x) - 105*e^(-7*x) - 105*I
)) + A*(28*e^(-x)/(245*e^(-x) + 735*I*e^(-2*x) - 1225*e^(-3*x) - 1225*I*e^(
-4*x) + 735*e^(-5*x) + 245*I*e^(-6*x) - 35*e^(-7*x) - 35*I) + 84*I*e^(-2*x)
/(245*e^(-x) + 735*I*e^(-2*x) - 1225*e^(-3*x) - 1225*I*e^(-4*x) + 735*e^(-5
*x) + 245*I*e^(-6*x) - 35*e^(-7*x) - 35*I) - 140*e^(-3*x)/(245*e^(-x) + 735
*I*e^(-2*x) - 1225*e^(-3*x) - 1225*I*e^(-4*x) + 735*e^(-5*x) + 245*I*e^(-6*
x) - 35*e^(-7*x) - 35*I) - 4*I/(245*e^(-x) + 735*I*e^(-2*x) - 1225*e^(-3*x)
 - 1225*I*e^(-4*x) + 735*e^(-5*x) + 245*I*e^(-6*x) - 35*e^(-7*x) - 35*I))
```

Fricas [A] time = 1.83645, size = 305, normalized size = 3.35

$$-\frac{280\,Be^{(4\,x)}+140\,(3\,A+2i\,B)e^{(3\,x)}-(-252i\,A+336\,B)e^{(2\,x)}-28\,(3\,A+4i\,B)e^{x}-12i\,A+16\,B}{105\,e^{(7\,x)}+735i\,e^{(6\,x)}-2205\,e^{(5\,x)}-3675i\,e^{(4\,x)}+3675\,e^{(3\,x)}+2205i\,e^{(2\,x)}-735\,e^{x}-105i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -(280*B*e^(4*x) + 140*(3*A + 2*I*B)*e^(3*x) - (-252*I*A + 336*B)*e^(2*x) -
28*(3*A + 4*I*B)*e^x - 12*I*A + 16*B)/(105*e^(7*x) + 735*I*e^(6*x) - 2205*e
^(5*x) - 3675*I*e^(4*x) + 3675*e^(3*x) + 2205*I*e^(2*x) - 735*e^x - 105*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14118, size = 81, normalized size = 0.89

$$-\frac{280\,Be^{(4\,x)}+420\,Ae^{(3\,x)}+280i\,Be^{(3\,x)}+252i\,Ae^{(2\,x)}-336\,Be^{(2\,x)}-84\,Ae^{x}-112i\,Be^{x}-12i\,A+16\,B}{105\,(e^{x}+i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="giac")
```

```
[Out] -1/105*(280*B*e^(4*x) + 420*A*e^(3*x) + 280*I*B*e^(3*x) + 252*I*A*e^(2*x) -
 336*B*e^(2*x) - 84*A*e^x - 112*I*B*e^x - 12*I*A + 16*B)/(e^x + I)^7
```

3.119 $\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx$

Optimal. Leaf size=27

$$-Bx+\frac{(-B+iA)\cosh(x)}{-\sinh(x)+i}$$

```
[Out] -(B*x) + ((I*A - B)*Cosh[x])/(I - Sinh[x])
```

Rubi [A] time = 0.042611, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2735, 2648}

$$-Bx+\frac{(-B+iA)\cosh(x)}{-\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x]),x]
```

```
[Out] -(B*x) + ((I*A - B)*Cosh[x])/(I - Sinh[x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx &= -Bx+(A+iB)\int \frac{1}{i-\sinh(x)}\,dx\\ &= -Bx+\frac{(iA-B)\cosh(x)}{i-\sinh(x)}\end{aligned}$$

Mathematica [B] time = 0.0753079, size = 59, normalized size = 2.19

$$\frac{\left(-\sinh\left(\frac{x}{2}\right)+i\cosh\left(\frac{x}{2}\right)\right)\left(Bx\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)(2A+B(x+2i))\right)}{\sinh(x)-i}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x]),x]
```

```
[Out] ((I*Cosh[x/2] - Sinh[x/2])*(B*x*Cosh[x/2] + I*(2*A + B*(2*I + x))*Sinh[x/2]
))/(-I + Sinh[x])
```

Maple [A] time = 0.03, size = 46, normalized size = 1.7

$$-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\frac{B}{\tanh(x/2)-i}-2iA\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x)),x)
```

```
[Out] -B*ln(tanh(1/2*x)+1)+B*ln(tanh(1/2*x)-1)+2/(tanh(1/2*x)-I)*B-2*I/(tanh(1/2*
x)-I)*A
```

Maxima [A] time = 1.19698, size = 36, normalized size = 1.33

$$-B\left(x-\frac{2i}{e^{(-x)}+i}\right)+\frac{2A}{e^{(-x)}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(x - 2*I/(e^(-x) + I)) + 2*A/(e^(-x) + I)
```

Fricas [A] time = 1.7196, size = 59, normalized size = 2.19

$$-\frac{Bxe^{x}-iBx-2A-2iB}{e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*e^x - I*B*x - 2*A - 2*I*B)/(e^x - I)
```

Sympy [A] time = 0.209746, size = 15, normalized size = 0.56

$$-Bx+\frac{2A+2iB}{e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x)
```

```
[Out] -B*x + (2*A + 2*I*B)/(exp(x) - I)
```

Giac [A] time = 1.29128, size = 24, normalized size = 0.89

$$-Bx+\frac{2(A+iB)}{e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*(A + I*B)/(e^x - I)
```

3.120 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx$

Optimal. Leaf size=49

$$\frac{(A-2iB)\cosh(x)}{3(-\sinh(x)+i)}+\frac{(-B+iA)\cosh(x)}{3(-\sinh(x)+i)^2}$$

```
[Out] ((I*A - B)*Cosh[x])/(3*(I - Sinh[x])^2) + ((A - (2*I)*B)*Cosh[x])/(3*(I - S
inh[x]))
```

Rubi [A] time = 0.0435185, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2750, 2648}

$$\frac{(A-2iB)\cosh(x)}{3(-\sinh(x)+i)}+\frac{(-B+iA)\cosh(x)}{3(-\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(3*(I - Sinh[x])^2) + ((A - (2*I)*B)*Cosh[x])/(3*(I - S
inh[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx &= \frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{1}{3}(-iA-2B)\int \frac{1}{i-\sinh(x)}\,dx\\ &= \frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{(A-2iB)\cosh(x)}{3(i-\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.0279167, size = 32, normalized size = 0.65

$$\frac{\cosh(x)(-(A-2iB)\sinh(x)+2iA+B)}{3(\sinh(x)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*((2*I)*A + B - (A - (2*I)*B)*Sinh[x]))/(3*(-I + Sinh[x])^2)
```

Maple [A] time = 0.024, size = 52, normalized size = 1.1

$$-(2\,iA-2\,B)\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}-2\,\frac{A}{\tanh\left(x/2\right)-i}-\frac{-4\,iB-4\,A}{3}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x))^2,x)
```

```
[Out] -(2*I*A-2*B)/(tanh(1/2*x)-I)^2-2*A/(tanh(1/2*x)-I)-2/3*(-2*I*B-2*A)/(tanh(1
/2*x)-I)^3
```

Maxima [B] time = 1.30827, size = 190, normalized size = 3.88

$$-A\left(\frac{6\,e^{(-x)}}{9\,e^{(-x)}-9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}+3i}+\frac{2i}{9\,e^{(-x)}-9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}+3i}\right)+\frac{1}{2}\,B\left(\frac{12i\,e^{(-x)}}{9\,e^{(-x)}-9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}+3i}+\frac{}{9\,e^{(-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -A*(6*e^(-x)/(9*e^(-x) - 9*I*e^(-2*x) - 3*e^(-3*x) + 3*I) + 2*I/(9*e^(-x) -
 9*I*e^(-2*x) - 3*e^(-3*x) + 3*I)) + 1/2*B*(12*I*e^(-x)/(9*e^(-x) - 9*I*e^(
-2*x) - 3*e^(-3*x) + 3*I) + 12*e^(-2*x)/(9*e^(-x) - 9*I*e^(-2*x) - 3*e^(-3*
x) + 3*I) - 8/(9*e^(-x) - 9*I*e^(-2*x) - 3*e^(-3*x) + 3*I))
```

Fricas [A] time = 1.8014, size = 119, normalized size = 2.43

$$-\frac{6\,Be^{(2\,x)}+6\,(A-i\,B)e^{x}-2i\,A-4\,B}{3\,e^{(3\,x)}-9i\,e^{(2\,x)}-9\,e^{x}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(6*B*e^(2*x) + 6*(A - I*B)*e^x - 2*I*A - 4*B)/(3*e^(3*x) - 9*I*e^(2*x) - 9
*e^x + 3*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29165, size = 43, normalized size = 0.88

$$-\frac{6\,Be^{(2\,x)} + 6\,Ae^{x} - 6i\,Be^{x} - 2i\,A - 4\,B}{3\,(e^{x} - i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*B*e^(2*x) + 6*A*e^x - 6*I*B*e^x - 2*I*A - 4*B)/(e^x - I)^3
```

3.121 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx$

Optimal. Leaf size=76

$$-\frac{(3B+2iA)\cosh(x)}{15(-\sinh(x)+i)}+\frac{(2A-3iB)\cosh(x)}{15(-\sinh(x)+i)^2}+\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}$$

```
[Out] ((I*A - B)*Cosh[x])/(5*(I - Sinh[x])^3) + ((2*A - (3*I)*B)*Cosh[x])/(15*(I
- Sinh[x])^2) - (((2*I)*A + 3*B)*Cosh[x])/(15*(I - Sinh[x]))
```

Rubi [A] time = 0.0585895, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2750, 2650, 2648}

$$-\frac{(3B+2iA)\cosh(x)}{15(-\sinh(x)+i)}+\frac{(2A-3iB)\cosh(x)}{15(-\sinh(x)+i)^2}+\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(5*(I - Sinh[x])^3) + ((2*A - (3*I)*B)*Cosh[x])/(15*(I
- Sinh[x])^2) - (((2*I)*A + 3*B)*Cosh[x])/(15*(I - Sinh[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx &= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{1}{5}(-2iA-3B)\int \frac{1}{(i-\sinh(x))^2}\,dx\\
&= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2}+\frac{1}{15}(-2A+3iB)\int \frac{1}{i-\sinh(x)}\,dx\\
&= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2}-\frac{(2iA+3B)\cosh(x)}{15(i-\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.215178, size = 92, normalized size = 1.21

$$-\frac{5(2A-3iB)\cosh\left(\frac{3x}{2}\right)-20iA\sinh\left(\frac{x}{2}\right)+2iA\sinh\left(\frac{5x}{2}\right)-15B\sinh\left(\frac{x}{2}\right)+3B\sinh\left(\frac{5x}{2}\right)+15iB\cosh\left(\frac{x}{2}\right)}{30\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
```

```
[Out] -((15*I)*B*Cosh[x/2] + 5*(2*A - (3*I)*B)*Cosh[(3*x)/2] - (20*I)*A*Sinh[x/2]
 - 15*B*Sinh[x/2] + (2*I)*A*Sinh[(5*x)/2] + 3*B*Sinh[(5*x)/2])/(30*(Cosh[x/
2] + I*Sinh[x/2])^5)
```

Maple [A] time = 0.035, size = 91, normalized size = 1.2

$$-(4A+2iB)\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+2iA\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}-\frac{-8iA+8B}{5}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-5}-\frac{-8A-8iB}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x))^3,x)
```

```
[Out] -(4*A+2*I*B)/(tanh(1/2*x)-I)^2+2*I*A/(tanh(1/2*x)-I)-2/5*(-4*I*A+4*B)/(tanh
(1/2*x)-I)^5-1/2*(-8*A-8*I*B)/(tanh(1/2*x)-I)^4-2/3*(8*I*A-6*B)/(tanh(1/2*x
)-I)^3
```

Maxima [B] time = 1.26862, size = 379, normalized size = 4.99

$$-A\left(-\frac{20ie^{(-x)}}{75e^{(-x)}-150ie^{(-2x)}-150e^{(-3x)}+75ie^{(-4x)}+15e^{(-5x)}+15i}-\frac{40e^{(-2x)}}{75e^{(-x)}-150ie^{(-2x)}-150e^{(-3x)}+75ie^{(-4x)}+15e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -A*(-20*I*e^(-x)/(75*e^(-x) - 150*I*e^(-2*x) - 150*e^(-3*x) + 75*I*e^(-4*x)
 + 15*e^(-5*x) + 15*I) - 40*e^(-2*x)/(75*e^(-x) - 150*I*e^(-2*x) - 150*e^(-
3*x) + 75*I*e^(-4*x) + 15*e^(-5*x) + 15*I) + 4/(75*e^(-x) - 150*I*e^(-2*x)
- 150*e^(-3*x) + 75*I*e^(-4*x) + 15*e^(-5*x) + 15*I)) + 1/2*B*(20*e^(-x)/(2
5*e^(-x) - 50*I*e^(-2*x) - 50*e^(-3*x) + 25*I*e^(-4*x) + 5*e^(-5*x) + 5*I)
- 20*I*e^(-2*x)/(25*e^(-x) - 50*I*e^(-2*x) - 50*e^(-3*x) + 25*I*e^(-4*x) +
5*e^(-5*x) + 5*I) - 20*e^(-3*x)/(25*e^(-x) - 50*I*e^(-2*x) - 50*e^(-3*x) +
25*I*e^(-4*x) + 5*e^(-5*x) + 5*I) + 4*I/(25*e^(-x) - 50*I*e^(-2*x) - 50*e^(
-3*x) + 25*I*e^(-4*x) + 5*e^(-5*x) + 5*I))
```

Fricas [A] time = 1.73298, size = 208, normalized size = 2.74

$$\frac{30Be^{(3x)}+10(4A-3iB)e^{(2x)}+(-20iA-30B)e^{x}-4A+6iB}{15e^{(5x)}-75ie^{(4x)}-150e^{(3x)}+150ie^{(2x)}+75e^{x}-15i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (30*B*e^(3*x) + 10*(4*A - 3*I*B)*e^(2*x) + (-20*I*A - 30*B)*e^x - 4*A + 6*I
*B)/(15*e^(5*x) - 75*I*e^(4*x) - 150*e^(3*x) + 150*I*e^(2*x) + 75*e^x - 15*
I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3382, size = 62, normalized size = 0.82

$$\frac{30\,Be^{(3\,x)} + 40\,Ae^{(2\,x)} - 30i\,Be^{(2\,x)} - 20i\,Ae^{x} - 30\,Be^{x} - 4\,A + 6i\,B}{15\,(e^{x} - i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/15*(30*B*e^(3*x) + 40*A*e^(2*x) - 30*I*B*e^(2*x) - 20*I*A*e^x - 30*B*e^x
- 4*A + 6*I*B)/(e^x - I)^5
```

3.122 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx$

Optimal. Leaf size=101

$$-\frac{2(3A-4iB)\cosh(x)}{105(-\sinh(x)+i)}-\frac{2(4B+3iA)\cosh(x)}{105(-\sinh(x)+i)^2}+\frac{(3A-4iB)\cosh(x)}{35(-\sinh(x)+i)^3}+\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}$$

```
[Out] ((I*A - B)*Cosh[x])/(7*(I - Sinh[x])^4) + ((3*A - (4*I)*B)*Cosh[x])/(35*(I
- Sinh[x])^3) - (2*((3*I)*A + 4*B)*Cosh[x])/(105*(I - Sinh[x])^2) - (2*(3*A
 - (4*I)*B)*Cosh[x])/(105*(I - Sinh[x]))
```

Rubi [A] time = 0.0699008, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2750, 2650, 2648}

$$-\frac{2(3A-4iB)\cosh(x)}{105(-\sinh(x)+i)}-\frac{2(4B+3iA)\cosh(x)}{105(-\sinh(x)+i)^2}+\frac{(3A-4iB)\cosh(x)}{35(-\sinh(x)+i)^3}+\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(7*(I - Sinh[x])^4) + ((3*A - (4*I)*B)*Cosh[x])/(35*(I
- Sinh[x])^3) - (2*((3*I)*A + 4*B)*Cosh[x])/(105*(I - Sinh[x])^2) - (2*(3*A
 - (4*I)*B)*Cosh[x])/(105*(I - Sinh[x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}dx &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{1}{7}(-3iA-4B)\int \frac{1}{(i-\sinh(x))^3}dx\\ &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{1}{35}(2(3A-4iB))\int \frac{1}{(i-\sinh(x))^2}dx\\ &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}+\frac{1}{105}(2(3iA+4B))\int \frac{1}{i-\sin}\\ &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}-\frac{2(3A-4iB)\cosh(x)}{105(i-\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.0487166, size = 63, normalized size = 0.62

$$\frac{\cosh(x)\left((6A-8iB)\sinh^3(x)+(-32B-24iA)\sinh^2(x)+(-39A+52iB)\sinh(x)+36iA+13B\right)}{105(\sinh(x)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
```

```
[Out] (Cosh[x]*((36*I)*A + 13*B + (-39*A + (52*I)*B)*Sinh[x] + ((-24*I)*A - 32*B)
*Sinh[x]^2 + (6*A - (8*I)*B)*Sinh[x]^3))/(105*(-I + Sinh[x])^4)
```

Maple [A] time = 0.033, size = 128, normalized size = 1.3

$$-\frac{32\,iA-24\,B}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-4}-\frac{16\,A+16\,iB}{7}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-7}+2\,\frac{A}{\tanh(x/2)-i}-(-6\,iA+2\,B)\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x))^4,x)
```

```
[Out] -1/2*(32*I*A-24*B)/(tanh(1/2*x)-I)^4-2/7*(8*A+8*I*B)/(tanh(1/2*x)-I)^7+2*A/
(tanh(1/2*x)-I)-(-6*I*A+2*B)/(tanh(1/2*x)-I)^2-2/5*(-36*A-32*I*B)/(tanh(1/2
*x)-I)^5-1/3*(-24*I*A+24*B)/(tanh(1/2*x)-I)^6-2/3*(18*A+10*I*B)/(tanh(1/2*x
)-I)^3
```

Maxima [B] time = 1.50314, size = 632, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 1/2*B*(-224*I*e^(-x)/(735*e^(-x) - 2205*I*e^(-2*x) - 3675*e^(-3*x) + 3675*I
*e^(-4*x) + 2205*e^(-5*x) - 735*I*e^(-6*x) - 105*e^(-7*x) + 105*I) - 672*e^
(-2*x)/(735*e^(-x) - 2205*I*e^(-2*x) - 3675*e^(-3*x) + 3675*I*e^(-4*x) + 22
05*e^(-5*x) - 735*I*e^(-6*x) - 105*e^(-7*x) + 105*I) + 560*I*e^(-3*x)/(735*
e^(-x) - 2205*I*e^(-2*x) - 3675*e^(-3*x) + 3675*I*e^(-4*x) + 2205*e^(-5*x)
- 735*I*e^(-6*x) - 105*e^(-7*x) + 105*I) + 560*e^(-4*x)/(735*e^(-x) - 2205*
I*e^(-2*x) - 3675*e^(-3*x) + 3675*I*e^(-4*x) + 2205*e^(-5*x) - 735*I*e^(-6*
x) - 105*e^(-7*x) + 105*I) + 32/(735*e^(-x) - 2205*I*e^(-2*x) - 3675*e^(-3*
```

```
x) + 3675*I*e^(-4*x) + 2205*e^(-5*x) - 735*I*e^(-6*x) - 105*e^(-7*x) + 105*
I)) + A*(28*e^(-x)/(245*e^(-x) - 735*I*e^(-2*x) - 1225*e^(-3*x) + 1225*I*e^
(-4*x) + 735*e^(-5*x) - 245*I*e^(-6*x) - 35*e^(-7*x) + 35*I) - 84*I*e^(-2*x
)/(245*e^(-x) - 735*I*e^(-2*x) - 1225*e^(-3*x) + 1225*I*e^(-4*x) + 735*e^(-
5*x) - 245*I*e^(-6*x) - 35*e^(-7*x) + 35*I) - 140*e^(-3*x)/(245*e^(-x) - 73
5*I*e^(-2*x) - 1225*e^(-3*x) + 1225*I*e^(-4*x) + 735*e^(-5*x) - 245*I*e^(-6
*x) - 35*e^(-7*x) + 35*I) + 4*I/(245*e^(-x) - 735*I*e^(-2*x) - 1225*e^(-3*x
) + 1225*I*e^(-4*x) + 735*e^(-5*x) - 245*I*e^(-6*x) - 35*e^(-7*x) + 35*I))
```

Fricas [A] time = 1.72348, size = 304, normalized size = 3.01

$$-\frac{280\,Be^{(4\,x)} + 140\,(3\,A - 2i\,B)e^{(3\,x)} - (252i\,A + 336\,B)e^{(2\,x)} - 28\,(3\,A - 4i\,B)e^{x} + 12i\,A + 16\,B}{105\,e^{(7\,x)} - 735i\,e^{(6\,x)} - 2205\,e^{(5\,x)} + 3675i\,e^{(4\,x)} + 3675\,e^{(3\,x)} - 2205i\,e^{(2\,x)} - 735\,e^{x} + 105i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -(280*B*e^(4*x) + 140*(3*A - 2*I*B)*e^(3*x) - (252*I*A + 336*B)*e^(2*x) - 2
8*(3*A - 4*I*B)*e^x + 12*I*A + 16*B)/(105*e^(7*x) - 735*I*e^(6*x) - 2205*e^
(5*x) + 3675*I*e^(4*x) + 3675*e^(3*x) - 2205*I*e^(2*x) - 735*e^x + 105*I)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28593, size = 81, normalized size = 0.8

$$-\frac{280\,Be^{(4\,x)} + 420\,Ae^{(3\,x)} - 280i\,Be^{(3\,x)} - 252i\,Ae^{(2\,x)} - 336\,Be^{(2\,x)} - 84\,Ae^{x} + 112i\,Be^{x} + 12i\,A + 16\,B}{105\,(e^{x} - i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="giac")
```

```
[Out] -1/105*(280*B*e^(4*x) + 420*A*e^(3*x) - 280*I*B*e^(3*x) - 252*I*A*e^(2*x) -
 336*B*e^(2*x) - 84*A*e^x + 112*I*B*e^x + 12*I*A + 16*B)/(e^x - I)^7
```

3.123 $\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{2}(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

```
[Out] (Sqrt[2]*(I*A - B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]]
)])/Sqrt[a] + (2*B*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rubi [A] time = 0.0659672, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2751, 2649, 206}

$$\frac{\sqrt{2}(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] (Sqrt[2]*(I*A - B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]]
)])/Sqrt[a] + (2*B*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx &= \frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}+(A+iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx\\
&= \frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}+(2(iA-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)\\
&= \frac{\sqrt{2}(iA-B)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.11868, size = 85, normalized size = 1.29

$$\frac{2\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left((1+i)\sqrt[4]{-1}(B-iA)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{4}\right)+i}{\sqrt{2}}\right)-iB\sinh\left(\frac{x}{2}\right)+B\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] (2*(Cosh[x/2] + I*Sinh[x/2])*((1 + I)*(-1)^(1/4)*((-I)*A + B)*ArcTan[(I + T
anh[x/4])/Sqrt[2]] + B*Cosh[x/2] - I*B*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int (A+B\sinh(x))\frac{1}{\sqrt{a+ia\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x)
```

```
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x)+A}{\sqrt{i\,a\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)
```

Fricas [B] time = 1.80588, size = 738, normalized size = 11.18

$$\frac{\sqrt{\frac{1}{2}}\sqrt{i\,ae^{(2x)}+2\,ae^{x}-i\,a}(-4i\,Be^{x}+4\,B)e^{\left(-\frac{1}{2}\,x\right)}-2\sqrt{2}(ae^{x}-i\,a)\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}\log\left(\frac{\sqrt{\frac{1}{2}}\sqrt{i\,ae^{(2x)}+2\,ae^{x}-i\,a}(-4i\,A+4\,B)e^{\left(-\frac{1}{2}\,x\right)}+2\sqrt{2}(a}{(-4i\,A+4\,B)e^{x}-4\,A-4i\,B}\right.}{2\,(ae^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*(-4*I*B*e^x + 4*B)*e^(-1/2
*x) - 2*sqrt(2)*(a*e^x - I*a)*sqrt(-(A^2 + 2*I*A*B - B^2)/a)*log((sqrt(1/2)
*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*(-4*I*A + 4*B)*e^(-1/2*x) + 2*sqrt(2)*(a
*e^x - I*a)*sqrt(-(A^2 + 2*I*A*B - B^2)/a))/((-4*I*A + 4*B)*e^x - 4*A - 4*I
*B)) + 2*sqrt(2)*(a*e^x - I*a)*sqrt(-(A^2 + 2*I*A*B - B^2)/a)*log((sqrt(1/2
```

```
)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*(-4*I*A + 4*B)*e^(-1/2*x) - 2*sqrt(2)*(
a*e^x - I*a)*sqrt(-(A^2 + 2*I*A*B - B^2)/a))/((-4*I*A + 4*B)*e^x - 4*A - 4*
I*B)))/(a*e^x - I*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B \sinh (x)}{\sqrt{a \left(i \sinh (x) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(1/2),x)
```

```
[Out] Integral((A + B*sinh(x))/sqrt(a*(I*sinh(x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sinh (x) + A}{\sqrt{i\, a \sinh (x) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)
```

3.124 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{(3B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

```
[Out] ((I*A + 3*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/(2
*Sqrt[2]*a^(3/2)) + ((I*A - B)*Cosh[x])/(2*(a + I*a*Sinh[x])^(3/2))
```

Rubi [A] time = 0.0768225, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2750, 2649, 206}

$$\frac{(3B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
```

```
[Out] ((I*A + 3*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/(2
*Sqrt[2]*a^(3/2)) + ((I*A - B)*Cosh[x])/(2*(a + I*a*Sinh[x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx &= \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}} + \frac{(A-3iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx}{4a} \\ &= \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}} + \frac{(iA+3B)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)}{2a} \\ &= \frac{(iA+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}a^{3/2}} + \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.230316, size = 105, normalized size = 1.33

$$\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left((A+iB)\sinh\left(\frac{x}{2}\right)+i(A+iB)\cosh\left(\frac{x}{2}\right)+(1+i)\sqrt[4]{-1}(A-3iB)(\sinh(x)-i)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{4}\right)+i}{\sqrt{2}}\right)\right)}{2(a+ia\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
```

```
[Out] ((Cosh[x/2] + I*Sinh[x/2])*(I*(A + I*B)*Cosh[x/2] + (A + I*B)*Sinh[x/2] + (
1 + I)*(-1)^(1/4)*(A - (3*I)*B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(-I + Sinh[
x])))/(2*(a + I*a*Sinh[x])^(3/2))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int (A+B\sinh(x))(a+ia\sinh(x))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
```

```
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x)+A}{(i\,a\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

Fricas [B] time = 1.93149, size = 936, normalized size = 11.85

$$\frac{\sqrt{\frac{1}{2}}\left((-2iA+2B)e^{(2x)}+2(A+iB)e^{x}\right)\sqrt{i\,ae^{(2x)}+2\,ae^{x}-i\,ae}^{\left(-\frac{1}{2}x\right)}+\sqrt{\frac{1}{2}}\left(a^2e^{(3x)}-3i\,a^2e^{(2x)}-3\,a^2e^{x}+i\,a^2\right)\sqrt{-\frac{A^2-6iA\cdots}{a^{\cdots}}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-2*I*A + 2*B)*e^(2*x) + 2*(A + I*B)*e^x)*sqrt(I*a*e^(2*x) + 2*
a*e^x - I*a)*e^(-1/2*x) + sqrt(1/2)*(a^2*e^(3*x) - 3*I*a^2*e^(2*x) - 3*a^2*
e^x + I*a^2)*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3)*log((sqrt(1/2)*sqrt(I*a*e^(
2*x) + 2*a*e^x - I*a)*(I*A + 3*B)*e^(-1/2*x) + sqrt(1/2)*(a^2*e^x - I*a^2)*
sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3))/((I*A + 3*B)*e^x + A - 3*I*B)) - sqrt(1
/2)*(a^2*e^(3*x) - 3*I*a^2*e^(2*x) - 3*a^2*e^x + I*a^2)*sqrt(-(A^2 - 6*I*A*
B - 9*B^2)/a^3)*log((sqrt(1/2)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*(I*A + 3*B
)*e^(-1/2*x) - sqrt(1/2)*(a^2*e^x - I*a^2)*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^
3))/((I*A + 3*B)*e^x + A - 3*I*B)))/(2*a^2*e^(3*x) - 6*I*a^2*e^(2*x) - 6*a^
2*e^x + 2*I*a^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x) + A}{(i\,a\sinh(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

3.125 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{(5B+3iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}a^{5/2}}+\frac{(5B+3iA)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

```
[Out] (((3*I)*A + 5*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])]
)/(16*Sqrt[2]*a^(5/2)) + ((I*A - B)*Cosh[x])/(4*(a + I*a*Sinh[x])^(5/2)) +
(((3*I)*A + 5*B)*Cosh[x])/(16*a*(a + I*a*Sinh[x])^(3/2))
```

Rubi [A] time = 0.100081, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2750, 2650, 2649, 206}

$$\frac{(5B+3iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}a^{5/2}}+\frac{(5B+3iA)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
```

```
[Out] (((3*I)*A + 5*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])]
)/(16*Sqrt[2]*a^(5/2)) + ((I*A - B)*Cosh[x])/(4*(a + I*a*Sinh[x])^(5/2)) +
(((3*I)*A + 5*B)*Cosh[x])/(16*a*(a + I*a*Sinh[x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx &= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3A-5iB)\int \frac{1}{(a+ia\sinh(x))^{3/2}}\,dx}{8a} \\
&= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}} + \frac{(3A-5iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx}{32a^2} \\
&= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}} + \frac{(3iA+5B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)}{16a^2} \\
&= \frac{(3iA+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}a^{5/2}} + \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.211919, size = 184, normalized size = 1.67

$$
\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(8(A+iB)\sinh\left(\frac{x}{2}\right)+2(5B+3iA)\sinh\left(\frac{x}{2}\right)(\sinh(x)-i)+(5B+3iA)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3+\right.}{16(a+ia\sinh(x))^{5/}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
```

```
[Out] ((Cosh[x/2] + I*Sinh[x/2])*((4*I)*(A + I*B)*(Cosh[x/2] + I*Sinh[x/2]) + ((3
*I)*A + 5*B)*(Cosh[x/2] + I*Sinh[x/2])^3 + (1 - I)*(-1)^(1/4)*(3*A - (5*I)*
B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(Cosh[x/2] + I*Sinh[x/2])^4 + 8*(A + I*B
)*Sinh[x/2] + 2*((3*I)*A + 5*B)*Sinh[x/2]*(-I + Sinh[x])))/(16*(a + I*a*Sin
h[x])^(5/2))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (A+B\sinh(x))\,(a+ia\sinh(x))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

```
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x)+A}{(i\,a\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

Fricas [B] time = 1.94748, size = 1218, normalized size = 11.07

$$8\sqrt{\frac{1}{2}}\left((-3iA-5B)e^{(4x)}-(11A+3iB)e^{(3x)}+(-11iA+3B)e^{(2x)}-(3A-5iB)e^{x}\right)\sqrt{iae^{(2x)}+2ae^{x}-iae}^{\left(-\frac{1}{2}x\right)}+\sqrt{\frac{1}{2}}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*sqrt(1/2)*((-3*I*A - 5*B)*e^(4*x) - (11*A + 3*I*B)*e^(3*x) + (-11*I*
A + 3*B)*e^(2*x) - (3*A - 5*I*B)*e^x)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*e^(
-1/2*x) + sqrt(1/2)*(4*a^3*e^(5*x) - 20*I*a^3*e^(4*x) - 40*a^3*e^(3*x) + 40
*I*a^3*e^(2*x) + 20*a^3*e^x - 4*I*a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^
5)*log((sqrt(1/2)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)*(3*I*A + 5*B)*e^(-1/2*x
) + sqrt(1/2)*(a^3*e^x - I*a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5))/((3
*I*A + 5*B)*e^x + 3*A - 5*I*B)) - sqrt(1/2)*(4*a^3*e^(5*x) - 20*I*a^3*e^(4*
x) - 40*a^3*e^(3*x) + 40*I*a^3*e^(2*x) + 20*a^3*e^x - 4*I*a^3)*sqrt(-(9*A^2
 - 30*I*A*B - 25*B^2)/a^5)*log((sqrt(1/2)*sqrt(I*a*e^(2*x) + 2*a*e^x - I*a)
*(3*I*A + 5*B)*e^(-1/2*x) - sqrt(1/2)*(a^3*e^x - I*a^3)*sqrt(-(9*A^2 - 30*I
*A*B - 25*B^2)/a^5))/((3*I*A + 5*B)*e^x + 3*A - 5*I*B)))/(8*a^3*e^(5*x) - 4
0*I*a^3*e^(4*x) - 80*a^3*e^(3*x) + 80*I*a^3*e^(2*x) + 40*a^3*e^x - 8*I*a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x)+A}{(ia\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

3.126 $\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=259

$$-\frac{2i\left(a^2+b^2\right)\left(15a^2B+56aAb-25b^2B\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{105b\sqrt{a+b\sinh(x)}}+\frac{2}{105}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a}$$

```
[Out] (2*(56*a*A*b + 15*a^2*B - 25*b^2*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/105 + (2*(
7*A*b + 5*a*B)*Cosh[x]*(a + b*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + b*Sinh
[x])^(5/2))/7 + (((2*I)/105)*(161*a^2*A*b - 63*A*b^3 + 15*a^3*B - 145*a*b^2
*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt
[(a + b*Sinh[x])/(a - I*b)]) - (((2*I)/105)*(a^2 + b^2)*(56*a*A*b + 15*a^2*
B - 25*b^2*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rubi [A] time = 0.446324, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2}{105}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}-\frac{2i\left(a^2+b^2\right)\left(15a^2B+56aAb-25b^2B\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{105b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```



```
[Out] (2*(56*a*A*b + 15*a^2*B - 25*b^2*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/105 + (2*(
7*A*b + 5*a*B)*Cosh[x]*(a + b*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + b*Sinh
[x])^(5/2))/7 + (((2*I)/105)*(161*a^2*A*b - 63*A*b^3 + 15*a^3*B - 145*a*b^2
*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt
[(a + b*Sinh[x])/(a - I*b)]) - (((2*I)/105)*(a^2 + b^2)*(56*a*A*b + 15*a^2*
B - 25*b^2*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+\frac{2}{7}\int (a+b\sinh(x))^{3/2}\left(\frac{1}{2}(7aA-5bB)+\frac{1}{2}(7$$

$$= \frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7Ab+5aB)\cos$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7Ab+5aB)\cos$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7Ab+5aB)\cos$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7Ab+5aB)\cos$$

Mathematica [A] time = 0.771175, size = 241, normalized size = 0.93

$$\frac{\cosh(x)(a+b\sinh(x))\left(90a^2B+6b\sinh(x)(15aB+7Ab)+154aAb+15b^2B\cosh(2x)-65b^2B\right)+\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\left(b\left(105a^3\right.\right.}{}}{105\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (((2*I)*(b*(105*a^3*A - 119*a*A*b^2 - 135*a^2*b*B + 25*b^3*B)*EllipticF[(Pi
 - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + (161*a^2*A*b - 63*A*b^3 + 15*a^3*B -
 145*a*b^2*B)*((a - I*b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]
- a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]))*Sqrt[(a + b*Sinh[x]
```

```
)/(a - I*b)])/b + Cosh[x]*(a + b*Sinh[x])*(154*a*A*b + 90*a^2*B - 65*b^2*B
+ 15*b^2*B*Cosh[2*x] + 6*b*(7*A*b + 15*a*B)*Sinh[x]))/(105*Sqrt[a + b*Sinh[
x]])
```

Maple [B] time = 0.108, size = 1893, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

```
[Out] 2/105*(56*I*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*
((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I
*b-a)/(I*b+a))^(1/2))*a*b^4+63*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x)
)*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/
(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^5-25*I*B*(-(a+b*sinh(x))/(I*b-a)
)^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Ellipti
cF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^5+98*A*a*b^4*
sinh(x)+90*B*a^2*b^3*sinh(x)+60*B*a*b^4*sinh(x)^4+98*A*a*b^4*sinh(x)^3+90*B
*a^2*b^3*sinh(x)^3+77*A*a^2*b^3*sinh(x)^2+45*B*a^3*b^2*sinh(x)^2+35*B*a*b^4
*sinh(x)^2-15*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2
)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-
(I*b-a)/(I*b+a))^(1/2))*a^5-63*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x)
)*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/
(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^5-10*I*B*(-(a+b*sinh(x))/(I*b-a)
)^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Ellipti
cF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^3+56*I*A*
(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b
/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))
^(1/2))*a^3*b^2+77*A*a^2*b^3+45*B*a^3*b^2-25*B*a*b^4+15*I*B*(-(a+b*sinh(x))
/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)
*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4*b+1
5*B*b^5*sinh(x)^5+21*A*b^5*sinh(x)^4-10*B*b^5*sinh(x)^3+21*A*b^5*sinh(x)^2-
25*B*b^5*sinh(x)+42*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a)
)^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1
/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^3-98*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*(
(I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b
*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^3-120*B*(-(a+b*sin
h(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^
(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^
3*b^2-120*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((
I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b
-a)/(I*b+a))^(1/2))*a*b^4+130*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))
*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(
I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3*b^2+145*B*(-(a+b*sinh(x))/(I*b-
a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Ellip
ticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^4+105*A*(
-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/
(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^
(1/2))*a^4*b-161*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(
1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2)
,(-(I*b-a)/(I*b+a))^(1/2))*a^4*b)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x) + A)(b\sinh(x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(Bb^2\sinh(x)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\sinh(x)^2 + \left(Ba^2 + 2\,Aab\right)\sinh(x)\right)\sqrt{b\sinh(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*sinh(x)^3 + A*a^2 + (2*B*a*b + A*b^2)*sinh(x)^2 + (B*a^2 +
2*A*a*b)*sinh(x))*sqrt(b*sinh(x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(5/2)*(A+B*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x) + A)(b\sinh(x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

3.127 $\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=207

$$-\frac{2i\left(a^2+b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{15b\sqrt{a+b\sinh(x)}}+\frac{2i\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] (2*(5*A*b + 3*a*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*B*Cosh[x]*(a + b*Si
nh[x])^(3/2))/5 + (((2*I)/15)*(20*a*A*b + 3*a^2*B - 9*b^2*B)*EllipticE[Pi/4
 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])/(
a - I*b)]) - (((2*I)/15)*(a^2 + b^2)*(5*A*b + 3*a*B)*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```

Rubi [A] time = 0.311112, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2i\left(a^2+b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15b\sqrt{a+b\sinh(x)}}+\frac{2i\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{15}\text{co}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```


```
[Out] (2*(5*A*b + 3*a*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*B*Cosh[x]*(a + b*Si
nh[x])^(3/2))/5 + (((2*I)/15)*(20*a*A*b + 3*a^2*B - 9*b^2*B)*EllipticE[Pi/4
 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])/(
a - I*b)]) - (((2*I)/15)*(a^2 + b^2)*(5*A*b + 3*a*B)*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```


Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2}{5}\int \sqrt{a+b\sinh(x)}\left(\frac{1}{2}(5aA-3bB)+\frac{1}{2}(5A\right.$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{4}{15}$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} - \frac{((a}{}$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{((2}{}$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2i}{}$$

Mathematica [A] time = 0.549874, size = 196, normalized size = 0.95

$$\frac{2\left(\cosh(x)(a+b\sinh(x))(6aB+5Ab+3bB\sinh(x)) + \frac{i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\left(b\left(15a^2A-12abB-5Ab^2\right)\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+\left(3a^2B+20aAb\right.\right.}{b}\right.}{15\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*((I*(b*(15*a^2*A - 5*A*b^2 - 12*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2*
I)*b)/(a - I*b)] + (20*a*A*b + 3*a^2*B - 9*b^2*B)*((a - I*b)*EllipticE[(Pi
- (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)
*b)/(a - I*b)]))*Sqrt[(a + b*Sinh[x])/(a - I*b)])/b + Cosh[x]*(a + b*Sinh[x
])*(5*A*b + 6*a*B + 3*b*B*Sinh[x])))/(15*Sqrt[a + b*Sinh[x]])
```

Maple [B] time = 0.157, size = 1037, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

```
[Out] ((a+b*sinh(x))*cosh(x)^2)^(1/2)*(B*b^2*(2/5/b*sinh(x)*((a+b*sinh(x))*cosh(x
)^2)^(1/2)-8/15*a/b^2*((a+b*sinh(x))*cosh(x)^2)^(1/2)-4/15*a/b*(a/b-I)*((-b
*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*
b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-
a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+2*(-3/5+8/15*a^2/b^2)*(a/b-I)*((-b*sinh(
x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^
(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((-a/b-I)*EllipticE(((-b*sinh(x)-a)/(
I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-a)/(I*b-a))
^(1/2),((a-I*b)/(I*b+a))^(1/2))))+(A*b^2+2*B*a*b)*(2/3/b*((a+b*sinh(x))*cos
h(x)^2)^(1/2)-2/3*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*
b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*E
llipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))-4/3*a/b*(a
/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh
(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((-a/b-I)*EllipticE((
(-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sin
h(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))))+2*(2*A*a*b+B*a^2)*(a/b-I)
*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*
b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((-a/b-I)*EllipticE(((-b*s
inh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-
a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))+2*a^2*A*(a/b-I)*((-b*sinh(x)-a)
/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)
/((a+b*sinh(x))*cosh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),(
(a-I*b)/(I*b+a))^(1/2)))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x) + A)(b\sinh(x) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\sinh(x)^{2} + Aa + (Ba + Ab)\sinh(x)\right)\sqrt{b\sinh(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*b*sinh(x)^2 + A*a + (B*a + A*b)*sinh(x))*sqrt(b*sinh(x) + a), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sinh(x))\,(a + b\sinh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(3/2)*(A+B*sinh(x)),x)
```

```
[Out] Integral((A + B*sinh(x))*(a + b*sinh(x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x) + A)(b\sinh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

3.128 $\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx$

Optimal. Leaf size=164

$$-\frac{2iB\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{3b\sqrt{a+b\sinh(x)}}+\frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}$$

```
[Out] (2*B*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((2*I)/3)*(3*A*b + a*B)*EllipticE[Pi
/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])
/(a - I*b)]) - (((2*I)/3)*(a^2 + b^2)*B*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*
a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rubi [A] time = 0.21153, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2iB\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\sqrt{a+b\sinh(x)}}+\frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*B*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((2*I)/3)*(3*A*b + a*B)*EllipticE[Pi
/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])
/(a - I*b)]) - (((2*I)/3)*(a^2 + b^2)*B*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*
a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx = \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{3}\int \frac{\frac{1}{2}(3aA-bB)+\frac{1}{2}(3Ab+aB)\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} - \frac{\left(\left(a^2+b^2\right)B\right)\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{3b} + \frac{(3Ab+aB)\int \sqrt{\ldots}}{3\ldots}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{\left((3Ab+aB)\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\ldots}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2i(3Ab+aB)E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \ldots$$

Mathematica [A] time = 0.401473, size = 151, normalized size = 0.92

$$\frac{-2iB\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+2(b+ia)(aB+3Ab)\sqrt{\frac{a+b\sinh(x)}{a-ib}}E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+2b\ldots}{3b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*b*B*Cosh[x]*(a + b*Sinh[x]) + 2*(I*a + b)*(3*A*b + a*B)*EllipticE[(Pi -
(2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(
a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*
Sinh[x])/(a - I*b)])/(3*b*Sqrt[a + b*Sinh[x]])
```

Maple [B] time = 0.093, size = 897, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

```
[Out] 2/3*(I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+s
inh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)
/(I*b+a))^(1/2))*a^2*b+I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I
*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a
))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3+3*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*(
(I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b
*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b+3*A*(-(a+b*sinh(x)
)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2
)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3-3*
A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))
*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a
))^(1/2))*a^2*b-3*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^
(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2
),(-(I*b-a)/(I*b+a))^(1/2))*b^3-B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x
))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))
/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3-B*(-(a+b*sinh(x))/(I*b-a))^(1
/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((
-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^2+B*b^3*sinh(x)
^3+B*a*b^2*sinh(x)^2+B*b^3*sinh(x)+B*a*b^2)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x) + A)\sqrt{b\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\sinh(x) + A)\sqrt{b\sinh(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + B\sinh(x))\sqrt{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(1/2)*(A+B*sinh(x)),x)
```

```
[Out] Integral((A + B*sinh(x))*sqrt(a + b*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sinh(x) + A)\sqrt{b\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

3.129 $\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=55

$$\frac{Bx}{b}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[Out] (B*x)/b - (2*(A*b - a*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqr
t[a^2 + b^2])
```

Rubi [A] time = 0.0750333, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2735, 2660, 618, 206}

$$\frac{Bx}{b}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b - (2*(A*b - a*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqr
t[a^2 + b^2])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{Bx}{b} - \frac{(i(iAb-iaB))\int \frac{1}{a+b\sinh(x)}\,dx}{b}$$
$$= \frac{Bx}{b} - \frac{(2i(iAb-iaB))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{Bx}{b} + \frac{(4i(iAb-iaB))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{Bx}{b} - \frac{2(Ab-aB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

Mathematica [A] time = 0.102054, size = 61, normalized size = 1.11

$$\frac{\frac{2(Ab-aB)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x + (2*(A*b - a*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
 - b^2])/b
```

Maple [B] time = 0.02, size = 101, normalized size = 1.8

$$\frac{B}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{B}{b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\,\frac{A}{\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tanh(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)-2\,\frac{aB}{b\sqrt{a^2+b^2}}\text{Artanh}\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] B/b*ln(tanh(1/2*x)+1)-B/b*ln(tanh(1/2*x)-1)+2/(a^2+b^2)^(1/2)*arctanh(1/2*(
2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A-2/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*
a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*a*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87912, size = 387, normalized size = 7.04

$$-\frac{(Ba-Ab)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2(b\cosh(x)+a)\sinh(x)-b}\right)-\left(Ba^2+\cdots\right.}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((B*a - A*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x)/(a^2*b + b^3)
```

Sympy [A] time = 97.3601, size = 469, normalized size = 8.53

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx\right) & \text{for } a = \cdots \\ -\frac{2Ab^2\tanh\left(\frac{x}{2}\right)}{-b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} - \frac{Bb^2x\tanh\left(\frac{x}{2}\right)}{-b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} + \frac{iBbx\sqrt{b^2}}{-b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} - \frac{2iBb\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}{-b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} & \text{for } a = \cdots \\ \frac{2Ab^2\tanh\left(\frac{x}{2}\right)}{b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} + \frac{Bb^2x\tanh\left(\frac{x}{2}\right)}{b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} + \frac{iBbx\sqrt{b^2}}{b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} - \frac{2iBb\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}{b^3\tanh\left(\frac{x}{2}\right)+ib^2\sqrt{b^2}} & \text{for } a = \cdots \\ \frac{Ax+B\cosh(x)}{a} & \text{for } b = \cdots \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx}{b} & \text{for } a = \cdots \\ -\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}} + \frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}} + \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} + \frac{Bx}{b} & \text{otherwi}\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), (-2*A*b**2*t
anh(x/2)/(-b**3*tanh(x/2) + I*b**2*sqrt(b**2)) - B*b**2*x*tanh(x/2)/(-b**3*
tanh(x/2) + I*b**2*sqrt(b**2)) + I*B*b*x*sqrt(b**2)/(-b**3*tanh(x/2) + I*b*
*2*sqrt(b**2)) - 2*I*B*b*sqrt(b**2)*tanh(x/2)/(-b**3*tanh(x/2) + I*b**2*sqr
t(b**2)), Eq(a, -sqrt(-b**2))), (2*A*b**2*tanh(x/2)/(b**3*tanh(x/2) + I*b**
2*sqrt(b**2)) + B*b**2*x*tanh(x/2)/(b**3*tanh(x/2) + I*b**2*sqrt(b**2)) + I
*B*b*x*sqrt(b**2)/(b**3*tanh(x/2) + I*b**2*sqrt(b**2)) - 2*I*B*b*sqrt(b**2)
*tanh(x/2)/(b**3*tanh(x/2) + I*b**2*sqrt(b**2)), Eq(a, sqrt(-b**2))), ((A*x
 + B*cosh(x))/a, Eq(b, 0)), ((A*log(tanh(x/2)) + B*x)/b, Eq(a, 0)), (-A*log
(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + A*log(tanh(x/2)
 - b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + B*a*log(tanh(x/2) - b/a -
 sqrt(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) - B*a*log(tanh(x/2) - b/a + sqr
t(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) + B*x/b, True))
```

Giac [A] time = 1.22866, size = 101, normalized size = 1.84

$$\frac{Bx}{b} - \frac{(Ba-Ab)\log\left(\frac{\left|2be^x+2a-2\sqrt{a^2+b^2}\right|}{\left|2be^x+2a+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a - A*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b)
```

3.130 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{2(aA+bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\cosh(x)(Ab-aB)}{\left(a^2+b^2\right)(a+b\sinh(x))}$$

```
[Out] (-2*(a*A + b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/
2) - ((A*b - a*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.0828692, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2754, 12, 2660, 618, 206}

$$-\frac{2(aA+bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\cosh(x)(Ab-aB)}{\left(a^2+b^2\right)(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/
2) - ((A*b - a*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\int \frac{-aA-bB}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{(aA+bB)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{(2(aA+bB))\,\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{(4(aA+bB))\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2(aA+bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.152572, size = 82, normalized size = 1.11

$$
\frac{\frac{2(aA+bB)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\cosh(x)(aB-Ab)}{a+b\sinh(x)}}{a^2+b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*(a*A + b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] + ((-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x]))/(a^2 + b^2)
```

Maple [A] time = 0.026, size = 113, normalized size = 1.5

$$
-2\,\frac{1}{a\left(\tanh\left(x/2\right)\right)^2-2\,\tanh\left(x/2\right)b-a}\left(-\frac{b\left(Ab-aB\right)\tanh\left(x/2\right)}{a\left(a^2+b^2\right)}-\frac{Ab-aB}{a^2+b^2}\right)+2\,\frac{Aa+bB}{\left(a^2+b^2\right)^{3/2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tanh(}{\sqrt{a^2}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^2,x)
```

```
[Out] -2*(-b*(A*b-B*a)/a/(a^2+b^2)*tanh(1/2*x)-(A*b-B*a)/(a^2+b^2))/(a*tanh(1/2*x
)^2-2*tanh(1/2*x)*b-a)+2*(A*a+B*b)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/
2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88599, size = 1065, normalized size = 14.39

$$-\frac{2\,Ba^3b - 2\,Aa^2b^2 + 2\,Bab^3 - 2\,Ab^4 - \left(Aab^2 + Bb^3 - \left(Aab^2 + Bb^3\right)\cosh\left(x\right)^2 - \left(Aab^2 + Bb^3\right)\sinh\left(x\right)^2 - 2\left(Aa^2b + Bab\right.\right.}{a^4b^2 + 2\,a^2b^4 + b^6 - \left(a^4b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*B*a^3*b - 2*A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4 - (A*a*b^2 + B*b^3 - (A*a*b
^2 + B*b^3)*cosh(x)^2 - (A*a*b^2 + B*b^3)*sinh(x)^2 - 2*(A*a^2*b + B*a*b^2)
*cosh(x) - 2*(A*a^2*b + B*a*b^2 + (A*a*b^2 + B*b^3)*cosh(x))*sinh(x))*sqrt(
a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x)
 - b)) - 2*(B*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*cosh(x) - 2*(B*a^4 - A*a
^3*b + B*a^2*b^2 - A*a*b^3)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2
+ 2*a^2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^2 - 2*(a
^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 + (a^4*b^2
 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36898, size = 161, normalized size = 2.18

$$\frac{\left(Aa + Bb\right)\log\left(\frac{\left|2\,be^{x} + 2\,a - 2\,\sqrt{a^2 + b^2}\right|}{\left|2\,be^{x} + 2\,a + 2\,\sqrt{a^2 + b^2}\right|}\right)}{\left(a^2 + b^2\right)^{\frac{3}{2}}} - \frac{2\left(Ba^2e^{x} - Aabe^{x} - Bab + Ab^2\right)}{\left(a^2b + b^3\right)\left(be^{(2\,x)} + 2\,ae^{x} - b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (A*a + B*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a +
2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(B*a^2*e^x - A*a*b*e^x - B*a*b +
A*b^2)/((a^2*b + b^3)*(b*e^(2*x) + 2*a*e^x - b))
```

3.131 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx$

Optimal. Leaf size=128

$$-\frac{\left(2a^2A+3abB-Ab^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\cosh(x)\left(a^2(-B)+3aAb+2b^2B\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{\cosh(x)(Ab-aB)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2}$$

```
[Out] -(((2*a^2*A - A*b^2 + 3*a*b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/
(a^2 + b^2)^(5/2)) - ((A*b - a*B)*Cosh[x])/(2*(a^2 + b^2)*(a + b*Sinh[x])^2
) - ((3*a*A*b - a^2*B + 2*b^2*B)*Cosh[x])/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```

Rubi [A] time = 0.17506, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2754, 12, 2660, 618, 206}

$$-\frac{\left(2a^2A+3abB-Ab^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\cosh(x)\left(a^2(-B)+3aAb+2b^2B\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{\cosh(x)(Ab-aB)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```


```
[Out] -(((2*a^2*A - A*b^2 + 3*a*b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/
(a^2 + b^2)^(5/2)) - ((A*b - a*B)*Cosh[x])/(2*(a^2 + b^2)*(a + b*Sinh[x])^2
) - ((3*a*A*b - a^2*B + 2*b^2*B)*Cosh[x])/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx = -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\int \frac{-2(aA+bB)+(Ab-aB)\sinh(x)}{(a+b\sinh(x))^2}\,dx}{2\left(a^2+b^2\right)}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\int \frac{2a^2A-Ab^2+3abB}{a+b\sinh(x)}\,dx}{2\left(a^2+b^2\right)^2}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\left(2a^2A-Ab^2+3abB\right)\int \frac{}{a+b}}{2\left(a^2+b^2\right)^2}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\left(2a^2A-Ab^2+3abB\right)\mathrm{Subs}}{\left(a^2\right.}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} - \frac{\left(2\left(2a^2A-Ab^2+3abB\right)\right)\mathrm{S}}{}$$

$$= -\frac{\left(2a^2A-Ab^2+3abB\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+\right.}{2\left(a^2+b^2\right)^2(}$$

Mathematica [A] time = 0.270176, size = 131, normalized size = 1.02

$$\frac{\frac{2\left(2a^2A+3abB-Ab^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{\cosh(x)\left(a^2B-3aAb-2b^2B\right)}{a+b\sinh(x)} + \frac{\left(a^2+b^2\right)\cosh(x)(aB-Ab)}{(a+b\sinh(x))^2}}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```

```
[Out] ((2*(2*a^2*A - A*b^2 + 3*a*b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])
/Sqrt[-a^2 - b^2] + ((a^2 + b^2)*(-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x])^2
+ ((-3*a*A*b + a^2*B - 2*b^2*B)*Cosh[x])/(a + b*Sinh[x]))/(2*(a^2 + b^2)^2)
```

Maple [B] time = 0.036, size = 314, normalized size = 2.5

$$-2\,\frac{1}{\left(a\,(\tanh(x/2))^2-2\,\tanh(x/2)\,b-a\right)^2}\left(-1/2\,\frac{b\left(5\,Aa^2b+2\,Ab^3-3\,a^3B\right)(\tanh(x/2))^3}{a\left(a^4+2\,a^2b^2+b^4\right)}-1/2\,\frac{\left(4\,Aa^4b-7\,Aa^2b^3-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*b*(5*A*a^2*b+2*A*b^3-3*B*a^3)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*x)^3-
1/2*(4*A*a^4*b-7*A*a^2*b^3-2*A*b^5-2*B*a^5+5*B*a^3*b^2-2*B*a*b^4)/(a^4+2*a^
2*b^2+b^4)/a^2*tanh(1/2*x)^2+1/2*b*(11*A*a^2*b+2*A*b^3-5*B*a^3+4*B*a*b^2)/(
a^4+2*a^2*b^2+b^4)/a*tanh(1/2*x)+1/2*(4*A*a^2*b+A*b^3-2*B*a^3+B*a*b^2)/(a^4
+2*a^2*b^2+b^4))/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2+(2*A*a^2-A*b^2+3*B*a
*b)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(
a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03506, size = 3617, normalized size = 28.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 - 2*B*a^2*b^4 - 6*A*a*b^5 - 4*B*b^6 - 2*(2*
A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - 2*(2*A
*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + 2*(2*B*
a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 + 2*B
*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 - 3*B*
a^2*b^4 + 3*A*a*b^5 + 2*B*b^6 - 3*(2*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 +
3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^2 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5 +
(2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^4 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*
b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x)^3 + 4*(2*A
*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x
))*sinh(x)^3 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
)*cosh(x)^2 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
+ 3*(2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^2 + 6*(2*A*a^3*b^2 + 3*B*a^2*
b^3 - A*a*b^4)*cosh(x))*sinh(x)^2 - 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)
*cosh(x) - 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 - (2*A*a^2*b^3 + 3*B*a*b^
4 - A*b^5)*cosh(x)^3 - 3*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x)^2 -
(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x
))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) - 2*(4*B*a^5*b - 10*A*a^4*b^2 - B*a^3*b^3 - 11*A*a^2*b^4 - 5
*B*a*b^5 - A*b^6)*cosh(x) - 2*(4*B*a^5*b - 10*A*a^4*b^2 - B*a^3*b^3 - 11*A*
a^2*b^4 - 5*B*a*b^5 - A*b^6 + 3*(2*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*
B*a*b^5 - A*b^6)*cosh(x)^2 - 2*(2*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3
*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 + 2*B*b^6)*cosh(x))*sinh(x))/(a^6*b^3 + 3*a^
4*b^5 + 3*a^2*b^7 + b^9 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x)^4
 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*sinh(x)^4 + 4*(a^7*b^2 + 3*a^5*b
^4 + 3*a^3*b^6 + a*b^8)*cosh(x)^3 + 4*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*
b^8 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x))*sinh(x)^3 + 2*(2*a^8
```

```
*b + 5*a^6*b^3 + 3*a^4*b^5 - a^2*b^7 - b^9)*cosh(x)^2 + 2*(2*a^8*b + 5*a^6*
b^3 + 3*a^4*b^5 - a^2*b^7 - b^9 + 3*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)
*cosh(x)^2 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2
 - 4*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x) - 4*(a^7*b^2 + 3*a^5
*b^4 + 3*a^3*b^6 + a*b^8 - (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x)^
3 - 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x)^2 - (2*a^8*b + 5*a^
6*b^3 + 3*a^4*b^5 - a^2*b^7 - b^9)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27497, size = 377, normalized size = 2.95

$$-\frac{\left(2\,Aa^2+3\,Bab-Ab^2\right)\log\left(\frac{\left|-2\,be^x-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|-2\,be^x-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\,Aa^2b^2e^{(3\,x)}+3\,Bab^3e^{(3\,x)}-Ab^4e^{(3\,x)}-2\,Ba^4e^{(2\,x)}+6\,Aa^3be^{(2\,x)}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*A*a^2 + 3*B*a*b - A*b^2)*log(abs(-2*b*e^x - 2*a - 2*sqrt(a^2 + b^2)
)/abs(-2*b*e^x - 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^
2 + b^2)) + (2*A*a^2*b^2*e^(3*x) + 3*B*a*b^3*e^(3*x) - A*b^4*e^(3*x) - 2*B*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) + 5*B*a^2*b^2*e^(2*x) - 3*A*a*b^3*e^(2*x) -
 2*B*b^4*e^(2*x) + 4*B*a^3*b*e^x - 10*A*a^2*b^2*e^x - 5*B*a*b^3*e^x - A*b^4
*e^x - B*a^2*b^2 + 3*A*a*b^3 + 2*B*b^4)/((a^4*b + 2*a^2*b^3 + b^5)*(b*e^(2*
x) + 2*a*e^x - b)^2)
```

3.132 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{\cosh(x)\left(11a^2Ab-2a^3B+13ab^2B-4Ab^3\right)}{6\left(a^2+b^2\right)^3(a+b\sinh(x))}-\frac{\cosh(x)\left(-2a^2B+\right.}{6\left(a^2+b^2\right)^2(a}$$

```
[Out] -(((2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2 + b^2)^(7/2)) - ((A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a
+ b*Sinh[x])^3) - ((5*a*A*b - 2*a^2*B + 3*b^2*B)*Cosh[x])/(6*(a^2 + b^2)^2*
(a + b*Sinh[x])^2) - ((11*a^2*A*b - 4*A*b^3 - 2*a^3*B + 13*a*b^2*B)*Cosh[x]
)/(6*(a^2 + b^2)^3*(a + b*Sinh[x]))
```

Rubi [A] time = 0.32539, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2754, 12, 2660, 618, 206}

$$-\frac{\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{\cosh(x)\left(11a^2Ab-2a^3B+13ab^2B-4Ab^3\right)}{6\left(a^2+b^2\right)^3(a+b\sinh(x))}-\frac{\cosh(x)\left(-2a^2B+\right.}{6\left(a^2+b^2\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

```
[Out] -(((2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2 + b^2)^(7/2)) - ((A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a
+ b*Sinh[x])^3) - ((5*a*A*b - 2*a^2*B + 3*b^2*B)*Cosh[x])/(6*(a^2 + b^2)^2*
(a + b*Sinh[x])^2) - ((11*a^2*A*b - 4*A*b^3 - 2*a^3*B + 13*a*b^2*B)*Cosh[x]
)/(6*(a^2 + b^2)^3*(a + b*Sinh[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\int \frac{-3(aA+bB)+2(Ab-aB)\sinh(x)}{(a+b\sinh(x))^3}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} + \frac{\int \frac{2\left(3a^2A-2Ab^2+5abB\right)-\left(5aAb-2\right.}{(a+b\sinh(x))}}{6\left(a^2+b^2\right.}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3-2a^3B+\right.}{6\left(a^2+b^2\right)^3(a+b}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3-2a^3B+\right.}{6\left(a^2+b^2\right)^3(a+b}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3-2a^3B+\right.}{6\left(a^2+b^2\right)^3(a+b}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3-2a^3B+\right.}{6\left(a^2+b^2\right)^3(a+b}$$

$$= -\frac{\left(2a^3A-3aAb^2+4a^2bB-b^3B\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{(5aA}{6(a}$$

Mathematica [A] time = 0.461131, size = 189, normalized size = 1.01

$$\frac{\frac{6\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{2\left(a^2+b^2\right)^2\cosh(x)(aB-Ab)}{(a+b\sinh(x))^3} + \frac{\left(a^2+b^2\right)\cosh(x)\left(2a^2B-5aAb-3b^2B\right)}{(a+b\sinh(x))^2} + \frac{\cosh(x)\left(-11a^2Ab+2a^3B-13a\right.}{a+b\sinh(x)}}{6\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

```
[Out] ((6*(2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt
[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*(a^2 + b^2)^2*(-(A*b) + a*B)*Cosh[x])/
(a + b*Sinh[x])^3 + ((a^2 + b^2)*(-5*a*A*b + 2*a^2*B - 3*b^2*B)*Cosh[x])/(a
 + b*Sinh[x])^2 + ((-11*a^2*A*b + 4*A*b^3 + 2*a^3*B - 13*a*b^2*B)*Cosh[x])/
(a + b*Sinh[x]))/(6*(a^2 + b^2)^3)
```

Maple [B] time = 0.041, size = 633, normalized size = 3.4

$$-2\,{\frac{1}{\left(a\left(\tanh\left(x/2\right)\right)^{2}-2\,\tanh\left(x/2\right)b-a\right)^{3}}\left(-1/2\,{\frac{b\left(9\,Aa^{4}b+6\,Aa^{2}b^{3}+2\,Ab^{5}-4\,Ba^{5}+Ba^{3}b^{2}\right)\left(\tanh\left(x/2\right)\right)^{5}}{a\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}}-1/2\,{\frac{\left(6\,Aa\right.}{}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^4,x)
```

```
[Out] -2*(-1/2*b*(9*A*a^4*b+6*A*a^2*b^3+2*A*b^5-4*B*a^5+B*a^3*b^2)/a/(a^6+3*a^4*b
^2+3*a^2*b^4+b^6)*tanh(1/2*x)^5-1/2*(6*A*a^6*b-27*A*a^4*b^3-12*A*a^2*b^5-4*
A*b^7-2*B*a^7+14*B*a^5*b^2-11*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4
+b^6)/a^2*tanh(1/2*x)^4+1/3/a^3*b*(54*A*a^6*b-21*A*a^4*b^3-4*A*a^2*b^5-4*A*
b^7-18*B*a^7+42*B*a^5*b^2-17*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+
b^6)*tanh(1/2*x)^3+1/a^2*(6*A*a^6*b-20*A*a^4*b^3-3*A*a^2*b^5-2*A*b^7-2*B*a^
7+10*B*a^5*b^2-14*B*a^3*b^4-B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2
*x)^2-1/2/a*b*(27*A*a^4*b+4*A*a^2*b^3+2*A*b^5-8*B*a^5+19*B*a^3*b^2+2*B*a*b^
4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*x)-1/6*(18*A*a^4*b+5*A*a^2*b^3+2*
A*b^5-6*B*a^5+10*B*a^3*b^2+B*a*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6))/(a*tanh(
1/2*x)^2-2*tanh(1/2*x)*b-a)^3+(2*A*a^3-3*A*a*b^2+4*B*a^2*b-B*b^3)/(a^6+3*a^
4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2
+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.33391, size = 8690, normalized size = 46.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 - 22*B*a^3*b^5 - 14*A*a^2*b^6 - 26*B*a*b^7
 + 8*A*b^8 + 6*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a
*b^7 - B*b^8)*cosh(x)^5 + 6*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^
2*b^6 - 3*A*a*b^7 - B*b^8)*sinh(x)^5 + 30*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^
4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x)^4 + 30*(2*A*a^6*b^2 +
4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7 + (2*A*a^5*b^
3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 - B*b^8)*cosh(x))*sin
h(x)^4 - 4*(4*B*a^8 - 22*A*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^
4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*b^7)*cosh(x)^3 - 4*(4*B*a^8 - 22*A
*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^
2*b^6 - 12*A*a*b^7 - 15*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^
6 - 3*A*a*b^7 - B*b^8)*cosh(x)^2 - 30*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^
```

4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x))*sinh(x)^3 + 12*(4*B*a^7*b - 17*A*a^6*b^2 - 13*B*a^5*b^3 - 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 + 4*B*a*b^7 - 2*A*b^8)*cosh(x)^2 + 12*(4*B*a^7*b - 17*A*a^6*b^2 - 13*B*a^5*b^3 - 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 + 4*B*a*b^7 - 2*A*b^8 + 5*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 - B*b^8)*cosh(x)^3 + 15*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x)^2 - (4*B*a^8 - 22*A*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*b^7)*cosh(x))*sinh(x)^2 + 3*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7 - (2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^6 - (2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*sinh(x)^6 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^5 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^5 - 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^4 - 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7 + 5*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 10*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x))*sinh(x)^4 - 4*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6)*cosh(x)^3 - 4*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6 + 5*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^3 + 15*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^2 + 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x))*sinh(x)^3 + 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^2 + 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7 - 5*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^4 - 20*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^3 - 6*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^2 - 4*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6)*cosh(x))*sinh(x)^2 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x) - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^5 + 5*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^4 + 2*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^3 + 2*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6)*cosh(x)^2 - (8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 - 18*B*a^4*b^4 - 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 - B*b^8)*cosh(x) - 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 - 18*B*a^4*b^4 - 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 - B*b^8 - 5*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 - B*b^8)*cosh(x)^4 - 20*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*b^7)*cosh(x)^2 - 4*(4*B*a^7*b - 17*A*a^6*b^2 - 13*B*a^5*b^3 - 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 + 4*B*a*b^7 - 2*A*b^8)*cosh(x))*sinh(x))/(a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12 - (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^6 - (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*sinh(x)^6 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x)^5 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11 + (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x))*sinh(x)^5 - 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x)^4 - 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12 + 5*(a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^2 + 10*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x))*sinh(x)^4 - 4*(2*a^11*b + 5*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*b^11)*cosh(x)^3 - 4*(2*a^11*b + 5*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*b^11 + 5*(a^8*b^4 + 4*a^6*b^6

```
 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^3 + 15*(a^9*b^3 + 4*a^7*b^5 + 6*a
^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x)^2 + 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^
6*b^6 + 10*a^4*b^8 - b^12)*cosh(x))*sinh(x)^3 + 3*(4*a^10*b^2 + 15*a^8*b^4
+ 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x)^2 + 3*(4*a^10*b^2 + 15*a^8*b^4 +
20*a^6*b^6 + 10*a^4*b^8 - b^12 - 5*(a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2
*b^10 + b^12)*cosh(x)^4 - 20*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 +
 a*b^11)*cosh(x)^3 - 6*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 -
 b^12)*cosh(x)^2 - 4*(2*a^11*b + 5*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*
b^11)*cosh(x))*sinh(x)^2 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 +
 a*b^11)*cosh(x) - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11
+ (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^5 + 5*(a^9*
b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x)^4 + 2*(4*a^10*b^2
 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x)^3 + 2*(2*a^11*b + 5
*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*b^11)*cosh(x)^2 - (4*a^10*b^2 + 15
*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29712, size = 644, normalized size = 3.44

$$\frac{\left(2\,Aa^3+4\,Ba^2b-3\,Aab^2-Bb^3\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{6\,Aa^3b^3e^{(5\,x)}+12\,Ba^2b^4e^{(5\,x)}-9\,Aab^5e^{(5\,x)}-3\,Bb^6e^{(5\,x)}+30\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a^3 + 4*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/3*(6*A*a^3*b^3*e^(5*x) + 12*B*a^2*b^4*
e^(5*x) - 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x) + 30*A*a^4*b^2*e^(4*x) + 60*B
*a^3*b^3*e^(4*x) - 45*A*a^2*b^4*e^(4*x) - 15*B*a*b^5*e^(4*x) - 8*B*a^6*e^(3
*x) + 44*A*a^5*b*e^(3*x) + 64*B*a^4*b^2*e^(3*x) - 82*A*a^3*b^3*e^(3*x) - 78
*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) + 24*B*a^5*b*e^(2*x) - 102*A*a^4*b^
2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a^2*b^4*e^(2*x) + 24*B*a*b^5*e^(2*
x) - 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x + 60*A*a^3*b^3*e^x + 66*B*a^2*b^4*
e^x - 15*A*a*b^5*e^x + 3*B*b^6*e^x + 2*B*a^3*b^3 - 11*A*a^2*b^4 - 13*B*a*b^
5 + 4*A*b^6)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*(b*e^(2*x) + 2*a*e^x -
b)^3)
```

3.133 $\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2B\left(a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{Bx}{b}$$

```
[Out] (B*x)/b + (2*(a^2 - b^2)*B*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b
*Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0837459, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2735, 2660, 618, 206}

$$\frac{2B\left(a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b + (2*(a^2 - b^2)*B*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b
*Sqrt[a^2 + b^2])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{Bx}{b}-\frac{\left(i\left(-iaB+\frac{ib^2B}{a}\right)\right)\int \frac{1}{a+b\sinh(x)}\,dx}{b}$$
$$=\frac{Bx}{b}-\frac{\left(2i\left(-iaB+\frac{ib^2B}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$=\frac{Bx}{b}+\frac{\left(4i\left(-iaB+\frac{ib^2B}{a}\right)\right)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$=\frac{Bx}{b}+\frac{2\left(a^2-b^2\right)B\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}$$

Mathematica [A] time = 0.0692626, size = 66, normalized size = 1.1

$$\frac{B\left(ax-\frac{2\left(a^2-b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*(a*x - (2*(a^2 - b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2]))/(a*b)
```

Maple [A] time = 0.02, size = 105, normalized size = 1.8

$$\frac{B}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{B}{b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-2\frac{aB}{b\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\frac{2a\tanh(x/2)-2b}{\sqrt{a^2+b^2}}\right)+2\frac{bB}{a\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] B/b*ln(tanh(1/2*x)+1)-B/b*ln(tanh(1/2*x)-1)-2/b/(a^2+b^2)^(1/2)*arctanh(1/2
*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*a*B+2*B/a*b/(a^2+b^2)^(1/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78377, size = 398, normalized size = 6.63

$$-\frac{\left(Ba^2 - Bb^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right) - (B}{a^3b+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((B*a^2 - B*b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*
b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)
*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*
(b*cosh(x) + a)*sinh(x) - b)) - (B*a^3 + B*a*b^2)*x)/(a^3*b + a*b^3)
```

Sympy [A] time = 103.155, size = 350, normalized size = 5.83

$$\begin{cases} \text{NaN} & \text{for}\ a \\ \frac{B\cosh(x)}{a} & \text{for}\ b \\ \frac{Bbx}{b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)} - \frac{4Bb\tanh\left(\frac{x}{2}\right)}{b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)} + \frac{iBx\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}{b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)} & \text{for}\ a \\ -\frac{Bbx}{-b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)} + \frac{4Bb\tanh\left(\frac{x}{2}\right)}{-b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)} + \frac{iBx\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}{-b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)} & \text{for}\ a \\ \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} + \frac{Bx}{b} - \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{a\sqrt{a^2+b^2}} + \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{a\sqrt{a^2+b^2}} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*cosh(x)/a, Eq(b, 0)), (B*b*x/(b**2
 + I*b*sqrt(b**2)*tanh(x/2)) - 4*B*b*tanh(x/2)/(b**2 + I*b*sqrt(b**2)*tanh(
x/2)) + I*B*x*sqrt(b**2)*tanh(x/2)/(b**2 + I*b*sqrt(b**2)*tanh(x/2)), Eq(a,
 -sqrt(-b**2))), (-B*b*x/(-b**2 + I*b*sqrt(b**2)*tanh(x/2)) + 4*B*b*tanh(x/
2)/(-b**2 + I*b*sqrt(b**2)*tanh(x/2)) + I*B*x*sqrt(b**2)*tanh(x/2)/(-b**2 +
 I*b*sqrt(b**2)*tanh(x/2)), Eq(a, sqrt(-b**2))), (B*a*log(tanh(x/2) - b/a -
 sqrt(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) - B*a*log(tanh(x/2) - b/a + sqr
t(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) + B*x/b - B*b*log(tanh(x/2) - b/a -
 sqrt(a**2 + b**2)/a)/(a*sqrt(a**2 + b**2)) + B*b*log(tanh(x/2) - b/a + sqr
t(a**2 + b**2)/a)/(a*sqrt(a**2 + b**2)), True))
```

Giac [A] time = 1.26471, size = 111, normalized size = 1.85

$$\frac{Bx}{b} - \frac{\left(Ba^2 - Bb^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a^2 - B*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*
e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b)
```

3.134 $\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

[Out] (B*x)/b

Rubi [A] time = 0.001466, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

[In] Int[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]

[Out] (B*x)/b

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

Mathematica [A] time = 0.0004363, size = 6, normalized size = 1.

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

[In] Integrate[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]

[Out] (B*x)/b

Maple [A] time = 0.002, size = 7, normalized size = 1.2

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] B*x/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71968, size = 9, normalized size = 1.5

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

Sympy [A] time = 0.437974, size = 3, normalized size = 0.5

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] B*x/b
```

Giac [A] time = 1.16328, size = 8, normalized size = 1.33

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b
```

3.135 $\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx$

Optimal. Leaf size=12

$$-\frac{\cosh(x)}{a\sinh(x)+b}$$

```
[Out] -(Cosh[x]/(b + a*Sinh[x]))
```

Rubi [A] time = 0.0321646, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2754, 8}

$$-\frac{\cosh(x)}{a\sinh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
```

```
[Out] -(Cosh[x]/(b + a*Sinh[x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx &= -\frac{\cosh(x)}{b+a\sinh(x)}-\frac{\int 0\,dx}{a^2+b^2}\\ &= -\frac{\cosh(x)}{b+a\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.03527, size = 12, normalized size = 1.

$$-\frac{\cosh(x)}{a\sinh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
```

```
[Out] -(Cosh[x]/(b + a*Sinh[x]))
```

Maple [B] time = 0.029, size = 36, normalized size = 3.

$$-2\,\frac{1}{(\tanh(x/2))^2 b-2\,a\tanh(x/2)-b}\left(-\frac{a\tanh(x/2)}{b}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*sinh(x))/(b+a*sinh(x))^2,x)
```

```
[Out] -2*(-a/b*tanh(1/2*x)-1)/(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.63292, size = 158, normalized size = 13.17

$$\frac{2\,(b\cosh(x)+b\sinh(x)-a)}{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 2*(b*cosh(x) + b*sinh(x) - a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x
) - a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23304, size = 41, normalized size = 3.42

$$\frac{2\,(be^x-a)}{\left(ae^{(2\,x)}+2\,be^x-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(b*e^x - a)/((a*e^(2*x) + 2*b*e^x - a)*a)
```

3.136 $\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx$

Optimal. Leaf size=34

$$\frac{4x}{\sqrt{5}}-x-\frac{8\tanh^{-1}\left(\frac{\cosh(x)}{\sinh(x)+\sqrt{5}+2}\right)}{\sqrt{5}}$$

```
[Out] -x + (4*x)/Sqrt[5] - (8*ArcTanh[Cosh[x]/(2 + Sqrt[5] + Sinh[x])])/Sqrt[5]
```

Rubi [A] time = 0.037192, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2735, 2657}

$$\frac{4x}{\sqrt{5}}-x-\frac{8\tanh^{-1}\left(\frac{\cosh(x)}{\sinh(x)+\sqrt{5}+2}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Sinh[x])/(2 + Sinh[x]),x]
```

```
[Out] -x + (4*x)/Sqrt[5] - (8*ArcTanh[Cosh[x]/(2 + Sqrt[5] + Sinh[x])])/Sqrt[5]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx &= -x+4\int \frac{1}{2+\sinh(x)}\,dx\\ &= -x+\frac{4x}{\sqrt{5}}-\frac{8\tanh^{-1}\left(\frac{\cosh(x)}{2+\sqrt{5}+\sinh(x)}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.0855739, size = 28, normalized size = 0.82

$$-x-\frac{8\tanh^{-1}\left(\frac{1-2\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Sinh[x])/(2 + Sinh[x]),x]
```

```
[Out] -x - (8*ArcTanh[(1 - 2*Tanh[x/2])/Sqrt[5]])/Sqrt[5]
```

Maple [A] time = 0.016, size = 37, normalized size = 1.1

$$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{8\sqrt{5}}{5}\mathrm{Artanh}\left(\frac{\sqrt{5}}{5}\left(2\tanh\left(x/2\right)-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-sinh(x))/(2+sinh(x)),x)
```

```
[Out] -ln(tanh(1/2*x)+1)+ln(tanh(1/2*x)-1)+8/5*5^(1/2)*arctanh(1/5*(2*tanh(1/2*x)
-1)*5^(1/2))
```

Maxima [A] time = 1.56699, size = 46, normalized size = 1.35

$$\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-e^{(-x)}+2}{\sqrt{5}+e^{(-x)}-2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="maxima")
```

```
[Out] 4/5*sqrt(5)*log(-(sqrt(5) - e^(-x) + 2)/(sqrt(5) + e^(-x) - 2)) - x
```

Fricas [A] time = 1.75286, size = 139, normalized size = 4.09

$$\frac{4}{5}\sqrt{5}\log\left(-\frac{\left(2\sqrt{5}-5\right)\cosh\left(x\right)-2\left(\sqrt{5}-2\right)\sinh\left(x\right)+\sqrt{5}-2}{\sinh\left(x\right)+2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="fricas")
```

```
[Out] 4/5*sqrt(5)*log(-((2*sqrt(5) - 5)*cosh(x) - 2*(sqrt(5) - 2)*sinh(x) + sqrt(
5) - 2)/(sinh(x) + 2)) - x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20581, size = 45, normalized size = 1.32

$$\frac{4}{5}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+2e^{x}+4\right|}{2\left(\sqrt{5}+e^{x}+2\right)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="giac")
```

```
[Out] 4/5*sqrt(5)*log(1/2*abs(-2*sqrt(5) + 2*e^x + 4)/(sqrt(5) + e^x + 2)) - x
```

3.137 $\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=136

$$\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{b\sqrt{a+b\sinh(x)}}+\frac{2iB\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] ((2*I)*B*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b
*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*(A*b - a*B)*EllipticF[Pi/4 - (I/
2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[
x]])
```

Rubi [A] time = 0.120167, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}+\frac{2iB\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
```


```
[Out] ((2*I)*B*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b
*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*(A*b - a*B)*EllipticF[Pi/4 - (I/
2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[
x]])
```


Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx &= \frac{B\int\sqrt{a+b\sinh(x)}\,dx}{b}+\frac{(i(-iAb+iaB))\int\frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b}\\ &= \frac{\left(B\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{\left(i(-iAb+iaB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{b\sqrt{a+b\sinh(x)}}\\ &= \frac{2iBE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2i(Ab-aB)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}\end{aligned}$$

Mathematica [A] time = 0.503481, size = 109, normalized size = 0.8

$$\frac{2\sqrt{\frac{a+b\sinh(x)}{a-ib}}\left(i(Ab-aB)\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+B(b+ia)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*((I*a + b)*B*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + I*(A*b
- a*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sinh[
x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Maple [A] time = 0.092, size = 266, normalized size = 2.

$$-2\,\frac{ib-a}{b^2\cosh(x)\sqrt{a+b\sinh(x)}}\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))\,b}{ib+a}}\sqrt{\frac{(i+\sinh(x))\,b}{ib-a}}\left(-iB\text{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x)
```

```
[Out] -2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)*(-I*B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(
-(I*b-a)/(I*b+a))^(1/2))*b+I*B*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b+A*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))*b-B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x) + A}{\sqrt{b\sinh(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\sinh(x) + A}{\sqrt{b\sinh(x) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sinh(x)}{\sqrt{a + b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral((A + B*sinh(x))/sqrt(a + b*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x) + A}{\sqrt{b\sinh(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

3.138 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2iB\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{b\sqrt{a+b\sinh(x)}}-\frac{2\cosh(x)(Ab-aB)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] (-2*(A*b - a*B)*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*(A*b -
a*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*(a^
2 + b^2)*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*B*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```

Rubi [A] time = 0.226655, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\cosh(x)(Ab-aB)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2iB\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*(A*b -
a*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*(a^
2 + b^2)*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*B*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx &= -\frac{2(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}-\frac{2\int\frac{\frac{1}{2}(-aA-bB)-\frac{1}{2}(Ab-aB)\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{a^2+b^2}\\
&= -\frac{2(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{B\int\frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b}+\frac{(Ab-aB)\int\sqrt{a+b\sinh(x)}\,dx}{b\left(a^2+b^2\right)}\\
&= -\frac{2(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\left((Ab-aB)\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{\left(B\sqrt{\frac{a+b\sinh(\ldots}{a-ib}}\ldots\right.}{b\sqrt{a\ldots}}\\
&= -\frac{2(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2iBF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\ldots}{b\sqrt{a+b\sin\ldots}}
\end{aligned}
$$

Mathematica [A] time = 0.618416, size = 159, normalized size = 0.9

$$
\frac{2iB\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+2b\cosh(x)(aB-Ab)+\frac{2i(Ab-aB)(a+b\sinh(x))E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}}{b\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*(-(A*b) + a*B)*Cosh[x] + ((2*I)*(A*b - a*B)*EllipticE[(Pi - (2*I)*x)/4
, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x]))/Sqrt[(a + b*Sinh[x])/(a - I*b)] +
(2*I)*(a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[
(a + b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Maple [B] time = 0.163, size = 517, normalized size = 2.9

$$\frac{1}{\cosh(x)}\sqrt{(a+b\sinh(x))(\cosh(x))^2}\left(2\frac{B}{b\sqrt{(a+b\sinh(x))(\cosh(x))^2}}\left(\frac{a}{b}-i\right)\sqrt{\frac{-b\sinh(x)-a}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x)
```

```
[Out] ((a+b*sinh(x))*cosh(x)^2)^(1/2)*(2*B/b*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/
2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x
))*cosh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+
a))^(1/2))+(A*b-B*a)/b*(-2*b*cosh(x)^2/(a^2+b^2)/((a+b*sinh(x))*cosh(x)^2)^
(1/2)+2*a/(a^2+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)
*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+2*b/(a^2
+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*
((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((-a/b-I)*Ell
ipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(
((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))))/cosh(x)/(a+b*sin
h(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x)+A}{(b\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sinh(x)+A)\sqrt{b\sinh(x)+a}}{b^2\sinh(x)^2+2\,ab\sinh(x)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*sinh(x) + A)*sqrt(b*sinh(x) + a)/(b^2*sinh(x)^2 + 2*a*b*sinh(x)
 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x) + A}{(b\sinh(x) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

3.139 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{b+ia}\right)}{3b\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}$$

```
[Out] (-2*(A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (2*(4*a*A*
b - a^2*B + 3*b^2*B)*Cosh[x])/(3*(a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((2*
I)/3)*(4*a*A*b - a^2*B + 3*b^2*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)
]*Sqrt[a + b*Sinh[x]])/(b*(a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(a - I*b)]) -
(((2*I)/3)*(A*b - a*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
 b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rubi [A] time = 0.334651, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (2*(4*a*A*
b - a^2*B + 3*b^2*B)*Cosh[x])/(3*(a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((2*
I)/3)*(4*a*A*b - a^2*B + 3*b^2*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)
]*Sqrt[a + b*Sinh[x]])/(b*(a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(a - I*b)]) -
(((2*I)/3)*(A*b - a*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
 b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aA+bB)+\frac{1}{2}(Ab-aB)\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2A-Ab^2+4abB\right)+\frac{1}{4}(4aA\ldots}{\sqrt{a+b\sinh\ldots}}}{3\left(a^2+b^{\ldots}\right.}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} - \frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{3b\left(a^2+b^2\right)}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{\left(\left(4aAb-a^2B+3b^2B\right)\sqrt{a\ldots}\right.}{3b\left(a^2+\ldots\right.}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{2i\left(4aAb-a^2B+3b^2B\right)E\ldots}{3b\left(a^2+b^2\ldots\right.}$$

Mathematica [A] time = 0.776822, size = 236, normalized size = 0.94

$$\frac{2i\left(\sqrt{\frac{a+b\sinh(x)}{a-ib}}(a+b\sinh(x))\left(b\left(3a^2A+4abB-Ab^2\right)\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+\left(a^2(-B)+4aAb+3b^2B\right)\left((a-ib)\ldots\right.\right.\right.}{3b\left(a^2+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (((2*I)/3)*((b*(3*a^2*A - A*b^2 + 4*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2
*I)*b)/(a - I*b)] + (4*a*A*b - a^2*B + 3*b^2*B)*((a - I*b)*EllipticE[(Pi -
(2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b
)/(a - I*b)]))*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a - I*b)] + I*b*Cosh[x
```

```
]*(-((a^2 + b^2)*(-(A*b) + a*B)) - (-4*a*A*b + a^2*B - 3*b^2*B)*(a + b*Sinh
[x]))))/(b*(a^2 + b^2)^2*(a + b*Sinh[x])^(3/2))
```

Maple [B] time = 0.239, size = 806, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x)
```

```
[Out] ((a+b*sinh(x))*cosh(x)^2)^(1/2)*((A*b-B*a)/b*(-2/3/b/(a^2+b^2)*((a+b*sinh(x
))*cosh(x)^2)^(1/2)/(sinh(x)+a/b)^2-8/3*b*cosh(x)^2/(a^2+b^2)^2*a/((a+b*sin
h(x))*cosh(x)^2)^(1/2)+2*(3*a^2-b^2)/(3*a^4+6*a^2*b^2+3*b^4)*(a/b-I)*((-b*s
inh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-
a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a)
)^(1/2),((a-I*b)/(I*b+a))^(1/2))+8/3*a*b/(a^2+b^2)^2*(a/b-I)*((-b*sinh(x)-a
)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2
)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((-a/b-I)*EllipticE(((-b*sinh(x)-a)/(I*b-
a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/
2),((a-I*b)/(I*b+a))^(1/2))))+B/b*(-2*b*cosh(x)^2/(a^2+b^2)/((a+b*sinh(x))*
cosh(x)^2)^(1/2)+2*a/(a^2+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-s
inh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(
x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2
))+2*b/(a^2+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b
+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((
-a/b-I)*EllipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I
*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))))/cosh(
x)/(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sinh(x) + A}{(b\sinh(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sinh(x) + A)\sqrt{b\sinh(x) + a}}{b^3\sinh(x)^3 + 3\,ab^2\sinh(x)^2 + 3\,a^2b\sinh(x) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*sinh(x) + A)*sqrt(b*sinh(x) + a)/(b^3*sinh(x)^3 + 3*a*b^2*sinh(
x)^2 + 3*a^2*b*sinh(x) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B \sinh(x) + A}{(b \sinh(x) + a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

3.140 $\int \left(a \sinh^2(x)\right)^{5/2} dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\coth(x)\sqrt{a\sinh^2(x)}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}-\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}$$

```
[Out] (8*a^2*Coth[x]*Sqrt[a*Sinh[x]^2])/15 - (4*a*Coth[x]*(a*Sinh[x]^2)^(3/2))/15
 + (Coth[x]*(a*Sinh[x]^2)^(5/2))/5
```

Rubi [A] time = 0.0371284, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3203, 3207, 2638}

$$\frac{8}{15}a^2\coth(x)\sqrt{a\sinh^2(x)}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}-\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Coth[x]*Sqrt[a*Sinh[x]^2])/15 - (4*a*Coth[x]*(a*Sinh[x]^2)^(3/2))/15
 + (Coth[x]*(a*Sinh[x]^2)^(5/2))/5
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a\sinh^2(x)\right)^{5/2} dx &= \frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}-\frac{1}{5}(4a)\int \left(a\sinh^2(x)\right)^{3/2} dx\\ &= -\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\right)\int \sqrt{a\sinh^2(x)}\,dx\\ &= -\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\mathbf{csch}(x)\sqrt{a\sinh^2(x)}\right)\int \text{si}\\ &= \frac{8}{15}a^2\coth(x)\sqrt{a\sinh^2(x)}-\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0408078, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2(150\cosh(x)-25\cosh(3x)+3\cosh(5x))\mathbf{csch}(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(5/2),x]
```

```
[Out] (a^2*(150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x]*Sqrt[a*Sinh[x]^2])/
240
```

Maple [A] time = 0.045, size = 32, normalized size = 0.6

$$\frac{a^3\sinh(x)\cosh(x)\left(3(\sinh(x))^4-4(\sinh(x))^2+8\right)}{15}\frac{1}{\sqrt{a(\sinh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(5/2),x)
```

```
[Out] 1/15*a^3*sinh(x)*cosh(x)*(3*sinh(x)^4-4*sinh(x)^2+8)/(a*sinh(x)^2)^(1/2)
```

Maxima [A] time = 1.81499, size = 72, normalized size = 1.36

$$-\frac{1}{160}a^{\frac{5}{2}}e^{(5x)}+\frac{5}{96}a^{\frac{5}{2}}e^{(3x)}-\frac{5}{16}a^{\frac{5}{2}}e^{(-x)}+\frac{5}{96}a^{\frac{5}{2}}e^{(-3x)}-\frac{1}{160}a^{\frac{5}{2}}e^{(-5x)}-\frac{5}{16}a^{\frac{5}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) + 5/96*
a^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) - 5/16*a^(5/2)*e^x
```

Fricas [B] time = 1.78509, size = 1462, normalized size = 27.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cosh
(x)^2 - 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 - 5*a^2*cosh(x))*e^x*sin
h(x)^7 + 10*(63*a^2*cosh(x)^4 - 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6 +
4*(189*a^2*cosh(x)^5 - 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)^5 +
 10*(63*a^2*cosh(x)^6 - 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 + 15*a^2)*e^x
*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 - 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^3 + 15
*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 - 140*a^2*cosh(x)^6 + 450
*a^2*cosh(x)^4 + 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a^2*cosh(
```

```
x)^9 - 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 + 60*a^2*cosh(x)^3 - 5*a^2*cosh(
x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 - 25*a^2*cosh(x)^8 + 150*a^2*cosh(x)^6
+ 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 + 3*a^2)*e^x)*sqrt(a*e^(4*x) - 2*a*e
^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) - 1)*sinh(x)^5 - cosh(x)^5
 + 5*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(2*x) - cosh(x
)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) - cosh(x)^3)*sinh(x)^2 + 5*(cosh(x)^
4*e^(2*x) - cosh(x)^4)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31344, size = 162, normalized size = 3.06

$$\frac{1}{480}\left(3\,a^2e^{(5\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^x\right)-25\,a^2e^{(3\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^x\right)+150\,a^2e^x\mathrm{sgn}\left(e^{(3\,x)}-e^x\right)+\left(150\,a^2e^{(4\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^x\right)-25\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*e^(5*x)*sgn(e^(3*x) - e^x) - 25*a^2*e^(3*x)*sgn(e^(3*x) - e^x)
 + 150*a^2*e^x*sgn(e^(3*x) - e^x) + (150*a^2*e^(4*x)*sgn(e^(3*x) - e^x) - 2
5*a^2*e^(2*x)*sgn(e^(3*x) - e^x) + 3*a^2*sgn(e^(3*x) - e^x))*e^(-5*x))*sqrt
(a)
```

3.141 $\int \left(a \sinh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=34

$$\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}$$

```
[Out] (-2*a*Coth[x]*Sqrt[a*Sinh[x]^2])/3 + (Coth[x]*(a*Sinh[x]^2)^(3/2))/3
```

Rubi [A] time = 0.0239005, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3203, 3207, 2638}

$$\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(3/2),x]
```

```
[Out] (-2*a*Coth[x]*Sqrt[a*Sinh[x]^2])/3 + (Coth[x]*(a*Sinh[x]^2)^(3/2))/3
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a\sinh^2(x)\right)^{3/2} dx &= \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{1}{3}(2a)\int \sqrt{a\sinh^2(x)}\,dx\\
&= \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{1}{3}\left(2a\operatorname{csch}(x)\sqrt{a\sinh^2(x)}\right)\int \sinh(x)\,dx\\
&= -\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}+\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0361036, size = 26, normalized size = 0.76

$$\frac{1}{12}a(\cosh(3x)-9\cosh(x))\operatorname{csch}(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(3/2),x]
```

```
[Out] (a*(-9*Cosh[x] + Cosh[3*x])*Csch[x]*Sqrt[a*Sinh[x]^2])/12
```

Maple [A] time = 0.035, size = 24, normalized size = 0.7

$$\frac{a^2 \sinh(x) \cosh(x) \left((\sinh(x))^2 - 2\right)}{3} \frac{1}{\sqrt{a (\sinh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(3/2),x)
```

```
[Out] 1/3*a^2*sinh(x)*cosh(x)*(sinh(x)^2-2)/(a*sinh(x)^2)^(1/2)
```

Maxima [A] time = 1.81804, size = 47, normalized size = 1.38

$$-\frac{1}{24} a^{\frac{3}{2}} e^{(3x)} + \frac{3}{8} a^{\frac{3}{2}} e^{(-x)} - \frac{1}{24} a^{\frac{3}{2}} e^{(-3x)} + \frac{3}{8} a^{\frac{3}{2}} e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*a^(3/2)*e^(3*x) + 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a^
(3/2)*e^x
```

Fricas [B] time = 1.79921, size = 672, normalized size = 19.76

$$\frac{\left(6 a \cosh(x) e^{x} \sinh(x)^5 + a e^{x} \sinh(x)^6 + 3\left(5 a \cosh(x)^2 - 3 a\right) e^{x} \sinh(x)^4 + 4\left(5 a \cosh(x)^3 - 9 a \cosh(x)\right) e^{x} \sinh(\right.}{24\left(\cosh(x)^3 e^{(2x)} + \left(e^{(2x)} - 1\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 - 3*a)
*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*cos
h(x)^4 - 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 - 6*a*cosh(x)
^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 - 9*a*cosh(x)^4 - 9*a*cosh(x)^
2 + a)*e^x)*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x) + (
e^(2*x) - 1)*sinh(x)^3 - cosh(x)^3 + 3*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^
2 + 3*(cosh(x)^2*e^(2*x) - cosh(x)^2)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sinh^2(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**2)**(3/2), x)
```

Giac [B] time = 1.18584, size = 95, normalized size = 2.79

$$-\frac{1}{24}\left(\left(9\,e^{(2\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)-\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)\right)e^{(-3\,x)}-e^{(3\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)+9\,e^{x}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x)*sgn(e^(3*x) - e^x) - sgn(e^(3*x) - e^x))*e^(-3*x) - e^(3*
x)*sgn(e^(3*x) - e^x) + 9*e^x*sgn(e^(3*x) - e^x))*a^(3/2)
```

3.142 $\int \sqrt{a \sinh^2(x)}\,dx$

Optimal. Leaf size=13

$$\coth(x)\sqrt{a\sinh^2(x)}$$

```
[Out] Coth[x]*Sqrt[a*Sinh[x]^2]
```

Rubi [A] time = 0.0128127, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 2638}

$$\coth(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sinh[x]^2],x]
```

```
[Out] Coth[x]*Sqrt[a*Sinh[x]^2]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\sinh^2(x)}\,dx &= \left(\operatorname{csch}(x)\sqrt{a\sinh^2(x)}\right)\int \sinh(x)\,dx\\ &= \coth(x)\sqrt{a\sinh^2(x)}\end{aligned}$$

Mathematica [A] time = 0.004513, size = 13, normalized size = 1.

$$\coth(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sinh[x]^2],x]
```

```
[Out] Coth[x]*Sqrt[a*Sinh[x]^2]
```

Maple [A] time = 0.03, size = 15, normalized size = 1.2

$$a\sinh\left(x\right)\cosh\left(x\right)\frac{1}{\sqrt{a\left(\sinh\left(x\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(1/2),x)
```

```
[Out] 1/(a*sinh(x)^2)^(1/2)*a*sinh(x)*cosh(x)
```

Maxima [A] time = 1.82899, size = 23, normalized size = 1.77

$$-\frac{1}{2}\sqrt{a}e^{(-x)}-\frac{1}{2}\sqrt{a}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*e^(-x) - 1/2*sqrt(a)*e^x
```

Fricas [B] time = 1.74253, size = 216, normalized size = 16.62

$$\frac{\left(2\,\cosh\left(x\right)e^{x}\sinh\left(x\right)+e^{x}\sinh\left(x\right)^{2}+\left(\cosh\left(x\right)^{2}+1\right)e^{x}\right)\sqrt{ae^{(4\,x)}-2\,ae^{(2\,x)}+a}e^{(-x)}}{2\left(\cosh\left(x\right)e^{(2\,x)}+\left(e^{(2\,x)}-1\right)\sinh\left(x\right)-\cosh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)*sqrt(a*e^
(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) - 1)*sinh(x) -
cosh(x))
```

Sympy [A] time = 0.609714, size = 19, normalized size = 1.46

$$\frac{\sqrt{a}\sqrt{\sinh^{2}\left(x\right)}\cosh\left(x\right)}{\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**2)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(sinh(x)**2)*cosh(x)/sinh(x)
```

Giac [B] time = 1.19944, size = 46, normalized size = 3.54

$$\frac{1}{2}\left(e^{(-x)}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)+e^{x}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x)*sgn(e^(3*x) - e^x) + e^x*sgn(e^(3*x) - e^x))*sqrt(a)
```

3.143 $\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{\sqrt{a\sinh^2(x)}}$$

```
[Out] -((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[a*Sinh[x]^2])
```

Rubi [A] time = 0.0133405, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3770}

$$-\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{\sqrt{a\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sinh[x]^2],x]
```

```
[Out] -((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[a*Sinh[x]^2])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx &= \frac{\sinh(x)\int \text{csch}(x)\,dx}{\sqrt{a\sinh^2(x)}}\\ &= -\frac{\tanh^{-1}(\cosh(x))\sinh(x)}{\sqrt{a\sinh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0056344, size = 20, normalized size = 1.18

$$\frac{\sinh(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sinh[x]^2],x]
```

```
[Out] (Log[Tanh[x/2]]*Sinh[x])/Sqrt[a*Sinh[x]^2]
```

Maple [B] time = 0.053, size = 49, normalized size = 2.9

$$-\frac{\sinh(x)}{\cosh(x)}\sqrt{a(\cosh(x))^2}\ln\left(2\,\frac{\sqrt{a}\sqrt{a(\cosh(x))^2}+a}{\sinh(x)}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{a(\sinh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(1/2),x)
```

```
[Out] -sinh(x)*(a*cosh(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2)+a)/s
inh(x))/cosh(x)/(a*sinh(x)^2)^(1/2)
```

Maxima [A] time = 1.80917, size = 32, normalized size = 1.88

$$\frac{\log\left(e^{(-x)}+1\right)}{\sqrt{a}}-\frac{\log\left(e^{(-x)}-1\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(-x) + 1)/sqrt(a) - log(e^(-x) - 1)/sqrt(a)
```

Fricas [B] time = 1.71426, size = 312, normalized size = 18.35

$$\left[\frac{\sqrt{ae^{(4x)}-2\,ae^{(2x)}+a}\log\left(\frac{\cosh(x)+\sinh(x)-1}{\cosh(x)+\sinh(x)+1}\right)}{ae^{(2x)}-a},\ \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{ae^{(4x)}-2\,ae^{(2x)}+a}\sqrt{-a}}{a\cosh(x)e^{(2x)}-a\cosh(x)+\left(ae^{(2x)}-a\right)\sinh(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*log((cosh(x) + sinh(x) - 1)/(cosh(x) + s
inh(x) + 1))/(a*e^(2*x) - a), 2*sqrt(-a)*arctan(sqrt(a*e^(4*x) - 2*a*e^(2*x
) + a)*sqrt(-a)/(a*cosh(x)*e^(2*x) - a*cosh(x) + (a*e^(2*x) - a)*sinh(x)))/
a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\sinh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(x)**2), x)
```

Giac [B] time = 1.26213, size = 61, normalized size = 3.59

$$-\frac{\log\left(e^{x}+1\right)}{\sqrt{a}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)}+\frac{\log\left(|e^{x}-1|\right)}{\sqrt{a}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(e^x + 1)/(sqrt(a)*sgn(e^(3*x) - e^x)) + log(abs(e^x - 1))/(sqrt(a)*sgn
(e^(3*x) - e^x))
```

3.144 $\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}$$

```
[Out] -Coth[x]/(2*a*Sqrt[a*Sinh[x]^2]) + (ArcTanh[Cosh[x]]*Sinh[x])/(2*a*Sqrt[a*S
inh[x]^2])
```

Rubi [A] time = 0.0262079, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3204, 3207, 3770}

$$\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(-3/2),x]
```

```
[Out] -Coth[x]/(2*a*Sqrt[a*Sinh[x]^2]) + (ArcTanh[Cosh[x]]*Sinh[x])/(2*a*Sqrt[a*S
inh[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} - \frac{\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx}{2a}$$
$$= -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} - \frac{\sinh(x)\int \operatorname{csch}(x)\,dx}{2a\sqrt{a\sinh^2(x)}}$$
$$= -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} + \frac{\tanh^{-1}(\cosh(x))\sinh(x)}{2a\sqrt{a\sinh^2(x)}}$$

Mathematica [A] time = 0.0375964, size = 44, normalized size = 1.05

$$-\frac{\sinh^3(x)\left(\operatorname{csch}^2\left(\frac{x}{2}\right) + \operatorname{sech}^2\left(\frac{x}{2}\right) + 4\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{8\left(a\sinh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(-3/2),x]
```

```
[Out] -((Csch[x/2]^2 + 4*Log[Tanh[x/2]] + Sech[x/2]^2)*Sinh[x]^3)/(8*(a*Sinh[x]^2
)^(3/2))
```

Maple [B] time = 0.05, size = 71, normalized size = 1.7

$$-\frac{1}{2\cosh(x)\sinh(x)}\sqrt{a(\cosh(x))^2}\left(-\ln\left(2\frac{\sqrt{a}\sqrt{a(\cosh(x))^2}+a}{\sinh(x)}\right)a(\sinh(x))^2 + \sqrt{a}\sqrt{a(\cosh(x))^2}\right)a^{-\frac{5}{2}}\frac{1}{\sqrt{a(\sinh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(3/2),x)
```

```
[Out] -1/2/a^(5/2)/sinh(x)*(a*cosh(x)^2)^(1/2)*(-ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2
)+a)/sinh(x))*a*sinh(x)^2+a^(1/2)*(a*cosh(x)^2)^(1/2))/cosh(x)/(a*sinh(x)^2
)^(1/2)
```

Maxima [A] time = 1.75467, size = 84, normalized size = 2.

$$-\frac{e^{(-x)} + e^{(-3x)}}{2a^{\frac{3}{2}}e^{(-2x)} - a^{\frac{3}{2}}e^{(-4x)} - a^{\frac{3}{2}}} - \frac{\log\left(e^{(-x)}+1\right)}{2a^{\frac{3}{2}}} + \frac{\log\left(e^{(-x)}-1\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + e^(-3*x))/(2*a^(3/2)*e^(-2*x) - a^(3/2)*e^(-4*x) - a^(3/2)) - 1/
2*log(e^(-x) + 1)/a^(3/2) + 1/2*log(e^(-x) - 1)/a^(3/2)
```

Fricas [B] time = 1.81061, size = 927, normalized size = 22.07

$$\frac{\left(6 \cosh(x) e^{x} \sinh(x)^{2} + 2 e^{x} \sinh(x)^{3} + 2 \left(3 \cosh(x)^{2} + 1\right) e^{x} \sinh(x) + 2 \left(\cosh(x)^{3} + \cosh(x)\right) e^{x} - \left(4 \cosh(x) e^{x} s\right.}{2 \left(a^{2} \cosh(x)^{4} - \left(a^{2} e^{(2x)} - a^{2}\right) \sinh(x)^{4} - 2 a^{2} \cosh(x)^{2} - 4 \left(a^{2} \cosh(x) e^{(2x)} - a^{2} \cosh(x)\right) \sinh(x)^{3} + 2 \left(3 a^{2} co\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(x)*e^x*sinh(x)^2 + 2*e^x*sinh(x)^3 + 2*(3*cosh(x)^2 + 1)*e^x*si
nh(x) + 2*(cosh(x)^3 + cosh(x))*e^x - (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x
)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 - cosh(x))*e^x*sinh(
x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*e^x)*log((cosh(x) + sinh(x) + 1)/(cosh(x
) + sinh(x) - 1)))*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(x)^4
- (a^2*e^(2*x) - a^2)*sinh(x)^4 - 2*a^2*cosh(x)^2 - 4*(a^2*cosh(x)*e^(2*x)
- a^2*cosh(x))*sinh(x)^3 + 2*(3*a^2*cosh(x)^2 - a^2 - (3*a^2*cosh(x)^2 - a^
2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*cosh(x)^4 - 2*a^2*cosh(x)^2 + a^2)*e^(2*
x) + 4*(a^2*cosh(x)^3 - a^2*cosh(x) - (a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x)
)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sinh^{2}(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**2)**(-3/2), x)
```

Giac [B] time = 1.33836, size = 127, normalized size = 3.02

$$\frac{\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{\sqrt{a}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)} - \frac{\log\left(e^{(-x)}+e^{x}-2\right)}{\sqrt{a}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)} - \frac{4\left(e^{(-x)}+e^{x}\right)}{\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)\sqrt{a}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(log(e^(-x) + e^x + 2)/(sqrt(a)*sgn(e^(3*x) - e^x)) - log(e^(-x) + e^x
- 2)/(sqrt(a)*sgn(e^(3*x) - e^x)) - 4*(e^(-x) + e^x)/(((e^(-x) + e^x)^2 - 4
)*sqrt(a)*sgn(e^(3*x) - e^x)))/a
```

3.145 $\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}-\frac{3\sinh(x)\tanh^{-1}(\cosh(x))}{8a^2\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

```
[Out] -Coth[x]/(4*a*(a*Sinh[x]^2)^(3/2)) + (3*Coth[x])/(8*a^2*Sqrt[a*Sinh[x]^2])
- (3*ArcTanh[Cosh[x]]*Sinh[x])/(8*a^2*Sqrt[a*Sinh[x]^2])
```

Rubi [A] time = 0.0369936, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3204, 3207, 3770}

$$\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}-\frac{3\sinh(x)\tanh^{-1}(\cosh(x))}{8a^2\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(-5/2),x]
```

```
[Out] -Coth[x]/(4*a*(a*Sinh[x]^2)^(3/2)) + (3*Coth[x])/(8*a^2*Sqrt[a*Sinh[x]^2])
- (3*ArcTanh[Cosh[x]]*Sinh[x])/(8*a^2*Sqrt[a*Sinh[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} - \frac{3\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx}{4a}$$

$$= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} + \frac{3\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx}{8a^2}$$

$$= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} + \frac{(3\sinh(x))\int \operatorname{csch}(x)\,dx}{8a^2\sqrt{a\sinh^2(x)}}$$

$$= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} - \frac{3\tanh^{-1}(\cosh(x))\sinh(x)}{8a^2\sqrt{a\sinh^2(x)}}$$

Mathematica [A] time = 0.100122, size = 67, normalized size = 1.1

$$-\frac{\operatorname{csch}(x)\sqrt{a\sinh^2(x)}\left(\operatorname{csch}^4\left(\frac{x}{2}\right) - 6\operatorname{csch}^2\left(\frac{x}{2}\right) - \operatorname{sech}^4\left(\frac{x}{2}\right) - 6\operatorname{sech}^2\left(\frac{x}{2}\right) - 24\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{64a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(-5/2),x]
```

```
[Out] -(Csch[x]*(-6*Csch[x/2]^2 + Csch[x/2]^4 - 24*Log[Tanh[x/2]] - 6*Sech[x/2]^2
 - Sech[x/2]^4)*Sqrt[a*Sinh[x]^2])/(64*a^3)
```

Maple [A] time = 0.053, size = 89, normalized size = 1.5

$$\frac{1}{8\,(\sinh(x))^3\cosh(x)}\sqrt{a\,(\cosh(x))^2}\left(-3\,\ln\left(2\,\frac{\sqrt{a}\sqrt{a\,(\cosh(x))^2}+a}{\sinh(x)}\right)a\,(\sinh(x))^4 + 3\sqrt{a\,(\cosh(x))^2}\,(\sinh(x))^2\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(5/2),x)
```

```
[Out] 1/8/a^(7/2)/sinh(x)^3*(a*cosh(x)^2)^(1/2)*(-3*ln(2*(a^(1/2)*(a*cosh(x)^2)^(
1/2)+a)/sinh(x))*a*sinh(x)^4+3*(a*cosh(x)^2)^(1/2)*sinh(x)^2*a^(1/2)-2*a^(1
/2)*(a*cosh(x)^2)^(1/2))/cosh(x)/(a*sinh(x)^2)^(1/2)
```

Maxima [A] time = 1.82704, size = 130, normalized size = 2.13

$$\frac{3\,e^{(-x)} - 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} + 3\,e^{(-7\,x)}}{4\left(4\,a^{\frac{5}{2}}e^{(-2\,x)} - 6\,a^{\frac{5}{2}}e^{(-4\,x)} + 4\,a^{\frac{5}{2}}e^{(-6\,x)} - a^{\frac{5}{2}}e^{(-8\,x)} - a^{\frac{5}{2}}\right)} + \frac{3\,\log\left(e^{(-x)}+1\right)}{8\,a^{\frac{5}{2}}} - \frac{3\,\log\left(e^{(-x)}-1\right)}{8\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(-x) - 11*e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*a^(5/2)*e^(-2*x)
 - 6*a^(5/2)*e^(-4*x) + 4*a^(5/2)*e^(-6*x) - a^(5/2)*e^(-8*x) - a^(5/2)) +
3/8*log(e^(-x) + 1)/a^(5/2) - 3/8*log(e^(-x) - 1)/a^(5/2)
```

Fricas [B] time = 1.96298, size = 2514, normalized size = 41.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(42*cosh(x)*e^x*sinh(x)^6 + 6*e^x*sinh(x)^7 + 2*(63*cosh(x)^2 - 11)*e^
x*sinh(x)^5 + 10*(21*cosh(x)^3 - 11*cosh(x))*e^x*sinh(x)^4 + 2*(105*cosh(x)
^4 - 110*cosh(x)^2 - 11)*e^x*sinh(x)^3 + 2*(63*cosh(x)^5 - 110*cosh(x)^3 -
33*cosh(x))*e^x*sinh(x)^2 + 2*(21*cosh(x)^6 - 55*cosh(x)^4 - 33*cosh(x)^2 +
 3)*e^x*sinh(x) + 2*(3*cosh(x)^7 - 11*cosh(x)^5 - 11*cosh(x)^3 + 3*cosh(x))
*e^x + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*e^x
*sinh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 -
30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x)
)*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^x*sinh
(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*e^x*sinh(x) + (
cosh(x)^8 - 4*cosh(x)^6 + 6*cosh(x)^4 - 4*cosh(x)^2 + 1)*e^x)*log((cosh(x)
+ sinh(x) - 1)/(cosh(x) + sinh(x) + 1)))*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*
e^(-x)/(a^3*cosh(x)^8 - 4*a^3*cosh(x)^6 - (a^3*e^(2*x) - a^3)*sinh(x)^8 - 8
*(a^3*cosh(x)*e^(2*x) - a^3*cosh(x))*sinh(x)^7 + 6*a^3*cosh(x)^4 + 4*(7*a^3
*cosh(x)^2 - a^3 - (7*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^6 + 8*(7*a^3*co
sh(x)^3 - 3*a^3*cosh(x) - (7*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(2*x))*sinh(x
)^5 - 4*a^3*cosh(x)^2 + 2*(35*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + 3*a^3 - (3
5*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)^4 + 8*(7*a^3*c
osh(x)^5 - 10*a^3*cosh(x)^3 + 3*a^3*cosh(x) - (7*a^3*cosh(x)^5 - 10*a^3*cos
h(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 4*(7*a^3*cosh(x)^6 - 15*
a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 - a^3 - (7*a^3*cosh(x)^6 - 15*a^3*cosh(x)^4
 + 9*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^2 - (a^3*cosh(x)^8 - 4*a^3*cosh(
x)^6 + 6*a^3*cosh(x)^4 - 4*a^3*cosh(x)^2 + a^3)*e^(2*x) + 8*(a^3*cosh(x)^7
- 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 - a^3*cosh(x) - (a^3*cosh(x)^7 - 3*a^3*
cosh(x)^5 + 3*a^3*cosh(x)^3 - a^3*cosh(x))*e^(2*x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**2)**(5/2),x)
```

```
[Out] Integral((a*sinh(x)**2)**(-5/2), x)
```

Giac [B] time = 1.36389, size = 149, normalized size = 2.44

$$-\frac{3\,\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}+\frac{3\,\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}+\frac{3\,\sqrt{a}\left(e^{(-x)}+e^{x}\right)^{3}-20\,\sqrt{a}\left(e^{(-x)}+e^{x}\right)}{4\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}a^{3}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -3/16*log(e^(-x) + e^x + 2)/(a^(5/2)*sgn(e^(3*x) - e^x)) + 3/16*log(e^(-x)
+ e^x - 2)/(a^(5/2)*sgn(e^(3*x) - e^x)) + 1/4*(3*sqrt(a)*(e^(-x) + e^x)^3 -
 20*sqrt(a)*(e^(-x) + e^x))/(((e^(-x) + e^x)^2 - 4)^2*a^3*sgn(e^(3*x) - e^x
))
```

3.146 $\int \left(a\sinh^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=135

$$\frac{26}{77}ia^2\sqrt{i\sinh(x)}\operatorname{csch}^2(x)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{a\sinh^3(x)}+\frac{2}{15}a^2\sinh^5(x)\cosh(x)\sqrt{a\sinh^3(x)}-\frac{26}{165}a^2\sinh^3(x)\cosh$$

```
[Out] (-26*a^2*Coth[x]*Sqrt[a*Sinh[x]^3])/77 + ((26*I)/77)*a^2*Csch[x]^2*Elliptic
F[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3] + (78*a^2*Cosh[x]*Si
nh[x]*Sqrt[a*Sinh[x]^3])/385 - (26*a^2*Cosh[x]*Sinh[x]^3*Sqrt[a*Sinh[x]^3])
/165 + (2*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]^3])/15
```

Rubi [A] time = 0.0605775, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3207, 2635, 2642, 2641}

$$\frac{2}{15}a^2\sinh^5(x)\cosh(x)\sqrt{a\sinh^3(x)}-\frac{26}{165}a^2\sinh^3(x)\cosh(x)\sqrt{a\sinh^3(x)}+\frac{78}{385}a^2\sinh(x)\cosh(x)\sqrt{a\sinh^3(x)}-\frac{26}{77}a^2\mathrm{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(5/2),x]
```


```
[Out] (-26*a^2*Coth[x]*Sqrt[a*Sinh[x]^3])/77 + ((26*I)/77)*a^2*Csch[x]^2*Elliptic
F[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3] + (78*a^2*Cosh[x]*Si
nh[x]*Sqrt[a*Sinh[x]^3])/385 - (26*a^2*Cosh[x]*Sinh[x]^3*Sqrt[a*Sinh[x]^3])
/165 + (2*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]^3])/15
```


Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\sinh^3(x)\right)^{5/2} dx = \frac{\left(a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{15}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} - \frac{\left(13a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{11}{2}}(x)\,dx}{15\sinh^{\frac{3}{2}}(x)}$$

$$= -\frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} + \frac{\left(39a^2\sqrt{a\sinh^3(\right.}{55\,\mathrm{si}}$$

$$= \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh(x)\sin$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{}$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{}$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{26}{77}ia^2\mathrm{csch}^2(x)F\left(\frac{\pi}{4} - \frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\,\mathrm{cc}$$

Mathematica [A] time = 0.182696, size = 67, normalized size = 0.5

$$\frac{a^2\mathrm{csch}(x)\sqrt{a\sinh^3(x)}\left(-\frac{12480\,\mathrm{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)}{\sqrt{i\sinh(x)}} - 15465\cosh(x) + 3657\cosh(3x) - 749\cosh(5x) + 77\cosh(7x)\right)}{36960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(5/2),x]
```

```
[Out] (a^2*Csch[x]*(-15465*Cosh[x] + 3657*Cosh[3*x] - 749*Cosh[5*x] + 77*Cosh[7*x
] - (12480*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]])*Sqrt[a*Sinh[x]^
3])/36960
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \left(a\left(\sinh(x)\right)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(5/2),x)
```

```
[Out] int((a*sinh(x)^3)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sinh(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sinh(x)^3)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sinh(x)^3}a^2\sinh(x)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sinh(x)^3)*a^2*sinh(x)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sinh(x)^3\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sinh(x)^3)^(5/2), x)
```

3.147 $\int \left(a\sinh^3(x)\right)^{3/2} dx$

Optimal. Leaf size=83

$$\frac{2}{9}a\sinh^2(x)\cosh(x)\sqrt{a\sinh^3(x)}-\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)}+\frac{14ia\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}}$$

```
[Out] (-14*a*Cosh[x]*Sqrt[a*Sinh[x]^3])/45 + (((14*I)/15)*a*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2]*Sqrt[a*Sinh[x]^3])/Sqrt[I*Sinh[x]] + (2*a*Cosh[x]*Sinh[x]^2
*Sqrt[a*Sinh[x]^3])/9
```

Rubi [A] time = 0.0436829, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3207, 2635, 2640, 2639}

$$\frac{2}{9}a\sinh^2(x)\cosh(x)\sqrt{a\sinh^3(x)}-\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)}+\frac{14ia\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(3/2),x]
```

```
[Out] (-14*a*Cosh[x]*Sqrt[a*Sinh[x]^3])/45 + (((14*I)/15)*a*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2]*Sqrt[a*Sinh[x]^3])/Sqrt[I*Sinh[x]] + (2*a*Cosh[x]*Sinh[x]^2
*Sqrt[a*Sinh[x]^3])/9
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = \frac{\left(a\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{9}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}$$
$$= \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} - \frac{\left(7a\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{5}{2}}(x)\,dx}{9\sinh^{\frac{3}{2}}(x)}$$
$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} + \frac{\left(7a\sqrt{a\sinh^3(x)}\right)\int \sqrt{\sinh(x)}\,dx}{15\sinh^{\frac{3}{2}}(x)}$$
$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} + \frac{\left(7a\text{csch}(x)\sqrt{a\sinh^3(x)}\right)\int \sqrt{i\sin}}{15\sqrt{i\sinh(x)}}$$
$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{14ia\text{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sin}$$

Mathematica [A] time = 0.0644298, size = 57, normalized size = 0.69

$$\frac{1}{180}a\text{csch}(x)\sqrt{a\sinh^3(x)}\left(-38\sinh(2x)+5\sinh(4x)+168\sqrt{i\sinh(x)}\text{csch}(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(3/2),x]
```

```
[Out] (a*Csch[x]*Sqrt[a*Sinh[x]^3]*(168*Csch[x]*EllipticE[(Pi - (2*I)*x)/4, 2]*Sq
rt[I*Sinh[x]] - 38*Sinh[2*x] + 5*Sinh[4*x]))/180
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \left(a(\sinh(x))^3\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(3/2),x)
```

```
[Out] int((a*sinh(x)^3)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sinh(x)^3\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sinh(x)^3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a \sinh(x)^3} a \sinh(x)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sinh(x)^3)*a*sinh(x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sinh^3(x)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sinh(x)^3\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sinh(x)^3)^(3/2), x)
```

3.148 $\int \sqrt{a\sinh^3(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{a\sinh^3(x)}$$

```
[Out] (2*Coth[x]*Sqrt[a*Sinh[x]^3])/3 - ((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (I/2
)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3]
```

Rubi [A] time = 0.0326089, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3207, 2635, 2642, 2641}

$$\frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sinh[x]^3],x]
```


```
[Out] (2*Coth[x]*Sqrt[a*Sinh[x]^3])/3 - ((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (I/2
)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3]
```


Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\sinh^3(x)}\,dx = \frac{\sqrt{a\sinh^3(x)}\int \sinh^{\frac{3}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}$$
$$= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{\sqrt{a\sinh^3(x)}\int \frac{1}{\sqrt{\sinh(x)}}\,dx}{3\sinh^{\frac{3}{2}}(x)}$$
$$= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{1}{3}\left(\operatorname{csch}^2(x)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx$$
$$= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\operatorname{csch}^2(x)F\left(\frac{\pi}{4} - \frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}$$

Mathematica [C] time = 0.0834606, size = 60, normalized size = 0.97

$$\frac{2}{3}\sqrt{a\sinh^3(x)}\left(\coth(x) - \sqrt{2}\operatorname{csch}^2(x)\sqrt{-\sinh(x)(\sinh(x)+\cosh(x))}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2x)+\sinh(2x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (2*Sqrt[a*Sinh[x]^3]*(Coth[x] - Sqrt[2]*Csch[x]^2*Hypergeometric2F1[1/4, 1/
2, 5/4, Cosh[2*x] + Sinh[2*x]]*Sqrt[-(Sinh[x]*(Cosh[x] + Sinh[x]))]))/3
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \sqrt{a(\sinh(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(1/2),x)
```

```
[Out] int((a*sinh(x)^3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sinh(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sinh(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\sinh(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sinh(x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sinh^3(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sinh(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sinh(x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sinh(x)^3), x)
```

3.149 $\int \frac{1}{\sqrt{a \sinh^3(x)}}\,dx$

Optimal. Leaf size=60

$$-\frac{2\sinh(x)\cosh(x)}{\sqrt{a\sinh^3(x)}}+\frac{2i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

```
[Out] (-2*Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^3] + ((2*I)*EllipticE[Pi/4 - (I/2)*x, 2
]*Sinh[x]^2)/(Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rubi [A] time = 0.0298034, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3207, 2636, 2640, 2639}

$$-\frac{2\sinh(x)\cosh(x)}{\sqrt{a\sinh^3(x)}}+\frac{2i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (-2*Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^3] + ((2*I)*EllipticE[Pi/4 - (I/2)*x, 2
]*Sinh[x]^2)/(Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx = \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\sinh^3(x)}}$$
$$= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}} + \frac{\sinh^{\frac{3}{2}}(x)\int \sqrt{\sinh(x)}\,dx}{\sqrt{a\sinh^3(x)}}$$
$$= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}} + \frac{\sinh^2(x)\int \sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$
$$= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}} + \frac{2iE\left(\frac{\pi}{4} - \frac{ix}{2}\middle|2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

Mathematica [A] time = 0.0297168, size = 42, normalized size = 0.7

$$-\frac{2\sinh(x)\left(\cosh(x) - \sqrt{i\sinh(x)}E\left(\frac{1}{4}(\pi - 2ix)\middle|2\right)\right)}{\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (-2*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/Sqr
t[a*Sinh[x]^3]
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sinh(x)\right)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sinh(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sinh(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a \sinh(x)^3}}{a \sinh(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sinh(x)^3)/(a*sinh(x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \sinh^3(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \sinh(x)^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sinh(x)^3), x)
```

3.150 $\int \frac{1}{\left(a \sinh^3(x)\right)^{3/2}} dx$

Optimal. Leaf size=87

$$\frac{10i\sqrt{i\sinh(x)}\sinh(x)\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)}{21a\sqrt{a\sinh^3(x)}}+\frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}$$

```
[Out] (10*Cosh[x])/(21*a*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x])/(7*a*Sqrt[a*Sin
h[x]^3]) + (((10*I)/21)*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sinh[x
])/(a*Sqrt[a*Sinh[x]^3])
```

Rubi [A] time = 0.0419059, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3207, 2636, 2642, 2641}

$$\frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}+\frac{10i\sqrt{i\sinh(x)}\sinh(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(-3/2),x]
```

```
[Out] (10*Cosh[x])/(21*a*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x])/(7*a*Sqrt[a*Sin
h[x]^3]) + (((10*I)/21)*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sinh[x
])/(a*Sqrt[a*Sinh[x]^3])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx &= \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\sinh^3(x)}}\\
&= -\frac{2\coth(x)\mathbf{csch}(x)}{7a\sqrt{a\sinh^3(x)}} - \frac{\left(5\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\sinh^3(x)}}\\
&= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\mathbf{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{\left(5\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\sinh(x)}}\,dx}{21a\sqrt{a\sinh^3(x)}}\\
&= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\mathbf{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{\left(5\sqrt{i\sinh(x)}\sinh(x)\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx}{21a\sqrt{a\sinh^3(x)}}\\
&= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\mathbf{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{10iF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\sinh(x)}{21a\sqrt{a\sinh^3(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.0820777, size = 53, normalized size = 0.61

$$
\frac{2\left(5(i\sinh(x))^{3/2}\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)+5\cosh(x)-3\coth(x)\mathbf{csch}(x)\right)}{21a\sqrt{a\sinh^3(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(-3/2),x]
```

```
[Out] (2*(5*Cosh[x] - 3*Coth[x]*Csch[x] + 5*EllipticF[(Pi - (2*I)*x)/4, 2]*(I*Sin
h[x])^(3/2)))/(21*a*Sqrt[a*Sinh[x]^3])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$
\int \left(a\left(\sinh(x)\right)^3\right)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\left(a\sinh(x)^3\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sinh(x)^3)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sinh\left(x\right)^{3}}}{a^{2}\sinh\left(x\right)^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sinh(x)^3)/(a^2*sinh(x)^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sinh^{3}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sinh\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sinh(x)^3)^(-3/2), x)
```

3.151 $\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\sinh^3(x)}}-\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}}-\frac{154i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}}+\frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}}$$

```
[Out] (-154*Coth[x])/(585*a^2*Sqrt[a*Sinh[x]^3]) + (22*Coth[x]*Csch[x]^2)/(117*a^
2*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x]^4)/(13*a^2*Sqrt[a*Sinh[x]^3]) + (
154*Cosh[x]*Sinh[x])/(195*a^2*Sqrt[a*Sinh[x]^3]) - (((154*I)/195)*EllipticE
[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/(a^2*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rubi [A] time = 0.0607026, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3207, 2636, 2640, 2639}

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\sinh^3(x)}}-\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}}-\frac{154i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}}+\frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(-5/2),x]
```

```
[Out] (-154*Coth[x])/(585*a^2*Sqrt[a*Sinh[x]^3]) + (22*Coth[x]*Csch[x]^2)/(117*a^
2*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x]^4)/(13*a^2*Sqrt[a*Sinh[x]^3]) + (
154*Cosh[x]*Sinh[x])/(195*a^2*Sqrt[a*Sinh[x]^3]) - (((154*I)/195)*EllipticE
[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/(a^2*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} - \frac{\left(11\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\sinh^3(x)}}$$

$$= \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{\left(77\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} - \frac{\left(77\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{3}{2}}(x)}\,dx}{195a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}} - \frac{(77\,\mathrm{si}}{1}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}} - \frac{(77\,\mathrm{si}}{195a^2}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}} - \frac{154i}{195a^2}$$

Mathematica [A] time = 0.196194, size = 69, normalized size = 0.51

$$\frac{462\sinh(x)\cosh(x) - 2\coth(x)\left(45\operatorname{csch}^4(x) - 55\operatorname{csch}^2(x) + 77\right) + 462i(i\sinh(x))^{3/2}E\left(\frac{1}{4}(\pi - 2ix)\middle|2\right)}{585a^2\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(-5/2),x]
```

```
[Out] (-2*Coth[x]*(77 - 55*Csch[x]^2 + 45*Csch[x]^4) + (462*I)*EllipticE[(Pi - (2
*I)*x)/4, 2]*(I*Sinh[x])^(3/2) + 462*Cosh[x]*Sinh[x])/(585*a^2*Sqrt[a*Sinh[
x]^3])
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \left(a\left(\sinh(x)\right)^3\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sinh(x)^3)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sinh\left(x\right)^3}}{a^3\sinh\left(x\right)^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sinh(x)^3)/(a^3*sinh(x)^9), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sinh(x)^3)^(-5/2), x)
```

3.152 $\int \left(a \sinh^4(x)\right)^{5/2} dx$

Optimal. Leaf size=132

$$\frac{1}{10}a^2 \sinh^7(x)\cosh(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2 \sinh^5(x)\cosh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2 \sinh^3(x)\cosh(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2 \mathrm{s}$$

```
[Out] (63*a^2*Coth[x]*Sqrt[a*Sinh[x]^4])/256 - (63*a^2*x*Csch[x]^2*Sqrt[a*Sinh[x]
^4])/256 - (21*a^2*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/128 + (21*a^2*Cosh[x]
*Sinh[x]^3*Sqrt[a*Sinh[x]^4])/160 - (9*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]
^4])/80 + (a^2*Cosh[x]*Sinh[x]^7*Sqrt[a*Sinh[x]^4])/10
```

Rubi [A] time = 0.0488886, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{10}a^2 \sinh^7(x)\cosh(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2 \sinh^5(x)\cosh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2 \sinh^3(x)\cosh(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2 \mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(5/2),x]
```

```
[Out] (63*a^2*Coth[x]*Sqrt[a*Sinh[x]^4])/256 - (63*a^2*x*Csch[x]^2*Sqrt[a*Sinh[x]
^4])/256 - (21*a^2*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/128 + (21*a^2*Cosh[x]
*Sinh[x]^3*Sqrt[a*Sinh[x]^4])/160 - (9*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]
^4])/80 + (a^2*Cosh[x]*Sinh[x]^7*Sqrt[a*Sinh[x]^4])/10
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a\sinh^4(x)\right)^{5/2} dx = \left(a^2\mathbf{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^{10}(x)\,dx$$
$$= \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)} - \frac{1}{10}\left(9a^2\mathbf{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^8(x)\,dx$$
$$= -\frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)} + \frac{1}{80}\left(63a^2\mathbf{csch}^2(x\right.$$
$$= \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\cosh(x)\sinh$$
$$= -\frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2\cosh(x)\sinh$$
$$= \frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a}$$
$$= \frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)} - \frac{63}{256}a^2x\mathbf{csch}^2(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(}$$

Mathematica [A] time = 0.158968, size = 53, normalized size = 0.4

$$\frac{a(-2520x + 2100\sinh(2x) - 600\sinh(4x) + 150\sinh(6x) - 25\sinh(8x) + 2\sinh(10x))\mathbf{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(5/2),x]
```

```
[Out] (a*Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x]
+ 150*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

Maple [A] time = 0.115, size = 177, normalized size = 1.3

$$\frac{\sqrt{8}\,(-1+\cosh(2x))\,\sqrt{2}}{10240\,\sinh(2x)}\sqrt{a(-1+\cosh(2x))(\cosh(2x)+1)}a^{\frac{3}{2}}\left(8\sqrt{a(\sinh(2x))^2}\sqrt{a}(\sinh(2x))^4 - 50\sqrt{a(\sinh(2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(5/2),x)
```

```
[Out] 1/10240*8^(1/2)*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*2^(1/
2)*a^(3/2)*(8*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^4-50*(a*sinh(2*x)^2)^
(1/2)*a^(1/2)*cosh(2*x)*sinh(2*x)^2+160*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(
2*x)^2-325*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(1/2
)*a^(1/2)-315*ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/(a*(
-1+cosh(2*x))^2)^(1/2)
```

Maxima [A] time = 1.73714, size = 135, normalized size = 1.02

$$-\frac{63}{256}a^{\frac{5}{2}}x - \frac{1}{20480}\left(25a^{\frac{5}{2}}e^{(-2x)} - 150a^{\frac{5}{2}}e^{(-4x)} + 600a^{\frac{5}{2}}e^{(-6x)} - 2100a^{\frac{5}{2}}e^{(-8x)} + 2100a^{\frac{5}{2}}e^{(-12x)} - 600a^{\frac{5}{2}}e^{(-14x)} + 150a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -63/256*a^(5/2)*x - 1/20480*(25*a^(5/2)*e^(-2*x) - 150*a^(5/2)*e^(-4*x) + 6
00*a^(5/2)*e^(-6*x) - 2100*a^(5/2)*e^(-8*x) + 2100*a^(5/2)*e^(-12*x) - 600*
a^(5/2)*e^(-14*x) + 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) + 2*a^(5/2
)*e^(-20*x) - 2*a^(5/2))*e^(10*x)
```

Fricas [B] time = 2.0534, size = 4963, normalized size = 37.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*(
76*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 - 15*a^
2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 - 255*a^2*cosh(x)^2 +
 10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 - 425*a^2*cosh(x)^3 + 5
0*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 - 1275*a^2*cosh(
x)^4 + 300*a^2*cosh(x)^2 - 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2*cosh(
x)^7 - 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sin
h(x)^13 + 60*(4199*a^2*cosh(x)^8 - 7735*a^2*cosh(x)^6 + 4550*a^2*cosh(x)^4
- 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*cosh(x)^9 -
 9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 - 2730*a^2*cosh(x)^3 + 315*a^2*cos
h(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 - 546975*a^2*cosh(x)^8
+ 600600*a^2*cosh(x)^6 - 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(x)^2 - 2520*
a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 - 60775*a^2*cosh(x)^9
+ 85800*a^2*cosh(x)^7 - 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(x)^3 - 2520*a^
2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 - 36465*a^2*cosh(x
)^10 + 64350*a^2*cosh(x)^8 - 60060*a^2*cosh(x)^6 + 34650*a^2*cosh(x)^4 - 75
60*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*a^2*cosh(x)^13 -
3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 - 8580*a^2*cosh(x)^7 + 6930*a^2*co
sh(x)^5 - 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sinh(x)^7 + 60*(12
92*a^2*cosh(x)^14 - 7735*a^2*cosh(x)^12 + 20020*a^2*cosh(x)^10 - 30030*a^2*
cosh(x)^8 + 32340*a^2*cosh(x)^6 - 17640*a^2*x*cosh(x)^4 - 980*a^2*cosh(x)^2
 + 10*a^2)*e^(2*x)*sinh(x)^6 + 24*(1292*a^2*cosh(x)^15 - 8925*a^2*cosh(x)^1
3 + 27300*a^2*cosh(x)^11 - 50050*a^2*cosh(x)^9 + 69300*a^2*cosh(x)^7 - 5292
0*a^2*x*cosh(x)^5 - 4900*a^2*cosh(x)^3 + 150*a^2*cosh(x))*e^(2*x)*sinh(x)^5
 + 30*(323*a^2*cosh(x)^16 - 2550*a^2*cosh(x)^14 + 9100*a^2*cosh(x)^12 - 200
20*a^2*cosh(x)^10 + 34650*a^2*cosh(x)^8 - 35280*a^2*x*cosh(x)^6 - 4900*a^2*
cosh(x)^4 + 300*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^4 + 120*(19*a^2*cosh
(x)^17 - 170*a^2*cosh(x)^15 + 700*a^2*cosh(x)^13 - 1820*a^2*cosh(x)^11 + 38
50*a^2*cosh(x)^9 - 5040*a^2*x*cosh(x)^7 - 980*a^2*cosh(x)^5 + 100*a^2*cosh(
x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x)^3 + 5*(76*a^2*cosh(x)^18 - 765*a^2*co
sh(x)^16 + 3600*a^2*cosh(x)^14 - 10920*a^2*cosh(x)^12 + 27720*a^2*cosh(x)^1
0 - 45360*a^2*x*cosh(x)^8 - 11760*a^2*cosh(x)^6 + 1800*a^2*cosh(x)^4 - 180*
a^2*cosh(x)^2 + 5*a^2)*e^(2*x)*sinh(x)^2 + 10*(4*a^2*cosh(x)^19 - 45*a^2*co
sh(x)^17 + 240*a^2*cosh(x)^15 - 840*a^2*cosh(x)^13 + 2520*a^2*cosh(x)^11 -
5040*a^2*x*cosh(x)^9 - 1680*a^2*cosh(x)^7 + 360*a^2*cosh(x)^5 - 60*a^2*cosh
(x)^3 + 5*a^2*cosh(x))*e^(2*x)*sinh(x) + (2*a^2*cosh(x)^20 - 25*a^2*cosh(x)
^18 + 150*a^2*cosh(x)^16 - 600*a^2*cosh(x)^14 + 2100*a^2*cosh(x)^12 - 5040*
a^2*x*cosh(x)^10 - 2100*a^2*cosh(x)^8 + 600*a^2*cosh(x)^6 - 150*a^2*cosh(x)
^4 + 25*a^2*cosh(x)^2 - 2*a^2)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*
e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^10*e^(4*x) - 2*cosh(x)^10*e^(2
*x) + (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^10 + cosh(x)^10 + 10*(cosh(x)*e^(4*
x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^9 + 45*(cosh(x)^2*e^(4*x) - 2*cos
```

```
h(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^8 + 120*(cosh(x)^3*e^(4*x) - 2*cosh(x)^
3*e^(2*x) + cosh(x)^3)*sinh(x)^7 + 210*(cosh(x)^4*e^(4*x) - 2*cosh(x)^4*e^(
2*x) + cosh(x)^4)*sinh(x)^6 + 252*(cosh(x)^5*e^(4*x) - 2*cosh(x)^5*e^(2*x)
+ cosh(x)^5)*sinh(x)^5 + 210*(cosh(x)^6*e^(4*x) - 2*cosh(x)^6*e^(2*x) + cos
h(x)^6)*sinh(x)^4 + 120*(cosh(x)^7*e^(4*x) - 2*cosh(x)^7*e^(2*x) + cosh(x)^
7)*sinh(x)^3 + 45*(cosh(x)^8*e^(4*x) - 2*cosh(x)^8*e^(2*x) + cosh(x)^8)*sin
h(x)^2 + 10*(cosh(x)^9*e^(4*x) - 2*cosh(x)^9*e^(2*x) + cosh(x)^9)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27297, size = 154, normalized size = 1.17

$$-\frac{1}{20480}\left(5040\,a^2x - 2\,a^2e^{(10\,x)} + 25\,a^2e^{(8\,x)} - 150\,a^2e^{(6\,x)} + 600\,a^2e^{(4\,x)} - 2100\,a^2e^{(2\,x)} - \left(5754\,a^2e^{(10\,x)} - 2100\,a^2e^{(8\,x)} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/20480*(5040*a^2*x - 2*a^2*e^(10*x) + 25*a^2*e^(8*x) - 150*a^2*e^(6*x) +
600*a^2*e^(4*x) - 2100*a^2*e^(2*x) - (5754*a^2*e^(10*x) - 2100*a^2*e^(8*x)
+ 600*a^2*e^(6*x) - 150*a^2*e^(4*x) + 25*a^2*e^(2*x) - 2*a^2)*e^(-10*x))*sq
rt(a)
```

3.153 $\int \left(a\sinh^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$\frac{1}{6}a\sinh^3(x)\cosh(x)\sqrt{a\sinh^4(x)}-\frac{5}{24}a\sinh(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)}-\frac{5}{16}ax\text{csch}^2(x)\sqrt{a\sinh^4(x)}$$

```
[Out] (5*a*Coth[x]*Sqrt[a*Sinh[x]^4])/16 - (5*a*x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/16
 - (5*a*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/24 + (a*Cosh[x]*Sinh[x]^3*Sqrt[a
*Sinh[x]^4])/6
```

Rubi [A] time = 0.0320572, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{6}a\sinh^3(x)\cosh(x)\sqrt{a\sinh^4(x)}-\frac{5}{24}a\sinh(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)}-\frac{5}{16}ax\text{csch}^2(x)\sqrt{a\sinh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(3/2),x]
```

```
[Out] (5*a*Coth[x]*Sqrt[a*Sinh[x]^4])/16 - (5*a*x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/16
 - (5*a*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/24 + (a*Cosh[x]*Sinh[x]^3*Sqrt[a
*Sinh[x]^4])/6
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a\sinh^4(x)\right)^{3/2} dx = \left(a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^6(x)\,dx$$
$$= \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{1}{6}\left(5a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^4(x)\,dx$$
$$= -\frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} + \frac{1}{8}\left(5a\text{csch}^2(x)\sqrt{a\sin\right.$$
$$= \frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)} - \frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(}$$
$$= \frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)} - \frac{5}{16}ax\text{csch}^2(x)\sqrt{a\sinh^4(x)} - \frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{1}{6}$$

Mathematica [A] time = 0.0905928, size = 38, normalized size = 0.49

$$\frac{1}{192}(-60x + 45\sinh(2x) - 9\sinh(4x) + \sinh(6x))\text{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(3/2),x]
```

```
[Out] (Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[6
*x]))/192
```

Maple [B] time = 0.091, size = 131, normalized size = 1.7

$$\frac{\sqrt{8}\,(-1+\cosh(2x))\,\sqrt{2}}{384\,\sinh(2x)}\sqrt{a\,(-1+\cosh(2x))\,(\cosh(2x)+1)}\sqrt{a}\left(2\sqrt{a\,(\sinh(2x))^2}\sqrt{a}\,(\sinh(2x))^2 - 9\cosh(2x)\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(3/2),x)
```

```
[Out] 1/384*8^(1/2)*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*2^(1/2)
*a^(1/2)*(2*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2-9*cosh(2*x)*(a*sinh(2
*x)^2)^(1/2)*a^(1/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)-15*ln(a^(1/2)*cosh(2*
x)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/(a*(-1+cosh(2*x))^2)^(1/2)
```

Maxima [A] time = 1.77556, size = 85, normalized size = 1.09

$$-\frac{5}{16}a^{\frac{3}{2}}x - \frac{1}{384}\left(9a^{\frac{3}{2}}e^{(-2x)} - 45a^{\frac{3}{2}}e^{(-4x)} + 45a^{\frac{3}{2}}e^{(-8x)} - 9a^{\frac{3}{2}}e^{(-10x)} + a^{\frac{3}{2}}e^{(-12x)} - a^{\frac{3}{2}}\right)e^{(6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -5/16*a^(3/2)*x - 1/384*(9*a^(3/2)*e^(-2*x) - 45*a^(3/2)*e^(-4*x) + 45*a^(3
/2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) + a^(3/2)*e^(-12*x) - a^(3/2))*e^(6*x)
```

Fricas [B] time = 1.84462, size = 2068, normalized size = 26.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*cos
h(x)^2 - 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 - 9*a*cosh(x))*e^(2*x
)*sinh(x)^9 + 45*(11*a*cosh(x)^4 - 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8 + 7
2*(11*a*cosh(x)^5 - 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6*(15
4*a*cosh(x)^6 - 315*a*cosh(x)^4 + 210*a*cosh(x)^2 - 20*a*x)*e^(2*x)*sinh(x)
^6 + 36*(22*a*cosh(x)^7 - 63*a*cosh(x)^5 + 70*a*cosh(x)^3 - 20*a*x*cosh(x))
*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 - 42*a*cosh(x)^6 + 70*a*cosh(x)^4 -
 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 - 54*a*cosh(x
)^7 + 126*a*cosh(x)^5 - 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*sinh(x)^3
+ 3*(22*a*cosh(x)^10 - 135*a*cosh(x)^8 + 420*a*cosh(x)^6 - 600*a*x*cosh(x)^
4 - 90*a*cosh(x)^2 + 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11 - 15*a*cosh
(x)^9 + 60*a*cosh(x)^7 - 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 + 3*a*cosh(x))*
e^(2*x)*sinh(x) + (a*cosh(x)^12 - 9*a*cosh(x)^10 + 45*a*cosh(x)^8 - 120*a*x
*cosh(x)^6 - 45*a*cosh(x)^4 + 9*a*cosh(x)^2 - a)*e^(2*x))*sqrt(a*e^(8*x) -
4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^6*e^(4*x) -
2*cosh(x)^6*e^(2*x) + (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(
cosh(x)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 + 15*(cosh(x)^2*e^
(4*x) - 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(cosh(x)^3*e^(4*x)
- 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(x)^4*e^(4*x) - 2*co
sh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^5*e^(4*x) - 2*cosh(x)^5
*e^(2*x) + cosh(x)^5)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sinh^4(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**4)**(3/2), x)
```

Giac [A] time = 1.20904, size = 68, normalized size = 0.87

$$\frac{1}{384}\left(\left(110\,e^{(6\,x)}-45\,e^{(4\,x)}+9\,e^{(2\,x)}-1\right)e^{(-6\,x)}-120\,x+e^{(6\,x)}-9\,e^{(4\,x)}+45\,e^{(2\,x)}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*x
) - 9*e^(4*x) + 45*e^(2*x))*a^(3/2)
```

3.154 $\int \sqrt{a\sinh^4(x)}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)}-\frac{1}{2}x\mathrm{csch}^2(x)\sqrt{a\sinh^4(x)}$$

```
[Out] (Coth[x]*Sqrt[a*Sinh[x]^4])/2 - (x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/2
```

Rubi [A] time = 0.0149813, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)}-\frac{1}{2}x\mathrm{csch}^2(x)\sqrt{a\sinh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sinh[x]^4],x]
```

```
[Out] (Coth[x]*Sqrt[a*Sinh[x]^4])/2 - (x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/2
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\sinh^4(x)}\,dx &= \left(\mathrm{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^2(x)\,dx\\ &= \frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)}-\frac{1}{2}\left(\mathrm{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int 1\,dx\\ &= \frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)}-\frac{1}{2}x\mathrm{csch}^2(x)\sqrt{a\sinh^4(x)}\end{aligned}$$

Mathematica [A] time = 0.0380094, size = 24, normalized size = 0.67

$$\frac{1}{2}\sqrt{a\sinh^4(x)}\left(\coth(x)-x\mathrm{csch}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sinh[x]^4],x]
```

```
[Out] ((Coth[x] - x*Csch[x]^2)*Sqrt[a*Sinh[x]^4])/2
```

Maple [B] time = 0.095, size = 90, normalized size = 2.5

$$\frac{\sqrt{8}\,(-1+\cosh(2x))\,\sqrt{2}}{16\,\sinh(2x)}\sqrt{a\,(-1+\cosh(2x))\,(\cosh(2x)+1)}\left(\sqrt{a\,(\sinh(2x))^2}\sqrt{a}-\ln\left(\sqrt{a}\cosh(2x)+\sqrt{a\,(\sinh(2x))^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(1/2),x)
```

```
[Out] 1/16*8^(1/2)*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*2^(1/2)*
((a*sinh(2*x)^2)^(1/2)*a^(1/2)-ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*
a)/a^(1/2)/sinh(2*x)/(a*(-1+cosh(2*x))^2)^(1/2)
```

Maxima [A] time = 1.87253, size = 36, normalized size = 1.

$$-\frac{1}{8}\left(\sqrt{a}e^{(-4x)}-\sqrt{a}\right)e^{(2x)}-\frac{1}{2}\sqrt{a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) - 1/2*sqrt(a)*x
```

Fricas [B] time = 1.7905, size = 551, normalized size = 15.31

$$\frac{\left(4\cosh(x)\,e^{(2x)}\sinh(x)^3+e^{(2x)}\sinh(x)^4+2\left(3\cosh(x)^2-2x\right)e^{(2x)}\sinh(x)^2+4\left(\cosh(x)^3-2x\cosh(x)\right)e^{(2x)}\sinh(\ldots\right.}{8\left(\cosh(x)^2e^{(4x)}-2\cosh(x)^2e^{(2x)}+\left(e^{(4x)}-2e^{(2x)}+1\right)\sinh(x)^2+\cosh(x)^2+2\left(\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 - 2*x
)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh(x
)^4 - 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x
) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) - 2*cosh(x)^2*e^(2*x) + (e
^(4*x) - 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) - 2*cosh
(x)*e^(2*x) + cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sinh^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sinh(x)**4), x)
```

Giac [A] time = 1.22562, size = 35, normalized size = 0.97

$$\frac{1}{8}\left(\left(2\,e^{(2\,x)}-1\right)e^{(-2\,x)}-4\,x+e^{(2\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))*sqrt(a)
```

3.155 $\int \frac{1}{\sqrt{a \sinh^4(x)}} dx$

Optimal. Leaf size=16

$$-\frac{\sinh(x)\cosh(x)}{\sqrt{a\sinh^4(x)}}$$

[Out] -((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])

Rubi [A] time = 0.0136318, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 3767, 8}

$$-\frac{\sinh(x)\cosh(x)}{\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*Sinh[x]^4],x]

[Out] -((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a\sinh^4(x)}} dx &= \frac{\sinh^2(x)\int \operatorname{csch}^2(x)\,dx}{\sqrt{a\sinh^4(x)}}\\
&= -\frac{\left(i\sinh^2(x)\right)\operatorname{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)}{\sqrt{a\sinh^4(x)}}\\
&= -\frac{\cosh(x)\sinh(x)}{\sqrt{a\sinh^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0056078, size = 16, normalized size = 1.

$$-\frac{\sinh(x)\cosh(x)}{\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sinh[x]^4],x]
```

```
[Out] -((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])
```

Maple [B] time = 0.083, size = 56, normalized size = 3.5

$$-\frac{\sqrt{8}\sqrt{2}}{4\,a\sinh(2\,x)}\sqrt{a\,(-1+\cosh(2\,x))\,(\cosh(2\,x)+1)}\sqrt{a\,(\sinh(2\,x))^2}\frac{1}{\sqrt{a\,(-1+\cosh(2\,x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(1/2),x)
```

```
[Out] -1/4*8^(1/2)*2^(1/2)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/a*(a*sinh(2*x)^
2)^(1/2)/sinh(2*x)/(a*(-1+cosh(2*x))^2)^(1/2)
```

Maxima [A] time = 1.78469, size = 24, normalized size = 1.5

$$\frac{2}{\sqrt{a}e^{(-2\,x)}-\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/(sqrt(a)*e^(-2*x) - sqrt(a))
```

Fricas [B] time = 1.68028, size = 338, normalized size = 21.12

$$-\frac{2\sqrt{ae^{(8\,x)}-4\,ae^{(6\,x)}+6\,ae^{(4\,x)}-4\,ae^{(2\,x)}+a}}{a\cosh(x)^2+\left(ae^{(4\,x)}-2\,ae^{(2\,x)}+a\right)\sinh(x)^2+\left(a\cosh(x)^2-a\right)e^{(4\,x)}-2\left(a\cosh(x)^2-a\right)e^{(2\,x)}+2\left(a\cosh(x)\,e^{(4\,x)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(a*cosh(x)
^2 + (a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 - a)*e^(4*x) -
2*(a*cosh(x)^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x) +
a*cosh(x))*sinh(x) - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \sinh^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(x)**4), x)
```

Giac [A] time = 1.31557, size = 18, normalized size = 1.12

$$-\frac{2}{\sqrt{a}\left(e^{(2x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(a)*(e^(2*x) - 1))
```

3.156 $\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sinh(x)\cosh(x)}{a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}+\frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}$$

```
[Out] (2*Cosh[x]^2*Coth[x])/(3*a*Sqrt[a*Sinh[x]^4]) - (Cosh[x]^2*Coth[x]^3)/(5*a*
Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a*Sqrt[a*Sinh[x]^4])
```

Rubi [A] time = 0.0220898, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3767}

$$-\frac{\sinh(x)\cosh(x)}{a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}+\frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(-3/2),x]
```

```
[Out] (2*Cosh[x]^2*Coth[x])/(3*a*Sqrt[a*Sinh[x]^4]) - (Cosh[x]^2*Coth[x]^3)/(5*a*
Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a*Sqrt[a*Sinh[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx &= \frac{\sinh^2(x)\int \operatorname{csch}^6(x)\,dx}{a\sqrt{a\sinh^4(x)}}\\ &= -\frac{\left(i\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-i\coth(x)\right)}{a\sqrt{a\sinh^4(x)}}\\ &= \frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}-\frac{\cosh(x)\sinh(x)}{a\sqrt{a\sinh^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.038011, size = 34, normalized size = 0.5

$$-\frac{\sinh^5(x)\cosh(x)\left(3\text{csch}^4(x)-4\text{csch}^2(x)+8\right)}{15\left(a\sinh^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(-3/2),x]
```

```
[Out] -(Cosh[x]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x]^5)/(15*(a*Sinh[x]^4)^(3/2
))
```

Maple [A] time = 0.086, size = 80, normalized size = 1.2

$$-\frac{\sqrt{8}\sqrt{2}\left(2\,(\cosh(2x))^2-6\,\cosh(2x)+7\right)}{15\,a^2\,(-1+\cosh(2x))^2\sinh(2x)}\sqrt{a\,(\sinh(2x))^2}\sqrt{a\,(-1+\cosh(2x))\,(\cosh(2x)+1)}\frac{1}{\sqrt{a\,(-1+\cosh(2x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(3/2),x)
```

```
[Out] -1/15*8^(1/2)/a^2*2^(1/2)*(2*cosh(2*x)^2-6*cosh(2*x)+7)*(a*sinh(2*x)^2)^(1/
2)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/(-1+cosh(2*x))^2/sinh(2*x)/(a*(-1
+cosh(2*x))^2)^(1/2)
```

Maxima [B] time = 1.93976, size = 231, normalized size = 3.4

$$-\frac{16\,e^{(-2x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2x)}-10\,a^{\frac{3}{2}}e^{(-4x)}+10\,a^{\frac{3}{2}}e^{(-6x)}-5\,a^{\frac{3}{2}}e^{(-8x)}+a^{\frac{3}{2}}e^{(-10x)}-a^{\frac{3}{2}}\right)}+\frac{32\,e^{(-4x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2x)}-10\,a^{\frac{3}{2}}e^{(-4x)}+10\,a^{\frac{3}{2}}e^{(-6x)}-5\,a^{\frac{3}{2}}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6
*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 32/3*e^(-4*x)/(5*
a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)*e^
(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) - 10*a^(3
/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x)
 - a^(3/2))
```

Fricas [B] time = 2.04043, size = 3163, normalized size = 46.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x)
^2 - 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x) + (1
0*cosh(x)^4 - 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*
e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) - 2*a^2*
e^(2*x) + a^2)*sinh(x)^10 - 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x) - 2*a
^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 - a^2 + (9
*a^2*cosh(x)^2 - a^2)*e^(4*x) - 2*(9*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^
8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*cosh(x)^3
 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x
)^7 + 10*(21*a^2*cosh(x)^4 - 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cosh(x)^4 - 1
4*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(21*a^2*cosh(x)^4 - 14*a^2*cosh(x)^2 + a
^2)*e^(2*x))*sinh(x)^6 - 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5 - 70*a^2*co
sh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^3 + 15*a^2*co
sh(x))*e^(4*x) - 2*(63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^3 + 15*a^2*cosh(x))*e
^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x
)^2 - a^2 + (21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*
e^(4*x) - 2*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*
e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^
5 + 5*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^5 + 5*
a^2*cosh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^5
 + 5*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 5*(9*a^2*cosh(x)^8 -
 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh(x)^2 + a^2 + (9*a^2*cosh
(x)^8 - 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh(x)^2 + a^2)*e^(4*
x) - 2*(9*a^2*cosh(x)^8 - 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh
(x)^2 + a^2)*e^(2*x))*sinh(x)^2 - a^2 + (a^2*cosh(x)^10 - 5*a^2*cosh(x)^8 +
 10*a^2*cosh(x)^6 - 10*a^2*cosh(x)^4 + 5*a^2*cosh(x)^2 - a^2)*e^(4*x) - 2*(
a^2*cosh(x)^10 - 5*a^2*cosh(x)^8 + 10*a^2*cosh(x)^6 - 10*a^2*cosh(x)^4 + 5*
a^2*cosh(x)^2 - a^2)*e^(2*x) + 10*(a^2*cosh(x)^9 - 4*a^2*cosh(x)^7 + 6*a^2*
cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^9 - 4*a^2*cosh(x)^
7 + 6*a^2*cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) - 2*(a^2*cosh(
x)^9 - 4*a^2*cosh(x)^7 + 6*a^2*cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x))*e
^(2*x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**4)**(-3/2), x)
```

Giac [A] time = 1.27163, size = 47, normalized size = 0.69

$$-\frac{16\left(10\sqrt{a}e^{(4x)} - 5\sqrt{a}e^{(2x)} + \sqrt{a}\right)}{15\,a^2\left(e^{(2x)} - 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*(10*sqrt(a)*e^(4*x) - 5*sqrt(a)*e^(2*x) + sqrt(a))/(a^2*(e^(2*x) - 1
)^5)
```

3.157 $\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}}-\frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}}$$

```
[Out] (4*Cosh[x]^2*Coth[x])/(3*a^2*Sqrt[a*Sinh[x]^4]) - (6*Cosh[x]^2*Coth[x]^3)/(
5*a^2*Sqrt[a*Sinh[x]^4]) + (4*Cosh[x]^2*Coth[x]^5)/(7*a^2*Sqrt[a*Sinh[x]^4]
) - (Cosh[x]^2*Coth[x]^7)/(9*a^2*Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a^
2*Sqrt[a*Sinh[x]^4])
```

Rubi [A] time = 0.0326209, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3767}

$$-\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}}-\frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(-5/2),x]
```

```
[Out] (4*Cosh[x]^2*Coth[x])/(3*a^2*Sqrt[a*Sinh[x]^4]) - (6*Cosh[x]^2*Coth[x]^3)/(
5*a^2*Sqrt[a*Sinh[x]^4]) + (4*Cosh[x]^2*Coth[x]^5)/(7*a^2*Sqrt[a*Sinh[x]^4]
) - (Cosh[x]^2*Coth[x]^7)/(9*a^2*Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a^
2*Sqrt[a*Sinh[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = \frac{\sinh^2(x)\int \operatorname{csch}^{10}(x)\,dx}{a^2\sqrt{a\sinh^4(x)}}$$
$$= -\frac{\left(i\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)dx, x, -i\coth(x)\right)}{a^2\sqrt{a\sinh^4(x)}}$$
$$= \frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}} - \frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}} + \frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}} - \frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}} - \frac{\cosh(x}{a^2\sqrt{a\,\mathrm{s}}}$$

Mathematica [A] time = 0.058047, size = 47, normalized size = 0.4

$$-\frac{\sinh(x)\cosh(x)\left(35\operatorname{csch}^8(x)-40\operatorname{csch}^6(x)+48\operatorname{csch}^4(x)-64\operatorname{csch}^2(x)+128\right)}{315a^2\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(-5/2),x]
```

```
[Out] -(Cosh[x]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 - 40*Csch[x]^6 + 35*Csch[x]^8)
*Sinh[x])/(315*a^2*Sqrt[a*Sinh[x]^4])
```

Maple [A] time = 0.086, size = 96, normalized size = 0.8

$$-\frac{4\sqrt{8}\sqrt{2}\left(8\left(\cosh(2x)\right)^4-40\left(\cosh(2x)\right)^3+84\left(\cosh(2x)\right)^2-100\cosh(2x)+83\right)}{315a^3\left(-1+\cosh(2x)\right)^4\sinh(2x)}\sqrt{a\left(\sinh(2x)\right)^2}\sqrt{a\left(-1+\cosh(2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(5/2),x)
```

```
[Out] -4/315*8^(1/2)*2^(1/2)/a^3*(8*cosh(2*x)^4-40*cosh(2*x)^3+84*cosh(2*x)^2-100
*cosh(2*x)+83)*(a*sinh(2*x)^2)^(1/2)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)
/(-1+cosh(2*x))^4/sinh(2*x)/(a*(-1+cosh(2*x))^2)^(1/2)
```

Maxima [B] time = 2.01625, size = 630, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(
-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-12*x)
 + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(5/2)
) + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)
*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-1
2*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(
```

```
5/2)) - 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(
5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e
^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) -
 a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a
^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)
*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x)
 - a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2
)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-
12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^
(5/2))
```

Fricas [B] time = 2.56442, size = 9072, normalized size = 76.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(1008*cosh(x)*e^(2*x)*sinh(x)^7 + 126*e^(2*x)*sinh(x)^8 + 84*(42*c
osh(x)^2 - 1)*e^(2*x)*sinh(x)^6 + 504*(14*cosh(x)^3 - cosh(x))*e^(2*x)*sinh
(x)^5 + 36*(245*cosh(x)^4 - 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(147*c
osh(x)^5 - 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(392*cosh(x)^6 -
 140*cosh(x)^4 + 24*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^2 + 18*(56*cosh(x)^7 - 2
8*cosh(x)^5 + 8*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x) + (126*cosh(x)^8 - 84*
cosh(x)^6 + 36*cosh(x)^4 - 9*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e
^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(a^3*cosh(x)^18 - 9*a^3*co
sh(x)^16 + (a^3*e^(4*x) - 2*a^3*e^(2*x) + a^3)*sinh(x)^18 + 18*(a^3*cosh(x)
*e^(4*x) - 2*a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^17 + 36*a^3*cosh(x)
^14 + 9*(17*a^3*cosh(x)^2 - a^3 + (17*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(17*
a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^16 + 48*(17*a^3*cosh(x)^3 - 3*a^3*cos
h(x) + (17*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^3 - 3
*a^3*cosh(x))*e^(2*x))*sinh(x)^15 - 84*a^3*cosh(x)^12 + 36*(85*a^3*cosh(x)^
4 - 30*a^3*cosh(x)^2 + a^3 + (85*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + a^3)*e^
(4*x) - 2*(85*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^14 +
 504*(17*a^3*cosh(x)^5 - 10*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^5
 - 10*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^5 - 10*a^3*c
osh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^13 + 126*a^3*cosh(x)^10 + 84*(221*
a^3*cosh(x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3 + (221*a^3*cosh(
x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(221*a^3*cos
h(x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^12 +
144*(221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 - 7*a^3*cosh(
x) + (221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 - 7*a^3*cosh
(x))*e^(4*x) - 2*(221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3
- 7*a^3*cosh(x))*e^(2*x))*sinh(x)^11 - 126*a^3*cosh(x)^8 + 18*(2431*a^3*cos
h(x)^8 - 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 - 308*a^3*cosh(x)^2 + 7*a^
3 + (2431*a^3*cosh(x)^8 - 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 - 308*a^3
*cosh(x)^2 + 7*a^3)*e^(4*x) - 2*(2431*a^3*cosh(x)^8 - 4004*a^3*cosh(x)^6 +
2002*a^3*cosh(x)^4 - 308*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^10 + 4*(12
155*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 - 4620*a^3*co
sh(x)^3 + 315*a^3*cosh(x) + (12155*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18
018*a^3*cosh(x)^5 - 4620*a^3*cosh(x)^3 + 315*a^3*cosh(x))*e^(4*x) - 2*(1215
5*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 - 4620*a^3*cosh
(x)^3 + 315*a^3*cosh(x))*e^(2*x))*sinh(x)^9 + 84*a^3*cosh(x)^6 + 18*(2431*a
^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 - 2310*a^3*cosh(x)^
4 + 315*a^3*cosh(x)^2 - 7*a^3 + (2431*a^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 +
 6006*a^3*cosh(x)^6 - 2310*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 - 7*a^3)*e^(4*
```

```
x) - 2*(2431*a^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 - 231
0*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 - 7*a^3)*e^(2*x))*sinh(x)^8 + 144*(221*
a^3*cosh(x)^11 - 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^7 - 462*a^3*cosh(x)^5
+ 105*a^3*cosh(x)^3 - 7*a^3*cosh(x) + (221*a^3*cosh(x)^11 - 715*a^3*cosh(x)
^9 + 858*a^3*cosh(x)^7 - 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 - 7*a^3*cosh
(x))*e^(4*x) - 2*(221*a^3*cosh(x)^11 - 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^
7 - 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 - 7*a^3*cosh(x))*e^(2*x))*sinh(x)
^7 - 36*a^3*cosh(x)^4 + 84*(221*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*
a^3*cosh(x)^8 - 924*a^3*cosh(x)^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 +
a^3 + (221*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 - 924*a
^3*cosh(x)^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(221
*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 - 924*a^3*cosh(x)
^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^6 + 504*(
17*a^3*cosh(x)^13 - 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)
^7 + 63*a^3*cosh(x)^5 - 14*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^13
 - 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)^7 + 63*a^3*cosh(
x)^5 - 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^13 - 78*
a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)^7 + 63*a^3*cosh(x)^5 -
 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 9*a^3*cosh(x)^2 + 36*
(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 - 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 - a^3 +
(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 - 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 - a^3)*e
^(4*x) - 2*(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 -
1155*a^3*cosh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)
^2 - a^3)*e^(2*x))*sinh(x)^4 + 48*(17*a^3*cosh(x)^15 - 105*a^3*cosh(x)^13 +
 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 - 147*a^3*cosh(
x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x) + (17*a^3*cosh(x)^15 - 105*a^3*cosh
(x)^13 + 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 - 147*a
^3*cosh(x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x
)^15 - 105*a^3*cosh(x)^13 + 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^
3*cosh(x)^7 - 147*a^3*cosh(x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(2*x)
)*sinh(x)^3 - a^3 + 9*(17*a^3*cosh(x)^16 - 120*a^3*cosh(x)^14 + 364*a^3*cos
h(x)^12 - 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 - 392*a^3*cosh(x)^6 + 140*
a^3*cosh(x)^4 - 24*a^3*cosh(x)^2 + a^3 + (17*a^3*cosh(x)^16 - 120*a^3*cosh(
x)^14 + 364*a^3*cosh(x)^12 - 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 - 392*a
^3*cosh(x)^6 + 140*a^3*cosh(x)^4 - 24*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(17*
a^3*cosh(x)^16 - 120*a^3*cosh(x)^14 + 364*a^3*cosh(x)^12 - 616*a^3*cosh(x)^
10 + 630*a^3*cosh(x)^8 - 392*a^3*cosh(x)^6 + 140*a^3*cosh(x)^4 - 24*a^3*cos
h(x)^2 + a^3)*e^(2*x))*sinh(x)^2 + (a^3*cosh(x)^18 - 9*a^3*cosh(x)^16 + 36*
a^3*cosh(x)^14 - 84*a^3*cosh(x)^12 + 126*a^3*cosh(x)^10 - 126*a^3*cosh(x)^8
 + 84*a^3*cosh(x)^6 - 36*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2
*(a^3*cosh(x)^18 - 9*a^3*cosh(x)^16 + 36*a^3*cosh(x)^14 - 84*a^3*cosh(x)^12
 + 126*a^3*cosh(x)^10 - 126*a^3*cosh(x)^8 + 84*a^3*cosh(x)^6 - 36*a^3*cosh(
x)^4 + 9*a^3*cosh(x)^2 - a^3)*e^(2*x) + 18*(a^3*cosh(x)^17 - 8*a^3*cosh(x)^
15 + 28*a^3*cosh(x)^13 - 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)^9 - 56*a^3*cosh
(x)^7 + 28*a^3*cosh(x)^5 - 8*a^3*cosh(x)^3 + a^3*cosh(x) + (a^3*cosh(x)^17
- 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 - 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)
^9 - 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 - 8*a^3*cosh(x)^3 + a^3*cosh(x))*e
^(4*x) - 2*(a^3*cosh(x)^17 - 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 - 56*a^3*
cosh(x)^11 + 70*a^3*cosh(x)^9 - 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 - 8*a^3
*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sinh(x)**4)**(-5/2), x)
```

Giac [A] time = 1.39216, size = 72, normalized size = 0.61

$$-\frac{256\left(126\sqrt{a}e^{(8x)} - 84\sqrt{a}e^{(6x)} + 36\sqrt{a}e^{(4x)} - 9\sqrt{a}e^{(2x)} + \sqrt{a}\right)}{315\,a^3\left(e^{(2x)} - 1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*(126*sqrt(a)*e^(8*x) - 84*sqrt(a)*e^(6*x) + 36*sqrt(a)*e^(4*x) - 9
*sqrt(a)*e^(2*x) + sqrt(a))/(a^3*(e^(2*x) - 1)^9)
```

3.158 $\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=50

$$-\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{1}{6}i\sinh(x)\cosh^5(x)-\frac{5}{24}i\sinh(x)\cosh^3(x)-\frac{5}{16}i\sinh(x)\cosh(x)$$

```
[Out] ((-5*I)/16)*x + Cosh[x]^7/7 - ((5*I)/16)*Cosh[x]*Sinh[x] - ((5*I)/24)*Cosh[
x]^3*Sinh[x] - (I/6)*Cosh[x]^5*Sinh[x]
```

Rubi [A] time = 0.0549461, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$-\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{1}{6}i\sinh(x)\cosh^5(x)-\frac{5}{24}i\sinh(x)\cosh^3(x)-\frac{5}{16}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^8/(I + Sinh[x]),x]
```

```
[Out] ((-5*I)/16)*x + Cosh[x]^7/7 - ((5*I)/16)*Cosh[x]*Sinh[x] - ((5*I)/24)*Cosh[
x]^3*Sinh[x] - (I/6)*Cosh[x]^5*Sinh[x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx &= \frac{\cosh^7(x)}{7}-i\int \cosh^6(x)\,dx\\
&= \frac{\cosh^7(x)}{7}-\frac{1}{6}i\cosh^5(x)\sinh(x)-\frac{5}{6}i\int \cosh^4(x)\,dx\\
&= \frac{\cosh^7(x)}{7}-\frac{5}{24}i\cosh^3(x)\sinh(x)-\frac{1}{6}i\cosh^5(x)\sinh(x)-\frac{5}{8}i\int \cosh^2(x)\,dx\\
&= \frac{\cosh^7(x)}{7}-\frac{5}{16}i\cosh(x)\sinh(x)-\frac{5}{24}i\cosh^3(x)\sinh(x)-\frac{1}{6}i\cosh^5(x)\sinh(x)-\frac{5}{16}i\int 1\,dx\\
&= -\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{5}{16}i\cosh(x)\sinh(x)-\frac{5}{24}i\cosh^3(x)\sinh(x)-\frac{1}{6}i\cosh^5(x)\sinh(x)
\end{aligned}$$

Mathematica [B] time = 0.150883, size = 219, normalized size = 4.38

$$\frac{\cosh^9(x)\left(48\sqrt{1+i\sinh(x)}\sinh^7(x)-8i\sqrt{1+i\sinh(x)}\sinh^6(x)+200\sqrt{1+i\sinh(x)}\sinh^5(x)-38i\sqrt{1+i\sinh(x)}\sinh\right.}{336}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^8/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]^9*((6*I)*(35*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[x
]] + 8*Sqrt[1 + I*Sinh[x]]) + 279*Sqrt[1 + I*Sinh[x]]*Sinh[x] - (87*I)*Sqrt
[1 + I*Sinh[x]]*Sinh[x]^2 + 326*Sqrt[1 + I*Sinh[x]]*Sinh[x]^3 - (38*I)*Sqrt
[1 + I*Sinh[x]]*Sinh[x]^4 + 200*Sqrt[1 + I*Sinh[x]]*Sinh[x]^5 - (8*I)*Sqrt[
1 + I*Sinh[x]]*Sinh[x]^6 + 48*Sqrt[1 + I*Sinh[x]]*Sinh[x]^7))/(336*Sqrt[1 +
 I*Sinh[x]]*(-I + Sinh[x])^4*(I + Sinh[x])^5)
```

Maple [B] time = 0.066, size = 292, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^8/(I+sinh(x)),x)
```

```
[Out] -11/16/(tanh(1/2*x)+1)^2+5/16/(tanh(1/2*x)+1)+9/8/(tanh(1/2*x)+1)^3-5/16/(t
anh(1/2*x)-1)-11/16/(tanh(1/2*x)-1)^2-9/8/(tanh(1/2*x)-1)^3-11/16*I/(tanh(1
/2*x)+1)-7/6*I/(tanh(1/2*x)+1)^3-11/16*I/(tanh(1/2*x)-1)-7/6*I/(tanh(1/2*x)
-1)^3+19/16*I/(tanh(1/2*x)+1)^2+1/6*I/(tanh(1/2*x)+1)^6-1/2*I/(tanh(1/2*x)+
1)^5+5/16*I*ln(tanh(1/2*x)-1)-I/(tanh(1/2*x)-1)^4-19/16*I/(tanh(1/2*x)-1)^2
+I/(tanh(1/2*x)+1)^4-1/2*I/(tanh(1/2*x)-1)^5-1/6*I/(tanh(1/2*x)-1)^6-5/16*I
*ln(tanh(1/2*x)+1)-1/7/(tanh(1/2*x)-1)^7-5/4/(tanh(1/2*x)-1)^4+1/(tanh(1/2*
x)+1)^5-1/2/(tanh(1/2*x)-1)^6-5/4/(tanh(1/2*x)+1)^4-1/2/(tanh(1/2*x)+1)^6-1
/(tanh(1/2*x)-1)^5+1/7/(tanh(1/2*x)+1)^7
```

Maxima [B] time = 1.35429, size = 122, normalized size = 2.44

$$-\frac{1}{5376}\left(14i\,e^{(-x)}-42\,e^{(-2\,x)}+126i\,e^{(-3\,x)}-126\,e^{(-4\,x)}+630i\,e^{(-5\,x)}-210\,e^{(-6\,x)}-6\right)e^{(7\,x)}-\frac{5}{16}i\,x+\frac{5}{128}\,e^{(-x)}+\frac{15}{128}i\,e^{(-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/5376*(14*I*e^(-x) - 42*e^(-2*x) + 126*I*e^(-3*x) - 126*e^(-4*x) + 630*I*
e^(-5*x) - 210*e^(-6*x) - 6)*e^(7*x) - 5/16*I*x + 5/128*e^(-x) + 15/128*I*e
^(-2*x) + 3/128*e^(-3*x) + 3/128*I*e^(-4*x) + 1/128*e^(-5*x) + 1/384*I*e^(-
6*x) + 1/896*e^(-7*x)
```

Fricas [B] time = 1.8912, size = 301, normalized size = 6.02

$$\frac{1}{2688}\left(-840i\,xe^{(7\,x)}+3\,e^{(14\,x)}-7i\,e^{(13\,x)}+21\,e^{(12\,x)}-63i\,e^{(11\,x)}+63\,e^{(10\,x)}-315i\,e^{(9\,x)}+105\,e^{(8\,x)}+105\,e^{(6\,x)}+315i\,e^{(5\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2688*(-840*I*x*e^(7*x) + 3*e^(14*x) - 7*I*e^(13*x) + 21*e^(12*x) - 63*I*e
^(11*x) + 63*e^(10*x) - 315*I*e^(9*x) + 105*e^(8*x) + 105*e^(6*x) + 315*I*e
^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*x) + 7*I*e^x + 3)*e^(-7*x)
```

Sympy [B] time = 0.729779, size = 124, normalized size = 2.48

$$-\frac{5ix}{16}+\frac{e^{7x}}{896}-\frac{ie^{6x}}{384}+\frac{e^{5x}}{128}-\frac{3ie^{4x}}{128}+\frac{3e^{3x}}{128}-\frac{15ie^{2x}}{128}+\frac{5e^{x}}{128}+\frac{5e^{-x}}{128}+\frac{15ie^{-2x}}{128}+\frac{3e^{-3x}}{128}+\frac{3ie^{-4x}}{128}+\frac{e^{-5x}}{128}+\frac{ie^{-6x}}{384}+\frac{e^{-7x}}{896}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**8/(I+sinh(x)),x)
```

```
[Out] -5*I*x/16 + exp(7*x)/896 - I*exp(6*x)/384 + exp(5*x)/128 - 3*I*exp(4*x)/128
 + 3*exp(3*x)/128 - 15*I*exp(2*x)/128 + 5*exp(x)/128 + 5*exp(-x)/128 + 15*I
*exp(-2*x)/128 + 3*exp(-3*x)/128 + 3*I*exp(-4*x)/128 + exp(-5*x)/128 + I*ex
p(-6*x)/384 + exp(-7*x)/896
```

Giac [B] time = 1.20025, size = 116, normalized size = 2.32

$$\frac{1}{2688}\left(105\,e^{(6\,x)}+315i\,e^{(5\,x)}+63\,e^{(4\,x)}+63i\,e^{(3\,x)}+21\,e^{(2\,x)}+7i\,e^{x}+3\right)e^{(-7\,x)}-\frac{5}{16}i\,x+\frac{1}{896}\,e^{(7\,x)}-\frac{1}{384}i\,e^{(6\,x)}+\frac{1}{128}\,e^{(5\,x)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2688*(105*e^(6*x) + 315*I*e^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*x
) + 7*I*e^x + 3)*e^(-7*x) - 5/16*I*x + 1/896*e^(7*x) - 1/384*I*e^(6*x) + 1/
128*e^(5*x) - 3/128*I*e^(4*x) + 3/128*e^(3*x) - 15/128*I*e^(2*x) + 5/128*e^
x
```

3.159 $\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{6}(-\sinh(x)+i)^6-\frac{4}{5}i(-\sinh(x)+i)^5-(-\sinh(x)+i)^4$$

```
[Out] -(I - Sinh[x])^4 - ((4*I)/5)*(I - Sinh[x])^5 + (I - Sinh[x])^6/6
```

Rubi [A] time = 0.0451925, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\frac{1}{6}(-\sinh(x)+i)^6-\frac{4}{5}i(-\sinh(x)+i)^5-(-\sinh(x)+i)^4$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^7/(I + Sinh[x]),x]
```

```
[Out] -(I - Sinh[x])^4 - ((4*I)/5)*(I - Sinh[x])^5 + (I - Sinh[x])^6/6
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx &= -\operatorname{Subst}\left(\int (i-x)^3(i+x)^2\,dx, x, \sinh(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(-4(i-x)^3-4i(i-x)^4+(i-x)^5\right)\,dx, x, \sinh(x)\right)\\
&= -(i-\sinh(x))^4-\frac{4}{5}i(i-\sinh(x))^5+\frac{1}{6}(i-\sinh(x))^6
\end{aligned}$$

Mathematica [A] time = 0.029949, size = 42, normalized size = 0.98

$$\frac{1}{30}\sinh(x)\left(5\sinh^5(x)-6i\sinh^4(x)+15\sinh^3(x)-20i\sinh^2(x)+15\sinh(x)-30i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^7/(I + Sinh[x]),x]
```

```
[Out] (Sinh[x]*(-30*I + 15*Sinh[x] - (20*I)*Sinh[x]^2 + 15*Sinh[x]^3 - (6*I)*Sinh
[x]^4 + 5*Sinh[x]^5))/30
```

Maple [B] time = 0.057, size = 142, normalized size = 3.3

$$\frac{11}{16}-\frac{7i}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{7}{8}-\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}-\frac{1}{2}-\frac{i}{5}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-5}-\frac{5}{16}-i\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{11}{12}-\frac{11i}{12}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(I+sinh(x)),x)
```

```
[Out] (11/16-7/8*I)/(tanh(1/2*x)+1)^2+(7/8-1/2*I)/(tanh(1/2*x)+1)^4+(-1/2+1/5*I)/
(tanh(1/2*x)+1)^5+(-5/16+I)/(tanh(1/2*x)+1)+(-11/12+11/12*I)/(tanh(1/2*x)+1
)^3+1/6/(tanh(1/2*x)+1)^6+(11/12+11/12*I)/(tanh(1/2*x)-1)^3+(11/16+7/8*I)/(
tanh(1/2*x)-1)^2+(7/8+1/2*I)/(tanh(1/2*x)-1)^4+(1/2+1/5*I)/(tanh(1/2*x)-1)^
5+(5/16+I)/(tanh(1/2*x)-1)+1/6/(tanh(1/2*x)-1)^6
```

Maxima [B] time = 1.24185, size = 101, normalized size = 2.35

$$-\frac{1}{1920}\left(12i\,e^{(-x)}-30\,e^{(-2\,x)}+100i\,e^{(-3\,x)}-75\,e^{(-4\,x)}+600i\,e^{(-5\,x)}-5\right)e^{(6\,x)}+\frac{5}{16}i\,e^{(-x)}+\frac{5}{128}\,e^{(-2\,x)}+\frac{5}{96}i\,e^{(-3\,x)}+\frac{1}{64}\,e^{(-4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*I*e^(-x) - 30*e^(-2*x) + 100*I*e^(-3*x) - 75*e^(-4*x) + 600*I*e
^(-5*x) - 5)*e^(6*x) + 5/16*I*e^(-x) + 5/128*e^(-2*x) + 5/96*I*e^(-3*x) + 1
/64*e^(-4*x) + 1/160*I*e^(-5*x) + 1/384*e^(-6*x)
```

Fricas [B] time = 1.85776, size = 240, normalized size = 5.58

$$\frac{1}{1920}\left(5\,e^{(12\,x)}-12i\,e^{(11\,x)}+30\,e^{(10\,x)}-100i\,e^{(9\,x)}+75\,e^{(8\,x)}-600i\,e^{(7\,x)}+600i\,e^{(5\,x)}+75\,e^{(4\,x)}+100i\,e^{(3\,x)}+30\,e^{(2\,x)}+12i\,e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*e^(12*x) - 12*I*e^(11*x) + 30*e^(10*x) - 100*I*e^(9*x) + 75*e^(8*
x) - 600*I*e^(7*x) + 600*I*e^(5*x) + 75*e^(4*x) + 100*I*e^(3*x) + 30*e^(2*x
) + 12*I*e^x + 5)*e^(-6*x)
```

Sympy [B] time = 0.59726, size = 100, normalized size = 2.33

$$\frac{e^{6x}}{384}-\frac{ie^{5x}}{160}+\frac{e^{4x}}{64}-\frac{5ie^{3x}}{96}+\frac{5e^{2x}}{128}-\frac{5ie^{x}}{16}+\frac{5ie^{-x}}{16}+\frac{5e^{-2x}}{128}+\frac{5ie^{-3x}}{96}+\frac{e^{-4x}}{64}+\frac{ie^{-5x}}{160}+\frac{e^{-6x}}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**7/(I+sinh(x)),x)
```

```
[Out] exp(6*x)/384 - I*exp(5*x)/160 + exp(4*x)/64 - 5*I*exp(3*x)/96 + 5*exp(2*x)/
128 - 5*I*exp(x)/16 + 5*I*exp(-x)/16 + 5*exp(-2*x)/128 + 5*I*exp(-3*x)/96 +
 exp(-4*x)/64 + I*exp(-5*x)/160 + exp(-6*x)/384
```

Giac [B] time = 1.18407, size = 96, normalized size = 2.23

$$-\frac{1}{1920}\left(-600 i e^{(5x)} - 75 e^{(4x)} - 100 i e^{(3x)} - 30 e^{(2x)} - 12 i e^{x} - 5\right) e^{(-6x)} + \frac{1}{384} e^{(6x)} - \frac{1}{160} i e^{(5x)} + \frac{1}{64} e^{(4x)} - \frac{5}{96} i e^{(3x)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/1920*(-600*I*e^(5*x) - 75*e^(4*x) - 100*I*e^(3*x) - 30*e^(2*x) - 12*I*e^
x - 5)*e^(-6*x) + 1/384*e^(6*x) - 1/160*I*e^(5*x) + 1/64*e^(4*x) - 5/96*I*e
^(3*x) + 5/128*e^(2*x) - 5/16*I*e^x
```

3.160 $\int \frac{\cosh^6(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=38

$$-\frac{3ix}{8}+\frac{\cosh^5(x)}{5}-\frac{1}{4}i\sinh(x)\cosh^3(x)-\frac{3}{8}i\sinh(x)\cosh(x)$$

```
[Out] ((-3*I)/8)*x + Cosh[x]^5/5 - ((3*I)/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Si
nh[x]
```

Rubi [A] time = 0.047439, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$-\frac{3ix}{8}+\frac{\cosh^5(x)}{5}-\frac{1}{4}i\sinh(x)\cosh^3(x)-\frac{3}{8}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(I + Sinh[x]),x]
```


```
[Out] ((-3*I)/8)*x + Cosh[x]^5/5 - ((3*I)/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Si
nh[x]
```


Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^6(x)}{i+\sinh(x)} \, dx &= \frac{\cosh^5(x)}{5}-i\int \cosh^4(x) \, dx\\
&= \frac{\cosh^5(x)}{5}-\frac{1}{4}i\cosh^3(x)\sinh(x)-\frac{3}{4}i\int \cosh^2(x) \, dx\\
&= \frac{\cosh^5(x)}{5}-\frac{3}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)-\frac{3}{8}i\int 1 \, dx\\
&= -\frac{3ix}{8}+\frac{\cosh^5(x)}{5}-\frac{3}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)
\end{aligned}$$

Mathematica [B] time = 0.240143, size = 131, normalized size = 3.45

$$-\frac{i\cosh^7(x)\left(8\sinh^5(x)-2i\sinh^4(x)+26\sinh^3(x)-9i\sinh^2(x)+33\sinh(x)+\frac{30i\sqrt{1-i\sinh(x)}\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{1+i\sinh(x)}}+8i\right)}{40\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^8\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(I + Sinh[x]),x]
```

```
[Out] ((-I/40)*Cosh[x]^7*(8*I + ((30*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[
1 - I*Sinh[x]])/Sqrt[1 + I*Sinh[x]] + 33*Sinh[x] - (9*I)*Sinh[x]^2 + 26*Sin
h[x]^3 - (2*I)*Sinh[x]^4 + 8*Sinh[x]^5))/((Cosh[x/2] - I*Sinh[x/2])^8*(Cosh
[x/2] + I*Sinh[x/2])^6)
```

Maple [B] time = 0.052, size = 210, normalized size = 5.5

$$-\frac{3i}{8}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}-\frac{i}{4}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-4}+\frac{3}{4}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{3i}{8}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(I+sinh(x)),x)
```

```
[Out] -3/8*I*ln(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^4-1/4*I/(tanh(1/2*x)-1)^4+3/4/
(tanh(1/2*x)+1)^3+3/8*I*ln(tanh(1/2*x)-1)+3/8/(tanh(1/2*x)+1)-7/8*I/(tanh(1
/2*x)-1)^2-5/8/(tanh(1/2*x)+1)^2-5/8*I/(tanh(1/2*x)+1)+1/5/(tanh(1/2*x)+1)^
5-1/2*I/(tanh(1/2*x)-1)^3-1/2/(tanh(1/2*x)-1)^4+7/8*I/(tanh(1/2*x)+1)^2-3/8
/(tanh(1/2*x)-1)+1/4*I/(tanh(1/2*x)+1)^4-5/8/(tanh(1/2*x)-1)^2-1/2*I/(tanh(
1/2*x)+1)^3-3/4/(tanh(1/2*x)-1)^3-5/8*I/(tanh(1/2*x)-1)-1/5/(tanh(1/2*x)-1)
^5
```

Maxima [B] time = 1.30051, size = 89, normalized size = 2.34

$$-\frac{1}{320}\left(5ie^{(-x)}-10e^{(-2x)}+40ie^{(-3x)}-20e^{(-4x)}-2\right)e^{(5x)}-\frac{3}{8}ix+\frac{1}{16}e^{(-x)}+\frac{1}{8}ie^{(-2x)}+\frac{1}{32}e^{(-3x)}+\frac{1}{64}ie^{(-4x)}+\frac{1}{160}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/320*(5*I*e^(-x) - 10*e^(-2*x) + 40*I*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x)
 - 3/8*I*x + 1/16*e^(-x) + 1/8*I*e^(-2*x) + 1/32*e^(-3*x) + 1/64*I*e^(-4*x)
 + 1/160*e^(-5*x)
```

Fricas [B] time = 1.75102, size = 213, normalized size = 5.61

$$\frac{1}{320}\left(-120ixe^{(5x)}+2e^{(10x)}-5ie^{(9x)}+10e^{(8x)}-40ie^{(7x)}+20e^{(6x)}+20e^{(4x)}+40ie^{(3x)}+10e^{(2x)}+5ie^{x}+2\right)e^{(-5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/320*(-120*I*x*e^(5*x) + 2*e^(10*x) - 5*I*e^(9*x) + 10*e^(8*x) - 40*I*e^(7
*x) + 20*e^(6*x) + 20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*e^
(-5*x)
```

Sympy [B] time = 0.481323, size = 82, normalized size = 2.16

$$-\frac{3ix}{8}+\frac{e^{5x}}{160}-\frac{ie^{4x}}{64}+\frac{e^{3x}}{32}-\frac{ie^{2x}}{8}+\frac{e^{x}}{16}+\frac{e^{-x}}{16}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{32}+\frac{ie^{-4x}}{64}+\frac{e^{-5x}}{160}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(I+sinh(x)),x)
```

```
[Out] -3*I*x/8 + exp(5*x)/160 - I*exp(4*x)/64 + exp(3*x)/32 - I*exp(2*x)/8 + exp(
x)/16 + exp(-x)/16 + I*exp(-2*x)/8 + exp(-3*x)/32 + I*exp(-4*x)/64 + exp(-5
*x)/160
```

Giac [B] time = 1.29176, size = 84, normalized size = 2.21

$$\frac{1}{320}\left(20\,e^{(4\,x)}+40i\,e^{(3\,x)}+10\,e^{(2\,x)}+5i\,e^{x}+2\right)e^{(-5\,x)}-\frac{3}{8}i\,x+\frac{1}{160}\,e^{(5\,x)}-\frac{1}{64}i\,e^{(4\,x)}+\frac{1}{32}\,e^{(3\,x)}-\frac{1}{8}i\,e^{(2\,x)}+\frac{1}{16}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/320*(20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*e^(-5*x) - 3/8
*I*x + 1/160*e^(5*x) - 1/64*I*e^(4*x) + 1/32*e^(3*x) - 1/8*I*e^(2*x) + 1/16
*e^x
```

3.161 $\int \frac{\cosh^5(x)}{i+\sinh(x)} dx$

Optimal. Leaf size=33

$$\frac{\sinh^4(x)}{4} - \frac{1}{3}i\sinh^3(x) + \frac{\sinh^2(x)}{2} - i\sinh(x)$$

```
[Out] (-I)*Sinh[x] + Sinh[x]^2/2 - (I/3)*Sinh[x]^3 + Sinh[x]^4/4
```

Rubi [A] time = 0.0391253, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\frac{\sinh^4(x)}{4} - \frac{1}{3}i\sinh^3(x) + \frac{\sinh^2(x)}{2} - i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(I + Sinh[x]),x]
```

```
[Out] (-I)*Sinh[x] + Sinh[x]^2/2 - (I/3)*Sinh[x]^3 + Sinh[x]^4/4
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^5(x)}{i+\sinh(x)} dx &= \text{Subst}\left(\int (i-x)^2(i+x)\, dx, x, \sinh(x)\right) \\
&= \text{Subst}\left(\int \left(-i+x-ix^2+x^3\right) dx, x, \sinh(x)\right) \\
&= -i\sinh(x) + \frac{\sinh^2(x)}{2} - \frac{1}{3}i\sinh^3(x) + \frac{\sinh^4(x)}{4}
\end{aligned}$$

Mathematica [A] time = 0.0200937, size = 28, normalized size = 0.85

$$\frac{1}{12}\sinh(x)\left(3\sinh^3(x) - 4i\sinh^2(x) + 6\sinh(x) - 12i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(I + Sinh[x]),x]
```

```
[Out] (Sinh[x]*(-12*I + 6*Sinh[x] - (4*I)*Sinh[x]^2 + 3*Sinh[x]^3))/12
```

Maple [B] time = 0.043, size = 94, normalized size = 2.9

$$\frac{5}{8}-\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{2}-\frac{i}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}-\frac{3}{8}-i\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-4}+\frac{5}{8}+\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(I+sinh(x)),x)
```

```
[Out] (5/8-1/2*I)/(tanh(1/2*x)+1)^2+(-1/2+1/3*I)/(tanh(1/2*x)+1)^3+(-3/8+I)/(tanh
(1/2*x)+1)+1/4/(tanh(1/2*x)+1)^4+(5/8+1/2*I)/(tanh(1/2*x)-1)^2+(1/2+1/3*I)/
(tanh(1/2*x)-1)^3+(3/8+I)/(tanh(1/2*x)-1)+1/4/(tanh(1/2*x)-1)^4
```

Maxima [B] time = 1.11092, size = 69, normalized size = 2.09

$$-\frac{1}{192}\left(8i\,e^{(-x)}-12\,e^{(-2\,x)}+72i\,e^{(-3\,x)}-3\right)e^{(4\,x)}+\frac{3}{8}i\,e^{(-x)}+\frac{1}{16}\,e^{(-2\,x)}+\frac{1}{24}i\,e^{(-3\,x)}+\frac{1}{64}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*I*e^(-x) - 12*e^(-2*x) + 72*I*e^(-3*x) - 3)*e^(4*x) + 3/8*I*e^(-x
) + 1/16*e^(-2*x) + 1/24*I*e^(-3*x) + 1/64*e^(-4*x)
```

Fricas [B] time = 1.80942, size = 151, normalized size = 4.58

$$\frac{1}{192}\left(3\,e^{(8\,x)}-8i\,e^{(7\,x)}+12\,e^{(6\,x)}-72i\,e^{(5\,x)}+72i\,e^{(3\,x)}+12\,e^{(2\,x)}+8i\,e^{x}+3\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*e^(8*x) - 8*I*e^(7*x) + 12*e^(6*x) - 72*I*e^(5*x) + 72*I*e^(3*x) +
 12*e^(2*x) + 8*I*e^x + 3)*e^(-4*x)
```

Sympy [B] time = 0.370981, size = 63, normalized size = 1.91

$$\frac{e^{4x}}{64}-\frac{ie^{3x}}{24}+\frac{e^{2x}}{16}-\frac{3ie^{x}}{8}+\frac{3ie^{-x}}{8}+\frac{e^{-2x}}{16}+\frac{ie^{-3x}}{24}+\frac{e^{-4x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(I+sinh(x)),x)
```

```
[Out] exp(4*x)/64 - I*exp(3*x)/24 + exp(2*x)/16 - 3*I*exp(x)/8 + 3*I*exp(-x)/8 +
exp(-2*x)/16 + I*exp(-3*x)/24 + exp(-4*x)/64
```

Giac [B] time = 1.2244, size = 63, normalized size = 1.91

$$-\frac{1}{192}\left(-72i\,e^{(3x)}-12\,e^{(2x)}-8i\,e^{x}-3\right)e^{(-4x)}+\frac{1}{64}\,e^{(4x)}-\frac{1}{24}i\,e^{(3x)}+\frac{1}{16}\,e^{(2x)}-\frac{3}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/192*(-72*I*e^(3*x) - 12*e^(2*x) - 8*I*e^x - 3)*e^(-4*x) + 1/64*e^(4*x) -
 1/24*I*e^(3*x) + 1/16*e^(2*x) - 3/8*I*e^x
```

3.162 $\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\sinh(x)\cosh(x)$$

```
[Out] (-I/2)*x + Cosh[x]^3/3 - (I/2)*Cosh[x]*Sinh[x]
```

Rubi [A] time = 0.0413358, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$-\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (-I/2)*x + Cosh[x]^3/3 - (I/2)*Cosh[x]*Sinh[x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx &= \frac{\cosh^3(x)}{3}-i\int \cosh^2(x)\,dx\\
&= \frac{\cosh^3(x)}{3}-\frac{1}{2}i\cosh(x)\sinh(x)-\frac{1}{2}i\int 1\,dx\\
&= -\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\cosh(x)\sinh(x)
\end{aligned}$$

Mathematica [B] time = 0.148906, size = 93, normalized size = 3.58

$$\frac{\cosh^5(x)\left(2\sinh^3(x)-i\sinh^2(x)+5\sinh(x)+\frac{6i\sqrt{1-i\sinh(x)}\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{1+i\sinh(x)}}+2i\right)}{6(\sinh(x)-i)^2(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]^5*(2*I + ((6*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sin
h[x]])/Sqrt[1 + I*Sinh[x]] + 5*Sinh[x] - I*Sinh[x]^2 + 2*Sinh[x]^3))/(6*(-I
 + Sinh[x])^2*(I + Sinh[x])^3)
```

Maple [B] time = 0.036, size = 126, normalized size = 4.9

$$-\frac{i}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(I+sinh(x)),x)
```

```
[Out] -1/2*I*ln(tanh(1/2*x)+1)+1/2/(tanh(1/2*x)+1)-1/2*I/(tanh(1/2*x)+1)-1/2/(tan
h(1/2*x)+1)^2+1/2*I/(tanh(1/2*x)+1)^2+1/3/(tanh(1/2*x)+1)^3+1/2*I*ln(tanh(1
/2*x)-1)-1/2/(tanh(1/2*x)-1)^2-1/2*I/(tanh(1/2*x)-1)^2-1/2/(tanh(1/2*x)-1)-
1/2*I/(tanh(1/2*x)-1)-1/3/(tanh(1/2*x)-1)^3
```

Maxima [B] time = 1.24569, size = 57, normalized size = 2.19

$$-\frac{1}{48}\left(6i\,e^{(-x)}-6\,e^{(-2\,x)}-2\right)e^{(3\,x)}-\frac{1}{2}i\,x+\frac{1}{8}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}+\frac{1}{24}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/48*(6*I*e^(-x) - 6*e^(-2*x) - 2)*e^(3*x) - 1/2*I*x + 1/8*e^(-x) + 1/8*I*
e^(-2*x) + 1/24*e^(-3*x)
```

Fricas [B] time = 1.84849, size = 128, normalized size = 4.92

$$\frac{1}{24}\left(-12i\,xe^{(3\,x)}+e^{(6\,x)}-3i\,e^{(5\,x)}+3\,e^{(4\,x)}+3\,e^{(2\,x)}+3i\,e^{x}+1\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-12*I*x*e^(3*x) + e^(6*x) - 3*I*e^(5*x) + 3*e^(4*x) + 3*e^(2*x) + 3*I
*e^x + 1)*e^(-3*x)
```

Sympy [B] time = 0.280378, size = 48, normalized size = 1.85

$$-\frac{ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}+\frac{e^{x}}{8}+\frac{e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(I+sinh(x)),x)
```

```
[Out] -I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 + exp(x)/8 + exp(-x)/8 + I*exp(-2*x)/8
+ exp(-3*x)/24
```

Giac [B] time = 1.33892, size = 51, normalized size = 1.96

$$\frac{1}{24}\left(3e^{(2x)}+3ie^{x}+1\right)e^{(-3x)}-\frac{1}{2}ix+\frac{1}{24}e^{(3x)}-\frac{1}{8}ie^{(2x)}+\frac{1}{8}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(3*e^(2*x) + 3*I*e^x + 1)*e^(-3*x) - 1/2*I*x + 1/24*e^(3*x) - 1/8*I*e^
(2*x) + 1/8*e^x
```

3.163 $\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^2(x)}{2} - i\sinh(x)$$

```
[Out] (-I)*Sinh[x] + Sinh[x]^2/2
```

Rubi [A] time = 0.0342303, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2667}

$$\frac{\sinh^2(x)}{2} - i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (-I)*Sinh[x] + Sinh[x]^2/2
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx &= -\text{Subst}\left(\int (i-x)\,dx, x, \sinh(x)\right)\\ &= -i\sinh(x) + \frac{\sinh^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0102805, size = 12, normalized size = 0.8

$$\frac{1}{2}\sinh(x)(\sinh(x)-2i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (Sinh[x]*(-2*I + Sinh[x]))/2
```

Maple [A] time = 0.014, size = 13, normalized size = 0.9

$$-i\sinh(x) + \frac{(\sinh(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(I+sinh(x)),x)
```

```
[Out] -I*sinh(x)+1/2*sinh(x)^2
```

Maxima [B] time = 1.06961, size = 36, normalized size = 2.4

$$\frac{1}{8}\left(-4i\,e^{(-x)}+1\right)e^{(2\,x)}+\frac{1}{2}i\,e^{(-x)}+\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*I*e^(-x) + 1)*e^(2*x) + 1/2*I*e^(-x) + 1/8*e^(-2*x)
```

Fricas [A] time = 1.7818, size = 70, normalized size = 4.67

$$\frac{1}{8}\left(e^{(4\,x)}-4i\,e^{(3\,x)}+4i\,e^{x}+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(e^(4*x) - 4*I*e^(3*x) + 4*I*e^x + 1)*e^(-2*x)
```

Sympy [B] time = 0.207659, size = 27, normalized size = 1.8

$$\frac{e^{2x}}{8}-\frac{ie^{x}}{2}+\frac{ie^{-x}}{2}+\frac{e^{-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(I+sinh(x)),x)
```

```
[Out] exp(2*x)/8 - I*exp(x)/2 + I*exp(-x)/2 + exp(-2*x)/8
```

Giac [B] time = 1.23027, size = 31, normalized size = 2.07

$$-\frac{1}{8}\,(-4i\,e^{x}-1)e^{(-2\,x)}+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/8*(-4*I*e^x - 1)*e^(-2*x) + 1/8*e^(2*x) - 1/2*I*e^x
```

3.164 $\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=8

$$\cosh(x) - ix$$

```
[Out] (-I)*x + Cosh[x]
```

Rubi [A] time = 0.0312799, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2682, 8}

$$\cosh(x) - ix$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-I)*x + Cosh[x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx &= \cosh(x) - i\int 1\,dx\\ &= -ix + \cosh(x)\end{aligned}$$

Mathematica [B] time = 0.0463424, size = 34, normalized size = 4.25

$$\cosh(x) + 2\sqrt{\cosh^2(x)}\mathbf{sech}(x)\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(I + Sinh[x]),x]
```

```
[Out] Cosh[x] + 2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x]
```

Maple [B] time = 0.028, size = 40, normalized size = 5.

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(I+sinh(x)),x)
```

```
[Out] -I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)+I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)-1
)
```

Maxima [B] time = 1.25592, size = 19, normalized size = 2.38

$$-i\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 1/2*e^(-x) + 1/2*e^x
```

Fricas [B] time = 1.69833, size = 53, normalized size = 6.62

$$\frac{1}{2}\left(-2i\,xe^{x}+e^{(2\,x)}+1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(-2*I*x*e^x + e^(2*x) + 1)*e^(-x)
```

Sympy [B] time = 0.144915, size = 14, normalized size = 1.75

$$-ix+\frac{e^{x}}{2}+\frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(I+sinh(x)),x)
```

```
[Out] -I*x + exp(x)/2 + exp(-x)/2
```

Giac [B] time = 1.23315, size = 19, normalized size = 2.38

$$-i\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="giac")

[Out] -I*x + 1/2*e^(-x) + 1/2*e^x
```

3.165 $\int \frac{\cosh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=7

$$\log(\sinh(x)+i)$$

```
[Out] Log[I + Sinh[x]]
```

Rubi [A] time = 0.0184888, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 31}

$$\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(I + Sinh[x]),x]
```

```
[Out] Log[I + Sinh[x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{i+\sinh(x)}\,dx &= \text{Subst}\left(\int \frac{1}{i+x}\,dx, x, \sinh(x)\right)\\ &= \log(i+\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0053324, size = 7, normalized size = 1.

$$\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(I + Sinh[x]),x]
```

```
[Out] Log[I + Sinh[x]]
```

Maple [A] time = 0.008, size = 7, normalized size = 1.

$$\ln\left(i+\sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(I+sinh(x)),x)
```

```
[Out] ln(I+sinh(x))
```

Maxima [A] time = 1.18454, size = 7, normalized size = 1.

$$\log\left(\sinh\left(x\right)+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] log(sinh(x) + I)
```

Fricas [B] time = 1.82319, size = 28, normalized size = 4.

$$-x+2\,\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -x + 2*log(e^x + I)
```

Sympy [A] time = 0.147357, size = 8, normalized size = 1.14

$$-x+2\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x)
```

```
[Out] -x + 2*log(exp(x) + I)
```

Giac [B] time = 1.3252, size = 15, normalized size = 2.14

$$-x+2\,\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -x + 2*log(e^x + I)
```

3.166 $\int \frac{\textbf{sech}(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{i}{2(\sinh(x)+i)}-\frac{1}{2}i\tan^{-1}(\sinh(x))$$

```
[Out] (-I/2)*ArcTan[Sinh[x]] - (I/2)/(I + Sinh[x])
```

Rubi [A] time = 0.034876, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2667, 44, 203}

$$-\frac{i}{2(\sinh(x)+i)}-\frac{1}{2}i\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + Sinh[x]),x]
```

```
[Out] (-I/2)*ArcTan[Sinh[x]] - (I/2)/(I + Sinh[x])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}(x)}{i+\sinh(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{(i-x)(i+x)^2}\,dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{i}{2(i+x)^2}+\frac{i}{2\left(1+x^2\right)}\right)dx, x, \sinh(x)\right)\\
&= -\frac{i}{2(i+\sinh(x))}-\frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{2}i\tan^{-1}(\sinh(x))-\frac{i}{2(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.021417, size = 18, normalized size = 0.75

$$-\frac{1}{2}i\left(\tan^{-1}(\sinh(x))+\frac{1}{\sinh(x)+i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + Sinh[x]),x]
```

```
[Out] (-I/2)*(ArcTan[Sinh[x]] + (I + Sinh[x])^(-1))
```

Maple [B] time = 0.029, size = 43, normalized size = 1.8

$$-\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}+\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}+\frac{1}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+sinh(x)),x)
```

```
[Out] -1/2*ln(tanh(1/2*x)-I)+I/(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)^2+1/2*ln(tanh(1/
2*x)+I)
```

Maxima [B] time = 1.19936, size = 55, normalized size = 2.29

$$\frac{2i\,e^{(-x)}}{-4i\,e^{(-x)}+2\,e^{(-2\,x)}-2}-\frac{1}{2}\log\left(e^{(-x)}+i\right)+\frac{1}{2}\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*I*e^(-x)/(-4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(e
^(-x) - I)
```

Fricas [B] time = 1.78173, size = 158, normalized size = 6.58

$$\frac{\left(e^{(2\,x)}+2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-\left(e^{(2\,x)}+2i\,e^{x}-1\right)\log\left(e^{x}-i\right)-2i\,e^{x}}{2\left(e^{(2\,x)}+2i\,e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) + 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) + 2*I*e^x - 1)*log(e^x
 - I) - 2*I*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)/(sinh(x) + I), x)
```

Giac [B] time = 1.29249, size = 69, normalized size = 2.88

$$-\frac{e^{(-x)} - e^{x} - 6i}{4\left(e^{(-x)} - e^{x} - 2i\right)} + \frac{1}{4}\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{1}{4}\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/4*(e^(-x) - e^x - 6*I)/(e^(-x) - e^x - 2*I) + 1/4*log(-e^(-x) + e^x + 2*
I) - 1/4*log(-e^(-x) + e^x - 2*I)
```

3.167 $\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=25

$$-\frac{2}{3}i\tanh(x)-\frac{i\operatorname{sech}(x)}{3(\sinh(x)+i)}$$

```
[Out] ((-I/3)*Sech[x])/(I + Sinh[x]) - ((2*I)/3)*Tanh[x]
```

Rubi [A] time = 0.0413912, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 3767, 8}

$$-\frac{2}{3}i\tanh(x)-\frac{i\operatorname{sech}(x)}{3(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(I + Sinh[x]),x]
```

```
[Out] ((-I/3)*Sech[x])/(I + Sinh[x]) - ((2*I)/3)*Tanh[x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx &= -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))}-\frac{2}{3}i\int \operatorname{sech}^2(x)\,dx\\ &= -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))}+\frac{2}{3}\operatorname{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)\\ &= -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))}-\frac{2}{3}i\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0322801, size = 22, normalized size = 0.88

$$-\frac{1}{3}i\left(2\tanh(x)+\frac{\operatorname{sech}(x)}{\sinh(x)+i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-I/3)*(Sech[x]/(I + Sinh[x]) + 2*Tanh[x])
```

Maple [B] time = 0.028, size = 49, normalized size = 2.

$$-\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}-\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}+\frac{2i}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}-\frac{3i}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(I+sinh(x)),x)
```

```
[Out] -1/2*I/(tanh(1/2*x)-I)-1/(tanh(1/2*x)+I)^2+2/3*I/(tanh(1/2*x)+I)^3-3/2*I/(t
anh(1/2*x)+I)
```

Maxima [B] time = 1.10298, size = 72, normalized size = 2.88

$$-\frac{8\,e^{(-x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}+\frac{4i}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -8*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 4*I/(-6*I*e^(-x)
- 6*I*e^(-3*x) + 3*e^(-4*x) - 3)
```

Fricas [A] time = 1.63876, size = 76, normalized size = 3.04

$$-\frac{8\,e^{x}+4i}{3\,e^{(4\,x)}+6i\,e^{(3\,x)}+6i\,e^{x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(8*e^x + 4*I)/(3*e^(4*x) + 6*I*e^(3*x) + 6*I*e^x - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**2/(sinh(x) + I), x)
```

Giac [A] time = 1.272, size = 39, normalized size = 1.56

$$\frac{1}{2\left(e^{x}-i\right)}-\frac{3\,e^{(2\,x)}+12i\,e^{x}-5}{6\left(e^{x}+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2/(e^x - I) - 1/6*(3*e^(2*x) + 12*I*e^x - 5)/(e^x + I)^3
```

3.168 $\int \frac{\text{sech}^3(x)}{i+\sinh(x)} dx$

Optimal. Leaf size=52

$$\frac{i}{8(-\sinh(x)+i)} - \frac{i}{4(\sinh(x)+i)} + \frac{1}{8(\sinh(x)+i)^2} - \frac{3}{8} i \tan^{-1}(\sinh(x))$$

```
[Out] ((-3*I)/8)*ArcTan[Sinh[x]] + (I/8)/(I - Sinh[x]) + 1/(8*(I + Sinh[x])^2) -
(I/4)/(I + Sinh[x])
```

Rubi [A] time = 0.0525301, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 203}

$$\frac{i}{8(-\sinh(x)+i)} - \frac{i}{4(\sinh(x)+i)} + \frac{1}{8(\sinh(x)+i)^2} - \frac{3}{8} i \tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/8)*ArcTan[Sinh[x]] + (I/8)/(I - Sinh[x]) + 1/(8*(I + Sinh[x])^2) -
(I/4)/(I + Sinh[x])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \text{Subst}\left(\int \frac{1}{(i-x)^2(i+x)^3}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{i}{8(-i+x)^2} - \frac{1}{4(i+x)^3} + \frac{i}{4(i+x)^2} - \frac{3i}{8\left(1+x^2\right)}\right)dx, x, \sinh(x)\right)$$
$$= \frac{i}{8(i-\sinh(x))} + \frac{1}{8(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))} - \frac{3}{8}i\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{3}{8}i\tan^{-1}(\sinh(x)) + \frac{i}{8(i-\sinh(x))} + \frac{1}{8(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))}$$

Mathematica [A] time = 0.0414301, size = 61, normalized size = 1.17

$$-\frac{i\operatorname{sech}^2(x)\left(3\sinh^3(x)\tan^{-1}(\sinh(x)) + \sinh^2(x)\left(3+3i\tan^{-1}(\sinh(x))\right) + 3\sinh(x)\left(\tan^{-1}(\sinh(x))+i\right) + 3i\tan^{-1}\right.}{8(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-I/8)*Sech[x]^2*(2 + (3*I)*ArcTan[Sinh[x]] + 3*(I + ArcTan[Sinh[x]])*Sinh
[x] + (3 + (3*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 3*ArcTan[Sinh[x]]*Sinh[x]^3))
/(I + Sinh[x])
```

Maple [B] time = 0.036, size = 91, normalized size = 1.8

$$\frac{i}{4}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1} - \frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2} - \frac{3}{8}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right) - \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4} + i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1} - i\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(I+sinh(x)),x)
```

```
[Out] 1/4*I/(tanh(1/2*x)-I)-1/4/(tanh(1/2*x)-I)^2-3/8*ln(tanh(1/2*x)-I)-1/2/(tanh
(1/2*x)+I)^4+I/(tanh(1/2*x)+I)-I/(tanh(1/2*x)+I)^3+3/2/(tanh(1/2*x)+I)^2+3/
8*ln(tanh(1/2*x)+I)
```

Maxima [B] time = 1.18619, size = 124, normalized size = 2.38

$$\frac{8\left(3i\,e^{(-x)} - 6\,e^{(-2x)} + 2i\,e^{(-3x)} + 6\,e^{(-4x)} + 3i\,e^{(-5x)}\right)}{-64i\,e^{(-x)} - 32\,e^{(-2x)} - 128i\,e^{(-3x)} + 32\,e^{(-4x)} - 64i\,e^{(-5x)} + 32\,e^{(-6x)} - 32} - \frac{3}{8}\log\left(e^{(-x)}+i\right) + \frac{3}{8}\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 8*(3*I*e^(-x) - 6*e^(-2*x) + 2*I*e^(-3*x) + 6*e^(-4*x) + 3*I*e^(-5*x))/(-64
*I*e^(-x) - 32*e^(-2*x) - 128*I*e^(-3*x) + 32*e^(-4*x) - 64*I*e^(-5*x) + 32
*e^(-6*x) - 32) - 3/8*log(e^(-x) + I) + 3/8*log(e^(-x) - I)
```

Fricas [B] time = 1.75851, size = 451, normalized size = 8.67

$$\frac{\left(3\,e^{(6\,x)}+6i\,e^{(5\,x)}+3\,e^{(4\,x)}+12i\,e^{(3\,x)}-3\,e^{(2\,x)}+6i\,e^{x}-3\right)\log\left(e^{x}+i\right)-\left(3\,e^{(6\,x)}+6i\,e^{(5\,x)}+3\,e^{(4\,x)}+12i\,e^{(3\,x)}-3\,e^{(2\,x)}+6i\right.}{8\,e^{(6\,x)}+16i\,e^{(5\,x)}+8\,e^{(4\,x)}+32i\,e^{(3\,x)}-8\,e^{(2\,x)}+16i\,e^{x}-8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((3*e^(6*x) + 6*I*e^(5*x) + 3*e^(4*x) + 12*I*e^(3*x) - 3*e^(2*x) + 6*I*e^x
- 3)*log(e^x + I) - (3*e^(6*x) + 6*I*e^(5*x) + 3*e^(4*x) + 12*I*e^(3*x) - 3
*e^(2*x) + 6*I*e^x - 3)*log(e^x - I) - 6*I*e^(5*x) + 12*e^(4*x) - 4*I*e^(3*
x) - 12*e^(2*x) - 6*I*e^x)/(8*e^(6*x) + 16*I*e^(5*x) + 8*e^(4*x) + 32*I*e^(
3*x) - 8*e^(2*x) + 16*I*e^x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**3/(sinh(x) + I), x)
```

Giac [B] time = 1.32803, size = 124, normalized size = 2.38

$$\frac{3\,e^{(-x)}-3\,e^{x}+10i}{16\left(e^{(-x)}-e^{x}+2i\right)}-\frac{9\left(e^{(-x)}-e^{x}\right)^2-52i\,e^{(-x)}+52i\,e^{x}-84}{32\left(e^{(-x)}-e^{x}-2i\right)^2}+\frac{3}{16}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{3}{16}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/16*(3*e^(-x) - 3*e^x + 10*I)/(e^(-x) - e^x + 2*I) - 1/32*(9*(e^(-x) - e^x
)^2 - 52*I*e^(-x) + 52*I*e^x - 84)/(e^(-x) - e^x - 2*I)^2 + 3/16*log(-e^(-x
) + e^x + 2*I) - 3/16*log(-e^(-x) + e^x - 2*I)
```

3.169 $\int \frac{\text{sech}^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=37

$$\frac{4}{15}i\tanh^3(x)-\frac{4}{5}i\tanh(x)-\frac{i\text{sech}^3(x)}{5(\sinh(x)+i)}$$

```
[Out] ((-I/5)*Sech[x]^3)/(I + Sinh[x]) - ((4*I)/5)*Tanh[x] + ((4*I)/15)*Tanh[x]^3
```

Rubi [A] time = 0.0437954, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2672, 3767}

$$\frac{4}{15}i\tanh^3(x)-\frac{4}{5}i\tanh(x)-\frac{i\text{sech}^3(x)}{5(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(I + Sinh[x]),x]
```

```
[Out] ((-I/5)*Sech[x]^3)/(I + Sinh[x]) - ((4*I)/5)*Tanh[x] + ((4*I)/15)*Tanh[x]^3
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^4(x)}{i+\sinh(x)}\,dx &= -\frac{i\text{sech}^3(x)}{5(i+\sinh(x))}-\frac{4}{5}i\int \text{sech}^4(x)\,dx\\ &= -\frac{i\text{sech}^3(x)}{5(i+\sinh(x))}+\frac{4}{5}\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\tanh(x)\right)\\ &= -\frac{i\text{sech}^3(x)}{5(i+\sinh(x))}-\frac{4}{5}i\tanh(x)+\frac{4}{15}i\tanh^3(x)\end{aligned}$$

Mathematica [A] time = 0.0642261, size = 35, normalized size = 0.95

$$-\frac{1}{15}i\left(8\tanh^3(x)+\frac{3\text{sech}^3(x)}{\sinh(x)+i}+12\tanh(x)\text{sech}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(I + Sinh[x]),x]
```

```
[Out] (-I/15)*((3*Sech[x]^3)/(I + Sinh[x]) + 12*Sech[x]^2*Tanh[x] + 8*Tanh[x]^3)
```

Maple [B] time = 0.04, size = 93, normalized size = 2.5

$$\frac{i}{6}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}-\frac{5i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}+\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}-\frac{2i}{5}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-5}+\frac{5i}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}-\frac{11i}{8}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(I+sinh(x)),x)
```

```
[Out] 1/6*I/(tanh(1/2*x)-I)^3-5/8*I/(tanh(1/2*x)-I)+1/4/(tanh(1/2*x)-I)^2-2/5*I/(
tanh(1/2*x)+I)^5+5/3*I/(tanh(1/2*x)+I)^3-11/8*I/(tanh(1/2*x)+I)+1/(tanh(1/2
*x)+I)^4-3/2/(tanh(1/2*x)+I)^2
```

Maxima [B] time = 1.21073, size = 277, normalized size = 7.49

$$-\frac{32\,e^{(-x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}+\frac{}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -32*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30
*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 32*I*e^(-2*x)/(-30*I*e^(-x)
 - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x
) + 15*e^(-8*x) - 15) - 96*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-
3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 16
*I/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x
) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15)
```

Fricas [B] time = 1.78141, size = 197, normalized size = 5.32

$$-\frac{96\,e^{(3\,x)}+32i\,e^{(2\,x)}+32\,e^{x}+16i}{15\,e^{(8\,x)}+30i\,e^{(7\,x)}+30\,e^{(6\,x)}+90i\,e^{(5\,x)}+90i\,e^{(3\,x)}-30\,e^{(2\,x)}+30i\,e^{x}-15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(96*e^(3*x) + 32*I*e^(2*x) + 32*e^x + 16*I)/(15*e^(8*x) + 30*I*e^(7*x) + 3
0*e^(6*x) + 90*I*e^(5*x) + 90*I*e^(3*x) - 30*e^(2*x) + 30*I*e^x - 15)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(I+sinh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27, size = 72, normalized size = 1.95

$$\frac{9\,e^{(2\,x)} - 24i\,e^{x} - 11}{24\,(e^{x} - i)^{3}} - \frac{45\,e^{(4\,x)} + 240i\,e^{(3\,x)} - 490\,e^{(2\,x)} - 320i\,e^{x} + 73}{120\,(e^{x} + i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(9*e^(2*x) - 24*I*e^x - 11)/(e^x - I)^3 - 1/120*(45*e^(4*x) + 240*I*e^
(3*x) - 490*e^(2*x) - 320*I*e^x + 73)/(e^x + I)^5
```

3.170 $\int \frac{\text{sech}^5(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=80

$$\frac{i}{8(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{32(-\sinh(x)+i)^2}+\frac{3}{32(\sinh(x)+i)^2}+\frac{i}{24(\sinh(x)+i)^3}-\frac{5}{16}i\tan^{-1}(\sinh(x))$$

```
[Out] ((-5*I)/16)*ArcTan[Sinh[x]] - 1/(32*(I - Sinh[x])^2) + (I/8)/(I - Sinh[x])
+ (I/24)/(I + Sinh[x])^3 + 3/(32*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x]
)
```

Rubi [A] time = 0.0666413, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 203}

$$\frac{i}{8(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{32(-\sinh(x)+i)^2}+\frac{3}{32(\sinh(x)+i)^2}+\frac{i}{24(\sinh(x)+i)^3}-\frac{5}{16}i\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(I + Sinh[x]),x]
```

```
[Out] ((-5*I)/16)*ArcTan[Sinh[x]] - 1/(32*(I - Sinh[x])^2) + (I/8)/(I - Sinh[x])
+ (I/24)/(I + Sinh[x])^3 + 3/(32*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x]
)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{sech}^5(x)}{i+\sinh(x)}\,dx = -\,\text{Subst}\left(\int \frac{1}{(i-x)^3(i+x)^4}\,dx, x, \sinh(x)\right)$$
$$= -\,\text{Subst}\left(\int\left(-\frac{1}{16(-i+x)^3} - \frac{i}{8(-i+x)^2} + \frac{i}{8(i+x)^4} + \frac{3}{16(i+x)^3} - \frac{3i}{16(i+x)^2} + \frac{5i}{16\left(1+x^2\right)}\right)dx,$$
$$= -\frac{1}{32(i-\sinh(x))^2} + \frac{i}{8(i-\sinh(x))} + \frac{i}{24(i+\sinh(x))^3} + \frac{3}{32(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x))} - \frac{5}{16}$$
$$= -\frac{5}{16} i\tan^{-1}(\sinh(x)) - \frac{1}{32(i-\sinh(x))^2} + \frac{i}{8(i-\sinh(x))} + \frac{i}{24(i+\sinh(x))^3} + \frac{3}{32(i+\sinh(x))^2} -$$

Mathematica [A] time = 0.0671052, size = 94, normalized size = 1.18

$$-\frac{i\text{sech}^4(x)\left(15\sinh^5(x)\tan^{-1}(\sinh(x)) + 15\sinh^4(x)\left(1+i\tan^{-1}(\sinh(x))\right) + 15\sinh^3(x)\left(2\tan^{-1}(\sinh(x))+i\right) + 5}{48(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(I + Sinh[x]),x]
```

```
[Out] ((-I/48)*Sech[x]^4*(8 + (15*I)*ArcTan[Sinh[x]] + 5*(5*I + 3*ArcTan[Sinh[x]]
)*Sinh[x] + 5*(5 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 15*(I + 2*ArcTan[Sinh
[x]])*Sinh[x]^3 + 15*(1 + I*ArcTan[Sinh[x]])*Sinh[x]^4 + 15*ArcTan[Sinh[x]]
*Sinh[x]^5))/(I + Sinh[x])
```

Maple [B] time = 0.042, size = 137, normalized size = 1.7

$$\frac{3i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1} - \frac{i}{4}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3} + \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-4} - \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2} - \frac{5}{16}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right) + i\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(I+sinh(x)),x)
```

```
[Out] 3/8*I/(tanh(1/2*x)-I)-1/4*I/(tanh(1/2*x)-I)^3+1/8/(tanh(1/2*x)-I)^4-1/2/(ta
nh(1/2*x)-I)^2-5/16*ln(tanh(1/2*x)-I)+I/(tanh(1/2*x)+I)^5+I/(tanh(1/2*x)+I)
-25/12*I/(tanh(1/2*x)+I)^3+1/3/(tanh(1/2*x)+I)^6-15/8/(tanh(1/2*x)+I)^4+15/
8/(tanh(1/2*x)+I)^2+5/16*ln(tanh(1/2*x)+I)
```

Maxima [B] time = 1.18321, size = 189, normalized size = 2.36

$$\frac{32\left(15i e^{(-x)} - 30 e^{(-2x)} + 40i e^{(-3x)} - 110 e^{(-4x)} + 18i e^{(-5x)} + 110 e^{(-6x)} + 40i e^{(-7x)} + 30 e^{(-8x)} + 15i\right.}{-1536i e^{(-x)} - 2304 e^{(-2x)} - 6144i e^{(-3x)} - 1536 e^{(-4x)} - 9216i e^{(-5x)} + 1536 e^{(-6x)} - 6144i e^{(-7x)} + 2304 e^{(-8x)} - 1536i e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 32*(15*I*e^(-x) - 30*e^(-2*x) + 40*I*e^(-3*x) - 110*e^(-4*x) + 18*I*e^(-5*x
) + 110*e^(-6*x) + 40*I*e^(-7*x) + 30*e^(-8*x) + 15*I*e^(-9*x))/(-1536*I*e^
(-x) - 2304*e^(-2*x) - 6144*I*e^(-3*x) - 1536*e^(-4*x) - 9216*I*e^(-5*x) +
1536*e^(-6*x) - 6144*I*e^(-7*x) + 2304*e^(-8*x) - 1536*I*e^(-9*x) + 768*e^(
```

```
-10*x) - 768) - 5/16*log(e^(-x) + I) + 5/16*log(e^(-x) - I)
```

Fricas [B] time = 2.14083, size = 801, normalized size = 10.01

$$\frac{\left(15\,e^{(10\,x)} + 30i\,e^{(9\,x)} + 45\,e^{(8\,x)} + 120i\,e^{(7\,x)} + 30\,e^{(6\,x)} + 180i\,e^{(5\,x)} - 30\,e^{(4\,x)} + 120i\,e^{(3\,x)} - 45\,e^{(2\,x)} + 30i\,e^{x} - 15\right)\log\left(e^{x} + i\right.}{48\,e^{(10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((15*e^(10*x) + 30*I*e^(9*x) + 45*e^(8*x) + 120*I*e^(7*x) + 30*e^(6*x) + 18
0*I*e^(5*x) - 30*e^(4*x) + 120*I*e^(3*x) - 45*e^(2*x) + 30*I*e^x - 15)*log(
e^x + I) - (15*e^(10*x) + 30*I*e^(9*x) + 45*e^(8*x) + 120*I*e^(7*x) + 30*e^
(6*x) + 180*I*e^(5*x) - 30*e^(4*x) + 120*I*e^(3*x) - 45*e^(2*x) + 30*I*e^x
- 15)*log(e^x - I) - 30*I*e^(9*x) + 60*e^(8*x) - 80*I*e^(7*x) + 220*e^(6*x)
 - 36*I*e^(5*x) - 220*e^(4*x) - 80*I*e^(3*x) - 60*e^(2*x) - 30*I*e^x)/(48*e
^(10*x) + 96*I*e^(9*x) + 144*e^(8*x) + 384*I*e^(7*x) + 96*e^(6*x) + 576*I*e
^(5*x) - 96*e^(4*x) + 384*I*e^(3*x) - 144*e^(2*x) + 96*I*e^x - 48)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(I+sinh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26678, size = 159, normalized size = 1.99

$$\frac{15\left(e^{(-x)} - e^{x}\right)^{2} + 76i\,e^{(-x)} - 76i\,e^{x} - 100}{64\left(e^{(-x)} - e^{x} + 2i\right)^{2}} - \frac{55\left(e^{(-x)} - e^{x}\right)^{3} - 402i\left(e^{(-x)} - e^{x}\right)^{2} - 1020\,e^{(-x)} + 1020\,e^{x} + 936i}{192\left(e^{(-x)} - e^{x} - 2i\right)^{3}} + \frac{5}{32}\log\left(-e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/64*(15*(e^(-x) - e^x)^2 + 76*I*e^(-x) - 76*I*e^x - 100)/(e^(-x) - e^x + 2
*I)^2 - 1/192*(55*(e^(-x) - e^x)^3 - 402*I*(e^(-x) - e^x)^2 - 1020*e^(-x) +
 1020*e^x + 936*I)/(e^(-x) - e^x - 2*I)^3 + 5/32*log(-e^(-x) + e^x + 2*I) -
 5/32*log(-e^(-x) + e^x - 2*I)
```

3.171 $\int \frac{\cosh^6(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=40

$$-\frac{5x}{8}-\frac{5}{12}i\cosh^3(x)+\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{8}\sinh(x)\cosh(x)$$

```
[Out] (-5*x)/8 - ((5*I)/12)*Cosh[x]^3 - (5*Cosh[x]*Sinh[x])/8 + Cosh[x]^5/(4*(I +
 Sinh[x]))
```

Rubi [A] time = 0.0738782, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2679, 2682, 2635, 8}

$$-\frac{5x}{8}-\frac{5}{12}i\cosh^3(x)+\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{8}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-5*x)/8 - ((5*I)/12)*Cosh[x]^3 - (5*Cosh[x]*Sinh[x])/8 + Cosh[x]^5/(4*(I +
 Sinh[x]))
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = \frac{\cosh^5(x)}{4(i+\sinh(x))} - \frac{5}{4}i\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx$$
$$= -\frac{5}{12}i\cosh^3(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))} - \frac{5}{4}\int \cosh^2(x)\,dx$$
$$= -\frac{5}{12}i\cosh^3(x) - \frac{5}{8}\cosh(x)\sinh(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))} - \frac{5\int 1\,dx}{8}$$
$$= -\frac{5x}{8} - \frac{5}{12}i\cosh^3(x) - \frac{5}{8}\cosh(x)\sinh(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))}$$

Mathematica [B] time = 0.17929, size = 121, normalized size = 3.02

$$-\frac{i\cosh^7(x)\left(6\sinh^4(x) - 10i\sinh^3(x) + 7\sinh^2(x) - 25i\sinh(x) + \frac{30\sqrt{1-i\sinh(x)}\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{1+i\sinh(x)}} + 16\right)}{24\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^8\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] ((-I/24)*Cosh[x]^7*(16 + (30*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I
*Sinh[x]])/Sqrt[1 + I*Sinh[x]] - (25*I)*Sinh[x] + 7*Sinh[x]^2 - (10*I)*Sinh
[x]^3 + 6*Sinh[x]^4))/((Cosh[x/2] - I*Sinh[x/2])^8*(Cosh[x/2] + I*Sinh[x/2]
)^6)
```

Maple [B] time = 0.054, size = 166, normalized size = 4.2

$$\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3} + \frac{2i}{3}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-3} + \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} + i\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \frac{3}{8}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} + i\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(I+sinh(x))^2,x)
```

```
[Out] 1/2/(tanh(1/2*x)+1)^3+2/3*I/(tanh(1/2*x)-1)^3+1/8/(tanh(1/2*x)+1)^2+I/(tanh
(1/2*x)+1)^2-3/8/(tanh(1/2*x)+1)+I/(tanh(1/2*x)-1)^2-1/4/(tanh(1/2*x)+1)^4-
5/8*ln(tanh(1/2*x)+1)+1/2/(tanh(1/2*x)-1)^3-2/3*I/(tanh(1/2*x)+1)^3-1/8/(ta
nh(1/2*x)-1)^2+I/(tanh(1/2*x)-1)-3/8/(tanh(1/2*x)-1)-I/(tanh(1/2*x)+1)+1/4/
(tanh(1/2*x)-1)^4+5/8*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.12836, size = 73, normalized size = 1.82

$$-\frac{1}{192}\left(16i\,e^{(-x)} + 24\,e^{(-2x)} + 48i\,e^{(-3x)} - 3\right)e^{(4x)} - \frac{5}{8}x - \frac{1}{4}i\,e^{(-x)} + \frac{1}{8}e^{(-2x)} - \frac{1}{12}i\,e^{(-3x)} - \frac{1}{64}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/192*(16*I*e^(-x) + 24*e^(-2*x) + 48*I*e^(-3*x) - 3)*e^(4*x) - 5/8*x - 1/
4*I*e^(-x) + 1/8*e^(-2*x) - 1/12*I*e^(-3*x) - 1/64*e^(-4*x)
```

Fricas [A] time = 2.07973, size = 177, normalized size = 4.42

$$-\frac{1}{192}\left(120\,xe^{(4\,x)} - 3\,e^{(8\,x)} + 16i\,e^{(7\,x)} + 24\,e^{(6\,x)} + 48i\,e^{(5\,x)} + 48i\,e^{(3\,x)} - 24\,e^{(2\,x)} + 16i\,e^{x} + 3\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/192*(120*x*e^(4*x) - 3*e^(8*x) + 16*I*e^(7*x) + 24*e^(6*x) + 48*I*e^(5*x
) + 48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x)
```

Sympy [A] time = 0.441586, size = 65, normalized size = 1.62

$$-\frac{5x}{8} + \frac{e^{4x}}{64} - \frac{ie^{3x}}{12} - \frac{e^{2x}}{8} - \frac{ie^{x}}{4} - \frac{ie^{-x}}{4} + \frac{e^{-2x}}{8} - \frac{ie^{-3x}}{12} - \frac{e^{-4x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(I+sinh(x))**2,x)
```

```
[Out] -5*x/8 + exp(4*x)/64 - I*exp(3*x)/12 - exp(2*x)/8 - I*exp(x)/4 - I*exp(-x)/
4 + exp(-2*x)/8 - I*exp(-3*x)/12 - exp(-4*x)/64
```

Giac [A] time = 1.26273, size = 68, normalized size = 1.7

$$-\frac{1}{192}\left(48i\,e^{(3\,x)} - 24\,e^{(2\,x)} + 16i\,e^{x} + 3\right)e^{(-4\,x)} - \frac{5}{8}\,x + \frac{1}{64}\,e^{(4\,x)} - \frac{1}{12}i\,e^{(3\,x)} - \frac{1}{8}\,e^{(2\,x)} - \frac{1}{4}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/192*(48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x) - 5/8*x + 1/64*e
^(4*x) - 1/12*I*e^(3*x) - 1/8*e^(2*x) - 1/4*I*e^x
```

3.172 $\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=14

$$-\frac{1}{3}(-\sinh(x)+i)^3$$

```
[Out] -(I - Sinh[x])^3/3
```

Rubi [A] time = 0.0335453, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 32}

$$-\frac{1}{3}(-\sinh(x)+i)^3$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] -(I - Sinh[x])^3/3
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} dx &= \text{Subst}\left(\int (i-x)^2 dx, x, \sinh(x)\right)\\ &= -\frac{1}{3}(i-\sinh(x))^3\end{aligned}$$

Mathematica [A] time = 0.0193989, size = 18, normalized size = 1.29

$$\frac{1}{6}\sinh(x)(-6i\sinh(x)+\cosh(2x)-7)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] ((-7 + Cosh[2*x] - (6*I)*Sinh[x])*Sinh[x])/6
```

Maple [B] time = 0.047, size = 70, normalized size = 5.

$$\frac{1}{2}-i\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+1+i\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+1-i\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}-\frac{1}{2}+i\left(\tanh\left(\frac{x}{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(I+sinh(x))^2,x)
```

```
[Out] (1/2-I)/(tanh(1/2*x)+1)^2+(1+I)/(tanh(1/2*x)+1)-1/3/(tanh(1/2*x)+1)^3+(1-I)
/(tanh(1/2*x)-1)-(1/2+I)/(tanh(1/2*x)-1)^2-1/3/(tanh(1/2*x)-1)^3
```

Maxima [B] time = 1.2142, size = 53, normalized size = 3.79

$$-\frac{1}{96}\left(24i\,e^{(-x)}+60\,e^{(-2\,x)}-4\right)e^{(3\,x)}+\frac{5}{8}\,e^{(-x)}-\frac{1}{4}i\,e^{(-2\,x)}-\frac{1}{24}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(24*I*e^(-x) + 60*e^(-2*x) - 4)*e^(3*x) + 5/8*e^(-x) - 1/4*I*e^(-2*x)
 - 1/24*e^(-3*x)
```

Fricas [B] time = 1.97653, size = 107, normalized size = 7.64

$$\frac{1}{24}\left(e^{(6\,x)}-6i\,e^{(5\,x)}-15\,e^{(4\,x)}+15\,e^{(2\,x)}-6i\,e^{x}-1\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(e^(6*x) - 6*I*e^(5*x) - 15*e^(4*x) + 15*e^(2*x) - 6*I*e^x - 1)*e^(-3*
x)
```

Sympy [B] time = 0.315745, size = 44, normalized size = 3.14

$$\frac{e^{3x}}{24}-\frac{ie^{2x}}{4}-\frac{5e^{x}}{8}+\frac{5e^{-x}}{8}-\frac{ie^{-2x}}{4}-\frac{e^{-3x}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(I+sinh(x))**2,x)
```

```
[Out] exp(3*x)/24 - I*exp(2*x)/4 - 5*exp(x)/8 + 5*exp(-x)/8 - I*exp(-2*x)/4 - exp
(-3*x)/24
```

Giac [B] time = 1.21791, size = 47, normalized size = 3.36

$$\frac{1}{24}\left(15\,e^{(2\,x)} - 6i\,e^{x} - 1\right)e^{(-3\,x)} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{4}i\,e^{(2\,x)} - \frac{5}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*e^(2*x) - 6*I*e^x - 1)*e^(-3*x) + 1/24*e^(3*x) - 1/4*I*e^(2*x) - 5
/8*e^x
```

3.173 $\int \frac{\cosh^4(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=30

$$-\frac{3x}{2} - \frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(\sinh(x)+i)}$$

```
[Out] (-3*x)/2 - ((3*I)/2)*Cosh[x] + Cosh[x]^3/(2*(I + Sinh[x]))
```

Rubi [A] time = 0.0675867, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2679, 2682, 8}

$$-\frac{3x}{2} - \frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-3*x)/2 - ((3*I)/2)*Cosh[x] + Cosh[x]^3/(2*(I + Sinh[x]))
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^4(x)}{(i+\sinh(x))^2} dx &= \frac{\cosh^3(x)}{2(i+\sinh(x))} - \frac{3}{2}i \int \frac{\cosh^2(x)}{i+\sinh(x)} dx \\ &= -\frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))} - \frac{3\int 1\,dx}{2} \\ &= -\frac{3x}{2} - \frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))} \end{aligned}$$

Mathematica [A] time = 0.0718262, size = 46, normalized size = 1.53

$$\frac{1}{2}(\sinh(x)-4i)\cosh(x)-3i\sqrt{\cosh^2(x)}\mathbf{sech}(x)\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-3*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x] + (Cosh[
x]*(-4*I + Sinh[x]))/2
```

Maple [B] time = 0.042, size = 82, normalized size = 2.7

$$\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-2i\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{3}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+2i\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(I+sinh(x))^2,x)
```

```
[Out] 1/2/(tanh(1/2*x)+1)-2*I/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2-3/2*ln(tanh(1
/2*x)+1)+1/2/(tanh(1/2*x)-1)+2*I/(tanh(1/2*x)-1)+1/2/(tanh(1/2*x)-1)^2+3/2*
ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.12467, size = 41, normalized size = 1.37

$$-\frac{1}{8}\left(8ie^{(-x)}-1\right)e^{(2x)}-\frac{3}{2}x-ie^{(-x)}-\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(8*I*e^(-x) - 1)*e^(2*x) - 3/2*x - I*e^(-x) - 1/8*e^(-2*x)
```

Fricas [A] time = 1.83379, size = 92, normalized size = 3.07

$$-\frac{1}{8}\left(12xe^{(2x)}-e^{(4x)}+8ie^{(3x)}+8ie^{x}+1\right)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*x*e^(2*x) - e^(4*x) + 8*I*e^(3*x) + 8*I*e^x + 1)*e^(-2*x)
```

Sympy [A] time = 0.264454, size = 29, normalized size = 0.97

$$-\frac{3x}{2}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] -3*x/2 + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

Giac [A] time = 1.31306, size = 35, normalized size = 1.17

$$-\frac{1}{8}\,(8i\,e^{x}+1)e^{(-2\,x)}-\frac{3}{2}\,x+\frac{1}{8}\,e^{(2\,x)}-i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/8*(8*I*e^x + 1)*e^(-2*x) - 3/2*x + 1/8*e^(2*x) - I*e^x
```

3.174 $\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=14

$$\sinh(x) - 2i\log(\sinh(x) + i)$$

```
[Out] (-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Rubi [A] time = 0.0377104, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\sinh(x) - 2i\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(I + Sinh[x])^2,x]

[Out] (-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx &= -\operatorname{Subst}\left(\int \frac{i-x}{i+x}\,dx, x, \sinh(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(-1+\frac{2i}{i+x}\right)dx, x, \sinh(x)\right)\\
&= -2i\log(i+\sinh(x)) + \sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.0124456, size = 14, normalized size = 1.

$$\sinh(x) - 2i\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Maple [B] time = 0.042, size = 53, normalized size = 3.8

$$2\,i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+2\,i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}-4\,i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(I+sinh(x))^2,x)
```

```
[Out] 2*I*ln(tanh(1/2*x)+1)-1/(tanh(1/2*x)+1)+2*I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x
)-1)-4*I*ln(tanh(1/2*x)+I)
```

Maxima [B] time = 1.23432, size = 31, normalized size = 2.21

$$-2i\,x-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}-4i\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^(-x) - I)
```

Fricas [B] time = 1.92183, size = 82, normalized size = 5.86

$$\frac{1}{2}\left(4i\,xe^{x}-8i\,e^{x}\log\left(e^{x}+i\right)+e^{(2\,x)}-1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*I*x*e^x - 8*I*e^x*log(e^x + I) + e^(2*x) - 1)*e^(-x)
```

Sympy [B] time = 9.33967, size = 26, normalized size = 1.86

$$2ix+\frac{e^{x}}{2}-4i\log\left(e^{x}+i\right)-\frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] 2*I*x + exp(x)/2 - 4*I*log(exp(x) + I) - exp(-x)/2
```

Giac [A] time = 1.31995, size = 28, normalized size = 2.

$$2ix - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x} - 4i\log\left(e^{x} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^x + I)
```

3.175 $\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=14

$$x-\frac{2\cosh(x)}{\sinh(x)+i}$$

```
[Out] x - (2*Cosh[x])/(I + Sinh[x])
```

Rubi [A] time = 0.0336892, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2680, 8}

$$x-\frac{2\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(I + Sinh[x])^2,x]

[Out] x - (2*Cosh[x])/(I + Sinh[x])
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx &= -\frac{2\cosh(x)}{i+\sinh(x)}+\int 1\,dx\\ &= x-\frac{2\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] time = 0.0538803, size = 69, normalized size = 4.93

$$\frac{2\cosh^3(x)\left(-1-\frac{\sqrt{1-i\sinh(x)}\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{1+i\sinh(x)}}\right)}{(\sinh(x)-i)(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*Cosh[x]^3*(-1 - (ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[x]]
)/Sqrt[1 + I*Sinh[x]]))/((-I + Sinh[x])*(I + Sinh[x])^2)
```

Maple [B] time = 0.04, size = 29, normalized size = 2.1

$$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-4\,(\tanh(x/2)+i)^{-1}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(I+sinh(x))^2,x)
```

```
[Out] ln(tanh(1/2*x)+1)-4/(tanh(1/2*x)+I)-ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.21545, size = 16, normalized size = 1.14

$$x+\frac{4i}{e^{(-x)}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] x + 4*I/(e^(-x) - I)
```

Fricas [A] time = 1.73517, size = 42, normalized size = 3.

$$\frac{xe^{x}+ix+4i}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (x*e^x + I*x + 4*I)/(e^x + I)
```

Sympy [A] time = 0.183728, size = 8, normalized size = 0.57

$$x+\frac{4i}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(I+sinh(x))**2,x)
```

```
[Out] x + 4*I/(exp(x) + I)
```

<u>Giac</u> [A] time = 1.27628, size = 14, normalized size = 1.

$$x + \frac{4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] x + 4*I/(e^x + I)
```

3.176 $\int \frac{\cosh(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{\sinh(x)+i}$$

```
[Out] -(I + Sinh[x])^(-1)
```

Rubi [A] time = 0.0198091, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 32}

$$-\frac{1}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(I + Sinh[x])^2,x]
```

```
[Out] -(I + Sinh[x])^(-1)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(x)}{(i+\sinh(x))^2} \, dx &= \text{Subst}\left(\int \frac{1}{(i+x)^2} \, dx, x, \sinh(x)\right) \\ &= -\frac{1}{i+\sinh(x)} \end{aligned}$$

Mathematica [A] time = 0.0099376, size = 10, normalized size = 1.

$$-\frac{1}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(I + Sinh[x])^2,x]
```

```
[Out] -(I + Sinh[x])^(-1)
```

Maple [A] time = 0.013, size = 10, normalized size = 1.

$$-(i+\sinh(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(I+sinh(x))^2,x)
```

```
[Out] -1/(I+sinh(x))
```

Maxima [A] time = 1.22747, size = 11, normalized size = 1.1

$$-\frac{1}{\sinh(x)+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(sinh(x) + I)
```

Fricas [A] time = 1.73557, size = 43, normalized size = 4.3

$$-\frac{2\,e^{x}}{e^{(2\,x)}+2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*e^x/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] time = 0.211546, size = 19, normalized size = 1.9

$$-\frac{2e^{x}}{e^{2x}+2ie^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))**2,x)
```

```
[Out] -2*exp(x)/(exp(2*x) + 2*I*exp(x) - 1)
```

Giac [A] time = 1.29195, size = 14, normalized size = 1.4

$$-\frac{2\,e^{x}}{(e^{x}+i)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*e^x/(e^x + I)^2
```

3.177 $\int \frac{\text{sech}(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{4(\sinh(x)+i)}-\frac{i}{4(\sinh(x)+i)^2}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

```
[Out] -ArcTan[Sinh[x]]/4 - (I/4)/(I + Sinh[x])^2 - 1/(4*(I + Sinh[x]))
```

Rubi [A] time = 0.0391219, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2667, 44, 203}

$$-\frac{1}{4(\sinh(x)+i)}-\frac{i}{4(\sinh(x)+i)^2}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + Sinh[x])^2,x]
```

```
[Out] -ArcTan[Sinh[x]]/4 - (I/4)/(I + Sinh[x])^2 - 1/(4*(I + Sinh[x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}(x)}{(i+\sinh(x))^2}\,dx &= -\text{Subst}\left(\int \frac{1}{(i-x)(i+x)^3}\,dx,x,\sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{i}{2(i+x)^3}-\frac{1}{4(i+x)^2}+\frac{1}{4\left(1+x^2\right)}\right)dx,x,\sinh(x)\right)\\
&= -\frac{i}{4(i+\sinh(x))^2}-\frac{1}{4(i+\sinh(x))}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(x)\right)\\
&= -\frac{1}{4}\tan^{-1}(\sinh(x))-\frac{i}{4(i+\sinh(x))^2}-\frac{1}{4(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.042567, size = 26, normalized size = 0.76

$$\frac{1}{4}\left(-\tan^{-1}(\sinh(x))-\frac{\sinh(x)+2i}{(\sinh(x)+i)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + Sinh[x])^2,x]
```

```
[Out] (-ArcTan[Sinh[x]] - (2*I + Sinh[x])/(I + Sinh[x])^2)/4
```

Maple [B] time = 0.04, size = 70, normalized size = 2.1

$$\frac{i}{4}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4}-\frac{i}{4}\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{5i}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}-2\left(\tanh(x/2)+i\right)^{-3}+\frac{3}{2}\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+sinh(x))^2,x)
```

```
[Out] 1/4*I*ln(tanh(1/2*x)-I)+I/(tanh(1/2*x)+I)^4-1/4*I*ln(tanh(1/2*x)+I)-5/2*I/(
tanh(1/2*x)+I)^2-2/(tanh(1/2*x)+I)^3+3/2/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.12828, size = 95, normalized size = 2.79

$$-\frac{2\left(e^{(-x)}+4i\,e^{(-2x)}-e^{(-3x)}\right)}{16i\,e^{(-x)}-24\,e^{(-2x)}-16i\,e^{(-3x)}+4\,e^{(-4x)}+4}-\frac{1}{4}i\log\left(i\,e^{(-x)}+1\right)+\frac{1}{4}i\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(e^(-x) + 4*I*e^(-2*x) - e^(-3*x))/(16*I*e^(-x) - 24*e^(-2*x) - 16*I*e^(
-3*x) + 4*e^(-4*x) + 4) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) - 1)
```

Fricas [B] time = 1.84944, size = 298, normalized size = 8.76

$$\frac{\left(-i\,e^{(4x)}+4\,e^{(3x)}+6i\,e^{(2x)}-4\,e^{x}-i\right)\log\left(e^{x}+i\right)+\left(i\,e^{(4x)}-4\,e^{(3x)}-6i\,e^{(2x)}+4\,e^{x}+i\right)\log\left(e^{x}-i\right)-2\,e^{(3x)}-8i\,e^{(2x)}+2\,e}{4\,e^{(4x)}+16i\,e^{(3x)}-24\,e^{(2x)}-16i\,e^{x}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((-I*e^(4*x) + 4*e^(3*x) + 6*I*e^(2*x) - 4*e^x - I)*log(e^x + I) + (I*e^(4*
x) - 4*e^(3*x) - 6*I*e^(2*x) + 4*e^x + I)*log(e^x - I) - 2*e^(3*x) - 8*I*e^
(2*x) + 2*e^x)/(4*e^(4*x) + 16*I*e^(3*x) - 24*e^(2*x) - 16*I*e^x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(x)}{(\sinh(x)+i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)/(sinh(x) + I)**2, x)
```

Giac [B] time = 1.27089, size = 95, normalized size = 2.79

$$\frac{3i\left(e^{(-x)}-e^x\right)^2+20\,e^{(-x)}-20\,e^x-44i}{16\left(e^{(-x)}-e^x-2i\right)^2}-\frac{1}{8}i\log\left(i\,e^{(-x)}-i\,e^x+2\right)+\frac{1}{8}i\log\left(i\,e^{(-x)}-i\,e^x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(3*I*(e^(-x) - e^x)^2 + 20*e^(-x) - 20*e^x - 44*I)/(e^(-x) - e^x - 2*I
)^2 - 1/8*I*log(I*e^(-x) - I*e^x + 2) + 1/8*I*log(I*e^(-x) - I*e^x - 2)
```

3.178 $\int \frac{\text{sech}^2(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=37

$$-\frac{2\tanh(x)}{5}-\frac{\text{sech}(x)}{5(\sinh(x)+i)}-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

```
[Out] ((-I/5)*Sech[x])/(I + Sinh[x])^2 - Sech[x]/(5*(I + Sinh[x])) - (2*Tanh[x])/
5
```

Rubi [A] time = 0.0708574, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 3767, 8}

$$-\frac{2\tanh(x)}{5}-\frac{\text{sech}(x)}{5(\sinh(x)+i)}-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] ((-I/5)*Sech[x])/(I + Sinh[x])^2 - Sech[x]/(5*(I + Sinh[x])) - (2*Tanh[x])/
5
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^2(x)}{(i+\sinh(x))^2} \, dx &= -\frac{i\text{sech}(x)}{5(i+\sinh(x))^2}-\frac{3}{5}i\int \frac{\text{sech}^2(x)}{i+\sinh(x)} \, dx \\
&= -\frac{i\text{sech}(x)}{5(i+\sinh(x))^2}-\frac{\text{sech}(x)}{5(i+\sinh(x))}-\frac{2}{5}\int \text{sech}^2(x) \, dx \\
&= -\frac{i\text{sech}(x)}{5(i+\sinh(x))^2}-\frac{\text{sech}(x)}{5(i+\sinh(x))}-\frac{2}{5}i\,\text{Subst}\left(\int 1 \, dx,x,-i\tanh(x)\right) \\
&= -\frac{i\text{sech}(x)}{5(i+\sinh(x))^2}-\frac{\text{sech}(x)}{5(i+\sinh(x))}-\frac{2\tanh(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.0218572, size = 31, normalized size = 0.84

$$-\frac{\operatorname{sech}(x)(-5\sinh(x)+\sinh(3x)+4i\cosh(2x))}{10(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] -(Sech[x]*((4*I)*Cosh[2*x] - 5*Sinh[x] + Sinh[3*x]))/(10*(I + Sinh[x])^2)
```

Maple [B] time = 0.039, size = 70, normalized size = 1.9

$$-\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}-2i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4}+\frac{5i}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}-\frac{4}{5}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-5}+3\left(\tanh(x/2)+i\right)^{-3}-\frac{7}{4}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(I+sinh(x))^2,x)
```

```
[Out] -1/4/(tanh(1/2*x)-I)-2*I/(tanh(1/2*x)+I)^4+5/2*I/(tanh(1/2*x)+I)^2-4/5/(tan
h(1/2*x)+I)^5+3/(tanh(1/2*x)+I)^3-7/4/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.31682, size = 158, normalized size = 4.27

$$-\frac{16i\,e^{(-x)}}{20i\,e^{(-x)}-25\,e^{(-2x)}-25\,e^{(-4x)}-20i\,e^{(-5x)}+5\,e^{(-6x)}+5}+\frac{20\,e^{(-2x)}}{20i\,e^{(-x)}-25\,e^{(-2x)}-25\,e^{(-4x)}-20i\,e^{(-5x)}+5\,e^{(-6x)}+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -16*I*e^(-x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*I*e^(-5*x) + 5*e
^(-6*x) + 5) + 20*e^(-2*x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*I*
e^(-5*x) + 5*e^(-6*x) + 5) - 4/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 2
0*I*e^(-5*x) + 5*e^(-6*x) + 5)
```

Fricas [A] time = 1.73613, size = 132, normalized size = 3.57

$$-\frac{4\left(5\,e^{(2x)}+4i\,e^{x}-1\right)}{5\,e^{(6x)}+20i\,e^{(5x)}-25\,e^{(4x)}-25\,e^{(2x)}-20i\,e^{x}+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -4*(5*e^(2*x) + 4*I*e^x - 1)/(5*e^(6*x) + 20*I*e^(5*x) - 25*e^(4*x) - 25*e^
(2*x) - 20*I*e^x + 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{(\sinh(x)+i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**2/(sinh(x) + I)**2, x)
```

Giac [A] time = 1.23532, size = 55, normalized size = 1.49

$$-\frac{i}{4\,(e^x-i)}-\frac{-5i\,e^{(4\,x)}+30\,e^{(3\,x)}+80i\,e^{(2\,x)}-50\,e^x-11i}{20\,(e^x+i)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/4*I/(e^x - I) - 1/20*(-5*I*e^(4*x) + 30*e^(3*x) + 80*I*e^(2*x) - 50*e^x
- 11*I)/(e^x + I)^5
```

3.179 $\int \frac{\mathrm{sech}^3(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=60

$$\frac{1}{16(-\sinh(x)+i)}-\frac{3}{16(\sinh(x)+i)}-\frac{i}{8(\sinh(x)+i)^2}+\frac{1}{12(\sinh(x)+i)^3}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

```
[Out] -ArcTan[Sinh[x]]/4 + 1/(16*(I - Sinh[x])) + 1/(12*(I + Sinh[x])^3) - (I/8)/
(I + Sinh[x])^2 - 3/(16*(I + Sinh[x]))
```

Rubi [A] time = 0.0551326, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 203}

$$\frac{1}{16(-\sinh(x)+i)}-\frac{3}{16(\sinh(x)+i)}-\frac{i}{8(\sinh(x)+i)^2}+\frac{1}{12(\sinh(x)+i)^3}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] -ArcTan[Sinh[x]]/4 + 1/(16*(I - Sinh[x])) + 1/(12*(I + Sinh[x])^3) - (I/8)/
(I + Sinh[x])^2 - 3/(16*(I + Sinh[x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{sech}^3(x)}{(i+\sinh(x))^2}\,dx = \text{Subst}\left(\int \frac{1}{(i-x)^2(i+x)^4}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{16(-i+x)^2} - \frac{1}{4(i+x)^4} + \frac{i}{4(i+x)^3} + \frac{3}{16(i+x)^2} - \frac{1}{4\left(1+x^2\right)}\right) dx, x, \sinh(x)\right)$$
$$= \frac{1}{16(i-\sinh(x))} + \frac{1}{12(i+\sinh(x))^3} - \frac{i}{8(i+\sinh(x))^2} - \frac{3}{16(i+\sinh(x))} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.$$
$$= -\frac{1}{4}\tan^{-1}(\sinh(x)) + \frac{1}{16(i-\sinh(x))} + \frac{1}{12(i+\sinh(x))^3} - \frac{i}{8(i+\sinh(x))^2} - \frac{3}{16(i+\sinh(x))}$$

Mathematica [A] time = 0.0441689, size = 68, normalized size = 1.13

$$-\frac{\text{sech}^2(x)\left(6i\sinh^2(x) + 3\sinh^4(x)\tan^{-1}(\sinh(x)) + \sinh^3(x)\left(3+6i\tan^{-1}(\sinh(x))\right) + \sinh(x)\left(-1+6i\tan^{-1}(\sinh(x)\right.\right.}{12(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] -(Sech[x]^2*(4*I - 3*ArcTan[Sinh[x]] + (-1 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]
 + (6*I)*Sinh[x]^2 + (3 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^3 + 3*ArcTan[Sinh[
x]]*Sinh[x]^4))/(12*(I + Sinh[x])^2)
```

Maple [B] time = 0.05, size = 116, normalized size = 1.9

$$\frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2} + \frac{i}{4}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right) + \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1} + \frac{7i}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4} - \frac{2i}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-6} - \frac{i}{4}\ln\left(\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(I+sinh(x))^2,x)
```

```
[Out] 1/8*I/(tanh(1/2*x)-I)^2+1/4*I*ln(tanh(1/2*x)-I)+1/8/(tanh(1/2*x)-I)+7/2*I/(
tanh(1/2*x)+I)^4-2/3*I/(tanh(1/2*x)+I)^6-1/4*I*ln(tanh(1/2*x)+I)-23/8*I/(ta
nh(1/2*x)+I)^2+2/(tanh(1/2*x)+I)^5-11/3/(tanh(1/2*x)+I)^3+11/8/(tanh(1/2*x)
+I)
```

Maxima [B] time = 1.30324, size = 162, normalized size = 2.7

$$-\frac{8\left(3e^{(-x)} + 12ie^{(-2x)} - 13e^{(-3x)} + 8ie^{(-4x)} + 13e^{(-5x)} + 12ie^{(-6x)} - 3e^{(-7x)}\right)}{192ie^{(-x)} - 192e^{(-2x)} + 192ie^{(-3x)} - 480e^{(-4x)} - 192ie^{(-5x)} - 192e^{(-6x)} - 192ie^{(-7x)} + 48e^{(-8x)} + 48} - \frac{1}{4}i\log\left(ie^{(-x)} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -8*(3*e^(-x) + 12*I*e^(-2*x) - 13*e^(-3*x) + 8*I*e^(-4*x) + 13*e^(-5*x) + 1
2*I*e^(-6*x) - 3*e^(-7*x))/(192*I*e^(-x) - 192*e^(-2*x) + 192*I*e^(-3*x) -
480*e^(-4*x) - 192*I*e^(-5*x) - 192*e^(-6*x) - 192*I*e^(-7*x) + 48*e^(-8*x)
 + 48) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) - 1)
```

Fricas [B] time = 1.85669, size = 625, normalized size = 10.42

$$\frac{\left(-3i\,e^{(8\,x)}+12\,e^{(7\,x)}+12i\,e^{(6\,x)}+12\,e^{(5\,x)}+30i\,e^{(4\,x)}-12\,e^{(3\,x)}+12i\,e^{(2\,x)}-12\,e^{x}-3i\right)\log\left(e^{x}+i\right)+\left(3i\,e^{(8\,x)}-12\,e^{(7\,x)}-\right.}{12\,e^{(8\,x)}+48i\,e^{(7\,x)}-48\,e^{(6\,x)}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((-3*I*e^(8*x) + 12*e^(7*x) + 12*I*e^(6*x) + 12*e^(5*x) + 30*I*e^(4*x) - 12
*e^(3*x) + 12*I*e^(2*x) - 12*e^x - 3*I)*log(e^x + I) + (3*I*e^(8*x) - 12*e^
(7*x) - 12*I*e^(6*x) - 12*e^(5*x) - 30*I*e^(4*x) + 12*e^(3*x) - 12*I*e^(2*x
) + 12*e^x + 3*I)*log(e^x - I) - 6*e^(7*x) - 24*I*e^(6*x) + 26*e^(5*x) - 16
*I*e^(4*x) - 26*e^(3*x) - 24*I*e^(2*x) + 6*e^x)/(12*e^(8*x) + 48*I*e^(7*x)
- 48*e^(6*x) + 48*I*e^(5*x) - 120*e^(4*x) - 48*I*e^(3*x) - 48*e^(2*x) - 48*
I*e^x + 12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32622, size = 142, normalized size = 2.37

$$\frac{-i\,e^{(-x)}+i\,e^{x}+3}{8\left(e^{(-x)}-e^{x}+2i\right)}+\frac{11i\left(e^{(-x)}-e^{x}\right)^{3}+84\left(e^{(-x)}-e^{x}\right)^{2}-228i\,e^{(-x)}+228i\,e^{x}-240}{48\left(e^{(-x)}-e^{x}-2i\right)^{3}}-\frac{1}{8}i\,\log\left(-e^{(-x)}+e^{x}+2i\right)+\frac{1}{8}i\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*(-I*e^(-x) + I*e^x + 3)/(e^(-x) - e^x + 2*I) + 1/48*(11*I*(e^(-x) - e^x
)^3 + 84*(e^(-x) - e^x)^2 - 228*I*e^(-x) + 228*I*e^x - 240)/(e^(-x) - e^x -
 2*I)^3 - 1/8*I*log(-e^(-x) + e^x + 2*I) + 1/8*I*log(-e^(-x) + e^x - 2*I)
```

3.180 $\int \frac{\text{sech}^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=49

$$\frac{4\tanh^3(x)}{21}-\frac{4\tanh(x)}{7}-\frac{\text{sech}^3(x)}{7(\sinh(x)+i)}-\frac{i\,\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

```
[Out] ((-I/7)*Sech[x]^3)/(I + Sinh[x])^2 - Sech[x]^3/(7*(I + Sinh[x])) - (4*Tanh[
x])/7 + (4*Tanh[x]^3)/21
```

Rubi [A] time = 0.0784621, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2672, 3767}

$$\frac{4\tanh^3(x)}{21}-\frac{4\tanh(x)}{7}-\frac{\text{sech}^3(x)}{7(\sinh(x)+i)}-\frac{i\,\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] ((-I/7)*Sech[x]^3)/(I + Sinh[x])^2 - Sech[x]^3/(7*(I + Sinh[x])) - (4*Tanh[
x])/7 + (4*Tanh[x]^3)/21
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^4(x)}{(i+\sinh(x))^2}\,dx &= -\frac{i\,\text{sech}^3(x)}{7(i+\sinh(x))^2}-\frac{5}{7}i\int \frac{\text{sech}^4(x)}{i+\sinh(x)}\,dx\\
&= -\frac{i\,\text{sech}^3(x)}{7(i+\sinh(x))^2}-\frac{\text{sech}^3(x)}{7(i+\sinh(x))}-\frac{4}{7}\int \text{sech}^4(x)\,dx\\
&= -\frac{i\,\text{sech}^3(x)}{7(i+\sinh(x))^2}-\frac{\text{sech}^3(x)}{7(i+\sinh(x))}-\frac{4}{7}i\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\tanh(x)\right)\\
&= -\frac{i\,\text{sech}^3(x)}{7(i+\sinh(x))^2}-\frac{\text{sech}^3(x)}{7(i+\sinh(x))}-\frac{4\tanh(x)}{7}+\frac{4\tanh^3(x)}{21}
\end{aligned}$$

Mathematica [A] time = 0.03553, size = 47, normalized size = 0.96

$$-\frac{\text{sech}^3(x)(-14\sinh(x)-3\sinh(3x)+\sinh(5x)+8i\cosh(2x)+4i\cosh(4x))}{42(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] -(Sech[x]^3*((8*I)*Cosh[2*x] + (4*I)*Cosh[4*x] - 14*Sinh[x] - 3*Sinh[3*x] +
 Sinh[5*x]))/(42*(I + Sinh[x])^2)
```

Maple [B] time = 0.052, size = 116, normalized size = 2.4

$$-\frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+\frac{1}{12}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}-\frac{3}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}+2\,i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-6}-5\,i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4}+\frac{23}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(I+sinh(x))^2,x)
```

```
[Out] -1/8*I/(tanh(1/2*x)-I)^2+1/12/(tanh(1/2*x)-I)^3-3/8/(tanh(1/2*x)-I)+2*I/(ta
nh(1/2*x)+I)^6-5*I/(tanh(1/2*x)+I)^4+23/8*I/(tanh(1/2*x)+I)^2+4/7/(tanh(1/2
*x)+I)^7-4/(tanh(1/2*x)+I)^5+55/12/(tanh(1/2*x)+I)^3-13/8/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.16245, size = 428, normalized size = 8.73

$$-\frac{64i\,e^{(-x)}}{84i\,e^{(-x)}-63\,e^{(-2\,x)}+168i\,e^{(-3\,x)}-294\,e^{(-4\,x)}-294\,e^{(-6\,x)}-168i\,e^{(-7\,x)}-63\,e^{(-8\,x)}-84i\,e^{(-9\,x)}+21\,e^{(-10\,x)}+21}+\frac{}{84i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -64*I*e^(-x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(-4*x) - 2
94*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e^(-10*x) +
 21) + 48*e^(-2*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(-4*
x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e^(-1
0*x) + 21) - 128*I*e^(-3*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 2
94*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) +
 21*e^(-10*x) + 21) + 224*e^(-4*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3
*x) - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(
-9*x) + 21*e^(-10*x) + 21) - 16/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x)
 - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*
x) + 21*e^(-10*x) + 21)
```

Fricas [B] time = 1.76329, size = 259, normalized size = 5.29

$$-\frac{224\,e^{(4\,x)}+128i\,e^{(3\,x)}+48\,e^{(2\,x)}+64i\,e^{x}-16}{21\,e^{(10\,x)}+84i\,e^{(9\,x)}-63\,e^{(8\,x)}+168i\,e^{(7\,x)}-294\,e^{(6\,x)}-294\,e^{(4\,x)}-168i\,e^{(3\,x)}-63\,e^{(2\,x)}-84i\,e^{x}+21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(224*e^(4*x) + 128*I*e^(3*x) + 48*e^(2*x) + 64*I*e^x - 16)/(21*e^(10*x) +
84*I*e^(9*x) - 63*e^(8*x) + 168*I*e^(7*x) - 294*e^(6*x) - 294*e^(4*x) - 168
```

```
*I*e^(3*x) - 63*e^(2*x) - 84*I*e^x + 21)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26612, size = 88, normalized size = 1.8

$$-\frac{6i\,e^{(2\,x)}+15\,e^{x}-7i}{24\,(e^{x}-i)^{3}}-\frac{-42i\,e^{(6\,x)}+315\,e^{(5\,x)}+1015i\,e^{(4\,x)}-1750\,e^{(3\,x)}-1344i\,e^{(2\,x)}+511\,e^{x}+79i}{168\,(e^{x}+i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/24*(6*I*e^(2*x) + 15*e^x - 7*I)/(e^x - I)^3 - 1/168*(-42*I*e^(6*x) + 315
*e^(5*x) + 1015*I*e^(4*x) - 1750*e^(3*x) - 1344*I*e^(2*x) + 511*e^x + 79*I)
/(e^x + I)^7
```

3.181 $\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx$

Optimal. Leaf size=28

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0436273, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2667, 43}

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx &= -\left(i\,\mathrm{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, i\sinh(x)\right)\right)\\
&= -\left(i\,\mathrm{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, i\sinh(x)\right)\right)\\
&= i\log(i-\sinh(x))+\frac{2i}{1+i\sinh(x)}
\end{aligned}$$

Mathematica [A] time = 0.0858377, size = 45, normalized size = 1.61

$$\frac{2i\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\log(\cosh(x))+\sinh(x)\left(-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+i\log(\cosh(x))\right)+2}{\sinh(x)-i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
```

```
[Out] (2 + (2*I)*ArcTan[Tanh[x/2]] + Log[Cosh[x]] + (-2*ArcTan[Tanh[x/2]] + I*Log
[Cosh[x]])*Sinh[x])/(-I + Sinh[x])
```

Maple [B] time = 0.053, size = 56, normalized size = 2.

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2\,i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-4\,i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}-4\,(\tanh(x/2)-i)^{-1}-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1+I*sinh(x))^3,x)
```

```
[Out] -I*ln(tanh(1/2*x)+1)+2*I*ln(tanh(1/2*x)-I)-4*I/(tanh(1/2*x)-I)^2-4/(tanh(1/
2*x)-I)-I*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.19766, size = 45, normalized size = 1.61

$$i\,x-\frac{4\,e^{(-x)}}{2i\,e^{(-x)}+e^{(-2\,x)}-1}+2i\,\log\left(e^{(-x)}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] I*x - 4*e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) + 2*I*log(e^(-x) + I)
```

Fricas [B] time = 1.74555, size = 142, normalized size = 5.07

$$\frac{-i\,xe^{(2\,x)}-2\,(x-2)e^{x}+\left(2i\,e^{(2\,x)}+4\,e^{x}-2i\right)\log\left(e^{x}-i\right)+i\,x}{e^{(2\,x)}-2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x + (2*I*e^(2*x) + 4*e^x - 2*I)*log(e^x - I) +
I*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [A] time = 27.5753, size = 31, normalized size = 1.11

$$-ix+2i\log\left(e^{x}-i\right)+\frac{4e^{x}}{e^{2x}-2ie^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1+I*sinh(x))**3,x)
```

```
[Out] -I*x + 2*I*log(exp(x) - I) + 4*exp(x)/(exp(2*x) - 2*I*exp(x) - 1)
```

Giac [A] time = 1.24502, size = 36, normalized size = 1.29

$$\frac{4\,e^x}{(e^x - i)^2} - i\,\log(i\,e^x) + 2i\,\log(-i\,e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 4*e^x/(e^x - I)^2 - I*log(I*e^x) + 2*I*log(-I*e^x - 1)
```

3.182 $\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3} dx$

Optimal. Leaf size=20

$$\frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

[Out] ((I/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3

Rubi [A] time = 0.0330749, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2671}

$$\frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^2/(1 + I*Sinh[x])^3,x]

[Out] ((I/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = \frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

Mathematica [A] time = 0.0641376, size = 40, normalized size = 2.

$$-\frac{i\left(\cosh\left(\frac{3x}{2}\right)-3\cosh\left(\frac{x}{2}\right)\right)}{3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^2/(1 + I*Sinh[x])^3,x]

[Out] ((-I/3)*(-3*Cosh[x/2] + Cosh[(3*x)/2]))/(Cosh[x/2] + I*Sinh[x/2])^3

Maple [B] time = 0.047, size = 36, normalized size = 1.8

$$4i\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+2\left(\tanh(x/2)-i\right)^{-1}-\frac{8}{3}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1+I*sinh(x))^3,x)
```

```
[Out] 4*I/(tanh(1/2*x)-I)^2+2/(tanh(1/2*x)-I)-8/3/(tanh(1/2*x)-I)^3
```

Maxima [B] time = 1.11053, size = 72, normalized size = 3.6

$$\frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3}-\frac{2}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 6*e^(-2*x)/(-9*I*e^(-x) - 9*e^(-2*x) + 3*I*e^(-3*x) + 3) - 2/(-9*I*e^(-x) -
 9*e^(-2*x) + 3*I*e^(-3*x) + 3)
```

Fricas [B] time = 1.70145, size = 84, normalized size = 4.2

$$\frac{-6i\,e^{(2\,x)}+2i}{3\,e^{(3\,x)}-9i\,e^{(2\,x)}-9\,e^{x}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (-6*I*e^(2*x) + 2*I)/(3*e^(3*x) - 9*I*e^(2*x) - 9*e^x + 3*I)
```

Sympy [B] time = 0.330437, size = 32, normalized size = 1.6

$$\frac{-2ie^{2x}+\frac{2i}{3}}{e^{3x}-3ie^{2x}-3e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1+I*sinh(x))**3,x)
```

```
[Out] (-2*I*exp(2*x) + 2*I/3)/(exp(3*x) - 3*I*exp(2*x) - 3*exp(x) + I)
```

Giac [A] time = 1.25264, size = 22, normalized size = 1.1

$$-\frac{6i\,e^{(2\,x)}-2i}{3\,(e^{x}-i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -1/3*(6*I*e^(2*x) - 2*I)/(e^x - I)^3
```

3.183 $\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx$

Optimal. Leaf size=16

$$\frac{i}{2(1+i\sinh(x))^2}$$

[Out] (I/2)/(1 + I*Sinh[x])^2

Rubi [A] time = 0.0209054, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 32}

$$\frac{i}{2(1+i\sinh(x))^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(1 + I*Sinh[x])^3,x]

[Out] (I/2)/(1 + I*Sinh[x])^2

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx &= -\left(i\,\text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, i\sinh(x)\right)\right)\\ &= \frac{i}{2(1+i\sinh(x))^2}\end{aligned}$$

Mathematica [A] time = 0.0257987, size = 14, normalized size = 0.88

$$-\frac{i}{2(\sinh(x)-i)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]/(1 + I*Sinh[x])^3,x]

[Out] (-I/2)/(-I + Sinh[x])^2

Maple [A] time = 0.015, size = 13, normalized size = 0.8

$$\frac{\frac{i}{2}}{(1+i\sinh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1+I*sinh(x))^3,x)
```

```
[Out] 1/2*I/(1+I*sinh(x))^2
```

Maxima [A] time = 1.18221, size = 14, normalized size = 0.88

$$\frac{i}{2\,(i\,\sinh(x)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*I/(I*sinh(x) + 1)^2
```

Fricas [B] time = 1.68314, size = 86, normalized size = 5.38

$$-\frac{2i\,e^{(2\,x)}}{e^{(4\,x)}-4i\,e^{(3\,x)}-6\,e^{(2\,x)}+4i\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*I*e^(2*x)/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

Sympy [B] time = 0.388462, size = 37, normalized size = 2.31

$$-\frac{2ie^{2x}}{e^{4x}-4ie^{3x}-6e^{2x}+4ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))**3,x)
```

```
[Out] -2*I*exp(2*x)/(exp(4*x) - 4*I*exp(3*x) - 6*exp(2*x) + 4*I*exp(x) + 1)
```

Giac [A] time = 1.25265, size = 16, normalized size = 1.

$$-\frac{2i\,e^{(2\,x)}}{(e^{x}-i)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2*I*e^(2*x)/(e^x - I)^4
```

3.184 $\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rubi [A] time = 0.0403777, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2667, 43}

$$-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx &= i\,\mathrm{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, -i\sinh(x)\right)\\ &= i\,\mathrm{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, -i\sinh(x)\right)\\ &= -i\log(i+\sinh(x))-\frac{2i}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0861882, size = 45, normalized size = 1.73

$$\frac{-2i\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\log(\cosh(x))-2\sinh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-i\sinh(x)\log(\cosh(x))+2}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
```

```
[Out] (2 - (2*I)*ArcTan[Tanh[x/2]] + Log[Cosh[x]] - 2*ArcTan[Tanh[x/2]]*Sinh[x] -
 I*Log[Cosh[x]]*Sinh[x])/(I + Sinh[x])
```

Maple [B] time = 0.055, size = 56, normalized size = 2.2

$$i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+4\,i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}-2\,i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-4\,\left(\tanh\left(x/2\right)+i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1-I*sinh(x))^3,x)
```

```
[Out] I*ln(tanh(1/2*x)+1)+I*ln(tanh(1/2*x)-1)+4*I/(tanh(1/2*x)+I)^2-2*I*ln(tanh(1
/2*x)+I)-4/(tanh(1/2*x)+I)
```

Maxima [A] time = 1.13683, size = 45, normalized size = 1.73

$$-i\,x-\frac{4\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}-2i\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -I*x - 4*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - 2*I*log(e^(-x) - I)
```

Fricas [B] time = 1.89091, size = 142, normalized size = 5.46

$$\frac{i\,xe^{(2\,x)}-2\,(x-2)e^{x}+\left(-2i\,e^{(2\,x)}+4\,e^{x}+2i\right)\log\left(e^{x}+i\right)-i\,x}{e^{(2\,x)}+2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x + (-2*I*e^(2*x) + 4*e^x + 2*I)*log(e^x + I) -
I*x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [A] time = 27.5166, size = 31, normalized size = 1.19

$$ix-2i\log\left(e^{x}+i\right)+\frac{4e^{x}}{e^{2x}+2ie^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1-I*sinh(x))**3,x)
```

```
[Out] I*x - 2*I*log(exp(x) + I) + 4*exp(x)/(exp(2*x) + 2*I*exp(x) - 1)
```

Giac [A] time = 1.22038, size = 36, normalized size = 1.38

$$\frac{4\,e^{x}}{\left(e^{x}+i\right)^{2}}+i\,\log\left(-i\,e^{x}\right)-2i\,\log\left(i\,e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 4*e^x/(e^x + I)^2 + I*log(-I*e^x) - 2*I*log(I*e^x - 1)
```

3.185 $\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3} dx$

Optimal. Leaf size=20

$$-\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

```
[Out] ((-I/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3
```

Rubi [A] time = 0.0327605, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2671}

$$-\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
```

```
[Out] ((-I/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3} dx = -\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

Mathematica [A] time = 0.061717, size = 38, normalized size = 1.9

$$\frac{\cosh\left(\frac{3x}{2}\right)-3\cosh\left(\frac{x}{2}\right)}{3\left(\sinh\left(\frac{x}{2}\right)+i\cosh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-3*Cosh[x/2] + Cosh[(3*x)/2])/(3*(I*Cosh[x/2] + Sinh[x/2])^3)
```

Maple [B] time = 0.043, size = 36, normalized size = 1.8

$$-\frac{8}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}+2\left(\tanh(x/2)+i\right)^{-1}-4i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1-I*sinh(x))^3,x)
```

```
[Out] -8/3/(tanh(1/2*x)+I)^3+2/(tanh(1/2*x)+I)-4*I/(tanh(1/2*x)+I)^2
```

Maxima [B] time = 1.19423, size = 72, normalized size = 3.6

$$-\frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)} + 9\,e^{(-2\,x)} + 3i\,e^{(-3\,x)} - 3} + \frac{2}{-9i\,e^{(-x)} + 9\,e^{(-2\,x)} + 3i\,e^{(-3\,x)} - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -6*e^(-2*x)/(-9*I*e^(-x) + 9*e^(-2*x) + 3*I*e^(-3*x) - 3) + 2/(-9*I*e^(-x)
+ 9*e^(-2*x) + 3*I*e^(-3*x) - 3)
```

Fricas [B] time = 1.74558, size = 82, normalized size = 4.1

$$\frac{6i\,e^{(2\,x)} - 2i}{3\,e^{(3\,x)} + 9i\,e^{(2\,x)} - 9\,e^{x} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (6*I*e^(2*x) - 2*I)/(3*e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

Sympy [A] time = 0.332346, size = 32, normalized size = 1.6

$$\frac{2ie^{2x} - \frac{2i}{3}}{e^{3x} + 3ie^{2x} - 3e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1-I*sinh(x))**3,x)
```

```
[Out] (2*I*exp(2*x) - 2*I/3)/(exp(3*x) + 3*I*exp(2*x) - 3*exp(x) - I)
```

Giac [A] time = 1.26276, size = 22, normalized size = 1.1

$$-\frac{-6i\,e^{(2\,x)} + 2i}{3\,(e^{x} + i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -1/3*(-6*I*e^(2*x) + 2*I)/(e^x + I)^3
```

3.186 $\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx$

Optimal. Leaf size=16

$$-\frac{i}{2(1-i\sinh(x))^2}$$

```
[Out] (-I/2)/(1 - I*Sinh[x])^2
```

Rubi [A] time = 0.021534, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 32}

$$-\frac{i}{2(1-i\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-I/2)/(1 - I*Sinh[x])^2
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx &= i\,\text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, -i\sinh(x)\right)\\ &= -\frac{i}{2(1-i\sinh(x))^2}\end{aligned}$$

Mathematica [A] time = 0.0272961, size = 14, normalized size = 0.88

$$\frac{i}{2(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(1 - I*Sinh[x])^3,x]
```

```
[Out] (I/2)/(I + Sinh[x])^2
```

Maple [A] time = 0.017, size = 13, normalized size = 0.8

$$\frac{-\frac{i}{2}}{\left(1-i\sinh\left(x\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1-I*sinh(x))^3,x)
```

```
[Out] -1/2*I/(1-I*sinh(x))^2
```

Maxima [A] time = 1.21702, size = 14, normalized size = 0.88

$$-\frac{i}{2\left(-i\sinh\left(x\right)+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*I/(-I*sinh(x) + 1)^2
```

Fricas [B] time = 1.75141, size = 85, normalized size = 5.31

$$\frac{2i\,e^{(2\,x)}}{e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 2*I*e^(2*x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [B] time = 0.385706, size = 36, normalized size = 2.25

$$\frac{2ie^{2x}}{e^{4x}+4ie^{3x}-6e^{2x}-4ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))**3,x)
```

```
[Out] 2*I*exp(2*x)/(exp(4*x) + 4*I*exp(3*x) - 6*exp(2*x) - 4*I*exp(x) + 1)
```

Giac [A] time = 1.21441, size = 16, normalized size = 1.

$$\frac{2i\,e^{(2\,x)}}{\left(e^{x}+i\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 2*I*e^(2*x)/(e^x + I)^4
```

3.187 $\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=138

$$\frac{\left(a^2+3b^2\right)\sinh^4(x)}{4b^3}-\frac{a\left(a^2+3b^2\right)\sinh^3(x)}{3b^4}+\frac{\left(3a^2b^2+a^4+3b^4\right)\sinh^2(x)}{2b^5}-\frac{a\left(3a^2b^2+a^4+3b^4\right)\sinh(x)}{b^6}+\frac{\left(a^2+b^2\right.}{}$$

```
[Out] ((a^2 + b^2)^3*Log[a + b*Sinh[x]])/b^7 - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[
x])/b^6 + ((a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x]^2)/(2*b^5) - (a*(a^2 + 3*b^2)*
Sinh[x]^3)/(3*b^4) + ((a^2 + 3*b^2)*Sinh[x]^4)/(4*b^3) - (a*Sinh[x]^5)/(5*b
^2) + Sinh[x]^6/(6*b)
```

Rubi [A] time = 0.138243, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{\left(a^2+3b^2\right)\sinh^4(x)}{4b^3}-\frac{a\left(a^2+3b^2\right)\sinh^3(x)}{3b^4}+\frac{\left(3a^2b^2+a^4+3b^4\right)\sinh^2(x)}{2b^5}-\frac{a\left(3a^2b^2+a^4+3b^4\right)\sinh(x)}{b^6}+\frac{\left(a^2+b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^7/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)^3*Log[a + b*Sinh[x]])/b^7 - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[
x])/b^6 + ((a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x]^2)/(2*b^5) - (a*(a^2 + 3*b^2)*
Sinh[x]^3)/(3*b^4) + ((a^2 + 3*b^2)*Sinh[x]^4)/(4*b^3) - (a*Sinh[x]^5)/(5*b
^2) + Sinh[x]^6/(6*b)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{\left(-b^2-x^2\right)^3}{a+x}\,dx, x, b\sinh(x)\right)}{b^7}\\
&= -\frac{\operatorname{Subst}\left(\int \left(a^5\left(1+\frac{3b^2\left(a^2+b^2\right)}{a^4}\right)-\left(a^4+3a^2b^2+3b^4\right)x+a\left(a^2+3b^2\right)x^2-\left(a^2+3b^2\right)x^3+ax^4-\right.\right.}{b^7}\\
&= \frac{\left(a^2+b^2\right)^3\log(a+b\sinh(x))}{b^7}-\frac{a\left(a^4+3a^2b^2+3b^4\right)\sinh(x)}{b^6}+\frac{\left(a^4+3a^2b^2+3b^4\right)\sinh^2(x)}{2b^5}-
\end{aligned}$$

Mathematica [A] time = 0.156796, size = 121, normalized size = 0.88

$$\frac{-20ab^3\left(a^2+3b^2\right)\sinh^3(x)+30b^2\left(a^2+b^2\right)^2\sinh^2(x)-60ab\left(3a^2b^2+a^4+3b^4\right)\sinh(x)+15b^4\left(a^2+b^2\right)\cosh^4(x)+60}{60b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^7/(a + b*Sinh[x]),x]
```

```
[Out] (15*b^4*(a^2 + b^2)*Cosh[x]^4 + 10*b^6*Cosh[x]^6 + 60*(a^2 + b^2)^3*Log[a +
 b*Sinh[x]] - 60*a*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x] + 30*b^2*(a^2 + b^2)
^2*Sinh[x]^2 - 20*a*b^3*(a^2 + 3*b^2)*Sinh[x]^3 - 12*a*b^5*Sinh[x]^5)/(60*b
^7)
```

Maple [B] time = 0.046, size = 837, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(a+b*sinh(x)),x)
```

```
[Out] -3/b^3*ln(tanh(1/2*x)+1)*a^2-3/b^3*ln(tanh(1/2*x)-1)*a^2+1/b*ln(a*tanh(1/2*
x)^2-2*tanh(1/2*x)*b-a)+1/2/b/(tanh(1/2*x)-1)^5+9/8/b/(tanh(1/2*x)-1)^4+1/6
/b/(tanh(1/2*x)+1)^6+1/6/b/(tanh(1/2*x)-1)^6-1/2/b/(tanh(1/2*x)+1)^5+9/8/b/
(tanh(1/2*x)+1)^4-17/12/b/(tanh(1/2*x)+1)^3+29/16/b/(tanh(1/2*x)+1)^2-19/16
/b/(tanh(1/2*x)+1)+17/12/b/(tanh(1/2*x)-1)^3+29/16/b/(tanh(1/2*x)-1)^2+19/1
6/b/(tanh(1/2*x)-1)-1/b*ln(tanh(1/2*x)+1)-1/b*ln(tanh(1/2*x)-1)+13/8/b^3/(t
anh(1/2*x)+1)^2*a^2+1/b^6/(tanh(1/2*x)+1)*a^5-1/2/b^5/(tanh(1/2*x)+1)*a^4+3
/b^4/(tanh(1/2*x)+1)*a^3+1/5/b^2/(tanh(1/2*x)-1)^5*a+1/4/b^3/(tanh(1/2*x)-1
)^4*a^2+1/5/b^2/(tanh(1/2*x)+1)^5*a+1/4/b^3/(tanh(1/2*x)+1)^4*a^2-1/2/b^2/(
tanh(1/2*x)+1)^4*a+1/3/b^4/(tanh(1/2*x)+1)^3*a^3-1/2/b^3/(tanh(1/2*x)+1)^3*
a^2+5/4/b^2/(tanh(1/2*x)+1)^3*a+3/b^4/(tanh(1/2*x)-1)*a^3+1/b^7*ln(a*tanh(1
/2*x)^2-2*tanh(1/2*x)*b-a)*a^6+3/b^5*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*
a^4+3/b^3*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^2+1/2/b^2/(tanh(1/2*x)-1)
^4*a+1/3/b^4/(tanh(1/2*x)-1)^3*a^3+1/2/b^3/(tanh(1/2*x)-1)^3*a^2+5/4/b^2/(t
anh(1/2*x)-1)^3*a-1/b^7*ln(tanh(1/2*x)-1)*a^6-3/b^5*ln(tanh(1/2*x)-1)*a^4+1
/2/b^5/(tanh(1/2*x)-1)^2*a^4+1/2/b^4/(tanh(1/2*x)-1)^2*a^3+13/8/b^3/(tanh(1
/2*x)-1)^2*a^2+1/b^6/(tanh(1/2*x)-1)*a^5+1/2/b^5/(tanh(1/2*x)-1)*a^4-1/b^7*
ln(tanh(1/2*x)+1)*a^6-3/b^5*ln(tanh(1/2*x)+1)*a^4+1/2/b^5/(tanh(1/2*x)+1)^2
*a^4-1/2/b^4/(tanh(1/2*x)+1)^2*a^3+11/8/b^3/(tanh(1/2*x)-1)*a^2+3/b^2/(tanh
(1/2*x)-1)*a-11/8/b^2/(tanh(1/2*x)+1)^2*a-11/8/b^3/(tanh(1/2*x)+1)*a^2+3/b^
2/(tanh(1/2*x)+1)*a+11/8/b^2/(tanh(1/2*x)-1)^2*a
```

Maxima [B] time = 1.20597, size = 416, normalized size = 3.01

$$-\frac{\left(12\,ab^4e^{(-x)}-5\,b^5-30\left(a^2b^3+2\,b^5\right)e^{(-2\,x)}+20\left(4\,a^3b^2+9\,ab^4\right)e^{(-3\,x)}-15\left(16\,a^4b+40\,a^2b^3+29\,b^5\right)e^{(-4\,x)}+120\left(8\,a^5+\right.\right.}{1920\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 + 2*b^5)*e^(-2*x) + 20*(4*a^
3*b^2 + 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-4*x) +
120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 + 1/1920*(12*a*b^
4*e^(-5*x) + 5*b^5*e^(-6*x) + 120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-x) +
15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 + 9*a*b^4)*e^(
-3*x) + 30*(a^2*b^3 + 2*b^5)*e^(-4*x))/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*x/b^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*a*e^(-x) + b*e^(-2
*x) - b)/b^7
```

Fricas [B] time = 2.05305, size = 5296, normalized size = 38.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*b
^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^10 + 6*(55*b^
6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 + 10*b^6)*sinh(x)^10 - 20*(4*a^3
*b^3 + 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 - 4*a
^3*b^3 - 9*a*b^5 + 15*(a^2*b^4 + 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^4*b^2
 + 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5*cosh(
x)^3 + 16*a^4*b^2 + 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 + 2*b^6)*cosh(x)^2 -
12*(4*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*x*cosh(x)^6 - 120*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^7 +
 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b - 22*a^3*b^3 - 19*a*b
^5 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^2 + (
16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 + 12*a*b^5*cosh(x) + 1
2*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 + 2*b^6)*cosh(x)^
4 - 140*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 + 40*a^2*b^4 + 29*
b^6)*cosh(x)^2 - 160*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 70*(8*a^5*b +
22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 + 120*(8*a^5*b + 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b^5*cosh(x)^6 + 40
*a^5*b + 110*a^3*b^3 + 95*a*b^5 + 315*(a^2*b^4 + 2*b^6)*cosh(x)^5 - 105*(4*
a^3*b^3 + 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x
)^3 - 480*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x) - 105*(8*a^5*b + 22
*a^3*b^3 + 19*a*b^5)*cosh(x)^2)*sinh(x)^5 + 15*(16*a^4*b^2 + 40*a^2*b^4 + 2
9*b^6)*cosh(x)^4 + 15*(165*b^6*cosh(x)^8 - 264*a*b^5*cosh(x)^7 + 420*(a^2*b
^4 + 2*b^6)*cosh(x)^6 + 16*a^4*b^2 + 40*a^2*b^4 + 29*b^6 - 168*(4*a^3*b^3 +
 9*a*b^5)*cosh(x)^5 + 70*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^4 - 192
0*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^2 - 280*(8*a^5*b + 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^3 + 40*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x))*si
nh(x)^4 + 20*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 20*(55*b^6*cosh(x)^9 - 99*a*
b^5*cosh(x)^8 + 180*(a^2*b^4 + 2*b^6)*cosh(x)^7 - 84*(4*a^3*b^3 + 9*a*b^5)*
cosh(x)^6 + 4*a^3*b^3 + 9*a*b^5 + 42*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cos
h(x)^5 - 1920*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^3 - 210*(8*a^5*
b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^4 + 60*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)
*cosh(x)^2 + 3*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^3 + 30*(
a^2*b^4 + 2*b^6)*cosh(x)^2 + 30*(11*b^6*cosh(x)^10 - 22*a*b^5*cosh(x)^9 + 4
5*(a^2*b^4 + 2*b^6)*cosh(x)^8 - 24*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^7 + 14*(16
*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^6 + a^2*b^4 + 2*b^6 - 960*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^4 - 84*(8*a^5*b + 22*a^3*b^3 + 19*a*b^
5)*cosh(x)^5 + 40*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^3 + 3*(16*a^4*b
^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^2 + 2*(4*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh
(x)^2 + 1920*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5*sinh(x) + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^
```

```
4 + b^6)*cosh(x)^4*sinh(x)^2 + 20*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(
x)^3*sinh(x)^3 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2*sinh(x)^4
 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*sinh(x)^6)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)))
 + 12*(5*b^6*cosh(x)^11 - 11*a*b^5*cosh(x)^10 + 25*(a^2*b^4 + 2*b^6)*cosh(x
)^9 - 15*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^8 + 10*(16*a^4*b^2 + 40*a^2*b^4 + 29
*b^6)*cosh(x)^7 - 960*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 - 70*
(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^6 + a*b^5 + 50*(8*a^5*b + 22*a^3*
b^3 + 19*a*b^5)*cosh(x)^4 + 5*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^3
+ 5*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^2 + 5*(a^2*b^4 + 2*b^6)*cosh(x))*sinh(x))
/(b^7*cosh(x)^6 + 6*b^7*cosh(x)^5*sinh(x) + 15*b^7*cosh(x)^4*sinh(x)^2 + 20
*b^7*cosh(x)^3*sinh(x)^3 + 15*b^7*cosh(x)^2*sinh(x)^4 + 6*b^7*cosh(x)*sinh(
x)^5 + b^7*sinh(x)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**7/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25783, size = 343, normalized size = 2.49

$$5\,b^5\left(e^{(-x)}-e^x\right)^6+12\,ab^4\left(e^{(-x)}-e^x\right)^5+30\,a^2b^3\left(e^{(-x)}-e^x\right)^4+90\,b^5\left(e^{(-x)}-e^x\right)^4+80\,a^3b^2\left(e^{(-x)}-e^x\right)^3+240\,ab^4\left(e^{(-x)}-e^x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*b^5*(e^(-x) - e^x)^6 + 12*a*b^4*(e^(-x) - e^x)^5 + 30*a^2*b^3*(e^
(-x) - e^x)^4 + 90*b^5*(e^(-x) - e^x)^4 + 80*a^3*b^2*(e^(-x) - e^x)^3 + 240
*a*b^4*(e^(-x) - e^x)^3 + 240*a^4*b*(e^(-x) - e^x)^2 + 720*a^2*b^3*(e^(-x)
- e^x)^2 + 720*b^5*(e^(-x) - e^x)^2 + 960*a^5*(e^(-x) - e^x) + 2880*a^3*b^2
*(e^(-x) - e^x) + 2880*a*b^4*(e^(-x) - e^x))/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^7
```

3.188 $\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=145

$$-\frac{ax\left(20a^2b^2+8a^4+15b^4\right)}{8b^6}-\frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3}+\frac{\cosh(x)\left(8\right.}{}$$

```
[Out] -(a*(8*a^4 + 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) - (2*(a^2 + b^2)^(5/2)*ArcTanh
[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/b^6 + Cosh[x]^5/(5*b) + (Cosh[x]^3*(4*
(a^2 + b^2) - 3*a*b*Sinh[x]))/(12*b^3) + (Cosh[x]*(8*(a^2 + b^2)^2 - a*b*(4
*a^2 + 7*b^2)*Sinh[x]))/(8*b^5)
```

Rubi [A] time = 0.414462, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2695, 2865, 2735, 2660, 618, 206}

$$-\frac{ax\left(20a^2b^2+8a^4+15b^4\right)}{8b^6}-\frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3}+\frac{\cosh(x)\left(8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] -(a*(8*a^4 + 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) - (2*(a^2 + b^2)^(5/2)*ArcTanh
[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/b^6 + Cosh[x]^5/(5*b) + (Cosh[x]^3*(4*
(a^2 + b^2) - 3*a*b*Sinh[x]))/(12*b^3) + (Cosh[x]*(8*(a^2 + b^2)^2 - a*b*(4
*a^2 + 7*b^2)*Sinh[x]))/(8*b^5)
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx &= \frac{\cosh^5(x)}{5b}+\frac{i\int \frac{\cosh^4(x)(-ib+ia\sinh(x))}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{\cosh^5(x)}{5b}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3}-\frac{i\int \frac{\cosh^2(x)\left(ib\left(a^2+4b^2\right)-ia\left(4a^2+7b^2\right)\sinh(x)\right)}{a+b\sinh(x)}\,dx}{4b^3}\\
&= \frac{\cosh^5(x)}{5b}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3}+\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right.}{8b^5}\\
&= -\frac{a\left(8a^4+20a^2b^2+15b^4\right)x}{8b^6}+\frac{\cosh^5(x)}{5b}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3}+\frac{\cosh(x)\left(8\left(a^2+\right.\right.}{}\\
&= -\frac{a\left(8a^4+20a^2b^2+15b^4\right)x}{8b^6}+\frac{\cosh^5(x)}{5b}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3}+\frac{\cosh(x)\left(8\left(a^2+\right.\right.}{}\\
&= -\frac{a\left(8a^4+20a^2b^2+15b^4\right)x}{8b^6}+\frac{\cosh^5(x)}{5b}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3}+\frac{\cosh(x)\left(8\left(a^2+\right.\right.}{}\\
&= -\frac{a\left(8a^4+20a^2b^2+15b^4\right)x}{8b^6}-\frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6}+\frac{\cosh^5(x)}{5b}+\frac{\cosh^3(x)\left(4\left(a^2+\right.\right.}{12}
\end{aligned}
$$

Mathematica [C] time = 5.7785, size = 857, normalized size = 5.91

$$
\frac{\cosh(x)\left(240(a+ib)^2\sqrt{b^2}\tanh^{-1}\left(\frac{\sqrt{a-ib}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}}{\sqrt{a+ib}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}}\right)\sqrt{i\sinh(x)+1}(a-ib)^3+\sqrt{a+ib}\left(24\sqrt{a-ib}b^4\sqrt{b^2}\sqrt{i\sinh(x)+1}\sqrt{\right.\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]*(240*(a - I*b)^3*(a + I*b)^2*Sqrt[b^2]*ArcTanh[(Sqrt[a - I*b]*Sqrt
[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/
(a + I*b))])]*Sqrt[1 + I*Sinh[x]] + Sqrt[a + I*b]*((-240*I)*Sqrt[a - I*b]*b
*(a^2 + b^2)^2*ArcTan[(Sqrt[(-I)*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(
Sqrt[I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] - 15*
a*Sqrt[a - I*b]*b*Sqrt[b^2]*(4*a^2 + 9*b^2)*Sqrt[1 + I*Sinh[x]]*Sinh[x]*Sqr
t[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] + 8
*Sqrt[a - I*b]*(b^2)^(3/2)*(5*a^2 + 11*b^2)*Sqrt[1 + I*Sinh[x]]*Sinh[x]^2*S
qrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] -
 30*a*Sqrt[a - I*b]*b^3*Sqrt[b^2]*Sqrt[1 + I*Sinh[x]]*Sinh[x]^3*Sqrt[-((b*(
-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] + 24*Sqrt[a
 - I*b]*b^4*Sqrt[b^2]*Sqrt[1 + I*Sinh[x]]*Sinh[x]^4*Sqrt[-((b*(-I + Sinh[x]
))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] + 2*Sqrt[b^2]*Sqrt[-((b
*(-I + Sinh[x]))/(a + I*b))]*(-15*(-1)^(3/4)*Sqrt[b]*(8*a^4 - (4*I)*a^3*b +
 16*a^2*b^2 - (7*I)*a*b^3 + 8*b^4)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-
((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 4*Sqrt[a - I*b]*(15*a^4 + 35*a^2
*b^2 + 23*b^4)*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]))))
/(120*Sqrt[a - I*b]*Sqrt[a + I*b]*b^5*Sqrt[b^2]*Sqrt[1 + I*Sinh[x]]*Sqrt[-(
(b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])
```

Maple [B] time = 0.037, size = 674, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(a+b*sinh(x)),x)
```

```
[Out] 2/b^6/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*a^
6+6*a^4/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/
2))+6*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(
1/2))-1/5/b/(tanh(1/2*x)-1)^5-1/2/b/(tanh(1/2*x)-1)^4+1/5/b/(tanh(1/2*x)+1)
^5-1/2/b/(tanh(1/2*x)+1)^4+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2
*b)/(a^2+b^2)^(1/2))+13/12/b/(tanh(1/2*x)+1)^3-9/8/b/(tanh(1/2*x)+1)^2+15/8
/b/(tanh(1/2*x)+1)-13/12/b/(tanh(1/2*x)-1)^3-9/8/b/(tanh(1/2*x)-1)^2-15/8/b
/(tanh(1/2*x)-1)-1/2/b^3/(tanh(1/2*x)+1)^2*a^2+1/b^5/(tanh(1/2*x)+1)*a^4-1/
2/b^4/(tanh(1/2*x)+1)*a^3+1/4/b^2/(tanh(1/2*x)+1)^4*a+1/3/b^3/(tanh(1/2*x)+
1)^3*a^2-1/2/b^2/(tanh(1/2*x)+1)^3*a-1/2/b^4/(tanh(1/2*x)-1)*a^3-1/4/b^2/(t
anh(1/2*x)-1)^4*a-1/3/b^3/(tanh(1/2*x)-1)^3*a^2-1/2/b^2/(tanh(1/2*x)-1)^3*a
-1/2/b^4/(tanh(1/2*x)-1)^2*a^3-1/2/b^3/(tanh(1/2*x)-1)^2*a^2-1/b^5/(tanh(1/
2*x)-1)*a^4+1/2/b^4/(tanh(1/2*x)+1)^2*a^3-5/2/b^3/(tanh(1/2*x)-1)*a^2-9/8/b
^2/(tanh(1/2*x)-1)*a+5/2*a^3/b^4*ln(tanh(1/2*x)-1)+15/8*a/b^2*ln(tanh(1/2*x
)-1)+11/8/b^2/(tanh(1/2*x)+1)^2*a+5/2/b^3/(tanh(1/2*x)+1)*a^2-9/8/b^2/(tanh
(1/2*x)+1)*a-5/2*a^3/b^4*ln(tanh(1/2*x)+1)-15/8*a/b^2*ln(tanh(1/2*x)+1)-11/
8/b^2/(tanh(1/2*x)-1)^2*a-a^5/b^6*ln(tanh(1/2*x)+1)+a^5/b^6*ln(tanh(1/2*x)-
1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92581, size = 3822, normalized size = 26.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^5
*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 + 7*b^5)*cosh(x)^8 + 5*(54*b^5*
cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 + 14*b^5)*sinh(x)^8 - 120*(a^3*b^2
 + 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*b
^2 - 12*a*b^4 + 4*(4*a^2*b^3 + 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 + 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x
)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b + 54*a^2*b^3 + 3
3*b^5 + 14*(4*a^2*b^3 + 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 + 2*a*b^4)*cosh(x))*
sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4 +
 280*(4*a^2*b^3 + 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 + 2*a*b^4)*cosh(x)^2 - 6
0*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*c
osh(x))*sinh(x)^5 + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^4 +
10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 + 48*a^4*b + 108*a^2*b^3 + 66*b
^5 + 70*(4*a^2*b^3 + 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 + 2*a*b^4)*cosh(x)^3 -
 60*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b + 18*a^2*b^3 +
11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 + 2*a*b^4)*cosh(x)^3 + 20*(36*b
^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 + 7*b^5)*cosh(x)^5 + 6*a^
3*b^2 + 12*a*b^4 - 210*(a^3*b^2 + 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 + 20*a^3*b
^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^3 +
 12*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 + 10*(4*a^2*b^3 + 7*
b^5)*cosh(x)^2 + 10*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3
+ 7*b^5)*cosh(x)^6 - 252*(a^3*b^2 + 2*a*b^4)*cosh(x)^5 + 4*a^2*b^3 + 7*b^5
- 120*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b + 18*a^2*b^
3 + 11*b^5)*cosh(x)^4 + 36*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*(
a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x)^2 + 960*((a^4 + 2*a^2*b^2 + b^4)*cosh(x
)^5 + 5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 + 2*a^2*b^2 + b
^4)*cosh(x)^3*sinh(x)^2 + 10*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 +
5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 + 2*a^2*b^2 + b^4)*sinh(
x)^5)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) + 5*(12*b^5*cosh(x)^9 - 27*a*b^4*cosh(x)^8 + 16*(4*a^2*b^
3 + 7*b^5)*cosh(x)^7 - 168*(a^3*b^2 + 2*a*b^4)*cosh(x)^6 - 120*(8*a^5 + 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^4 + 72*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x
)^5 + 3*a*b^4 + 48*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^3 + 72*(a^3*b^2
+ 2*a*b^4)*cosh(x)^2 + 4*(4*a^2*b^3 + 7*b^5)*cosh(x))*sinh(x))/(b^6*cosh(x)
^5 + 5*b^6*cosh(x)^4*sinh(x) + 10*b^6*cosh(x)^3*sinh(x)^2 + 10*b^6*cosh(x)^
2*sinh(x)^3 + 5*b^6*cosh(x)*sinh(x)^4 + b^6*sinh(x)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22981, size = 389, normalized size = 2.68

$$\frac{6\,b^4e^{(5\,x)} - 15\,ab^3e^{(4\,x)} + 40\,a^2b^2e^{(3\,x)} + 70\,b^4e^{(3\,x)} - 120\,a^3be^{(2\,x)} - 240\,ab^3e^{(2\,x)} + 480\,a^4e^x + 1080\,a^2b^2e^x + 660\,b^4e^x}{960\,b^5} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) + 70*b^4*e^(3*
x) - 120*a^3*b*e^(2*x) - 240*a*b^3*e^(2*x) + 480*a^4*e^x + 1080*a^2*b^2*e^x
 + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*(15
*a*b^4*e^x + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(a^3*
b^2 + 2*a*b^4)*e^(3*x) + 10*(4*a^2*b^3 + 7*b^5)*e^(2*x))*e^(-5*x)/b^6 + (a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))
/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
```

3.189 $\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=81

$$\frac{(a^2+2b^2)\sinh^2(x)}{2b^3}-\frac{a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5}-\frac{a\sinh^3(x)}{3b^2}+\frac{\sinh^4(x)}{4b}$$

```
[Out] ((a^2 + b^2)^2*Log[a + b*Sinh[x]])/b^5 - (a*(a^2 + 2*b^2)*Sinh[x])/b^4 + ((
a^2 + 2*b^2)*Sinh[x]^2)/(2*b^3) - (a*Sinh[x]^3)/(3*b^2) + Sinh[x]^4/(4*b)
```

Rubi [A] time = 0.0909712, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{(a^2+2b^2)\sinh^2(x)}{2b^3}-\frac{a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5}-\frac{a\sinh^3(x)}{3b^2}+\frac{\sinh^4(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)^2*Log[a + b*Sinh[x]])/b^5 - (a*(a^2 + 2*b^2)*Sinh[x])/b^4 + ((
a^2 + 2*b^2)*Sinh[x]^2)/(2*b^3) - (a*Sinh[x]^3)/(3*b^2) + Sinh[x]^4/(4*b)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx&=\frac{\text{Subst}\left(\int \frac{(-b^2-x^2)^2}{a+x}\,dx,x,b\sinh(x)\right)}{b^5}\\&=\frac{\text{Subst}\left(\int \left(-a(a^2+2b^2)+(a^2+2b^2)x-ax^2+x^3+\frac{(a^2+b^2)^2}{a+x}\right)dx,x,b\sinh(x)\right)}{b^5}\\&=\frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5}-\frac{a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^2+2b^2)\sinh^2(x)}{2b^3}-\frac{a\sinh^3(x)}{3b^2}+\frac{\sinh^4(x)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0877441, size = 76, normalized size = 0.94

$$\frac{6b^2(a^2+b^2)\sinh^2(x)-12ab(a^2+2b^2)\sinh(x)+12(a^2+b^2)^2\log(a+b\sinh(x))-4ab^3\sinh^3(x)+3b^4\cosh^4(x)}{12b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] (3*b^4*Cosh[x]^4 + 12*(a^2 + b^2)^2*Log[a + b*Sinh[x]] - 12*a*b*(a^2 + 2*b^
2)*Sinh[x] + 6*b^2*(a^2 + b^2)*Sinh[x]^2 - 4*a*b^3*Sinh[x]^3)/(12*b^5)
```

Maple [B] time = 0.033, size = 447, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+b*sinh(x)),x)
```

```
[Out] -2/b^3*ln(tanh(1/2*x)+1)*a^2-2/b^3*ln(tanh(1/2*x)-1)*a^2+1/b*ln(a*tanh(1/2*
x)^2-2*tanh(1/2*x)*b-a)+1/4/b/(tanh(1/2*x)-1)^4+1/4/b/(tanh(1/2*x)+1)^4-1/2
/b/(tanh(1/2*x)+1)^3+9/8/b/(tanh(1/2*x)+1)^2-7/8/b/(tanh(1/2*x)+1)+1/2/b/(t
anh(1/2*x)-1)^3+9/8/b/(tanh(1/2*x)-1)^2+7/8/b/(tanh(1/2*x)-1)-1/b*ln(tanh(1
/2*x)+1)-1/b*ln(tanh(1/2*x)-1)+1/2/b^3/(tanh(1/2*x)+1)^2*a^2+1/b^4/(tanh(1/
2*x)+1)*a^3+1/3/b^2/(tanh(1/2*x)+1)^3*a+1/b^4/(tanh(1/2*x)-1)*a^3+1/b^5*ln(
a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^4+2/b^3*ln(a*tanh(1/2*x)^2-2*tanh(1/2*
x)*b-a)*a^2+1/3/b^2/(tanh(1/2*x)-1)^3*a-1/b^5*ln(tanh(1/2*x)-1)*a^4+1/2/b^3
/(tanh(1/2*x)-1)^2*a^2-1/b^5*ln(tanh(1/2*x)+1)*a^4+1/2/b^3/(tanh(1/2*x)-1)*
a^2+2/b^2/(tanh(1/2*x)-1)*a-1/2/b^2/(tanh(1/2*x)+1)^2*a-1/2/b^3/(tanh(1/2*x
)+1)*a^2+2/b^2/(tanh(1/2*x)+1)*a+1/2/b^2/(tanh(1/2*x)-1)^2*a
```

Maxima [B] time = 1.25078, size = 243, normalized size = 3.

$$-\frac{\left(8\,ab^2e^{(-x)} - 3\,b^3 - 12\left(2\,a^2b + 3\,b^3\right)e^{(-2\,x)} + 24\left(4\,a^3 + 7\,ab^2\right)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,b^4} + \frac{8\,ab^2e^{(-3\,x)} + 3\,b^3e^{(-4\,x)} + 24\left(4\,a^3 + 7\,ab^2\right.}{192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b + 3*b^3)*e^(-2*x) + 24*(4*a^3
+ 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 + 1/192*(8*a*b^2*e^(-3*x) + 3*b^3*e^(-4*x)
 + 24*(4*a^3 + 7*a*b^2)*e^(-x) + 12*(2*a^2*b + 3*b^3)*e^(-2*x))/b^4 + (a^4
+ 2*a^2*b^2 + b^4)*x/b^5 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*a*e^(-x) + b*e^(-
2*x) - b)/b^5
```

Fricas [B] time = 1.92069, size = 2228, normalized size = 27.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*cos
h(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 + 3*b^4)*cosh(x)^6 + 4*(21*b^4*cosh
```

```
(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 + 9*b^4)*sinh(x)^6 - 192*(a^4 + 2*a^2*
b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b + 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*cosh(
x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b - 7*a*b^3 + 3*(2*a^2*b^2 + 3*b^4)*cosh(x
))*sinh(x)^5 + 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cosh(x)^3
 + 90*(2*a^2*b^2 + 3*b^4)*cosh(x)^2 - 96*(a^4 + 2*a^2*b^2 + b^4)*x - 60*(4*
a^3*b + 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 + 24*(4*a^3*b + 7*a*b^3)*cosh(x
)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 + 12*a^3*b + 21*a*b^3 + 30*(
2*a^2*b^2 + 3*b^4)*cosh(x)^3 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x) - 30*(4
*a^3*b + 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 + 3*b^4)*cosh(x)^2 +
 12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 + 3*b^4)*cosh(x)^
4 + 2*a^2*b^2 + 3*b^4 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 20*(4*a^3*
b + 7*a*b^3)*cosh(x)^3 + 6*(4*a^3*b + 7*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a
^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3*sinh(
x) + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4 + 2*a^2*b^2 + b
^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^4)*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3*cosh(x)^6 + 9*(2*
a^2*b^2 + 3*b^4)*cosh(x)^5 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^3 - 15*(4
*a^3*b + 7*a*b^3)*cosh(x)^4 + a*b^3 + 9*(4*a^3*b + 7*a*b^3)*cosh(x)^2 + 3*(
2*a^2*b^2 + 3*b^4)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(
x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2494, size = 188, normalized size = 2.32

$$\frac{3\,b^3\left(e^{(-x)}-e^x\right)^4+8\,ab^2\left(e^{(-x)}-e^x\right)^3+24\,a^2b\left(e^{(-x)}-e^x\right)^2+48\,b^3\left(e^{(-x)}-e^x\right)^2+96\,a^3\left(e^{(-x)}-e^x\right)+192\,ab^2\left(e^{(-x)}-e^x\right)}{192\,b^4}+\frac{(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b^3*(e^(-x) - e^x)^4 + 8*a*b^2*(e^(-x) - e^x)^3 + 24*a^2*b*(e^(-x)
 - e^x)^2 + 48*b^3*(e^(-x) - e^x)^2 + 96*a^3*(e^(-x) - e^x) + 192*a*b^2*(e^
(-x) - e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(-b*(e^(-x) - e^x) + 2*a)
)/b^5
```

3.190 $\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=97

$$-\frac{ax\left(2a^2+3b^2\right)}{2b^4}-\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}+\frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3}+\frac{\cosh^3(x)}{3b}$$

```
[Out] -(a*(2*a^2 + 3*b^2)*x)/(2*b^4) - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tanh[x
/2])/Sqrt[a^2 + b^2]])/b^4 + Cosh[x]^3/(3*b) + (Cosh[x]*(2*(a^2 + b^2) - a*
b*Sinh[x]))/(2*b^3)
```

Rubi [A] time = 0.240779, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2695, 2865, 2735, 2660, 618, 206}

$$-\frac{ax\left(2a^2+3b^2\right)}{2b^4}-\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}+\frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3}+\frac{\cosh^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -(a*(2*a^2 + 3*b^2)*x)/(2*b^4) - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tanh[x
/2])/Sqrt[a^2 + b^2]])/b^4 + Cosh[x]^3/(3*b) + (Cosh[x]*(2*(a^2 + b^2) - a*
b*Sinh[x]))/(2*b^3)
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \frac{\cosh^3(x)}{3b} + \frac{i\int \frac{\cosh^2(x)(-ib+ia\sinh(x))}{a+b\sinh(x)}\,dx}{b}$$

$$= \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3} - \frac{i\int \frac{ib\left(a^2+2b^2\right)-ia\left(2a^2+3b^2\right)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^3}$$

$$= -\frac{a\left(2a^2+3b^2\right)x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3} + \frac{\left(a^2+b^2\right)^2\int \frac{1}{a+b\sinh(x)}\,dx}{b^4}$$

$$= -\frac{a\left(2a^2+3b^2\right)x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3} + \frac{\left(2\left(a^2+b^2\right)^2\right)\text{Subst}\left(\int \frac{1}{a+2b\ldots}\right.}{b^4}$$

$$= -\frac{a\left(2a^2+3b^2\right)x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3} - \frac{\left(4\left(a^2+b^2\right)^2\right)\text{Subst}\left(\int \frac{\ldots}{4(a^2+\ldots}\right.}{b\ldots}$$

$$= -\frac{a\left(2a^2+3b^2\right)x}{2b^4} - \frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sin\ldots\right.}{2b^3}$$

Mathematica [C] time = 3.49875, size = 651, normalized size = 6.71

$$\frac{b\cosh^3(x)\left(\sqrt{a+ib}\left(2\sqrt{b^2}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\left(\sqrt{a-ib}\left(3a^2+4b^2\right)\sqrt{1+i\sinh(x)}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}} - 3i\sqrt{b}\left((1+i)\sqrt{2}a^2-(-1)^{3/4}a\ldots\right.\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (b*Cosh[x]^3*(12*(a - I*b)^2*(a + I*b)*Sqrt[b^2]*ArcTanh[(Sqrt[a - I*b]*Sqr
t[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))
/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] + Sqrt[a + I*b]*((-12*I)*Sqrt[a - I*b]*b
*(a^2 + b^2)*ArcTan[(Sqrt[(-I)*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sq
rt[I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] - 3*a*S
qrt[a - I*b]*b*Sqrt[b^2]*Sqrt[1 + I*Sinh[x]]*Sinh[x]*Sqrt[-((b*(-I + Sinh[x
]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] + 2*Sqrt[a - I*b]*(b^2
```

```
)^(3/2)*Sqrt[1 + I*Sinh[x]]*Sinh[x]^2*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]
*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] + 2*Sqrt[b^2]*Sqrt[-((b*(-I + Sinh[x]
))/(a + I*b))]*((-3*I)*Sqrt[b]*((1 + I)*Sqrt[2]*a^2 - (-1)^(3/4)*a*b + (1 +
 I)*Sqrt[2]*b^2)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))
/(a - I*b))])/Sqrt[b]] + Sqrt[a - I*b]*(3*a^2 + 4*b^2)*Sqrt[1 + I*Sinh[x]]*
Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]))))/(6*(a - I*b)^(3/2)*(a + I*b)^(3/2)
*(b^2)^(3/2)*Sqrt[1 + I*Sinh[x]]*(-((b*(-I + Sinh[x]))/(a + I*b)))^(3/2)*(-
((b*(I + Sinh[x]))/(a - I*b)))^(3/2))
```

Maple [B] time = 0.032, size = 336, normalized size = 3.5

$$\frac{1}{3b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-3}+\frac{a}{2b^{2}}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{2b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{a^{2}}{b^{3}}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{a}{2b^{2}}\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*sinh(x)),x)
```

```
[Out] 1/3/b/(tanh(1/2*x)+1)^3+1/2/b^2/(tanh(1/2*x)+1)^2*a-1/2/b/(tanh(1/2*x)+1)^2
+1/b^3/(tanh(1/2*x)+1)*a^2-1/2/b^2/(tanh(1/2*x)+1)*a+3/2/b/(tanh(1/2*x)+1)-
a^3/b^4*ln(tanh(1/2*x)+1)-3/2*a/b^2*ln(tanh(1/2*x)+1)-1/3/b/(tanh(1/2*x)-1)
^3-1/2/b^2/(tanh(1/2*x)-1)^2*a-1/2/b/(tanh(1/2*x)-1)^2-1/b^3/(tanh(1/2*x)-1
)*a^2-1/2/b^2/(tanh(1/2*x)-1)*a-3/2/b/(tanh(1/2*x)-1)+a^3/b^4*ln(tanh(1/2*x
)-1)+3/2*a/b^2*ln(tanh(1/2*x)-1)+2*a^4/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a
*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+4*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan
h(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93333, size = 1523, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x)
- a*b^2)*sinh(x)^5 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b + 5*b^3)
*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b + 5*b^3)*sinh(x
)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2*a^3
 + 3*a*b^2)*x + 6*(4*a^2*b + 5*b^3)*cosh(x))*sinh(x)^3 + b^3 + 3*(4*a^2*b +
 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 + 4*a^2*b + 5*b
```

```
^3 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b + 5*b^3)*cosh(x)^2)*sinh(x
)^2 + 24*((a^2 + b^2)*cosh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2*sinh(x) + 3*(a^2
+ b^2)*cosh(x)*sinh(x)^2 + (a^2 + b^2)*sinh(x)^3)*sqrt(a^2 + b^2)*log((b^2*
cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) +
a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
+ b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 3*(2*b^3*co
sh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*b
 + 5*b^3)*cosh(x)^3 + a*b^2 + 2*(4*a^2*b + 5*b^3)*cosh(x))*sinh(x))/(b^4*co
sh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24197, size = 227, normalized size = 2.34

$$\frac{b^2e^{(3\,x)} - 3\,abe^{(2\,x)} + 12\,a^2e^x + 15\,b^2e^x}{24\,b^3} - \frac{\left(2\,a^3 + 3\,ab^2\right)x}{2\,b^4} + \frac{\left(3\,ab^2e^x + b^3 + 3\left(4\,a^2b + 5\,b^3\right)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^4} + \frac{\left(a^4 + 2\,a^2b^2 + b^4\right.}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 15*b^2*e^x)/b^3 - 1/2*(2*a
^3 + 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x + b^3 + 3*(4*a^2*b + 5*b^3)*e^(2*x)
)*e^(-3*x)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2
 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4)
```

3.191 $\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=38

$$\frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}$$

```
[Out] ((a^2 + b^2)*Log[a + b*Sinh[x]])/b^3 - (a*Sinh[x])/b^2 + Sinh[x]^2/(2*b)
```

Rubi [A] time = 0.0611395, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Sinh[x]])/b^3 - (a*Sinh[x])/b^2 + Sinh[x]^2/(2*b)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{a+x}\,dx,x,b\sinh(x)\right)}{b^3}\\ &= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2-b^2}{a+x}\right)dx,x,b\sinh(x)\right)}{b^3}\\ &= \frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0348453, size = 38, normalized size = 1.

$$-\frac{-(a^2+b^2)\log(a+b\sinh(x))+ab\sinh(x)-\frac{1}{2}b^2\sinh^2(x)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -((-((a^2 + b^2)*Log[a + b*Sinh[x]]) + a*b*Sinh[x] - (b^2*Sinh[x]^2)/2)/b^3
)
```

Maple [B] time = 0.028, size = 185, normalized size = 4.9

$$\frac{1}{2b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}-\frac{1}{2b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{a}{b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{a^2}{b^3}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*sinh(x)),x)
```

```
[Out] 1/2/b/(tanh(1/2*x)+1)^2-1/2/b/(tanh(1/2*x)+1)+1/b^2/(tanh(1/2*x)+1)*a-1/b^3
*ln(tanh(1/2*x)+1)*a^2-1/b*ln(tanh(1/2*x)+1)+1/2/b/(tanh(1/2*x)-1)^2+1/2/b/
(tanh(1/2*x)-1)+1/b^2/(tanh(1/2*x)-1)*a-1/b^3*ln(tanh(1/2*x)-1)*a^2-1/b*ln(
tanh(1/2*x)-1)+1/b^3*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^2+1/b*ln(a*tan
h(1/2*x)^2-2*tanh(1/2*x)*b-a)
```

Maxima [B] time = 1.29438, size = 109, normalized size = 2.87

$$-\frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2}+\frac{4\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^2}+\frac{\left(a^2+b^2\right)x}{b^3}+\frac{\left(a^2+b^2\right)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 + 1/8*(4*a*e^(-x) + b*e^(-2*x))/b^2 + (a^
2 + b^2)*x/b^3 + (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^3
```

Fricas [B] time = 1.80766, size = 629, normalized size = 16.55

$$\frac{b^2\cosh(x)^4+b^2\sinh(x)^4-4\,ab\cosh(x)^3-8\left(a^2+b^2\right)x\cosh(x)^2+4\left(b^2\cosh(x)-ab\right)\sinh(x)^3+4\,ab\cosh(x)+2\,(3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 + b^2)*x*cosh
(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*b^2*cosh(x)^
2 - 6*a*b*cosh(x) - 4*(a^2 + b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 + b^2)*cosh(
x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2)*log(2*(b*sinh
(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4*(a^2
 + b^2)*x*cosh(x) + a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) +
b^3*sinh(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15903, size = 82, normalized size = 2.16

$$\frac{b\left(e^{(-x)} - e^{x}\right)^{2} + 4\,a\left(e^{(-x)} - e^{x}\right)}{8\,b^{2}} + \frac{\left(a^{2} + b^{2}\right)\log\left(\left|-b\left(e^{(-x)} - e^{x}\right) + 2\,a\right|\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*(e^(-x) - e^x)^2 + 4*a*(e^(-x) - e^x))/b^2 + (a^2 + b^2)*log(abs(-b*
(e^(-x) - e^x) + 2*a))/b^3
```

3.192 $\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

```
[Out] -((a*x)/b^2) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/b^2 + Cosh[x]/b
```

Rubi [A] time = 0.108934, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2695, 2735, 2660, 618, 206}

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -((a*x)/b^2) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/b^2 + Cosh[x]/b
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx &= \frac{\cosh(x)}{b}+\frac{i\int \frac{-ib+ia\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{\left(2\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}-\frac{\left(4\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}+\frac{\cosh(x)}{b}
\end{aligned}$$

Mathematica [C] time = 1.02959, size = 429, normalized size = 7.94

$$\frac{\cosh(x)\left(2\sqrt{b^2}(a-ib)\sqrt{1+i\sinh(x)}\tanh^{-1}\left(\frac{\sqrt{a-ib}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}}{\sqrt{a+ib}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}}\right)+\sqrt{a+ib}\left(\sqrt{b^2}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\left(\sqrt{a-ib}\sqrt{1+i\sinh(x)}\sqrt{\cdots}\right.\right.\right.}{b\sqrt{b^2}\sqrt{a-ib}\sqrt{a+ib}\sqrt{1+i\sinh(x\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]*(2*(a - I*b)*Sqrt[b^2]*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[
x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sq
rt[1 + I*Sinh[x]] + Sqrt[a + I*b]*((-2*I)*Sqrt[a - I*b]*b*ArcTan[(Sqrt[(-I)
*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqrt[I*b]*Sqrt[-((b*(-I + Sinh[x
]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] + Sqrt[b^2]*Sqrt[-((b*(-I + Sinh[x]))
/(a + I*b))]*(-2*(-1)^(3/4)*Sqrt[b]*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[
-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + Sqrt[a - I*b]*Sqrt[1 + I*Sinh[x
]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]))))/(Sqrt[a - I*b]*Sqrt[a + I*b]*b*
Sqrt[b^2]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-(
(b*(I + Sinh[x]))/(a - I*b))])
```

Maple [B] time = 0.024, size = 126, normalized size = 2.3

$$\frac{1}{b}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-\frac{a}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{b}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}+\frac{a}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\frac{a^2}{b^2\sqrt{a^2+b^2}}\text{Artanh}\left(1/\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sinh(x)),x)
```

```
[Out] 1/b/(tanh(1/2*x)+1)-a/b^2*ln(tanh(1/2*x)+1)-1/b/(tanh(1/2*x)-1)+a/b^2*ln(ta
nh(1/2*x)-1)+2*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a
```

```
^2+b^2)^(1/2))+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.74153, size = 504, normalized size = 9.33

$$-\frac{2\,ax\cosh(x) - b\cosh(x)^2 - b\sinh(x)^2 - 2\sqrt{a^2+b^2}(\cosh(x)+\sinh(x))\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b\cdots}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh\cdots}\right.}{2\,(b^2\cosh(x)+b^2\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 + b^2)*(cosh(x
) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b
^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(
x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(
x) - b)) + 2*(a*x - b*cosh(x))*sinh(x) - b)/(b^2*cosh(x) + b^2*sinh(x))
```

Sympy [A] time = 177.406, size = 398, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(log(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x
/2))/(tanh(x/2)**2 - 1) - 2/(tanh(x/2)**2 - 1)), Eq(a, 0) & Eq(b, 0)), ((-x
*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)), ((log(tan
h(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x/2))/(tanh(x/2)**2 - 1)
 - 2/(tanh(x/2)**2 - 1))/b, Eq(a, 0)), (-a*x*tanh(x/2)**2/(b**2*tanh(x/2)**
2 - b**2) + a*x/(b**2*tanh(x/2)**2 - b**2) - b*tanh(x/2)**2/(b**2*tanh(x/2)
**2 - b**2) - b/(b**2*tanh(x/2)**2 - b**2) - sqrt(a**2 + b**2)*log(tanh(x/2
) - b/a - sqrt(a**2 + b**2)/a)*tanh(x/2)**2/(b**2*tanh(x/2)**2 - b**2) + sq
rt(a**2 + b**2)*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(b**2*tanh(x/2)*
*2 - b**2) + sqrt(a**2 + b**2)*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)*t
anh(x/2)**2/(b**2*tanh(x/2)**2 - b**2) - sqrt(a**2 + b**2)*log(tanh(x/2) -
b/a + sqrt(a**2 + b**2)/a)/(b**2*tanh(x/2)**2 - b**2), True))
```

Giac [A] time = 1.26692, size = 112, normalized size = 2.07

$$-\frac{ax}{b^2} + \frac{e^{(-x)}}{2b} + \frac{e^x}{2b} + \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2be^x+2a-2\sqrt{a^2+b^2}\right|}{\left|2be^x+2a+2\sqrt{a^2+b^2}\right|}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b + sqrt(a^2 + b^2)*log(abs(2*b*e^x + 2*a
 - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/b^2
```

3.193 $\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sinh(x))}{b}$$

```
[Out] Log[a + b*Sinh[x]]/b
```

Rubi [A] time = 0.0249298, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 31}

$$\frac{\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[a + b*Sinh[x]]/b
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(x)}{a+b\sinh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{b}\\ &= \frac{\log(a+b\sinh(x))}{b} \end{aligned}$$

Mathematica [A] time = 0.0053467, size = 11, normalized size = 1.

$$\frac{\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[a + b*Sinh[x]]/b
```

Maple [A] time = 0.01, size = 12, normalized size = 1.1

$$\frac{\ln \left(a + b \sinh \left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*sinh(x)),x)
```

```
[Out] ln(a+b*sinh(x))/b
```

Maxima [A] time = 1.06699, size = 15, normalized size = 1.36

$$\frac{\log \left(b \sinh \left(x\right) + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] log(b*sinh(x) + a)/b
```

Fricas [B] time = 2.02641, size = 72, normalized size = 6.55

$$-\frac{x - \log \left(\frac{2 \left(b \sinh(x) + a\right)}{\cosh(x) - \sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(x - log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))))/b
```

Sympy [A] time = 0.424653, size = 14, normalized size = 1.27

$$\begin{cases} \frac{\log \left(\frac{a}{b} + \sinh \left(x\right)\right)}{b} & \text{for}\: b \neq 0 \\ \frac{\sinh \left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((log(a/b + sinh(x))/b, Ne(b, 0)), (sinh(x)/a, True))
```

Giac [A] time = 1.26256, size = 30, normalized size = 2.73

$$\frac{\log \left(\left|-b \left(e^{(-x)} - e^{x}\right) + 2 \, a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b
```

3.194 $\int \frac{\mathbf{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=48

$$\frac{b\log(a+b\sinh(x))}{a^2+b^2}+\frac{a\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}$$

```
[Out] (a*ArcTan[Sinh[x]])/(a^2 + b^2) - (b*Log[Cosh[x]])/(a^2 + b^2) + (b*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Rubi [A] time = 0.060295, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2668, 706, 31, 635, 204, 260}

$$\frac{b\log(a+b\sinh(x))}{a^2+b^2}+\frac{a\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Sinh[x]),x]
```


```
[Out] (a*ArcTan[Sinh[x]])/(a^2 + b^2) - (b*Log[Cosh[x]])/(a^2 + b^2) + (b*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx &= -\left(b\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\right)\\
&= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{a^2+b^2} + \frac{b\,\mathrm{Subst}\left(\int \frac{-a+x}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{b\log(a+b\sinh(x))}{a^2+b^2} + \frac{b\,\mathrm{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2} - \frac{(ab)\,\mathrm{Subst}\left(\int \frac{1}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{a\tan^{-1}(\sinh(x))}{a^2+b^2} - \frac{b\log(\cosh(x))}{a^2+b^2} + \frac{b\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [B] time = 0.0953978, size = 99, normalized size = 2.06

$$-\frac{b\left(\left(\sqrt{-b^2}-a\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{2\sqrt{-b^2}\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Sinh[x]),x]
```

```
[Out] -(b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[a + b
*Sinh[x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/(2*Sqrt[-b^2]*(a
^2 + b^2))
```

Maple [A] time = 0.021, size = 71, normalized size = 1.5

$$-\frac{b}{a^2+b^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)+2\,\frac{a\arctan\left(\tanh\left(x/2\right)\right)}{a^2+b^2}+\frac{b}{a^2+b^2}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-2\,\tanh\left(x/2\right)b-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*sinh(x)),x)
```

```
[Out] -1/(a^2+b^2)*b*ln(tanh(1/2*x)^2+1)+2/(a^2+b^2)*a*arctan(tanh(1/2*x))+b/(a^2
+b^2)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)
```

Maxima [A] time = 1.84252, size = 89, normalized size = 1.85

$$-\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}+\frac{b\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}-\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(e^(-x))/(a^2 + b^2) + b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
+ b^2) - b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Fricas [A] time = 2.15307, size = 177, normalized size = 3.69

$$\frac{2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+b\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-b\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^{2}+b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a*arctan(cosh(x) + sinh(x)) + b*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) - b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*sinh(x)), x)
```

Giac [A] time = 1.2414, size = 120, normalized size = 2.5

$$\frac{b^{2}\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a^{2}b+b^{3}}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)a}{2\left(a^{2}+b^{2}\right)}-\frac{b\log\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)}{2\left(a^{2}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) - 1/2*b*log((e^(-x) - e^x)^2 + 4)/(a
^2 + b^2)
```

3.195 $\int \frac{\text{sech}^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=59

$$\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

```
[Out] (-2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (Se
ch[x]*(b + a*Sinh[x]))/(a^2 + b^2)
```

Rubi [A] time = 0.0789683, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2696, 12, 2660, 618, 206}

$$\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (Se
ch[x]*(b + a*Sinh[x]))/(a^2 + b^2)
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx &= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{\int \frac{b^2}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{b^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}-\frac{\left(4b^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}
\end{aligned}
$$

Mathematica [A] time = 0.166832, size = 67, normalized size = 1.14

$$
\frac{\frac{2b^2\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+a\tanh(x)+b\operatorname{sech}(x)}{a^2+b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] ((2*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + b*Se
ch[x] + a*Tanh[x])/(a^2 + b^2)
```

Maple [A] time = 0.026, size = 71, normalized size = 1.2

$$
-2\,\frac{-a\tanh(x/2)-b}{\left(a^2+b^2\right)\left((\tanh(x/2))^2+1\right)}+2\,\frac{b^2}{\left(a^2+b^2\right)^{3/2}}\operatorname{Artanh}\left(1/2\,\frac{2\,a\tanh(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*sinh(x)),x)
```

```
[Out] -2/(a^2+b^2)*(-a*tanh(1/2*x)-b)/(tanh(1/2*x)^2+1)+2*b^2/(a^2+b^2)^(3/2)*arc
tanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05738, size = 693, normalized size = 11.75

$$-\frac{2\,a^3+2\,ab^2-\left(b^2\cosh\left(x\right)^2+2\,b^2\cosh\left(x\right)\sinh\left(x\right)+b^2\sinh\left(x\right)^2+b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2 \ldots}{b\cosh(x)^2+b\sinh(x \ldots}\right.}{a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cosh\left(x\right)^2+2\left(a^4+2\,a^2b^2+b^4\right) \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
+ b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x))/(a^
4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{sech}^2\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*sinh(x)), x)
```

Giac [A] time = 1.32447, size = 117, normalized size = 1.98

$$\frac{b^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\,(be^x-a)}{\left(a^2+b^2\right)\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

3.196 $\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=87

$$\frac{b^3\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{a(a^2+3b^2)\tan^{-1}(\sinh(x))}{2(a^2+b^2)^2}-\frac{b^3\log(\cosh(x))}{(a^2+b^2)^2}+\frac{\operatorname{sech}^2(x)(a\sinh(x)+b)}{2(a^2+b^2)}$$

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) - (b^3*Log[Cosh[x]])/(a
^2 + b^2)^2 + (b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(b + a*Si
nh[x]))/(2*(a^2 + b^2))
```

Rubi [A] time = 0.122492, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2668, 741, 801, 635, 203, 260}

$$\frac{b^3\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{a(a^2+3b^2)\tan^{-1}(\sinh(x))}{2(a^2+b^2)^2}-\frac{b^3\log(\cosh(x))}{(a^2+b^2)^2}+\frac{\operatorname{sech}^2(x)(a\sinh(x)+b)}{2(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) - (b^3*Log[Cosh[x]])/(a
^2 + b^2)^2 + (b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(b + a*Si
nh[x]))/(2*(a^2 + b^2))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = b^3\,\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b\,\text{Subst}\left(\int \frac{a^2+2b^2+ax}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}$$

$$= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2b^2}{\left(a^2+b^2\right)(a+x)} + \frac{-a^3-3ab^2+2b^2x}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right) dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}$$

$$= \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b\,\text{Subst}\left(\int \frac{-a^3-3ab^2+2b^2x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)^2}$$

$$= \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b^3\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^2} + \frac{\left(ab\left(a^2+\right.\right.}{}$$

$$= \frac{a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2} - \frac{b^3\log(\cosh(x))}{\left(a^2+b^2\right)^2} + \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)}$$

Mathematica [A] time = 0.124474, size = 77, normalized size = 0.89

$$\frac{b\left(a^2+b^2\right)\operatorname{sech}^2(x)+2a\left(a^2+3b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+a\left(a^2+b^2\right)\tanh(x)\operatorname{sech}(x)+2b^3(\log(a+b\sinh(x))-\log(\cosh}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*(a^2 + 3*b^2)*ArcTan[Tanh[x/2]] + 2*b^3*(-Log[Cosh[x]] + Log[a + b*Sin
h[x]]) + b*(a^2 + b^2)*Sech[x]^2 + a*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 +
 b^2)^2)
```

Maple [B] time = 0.033, size = 353, normalized size = 4.1

$$-\frac{a^3}{a^4+2a^2b^2+b^4}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2} - \frac{ab^2}{a^4+2a^2b^2+b^4}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2} - 2\,\frac{}{\left(a^4+2a^2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*sinh(x)),x)
```

```
[Out] -1/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^3-1/(a^4+2*a^2*b
^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a*b^2-2/(a^4+2*a^2*b^2+b^4)/(tanh
(1/2*x)^2+1)^2*tanh(1/2*x)^2*a^2*b-2/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^
2*tanh(1/2*x)^2*b^3+1/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a
^3+1/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a*b^2-1/(a^4+2*a^2
*b^2+b^4)*b^3*ln(tanh(1/2*x)^2+1)+1/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*x))
*a^3+3/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*x))*a*b^2+b^3/(a^4+2*a^2*b^2+b^4
)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)
```

Maxima [A] time = 1.84185, size = 215, normalized size = 2.47

$$\frac{b^3 \log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{b^3 \log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{\left(a^3 + 3\,ab^2\right)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{ae^{(-x)} + 2\,be^{(-2\,x)} - ae^{(-3}}{a^2 + b^2 + 2\left(a^2 + b^2\right)e^{(-2\,x)} + \left(a^2 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(e^(
-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 + 3*a*b^2)*arctan(e^(-x))/(a^4 +
2*a^2*b^2 + b^4) + (a*e^(-x) + 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2 + 2*(a
^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Fricas [B] time = 2.31088, size = 1724, normalized size = 19.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 + 2*(a^2*b + b^3)*cosh(x
)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 + 3*a*b
^2)*cosh(x)^4 + 4*(a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 3*a*b^2)*sinh(
x)^4 + a^3 + 3*a*b^2 + 2*(a^3 + 3*a*b^2)*cosh(x)^2 + 2*(a^3 + 3*a*b^2 + 3*(
a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a*b^2)*cosh(x)^3 + (a^3 +
 3*a*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)*cosh(
x) + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(
x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*c
osh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (b^3*cosh(x)^
4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*
b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*l
og(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(x)^
2 - 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 +
4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(
x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4
 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a
^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{a + b \sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*sinh(x)), x)
```

Giac [B] time = 1.24736, size = 289, normalized size = 3.32

$$\frac{b^4 \log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{a^4 b + 2\,a^2 b^3 + b^5} - \frac{b^3 \log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^4 + 2\,a^2 b^2 + b^4\right)} + \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(a^3 + 3\,ab^2\right)}{4\left(a^4 + 2\,a^2 b^2 + b^4\right)} + \frac{b^3\left(e^{(-x)} - e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - 1/2*b^3*l
og((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(1/2*(
e^(2*x) - 1)*e^(-x)))*(a^3 + 3*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(b^3*(e
^(-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 4*a^2*b +
8*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

3.197 $\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=100

$$-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{\operatorname{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}+\frac{\operatorname{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{3\left(a^2+b^2\right)^2}$$

```
[Out] (-2*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (Se
ch[x]^3*(b + a*Sinh[x]))/(3*(a^2 + b^2)) + (Sech[x]*(3*b^3 + a*(2*a^2 + 5*b
^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rubi [A] time = 0.213126, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2696, 2866, 12, 2660, 618, 206}

$$-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{\operatorname{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}+\frac{\operatorname{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (Se
ch[x]^3*(b + a*Sinh[x]))/(3*(a^2 + b^2)) + (Sech[x]*(3*b^3 + a*(2*a^2 + 5*b
^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx &= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\int \frac{\operatorname{sech}^2(x)\left(-2a^2-3b^2-2ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{3\left(a^2+b^2\right)}\\
&= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3\left(a^2+b^2\right)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\int \frac{3b^4}{a+b\sinh(x)}\,dx}{3\left(a^2+b^2\right)^2}\\
&= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3\left(a^2+b^2\right)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{b^4\int \frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}\\
&= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3\left(a^2+b^2\right)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\left(2b^4\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x\right.}{\left(a^2+b^2\right)^2}\\
&= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3\left(a^2+b^2\right)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} - \frac{\left(4b^4\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,\right.}{\left(a^2+b^2\right)^2}\\
&= -\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} + \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3\left(a^2+b^2\right)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}
\end{aligned}
$$

Mathematica [A] time = 0.332811, size = 102, normalized size = 1.02

$$
\frac{a\left(2a^2+5b^2\right)\tanh(x)+\frac{6b^4\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\left(a^2+b^2\right)\operatorname{sech}^3(x)(a\sinh(x)+b)+3b^3\operatorname{sech}(x)}{3\left(a^2+b^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((6*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 3*b^
3*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) + a*(2*a^2 + 5*b^2)*Tanh[
```

```
x])/(3*(a^2 + b^2)^2)
```

Maple [B] time = 0.034, size = 182, normalized size = 1.8

$$-2\frac{\left(-a^3-2\,ab^2\right)(\tanh(x/2))^5+\left(-a^2b-2\,b^3\right)(\tanh(x/2))^4+\left(-2/3\,a^3-8/3\,ab^2\right)(\tanh(x/2))^3-2\,b^3(\tanh(x/2))^2+}{\left(a^4+2\,a^2b^2+b^4\right)\left((\tanh(x/2))^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*sinh(x)),x)
```

```
[Out] -2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tanh(1/2*x)^5+(-a^2*b-2*b^3)*tanh(1/
2*x)^4+(-2/3*a^3-8/3*a*b^2)*tanh(1/2*x)^3-2*b^3*tanh(1/2*x)^2+(-a^3-2*a*b^2
)*tanh(1/2*x)-1/3*a^2*b-4/3*b^3)/(tanh(1/2*x)^2+1)^3+2*b^4/(a^4+2*a^2*b^2+b
^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22651, size = 2807, normalized size = 28.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^2*b^3 + b^5)*cosh(x)^5 + 6*(a^2*b^3 + b^5)*sinh(x)^5 - 4*a^5 - 14
*a^3*b^2 - 10*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b^4 -
5*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5)*cosh
(x)^3 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 + b^5)*cosh(x)^2 - 6*(
a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(x
)^2 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 + b^5)*cosh(x)^3 + 3*(a^3*
b^2 + a*b^4)*cosh(x)^2 - (2*a^4*b + 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2 +
 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 + 3*b^4*cosh(x)
^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*sinh(x)^4 + b^4 + 4*(5*b^4
*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^
2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh
(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) + 6*(a^2*b^3 + b^5)*cosh(x) + 6*(a^2*b^3 + b^5 + 5*(a^2*b^
3 + b^5)*cosh(x)^4 - 4*(a^3*b^2 + a*b^4)*cosh(x)^3 + 2*(2*a^4*b + 7*a^2*b^3
 + 5*b^5)*cosh(x)^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^6
```

```
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^6 +
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 3*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 6*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{a + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*sinh(x)), x)
```

Giac [A] time = 1.36113, size = 243, normalized size = 2.43

$$\frac{b^4 \log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)\sqrt{a^2+b^2}} + \frac{2\left(3\,b^3e^{(5\,x)} - 3\,ab^2e^{(4\,x)} + 4\,a^2be^{(3\,x)} + 10\,b^3e^{(3\,x)} - 6\,a^3e^{(2\,x)} - 12\,ab^2e^{(2\,x)} + 3\,b^3e^x - 2\,a^3 - 5\right.}{3\left(a^4 + 2\,a^2b^2 + b^4\right)\left(e^{(2\,x)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3*(3*b^3*e^(5*x)
- 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) + 10*b^3*e^(3*x) - 6*a^3*e^(2*x) - 12*a
*b^2*e^(2*x) + 3*b^3*e^x - 2*a^3 - 5*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*
x) + 1)^3)
```

3.198 $\int \frac{\text{sech}^5(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=135

$$\frac{b^5\log(a+b\sinh(x))}{(a^2+b^2)^3}+\frac{a(10a^2b^2+3a^4+15b^4)\tan^{-1}(\sinh(x))}{8(a^2+b^2)^3}-\frac{b^5\log(\cosh(x))}{(a^2+b^2)^3}+\frac{\text{sech}^4(x)(a\sinh(x)+b)}{4(a^2+b^2)}+\text{sech}$$

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/(8*(a^2 + b^2)^3) - (b^5*
Log[Cosh[x]])/(a^2 + b^2)^3 + (b^5*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (Sec
h[x]^4*(b + a*Sinh[x]))/(4*(a^2 + b^2)) + (Sech[x]^2*(4*b^3 + a*(3*a^2 + 7*
b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```

Rubi [A] time = 0.19725, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2668, 741, 823, 801, 635, 203, 260}

$$\frac{b^5\log(a+b\sinh(x))}{(a^2+b^2)^3}+\frac{a(10a^2b^2+3a^4+15b^4)\tan^{-1}(\sinh(x))}{8(a^2+b^2)^3}-\frac{b^5\log(\cosh(x))}{(a^2+b^2)^3}+\frac{\text{sech}^4(x)(a\sinh(x)+b)}{4(a^2+b^2)}+\text{sech}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/(8*(a^2 + b^2)^3) - (b^5*
Log[Cosh[x]])/(a^2 + b^2)^3 + (b^5*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (Sec
h[x]^4*(b + a*Sinh[x]))/(4*(a^2 + b^2)) + (Sech[x]^2*(4*b^3 + a*(3*a^2 + 7*
b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = -\left(b^5 \operatorname{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)^3}\,dx, x, b\sinh(x)\right)\right)$$

$$= \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{b^3 \operatorname{Subst}\left(\int \frac{3a^2+4b^2+3ax}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)}{4\left(a^2+b^2\right)}$$

$$= \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a\left(3a^2+7b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} + \frac{b \operatorname{Subst}\left(\int \frac{-3a^4-7a^2b^2-8b^4-a(3a}{(a+x)\left(-b^2-x^2\right)}\right.}{8\left(a^2+b\right.}$$

$$= \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a\left(3a^2+7b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} + \frac{b \operatorname{Subst}\left(\int \left(\frac{8b^4}{\left(a^2+b^2\right)(a+x)} + \frac{3a^5}{}\right.\right.}{8(}$$

$$= \frac{b^5 \log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a\left(3a^2+7b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} + \frac{b\operatorname{Sul}}{}$$

$$= \frac{b^5 \log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a\left(3a^2+7b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} - \frac{b^5\operatorname{Su}}{}$$

$$= \frac{a\left(3a^4+10a^2b^2+15b^4\right)\tan^{-1}(\sinh(x))}{8\left(a^2+b^2\right)^3} - \frac{b^5 \log(\cosh(x))}{\left(a^2+b^2\right)^3} + \frac{b^5 \log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{\operatorname{sech}^4(x)(b+}{4\left(a^2+\right.}$$

Mathematica [A] time = 0.225598, size = 135, normalized size = 1.

$$\frac{4b^3\left(a^2+b^2\right)\operatorname{sech}^2(x) + 2b\left(a^2+b^2\right)^2\operatorname{sech}^4(x) + \left(20a^3b^2+6a^5+30ab^4\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + 2a\left(a^2+b^2\right)^2\tanh(x)\operatorname{sech}}{8\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] ((6*a^5 + 20*a^3*b^2 + 30*a*b^4)*ArcTan[Tanh[x/2]] + 8*b^5*(-Log[Cosh[x]] +
 Log[a + b*Sinh[x]]) + 4*b^3*(a^2 + b^2)*Sech[x]^2 + 2*b*(a^2 + b^2)^2*Sech
[x]^4 + a*(3*a^4 + 10*a^2*b^2 + 7*b^4)*Sech[x]*Tanh[x] + 2*a*(a^2 + b^2)^2*
Sech[x]^3*Tanh[x])/(8*(a^2 + b^2)^3)
```

Maple [B] time = 0.043, size = 1140, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(a+b*sinh(x)),x)
```

```
[Out] -4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*b^5+5/4/
(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^5+5/2/(a^6+
3*a^4*b^2+3*a^2*b^4+b^6)*arctan(tanh(1/2*x))*a^3*b^2+15/4/(a^6+3*a^4*b^2+3*
a^2*b^4+b^6)*arctan(tanh(1/2*x))*a*b^4-5/4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(t
anh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^5-4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1
/2*x)^2+1)^4*tanh(1/2*x)^6*b^5+3/4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*
x)^2+1)^4*tanh(1/2*x)^5*a^5-4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+
1)^4*tanh(1/2*x)^4*b^5-3/4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^
4*tanh(1/2*x)^3*a^5+b^5/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*ln(a*tanh(1/2*x)^2-2*
tanh(1/2*x)*b-a)-1/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*b^5*ln(tanh(1/2*x)^2+1)+3/
4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*arctan(tanh(1/2*x))*a^5-2/(a^6+3*a^4*b^2+3*
a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^4*b-6/(a^6+3*a^4*b^2+3*a^2
*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^2*b^3+7/2/(a^6+3*a^4*b^2+3*a^
2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^3*b^2+9/4/(a^6+3*a^4*b^2+3*a^2
*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a*b^4-7/2/(a^6+3*a^4*b^2+3*a^2*b^
4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^3*b^2-9/4/(a^6+3*a^4*b^2+3*a^2*b
^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a*b^4+1/2/(a^6+3*a^4*b^2+3*a^2*b^
4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^3*b^2-1/4/(a^6+3*a^4*b^2+3*a^2*b
^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a*b^4-4/(a^6+3*a^4*b^2+3*a^2*b^4+
b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a^2*b^3-1/2/(a^6+3*a^4*b^2+3*a^2*b^4
+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^3*b^2+1/4/(a^6+3*a^4*b^2+3*a^2*b^
4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a*b^4-2/(a^6+3*a^4*b^2+3*a^2*b^4+b
^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^4*b-6/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)
/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^2*b^3
```

Maxima [B] time = 1.86502, size = 466, normalized size = 3.45

$$\frac{b^5 \log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{b^5 \log\left(e^{(-2\,x)} + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{\left(3\,a^5 + 10\,a^3b^2 + 15\,ab^4\right)\arctan\left(e^{(-x)}\right)}{4\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} + \frac{8\,b^3e^{(-2\,x)} + 8}{4\left(a^4 + 2\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^5*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 b^5*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^5 + 1
0*a^3*b^2 + 15*a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 + 7*a*b^2)*e^(-x) + (11*a^3 +
```

```
 15*a*b^2)*e^(-3*x) + 16*(a^2*b + 2*b^3)*e^(-4*x) - (11*a^3 + 15*a*b^2)*e^(
-5*x) - (3*a^3 + 7*a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2
*b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*
b^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

Fricas [B] time = 2.85386, size = 6700, normalized size = 49.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^7 + (3*a^5 + 10*a^3*b^2 + 7*a*b
^4)*sinh(x)^7 + 8*(a^2*b^3 + b^5)*cosh(x)^6 + (8*a^2*b^3 + 8*b^5 + 7*(3*a^5
 + 10*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 26*a^3*b^2 + 15*a*b
^4)*cosh(x)^5 + (11*a^5 + 26*a^3*b^2 + 15*a*b^4 + 21*(3*a^5 + 10*a^3*b^2 +
7*a*b^4)*cosh(x)^2 + 48*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^5 + 16*(a^4*b + 3*
a^2*b^3 + 2*b^5)*cosh(x)^4 + (16*a^4*b + 48*a^2*b^3 + 32*b^5 + 35*(3*a^5 +
10*a^3*b^2 + 7*a*b^4)*cosh(x)^3 + 120*(a^2*b^3 + b^5)*cosh(x)^2 + 5*(11*a^5
 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 26*a^3*b^2 + 15*a*
b^4)*cosh(x)^3 - (11*a^5 + 26*a^3*b^2 + 15*a*b^4 - 35*(3*a^5 + 10*a^3*b^2 +
 7*a*b^4)*cosh(x)^4 - 160*(a^2*b^3 + b^5)*cosh(x)^3 - 10*(11*a^5 + 26*a^3*b
^2 + 15*a*b^4)*cosh(x)^2 - 64*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^
3 + 8*(a^2*b^3 + b^5)*cosh(x)^2 + (21*(3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x
)^5 + 8*a^2*b^3 + 8*b^5 + 120*(a^2*b^3 + b^5)*cosh(x)^4 + 10*(11*a^5 + 26*a
^3*b^2 + 15*a*b^4)*cosh(x)^3 + 96*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3
*(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x)^8 + 8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^
7 + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(3*a^5 + 10*a^3*b^2 + 15*
a*b^4)*cosh(x)^6 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 7*(3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*co
sh(x)^3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 + 10
*a^3*b^2 + 15*a*b^4 + 6*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(9*a^
5 + 30*a^3*b^2 + 45*a*b^4 + 35*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 +
30*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 10*
a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 10*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^
3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 10*a^
3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^
6 + 3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 15*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh
(x)^4 + 9*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5
+ 10*a^3*b^2 + 15*a*b^4)*cosh(x)^7 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh
(x)^5 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (3*a^5 + 10*a^3*b^2 +
 15*a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (3*a^5 + 10*a^3*b^
2 + 7*a*b^4)*cosh(x) + 4*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sin
h(x)^8 + 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b^5*cos
h(x)^2 + b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^5 + b
^5 + 2*(35*b^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*c
osh(x)^5 + 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cosh(x)^6
 + 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(x)^7 +
 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))) - 4*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b
^5*sinh(x)^8 + 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b
^5*cosh(x)^2 + b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)
^5 + b^5 + 2*(35*b^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7
*b^5*cosh(x)^5 + 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cos
h(x)^6 + 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(
```

```
x)^7 + 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*cosh
(x)/(cosh(x) - sinh(x))) + (7*(3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^6 + 48
*(a^2*b^3 + b^5)*cosh(x)^5 - 3*a^5 - 10*a^3*b^2 - 7*a*b^4 + 5*(11*a^5 + 26*
a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 64*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x)^3 -
3*(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 16*(a^2*b^3 + b^5)*cosh(x))*
sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*sinh(x)^8 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 4*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6 + 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2
)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 8*(7*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*
sinh(x)^5 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 2*(3*a^6 + 9*
a^4*b^2 + 9*a^2*b^4 + 3*b^6 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x
)^4 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x
)^3 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 4*(7*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 15*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh
(x)^7 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x
))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**5/(a + b*sinh(x)), x)
```

Giac [B] time = 1.23793, size = 498, normalized size = 3.69

$$\frac{b^6\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2a\right|\right)}{a^6b+3a^4b^3+3a^2b^5+b^7}-\frac{b^5\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)\left(3a^5+10a^3b^2+15\right.}{16\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)
 - 1/2*b^5*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 + 10*a^3*b^2 + 15*a*b
^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(3*b^5*(e^(-x) - e^x)^4 - 3*a
^5*(e^(-x) - e^x)^3 - 10*a^3*b^2*(e^(-x) - e^x)^3 - 7*a*b^4*(e^(-x) - e^x)^
3 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 32*b^5*(e^(-x) - e^x)^2 - 20*a^5*(e^(-x) -
 e^x) - 56*a^3*b^2*(e^(-x) - e^x) - 36*a*b^4*(e^(-x) - e^x) + 16*a^4*b + 64
*a^2*b^3 + 96*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) - e^x)^2 +
 4)^2)
```

3.199 $\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=146

$$-\frac{2b^6\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}+\frac{\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{15\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}(x)\left(a\left(26a^2b^2+8\right.\right.}{15\left(\right.}$$

```
[Out] (-2*b^6*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (Se
ch[x]^5*(b + a*Sinh[x]))/(5*(a^2 + b^2)) + (Sech[x]^3*(5*b^3 + a*(4*a^2 + 9
*b^2)*Sinh[x]))/(15*(a^2 + b^2)^2) + (Sech[x]*(15*b^5 + a*(8*a^4 + 26*a^2*b
^2 + 33*b^4)*Sinh[x]))/(15*(a^2 + b^2)^3)
```

Rubi [A] time = 0.41731, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2696, 2866, 12, 2660, 618, 206}

$$-\frac{2b^6\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}+\frac{\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{15\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}(x)\left(a\left(26a^2b^2+8\right.\right.}{15\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^6*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (Se
ch[x]^5*(b + a*Sinh[x]))/(5*(a^2 + b^2)) + (Sech[x]^3*(5*b^3 + a*(4*a^2 + 9
*b^2)*Sinh[x]))/(15*(a^2 + b^2)^2) + (Sech[x]*(15*b^5 + a*(8*a^4 + 26*a^2*b
^2 + 33*b^4)*Sinh[x]))/(15*(a^2 + b^2)^3)
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} - \frac{\int \frac{\operatorname{sech}^4(x)\left(-4a^2-5b^2-4ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{5(a^2+b^2)}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\int \frac{\operatorname{sech}^2(x)\left(8a^4+18a^2b^2+15b^4+2a\right.}{a+b\sinh(x}}{15(a^2+b}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a\left(8a^4+2\right.\right.}{15(a^2+}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a\left(8a^4+2\right.\right.}{15(a^2+}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a\left(8a^4+2\right.\right.}{15(a^2+}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a\left(8a^4+2\right.\right.}{15(a^2+}$$

$$= -\frac{2b^6\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15(a^2+b^2)^2}$$

Mathematica [A] time = 0.444767, size = 146, normalized size = 1.

$$\frac{a\left(26a^2b^2+8a^4+33b^4\right)\tanh(x) + \frac{30b^6\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 3\left(a^2+b^2\right)^2\operatorname{sech}^5(x)(a\sinh(x)+b) + \left(a^2+b^2\right)\operatorname{sech}^3(x)\left(a\right.}{15\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] ((30*b^6*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 15*
b^5*Sech[x] + 3*(a^2 + b^2)^2*Sech[x]^5*(b + a*Sinh[x]) + (a^2 + b^2)*Sech[
x]^3*(5*b^3 + a*(4*a^2 + 9*b^2)*Sinh[x]) + a*(8*a^4 + 26*a^2*b^2 + 33*b^4)*
Tanh[x])/(15*(a^2 + b^2)^3)
```

Maple [B] time = 0.048, size = 350, normalized size = 2.4

$$-2\frac{1}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left((\tanh(x/2))^2+1\right)^5}\left(\left(-a^5-3a^3b^2-3ab^4\right)(\tanh(x/2))^9+\left(-a^4b-3a^2b^3-3b^5\right)(\tanh(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^6/(a+b*sinh(x)),x)
```

```
[Out] -2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^5-3*a^3*b^2-3*a*b^4)*tanh(1/2*x)^9+(-
a^4*b-3*a^2*b^3-3*b^5)*tanh(1/2*x)^8+(-4/3*a^5-16/3*a^3*b^2-8*a*b^4)*tanh(1
/2*x)^7+(-2*a^2*b^3-6*b^5)*tanh(1/2*x)^6+(-58/15*a^5-166/15*a^3*b^2-66/5*a*
b^4)*tanh(1/2*x)^5+(-2*a^4*b-16/3*a^2*b^3-28/3*b^5)*tanh(1/2*x)^4+(-4/3*a^5
-16/3*a^3*b^2-8*a*b^4)*tanh(1/2*x)^3+(-2/3*a^2*b^3-14/3*b^5)*tanh(1/2*x)^2+
(-a^5-3*a^3*b^2-3*a*b^4)*tanh(1/2*x)-1/5*a^4*b-11/15*a^2*b^3-23/15*b^5)/(ta
nh(1/2*x)^2+1)^5+2*b^6/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctan
h(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39346, size = 7626, normalized size = 52.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/15*(30*(a^2*b^5 + b^7)*cosh(x)^9 + 30*(a^2*b^5 + b^7)*sinh(x)^9 - 30*(a^3
*b^4 + a*b^6)*cosh(x)^8 - 30*(a^3*b^4 + a*b^6 - 9*(a^2*b^5 + b^7)*cosh(x))*
sinh(x)^8 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^7 + 40*(a^4*b^3 + 5*a^
2*b^5 + 4*b^7 + 27*(a^2*b^5 + b^7)*cosh(x)^2 - 6*(a^3*b^4 + a*b^6)*cosh(x))
*sinh(x)^7 - 16*a^7 - 68*a^5*b^2 - 118*a^3*b^4 - 66*a*b^6 - 60*(a^5*b^2 + 4
*a^3*b^4 + 3*a*b^6)*cosh(x)^6 - 20*(3*a^5*b^2 + 12*a^3*b^4 + 9*a*b^6 - 126*
(a^2*b^5 + b^7)*cosh(x)^3 + 42*(a^3*b^4 + a*b^6)*cosh(x)^2 - 14*(a^4*b^3 +
5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^6 + 4*(24*a^6*b + 92*a^4*b^3 + 157*a^2*
b^5 + 89*b^7)*cosh(x)^5 + 4*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7 +
 945*(a^2*b^5 + b^7)*cosh(x)^4 - 420*(a^3*b^4 + a*b^6)*cosh(x)^3 + 210*(a^4
```

*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 90*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*co
sh(x))*sinh(x)^5 - 20*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x)^
4 - 20*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6 - 189*(a^2*b^5 + b^7)*co
sh(x)^5 + 105*(a^3*b^4 + a*b^6)*cosh(x)^4 - 70*(a^4*b^3 + 5*a^2*b^5 + 4*b^7
)*cosh(x)^3 + 45*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^2 - (24*a^6*b + 92
*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x))*sinh(x)^4 + 40*(a^4*b^3 + 5*a^2*b
^5 + 4*b^7)*cosh(x)^3 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7 + 63*(a^2*b^5 + b^7
)*cosh(x)^6 - 42*(a^3*b^4 + a*b^6)*cosh(x)^5 + 35*(a^4*b^3 + 5*a^2*b^5 + 4*
b^7)*cosh(x)^4 - 30*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^3 + (24*a^6*b +
 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^2 - 2*(8*a^7 + 31*a^5*b^2 + 47*
a^3*b^4 + 24*a*b^6)*cosh(x))*sinh(x)^3 - 20*(4*a^7 + 17*a^5*b^2 + 28*a^3*b^
4 + 15*a*b^6)*cosh(x)^2 + 20*(54*(a^2*b^5 + b^7)*cosh(x)^7 - 4*a^7 - 17*a^5
*b^2 - 28*a^3*b^4 - 15*a*b^6 - 42*(a^3*b^4 + a*b^6)*cosh(x)^6 + 42*(a^4*b^3
 + 5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 45*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x
)^4 + 2*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^3 - 6*(8*a^7
 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x)^2 + 6*(a^4*b^3 + 5*a^2*b^5 +
 4*b^7)*cosh(x))*sinh(x)^2 + 15*(b^6*cosh(x)^10 + 10*b^6*cosh(x)*sinh(x)^9
+ b^6*sinh(x)^10 + 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^6 + 10*b^6*cosh(x)^4 +
5*(9*b^6*cosh(x)^2 + b^6)*sinh(x)^8 + 5*b^6*cosh(x)^2 + 40*(3*b^6*cosh(x)^3
 + b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 + 14*b^6*cosh(x)^2 + b^6)*
sinh(x)^6 + b^6 + 4*(63*b^6*cosh(x)^5 + 70*b^6*cosh(x)^3 + 15*b^6*cosh(x))*
sinh(x)^5 + 10*(21*b^6*cosh(x)^6 + 35*b^6*cosh(x)^4 + 15*b^6*cosh(x)^2 + b^
6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 + 7*b^6*cosh(x)^5 + 5*b^6*cosh(x)^3 + b^
6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8 + 28*b^6*cosh(x)^6 + 30*b^6*cosh(
x)^4 + 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2 + 10*(b^6*cosh(x)^9 + 4*b^6*cosh(x
)^7 + 6*b^6*cosh(x)^5 + 4*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x))*sqrt(a^2 +
b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(
b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))
/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b))
 + 30*(a^2*b^5 + b^7)*cosh(x) + 10*(27*(a^2*b^5 + b^7)*cosh(x)^8 - 24*(a^3*
b^4 + a*b^6)*cosh(x)^7 + 3*a^2*b^5 + 3*b^7 + 28*(a^4*b^3 + 5*a^2*b^5 + 4*b^
7)*cosh(x)^6 - 36*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^5 + 2*(24*a^6*b +
 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^4 - 8*(8*a^7 + 31*a^5*b^2 + 47*
a^3*b^4 + 24*a*b^6)*cosh(x)^3 + 12*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^2
- 4*(4*a^7 + 17*a^5*b^2 + 28*a^3*b^4 + 15*a*b^6)*cosh(x))*sinh(x))/((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^10 + 10*(a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^9 + (a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*sinh(x)^10 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8)*cosh(x)^8 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 9
*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^8 + a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 40*(3*(a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*cosh(x))*sinh(x)^7 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*cosh(x)^6 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 21*(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 14*(a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^5 + 70*(a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^3 + 15*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)*cosh(x))*sinh(x)^5 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*cosh(x)^4 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 21
*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^6 + 35*(a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 15*(a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^7 + 7*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x))*sinh(
x)^3 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2 + 5*(9*(
a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^8 + a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 28*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6

```
 + b^8)*cosh(x)^6 + 30*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh
(x)^4 + 12*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(
x)^2 + 10*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^9 + 4*(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^7 + 6*(a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^5 + 4*(a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**6/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28309, size = 436, normalized size = 2.99

$$\frac{b^6 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(15\,b^5e^{(9\,x)}-15\,ab^4e^{(8\,x)}+20\,a^2b^3e^{(7\,x)}+80\,b^5e^{(7\,x)}-30\,a^3b^2e^{(6\,x)}-90\,ab^4e^{(6\,x)}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/15*(1
5*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2*b^3*e^(7*x) + 80*b^5*e^(7*x) - 30
*a^3*b^2*e^(6*x) - 90*a*b^4*e^(6*x) + 48*a^4*b*e^(5*x) + 136*a^2*b^3*e^(5*x
) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) - 230*a^3*b^2*e^(4*x) - 240*a*b^4*e^(4
*x) + 20*a^2*b^3*e^(3*x) + 80*b^5*e^(3*x) - 40*a^5*e^(2*x) - 130*a^3*b^2*e^
(2*x) - 150*a*b^4*e^(2*x) + 15*b^5*e^x - 8*a^5 - 26*a^3*b^2 - 33*a*b^4)/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^5)
```

3.200 $\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=94

$$\frac{3x\left(2a^2+b^2\right)}{2b^4}+\frac{6a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

```
[Out] (3*(2*a^2 + b^2)*x)/(2*b^4) + (6*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/b^4 - (3*Cosh[x]*(2*a - b*Sinh[x]))/(2*b^3) - Cosh[x]^3
/(b*(a + b*Sinh[x]))
```

Rubi [A] time = 0.220092, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2693, 2865, 2735, 2660, 618, 206}

$$\frac{3x\left(2a^2+b^2\right)}{2b^4}+\frac{6a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (3*(2*a^2 + b^2)*x)/(2*b^4) + (6*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/b^4 - (3*Cosh[x]*(2*a - b*Sinh[x]))/(2*b^3) - Cosh[x]^3
/(b*(a + b*Sinh[x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\cosh^3(x)}{b(a+b\sinh(x))} + \frac{3\int \frac{\cosh^2(x)\sinh(x)}{a+b\sinh(x)}\,dx}{b}$$

$$= -\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} + \frac{(3i)\int \frac{iab-i(2a^2+b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^3}$$

$$= \frac{3(2a^2+b^2)x}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} - \frac{(3a(a^2+b^2))\int \frac{1}{a+b\sinh(x)}\,dx}{b^4}$$

$$= \frac{3(2a^2+b^2)x}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} - \frac{(6a(a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{a+2bx-a\ldots}\right.}{b^4}$$

$$= \frac{3(2a^2+b^2)x}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} + \frac{(12a(a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{4(a^2+b\ldots}\right.}{b^4}$$

$$= \frac{3(2a^2+b^2)x}{2b^4} + \frac{6a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x\ldots}$$

Mathematica [C] time = 6.278, size = 2901, normalized size = 30.86

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((-I)*Cosh[x]^3*((I*b*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(5/2)*((
I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((((-I)*a*b)/(a - I*b) - b^2
/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a + b*Sinh[x])) - ((16*
Sqrt[2]*(a - I*b)*b^2*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(5/2)*Sq
rt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)
/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)*((5*(1/(2*(1 - ((I/2)*(a - I*
b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2) + (1 - ((I/2)*(a - I
*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(-1)))/8 + (((15*I)/32
)*b^3*(((-I)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b + ((
```

```
a - I*b)^2*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^2)/(3*b^2) + ((-1)^
(1/4)*Sqrt[2]*Sqrt[a - I*b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/
(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[((-I)*b)/(a - I
*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[b]*Sqrt[1 - ((I/2)*(a - I*b)*(((-I)*b)/
(a - I*b) - (b*Sinh[x])/(a - I*b)))/b])))/((a - I*b)^3*(((-I)*b)/(a - I*b)
- (b*Sinh[x])/(a - I*b))^3*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*S
inh[x])/(a - I*b)))/b)^2)))/(5*(a + I*b)*(a^2 + b^2)*Sqrt[((-I)*(a + I*b)*(
(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b]) - (3*a*b^2*((-4*Sqrt[2]*(((-I
)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(3/2)*Sqrt[(I*b)/(a + I*b) - (b*Sin
h[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a
 - I*b)))/b)^(5/2)*((3/(4*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Si
nh[x])/(a - I*b)))/b)^2) + (1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*S
inh[x])/(a - I*b)))/b)^(-1))/2 - (3*b^2*(((-I)*(a - I*b)*(((-I)*b)/(a - I*b
) - (b*Sinh[x])/(a - I*b)))/b + ((-1)^(1/4)*Sqrt[2]*Sqrt[a - I*b]*ArcSin[((
-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(
Sqrt[2]*Sqrt[b])]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[b
]*Sqrt[1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b
])))/(8*(a - I*b)^2*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^2*(1 - ((I
/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2)))/(3*(a +
 I*b)*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b]) -
 (I*((I*a*b)/(a - I*b) + b^2/(a - I*b))*(((-I)*((I*a*b)/(a - I*b) + b^2/(a
- I*b))*(((-I)*((I*a*b)/(a + I*b) - b^2/(a + I*b))*(((-2*I)*((I*a*b)/(a + I
*b) - b^2/(a + I*b))*ArcTan[(Sqrt[((-I)*a*b)/(a + I*b) + b^2/(a + I*b)]*Sqr
t[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[(I*a*b)/(a - I*b) + b^
2/(a - I*b)]*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)])])/(b*Sqrt[(I*a*
b)/(a - I*b) + b^2/(a - I*b)]*Sqrt[((-I)*a*b)/(a + I*b) + b^2/(a + I*b)]) +
 ((2*I)*Sqrt[a - I*b]*ArcTanh[(Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*S
inh[x])/(a - I*b)])/(Sqrt[a + I*b]*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a +
I*b)])])/(Sqrt[a + I*b]*b)))/b + ((2*I)*Sqrt[2]*(a - I*b)*Sqrt[((-I)*b)/(a
- I*b) - (b*Sinh[x])/(a - I*b)]*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b
)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(
3/2)*(-(((-1)^(3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/
(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - I
*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)]*(1 - ((I/2)*(a - I*b)*
(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(3/2))) + 1/(2*(1 - ((I/2)
*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b))))/((a + I*b)*b
*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b])))/b -
(4*Sqrt[2]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)]*Sqrt[(I*b)/(a +
 I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) -
(b*Sinh[x])/(a - I*b)))/b)^(5/2)*((-3*(-1)^(3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)
*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*S
qrt[b])])/(4*Sqrt[2]*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a
 - I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))
)/b)^(5/2)) + (3/(2*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])
/(a - I*b)))/b)^2) + (1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x]
)/(a - I*b)))/b)^(-1))/4))/((a + I*b)*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b)
 - (b*Sinh[x])/(a + I*b)))/b])))/b))/(a^2 + b^2))/((((-I)*a*b)/(a - I*b) -
b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b)))))/((1 - (a + b*Sinh[
x])/(a - I*b))^(3/2)*(1 - (a + b*Sinh[x])/(a + I*b))^(3/2))
```

Maple [B] time = 0.058, size = 290, normalized size = 3.1

$$-\frac{1}{2b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2b^2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}-2\,\frac{a}{b^3\left(\tanh\left(x/2\right)+1\right)}+3\,\frac{\ln\left(\tanh\left(x/2\right)+1\right)a^2}{b^4}+\frac{3}{2b^2}\ln\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] -1/2/b^2/(tanh(1/2*x)+1)^2+1/2/b^2/(tanh(1/2*x)+1)-2/b^3/(tanh(1/2*x)+1)*a+
3/b^4*ln(tanh(1/2*x)+1)*a^2+3/2/b^2*ln(tanh(1/2*x)+1)+1/2/b^2/(tanh(1/2*x)-
1)^2+1/2/b^2/(tanh(1/2*x)-1)+2/b^3/(tanh(1/2*x)-1)*a-3/b^4*ln(tanh(1/2*x)-1
)*a^2-3/2/b^2*ln(tanh(1/2*x)-1)+2/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a
*tanh(1/2*x)+2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/a*tanh(1/2*x)+2/b^3/(a*t
anh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^2+2/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)
-6/b^4*a*(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1482, size = 2184, normalized size = 23.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 6*a*b^2*cosh(x)^5 + 6*(b^3*cosh(x) - a
*b^2)*sinh(x)^5 - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x)^4 + (15*b
^3*cosh(x)^2 - 30*a*b^2*cosh(x) - 16*a^2*b - b^3 + 12*(2*a^2*b + b^3)*x)*si
nh(x)^4 + 6*a*b^2*cosh(x) + 8*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*cosh(
x)^3 + 4*(5*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 + 4*a^3 + 4*a*b^2 + 6*(2*a^3
 + a*b^2)*x - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x)^3 +
b^3 - (32*a^2*b + 17*b^3 + 12*(2*a^2*b + b^3)*x)*cosh(x)^2 + (15*b^3*cosh(x
)^4 - 60*a*b^2*cosh(x)^3 - 32*a^2*b - 17*b^3 - 6*(16*a^2*b + b^3 - 12*(2*a^
2*b + b^3)*x)*cosh(x)^2 - 12*(2*a^2*b + b^3)*x + 24*(2*a^3 + 2*a*b^2 + 3*(2
*a^3 + a*b^2)*x)*cosh(x))*sinh(x)^2 + 24*(a*b*cosh(x)^4 + a*b*sinh(x)^4 + 2
*a^2*cosh(x)^3 - a*b*cosh(x)^2 + 2*(2*a*b*cosh(x) + a^2)*sinh(x)^3 + (6*a*b
*cosh(x)^2 + 6*a^2*cosh(x) - a*b)*sinh(x)^2 + 2*(2*a*b*cosh(x)^3 + 3*a^2*co
sh(x)^2 - a*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*si
nh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*s
qrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*
a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(3*b^3*cosh(x)^5 - 15*a*b^2
*cosh(x)^4 - 2*(16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x)^3 + 3*a*b^2
+ 12*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*cosh(x)^2 - (32*a^2*b + 17*b^3
 + 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + b^5*sinh(x)^4 +
 2*a*b^4*cosh(x)^3 - b^5*cosh(x)^2 + 2*(2*b^5*cosh(x) + a*b^4)*sinh(x)^3 +
(6*b^5*cosh(x)^2 + 6*a*b^4*cosh(x) - b^5)*sinh(x)^2 + 2*(2*b^5*cosh(x)^3 +
3*a*b^4*cosh(x)^2 - b^5*cosh(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25865, size = 240, normalized size = 2.55

$$\frac{3\left(2\,a^2+b^2\right)x}{2\,b^4}-\frac{3\left(a^3+ab^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4}+\frac{b^2e^{(2\,x)}-8\,abe^x}{8\,b^4}+\frac{\left(6\,ab^2e^x+b^3+8\left(2\,a^3+ab^2\right)e^{(3\,x)}-\left(32\,a^2b\right.\right.}{8\left(be^{(2\,x)}+2\,ae^x-b\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3/2*(2*a^2 + b^2)*x/b^4 - 3*(a^3 + a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^
2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 1
/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(6*a*b^2*e^x + b^3 + 8*(2*a^3 + a*b^
2)*e^(3*x) - (32*a^2*b + 17*b^3)*e^(2*x))*e^(-2*x)/((b*e^(2*x) + 2*a*e^x -
b)*b^4)
```

3.201 $\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=40

$$-\frac{a^2+b^2}{b^3(a+b\sinh(x))}-\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}$$

```
[Out] (-2*a*Log[a + b*Sinh[x]])/b^3 + Sinh[x]/b^2 - (a^2 + b^2)/(b^3*(a + b*Sinh[
x]))
```

Rubi [A] time = 0.059733, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$-\frac{a^2+b^2}{b^3(a+b\sinh(x))}-\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Sinh[x]])/b^3 + Sinh[x]/b^2 - (a^2 + b^2)/(b^3*(a + b*Sinh[
x]))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{(a+x)^2}\,dx, x, b\sinh(x)\right)}{b^3}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{-a^2-b^2}{(a+x)^2}+\frac{2a}{a+x}\right)dx, x, b\sinh(x)\right)}{b^3}\\ &= -\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}-\frac{a^2+b^2}{b^3(a+b\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.0702072, size = 37, normalized size = 0.92

$$-\frac{\frac{a^2+b^2}{a+b\sinh(x)}+2a\log(a+b\sinh(x))-b\sinh(x)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] -((2*a*Log[a + b*Sinh[x]] - b*Sinh[x] + (a^2 + b^2)/(a + b*Sinh[x]))/b^3)
```

Maple [B] time = 0.052, size = 141, normalized size = 3.5

$$2\,\frac{a\ln\left(\tanh\left(x/2\right)+1\right)}{b^{3}}-\frac{1}{b^{2}}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+2\,\frac{a\ln\left(\tanh\left(x/2\right)-1\right)}{b^{3}}-\frac{1}{b^{2}}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}-2\,\frac{a\tan\ldots}{b^{2}\left(a\left(\tanh\left(x/2\right)\right)^{2}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] 2*a/b^3*ln(tanh(1/2*x)+1)-1/b^2/(tanh(1/2*x)+1)+2*a/b^3*ln(tanh(1/2*x)-1)-1
/b^2/(tanh(1/2*x)-1)-2/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a*tanh(1/2*x
)-2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/a*tanh(1/2*x)-2/b^3*a*ln(a*tanh(1/2
*x)^2-2*tanh(1/2*x)*b-a)
```

Maxima [B] time = 1.1072, size = 138, normalized size = 3.45

$$\frac{2\,abe^{(-x)}+b^{2}-\left(4\,a^{2}+5\,b^{2}\right)e^{(-2\,x)}}{2\left(b^{4}e^{(-x)}+2\,ab^{3}e^{(-2\,x)}-b^{4}e^{(-3\,x)}\right)}-\frac{2\,ax}{b^{3}}-\frac{e^{(-x)}}{2\,b^{2}}-\frac{2\,a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*e^(-x) + b^2 - (4*a^2 + 5*b^2)*e^(-2*x))/(b^4*e^(-x) + 2*a*b^3*e
^(-2*x) - b^4*e^(-3*x)) - 2*a*x/b^3 - 1/2*e^(-x)/b^2 - 2*a*log(-2*a*e^(-x)
+ b*e^(-2*x) - b)/b^3
```

Fricas [B] time = 2.16806, size = 999, normalized size = 24.98

$$\frac{b^{2}\cosh\left(x\right)^{4}+b^{2}\sinh\left(x\right)^{4}+2\left(2\,abx+ab\right)\cosh\left(x\right)^{3}+2\left(2\,abx+2\,b^{2}\cosh\left(x\right)+ab\right)\sinh\left(x\right)^{3}+2\left(4\,a^{2}x-2\,a^{2}-3\,b^{2}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 2*(2*a*b*x + a*b)*cosh(x)^3 + 2*(2*a*b
*x + 2*b^2*cosh(x) + a*b)*sinh(x)^3 + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x)^2
 + 2*(3*b^2*cosh(x)^2 + 4*a^2*x - 2*a^2 - 3*b^2 + 3*(2*a*b*x + a*b)*cosh(x)
)*sinh(x)^2 + b^2 - 2*(2*a*b*x + a*b)*cosh(x) - 4*(a*b*cosh(x)^3 + a*b*sinh
(x)^3 + 2*a^2*cosh(x)^2 - a*b*cosh(x) + (3*a*b*cosh(x) + 2*a^2)*sinh(x)^2 +
 (3*a*b*cosh(x)^2 + 4*a^2*cosh(x) - a*b)*sinh(x))*log(2*(b*sinh(x) + a)/(co
sh(x) - sinh(x))) + 2*(2*b^2*cosh(x)^3 - 2*a*b*x + 3*(2*a*b*x + a*b)*cosh(x
```

```
)^2 - a*b + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x))*sinh(x))/(b^4*cosh(x)^3 +
b^4*sinh(x)^3 + 2*a*b^3*cosh(x)^2 - b^4*cosh(x) + (3*b^4*cosh(x) + 2*a*b^3)
*sinh(x)^2 + (3*b^4*cosh(x)^2 + 4*a*b^3*cosh(x) - b^4)*sinh(x))
```

Sympy [A] time = 2.02208, size = 209, normalized size = 5.22

$$\begin{cases} \tilde{\infty}\left(2\sinh(x) - \frac{\cosh^2(x)}{\sinh(x)}\right) & \text{for } a \\ \frac{2\sinh(x) - \frac{\cosh^2(x)}{\sinh(x)}}{b^2} & \text{for } a \\ \frac{-\frac{2\sinh^3(x)}{3} + \sinh(x)\cosh^2(x)}{a^2} & \text{for } b \\ -\frac{2a^3\log\left(\frac{a}{b} + \sinh(x)\right)}{a^2b^3 + ab^4\sinh(x)} - \frac{2a^3}{a^2b^3 + ab^4\sinh(x)} - \frac{2a^2b\log\left(\frac{a}{b} + \sinh(x)\right)\sinh(x)}{a^2b^3 + ab^4\sinh(x)} + \frac{ab^2\sinh^2(x)}{a^2b^3 + ab^4\sinh(x)} - \frac{b^3\sinh^3(x)}{a^2b^3 + ab^4\sinh(x)} + \frac{b^3\sinh(x)\cosh^2(x)}{a^2b^3 + ab^4\sinh(x)} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*(2*sinh(x) - cosh(x)**2/sinh(x)), Eq(a, 0) & Eq(b, 0)), ((2*
sinh(x) - cosh(x)**2/sinh(x))/b**2, Eq(a, 0)), ((-2*sinh(x)**3/3 + sinh(x)*
cosh(x)**2)/a**2, Eq(b, 0)), (-2*a**3*log(a/b + sinh(x))/(a**2*b**3 + a*b**
4*sinh(x)) - 2*a**3/(a**2*b**3 + a*b**4*sinh(x)) - 2*a**2*b*log(a/b + sinh(
x))*sinh(x)/(a**2*b**3 + a*b**4*sinh(x)) + a*b**2*sinh(x)**2/(a**2*b**3 + a
*b**4*sinh(x)) - b**3*sinh(x)**3/(a**2*b**3 + a*b**4*sinh(x)) + b**3*sinh(x
)*cosh(x)**2/(a**2*b**3 + a*b**4*sinh(x)), True))
```

Giac [B] time = 1.21533, size = 111, normalized size = 2.78

$$-\frac{e^{(-x)} - e^x}{2\,b^2} - \frac{2\,a\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{b^3} + \frac{2\left(ab\left(e^{(-x)} - e^x\right) - a^2 + b^2\right)}{\left(b\left(e^{(-x)} - e^x\right) - 2\,a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/b^2 - 2*a*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^3 + 2*(a*
b*(e^(-x) - e^x) - a^2 + b^2)/((b*(e^(-x) - e^x) - 2*a)*b^3)
```

3.202 $\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=62

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2 + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]) - Cosh[x]/(b*(a + b*Sinh[x]))
```

Rubi [A] time = 0.111346, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2693, 2735, 2660, 618, 206}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] x/b^2 + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]) - Cosh[x]/(b*(a + b*Sinh[x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}
\end{aligned}$$

Mathematica [C] time = 3.40129, size = 659, normalized size = 10.63

$$\cosh(x)\left(\sqrt{a+ib}\left(\sqrt{b^2}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\left(2\sqrt[4]{-1}a\sqrt{b}(b+ia)\sin^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-ib}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}}{\sqrt{b}}\right)-\sqrt{a-ib}\left(a^2+b^2\right)\sqrt{1+i\sinh(x)}\cdots\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*((2*I)*a*Sqrt[b^2]*(I*a + b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I +
 Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))
])]*Sqrt[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*((2*I)*a^2*Sqrt[a -
 I*b]*b*ArcTan[(Sqrt[(-I)*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqrt[I*
b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] + 2*Sinh[x]*
(I*a*Sqrt[a - I*b]*b^2*ArcTan[(Sqrt[(-I)*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I
*b))])/(Sqrt[I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x
]] + (-1)^(1/4)*b^(3/2)*Sqrt[b^2]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*
b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]]*Sqrt[-((b*(-I + Sinh[x]))
/(a + I*b))]) + Sqrt[b^2]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*(2*(-1)^(1/
4)*a*Sqrt[b]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sin
h[x]))/(a - I*b))])/Sqrt[b]] - Sqrt[a - I*b]*(a^2 + b^2)*Sqrt[1 + I*Sinh[x]
]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]))))/((a - I*b)^(3/2)*(a + I*b)^(3/2)
*b*Sqrt[b^2]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt
[-((b*(I + Sinh[x]))/(a - I*b))]*(a + b*Sinh[x]))
```

Maple [B] time = 0.043, size = 119, normalized size = 1.9

$$\frac{1}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{b^2}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2\frac{\tanh(x/2)}{\left(a(\tanh(x/2))^2-2\tanh(x/2)b-a\right)a}+2\frac{1}{b\left(a(\tanh(x/2))^2-2\tanh\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] 1/b^2*ln(tanh(1/2*x)+1)-1/b^2*ln(tanh(1/2*x)-1)+2/(a*tanh(1/2*x)^2-2*tanh(1
/2*x)*b-a)/a*tanh(1/2*x)+2/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-2/b^2*a/(a
^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10069, size = 933, normalized size = 15.05

$$-\frac{(a^2b+b^3)x\cosh(x)^2+(a^2b+b^3)x\sinh(x)^2-2a^2b-2b^3+(ab\cosh(x)^2+ab\sinh(x)^2+2a^2\cosh(x)-ab+2(a}{a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*x*cosh(x)^2 + (a^2*b + b^3)*x*sinh(x)^2 - 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) - b)) - (a^2*b + b^3)*x + 2*(a^3 + a*b^2 + (a^3 + a*b^2)*
x)*cosh(x) + 2*(a^3 + a*b^2 + (a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*x)*si
nh(x))/(a^2*b^3 + b^5 - (a^2*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)
^2 - 2*(a^3*b^2 + a*b^4)*cosh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cos
h(x))*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14828, size = 131, normalized size = 2.11

$$-\frac{a\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}b^{2}}+\frac{x}{b^{2}}+\frac{2\,(ae^{x}-b)}{\left(be^{(2\,x)}+2\,ae^{x}-b\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + x/b^2 + 2*(a*e^x - b)/((b*e^(2*x) + 2*a*
e^x - b)*b^2)
```

3.203 $\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{b(a+b\sinh(x))}$$

[Out] -(1/(b*(a + b*Sinh[x])))

Rubi [A] time = 0.0253128, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 32}

$$-\frac{1}{b(a+b\sinh(x))}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(a + b*Sinh[x])^2,x]

[Out] -(1/(b*(a + b*Sinh[x])))

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, b\sinh(x)\right)}{b}\\ &= -\frac{1}{b(a+b\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.0131484, size = 13, normalized size = 1.

$$-\frac{1}{b(a+b\sinh(x))}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]/(a + b*Sinh[x])^2,x]

[Out] -(1/(b*(a + b*Sinh[x])))

Maple [A] time = 0.019, size = 14, normalized size = 1.1

$$-\frac{1}{b\left(a+b\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*sinh(x))^2,x)
```

```
[Out] -1/b/(a+b*sinh(x))
```

Maxima [A] time = 1.2087, size = 18, normalized size = 1.38

$$-\frac{1}{(b\sinh\left(x\right)+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sinh(x) + a)*b)
```

Fricas [B] time = 1.95935, size = 149, normalized size = 11.46

$$-\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{b^2\cosh\left(x\right)^2+b^2\sinh\left(x\right)^2+2\,ab\cosh\left(x\right)-b^2+2\left(b^2\cosh\left(x\right)+ab\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) - b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

Sympy [A] time = 1.01477, size = 32, normalized size = 2.46

$$\begin{cases} \frac{\tilde{\infty}}{\sinh\left(x\right)} & \text{for}\: a = 0 \wedge b = 0 \\ \tilde{\infty}\sinh\left(x\right) & \text{for}\: a = -b\sinh\left(x\right) \\ \frac{\sinh\left(x\right)}{a^2} & \text{for}\: b = 0 \\ -\frac{1}{ab+b^2\sinh\left(x\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo/sinh(x), Eq(a, 0) & Eq(b, 0)), (zoo*sinh(x), Eq(a, -b*sinh(x
))), (sinh(x)/a**2, Eq(b, 0)), (-1/(a*b + b**2*sinh(x)), True))
```

Giac [A] time = 1.12229, size = 30, normalized size = 2.31

$$\frac{2}{\left(b\left(e^{(-x)} - e^{x}\right) - 2\,a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2/((b*(e^(-x) - e^x) - 2*a)*b)
```

3.204 $\int \frac{\mathbf{sech}(x)}{(a+b\sinh(x))^2} dx$

Optimal. Leaf size=79

$$-\frac{b}{(a^2+b^2)(a+b\sinh(x))}+\frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{(a^2-b^2)\tan^{-1}(\sinh(x))}{(a^2+b^2)^2}-\frac{2ab\log(\cosh(x))}{(a^2+b^2)^2}$$

```
[Out] ((a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^2 - (2*a*b*Log[Cosh[x]])/(a^2 + b
^2)^2 + (2*a*b*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 - b/((a^2 + b^2)*(a + b*Si
nh[x]))
```

Rubi [A] time = 0.102935, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2668, 710, 801, 635, 203, 260}

$$-\frac{b}{(a^2+b^2)(a+b\sinh(x))}+\frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{(a^2-b^2)\tan^{-1}(\sinh(x))}{(a^2+b^2)^2}-\frac{2ab\log(\cosh(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Sinh[x])^2,x]
```



```
[Out] ((a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^2 - (2*a*b*Log[Cosh[x]])/(a^2 + b
^2)^2 + (2*a*b*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 - b/((a^2 + b^2)*(a + b*Si
nh[x]))
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx &= -\left(b\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\right)\\
&= -\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\,\mathrm{Subst}\left(\int \frac{a-x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\,\mathrm{Subst}\left(\int \left(-\frac{2a}{\left(a^2+b^2\right)(a+x)} + \frac{-a^2+b^2+2ax}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\,\mathrm{Subst}\left(\int \frac{-a^2+b^2+2ax}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^2}\\
&= \frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^2} +\\
&= \frac{\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{2ab\log(\cosh(x))}{\left(a^2+b^2\right)^2} + \frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)(a+b\sin}
\end{aligned}$$

Mathematica [A] time = 0.442941, size = 121, normalized size = 1.53

$$-\frac{b\left(\frac{2\left(a^2+b^2\right)}{a+b\sinh(x)} + \left(\frac{b^2-a^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right) + \left(\frac{a^2-b^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right) - 4a\log(a+b\sinh(x))\right)}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -(b*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 4*a*Log[
a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]
 + (2*(a^2 + b^2))/(a + b*Sinh[x])))/(2*(a^2 + b^2)^2)
```

Maple [B] time = 0.046, size = 201, normalized size = 2.5

$$-2\,\frac{ab\ln\left(\left(\tanh(x/2)\right)^2+1\right)}{a^4+2\,a^2b^2+b^4} + 2\,\frac{\arctan(\tanh(x/2))\,a^2}{a^4+2\,a^2b^2+b^4} - 2\,\frac{\arctan(\tanh(x/2))\,b^2}{a^4+2\,a^2b^2+b^4} - 2\,\frac{ab^2\tanh(x/2}{\left(a^2+b^2\right)^2\left(a\left(\tanh(x/2)\right)^2-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*sinh(x))^2,x)
```

```
[Out] -2/(a^4+2*a^2*b^2+b^4)*a*b*ln(tanh(1/2*x)^2+1)+2/(a^4+2*a^2*b^2+b^4)*arctan
(tanh(1/2*x))*a^2-2/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*x))*b^2-2*b^2/(a^2+
b^2)^2*a*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-2*b^4/(a^2+b^2)^2/
a*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2*b/(a^2+b^2)^2*a*ln(a*ta
nh(1/2*x)^2-2*tanh(1/2*x)*b-a)
```

Maxima [A] time = 1.74772, size = 201, normalized size = 2.54

$$\frac{2\,ab\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,ab\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(a^2-b^2\right)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,be^{(-x)}}{a^2b+b^3+2\left(a^3+ab^2\right)e^{(-x)}-\left(a^2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - 2*a*b*log
(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a^2 - b^2)*arctan(e^(-x))/(a^4
+ 2*a^2*b^2 + b^4) - 2*b*e^(-x)/(a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-x) - (a^
2*b + b^3)*e^(-2*x))
```

Fricas [B] time = 2.22788, size = 963, normalized size = 12.19

$$\frac{2\left(\left(a^2b-b^3-\left(a^2b-b^3\right)\cosh\left(x\right)^2-\left(a^2b-b^3\right)\sinh\left(x\right)^2-2\left(a^3-ab^2\right)\cosh\left(x\right)-2\left(a^3-ab^2+\left(a^2b-b^3\right)\cosh\left(x\right)\right)\sinh\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 2*((a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a^
3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*arcta
n(cosh(x) + sinh(x)) + (a^2*b + b^3)*cosh(x) - (a*b^2*cosh(x)^2 + a*b^2*sin
h(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*log(2
*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (a*b^2*cosh(x)^2 + a*b^2*sinh(x)^2
+ 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + (a^2*b + b^3)*sinh(x))/(a^4*b + 2*a^2*b^3 + b^5 -
(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 -
 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b
+ 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(x\right)}{\left(a+b\sinh\left(x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)/(a + b*sinh(x))**2, x)
```

Giac [B] time = 1.13246, size = 251, normalized size = 3.18

$$\frac{2\,ab^2\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{ab\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(a^2-b^2\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{2\left(ab\right.}{\left(a^4+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*l
og((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(pi + 2*arctan(1/2*(
e^(2*x) - 1)*e^(-x)))*(a^2 - b^2)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a*b^2*(e^(-x
) - e^x) - 3*a^2*b - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x) - 2*a)
)
```

3.205 $\int \frac{\text{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{6ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{b\text{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\text{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{\left(a^2+b^2\right)^2}$$

```
[Out] (-6*a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (
b*Sech[x])/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]*(3*a*b + (a^2 - 2*b^2)*
Sinh[x]))/(a^2 + b^2)^2
```

Rubi [A] time = 0.170352, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2694, 2866, 12, 2660, 618, 206}

$$-\frac{6ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{b\text{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\text{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (-6*a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (
b*Sech[x])/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]*(3*a*b + (a^2 - 2*b^2)*
Sinh[x]))/(a^2 + b^2)^2
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\int \frac{\operatorname{sech}^2(x)(-a+2b\sinh(x))}{a+b\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+\left(a^2-2b^2\right)\sinh(x)\right)}{\left(a^2+b^2\right)^2} + \frac{\int \frac{3ab^2}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}$$

$$= -\frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+\left(a^2-2b^2\right)\sinh(x)\right)}{\left(a^2+b^2\right)^2} + \frac{\left(3ab^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}$$

$$= -\frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+\left(a^2-2b^2\right)\sinh(x)\right)}{\left(a^2+b^2\right)^2} + \frac{\left(6ab^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax}\right.}{\left(a^2+b^2\right)}$$

$$= -\frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+\left(a^2-2b^2\right)\sinh(x)\right)}{\left(a^2+b^2\right)^2} - \frac{\left(12ab^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)}\right.}{\left(a^2\right.}$$

$$= -\frac{6ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+\left(a^2-2b^2\right)\sinh(x)\right)}{\left(a^2+b^2\right)^2}$$

Mathematica [A] time = 0.241977, size = 94, normalized size = 1.01

$$\frac{\frac{6ab^2\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + a^2\tanh(x) - \frac{b^3\cosh(x)}{a+b\sinh(x)} + 2ab\operatorname{sech}(x) - b^2\tanh(x)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 2*
a*b*Sech[x] - (b^3*Cosh[x])/(a + b*Sinh[x]) + a^2*Tanh[x] - b^2*Tanh[x])/(a
^2 + b^2)^2
```

Maple [A] time = 0.041, size = 138, normalized size = 1.5

$$-2\,\frac{\left(-a^2+b^2\right)\tanh(x/2)-2\,ab}{\left(a^4+2\,a^2b^2+b^4\right)\left((\tanh(x/2))^2+1\right)}-2\,\frac{b^2}{\left(a^2+b^2\right)^2}\left(\frac{1}{a\,(\tanh(x/2))^2-2\,\tanh(x/2)\,b-a}\left(-\frac{b^2\tanh(x/2)}{a}-b\right)-3\,-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] -2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(tanh(1/2*x)^2+1)-2*b
^2/(a^2+b^2)^2*((-b^2/a*tanh(1/2*x)-b)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-
3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15832, size = 1916, normalized size = 20.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 6*(a^3*b^2 + a*b^4)*cosh(x)^3 + 6*(a^3*b^2
+ a*b^4)*sinh(x)^3 + 6*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b + a^2*b^3 + 3
*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^2 + 3*(a*b^3*cosh(x)^4 + a*b^3*sinh(x)^
4 + 2*a^2*b^2*cosh(x)^3 + 2*a^2*b^2*cosh(x) - a*b^3 + 2*(2*a*b^3*cosh(x) +
a^2*b^2)*sinh(x)^3 + 6*(a*b^3*cosh(x)^2 + a^2*b^2*cosh(x))*sinh(x)^2 + 2*(2
*a*b^3*cosh(x)^3 + 3*a^2*b^2*cosh(x)^2 + a^2*b^2)*sinh(x))*sqrt(a^2 + b^2)*
log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*c
osh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*c
osh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*
(2*a^5 + a^3*b^2 - a*b^4)*cosh(x) - 2*(2*a^5 + a^3*b^2 - a*b^4 - 9*(a^3*b^2
 + a*b^4)*cosh(x)^2 - 6*(a^4*b + a^2*b^3)*cosh(x))*sinh(x))/(a^6*b + 3*a^4*
b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (
a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^4 - 2*(a^7 + 3*a^5*b^2 + 3*a^3
*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 - 6*((a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x
))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x) - 2*(a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**2/(a + b*sinh(x))**2, x)
```

Giac [A] time = 1.15158, size = 225, normalized size = 2.42

$$\frac{3\,ab^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,ab^2e^{(3\,x)}+3\,a^2be^{(2\,x)}-2\,a^3e^x+ab^2e^x+a^2b-2\,b^3\right)}{\left(a^4+2\,a^2b^2+b^4\right)\left(be^{(4\,x)}+2\,ae^{(3\,x)}+2\,ae^x-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3*a*b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a*b^2*e^(3
*x) + 3*a^2*b*e^(2*x) - 2*a^3*e^x + a*b^2*e^x + a^2*b - 2*b^3)/((a^4 + 2*a^
2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x - b))
```

3.206 $\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=136

$$\frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{\left(6a^2b^2+a^4-3b^4\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^3}-\frac{4ab^3\log(\cosh(x))}{\left(a^2+b^2\right)^3}+\frac{\text{sech}}{2\left(a^2\right.}$$

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^3) - (4*a*b^3*Lo
g[Cosh[x]])/(a^2 + b^2)^3 + (4*a*b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (b
*(a^2 - 3*b^2))/(2*(a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(b + a*Sinh[
x]))/(2*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [A] time = 0.164562, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2668, 741, 801, 635, 203, 260}

$$\frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{\left(6a^2b^2+a^4-3b^4\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^3}-\frac{4ab^3\log(\cosh(x))}{\left(a^2+b^2\right)^3}+\frac{\text{sech}}{2\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Sinh[x])^2,x]
```


```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^3) - (4*a*b^3*Lo
g[Cosh[x]])/(a^2 + b^2)^3 + (4*a*b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (b
*(a^2 - 3*b^2))/(2*(a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(b + a*Sinh[
x]))/(2*(a^2 + b^2)*(a + b*Sinh[x]))
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\,\text{Subst}\left(\int \frac{a^2+3b^2+2ax}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}$$

$$= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\,\text{Subst}\left(\int \left(\frac{a^2-3b^2}{\left(a^2+b^2\right)(a+x)^2} - \frac{8ab^2}{\left(a^2+b^2\right)^2(a+x)} + \frac{-a^4-6a^2b^2+3b^4+8ab^2x}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right) dx$$

$$= \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\,\text{Subs}}{}$$

$$= \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\left(4ab^3\right.}{}$$

$$= \frac{\left(a^4+6a^2b^2-3b^4\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^3} - \frac{4ab^3\log(\cosh(x))}{\left(a^2+b^2\right)^3} + \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{}{2\left(a^2+\right.}$$

Mathematica [A] time = 2.44154, size = 260, normalized size = 1.91

$$-\frac{\frac{b\left(\frac{2a\left(a^2+b^2\right)\left(\left(\sqrt{-b^2}-a\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{\sqrt{-b^2}}+\left(3b^2-a^2\right)\left(\frac{2\left(a^2+b^2\right)}{a+b\sinh(x)}+\left(\frac{b^2-a^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)+\left(\right.\right.\right.}{\left(a^2+b^2\right)^2}}{4\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] -((-2*Sech[x]^2*(b + a*Sinh[x]))/(a + b*Sinh[x]) + (b*((2*a*(a^2 + b^2)*((-
a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[a + b*Sinh[x
]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] + (-a^2 + 3*
b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 4*a*Log
[a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]
] + (2*(a^2 + b^2))/(a + b*Sinh[x]))))/(a^2 + b^2)^2)/(4*(a^2 + b^2))
```

Maple [B] time = 0.069, size = 548, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] -1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^4+1/(a
^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*b^4-4/(a^4+2*
a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2*a^3*b-4/(a^4+2*a^2
*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2*a*b^3+1/(a^4+2*a^2*b^
2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a^4-1/(a^4+2*a^2*b^2+b^4)/
(a^2+b^2)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*b^4-4/(a^4+2*a^2*b^2+b^4)/(a^2+b^
2)*a*b^3*ln(tanh(1/2*x)^2+1)+1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/
2*x))*a^4+6/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*a^2*b^2-3/(a^
4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*b^4-2*b^4/(a^2+b^2)^3*a*tanh
(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-2*b^6/(a^2+b^2)^3/a*tanh(1/2*x)
/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+4*b^3/(a^2+b^2)^3*a*ln(a*tanh(1/2*x)^2
-2*tanh(1/2*x)*b-a)
```

Maxima [B] time = 1.87928, size = 506, normalized size = 3.72

$$\frac{4\,ab^3\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,ab^3\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(a^4+6\,a^2b^2-3\,b^4\right)\arctan\left(e^{(-x)}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{}{a^4b+2\,a^2b^3+b^5+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 4*a*b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) - 4*a*b^3*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4 +
 6*a^2*b^2 - 3*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((
a^2*b - 3*b^3)*e^(-x) + 2*(a^3 + a*b^2)*e^(-2*x) + 2*(3*a^2*b - b^3)*e^(-3*
x) - 2*(a^3 + a*b^2)*e^(-4*x) + (a^2*b - 3*b^3)*e^(-5*x))/(a^4*b + 2*a^2*b^
3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b^5)*e^
(-2*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e
^(-4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3 + b^5)*
e^(-6*x))
```

Fricas [B] time = 2.82989, size = 6197, normalized size = 45.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^5 + (a^4*b - 2*a^2*b^3 - 3*b^5)*sinh(
x)^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + (2*a^5 + 4*a^3*b^2 + 2*a*b^4
 + 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^4 + 2*(3*a^4*b + 2*a^2*b^
3 - b^5)*cosh(x)^3 + 2*(3*a^4*b + 2*a^2*b^3 - b^5 + 5*(a^4*b - 2*a^2*b^3 -
3*b^5)*cosh(x)^2 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 2*(a^5
```

```
+ 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 - 5*(a^4*b - 2*
a^2*b^3 - 3*b^5)*cosh(x)^3 - 6*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 3*(3*a
^4*b + 2*a^2*b^3 - b^5)*cosh(x))*sinh(x)^2 + ((a^4*b + 6*a^2*b^3 - 3*b^5)*c
osh(x)^6 + (a^4*b + 6*a^2*b^3 - 3*b^5)*sinh(x)^6 + 2*(a^5 + 6*a^3*b^2 - 3*a
*b^4)*cosh(x)^5 + 2*(a^5 + 6*a^3*b^2 - 3*a*b^4 + 3*(a^4*b + 6*a^2*b^3 - 3*b
^5)*cosh(x))*sinh(x)^5 - a^4*b - 6*a^2*b^3 + 3*b^5 + (a^4*b + 6*a^2*b^3 - 3
*b^5)*cosh(x)^4 + (a^4*b + 6*a^2*b^3 - 3*b^5 + 15*(a^4*b + 6*a^2*b^3 - 3*b^
5)*cosh(x)^2 + 10*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x)^4 + 4*(a^5 +
 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 + 4*(a^5 + 6*a^3*b^2 - 3*a*b^4 + 5*(a^4*b +
 6*a^2*b^3 - 3*b^5)*cosh(x)^3 + 5*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^2 + (
a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - (a^4*b + 6*a^2*b^3 - 3*b^5)
*cosh(x)^2 - (a^4*b + 6*a^2*b^3 - 3*b^5 - 15*(a^4*b + 6*a^2*b^3 - 3*b^5)*co
sh(x)^4 - 20*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 - 6*(a^4*b + 6*a^2*b^3 -
 3*b^5)*cosh(x)^2 - 12*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x)^2 + 2*(
a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x) + 2*(3*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(
x)^5 + a^5 + 6*a^3*b^2 - 3*a*b^4 + 5*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^4
+ 2*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^3 + 6*(a^5 + 6*a^3*b^2 - 3*a*b^4)*c
osh(x)^2 - (a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))*arctan(cosh(x) + s
inh(x)) + (a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x) + 4*(a*b^4*cosh(x)^6 + a*b^4*
sinh(x)^6 + 2*a^2*b^3*cosh(x)^5 + a*b^4*cosh(x)^4 + 4*a^2*b^3*cosh(x)^3 - a
*b^4*cosh(x)^2 + 2*a^2*b^3*cosh(x) + 2*(3*a*b^4*cosh(x) + a^2*b^3)*sinh(x)^
5 - a*b^4 + (15*a*b^4*cosh(x)^2 + 10*a^2*b^3*cosh(x) + a*b^4)*sinh(x)^4 + 4
*(5*a*b^4*cosh(x)^3 + 5*a^2*b^3*cosh(x)^2 + a*b^4*cosh(x) + a^2*b^3)*sinh(x
)^3 + (15*a*b^4*cosh(x)^4 + 20*a^2*b^3*cosh(x)^3 + 6*a*b^4*cosh(x)^2 + 12*a
^2*b^3*cosh(x) - a*b^4)*sinh(x)^2 + 2*(3*a*b^4*cosh(x)^5 + 5*a^2*b^3*cosh(x
)^4 + 2*a*b^4*cosh(x)^3 + 6*a^2*b^3*cosh(x)^2 - a*b^4*cosh(x) + a^2*b^3)*si
nh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - 4*(a*b^4*cosh(x)^6 + a*
b^4*sinh(x)^6 + 2*a^2*b^3*cosh(x)^5 + a*b^4*cosh(x)^4 + 4*a^2*b^3*cosh(x)^3
 - a*b^4*cosh(x)^2 + 2*a^2*b^3*cosh(x) + 2*(3*a*b^4*cosh(x) + a^2*b^3)*sinh
(x)^5 - a*b^4 + (15*a*b^4*cosh(x)^2 + 10*a^2*b^3*cosh(x) + a*b^4)*sinh(x)^4
 + 4*(5*a*b^4*cosh(x)^3 + 5*a^2*b^3*cosh(x)^2 + a*b^4*cosh(x) + a^2*b^3)*si
nh(x)^3 + (15*a*b^4*cosh(x)^4 + 20*a^2*b^3*cosh(x)^3 + 6*a*b^4*cosh(x)^2 +
12*a^2*b^3*cosh(x) - a*b^4)*sinh(x)^2 + 2*(3*a*b^4*cosh(x)^5 + 5*a^2*b^3*co
sh(x)^4 + 2*a*b^4*cosh(x)^3 + 6*a^2*b^3*cosh(x)^2 - a*b^4*cosh(x) + a^2*b^3
)*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (a^4*b - 2*a^2*b^3 - 3*b^5
+ 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^4 + 8*(a^5 + 2*a^3*b^2 + a*b^4)*cos
h(x)^3 + 6*(3*a^4*b + 2*a^2*b^3 - b^5)*cosh(x)^2 - 4*(a^5 + 2*a^3*b^2 + a*b
^4)*cosh(x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4
*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^6 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*s
inh(x)^6 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 - 2*(a^7 + 3*a
^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x
))*sinh(x)^5 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (a^6*b + 3
*a^4*b^3 + 3*a^2*b^5 + b^7 + 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(
x)^2 + 10*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^4 - 4*(a^7
 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6 + 5*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 5*(a^7 + 3*
a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b
^7)*cosh(x))*sinh(x)^3 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 +
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*cosh(x)^4 - 20*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 6*(a^
6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 - 12*(a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^5 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^
4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 6*(a^7 + 3*a^5*b^2
+ 3*a^3*b^4 + a*b^6)*cosh(x)^2 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**3/(a + b*sinh(x))**2, x)
```

Giac [B] time = 1.17612, size = 398, normalized size = 2.93

$$\frac{4\,ab^4\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7} - \frac{2\,ab^3\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)\left(a^4 + 6\,a^2b^2 - 3\,b^4\right)}{4\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 4*a*b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 2*a*b^3*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^4 + 6*a^2*b^2 - 3*b^4)
/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^2*b*(e^(-x) - e^x)^2 - 3*b^3*(e^(
-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 8*a^2*b - 8*
b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a*(e^(-x) - e^x)^2 +
4*b*(e^(-x) - e^x) - 8*a))
```

3.207 $\int \frac{\mathrm{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=144

$$-\frac{10ab^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{b\,\mathrm{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\mathrm{sech}^3(x)\left(\left(a^2-4b^2\right)\sinh(x)+5ab\right)}{3\left(a^2+b^2\right)^2}+\frac{\mathrm{sech}(x)\left(\left(9a^2b^2+2\right.\right.}{3(}$$

```
[Out] (-10*a*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) -
(b*Sech[x]^3)/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]^3*(5*a*b + (a^2 - 4*
b^2)*Sinh[x]))/(3*(a^2 + b^2)^2) + (Sech[x]*(15*a*b^3 + (2*a^4 + 9*a^2*b^2
- 8*b^4)*Sinh[x]))/(3*(a^2 + b^2)^3)
```

Rubi [A] time = 0.310433, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2694, 2866, 12, 2660, 618, 206}

$$-\frac{10ab^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{b\,\mathrm{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\mathrm{sech}^3(x)\left(\left(a^2-4b^2\right)\sinh(x)+5ab\right)}{3\left(a^2+b^2\right)^2}+\frac{\mathrm{sech}(x)\left(\left(9a^2b^2+2\right.\right.}{3(}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-10*a*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) -
(b*Sech[x]^3)/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]^3*(5*a*b + (a^2 - 4*
b^2)*Sinh[x]))/(3*(a^2 + b^2)^2) + (Sech[x]*(15*a*b^3 + (2*a^4 + 9*a^2*b^2
- 8*b^4)*Sinh[x]))/(3*(a^2 + b^2)^3)
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\int \frac{\operatorname{sech}^4(x)(-a+4b\sinh(x))}{a+b\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\int \frac{\operatorname{sech}^2(x)\left(a\left(2a^2+7b^2\right)+2b\left(a^2-4b^2\right.\right.}{a+b\sinh(x)}}{3\left(a^2+b^2\right)^2}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15ab^3+\left(2a^4+9a\right.\right.}{3\left(a^2+b^2\right.}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15ab^3+\left(2a^4+9a\right.\right.}{3\left(a^2+b^2\right.}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15ab^3+\left(2a^4+9a\right.\right.}{3\left(a^2+b^2\right.}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15ab^3+\left(2a^4+9a\right.\right.}{3\left(a^2+b^2\right.}$$

$$= -\frac{10ab^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}$$

Mathematica [A] time = 0.430103, size = 137, normalized size = 0.95

$$\frac{\left(9a^2b^2+2a^4-5b^4\right)\tanh(x)+\frac{30ab^4\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\left(a^2+b^2\right)\operatorname{sech}^3(x)\left(\left(a^2-b^2\right)\sinh(x)+2ab\right)+12ab^3\operatorname{sech}(x)-\frac{3b^5}{a+b}}{3\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((30*a*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 1
2*a*b^3*Sech[x] - (3*b^5*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b^2)*Sech[x]^3*(
2*a*b + (a^2 - b^2)*Sinh[x]) + (2*a^4 + 9*a^2*b^2 - 5*b^4)*Tanh[x])/(3*(a^2
 + b^2)^3)
```

Maple [A] time = 0.072, size = 266, normalized size = 1.9

$$-2\,\frac{\left(-a^4-3\,a^2b^2+2\,b^4\right)(\tanh(x/2))^5+\left(-2\,a^3b-6\,ab^3\right)(\tanh(x/2))^4+\left(-2/3\,a^4-6\,a^2b^2+8/3\,b^4\right)(\tanh(x/2))^3-8}{\left(a^2+b^2\right)\left(a^4+2\,a^2b^2+b^4\right)\left((\tanh(x/2))^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] -2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4-3*a^2*b^2+2*b^4)*tanh(1/2*x)^5+(-2*
a^3*b-6*a*b^3)*tanh(1/2*x)^4+(-2/3*a^4-6*a^2*b^2+8/3*b^4)*tanh(1/2*x)^3-8*a
*b^3*tanh(1/2*x)^2+(-a^4-3*a^2*b^2+2*b^4)*tanh(1/2*x)-2/3*a^3*b-14/3*a*b^3)
/(tanh(1/2*x)^2+1)^3-2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2/a*tanh(1/2*
x)-b)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-5*a/(a^2+b^2)^(1/2)*arctanh(1/2*(
2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.45743, size = 7135, normalized size = 49.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(30*(a^3*b^4 + a*b^6)*cosh(x)^7 + 30*(a^3*b^4 + a*b^6)*sinh(x)^7 + 4*a
^6*b + 22*a^4*b^3 + 2*a^2*b^5 - 16*b^7 + 30*(a^4*b^3 + a^2*b^5)*cosh(x)^6 +
 30*(a^4*b^3 + a^2*b^5 + 7*(a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^6 - 10*(2*a^5
*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^5 - 10*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6
 - 63*(a^3*b^4 + a*b^6)*cosh(x)^2 - 18*(a^4*b^3 + a^2*b^5)*cosh(x))*sinh(x)
^5 + 10*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^4 + 10*(2*a^6*b + 13*a^
4*b^3 + 11*a^2*b^5 + 105*(a^3*b^4 + a*b^6)*cosh(x)^3 + 45*(a^4*b^3 + a^2*b^
5)*cosh(x)^2 - 5*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x))*sinh(x)^4 - 2*(
12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 - 13*a*b^6)*cosh(x)^3 - 2*(12*a^7 + 56*a^5
*b^2 + 31*a^3*b^4 - 13*a*b^6 - 525*(a^3*b^4 + a*b^6)*cosh(x)^4 - 300*(a^4*b
^3 + a^2*b^5)*cosh(x)^3 + 50*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^2 -
20*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x))*sinh(x)^3 + 2*(4*a^6*b + 37
```

```
*a^4*b^3 + 17*a^2*b^5 - 16*b^7)*cosh(x)^2 + 2*(4*a^6*b + 37*a^4*b^3 + 17*a^
2*b^5 - 16*b^7 + 315*(a^3*b^4 + a*b^6)*cosh(x)^5 + 225*(a^4*b^3 + a^2*b^5)*
cosh(x)^4 - 50*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^3 + 30*(2*a^6*b +
13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^2 - 3*(12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 -
13*a*b^6)*cosh(x))*sinh(x)^2 + 15*(a*b^5*cosh(x)^8 + a*b^5*sinh(x)^8 + 2*a^
2*b^4*cosh(x)^7 + 2*a*b^5*cosh(x)^6 + 6*a^2*b^4*cosh(x)^5 + 6*a^2*b^4*cosh(
x)^3 - 2*a*b^5*cosh(x)^2 + 2*(4*a*b^5*cosh(x) + a^2*b^4)*sinh(x)^7 + 2*a^2*
b^4*cosh(x) + 2*(14*a*b^5*cosh(x)^2 + 7*a^2*b^4*cosh(x) + a*b^5)*sinh(x)^6
- a*b^5 + 2*(28*a*b^5*cosh(x)^3 + 21*a^2*b^4*cosh(x)^2 + 6*a*b^5*cosh(x) +
3*a^2*b^4)*sinh(x)^5 + 10*(7*a*b^5*cosh(x)^4 + 7*a^2*b^4*cosh(x)^3 + 3*a*b^
5*cosh(x)^2 + 3*a^2*b^4*cosh(x))*sinh(x)^4 + 2*(28*a*b^5*cosh(x)^5 + 35*a^2
*b^4*cosh(x)^4 + 20*a*b^5*cosh(x)^3 + 30*a^2*b^4*cosh(x)^2 + 3*a^2*b^4)*sin
h(x)^3 + 2*(14*a*b^5*cosh(x)^6 + 21*a^2*b^4*cosh(x)^5 + 15*a*b^5*cosh(x)^4
+ 30*a^2*b^4*cosh(x)^3 + 9*a^2*b^4*cosh(x) - a*b^5)*sinh(x)^2 + 2*(4*a*b^5*
cosh(x)^7 + 7*a^2*b^4*cosh(x)^6 + 6*a*b^5*cosh(x)^5 + 15*a^2*b^4*cosh(x)^4
+ 9*a^2*b^4*cosh(x)^2 - 2*a*b^5*cosh(x) + a^2*b^4)*sinh(x))*sqrt(a^2 + b^2)
*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*
cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*
cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2
*(4*a^7 + 22*a^5*b^2 + 17*a^3*b^4 - a*b^6)*cosh(x) - 2*(4*a^7 + 22*a^5*b^2
+ 17*a^3*b^4 - a*b^6 - 105*(a^3*b^4 + a*b^6)*cosh(x)^6 - 90*(a^4*b^3 + a^2*
b^5)*cosh(x)^5 + 25*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^4 - 20*(2*a^6
*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^3 + 3*(12*a^7 + 56*a^5*b^2 + 31*a^3*b
^4 - 13*a*b^6)*cosh(x)^2 - 2*(4*a^6*b + 37*a^4*b^3 + 17*a^2*b^5 - 16*b^7)*c
osh(x))*sinh(x))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 - (a^8*b
+ 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^8 - (a^8*b + 4*a^6*b^3 +
 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sinh(x)^8 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 +
4*a^3*b^6 + a*b^8)*cosh(x)^7 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 +
 a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(
x)^7 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^6 - 2*(a
^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 + 14*(a^8*b + 4*a^6*b^3 + 6*
a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a
^3*b^6 + a*b^8)*cosh(x))*sinh(x)^6 - 6*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3
*b^6 + a*b^8)*cosh(x)^5 - 2*(3*a^9 + 12*a^7*b^2 + 18*a^5*b^4 + 12*a^3*b^6 +
 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3 +
 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 + 6*(a^8*b
+ 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(x)^5 - 10*(7*(a^8*
b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^4 + 7*(a^9 + 4*a^7*b^2
 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*
b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b
^6 + a*b^8)*cosh(x))*sinh(x)^4 - 6*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6
 + a*b^8)*cosh(x)^3 - 2*(3*a^9 + 12*a^7*b^2 + 18*a^5*b^4 + 12*a^3*b^6 + 3*a
*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^5 + 35*
(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 + 20*(a^8*b + 4
*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3 + 30*(a^9 + 4*a^7*b^2 + 6
*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2)*sinh(x)^3 + 2*(a^8*b + 4*a^6*b^3 +
 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5
+ 4*a^2*b^7 + b^9 - 14*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*co
sh(x)^6 - 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^5 -
15*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^4 - 30*(a^9 +
4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 - 9*(a^9 + 4*a^7*b^2 +
 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2 - 2*(a^9 + 4*a^7*b^2 + 6
*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x) - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(
x)^7 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^6 + 6*(a
^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^5 + 15*(a^9 + 4*a^7
*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 + 9*(a^9 + 4*a^7*b^2 + 6*a^
5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4
*a^2*b^7 + b^9)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**4/(a + b*sinh(x))**2, x)
```

Giac [B] time = 1.17021, size = 387, normalized size = 2.69

$$\frac{5\,ab^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(ab^4e^x-b^5\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}+\frac{2\left(12\,ab^3e^{(5\,x)}-9\,a^2b^2e^{(4\,x)}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 5*a*b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(
a*b^4*e^x - b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*x) + 2*a*e^x
- b)) + 2/3*(12*a*b^3*e^(5*x) - 9*a^2*b^2*e^(4*x) + 3*b^4*e^(4*x) + 8*a^3*b
*e^(3*x) + 32*a*b^3*e^(3*x) - 6*a^4*e^(2*x) - 18*a^2*b^2*e^(2*x) + 12*b^4*e
^(2*x) + 12*a*b^3*e^x - 2*a^4 - 9*a^2*b^2 + 5*b^4)/((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

3.208 $\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{5}i\tanh^5(x)-\frac{1}{5}\text{sech}^5(x)+\frac{2\text{sech}^3(x)}{3}-\text{sech}(x)$$

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5 - (I/5)*Tanh[x]^5
```

Rubi [A] time = 0.0789464, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{1}{5}i\tanh^5(x)-\frac{1}{5}\text{sech}^5(x)+\frac{2\text{sech}^3(x)}{3}-\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(I + Sinh[x]),x]
```

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5 - (I/5)*Tanh[x]^5
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = -\left(i\int \operatorname{sech}^2(x)\tanh^4(x)\,dx\right) + \int \operatorname{sech}(x)\tanh^5(x)\,dx$$
$$= -\operatorname{Subst}\left(\int x^4\,dx, x, i\tanh(x)\right) - \operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \operatorname{sech}(x)\right)$$
$$= -\frac{1}{5}i\tanh^5(x) - \operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \operatorname{sech}(x)\right)$$
$$= -\operatorname{sech}(x) + \frac{2\operatorname{sech}^3(x)}{3} - \frac{\operatorname{sech}^5(x)}{5} - \frac{1}{5}i\tanh^5(x)$$

Mathematica [B] time = 0.135313, size = 96, normalized size = 3.1

$$-\frac{64i\sinh(x)+178i\sinh(2x)-192i\sinh(3x)+89i\sinh(4x)-534\cosh(x)+288\cosh(2x)-178\cosh(3x)+24\cosh}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(I + Sinh[x]),x]
```

```
[Out] -(200 - 534*Cosh[x] + 288*Cosh[2*x] - 178*Cosh[3*x] + 24*Cosh[4*x] + (64*I)
*Sinh[x] + (178*I)*Sinh[2*x] - (192*I)*Sinh[3*x] + (89*I)*Sinh[4*x])/(960*(
Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] time = 0.07, size = 93, normalized size = 3.

$$\frac{3i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}+\frac{i}{6}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3}+\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+\frac{i}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}-\frac{2i}{5}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-5}-\frac{3i}{8}\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(I+sinh(x)),x)
```

```
[Out] 3/8*I/(tanh(1/2*x)-I)+1/6*I/(tanh(1/2*x)-I)^3+1/4/(tanh(1/2*x)-I)^2+1/3*I/(
tanh(1/2*x)+I)^3-2/5*I/(tanh(1/2*x)+I)^5-3/8*I/(tanh(1/2*x)+I)+1/(tanh(1/2*
x)+I)^4+1/2/(tanh(1/2*x)+I)^2
```

Maxima [B] time = 1.12585, size = 558, normalized size = 18.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 18*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*
e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 42*I*e^(-2*x)/(-30*I*e^(-x)
- 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x)
 + 15*e^(-8*x) - 15) - 26*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3
*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 50*
I*e^(-4*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30
*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) - 10*e^(-5*x)/(-30*I*e^(-x) -
 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x)
```

```
+ 15*e^(-8*x) - 15) + 30*I*e^(-6*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-
3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) - 30
*e^(-7*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*
e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 6*I/(-30*I*e^(-x) - 30*e^(-2
*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-
8*x) - 15)
```

Fricas [B] time = 2.07948, size = 271, normalized size = 8.74

$$-\frac{30\,e^{(7\,x)} + 30i\,e^{(6\,x)} + 10\,e^{(5\,x)} + 50i\,e^{(4\,x)} + 26\,e^{(3\,x)} + 42i\,e^{(2\,x)} - 18\,e^{x} + 6i}{15\,e^{(8\,x)} + 30i\,e^{(7\,x)} + 30\,e^{(6\,x)} + 90i\,e^{(5\,x)} + 90i\,e^{(3\,x)} - 30\,e^{(2\,x)} + 30i\,e^{x} - 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(30*e^(7*x) + 30*I*e^(6*x) + 10*e^(5*x) + 50*I*e^(4*x) + 26*e^(3*x) + 42*I
*e^(2*x) - 18*e^x + 6*I)/(15*e^(8*x) + 30*I*e^(7*x) + 30*e^(6*x) + 90*I*e^(
5*x) + 90*I*e^(3*x) - 30*e^(2*x) + 30*I*e^x - 15)
```

Sympy [B] time = 1.30056, size = 116, normalized size = 3.74

$$\frac{-2e^{7x} - 2ie^{6x} - \frac{2e^{5x}}{3} - \frac{10ie^{4x}}{3} - \frac{26e^{3x}}{15} - \frac{14ie^{2x}}{5} + \frac{6e^{x}}{5} - \frac{2i}{5}}{e^{8x} + 2ie^{7x} + 2e^{6x} + 6ie^{5x} + 6ie^{3x} - 2e^{2x} + 2ie^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(I+sinh(x)),x)
```

```
[Out] (-2*exp(7*x) - 2*I*exp(6*x) - 2*exp(5*x)/3 - 10*I*exp(4*x)/3 - 26*exp(3*x)/
15 - 14*I*exp(2*x)/5 + 6*exp(x)/5 - 2*I/5)/(exp(8*x) + 2*I*exp(7*x) + 2*exp
(6*x) + 6*I*exp(5*x) + 6*I*exp(3*x) - 2*exp(2*x) + 2*I*exp(x) - 1)
```

Giac [B] time = 1.15486, size = 72, normalized size = 2.32

$$-\frac{15\,e^{(2\,x)} - 24i\,e^{x} - 13}{24\,{\left(e^{x} - i\right)}^{3}} - \frac{165\,e^{(4\,x)} + 480i\,e^{(3\,x)} - 650\,e^{(2\,x)} - 400i\,e^{x} + 113}{120\,{\left(e^{x} + i\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/24*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3 - 1/120*(165*e^(4*x) + 480*I
*e^(3*x) - 650*e^(2*x) - 400*I*e^x + 113)/(e^x + I)^5
```

3.209 $\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{4}i\tanh^4(x)+\frac{3}{8}\tan^{-1}(\sinh(x))-\frac{1}{4}\tanh^3(x)\mathrm{sech}(x)-\frac{3}{8}\tanh(x)\mathrm{sech}(x)$$

```
[Out] (3*ArcTan[Sinh[x]])/8 - (3*Sech[x]*Tanh[x])/8 - (Sech[x]*Tanh[x]^3)/4 - (I/
4)*Tanh[x]^4
```

Rubi [A] time = 0.091929, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{1}{4}i\tanh^4(x)+\frac{3}{8}\tan^{-1}(\sinh(x))-\frac{1}{4}\tanh^3(x)\mathrm{sech}(x)-\frac{3}{8}\tanh(x)\mathrm{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 - (3*Sech[x]*Tanh[x])/8 - (Sech[x]*Tanh[x]^3)/4 - (I/
4)*Tanh[x]^4
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh^3(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^4(x)\,dx\\
&= -\frac{1}{4}\operatorname{sech}(x)\tanh^3(x) - i\,\text{Subst}\left(\int x^3\,dx, x, i\tanh(x)\right)+\frac{3}{4}\int \operatorname{sech}(x)\tanh^2(x)\,dx\\
&= -\frac{3}{8}\operatorname{sech}(x)\tanh(x)-\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-\frac{1}{4}i\tanh^4(x)+\frac{3}{8}\int \operatorname{sech}(x)\,dx\\
&= \frac{3}{8}\tan^{-1}(\sinh(x))-\frac{3}{8}\operatorname{sech}(x)\tanh(x)-\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-\frac{1}{4}i\tanh^4(x)
\end{aligned}
$$

Mathematica [A] time = 0.0772882, size = 42, normalized size = 1.17

$$\frac{1}{8}\left(3\tan^{-1}(\sinh(x))-\frac{5\sinh^2(x)+i\sinh(x)+2}{(\sinh(x)-i)(\sinh(x)+i)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]] - (2 + I*Sinh[x] + 5*Sinh[x]^2)/((-I + Sinh[x])*(I + Sin
h[x])^2))/8
```

Maple [B] time = 0.059, size = 79, normalized size = 2.2

$$-\frac{3i}{8}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\frac{i}{4}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}+\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}-\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4}+\frac{3i}{8}\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(I+sinh(x)),x)
```

```
[Out] -3/8*I*ln(tanh(1/2*x)-I)+1/4*I/(tanh(1/2*x)-I)^2+1/4/(tanh(1/2*x)-I)-1/2*I/
(tanh(1/2*x)+I)^4+3/8*I*ln(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)^3+1/2/(tanh(1/2
*x)+I)
```

Maxima [B] time = 1.03303, size = 128, normalized size = 3.56

$$\frac{5e^{(-x)}+2ie^{(-2x)}-2e^{(-3x)}-2ie^{(-4x)}+5e^{(-5x)}}{-8ie^{(-x)}-4e^{(-2x)}-16ie^{(-3x)}+4e^{(-4x)}-8ie^{(-5x)}+4e^{(-6x)}-4}+\frac{3}{8}i\log\left(ie^{(-x)}+1\right)-\frac{3}{8}i\log\left(ie^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] (5*e^(-x) + 2*I*e^(-2*x) - 2*e^(-3*x) - 2*I*e^(-4*x) + 5*e^(-5*x))/(-8*I*e^
(-x) - 4*e^(-2*x) - 16*I*e^(-3*x) + 4*e^(-4*x) - 8*I*e^(-5*x) + 4*e^(-6*x)
- 4) + 3/8*I*log(I*e^(-x) + 1) - 3/8*I*log(I*e^(-x) - 1)
```

Fricas [B] time = 2.08846, size = 455, normalized size = 12.64

$$\frac{\left(3i\,e^{(6\,x)} - 6\,e^{(5\,x)} + 3i\,e^{(4\,x)} - 12\,e^{(3\,x)} - 3i\,e^{(2\,x)} - 6\,e^{x} - 3i\right)\log\left(e^{x} + i\right) + \left(-3i\,e^{(6\,x)} + 6\,e^{(5\,x)} - 3i\,e^{(4\,x)} + 12\,e^{(3\,x)} + 3i\,e^{(2\,x}}{8\,e^{(6\,x)} + 16i\,e^{(5\,x)} + 8\,e^{(4\,x)} + 32i\,e^{(3\,x)} - 8\,e^{(2\,x)} + 16i\,e^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((3*I*e^(6*x) - 6*e^(5*x) + 3*I*e^(4*x) - 12*e^(3*x) - 3*I*e^(2*x) - 6*e^x
- 3*I)*log(e^x + I) + (-3*I*e^(6*x) + 6*e^(5*x) - 3*I*e^(4*x) + 12*e^(3*x)
+ 3*I*e^(2*x) + 6*e^x + 3*I)*log(e^x - I) - 10*e^(5*x) - 4*I*e^(4*x) + 4*e^
(3*x) + 4*I*e^(2*x) - 10*e^x)/(8*e^(6*x) + 16*I*e^(5*x) + 8*e^(4*x) + 32*I*
e^(3*x) - 8*e^(2*x) + 16*I*e^x - 8)
```

Sympy [B] time = 0.806312, size = 97, normalized size = 2.69

$$\frac{-\frac{5e^{5x}}{4} - \frac{ie^{4x}}{2} + \frac{e^{3x}}{2} + \frac{ie^{2x}}{2} - \frac{5e^{x}}{4}}{e^{6x} + 2ie^{5x} + e^{4x} + 4ie^{3x} - e^{2x} + 2ie^{x} - 1} + \operatorname{RootSum}\left(64z^{2} + 9, \left(i \mapsto i\log\left(\frac{8i}{3} + e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(I+sinh(x)),x)
```

```
[Out] (-5*exp(5*x)/4 - I*exp(4*x)/2 + exp(3*x)/2 + I*exp(2*x)/2 - 5*exp(x)/4)/(ex
p(6*x) + 2*I*exp(5*x) + exp(4*x) + 4*I*exp(3*x) - exp(2*x) + 2*I*exp(x) - 1
) + RootSum(64*_z**2 + 9, Lambda(_i, _i*log(8*_i/3 + exp(x))))
```

Giac [B] time = 1.13761, size = 124, normalized size = 3.44

$$\frac{3i\,e^{(-x)} - 3i\,e^{x} - 2}{16\left(e^{(-x)} - e^{x} + 2i\right)} - \frac{9i\left(e^{(-x)} - e^{x}\right)^{2} + 4\,e^{(-x)} - 4\,e^{x} + 12i}{32\left(e^{(-x)} - e^{x} - 2i\right)^{2}} + \frac{3}{16}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{3}{16}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/16*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/32*(9*I*(e^(-x) -
e^x)^2 + 4*e^(-x) - 4*e^x + 12*I)/(e^(-x) - e^x - 2*I)^2 + 3/16*I*log(-e^(-
x) + e^x + 2*I) - 3/16*I*log(-e^(-x) + e^x - 2*I)
```

3.210 $\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{3}i\tanh^3(x)+\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

```
[Out] -Sech[x] + Sech[x]^3/3 - (I/3)*Tanh[x]^3
```

Rubi [A] time = 0.0762514, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2607, 30, 2606}

$$-\frac{1}{3}i\tanh^3(x)+\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(I + Sinh[x]),x]
```

```
[Out] -Sech[x] + Sech[x]^3/3 - (I/3)*Tanh[x]^3
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = -\left(i\int \operatorname{sech}^2(x)\tanh^2(x)\,dx\right) + \int \operatorname{sech}(x)\tanh^3(x)\,dx$$
$$= \operatorname{Subst}\left(\int x^2\,dx, x, i\tanh(x)\right) + \operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \operatorname{sech}(x)\right)$$
$$= -\operatorname{sech}(x) + \frac{\operatorname{sech}^3(x)}{3} - \frac{1}{3}i\tanh^3(x)$$

Mathematica [B] time = 0.0590886, size = 67, normalized size = 2.91

$$\frac{4i\sinh(x)-\cosh(2x)+(5-5i\sinh(x))\cosh(x)-3}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-3 - Cosh[2*x] + Cosh[x]*(5 - (5*I)*Sinh[x]) + (4*I)*Sinh[x])/(6*(Cosh[x/2
] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [B] time = 0.045, size = 47, normalized size = 2.

$$\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1} - \frac{2i}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3} - \frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1} + \left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(I+sinh(x)),x)
```

```
[Out] 1/2*I/(tanh(1/2*x)-I)-2/3*I/(tanh(1/2*x)+I)^3-1/2*I/(tanh(1/2*x)+I)+1/(tanh
(1/2*x)+I)^2
```

Maxima [B] time = 1.11683, size = 147, normalized size = 6.39

$$\frac{2\,e^{(-x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3} + \frac{6i\,e^{(-2\,x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3} - \frac{6\,e^{(-3\,x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3} + \frac{}{-6i\,e^{(-x)}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 6*I*e^(-2*x)/(-6*I
*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) - 6*e^(-3*x)/(-6*I*e^(-x) - 6*I*e^
(-3*x) + 3*e^(-4*x) - 3) + 2*I/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3
)
```

Fricas [B] time = 2.03306, size = 111, normalized size = 4.83

$$-\frac{6\,e^{(3\,x)}+6i\,e^{(2\,x)}-2\,e^{x}+2i}{3\,e^{(4\,x)}+6i\,e^{(3\,x)}+6i\,e^{x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(6*e^(3*x) + 6*I*e^(2*x) - 2*e^x + 2*I)/(3*e^(4*x) + 6*I*e^(3*x) + 6*I*e^x
 - 3)
```

Sympy [B] time = 0.379415, size = 48, normalized size = 2.09

$$\frac{-2e^{3x} - 2ie^{2x} + \frac{2e^{x}}{3} - \frac{2i}{3}}{e^{4x} + 2ie^{3x} + 2ie^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(I+sinh(x)),x)
```

```
[Out] (-2*exp(3*x) - 2*I*exp(2*x) + 2*exp(x)/3 - 2*I/3)/(exp(4*x) + 2*I*exp(3*x)
+ 2*I*exp(x) - 1)
```

Giac [A] time = 1.12204, size = 39, normalized size = 1.7

$$-\frac{1}{2\,(e^{x} - i)} - \frac{9\,e^{(2\,x)} + 12i\,e^{x} - 7}{6\,(e^{x} + i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2/(e^x - I) - 1/6*(9*e^(2*x) + 12*I*e^x - 7)/(e^x + I)^3
```

3.211 $\int \frac{\tanh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}i\mathrm{sech}^2(x)+\frac{1}{2}\tan^{-1}(\sinh(x))-\frac{1}{2}\tanh(x)\mathrm{sech}(x)$$

```
[Out] ArcTan[Sinh[x]]/2 + (I/2)*Sech[x]^2 - (Sech[x]*Tanh[x])/2
```

Rubi [A] time = 0.0551926, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2706, 2606, 30, 2611, 3770}

$$\frac{1}{2}i\mathrm{sech}^2(x)+\frac{1}{2}\tan^{-1}(\sinh(x))-\frac{1}{2}\tanh(x)\mathrm{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(I + Sinh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/2 + (I/2)*Sech[x]^2 - (Sech[x]*Tanh[x])/2
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^2(x)\,dx\\ &= -\frac{1}{2}\operatorname{sech}(x)\tanh(x)+i\,\mathrm{Subst}\left(\int x\,dx, x, \operatorname{sech}(x)\right)+\frac{1}{2}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{2}\tan^{-1}(\sinh(x))+\frac{1}{2}i\operatorname{sech}^2(x)-\frac{1}{2}\operatorname{sech}(x)\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0204506, size = 20, normalized size = 0.77

$$\frac{1}{2}\tan^{-1}(\sinh(x))-\frac{1}{2(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(I + Sinh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/2 - 1/(2*(I + Sinh[x]))
```

Maple [B] time = 0.032, size = 45, normalized size = 1.7

$$-\frac{i}{2}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}+\frac{i}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(I+sinh(x)),x)
```

```
[Out] -1/2*I*ln(tanh(1/2*x)-I)-I/(tanh(1/2*x)+I)^2+1/2*I*ln(tanh(1/2*x)+I)+1/(tan
h(1/2*x)+I)
```

Maxima [B] time = 1.16807, size = 57, normalized size = 2.19

$$\frac{e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}+\frac{1}{2}i\,\log\left(i\,e^{(-x)}+1\right)-\frac{1}{2}i\,\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) + 1/2*I*log(I*e^(-x) + 1) - 1/2*I*log(I
*e^(-x) - 1)
```

Fricas [B] time = 2.13283, size = 157, normalized size = 6.04

$$\frac{\left(i\,e^{(2\,x)}-2\,e^{x}-i\right)\log\left(e^{x}+i\right)+\left(-i\,e^{(2\,x)}+2\,e^{x}+i\right)\log\left(e^{x}-i\right)-2\,e^{x}}{2\left(e^{(2\,x)}+2i\,e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*e^(2*x) - 2*e^x - I)*log(e^x + I) + (-I*e^(2*x) + 2*e^x + I)*log(e^
x - I) - 2*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [A] time = 0.269696, size = 32, normalized size = 1.23

$$\operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+e^x\right)\right)\right)-\frac{e^x}{e^{2x}+2ie^x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x)))) - exp(x)/(exp(2*x)
+ 2*I*exp(x) - 1)
```

Giac [B] time = 1.08919, size = 72, normalized size = 2.77

$$\frac{-i\,e^{(-x)}+i\,e^x+2}{4\left(e^{(-x)}-e^x-2i\right)}+\frac{1}{4}i\,\log\left(-e^{(-x)}+e^x+2i\right)-\frac{1}{4}i\,\log\left(-e^{(-x)}+e^x-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/4*(-I*e^(-x) + I*e^x + 2)/(e^(-x) - e^x - 2*I) + 1/4*I*log(-e^(-x) + e^x
+ 2*I) - 1/4*I*log(-e^(-x) + e^x - 2*I)
```

3.212 $\int \frac{\coth(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=19

$$i\log(\sinh(x)+i)-i\log(\sinh(x))$$

```
[Out] (-I)*Log[Sinh[x]] + I*Log[I + Sinh[x]]
```

Rubi [A] time = 0.0324123, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2707, 36, 29, 31}

$$i\log(\sinh(x)+i)-i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(I + Sinh[x]),x]
```

```
[Out] (-I)*Log[Sinh[x]] + I*Log[I + Sinh[x]]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{i+\sinh(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x(i+x)}\,dx, x, \sinh(x)\right)\\ &= -\left(i\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh(x)\right)\right)+i\,\text{Subst}\left(\int \frac{1}{i+x}\,dx, x, \sinh(x)\right)\\ &= -i\log(\sinh(x))+i\log(i+\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0082583, size = 19, normalized size = 1.

$$i\log(\sinh(x)+i)-i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(I + Sinh[x]),x]
```

```
[Out] (-I)*Log[Sinh[x]] + I*Log[I + Sinh[x]]
```

Maple [A] time = 0.022, size = 17, normalized size = 0.9

$$-i\ln(\sinh(x)) + i\ln(i + \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(I+sinh(x)),x)
```

```
[Out] -I*ln(sinh(x))+I*ln(I+sinh(x))
```

Maxima [B] time = 1.03739, size = 38, normalized size = 2.

$$-i\,\log\left(e^{(-x)} + 1\right) + 2i\,\log\left(e^{(-x)} - i\right) - i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -I*log(e^(-x) + 1) + 2*I*log(e^(-x) - I) - I*log(e^(-x) - 1)
```

Fricas [A] time = 2.04954, size = 54, normalized size = 2.84

$$-i\,\log\left(e^{(2\,x)} - 1\right) + 2i\,\log\left(e^{x} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -I*log(e^(2*x) - 1) + 2*I*log(e^x + I)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x)
```

```
[Out] Exception raised: PolynomialError
```

Giac [A] time = 1.11244, size = 31, normalized size = 1.63

$$-i\log(e^x+1)+2i\log(e^x+i)-i\log(|e^x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -I*log(e^x + 1) + 2*I*log(e^x + I) - I*log(abs(e^x - 1))
```

3.213 $\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=12

$$-\tanh^{-1}(\cosh(x)) + i\coth(x)$$

```
[Out] -ArcTanh[Cosh[x]] + I*Coth[x]
```

Rubi [A] time = 0.0444647, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 3767, 8, 3770}

$$-\tanh^{-1}(\cosh(x)) + i\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(I + Sinh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + I*Coth[x]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{csch}^2(x)\,dx\right) + \int \operatorname{csch}(x)\,dx \\
&= -\tanh^{-1}(\cosh(x)) - \text{Subst}(\int 1\,dx, x, -i\coth(x)) \\
&= -\tanh^{-1}(\cosh(x)) + i\coth(x)
\end{aligned}$$

Mathematica [B] time = 0.0346959, size = 32, normalized size = 2.67

$$\frac{1}{2}i\tanh\left(\frac{x}{2}\right) + \frac{1}{2}i\coth\left(\frac{x}{2}\right) + \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(I + Sinh[x]),x]
```

```
[Out] (I/2)*Coth[x/2] + Log[Tanh[x/2]] + (I/2)*Tanh[x/2]
```

Maple [A] time = 0.026, size = 23, normalized size = 1.9

$$\frac{i}{2}\tanh\left(\frac{x}{2}\right)+\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(I+sinh(x)),x)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/2*I/tanh(1/2*x)+ln(tanh(1/2*x))
```

Maxima [B] time = 1.07307, size = 36, normalized size = 3.

$$-\frac{2i}{e^{(-2x)}-1}-\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*I/(e^(-2*x) - 1) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [B] time = 2.0131, size = 108, normalized size = 9.

$$-\frac{\left(e^{(2x)}-1\right)\log\left(e^{x}+1\right)-\left(e^{(2x)}-1\right)\log\left(e^{x}-1\right)-2i}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -((e^(2*x) - 1)*log(e^x + 1) - (e^(2*x) - 1)*log(e^x - 1) - 2*I)/(e^(2*x) -
 1)
```

Sympy [B] time = 0.217057, size = 22, normalized size = 1.83

$$\log\left(e^{x}-1\right)-\log\left(e^{x}+1\right)+\frac{2i}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(I+sinh(x)),x)
```

```
[Out] log(exp(x) - 1) - log(exp(x) + 1) + 2*I/(exp(2*x) - 1)
```

Giac [B] time = 1.12792, size = 32, normalized size = 2.67

$$\frac{2i}{e^{(2x)} - 1} - \log(e^x + 1) + \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 2*I/(e^(2*x) - 1) - log(e^x + 1) + log(abs(e^x - 1))
```

3.214 $\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=15

$$-\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)$$

```
[Out] -Csch[x] + (I/2)*Csch[x]^2
```

Rubi [A] time = 0.0585608, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2606, 30, 8}

$$-\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(I + Sinh[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx &= -\left(i\int \coth(x)\text{csch}^2(x)\,dx\right)+\int \coth(x)\text{csch}(x)\,dx\\ &= -(i\,\text{Subst}(\int 1\,dx,x,-i\text{csch}(x)))-i\,\text{Subst}(\int x\,dx,x,-i\text{csch}(x))\\ &= -\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)\end{aligned}$$

Mathematica [A] time = 0.0114535, size = 15, normalized size = 1.

$$-\operatorname{csch}(x) + \frac{1}{2} i \operatorname{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(I + Sinh[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2
```

Maple [B] time = 0.038, size = 34, normalized size = 2.3

$$\frac{1}{2}\tanh\left(\frac{x}{2}\right) + \frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} - \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(I+sinh(x)),x)
```

```
[Out] 1/2*tanh(1/2*x)+1/8*I*tanh(1/2*x)^2+1/8*I/tanh(1/2*x)^2-1/2/tanh(1/2*x)
```

Maxima [B] time = 1.04582, size = 90, normalized size = 6.

$$\frac{2\,e^{(-x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{2\,e^{(-3\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*I*e^(-2*x)/(2*e^(-2*x) - e^(-4*x)
- 1) - 2*e^(-3*x)/(2*e^(-2*x) - e^(-4*x) - 1)
```

Fricas [B] time = 1.98198, size = 84, normalized size = 5.6

$$-\frac{2\,e^{(3\,x)} - 2i\,e^{(2\,x)} - 2\,e^{x}}{e^{(4\,x)} - 2\,e^{(2\,x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(2*e^(3*x) - 2*I*e^(2*x) - 2*e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [B] time = 0.256396, size = 32, normalized size = 2.13

$$\frac{-2e^{3x} + 2ie^{2x} + 2e^{x}}{e^{4x} - 2e^{2x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(I+sinh(x)),x)
```

```
[Out] (-2*exp(3*x) + 2*I*exp(2*x) + 2*exp(x))/(exp(4*x) - 2*exp(2*x) + 1)
```

Giac [B] time = 1.14897, size = 32, normalized size = 2.13

$$\frac{2\,e^{(-x)} - 2\,e^{x} + 2i}{\left(e^{(-x)} - e^{x}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] (2*e^(-x) - 2*e^x + 2*I)/(e^(-x) - e^x)^2
```

3.215 $\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=26

$$\frac{1}{3}i\coth^3(x)-\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\text{csch}(x)$$

```
[Out] -ArcTanh[Cosh[x]]/2 + (I/3)*Coth[x]^3 - (Coth[x]*Csch[x])/2
```

Rubi [A] time = 0.0779558, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{1}{3}i\coth^3(x)-\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(I + Sinh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/2 + (I/3)*Coth[x]^3 - (Coth[x]*Csch[x])/2
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = -\left(i\int \coth^2(x)\mathrm{csch}^2(x)\,dx\right)+\int \coth^2(x)\mathrm{csch}(x)\,dx$$
$$= -\frac{1}{2}\coth(x)\mathrm{csch}(x)+\frac{1}{2}\int \mathrm{csch}(x)\,dx-\mathrm{Subst}\left(\int x^2\,dx,x,i\coth(x)\right)$$
$$= -\frac{1}{2}\tanh^{-1}(\cosh(x))+\frac{1}{3}i\coth^3(x)-\frac{1}{2}\coth(x)\mathrm{csch}(x)$$

Mathematica [B] time = 0.0366139, size = 100, normalized size = 3.85

$$\frac{1}{6}i\tanh\left(\frac{x}{2}\right)+\frac{1}{6}i\coth\left(\frac{x}{2}\right)-\frac{1}{8}\mathrm{csch}^2\left(\frac{x}{2}\right)-\frac{1}{8}\mathrm{sech}^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{24}i\coth\left(\frac{x}{2}\right)\mathrm{csch}^2\left(\frac{x}{2}\right)-\frac{1}{24}i\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(I + Sinh[x]),x]
```

```
[Out] (I/6)*Coth[x/2] - Csch[x/2]^2/8 + (I/24)*Coth[x/2]*Csch[x/2]^2 + Log[Tanh[x
/2]]/2 - Sech[x/2]^2/8 + (I/6)*Tanh[x/2] - (I/24)*Sech[x/2]^2*Tanh[x/2]
```

Maple [B] time = 0.046, size = 59, normalized size = 2.3

$$\frac{i}{8}\tanh\left(\frac{x}{2}\right)+\frac{i}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+\frac{i}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}+\frac{1}{2}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(I+sinh(x)),x)
```

```
[Out] 1/8*I*tanh(1/2*x)+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2-1/8/tanh(1/2*x)^2+
1/8*I/tanh(1/2*x)+1/24*I/tanh(1/2*x)^3+1/2*ln(tanh(1/2*x))
```

Maxima [B] time = 1.05202, size = 82, normalized size = 3.15

$$\frac{3\,e^{(-x)}-6i\,e^{(-4\,x)}-3\,e^{(-5\,x)}-2i}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*e^(-x) - 6*I*e^(-4*x) - 3*e^(-5*x) - 2*I)/(3*e^(-2*x) - 3*e^(-4*x) +
 e^(-6*x) - 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Fricas [B] time = 2.01305, size = 263, normalized size = 10.12

$$-\frac{3\left(e^{(6\,x)}-3\,e^{(4\,x)}+3\,e^{(2\,x)}-1\right)\log\left(e^x+1\right)-3\left(e^{(6\,x)}-3\,e^{(4\,x)}+3\,e^{(2\,x)}-1\right)\log\left(e^x-1\right)+6\,e^{(5\,x)}-12i\,e^{(4\,x)}-6\,e^x-4i}{6\left(e^{(6\,x)}-3\,e^{(4\,x)}+3\,e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)*log(e^x + 1) - 3*(e^(6*x) - 3
*e^(4*x) + 3*e^(2*x) - 1)*log(e^x - 1) + 6*e^(5*x) - 12*I*e^(4*x) - 6*e^x -
 4*I)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [B] time = 0.478784, size = 58, normalized size = 2.23

$$\frac{-e^{5x}+2ie^{4x}+e^{x}+\frac{2i}{3}}{e^{6x}-3e^{4x}+3e^{2x}-1}+\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(I+sinh(x)),x)
```

```
[Out] (-exp(5*x) + 2*I*exp(4*x) + exp(x) + 2*I/3)/(exp(6*x) - 3*exp(4*x) + 3*exp(
2*x) - 1) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Giac [B] time = 1.15295, size = 59, normalized size = 2.27

$$-\frac{3\,e^{(5\,x)}-6i\,e^{(4\,x)}-3\,e^{x}-2i}{3\,\left(e^{(2\,x)}-1\right)^{3}}-\frac{1}{2}\,\log\left(e^{x}+1\right)+\frac{1}{2}\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/3*(3*e^(5*x) - 6*I*e^(4*x) - 3*e^x - 2*I)/(e^(2*x) - 1)^3 - 1/2*log(e^x
+ 1) + 1/2*log(abs(e^x - 1))
```

3.216 $\int \frac{\coth^5(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=23

$$\frac{1}{4}i\coth^4(x) - \frac{\text{csch}^3(x)}{3} - \text{csch}(x)$$

```
[Out] (I/4)*Coth[x]^4 - Csch[x] - Csch[x]^3/3
```

Rubi [A] time = 0.0768823, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2607, 30, 2606}

$$\frac{1}{4}i\coth^4(x) - \frac{\text{csch}^3(x)}{3} - \text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(I + Sinh[x]),x]
```

```
[Out] (I/4)*Coth[x]^4 - Csch[x] - Csch[x]^3/3
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = -\left(i\int \coth^3(x)\text{csch}^2(x)\,dx\right)+\int \coth^3(x)\text{csch}(x)\,dx$$
$$= i\,\text{Subst}\left(\int x^3\,dx, x, i\coth(x)\right)+i\,\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, -i\text{csch}(x)\right)$$
$$= \frac{1}{4}i\coth^4(x)-\text{csch}(x)-\frac{\text{csch}^3(x)}{3}$$

Mathematica [A] time = 0.0096195, size = 33, normalized size = 1.43

$$\frac{1}{4}i\text{csch}^4(x)-\frac{\text{csch}^3(x)}{3}+\frac{1}{2}i\text{csch}^2(x)-\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(I + Sinh[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 + (I/4)*Csch[x]^4
```

Maple [B] time = 0.066, size = 68, normalized size = 3.

$$\frac{3}{8}\tanh\left(\frac{x}{2}\right)+\frac{i}{64}\left(\tanh\left(\frac{x}{2}\right)\right)^4+\frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{i}{16}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{i}{16}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}-\frac{3}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{1}{24}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(I+sinh(x)),x)
```

```
[Out] 3/8*tanh(1/2*x)+1/64*I*tanh(1/2*x)^4+1/24*tanh(1/2*x)^3+1/16*I*tanh(1/2*x)^
2+1/16*I/tanh(1/2*x)^2-3/8/tanh(1/2*x)-1/24/tanh(1/2*x)^3+1/64*I/tanh(1/2*x
)^4
```

Maxima [B] time = 1.06873, size = 277, normalized size = 12.04

$$\frac{2\,e^{(-x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{2i\,e^{(-2\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{10\,e^{(-3\,x)}}{3\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-2*
x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 10/3*e^(-3*x)/(4
*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 10/3*e^(-5*x)/(4*e^(-
2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-6*x)/(4*e^(-2*x) -
 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*e^(-7*x)/(4*e^(-2*x) - 6*e^(-4
*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Fricas [B] time = 1.97264, size = 176, normalized size = 7.65

$$-\frac{6\,e^{(7\,x)}-6i\,e^{(6\,x)}-10\,e^{(5\,x)}+10\,e^{(3\,x)}-6i\,e^{(2\,x)}-6\,e^{x}}{3\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*e^(7*x) - 6*I*e^(6*x) - 10*e^(5*x) + 10*e^(3*x) - 6*I*e^(2*x) - 6*e
^x)/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [B] time = 0.754139, size = 71, normalized size = 3.09

$$\frac{-2e^{7x}+2ie^{6x}+\frac{10e^{5x}}{3}-\frac{10e^{3x}}{3}+2ie^{2x}+2e^{x}}{e^{8x}-4e^{6x}+6e^{4x}-4e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(I+sinh(x)),x)
```

```
[Out] (-2*exp(7*x) + 2*I*exp(6*x) + 10*exp(5*x)/3 - 10*exp(3*x)/3 + 2*I*exp(2*x)
+ 2*exp(x))/(exp(8*x) - 4*exp(6*x) + 6*exp(4*x) - 4*exp(2*x) + 1)
```

Giac [B] time = 1.13416, size = 69, normalized size = 3.

$$\frac{6\left(e^{(-x)}-e^{x}\right)^{3}+6i\left(e^{(-x)}-e^{x}\right)^{2}+8\,e^{(-x)}-8\,e^{x}+12i}{3\left(e^{(-x)}-e^{x}\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/3*(6*(e^(-x) - e^x)^3 + 6*I*(e^(-x) - e^x)^2 + 8*e^(-x) - 8*e^x + 12*I)/(
e^(-x) - e^x)^4
```

3.217 $\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=36

$$\frac{1}{5}i\coth^5(x)-\frac{3}{8}\tanh^{-1}(\cosh(x))-\frac{1}{4}\coth^3(x)\mathbf{csch}(x)-\frac{3}{8}\coth(x)\mathbf{csch}(x)$$

```
[Out] (-3*ArcTanh[Cosh[x]])/8 + (I/5)*Coth[x]^5 - (3*Coth[x]*Csch[x])/8 - (Coth[x
]^3*Csch[x])/4
```

Rubi [A] time = 0.0969328, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{1}{5}i\coth^5(x)-\frac{3}{8}\tanh^{-1}(\cosh(x))-\frac{1}{4}\coth^3(x)\mathbf{csch}(x)-\frac{3}{8}\coth(x)\mathbf{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(I + Sinh[x]),x]
```

```
[Out] (-3*ArcTanh[Cosh[x]])/8 + (I/5)*Coth[x]^5 - (3*Coth[x]*Csch[x])/8 - (Coth[x
]^3*Csch[x])/4
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx &= -\left(i\int \coth^4(x)\text{csch}^2(x)\,dx\right)+\int \coth^4(x)\text{csch}(x)\,dx\\
&= -\frac{1}{4}\coth^3(x)\text{csch}(x)+\frac{3}{4}\int \coth^2(x)\text{csch}(x)\,dx+\text{Subst}\left(\int x^4\,dx,x,i\coth(x)\right)\\
&= \frac{1}{5}i\coth^5(x)-\frac{3}{8}\coth(x)\text{csch}(x)-\frac{1}{4}\coth^3(x)\text{csch}(x)+\frac{3}{8}\int \text{csch}(x)\,dx\\
&= -\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{5}i\coth^5(x)-\frac{3}{8}\coth(x)\text{csch}(x)-\frac{1}{4}\coth^3(x)\text{csch}(x)
\end{aligned}
$$

Mathematica [B] time = 0.0402153, size = 164, normalized size = 4.56

$$
\frac{1}{10}i\tanh\left(\frac{x}{2}\right)+\frac{1}{10}i\coth\left(\frac{x}{2}\right)-\frac{1}{64}\text{csch}^4\left(\frac{x}{2}\right)-\frac{5}{32}\text{csch}^2\left(\frac{x}{2}\right)+\frac{1}{64}\text{sech}^4\left(\frac{x}{2}\right)-\frac{5}{32}\text{sech}^2\left(\frac{x}{2}\right)+\frac{3}{8}\log\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{160}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(I + Sinh[x]),x]
```

```
[Out] (I/10)*Coth[x/2] - (5*Csch[x/2]^2)/32 + ((7*I)/160)*Coth[x/2]*Csch[x/2]^2 -
 Csch[x/2]^4/64 + (I/160)*Coth[x/2]*Csch[x/2]^4 + (3*Log[Tanh[x/2]])/8 - (5
*Sech[x/2]^2)/32 + Sech[x/2]^4/64 + (I/10)*Tanh[x/2] - ((7*I)/160)*Sech[x/2
]^2*Tanh[x/2] + (I/160)*Sech[x/2]^4*Tanh[x/2]
```

Maple [B] time = 0.07, size = 93, normalized size = 2.6

$$
\frac{i}{16}\tanh\left(\frac{x}{2}\right)+\frac{i}{160}\left(\tanh\left(\frac{x}{2}\right)\right)^5+\frac{1}{64}\left(\tanh\left(\frac{x}{2}\right)\right)^4+\frac{i}{32}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{i}{16}\left(\tanh\left(\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(I+sinh(x)),x)
```

```
[Out] 1/16*I*tanh(1/2*x)+1/160*I*tanh(1/2*x)^5+1/64*tanh(1/2*x)^4+1/32*I*tanh(1/2
*x)^3+1/8*tanh(1/2*x)^2-1/8/tanh(1/2*x)^2+1/16*I/tanh(1/2*x)+1/160*I/tanh(1
/2*x)^5+1/32*I/tanh(1/2*x)^3+3/8*ln(tanh(1/2*x))-1/64/tanh(1/2*x)^4
```

Maxima [B] time = 1.08917, size = 123, normalized size = 3.42

$$
\frac{25\,e^{(-x)}-10\,e^{(-3\,x)}-80i\,e^{(-4\,x)}+10\,e^{(-7\,x)}-40i\,e^{(-8\,x)}-25\,e^{(-9\,x)}-8i}{20\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}-\frac{3}{8}\log\left(e^{(-x)}+1\right)+\frac{3}{8}\log\left(e^{(-x)}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/20*(25*e^(-x) - 10*e^(-3*x) - 80*I*e^(-4*x) + 10*e^(-7*x) - 40*I*e^(-8*x)
 - 25*e^(-9*x) - 8*I)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
+ e^(-10*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

Fricas [B] time = 2.13792, size = 437, normalized size = 12.14

$$-\frac{15\left(e^{(10x)} - 5e^{(8x)} + 10e^{(6x)} - 10e^{(4x)} + 5e^{(2x)} - 1\right)\log\left(e^{x} + 1\right) - 15\left(e^{(10x)} - 5e^{(8x)} + 10e^{(6x)} - 10e^{(4x)} + 5e^{(2x)} - 1\right)}{40\left(e^{(10x)} - 5e^{(8x)} + 10e^{(6x)} - 10e^{(4x)} + 5e^{(2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/40*(15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*x) - 1)*
log(e^x + 1) - 15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*
x) - 1)*log(e^x - 1) + 50*e^(9*x) - 80*I*e^(8*x) - 20*e^(7*x) - 160*I*e^(4*
x) + 20*e^(3*x) - 50*e^x - 16*I)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^
(4*x) + 5*e^(2*x) - 1)
```

Sympy [B] time = 1.37699, size = 104, normalized size = 2.89

$$\frac{3\log\left(e^{x} - 1\right)}{8} - \frac{3\log\left(e^{x} + 1\right)}{8} + \frac{-\frac{5e^{9x}}{4} + 2ie^{8x} + \frac{e^{7x}}{2} + 4ie^{4x} - \frac{e^{3x}}{2} + \frac{5e^{x}}{4} + \frac{2i}{5}}{e^{10x} - 5e^{8x} + 10e^{6x} - 10e^{4x} + 5e^{2x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(I+sinh(x)),x)
```

```
[Out] 3*log(exp(x) - 1)/8 - 3*log(exp(x) + 1)/8 + (-5*exp(9*x)/4 + 2*I*exp(8*x) +
 exp(7*x)/2 + 4*I*exp(4*x) - exp(3*x)/2 + 5*exp(x)/4 + 2*I/5)/(exp(10*x) -
5*exp(8*x) + 10*exp(6*x) - 10*exp(4*x) + 5*exp(2*x) - 1)
```

Giac [B] time = 1.12559, size = 84, normalized size = 2.33

$$-\frac{25e^{(9x)} - 40ie^{(8x)} - 10e^{(7x)} - 80ie^{(4x)} + 10e^{(3x)} - 25e^{x} - 8i}{20\left(e^{(2x)} - 1\right)^{5}} - \frac{3}{8}\log\left(e^{x} + 1\right) + \frac{3}{8}\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/20*(25*e^(9*x) - 40*I*e^(8*x) - 10*e^(7*x) - 80*I*e^(4*x) + 10*e^(3*x) -
 25*e^x - 8*I)/(e^(2*x) - 1)^5 - 3/8*log(e^x + 1) + 3/8*log(abs(e^x - 1))
```

3.218 $\int \frac{\tanh^4(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=47

$$\frac{2\tanh^7(x)}{7} - \frac{\tanh^5(x)}{5} + \frac{2}{7}i\text{sech}^7(x) - \frac{4}{5}i\text{sech}^5(x) + \frac{2}{3}i\text{sech}^3(x)$$

```
[Out] ((2*I)/3)*Sech[x]^3 - ((4*I)/5)*Sech[x]^5 + ((2*I)/7)*Sech[x]^7 - Tanh[x]^5
/5 + (2*Tanh[x]^7)/7
```

Rubi [A] time = 0.122853, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2711, 2607, 14, 2606, 270, 30}

$$\frac{2\tanh^7(x)}{7} - \frac{\tanh^5(x)}{5} + \frac{2}{7}i\text{sech}^7(x) - \frac{4}{5}i\text{sech}^5(x) + \frac{2}{3}i\text{sech}^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] ((2*I)/3)*Sech[x]^3 - ((4*I)/5)*Sech[x]^5 + ((2*I)/7)*Sech[x]^7 - Tanh[x]^5
/5 + (2*Tanh[x]^7)/7
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = \int \left(-\text{sech}^4(x)\tanh^4(x) - 2i\text{sech}^3(x)\tanh^5(x) + \text{sech}^2(x)\tanh^6(x)\right)dx$$
$$= -\left(2i\int \text{sech}^3(x)\tanh^5(x)\,dx\right) - \int \text{sech}^4(x)\tanh^4(x)\,dx + \int \text{sech}^2(x)\tanh^6(x)\,dx$$
$$= i\,\text{Subst}\left(\int x^6\,dx, x, i\tanh(x)\right) + i\,\text{Subst}\left(\int x^4\left(1+x^2\right)dx, x, i\tanh(x)\right) + 2i\,\text{Subst}\left(\int x^2\left(-\right.\right.$$
$$= \frac{\tanh^7(x)}{7} + i\,\text{Subst}\left(\int \left(x^4+x^6\right)dx, x, i\tanh(x)\right) + 2i\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \text{sech}(\right.$$
$$= \frac{2}{3}i\text{sech}^3(x) - \frac{4}{5}i\text{sech}^5(x) + \frac{2}{7}i\text{sech}^7(x) - \frac{\tanh^5(x)}{5} + \frac{2\tanh^7(x)}{7}$$

Mathematica [B] time = 0.146499, size = 112, normalized size = 2.38

$$-\frac{1232\sinh(x) + 824\sinh(2x) - 1896\sinh(3x) + 412\sinh(4x) + 72\sinh(5x) + 1442i\cosh(x) - 1664i\cosh(2x) + 309}{13440\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^7\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] -(-672*I + (1442*I)*Cosh[x] - (1664*I)*Cosh[2*x] + (309*I)*Cosh[3*x] + (288
*I)*Cosh[4*x] - (103*I)*Cosh[5*x] + 1232*Sinh[x] + 824*Sinh[2*x] - 1896*Sin
h[3*x] + 412*Sinh[4*x] + 72*Sinh[5*x])/(13440*(Cosh[x/2] - I*Sinh[x/2])^7*(
Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] time = 0.093, size = 116, normalized size = 2.5

$$-\frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2} + \frac{1}{12}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-3} + \frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1} + 2i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-6} - i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4} - \frac{i}{8}\left(\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(I+sinh(x))^2,x)
```

```
[Out] -1/8*I/(tanh(1/2*x)-I)^2+1/12/(tanh(1/2*x)-I)^3+1/8/(tanh(1/2*x)-I)+2*I/(ta
nh(1/2*x)+I)^6-I/(tanh(1/2*x)+I)^4-1/8*I/(tanh(1/2*x)+I)^2+4/7/(tanh(1/2*x)
+I)^7-12/5/(tanh(1/2*x)+I)^5-1/12/(tanh(1/2*x)+I)^3-1/8/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.11311, size = 774, normalized size = 16.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 72*I*e^(-x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) -
 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-1
0*x) + 105) - 264*e^(-2*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) -
1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9
*x) + 105*e^(-10*x) + 105) - 136*I*e^(-3*x)/(420*I*e^(-x) - 315*e^(-2*x) +
840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8
*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 28*e^(-4*x)/(420*I*e^(-x) - 3
15*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*
x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) - 168*I*e^(-5*x)/
(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*
x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105)
- 280*e^(-6*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x
) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^
(-10*x) + 105) - 280*I*e^(-7*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*
x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*
e^(-9*x) + 105*e^(-10*x) + 105) + 210*e^(-8*x)/(420*I*e^(-x) - 315*e^(-2*x)
 + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^
(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 18/(420*I*e^(-x) - 315*e^(
-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 3
15*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105)
```

Fricas [B] time = 2.00328, size = 351, normalized size = 7.47

$$-\frac{210\,e^{(8\,x)}+280i\,e^{(7\,x)}-280\,e^{(6\,x)}+168i\,e^{(5\,x)}+28\,e^{(4\,x)}+136i\,e^{(3\,x)}-264\,e^{(2\,x)}-72i\,e^{x}+18}{105\,e^{(10\,x)}+420i\,e^{(9\,x)}-315\,e^{(8\,x)}+840i\,e^{(7\,x)}-1470\,e^{(6\,x)}-1470\,e^{(4\,x)}-840i\,e^{(3\,x)}-315\,e^{(2\,x)}-420i\,e^{x}+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(210*e^(8*x) + 280*I*e^(7*x) - 280*e^(6*x) + 168*I*e^(5*x) + 28*e^(4*x) +
136*I*e^(3*x) - 264*e^(2*x) - 72*I*e^x + 18)/(105*e^(10*x) + 420*I*e^(9*x)
- 315*e^(8*x) + 840*I*e^(7*x) - 1470*e^(6*x) - 1470*e^(4*x) - 840*I*e^(3*x)
 - 315*e^(2*x) - 420*I*e^x + 105)
```

Sympy [B] time = 2.53294, size = 139, normalized size = 2.96

$$\frac{-2e^{8x}-\frac{8ie^{7x}}{3}+\frac{8e^{6x}}{3}-\frac{8ie^{5x}}{5}-\frac{4e^{4x}}{15}-\frac{136ie^{3x}}{105}+\frac{88e^{2x}}{35}+\frac{24ie^{x}}{35}-\frac{6}{35}}{e^{10x}+4ie^{9x}-3e^{8x}+8ie^{7x}-14e^{6x}-14e^{4x}-8ie^{3x}-3e^{2x}-4ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] (-2*exp(8*x) - 8*I*exp(7*x)/3 + 8*exp(6*x)/3 - 8*I*exp(5*x)/5 - 4*exp(4*x)/
15 - 136*I*exp(3*x)/105 + 88*exp(2*x)/35 + 24*I*exp(x)/35 - 6/35)/(exp(10*x
) + 4*I*exp(9*x) - 3*exp(8*x) + 8*I*exp(7*x) - 14*exp(6*x) - 14*exp(4*x) -
8*I*exp(3*x) - 3*exp(2*x) - 4*I*exp(x) + 1)
```

Giac [B] time = 1.16407, size = 88, normalized size = 1.87

$$-\frac{-6i\,e^{(2\,x)} - 9\,e^{x} + 5i}{24\,(e^{x} - i)^{3}} - \frac{210i\,e^{(6\,x)} - 105\,e^{(5\,x)} + 175i\,e^{(4\,x)} - 910\,e^{(3\,x)} - 756i\,e^{(2\,x)} + 427\,e^{x} + 31i}{840\,(e^{x} + i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/24*(-6*I*e^(2*x) - 9*e^x + 5*I)/(e^x - I)^3 - 1/840*(210*I*e^(6*x) - 105
*e^(5*x) + 175*I*e^(4*x) - 910*e^(3*x) - 756*I*e^(2*x) + 427*e^x + 31*I)/(e
^x + I)^7
```

3.219 $\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{i}{16(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{4(\sinh(x)+i)^2}+\frac{i}{12(\sinh(x)+i)^3}-\frac{1}{8}i\tan^{-1}(\sinh(x))$$

```
[Out] (-I/8)*ArcTan[Sinh[x]] - (I/16)/(I - Sinh[x]) + (I/12)/(I + Sinh[x])^3 - 1/
(4*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x])
```

Rubi [A] time = 0.0645879, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2707, 88, 203}

$$-\frac{i}{16(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{4(\sinh(x)+i)^2}+\frac{i}{12(\sinh(x)+i)^3}-\frac{1}{8}i\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-I/8)*ArcTan[Sinh[x]] - (I/16)/(I - Sinh[x]) + (I/12)/(I + Sinh[x])^3 - 1/
(4*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx &= \text{Subst}\left(\int \frac{x^3}{(i-x)^2(i+x)^4}\,dx,x,\sinh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{16(-i+x)^2}-\frac{i}{4(i+x)^4}+\frac{1}{2(i+x)^3}+\frac{3i}{16(i+x)^2}-\frac{i}{8\left(1+x^2\right)}\right)dx,x,\sinh(x)\right)\\
&= -\frac{i}{16(i-\sinh(x))}+\frac{i}{12(i+\sinh(x))^3}-\frac{1}{4(i+\sinh(x))^2}-\frac{3i}{16(i+\sinh(x))}-\frac{1}{8}i\,\text{Subst}\left(\int \frac{1}{1+x^2}\,d
\right.\\
&= -\frac{1}{8}i\tan^{-1}(\sinh(x))-\frac{i}{16(i-\sinh(x))}+\frac{i}{12(i+\sinh(x))^3}-\frac{1}{4(i+\sinh(x))^2}-\frac{3i}{16(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.0997977, size = 52, normalized size = 0.79

$$\frac{1}{48}\left(\frac{2\left(-3i\sinh^3(x)-6\sinh^2(x)-7i\sinh(x)+2\right)}{(\sinh(x)-i)(\sinh(x)+i)^3}-6i\tan^{-1}(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] ((-6*I)*ArcTan[Sinh[x]] + (2*(2 - (7*I)*Sinh[x] - 6*Sinh[x]^2 - (3*I)*Sinh[
x]^3))/((-I + Sinh[x])*(I + Sinh[x])^3))/48
```

Maple [B] time = 0.078, size = 114, normalized size = 1.7

$$-\frac{i}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-2}-\frac{1}{8}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+2i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-5}-\frac{2i}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}-\frac{i}{8}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(I+sinh(x))^2,x)
```

```
[Out] -1/8*I/(tanh(1/2*x)-I)+1/8/(tanh(1/2*x)-I)^2-1/8*ln(tanh(1/2*x)-I)+2*I/(tan
h(1/2*x)+I)^5-2/3*I/(tanh(1/2*x)+I)^3-1/8*I/(tanh(1/2*x)+I)+2/3/(tanh(1/2*x
)+I)^6-2/(tanh(1/2*x)+I)^4-1/8/(tanh(1/2*x)+I)^2+1/8*ln(tanh(1/2*x)+I)
```

Maxima [B] time = 1.11928, size = 155, normalized size = 2.35

$$\frac{-3ie^{(-x)}-12e^{(-2x)}-19ie^{(-3x)}+40e^{(-4x)}+19ie^{(-5x)}-12e^{(-6x)}+3ie^{(-7x)}}{48ie^{(-x)}-48e^{(-2x)}+48ie^{(-3x)}-120e^{(-4x)}-48ie^{(-5x)}-48e^{(-6x)}-48ie^{(-7x)}+12e^{(-8x)}+12}-\frac{1}{8}\log\left(e^{(-x)}+i\right)+\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (-3*I*e^(-x) - 12*e^(-2*x) - 19*I*e^(-3*x) + 40*e^(-4*x) + 19*I*e^(-5*x) -
12*e^(-6*x) + 3*I*e^(-7*x))/(48*I*e^(-x) - 48*e^(-2*x) + 48*I*e^(-3*x) - 12
0*e^(-4*x) - 48*I*e^(-5*x) - 48*e^(-6*x) - 48*I*e^(-7*x) + 12*e^(-8*x) + 12
) - 1/8*log(e^(-x) + I) + 1/8*log(e^(-x) - I)
```

Fricas [B] time = 2.02832, size = 621, normalized size = 9.41

$$\frac{\left(3e^{(8x)}+12ie^{(7x)}-12e^{(6x)}+12ie^{(5x)}-30e^{(4x)}-12ie^{(3x)}-12e^{(2x)}-12ie^{x}+3\right)\log\left(e^{x}+i\right)-\left(3e^{(8x)}+12ie^{(7x)}-12\right.}{24e^{(8x)}+96ie^{(7x)}-96e^{(6x)}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((3*e^(8*x) + 12*I*e^(7*x) - 12*e^(6*x) + 12*I*e^(5*x) - 30*e^(4*x) - 12*I*
e^(3*x) - 12*e^(2*x) - 12*I*e^x + 3)*log(e^x + I) - (3*e^(8*x) + 12*I*e^(7*
x) - 12*e^(6*x) + 12*I*e^(5*x) - 30*e^(4*x) - 12*I*e^(3*x) - 12*e^(2*x) - 1
```

```
2*I*e^x + 3)*log(e^x - I) - 6*I*e^(7*x) - 24*e^(6*x) - 38*I*e^(5*x) + 80*e^
(4*x) + 38*I*e^(3*x) - 24*e^(2*x) + 6*I*e^x)/(24*e^(8*x) + 96*I*e^(7*x) - 9
6*e^(6*x) + 96*I*e^(5*x) - 240*e^(4*x) - 96*I*e^(3*x) - 96*e^(2*x) - 96*I*e
^x + 24)
```

Sympy [B] time = 1.5879, size = 129, normalized size = 1.95

$$\frac{-\frac{ie^{7x}}{4}-e^{6x}-\frac{19ie^{5x}}{12}+\frac{10e^{4x}}{3}+\frac{19ie^{3x}}{12}-e^{2x}+\frac{ie^{x}}{4}}{e^{8x}+4ie^{7x}-4e^{6x}+4ie^{5x}-10e^{4x}-4ie^{3x}-4e^{2x}-4ie^{x}+1}-\frac{\log\left(e^{x}-i\right)}{8}+\frac{\log\left(e^{x}+i\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] (-I*exp(7*x)/4 - exp(6*x) - 19*I*exp(5*x)/12 + 10*exp(4*x)/3 + 19*I*exp(3*x
)/12 - exp(2*x) + I*exp(x)/4)/(exp(8*x) + 4*I*exp(7*x) - 4*exp(6*x) + 4*I*e
xp(5*x) - 10*exp(4*x) - 4*I*exp(3*x) - 4*exp(2*x) - 4*I*exp(x) + 1) - log(e
xp(x) - I)/8 + log(exp(x) + I)/8
```

Giac [B] time = 1.18423, size = 138, normalized size = 2.09

$$\frac{e^{(-x)}-e^{x}}{16\left(e^{(-x)}-e^{x}+2i\right)}-\frac{11\left(e^{(-x)}-e^{x}\right)^{3}-102i\left(e^{(-x)}-e^{x}\right)^{2}-180\,e^{(-x)}+180\,e^{x}+104i}{96\left(e^{(-x)}-e^{x}-2i\right)^{3}}+\frac{1}{16}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{16}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(e^(-x) - e^x)/(e^(-x) - e^x + 2*I) - 1/96*(11*(e^(-x) - e^x)^3 - 102*
I*(e^(-x) - e^x)^2 - 180*e^(-x) + 180*e^x + 104*I)/(e^(-x) - e^x - 2*I)^3 +
 1/16*log(-e^(-x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

3.220 $\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^5(x)}{5}-\frac{\tanh^3(x)}{3}-\frac{2}{5}i\text{sech}^5(x)+\frac{2}{3}i\text{sech}^3(x)$$

```
[Out] ((2*I)/3)*Sech[x]^3 - ((2*I)/5)*Sech[x]^5 - Tanh[x]^3/3 + (2*Tanh[x]^5)/5
```

Rubi [A] time = 0.118239, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2711, 2607, 14, 2606, 30}

$$\frac{2\tanh^5(x)}{5}-\frac{\tanh^3(x)}{3}-\frac{2}{5}i\text{sech}^5(x)+\frac{2}{3}i\text{sech}^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] ((2*I)/3)*Sech[x]^3 - ((2*I)/5)*Sech[x]^5 - Tanh[x]^3/3 + (2*Tanh[x]^5)/5
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = -\int \left(\operatorname{sech}^4(x)\tanh^2(x) + 2i\operatorname{sech}^3(x)\tanh^3(x) - \operatorname{sech}^2(x)\tanh^4(x)\right)\,dx$$
$$= -\left(2i\int \operatorname{sech}^3(x)\tanh^3(x)\,dx\right) - \int \operatorname{sech}^4(x)\tanh^2(x)\,dx + \int \operatorname{sech}^2(x)\tanh^4(x)\,dx$$
$$= -\left(i\,\mathrm{Subst}\left(\int x^4\,dx, x, i\tanh(x)\right)\right) - i\,\mathrm{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, i\tanh(x)\right) - 2i\,\mathrm{Subst}\left(\int x^2\left(-\right.\right.$$
$$= \frac{\tanh^5(x)}{5} - i\,\mathrm{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, i\tanh(x)\right) - 2i\,\mathrm{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \operatorname{sech}(x)\right)$$
$$= \frac{2}{3}i\operatorname{sech}^3(x) - \frac{2}{5}i\operatorname{sech}^5(x) - \frac{\tanh^3(x)}{3} + \frac{2\tanh^5(x)}{5}$$

Mathematica [B] time = 0.0923001, size = 84, normalized size = 2.27

$$\frac{140\sinh(x) - 44\sinh(2x) - 4\sinh(3x) - 55i\cosh(x) - 16i\cosh(2x) + 11i\cosh(3x) + 80i}{240\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (80*I - (55*I)*Cosh[x] - (16*I)*Cosh[2*x] + (11*I)*Cosh[3*x] + 140*Sinh[x]
- 44*Sinh[2*x] - 4*Sinh[3*x])/(240*(Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2] +
 I*Sinh[x/2]))
```

Maple [B] time = 0.059, size = 70, normalized size = 1.9

$$\frac{1}{4}\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1} + 2i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4} - \frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2} + \frac{4}{5}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-5} - \frac{5}{3}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3} - \frac{1}{4}\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(I+sinh(x))^2,x)
```

```
[Out] 1/4/(tanh(1/2*x)-I)+2*I/(tanh(1/2*x)+I)^4-1/2*I/(tanh(1/2*x)+I)^2+4/5/(tanh
(1/2*x)+I)^5-5/3/(tanh(1/2*x)+I)^3-1/4/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.08837, size = 266, normalized size = 7.19

$$\frac{8i\,e^{(-x)}}{60i\,e^{(-x)} - 75\,e^{(-2\,x)} - 75\,e^{(-4\,x)} - 60i\,e^{(-5\,x)} + 15\,e^{(-6\,x)} + 15} - \frac{40\,e^{(-2\,x)}}{60i\,e^{(-x)} - 75\,e^{(-2\,x)} - 75\,e^{(-4\,x)} - 60i\,e^{(-5\,x)} + 15\,e^{(-6\,x)} + 15} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 8*I*e^(-x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^
(-6*x) + 15) - 40*e^(-2*x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*
e^(-5*x) + 15*e^(-6*x) + 15) - 40*I*e^(-3*x)/(60*I*e^(-x) - 75*e^(-2*x) - 7
5*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 30*e^(-4*x)/(60*I*e^(-x) -
 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 2/(60*I*e^
```

```
(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15)
```

Fricas [B] time = 2.01259, size = 171, normalized size = 4.62

$$-\frac{30\,e^{(4\,x)} + 40i\,e^{(3\,x)} - 40\,e^{(2\,x)} - 8i\,e^{x} + 2}{15\,e^{(6\,x)} + 60i\,e^{(5\,x)} - 75\,e^{(4\,x)} - 75\,e^{(2\,x)} - 60i\,e^{x} + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(30*e^(4*x) + 40*I*e^(3*x) - 40*e^(2*x) - 8*I*e^x + 2)/(15*e^(6*x) + 60*I*
e^(5*x) - 75*e^(4*x) - 75*e^(2*x) - 60*I*e^x + 15)
```

Sympy [B] time = 0.823135, size = 71, normalized size = 1.92

$$\frac{-2e^{4x} - \frac{8ie^{3x}}{3} + \frac{8e^{2x}}{3} + \frac{8ie^{x}}{15} - \frac{2}{15}}{e^{6x} + 4ie^{5x} - 5e^{4x} - 5e^{2x} - 4ie^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(I+sinh(x))**2,x)
```

```
[Out] (-2*exp(4*x) - 8*I*exp(3*x)/3 + 8*exp(2*x)/3 + 8*I*exp(x)/15 - 2/15)/(exp(6
*x) + 4*I*exp(5*x) - 5*exp(4*x) - 5*exp(2*x) - 4*I*exp(x) + 1)
```

Giac [A] time = 1.13342, size = 55, normalized size = 1.49

$$\frac{i}{4\,(e^{x} - i)} - \frac{15i\,e^{(4\,x)} + 30\,e^{(3\,x)} + 40i\,e^{(2\,x)} - 50\,e^{x} - 7i}{60\,(e^{x} + i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/4*I/(e^x - I) - 1/60*(15*I*e^(4*x) + 30*e^(3*x) + 40*I*e^(2*x) - 50*e^x -
 7*I)/(e^x + I)^5
```

3.221 $\int \frac{\tanh(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=36

$$-\frac{i}{4(\sinh(x)+i)}-\frac{1}{4(\sinh(x)+i)^2}-\frac{1}{4}i\tan^{-1}(\sinh(x))$$

```
[Out] (-I/4)*ArcTan[Sinh[x]] - 1/(4*(I + Sinh[x])^2) - (I/4)/(I + Sinh[x])
```

Rubi [A] time = 0.0365092, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2707, 77, 203}

$$-\frac{i}{4(\sinh(x)+i)}-\frac{1}{4(\sinh(x)+i)^2}-\frac{1}{4}i\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(I + Sinh[x])^2,x]
```

```
[Out] (-I/4)*ArcTan[Sinh[x]] - 1/(4*(I + Sinh[x])^2) - (I/4)/(I + Sinh[x])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{(i+\sinh(x))^2} \, dx &= -\text{Subst}\left(\int \frac{x}{(i-x)(i+x)^3} \, dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{1}{2(i+x)^3}-\frac{i}{4(i+x)^2}+\frac{i}{4\left(1+x^2\right)}\right) dx, x, \sinh(x)\right)\\
&= -\frac{1}{4(i+\sinh(x))^2}-\frac{i}{4(i+\sinh(x))}-\frac{1}{4}i\,\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sinh(x)\right)\\
&= -\frac{1}{4}i\tan^{-1}(\sinh(x))-\frac{1}{4(i+\sinh(x))^2}-\frac{i}{4(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.0303892, size = 29, normalized size = 0.81

$$-\frac{i\left(\sinh(x)+(\sinh(x)+i)^2\tan^{-1}(\sinh(x))\right)}{4(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(I + Sinh[x])^2,x]
```

```
[Out] ((-I/4)*(Sinh[x] + ArcTan[Sinh[x]]*(I + Sinh[x])^2))/(I + Sinh[x])^2
```

Maple [B] time = 0.051, size = 66, normalized size = 1.8

$$-\frac{1}{4}\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+2i\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-3}-\frac{i}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}+\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-4}-\frac{3}{2}\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-2}+\frac{1}{4}\ln\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(I+sinh(x))^2,x)
```

```
[Out] -1/4*ln(tanh(1/2*x)-I)+2*I/(tanh(1/2*x)+I)^3-1/2*I/(tanh(1/2*x)+I)+1/(tanh(
1/2*x)+I)^4-3/2/(tanh(1/2*x)+I)^2+1/4*ln(tanh(1/2*x)+I)
```

Maxima [B] time = 1.07141, size = 82, normalized size = 2.28

$$\frac{-i e^{(-x)}+i e^{(-3x)}}{8i e^{(-x)}-12 e^{(-2x)}-8i e^{(-3x)}+2 e^{(-4x)}+2}-\frac{1}{4}\log\left(e^{(-x)}+i\right)+\frac{1}{4}\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (-I*e^(-x) + I*e^(-3*x))/(8*I*e^(-x) - 12*e^(-2*x) - 8*I*e^(-3*x) + 2*e^(-4
*x) + 2) - 1/4*log(e^(-x) + I) + 1/4*log(e^(-x) - I)
```

Fricas [B] time = 2.06011, size = 284, normalized size = 7.89

$$\frac{\left(e^{(4x)}+4i e^{(3x)}-6 e^{(2x)}-4i e^{x}+1\right)\log\left(e^{x}+i\right)-\left(e^{(4x)}+4i e^{(3x)}-6 e^{(2x)}-4i e^{x}+1\right)\log\left(e^{x}-i\right)-2i e^{(3x)}+2i e^{x}}{4 e^{(4x)}+16i e^{(3x)}-24 e^{(2x)}-16i e^{x}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x + I) - (e^(4*x)
+ 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x - I) - 2*I*e^(3*x) + 2*I*e
^x)/(4*e^(4*x) + 16*I*e^(3*x) - 24*e^(2*x) - 16*I*e^x + 4)
```

Sympy [B] time = 0.486307, size = 60, normalized size = 1.67

$$\frac{-\frac{ie^{3x}}{2}+\frac{ie^{x}}{2}}{e^{4x}+4ie^{3x}-6e^{2x}-4ie^{x}+1}-\frac{\log\left(e^{x}-i\right)}{4}+\frac{\log\left(e^{x}+i\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))**2,x)
```

```
[Out] (-I*exp(3*x)/2 + I*exp(x)/2)/(exp(4*x) + 4*I*exp(3*x) - 6*exp(2*x) - 4*I*ex
p(x) + 1) - log(exp(x) - I)/4 + log(exp(x) + I)/4
```

Giac [B] time = 1.13077, size = 89, normalized size = 2.47

$$-\frac{3\left(e^{(-x)}-e^{x}\right)^{2}-20i\,e^{(-x)}+20i\,e^{x}-12}{16\left(e^{(-x)}-e^{x}-2i\right)^{2}}+\frac{1}{8}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{8}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/16*(3*(e^(-x) - e^x)^2 - 20*I*e^(-x) + 20*I*e^x - 12)/(e^(-x) - e^x - 2*
I)^2 + 1/8*log(-e^(-x) + e^x + 2*I) - 1/8*log(-e^(-x) + e^x - 2*I)
```

3.222 $\int \frac{\coth(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=25

$$-\frac{i}{\sinh(x)+i} - \log(\sinh(x)) + \log(\sinh(x)+i)$$

```
[Out] -Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])
```

Rubi [A] time = 0.0400701, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2707, 44}

$$-\frac{i}{\sinh(x)+i} - \log(\sinh(x)) + \log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(I + Sinh[x])^2,x]
```

```
[Out] -Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{(i+\sinh(x))^2} dx &= \text{Subst}\left(\int \frac{1}{x(i+x)^2} dx, x, \sinh(x)\right) \\
&= \text{Subst}\left(\int \left(-\frac{1}{x} + \frac{i}{(i+x)^2} + \frac{1}{i+x}\right) dx, x, \sinh(x)\right) \\
&= -\log(\sinh(x)) + \log(i+\sinh(x)) - \frac{i}{i+\sinh(x)}
\end{aligned}$$

Mathematica [A] time = 0.026686, size = 25, normalized size = 1.

$$-\frac{i}{\sinh(x)+i} - \log(\sinh(x)) + \log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(I + Sinh[x])^2,x]
```

```
[Out] -Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])
```

Maple [A] time = 0.033, size = 23, normalized size = 0.9

$$-\ln(\sinh(x)) + \ln(i + \sinh(x)) - \frac{i}{i + \sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(I+sinh(x))^2,x)
```

```
[Out] -ln(sinh(x))+ln(I+sinh(x))-I/(I+sinh(x))
```

Maxima [B] time = 1.11766, size = 65, normalized size = 2.6

$$\frac{2i\,e^{(-x)}}{-2i\,e^{(-x)} + e^{(-2\,x)} - 1} - \log\left(e^{(-x)} + 1\right) + 2\,\log\left(e^{(-x)} - i\right) - \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*I*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - log(e^(-x) + 1) + 2*log(e^(-x) -
I) - log(e^(-x) - 1)
```

Fricas [B] time = 2.0516, size = 162, normalized size = 6.48

$$-\frac{\left(e^{(2\,x)} + 2i\,e^{x} - 1\right)\log\left(e^{(2\,x)} - 1\right) - 2\left(e^{(2\,x)} + 2i\,e^{x} - 1\right)\log\left(e^{x} + i\right) + 2i\,e^{x}}{e^{(2\,x)} + 2i\,e^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((e^(2*x) + 2*I*e^x - 1)*log(e^(2*x) - 1) - 2*(e^(2*x) + 2*I*e^x - 1)*log(
e^x + I) + 2*I*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] time = 0.513079, size = 36, normalized size = 1.44

$$2\log\left(e^{x} + i\right) - \log\left(e^{2x} - 1\right) - \frac{2ie^{x}}{e^{2x} + 2ie^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))**2,x)
```

```
[Out] 2*log(exp(x) + I) - log(exp(2*x) - 1) - 2*I*exp(x)/(exp(2*x) + 2*I*exp(x) -
 1)
```

Giac [A] time = 1.12059, size = 45, normalized size = 1.8

$$-\frac{2i\,e^{x}}{\left(e^{x}+i\right)^{2}}-\log\left(e^{x}+1\right)+2\,\log\left(e^{x}+i\right)-\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*I*e^x/(e^x + I)^2 - log(e^x + 1) + 2*log(e^x + I) - log(abs(e^x - 1))
```

3.223 $\int \frac{\coth^2(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=26

$$\coth(x) + 2i\tanh^{-1}(\cosh(x)) + \frac{2i\coth(x)}{-\operatorname{csch}(x) + i}$$

```
[Out] (2*I)*ArcTanh[Cosh[x]] + Coth[x] + ((2*I)*Coth[x])/(I - Csch[x])
```

Rubi [A] time = 0.0677855, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2709, 3770, 3767, 8, 3777}

$$\coth(x) + 2i\tanh^{-1}(\cosh(x)) + \frac{2i\coth(x)}{-\operatorname{csch}(x) + i}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*ArcTanh[Cosh[x]] + Coth[x] + ((2*I)*Coth[x])/(I - Csch[x])
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = \int \left(2 - 2i\text{csch}(x) - \text{csch}^2(x) + \frac{2i}{-i+\text{csch}(x)}\right) dx$$
$$= 2x - 2i\int \text{csch}(x)\,dx + 2i\int \frac{1}{-i+\text{csch}(x)}\,dx - \int \text{csch}^2(x)\,dx$$
$$= 2x + 2i\tanh^{-1}(\cosh(x)) + \frac{2i\coth(x)}{i-\text{csch}(x)} + i\,\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right) + 2i\int i\,dx$$
$$= 2i\tanh^{-1}(\cosh(x)) + \coth(x) + \frac{2i\coth(x)}{i-\text{csch}(x)}$$

Mathematica [B] time = 0.138201, size = 66, normalized size = 2.54

$$\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right) + \coth\left(\frac{x}{2}\right) - 4i\log\left(\sinh\left(\frac{x}{2}\right)\right) + 4i\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (Coth[x/2] + (4*I)*Log[Cosh[x/2]] - (4*I)*Log[Sinh[x/2]] + (8*Sinh[x/2])/(C
osh[x/2] - I*Sinh[x/2]) + Tanh[x/2])/2
```

Maple [A] time = 0.045, size = 35, normalized size = 1.4

$$\frac{1}{2}\tanh\left(\frac{x}{2}\right) - 2\,i\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} + 4\,(\tanh(x/2) + i)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(I+sinh(x))^2,x)
```

```
[Out] 1/2*tanh(1/2*x)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)+4/(tanh(1/2*x)+I)
```

Maxima [B] time = 1.13042, size = 73, normalized size = 2.81

$$\frac{2\,e^{(-x)} + 4i\,e^{(-2\,x)} - 6i}{e^{(-x)} + i\,e^{(-2\,x)} - e^{(-3\,x)} - i} + 2i\,\log\left(e^{(-x)} + 1\right) - 2i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (2*e^(-x) + 4*I*e^(-2*x) - 6*I)/(e^(-x) + I*e^(-2*x) - e^(-3*x) - I) + 2*I*
log(e^(-x) + 1) - 2*I*log(e^(-x) - 1)
```

Fricas [B] time = 2.08805, size = 232, normalized size = 8.92

$$\frac{\left(2i\,e^{(3\,x)} - 2\,e^{(2\,x)} - 2i\,e^{x} + 2\right)\log\left(e^{x} + 1\right) + \left(-2i\,e^{(3\,x)} + 2\,e^{(2\,x)} + 2i\,e^{x} - 2\right)\log\left(e^{x} - 1\right) - 4i\,e^{(2\,x)} + 2\,e^{x} + 6i}{e^{(3\,x)} + i\,e^{(2\,x)} - e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((2*I*e^(3*x) - 2*e^(2*x) - 2*I*e^x + 2)*log(e^x + 1) + (-2*I*e^(3*x) + 2*e
^(2*x) + 2*I*e^x - 2)*log(e^x - 1) - 4*I*e^(2*x) + 2*e^x + 6*I)/(e^(3*x) +
I*e^(2*x) - e^x - I)
```

Sympy [B] time = 0.47057, size = 49, normalized size = 1.88

$$\frac{-4ie^{2x} + 2e^{x} + 6i}{e^{3x} + ie^{2x} - e^{x} - i} + 2\operatorname{RootSum}\left(z^{2} + 1, \left(i \mapsto i\log\left(-ii + e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(I+sinh(x))**2,x)
```

```
[Out] (-4*I*exp(2*x) + 2*exp(x) + 6*I)/(exp(3*x) + I*exp(2*x) - exp(x) - I) + 2*R
ootSum(_z**2 + 1, Lambda(_i, _i*log(-_i*I + exp(x))))
```

Giac [B] time = 1.15333, size = 63, normalized size = 2.42

$$\frac{-4i\,e^{(2\,x)} + 2\,e^{x} + 6i}{e^{(3\,x)} + i\,e^{(2\,x)} - e^{x} - i} + 2i\,\log\left(e^{x} + 1\right) - 2i\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] (-4*I*e^(2*x) + 2*e^x + 6*I)/(e^(3*x) + I*e^(2*x) - e^x - I) + 2*I*log(e^x
+ 1) - 2*I*log(abs(e^x - 1))
```

3.224 $\int \frac{\coth^3(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=29

$$\frac{\operatorname{csch}^2(x)}{2} + 2i\operatorname{csch}(x) + 2\log(\sinh(x)) - 2\log(\sinh(x) + i)$$

```
[Out] (2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

Rubi [A] time = 0.0480786, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2707, 77}

$$\frac{\operatorname{csch}^2(x)}{2} + 2i\operatorname{csch}(x) + 2\log(\sinh(x)) - 2\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{(i+\sinh(x))^2} dx &= -\operatorname{Subst}\left(\int \frac{i-x}{x^3(i+x)} dx, x, \sinh(x)\right) \\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{x^3} + \frac{2i}{x^2} - \frac{2}{x} + \frac{2}{i+x}\right) dx, x, \sinh(x)\right) \\
&= 2i\operatorname{csch}(x) + \frac{\operatorname{csch}^2(x)}{2} + 2\log(\sinh(x)) - 2\log(i+\sinh(x))
\end{aligned}$$

Mathematica [A] time = 0.016163, size = 29, normalized size = 1.

$$\frac{\operatorname{csch}^2(x)}{2} + 2i\operatorname{csch}(x) + 2\log(\sinh(x)) - 2\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

Maple [A] time = 0.06, size = 51, normalized size = 1.8

$$-i\tanh\left(\frac{x}{2}\right)+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+i\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+\frac{1}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+2\ln\left(\tanh\left(x/2\right)\right)-4\ln\left(\tanh\left(x/2\right)+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(I+sinh(x))^2,x)
```

```
[Out] -I*tanh(1/2*x)+1/8*tanh(1/2*x)^2+I/tanh(1/2*x)+1/8/tanh(1/2*x)^2+2*ln(tanh(
1/2*x))-4*ln(tanh(1/2*x)+I)
```

Maxima [B] time = 1.14238, size = 86, normalized size = 2.97

$$\frac{-4i\,e^{(-x)}-2\,e^{(-2\,x)}+4i\,e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+2\,\log\left(e^{(-x)}+1\right)-4\,\log\left(e^{(-x)}-i\right)+2\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (-4*I*e^(-x) - 2*e^(-2*x) + 4*I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 2*l
og(e^(-x) + 1) - 4*log(e^(-x) - I) + 2*log(e^(-x) - 1)
```

Fricas [B] time = 2.06189, size = 207, normalized size = 7.14

$$\frac{2\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)\log\left(e^{(2\,x)}-1\right)-4\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)\log\left(e^{x}+i\right)+4i\,e^{(3\,x)}+2\,e^{(2\,x)}-4i\,e^{x}}{e^{(4\,x)}-2\,e^{(2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*(e^(4*x) - 2*e^(2*x) + 1)*log(e^(2*x) - 1) - 4*(e^(4*x) - 2*e^(2*x) + 1)
*log(e^x + I) + 4*I*e^(3*x) + 2*e^(2*x) - 4*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1
)
```

Sympy [A] time = 0.688085, size = 53, normalized size = 1.83

$$\frac{4ie^{3x}+2e^{2x}-4ie^{x}}{e^{4x}-2e^{2x}+1}-4\log\left(e^{x}+i\right)+2\log\left(e^{2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(I+sinh(x))**2,x)
```

```
[Out] (4*I*exp(3*x) + 2*exp(2*x) - 4*I*exp(x))/(exp(4*x) - 2*exp(2*x) + 1) - 4*lo
g(exp(x) + I) + 2*log(exp(2*x) - 1)
```

Giac [B] time = 1.12132, size = 72, normalized size = 2.48

$$\frac{4i\,e^{(3\,x)} + 2\,e^{(2\,x)} - 4i\,e^{x}}{(e^{x}+1)^{2}(e^{x}-1)^{2}} + 2\,\log\left(e^{x}+1\right) - 4\,\log\left(e^{x}+i\right) + 2\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] (4*I*e^(3*x) + 2*e^(2*x) - 4*I*e^x)/((e^x + 1)^2*(e^x - 1)^2) + 2*log(e^x +
 1) - 4*log(e^x + I) + 2*log(abs(e^x - 1))
```

3.225 $\int \frac{\coth^4(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=28

$$\frac{\coth^3(x)}{3} - 2\coth(x) - i\tanh^{-1}(\cosh(x)) + i\coth(x)\text{csch}(x)$$

```
[Out] (-I)*ArcTanh[Cosh[x]] - 2*Coth[x] + Coth[x]^3/3 + I*Coth[x]*Csch[x]
```

Rubi [A] time = 0.094395, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2708, 2757, 3767, 8, 3768, 3770}

$$\frac{\coth^3(x)}{3} - 2\coth(x) - i\tanh^{-1}(\cosh(x)) + i\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-I)*ArcTanh[Cosh[x]] - 2*Coth[x] + Coth[x]^3/3 + I*Coth[x]*Csch[x]
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = \int \operatorname{csch}^4(x)(i-\sinh(x))^2\,dx$$
$$= \int \left(\operatorname{csch}^2(x) - 2i\operatorname{csch}^3(x) - \operatorname{csch}^4(x)\right)dx$$
$$= -\left(2i\int \operatorname{csch}^3(x)\,dx\right) + \int \operatorname{csch}^2(x)\,dx - \int \operatorname{csch}^4(x)\,dx$$
$$= i\coth(x)\operatorname{csch}(x) + i\int \operatorname{csch}(x)\,dx - i\operatorname{Subst}(\int 1\,dx, x, -i\coth(x)) - i\operatorname{Subst}\left(\int \left(1+x^2\right)dx, x\right.$$
$$= -i\tanh^{-1}(\cosh(x)) - 2\coth(x) + \frac{\coth^3(x)}{3} + i\coth(x)\operatorname{csch}(x)$$

Mathematica [B] time = 0.0541919, size = 107, normalized size = 3.82

$$-\frac{5}{6}\tanh\left(\frac{x}{2}\right) - \frac{5}{6}\coth\left(\frac{x}{2}\right) + \frac{1}{4}i\operatorname{csch}^2\left(\frac{x}{2}\right) + \frac{1}{4}i\operatorname{sech}^2\left(\frac{x}{2}\right) + i\log\left(\sinh\left(\frac{x}{2}\right)\right) - i\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{1}{24}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-5*Coth[x/2])/6 + (I/4)*Csch[x/2]^2 + (Coth[x/2]*Csch[x/2]^2)/24 - I*Log[C
osh[x/2]] + I*Log[Sinh[x/2]] + (I/4)*Sech[x/2]^2 - (5*Tanh[x/2])/6 - (Sech[
x/2]^2*Tanh[x/2])/24
```

Maple [B] time = 0.051, size = 58, normalized size = 2.1

$$-\frac{7}{8}\tanh\left(\frac{x}{2}\right) + \frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{i}{4}\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \frac{i}{4}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2} - \frac{7}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} + \frac{1}{24}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3} + i\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(I+sinh(x))^2,x)
```

```
[Out] -7/8*tanh(1/2*x)+1/24*tanh(1/2*x)^3-1/4*I*tanh(1/2*x)^2+1/4*I/tanh(1/2*x)^2
-7/8/tanh(1/2*x)+1/24/tanh(1/2*x)^3+I*ln(tanh(1/2*x))
```

Maxima [B] time = 1.02983, size = 90, normalized size = 3.21

$$\frac{-6i\,e^{(-x)} - 24\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 6i\,e^{(-5\,x)} + 10}{3\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} - i\,\log\left(e^{(-x)} + 1\right) + i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(-6*I*e^(-x) - 24*e^(-2*x) + 6*e^(-4*x) + 6*I*e^(-5*x) + 10)/(3*e^(-2*x
) - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

Fricas [B] time = 2.03329, size = 302, normalized size = 10.79

$$\frac{\left(-3i\,e^{(6\,x)}+9i\,e^{(4\,x)}-9i\,e^{(2\,x)}+3i\right)\log\left(e^{x}+1\right)+\left(3i\,e^{(6\,x)}-9i\,e^{(4\,x)}+9i\,e^{(2\,x)}-3i\right)\log\left(e^{x}-1\right)+6i\,e^{(5\,x)}-6\,e^{(4\,x)}+24\,e^{(2\,x)}}{3\left(e^{(6\,x)}-3\,e^{(4\,x)}+3\,e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((-3*I*e^(6*x) + 9*I*e^(4*x) - 9*I*e^(2*x) + 3*I)*log(e^x + 1) + (3*I*e
^(6*x) - 9*I*e^(4*x) + 9*I*e^(2*x) - 3*I)*log(e^x - 1) + 6*I*e^(5*x) - 6*e^
(4*x) + 24*e^(2*x) - 6*I*e^x - 10)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [B] time = 0.550075, size = 66, normalized size = 2.36

$$\operatorname{RootSum}\left(z^{2}+1,\left(i\mapsto i\log\left(ii+e^{x}\right)\right)\right)+\frac{2ie^{5x}-2e^{4x}+8e^{2x}-2ie^{x}-\frac{10}{3}}{e^{6x}-3e^{4x}+3e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(I+sinh(x))**2,x)
```

```
[Out] RootSum(_z**2 + 1, Lambda(_i, _i*log(_i*I + exp(x)))) + (2*I*exp(5*x) - 2*e
xp(4*x) + 8*exp(2*x) - 2*I*exp(x) - 10/3)/(exp(6*x) - 3*exp(4*x) + 3*exp(2*
x) - 1)
```

Giac [B] time = 1.14062, size = 68, normalized size = 2.43

$$-\frac{-6i\,e^{(5\,x)}+6\,e^{(4\,x)}-24\,e^{(2\,x)}+6i\,e^{x}+10}{3\left(e^{(2\,x)}-1\right)^{3}}-i\,\log\left(e^{x}+1\right)+i\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*(-6*I*e^(5*x) + 6*e^(4*x) - 24*e^(2*x) + 6*I*e^x + 10)/(e^(2*x) - 1)^3
 - I*log(e^x + 1) + I*log(abs(e^x - 1))
```

3.226 $\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=27

$$\frac{\operatorname{csch}^4(x)}{4}+\frac{2}{3}i\operatorname{csch}^3(x)-\frac{\operatorname{csch}^2(x)}{2}$$

```
[Out] -Csch[x]^2/2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

Rubi [A] time = 0.0427058, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2707, 43}

$$\frac{\operatorname{csch}^4(x)}{4}+\frac{2}{3}i\operatorname{csch}^3(x)-\frac{\operatorname{csch}^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] -Csch[x]^2/2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx &= \operatorname{Subst}\left(\int \frac{(i-x)^2}{x^5}\,dx,x,\sinh(x)\right)\\ &= \operatorname{Subst}\left(\int \left(-\frac{1}{x^5}-\frac{2i}{x^4}+\frac{1}{x^3}\right)dx,x,\sinh(x)\right)\\ &= -\frac{1}{2}\operatorname{csch}^2(x)+\frac{2}{3}i\operatorname{csch}^3(x)+\frac{\operatorname{csch}^4(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.0123971, size = 27, normalized size = 1.

$$\frac{\operatorname{csch}^4(x)}{4}+\frac{2}{3}i\operatorname{csch}^3(x)-\frac{\operatorname{csch}^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] -Csch[x]^2/2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

Maple [B] time = 0.067, size = 68, normalized size = 2.5

$$\frac{i}{4}\tanh\left(\frac{x}{2}\right)+\frac{1}{64}\left(\tanh\left(\frac{x}{2}\right)\right)^{4}-\frac{i}{12}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}-\frac{3}{16}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-\frac{3}{16}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}-\frac{i}{4}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+\frac{i}{12}\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(I+sinh(x))^2,x)
```

```
[Out] 1/4*I*tanh(1/2*x)+1/64*tanh(1/2*x)^4-1/12*I*tanh(1/2*x)^3-3/16*tanh(1/2*x)^
2-3/16/tanh(1/2*x)^2-1/4*I/tanh(1/2*x)+1/12*I/tanh(1/2*x)^3+1/64/tanh(1/2*x
)^4
```

Maxima [B] time = 1.02155, size = 231, normalized size = 8.56

$$\frac{2\,e^{(-2\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{16i\,e^{(-3\,x)}}{3\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}-\frac{8\,e^{(-4\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 16/3*I*e
^(-3*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 8*e^(-4*x)/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 16/3*I*e^(-5*x)/(4*
e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 2*e^(-6*x)/(4*e^(-2*x)
 - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Fricas [B] time = 1.95788, size = 166, normalized size = 6.15

$$-\frac{6\,e^{(6\,x)}-16i\,e^{(5\,x)}-24\,e^{(4\,x)}+16i\,e^{(3\,x)}+6\,e^{(2\,x)}}{3\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*e^(6*x) - 16*I*e^(5*x) - 24*e^(4*x) + 16*I*e^(3*x) + 6*e^(2*x))/(e^
(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [B] time = 0.74758, size = 66, normalized size = 2.44

$$\frac{-2e^{6x}+\frac{16ie^{5x}}{3}+8e^{4x}-\frac{16ie^{3x}}{3}-2e^{2x}}{e^{8x}-4e^{6x}+6e^{4x}-4e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(I+sinh(x))**2,x)
```

```
[Out] (-2*exp(6*x) + 16*I*exp(5*x)/3 + 8*exp(4*x) - 16*I*exp(3*x)/3 - 2*exp(2*x))
/(exp(8*x) - 4*exp(6*x) + 6*exp(4*x) - 4*exp(2*x) + 1)
```

Giac [A] time = 1.10279, size = 51, normalized size = 1.89

$$-\frac{6\left(e^{(-x)}-e^{x}\right)^{2}+16i\,e^{(-x)}-16i\,e^{x}-12}{3\left(e^{(-x)}-e^{x}\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*(e^(-x) - e^x)^2 + 16*I*e^(-x) - 16*I*e^x - 12)/(e^(-x) - e^x)^4
```

3.227 $\int \frac{\coth^6(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=48

$$\frac{\coth^5(x)}{5} - \frac{2\coth^3(x)}{3} - \frac{1}{4}i\tanh^{-1}(\cosh(x)) + \frac{1}{2}i\coth(x)\text{csch}^3(x) + \frac{1}{4}i\coth(x)\text{csch}(x)$$

```
[Out] (-I/4)*ArcTanh[Cosh[x]] - (2*Coth[x]^3)/3 + Coth[x]^5/5 + (I/4)*Coth[x]*Csc
h[x] + (I/2)*Coth[x]*Csch[x]^3
```

Rubi [A] time = 0.0905616, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2709, 3767, 8, 3768, 3770}

$$\frac{\coth^5(x)}{5} - \frac{2\coth^3(x)}{3} - \frac{1}{4}i\tanh^{-1}(\cosh(x)) + \frac{1}{2}i\coth(x)\text{csch}^3(x) + \frac{1}{4}i\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-I/4)*ArcTanh[Cosh[x]] - (2*Coth[x]^3)/3 + Coth[x]^5/5 + (I/4)*Coth[x]*Csc
h[x] + (I/2)*Coth[x]*Csch[x]^3
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = \int \left(\operatorname{csch}^2(x) - 2i\operatorname{csch}^3(x) - 2i\operatorname{csch}^5(x) - \operatorname{csch}^6(x)\right)dx$$
$$= -\left(2i\int \operatorname{csch}^3(x)\,dx\right) - 2i\int \operatorname{csch}^5(x)\,dx + \int \operatorname{csch}^2(x)\,dx - \int \operatorname{csch}^6(x)\,dx$$
$$= i\coth(x)\operatorname{csch}(x) + \frac{1}{2}i\coth(x)\operatorname{csch}^3(x) + i\int \operatorname{csch}(x)\,dx - i\,\mathrm{Subst}\left(\int 1\,dx, x, -i\coth(x)\right) + i\,\mathrm{S}$$
$$= -i\tanh^{-1}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\operatorname{csch}(x) + \frac{1}{2}i\coth(x)\operatorname{csch}^3(x) - \frac{3}{4}i$$
$$= -\frac{1}{4}i\tanh^{-1}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\operatorname{csch}(x) + \frac{1}{2}i\coth(x)\operatorname{csch}^3(x)$$

Mathematica [B] time = 0.0582698, size = 175, normalized size = 3.65

$$-\frac{7}{30}\tanh\left(\frac{x}{2}\right) - \frac{7}{30}\coth\left(\frac{x}{2}\right) + \frac{1}{32}i\operatorname{csch}^4\left(\frac{x}{2}\right) + \frac{1}{16}i\operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{32}i\operatorname{sech}^4\left(\frac{x}{2}\right) + \frac{1}{16}i\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{4}i\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-7*Coth[x/2])/30 + (I/16)*Csch[x/2]^2 - (19*Coth[x/2]*Csch[x/2]^2)/480 + (
I/32)*Csch[x/2]^4 + (Coth[x/2]*Csch[x/2]^4)/160 - (I/4)*Log[Cosh[x/2]] + (I
/4)*Log[Sinh[x/2]] + (I/16)*Sech[x/2]^2 - (I/32)*Sech[x/2]^4 - (7*Tanh[x/2]
)/30 + (19*Sech[x/2]^2*Tanh[x/2])/480 + (Sech[x/2]^4*Tanh[x/2])/160
```

Maple [B] time = 0.073, size = 74, normalized size = 1.5

$$-\frac{3}{16}\tanh\left(\frac{x}{2}\right) + \frac{1}{160}\left(\tanh\left(\frac{x}{2}\right)\right)^5 - \frac{i}{32}\left(\tanh\left(\frac{x}{2}\right)\right)^4 - \frac{5}{96}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{3}{16}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} + \frac{1}{160}\left(\tanh\left(\frac{x}{2}\right)\right)^{-5} - \frac{5}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(I+sinh(x))^2,x)
```

```
[Out] -3/16*tanh(1/2*x)+1/160*tanh(1/2*x)^5-1/32*I*tanh(1/2*x)^4-5/96*tanh(1/2*x)
^3-3/16/tanh(1/2*x)+1/160/tanh(1/2*x)^5-5/96/tanh(1/2*x)^3+1/4*I*ln(tanh(1/
2*x))+1/32*I/tanh(1/2*x)^4
```

Maxima [B] time = 1.06903, size = 139, normalized size = 2.9

$$\frac{-15i\,e^{(-x)} - 80\,e^{(-2\,x)} - 90i\,e^{(-3\,x)} + 40\,e^{(-4\,x)} - 240\,e^{(-6\,x)} + 90i\,e^{(-7\,x)} + 60\,e^{(-8\,x)} + 15i\,e^{(-9\,x)} + 28}{30\left(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1\right)} - \frac{1}{4}i\log\left(e^{(-x)} + 1\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(-15*I*e^(-x) - 80*e^(-2*x) - 90*I*e^(-3*x) + 40*e^(-4*x) - 240*e^(-6*
x) + 90*I*e^(-7*x) + 60*e^(-8*x) + 15*I*e^(-9*x) + 28)/(5*e^(-2*x) - 10*e^(
-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) - 1/4*I*log(e^(-x) + 1) +
```

```
1/4*I*log(e^(-x) - 1)
```

Fricas [B] time = 2.11256, size = 527, normalized size = 10.98

$$\frac{\left(-15i\,e^{(10\,x)}+75i\,e^{(8\,x)}-150i\,e^{(6\,x)}+150i\,e^{(4\,x)}-75i\,e^{(2\,x)}+15i\right)\log\left(e^{x}+1\right)+\left(15i\,e^{(10\,x)}-75i\,e^{(8\,x)}+150i\,e^{(6\,x)}-150i\,e^{(4\,x}}{60\left(e^{(10\,x)}-5\,e^{(8\,x)}+10\,e^{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/60*((-15*I*e^(10*x) + 75*I*e^(8*x) - 150*I*e^(6*x) + 150*I*e^(4*x) - 75*I
*e^(2*x) + 15*I)*log(e^x + 1) + (15*I*e^(10*x) - 75*I*e^(8*x) + 150*I*e^(6*
x) - 150*I*e^(4*x) + 75*I*e^(2*x) - 15*I)*log(e^x - 1) + 30*I*e^(9*x) - 120
*e^(8*x) + 180*I*e^(7*x) + 480*e^(6*x) - 80*e^(4*x) - 180*I*e^(3*x) + 160*e
^(2*x) - 30*I*e^x - 56)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5
*e^(2*x) - 1)
```

Sympy [B] time = 1.52104, size = 117, normalized size = 2.44

$$\operatorname{RootSum}\left(16z^{2}+1,\left(i\mapsto i\log\left(4ii+e^{x}\right)\right)\right)+\frac{\frac{ie^{9x}}{2}-2e^{8x}+3ie^{7x}+8e^{6x}-\frac{4e^{4x}}{3}-3ie^{3x}+\frac{8e^{2x}}{3}-\frac{ie^{x}}{2}-\frac{14}{15}}{e^{10x}-5e^{8x}+10e^{6x}-10e^{4x}+5e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(I+sinh(x))**2,x)
```

```
[Out] RootSum(16*_z**2 + 1, Lambda(_i, _i*log(4*_i*I + exp(x)))) + (I*exp(9*x)/2
- 2*exp(8*x) + 3*I*exp(7*x) + 8*exp(6*x) - 4*exp(4*x)/3 - 3*I*exp(3*x) + 8*
exp(2*x)/3 - I*exp(x)/2 - 14/15)/(exp(10*x) - 5*exp(8*x) + 10*exp(6*x) - 10
*exp(4*x) + 5*exp(2*x) - 1)
```

Giac [B] time = 1.14469, size = 100, normalized size = 2.08

$$-\frac{-15i\,e^{(9\,x)}+60\,e^{(8\,x)}-90i\,e^{(7\,x)}-240\,e^{(6\,x)}+40\,e^{(4\,x)}+90i\,e^{(3\,x)}-80\,e^{(2\,x)}+15i\,e^{x}+28}{30\left(e^{(2\,x)}-1\right)^{5}}-\frac{1}{4}i\,\log\left(e^{x}+1\right)+\frac{1}{4}i\,\log\left(|e^{x}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/30*(-15*I*e^(9*x) + 60*e^(8*x) - 90*I*e^(7*x) - 240*e^(6*x) + 40*e^(4*x)
 + 90*I*e^(3*x) - 80*e^(2*x) + 15*I*e^x + 28)/(e^(2*x) - 1)^5 - 1/4*I*log(e
^x + 1) + 1/4*I*log(abs(e^x - 1))
```

3.228 $\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=124

$$-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}-\frac{a^3\tanh(x)}{\left(a^2+b^2\right)^2}+\frac{b\text{sech}^3(x)}{3\left(a^2+b^2\right)}-\frac{a^2b\text{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{b\text{sech}(x)}{a^2+b^2}$$

```
[Out] (-2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (a^
2*b*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x])/(a^2 + b^2) + (b*Sech[x]^3)/(3*(a^
2 + b^2)) - (a^3*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x]^3)/(3*(a^2 + b^2))
```

Rubi [A] time = 0.184068, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2660, 618, 206}

$$-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}-\frac{a^3\tanh(x)}{\left(a^2+b^2\right)^2}+\frac{b\text{sech}^3(x)}{3\left(a^2+b^2\right)}-\frac{a^2b\text{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{b\text{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (-2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (a^
2*b*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x])/(a^2 + b^2) + (b*Sech[x]^3)/(3*(a^
2 + b^2)) - (a^3*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x]^3)/(3*(a^2 + b^2))
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = -\frac{a\int \operatorname{sech}^2(x)\tanh^2(x)\,dx}{a^2+b^2}+\frac{a^2\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx}{a^2+b^2}+\frac{b\int \operatorname{sech}(x)\tanh^3(x)\,dx}{a^2+b^2}$$

$$= -\frac{a^3\int \operatorname{sech}^2(x)\,dx}{\left(a^2+b^2\right)^2}+\frac{a^4\int \frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}+\frac{\left(a^2b\right)\int \operatorname{sech}(x)\tanh(x)\,dx}{\left(a^2+b^2\right)^2}-\frac{(ia)\operatorname{Subst}\left(\int x^2\,dx,x,i\operatorname{ta}\right.}{a^2+b^2}$$

$$= -\frac{b\operatorname{sech}(x)}{a^2+b^2}+\frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}-\frac{\left(ia^3\right)\operatorname{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)}{\left(a^2+b^2\right)^2}+\frac{\left(2a^4\right)\operatorname{Subst}\left(\int \frac{}{a+2}\right.}{\left(a^2\right.}$$

$$= -\frac{a^2b\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}+\frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)}-\frac{a^3\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}-\frac{\left(4a^4\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,d\right.}{\left(a^2+b\right.}$$

$$= -\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{a^2b\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}+\frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)}-\frac{a^3\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}$$

Mathematica [A] time = 0.351928, size = 108, normalized size = 0.87

$$\frac{-a\left(4a^2+b^2\right)\tanh(x)-3b\left(2a^2+b^2\right)\operatorname{sech}(x)+\frac{6a^4\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\left(a^2+b^2\right)\operatorname{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((6*a^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 3*b*
(2*a^2 + b^2)*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) - a*(4*a^2 +
b^2)*Tanh[x])/(3*(a^2 + b^2)^2)
```

Maple [A] time = 0.039, size = 163, normalized size = 1.3

$$2\,\frac{-a^3\,(\tanh(x/2))^5-a^2b\,(\tanh(x/2))^4+\left(-10/3\,a^3-4/3\,ab^2\right)(\tanh(x/2))^3+\left(-4\,a^2b-2\,b^3\right)(\tanh(x/2))^2-a^3\tanh}{\left(a^2+b^2\right)^2\left((\tanh(x/2))^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sinh(x)),x)
```

```
[Out] 2/(a^2+b^2)^2*(-a^3*tanh(1/2*x)^5-a^2*b*tanh(1/2*x)^4+(-10/3*a^3-4/3*a*b^2)
*tanh(1/2*x)^3+(-4*a^2*b-2*b^3)*tanh(1/2*x)^2-a^3*tanh(1/2*x)-5/3*a^2*b-2/3
*b^3)/(tanh(1/2*x)^2+1)^3+32*a^4/(16*a^4+32*a^2*b^2+16*b^4)/(a^2+b^2)^(1/2)
*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.24552, size = 2944, normalized size = 23.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b + 3*a^2*b^3 + b^
5)*sinh(x)^5 - 8*a^5 - 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^4)
*cosh(x)^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b + 3*a^2*b^3 + b^5)*c
osh(x))*sinh(x)^4 + 4*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^3 + 4*(4*a^4*b +
5*a^2*b^3 + b^5 + 15*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 - 6*(2*a^5 + 3*a
^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + a^3*b^2)*cosh(x)^2 - 12*(a^5
 + a^3*b^2 - 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^3 + 3*(2*a^5 + 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 - 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh
(x)^3 + 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a
```

```
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
+ b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) - b)) + 6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*a^4*b + 3*a^2*b^
3 + b^5 + 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 + 3*a^3*b^2 +
a*b^4)*cosh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^2 - 4*(a^5 + a^3*b
^2)*cosh(x))*sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
+ 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x))*sinh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 +
3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^4 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 +
6*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh
(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sinh(x)), x)
```

Giac [A] time = 1.19481, size = 266, normalized size = 2.15

$$\frac{a^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}} - \frac{2\left(6\,a^2be^{(5\,x)}+3\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}-3\,ab^2e^{(4\,x)}+8\,a^2be^{(3\,x)}+2\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}+6\,a^2be^x\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(6*a^2*b*e^(5*x
) + 3*b^3*e^(5*x) - 6*a^3*e^(4*x) - 3*a*b^2*e^(4*x) + 8*a^2*b*e^(3*x) + 2*b
^3*e^(3*x) - 6*a^3*e^(2*x) + 6*a^2*b*e^x + 3*b^3*e^x - 4*a^3 - a*b^2)/((a^4
 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

3.229 $\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=88

$$-\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{b\left(3a^2+b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}+\frac{a^3\log(\cosh(x))}{\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)}$$

```
[Out] (b*(3*a^2 + b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) + (a^3*Log[Cosh[x]])/(a
^2 + b^2)^2 - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(a - b*Si
nh[x]))/(2*(a^2 + b^2))
```

Rubi [A] time = 0.175015, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2721, 1647, 801, 635, 203, 260}

$$-\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{b\left(3a^2+b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}+\frac{a^3\log(\cosh(x))}{\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (b*(3*a^2 + b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) + (a^3*Log[Cosh[x]])/(a
^2 + b^2)^2 - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(a - b*Si
nh[x]))/(2*(a^2 + b^2))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = \text{Subst}\left(\int \frac{x^3}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2+b^2}+\frac{b^2\left(2a^2+b^2\right)x}{a^2+b^2}}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2b^2}$$

$$= \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{2a^3b^2}{\left(a^2+b^2\right)^2(a+x)} - \frac{b^2\left(3a^2b^2+b^4+2a^3x\right)}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)dx, x, b\sinh(x)\right)}{2b^2}$$

$$= -\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} + \frac{\text{Subst}\left(\int \frac{3a^2b^2+b^4+2a^3x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)^2}$$

$$= -\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} + \frac{a^3\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^2} + \frac{\left(b^2\left(3a^2\right.\right.}{}$$

$$= \frac{b\left(3a^2+b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2} + \frac{a^3\log(\cosh(x))}{\left(a^2+b^2\right)^2} - \frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)}$$

Mathematica [C] time = 0.175322, size = 153, normalized size = 1.74

$$\frac{a\operatorname{sech}^2(x)}{2\left(a^2+b^2\right)} - \frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\left(a^3-i\left(2a^2b+b^3\right)\right)\log(-\sinh(x)+i)}{2\left(a^2+b^2\right)^2} + \frac{\left(a^3+i\left(2a^2b+b^3\right)\right)\log(\sinh(x)+i)}{2\left(a^2+b^2\right)^2} - \frac{b\,t}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -(b*ArcTan[Sinh[x]])/(2*(a^2 + b^2)) + ((a^3 - I*(2*a^2*b + b^3))*Log[I - S
inh[x]])/(2*(a^2 + b^2)^2) + ((a^3 + I*(2*a^2*b + b^3))*Log[I + Sinh[x]])/(
2*(a^2 + b^2)^2) - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (a*Sech[x]^2)/(
2*(a^2 + b^2)) - (b*Sech[x]*Tanh[x])/(2*(a^2 + b^2))
```

Maple [B] time = 0.036, size = 357, normalized size = 4.1

$$\frac{a^2b}{a^4+2a^2b^2+b^4}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2} + \frac{b^3}{a^4+2a^2b^2+b^4}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-2} - 2\frac{}{\left(a^4+2a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sinh(x)),x)
```

```
[Out] 1/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^2*b+1/(a^4+2*a^2*
b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*b^3-2/(a^4+2*a^2*b^2+b^4)/(tanh(
1/2*x)^2+1)^2*tanh(1/2*x)^2*a^3-2/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*t
anh(1/2*x)^2*a*b^2-1/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a^
2*b-1/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*b^3+1/(a^4+2*a^2*
b^2+b^4)*a^3*ln(tanh(1/2*x)^2+1)+3/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*x))*
a^2*b+1/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*x))*b^3-8*a^3/(8*a^4+16*a^2*b^2
+8*b^4)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)
```

Maxima [A] time = 1.57276, size = 216, normalized size = 2.45

$$-\frac{a^3 \log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{a^3 \log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{\left(3\,a^2b + b^3\right)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{be^{(-x)} - 2\,ae^{(-2\,x)} - be^{(-}}{a^2 + b^2 + 2\left(a^2 + b^2\right)e^{(-2\,x)} + \left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + a^3*log(e^
(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (3*a^2*b + b^3)*arctan(e^(-x))/(a^4 +
 2*a^2*b^2 + b^4) - (b*e^(-x) - 2*a*e^(-2*x) - b*e^(-3*x))/(a^2 + b^2 + 2*(
a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Fricas [B] time = 2.26805, size = 1725, normalized size = 19.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*cosh(x)^3 + (a^2*b + b^3)*sinh(x)^3 - 2*(a^3 + a*b^2)*cosh(
x)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(x))*sinh(x)^2 - ((3*a^2*b +
b^3)*cosh(x)^4 + 4*(3*a^2*b + b^3)*cosh(x)*sinh(x)^3 + (3*a^2*b + b^3)*sinh
(x)^4 + 3*a^2*b + b^3 + 2*(3*a^2*b + b^3)*cosh(x)^2 + 2*(3*a^2*b + b^3 + 3*
(3*a^2*b + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((3*a^2*b + b^3)*cosh(x)^3 + (3*a^
2*b + b^3)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b + b^3)*cosh
(x) + (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh
(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*
cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^3*cosh(x)
^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3
*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))*
log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(x)
^2 + 4*(a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 +
 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh
(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^
4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((
a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sinh(x)), x)
```

Giac [B] time = 1.1375, size = 285, normalized size = 3.24

$$-\frac{a^3 b\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{a^4 b + 2\,a^2 b^3 + b^5} + \frac{a^3\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^4 + 2\,a^2 b^2 + b^4\right)} + \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^2 b + b^3\right)}{4\left(a^4 + 2\,a^2 b^2 + b^4\right)} - \frac{a^3\left(e^{(-x)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a^
3*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*(3*a^2*b + b^3)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^3
*(e^(-x) - e^x)^2 - 2*a^2*b*(e^(-x) - e^x) - 2*b^3*(e^(-x) - e^x) - 4*a*b^2
)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

3.230 $\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=69

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\text{sech}(x)}{a^2+b^2}$$

```
[Out] (-2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (b*
Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Rubi [A] time = 0.0914194, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2727, 3767, 8, 2606, 2660, 618, 206}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\text{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (b*
Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx &= -\frac{a\int \operatorname{sech}^2(x)\,dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}+\frac{b\int \operatorname{sech}(x)\tanh(x)\,dx}{a^2+b^2}\\
&= -\frac{(ia)\operatorname{Subst}(\int 1\,dx,x,-i\tanh(x))}{a^2+b^2}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{b\operatorname{Subst}(\int 1\,dx,x}{a^2+b^2}\\
&= -\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}-\frac{\left(4a^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.188303, size = 69, normalized size = 1.

$$\frac{a\left(\frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\tanh(x)\right)-b\operatorname{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-(b*Sech[x]) + a*((2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a
^2 - b^2] - Tanh[x]))/(a^2 + b^2)
```

Maple [A] time = 0.029, size = 84, normalized size = 1.2

$$2\,\frac{-a\tanh(x/2)-b}{\left(a^2+b^2\right)\left((\tanh(x/2))^2+1\right)}+8\,\frac{a^2}{\left(4\,a^2+4\,b^2\right)\sqrt{a^2+b^2}}\operatorname{Artanh}\left(1/2\,\frac{2\,a\tanh(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sinh(x)),x)
```

```
[Out] 2/(a^2+b^2)*(-a*tanh(1/2*x)-b)/(tanh(1/2*x)^2+1)+8*a^2/(4*a^2+4*b^2)/(a^2+b
^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05387, size = 691, normalized size = 10.01

$$\frac{2\,a^{3} + 2\,ab^{2} + \left(a^{2}\cosh\left(x\right)^{2} + 2\,a^{2}\cosh\left(x\right)\sinh\left(x\right) + a^{2}\sinh\left(x\right)^{2} + a^{2}\right)\sqrt{a^{2} + b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2} + b^{2}\sinh(x)^{2} + 2\,ab\cosh(x) + 2\ldots}{b\cosh(x)^{2} + b\sinh\ldots}\right.}{a^{4} + 2\,a^{2}b^{2} + b^{4} + \left(a^{4} + 2\,a^{2}b^{2} + b^{4}\right)\cosh\left(x\right)^{2} + 2\left(a^{4} + 2\,a^{2}b^{2} + b\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 +
 a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x))/(a^4
 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2
 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^{2}\left(x\right)}{a + b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sinh(x)), x)
```

Giac [A] time = 1.19777, size = 117, normalized size = 1.7

$$\frac{a^{2}\log\left(\frac{\left|2\,be^{x} + 2\,a - 2\,\sqrt{a^{2} + b^{2}}\right|}{\left|2\,be^{x} + 2\,a + 2\,\sqrt{a^{2} + b^{2}}\right|}\right)}{\left(a^{2} + b^{2}\right)^{\frac{3}{2}}} - \frac{2\,\left(be^{x} - a\right)}{\left(a^{2} + b^{2}\right)\left(e^{\left(2\,x\right)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

3.231 $\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=48

$$-\frac{a\log(a+b\sinh(x))}{a^2+b^2}+\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{a\log(\cosh(x))}{a^2+b^2}$$

```
[Out] (b*ArcTan[Sinh[x]])/(a^2 + b^2) + (a*Log[Cosh[x]])/(a^2 + b^2) - (a*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Rubi [A] time = 0.0688227, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2721, 801, 635, 203, 260}

$$-\frac{a\log(a+b\sinh(x))}{a^2+b^2}+\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{a\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sinh[x]),x]
```


```
[Out] (b*ArcTan[Sinh[x]])/(a^2 + b^2) + (a*Log[Cosh[x]])/(a^2 + b^2) - (a*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```


Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx &= -\mathrm{Subst}\left(\int \frac{x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\\
&= -\mathrm{Subst}\left(\int \left(\frac{a}{\left(a^2+b^2\right)(a+x)}+\frac{-b^2-ax}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx, x, b\sinh(x)\right)\\
&= -\frac{a\log(a+b\sinh(x))}{a^2+b^2}-\frac{\mathrm{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{a\log(a+b\sinh(x))}{a^2+b^2}+\frac{a\,\mathrm{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{a\log(\cosh(x))}{a^2+b^2}-\frac{a\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}
$$

Mathematica [A] time = 0.0579387, size = 36, normalized size = 0.75

$$\frac{-a\log(a+b\sinh(x))+a\log(\cosh(x))+2b\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sinh[x]),x]
```

```
[Out] (2*b*ArcTan[Tanh[x/2]] + a*Log[Cosh[x]] - a*Log[a + b*Sinh[x]])/(a^2 + b^2)
```

Maple [A] time = 0.024, size = 84, normalized size = 1.8

$$2\,\frac{a\ln\left((\tanh(x/2))^2+1\right)}{2\,a^2+2\,b^2}+4\,\frac{b\arctan(\tanh(x/2))}{2\,a^2+2\,b^2}-2\,\frac{a\ln\left(a(\tanh(x/2))^2-2\,\tanh(x/2)\,b-a\right)}{2\,a^2+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sinh(x)),x)
```

```
[Out] 2/(2*a^2+2*b^2)*a*ln(tanh(1/2*x)^2+1)+4/(2*a^2+2*b^2)*b*arctan(tanh(1/2*x))
-2*a/(2*a^2+2*b^2)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)
```

Maxima [A] time = 1.53292, size = 89, normalized size = 1.85

$$-\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2}-\frac{a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}+\frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(e^(-x))/(a^2 + b^2) - a*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
+ b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Fricas [A] time = 2.12042, size = 177, normalized size = 3.69

$$\frac{2\,b\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-a\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+a\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*b*arctan(cosh(x) + sinh(x)) - a*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) + a*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*sinh(x)), x)
```

Giac [A] time = 1.13206, size = 120, normalized size = 2.5

$$-\frac{ab\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a^2b+b^3}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)b}{2\left(a^2+b^2\right)}+\frac{a\log\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)/(
a^2 + b^2)
```

3.232 $\int \frac{\coth(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\log(\sinh(x))}{a}-\frac{\log(a+b\sinh(x))}{a}$$

```
[Out] Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Rubi [A] time = 0.0408944, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\sinh(x))}{a}-\frac{\log(a+b\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+b\sinh(x)}\,dx&=\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx,x,b\sinh(x)\right)\\&=\frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,b\sinh(x)\right)}{a}-\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{a}\\&=\frac{\log(\sinh(x))}{a}-\frac{\log(a+b\sinh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.0095571, size = 20, normalized size = 1.

$$\frac{\log(\sinh(x))}{a}-\frac{\log(a+b\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Maple [A] time = 0.02, size = 21, normalized size = 1.1

$$\frac{\ln(\sinh(x))}{a}-\frac{\ln(a+b\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x)),x)
```

```
[Out] ln(sinh(x))/a-ln(a+b*sinh(x))/a
```

Maxima [B] time = 1.06044, size = 62, normalized size = 3.1

$$-\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a}+\frac{\log\left(e^{(-x)}+1\right)}{a}+\frac{\log\left(e^{(-x)}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -log(-2*a*e^(-x) + b*e^(-2*x) - b)/a + log(e^(-x) + 1)/a + log(e^(-x) - 1)/
a
```

Fricas [A] time = 2.0267, size = 116, normalized size = 5.8

$$-\frac{\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(x) - sin
h(x))))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*sinh(x)), x)
```

Giac [A] time = 1.12772, size = 53, normalized size = 2.65

$$-\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a}+\frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -log(abs(-b*(e^(-x) - e^x) + 2*a))/a + log(abs(-e^(-x) + e^x))/a
```

3.233 $\int \frac{\coth^2(x)}{a+b\sinh(x)} dx$

Optimal. Leaf size=56

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/a^2 - Coth[x]/a
```

Rubi [A] time = 0.234705, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 206}

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/a^2 - Coth[x]/a
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx &= \int \frac{\operatorname{csch}^2(x)\left(1+\sinh^2(x)\right)}{a+b\sinh(x)}\,dx\\
&= -\frac{\coth(x)}{a}+\frac{i\int \frac{\operatorname{csch}(x)(ib-ia\sinh(x))}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{\coth(x)}{a}-\frac{b\int \operatorname{csch}(x)\,dx}{a^2}+\frac{\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2}\\
&= \frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}+\frac{\left(2\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}-\frac{\left(4\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\coth(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.175744, size = 82, normalized size = 1.46

$$
-\frac{\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)\left(\sinh(x)\left(2\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+b\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)+a\cosh(x)\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -(Csch[x/2]*Sech[x/2]*(a*Cosh[x] + (2*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[x
/2])/Sqrt[-a^2 - b^2]] + b*Log[Tanh[x/2]])*Sinh[x]))/(2*a^2)
```

Maple [B] time = 0.026, size = 107, normalized size = 1.9

$$-\frac{1}{2a}\tanh\left(\frac{x}{2}\right)-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{b}{a^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\frac{1}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\frac{2a\tanh(x/2)-2b}{\sqrt{a^2+b^2}}\right)+2\frac{b^2}{a^2\sqrt{a^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sinh(x)),x)
```

```
[Out] -1/2/a*tanh(1/2*x)-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))+2/(a^2+b^2)^(1
/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+2*b^2/a^2/(a^2+b^2)^
(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.23027, size = 705, normalized size = 12.59

$$\frac{\sqrt{a^2+b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2(b\cosh(x)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log((b^2*c
osh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a
*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 +
 b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (b*cosh(x)^2
 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(x) + 1) - (b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(x) - 1
) - 2*a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth^2(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sinh(x)), x)
```

Giac [A] time = 1.14298, size = 128, normalized size = 2.29

$$\frac{b\log\left(e^{x}+1\right)}{a^{2}}-\frac{b\log\left(\left|e^{x}-1\right|\right)}{a^{2}}+\frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{a^{2}}-\frac{2}{a\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b*log(e^x + 1)/a^2 - b*log(abs(e^x - 1))/a^2 + sqrt(a^2 + b^2)*log(abs(2*b*
e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/a^2
- 2/(a*(e^(2*x) - 1))
```

3.234 $\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=52

$$\frac{(a^2+b^2)\log(\sinh(x))}{a^3}-\frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}+\frac{b\operatorname{csch}(x)}{a^2}-\frac{\operatorname{csch}^2(x)}{2a}$$

```
[Out] (b*Csch[x])/a^2 - Csch[x]^2/(2*a) + ((a^2 + b^2)*Log[Sinh[x]])/a^3 - ((a^2
+ b^2)*Log[a + b*Sinh[x]])/a^3
```

Rubi [A] time = 0.0947691, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2721, 894}

$$\frac{(a^2+b^2)\log(\sinh(x))}{a^3}-\frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}+\frac{b\operatorname{csch}(x)}{a^2}-\frac{\operatorname{csch}^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (b*Csch[x])/a^2 - Csch[x]^2/(2*a) + ((a^2 + b^2)*Log[Sinh[x]])/a^3 - ((a^2
+ b^2)*Log[a + b*Sinh[x]])/a^3
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)}\,dx,x,b\sinh(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(-\frac{b^2}{ax^3}+\frac{b^2}{a^2x^2}+\frac{-a^2-b^2}{a^3x}+\frac{a^2+b^2}{a^3(a+x)}\right)dx,x,b\sinh(x)\right)\\ &= \frac{b\operatorname{csch}(x)}{a^2}-\frac{\operatorname{csch}^2(x)}{2a}+\frac{(a^2+b^2)\log(\sinh(x))}{a^3}-\frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0609568, size = 45, normalized size = 0.87

$$\frac{2(a^2+b^2)(\log(\sinh(x))-\log(a+b\sinh(x)))-a^2\operatorname{csch}^2(x)+2ab\operatorname{csch}(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + b^2)*(Log[Sinh[x]] - Log[a + b*Si
nh[x]]))/(2*a^3)
```

Maple [B] time = 0.034, size = 120, normalized size = 2.3

$$-\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-\frac{b}{2a^{2}}\tanh\left(\frac{x}{2}\right)-\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{b^{2}}{a^{3}}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{b}{2a^{2}}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{1}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sinh(x)),x)
```

```
[Out] -1/8/a*tanh(1/2*x)^2-1/2/a^2*tanh(1/2*x)*b-1/8/a/tanh(1/2*x)^2+1/a*ln(tanh(
1/2*x))+1/a^3*ln(tanh(1/2*x))*b^2+1/2*b/a^2/tanh(1/2*x)-1/a*ln(a*tanh(1/2*x
)^2-2*tanh(1/2*x)*b-a)-1/a^3*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^2
```

Maxima [B] time = 1.02386, size = 157, normalized size = 3.02

$$-\frac{2\left(be^{(-x)}-ae^{(-2x)}-be^{(-3x)}\right)}{2a^{2}e^{(-2x)}-a^{2}e^{(-4x)}-a^{2}}-\frac{\left(a^{2}+b^{2}\right)\log\left(-2ae^{(-x)}+be^{(-2x)}-b\right)}{a^{3}}+\frac{\left(a^{2}+b^{2}\right)\log\left(e^{(-x)}+1\right)}{a^{3}}+\frac{\left(a^{2}+b^{2}\right)\log\left(e^{(-x)}-\right.}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*(b*e^(-x) - a*e^(-2*x) - b*e^(-3*x))/(2*a^2*e^(-2*x) - a^2*e^(-4*x) - a^
2) - (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^3 + (a^2 + b^2)*log(e^
(-x) + 1)/a^3 + (a^2 + b^2)*log(e^(-x) - 1)/a^3
```

Fricas [B] time = 2.17359, size = 1165, normalized size = 22.4

$$\frac{2\,ab\cosh\left(x\right)^{3}+2\,ab\sinh\left(x\right)^{3}-2\,a^{2}\cosh\left(x\right)^{2}-2\,ab\cosh\left(x\right)+2\left(3\,ab\cosh\left(x\right)-a^{2}\right)\sinh\left(x\right)^{2}-\left(\left(a^{2}+b^{2}\right)\cosh\left(x\right)^{4}+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 - 2*a*b*cosh(x) + 2*(3
*a*b*cosh(x) - a^2)*sinh(x)^2 - ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh
(x)*sinh(x)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2
 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(
x)^3 - (a^2 + b^2)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(
x))) + ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*sinh(x)^3 + (a^2 + b^
2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^2)*cosh(x)^2 - a^2 -
 b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3 - (a^2 + b^2)*cosh(x
```

```
))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)^2 - 2*a^2
*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sin
h(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 + 4*(a
^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*sinh(x)), x)
```

Giac [B] time = 1.1421, size = 169, normalized size = 3.25

$$\frac{(a^2+b^2)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{a^3} - \frac{(a^2b+b^3)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2a\right|\right)}{a^3b} - \frac{3a^2\left(e^{(-x)}-e^x\right)^2+3b^2\left(e^{(-x)}-e^x\right)^2+4ab\left(e^{(-x)}-e^x\right)+4a^2}{2a^3\left(e^{(-x)}-e^x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(-e^(-x) + e^x))/a^3 - (a^2*b + b^3)*log(abs(-b*(e^(-x)
- e^x) + 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) - e^x)^2 + 3*b^2*(e^(-x) - e^x)
^2 + 4*a*b*(e^(-x) - e^x) + 4*a^2)/(a^3*(e^(-x) - e^x)^2)
```

3.235 $\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=108

$$-\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4}-\frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3}+\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\text{coth}$$

```
[Out] (b*(3*a^2 + 2*b^2)*ArcTanh[Cosh[x]])/(2*a^4) - (2*(a^2 + b^2)^(3/2)*ArcTanh
[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^4 - ((4*a^2 + 3*b^2)*Coth[x])/(3*a^3
) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)
```

Rubi [A] time = 0.413013, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2725, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4}-\frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3}+\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\text{coth}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (b*(3*a^2 + 2*b^2)*ArcTanh[Cosh[x]])/(2*a^4) - (2*(a^2 + b^2)^(3/2)*ArcTanh
[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^4 - ((4*a^2 + 3*b^2)*Coth[x])/(3*a^3
) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a} + \frac{\int \frac{\text{csch}^2(x)\left(2\left(4a^2+3b^2\right)-ab\sinh(x)+3\left(2a^2+b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{6a^2}$$

$$= -\frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a} + \frac{i\int \frac{\text{csch}(x)\left(3ib\left(3a^2+2b^2\right)-3ia\left(2a^2+b^2\right.\right.}{a+b\sinh(x)}}{6a^3}$$

$$= -\frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a} + \frac{\left(a^2+b^2\right)^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^4} - \frac{(b}{}$$

$$= \frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$= \frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$= \frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4} - \frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3} + \frac{b}{}$$

Mathematica [A] time = 0.422632, size = 176, normalized size = 1.63

$$\frac{-4a\left(4a^2+3b^2\right)\tanh\left(\frac{x}{2}\right)-4a\left(4a^2+3b^2\right)\coth\left(\frac{x}{2}\right)-12b\left(3a^2+2b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)+48\left(-a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b-a\tan}{\sqrt{-a^2}}\right.}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (48*(-a^2 - b^2)^(3/2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] - 4*a*(4*
a^2 + 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 - 12*b*(3*a^2 + 2*b^2)*Log[Tan
h[x/2]] + 3*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2
]^4*Sinh[x])/2 - 4*a*(4*a^2 + 3*b^2)*Tanh[x/2])/(24*a^4)
```

Maple [B] time = 0.036, size = 232, normalized size = 2.2

$$-\frac{1}{24a}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}-\frac{b}{8a^{2}}\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-\frac{5}{8a}\tanh\left(\frac{x}{2}\right)-\frac{b^{2}}{2a^{3}}\tanh\left(\frac{x}{2}\right)-\frac{1}{24a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3}-\frac{5}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-\frac{b^{2}}{2a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*sinh(x)),x)
```

```
[Out] -1/24/a*tanh(1/2*x)^3-1/8/a^2*b*tanh(1/2*x)^2-5/8/a*tanh(1/2*x)-1/2/a^3*b^2
*tanh(1/2*x)-1/24/a/tanh(1/2*x)^3-5/8/a/tanh(1/2*x)-1/2/a^3/tanh(1/2*x)*b^2
+1/8/a^2*b/tanh(1/2*x)^2-3/2/a^2*b*ln(tanh(1/2*x))-1/a^4*b^3*ln(tanh(1/2*x)
)+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+4*b^
2/a^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+2/
a^4*b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.47033, size = 3380, normalized size = 31.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^2*b*cosh(x)^5 + 6*a^2*b*sinh(x)^5 - 12*(2*a^3 + a*b^2)*cosh(x)^4 +
 6*(5*a^2*b*cosh(x) - 4*a^3 - 2*a*b^2)*sinh(x)^4 - 6*a^2*b*cosh(x) + 12*(5*
a^2*b*cosh(x)^2 - 4*(2*a^3 + a*b^2)*cosh(x))*sinh(x)^3 - 16*a^3 - 12*a*b^2
+ 24*(a^3 + a*b^2)*cosh(x)^2 + 12*(5*a^2*b*cosh(x)^3 + 2*a^3 + 2*a*b^2 - 6*
(2*a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^2 + b^2)*cosh(x)^6 + 6*(a^2 +
b^2)*cosh(x)*sinh(x)^5 + (a^2 + b^2)*sinh(x)^6 - 3*(a^2 + b^2)*cosh(x)^4 +
3*(5*(a^2 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 + b^2)*cosh(x
)^3 - 3*(a^2 + b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2 + 3*(5*(a^
2 + b^2)*cosh(x)^4 - 6*(a^2 + b^2)*cosh(x)^2 + a^2 + b^2)*sinh(x)^2 - a^2 -
 b^2 + 6*((a^2 + b^2)*cosh(x)^5 - 2*(a^2 + b^2)*cosh(x)^3 + (a^2 + b^2)*cos
```

```
h(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) - b)) + 3*((3*a^2*b + 2*b^3)*cosh(x)^6 + 6*(3*a^2*b +
2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b + 2*b^3)*sinh(x)^6 - 3*(3*a^2*b + 2*b^3
)*cosh(x)^4 - 3*(3*a^2*b + 2*b^3 - 5*(3*a^2*b + 2*b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(5*(3*a^2*b + 2*b^3)*cosh(x)^3 - 3*(3*a^2*b + 2*b^3)*cosh(x))*sinh(x)^
3 - 3*a^2*b - 2*b^3 + 3*(3*a^2*b + 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b + 2*b^3
)*cosh(x)^4 + 3*a^2*b + 2*b^3 - 6*(3*a^2*b + 2*b^3)*cosh(x)^2)*sinh(x)^2 +
6*((3*a^2*b + 2*b^3)*cosh(x)^5 - 2*(3*a^2*b + 2*b^3)*cosh(x)^3 + (3*a^2*b +
 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*((3*a^2*b + 2*b^3)
*cosh(x)^6 + 6*(3*a^2*b + 2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b + 2*b^3)*sinh
(x)^6 - 3*(3*a^2*b + 2*b^3)*cosh(x)^4 - 3*(3*a^2*b + 2*b^3 - 5*(3*a^2*b + 2
*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^2*b + 2*b^3)*cosh(x)^3 - 3*(3*a^2*b
+ 2*b^3)*cosh(x))*sinh(x)^3 - 3*a^2*b - 2*b^3 + 3*(3*a^2*b + 2*b^3)*cosh(x)
^2 + 3*(5*(3*a^2*b + 2*b^3)*cosh(x)^4 + 3*a^2*b + 2*b^3 - 6*(3*a^2*b + 2*b^
3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^2*b + 2*b^3)*cosh(x)^5 - 2*(3*a^2*b + 2*b
^3)*cosh(x)^3 + (3*a^2*b + 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) -
 1) + 6*(5*a^2*b*cosh(x)^4 - 8*(2*a^3 + a*b^2)*cosh(x)^3 - a^2*b + 8*(a^3 +
 a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*si
nh(x)^6 - 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sin
h(x)^4 - a^4 + 4*(5*a^4*cosh(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cos
h(x)^4 - 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x
)^3 + a^4*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^4(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*sinh(x)), x)
```

Giac [B] time = 1.13321, size = 262, normalized size = 2.43

$$\frac{\left(3\,a^2b+2\,b^3\right)\log\left(e^x+1\right)}{2\,a^4} - \frac{\left(3\,a^2b+2\,b^3\right)\log\left(\left|e^x-1\right|\right)}{2\,a^4} + \frac{\left(a^4+2\,a^2b^2+b^4\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^4} + \frac{3\,abe^{(5\,x)}-12\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*b + 2*b^3)*log(e^x + 1)/a^4 - 1/2*(3*a^2*b + 2*b^3)*log(abs(e^x
- 1))/a^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^
2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/3*(3*
a*b*e^(5*x) - 12*a^2*e^(4*x) - 6*b^2*e^(4*x) + 12*a^2*e^(2*x) + 12*b^2*e^(2
*x) - 3*a*b*e^x - 8*a^2 - 6*b^2)/(a^3*(e^(2*x) - 1)^3)
```

3.236 $\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=224

$$-\frac{2a^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{\left(a^2-b^2\right)\tanh^3(x)}{3\left(a^2+b^2\right)^2}-\frac{\left(-3a^2b^2+2a^4-b^4\right)\tanh(x)}{\left(a^2+b^2\right)^3}+\frac{\left(a^2-b^2\right.}{\left(a^2+\right.}$$

```
[Out] (-2*a^5*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (8*
a^3*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) - (4*
a^3*b*Sech[x])/(a^2 + b^2)^3 + (2*a*b*Sech[x]^3)/(3*(a^2 + b^2)^2) - (a^4*b
*Cosh[x])/((a^2 + b^2)^3*(a + b*Sinh[x])) + ((a^2 - b^2)*Tanh[x])/(a^2 + b^
2)^2 - ((2*a^4 - 3*a^2*b^2 - b^4)*Tanh[x])/(a^2 + b^2)^3 - ((a^2 - b^2)*Tan
h[x]^3)/(3*(a^2 + b^2)^2)
```

Rubi [A] time = 0.434703, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2731, 2664, 12, 2660, 618, 206, 2669, 3767, 8}

$$-\frac{2a^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{\left(a^2-b^2\right)\tanh^3(x)}{3\left(a^2+b^2\right)^2}-\frac{\left(-3a^2b^2+2a^4-b^4\right)\tanh(x)}{\left(a^2+b^2\right)^3}+\frac{\left(a^2-b^2\right.}{\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^5*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (8*
a^3*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) - (4*
a^3*b*Sech[x])/(a^2 + b^2)^3 + (2*a*b*Sech[x]^3)/(3*(a^2 + b^2)^2) - (a^4*b
*Cosh[x])/((a^2 + b^2)^3*(a + b*Sinh[x])) + ((a^2 - b^2)*Tanh[x])/(a^2 + b^
2)^2 - ((2*a^4 - 3*a^2*b^2 - b^4)*Tanh[x])/(a^2 + b^2)^3 - ((a^2 - b^2)*Tan
h[x]^3)/(3*(a^2 + b^2)^2)
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \int\left(\frac{a^4}{\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{4a^3b^2}{\left(a^2+b^2\right)^3(a+b\sinh(x))} + \frac{\operatorname{sech}^4(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(\right.}{\left(a^2+b^2\right)^2}\right.$$

$$= \frac{\int \operatorname{sech}^2(x)\left(-2a^4\left(1-\frac{3a^2b^2+b^4}{2a^4}\right)+4a^3b\sinh(x)\right)dx}{\left(a^2+b^2\right)^3} - \frac{\left(4a^3b^2\right)\int\frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^3} + \frac{\int \operatorname{sech}^4(x)\left(a^2\right.}{}$$

$$= -\frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^4b\cosh(x)}{\left(a^2+b^2\right)^3(a+b\sinh(x))} + \frac{a^4\int\frac{a}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^3} - \frac{\left(8a^3b^2\right)\mathrm{Subs}}{}$$

$$= -\frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^4b\cosh(x)}{\left(a^2+b^2\right)^3(a+b\sinh(x))} + \frac{a^5\int\frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^3} + \frac{\left(16a^3b^2\right)\mathrm{Sub}}{}$$

$$= \frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^4b\cosh(x)}{\left(a^2+b^2\right)^3(a+b\sinh(x))} + \frac{\left(a^2-\right.}{(a}$$

$$= \frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^4b\cosh(x)}{\left(a^2+b^2\right)^3(a+b\sinh(x))} + \frac{\left(a^2-\right.}{(a}$$

$$= -\frac{2a^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} + \frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} - \frac{}{\left(a^2+\right.}$$

Mathematica [A] time = 0.389672, size = 144, normalized size = 0.64

$$\frac{\left(9a^2b^2-4a^4+b^4\right)\tanh(x) + \frac{6a^3\left(a^2-4b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \left(a^2+b^2\right)\operatorname{sech}^3(x)\left(\left(a^2-b^2\right)\sinh(x)+2ab\right) - 12a^3b\operatorname{sech}(x) -}{3\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a^3*(a^2 - 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
 - b^2] - 12*a^3*b*Sech[x] - (3*a^4*b*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b^2
)*Sech[x]^3*(2*a*b + (a^2 - b^2)*Sinh[x]) + (-4*a^4 + 9*a^2*b^2 + b^4)*Tanh
[x])/(3*(a^2 + b^2)^3)
```

Maple [A] time = 0.062, size = 262, normalized size = 1.2

$$2\,\frac{\left(-a^4+3\,a^2b^2\right)(\tanh(x/2))^5+\left(-2\,a^3b+2\,ab^3\right)(\tanh(x/2))^4+\left(-10/3\,a^4+6\,a^2b^2+4/3\,b^4\right)(\tanh(x/2))^3-8\,a^3b\,(\tanh}{\left(a^2+b^2\right)\left(a^4+2\,a^2b^2+b^4\right)\left((\tanh(x/2))^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] 2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4+3*a^2*b^2)*tanh(1/2*x)^5+(-2*a^3*b+2
*a*b^3)*tanh(1/2*x)^4+(-10/3*a^4+6*a^2*b^2+4/3*b^4)*tanh(1/2*x)^3-8*a^3*b*t
anh(1/2*x)^2+(-a^4+3*a^2*b^2)*tanh(1/2*x)-10/3*a^3*b+2/3*a*b^3)/(tanh(1/2*x
)^2+1)^3-2*a^3/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2*tanh(1/2*x)-a*b)/(a*tan
h(1/2*x)^2-2*tanh(1/2*x)*b-a)-(a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72786, size = 8182, normalized size = 36.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^7 + 6*(a^7 - 3*a^5*b^2 - 4*a^
3*b^4)*sinh(x)^7 - 14*a^6*b + 4*a^4*b^3 + 20*a^2*b^5 + 2*b^7 - 6*(7*a^6*b +
 10*a^4*b^3 + 4*a^2*b^5 + b^7)*cosh(x)^6 - 6*(7*a^6*b + 10*a^4*b^3 + 4*a^2*
b^5 + b^7 - 7*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x))*sinh(x)^6 + 2*(21*a^7
- a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)*cosh(x)^5 + 2*(21*a^7 - a^5*b^2 - 20*a^3*
b^4 + 2*a*b^6 + 63*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^2 - 18*(7*a^6*b +
10*a^4*b^3 + 4*a^2*b^5 + b^7)*cosh(x))*sinh(x)^5 - 2*(41*a^6*b + 34*a^4*b^3
 - 10*a^2*b^5 - 3*b^7)*cosh(x)^4 - 2*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 -
3*b^7 - 105*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^3 + 45*(7*a^6*b + 10*a^4*
b^3 + 4*a^2*b^5 + b^7)*cosh(x)^2 - 5*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b
^6)*cosh(x))*sinh(x)^4 + 2*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cos
h(x)^3 + 2*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6 + 105*(a^7 - 3*a^5*b
^2 - 4*a^3*b^4)*cosh(x)^4 - 60*(7*a^6*b + 10*a^4*b^3 + 4*a^2*b^5 + b^7)*cos
h(x)^3 + 10*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)*cosh(x)^2 - 4*(41*a^6
*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x))*sinh(x)^3 - 2*(35*a^6*b + 26
*a^4*b^3 - 8*a^2*b^5 + b^7)*cosh(x)^2 - 2*(35*a^6*b + 26*a^4*b^3 - 8*a^2*b^
5 + b^7 - 63*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^5 + 45*(7*a^6*b + 10*a^4
*b^3 + 4*a^2*b^5 + b^7)*cosh(x)^4 - 10*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a
*b^6)*cosh(x)^3 + 6*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x)^2
- 3*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cosh(x))*sinh(x)^2 - 3*((a
^5*b - 4*a^3*b^3)*cosh(x)^8 + (a^5*b - 4*a^3*b^3)*sinh(x)^8 + 2*(a^6 - 4*a^
4*b^2)*cosh(x)^7 + 2*(a^6 - 4*a^4*b^2 + 4*(a^5*b - 4*a^3*b^3)*cosh(x))*sinh
(x)^7 + 2*(a^5*b - 4*a^3*b^3)*cosh(x)^6 + 2*(a^5*b - 4*a^3*b^3 + 14*(a^5*b
- 4*a^3*b^3)*cosh(x)^2 + 7*(a^6 - 4*a^4*b^2)*cosh(x))*sinh(x)^6 - a^5*b + 4
*a^3*b^3 + 6*(a^6 - 4*a^4*b^2)*cosh(x)^5 + 2*(3*a^6 - 12*a^4*b^2 + 28*(a^5*
b - 4*a^3*b^3)*cosh(x)^3 + 21*(a^6 - 4*a^4*b^2)*cosh(x)^2 + 6*(a^5*b - 4*a^
3*b^3)*cosh(x))*sinh(x)^5 + 10*(7*(a^5*b - 4*a^3*b^3)*cosh(x)^4 + 7*(a^6 -
4*a^4*b^2)*cosh(x)^3 + 3*(a^5*b - 4*a^3*b^3)*cosh(x)^2 + 3*(a^6 - 4*a^4*b^2
)*cosh(x))*sinh(x)^4 + 6*(a^6 - 4*a^4*b^2)*cosh(x)^3 + 2*(3*a^6 - 12*a^4*b^
2 + 28*(a^5*b - 4*a^3*b^3)*cosh(x)^5 + 35*(a^6 - 4*a^4*b^2)*cosh(x)^4 + 20*
```

```
(a^5*b - 4*a^3*b^3)*cosh(x)^3 + 30*(a^6 - 4*a^4*b^2)*cosh(x)^2)*sinh(x)^3 -
 2*(a^5*b - 4*a^3*b^3)*cosh(x)^2 + 2*(14*(a^5*b - 4*a^3*b^3)*cosh(x)^6 - a^
5*b + 4*a^3*b^3 + 21*(a^6 - 4*a^4*b^2)*cosh(x)^5 + 15*(a^5*b - 4*a^3*b^3)*c
osh(x)^4 + 30*(a^6 - 4*a^4*b^2)*cosh(x)^3 + 9*(a^6 - 4*a^4*b^2)*cosh(x))*si
nh(x)^2 + 2*(a^6 - 4*a^4*b^2)*cosh(x) + 2*(4*(a^5*b - 4*a^3*b^3)*cosh(x)^7
 + 7*(a^6 - 4*a^4*b^2)*cosh(x)^6 + a^6 - 4*a^4*b^2 + 6*(a^5*b - 4*a^3*b^3)*c
osh(x)^5 + 15*(a^6 - 4*a^4*b^2)*cosh(x)^4 + 9*(a^6 - 4*a^4*b^2)*cosh(x)^2 -
 2*(a^5*b - 4*a^3*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh
(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(11*a^7 + 5*a^5*b^
2 - 8*a^3*b^4 - 2*a*b^6)*cosh(x) + 2*(11*a^7 + 5*a^5*b^2 - 8*a^3*b^4 - 2*a*
b^6 + 21*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^6 - 18*(7*a^6*b + 10*a^4*b^3
 + 4*a^2*b^5 + b^7)*cosh(x)^5 + 5*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)
*cosh(x)^4 - 4*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x)^3 + 3*(
21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cosh(x)^2 - 2*(35*a^6*b + 26*a^
4*b^3 - 8*a^2*b^5 + b^7)*cosh(x))*sinh(x))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
 4*a^2*b^7 + b^9 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x
)^8 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sinh(x)^8 - 2*(a^9
 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^7 - 2*(a^9 + 4*a^7*b^2
 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2
*b^7 + b^9)*cosh(x))*sinh(x)^7 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b
^7 + b^9)*cosh(x)^6 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 +
14*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 7*(a^9 + 4
*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^6 - 6*(a^9 + 4*a
^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^5 - 2*(3*a^9 + 12*a^7*b^2 +
 18*a^5*b^4 + 12*a^3*b^6 + 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*
a^2*b^7 + b^9)*cosh(x)^3 + 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*
b^8)*cosh(x)^2 + 6*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x
))*sinh(x)^5 - 10*(7*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh
(x)^4 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 + 3*(
a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 3*(a^9 + 4*a^7
*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^4 - 6*(a^9 + 4*a^7*b
^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 - 2*(3*a^9 + 12*a^7*b^2 + 18*
a^5*b^4 + 12*a^3*b^6 + 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*
b^7 + b^9)*cosh(x)^5 + 35*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)
*cosh(x)^4 + 20*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3
 + 30*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2)*sinh(x)^
3 + 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 2*(a^8*
b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 - 14*(a^8*b + 4*a^6*b^3 + 6*a^4
*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^6 - 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3
*b^6 + a*b^8)*cosh(x)^5 - 15*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b
^9)*cosh(x)^4 - 30*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x
)^3 - 9*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^
2 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x) - 2*(a^9 +
4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^
5 + 4*a^2*b^7 + b^9)*cosh(x)^7 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6
 + a*b^8)*cosh(x)^6 + 6*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*c
osh(x)^5 + 15*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 +
 9*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 - 2*(a^8*b +
 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(x)}{(a + b\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sinh(x))**2, x)
```

Giac [A] time = 1.22414, size = 394, normalized size = 1.76

$$\frac{\left(a^5-4\,a^3b^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(a^5e^x-a^4b\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}-\frac{2\left(12\,a^3be^{(5\,x)}-6\,a^4e^{(4\,x)}+9\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^5 - 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x +
2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b
^2)) + 2*(a^5*e^x - a^4*b)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*x)
+ 2*a*e^x - b)) - 2/3*(12*a^3*b*e^(5*x) - 6*a^4*e^(4*x) + 9*a^2*b^2*e^(4*x)
 + 3*b^4*e^(4*x) + 16*a^3*b*e^(3*x) - 8*a*b^3*e^(3*x) - 6*a^4*e^(2*x) + 18*
a^2*b^2*e^(2*x) + 12*a^3*b*e^x - 4*a^4 + 9*a^2*b^2 + b^4)/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

3.237 $\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=135

$$\frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{ab\left(3a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{a^2\left(a^2-3b^2\right)\log(\cosh(x)}{\left(a^2+b^2\right)^3}$$

```
[Out] (a*b*(3*a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^3 + (a^2*(a^2 - 3*b^2)*Log[
Cosh[x]])/(a^2 + b^2)^3 - (a^2*(a^2 - 3*b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2
)^3 + a^3/((a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(a^2 - b^2 - 2*a*b*S
inh[x]))/(2*(a^2 + b^2)^2)
```

Rubi [A] time = 0.355971, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2721, 1647, 1629, 635, 203, 260}

$$\frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{ab\left(3a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{a^2\left(a^2-3b^2\right)\log(\cosh(x)}{\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (a*b*(3*a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^3 + (a^2*(a^2 - 3*b^2)*Log[
Cosh[x]])/(a^2 + b^2)^3 - (a^2*(a^2 - 3*b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2
)^3 + a^3/((a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(a^2 - b^2 - 2*a*b*S
inh[x]))/(2*(a^2 + b^2)^2)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Subst}\left(\int \frac{x^3}{(a+x)^2\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^4}{\left(a^2+b^2\right)^2}+\frac{2a^2b^2x}{a^2+b^2}-\frac{2ab^4x^2}{\left(a^2+b^2\right)^2}}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2b^2}$$

$$= \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} - \frac{\text{Subst}\left(\int\left(\frac{2a^3b^2}{\left(a^2+b^2\right)^2(a+x)^2}+\frac{2a^2b^2\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3(a+x)}+\frac{2ab^2\left(-b^2\left(3a^2-b^2\right)-a\right.}{\left(a^2+b^2\right)^3\left(b^2+\right.}\right.\right.}{2b^2}$$

$$= -\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\,\text{sin}\right.}{2\left(a^2+b^2\right)^2}$$

$$= -\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\,\text{sin}\right.}{2\left(a^2+b^2\right)^2}$$

$$= \frac{ab\left(3a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^2\left(a^2-3b^2\right)\log(\cosh(x))}{\left(a^2+b^2\right)^3} - \frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}$$

Mathematica [C] time = 0.661667, size = 150, normalized size = 1.11

$$\frac{\frac{2a^3\left(a^2+b^2\right)}{a+b\sinh(x)} + \left(a^4-b^4\right)\text{sech}^2(x) - 2a^2\left(a^2-3b^2\right)\log(a+b\sinh(x)) - 2ab\left(a^2+b^2\right)\tan^{-1}(\sinh(x)) - 2ab\left(a^2+b^2\right)\text{tanl}}{2\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^2*(a - I*b)*(a - (3*I)*b)*Log[I - S
inh[x]] + a^2*(a + I*b)*(a + (3*I)*b)*Log[I + Sinh[x]] - 2*a^2*(a^2 - 3*b^2
)*Log[a + b*Sinh[x]] + (a^4 - b^4)*Sech[x]^2 + (2*a^3*(a^2 + b^2))/(a + b*S
inh[x]) - 2*a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 + b^2)^3)
```

Maple [B] time = 0.062, size = 491, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] 2/(a^2+b^2)^3/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^3*b+2/(a^2+b^2)^3/(tanh(1
/2*x)^2+1)^2*tanh(1/2*x)^3*a*b^3-2/(a^2+b^2)^3/(tanh(1/2*x)^2+1)^2*tanh(1/2
*x)^2*a^4+2/(a^2+b^2)^3/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2*b^4-2/(a^2+b^2)^3
/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a^3*b-2/(a^2+b^2)^3/(tanh(1/2*x)^2+1)^2*ta
nh(1/2*x)*a*b^3+1/(a^2+b^2)^3*ln(tanh(1/2*x)^2+1)*a^4-3/(a^2+b^2)^3*ln(tanh
(1/2*x)^2+1)*a^2*b^2+6/(a^2+b^2)^3*arctan(tanh(1/2*x))*a^3*b-2/(a^2+b^2)^3*
a*arctan(tanh(1/2*x))*b^3+2*a^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*tanh(1/2*x)/(
a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b+2*a^2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ta
nh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^3-a^4/(a^4+2*a^2*b^2+b^4)/(
a^2+b^2)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+3*a^2/(a^4+2*a^2*b^2+b^4)/(a
^2+b^2)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^2
```

Maxima [B] time = 1.59428, size = 506, normalized size = 3.75

$$-\frac{2\left(3a^3b-ab^3\right)\arctan\left(e^{(-x)}\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(a^4-3a^2b^2\right)\log\left(-2ae^{(-x)}+be^{(-2x)}-b\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\left(a^4-3a^2b^2\right)\log\left(e^{(-2x)}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{}{a^4b+2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(3*a^3*b - a*b^3)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (
a^4 - 3*a^2*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6) + (a^4 - 3*a^2*b^2)*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6) + 2*(4*a^3*e^(-3*x) + (a^3 - a*b^2)*e^(-x) - (a^2*b + b^3)*e^(-2
*x) + (a^2*b + b^3)*e^(-4*x) + (a^3 - a*b^2)*e^(-5*x))/(a^4*b + 2*a^2*b^3 +
 b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b^5)*e^(-2
*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-
4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(
-6*x))
```

Fricas [B] time = 2.79176, size = 6589, normalized size = 48.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*(a^5 - a*b^4)*cosh(x)^5 + 2*(a^5 - a*b^4)*sinh(x)^5 - 2*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5 - 5*(a^5 - a*b^4)*cosh(x)
)*sinh(x)^4 + 8*(a^5 + a^3*b^2)*cosh(x)^3 + 4*(2*a^5 + 2*a^3*b^2 + 5*(a^5 -
 a*b^4)*cosh(x)^2 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 + 2*(a^4
*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5 + 10*(a^5 - a*
```

```
b^4)*cosh(x)^3 - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 12*(a^5 + a^3*b^2)
*cosh(x))*sinh(x)^2 + 2*((3*a^3*b^2 - a*b^4)*cosh(x)^6 + (3*a^3*b^2 - a*b^4
)*sinh(x)^6 + 2*(3*a^4*b - a^2*b^3)*cosh(x)^5 + 2*(3*a^4*b - a^2*b^3 + 3*(3
*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^5 - 3*a^3*b^2 + a*b^4 + (3*a^3*b^2 - a*b
^4)*cosh(x)^4 + (3*a^3*b^2 - a*b^4 + 15*(3*a^3*b^2 - a*b^4)*cosh(x)^2 + 10*
(3*a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^4*b - a^2*b^3)*cosh(x)^3 +
4*(3*a^4*b - a^2*b^3 + 5*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 5*(3*a^4*b - a^2*b
^3)*cosh(x)^2 + (3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - (3*a^3*b^2 - a*b^4
)*cosh(x)^2 - (3*a^3*b^2 - a*b^4 - 15*(3*a^3*b^2 - a*b^4)*cosh(x)^4 - 20*(3
*a^4*b - a^2*b^3)*cosh(x)^3 - 6*(3*a^3*b^2 - a*b^4)*cosh(x)^2 - 12*(3*a^4*b
 - a^2*b^3)*cosh(x))*sinh(x)^2 + 2*(3*a^4*b - a^2*b^3)*cosh(x) + 2*(3*(3*a^
3*b^2 - a*b^4)*cosh(x)^5 + 3*a^4*b - a^2*b^3 + 5*(3*a^4*b - a^2*b^3)*cosh(x
)^4 + 2*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 6*(3*a^4*b - a^2*b^3)*cosh(x)^2 - (
3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 2*(a^5 - a
*b^4)*cosh(x) - ((a^4*b - 3*a^2*b^3)*cosh(x)^6 + (a^4*b - 3*a^2*b^3)*sinh(x
)^6 + 2*(a^5 - 3*a^3*b^2)*cosh(x)^5 + 2*(a^5 - 3*a^3*b^2 + 3*(a^4*b - 3*a^2
*b^3)*cosh(x))*sinh(x)^5 - a^4*b + 3*a^2*b^3 + (a^4*b - 3*a^2*b^3)*cosh(x)^
4 + (a^4*b - 3*a^2*b^3 + 15*(a^4*b - 3*a^2*b^3)*cosh(x)^2 + 10*(a^5 - 3*a^3
*b^2)*cosh(x))*sinh(x)^4 + 4*(a^5 - 3*a^3*b^2)*cosh(x)^3 + 4*(a^5 - 3*a^3*b
^2 + 5*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 5*(a^5 - 3*a^3*b^2)*cosh(x)^2 + (a^4
*b - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^4*b - 3*a^2*b^3)*cosh(x)^2 - (a^4*b
 - 3*a^2*b^3 - 15*(a^4*b - 3*a^2*b^3)*cosh(x)^4 - 20*(a^5 - 3*a^3*b^2)*cosh
(x)^3 - 6*(a^4*b - 3*a^2*b^3)*cosh(x)^2 - 12*(a^5 - 3*a^3*b^2)*cosh(x))*sin
h(x)^2 + 2*(a^5 - 3*a^3*b^2)*cosh(x) + 2*(3*(a^4*b - 3*a^2*b^3)*cosh(x)^5 +
 a^5 - 3*a^3*b^2 + 5*(a^5 - 3*a^3*b^2)*cosh(x)^4 + 2*(a^4*b - 3*a^2*b^3)*co
sh(x)^3 + 6*(a^5 - 3*a^3*b^2)*cosh(x)^2 - (a^4*b - 3*a^2*b^3)*cosh(x))*sinh
(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + ((a^4*b - 3*a^2*b^3)*cosh
(x)^6 + (a^4*b - 3*a^2*b^3)*sinh(x)^6 + 2*(a^5 - 3*a^3*b^2)*cosh(x)^5 + 2*(
a^5 - 3*a^3*b^2 + 3*(a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^5 - a^4*b + 3*a^2*
b^3 + (a^4*b - 3*a^2*b^3)*cosh(x)^4 + (a^4*b - 3*a^2*b^3 + 15*(a^4*b - 3*a^
2*b^3)*cosh(x)^2 + 10*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^4 + 4*(a^5 - 3*a^3
*b^2)*cosh(x)^3 + 4*(a^5 - 3*a^3*b^2 + 5*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 5*
(a^5 - 3*a^3*b^2)*cosh(x)^2 + (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^4
*b - 3*a^2*b^3)*cosh(x)^2 - (a^4*b - 3*a^2*b^3 - 15*(a^4*b - 3*a^2*b^3)*cos
h(x)^4 - 20*(a^5 - 3*a^3*b^2)*cosh(x)^3 - 6*(a^4*b - 3*a^2*b^3)*cosh(x)^2 -
 12*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^2 + 2*(a^5 - 3*a^3*b^2)*cosh(x) + 2*
(3*(a^4*b - 3*a^2*b^3)*cosh(x)^5 + a^5 - 3*a^3*b^2 + 5*(a^5 - 3*a^3*b^2)*co
sh(x)^4 + 2*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 6*(a^5 - 3*a^3*b^2)*cosh(x)^2 -
 (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) +
 2*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*cosh(x)^4 - 4*(a^4*b + 2*a^2*b^3 + b^5)*c
osh(x)^3 + 12*(a^5 + a^3*b^2)*cosh(x)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(
x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^6 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^6
- 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 - 2*(a^7 + 3*a^5*b^2 +
3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x
)^5 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7 + 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + 10
*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^4 - 4*(a^7 + 3*a^5*
b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6
 + 5*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 5*(a^7 + 3*a^5*b^2 +
 3*a^3*b^4 + a*b^6)*cosh(x)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(
x))*sinh(x)^3 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^6*b +
3*a^4*b^3 + 3*a^2*b^5 + b^7 - 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x)^4 - 20*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 6*(a^6*b + 3*a
^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 - 12*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x) -
2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*cosh(x)^5 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 2*(a^
6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b
```

```
^4 + a*b^6)*cosh(x)^2 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh
(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sinh(x))**2, x)
```

Giac [B] time = 1.17526, size = 414, normalized size = 3.07

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)\left(3a^3b-ab^3\right)}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)} + \frac{\left(a^4-3a^2b^2\right)\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)} - \frac{\left(a^4b-3a^2b^3\right)\log\left(\left|-b\left(e^{(-x)}-e\right.\right.\right.}{a^6b+3a^4b^3+3a^2b^5+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^3*b - a*b^3)/(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*b^2)*log((e^(-x) - e^x)^2 + 4)/
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4*b - 3*a^2*b^3)*log(abs(-b*(e^(-x
) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 2*(a^3*(e^(-x) - e
^x)^2 - a*b^2*(e^(-x) - e^x)^2 + a^2*b*(e^(-x) - e^x) + b^3*(e^(-x) - e^x)
+ 6*a^3 - 2*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a*(e^(-
x) - e^x)^2 + 4*b*(e^(-x) - e^x) - 8*a))
```

3.238 $\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=144

$$-\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\left(a^2-b^2\right)\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{2ab\,\mathrm{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}$$

```
[Out] (-2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (4*
a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (2*a*
b*Sech[x])/(a^2 + b^2)^2 - (a^2*b*Cosh[x])/((a^2 + b^2)^2*(a + b*Sinh[x]))
- ((a^2 - b^2)*Tanh[x])/(a^2 + b^2)^2
```

Rubi [A] time = 0.248748, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2731, 2664, 12, 2660, 618, 206, 2669, 3767, 8}

$$-\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\left(a^2-b^2\right)\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{2ab\,\mathrm{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (4*
a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (2*a*
b*Sech[x])/(a^2 + b^2)^2 - (a^2*b*Cosh[x])/((a^2 + b^2)^2*(a + b*Sinh[x]))
- ((a^2 - b^2)*Tanh[x])/(a^2 + b^2)^2
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = -\int\left(-\frac{a^2}{\left(a^2+b^2\right)(a+b\sinh(x))^2}+\frac{2ab^2}{\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{\text{sech}^2(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\right.}{\left(a^2+b^2\right)^2}\right.$$

$$= -\frac{\int \text{sech}^2(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(x)\right)dx}{\left(a^2+b^2\right)^2}-\frac{\left(2ab^2\right)\int\frac{1}{a+b\sinh(x)}dx}{\left(a^2+b^2\right)^2}+\frac{a^2\int\frac{1}{(a+b\sinh(x))^2}dx}{a^2+b^2}$$

$$= -\frac{2ab\text{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{a^2\int\frac{a}{a+b\sinh(x)}dx}{\left(a^2+b^2\right)^2}-\frac{\left(4ab^2\right)\text{Subst}\left(\int\frac{1}{a+2bx-ax^2}\right.}{\left(a^2+b^2\right)^2}$$

$$= -\frac{2ab\text{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{a^3\int\frac{1}{a+b\sinh(x)}dx}{\left(a^2+b^2\right)^2}+\frac{\left(8ab^2\right)\text{Subst}\left(\int\frac{1}{4\left(a^2+b^2\right)-x^2}\right.}{\left(a^2+\right.}$$

$$= \frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{2ab\text{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{\left(a^2-b^2\right)\tanh(x)}{\left(a^2+b^2\right)^2}+$$

$$= \frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{2ab\text{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{\left(a^2-b^2\right)\tanh(x)}{\left(a^2+b^2\right)^2}-$$

$$= -\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{2ab\text{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sin}$$

Mathematica [A] time = 0.264834, size = 100, normalized size = 0.69

$$\frac{\left(b^2-a^2\right)\tanh(x)+\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{a^2b\cosh(x)}{a+b\sinh(x)}-2ab\text{sech}(x)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*a*(a^2 - 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 -
 b^2] - 2*a*b*Sech[x] - (a^2*b*Cosh[x])/(a + b*Sinh[x]) + (-a^2 + b^2)*Tanh
[x])/(a^2 + b^2)^2
```

Maple [A] time = 0.05, size = 142, normalized size = 1.

$$2\,\frac{\left(-a^2+b^2\right)\tanh(x/2)-2\,ab}{\left(a^4+2\,a^2b^2+b^4\right)\left((\tanh(x/2))^2+1\right)}-2\,\frac{a}{\left(a^2+b^2\right)^2}\left(\frac{-b^2\tanh(x/2)-ab}{a\,(\tanh(x/2))^2-2\,\tanh(x/2)\,b-a}-\frac{a^2-2\,b^2}{\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] 2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(tanh(1/2*x)^2+1)-2*a/
(a^2+b^2)^2*((-b^2*tanh(1/2*x)-a*b)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-(a^
```

```
2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14812, size = 2144, normalized size = 14.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (4*a^4*b + 2*a^2*b^3 - 2*b^5 - 2*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x)^3 - 2*(a
^5 - a^3*b^2 - 2*a*b^4)*sinh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^2
 + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - 3*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x))*sinh
(x)^2 + ((a^3*b - 2*a*b^3)*cosh(x)^4 + (a^3*b - 2*a*b^3)*sinh(x)^4 - a^3*b
+ 2*a*b^3 + 2*(a^4 - 2*a^2*b^2)*cosh(x)^3 + 2*(a^4 - 2*a^2*b^2 + 2*(a^3*b -
 2*a*b^3)*cosh(x))*sinh(x)^3 + 6*((a^3*b - 2*a*b^3)*cosh(x)^2 + (a^4 - 2*a^
2*b^2)*cosh(x))*sinh(x)^2 + 2*(a^4 - 2*a^2*b^2)*cosh(x) + 2*(a^4 - 2*a^2*b^
2 + 2*(a^3*b - 2*a*b^3)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x))
*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*s
inh(x) - b)) - 6*(a^5 + a^3*b^2)*cosh(x) - 2*(3*a^5 + 3*a^3*b^2 + 3*(a^5 -
a^3*b^2 - 2*a*b^4)*cosh(x)^2 - 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x))*sinh(
x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*cosh(x)^4 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^4 - 2*(a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4
 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 - 6*(
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^
2)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sinh(x))**2, x)
```

Giac [A] time = 1.18394, size = 244, normalized size = 1.69

$$\frac{\left(a^3 - 2\,ab^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)\sqrt{a^2 + b^2}} + \frac{2\,\left(a^3e^{(3\,x)} - 2\,ab^2e^{(3\,x)} - 4\,a^2be^{(2\,x)} - b^3e^{(2\,x)} + 3\,a^3e^x - 2\,a^2b + b^3\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)\left(be^{(4\,x)} + 2\,ae^{(3\,x)} + 2\,ae^x - b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^3 - 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*
a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^3*
e^(3*x) - 2*a*b^2*e^(3*x) - 4*a^2*b*e^(2*x) - b^3*e^(2*x) + 3*a^3*e^x - 2*a
^2*b + b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x - b
))
```

3.239 $\int \frac{\tanh(x)}{(a+b\sinh(x))^2} dx$

Optimal. Leaf size=85

$$\frac{a}{(a^2+b^2)(a+b\sinh(x))} - \frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{2ab\tan^{-1}(\sinh(x))}{(a^2+b^2)^2} + \frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2}$$

```
[Out] (2*a*b*ArcTan[Sinh[x]])/(a^2 + b^2)^2 + ((a^2 - b^2)*Log[Cosh[x]])/(a^2 + b
^2)^2 - ((a^2 - b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + a/((a^2 + b^2)*(a
+ b*Sinh[x]))
```

Rubi [A] time = 0.103188, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2721, 801, 635, 203, 260}

$$\frac{a}{(a^2+b^2)(a+b\sinh(x))} - \frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{2ab\tan^{-1}(\sinh(x))}{(a^2+b^2)^2} + \frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*a*b*ArcTan[Sinh[x]])/(a^2 + b^2)^2 + ((a^2 - b^2)*Log[Cosh[x]])/(a^2 + b
^2)^2 - ((a^2 - b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + a/((a^2 + b^2)*(a
+ b*Sinh[x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = -\operatorname{Subst}\left(\int \frac{x}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)$$

$$= -\operatorname{Subst}\left(\int \left(\frac{a}{\left(a^2+b^2\right)(a+x)^2} + \frac{a^2-b^2}{\left(a^2+b^2\right)^2(a+x)} + \frac{-2ab^2-\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right) dx, x, b\sinh(x)\right.$$

$$= -\frac{\left(a^2-b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{a}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\operatorname{Subst}\left(\int \frac{-2ab^2-\left(a^2-b^2\right)x}{b^2+x^2}\,dx, x, b\,\mathrm{si}\right.}{\left(a^2+b^2\right)^2}$$

$$= -\frac{\left(a^2-b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{a}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\left(2ab^2\right)\operatorname{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\,\mathrm{sir}\right.}{\left(a^2+b^2\right)^2}$$

$$= \frac{2ab\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\left(a^2-b^2\right)\log(\cosh(x))}{\left(a^2+b^2\right)^2} - \frac{\left(a^2-b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{a}{\left(a^2+b^2\right)(a+}$$

Mathematica [C] time = 0.225366, size = 146, normalized size = 1.72

$$\frac{a\left(2\left(\left(b^2-a^2\right)\log(a+b\sinh(x))+a^2+b^2\right)+(a-ib)^2\log(-\sinh(x)+i)+(a+ib)^2\log(\sinh(x)+i)\right)+b\sinh(x)\left(2\right.}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (a*((a - I*b)^2*Log[I - Sinh[x]] + (a + I*b)^2*Log[I + Sinh[x]] + 2*(a^2 +
b^2 + (-a^2 + b^2)*Log[a + b*Sinh[x]])) + b*((a - I*b)^2*Log[I - Sinh[x]] +
 (a + I*b)^2*Log[I + Sinh[x]] + 2*(-a^2 + b^2)*Log[a + b*Sinh[x]])*Sinh[x])
/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```

Maple [B] time = 0.042, size = 248, normalized size = 2.9

$$2\,\frac{\ln\left((\tanh(x/2))^2+1\right)a^2}{2a^4+4a^2b^2+2b^4} - 2\,\frac{\ln\left((\tanh(x/2))^2+1\right)b^2}{2a^4+4a^2b^2+2b^4} + 8\,\frac{ab\arctan(\tanh(x/2))}{2a^4+4a^2b^2+2b^4} + 2\,\frac{\tanh(x/2)\,a^2}{\left(a^2+b^2\right)^2\left(a(\tanh(x/2))^2-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sinh(x))^2,x)
```

```
[Out] 2/(2*a^4+4*a^2*b^2+2*b^4)*ln(tanh(1/2*x)^2+1)*a^2-2/(2*a^4+4*a^2*b^2+2*b^4)
*ln(tanh(1/2*x)^2+1)*b^2+8/(2*a^4+4*a^2*b^2+2*b^4)*a*b*arctan(tanh(1/2*x))+
2/(a^2+b^2)^2*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^2*b+2/(a^2+
b^2)^2*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^3-1/(a^2+b^2)^2*ln
(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^2+1/(a^2+b^2)^2*ln(a*tanh(1/2*x)^2-2*
tanh(1/2*x)*b-a)*b^2
```

Maxima [A] time = 1.57067, size = 209, normalized size = 2.46

$$-\frac{4\,ab\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,ae^{(-x)}}{a^2b+b^3+2\left(a^3+ab^2\right)e^{(-x)}-\left(a^2b+b^3\right)e^{(-2\,x)}}-\frac{\left(a^2-b^2\right)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2-b^2\right.}{a^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -4*a*b*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + 2*a*e^(-x)/(a^2*b + b^3 + 2
*(a^3 + a*b^2)*e^(-x) - (a^2*b + b^3)*e^(-2*x)) - (a^2 - b^2)*log(-2*a*e^(-
x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(e^(-2*x) + 1
)/(a^4 + 2*a^2*b^2 + b^4)
```

Fricas [B] time = 2.19909, size = 1029, normalized size = 12.11

$$-\frac{4\left(ab^2\cosh(x)^2+ab^2\sinh(x)^2+2\,a^2b\cosh(x)-ab^2+2\left(ab^2\cosh(x)+a^2b\right)\sinh(x)\right)\arctan\left(\cosh(x)+\sinh(x)\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(4*(a*b^2*cosh(x)^2 + a*b^2*sinh(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2
*cosh(x) + a^2*b)*sinh(x))*arctan(cosh(x) + sinh(x)) + 2*(a^3 + a*b^2)*cosh
(x) + (a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*
(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*lo
g(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^2*b - b^3 - (a^2*b - b^3)*cos
h(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2
 + (a^2*b - b^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(
a^3 + a*b^2)*sinh(x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*
cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^
4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)
)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)/(a + b*sinh(x))**2, x)
```

Giac [B] time = 1.17057, size = 269, normalized size = 3.16

$$\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)ab}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2-b^2\right)\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{\left(a^2b-b^3\right)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}+\frac{a^2b(e}{(a^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a*b/(a^4 + 2*a^2*b^2 + b^4) + 1/2
*(a^2 - b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - (a^2*b - b
^3)*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + (a^2*b*(e
^(-x) - e^x) - b^3*(e^(-x) - e^x) - 4*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(
-x) - e^x) - 2*a))
```

3.240 $\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=32

$$-\frac{\log(a+b\sinh(x))}{a^2}+\frac{\log(\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}$$

```
[Out] Log[Sinh[x]]/a^2 - Log[a + b*Sinh[x]]/a^2 + 1/(a*(a + b*Sinh[x]))
```

Rubi [A] time = 0.0530725, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2721, 44}

$$-\frac{\log(a+b\sinh(x))}{a^2}+\frac{\log(\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] Log[Sinh[x]]/a^2 - Log[a + b*Sinh[x]]/a^2 + 1/(a*(a + b*Sinh[x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx &= \text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, b\sinh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{1}{a(a+x)^2}-\frac{1}{a^2(a+x)}\right)dx, x, b\sinh(x)\right)\\ &= \frac{\log(\sinh(x))}{a^2}-\frac{\log(a+b\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.0467081, size = 27, normalized size = 0.84

$$\frac{\frac{a}{a+b\sinh(x)}-\log(a+b\sinh(x))+\log(\sinh(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (Log[Sinh[x]] - Log[a + b*Sinh[x]] + a/(a + b*Sinh[x]))/a^2
```

Maple [A] time = 0.033, size = 33, normalized size = 1.

$$\frac{\ln(\sinh(x))}{a^2} - \frac{\ln(a+b\sinh(x))}{a^2} + \frac{1}{a(a+b\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x))^2,x)
```

```
[Out] ln(sinh(x))/a^2-ln(a+b*sinh(x))/a^2+1/a/(a+b*sinh(x))
```

Maxima [B] time = 1.02619, size = 101, normalized size = 3.16

$$\frac{2e^{(-x)}}{2a^2e^{(-x)} - abe^{(-2x)} + ab} - \frac{\log\left(-2ae^{(-x)} + be^{(-2x)} - b\right)}{a^2} + \frac{\log\left(e^{(-x)} + 1\right)}{a^2} + \frac{\log\left(e^{(-x)} - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(2*a^2*e^(-x) - a*b*e^(-2*x) + a*b) - log(-2*a*e^(-x) + b*e^(-2*x)
 - b)/a^2 + log(e^(-x) + 1)/a^2 + log(e^(-x) - 1)/a^2
```

Fricas [B] time = 2.1266, size = 477, normalized size = 14.91

$$\frac{2a\cosh(x) - \left(b\cosh(x)^2 + b\sinh(x)^2 + 2a\cosh(x) + 2(b\cosh(x)+a)\sinh(x) - b\right)\log\left(\frac{2(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right) + \left(b\cosh\right.}{a^2b\cosh(x)^2 + a^2b\sinh(x)^2 + 2a^3\cosh(x) - a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a*cosh(x) - (b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (b*cosh(x)^2 + b
*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)*log(2*sinh(x)/(co
sh(x) - sinh(x))) + 2*a*sinh(x))/(a^2*b*cosh(x)^2 + a^2*b*sinh(x)^2 + 2*a^3
*cosh(x) - a^2*b + 2*(a^2*b*cosh(x) + a^3)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)/(a + b*sinh(x))**2, x)
```

Giac [B] time = 1.13401, size = 101, normalized size = 3.16

$$-\frac{\log\left(\left|-b\left(e^{(-x)} - e^{x}\right) + 2\,a\right|\right)}{a^{2}} + \frac{\log\left(\left|-e^{(-x)} + e^{x}\right|\right)}{a^{2}} + \frac{b\left(e^{(-x)} - e^{x}\right) - 4\,a}{\left(b\left(e^{(-x)} - e^{x}\right) - 2\,a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -log(abs(-b*(e^(-x) - e^x) + 2*a))/a^2 + log(abs(-e^(-x) + e^x))/a^2 + (b*(
e^(-x) - e^x) - 4*a)/((b*(e^(-x) - e^x) - 2*a)*a^2)
```

3.241 $\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=80

$$-\frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2\coth(x)}{a^2}+\frac{\coth(x)}{a(a+b\sinh(x))}$$

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) - (2*Coth[x])/a^2 + Coth[x]/(a*(a + b*
Sinh[x]))
```

Rubi [A] time = 0.402169, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 206}

$$-\frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2\coth(x)}{a^2}+\frac{\coth(x)}{a(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) - (2*Coth[x])/a^2 + Coth[x]/(a*(a + b*
Sinh[x]))
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx &= \int \frac{\operatorname{csch}^2(x)\left(1+\sinh^2(x)\right)}{(a+b\sinh(x))^2}\,dx\\
&= \frac{\coth(x)}{a(a+b\sinh(x))}+\frac{\int \frac{\operatorname{csch}^2(x)\left(2\left(a^2+b^2\right)+\left(a^2+b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}\\
&= -\frac{2\coth(x)}{a^2}+\frac{\coth(x)}{a(a+b\sinh(x))}+\frac{i\int \frac{\operatorname{csch}(x)\left(2ib\left(a^2+b^2\right)-ia\left(a^2+b^2\right)\sinh(x)\right)}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}\\
&= -\frac{2\coth(x)}{a^2}+\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{(2b)\int \operatorname{csch}(x)\,dx}{a^3}+\frac{\left(a^2+2b^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{a^3}\\
&= \frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2\coth(x)}{a^2}+\frac{\coth(x)}{a(a+b\sinh(x))}+\frac{\left(2\left(a^2+2b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\mathrm{t}\right.}{a^3}\\
&= \frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2\coth(x)}{a^2}+\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\left(4\left(a^2+2b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,\right.}{a^3}\\
&= \frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{2\coth(x)}{a^2}+\frac{\coth(x)}{a(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.477892, size = 102, normalized size = 1.27

$$-\frac{-\frac{4\left(a^2+2b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{2ab\cosh(x)}{a+b\sinh(x)}+a\tanh\left(\frac{x}{2}\right)+a\coth\left(\frac{x}{2}\right)+4b\log\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] -((-4*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 -
 b^2] + a*Coth[x/2] + 4*b*Log[Tanh[x/2]] + (2*a*b*Cosh[x])/(a + b*Sinh[x])
+ a*Tanh[x/2])/(2*a^3)
```

Maple [B] time = 0.041, size = 170, normalized size = 2.1

$$-\frac{1}{2a^2}\tanh\left(\frac{x}{2}\right)-\frac{1}{2a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-2\frac{b\ln(\tanh(x/2))}{a^3}+2\frac{b^2\tanh(x/2)}{a^3\left(a(\tanh(x/2))^2-2\tanh(x/2)b-a\right)}+2\frac{}{a^2\left(a(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] -1/2/a^2*tanh(1/2*x)-1/2/a^2/tanh(1/2*x)-2/a^3*b*ln(tanh(1/2*x))+2/a^3/(a*t
anh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^2*tanh(1/2*x)+2/a^2/(a*tanh(1/2*x)^2-2*ta
nh(1/2*x)*b-a)*b+2/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2
+b^2)^(1/2))+4/a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b
^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.74811, size = 3102, normalized size = 38.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*sinh(x
)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 + a^2*b^2
)*cosh(x))*sinh(x)^2 + ((a^2*b + 2*b^3)*cosh(x)^4 + (a^2*b + 2*b^3)*sinh(x)
^4 + 2*(a^3 + 2*a*b^2)*cosh(x)^3 + 2*(a^3 + 2*a*b^2 + 2*(a^2*b + 2*b^3)*cos
h(x))*sinh(x)^3 + a^2*b + 2*b^3 - 2*(a^2*b + 2*b^3)*cosh(x)^2 - 2*(a^2*b +
2*b^3 - 3*(a^2*b + 2*b^3)*cosh(x)^2 - 3*(a^3 + 2*a*b^2)*cosh(x))*sinh(x)^2
- 2*(a^3 + 2*a*b^2)*cosh(x) + 2*(2*(a^2*b + 2*b^3)*cosh(x)^3 - a^3 - 2*a*b^
2 + 3*(a^3 + 2*a*b^2)*cosh(x)^2 - 2*(a^2*b + 2*b^3)*cosh(x))*sinh(x))*sqrt(
a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x)
```

```
 + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x)
 - b)) - 6*(a^4 + a^2*b^2)*cosh(x) + 2*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2*b^
2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*cosh(x)^3 + 2*(a^3*b
 + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 2*(a^2*b^2 + b^4)*cosh(x)
^2 - 2*(a^2*b^2 + b^4 - 3*(a^2*b^2 + b^4)*cosh(x)^2 - 3*(a^3*b + a*b^3)*cos
h(x))*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 - 2*(a^2*b^2
 + b^4)*cosh(x)^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 + 2*(a^2*b^2 + b^4)*cosh(x)
)*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2
*b^2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*cosh(x)^3 + 2*(a^
3*b + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 2*(a^2*b^2 + b^4)*cosh
(x)^2 - 2*(a^2*b^2 + b^4 - 3*(a^2*b^2 + b^4)*cosh(x)^2 - 3*(a^3*b + a*b^3)*
cosh(x))*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 - 2*(a^2*
b^2 + b^4)*cosh(x)^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 + 2*(a^2*b^2 + b^4)*cosh
(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(3*a^4 + 3*a^2*b^2 - 3*(a^4 +
a^2*b^2)*cosh(x)^2 + 4*(a^3*b + a*b^3)*cosh(x))*sinh(x))/(a^5*b + a^3*b^3 +
 (a^5*b + a^3*b^3)*cosh(x)^4 + (a^5*b + a^3*b^3)*sinh(x)^4 + 2*(a^6 + a^4*b
^2)*cosh(x)^3 + 2*(a^6 + a^4*b^2 + 2*(a^5*b + a^3*b^3)*cosh(x))*sinh(x)^3 -
 2*(a^5*b + a^3*b^3)*cosh(x)^2 - 2*(a^5*b + a^3*b^3 - 3*(a^5*b + a^3*b^3)*c
osh(x)^2 - 3*(a^6 + a^4*b^2)*cosh(x))*sinh(x)^2 - 2*(a^6 + a^4*b^2)*cosh(x)
 - 2*(a^6 + a^4*b^2 - 2*(a^5*b + a^3*b^3)*cosh(x)^3 - 3*(a^6 + a^4*b^2)*cos
h(x)^2 + 2*(a^5*b + a^3*b^3)*cosh(x))*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)**2/(a + b*sinh(x))**2, x)
```

Giac [A] time = 1.18626, size = 200, normalized size = 2.5

$$\frac{2\,b\log\left(e^{x}+1\right)}{a^{3}}-\frac{2\,b\log\left(|e^{x}-1|\right)}{a^{3}}+\frac{\left(a^{2}+2\,b^{2}\right)\log\left(\frac{|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}|}{|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}|}\right)}{\sqrt{a^{2}+b^{2}}a^{3}}+\frac{2\left(ae^{(3\,x)}-2\,be^{(2\,x)}-3\,ae^{x}+2\,b\right)}{\left(be^{(4\,x)}+2\,ae^{(3\,x)}-2\,be^{(2\,x)}-2\,ae^{x}+b\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3 + (a^2 + 2*b^2)*log(abs(2*
b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(s
qrt(a^2 + b^2)*a^3) + 2*(a*e^(3*x) - 2*b*e^(2*x) - 3*a*e^x + 2*b)/((b*e^(4*
x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x + b)*a^2)
```

3.242 $\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=76

$$\frac{a^2+b^2}{a^3(a+b\sinh(x))}+\frac{\left(a^2+3b^2\right)\log(\sinh(x))}{a^4}-\frac{\left(a^2+3b^2\right)\log(a+b\sinh(x))}{a^4}+\frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}$$

```
[Out] (2*b*Csch[x])/a^3 - Csch[x]^2/(2*a^2) + ((a^2 + 3*b^2)*Log[Sinh[x]])/a^4 -
((a^2 + 3*b^2)*Log[a + b*Sinh[x]])/a^4 + (a^2 + b^2)/(a^3*(a + b*Sinh[x]))
```

Rubi [A] time = 0.110462, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2721, 894}

$$\frac{a^2+b^2}{a^3(a+b\sinh(x))}+\frac{\left(a^2+3b^2\right)\log(\sinh(x))}{a^4}-\frac{\left(a^2+3b^2\right)\log(a+b\sinh(x))}{a^4}+\frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*Csch[x])/a^3 - Csch[x]^2/(2*a^2) + ((a^2 + 3*b^2)*Log[Sinh[x]])/a^4 -
((a^2 + 3*b^2)*Log[a + b*Sinh[x]])/a^4 + (a^2 + b^2)/(a^3*(a + b*Sinh[x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx&=-\text{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)^2}\,dx,x,b\sinh(x)\right)\\&=-\text{Subst}\left(\int \left(-\frac{b^2}{a^2x^3}+\frac{2b^2}{a^3x^2}+\frac{-a^2-3b^2}{a^4x}+\frac{a^2+b^2}{a^3(a+x)^2}+\frac{a^2+3b^2}{a^4(a+x)}\right)dx,x,b\sinh(x)\right)\\&=\frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}+\frac{\left(a^2+3b^2\right)\log(\sinh(x))}{a^4}-\frac{\left(a^2+3b^2\right)\log(a+b\sinh(x))}{a^4}+\frac{a^2 \ldots}{a^3(a+b \ldots}\end{aligned}$$

Mathematica [A] time = 0.197432, size = 73, normalized size = 0.96

$$\frac{\frac{2a\left(a^2+b^2\right)}{a+b\sinh(x)}+2\left(a^2+3b^2\right)\log(\sinh(x))-2\left(a^2+3b^2\right)\log(a+b\sinh(x))-a^2\text{csch}^2(x)+4ab\text{csch}(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (4*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + 3*b^2)*Log[Sinh[x]] - 2*(a^2 + 3*
b^2)*Log[a + b*Sinh[x]] + (2*a*(a^2 + b^2))/(a + b*Sinh[x]))/(2*a^4)
```

Maple [B] time = 0.056, size = 184, normalized size = 2.4

$$-\frac{1}{8a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{b}{a^3}\tanh\left(\frac{x}{2}\right)-\frac{1}{8a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{1}{a^2}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+3\frac{\ln(\tanh(x/2))b^2}{a^4}+\frac{b}{a^3}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] -1/8/a^2*tanh(1/2*x)^2-1/a^3*tanh(1/2*x)*b-1/8/a^2/tanh(1/2*x)^2+1/a^2*ln(t
anh(1/2*x))+3/a^4*ln(tanh(1/2*x))*b^2+b/a^3/tanh(1/2*x)+2/a^2*tanh(1/2*x)/(
a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b+2/a^4*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*t
anh(1/2*x)*b-a)*b^3-1/a^2*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-3/a^4*ln(a*
tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^2
```

Maxima [B] time = 1.05053, size = 273, normalized size = 3.59

$$\frac{2\left(3abe^{(-2x)}-3abe^{(-4x)}+\left(a^2+3b^2\right)e^{(-x)}-2\left(2a^2+3b^2\right)e^{(-3x)}+\left(a^2+3b^2\right)e^{(-5x)}\right)}{2a^4e^{(-x)}-3a^3be^{(-2x)}-4a^4e^{(-3x)}+3a^3be^{(-4x)}+2a^4e^{(-5x)}-a^3be^{(-6x)}+a^3b}-\frac{\left(a^2+3b^2\right)\log\left(-2ae^{(-x)}+be^{(-2x}\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) + (a^2 + 3*b^2)*e^(-x) - 2*(2*a^2 + 3*b^
2)*e^(-3*x) + (a^2 + 3*b^2)*e^(-5*x))/(2*a^4*e^(-x) - 3*a^3*b*e^(-2*x) - 4*
a^4*e^(-3*x) + 3*a^3*b*e^(-4*x) + 2*a^4*e^(-5*x) - a^3*b*e^(-6*x) + a^3*b)
- (a^2 + 3*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^4 + (a^2 + 3*b^2)*log(e
^(-x) + 1)/a^4 + (a^2 + 3*b^2)*log(e^(-x) - 1)/a^4
```

Fricas [B] time = 2.29207, size = 3726, normalized size = 49.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (6*a^2*b*cosh(x)^4 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2)*sinh(x
)^5 - 6*a^2*b*cosh(x)^2 + 2*(3*a^2*b + 5*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^4
 - 4*(2*a^3 + 3*a*b^2)*cosh(x)^3 + 4*(6*a^2*b*cosh(x) - 2*a^3 - 3*a*b^2 + 5
*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^3 + 2*(18*a^2*b*cosh(x)^2 + 10*(a^3 + 3
*a*b^2)*cosh(x)^3 - 3*a^2*b - 6*(2*a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a
^3 + 3*a*b^2)*cosh(x) - ((a^2*b + 3*b^3)*cosh(x)^6 + (a^2*b + 3*b^3)*sinh(x
```

```
)^6 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2 + 3*(a^2*b + 3*b^3)*co
sh(x))*sinh(x)^5 - 3*(a^2*b + 3*b^3)*cosh(x)^4 - (3*a^2*b + 9*b^3 - 15*(a^2
*b + 3*b^3)*cosh(x)^2 - 10*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^4 - 4*(a^3 + 3*
a*b^2)*cosh(x)^3 + 4*(5*(a^2*b + 3*b^3)*cosh(x)^3 - a^3 - 3*a*b^2 + 5*(a^3
+ 3*a*b^2)*cosh(x)^2 - 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^3 - a^2*b - 3*b^3
 + 3*(a^2*b + 3*b^3)*cosh(x)^2 + (15*(a^2*b + 3*b^3)*cosh(x)^4 + 20*(a^3 +
3*a*b^2)*cosh(x)^3 + 3*a^2*b + 9*b^3 - 18*(a^2*b + 3*b^3)*cosh(x)^2 - 12*(a
^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a^3 + 3*a*b^2)*cosh(x) + 2*(3*(a^2*b
+ 3*b^3)*cosh(x)^5 + 5*(a^3 + 3*a*b^2)*cosh(x)^4 - 6*(a^2*b + 3*b^3)*cosh(x
)^3 + a^3 + 3*a*b^2 - 6*(a^3 + 3*a*b^2)*cosh(x)^2 + 3*(a^2*b + 3*b^3)*cosh(
x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + ((a^2*b + 3*b^3)*
cosh(x)^6 + (a^2*b + 3*b^3)*sinh(x)^6 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^
3 + 3*a*b^2 + 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^5 - 3*(a^2*b + 3*b^3)*cosh
(x)^4 - (3*a^2*b + 9*b^3 - 15*(a^2*b + 3*b^3)*cosh(x)^2 - 10*(a^3 + 3*a*b^2
)*cosh(x))*sinh(x)^4 - 4*(a^3 + 3*a*b^2)*cosh(x)^3 + 4*(5*(a^2*b + 3*b^3)*c
osh(x)^3 - a^3 - 3*a*b^2 + 5*(a^3 + 3*a*b^2)*cosh(x)^2 - 3*(a^2*b + 3*b^3)*
cosh(x))*sinh(x)^3 - a^2*b - 3*b^3 + 3*(a^2*b + 3*b^3)*cosh(x)^2 + (15*(a^2
*b + 3*b^3)*cosh(x)^4 + 20*(a^3 + 3*a*b^2)*cosh(x)^3 + 3*a^2*b + 9*b^3 - 18
*(a^2*b + 3*b^3)*cosh(x)^2 - 12*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a^3
 + 3*a*b^2)*cosh(x) + 2*(3*(a^2*b + 3*b^3)*cosh(x)^5 + 5*(a^3 + 3*a*b^2)*co
sh(x)^4 - 6*(a^2*b + 3*b^3)*cosh(x)^3 + a^3 + 3*a*b^2 - 6*(a^3 + 3*a*b^2)*c
osh(x)^2 + 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sin
h(x))) + 2*(12*a^2*b*cosh(x)^3 + 5*(a^3 + 3*a*b^2)*cosh(x)^4 - 6*a^2*b*cosh
(x) + a^3 + 3*a*b^2 - 6*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x))/(a^4*b*cosh(x
)^6 + a^4*b*sinh(x)^6 + 2*a^5*cosh(x)^5 - 3*a^4*b*cosh(x)^4 - 4*a^5*cosh(x)
^3 + 3*a^4*b*cosh(x)^2 + 2*a^5*cosh(x) + 2*(3*a^4*b*cosh(x) + a^5)*sinh(x)^
5 - a^4*b + (15*a^4*b*cosh(x)^2 + 10*a^5*cosh(x) - 3*a^4*b)*sinh(x)^4 + 4*(
5*a^4*b*cosh(x)^3 + 5*a^5*cosh(x)^2 - 3*a^4*b*cosh(x) - a^5)*sinh(x)^3 + (1
5*a^4*b*cosh(x)^4 + 20*a^5*cosh(x)^3 - 18*a^4*b*cosh(x)^2 - 12*a^5*cosh(x)
+ 3*a^4*b)*sinh(x)^2 + 2*(3*a^4*b*cosh(x)^5 + 5*a^5*cosh(x)^4 - 6*a^4*b*cos
h(x)^3 - 6*a^5*cosh(x)^2 + 3*a^4*b*cosh(x) + a^5)*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)**3/(a + b*sinh(x))**2, x)
```

Giac [B] time = 1.15317, size = 257, normalized size = 3.38

$$\frac{\left(a^2+3\,b^2\right)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{a^4}-\frac{\left(a^2b+3\,b^3\right)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b}+\frac{a^2b\left(e^{(-x)}-e^x\right)+3\,b^3\left(e^{(-x)}-e^x\right)-4\,a^3-8\,a}{\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + 3*b^2)*log(abs(-e^(-x) + e^x))/a^4 - (a^2*b + 3*b^3)*log(abs(-b*(e^(
-x) - e^x) + 2*a))/(a^4*b) + (a^2*b*(e^(-x) - e^x) + 3*b^3*(e^(-x) - e^x) -
```

```
 4*a^3 - 8*a*b^2)/((b*(e^(-x) - e^x) - 2*a)*a^4) - 1/2*(3*a^2*(e^(-x) - e^x
)^2 + 9*b^2*(e^(-x) - e^x)^2 + 8*a*b*(e^(-x) - e^x) + 4*a^2)/(a^4*(e^(-x) -
 e^x)^2)
```

3.243 $\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=159

$$-\frac{2\sqrt{a^2+b^2}\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4}+\frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5}+\frac{\left(a^2+2b^2\right)\mathrm{co}}{a}$$

```
[Out] (b*(3*a^2 + 4*b^2)*ArcTanh[Cosh[x]])/a^5 - (2*Sqrt[a^2 + b^2]*(a^2 + 4*b^2)
*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - ((7*a^2 + 12*b^2)*Coth[x
])/(3*a^4) + ((a^2 + 2*b^2)*Coth[x]*Csch[x])/(a^3*b) - ((3 + (4*b^2)/a^2)*C
oth[x]*Csch[x])/(3*b*(a + b*Sinh[x])) - (Coth[x]*Csch[x]^2)/(3*a*(a + b*Sin
h[x]))
```

Rubi [A] time = 0.670978, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2\sqrt{a^2+b^2}\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4}+\frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5}+\frac{\left(a^2+2b^2\right)\mathrm{co}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (b*(3*a^2 + 4*b^2)*ArcTanh[Cosh[x]])/a^5 - (2*Sqrt[a^2 + b^2]*(a^2 + 4*b^2)
*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - ((7*a^2 + 12*b^2)*Coth[x
])/(3*a^4) + ((a^2 + 2*b^2)*Coth[x]*Csch[x])/(a^3*b) - ((3 + (4*b^2)/a^2)*C
oth[x]*Csch[x])/(3*b*(a + b*Sinh[x])) - (Coth[x]*Csch[x]^2)/(3*a*(a + b*Sin
h[x]))
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\mathrm{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\mathrm{csch}^2(x)}{3a(a+b\sinh(x))} - \frac{\int \frac{\mathrm{csch}^3(x)\left(6\left(a^2+2b^2\right)-ab\sinh(x)+\left(3a^2+8b^2\right)\sinh^2(x\cdots}{a+b\sinh(x)}}{3a^2b}$$

$$= \frac{\left(a^2+2b^2\right)\coth(x)\mathrm{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\mathrm{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\mathrm{csch}^2(x)}{3a(a+b\sinh(x))} - \frac{i\int \frac{\mathrm{csch}^2(x)(2ib(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4} + \frac{\left(a^2+2b^2\right)\coth(x)\mathrm{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\mathrm{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\cdots}{3a(a+b\cdots}$$

$$= -\frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4} + \frac{\left(a^2+2b^2\right)\coth(x)\mathrm{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\mathrm{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\cdots}{3a(a+b\cdots}$$

$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} - \frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4} + \frac{\left(a^2+2b^2\right)\coth(x)\mathrm{csch}(x)}{a^3b} - \frac{(3+\cdots}{3\cdots}$$

$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} - \frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4} + \frac{\left(a^2+2b^2\right)\coth(x)\mathrm{csch}(x)}{a^3b} - \frac{(3+\cdots}{3\cdots}$$

$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} - \frac{2\sqrt{a^2+b^2}\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{\left(7a^2+12b^2\right)\cdots}{3a^4}$$

Mathematica [A] time = 0.818166, size = 214, normalized size = 1.35

$$\frac{-4a\left(4a^2+9b^2\right)\tanh\left(\frac{x}{2}\right) - 4a\left(4a^2+9b^2\right)\coth\left(\frac{x}{2}\right) - 24b\left(3a^2+4b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{48\left(5a^2b^2+a^4+4b^4\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\cdots\right.}{\sqrt{-a^2-b^2}}\cdots}{24a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((48*(a^4 + 5*a^2*b^2 + 4*b^4)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2] - 4*a*(4*a^2 + 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 - 24
*b*(3*a^2 + 4*b^2)*Log[Tanh[x/2]] + 6*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*S
inh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2 - (24*a*b*(a^2 + b^2)*Cosh[x])/(a
+ b*Sinh[x]) - 4*a*(4*a^2 + 9*b^2)*Tanh[x/2])/(24*a^5)
```

Maple [B] time = 0.056, size = 357, normalized size = 2.3

$$-\frac{1}{24\,a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^3 - \frac{b}{4\,a^3}\left(\tanh\left(\frac{x}{2}\right)\right)^2 - \frac{5}{8\,a^2}\tanh\left(\frac{x}{2}\right) - \frac{3\,b^2}{2\,a^4}\tanh\left(\frac{x}{2}\right) - \frac{1}{24\,a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-3} - \frac{5}{8\,a^2}\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] -1/24/a^2*tanh(1/2*x)^3-1/4/a^3*b*tanh(1/2*x)^2-5/8/a^2*tanh(1/2*x)-3/2/a^4
*b^2*tanh(1/2*x)-1/24/a^2/tanh(1/2*x)^3-5/8/a^2/tanh(1/2*x)-3/2/a^4/tanh(1/
2*x)*b^2+1/4/a^3*b/tanh(1/2*x)^2-3/a^3*b*ln(tanh(1/2*x))-4/a^5*b^3*ln(tanh(
1/2*x))+2/a^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^2*tanh(1/2*x)+2/a^5/(a*
tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*tanh(1/2*x)*b^4+2/a^2/(a*tanh(1/2*x)^2-2*t
```

```
anh(1/2*x)*b-a)*b+2/a^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^3+2/a/(a^2+b^
2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+10/a^3/(a^2+b^2
)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*b^2+8/a^5/(a^2+b
^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.02804, size = 9129, normalized size = 57.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^4 + 2*a^2*b^2)*cosh(x)^7 + 6*(a^4 + 2*a^2*b^2)*sinh(x)^7 - 6*(a^3
*b + 4*a*b^3)*cosh(x)^6 - 6*(a^3*b + 4*a*b^3 - 7*(a^4 + 2*a^2*b^2)*cosh(x))
*sinh(x)^6 - 6*(7*a^4 + 10*a^2*b^2)*cosh(x)^5 - 6*(7*a^4 + 10*a^2*b^2 - 21*
(a^4 + 2*a^2*b^2)*cosh(x)^2 + 6*(a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^5 + 6*(7
*a^3*b + 12*a*b^3)*cosh(x)^4 + 6*(7*a^3*b + 12*a*b^3 + 35*(a^4 + 2*a^2*b^2)
*cosh(x)^3 - 15*(a^3*b + 4*a*b^3)*cosh(x)^2 - 5*(7*a^4 + 10*a^2*b^2)*cosh(x
))*sinh(x)^4 + 14*a^3*b + 24*a*b^3 + 42*(a^4 + 2*a^2*b^2)*cosh(x)^3 + 6*(35
*(a^4 + 2*a^2*b^2)*cosh(x)^4 + 7*a^4 + 14*a^2*b^2 - 20*(a^3*b + 4*a*b^3)*co
sh(x)^3 - 10*(7*a^4 + 10*a^2*b^2)*cosh(x)^2 + 4*(7*a^3*b + 12*a*b^3)*cosh(x
))*sinh(x)^3 - 2*(25*a^3*b + 36*a*b^3)*cosh(x)^2 + 2*(63*(a^4 + 2*a^2*b^2)*
cosh(x)^5 - 45*(a^3*b + 4*a*b^3)*cosh(x)^4 - 25*a^3*b - 36*a*b^3 - 30*(7*a^
4 + 10*a^2*b^2)*cosh(x)^3 + 18*(7*a^3*b + 12*a*b^3)*cosh(x)^2 + 63*(a^4 + 2
*a^2*b^2)*cosh(x))*sinh(x)^2 + 3*((a^2*b + 4*b^3)*cosh(x)^8 + (a^2*b + 4*b^
3)*sinh(x)^8 + 2*(a^3 + 4*a*b^2)*cosh(x)^7 + 2*(a^3 + 4*a*b^2 + 4*(a^2*b +
4*b^3)*cosh(x))*sinh(x)^7 - 4*(a^2*b + 4*b^3)*cosh(x)^6 - 2*(2*a^2*b + 8*b^
3 - 14*(a^2*b + 4*b^3)*cosh(x)^2 - 7*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^6 - 6
*(a^3 + 4*a*b^2)*cosh(x)^5 + 2*(28*(a^2*b + 4*b^3)*cosh(x)^3 - 3*a^3 - 12*a
*b^2 + 21*(a^3 + 4*a*b^2)*cosh(x)^2 - 12*(a^2*b + 4*b^3)*cosh(x))*sinh(x)^5
 + 6*(a^2*b + 4*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 4*b^3)*cosh(x)^4 + 35*(a^3
+ 4*a*b^2)*cosh(x)^3 + 3*a^2*b + 12*b^3 - 30*(a^2*b + 4*b^3)*cosh(x)^2 - 15
*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^4 + 6*(a^3 + 4*a*b^2)*cosh(x)^3 + 2*(28*(
a^2*b + 4*b^3)*cosh(x)^5 + 35*(a^3 + 4*a*b^2)*cosh(x)^4 - 40*(a^2*b + 4*b^3
)*cosh(x)^3 + 3*a^3 + 12*a*b^2 - 30*(a^3 + 4*a*b^2)*cosh(x)^2 + 12*(a^2*b +
 4*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*b^3 - 4*(a^2*b + 4*b^3)*cosh(x)^2 +
2*(14*(a^2*b + 4*b^3)*cosh(x)^6 + 21*(a^3 + 4*a*b^2)*cosh(x)^5 - 30*(a^2*b
+ 4*b^3)*cosh(x)^4 - 30*(a^3 + 4*a*b^2)*cosh(x)^3 - 2*a^2*b - 8*b^3 + 18*(a
^2*b + 4*b^3)*cosh(x)^2 + 9*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^2 - 2*(a^3 + 4
*a*b^2)*cosh(x) + 2*(4*(a^2*b + 4*b^3)*cosh(x)^7 + 7*(a^3 + 4*a*b^2)*cosh(x
)^6 - 12*(a^2*b + 4*b^3)*cosh(x)^5 - 15*(a^3 + 4*a*b^2)*cosh(x)^4 + 12*(a^2
*b + 4*b^3)*cosh(x)^3 - a^3 - 4*a*b^2 + 9*(a^3 + 4*a*b^2)*cosh(x)^2 - 4*(a^
2*b + 4*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
```

```
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(11*a^4 + 18*a^2*b^2)*cosh(x
) + 3*((3*a^2*b^2 + 4*b^4)*cosh(x)^8 + (3*a^2*b^2 + 4*b^4)*sinh(x)^8 + 2*(3
*a^3*b + 4*a*b^3)*cosh(x)^7 + 2*(3*a^3*b + 4*a*b^3 + 4*(3*a^2*b^2 + 4*b^4)*
cosh(x))*sinh(x)^7 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x)^6 - 2*(6*a^2*b^2 + 8*b^4
 - 14*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 - 7*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x
)^6 - 6*(3*a^3*b + 4*a*b^3)*cosh(x)^5 - 2*(9*a^3*b + 12*a*b^3 - 28*(3*a^2*b
^2 + 4*b^4)*cosh(x)^3 - 21*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 +
4*b^4)*cosh(x))*sinh(x)^5 + 6*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 2*(35*(3*a^2*
b^2 + 4*b^4)*cosh(x)^4 + 9*a^2*b^2 + 12*b^4 + 35*(3*a^3*b + 4*a*b^3)*cosh(x
)^3 - 30*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 - 15*(3*a^3*b + 4*a*b^3)*cosh(x))*si
nh(x)^4 + 3*a^2*b^2 + 4*b^4 + 6*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 2*(28*(3*a^
2*b^2 + 4*b^4)*cosh(x)^5 + 35*(3*a^3*b + 4*a*b^3)*cosh(x)^4 + 9*a^3*b + 12*
a*b^3 - 40*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^2
 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^3 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x
)^2 + 2*(14*(3*a^2*b^2 + 4*b^4)*cosh(x)^6 + 21*(3*a^3*b + 4*a*b^3)*cosh(x)^
5 - 30*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 - 6*a^2*b^2 - 8*b^4 - 30*(3*a^3*b + 4*
a*b^3)*cosh(x)^3 + 18*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 + 9*(3*a^3*b + 4*a*b^3)
*cosh(x))*sinh(x)^2 - 2*(3*a^3*b + 4*a*b^3)*cosh(x) + 2*(4*(3*a^2*b^2 + 4*b
^4)*cosh(x)^7 + 7*(3*a^3*b + 4*a*b^3)*cosh(x)^6 - 12*(3*a^2*b^2 + 4*b^4)*co
sh(x)^5 - 15*(3*a^3*b + 4*a*b^3)*cosh(x)^4 - 3*a^3*b - 4*a*b^3 + 12*(3*a^2*
b^2 + 4*b^4)*cosh(x)^3 + 9*(3*a^3*b + 4*a*b^3)*cosh(x)^2 - 4*(3*a^2*b^2 + 4
*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*((3*a^2*b^2 + 4*b^4)
*cosh(x)^8 + (3*a^2*b^2 + 4*b^4)*sinh(x)^8 + 2*(3*a^3*b + 4*a*b^3)*cosh(x)^
7 + 2*(3*a^3*b + 4*a*b^3 + 4*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^7 - 4*(3*
a^2*b^2 + 4*b^4)*cosh(x)^6 - 2*(6*a^2*b^2 + 8*b^4 - 14*(3*a^2*b^2 + 4*b^4)*
cosh(x)^2 - 7*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^6 - 6*(3*a^3*b + 4*a*b^3
)*cosh(x)^5 - 2*(9*a^3*b + 12*a*b^3 - 28*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 - 21
*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^5
+ 6*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 2*(35*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 9
*a^2*b^2 + 12*b^4 + 35*(3*a^3*b + 4*a*b^3)*cosh(x)^3 - 30*(3*a^2*b^2 + 4*b^
4)*cosh(x)^2 - 15*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 + 3*a^2*b^2 + 4*b^
4 + 6*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 2*(28*(3*a^2*b^2 + 4*b^4)*cosh(x)^5 +
 35*(3*a^3*b + 4*a*b^3)*cosh(x)^4 + 9*a^3*b + 12*a*b^3 - 40*(3*a^2*b^2 + 4*
b^4)*cosh(x)^3 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 + 4*b^4)*
cosh(x))*sinh(x)^3 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 + 2*(14*(3*a^2*b^2 + 4
*b^4)*cosh(x)^6 + 21*(3*a^3*b + 4*a*b^3)*cosh(x)^5 - 30*(3*a^2*b^2 + 4*b^4)
*cosh(x)^4 - 6*a^2*b^2 - 8*b^4 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 18*(3*a
^2*b^2 + 4*b^4)*cosh(x)^2 + 9*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 - 2*(3
*a^3*b + 4*a*b^3)*cosh(x) + 2*(4*(3*a^2*b^2 + 4*b^4)*cosh(x)^7 + 7*(3*a^3*b
 + 4*a*b^3)*cosh(x)^6 - 12*(3*a^2*b^2 + 4*b^4)*cosh(x)^5 - 15*(3*a^3*b + 4*
a*b^3)*cosh(x)^4 - 3*a^3*b - 4*a*b^3 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 + 9
*(3*a^3*b + 4*a*b^3)*cosh(x)^2 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x))*lo
g(cosh(x) + sinh(x) - 1) + 2*(21*(a^4 + 2*a^2*b^2)*cosh(x)^6 - 18*(a^3*b +
4*a*b^3)*cosh(x)^5 - 15*(7*a^4 + 10*a^2*b^2)*cosh(x)^4 - 11*a^4 - 18*a^2*b^
2 + 12*(7*a^3*b + 12*a*b^3)*cosh(x)^3 + 63*(a^4 + 2*a^2*b^2)*cosh(x)^2 - 2*
(25*a^3*b + 36*a*b^3)*cosh(x))*sinh(x))/(a^5*b*cosh(x)^8 + a^5*b*sinh(x)^8
+ 2*a^6*cosh(x)^7 - 4*a^5*b*cosh(x)^6 - 6*a^6*cosh(x)^5 + 6*a^5*b*cosh(x)^4
 + 6*a^6*cosh(x)^3 - 4*a^5*b*cosh(x)^2 + 2*(4*a^5*b*cosh(x) + a^6)*sinh(x)^
7 - 2*a^6*cosh(x) + 2*(14*a^5*b*cosh(x)^2 + 7*a^6*cosh(x) - 2*a^5*b)*sinh(x
)^6 + a^5*b + 2*(28*a^5*b*cosh(x)^3 + 21*a^6*cosh(x)^2 - 12*a^5*b*cosh(x) -
 3*a^6)*sinh(x)^5 + 2*(35*a^5*b*cosh(x)^4 + 35*a^6*cosh(x)^3 - 30*a^5*b*cos
h(x)^2 - 15*a^6*cosh(x) + 3*a^5*b)*sinh(x)^4 + 2*(28*a^5*b*cosh(x)^5 + 35*a
^6*cosh(x)^4 - 40*a^5*b*cosh(x)^3 - 30*a^6*cosh(x)^2 + 12*a^5*b*cosh(x) + 3
*a^6)*sinh(x)^3 + 2*(14*a^5*b*cosh(x)^6 + 21*a^6*cosh(x)^5 - 30*a^5*b*cosh(
x)^4 - 30*a^6*cosh(x)^3 + 18*a^5*b*cosh(x)^2 + 9*a^6*cosh(x) - 2*a^5*b)*sin
h(x)^2 + 2*(4*a^5*b*cosh(x)^7 + 7*a^6*cosh(x)^6 - 12*a^5*b*cosh(x)^5 - 15*a
^6*cosh(x)^4 + 12*a^5*b*cosh(x)^3 + 9*a^6*cosh(x)^2 - 4*a^5*b*cosh(x) - a^6
)*sinh(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20544, size = 327, normalized size = 2.06

$$\frac{\left(3\,a^{2}b+4\,b^{3}\right)\log\left(e^{x}+1\right)}{a^{5}}-\frac{\left(3\,a^{2}b+4\,b^{3}\right)\log\left(\left|e^{x}-1\right|\right)}{a^{5}}+\frac{\left(a^{4}+5\,a^{2}b^{2}+4\,b^{4}\right)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}a^{5}}+\frac{2\left(a^{3}e^{x}+ab^{2}e^{x}-\right.}{\left(be^{(2\,x)}+2\,ae^{x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^2*b + 4*b^3)*log(e^x + 1)/a^5 - (3*a^2*b + 4*b^3)*log(abs(e^x - 1))/a^
5 + (a^4 + 5*a^2*b^2 + 4*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/ab
s(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5) + 2*(a^3*e^x +
a*b^2*e^x - a^2*b - b^3)/((b*e^(2*x) + 2*a*e^x - b)*a^4) + 2/3*(3*a*b*e^(5*
x) - 6*a^2*e^(4*x) - 9*b^2*e^(4*x) + 6*a^2*e^(2*x) + 18*b^2*e^(2*x) - 3*a*b
*e^x - 4*a^2 - 9*b^2)/(a^4*(e^(2*x) - 1)^3)
```

3.244 $\int \coth(x)\sqrt{a+b\sinh(x)}\,dx$

Optimal. Leaf size=37

$$2\sqrt{a+b\sinh(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Rubi [A] time = 0.0624441, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2721, 50, 63, 207}

$$2\sqrt{a+b\sinh(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]],x]
```

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \coth(x)\sqrt{a+b\sinh(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\sinh(x)\right)\\ &= 2\sqrt{a+b\sinh(x)} + a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sinh(x)\right)\\ &= 2\sqrt{a+b\sinh(x)} + (2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sinh(x)}\right)\\ &= -2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right) + 2\sqrt{a+b\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.0217951, size = 37, normalized size = 1.

$$2\sqrt{a+b\sinh(x)} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]],x]
```

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Maple [A] time = 0.014, size = 30, normalized size = 0.8

$$-2\,\text{Artanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)\sqrt{a} + 2\sqrt{a+b\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sinh(x))^(1/2),x)
```

```
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*sinh(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(x)+a}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

Fricas [B] time = 4.91948, size = 1071, normalized size = 28.95

$$\left[\frac{1}{2}\sqrt{a}\log\left(-\frac{b^2\cosh(x)^4 + b^2\sinh(x)^4 + 16\,ab\cosh(x)^3 + 4\left(b^2\cosh(x) + 4\,ab\right)\sinh(x)^3 - 16\,ab\cosh(x) + 2\left(16\,a^2 - b\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^
2*cosh(x) + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^2
+ 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*cosh
(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b*cos
h(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*sqrt(
a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^2)*cos
h(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*
sqrt(b*sinh(x) + a), sqrt(-a)*arctan(4*sqrt(b*sinh(x) + a)*sqrt(-a)*(cosh(x
) + sinh(x))/(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2*a)
*sinh(x) - b)) + 2*sqrt(b*sinh(x) + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh(x)} \coth(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x))*coth(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(x) + a} \coth(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

3.245 $\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0589528, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2721, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sinh(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sinh(x)}\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0126825, size = 24, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.016, size = 19, normalized size = 0.8

$$-2\frac{1}{\sqrt{a}}\operatorname{Artanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x))^(1/2),x)
```

```
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

Fricas [B] time = 2.58261, size = 1099, normalized size = 45.79

$$\left[\log\left(\frac{b^2\cosh(x)^4+b^2\sinh(x)^4+16\,ab\cosh(x)^3+4\left(b^2\cosh(x)+4\,ab\right)\sinh(x)^3-16\,ab\cosh(x)+2\left(16\,a^2-b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+24\,ab\cosh(x)+16\,a^2\cdots\right.}{\cosh(x)^4+4\cosh(x)\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x)
 + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^2 + 2*(3*b^
2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*cosh(x)^3 + b
*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b*cosh(x) + (3
*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*sqrt(a) + b^2
+ 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^2)*cosh(x))*sin
h(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*si
```

```
nh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1))/sqrt(a), sqrt
(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2
*a)*sinh(x) - b)*sqrt(b*sinh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^
2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a + b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

3.246 $\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=51

$$\frac{B\log(a+b\sinh(x))}{b}-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
a + b*Sinh[x]])/b
```

Rubi [A] time = 0.134812, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4401, 2660, 618, 206, 2668, 31}

$$\frac{B\log(a+b\sinh(x))}{b}-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
a + b*Sinh[x]])/b
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx &= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\cosh(x)}{a+b\sinh(x)}\right)dx\\
&= A\int \frac{1}{a+b\sinh(x)}\,dx+B\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx\\
&= (2A)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{B\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{b}\\
&= \frac{B\log(a+b\sinh(x))}{b}-(4A)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\\
&= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(a+b\sinh(x))}{b}
\end{aligned}$$

Mathematica [A] time = 0.0762386, size = 59, normalized size = 1.16

$$\frac{2A\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{B\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*Log[
a + b*Sinh[x]])/b
```

Maple [A] time = 0.025, size = 88, normalized size = 1.7

$$-\frac{B}{b}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{B}{b}\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{B}{b}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-2\tanh(x/2)\,b-a\right)+2\frac{A}{\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] -B/b*ln(tanh(1/2*x)+1)-B/b*ln(tanh(1/2*x)-1)+1/b*B*ln(a*tanh(1/2*x)^2-2*tan
h(1/2*x)*b-a)+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)
^(1/2))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.09345, size = 458, normalized size = 8.98

$$\frac{\sqrt{a^2+b^2}Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-\left(Ba^2+Bb^2\right)}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x + (B*a^2 + B*b^2)*log(2*(b*sinh(x) + a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

Sympy [A] time = 99.5566, size = 695, normalized size = 13.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2))), Eq(a, 0) & Eq(b, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2
) + 1) + B*log(tanh(x/2)))/b, Eq(a, 0)), (-2*A*tanh(x/2)/(-b*tanh(x/2) + I*
b) - B*x*tanh(x/2)/(-b*tanh(x/2) + I*b) + I*B*x/(-b*tanh(x/2) + I*b) + 2*B*
log(tanh(x/2) + 1)*tanh(x/2)/(-b*tanh(x/2) + I*b) - 2*I*B*log(tanh(x/2) + 1
)/(-b*tanh(x/2) + I*b) - 2*B*log(tanh(x/2) - I)*tanh(x/2)/(-b*tanh(x/2) + I
*b) + 2*I*B*log(tanh(x/2) - I)/(-b*tanh(x/2) + I*b), Eq(a, -I*b)), (2*A*tan
h(x/2)/(b*tanh(x/2) + I*b) + B*x*tanh(x/2)/(b*tanh(x/2) + I*b) + I*B*x/(b*t
anh(x/2) + I*b) - 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(b*tanh(x/2) + I*b) - 2*
I*B*log(tanh(x/2) + 1)/(b*tanh(x/2) + I*b) + 2*B*log(tanh(x/2) + I)*tanh(x/
2)/(b*tanh(x/2) + I*b) + 2*I*B*log(tanh(x/2) + I)/(b*tanh(x/2) + I*b), Eq(a
, I*b)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (-A*b*sqrt(a**2 + b**2)*log(tanh(
x/2) - b/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) + A*b*sqrt(a**2 + b**2)*l
og(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(a**2*b + b**3) + B*a**2*x/(a**2*
b + b**3) - 2*B*a**2*log(tanh(x/2) + 1)/(a**2*b + b**3) + B*a**2*log(tanh(x
/2) - b/a - sqrt(a**2 + b**2)/a)/(a**2*b + b**3) + B*a**2*log(tanh(x/2) - b
/a + sqrt(a**2 + b**2)/a)/(a**2*b + b**3) + B*b**2*x/(a**2*b + b**3) - 2*B*
b**2*log(tanh(x/2) + 1)/(a**2*b + b**3) + B*b**2*log(tanh(x/2) - b/a - sqrt
(a**2 + b**2)/a)/(a**2*b + b**3) + B*b**2*log(tanh(x/2) - b/a + sqrt(a**2 +
 b**2)/a)/(a**2*b + b**3), True))
```

Giac [A] time = 1.13363, size = 117, normalized size = 2.29

$$\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}}-\frac{Bx}{b}+\frac{B\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2) - B*x/b + B*log(abs(b*e^(2*x) + 2*a*e^x - b))/b
```

3.247 $\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=25

$$B\log(\sinh(x)+i)-\frac{A\cosh(x)}{1-i\sinh(x)}$$

```
[Out] B*Log[I + Sinh[x]] - (A*Cosh[x])/(1 - I*Sinh[x])
```

Rubi [A] time = 0.0830601, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4401, 2648, 2667, 31}

$$B\log(\sinh(x)+i)-\frac{A\cosh(x)}{1-i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(I + Sinh[x]),x]
```

```
[Out] B*Log[I + Sinh[x]] - (A*Cosh[x])/(1 - I*Sinh[x])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = \int \left(\frac{iA}{-1+i\sinh(x)} + \frac{iB\cosh(x)}{-1+i\sinh(x)}\right)dx$$
$$= (iA)\int \frac{1}{-1+i\sinh(x)}\,dx + (iB)\int \frac{\cosh(x)}{-1+i\sinh(x)}\,dx$$
$$= -\frac{A\cosh(x)}{1-i\sinh(x)} + B\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, i\sinh(x)\right)$$
$$= B\log(i+\sinh(x)) - \frac{A\cosh(x)}{1-i\sinh(x)}$$

Mathematica [A] time = 0.0658703, size = 48, normalized size = 1.92

$$-\frac{2iA\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)} - 2iB\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + B\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(I + Sinh[x]),x]
```

```
[Out] (-2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]] - ((2*I)*A*Sinh[x/2])/(Cosh[x/2
] - I*Sinh[x/2])
```

Maple [A] time = 0.032, size = 46, normalized size = 1.8

$$-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + 2B\ln\left(\tanh\left(x/2\right)+i\right) - 2iA\left(\tanh\left(\frac{x}{2}\right)+i\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(I+sinh(x)),x)
```

```
[Out] -B*ln(tanh(1/2*x)+1)-B*ln(tanh(1/2*x)-1)+2*B*ln(tanh(1/2*x)+I)-2*I/(tanh(1/
2*x)+I)*A
```

Maxima [A] time = 1.03481, size = 26, normalized size = 1.04

$$B\log\left(\sinh\left(x\right)+i\right) - \frac{2A}{e^{(-x)}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] B*log(sinh(x) + I) - 2*A/(e^(-x) - I)
```

Fricas [A] time = 2.04706, size = 90, normalized size = 3.6

$$-\frac{Bxe^{x} + iBx - 2\left(Be^{x}+iB\right)\log\left(e^{x}+i\right) + 2A}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*e^x + I*B*x - 2*(B*e^x + I*B)*log(e^x + I) + 2*A)/(e^x + I)
```

Sympy [A] time = 1.3108, size = 20, normalized size = 0.8

$$-\frac{2A}{e^{x}+i}-Bx+2B\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x)
```

```
[Out] -2*A/(exp(x) + I) - B*x + 2*B*log(exp(x) + I)
```

Giac [A] time = 1.13634, size = 30, normalized size = 1.2

$$-Bx+2\,B\log\left(e^{x}+i\right)-\frac{2\,A}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*B*log(e^x + I) - 2*A/(e^x + I)
```

3.248 $\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx$

Optimal. Leaf size=27

$$\frac{A\cosh(x)}{1+i\sinh(x)} - B\log(-\sinh(x)+i)$$

```
[Out] -(B*Log[I - Sinh[x]]) + (A*Cosh[x])/(1 + I*Sinh[x])
```

Rubi [A] time = 0.0865139, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4401, 2648, 2667, 31}

$$\frac{A\cosh(x)}{1+i\sinh(x)} - B\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(I - Sinh[x]),x]
```

```
[Out] -(B*Log[I - Sinh[x]]) + (A*Cosh[x])/(1 + I*Sinh[x])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx &= \int \left(-\frac{iA}{1+i\sinh(x)}-\frac{iB\cosh(x)}{1+i\sinh(x)}\right)dx \\
&= -\left((iA)\int \frac{1}{1+i\sinh(x)}\,dx\right)-(iB)\int \frac{\cosh(x)}{1+i\sinh(x)}\,dx \\
&= \frac{A\cosh(x)}{1+i\sinh(x)}-B\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,i\sinh(x)\right) \\
&= -B\log(i-\sinh(x))+\frac{A\cosh(x)}{1+i\sinh(x)}
\end{aligned}
$$

Mathematica [B] time = 0.0881459, size = 81, normalized size = 3.

$$
-\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(\sinh\left(\frac{x}{2}\right)\left(2A+2iB\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+B\log(\cosh(x))\right)+B\cosh\left(\frac{x}{2}\right)\left(2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-\right.\right.}{\sinh(x)-i}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(I - Sinh[x]),x]
```

```
[Out] -(((Cosh[x/2] + I*Sinh[x/2])*(B*Cosh[x/2]*(2*ArcTan[Tanh[x/2]] - I*Log[Cosh
[x]]) + (2*A + (2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]])*Sinh[x/2]))/(-I
+ Sinh[x]))
```

Maple [A] time = 0.037, size = 44, normalized size = 1.6

$$
B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\,iA\left(\tanh\left(\frac{x}{2}\right)-i\right)^{-1}-2\,B\ln\left(\tanh\left(x/2\right)-i\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(I-sinh(x)),x)
```

```
[Out] B*ln(tanh(1/2*x)+1)-2*I/(tanh(1/2*x)-I)*A-2*B*ln(tanh(1/2*x)-I)+B*ln(tanh(1
/2*x)-1)
```

Maxima [A] time = 1.05507, size = 27, normalized size = 1.

$$
-B\log\left(\sinh\left(x\right)-i\right)+\frac{2\,A}{e^{(-x)}+i}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*log(sinh(x) - I) + 2*A/(e^(-x) + I)
```

Fricas [A] time = 2.08174, size = 89, normalized size = 3.3

$$
\frac{Bxe^{x}-i\,Bx-2\left(Be^{x}-i\,B\right)\log\left(e^{x}-i\right)+2\,A}{e^{x}-i}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*e^x - I*B*x - 2*(B*e^x - I*B)*log(e^x - I) + 2*A)/(e^x - I)
```

Sympy [A] time = 0.366195, size = 20, normalized size = 0.74

$$\frac{2A}{e^x - i} + Bx - 2B\log\left(e^x - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x)
```

```
[Out] 2*A/(exp(x) - I) + B*x - 2*B*log(exp(x) - I)
```

Giac [A] time = 1.14116, size = 28, normalized size = 1.04

$$Bx - 2B\log\left(e^x - i\right) + \frac{2A}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="giac")
```

```
[Out] B*x - 2*B*log(e^x - I) + 2*A/(e^x - I)
```

3.249 $\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}+\frac{bB\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{aB\log(\cosh(x))}{a^2+b^2}$$

```
[Out] (b*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] + (a*B*Log[Cosh[x]])/(a^2 + b^2) - (a*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

Rubi [A] time = 0.177193, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4401, 2660, 618, 206, 2721, 801, 635, 203, 260}

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}+\frac{bB\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{aB\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (b*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] + (a*B*Log[Cosh[x]])/(a^2 + b^2) - (a*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx &= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\tanh(x)}{a+b\sinh(x)}\right)dx\\
&= A\int \frac{1}{a+b\sinh(x)}\,dx+B\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx\\
&= (2A)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)-B\,\text{Subst}\left(\int \frac{x}{(a+x)\left(-b^2-x^2\right)}\,dx,x,b\sinh(x)\right.\\
&= -\left((4A)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\right)-B\,\text{Subst}\left(\int\left(\frac{a}{\left(a^2+b^2\right)(a+x)}+\right.\right.\\
&= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}-\frac{B\,\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}+\frac{(aB)\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{a^2+b^2}+\frac{\left(b^2 \right.}{}\\
&= \frac{bB\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{aB\log(\cosh(x))}{a^2+b^2}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.354621, size = 132, normalized size = 1.48

$$-\frac{\cosh(x)(A+B\tanh(x))\left(2A\left(a^2+b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+2bB\sqrt{-a^2-b^2}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+aB\sqrt{-a^2-b^2}(\log(\cosh(\right.}{\left(-a^2-b^2\right)^{3/2}(A\cosh(x)+B\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

```
[Out] -((Cosh[x]*(2*b*Sqrt[-a^2 - b^2]*B*ArcTan[Tanh[x/2]] + 2*A*(a^2 + b^2)*ArcT
an[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Sqrt[-a^2 - b^2]*B*(Log[Cosh[x]]
 - Log[a + b*Sinh[x]]))*(A + B*Tanh[x]))/((-a^2 - b^2)^(3/2)*(A*Cosh[x] + B
*Sinh[x])))
```

Maple [A] time = 0.032, size = 150, normalized size = 1.7

$$\frac{aB}{a^2+b^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)+2\,\frac{Bb\arctan\left(\tanh\left(x/2\right)\right)}{a^2+b^2}-\frac{aB}{a^2+b^2}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-2\,\tanh\left(x/2\right)b-a\right)+2\,\frac{a}{(a^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tanh(x))/(a+b*sinh(x)),x)
```

```
[Out] B/(a^2+b^2)*a*ln(tanh(1/2*x)^2+1)+2*B/(a^2+b^2)*b*arctan(tanh(1/2*x))-1/(a^
2+b^2)*a*B*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2/(a^2+b^2)^(3/2)*arctanh(
1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*a^2*A+2/(a^2+b^2)^(3/2)*arctanh(
1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 15.2367, size = 510, normalized size = 5.73

$$\frac{2\,Bb\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-Ba\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+Ba\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2+b^2}A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh}{}\right.}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*B*b*arctan(cosh(x) + sinh(x)) - B*a*log(2*(b*sinh(x) + a)/(cosh(x) - sin
h(x))) + B*a*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((b^
2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2 + b^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\tanh(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*tanh(x))/(a + b*sinh(x)), x)
```

Giac [A] time = 1.17797, size = 166, normalized size = 1.87

$$\frac{2\,Bb\arctan\left(e^{x}\right)}{a^{2}+b^{2}}+\frac{Ba\log\left(e^{(2\,x)}+1\right)}{a^{2}+b^{2}}-\frac{Ba\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a^{2}+b^{2}}+\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*B*b*arctan(e^x)/(a^2 + b^2) + B*a*log(e^(2*x) + 1)/(a^2 + b^2) - B*a*log(
abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

3.250 $\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=60

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{B\log(a+b\sinh(x))}{a}+\frac{B\log(\sinh(x))}{a}$$

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
Sinh[x]])/a - (B*Log[a + b*Sinh[x]])/a
```

Rubi [A] time = 0.150008, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {4401, 2660, 618, 206, 2721, 36, 29, 31}

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{B\log(a+b\sinh(x))}{a}+\frac{B\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Coth[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
Sinh[x]])/a - (B*Log[a + b*Sinh[x]])/a
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
```

```
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx &= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\coth(x)}{a+b\sinh(x)}\right)dx\\ &= A\int \frac{1}{a+b\sinh(x)}\,dx+B\int \frac{\coth(x)}{a+b\sinh(x)}\,dx\\ &= (2A)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+B\,\mathrm{Subst}\left(\int \frac{1}{x(a+x)}\,dx,x,b\sinh(x)\right)\\ &= -\left((4A)\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\right)+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,b\sinh(x)\right)}{a}-\\ &= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(\sinh(x))}{a}-\frac{B\log(a+b\sinh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.15923, size = 65, normalized size = 1.08

$$\frac{2A\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{B(\log(\sinh(x))-\log(a+b\sinh(x)))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Coth[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*(Log
[Sinh[x]] - Log[a + b*Sinh[x]]))/a
```

Maple [A] time = 0.033, size = 73, normalized size = 1.2

$$\frac{B}{a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{B}{a}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2-2\tanh(x/2)\,b-a\right)+2\frac{A}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tanh(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*coth(x))/(a+b*sinh(x)),x)
```

```
[Out] B/a*ln(tanh(1/2*x))-1/a*B*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2/(a^2+b^2)
^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.23613, size = 502, normalized size = 8.37

$$\frac{\sqrt{a^2+b^2}Aa\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-\left(Ba^2+Bb^2\right)}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) - (B*a^2 + B*b^2)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) + (B*a^2 + B*b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^3 + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*coth(x))/(a + b*sinh(x)), x)
```

Giac [A] time = 1.17266, size = 138, normalized size = 2.3

$$\frac{A\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}+\frac{B\log\left(e^x+1\right)}{a}-\frac{B\log\left(\left|be^{(2\,x)}+2\,ae^x-b\right|\right)}{a}+\frac{B\log\left(\left|e^x-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2) + B*log(e^x + 1)/a - B*log(abs(b*e^(2*x) + 2*a*e^
x - b))/a + B*log(abs(e^x - 1))/a
```

3.251 $\int \frac{A+B\,\mathbf{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2}+\frac{aB\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{bB\log(\cosh(x))}{a^2+b^2}$$

```
[Out] (a*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] - (b*B*Log[Cosh[x]])/(a^2 + b^2) + (b*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

Rubi [A] time = 0.251957, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {4226, 4401, 2660, 618, 206, 2668, 706, 31, 635, 204, 260}

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2}+\frac{aB\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{bB\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sech[x])/(a + b*Sinh[x]),x]
```

```
[Out] (a*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] - (b*B*Log[Cosh[x]])/(a^2 + b^2) + (b*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

Rule 4226

```
Int[(u_)*((A_) + (B_.)*sec[(a_.) + (b_.)*(x_)]), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Cos[a + b*x]))/Cos[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+B\text{sech}(x)}{a+b\sinh(x)}\,dx = \int \frac{(B+A\cosh(x))\text{sech}(x)}{a+b\sinh(x)}\,dx$$
$$= \int \left(\frac{A}{a+b\sinh(x)} + \frac{B\text{sech}(x)}{a+b\sinh(x)}\right) dx$$
$$= A\int \frac{1}{a+b\sinh(x)}\,dx + B\int \frac{\text{sech}(x)}{a+b\sinh(x)}\,dx$$
$$= (2A)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - (bB)\,\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sin$$
$$= -\left((4A)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)\right) + \frac{(bB)\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh}{a^2+b^2}$$
$$= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{bB\log(a+b\sinh(x))}{a^2+b^2} + \frac{(bB)\,\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2} - $$
$$= \frac{aB\tan^{-1}(\sinh(x))}{a^2+b^2} - \frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{bB\log(\cosh(x))}{a^2+b^2} + \frac{bB\log(a+b\sinh(x))}{a^2+b^2}$$

Mathematica [A] time = 0.249137, size = 93, normalized size = 1.04

$$\frac{2A\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{2aB\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2} - \frac{bB(\log(\cosh(x))-\log(a+b\sinh(x)))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sech[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*B*ArcTan[Tanh[x/2]])/(a^2 + b^2) + (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[
-a^2 - b^2]])/Sqrt[-a^2 - b^2] - (b*B*(Log[Cosh[x]] - Log[a + b*Sinh[x]]))/
(a^2 + b^2)
```

Maple [A] time = 0.032, size = 150, normalized size = 1.7

$$-\frac{Bb}{a^2+b^2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right) + 2\frac{aB\arctan(\tanh(x/2))}{a^2+b^2} + \frac{Bb}{a^2+b^2}\ln\left(a\left(\tanh\left(\frac{x}{2}\right)\right)^2 - 2\tanh(x/2)\,b - a\right) + 2\frac{}{(a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sech(x))/(a+b*sinh(x)),x)
```

```
[Out] -B/(a^2+b^2)*b*ln(tanh(1/2*x)^2+1)+2*B/(a^2+b^2)*a*arctan(tanh(1/2*x))+1/(a
^2+b^2)*b*B*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2/(a^2+b^2)^(3/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*a^2*A+2/(a^2+b^2)^(3/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 26.1958, size = 510, normalized size = 5.73

$$\frac{2\,Ba\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+Bb\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-Bb\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2+b^2}A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2 \cdots}{\cdots}\right.}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*B*a*arctan(cosh(x) + sinh(x)) + B*b*log(2*(b*sinh(x) + a)/(cosh(x) - sin
h(x))) - B*b*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((b^
2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2 + b^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\operatorname{sech}\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*sech(x))/(a + b*sinh(x)), x)
```

Giac [A] time = 1.16818, size = 166, normalized size = 1.87

$$\frac{2\,Ba\arctan\left(e^{x}\right)}{a^2+b^2}-\frac{Bb\log\left(e^{(2\,x)}+1\right)}{a^2+b^2}+\frac{Bb\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a^2+b^2}+\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(e^x)/(a^2 + b^2) - B*b*log(e^(2*x) + 1)/(a^2 + b^2) + B*b*log(
abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

3.252 $\int \frac{A+B\mathbf{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=58

$$-\frac{2(aA-bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{B\tanh^{-1}(\cosh(x))}{a}$$

```
[Out] -((B*ArcTanh[Cosh[x]])/a) - (2*(a*A - b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a
^2 + b^2]])/(a*Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.157249, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2828, 3001, 3770, 2660, 618, 206}

$$-\frac{2(aA-bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{B\tanh^{-1}(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Csch[x])/(a + b*Sinh[x]),x]
```

```
[Out] -((B*ArcTanh[Cosh[x]])/a) - (2*(a*A - b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a
^2 + b^2]])/(a*Sqrt[a^2 + b^2])
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx &= -\left(i\int \frac{\text{csch}(x)(iB+iA\sinh(x))}{a+b\sinh(x)}\,dx\right)\\ &= \frac{B\int \text{csch}(x)\,dx}{a}+\frac{(aA-bB)\int \frac{1}{a+b\sinh(x)}\,dx}{a}\\ &= -\frac{B\tanh^{-1}(\cosh(x))}{a}+\frac{(2(aA-bB))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{B\tanh^{-1}(\cosh(x))}{a}-\frac{(4(aA-bB))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{B\tanh^{-1}(\cosh(x))}{a}-\frac{2(aA-bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.115828, size = 67, normalized size = 1.16

$$\frac{\frac{2(aA-bB)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Csch[x])/(a + b*Sinh[x]),x]
```

```
[Out] ((2*(a*A - b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] + B*Log[Tanh[x/2]])/a
```

Maple [A] time = 0.03, size = 86, normalized size = 1.5

$$\frac{B}{a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\,\frac{A}{\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tanh(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)-2\,\frac{Bb}{a\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tanh(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*csch(x))/(a+b*sinh(x)),x)
```

```
[Out] B/a*ln(tanh(1/2*x))+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^
2+b^2)^(1/2))*A-2*B/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(
a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.06544, size = 482, normalized size = 8.31

$$-\frac{(Aa-Bb)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)+\left(Ba\right.}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((A*a - B*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) - b)) + (B*a^2 + B*b^2)*log(cosh(x) + sinh(x) + 1) - (B
*a^2 + B*b^2)*log(cosh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*csch(x))/(a + b*sinh(x)), x)
```

Giac [A] time = 1.17163, size = 122, normalized size = 2.1

$$-\frac{B\log(e^x+1)}{a}+\frac{B\log(|e^x-1|)}{a}+\frac{(Aa-Bb)\log\left(\frac{|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}|}{|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*log(e^x + 1)/a + B*log(abs(e^x - 1))/a + (A*a - B*b)*log(abs(2*b*e^x + 2
*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 +
 b^2)*a)
```

3.253 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx$

Optimal. Leaf size=81

$$-\frac{2(Ac-aC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{ce\sqrt{a^2+c^2}}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

```
[Out] (C*x)/c - (2*(A*c - a*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]]
)/(c*Sqrt[a^2 + c^2]*e) + (B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Rubi [A] time = 0.168539, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4376, 2735, 2660, 618, 204, 2668, 31}

$$-\frac{2(Ac-aC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{ce\sqrt{a^2+c^2}}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
```

```
[Out] (C*x)/c - (2*(A*c - a*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]]
)/(c*Sqrt[a^2 + c^2]*e) + (B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx &= B\int \frac{\cosh(d+ex)}{a+c\sinh(d+ex)}\,dx+\int \frac{A+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx\\ &= \frac{Cx}{c}-\frac{(i(iAc-iaC))\int \frac{1}{a+c\sinh(d+ex)}\,dx}{c}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,c\sinh(d\right.}{ce}\\ &= \frac{Cx}{c}+\frac{B\log(a+c\sinh(d+ex))}{ce}-\frac{(2i(Ac-aC))\,\mathrm{Subst}\left(\int \frac{1}{a-2icx+ax^2}\,dx,\right.}{ce}\\ &= \frac{Cx}{c}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{(4i(Ac-aC))\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2+c^2)-x^2}\right.}{ce}\\ &= \frac{Cx}{c}-\frac{2(Ac-aC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{c\sqrt{a^2+c^2}e}+\frac{B\log(a+c\sinh(d+ex))}{ce}\end{aligned}$$

Mathematica [A] time = 0.283816, size = 85, normalized size = 1.05

$$\frac{\frac{2(Ac-aC)\tan^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}}+B\log(a+c\sinh(d+ex))+C(d+ex)}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
```

```
[Out] (C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 -
c^2]])/Sqrt[-a^2 - c^2] + B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Maple [B] time = 0.056, size = 213, normalized size = 2.6

$$\frac{B}{ec}\ln\left(a\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2-2\,c\tanh(1/2\,ex+d/2)-a\right)+2\,\frac{A}{e\sqrt{a^2+c^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tanh(1/2\,ex+d/2)-2\,c}{\sqrt{a^2+c^2}}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x)
```

```
[Out] 1/e/c*B*ln(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)+2/e/(a^2+c^2)
^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2))*A-2/e/c/(
a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2))*C
*a-1/e/c*ln(tanh(1/2*e*x+1/2*d)+1)*B+1/e/c*ln(tanh(1/2*e*x+1/2*d)+1)*C-1/e/
c*ln(tanh(1/2*e*x+1/2*d)-1)*B-1/e/c*ln(tanh(1/2*e*x+1/2*d)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13117, size = 616, normalized size = 7.6

$$-\frac{\left((B-C)a^2+(B-C)c^2\right)ex+(Ca-Ac)\sqrt{a^2+c^2}\log\left(\frac{c^2\cosh(ex+d)^2+c^2\sinh(ex+d)^2+2\,ac\cosh(ex+d)+2\,a^2+c^2+2\left(c^2\cosh(ex+d)+ac\right)\sinh(}{c\cosh(ex+d)^2+c\sinh(ex+d)^2+2\,a\cosh(ex+d)+2\,(c\cosh(ex}\right.}{\left(a^2c+c^3\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="f
ricas")
```

```
[Out] -(((B - C)*a^2 + (B - C)*c^2)*e*x + (C*a - A*c)*sqrt(a^2 + c^2)*log((c^2*co
sh(e*x + d)^2 + c^2*sinh(e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2
*(c^2*cosh(e*x + d) + a*c)*sinh(e*x + d) - 2*sqrt(a^2 + c^2)*(c*cosh(e*x +
d) + c*sinh(e*x + d) + a))/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cos
h(e*x + d) + 2*(c*cosh(e*x + d) + a)*sinh(e*x + d) - c)) - (B*a^2 + B*c^2)*
log(2*(c*sinh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d))))/((a^2*c + c^3
)*e)
```

Sympy [A] time = 130.497, size = 1358, normalized size = 16.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cosh(d) + C*sinh(d))/sinh(d), Eq(a, 0) & Eq(c, 0) &
 Eq(e, 0)), ((A*log(tanh(d/2 + e*x/2))/e + B*x - 2*B*log(tanh(d/2 + e*x/2)
+ 1)/e + B*log(tanh(d/2 + e*x/2))/e + C*x)/c, Eq(a, 0)), (-2*A*tanh(d/2 + e
*x/2)/(-c*e*tanh(d/2 + e*x/2) + I*c*e) - B*e*x*tanh(d/2 + e*x/2)/(-c*e*tanh
(d/2 + e*x/2) + I*c*e) + I*B*e*x/(-c*e*tanh(d/2 + e*x/2) + I*c*e) + 2*B*log
```

```
(tanh(d/2 + e*x/2) + 1)*tanh(d/2 + e*x/2)/(-c*e*tanh(d/2 + e*x/2) + I*c*e)
- 2*I*B*log(tanh(d/2 + e*x/2) + 1)/(-c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*B*l
og(tanh(d/2 + e*x/2) - I)*tanh(d/2 + e*x/2)/(-c*e*tanh(d/2 + e*x/2) + I*c*e
) + 2*I*B*log(tanh(d/2 + e*x/2) - I)/(-c*e*tanh(d/2 + e*x/2) + I*c*e) - C*e
*x*tanh(d/2 + e*x/2)/(-c*e*tanh(d/2 + e*x/2) + I*c*e) + I*C*e*x/(-c*e*tanh(
d/2 + e*x/2) + I*c*e) - 2*I*C*tanh(d/2 + e*x/2)/(-c*e*tanh(d/2 + e*x/2) + I
*c*e), Eq(a, -I*c)), (2*A*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e)
 + B*e*x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + I*B*e*x/(c*e*t
anh(d/2 + e*x/2) + I*c*e) - 2*B*log(tanh(d/2 + e*x/2) + 1)*tanh(d/2 + e*x/2
)/(c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*I*B*log(tanh(d/2 + e*x/2) + 1)/(c*e*t
anh(d/2 + e*x/2) + I*c*e) + 2*B*log(tanh(d/2 + e*x/2) + I)*tanh(d/2 + e*x/2
)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + 2*I*B*log(tanh(d/2 + e*x/2) + I)/(c*e*t
anh(d/2 + e*x/2) + I*c*e) + C*e*x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2)
+ I*c*e) + I*C*e*x/(c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*I*C*tanh(d/2 + e*x/2
)/(c*e*tanh(d/2 + e*x/2) + I*c*e), Eq(a, I*c)), ((A*x + B*sinh(d + e*x)/e +
 C*cosh(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B*cosh(d) + C*sinh(d))/(a + c*sin
h(d)), Eq(e, 0)), (-A*c*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2) - c/a - sqr
t(a**2 + c**2)/a)/(a**2*c*e + c**3*e) + A*c*sqrt(a**2 + c**2)*log(tanh(d/2
+ e*x/2) - c/a + sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e) + B*a**2*e*x/(a**
2*c*e + c**3*e) - 2*B*a**2*log(tanh(d/2 + e*x/2) + 1)/(a**2*c*e + c**3*e) +
 B*a**2*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3
*e) + B*a**2*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c**2)/a)/(a**2*c*e +
 c**3*e) + B*c**2*e*x/(a**2*c*e + c**3*e) - 2*B*c**2*log(tanh(d/2 + e*x/2)
+ 1)/(a**2*c*e + c**3*e) + B*c**2*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 +
 c**2)/a)/(a**2*c*e + c**3*e) + B*c**2*log(tanh(d/2 + e*x/2) - c/a + sqrt(a
**2 + c**2)/a)/(a**2*c*e + c**3*e) + C*a**2*e*x/(a**2*c*e + c**3*e) + C*a*s
qrt(a**2 + c**2)*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/a)/(a**2*c
*e + c**3*e) - C*a*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**
2 + c**2)/a)/(a**2*c*e + c**3*e) + C*c**2*e*x/(a**2*c*e + c**3*e), True))
```

Giac [A] time = 1.19282, size = 181, normalized size = 2.23

$$-\frac{(xe+d)(B-C)e^{(-1)}}{c}+\frac{Be^{(-1)}\log\left(\left|ce^{(2\,xe+2\,d)}+2\,ae^{(xe+d)}-c\right|\right)}{c}-\frac{(Ca-Ac)e^{(-1)}\log\left(\frac{\left|2\,ce^{(xe+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(xe+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\sqrt{a^2+c^2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="g
iac")
```

```
[Out] -(x*e + d)*(B - C)*e^(-1)/c + B*e^(-1)*log(abs(c*e^(2*x*e + 2*d) + 2*a*e^(x
*e + d) - c))/c - (C*a - A*c)*e^(-1)*log(abs(2*c*e^(x*e + d) + 2*a - 2*sqrt
(a^2 + c^2))/abs(2*c*e^(x*e + d) + 2*a + 2*sqrt(a^2 + c^2)))/(sqrt(a^2 + c^
2)*c)
```

3.254 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$

Optimal. Leaf size=113

$$-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{3/2}}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

```
[Out] (-2*(a*A + c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]])/((a^2 +
 c^2)^(3/2)*e) - B/(c*e*(a + c*Sinh[d + e*x])) - ((A*c - a*C)*Cosh[d + e*x]
)/((a^2 + c^2)*e*(a + c*Sinh[d + e*x]))
```

Rubi [A] time = 0.169336, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{3/2}}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]])/((a^2 +
 c^2)^(3/2)*e) - B/(c*e*(a + c*Sinh[d + e*x])) - ((A*c - a*C)*Cosh[d + e*x]
)/((a^2 + c^2)*e*(a + c*Sinh[d + e*x]))
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx + \int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$$

$$= -\frac{(Ac-aC)\cosh(d+ex)}{\left(a^2+c^2\right)e(a+c\sinh(d+ex))} - \frac{\int \frac{-aA-cC}{a+c\sinh(d+ex)}\,dx}{a^2+c^2} + \frac{B\,\text{Subst}\left(\int \frac{1}{(a+x}\right.}{}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{\left(a^2+c^2\right)e(a+c\sinh(d+ex))} + \frac{(aA+cC)\int}{a}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{\left(a^2+c^2\right)e(a+c\sinh(d+ex))} - \frac{(2i(aA+cC}{}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{\left(a^2+c^2\right)e(a+c\sinh(d+ex))} + \frac{(4i(aA+cC}{}$$

$$= -\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{3/2}e} - \frac{B}{ce(a+c\sinh(d+ex))} - \frac{(}{\left(a^2\right.}$$

Mathematica [A] time = 0.52467, size = 113, normalized size = 1.

$$\frac{\frac{2(aA+cC)\tan^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} - \frac{B\left(a^2+c^2\right)+c(Ac-aC)\cosh(d+ex)}{c(a+c\sinh(d+ex))}}{e\left(a^2+c^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,x
]
```

```
[Out] ((2*(a*A + c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 - c^2]])/Sqrt[-a
^2 - c^2] - (B*(a^2 + c^2) + c*(A*c - a*C)*Cosh[d + e*x])/(c*(a + c*Sinh[d
+ e*x])))/((a^2 + c^2)*e)
```

Maple [A] time = 0.092, size = 151, normalized size = 1.3

$$\frac{1}{e}\left(-2\frac{1}{a\left(\tanh\left(1/2\,ex+d/2\right)\right)^{2}-2\,c\tanh\left(1/2\,ex+d/2\right)-a}\left(-\frac{\left(Ac^{2}-Ba^{2}-Bc^{2}-Cac\right)\tanh\left(1/2\,ex+d/2\right)}{a\left(a^{2}+c^{2}\right)}-\frac{Ac-Ca}{a^{2}+c^{2}}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x)
```

```
[Out] 1/e*(-2*(-(A*c^2-B*a^2-B*c^2-C*a*c)/a/(a^2+c^2)*tanh(1/2*e*x+1/2*d)-(A*c-C*
a)/(a^2+c^2))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)+2*(A*a+C*
c)/(a^2+c^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2
)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16489, size = 1339, normalized size = 11.85

$$\frac{2\,Ca^{3}c-2\,Aa^{2}c^{2}+2\,Cac^{3}-2\,Ac^{4}-\left(Aac^{2}+Cc^{3}-\left(Aac^{2}+Cc^{3}\right)\cosh\left(ex+d\right)^{2}-\left(Aac^{2}+Cc^{3}\right)\sinh\left(ex+d\right)^{2}-2\left(Aa^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"fricas")
```

```
[Out] (2*C*a^3*c - 2*A*a^2*c^2 + 2*C*a*c^3 - 2*A*c^4 - (A*a*c^2 + C*c^3 - (A*a*c^
2 + C*c^3)*cosh(e*x + d)^2 - (A*a*c^2 + C*c^3)*sinh(e*x + d)^2 - 2*(A*a^2*c
 + C*a*c^2)*cosh(e*x + d) - 2*(A*a^2*c + C*a*c^2 + (A*a*c^2 + C*c^3)*cosh(e
*x + d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh
(e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*
c)*sinh(e*x + d) - 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a
))/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e
```

```
*x + d) + a)*sinh(e*x + d) - c)) - 2*((B + C)*a^4 - A*a^3*c + (2*B + C)*a^2
*c^2 - A*a*c^3 + B*c^4)*cosh(e*x + d) - 2*((B + C)*a^4 - A*a^3*c + (2*B + C
)*a^2*c^2 - A*a*c^3 + B*c^4)*sinh(e*x + d))/((a^4*c^2 + 2*a^2*c^4 + c^6)*e*
cosh(e*x + d)^2 + (a^4*c^2 + 2*a^2*c^4 + c^6)*e*sinh(e*x + d)^2 + 2*(a^5*c
+ 2*a^3*c^3 + a*c^5)*e*cosh(e*x + d) - (a^4*c^2 + 2*a^2*c^4 + c^6)*e + 2*((
a^4*c^2 + 2*a^2*c^4 + c^6)*e*cosh(e*x + d) + (a^5*c + 2*a^3*c^3 + a*c^5)*e)
*sinh(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18165, size = 262, normalized size = 2.32

$$\frac{(Aa+Cc)\log\left(\frac{\left|2\,ce^{(xe+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(xe+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\left(a^2e+c^2e\right)\sqrt{a^2+c^2}}-\frac{2\left(Ba^2e^{(xe+d)}+Ca^2e^{(xe+d)}-Aace^{(xe+d)}+Bc^2e^{(xe+d)}-Cac+Ac^2\right)}{\left(a^2ce+c^3e\right)\left(ce^{(2\,xe+2\,d)}+2\,ae^{(xe+d)}-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"giac")
```

```
[Out] (A*a + C*c)*log(abs(2*c*e^(x*e + d) + 2*a - 2*sqrt(a^2 + c^2))/abs(2*c*e^(x
*e + d) + 2*a + 2*sqrt(a^2 + c^2)))/((a^2*e + c^2*e)*sqrt(a^2 + c^2)) - 2*(
B*a^2*e^(x*e + d) + C*a^2*e^(x*e + d) - A*a*c*e^(x*e + d) + B*c^2*e^(x*e +
d) - C*a*c + A*c^2)/((a^2*c*e + c^3*e)*(c*e^(2*x*e + 2*d) + 2*a*e^(x*e + d)
 - c))
```

3.255 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(2a^2A+3acC-Ac^2\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{5/2}}-\frac{\left(a^2(-C)+3aAc+2c^2C\right)\cosh(d+ex)}{2e\left(a^2+c^2\right)^2(a+c\sinh(d+ex))}-\frac{(Ac-aC)\cosh(d+ex}{2e\left(a^2+c^2\right)(a+c\sinh(d+}$$

```
[Out] -(((2*a^2*A - A*c^2 + 3*a*c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2]])/Sqrt[a^2 +
 c^2]])/((a^2 + c^2)^(5/2)*e)) - B/(2*c*e*(a + c*Sinh[d + e*x])^2) - ((A*c
- a*C)*Cosh[d + e*x])/(2*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^2) - ((3*a*A*c
 - a^2*C + 2*c^2*C)*Cosh[d + e*x])/(2*(a^2 + c^2)^2*e*(a + c*Sinh[d + e*x])
)
```

Rubi [A] time = 0.269667, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$-\frac{\left(2a^2A+3acC-Ac^2\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{5/2}}-\frac{\left(a^2(-C)+3aAc+2c^2C\right)\cosh(d+ex)}{2e\left(a^2+c^2\right)^2(a+c\sinh(d+ex))}-\frac{(Ac-aC)\cosh(d+ex}{2e\left(a^2+c^2\right)(a+c\sinh(d+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,x]
```

```
[Out] -(((2*a^2*A - A*c^2 + 3*a*c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2]])/Sqrt[a^2 +
 c^2]])/((a^2 + c^2)^(5/2)*e)) - B/(2*c*e*(a + c*Sinh[d + e*x])^2) - ((A*c
- a*C)*Cosh[d + e*x])/(2*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^2) - ((3*a*A*c
 - a^2*C + 2*c^2*C)*Cosh[d + e*x])/(2*(a^2 + c^2)^2*e*(a + c*Sinh[d + e*x])
)
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx+\int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx$$

$$=-\frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2}-\frac{\int \frac{-2(aA+cC)+(Ac-aC)\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx}{2\left(a^2+c^2\right)}+$$

$$=-\frac{B}{2ce(a+c\sinh(d+ex))^2}-\frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2}-\frac{(3aAc}{2\left(a^2\right.}$$

$$=-\frac{B}{2ce(a+c\sinh(d+ex))^2}-\frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2}-\frac{(3aAc}{2\left(a^2\right.}$$

$$=-\frac{B}{2ce(a+c\sinh(d+ex))^2}-\frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2}-\frac{(3aAc}{2\left(a^2\right.}$$

$$=-\frac{B}{2ce(a+c\sinh(d+ex))^2}-\frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2}-\frac{(3aAc}{2\left(a^2\right.}$$

$$=-\frac{\left(2a^2A-Ac^2+3acC\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{5/2}e}-\frac{B}{2ce(a+c\sinh(d+}$$

Mathematica [A] time = 0.679457, size = 170, normalized size = 0.94

$$\frac{-\frac{(a^2+c^2)(B(a^2+c^2)+c(Ac-aC)\cosh(d+ex))}{c(a+c\sinh(d+ex))^2}+\frac{2(2a^2A+3acC-Ac^2)\tan^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}}+\frac{(a^2C-3aAc-2c^2C)\cosh(d+ex)}{a+c\sinh(d+ex)}}{2e\left(a^2+c^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,x
]
```

```
[Out] ((2*(2*a^2*A - A*c^2 + 3*a*c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2
- c^2]])/Sqrt[-a^2 - c^2] - ((a^2 + c^2)*(B*(a^2 + c^2) + c*(A*c - a*C)*Cos
h[d + e*x]))/(c*(a + c*Sinh[d + e*x])^2) + ((-3*a*A*c + a^2*C - 2*c^2*C)*Co
sh[d + e*x])/(a + c*Sinh[d + e*x]))/(2*(a^2 + c^2)^2*e)
```

Maple [B] time = 0.116, size = 416, normalized size = 2.3

$$\frac{1}{e}\left(-2\frac{1}{\left(a\left(\tanh(1/2\,ex+d/2)\right)^2-2\,c\tanh(1/2\,ex+d/2)-a\right)^2}\left(-1/2\,\frac{\left(5\,Aa^2c^2+2\,Ac^4-2\,Ba^4-4\,Ba^2c^2-2\,Bc^4-3\,Ca^3c\right)}{a\left(a^4+2\,a^2c^2+c^4\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x)
```

```
[Out] 1/e*(-2*(-1/2*(5*A*a^2*c^2+2*A*c^4-2*B*a^4-4*B*a^2*c^2-2*B*c^4-3*C*a^3*c)/a
/(a^4+2*a^2*c^2+c^4)*tanh(1/2*e*x+1/2*d)^3-1/2*(4*A*a^4*c-7*A*a^2*c^3-2*A*c
^5+2*B*a^4*c+4*B*a^2*c^3+2*B*c^5-2*C*a^5+5*C*a^3*c^2-2*C*a*c^4)/(a^4+2*a^2*
c^2+c^4)/a^2*tanh(1/2*e*x+1/2*d)^2+1/2*(11*A*a^2*c^2+2*A*c^4-2*B*a^4-4*B*a^
2*c^2-2*B*c^4-5*C*a^3*c+4*C*a*c^3)/(a^4+2*a^2*c^2+c^4)/a*tanh(1/2*e*x+1/2*d
)+1/2*(4*A*a^2*c+A*c^3-2*C*a^3+C*a*c^2)/(a^4+2*a^2*c^2+c^4))/(a*tanh(1/2*e*
x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)^2+(2*A*a^2-A*c^2+3*C*a*c)/(a^4+2*a^2*
c^2+c^4)/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2
)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3097, size = 4178, normalized size = 23.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"fricas")
```

```
[Out] -1/2*(2*C*a^4*c^2 - 6*A*a^3*c^3 - 2*C*a^2*c^4 - 6*A*a*c^5 - 4*C*c^6 - 2*(2*
A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*cosh(e*x + d)^3 -
2*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*sinh(e*x + d)
^3 + 2*(2*(B + C)*a^6 - 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(
2*B - C)*a^2*c^4 + 3*A*a*c^5 + 2*(B + C)*c^6)*cosh(e*x + d)^2 + 2*(2*(B + C
)*a^6 - 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a^2*c^4
 + 3*A*a*c^5 + 2*(B + C)*c^6 - 3*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3
*C*a*c^5 - A*c^6)*cosh(e*x + d))*sinh(e*x + d)^2 + (2*A*a^2*c^3 + 3*C*a*c^4
 - A*c^5 + (2*A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d)^4 + (2*A*a^2*c^3
 + 3*C*a*c^4 - A*c^5)*sinh(e*x + d)^4 + 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*
c^4)*cosh(e*x + d)^3 + 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4 + (2*A*a^2*c^
3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d))*sinh(e*x + d)^3 + 2*(4*A*a^4*c + 6*C*
a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cosh(e*x + d)^2 + 2*(4*A*a^4*c +
 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 + 3*(2*A*a^2*c^3 + 3*C*a*c^4
 - A*c^5)*cosh(e*x + d)^2 + 6*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(e*
x + d))*sinh(e*x + d)^2 - 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(e*x
+ d) - 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4 - (2*A*a^2*c^3 + 3*C*a*c^4 -
A*c^5)*cosh(e*x + d)^3 - 3*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(e*x +
 d)^2 - (4*A*a^4*c + 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cosh(e*
x + d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh(
e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*c
)*sinh(e*x + d) + 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a)
)/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e*
x + d) + a)*sinh(e*x + d) - c)) - 2*(4*C*a^5*c - 10*A*a^4*c^2 - C*a^3*c^3 -
 11*A*a^2*c^4 - 5*C*a*c^5 - A*c^6)*cosh(e*x + d) - 2*(4*C*a^5*c - 10*A*a^4*
c^2 - C*a^3*c^3 - 11*A*a^2*c^4 - 5*C*a*c^5 - A*c^6 + 3*(2*A*a^4*c^2 + 3*C*a
^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*cosh(e*x + d)^2 - 2*(2*(B + C)*a^6
- 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a^2*c^4 + 3*A
*a*c^5 + 2*(B + C)*c^6)*cosh(e*x + d))*sinh(e*x + d))/((a^6*c^3 + 3*a^4*c^5
 + 3*a^2*c^7 + c^9)*e*cosh(e*x + d)^4 + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 +
c^9)*e*sinh(e*x + d)^4 + 4*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e*cosh
(e*x + d)^3 + 2*(2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*e*cosh(e*
x + d)^2 + 4*((a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e*cosh(e*x + d) + (a^
7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e)*sinh(e*x + d)^3 - 4*(a^7*c^2 + 3*
a^5*c^4 + 3*a^3*c^6 + a*c^8)*e*cosh(e*x + d) + 2*(3*(a^6*c^3 + 3*a^4*c^5 +
3*a^2*c^7 + c^9)*e*cosh(e*x + d)^2 + 6*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a
*c^8)*e*cosh(e*x + d) + (2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*e
)*sinh(e*x + d)^2 + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e + 4*((a^6*c^3
 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e*cosh(e*x + d)^3 + 3*(a^7*c^2 + 3*a^5*c^4
+ 3*a^3*c^6 + a*c^8)*e*cosh(e*x + d)^2 + (2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 -
 a^2*c^7 - c^9)*e*cosh(e*x + d) - (a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)
*e)*sinh(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17361, size = 586, normalized size = 3.26

$$-\frac{\left(2\,Aa^2+3\,Cac-Ac^2\right)\log\left(\frac{\left|-2\,ce^{(xe+d)}-2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|-2\,ce^{(xe+d)}-2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{2\,\left(a^4e+2\,a^2c^2e+c^4e\right)\sqrt{a^2+c^2}}+\frac{2\,Aa^2c^2e^{(3\,xe+3\,d)}+3\,Cac^3e^{(3\,xe+3\,d)}-Ac^4e^{(3\,xe+3\,d)}-2\,Ba^4e^{(2\,xe+2\,d)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"giac")
```

```
[Out] -1/2*(2*A*a^2 + 3*C*a*c - A*c^2)*log(abs(-2*c*e^(x*e + d) - 2*a - 2*sqrt(a^
2 + c^2))/abs(-2*c*e^(x*e + d) - 2*a + 2*sqrt(a^2 + c^2)))/((a^4*e + 2*a^2*
c^2*e + c^4*e)*sqrt(a^2 + c^2)) + (2*A*a^2*c^2*e^(3*x*e + 3*d) + 3*C*a*c^3*
e^(3*x*e + 3*d) - A*c^4*e^(3*x*e + 3*d) - 2*B*a^4*e^(2*x*e + 2*d) - 2*C*a^4
*e^(2*x*e + 2*d) + 6*A*a^3*c*e^(2*x*e + 2*d) - 4*B*a^2*c^2*e^(2*x*e + 2*d)
+ 5*C*a^2*c^2*e^(2*x*e + 2*d) - 3*A*a*c^3*e^(2*x*e + 2*d) - 2*B*c^4*e^(2*x*
e + 2*d) - 2*C*c^4*e^(2*x*e + 2*d) + 4*C*a^3*c*e^(x*e + d) - 10*A*a^2*c^2*e
^(x*e + d) - 5*C*a*c^3*e^(x*e + d) - A*c^4*e^(x*e + d) - C*a^2*c^2 + 3*A*a*
c^3 + 2*C*c^4)/((a^4*c*e + 2*a^2*c^3*e + c^5*e)*(c*e^(2*x*e + 2*d) + 2*a*e^
(x*e + d) - c)^2)
```

3.256 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$

Optimal. Leaf size=250

$$-\frac{\left(2a^3A+4a^2cC-3aAc^2-c^3C\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{7/2}}-\frac{\left(11a^2Ac-2a^3C+13ac^2C-4Ac^3\right)\cosh(d+ex)}{6e\left(a^2+c^2\right)^3(a+c\sinh(d+ex))}-\frac{(-2a}{6e}$$

```
[Out] -(((2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTanh[(c - a*Tanh[(d + e*x)/
2])/Sqrt[a^2 + c^2]])/((a^2 + c^2)^(7/2)*e)) - B/(3*c*e*(a + c*Sinh[d + e*x
])^3) - ((A*c - a*C)*Cosh[d + e*x])/(3*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^
3) - ((5*a*A*c - 2*a^2*C + 3*c^2*C)*Cosh[d + e*x])/(6*(a^2 + c^2)^2*e*(a +
c*Sinh[d + e*x])^2) - ((11*a^2*A*c - 4*A*c^3 - 2*a^3*C + 13*a*c^2*C)*Cosh[d
 + e*x])/(6*(a^2 + c^2)^3*e*(a + c*Sinh[d + e*x]))
```

Rubi [A] time = 0.439518, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$-\frac{\left(2a^3A+4a^2cC-3aAc^2-c^3C\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{7/2}}-\frac{\left(11a^2Ac-2a^3C+13ac^2C-4Ac^3\right)\cosh(d+ex)}{6e\left(a^2+c^2\right)^3(a+c\sinh(d+ex))}-\frac{(-2a}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,x]
```

```
[Out] -(((2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTanh[(c - a*Tanh[(d + e*x)/
2])/Sqrt[a^2 + c^2]])/((a^2 + c^2)^(7/2)*e)) - B/(3*c*e*(a + c*Sinh[d + e*x
])^3) - ((A*c - a*C)*Cosh[d + e*x])/(3*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^
3) - ((5*a*A*c - 2*a^2*C + 3*c^2*C)*Cosh[d + e*x])/(6*(a^2 + c^2)^2*e*(a +
c*Sinh[d + e*x])^2) - ((11*a^2*A*c - 4*A*c^3 - 2*a^3*C + 13*a*c^2*C)*Cosh[d
 + e*x])/(6*(a^2 + c^2)^3*e*(a + c*Sinh[d + e*x]))
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx + \int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$$

$$= -\frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} - \frac{\int \frac{-3(aA+cC)+2(Ac-aC)\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx}{3\left(a^2+c^2\right)} +$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} - \frac{(5aAc}{6\left(a^2\right.}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} - \frac{(5aAc}{6\left(a^2\right.}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} - \frac{(5aAc}{6\left(a^2\right.}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} - \frac{(5aAc}{6\left(a^2\right.}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} - \frac{(5aAc}{6\left(a^2\right.}$$

$$= -\frac{\left(2a^3A-3aAc^2+4a^2cC-c^3C\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{7/2}e} - \frac{}{3ce(a+c}$$

Mathematica [A] time = 1.53869, size = 235, normalized size = 0.94

$$\frac{-\frac{2\left(a^2+c^2\right)^2\left(B\left(a^2+c^2\right)+c(Ac-aC)\cosh(d+ex)\right)}{c(a+c\sinh(d+ex))^3} + \frac{6\left(2a^3A+4a^2cC-3aAc^2-c^3C\right)\tan^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} + \frac{\left(a^2+c^2\right)\left(2a^2C-5aAc-3c^2C\right)\cosh(d+ex)}{(a+c\sinh(d+ex))^2} + \cdots}{6e\left(a^2+c^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,x
]
```

```
[Out] ((6*(2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTan[(c - a*Tanh[(d + e*x)/
2])/Sqrt[-a^2 - c^2]])/Sqrt[-a^2 - c^2] - (2*(a^2 + c^2)^2*(B*(a^2 + c^2) +
 c*(A*c - a*C)*Cosh[d + e*x]))/(c*(a + c*Sinh[d + e*x])^3) + ((a^2 + c^2)*(
-5*a*A*c + 2*a^2*C - 3*c^2*C)*Cosh[d + e*x])/(a + c*Sinh[d + e*x])^2 + ((-1
1*a^2*A*c + 4*A*c^3 + 2*a^3*C - 13*a*c^2*C)*Cosh[d + e*x])/(a + c*Sinh[d +
e*x]))/(6*(a^2 + c^2)^3*e)
```

Maple [B] time = 0.134, size = 844, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x)
```

```
[Out] 1/e*(-2*(-1/2*(9*A*a^4*c^2+6*A*a^2*c^4+2*A*c^6-2*B*a^6-6*B*a^4*c^2-6*B*a^2*
c^4-2*B*c^6-4*C*a^5*c+C*a^3*c^3)/a/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e
*x+1/2*d)^5-1/2*(6*A*a^6*c-27*A*a^4*c^3-12*A*a^2*c^5-4*A*c^7+4*B*a^6*c+12*B
*a^4*c^3+12*B*a^2*c^5+4*B*c^7-2*C*a^7+14*C*a^5*c^2-11*C*a^3*c^4-2*C*a*c^6)/
(a^6+3*a^4*c^2+3*a^2*c^4+c^6)/a^2*tanh(1/2*e*x+1/2*d)^4+1/3/a^3*(54*A*a^6*c
^2-21*A*a^4*c^4-4*A*a^2*c^6-4*A*c^8-6*B*a^8-14*B*a^6*c^2-6*B*a^4*c^4+6*B*a^
2*c^6+4*B*c^8-18*C*a^7*c+42*C*a^5*c^3-17*C*a^3*c^5-2*C*a*c^7)/(a^6+3*a^4*c^
2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^3+1/a^2*(6*A*a^6*c-20*A*a^4*c^3-3*A*a^
2*c^5-2*A*c^7+2*B*a^6*c+6*B*a^4*c^3+6*B*a^2*c^5+2*B*c^7-2*C*a^7+10*C*a^5*c^
2-14*C*a^3*c^4-C*a*c^6)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^2
-1/2/a*(27*A*a^4*c^2+4*A*a^2*c^4+2*A*c^6-2*B*a^6-6*B*a^4*c^2-6*B*a^2*c^4-2*
B*c^6-8*C*a^5*c+19*C*a^3*c^3+2*C*a*c^5)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(
1/2*e*x+1/2*d)-1/6*(18*A*a^4*c+5*A*a^2*c^3+2*A*c^5-6*C*a^5+10*C*a^3*c^2+C*a
*c^4)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*
e*x+1/2*d)-a)^3+(2*A*a^3-3*A*a*c^2+4*C*a^2*c-C*c^3)/(a^6+3*a^4*c^2+3*a^2*c^
4+c^6)/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^
(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.64937, size = 9709, normalized size = 38.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"fricas")
```

```
[Out] 1/6*(4*C*a^5*c^3 - 22*A*a^4*c^4 - 22*C*a^3*c^5 - 14*A*a^2*c^6 - 26*C*a*c^7
+ 8*A*c^8 + 6*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*
c^7 - C*c^8)*cosh(e*x + d)^5 + 6*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3
*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*sinh(e*x + d)^5 + 30*(2*A*a^6*c^2 + 4*C*a^5
*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*c^7)*cosh(e*x + d)^4 + 3
0*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*
c^7 + (2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*
c^8)*cosh(e*x + d))*sinh(e*x + d)^4 - 4*(4*(B + C)*a^8 - 22*A*a^7*c + 4*(4*
B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 + 29*A*a^3*c^5 + (16
*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(e*x + d)^3 - 4*(4*(B + C)*a
^8 - 22*A*a^7*c + 4*(4*B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c
^4 + 29*A*a^3*c^5 + (16*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8 - 15*(2*A*
a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*cosh(e
*x + d)^2 - 30*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a
^2*c^6 - C*a*c^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 12*(4*C*a^7*c - 17*A*a^6
*c^2 - 13*C*a^5*c^3 - 11*A*a^4*c^4 - 13*C*a^3*c^5 + 4*A*a^2*c^6 + 4*C*a*c^7
 - 2*A*c^8)*cosh(e*x + d)^2 + 12*(4*C*a^7*c - 17*A*a^6*c^2 - 13*C*a^5*c^3 -
```

```
11*A*a^4*c^4 - 13*C*a^3*c^5 + 4*A*a^2*c^6 + 4*C*a*c^7 - 2*A*c^8 + 5*(2*A*a
^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*cosh(e*
x + d)^3 + 15*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^
2*c^6 - C*a*c^7)*cosh(e*x + d)^2 - (4*(B + C)*a^8 - 22*A*a^7*c + 4*(4*B - 7
*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 + 29*A*a^3*c^5 + (16*B +
39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(e*x + d))*sinh(e*x + d)^2 + 3*(2
*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7 - (2*A*a^3*c^4 + 4*C*a^2*c^5 -
3*A*a*c^6 - C*c^7)*cosh(e*x + d)^6 - (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^
6 - C*c^7)*sinh(e*x + d)^6 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C
*a*c^6)*cosh(e*x + d)^5 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*
c^6 + (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(e*x + d))*sinh(e
*x + d)^5 - 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*
A*a*c^6 + C*c^7)*cosh(e*x + d)^4 - 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3
*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7 + 5*(2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A
*a*c^6 - C*c^7)*cosh(e*x + d)^2 + 10*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c
^5 - C*a*c^6)*cosh(e*x + d))*sinh(e*x + d)^4 - 4*(4*A*a^6*c + 8*C*a^5*c^2 -
12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C*a*c^6)*cosh(e*x + d)^3 - 4
*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C
*a*c^6 + 5*(2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(e*x + d)^3
+ 15*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*x + d)^2 +
3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*
c^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*
a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(e*x + d)^2 + 3*(8*A*a^5*c^2
+ 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7 - 5*(2*A*a
^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(e*x + d)^4 - 20*(2*A*a^4*c^3
+ 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*x + d)^3 - 6*(8*A*a^5*c^2 +
16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(e*x + d
)^2 - 4*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^
5 + 3*C*a*c^6)*cosh(e*x + d))*sinh(e*x + d)^2 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^
4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*x + d) - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3
*A*a^2*c^5 - C*a*c^6 + (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh
(e*x + d)^5 + 5*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*
x + d)^4 + 2*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A
*a*c^6 + C*c^7)*cosh(e*x + d)^3 + 2*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3
- 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C*a*c^6)*cosh(e*x + d)^2 - (8*A*a^5*c^2 +
16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(e*x +
d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh(e*x
+ d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*c)*si
nh(e*x + d) + 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a))/(c
*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e*x +
d) + a)*sinh(e*x + d) - c)) - 6*(4*C*a^6*c^2 - 20*A*a^5*c^3 - 18*C*a^4*c^4
- 15*A*a^3*c^5 - 23*C*a^2*c^6 + 5*A*a*c^7 - C*c^8)*cosh(e*x + d) - 6*(4*C*a
^6*c^2 - 20*A*a^5*c^3 - 18*C*a^4*c^4 - 15*A*a^3*c^5 - 23*C*a^2*c^6 + 5*A*a*
c^7 - C*c^8 - 5*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*
a*c^7 - C*c^8)*cosh(e*x + d)^4 - 20*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4
+ 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*c^7)*cosh(e*x + d)^3 + 2*(4*(B + C)*a^8 -
22*A*a^7*c + 4*(4*B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 +
29*A*a^3*c^5 + (16*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(e*x + d)
^2 - 4*(4*C*a^7*c - 17*A*a^6*c^2 - 13*C*a^5*c^3 - 11*A*a^4*c^4 - 13*C*a^3*c
^5 + 4*A*a^2*c^6 + 4*C*a*c^7 - 2*A*c^8)*cosh(e*x + d))*sinh(e*x + d))/((a^8
*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)^6 + (a^8*
c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*sinh(e*x + d)^6 + 6*(a^9
*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d)^5 + 3*(4
*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d)^4
+ 6*((a^8*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)
+ (a^9*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e)*sinh(e*x + d)^5
+ 4*(2*a^11*c + 5*a^9*c^3 - 10*a^5*c^7 - 10*a^3*c^9 - 3*a*c^11)*e*cosh(e*x
+ d)^3 + 3*(5*(a^8*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh
(e*x + d)^2 + 10*(a^9*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*c
```

```
osh(e*x + d) + (4*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e
)*sinh(e*x + d)^4 - 3*(4*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 -
c^12)*e*cosh(e*x + d)^2 + 4*(5*(a^8*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^1
0 + c^12)*e*cosh(e*x + d)^3 + 15*(a^9*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c
^9 + a*c^11)*e*cosh(e*x + d)^2 + 3*(4*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 +
10*a^4*c^8 - c^12)*e*cosh(e*x + d) + (2*a^11*c + 5*a^9*c^3 - 10*a^5*c^7 - 1
0*a^3*c^9 - 3*a*c^11)*e)*sinh(e*x + d)^3 + 6*(a^9*c^3 + 4*a^7*c^5 + 6*a^5*c
^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d) + 3*(5*(a^8*c^4 + 4*a^6*c^6 + 6*a^
4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)^4 + 20*(a^9*c^3 + 4*a^7*c^5 + 6*
a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d)^3 + 6*(4*a^10*c^2 + 15*a^8*c^
4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d)^2 + 4*(2*a^11*c + 5*a^9
*c^3 - 10*a^5*c^7 - 10*a^3*c^9 - 3*a*c^11)*e*cosh(e*x + d) - (4*a^10*c^2 +
15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e)*sinh(e*x + d)^2 - (a^8*c^4
+ 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e + 6*((a^8*c^4 + 4*a^6*c^6 +
6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)^5 + 5*(a^9*c^3 + 4*a^7*c^5 +
 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d)^4 + 2*(4*a^10*c^2 + 15*a^8
*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d)^3 + 2*(2*a^11*c + 5*
a^9*c^3 - 10*a^5*c^7 - 10*a^3*c^9 - 3*a*c^11)*e*cosh(e*x + d)^2 - (4*a^10*c
^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d) + (a^9*c^
3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e)*sinh(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**4,x)

[Out] Timed out
```

Giac [B] time = 1.20456, size = 988, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"giac")

[Out] 1/2*(2*A*a^3 + 4*C*a^2*c - 3*A*a*c^2 - C*c^3)*log(abs(2*c*e^(x*e + d) + 2*a
 - 2*sqrt(a^2 + c^2))/abs(2*c*e^(x*e + d) + 2*a + 2*sqrt(a^2 + c^2)))/((a^6
*e + 3*a^4*c^2*e + 3*a^2*c^4*e + c^6*e)*sqrt(a^2 + c^2)) + 1/3*(6*A*a^3*c^3
*e^(5*x*e + 5*d) + 12*C*a^2*c^4*e^(5*x*e + 5*d) - 9*A*a*c^5*e^(5*x*e + 5*d)
 - 3*C*c^6*e^(5*x*e + 5*d) + 30*A*a^4*c^2*e^(4*x*e + 4*d) + 60*C*a^3*c^3*e^
(4*x*e + 4*d) - 45*A*a^2*c^4*e^(4*x*e + 4*d) - 15*C*a*c^5*e^(4*x*e + 4*d) -
 8*B*a^6*e^(3*x*e + 3*d) - 8*C*a^6*e^(3*x*e + 3*d) + 44*A*a^5*c*e^(3*x*e +
3*d) - 24*B*a^4*c^2*e^(3*x*e + 3*d) + 64*C*a^4*c^2*e^(3*x*e + 3*d) - 82*A*a
^3*c^3*e^(3*x*e + 3*d) - 24*B*a^2*c^4*e^(3*x*e + 3*d) - 78*C*a^2*c^4*e^(3*x
*e + 3*d) + 24*A*a*c^5*e^(3*x*e + 3*d) - 8*B*c^6*e^(3*x*e + 3*d) + 24*C*a^5
*c*e^(2*x*e + 2*d) - 102*A*a^4*c^2*e^(2*x*e + 2*d) - 102*C*a^3*c^3*e^(2*x*e
 + 2*d) + 36*A*a^2*c^4*e^(2*x*e + 2*d) + 24*C*a*c^5*e^(2*x*e + 2*d) - 12*A*
c^6*e^(2*x*e + 2*d) - 12*C*a^4*c^2*e^(x*e + d) + 60*A*a^3*c^3*e^(x*e + d) +
 66*C*a^2*c^4*e^(x*e + d) - 15*A*a*c^5*e^(x*e + d) + 3*C*c^6*e^(x*e + d) +
```

```
2*C*a^3*c^3 - 11*A*a^2*c^4 - 13*C*a*c^5 + 4*A*c^6)/((a^6*c*e + 3*a^4*c^3*e
+ 3*a^2*c^5*e + c^7*e)*(c*e^(2*x*e + 2*d) + 2*a*e^(x*e + d) - c)^3)
```

3.257 $\int \frac{x^3}{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=439

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{3x\text{PolyLog}\left(3\right.}{4\sqrt{a}}$$

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^3*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sq
rt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x^2*PolyLog[2, -((b*E^(2*x))
/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x*PolyLo
g[3, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a -
 b]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(
4*Sqrt[a]*Sqrt[a - b]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[
a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a +
 2*Sqrt[a]*Sqrt[a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b])
```

Rubi [A] time = 0.730253, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5629, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{3x\text{PolyLog}\left(3\right.}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sinh[x]^2),x]
```

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^3*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sq
rt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x^2*PolyLog[2, -((b*E^(2*x))
/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x*PolyLo
g[3, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a -
 b]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(
4*Sqrt[a]*Sqrt[a - b]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[
a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a +
 2*Sqrt[a]*Sqrt[a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b])
```

Rule 5629

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx = 2\int \frac{x^3}{2a-b+b\cosh(2x)}\,dx$$
$$= 4\int \frac{e^{2x}x^3}{b+2(2a-b)e^{2x}+be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x^3}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x^3}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}$$
$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{3\int x^2\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{2\sqrt{a}\sqrt{a-b}} + \frac{3}{}$$
$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2\text{Li}_2\left(-\frac{}{2a+}\right)}{4\sqrt{a}\sqrt{a}}$$
$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2\text{Li}_2\left(-\frac{}{2a+}\right)}{4\sqrt{a}\sqrt{a}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2\text{Li}_2\left(-\frac{}{2a+}\right)}{4\sqrt{a}\sqrt{a}}$$
$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2\text{Li}_2\left(-\frac{}{2a+}\right)}{4\sqrt{a}\sqrt{a}}$$

Mathematica [A] time = 0.837932, size = 319, normalized size = 0.73

$$\frac{-6x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)+6x^2\text{PolyLog}\left(2,\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}-2a+b}\right)+6x\text{PolyLog}\left(3,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)-6x\text{PolyLog}\left(3,\frac{}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sinh[x]^2),x]
```

```
[Out] (-4*x^3*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)] + 4*x^3*Log[
1 - (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] - 6*x^2*PolyLog[2, -((b
*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))] + 6*x^2*PolyLog[2, (b*E^(2*x)
)/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] + 6*x*PolyLog[3, -((b*E^(2*x))/(2*a +
 2*Sqrt[a]*Sqrt[a - b] - b))] - 6*x*PolyLog[3, (b*E^(2*x))/(-2*a + 2*Sqrt[a
]*Sqrt[a - b] + b)] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a -
b] - b))] + 3*PolyLog[4, (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)])/(
8*Sqrt[a]*Sqrt[a - b])
```

Maple [B] time = 0.059, size = 919, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*sinh(x)^2),x)
```

```
[Out] 1/2/(a*(a-b))^(1/2)*x^3*ln(1-b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))-1/4/(a*(
a-b))^(1/2)*x^4+3/4/(a*(a-b))^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*(a*(a-b))^(
1/2)-2*a+b))-3/4/(a*(a-b))^(1/2)*x*polylog(3,b*exp(2*x)/(2*(a*(a-b))^(1/2)-
2*a+b))+3/8/(a*(a-b))^(1/2)*polylog(4,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))
+1/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*x
^3-1/2/(-2*(a*(a-b))^(1/2)-2*a+b)*x^4+1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)
-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a*x^3-1/2/(a*(a-b))^(1/
2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b
*x^3-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*x^4+1/4/(a*(a-b))^(1/
2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x^4+3/2/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(
2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*x^2+3/2/(a*(a-b))^(1/2)/(-2*(a*(a-
b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a*x^2-3/4
/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-
b))^(1/2)-2*a+b))*b*x^2-3/2/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(3,b*exp(2*x)
/(-2*(a*(a-b))^(1/2)-2*a+b))*x-3/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+
b)*polylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a*x+3/4/(a*(a-b))^(1/2)
/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b)
)*b*x+3/4/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(4,b*exp(2*x)/(-2*(a*(a-b))^(1/
2)-2*a+b))+3/4/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(4,b*exp(2
*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a-3/8/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2
*a+b)*polylog(4,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*sinh(x)^2 + a), x)
```

Fricas [C] time = 2.46599, size = 3956, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
- a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((2*a
 - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x^3*sq
rt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2
*a + b)/b) + b)/b) - b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) +
 (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 3*b*x^2*sqrt((a^2 - a*b)
/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sin
h(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)
 + b)/b + 1) + 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2
*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2
```

```
*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 3*b*x^2*sqrt((a^2 - a*
b)/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*s
inh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b
) + b)/b + 1) - 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (
2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2
*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) - 6*b*x*sqrt((a^2 - a*b)
/b^2)*polylog(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*
sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)
/b)/b) - 6*b*x*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a
- b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*
sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) + 6*b*x*sqrt((a^2 - a*b)/b^2)*polylo
g(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqr
t((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) + 6*b*
x*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
+ 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*
b)/b^2) - 2*a + b)/b)/b) + 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4, ((2*a - b)*
cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^
2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) + 6*b*sqrt((a^2 - a*b
)/b^2)*polylog(4, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a -
b)/b)/b) - 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4, ((2*a - b)*cosh(x) + (2*a -
 b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sq
rt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) - 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4,
 -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((
a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b))/(a^2 - a
*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*sinh(x)**2),x)
```

```
[Out] Integral(x**3/(a + b*sinh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sinh(x)^2 + a), x)
```

3.258 $\int \frac{x^2}{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=327

$$\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{\text{PolyLog}\left(3,-\frac{}{2\sqrt{a}}\right.}{4\sqrt{a}\sqrt{a-b}}$$

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^2*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (x*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a
 - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - (x*PolyLog[2, -((b*E^(2*x))/(2*a +
2*Sqrt[a]*Sqrt[a - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - PolyLog[3, -((b*E^(
2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b]) + PolyLog
[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b
])
```

Rubi [A] time = 0.538426, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5629, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{\text{PolyLog}\left(3,-\frac{}{2\sqrt{a}}\right.}{4\sqrt{a}\sqrt{a-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sinh[x]^2),x]
```

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^2*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (x*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a
 - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - (x*PolyLog[2, -((b*E^(2*x))/(2*a +
2*Sqrt[a]*Sqrt[a - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - PolyLog[3, -((b*E^(
2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b]) + PolyLog
[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b
])
```

Rule 5629

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx = 2\int \frac{x^2}{2a-b+b\cosh(2x)}\,dx$$
$$= 4\int \frac{e^{2x}x^2}{b+2(2a-b)e^{2x}+be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x^2}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x^2}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\int x\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{\sqrt{a}\sqrt{a-b}} + \cdots$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\text{Li}_2\left(-\frac{\cdots}{2a+2\cdots}\right)}{2\sqrt{a}\sqrt{a\cdots}}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\text{Li}_2\left(-\frac{\cdots}{2a+2\cdots}\right)}{2\sqrt{a}\sqrt{a\cdots}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\text{Li}_2\left(-\frac{\cdots}{2a+2\cdots}\right)}{2\sqrt{a}\sqrt{a\cdots}}$$

Mathematica [A] time = 0.693783, size = 240, normalized size = 0.73

$$\frac{-2x\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)+2x\text{PolyLog}\left(2,\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}-2a+b}\right)+\text{PolyLog}\left(3,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)-\text{PolyLog}\left(3,\frac{b\cdots}{2\sqrt{a}\sqrt{a}\cdots}\right)\cdots}{4\sqrt{a}\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sinh[x]^2),x]
```

```
[Out] (-2*x^2*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)] + 2*x^2*Log[
1 - (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] - 2*x*PolyLog[2, -((b*E
^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))] + 2*x*PolyLog[2, (b*E^(2*x))/(-
2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] + PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[
a]*Sqrt[a - b] - b))] - PolyLog[3, (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b
] + b)])/(4*Sqrt[a]*Sqrt[a - b])
```

Maple [B] time = 0.049, size = 710, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sinh(x)^2),x)
```

```
[Out] -2/3/(-2*(a*(a-b))^(1/2)-2*a+b)*x^3+1/(-2*(a*(a-b))^(1/2)-2*a+b)*x^2*ln(1-b
*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/(-2*(a*(a-b))^(1/2)-2*a+b)*x*polylo
g(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-1/2/(-2*(a*(a-b))^(1/2)-2*a+b)*p
olylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-2/3/(a*(a-b))^(1/2)/(-2*(a*
(a-b))^(1/2)-2*a+b)*a*x^3+1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*x^
2*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/(a*(a-b))^(1/2)/(-2*(a*(a-b
```

```
))^(1/2)-2*a+b)*a*x*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-1/2/(a
*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*polylog(3,b*exp(2*x)/(-2*(a*(a-b
))^(1/2)-2*a+b))+1/3/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x^3-1/2/(
a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x^2*ln(1-b*exp(2*x)/(-2*(a*(a-b
))^(1/2)-2*a+b))-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x*polylog
(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/4/(a*(a-b))^(1/2)/(-2*(a*(a-b))
^(1/2)-2*a+b)*b*polylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-1/3/(a*(a-
b))^(1/2)*x^3+1/2/(a*(a-b))^(1/2)*x^2*ln(1-b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*
a+b))+1/2/(a*(a-b))^(1/2)*x*polylog(2,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))
-1/4/(a*(a-b))^(1/2)*polylog(3,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\sinh(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*sinh(x)^2 + a), x)
```

Fricas [C] time = 2.37248, size = 2967, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
- a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((2*a
 - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x^2*sq
rt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2
*a + b)/b) + b)/b) - b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) +
 (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 2*b*x*sqrt((a^2 - a*b)/b
^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(
x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) +
 b)/b + 1) + 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a -
 b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*s
qrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 2*b*x*sqrt((a^2 - a*b)/b^2
)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x)
)*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) + b)
/b + 1) - 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)
*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt(
(a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) - 2*b*sqrt((a^2 - a*b)/b^2)*poly
log(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*s
qrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) - 2
*b*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 -
a*b)/b^2) + 2*a - b)/b)/b) + 2*b*sqrt((a^2 - a*b)/b^2)*polylog(3, ((2*a - b
```

```
)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/
b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) + 2*b*sqrt((a^2 - a*
b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) +
 b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
b)/b)/b))/(a^2 - a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sinh(x)**2),x)
```

```
[Out] Integral(x**2/(a + b*sinh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sinh(x)^2 + a), x)
```

3.259 $\int \frac{x}{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=215

$$\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}$$

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqrt[
a - b]) - (x*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqr
t[a]*Sqrt[a - b]) + PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] -
 b))]/(4*Sqrt[a]*Sqrt[a - b]) - PolyLog[2, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*S
qrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b])
```

Rubi [A] time = 0.329364, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5629, 3320, 2264, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sinh[x]^2),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqrt[
a - b]) - (x*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqr
t[a]*Sqrt[a - b]) + PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] -
 b))]/(4*Sqrt[a]*Sqrt[a - b]) - PolyLog[2, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*S
qrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b])
```

Rule 5629

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+b\sinh^2(x)}\,dx &= 2\int \frac{x}{2a-b+b\cosh(2x)}\,dx\\
&= 4\int \frac{e^{2x}x}{b+2(2a-b)e^{2x}+be^{4x}}\,dx\\
&= \frac{(2b)\int \frac{e^{2x}x}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}-\frac{(2b)\int \frac{e^{2x}x}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}\\
&= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{\int \log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{2\sqrt{a}\sqrt{a-b}}+\frac{\int \log
\end{aligned}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)}{x}dx,x,e^{2x}\right.}{4\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}}+\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-}}\right.}{4\sqrt{a}\sqrt{a-b}}$$

Mathematica [C] time = 0.771916, size = 576, normalized size = 2.68

$$-\frac{i\left(\text{PolyLog}\left(2,\frac{\left(2i\sqrt{a(b-a)}-2a+b\right)\left(\sqrt{a(b-a)}\tanh(x)+ia\right)}{b\sqrt{a(b-a)}\tanh(x)-iab}\right)-\text{PolyLog}\left(2,\frac{\left(-2i\sqrt{a(b-a)}-2a+b\right)\left(\sqrt{a(b-a)}\tanh(x)+ia\right)}{b\sqrt{a(b-a)}\tanh(x)-iab}\right)\right)-2i\cos^{-1}\left(1-\frac{2a}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sinh[x]^2),x]
```

```
[Out] -(4*x*ArcTan[(a*Coth[x])/Sqrt[-(a*(a - b))]] - (2*I)*ArcCos[1 - (2*a)/b]*Ar
cTan[(Sqrt[-a^2 + a*b]*Tanh[x])/a] + (ArcCos[1 - (2*a)/b] + 2*(ArcTan[(a*Co
th[x])/Sqrt[-(a*(a - b))]] + ArcTan[(Sqrt[-a^2 + a*b]*Tanh[x])/a]))*Log[(Sq
rt[2]*Sqrt[a*(-a + b)])/(Sqrt[b]*E^x*Sqrt[2*a - b + b*Cosh[2*x]])] + (ArcCo
s[1 - (2*a)/b] - 2*(ArcTan[(a*Coth[x])/Sqrt[-(a*(a - b))]] + ArcTan[(Sqrt[-
a^2 + a*b]*Tanh[x])/a]))*Log[(Sqrt[2]*Sqrt[a*(-a + b)]*E^x)/(Sqrt[b]*Sqrt[2
*a - b + b*Cosh[2*x]])] - (ArcCos[1 - (2*a)/b] + 2*ArcTan[(Sqrt[-a^2 + a*b]
*Tanh[x])/a])*Log[(2*a*((-I)*a + I*b + Sqrt[a*(-a + b)])*(-1 + Tanh[x]))/((
```

```
-I)*a*b + b*Sqrt[a*(-a + b)]*Tanh[x])] - (ArcCos[1 - (2*a)/b] - 2*ArcTan[(S
qrt[-a^2 + a*b]*Tanh[x])/a])*Log[(2*a*(I*a - I*b + Sqrt[a*(-a + b)])*(1 + T
anh[x]))/((-I)*a*b + b*Sqrt[a*(-a + b)]*Tanh[x])] + I*(-PolyLog[2, ((-2*a +
 b - (2*I)*Sqrt[a*(-a + b)])*(I*a + Sqrt[a*(-a + b)]*Tanh[x]))/((-I)*a*b +
b*Sqrt[a*(-a + b)]*Tanh[x])] + PolyLog[2, ((-2*a + b + (2*I)*Sqrt[a*(-a + b
)])*(I*a + Sqrt[a*(-a + b)]*Tanh[x]))/((-I)*a*b + b*Sqrt[a*(-a + b)]*Tanh[x
])]))/(4*Sqrt[a*(-a + b)])
```

Maple [B] time = 0.047, size = 505, normalized size = 2.4

$$x\ln\left(1-be^{2x}\left(-2\sqrt{a(a-b)}-2a+b\right)^{-1}\right)\left(-2\sqrt{a(a-b)}-2a+b\right)^{-1}+ax\ln\left(1-be^{2x}\left(-2\sqrt{a(a-b)}-2a+b\right)^{-1}\right)\frac{1}{\sqrt{a(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sinh(x)^2),x)
```

```
[Out] 1/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*x+
1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^
(1/2)-2*a+b))*a*x-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp
(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b*x-1/(-2*(a*(a-b))^(1/2)-2*a+b)*x^2-1/(a
*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*x^2+1/2/(a*(a-b))^(1/2)/(-2*(a*(
a-b))^(1/2)-2*a+b)*b*x^2+1/2/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x
)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b
)*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a-1/4/(a*(a-b))^(1/2)/(-
2*(a*(a-b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b
+1/2/(a*(a-b))^(1/2)*x*ln(1-b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))-1/2/(a*(a
-b))^(1/2)*x^2+1/4/(a*(a-b))^(1/2)*polylog(2,b*exp(2*x)/(2*(a*(a-b))^(1/2)-
2*a+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\sinh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*sinh(x)^2 + a), x)
```

Fricas [B] time = 2.25311, size = 1968, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 -
a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b
```

```
)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x*sqrt((a^
2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b
)/b) + b)/b) - b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) + (2*a -
b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqr
t((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + b*sqrt((a^2 - a*b)/b^2)*dilog(-(
((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^
2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) + b)/b + 1) +
 b*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/
b^2) + 2*a - b)/b) - b)/b + 1) - b*sqrt((a^2 - a*b)/b^2)*dilog(-(((2*a - b)
*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b
^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) + b)/b + 1) - b*sqrt((a^
2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x)
+ b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
 b)/b) - b)/b + 1))/(a^2 - a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + b\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)**2),x)
```

```
[Out] Integral(x/(a + b*sinh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*sinh(x)^2 + a), x)
```

3.260 $\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x} dx$

Optimal. Leaf size=47

$$-\frac{9}{4}\cosh(a)\mathrm{Chi}(bx)+\frac{1}{4}\cosh(3a)\mathrm{Chi}(3bx)-\frac{9}{4}\sinh(a)\mathrm{Shi}(bx)+\frac{1}{4}\sinh(3a)\mathrm{Shi}(3bx)$$

```
[Out] (-9*Cosh[a]*CoshIntegral[b*x])/4 + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (9*S
inh[a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rubi [A] time = 0.455922, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6742, 3303, 3298, 3301, 5448}

$$-\frac{9}{4}\cosh(a)\mathrm{Chi}(bx)+\frac{1}{4}\cosh(3a)\mathrm{Chi}(3bx)-\frac{9}{4}\sinh(a)\mathrm{Shi}(bx)+\frac{1}{4}\sinh(3a)\mathrm{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
```

```
[Out] (-9*Cosh[a]*CoshIntegral[b*x])/4 + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (9*S
inh[a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \int \left(-\frac{2\cosh(a+bx)}{x} + \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\right) dx$$
$$= -\left(2\int \frac{\cosh(a+bx)}{x}\,dx\right) + \int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx$$
$$= -\left((2\cosh(a))\int \frac{\cosh(bx)}{x}\,dx\right) - (2\sinh(a))\int \frac{\sinh(bx)}{x}\,dx + \int \left(-\frac{\cosh}{}\right.$$
$$= -2\cosh(a)\mathbf{Chi}(bx) - 2\sinh(a)\mathbf{Shi}(bx) - \frac{1}{4}\int \frac{\cosh(a+bx)}{x}\,dx + \frac{1}{4}\int \frac{\cos}{}$$
$$= -2\cosh(a)\mathbf{Chi}(bx) - 2\sinh(a)\mathbf{Shi}(bx) - \frac{1}{4}\cosh(a)\int \frac{\cosh(bx)}{x}\,dx + \frac{1}{4}\,\mathrm{c}$$
$$= -\frac{9}{4}\cosh(a)\mathbf{Chi}(bx) + \frac{1}{4}\cosh(3a)\mathbf{Chi}(3bx) - \frac{9}{4}\sinh(a)\mathbf{Shi}(bx) + \frac{1}{4}\sinh($$

Mathematica [A] time = 0.103917, size = 41, normalized size = 0.87

$$\frac{1}{4}(-9\cosh(a)\mathbf{Chi}(bx) + \cosh(3a)\mathbf{Chi}(3bx) - 9\sinh(a)\mathbf{Shi}(bx) + \sinh(3a)\mathbf{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
```

```
[Out] (-9*Cosh[a]*CoshIntegral[b*x] + Cosh[3*a]*CoshIntegral[3*b*x] - 9*Sinh[a]*S
inhIntegral[b*x] + Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Maple [A] time = 0.103, size = 47, normalized size = 1.

$$-\frac{e^{-3a}\mathrm{Ei}(1,3bx)}{8} + \frac{9e^{-a}\mathrm{Ei}(1,bx)}{8} + \frac{9e^{a}\mathrm{Ei}(1,-bx)}{8} - \frac{e^{3a}\mathrm{Ei}(1,-3bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x)
```

```
[Out] -1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)+9/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

Maxima [A] time = 1.30831, size = 57, normalized size = 1.21

$$\frac{1}{8}\,\mathrm{Ei}(3bx)e^{(3a)} - \frac{9}{8}\,\mathrm{Ei}(-bx)e^{(-a)} + \frac{1}{8}\,\mathrm{Ei}(-3bx)e^{(-3a)} - \frac{9}{8}\,\mathrm{Ei}(bx)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 9/8
*Ei(b*x)*e^a
```

Fricas [A] time = 2.0089, size = 204, normalized size = 4.34

$$\frac{1}{8}\left(\mathrm{Ei}\left(3\,bx\right)+\mathrm{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)-\frac{9}{8}\left(\mathrm{Ei}\left(bx\right)+\mathrm{Ei}\left(-bx\right)\right)\cosh\left(a\right)+\frac{1}{8}\left(\mathrm{Ei}\left(3\,bx\right)-\mathrm{Ei}\left(-3\,bx\right)\right)\sinh\left(3\,a\right)-\frac{9}{8}\left(\mathrm{Ei}\left(bx\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9/8*(Ei(b*x) + Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 9/8*(Ei(b*x) - Ei(-b*x))*sinh(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sinh^{2}\left(a+bx\right)-2\right)\cosh\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)**2)/x,x)
```

```
[Out] Integral((sinh(a + b*x)**2 - 2)*cosh(a + b*x)/x, x)
```

Giac [A] time = 1.12544, size = 57, normalized size = 1.21

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{\left(3\,a\right)}-\frac{9}{8}\,\mathrm{Ei}\left(-bx\right)e^{\left(-a\right)}+\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{\left(-3\,a\right)}-\frac{9}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 9/8
*Ei(b*x)*e^a
```

3.261 $\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$\frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

```
[Out] (3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - SinhIntegral[(3*Sqrt[
1 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rubi [A] time = 0.111941, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3298}

$$\frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - SinhIntegral[(3*Sqrt[
1 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{i\operatorname{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x} - \frac{i\sinh(3x)}{4x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} + \frac{3\operatorname{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$
$$= \frac{3\operatorname{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\operatorname{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

Mathematica [A] time = 0.0787227, size = 55, normalized size = 0.95

$$\frac{3\operatorname{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) - \operatorname{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - SinhIntegral[(3*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/(4*a)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\sinh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] -integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.262 $\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] -CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) + Log[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]/(2*a)
```

Rubi [A] time = 0.0807629, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3301}

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) + Log[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]/(2*a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$
$$= -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} + \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Mathematica [A] time = 0.0354142, size = 57, normalized size = 0.98

$$-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} + \frac{\log(1-ax)}{4a} - \frac{\log(ax+1)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) + Log[1 - a*x]/(4*a) -
 Log[1 + a*x]/(4*a)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\sinh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

```
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log(ax+1)}{4a} + \frac{\log(ax-1)}{4a} - \frac{1}{4}\int \frac{e^{\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}}{a^2x^2-1}\,dx - \frac{1}{4}\int \frac{e^{\left(-\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] -1/4*log(a*x + 1)/a + 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x + 1
)/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/sqr
t(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

3.263 $\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\mathrm{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] -(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi [A] time = 0.0384661, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6681, 3298}

$$-\frac{\mathrm{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\ &= -\frac{\mathrm{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0306777, size = 26, normalized size = 1.

$$-\frac{\mathrm{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\sinh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] -integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.264 $\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=39

$$\text{Unintegrable}\left(\frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/((1 - a*x)*(1 + a*x)), x]
```

Rubi [A] time = 0.037463, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Csch[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\text{Subst}\left(\int \frac{\operatorname{csch}(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 11.4382, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Maple [A] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\sinh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(a^2x^2-1)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="m
axima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{(a^2x^2-1)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="f
ricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(a^2x^2-1)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="g
iac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.265 $\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=41

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/((1 - a*x)*(1 + a*x)), x]
```

Rubi [A] time = 0.0799666, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Csch[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/
a)
```

Rubi steps

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\operatorname{csch}^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 36.221, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Maple [A] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\left(\sinh\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{ax+1}}{\sqrt{-ax+1}ae^{\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}-\sqrt{-ax+1}a}-\int\frac{\sqrt{ax+1}}{\left(a^2x^2-1\right)\sqrt{-ax+1}e^{\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}+\left(a^2x^2-1\right)\sqrt{-ax+1}}\,dx+\int\frac{}{\left(a^2x^2-1\right)\sqrt{-ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"maxima")
```

```
[Out] 2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - sqrt
(-a*x + 1)*a) - integrate(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(sq
rt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x) + integrate
(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(sqrt(-a*x + 1)/sqrt(a*x + 1
)) - (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"fricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int-\frac{1}{\left(a^2x^2-1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

3.266 $\int \sinh(a + b\log(cx^n))\,dx$

Optimal. Leaf size=54

$$\frac{x\sinh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\cosh(a+b\log(cx^n))}{1-b^2n^2}$$

```
[Out] -((b*n*x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2)) + (x*Sinh[a + b*Log[c*x^n]]
)/(1 - b^2*n^2)
```

Rubi [A] time = 0.0112868, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5517}

$$\frac{x\sinh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\cosh(a+b\log(cx^n))}{1-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2)) + (x*Sinh[a + b*Log[c*x^n]]
)/(1 - b^2*n^2)
```

Rule 5517

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Simp[(
x*Sinh[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] + Simp[(b*d*n*x*Cosh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] /; FreeQ[{a, b, c, d, n}, x] &&
NeQ[b^2*d^2*n^2 - 1, 0]
```

Rubi steps

$$\int \sinh(a+b\log(cx^n))\,dx=-\frac{bnx\cosh(a+b\log(cx^n))}{1-b^2n^2}+\frac{x\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

Mathematica [A] time = 0.0586636, size = 41, normalized size = 0.76

$$\frac{x(bn\cosh(a+b\log(cx^n))-\sinh(a+b\log(cx^n)))}{b^2n^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] (x*(b*n*Cosh[a + b*Log[c*x^n]] - Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \sinh(a+b\ln(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n)),x)
```

```
[Out] int(sinh(a+b*ln(c*x^n)),x)
```

Maxima [A] time = 1.11891, size = 70, normalized size = 1.3

$$\frac{c^b x e^{(b \log(x^n)+a)}}{2\,(bn+1)} + \frac{x e^{(-b \log(x^n)-a)}}{2\left(bc^b n - c^b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) + 1/2*x*e^(-b*log(x^n) - a)/(b*c^b*n
 - c^b)
```

Fricas [A] time = 2.04283, size = 123, normalized size = 2.28

$$\frac{bnx \cosh\left(bn \log\left(x\right) + b \log\left(c\right) + a\right) - x \sinh\left(bn \log\left(x\right) + b \log\left(c\right) + a\right)}{b^2 n^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*cosh(b*n*log(x) + b*log(c) + a) - x*sinh(b*n*log(x) + b*log(c) + a))
/(b^2*n^2 - 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.13202, size = 63, normalized size = 1.17

$$\frac{c^b x x^{bn} e^a}{2\,(bn+1)} + \frac{x e^{(-a)}}{2\,(bn-1) c^b x^{bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) + 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

3.267 $\int \sinh^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=88

$$\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{2b^2n^2x}{1-4b^2n^2}$$

```
[Out] (2*b^2*n^2*x)/(1 - 4*b^2*n^2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*
Log[c*x^n]])/(1 - 4*b^2*n^2) + (x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^2)
```

Rubi [A] time = 0.0206036, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5519, 8}

$$\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{2b^2n^2x}{1-4b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x)/(1 - 4*b^2*n^2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*
Log[c*x^n]])/(1 - 4*b^2*n^2) + (x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^2)
```

Rule 5519

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Sinh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (-Dist[(b^
2*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Sinh[d*(a + b*Log[c*x^n])]^
(p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a + b*
Log[c*x^n])]^(p - 1))/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sinh^2(a+b\log(cx^n))\,dx=-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}+\frac{(2b^2n}{1-}$$

$$=\frac{2b^2n^2x}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{x\sinh^2(a+b\log(cx}{1-4b^2n^2}$$

Mathematica [A] time = 0.100934, size = 55, normalized size = 0.62

$$-\frac{x\left(-2bn\sinh(2(a+b\log(cx^n)))+\cosh(2(a+b\log(cx^n)))+4b^2n^2-1\right)}{8b^2n^2-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] -((x*(-1 + 4*b^2*n^2 + Cosh[2*(a + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a + b*Log
[c*x^n])]))/(-2 + 8*b^2*n^2))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int (\sinh (a + b \ln (cx^n)))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^2,x)
```

Maxima [A] time = 1.16068, size = 90, normalized size = 1.02

$$\frac{c^{2b}xe^{(2b\log(x^n)+2a)}}{4(2bn+1)} - \frac{1}{2}x - \frac{xe^{(-2a)}}{4\left(2bc^{2b}n - c^{2b}\right)(x^n)^{2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) - 1/2*x - 1/4*x*e^(-2*a)/(
(2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

Fricas [A] time = 2.02648, size = 258, normalized size = 2.93

$$\frac{4bnx\cosh(bn\log(x) + b\log(c) + a)\sinh(bn\log(x) + b\log(c) + a) - x\cosh(bn\log(x) + b\log(c) + a)^2 - x\sinh(bn\log(x) + b\log(c) + a)^2 - (4b^2n^2-1)x}{2\left(4b^2n^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) + a)
^2 - (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.20731, size = 228, normalized size = 2.59

$$\frac{bc^{2\,b}nxx^{2\,bn}e^{(2\,a)}}{2\left(4\,b^2n^2-1\right)}-\frac{2\,b^2n^2x}{4\,b^2n^2-1}-\frac{c^{2\,b}xx^{2\,bn}e^{(2\,a)}}{4\left(4\,b^2n^2-1\right)}-\frac{bnxe^{(-2\,a)}}{2\left(4\,b^2n^2-1\right)c^{2\,b}x^{2\,bn}}+\frac{x}{2\left(4\,b^2n^2-1\right)}-\frac{xe^{(-2\,a)}}{4\left(4\,b^2n^2-1\right)c^{2\,b}x^{2\,bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 2*b^2*n^2*x/(4*b^2*n^
2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-2*
a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

3.268 $\int \sinh^3(a+b\log(cx^n))\,dx$

Optimal. Leaf size=149

$$\frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2}+\frac{6b^2n^2x\sinh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}-\frac{6b^3n^3x\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}-\frac{3bnx\sinh^2(a+b\log(cx^n))\,\mathrm{co}}{1-9b^2n^2}$$

```
[Out] (-6*b^3*n^3*x*Cosh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) + (6*b^2
*n^2*x*Sinh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/(1 - 9*b^2*n^2) + (x*Sinh[a +
b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2)
```

Rubi [A] time = 0.0405997, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5519, 5517}

$$\frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2}+\frac{6b^2n^2x\sinh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}-\frac{6b^3n^3x\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}-\frac{3bnx\sinh^2(a+b\log(cx^n))\,\mathrm{co}}{1-9b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x*Cosh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) + (6*b^2
*n^2*x*Sinh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/(1 - 9*b^2*n^2) + (x*Sinh[a +
b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2)
```

Rule 5519

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Sinh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (-Dist[(b^
2*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Sinh[d*(a + b*Log[c*x^n])]^
(p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a + b*
Log[c*x^n])]^(p - 1))/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rule 5517

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Simp[(
x*Sinh[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] + Simp[(b*d*n*x*Cosh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] /; FreeQ[{a, b, c, d, n}, x] &&
NeQ[b^2*d^2*n^2 - 1, 0]
```

Rubi steps

$$\int \sinh^3(a+b\log(cx^n))\,dx=-\frac{3bnx\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{1-9b^2n^2}+\frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2}+\frac{(6b^2n^2)}{}$$

$$=-\frac{6b^3n^3x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4}+\frac{6b^2n^2x\sinh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4}-\frac{3bnx\cosh(a+b\log(cx}{1-}$$

Mathematica [A] time = 0.502005, size = 120, normalized size = 0.81

$$\frac{x\left(-3bn\left(9b^2n^2-1\right)\cosh(a+b\log(cx^n))+3bn\left(b^2n^2-1\right)\cosh(3(a+b\log(cx^n)))-2\sinh(a+b\log(cx^n))\left(\left(b^2n^2-1\right)\right.}{36b^4n^4-40b^2n^2+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*(-3*b*n*(-1 + 9*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + 3*b*n*(-1 + b^2*n^2)*C
osh[3*(a + b*Log[c*x^n])] - 2*(1 - 13*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a +
b*Log[c*x^n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int (\sinh (a+b\ln (cx^n)))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^3,x)
```

Maxima [A] time = 1.21326, size = 154, normalized size = 1.03

$$\frac{c^{3\,b}xe^{(3\,b\log(x^n)+3\,a)}}{8\,(3\,bn+1)}-\frac{3\,c^{b}xe^{(b\log(x^n)+a)}}{8\,(bn+1)}-\frac{3\,xe^{(-b\log(x^n)-a)}}{8\left(bc^{b}n-c^{b}\right)}+\frac{xe^{(-3\,b\log(x^n)-3\,a)}}{8\left(3\,bc^{3\,b}n-c^{3\,b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) - 3/8*c^b*x*e^(b*log(x^n)
+ a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) + 1/8*x*e^(-3*b*
log(x^n) - 3*a)/(3*b*c^(3*b)*n - c^(3*b))
```

Fricas [A] time = 2.14739, size = 532, normalized size = 3.57

$$3\left(b^3n^3-bn\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+9\left(b^3n^3-bn\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 9*(b^3*n^3 - b
*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - (
b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^3 - 3*(9*b^3*n^3 - b*n)*x*co
sh(b*n*log(x) + b*log(c) + a) - 3*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^2 - (9*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4
- 10*b^2*n^2 + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.27506, size = 898, normalized size = 6.03

$$\frac{3\,b^3c^{3\,b}n^3xx^{3\,bn}e^{(3\,a)}}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}-\frac{27\,b^3c^bn^3xx^{bn}e^a}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}-\frac{b^2c^{3\,b}n^2xx^{3\,bn}e^{(3\,a)}}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}+\frac{27\,b^2c^bn^2xx^{bn}e^a}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}-\frac{3\,bc^{3\,b}nxx}{8\left(9\,b^4n^4-10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8
*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)*n
^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/8*b^2*c^b*n^2*x*x^
(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a
)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) + 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 +
1)*c^(3*b)*x^(3*b*n)) + 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 +
 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b
^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) + 1/8*b^2*n^2*x*
e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) - 3/8*c^b*x*x^(b*
n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) - 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c
^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n))
 - 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n))
```

3.269 $\int \sinh^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=191

$$\frac{12b^2n^2x\sinh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-16b^2n^2}-\frac{24b^3\cdots}{\cdots}$$

```
[Out] (24*b^4*n^4*x)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x*Cosh[a + b*Log
[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^
2*x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(1 - 16*b^2*n^2) + (x*Sinh[a +
 b*Log[c*x^n]]^4)/(1 - 16*b^2*n^2)
```

Rubi [A] time = 0.0512167, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5519, 8}

$$\frac{12b^2n^2x\sinh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-16b^2n^2}-\frac{24b^3\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x*Cosh[a + b*Log
[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^
2*x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(1 - 16*b^2*n^2) + (x*Sinh[a +
 b*Log[c*x^n]]^4)/(1 - 16*b^2*n^2)
```

Rule 5519

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Sinh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (-Dist[(b^
2*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Sinh[d*(a + b*Log[c*x^n])]^
(p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a + b*
Log[c*x^n])]^(p - 1))/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^4(a+b\log(cx^n))\,dx &= -\frac{4bnx\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{1-16b^2n^2}+\frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}+\frac{(12b^2\cdots}{\cdots}\\ &= -\frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}+\frac{12b^2n^2x\sinh^2(a+b\log(cx^n\cdots}{1-20b^2n^2+64b^4n^4}\\ &= \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4}-\frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}+\frac{12b^2n^2\cdots}{1-\cdots}\end{aligned}$$

Mathematica [A] time = 0.41787, size = 167, normalized size = 0.87

$$\frac{x\left(-128b^3n^3\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+16b^3n^3\sinh\left(4\left(a+b\log\left(cx^n\right)\right)\right)+\left(64b^2n^2-4\right)\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+\left(1-4\right.\right.}{8\left(64b^4n^4-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (-4 + 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x^
n])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] + 8*b*n*Sinh[2*(a + b*Log
[c*x^n])] - 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b*Lo
g[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2 + 6
4*b^4*n^4))
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int\left(\sinh\left(a+b\ln\left(cx^n\right)\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^4,x)
```

Maxima [A] time = 1.25793, size = 174, normalized size = 0.91

$$\frac{c^{4\,b}xe^{\left(4\,b\log\left(x^n\right)+4\,a\right)}}{16\,(4\,bn+1)}-\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+2\,a\right)}}{4\,(2\,bn+1)}+\frac{3}{8}\,x+\frac{xe^{\left(-2\,b\log\left(x^n\right)-2\,a\right)}}{4\left(2\,bc^{2\,b}n-c^{2\,b}\right)}-\frac{xe^{(-4\,a)}}{16\left(4\,bc^{4\,b}n-c^{4\,b}\right)\left(x^n\right)^{4\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) - 1/4*c^(2*b)*x*e^(2*b*lo
g(x^n) + 2*a)/(2*b*n + 1) + 3/8*x + 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^(2
*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b))
```

Fricas [A] time = 2.13351, size = 801, normalized size = 4.19

$$-\frac{\left(4\,b^2n^2-1\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4-16\left(4\,b^3n^3-bn\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\,\mathrm{l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] -1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3 -
 b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 +
 (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 - 4*(16*b^2*n^2 - 1)*x
```

```
*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*log(x)
 + b*log(c) + a)^2 - 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a)^
2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^3 - (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) +
a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2795, size = 1049, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 8*b^3*c
^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4*x
/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)/(6
4*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64*b^4
*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 -
20*b^2*n^2 + 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*
n^2 + 1) + 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2
*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n
)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x*x^(4*b*n
)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(
64*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 +
 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*
c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*x*e^(-2*a)/(
(64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e^(-4*a)/((64*b^4
*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

3.270 $\int x^m \sinh(a + b\log(cx^n))\,dx$

Optimal. Leaf size=73

$$\frac{(m+1)x^{m+1}\sinh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}-\frac{bnx^{m+1}\cosh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}$$

```
[Out] -((b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)) + ((1 + m)*
x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)
```

Rubi [A] time = 0.0237147, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5527}

$$\frac{(m+1)x^{m+1}\sinh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}-\frac{bnx^{m+1}\cosh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)) + ((1 + m)*
x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)
```

Rule 5527

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])])/(b^2*d
^2*e*n^2 - e*(m + 1)^2), x] + Simp[(b*d*n*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c
*x^n])])/(b^2*d^2*e*n^2 - e*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n},
x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rubi steps

$$\int x^m\sinh(a+b\log(cx^n))\,dx=-\frac{bnx^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}+\frac{(1+m)x^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

Mathematica [A] time = 0.132853, size = 54, normalized size = 0.74

$$\frac{x^{m+1}\left((m+1)\sinh(a+b\log(cx^n))-bn\cosh(a+b\log(cx^n))\right)}{(-bn+m+1)(bn+m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] (x^(1 + m)*(-(b*n*Cosh[a + b*Log[c*x^n]]) + (1 + m)*Sinh[a + b*Log[c*x^n]])
)/((1 + m - b*n)*(1 + m + b*n))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int x^m\sinh(a+b\ln(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n)),x)
```

Maxima [A] time = 1.11783, size = 86, normalized size = 1.18

$$\frac{c^b x e^{(b\log(x^n)+m\log(x)+a)}}{2\,(bn+m+1)} + \frac{x e^{(-b\log(x^n)+m\log(x)-a)}}{2\left(bc^b n - c^b(m+1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) + 1/2*x*e^(-b*log(x^n
) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

Fricas [A] time = 2.08741, size = 304, normalized size = 4.16

$$\frac{bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\cosh\left(m\log\left(x\right)\right)+bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(m\log\left(x\right)\right)-((m+1)}{b^2n^2-m^2-2\,m-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + b*n*x*cosh(b*n*log(
x) + b*log(c) + a)*sinh(m*log(x)) - ((m + 1)*x*cosh(m*log(x)) + (m + 1)*x*s
inh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m - 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.19303, size = 317, normalized size = 4.34

$$\frac{bc^b nxx^{bn}x^m e^a}{2\left(b^2n^2-m^2-2\,m-1\right)} - \frac{c^b mxx^{bn}x^m e^a}{2\left(b^2n^2-m^2-2\,m-1\right)} - \frac{c^b xx^{bn}x^m e^a}{2\left(b^2n^2-m^2-2\,m-1\right)} + \frac{bnxx^m e^{(-a)}}{2\left(b^2n^2-m^2-2\,m-1\right)c^b x^{bn}} + \frac{}{2\,(b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b*
n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2 -
 m^2 - 2*m - 1) + 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*
n)) + 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) + 1/2*x*x^
m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

3.271 $\int x^m \sinh^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=120

$$\frac{(m+1)x^{m+1}\sinh^2(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}+\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

```
[Out] (2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) - (2*b*n*x^(1 + m)*
Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2) + ((
1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 4*b^2*n^2)
```

Rubi [A] time = 0.0480274, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5529, 30}

$$\frac{(m+1)x^{m+1}\sinh^2(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}+\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) - (2*b*n*x^(1 + m)*
Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2) + ((
1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 4*b^2*n^2)
```

Rule 5529

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (-Dist[(b^2*d^2*n^2*p*(p - 1))/(b^
2*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2),
 x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a
+ b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{
a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2,
0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m\sinh^2(a+b\log(cx^n))\,dx &= -\frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-4b^2n^2}+\frac{(1+m)x^{1+m}\sinh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2}\\ &= \frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)}-\frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-4b^2n^2}\end{aligned}$$

Mathematica [A] time = 0.280334, size = 89, normalized size = 0.74

$$\frac{x^{m+1}\left(-2b(m+1)n\sinh(2(a+b\log(cx^n)))+(m+1)^2\cosh(2(a+b\log(cx^n)))+4b^2n^2-m^2-2m-1\right)}{2(m+1)(-2bn+m+1)(2bn+m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^(1 + m)*(-1 - 2*m - m^2 + 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n]
)] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n)*
(1 + m + 2*b*n))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x^{m} \left(\sinh \left(a + b\ln \left(cx^{n}\right)\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18709, size = 726, normalized size = 6.05

$$\left(m^{2} + 2\,m + 1\right)x \cosh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)^{2} \cosh \left(m \log \left(x\right)\right) + \left(4\,b^{2}n^{2} - m^{2} - 2\,m - 1\right)x \cosh \left(m \log \left(x\right)\right) + \left(\left(m^{2} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) + (
4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*log
(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2
- 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*m +
b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x) + b*
log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + (4*b^2
*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2 + 3*m^2
 + 3*m + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.27568, size = 1023, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 -
3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4
*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)
/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 2*b^2*n^2*x*x^m/(4*b^2
*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n)*x^m
*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*x*
x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1
/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*b*m*n*
x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x
^(2*b*n)) + m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4
*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2
*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3
*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m
^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^
3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)/((4*b^2*m*n^2
+ 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))
```

3.272 $\int x^m \sinh^3(a + b\log(cx^n))\,dx$

Optimal. Leaf size=203

$$\frac{(m+1)x^{m+1}\sinh^3(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}+\frac{6b^2(m+1)n^2x^{m+1}\sinh(a+b\log(cx^n))}{\left((m+1)^2-9b^2n^2\right)\left((m+1)^2-b^2n^2\right)}-\frac{6b^3n^3x^{m+1}\cosh(a+b\log(cx^n))}{\left((m+1)^2-9b^2n^2\right)\left((m+1)^2-b^2n^2\right)}-$$

```
[Out] (-6*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/(((1 + m)^2 - 9*b^2*n^2)*((1
+ m)^2 - b^2*n^2)) + (6*b^2*(1 + m)*n^2*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/(
((1 + m)^2 - 9*b^2*n^2)*((1 + m)^2 - b^2*n^2)) - (3*b*n*x^(1 + m)*Cosh[a +
b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 9*b^2*n^2) + ((1 + m)*
x^(1 + m)*Sinh[a + b*Log[c*x^n]]^3)/((1 + m)^2 - 9*b^2*n^2)
```

Rubi [A] time = 0.0862306, antiderivative size = 197, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5529, 5527}

$$\frac{(m+1)x^{m+1}\sinh^3(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}+\frac{6b^2(m+1)n^2x^{m+1}\sinh(a+b\log(cx^n))}{-10b^2(m+1)^2n^2+9b^4n^4+(m+1)^4}-\frac{6b^3n^3x^{m+1}\cosh(a+b\log(cx^n))}{-10b^2(m+1)^2n^2+9b^4n^4+(m+1)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^4 - 10*b^2*(1 + m)^2
*n^2 + 9*b^4*n^4) + (6*b^2*(1 + m)*n^2*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((
1 + m)^4 - 10*b^2*(1 + m)^2*n^2 + 9*b^4*n^4) - (3*b*n*x^(1 + m)*Cosh[a + b*
Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 9*b^2*n^2) + ((1 + m)*x^
(1 + m)*Sinh[a + b*Log[c*x^n]]^3)/((1 + m)^2 - 9*b^2*n^2)
```

Rule 5529

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (-Dist[(b^2*d^2*n^2*p*(p - 1))/(b^
2*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2),
 x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a
+ b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{
a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2,
0]
```

Rule 5527

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])])/(b^2*d
^2*e*n^2 - e*(m + 1)^2), x] + Simp[(b*d*n*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c
*x^n])])/(b^2*d^2*e*n^2 - e*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n},
x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rubi steps

$$\int x^m\sinh^3(a+b\log(cx^n))\,dx=-\frac{3bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{(1+m)^2-9b^2n^2}+\frac{(1+m)x^{1+m}\sinh^3(a+b\log}{(1+m)^2-9b^2n^2}$$

$$=-\frac{6b^3n^3x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^4-10b^2(1+m)^2n^2+9b^4n^4}+\frac{6b^2(1+m)n^2x^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^4-10b^2(1+m)^2n^2+9b^4n^4}-3bn$$

Mathematica [A] time = 1.32658, size = 292, normalized size = 1.44

$$\frac{1}{4}x^{m+1}\left(-\frac{3\cosh(bn\log(x))\left((m+1)\sinh\left(a+b\log\left(cx^n\right)-bn\log(x)\right)-bn\cosh\left(a+b\log\left(cx^n\right)-bn\log(x)\right)\right)}{(-bn+m+1)(bn+m+1)}-\frac{3\sinh$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^(1 + m)*((-3*Cosh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]]
) + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b
*n)) - (3*Sinh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]] - b
*n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n)) + (C
osh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)
*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*b*n)
) + (Sinh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] -
3*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3
*b*n))))/4
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int x^{m}\left(\sinh\left(a+b\ln\left(cx^{n}\right)\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n))^3,x)
```

Maxima [A] time = 1.23049, size = 186, normalized size = 0.92

$$\frac{c^{3\,b}xe^{\left(3\,b\log\left(x^{n}\right)+m\log(x)+3\,a\right)}}{8\,(3\,bn+m+1)}-\frac{3\,c^{b}xe^{\left(b\log\left(x^{n}\right)+m\log(x)+a\right)}}{8\,(bn+m+1)}-\frac{3\,xe^{\left(-b\log\left(x^{n}\right)+m\log(x)-a\right)}}{8\left(bc^{b}n-c^{b}(m+1)\right)}+\frac{xe^{\left(-3\,b\log\left(x^{n}\right)+m\log(x)-3\,a\right)}}{8\left(3\,bc^{3\,b}n-c^{3\,b}(m+1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) - 3/8*c^b*x
*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*log
(x) - a)/(b*c^b*n - c^b*(m + 1)) + 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3*a)
/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

Fricas [B] time = 2.16783, size = 1542, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^
3*cosh(m*log(x)) - 3*(9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x)
+ b*log(c) + a)*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m +
1)*x*cosh(m*log(x)) + (m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*sinh(m*
log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 9*((b^3*n^3 - (b*m^2 + 2*b*m +
 b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b^3*n^3 - (b*m^2
 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n
*log(x) + b*log(c) + a)^2 + 3*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*
x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) - (m^3 - 9*(b^2*m + b^2)
*n^2 + 3*m^2 + 3*m + 1)*x*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*n^2 + 3*m^
2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 - (m^3 - 9*(b^2*m + b^2)*n
^2 + 3*m^2 + 3*m + 1)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) +
3*((b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 -
(9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a))*sinh
(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6
*m^2 + 4*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44072, size = 4354, normalized size = 21.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^3*c^b*n^
3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x^m*e
^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^
3 + 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10
*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) -
 3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 -
20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b
)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 +
m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*c^b*m^2*n*x*x^(b*n)*x^m
*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2
*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1/8
*c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m
*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^(3*b)*m*n*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*c^b*m^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^
4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m
```

```
+ 1) + 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2
*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m^2*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*x^m*e^(3*a)
/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*
m^2 + 4*m + 1) - 27/8*b^3*n^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/
8*b^3*n^3*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
- 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 9/8*c^b*m^2*x*
x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n
^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*c^b*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 -
10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)
 + 3/8*c^(3*b)*m*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b
^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*m*n^2*x*x
^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 +
4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 1/8*b^2*m*n^2*x*x^m*e^(-3*a)/((9*b^
4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 +
4*m + 1)*c^(3*b)*x^(3*b*n)) - 9/8*c^b*m*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b
^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1
/8*c^(3*b)*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n
^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*m^2*n*x*x^m*e^(-a)
/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6
*m^2 + 4*m + 1)*c^b*x^(b*n)) - 27/8*b^2*n^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b
^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b
*x^(b*n)) - 3/8*b*m^2*n*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^
2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) +
1/8*b^2*n^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^
4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/8*c^b*x*x^
(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2
 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*m^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2
*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*
n)) + 3/4*b*m*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 +
m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 1/8*m^3*x*x^m*e^
(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m
^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/4*b*m*n*x*x^m*e^(-3*a)/((9*b^4
*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4
*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*m^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*
n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n
)) + 3/8*b*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/8*m^2*x*x^m*e^(-3
*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/8*b*n*x*x^m*e^(-3*a)/((9*b^4*n^4
- 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m +
1)*c^(3*b)*x^(3*b*n)) + 9/8*m*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/
8*m*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*x^m*e^(-a)/((9
*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2
 + 4*m + 1)*c^b*x^(b*n)) - 1/8*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2
- 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b
*n))
```

3.273 $\int x^m \sinh^4(a + b\log(cx^n))\,dx$

Optimal. Leaf size=266

$$\frac{(m+1)x^{m+1}\sinh^4(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}+\frac{12b^2(m+1)n^2x^{m+1}\sinh^2(a+b\log(cx^n))}{\left((m+1)^2-16b^2n^2\right)\left((m+1)^2-4b^2n^2\right)}-\frac{4bnx^{m+1}\sinh^3(a+b\log(cx^n))\cosh(}{(m+1)^2-16b^2n^2}$$

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2
*n^2)) - (24*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]
])/(((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2*n^2)) + (12*b^2*(1 + m)*n^2
*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/(((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 -
 4*b^2*n^2)) - (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n
]]^3)/((1 + m)^2 - 16*b^2*n^2) + ((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^
4)/((1 + m)^2 - 16*b^2*n^2)
```

Rubi [A] time = 0.128215, antiderivative size = 260, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5529, 30}

$$\frac{(m+1)x^{m+1}\sinh^4(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}+\frac{12b^2(m+1)n^2x^{m+1}\sinh^2(a+b\log(cx^n))}{-20b^2(m+1)^2n^2+64b^4n^4+(m+1)^4}-\frac{4bnx^{m+1}\sinh^3(a+b\log(cx^n))\cosh(}{(m+1)^2-16b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2
*n^2)) - (24*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]
])/((1 + m)^4 - 20*b^2*(1 + m)^2*n^2 + 64*b^4*n^4) + (12*b^2*(1 + m)*n^2*x^
(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^4 - 20*b^2*(1 + m)^2*n^2 + 64*b^
4*n^4) - (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/
((1 + m)^2 - 16*b^2*n^2) + ((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^4)/((1
 + m)^2 - 16*b^2*n^2)
```

Rule 5529

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (-Dist[(b^2*d^2*n^2*p*(p - 1))/(b^
2*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2),
 x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a
+ b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{
a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2,
0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^m \sinh^4(a+b\log(cx^n))\,dx = -\frac{4bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{(1+m)^2-16b^2n^2} + \frac{(1+m)x^{1+m}\sinh^4(a+b}{(1+m)^2-16b^2n}$$
$$= -\frac{24b^3n^3x^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^4-20b^2(1+m)^2n^2+64b^4n^4} + \frac{12b^2(1+m)n^2x^{1+m}\sin}{(1+m)^4-20b^2(1+}$$
$$= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^4-20b^2(1+m)^2n^2+64b^4n^4\right)} - \frac{24b^3n^3x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^4-20b^2(1+m)^2}$$

Mathematica [A] time = 3.44229, size = 311, normalized size = 1.17

$$\frac{1}{8}x^{m+1}\left(-\frac{4\sinh(2bn\log(x))\left((m+1)\sinh(2(a+b\log(cx^n)-bn\log(x)))-2bn\cosh(2(a+b\log(cx^n)-bn\log(x))\right)}{(-2bn+m+1)(2bn+m+1)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^(1 + m)*(3/(1 + m) - (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[x
] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1
+ m - 2*b*n)*(1 + m + 2*b*n)) - (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a -
b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]
))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cosh[4*(
a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*Log[c*
x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1 + m)*C
osh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[x] + b*L
og[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

Maple [F] time = 0.147, size = 0, normalized size = 0.

$$\int x^m (\sinh(a+b\ln(cx^n)))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31337, size = 2880, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] 1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*co
sh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) - 4*(m^4 + 4*m^3 - 16*(b^2*m
^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a
)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^
2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*
n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^4
+ 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m
^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b
*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^4 + 4*m^3 -
 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2*cosh(m*log(x)) - 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)
*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 - 4*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 -
2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sin
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b^3*m + b^3)*n^3 -
 (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3*cosh(
m*log(x)) - (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 - (16*(b^3*m
 + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^4 + 4*m^3 - 4*
(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^4 - 4*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m
 + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 2
0*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sinh(m*log(x)))/(m^5
+ 64*(b^4*m + b^4)*n^4 + 5*m^4 + 10*m^3 - 20*(b^2*m^3 + 3*b^2*m^2 + 3*b^2*m
 + b^2)*n^2 + 10*m^2 + 5*m + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.74208, size = 9293, normalized size = 34.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b
^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*
m^3 + 10*m^2 + 5*m + 1) - 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*m^2*n
^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n
^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*
n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 8*b^3*c^(2
*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*
m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4
+ 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(4*b)*m*n^2*x*x^(4*b
*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^
2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) +
1/2*b*c^(2*b)*m^3*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20
*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) + 8*b^2*c^(2*b)*m*n^2*x*x^(2*b*n)*x^m*e^(2*a)/(64
*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*
n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/16*c^(4*b)*m^4*x*
x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) - 3/4*b*c^(4*b)*m^2*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*c^(2*b)*m
^4*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + 3/2*b*c^(2*b)*m^2*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b
^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*
b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 15/2*b^2*m
^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
 m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4
*c^(4*b)*m^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1) - 3/4*b*c^(4*b)*m*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^
4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m
^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - c^(2*b)*m^3*x*x^(2*b*n)*x^m*
e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/2*b*c^
(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n
^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) + 8*b^3*m*n^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - b^3*m*n^3*x*x^m*e^(-4*a)/(
(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2
*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n))
 - 15*b^2*m*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
```

```
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) + 3/8*c^(4*b)*m^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*c^(2*b)*m^2*x*x^(
2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2
*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
 + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*
b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10
*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*m^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64
*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 2
0*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 8*b^3*n^3*x*x^m
*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m
^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)
*x^(2*b*n)) - 1/4*b^2*m^2*n^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - b^3*n^3*x*x^m*e^(-4*a)/((6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m
*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) +
 3/8*m^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2
 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1
5/2*b^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*
n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
+ 1/4*c^(4*b)*m*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2
*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^
3 + 10*m^2 + 5*m + 1) - c^(2*b)*m*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 6
4*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 -
20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b*m^3*n*x*x^m*e^(-2*a)/((64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 8*
b^2*m*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*
b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5
*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*m^3*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64
*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 2
0*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b^2*m*n^2*x
*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2
 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(
4*b)*x^(4*b*n)) + 3/2*m^3*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 1/16*c^(4*b)*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4
*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^
2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*c^(2*b)*x*x^(2*b*n)*x^m*e^(2*a)/(6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m
*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*m^4*x*x^m*e^(-
2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2
*b*n)) - 3/2*b*m^2*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 4*b^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m
*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 +
5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*m
^4*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2
*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
*c^(4*b)*x^(4*b*n)) + 3/4*b*m^2*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/4*b^2*n^2*x*x^m*e^(-
4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4
```

```
*b*n)) + 9/4*m^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2
*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m
+ 1) - m^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1)*c^(2*b)*x^(2*b*n)) - 3/2*b*m*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*
b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20
*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*m^3*x*x^m*e^
(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
- 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^
(4*b*n)) + 3/4*b*m*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/2*m*x*x^m/(64*b^4*m*n^4 + 64*b^4*
n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2
*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*m^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 +
 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
- 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 1/2*b*n*x*x^
m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b
)*x^(2*b*n)) + 3/8*m^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*
m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3
 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 1/4*b*n*x*x^m*e^(-4*a)/((64*b^4*m
*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 +
5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/8*x*
x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60
*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - m*x*x^m*e^(-
2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2
*b*n)) + 1/4*m*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
- 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^
2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/4*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^
4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b
^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*x^m*e^(-4*a
)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*
n))
```

3.274 $\int \frac{\sinh(a+b\log(cx^n))}{x} dx$

Optimal. Leaf size=18

$$\frac{\cosh(a+b\log(cx^n))}{bn}$$

```
[Out] Cosh[a + b*Log[c*x^n]]/(b*n)
```

Rubi [A] time = 0.0171739, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2638}

$$\frac{\cosh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]/x,x]

[Out] Cosh[a + b*Log[c*x^n]]/(b*n)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+b\log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\cosh(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [B] time = 0.014098, size = 37, normalized size = 2.06

$$\frac{\sinh(a)\sinh(b\log(cx^n))}{bn}+\frac{\cosh(a)\cosh(b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]/x,x]

[Out] (Cosh[a]*Cosh[b*Log[c*x^n]])/(b*n) + (Sinh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

Maple [A] time = 0.006, size = 19, normalized size = 1.1

$$\frac{\cosh(a+b\ln(cx^n))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))/x,x)
```

```
[Out] cosh(a+b*ln(c*x^n))/b/n
```

Maxima [A] time = 1.02403, size = 24, normalized size = 1.33

$$\frac{\cosh\left(b\log\left(cx^{n}\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] cosh(b*log(c*x^n) + a)/(b*n)
```

Fricas [A] time = 2.041, size = 53, normalized size = 2.94

$$\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] cosh(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [A] time = 1.72233, size = 37, normalized size = 2.06

$$\begin{cases} \log\left(x\right)\sinh\left(a\right) & \text{for}\ b=0\wedge\left(b=0\vee n=0\right)\\ \log\left(x\right)\sinh\left(a+b\log\left(c\right)\right) & \text{for}\ n=0\\ \frac{\cosh\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((log(x)*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*sinh(
a + b*log(c)), Eq(n, 0)), (cosh(a + b*n*log(x) + b*log(c))/(b*n), True))
```

Giac [B] time = 1.10892, size = 54, normalized size = 3.

$$\frac{\left(c^{2\,b}x^{bn}e^{\left(2\,a\right)}+\frac{1}{x^{bn}}\right)e^{\left(-a\right)}}{2\,bc^{b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 1/2*(c^(2*b)*x^(b*n)*e^(2*a) + 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

3.275 $\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=39

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\log(x)}{2}$$

```
[Out] -Log[x]/2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b*n)
```

Rubi [A] time = 0.0292156, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2635, 8}

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -Log[x]/2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \sinh^2(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\operatorname{Subst}\left(\int 1\,dx, x, \log\left(cx^n\right)\right)}{2n}\\ &= -\frac{\log(x)}{2}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}\end{aligned}$$

Mathematica [A] time = 0.0273888, size = 36, normalized size = 0.92

$$\frac{\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-2\left(a+b\log\left(cx^n\right)\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] (-2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

Maple [A] time = 0.01, size = 52, normalized size = 1.3

$$\frac{\cosh\left(a+b\ln\left(cx^{n}\right)\right)\sinh\left(a+b\ln\left(cx^{n}\right)\right)}{2bn}-\frac{\ln\left(cx^{n}\right)}{2n}-\frac{a}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^2/x,x)
```

```
[Out] 1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n-1/2*ln(c*x^n)/n-1/2/b/n*a
```

Maxima [A] time = 1.04754, size = 66, normalized size = 1.69

$$\frac{e^{\left(2\,b\log\left(cx^{n}\right)+2\,a\right)}}{8\,bn}-\frac{e^{\left(-2\,b\log\left(cx^{n}\right)-2\,a\right)}}{8\,bn}-\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) -
1/2*log(x)
```

Fricas [A] time = 2.06571, size = 123, normalized size = 3.15

$$-\frac{bn\log\left(x\right)-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -1/2*(b*n*log(x) - cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(
c) + a))/(b*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^{2}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(sinh(a + b*log(c*x**n))**2/x, x)
```

Giac [B] time = 1.21402, size = 109, normalized size = 2.79

$$-\frac{\left(4\,bc^{2\,b}ne^{\left(2\,a\right)}\log\left(x\right)-c^{4\,b}x^{2\,bn}e^{\left(4\,a\right)}-\frac{2\,c^{2\,b}x^{2\,bn}e^{\left(2\,a\right)}-1}{x^{2\,bn}}\right)e^{\left(-2\,a\right)}}{8\,bc^{2\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] -1/8*(4*b*c^(2*b)*n*e^(2*a)*log(x) - c^(4*b)*x^(2*b*n)*e^(4*a) - (2*c^(2*b)
*x^(2*b*n)*e^(2*a) - 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

3.276 $\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=43

$$\frac{\cosh^3(a+b\log(cx^n))}{3bn}-\frac{\cosh(a+b\log(cx^n))}{bn}$$

```
[Out] -(Cosh[a + b*Log[c*x^n]]/(b*n)) + Cosh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rubi [A] time = 0.0353946, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2633}

$$\frac{\cosh^3(a+b\log(cx^n))}{3bn}-\frac{\cosh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -(Cosh[a + b*Log[c*x^n]]/(b*n)) + Cosh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \sinh^3(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\
&= -\frac{\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh(a+b\log(cx^n))\right)}{bn}\\
&= -\frac{\cosh(a+b\log(cx^n))}{bn}+\frac{\cosh^3(a+b\log(cx^n))}{3bn}
\end{aligned}$$

Mathematica [A] time = 0.0138707, size = 45, normalized size = 1.05

$$\frac{\cosh(3(a+b\log(cx^n)))}{12bn}-\frac{3\cosh(a+b\log(cx^n))}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] (-3*Cosh[a + b*Log[c*x^n]])/(4*b*n) + Cosh[3*(a + b*Log[c*x^n])]/(12*b*n)
```

Maple [A] time = 0.007, size = 36, normalized size = 0.8

$$\frac{\cosh(a+b\ln(cx^n))}{bn}\left(-\frac{2}{3}+\frac{(\sinh(a+b\ln(cx^n)))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] 1/n/b*(-2/3+1/3*sinh(a+b*ln(c*x^n))^2)*cosh(a+b*ln(c*x^n))
```

Maxima [B] time = 1.03604, size = 116, normalized size = 2.7

$$\frac{e^{(3\,b\log(cx^n)+3\,a)}}{24\,bn}-\frac{3\,e^{(b\log(cx^n)+a)}}{8\,bn}-\frac{3\,e^{(-b\log(cx^n)-a)}}{8\,bn}+\frac{e^{(-3\,b\log(cx^n)-3\,a)}}{24\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) - 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8*
e^(-b*log(c*x^n) - a)/(b*n) + 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

Fricas [A] time = 2.09657, size = 208, normalized size = 4.84

$$\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-9\,\cosh\left(bn\log\left(x\right.\right.}{12\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a)^2 - 9*cosh(b*n*log(x) + b*log(c) + a))/(b*
n)
```

Sympy [A] time = 37.0679, size = 82, normalized size = 1.91

$$\begin{cases}\log\left(x\right)\sinh^{3}\left(a\right) & \text{for}\: b=0\wedge\left(b=0\vee n=0\right)\\ \log\left(x\right)\sinh^{3}\left(a+b\log\left(c\right)\right) & \text{for}\: n=0\\ \frac{\sinh^{2}\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)\cosh\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{bn}-\frac{2\cosh^{3}\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{3bn} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*si
nh(a + b*log(c))**3, Eq(n, 0)), (sinh(a + b*n*log(x) + b*log(c))**2*cosh(a
+ b*n*log(x) + b*log(c))/(b*n) - 2*cosh(a + b*n*log(x) + b*log(c))**3/(3*b*
n), True))
```

Giac [A] time = 1.18802, size = 109, normalized size = 2.53

$$\frac{\left(c^{6\,b}x^{3\,bn}e^{(6\,a)}-9\,c^{4\,b}x^{bn}e^{(4\,a)}-\frac{9\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1}{x^{3\,bn}}\right)e^{(-3\,a)}}{24\,bc^{3\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) - 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x^
(2*b*n)*e^(2*a) - 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

3.277 $\int \frac{\sinh^4(a+b\log(cx^n))}{x} \, dx$

Optimal. Leaf size=73

$$\frac{\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4bn}-\frac{3\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{8bn}+\frac{3\log(x)}{8}$$

```
[Out] (3*Log[x])/8 - (3*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(8*b*n) +
(Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(4*b*n)
```

Rubi [A] time = 0.0465652, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2635, 8}

$$\frac{\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4bn}-\frac{3\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{8bn}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (3*Log[x])/8 - (3*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(8*b*n) +
(Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(4*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \sinh^4(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{4bn}-\frac{3\,\text{Subst}\left(\int \sinh^2(a+bx)\,dx, x, \log(cx^n)\right)}{4n}\\ &= -\frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn}+\frac{\cosh(a+b\log(cx^n))\sinh^3(a+b\log(c\\ &= \frac{3\log(x)}{8}-\frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn}+\frac{\cosh(a+b\log(cx^n))\sinh^3(a\end{aligned}$$

Mathematica [A] time = 0.0467006, size = 51, normalized size = 0.7

$$\frac{12(a+b\log(cx^n))-8\sinh(2(a+b\log(cx^n)))+\sinh(4(a+b\log(cx^n)))}{32bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (12*(a + b*Log[c*x^n]) - 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[c
*x^n])])/(32*b*n)
```

Maple [A] time = 0.012, size = 84, normalized size = 1.2

$$\frac{\cosh\left(a+b\ln\left(cx^{n}\right)\right)\left(\sinh\left(a+b\ln\left(cx^{n}\right)\right)\right)^{3}}{4bn}-\frac{3\cosh\left(a+b\ln\left(cx^{n}\right)\right)\sinh\left(a+b\ln\left(cx^{n}\right)\right)}{8bn}+\frac{3\ln\left(cx^{n}\right)}{8n}+\frac{3a}{8bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] 1/4*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^3/b/n-3/8*cosh(a+b*ln(c*x^n))*s
inh(a+b*ln(c*x^n))/b/n+3/8*ln(c*x^n)/n+3/8/b/n*a
```

Maxima [A] time = 1.14188, size = 126, normalized size = 1.73

$$\frac{e^{\left(4\,b\log\left(cx^{n}\right)+4\,a\right)}}{64\,bn}-\frac{e^{\left(2\,b\log\left(cx^{n}\right)+2\,a\right)}}{8\,bn}+\frac{e^{\left(-2\,b\log\left(cx^{n}\right)-2\,a\right)}}{8\,bn}-\frac{e^{\left(-4\,b\log\left(cx^{n}\right)-4\,a\right)}}{64\,bn}+\frac{3}{8}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) - 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) +
1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n)
+ 3/8*log(x)
```

Fricas [A] time = 2.08291, size = 270, normalized size = 3.7

$$\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\,bn\log\left(x\right)+\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-4\right.}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log(x) + b*log
(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18396, size = 154, normalized size = 2.11

$$\frac{\left(24\,bc^{4\,b}ne^{(4\,a)}\log\left(x\right) + c^{8\,b}x^{4\,bn}e^{(8\,a)} - 8\,c^{6\,b}x^{2\,bn}e^{(6\,a)} - \frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 8\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] 1/64*(24*b*c^(4*b)*n*e^(4*a)*log(x) + c^(8*b)*x^(4*b*n)*e^(8*a) - 8*c^(6*b)
*x^(2*b*n)*e^(6*a) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) - 8*c^(2*b)*x^(2*b*n)*e^
(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

3.278 $\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=65

$$\frac{\cosh^5(a+b\log(cx^n))}{5bn}-\frac{2\cosh^3(a+b\log(cx^n))}{3bn}+\frac{\cosh(a+b\log(cx^n))}{bn}$$

```
[Out] Cosh[a + b*Log[c*x^n]]/(b*n) - (2*Cosh[a + b*Log[c*x^n]]^3)/(3*b*n) + Cosh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Rubi [A] time = 0.0415972, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2633}

$$\frac{\cosh^5(a+b\log(cx^n))}{5bn}-\frac{2\cosh^3(a+b\log(cx^n))}{3bn}+\frac{\cosh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Cosh[a + b*Log[c*x^n]]/(b*n) - (2*Cosh[a + b*Log[c*x^n]]^3)/(3*b*n) + Cosh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \sinh^5(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, \cosh(a+b\log(cx^n))\right)}{bn}\\ &= \frac{\cosh(a+b\log(cx^n))}{bn}-\frac{2\cosh^3(a+b\log(cx^n))}{3bn}+\frac{\cosh^5(a+b\log(cx^n))}{5bn}\end{aligned}$$

Mathematica [A] time = 0.0187452, size = 68, normalized size = 1.05

$$\frac{5\cosh(a+b\log(cx^n))}{8bn}-\frac{5\cosh(3(a+b\log(cx^n)))}{48bn}+\frac{\cosh(5(a+b\log(cx^n)))}{80bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (5*Cosh[a + b*Log[c*x^n]])/(8*b*n) - (5*Cosh[3*(a + b*Log[c*x^n])])/(48*b*n
) + Cosh[5*(a + b*Log[c*x^n])]/(80*b*n)
```

Maple [A] time = 0.012, size = 51, normalized size = 0.8

$$\frac{\cosh\left(a+b\ln\left(cx^{n}\right)\right)}{bn}\left(\frac{8}{15}+\frac{\left(\sinh\left(a+b\ln\left(cx^{n}\right)\right)\right)^{4}}{5}-\frac{4\left(\sinh\left(a+b\ln\left(cx^{n}\right)\right)\right)^{2}}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] 1/n/b*(8/15+1/5*sinh(a+b*ln(c*x^n))^4-4/15*sinh(a+b*ln(c*x^n))^2)*cosh(a+b*
ln(c*x^n))
```

Maxima [B] time = 1.10235, size = 176, normalized size = 2.71

$$\frac{e^{(5\,b\log(cx^{n})+5\,a)}}{160\,bn}-\frac{5\,e^{(3\,b\log(cx^{n})+3\,a)}}{96\,bn}+\frac{5\,e^{(b\log(cx^{n})+a)}}{16\,bn}+\frac{5\,e^{(-b\log(cx^{n})-a)}}{16\,bn}-\frac{5\,e^{(-3\,b\log(cx^{n})-3\,a)}}{96\,bn}+\frac{e^{(-5\,b\log(cx^{n})-5\,a)}}{160\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) - 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
+ 5/16*e^(b*log(c*x^n) + a)/(b*n) + 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/96
*e^(-3*b*log(c*x^n) - 3*a)/(b*n) + 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

Fricas [B] time = 2.11773, size = 421, normalized size = 6.48

$$3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{5}+15\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-25\cosh\left(bn\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/240*(3*cosh(b*n*log(x) + b*log(c) + a)^5 + 15*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a)^4 - 25*cosh(b*n*log(x) + b*log(c) + a)
^3 + 15*(2*cosh(b*n*log(x) + b*log(c) + a)^3 - 5*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + 150*cosh(b*n*log(x) + b*log(c) +
 a))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2113, size = 155, normalized size = 2.38

$$\frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)}-25\,c^{8\,b}x^{3\,bn}e^{(8\,a)}+150\,c^{6\,b}x^{bn}e^{(6\,a)}+\frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)}-25\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) - 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*c
^(6*b)*x^(b*n)*e^(6*a) + (150*c^(4*b)*x^(4*b*n)*e^(4*a) - 25*c^(2*b)*x^(2*b
*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

3.279 $\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=111

$$\frac{2\sinh^{\frac{3}{2}}(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{5bn}+\frac{6i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sinh[a + b*Log[c*x^n]]^(3/2))/(5*b*n)
```

Rubi [A] time = 0.0652123, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2635, 2640, 2639}

$$\frac{2\sinh^{\frac{3}{2}}(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{5bn}+\frac{6i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sinh[a + b*Log[c*x^n]]^(3/2))/(5*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\mathrm{Subst}\left(\int \sinh^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\cosh(a+b\log(cx^n))\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{5bn} - \frac{3\,\mathrm{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log\right.}{5n}$$
$$= \frac{2\cosh(a+b\log(cx^n))\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{5bn} - \frac{\left(3\sqrt{\sinh(a+b\log(cx^n))}\right)\mathrm{Subst}\left(\int\right.}{5n\sqrt{i\sinh(a+}}$$
$$= \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{5bn\sqrt{i\sinh(a+b\log(cx^n))}} + \frac{2\cosh(a+b\log(cx^n))\sin}{5bn}$$

Mathematica [A] time = 0.0883684, size = 96, normalized size = 0.86

$$\frac{\sinh(a+b\log(cx^n))\sinh(2(a+b\log(cx^n))) - 6\sqrt{i\sinh(a+b\log(cx^n))}E\left(\frac{1}{4}(-2ia-2ib\log(cx^n)+\pi)\middle|2\right)}{5bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*
Log[c*x^n]]] + Sinh[a + b*Log[c*x^n]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n*Sq
rt[Sinh[a + b*Log[c*x^n]]])
```

Maple [A] time = 0.054, size = 227, normalized size = 2.1

$$\frac{1}{bn\cosh(a+b\ln(cx^n))}\left(-\frac{6\sqrt{2}}{5}\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\mathrm{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^(5/2)/x,x)
```

```
[Out] 1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))
^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1
/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*
ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(
c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(c*x^n
))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

3.280 $\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=111

$$\frac{2\sqrt{\sinh(a+b\log(cx^n))}\cosh(a+b\log(cx^n))}{3bn}+\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*
Log[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(3*b*n)
```

Rubi [A] time = 0.0602746, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2635, 2642, 2641}

$$\frac{2\sqrt{\sinh(a+b\log(cx^n))}\cosh(a+b\log(cx^n))}{3bn}+\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}F\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*
Log[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(3*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx = \frac{\text{Subst}\left(\int \sinh^{\frac{3}{2}}(a+bx)dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\cosh(a+b\log(cx^n))\sqrt{\sinh(a+b\log(cx^n))}}{3bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}dx, x, \log(cx^n)\right)}{3n}$$
$$= \frac{2\cosh(a+b\log(cx^n))\sqrt{\sinh(a+b\log(cx^n))}}{3bn} - \frac{\sqrt{i\sinh(a+b\log(cx^n))}\,\text{Subst}\left(\int \frac{}{\sqrt{i\sin}}\right.}{3n\sqrt{\sinh(a+b\log(}}$$
$$= \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{2\cosh(a+b\log(cx^n))\sqrt{\sin}}{3bn}$$

Mathematica [C] time = 0.129836, size = 114, normalized size = 1.03

$$\frac{\sinh(2(a+b\log(cx^n))) - 2\sqrt{-\sinh(2(a+b\log(cx^n))) - \cosh(2(a+b\log(cx^n))) + 1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+b\log(c\right.}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(a
 + b*Log[c*x^n])]]*Sqrt[1 - Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[
c*x^n])]] + Sinh[2*(a + b*Log[c*x^n])])/(3*b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]
)
```

Maple [A] time = 0.055, size = 143, normalized size = 1.3

$$\frac{1}{bn\cosh(a+b\ln(cx^n))}\left(-\frac{i}{3}\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\text{EllipticF}\left(\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] 1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+2/3*sinh(a+b*ln(c*x^n))*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*
ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(b\log(cx^n)+a)^{\frac{3}{2}}}{x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

3.281 $\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=72

$$-\frac{2i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c
*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.043666, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2640, 2639}

$$-\frac{2i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c
*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}\\
&= \frac{\sqrt{\sinh(a+b\log(cx^n))}\operatorname{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{n\sqrt{i\sinh(a+b\log(cx^n))}}\\
&= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}
\end{aligned}$$

Mathematica [A] time = 0.0420745, size = 68, normalized size = 0.94

$$\frac{2\sqrt{i\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+b\log(cx^n))\right)\middle|2\right)}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^
n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Maple [A] time = 0.046, size = 146, normalized size = 2.

$$\frac{\sqrt{2}}{bn\cosh\left(a+b\ln\left(cx^{n}\right)\right)}\sqrt{-i\left(i+\sinh\left(a+b\ln\left(cx^{n}\right)\right)\right)}\sqrt{-i\left(-\sinh\left(a+b\ln\left(cx^{n}\right)\right)+i\right)}\sqrt{i\sinh\left(a+b\ln\left(cx^{n}\right)\right)}\left(2\,\mathrm{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] 1/n*(-I*(I+sinh(a+b*ln(c*x^n))))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))
)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sinh\left(b\log\left(cx^{n}\right)+a\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sinh(a + b*log(c*x**n)))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

3.282 $\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=72

$$-\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right),2\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log
[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.0439565, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2642, 2641}

$$-\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log
[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{n\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\\
&= -\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}
\end{aligned}$$

Mathematica [A] time = 0.0437464, size = 66, normalized size = 0.92

$$-\frac{2\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\text{EllipticF}\left(\frac{1}{4}\left(-2ia-2ib\log\left(cx^n\right)+\pi\right),2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
```

```
[Out] (-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Maple [A] time = 0.042, size = 120, normalized size = 1.7

$$\frac{i\sqrt{2}}{bn\cosh\left(a+b\ln\left(cx^{n}\right)\right)}\sqrt{-i\left(i+\sinh\left(a+b\ln\left(cx^{n}\right)\right)\right)}\sqrt{-i\left(-\sinh\left(a+b\ln\left(cx^{n}\right)\right)+i\right)}\sqrt{i\sinh\left(a+b\ln\left(cx^{n}\right)\right)}\mathrm{EllipticF}\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] I/n*(-I*(I+sinh(a+b*ln(c*x^n))))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(I+sinh(a+b*ln(c*x^n)))
)^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\sinh\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x\sqrt{\sinh\left(b\log\left(cx^{n}\right)+a\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(sinh(a + b*log(c*x**n)))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\sinh\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

3.283 $\int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=107

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}-\frac{2i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(b
*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.0566298, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2636, 2640, 2639}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}-\frac{2i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(b
*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{\sqrt{\sinh(a+b\log(cx^n))}\,\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x\right.}{n\sqrt{i\sinh(a+b\log(cx^n))}}$$

$$= -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

Mathematica [A] time = 0.0622233, size = 80, normalized size = 0.75

$$-\frac{2\left(\cosh(a+b\log(cx^n))-\sqrt{i\sinh(a+b\log(cx^n))}E\left(\frac{1}{4}(-2ia-2ib\log(cx^n)+\pi)\middle|2\right)\right)}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n]
)/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Maple [A] time = 0.013, size = 212, normalized size = 2.

$$\frac{1}{bn\cosh(a+b\ln(cx^n))}\left(2\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\text{EllipticE}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 1/n*(2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1/2)
,1/2*2^(1/2))-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^
n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*
ln(c*x^n))^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sinh(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\sinh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sinh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

3.284 $\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))} dx$

Optimal. Leaf size=111

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right),2\right)}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sinh[a + b*Log[c*x^n]]^(3/2)) + (((2*I)/
3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^
n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] time = 0.0610743, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2636, 2642, 2641}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sinh[a + b*Log[c*x^n]]^(3/2)) + (((2*I)/
3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^
n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\cosh(a+b\log(cx^n))}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n}$$
$$= -\frac{2\cosh(a+b\log(cx^n))}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\sqrt{i\sinh(a+b\log(cx^n))}\,\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx, x, \log\right.}{3n\sqrt{\sinh(a+b\log(cx^n))}}$$
$$= -\frac{2\cosh(a+b\log(cx^n))}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n}}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Mathematica [C] time = 0.127231, size = 122, normalized size = 1.1

$$-\frac{2\left(\sinh(a+b\log(cx^n))\sqrt{-\sinh(2(a+b\log(cx^n)))-\cosh(2(a+b\log(cx^n)))+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+b\log(cx^n}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*(Cosh[a + b*Log[c*x^n]] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a +
b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]*Sinh[a + b*Log[c*x^n]]*Sqrt[1
- Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Sinh[a
+ b*Log[c*x^n]]^(3/2))
```

Maple [A] time = 0.055, size = 144, normalized size = 1.3

$$-\frac{1}{3\,bn\cosh(a+b\ln(cx^n))}\left(i\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] -1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)
*(1+I*sinh(a+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1
-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+b*l
n(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sinh(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\sinh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sinh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)
```

3.285 $\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx$

Optimal. Leaf size=209

$$-\frac{5e^{-2a}x\left(cx^n\right)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^2}-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}-\frac{5e^{-3a}x\left(cx^n\right)^{-6/n}\csc^{-1}\left(\right.}{4\left(1-e\right.}$$

```
[Out] -(x*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/4 - (5*x*Sinh[a + (2*Log[c*x^n])/n]^(
5/2))/(4*E^(2*a)*(c*x^n)^(4/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^2) + (5*x*Si
nh[a + (2*Log[c*x^n])/n]^(5/2))/(12*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))) - (5*x
*ArcCsc[E^a*(c*x^n)^(2/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/(4*E^(3*a)*(c*
x^n)^(6/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))
```

Rubi [A] time = 0.160211, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5525, 5533, 353, 349, 345, 242, 277, 216}

$$-\frac{5e^{-2a}x\left(cx^n\right)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^2}-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}-\frac{5e^{-3a}x\left(cx^n\right)^{-6/n}\csc^{-1}\left(\right.}{4\left(1-e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] -(x*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/4 - (5*x*Sinh[a + (2*Log[c*x^n])/n]^(
5/2))/(4*E^(2*a)*(c*x^n)^(4/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^2) + (5*x*Si
nh[a + (2*Log[c*x^n])/n]^(5/2))/(12*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))) - (5*x
*ArcCsc[E^a*(c*x^n)^(2/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/(4*E^(3*a)*(c*
x^n)^(6/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))
```

Rule 5525

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5533

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 353

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &&
 IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 349

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*
(a + b*x^n)^p)/(m + 1), x] - Dist[(b*n*p)/(m + 1), Int[x^(m + n)*(a + b*x^n
```

```
)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && GtQ
[p, 0]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(x)}{n}\right)dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x\left(cx^n\right)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}\left(1-e^{-2a}x^{-4/n}\right)^{5/2}dx,x,cx^n\right)}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{\left(5x\left(cx^n\right)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}(\ldots\right.}{2n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}-\frac{\left(5e^{-2a}x\left(cx^n\right)^{-6/n}\sinh^{\frac{5}{2}}\left(a\ldots\right.\right.}{\ldots}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}-\frac{\left(5e^{-2a}x\left(cx^n\right)^{-6/n}\sinh^{\frac{5}{2}}\left(a\ldots\right.\right.}{4\ldots}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}+\frac{\left(5e^{-2a}x\left(cx^n\right)^{-6/n}\sinh^{\frac{5}{2}}\left(a\ldots\right.\right.}{4\ldots}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)-\frac{5e^{-2a}x\left(cx^n\right)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^2}+\frac{5x\sinh^{\frac{5}{2}}\left(a+\ldots\right.}{12\left(1-e^{-2a}\ldots\right.}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)-\frac{5e^{-2a}x\left(cx^n\right)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^2}+\frac{5x\sinh^{\frac{5}{2}}\left(a+\ldots\right.}{12\left(1-e^{-2a}\ldots\right.}$$

Mathematica [C] time = 0.4182, size = 86, normalized size = 0.41

$$\frac{1}{14}e^{2a}x(cx^n)^{4/n}\left(e^{2a}(cx^n)^{4/n}-1\right){}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-e^{2a}(cx^n)^{4/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] (E^(2*a)*x*(c*x^n)^(4/n)*(-1 + E^(2*a)*(c*x^n)^(4/n))*Hypergeometric2F1[2,
7/2, 9/2, 1 - E^(2*a)*(c*x^n)^(4/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/14
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int\left(\sinh\left(a+2\frac{\ln(cx^n)}{n}\right)\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
```

```
[Out] int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(5/2), x)
```

Fricas [A] time = 2.18024, size = 420, normalized size = 2.01

$$\frac{\left(15\sqrt{2}x^3\arctan\left(\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}-1}{x^2}}\right)e^{\left(\frac{3(an+2\log(c))}{2n}\right)}+2\sqrt{\frac{1}{2}}\left(2x^8e^{\left(\frac{4(an+2\log(c))}{n}\right)}-14x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}-3\right)\sqrt{\frac{x^4e^{\left(\frac{2(an-}{}\right.}}{}}\right.}{96x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log
(c))/n) - 1)/x^2))*e^(3/2*(a*n + 2*log(c))/n) + 2*sqrt(1/2)*(2*x^8*e^(4*(a*
n + 2*log(c))/n) - 14*x^4*e^(2*(a*n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(a*n
 + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*log(c
))/n)/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*ln(c*x**n)/n)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(5/2), x)
```

3.286 $\int \sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=103

$$\frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}+\frac{e^{-a}x\,(cx^n)^{-2/n}\csc^{-1}\left(e^a\,(cx^n)^{2/n}\right)\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{1-e^{-2a}\,(cx^n)^{-4/n}}}$$

```
[Out] (x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2 + (x*ArcCsc[E^a*(c*x^n)^(2/n)]*Sqrt[
Sinh[a + (2*Log[c*x^n])/n]])/(2*E^a*(c*x^n)^(2/n)*Sqrt[1 - 1/(E^(2*a)*(c*x^
n)^(4/n))])
```

Rubi [A] time = 0.0871008, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5525, 5533, 345, 242, 277, 216}

$$\frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}+\frac{e^{-a}x\,(cx^n)^{-2/n}\csc^{-1}\left(e^a\,(cx^n)^{2/n}\right)\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{1-e^{-2a}\,(cx^n)^{-4/n}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2 + (x*ArcCsc[E^a*(c*x^n)^(2/n)]*Sqrt[
Sinh[a + (2*Log[c*x^n])/n]])/(2*E^a*(c*x^n)^(2/n)*Sqrt[1 - 1/(E^(2*a)*(c*x^
n)^(4/n))])
```

Rule 5525

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5533

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sqrt{\sinh\left(a+\frac{2\log(x)}{n}\right)}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sqrt{1-e^{-2a}x^{-4/n}}\,dx,x,cx^n\right)}{n\sqrt{1-e^{-2a}(cx^n)^{-4/n}}}$$

$$= \frac{\left(x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int \sqrt{1-\frac{e^{-2a}}{x^2}}\,dx,x,(cx^n)^{2/n}\right)}{2\sqrt{1-e^{-2a}(cx^n)^{-4/n}}}$$

$$= -\frac{\left(x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1-e^{-2a}x^2}}{x^2}\,dx,x,(cx^n)^{-2/n}\right)}{2\sqrt{1-e^{-2a}(cx^n)^{-4/n}}}$$

$$= \frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}+\frac{\left(e^{-2a}x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-e \ldots}}\right.}{2\sqrt{1-e^{-2a}(cx^n)^{-4/n}}}$$

$$= \frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}+\frac{e^{-a}x(cx^n)^{-2/n}\sin^{-1}\left(e^{-a}(cx^n)^{-2/n}\right)\sqrt{\sinh\left(a+\frac{2\log(cx^n \ldots}{n}\right.}}{2\sqrt{1-e^{-2a}(cx^n)^{-4/n}}}$$

Mathematica [A] time = 0.268917, size = 74, normalized size = 0.72

$$\frac{1}{2}x\left(1-\frac{\tan^{-1}\left(\sqrt{e^{2a}(cx^n)^{4/n}-1}\right)}{\sqrt{e^{2a}(cx^n)^{4/n}-1}}\right)\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*(1 - ArcTan[Sqrt[-1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[-1 + E^(2*a)*(c*x^n)^
(4/n)])*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int \sqrt{\sinh\left(a + 2\,\frac{\ln\left(cx^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
```

```
[Out] int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sinh\left(a + \frac{2\,\log\left(cx^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(a + 2*log(c*x^n)/n)), x)
```

Fricas [A] time = 2.15805, size = 306, normalized size = 2.97

$$\frac{1}{4}\left(2\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}\right)e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}\right)e^{\left(-\frac{an+2\,\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(1/2*(a*n
 + 2*log(c))/n) - sqrt(2)*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n +
2*log(c))/n) - 1)/x^2))*e^(1/2*(a*n + 2*log(c))/n))*e^(-(a*n + 2*log(c))/n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sinh\left(a + \frac{2\log\left(cx^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*ln(c*x**n)/n)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(a + 2*log(c*x**n)/n)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.287 $\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{x\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(2*Sinh[a + (2*Log[c*x^n])/n]^(3/2))
```

Rubi [A] time = 0.0528222, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5525, 5533, 264}

$$-\frac{x\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] -(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(2*Sinh[a + (2*Log[c*x^n])/n]^(3/2))
```

Rule 5525

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5533

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x\left(cx^n\right)^{2/n}\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{3/2}}\,dx, x, cx^n\right)}{n\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A] time = 0.14718, size = 61, normalized size = 1.42

$$\frac{\sinh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)-\cosh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)}{x\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] (-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n]
)/n])/(x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])
```

Maple [F] time = 0.16, size = 0, normalized size = 0.

$$\int \left(\sinh\left(a+2\,\frac{\ln\left(cx^n\right)}{n}\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)
```

```
[Out] int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

Fricas [A] time = 2.43249, size = 167, normalized size = 3.88

$$-\frac{2\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n +
2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*ln(c*x**n)/n)**(3/2),x)
```

```
[Out] Integral(sinh(a + 2*log(c*x**n)/n)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sinh\left(a+\frac{2\,\log(cx^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

3.288 $\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=103

$$\frac{e^{-2a}x\,(cx^n)^{-4/n}\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(6*Sinh[a + (2*Log[c*x^n])/n]^(7/2)) +
 (x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(15*E^(2*a)*(c*x^n)^(4/n)*Sinh[a + (2*
Log[c*x^n])/n]^(7/2))
```

Rubi [A] time = 0.078949, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5525, 5533, 271, 264}

$$\frac{e^{-2a}x\,(cx^n)^{-4/n}\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] -(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(6*Sinh[a + (2*Log[c*x^n])/n]^(7/2)) +
 (x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(15*E^(2*a)*(c*x^n)^(4/n)*Sinh[a + (2*
Log[c*x^n])/n]^(7/2))
```

Rule 5525

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5533

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x\,(cx^n)^{6/n}\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{6}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{7/2}}\,dx, x, cx^n\right)}{n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{\left(2e^{-2a}x\,(cx^n)^{6/n}\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{10}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{7/2}}\,d\right.}{3n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} + \frac{e^{-2a}x\,(cx^n)^{-4/n}\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A] time = 0.260257, size = 121, normalized size = 1.17

$$\frac{\left(\left(5x^4+2\right)\sinh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)+\left(5x^4-2\right)\cosh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)\right)\left(\sinh\left(2a+\frac{4\log(cx^n)}{n}-4\log(x)\right)-c\right.}{15x^5\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] (((-2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (2 + 5*x^4)*Sinh[a -
 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n] +
Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Sinh[a + (2*Log[c*x^n])/n
]^(5/2))
```

Maple [F] time = 0.163, size = 0, normalized size = 0.

$$\int \left(\sinh\left(a+2\,\frac{\ln(cx^n)}{n}\right)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
```

```
[Out] int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

Fricas [A] time = 2.30082, size = 312, normalized size = 3.03

$$-\frac{8\sqrt{\frac{1}{2}}\left(5\,x^5 e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\,x\right)\sqrt{\frac{x^4 e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{15\left(x^{12}e^{\left(\frac{6\,(an+2\,\log(c))}{n}\right)}-3\,x^8 e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+3\,x^4 e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) - 2*x)*sqrt((x^4*e^(2*(a*n
+ 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*lo
g(c))/n) - 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/n)
- 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*ln(c*x**n)/n)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh\left(a+\frac{2\,\log(cx^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

3.289 $\int \sinh\left(\frac{a}{c+dx}\right) dx$

Optimal. Leaf size=36

$$\frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d}$$

```
[Out] -((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d
```

Rubi [A] time = 0.0467654, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5310, 5302, 3297, 3301}

$$\frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a/(c + d*x)],x]
```

```
[Out] -((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh\left(\frac{a}{c+dx}\right)dx = \frac{\text{Subst}\left(\int \sinh\left(\frac{a}{x}\right)dx, x, c+dx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh(ax)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{\cosh(ax)}{x}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= -\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d} + \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}$$

Mathematica [A] time = 0.0195202, size = 36, normalized size = 1.

$$\frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a/(c + d*x)],x]
```

```
[Out] -((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d
```

Maple [A] time = 0.009, size = 38, normalized size = 1.1

$$-\frac{a}{d}\left(-\frac{dx+c}{a}\sinh\left(\frac{a}{dx+c}\right) + \text{Chi}\left(\frac{a}{dx+c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(d*x+c)),x)
```

```
[Out] -1/d*a*(-1/a*(d*x+c)*sinh(a/(d*x+c))+Chi(a/(d*x+c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}ad\int \frac{xe^{\left(\frac{a}{dx+c}\right)}}{d^2x^2+2\,cdx+c^2}\,dx + \frac{1}{2}ad\int \frac{xe^{\left(-\frac{a}{dx+c}\right)}}{d^2x^2+2\,cdx+c^2}\,dx + \frac{1}{2}\,xe^{\left(\frac{a}{dx+c}\right)} - \frac{1}{2}\,xe^{\left(-\frac{a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*a*d
*integrate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*x*e^(a/(d
*x + c)) - 1/2*x*e^(-a/(d*x + c))
```

Fricas [A] time = 2.18674, size = 109, normalized size = 3.03

$$-\frac{a\text{Ei}\left(\frac{a}{dx+c}\right) + a\text{Ei}\left(-\frac{a}{dx+c}\right) - 2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*Ei(a/(d*x + c)) + a*Ei(-a/(d*x + c)) - 2*(d*x + c)*sinh(a/(d*x + c)
))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{a}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x)
```

```
[Out] Integral(sinh(a/(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(a/(d*x + c)), x)
```

3.290 $\int \sinh^2\left(\frac{a}{c+dx}\right) dx$

Optimal. Leaf size=39

$$\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d}-\frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

```
[Out] ((c + d*x)*Sinh[a/(c + d*x)]^2)/d - (a*SinhIntegral[(2*a)/(c + d*x)])/d
```

Rubi [A] time = 0.0636995, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5310, 5302, 3313, 12, 3298}

$$\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d}-\frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Sinh[a/(c + d*x)]^2)/d - (a*SinhIntegral[(2*a)/(c + d*x)])/d
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh^2\left(\frac{a}{c+dx}\right)dx = \frac{\text{Subst}\left(\int \sinh^2\left(\frac{a}{x}\right)dx, x, c+dx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh^2(ax)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} + \frac{(2ia)\,\text{Subst}\left(\int \frac{i\sinh(2ax)}{2x}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{\sinh(2ax)}{x}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

Mathematica [A] time = 0.0403094, size = 37, normalized size = 0.95

$$\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right) - a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a/(c + d*x)]^2,x]

[Out] ((c + d*x)*Sinh[a/(c + d*x)]^2 - a*SinhIntegral[(2*a)/(c + d*x)])/d
```

Maple [A] time = 0.016, size = 50, normalized size = 1.3

$$-\frac{a}{d}\left(\frac{dx+c}{2a} - \frac{dx+c}{2a}\cosh\left(2\frac{a}{dx+c}\right) + \text{Shi}\left(2\frac{a}{dx+c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(d*x+c))^2,x)

[Out] -1/d*a*(1/2/a*(d*x+c)-1/2/a*(d*x+c)*cosh(2*a/(d*x+c))+Shi(2*a/(d*x+c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}ad\int \frac{xe^{\left(\frac{2a}{dx+c}\right)}}{d^2x^2+2cdx+c^2}dx - \frac{1}{2}ad\int \frac{xe^{\left(-\frac{2a}{dx+c}\right)}}{d^2x^2+2cdx+c^2}dx + \frac{1}{4}xe^{\left(\frac{2a}{dx+c}\right)} + \frac{1}{4}xe^{\left(-\frac{2a}{dx+c}\right)} - \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*a*d*integrate(x*e^(2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/2*a
*d*integrate(x*e^(-2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*x*e^(
2*a/(d*x + c)) + 1/4*x*e^(-2*a/(d*x + c)) - 1/2*x
```

Fricas [A] time = 2.04376, size = 165, normalized size = 4.23

$$\frac{(dx+c)\cosh\left(\frac{a}{dx+c}\right)^2+(dx+c)\sinh\left(\frac{a}{dx+c}\right)^2-dx-a\mathrm{Ei}\left(\frac{2a}{dx+c}\right)+a\mathrm{Ei}\left(-\frac{2a}{dx+c}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((d*x + c)*cosh(a/(d*x + c))^2 + (d*x + c)*sinh(a/(d*x + c))^2 - d*x -
a*Ei(2*a/(d*x + c)) + a*Ei(-2*a/(d*x + c)))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{a}{dx+c}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(a/(d*x + c))^2, x)
```

3.291 $\int \sinh^3\left(\frac{a}{c+dx}\right) dx$

Optimal. Leaf size=59

$$\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d} - \frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d} + \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

```
[Out] (3*a*CoshIntegral[a/(c + d*x)])/(4*d) - (3*a*CoshIntegral[(3*a)/(c + d*x)])
/(4*d) + ((c + d*x)*Sinh[a/(c + d*x)]^3)/d
```

Rubi [A] time = 0.0851237, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5310, 5302, 3313, 3301}

$$\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d} - \frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d} + \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a/(c + d*x)]^3,x]
```

```
[Out] (3*a*CoshIntegral[a/(c + d*x)])/(4*d) - (3*a*CoshIntegral[(3*a)/(c + d*x)])
/(4*d) + ((c + d*x)*Sinh[a/(c + d*x)]^3)/d
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh^3\left(\frac{a}{c+dx}\right)dx = \frac{\text{Subst}\left(\int \sinh^3\left(\frac{a}{x}\right)dx, x, c+dx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh^3(ax)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d} + \frac{(3a)\,\text{Subst}\left(\int \left(\frac{\cosh(ax)}{4x} - \frac{\cosh(3ax)}{4x}\right)dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d} + \frac{(3a)\,\text{Subst}\left(\int \frac{\cosh(ax)}{x}dx, x, \frac{1}{c+dx}\right)}{4d} - \frac{(3a)\,\text{Subst}\left(\int \frac{\cosh(3ax)}{x}dx, x, \frac{1}{c+dx}\right)}{4d}$$
$$= \frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d} - \frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d} + \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

Mathematica [A] time = 0.05748, size = 54, normalized size = 0.92

$$\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right) - 3a\text{Chi}\left(\frac{3a}{c+dx}\right) + 4(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a/(c + d*x)]^3,x]
```

```
[Out] (3*a*CoshIntegral[a/(c + d*x)] - 3*a*CoshIntegral[(3*a)/(c + d*x)] + 4*(c +
 d*x)*Sinh[a/(c + d*x)]^3)/(4*d)
```

Maple [A] time = 0.013, size = 74, normalized size = 1.3

$$-\frac{a}{d}\left(\frac{3\,dx+3\,c}{4\,a}\sinh\left(\frac{a}{dx+c}\right) - \frac{3}{4}\text{Chi}\left(\frac{a}{dx+c}\right) - \frac{dx+c}{4\,a}\sinh\left(3\,\frac{a}{dx+c}\right) + \frac{3}{4}\text{Chi}\left(3\,\frac{a}{dx+c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(d*x+c))^3,x)
```

```
[Out] -1/d*a*(3/4/a*(d*x+c)*sinh(a/(d*x+c))-3/4*Chi(a/(d*x+c))-1/4/a*(d*x+c)*sinh
(3*a/(d*x+c))+3/4*Chi(3*a/(d*x+c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{8}ad\int \frac{xe^{\left(\frac{3a}{dx+c}\right)}}{d^2x^2+2cdx+c^2}dx - \frac{3}{8}ad\int \frac{xe^{\left(\frac{a}{dx+c}\right)}}{d^2x^2+2cdx+c^2}dx - \frac{3}{8}ad\int \frac{xe^{\left(-\frac{a}{dx+c}\right)}}{d^2x^2+2cdx+c^2}dx + \frac{3}{8}ad\int \frac{xe^{\left(-\frac{3a}{dx+c}\right)}}{d^2x^2+2cdx+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 3/8*a*d*integrate(x*e^(3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*a
*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*a*d*inte
grate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 3/8*a*d*integrate(
```

```
x*e^(-3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/8*x*e^(3*a/(d*x + c)
) - 3/8*x*e^(a/(d*x + c)) + 3/8*x*e^(-a/(d*x + c)) - 1/8*x*e^(-3*a/(d*x + c
))
```

Fricas [B] time = 2.13283, size = 269, normalized size = 4.56

$$\frac{2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)^{3}-3\,a\mathrm{Ei}\left(\frac{3\,a}{dx+c}\right)+3\,a\mathrm{Ei}\left(\frac{a}{dx+c}\right)+3\,a\mathrm{Ei}\left(-\frac{a}{dx+c}\right)-3\,a\mathrm{Ei}\left(-\frac{3\,a}{dx+c}\right)+6\left((dx+c)\cosh\left(\frac{a}{dx+c}\right)^{2}-dx-c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(d*x + c)*sinh(a/(d*x + c))^3 - 3*a*Ei(3*a/(d*x + c)) + 3*a*Ei(a/(d*
x + c)) + 3*a*Ei(-a/(d*x + c)) - 3*a*Ei(-3*a/(d*x + c)) + 6*((d*x + c)*cosh
(a/(d*x + c))^2 - d*x - c)*sinh(a/(d*x + c)))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{a}{dx+c}\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(a/(d*x + c))^3, x)
```

3.292 $\int \sinh\left(\frac{bx}{c+dx}\right) dx$

Optimal. Leaf size=74

$$\frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}-\frac{bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d}$$

```
[Out] (b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/d^2 + ((c + d*x)*Sinh[(b*
x)/(c + d*x)])/d - (b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.129571, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {5607, 3297, 3303, 3298, 3301}

$$\frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}-\frac{bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(b*x)/(c + d*x)],x]
```

```
[Out] (b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/d^2 + ((c + d*x)*Sinh[(b*
x)/(c + d*x)])/d - (b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/d^2
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sinh\left(\frac{bx}{c+dx}\right)dx &= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} + \frac{(bc)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}\\
&= \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} + \frac{\left(bc\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{bcx}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left(bc\sinh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin}{}\right.}{d^2}\\
&= \frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.282876, size = 70, normalized size = 0.95

$$
\frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right) - bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right) + d(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(b*x)/(c + d*x)],x]
```

```
[Out] (b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))] + d*(c + d*x)*Sinh[(b*x)/(
c + d*x)] - b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/d^2
```

Maple [A] time = 0.032, size = 113, normalized size = 1.5

$$
-\frac{dx+c}{2d}\mathrm{e}^{-\frac{bx}{dx+c}} - \frac{cb}{2d^2}\mathrm{e}^{-\frac{b}{d}}\,\text{Ei}\left(1, -\frac{cb}{d(dx+c)}\right) + \frac{x}{2}\mathrm{e}^{\frac{bx}{dx+c}} + \frac{c}{2d}\mathrm{e}^{\frac{bx}{dx+c}} - \frac{cb}{2d^2}\mathrm{e}^{\frac{b}{d}}\,\text{Ei}\left(1, \frac{cb}{d(dx+c)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x/(d*x+c)),x)
```

```
[Out] -1/2/d*exp(-b*x/(d*x+c))*(d*x+c)-1/2*c*b/d^2*exp(-b/d)*Ei(1,-b*c/d/(d*x+c))
+1/2*exp(b*x/(d*x+c))*x+1/2*c/d*exp(b*x/(d*x+c))-1/2*c*b/d^2*exp(b/d)*Ei(1,
b*c/d/(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
-\frac{1}{2}bc\int \frac{xe^{\left(\frac{bc}{d^2x+cd}\right)}}{d^2x^2e^{\frac{b}{d}} + 2cdxe^{\frac{b}{d}} + c^2e^{\frac{b}{d}}}dx - \frac{1}{2}bc\int \frac{xe^{\left(-\frac{bc}{d^2x+cd}+\frac{b}{d}\right)}}{d^2x^2+2cdx+c^2}dx - \frac{1}{2}\left(xe^{\left(\frac{bc}{d^2x+cd}\right)} - xe^{\left(-\frac{bc}{d^2x+cd}+\frac{2b}{d}\right)}\right)e^{\left(-\frac{b}{d}\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*b*c*integrate(x*e^(b*c/(d^2*x + c*d))/(d^2*x^2*e^(b/d) + 2*c*d*x*e^(b/
d) + c^2*e^(b/d)), x) - 1/2*b*c*integrate(x*e^(-b*c/(d^2*x + c*d) + b/d)/(d
^2*x^2 + 2*c*d*x + c^2), x) - 1/2*(x*e^(b*c/(d^2*x + c*d)) - x*e^(-b*c/(d^2
*x + c*d) + 2*b/d))*e^(-b/d)
```

Fricas [B] time = 1.97991, size = 532, normalized size = 7.19

$$-\frac{bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{b}{d}\right)\sinh\left(\frac{bx}{dx+c}\right)^2-\left(bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2+bc\mathrm{Ei}\left(\frac{bc}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)-2\left(d^2x+cd\right)\sinh}{2\left(d^2\cosh\left(\frac{bx}{dx+c}\right)^2-d^2\sinh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b/d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei(
-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 + b*c*Ei(b*c/(d^2*x + c*d)))*cosh
(b/d) - 2*(d^2*x + c*d)*sinh(b*x/(d*x + c)) - (b*c*Ei(-b*c/(d^2*x + c*d))*c
osh(b*x/(d*x + c))^2 - b*c*Ei(-b*c/(d^2*x + c*d))*sinh(b*x/(d*x + c))^2 - b
*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*sinh(
b*x/(d*x + c))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x)
```

```
[Out] Integral(sinh(b*x/(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{bx}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x/(d*x + c)), x)
```

3.293 $\int \sinh^2\left(\frac{bx}{c+dx}\right) dx$

Optimal. Leaf size=80

$$\frac{bc\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}-\frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d}$$

```
[Out] (b*c*CoshIntegral[(2*b*c)/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2 + ((c + d*x)*Si
nh[(b*x)/(c + d*x)]^2)/d - (b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(d*(c +
d*x))])/d^2
```

Rubi [A] time = 0.145793, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5607, 3313, 12, 3303, 3298, 3301}

$$\frac{bc\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}-\frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(b*x)/(c + d*x)]^2,x]
```

```
[Out] (b*c*CoshIntegral[(2*b*c)/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2 + ((c + d*x)*Si
nh[(b*x)/(c + d*x)]^2)/d - (b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(d*(c +
d*x))])/d^2
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh^2\left(\frac{bx}{c+dx}\right) dx = -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{(2ibc)\,\text{Subst}\left(\int \frac{i\sinh\left(\frac{2b}{d}-\frac{2bcx}{d}\right)}{2x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} + \frac{(bc)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{\left(bc\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left(bc\sinh\left(\frac{2b}{d}\right)\right)\text{Subs}}{}$$

$$= \frac{bc\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}$$

Mathematica [A] time = 0.358976, size = 85, normalized size = 1.06

$$\frac{2bc\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2bc}{d(c+dx)}\right) - 2bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right) + d\left((c+dx)\cosh\left(\frac{2bx}{c+dx}\right) - dx\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(b*x)/(c + d*x)]^2,x]
```

```
[Out] (d*(-(d*x) + (c + d*x)*Cosh[(2*b*x)/(c + d*x)]) + 2*b*c*CoshIntegral[(2*b*c
)/(d*(c + d*x))]*Sinh[(2*b)/d] - 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(
d*(c + d*x))])/(2*d^2)
```

Maple [A] time = 0.087, size = 120, normalized size = 1.5

$$-\frac{x}{2} + \frac{dx+c}{4d}e^{-2\frac{bx}{dx+c}} + \frac{cb}{2d^2}e^{-2\frac{b}{d}}\text{Ei}\left(1, -2\frac{cb}{d(dx+c)}\right) + \frac{x}{4}e^{2\frac{bx}{dx+c}} + \frac{c}{4d}e^{2\frac{bx}{dx+c}} - \frac{cb}{2d^2}e^{2\frac{b}{d}}\text{Ei}\left(1, 2\frac{cb}{d(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x/(d*x+c))^2,x)
```

```
[Out] -1/2*x+1/4/d*exp(-2*b*x/(d*x+c))*(d*x+c)+1/2*c*b/d^2*exp(-2*b/d)*Ei(1,-2*b*
c/d/(d*x+c))+1/4*exp(2*b*x/(d*x+c))*x+1/4*c/d*exp(2*b*x/(d*x+c))-1/2*c*b/d^
```

```
2*exp(2*b/d)*Ei(1,2*b*c/d/(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,bc\int\frac{xe^{\left(\frac{2\,bc}{d^2x+cd}\right)}}{d^2x^2e^{\left(\frac{2\,b}{d}\right)}+2\,cdxe^{\left(\frac{2\,b}{d}\right)}+c^2e^{\left(\frac{2\,b}{d}\right)}}\,dx-\frac{1}{2}\,bc\int\frac{xe^{\left(-\frac{2\,bc}{d^2x+cd}+\frac{2\,b}{d}\right)}}{d^2x^2+2\,cdx+c^2}\,dx+\frac{1}{4}\left(xe^{\left(\frac{2\,bc}{d^2x+cd}\right)}+xe^{\left(-\frac{2\,bc}{d^2x+cd}+\frac{4\,b}{d}\right)}\right)e^{\left(-\frac{2\,b}{d}\right)}-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*integrate(x*e^(2*b*c/(d^2*x + c*d))/(d^2*x^2*e^(2*b/d) + 2*c*d*x*e^
(2*b/d) + c^2*e^(2*b/d)), x) - 1/2*b*c*integrate(x*e^(-2*b*c/(d^2*x + c*d)
+ 2*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*(x*e^(2*b*c/(d^2*x + c*d)) + x
*e^(-2*b*c/(d^2*x + c*d) + 4*b/d))*e^(-2*b/d) - 1/2*x
```

Fricas [B] time = 2.11719, size = 589, normalized size = 7.36

$$-\frac{d^2x-\left(d^2x+cd\right)\cosh\left(\frac{bx}{dx+c}\right)^2+\left(bc\mathrm{Ei}\left(-\frac{2\,bc}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right)-d^2x-cd\right)\sinh\left(\frac{bx}{dx+c}\right)^2-\left(bc\mathrm{Ei}\left(-\frac{2\,bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2-b\right.}{2\left(d^2\cosh\left(\frac{bx}{dx+c}\right)^2-d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh(b*x/(d*x + c))^2 + (b*c*Ei(-2*b*c/(d^2*x +
 c*d))*cosh(2*b/d) - d^2*x - c*d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei(-2*b*c/(d
^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(2*b*c/(d^2*x + c*d)))*cosh(2*b/
d) - (b*c*Ei(-2*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(-2*b*c/(d
^2*x + c*d))*sinh(b*x/(d*x + c))^2 + b*c*Ei(2*b*c/(d^2*x + c*d)))*sinh(2*b/
d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*sinh(b*x/(d*x + c))^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sinh\left(\frac{bx}{dx+c}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x/(d*x + c))^2, x)
```

3.294 $\int \sinh^3\left(\frac{bx}{c+dx}\right) dx$

Optimal. Leaf size=143

$$-\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}-\frac{3bc\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\ldots$$

```
[Out] (-3*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/(4*d^2) + (3*b*c*Cosh[
(3*b)/d]*CoshIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(b*
x)/(c + d*x)]^3)/d + (3*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/(4
*d^2) - (3*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2)
```

Rubi [A] time = 0.246428, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5607, 3313, 3303, 3298, 3301}

$$-\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}-\frac{3bc\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(b*x)/(c + d*x)]^3,x]
```


```
[Out] (-3*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/(4*d^2) + (3*b*c*Cosh[
(3*b)/d]*CoshIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(b*
x)/(c + d*x)]^3)/d + (3*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/(4
*d^2) - (3*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2)
```


Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh^3\left(\frac{bx}{c+dx}\right) dx = -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} - \frac{(3bc)\,\text{Subst}\left(\int \left(-\frac{\cosh\left(\frac{3b}{d}-\frac{3bcx}{d}\right)}{4x} + \frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{4x}\right) dx, x, \frac{1}{c+dx}\right)}{d^2}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} + \frac{(3bc)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3b}{d}-\frac{3bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3bc)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x}\right.}{4d^2}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} - \frac{\left(3bc\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{\left(3bc\cosh\left(\frac{3b}{d}\right)\right)\text{Sub}}{}$$
$$= -\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2} + \frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} + \frac{3bc\sin}{}$$

Mathematica [A] time = 0.458828, size = 172, normalized size = 1.2

$$\frac{-3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right) + 3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right) - 3d^2x\sinh\left(\frac{bx}{c+dx}\right) + d^2x\sinh\left(\frac{3bx}{c+dx}\right) + 3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{}{d}\right.}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))] + 3*b*c*Cosh[(3*b)/d]*C
oshIntegral[(3*b*c)/(d*(c + d*x))] - 3*c*d*Sinh[(b*x)/(c + d*x)] - 3*d^2*x*
Sinh[(b*x)/(c + d*x)] + c*d*Sinh[(3*b*x)/(c + d*x)] + d^2*x*Sinh[(3*b*x)/(c
 + d*x)] + 3*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))] - 3*b*c*Sinh[(
3*b)/d]*SinhIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2)
```

Maple [A] time = 0.088, size = 228, normalized size = 1.6

$$-\frac{dx+c}{8d}e^{-3\frac{bx}{dx+c}} - \frac{3cb}{8d^2}e^{-3\frac{b}{d}}\text{Ei}\left(1,-3\frac{cb}{d(dx+c)}\right) + \frac{3dx+3c}{8d}e^{-\frac{bx}{dx+c}} + \frac{3cb}{8d^2}e^{-\frac{b}{d}}\text{Ei}\left(1,-\frac{cb}{d(dx+c)}\right) + \frac{x}{8}e^{3\frac{bx}{dx+c}} + \frac{c}{8d}e^{3\frac{b}{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x/(d*x+c))^3,x)
```

```
[Out] -1/8/d*exp(-3*b*x/(d*x+c))*(d*x+c)-3/8*c*b/d^2*exp(-3*b/d)*Ei(1,-3*b*c/d/(d
*x+c))+3/8/d*exp(-b*x/(d*x+c))*(d*x+c)+3/8*c*b/d^2*exp(-b/d)*Ei(1,-b*c/d/(d
```

```
*x+c))+1/8*exp(3*b*x/(d*x+c))*x+1/8*c/d*exp(3*b*x/(d*x+c))-3/8*c*b/d^2*exp(
3*b/d)*Ei(1,3*b*c/d/(d*x+c))-3/8*exp(b*x/(d*x+c))*x-3/8*c/d*exp(b*x/(d*x+c)
)+3/8*c*b/d^2*exp(b/d)*Ei(1,b*c/d/(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3}{8}bc\int\frac{xe^{\left(\frac{3bc}{d^2x+cd}\right)}}{d^2x^2e^{\left(\frac{3b}{d}\right)}+2cdxe^{\left(\frac{3b}{d}\right)}+c^2e^{\left(\frac{3b}{d}\right)}}dx+\frac{3}{8}bc\int\frac{xe^{\left(\frac{bc}{d^2x+cd}\right)}}{d^2x^2e^{\frac{b}{d}}+2cdxe^{\frac{b}{d}}+c^2e^{\frac{b}{d}}}dx+\frac{3}{8}bc\int\frac{xe^{\left(-\frac{bc}{d^2x+cd}+\frac{b}{d}\right)}}{d^2x^2+2cdx+c^2}dx-\frac{3}{8}bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -3/8*b*c*integrate(x*e^(3*b*c/(d^2*x + c*d))/(d^2*x^2*e^(3*b/d) + 2*c*d*x*e
^(3*b/d) + c^2*e^(3*b/d)), x) + 3/8*b*c*integrate(x*e^(b*c/(d^2*x + c*d))/(
d^2*x^2*e^(b/d) + 2*c*d*x*e^(b/d) + c^2*e^(b/d)), x) + 3/8*b*c*integrate(x*
e^(-b*c/(d^2*x + c*d) + b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*b*c*integr
ate(x*e^(-3*b*c/(d^2*x + c*d) + 3*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/8*
(x*e^(3*b*c/(d^2*x + c*d)) - 3*x*e^(b*c/(d^2*x + c*d) + 2*b/d) + 3*x*e^(-b*
c/(d^2*x + c*d) + 4*b/d) - x*e^(-3*b*c/(d^2*x + c*d) + 6*b/d))*e^(-3*b/d)
```

Fricas [B] time = 2.18035, size = 1524, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(3*b/d) - b*c*Ei(-b*c/(d^2*x + c*d
))*cosh(b/d))*sinh(b*x/(d*x + c))^4 + 2*(d^2*x + c*d)*sinh(b*x/(d*x + c))^3
 - 6*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(3*b/d) - b*c*
Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(b/d))*sinh(b*x/(d*x + c))
^2 + 3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^4 + b*c*Ei(3*b*c/(
d^2*x + c*d)))*cosh(3*b/d) - 3*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
c))^4 + b*c*Ei(b*c/(d^2*x + c*d)))*cosh(b/d) - 6*(d^2*x - (d^2*x + c*d)*cos
h(b*x/(d*x + c))^2 + c*d)*sinh(b*x/(d*x + c)) + 3*(b*c*Ei(-3*b*c/(d^2*x + c
*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
 c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-3*b*c/(d^2*x + c*d))*sinh(b*x/(d*x +
 c))^4 - b*c*Ei(3*b*c/(d^2*x + c*d)))*sinh(3*b/d) - 3*(b*c*Ei(-b*c/(d^2*x +
 c*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
 c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-b*c/(d^2*x + c*d))*sinh(b*x/(d*x + c
))^4 - b*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d*x + c))^4 - 2
*d^2*cosh(b*x/(d*x + c))^2*sinh(b*x/(d*x + c))^2 + d^2*sinh(b*x/(d*x + c))^
4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{bx}{dx+c}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x/(d*x + c))^3, x)
```

3.295 $\int \sinh\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=101

$$\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] ((b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c +
 d*x)*Sinh[(a + b*x)/(c + d*x)])/d - ((b*c - a*d)*Sinh[b/d]*SinhIntegral[(b
*c - a*d)/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.178836, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5607, 3297, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c +
 d*x)*Sinh[(a + b*x)/(c + d*x)])/d - ((b*c - a*d)*Sinh[b/d]*SinhIntegral[(b
*c - a*d)/(d*(c + d*x))])/d^2
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$
$$= \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left((bc-ad)\sin\right.}{}$$
$$= \frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [B] time = 0.706277, size = 373, normalized size = 3.69

$$\frac{(bc-ad)\left(\cosh\left(\frac{b}{d}\right)-\sinh\left(\frac{b}{d}\right)\right)\text{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)+(bc-ad)\left(\sinh\left(\frac{b}{d}\right)+\cosh\left(\frac{b}{d}\right)\right)\text{Chi}\left(\frac{ad-bc}{d(c+dx)}\right)+ad\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ac}{xd^2+c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)]*(Cosh[b/d] - Sinh[b/d]
) + (b*c - a*d)*CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cosh[b/d] + Sin
h[b/d]) + 2*c*d*Sinh[(a + b*x)/(c + d*x)] + 2*d^2*x*Sinh[(a + b*x)/(c + d*x
)] + b*c*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Cosh[b/
d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + b*c*Sinh[b/d]*SinhIntegral[
(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/(
d*(c + d*x))] + b*c*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - a*d
*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - b*c*Sinh[b/d]*SinhInte
gral[(b*c - a*d)/(c*d + d^2*x)] + a*d*Sinh[b/d]*SinhIntegral[(b*c - a*d)/(c
*d + d^2*x)])/(2*d^2)
```

Maple [B] time = 0.028, size = 347, normalized size = 3.4

$$-\frac{a}{2}e^{-\frac{bx+a}{dx+c}}\left(\frac{da}{dx+c}-\frac{cb}{dx+c}\right)^{-1}+\frac{cb}{2d}e^{-\frac{bx+a}{dx+c}}\left(\frac{da}{dx+c}-\frac{cb}{dx+c}\right)^{-1}+\frac{a}{2d}e^{-\frac{b}{d}}\text{Ei}\left(1,\frac{da-cb}{d(dx+c)}\right)-\frac{cb}{2d^2}e^{-\frac{b}{d}}\text{Ei}\left(1,\frac{da-cb}{d(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)/(d*x+c)),x)
```

```
[Out] -1/2*exp(-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a+1/2/d*exp(-(b*x+a)/(
d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*c*b+1/2/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x
+c))*a-1/2/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*c*b+1/2*d*exp((b*x+a)/(d
```

```
*x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*c*b+1/2*exp((b*x+
a)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+1/2/d*
exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d
*x+c))*c*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{bx+a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)/(d*x + c)), x)
```

Fricas [A] time = 2.04179, size = 344, normalized size = 3.41

$$\frac{\left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\mathrm{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)+2\left(d^2x+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)-\left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)-(bc-ad)\mathrm{Ei}\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a*
d)/(d^2*x + c*d)))*cosh(b/d) + 2*(d^2*x + c*d)*sinh((b*x + a)/(d*x + c)) -
((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)/(d
^2*x + c*d)))*sinh(b/d))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{bx+a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh((b*x + a)/(d*x + c)), x)
```

3.296 $\int \sinh^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] ((b*c - a*d)*CoshIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2
 + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*Cosh[(2*b)/d]*S
inhIntegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.18226, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5607, 3313, 12, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*CoshIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2
 + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*Cosh[(2*b)/d]*S
inhIntegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(2i(bc-ad))\,\text{Subst}\left(\int \frac{i\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$
$$= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$
$$= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad)\sin\right.}{}$$
$$= \frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

Mathematica [A] time = 0.848931, size = 112, normalized size = 1.05

$$\frac{2\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(ad-bc)}{d(c+dx)}\right)+2\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(ad-bc)}{d(c+dx)}\right)+d\left((c+dx)\cosh\left(\frac{2(a+bx)}{c+dx}\right)-dx\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] (d*(-(d*x) + (c + d*x)*Cosh[(2*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)*CoshI
ntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sinh[(2*b)/d] + 2*(b*c - a*d)*Cos
h[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(2*d^2)
```

Maple [B] time = 0.086, size = 358, normalized size = 3.4

$$-\frac{x}{2}+\frac{a}{4}e^{-2\frac{bx+a}{dx+c}}\left(\frac{da}{dx+c}-\frac{cb}{dx+c}\right)^{-1}-\frac{cb}{4d}e^{-2\frac{bx+a}{dx+c}}\left(\frac{da}{dx+c}-\frac{cb}{dx+c}\right)^{-1}-\frac{a}{2d}e^{-2\frac{b}{d}}\text{Ei}\left(1,2\frac{da-cb}{d(dx+c)}\right)+\frac{cb}{2d^2}e^{-2\frac{b}{d}}\text{Ei}\left(1,2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)/(d*x+c))^2,x)
```

```
[Out] -1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-1/4/d*exp(-2
*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*c*b-1/2/d*exp(-2*b/d)*Ei(1,2*(a
*d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2*b/d)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*c*b+1/
4*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*
c)*x*c*b+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+a)/(d*x+
c))/(a*d-b*c)*c^2*b+1/2/d*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*a-1/2/d^2
*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*c*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}x+\frac{1}{4}\int e^{\left(\frac{2\,bc}{d^2x+cd}-\frac{2\,a}{dx+c}-\frac{2\,b}{d}\right)}\,dx+\frac{1}{4}\int e^{\left(-\frac{2\,bc}{d^2x+cd}+\frac{2\,a}{dx+c}+\frac{2\,b}{d}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x)
+ 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

Fricas [B] time = 2.06622, size = 775, normalized size = 7.24

$$-\frac{d^2x-\left(d^2x+cd\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2-\left(d^2x-(bc-ad)\mathrm{Ei}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right)+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2-\left((bc-ad)\mathrm{Ei}\left(-\frac{2\,(bc-a}{d^2x+c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 - (d^2*x - (b*c - a
*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d*x
 + c))^2 - ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*
x + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) - ((b*
c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2 - (b*
c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2 + (b*
c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*x + a)/
(d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{bx+a}{dx+c}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sinh((b*x + a)/(d*x + c))^2, x)
```

3.297 $\int \sinh^3\left(\frac{a+bx}{c+dx}\right)dx$

Optimal. Leaf size=194

$$-\frac{3\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}+\frac{3\cosh\left(\frac{3b}{d}\right)(bc-ad)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\sinh}{}$$

```
[Out] (-3*(b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2)
+ (3*(b*c - a*d)*Cosh[(3*b)/d]*CoshIntegral[(3*(b*c - a*d))/(d*(c + d*x))])
/(4*d^2) + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*Sinh[
b/d]*SinhIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*Sinh
[(3*b)/d]*SinhIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d^2)
```

Rubi [A] time = 0.328356, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5607, 3313, 3303, 3298, 3301}

$$-\frac{3\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}+\frac{3\cosh\left(\frac{3b}{d}\right)(bc-ad)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)/(c + d*x)]^3,x]
```


```
[Out] (-3*(b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2)
+ (3*(b*c - a*d)*Cosh[(3*b)/d]*CoshIntegral[(3*(b*c - a*d))/(d*(c + d*x))])
/(4*d^2) + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*Sinh[
b/d]*SinhIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*Sinh
[(3*b)/d]*SinhIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d^2)
```


Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int\left(-\frac{\cosh\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{4x}+\frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{4x}\right)dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3(bc-ad))\,\text{Subst}}{}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left(3(bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{\left(3(bc-ad)\,\text{c}\right.}{}$$

$$= -\frac{3(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} + \frac{3(bc-ad)\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d}$$

Mathematica [B] time = 1.35566, size = 599, normalized size = 3.09

$$\frac{-3ad\sinh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)+3bc\sinh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)+3ad\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)-3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)+6\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (6*(b*c - a*d)*Cosh[(3*b)/d]*CoshIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))]
 - 3*b*c*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)] + 3*a*d*Cosh[b/d
]*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)] + 3*b*c*CoshIntegral[(b*c - a*d)/
(c*d + d^2*x)]*Sinh[b/d] - 3*a*d*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)]*Si
nh[b/d] - 3*(b*c - a*d)*CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cosh[b/
d] + Sinh[b/d]) - 6*c*d*Sinh[(a + b*x)/(c + d*x)] - 6*d^2*x*Sinh[(a + b*x)/
(c + d*x)] + 2*c*d*Sinh[(3*(a + b*x))/(c + d*x)] + 2*d^2*x*Sinh[(3*(a + b*x
))/(c + d*x)] - 3*b*c*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))]
+ 3*a*d*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - 3*b*c*Sinh[b
/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + 3*a*d*Sinh[b/d]*SinhIntegr
al[(-(b*c) + a*d)/(d*(c + d*x))] + 6*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*(-(b
*c) + a*d))/(d*(c + d*x))] - 6*a*d*Sinh[(3*b)/d]*SinhIntegral[(3*(-(b*c) +
a*d))/(d*(c + d*x))] - 3*b*c*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*
x)] + 3*a*d*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + 3*b*c*Sinh[
b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - 3*a*d*Sinh[b/d]*SinhIntegral
[(b*c - a*d)/(c*d + d^2*x)])/(8*d^2)
```

Maple [B] time = 0.084, size = 700, normalized size = 3.6

$$-\frac{a}{8}e^{-3\frac{bx+a}{dx+c}}\left(\frac{da}{dx+c}-\frac{cb}{dx+c}\right)^{-1}+\frac{cb}{8d}e^{-3\frac{bx+a}{dx+c}}\left(\frac{da}{dx+c}-\frac{cb}{dx+c}\right)^{-1}+\frac{3a}{8d}e^{-3\frac{b}{d}}\mathrm{Ei}\left(1,3\frac{da-cb}{d(dx+c)}\right)-\frac{3cb}{8d^2}e^{-3\frac{b}{d}}\mathrm{Ei}\left(1,3\frac{}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)/(d*x+c))^3,x)
```

```
[Out] -1/8*exp(-3*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a+1/8/d*exp(-3*(b*x+
a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*c*b+3/8/d*exp(-3*b/d)*Ei(1,3*(a*d-b*c
)/d/(d*x+c))*a-3/8/d^2*exp(-3*b/d)*Ei(1,3*(a*d-b*c)/d/(d*x+c))*c*b+3/8*exp(
-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-3/8/d*exp(-(b*x+a)/(d*x+c))/(
d*a/(d*x+c)-b*c/(d*x+c))*c*b-3/8/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*a+3/
8/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*c*b+1/8*d*exp(3*(b*x+a)/(d*x+c))/
(a*d-b*c)*x*a-1/8*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*x*c*b+1/8*exp(3*(b*x+a)/
(d*x+c))/(a*d-b*c)*c*a-1/8/d*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+3/8/d*e
xp(3*b/d)*Ei(1,-3*(a*d-b*c)/d/(d*x+c))*a-3/8/d^2*exp(3*b/d)*Ei(1,-3*(a*d-b*
c)/d/(d*x+c))*c*b-3/8*d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*a+3/8*exp((b*x+a)/
(d*x+c))/(a*d-b*c)*x*c*b-3/8*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c*a+3/8/d*exp((
b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b-3/8/d*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a
+3/8/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*c*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{bx+a}{dx+c}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)/(d*x + c))^3, x)
```

Fricas [B] time = 2.14485, size = 1536, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x +
c))^2*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^2 - 3*(b*c - a*d)*Ei(-3*(b*c -
a*d)/(d^2*x + c*d))*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^4 - 2*(d^2*x + c*
d)*sinh((b*x + a)/(d*x + c))^3 - 3*((b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x +
c*d))*cosh((b*x + a)/(d*x + c))^4 + (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*x + c
*d)))*cosh(3*b/d) + 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a
*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d) + 6*(d^2*x - (d^2*x + c*d)*co
sh((b*x + a)/(d*x + c))^2 + c*d)*sinh((b*x + a)/(d*x + c)) - 3*((b*c - a*d)
*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^4 - 2*(b*c - a*
d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2*sinh((b*x +
```

```
 a)/(d*x + c))^2 + (b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x +
 a)/(d*x + c))^4 - (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*x + c*d)))*sinh(3*b/d)
 - 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*
d)/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh((b*x + a)/(d*x + c))^4 - 2*d^2*cosh
((b*x + a)/(d*x + c))^2*sinh((b*x + a)/(d*x + c))^2 + d^2*sinh((b*x + a)/(d
*x + c))^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(\frac{bx+a}{dx+c}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sinh((b*x + a)/(d*x + c))^3, x)
```

3.298 $\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal. Leaf size=121

$$\frac{f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\operatorname{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)\operatorname{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

```
[Out] ((b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))
])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)])/d - ((b*c
- a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))])/d^2
```

Rubi [A] time = 0.259482, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5609, 5607, 3297, 3303, 3298, 3301}

$$\frac{f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\operatorname{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)\operatorname{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))
])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)])/d - ((b*c
- a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))])/d^2
```

Rule 5609

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)dx = \int \sinh\left(\frac{ce+af+(de+bf)x}{c+dx}\right)dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{((bc-ad)f)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{de+bf}{d}-\frac{(bc-ad)fx}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$
$$= \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{\left((bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)fx}{d}\right)}{x}dx, x, \frac{1}{c+d}\right.}{d^2}$$
$$= \frac{(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(bc-ad)f\sinh\Big(}{}$$

Mathematica [B] time = 1.43234, size = 449, normalized size = 3.71

$$\frac{f(bc-ad)\left(\cosh\left(\frac{bf}{d}+e\right)-\sinh\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)+f(bc-ad)\left(\sinh\left(\frac{bf}{d}+e\right)+\cosh\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{adf-bcf}{d(c+dx)}\right)+2d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*f*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))]*(Cosh[e + (b*f)/
d] - Sinh[e + (b*f)/d]) + (b*c - a*d)*f*CoshIntegral[(-(b*c*f) + a*d*f)/(d*
(c + d*x))]*(Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) + 2*c*d*Sinh[(c*e + a*f
 + d*e*x + b*f*x)/(c + d*x)] + 2*d^2*x*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c
+ d*x)] + b*c*f*Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x)
)] - a*d*f*Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] -
b*c*f*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + a*d*f
*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + b*c*f*Cosh
[e + (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] - a*d*f*Cosh[e
 + (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + b*c*f*Sinh[e +
 (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] - a*d*f*Sinh[e + (
b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))])/(2*d^2)
```

Maple [B] time = 0.043, size = 459, normalized size = 3.8

$$-\frac{af}{2}e^{-\frac{bfx+dex+af+ce}{dx+c}}\left(\frac{adf}{dx+c}-\frac{bcf}{dx+c}\right)^{-1}+\frac{bcf}{2d}e^{-\frac{bfx+dex+af+ce}{dx+c}}\left(\frac{adf}{dx+c}-\frac{bcf}{dx+c}\right)^{-1}+\frac{af}{2d}e^{-\frac{bf+de}{d}}\text{Ei}\left(1,\frac{f(da-cb)}{d(dx+c)}\right)-\frac{bcf}{2d^2}e^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c)),x)
```

```
[Out] -1/2*f*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*a+
1/2/d*f*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*c
*b+1/2/d*f*exp(-(b*f+d*e)/d)*Ei(1,(a*d-b*c)*f/d/(d*x+c))*a-1/2/d^2*f*exp(-(
b*f+d*e)/d)*Ei(1,(a*d-b*c)*f/d/(d*x+c))*c*b+1/2/d*f*exp((b*f*x+d*e*x+a*f+c*
e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*a-1/2/d^2*f*exp((b*f*x+d*e*x+a*
f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*c*b+1/2/d*f*exp((b*f+d*e)/d
)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*a-1/2/d^2*f*exp((b*
f+d*e)/d)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*c*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(e + \frac{(bx + a)f}{dx + c}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sinh(e + (b*x + a)*f/(d*x + c)), x)
```

Fricas [A] time = 2.04329, size = 417, normalized size = 3.45

$$\frac{\left((bc-ad)f\,\mathrm{Ei}\left(\frac{(bc-ad)f}{d^2x+cd}\right) + (bc-ad)f\,\mathrm{Ei}\left(-\frac{(bc-ad)f}{d^2x+cd}\right)\right)\cosh\left(\frac{de+bf}{d}\right) + 2\left(d^2x+cd\right)\sinh\left(\frac{ce+af+(de+bf)x}{dx+c}\right) - \left((bc-ad)f\,\mathrm{Ei}\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) + (b*c - a*d)*f*Ei(-(b*
c - a*d)*f/(d^2*x + c*d)))*cosh((d*e + b*f)/d) + 2*(d^2*x + c*d)*sinh((c*e
+ a*f + (d*e + b*f)*x)/(d*x + c)) - ((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x
+ c*d)) - (b*c - a*d)*f*Ei(-(b*c - a*d)*f/(d^2*x + c*d)))*sinh((d*e + b*f)/
d))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(e+\frac{(bx+a)f}{dx+c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(e + (b*x + a)*f/(d*x + c)), x)
```

3.299 $\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right) dx$

Optimal. Leaf size=129

$$\frac{f(bc-ad)\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{af+bfx+ce+}{c+dx}\right.}{d}$$

```
[Out] ((b*c - a*d)*f*CoshIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))]*Sinh[2*(e + (b
*f)/d)])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)]^2)/d
- ((b*c - a*d)*f*Cosh[2*(e + (b*f)/d)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c
 + d*x))])/d^2
```

Rubi [A] time = 0.279059, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5609, 5607, 3313, 12, 3303, 3298, 3301}

$$\frac{f(bc-ad)\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{af+bfx+ce+}{c+dx}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*f*CoshIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))]*Sinh[2*(e + (b
*f)/d)])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)]^2)/d
- ((b*c - a*d)*f*Cosh[2*(e + (b*f)/d)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c
 + d*x))])/d^2
```

Rule 5609

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)dx = \int \sinh^2\left(\frac{ce+af+(de+bf)x}{c+dx}\right)dx$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(2i(bc-ad)f)\,\text{Subst}\left(\int \frac{i\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-ad)fx}{d}\right)}{2x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{((bc-ad)f)\,\text{Subst}\left(\int \frac{\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-ad)fx}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{\left((bc-ad)f\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(bc-ad)fx}{d}\right)}{x}dx\right.}{d^2}$$

$$= \frac{(bc-ad)f\,\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)\sinh\left(2\left(e+\frac{bf}{d}\right)\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(bc-ad)f}{}$$

Mathematica [A] time = 2.00019, size = 136, normalized size = 1.05

$$\frac{2f(bc-ad)\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{2(adf-bcf)}{d(c+dx)}\right)+2f(bc-ad)\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{2(adf-bcf)}{d(c+dx)}\right)+d\left((c+dx)\cosh\left(\frac{2(af+bf}{c+}\right.\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

```
[Out] (d*(-(d*x) + (c + d*x)*Cosh[(2*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)]) + 2
*(b*c - a*d)*f*CoshIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))]*Sinh[2*(e
 + (b*f)/d)] + 2*(b*c - a*d)*f*Cosh[2*(e + (b*f)/d)]*SinhIntegral[(2*(-(b*c
*f) + a*d*f))/(d*(c + d*x))])/(2*d^2)
```

Maple [B] time = 0.097, size = 468, normalized size = 3.6

$$-\frac{x}{2}+\frac{af}{4}\mathrm{e}^{-2\frac{bfx+dex+af+ce}{dx+c}}\left(\frac{adf}{dx+c}-\frac{bcf}{dx+c}\right)^{-1}-\frac{bcf}{4d}\mathrm{e}^{-2\frac{bfx+dex+af+ce}{dx+c}}\left(\frac{adf}{dx+c}-\frac{bcf}{dx+c}\right)^{-1}-\frac{af}{2d}\mathrm{e}^{-2\frac{bf+de}{d}}\mathrm{Ei}\left(1,2\frac{f(da-cb}{d(dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c))^2,x)
```

```
[Out] -1/2*x+1/4*f*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)
*c*b)*a-1/4/d*f*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x
+c)*c*b)*c*b-1/2/d*f*exp(-2*(b*f+d*e)/d)*Ei(1,2*(a*d-b*c)*f/d/(d*x+c))*a+1/
2/d^2*f*exp(-2*(b*f+d*e)/d)*Ei(1,2*(a*d-b*c)*f/d/(d*x+c))*c*b+1/4/d*f*exp(2
*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*a-1/4/d^2*f
*exp(2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*c*b+1
/2/d*f*exp(2*(b*f+d*e)/d)*Ei(1,-2*(a*d-b*c)*f/d/(d*x+c)-2*(b*f+d*e)/d-2*(-b
*f-d*e)/d)*a-1/2/d^2*f*exp(2*(b*f+d*e)/d)*Ei(1,-2*(a*d-b*c)*f/d/(d*x+c)-2*(
b*f+d*e)/d-2*(-b*f-d*e)/d)*c*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}x+\frac{1}{4}\int e^{\left(\frac{2bcf}{d^2x+cd}-2e-\frac{2af}{dx+c}-\frac{2bf}{d}\right)}dx+\frac{1}{4}\int e^{\left(-\frac{2bcf}{d^2x+cd}+2e+\frac{2af}{dx+c}+\frac{2bf}{d}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/4*integrate(e^(2*b*c*f/(d^2*x + c*d) - 2*e - 2*a*f/(d*x + c) - 2
*b*f/d), x) + 1/4*integrate(e^(-2*b*c*f/(d^2*x + c*d) + 2*e + 2*a*f/(d*x +
c) + 2*b*f/d), x)
```

Fricas [B] time = 2.31664, size = 1018, normalized size = 7.89

$$-\frac{d^2x-\left(d^2x+cd\right)\cosh\left(\frac{ce+af+\left(de+bf\right)x}{dx+c}\right)^2+\left(\left(bc-ad\right)f\mathrm{Ei}\left(-\frac{2\left(bc-ad\right)f}{d^2x+cd}\right)\cosh\left(\frac{2\left(de+bf\right)}{d}\right)-d^2x-cd\right)\sinh\left(\frac{ce+af+\left(de+bf\right)x}{dx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 +
 ((b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh(2*(d*e + b*f)/d) -
d^2*x - c*d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 - ((b*c - a*d)*f
*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x +
 c))^2 - (b*c - a*d)*f*Ei(2*(b*c - a*d)*f/(d^2*x + c*d)))*cosh(2*(d*e + b*f
)/d) - ((b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f +
(d*e + b*f)*x)/(d*x + c))^2 - (b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*
d))*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + (b*c - a*d)*f*Ei(2*(b*c
 - a*d)*f/(d^2*x + c*d)))*sinh(2*(d*e + b*f)/d))/(d^2*cosh((c*e + a*f + (d*
e + b*f)*x)/(d*x + c))^2 - d^2*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^
```

```
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(e + \frac{(bx + a)f}{dx + c}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(e + (b*x + a)*f/(d*x + c))^2, x)
```

3.300 $\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)dx$

Optimal. Leaf size=226

$$-\frac{3f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\mathrm{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\mathrm{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)}{4d^2}$$

```
[Out] (-3*(b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*
x))])/(4*d^2) + (3*(b*c - a*d)*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(b*c
 - a*d)*f)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b
*f*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b
*c - a*d)*f)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*f*Sinh[3*(e + (b*f)/d
)]*SinhIntegral[(3*(b*c - a*d)*f)/(d*(c + d*x))])/(4*d^2)
```

Rubi [A] time = 0.476477, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5609, 5607, 3313, 3303, 3298, 3301}

$$-\frac{3f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\mathrm{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\mathrm{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```


```
[Out] (-3*(b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*
x))])/(4*d^2) + (3*(b*c - a*d)*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(b*c
 - a*d)*f)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b
*f*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b
*c - a*d)*f)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*f*Sinh[3*(e + (b*f)/d
)]*SinhIntegral[(3*(b*c - a*d)*f)/(d*(c + d*x))])/(4*d^2)
```


Rule 5609

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)dx = \int \sinh^3\left(\frac{ce+af+(de+bf)x}{c+dx}\right)dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(3(bc-ad)f)\,\text{Subst}\left(\int\left(-\frac{\cosh\left(3\left(e+\frac{bf}{d}\right)-\frac{3(bc-ad)fx}{d}\right)}{4x}+\frac{\cosh\left(e+\right.}{}\right.\right.}{d^2}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{(3(bc-ad)f)\,\text{Subst}\left(\int \frac{\cosh\left(3\left(e+\frac{bf}{d}\right)-\frac{3(bc-ad)fx}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{\left(3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)fx}{d}\right)}{x}dx, x,\right.}{4d^2}$$
$$= -\frac{3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2} + \frac{3(bc-ad)f\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\text{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}$$

Mathematica [B] time = 6.18071, size = 671, normalized size = 2.97

$$\frac{-6adf\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{3(adf-bcf)}{d(c+dx)}\right)+6bcf\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{3(adf-bcf)}{d(c+dx)}\right)+3f(bc-ad)\left(\sinh\left(\frac{bf}{d}+e\right)-\cosh\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```

```
[Out] (6*b*c*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(-(b*c*f) + a*d*f))/(d*(c +
d*x))] - 6*a*d*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(-(b*c*f) + a*d*f))/
(d*(c + d*x))] + 3*(b*c - a*d)*f*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))
]*(-Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) - 3*(b*c - a*d)*f*CoshIntegral[(
-(b*c*f) + a*d*f)/(d*(c + d*x))]*(Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) -
6*c*d*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)] - 6*d^2*x*Sinh[(c*e + a*f
 + d*e*x + b*f*x)/(c + d*x)] + 2*c*d*Sinh[(3*(c*e + a*f + d*e*x + b*f*x))/(
c + d*x)] + 2*d^2*x*Sinh[(3*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)] - 3*b*c
```

```
*f*Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + 3*a*d*f*
Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + 3*b*c*f*Sin
h[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] - 3*a*d*f*Sinh[e
 + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] - 3*b*c*f*Cosh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + 3*a*d*f*Cosh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] - 3*b*c*f*Sinh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + 3*a*d*f*Sinh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + 6*b*c*f*Sinh[3*(e
 + (b*f)/d)]*SinhIntegral[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))] - 6*a*d*f*S
inh[3*(e + (b*f)/d)]*SinhIntegral[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))])/(8
*d^2)
```

Maple [B] time = 0.105, size = 922, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c))^3,x)
```

```
[Out] -1/8*f*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*
a+1/8/d*f*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*
b)*c*b+3/8/d*f*exp(-3*(b*f+d*e)/d)*Ei(1,3*(a*d-b*c)*f/d/(d*x+c))*a-3/8/d^2*
f*exp(-3*(b*f+d*e)/d)*Ei(1,3*(a*d-b*c)*f/d/(d*x+c))*c*b+3/8*f*exp(-(b*f*x+d
*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*a-3/8/d*f*exp(-(b*f*x+
d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*c*b-3/8/d*f*exp(-(b*f
+d*e)/d)*Ei(1,(a*d-b*c)*f/d/(d*x+c))*a+3/8/d^2*f*exp(-(b*f+d*e)/d)*Ei(1,(a*
d-b*c)*f/d/(d*x+c))*c*b+1/8/d*f*exp(3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*
x+c)*a*f-f/d/(d*x+c)*c*b)*a-1/8/d^2*f*exp(3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/
(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*c*b+3/8/d*f*exp(3*(b*f+d*e)/d)*Ei(1,-3*(a*d
-b*c)*f/d/(d*x+c)-3*(b*f+d*e)/d-3*(-b*f-d*e)/d)*a-3/8/d^2*f*exp(3*(b*f+d*e)
/d)*Ei(1,-3*(a*d-b*c)*f/d/(d*x+c)-3*(b*f+d*e)/d-3*(-b*f-d*e)/d)*c*b-3/8/d*f
*exp((b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*a+3/8/d
^2*f*exp((b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*c*b
-3/8/d*f*exp((b*f+d*e)/d)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e
)/d)*a+3/8/d^2*f*exp((b*f+d*e)/d)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(
-b*f-d*e)/d)*c*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(e + \frac{(bx + a)f}{dx + c}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sinh(e + (b*x + a)*f/(d*x + c))^3, x)
```

Fricas [B] time = 2.55016, size = 2047, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f +
(d*e + b*f)*x)/(d*x + c))^2*cosh(3*(d*e + b*f)/d)*sinh((c*e + a*f + (d*e +
b*f)*x)/(d*x + c))^2 - 3*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*c
osh(3*(d*e + b*f)/d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - 2*(d^2
*x + c*d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^3 - 3*((b*c - a*d)*f*
Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x +
c))^4 + (b*c - a*d)*f*Ei(3*(b*c - a*d)*f/(d^2*x + c*d)))*cosh(3*(d*e + b*f)
/d) + 3*((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) + (b*c - a*d)*f*Ei(-
(b*c - a*d)*f/(d^2*x + c*d)))*cosh((d*e + b*f)/d) + 6*(d^2*x - (d^2*x + c*d
)*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + c*d)*sinh((c*e + a*f + (d
*e + b*f)*x)/(d*x + c)) - 3*((b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d
))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - 2*(b*c - a*d)*f*Ei(-3*(b
*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2*si
nh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + (b*c - a*d)*f*Ei(-3*(b*c - a*
d)*f/(d^2*x + c*d))*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - (b*c -
a*d)*f*Ei(3*(b*c - a*d)*f/(d^2*x + c*d)))*sinh(3*(d*e + b*f)/d) - 3*((b*c -
 a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) - (b*c - a*d)*f*Ei(-(b*c - a*d)*f/(
d^2*x + c*d)))*sinh((d*e + b*f)/d))/(d^2*cosh((c*e + a*f + (d*e + b*f)*x)/(
d*x + c))^4 - 2*d^2*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2*sinh((c*e
 + a*f + (d*e + b*f)*x)/(d*x + c))^2 + d^2*sinh((c*e + a*f + (d*e + b*f)*x)
/(d*x + c))^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh\left(e + \frac{(bx+a)f}{dx+c}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(e + (b*x + a)*f/(d*x + c))^3, x)
```

3.301 $\int e^{a+bx}\sinh^4(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] -E^(-3*a - 3*b*x)/(48*b) + E^(-a - b*x)/(4*b) + (3*E^(a + b*x))/(8*b) - E^(
3*a + 3*b*x)/(12*b) + E^(5*a + 5*b*x)/(80*b)
```

Rubi [A] time = 0.0395487, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 270}

$$-\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^4,x]
```

```
[Out] -E^(-3*a - 3*b*x)/(48*b) + E^(-a - b*x)/(4*b) + (3*E^(a + b*x))/(8*b) - E^(
3*a + 3*b*x)/(12*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\sinh^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{16x^4}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^4}\,dx,x,e^{a+bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(6+\frac{1}{x^4}-\frac{4}{x^2}-4x^2+x^4\right)dx,x,e^{a+bx}\right)}{16b}\\ &= -\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}\end{aligned}$$

Mathematica [A] time = 0.0407869, size = 62, normalized size = 0.75

$$\frac{e^{-3(a+bx)}\left(60e^{2(a+bx)}+90e^{4(a+bx)}-20e^{6(a+bx)}+3e^{8(a+bx)}-5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^4,x]
```

```
[Out] (-5 + 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) - 20*E^(6*(a + b*x)) + 3*E^(8
*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

Maple [A] time = 0.008, size = 77, normalized size = 0.9

$$\frac{1}{b}\left(\left(\frac{8}{15}+\frac{(\sinh(bx+a))^4}{5}-\frac{4\,(\sinh(bx+a))^2}{15}\right)\cosh(bx+a)+\frac{(\sinh(bx+a))^3\,(\cosh(bx+a))^2}{5}-\frac{\sinh(bx+a)\,(\cosh(}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a)^4,x)
```

```
[Out] 1/b*((8/15+1/5*sinh(b*x+a)^4-4/15*sinh(b*x+a)^2)*cosh(b*x+a)+1/5*sinh(b*x+a
)^3*cosh(b*x+a)^2-1/5*sinh(b*x+a)*cosh(b*x+a)^2+1/5*sinh(b*x+a))
```

Maxima [A] time = 1.17974, size = 92, normalized size = 1.11

$$\frac{e^{(5\,bx+5\,a)}}{80\,b}-\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{3\,e^{(bx+a)}}{8\,b}+\frac{e^{(-bx-a)}}{4\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/80*e^(5*b*x + 5*a)/b - 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b + 1/4*e
^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

Fricas [A] time = 2.21301, size = 325, normalized size = 3.92

$$-\frac{\cosh(bx+a)^4-16\,\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\,\cosh(bx+a)^2-10\right)\sinh(bx+a)^2-20\,\cosh(}{120\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(b*x + a)^2 - 20*cosh(b*x + a)^2 - 16*(c
osh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) - b*
sinh(b*x + a))
```

Sympy [A] time = 104.606, size = 139, normalized size = 1.67

$$\begin{cases} \frac{e^a e^{bx} \sinh^4(a+bx)}{5b} + \frac{4e^a e^{bx} \sinh^3(a+bx)\cosh(a+bx)}{5b} - \frac{4e^a e^{bx} \sinh^2(a+bx)\cosh^2(a+bx)}{5b} - \frac{8e^a e^{bx} \sinh(a+bx)\cosh^3(a+bx)}{15b} + \frac{8e^a e^{bx} \cosh^4(a+bx)}{15b} \\ xe^a \sinh^4(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**4/(5*b) + 4*exp(a)*exp(b*x)*sinh(
a + b*x)**3*cosh(a + b*x)/(5*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(5*b) - 8*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*b)
+ 8*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a)**
4, True))
```

Giac [A] time = 1.14718, size = 81, normalized size = 0.98

$$\frac{5\left(12\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+3\,e^{(5\,bx+5\,a)}-20\,e^{(3\,bx+3\,a)}+90\,e^{(bx+a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/240*(5*(12*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + 3*e^(5*b*x + 5*a) - 20
*e^(3*b*x + 3*a) + 90*e^(b*x + a))/b
```

3.302 $\int e^{a+bx} \sinh^3(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

```
[Out] E^(-2*a - 2*b*x)/(16*b) - (3*E^(2*a + 2*b*x))/(16*b) + E^(4*a + 4*b*x)/(32*
b) + (3*x)/8
```

Rubi [A] time = 0.0384831, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 266, 43}

$$\frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-2*a - 2*b*x)/(16*b) - (3*E^(2*a + 2*b*x))/(16*b) + E^(4*a + 4*b*x)/(32*
b) + (3*x)/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{a+bx}\sinh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{8x^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{x^3}\,dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{2a+2bx}\right)}{16b}$$
$$= \frac{e^{-2a-2bx}}{16b} - \frac{3e^{2a+2bx}}{16b} + \frac{e^{4a+4bx}}{32b} + \frac{3x}{8}$$

Mathematica [A] time = 0.0336144, size = 45, normalized size = 0.79

$$\frac{e^{-2(a+bx)} - 3e^{2(a+bx)} + \frac{1}{2}e^{4(a+bx)} + 6bx}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(-2*(a + b*x)) - 3*E^(2*(a + b*x)) + E^(4*(a + b*x))/2 + 6*b*x)/(16*b)
```

Maple [A] time = 0.006, size = 67, normalized size = 1.2

$$\frac{1}{b}\left(\left(\frac{(\sinh(bx+a))^3}{4} - \frac{3\sinh(bx+a)}{8}\right)\cosh(bx+a) + \frac{3bx}{8} + \frac{3a}{8} + \frac{(\sinh(bx+a))^2(\cosh(bx+a))^2}{4} - \frac{(\cosh(bx+}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*((1/4*sinh(b*x+a)^3-3/8*sinh(b*x+a))*cosh(b*x+a)+3/8*b*x+3/8*a+1/4*sinh
(b*x+a)^2*cosh(b*x+a)^2-1/4*cosh(b*x+a)^2)
```

Maxima [A] time = 1.11444, size = 72, normalized size = 1.26

$$\frac{3(bx+a)}{8b} + \frac{e^{(4bx+4a)}}{32b} - \frac{3e^{(2bx+2a)}}{16b} + \frac{e^{(-2bx-2a)}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b - 3/16*e^(2*b*x + 2*a)/b + 1/16*e^
(-2*b*x - 2*a)/b
```

Fricas [B] time = 2.05265, size = 259, normalized size = 4.54

$$\frac{3\,\cosh\left(bx+a\right)^3+9\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-\sinh\left(bx+a\right)^3+6\,(2\,bx-1)\cosh\left(bx+a\right)-3\left(4\,bx+\cosh\left(bx+a\right)^2\right.}{32\,(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a)^3
 + 6*(2*b*x - 1)*cosh(b*x + a) - 3*(4*b*x + cosh(b*x + a)^2 + 2)*sinh(b*x +
 a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 27.0309, size = 187, normalized size = 3.28

$$\begin{cases}\frac{3xe^ae^{bx}\sinh^3(a+bx)}{8}-\frac{3xe^ae^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8}-\frac{3xe^ae^{bx}\sinh(a+bx)\cosh^2(a+bx)}{8}+\frac{3xe^ae^{bx}\cosh^3(a+bx)}{8}+\frac{5e^ae^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8b}\\ xe^a\sinh^3(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*sin
h(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a +
b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 + 5*exp(a)*exp(b*x)*sinh
(a + b*x)**2*cosh(a + b*x)/(8*b) - exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b
*x)**2/(8*b) - exp(a)*exp(b*x)*cosh(a + b*x)**3/(4*b), Ne(b, 0)), (x*exp(a)
*sinh(a)**3, True))
```

Giac [A] time = 1.13775, size = 77, normalized size = 1.35

$$\frac{12\,bx-2\left(3\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)}+12\,a+e^{(4\,bx+4\,a)}-6\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*x
 + 4*a) - 6*e^(2*b*x + 2*a))/b
```

3.303 $\int e^{a+bx}\sinh^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{e^{-a-bx}}{4b}-\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

```
[Out] -E^(-a - b*x)/(4*b) - E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(12*b)
```

Rubi [A] time = 0.0312144, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 270}

$$-\frac{e^{-a-bx}}{4b}-\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-a - b*x)/(4*b) - E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(12*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\sinh^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{4x^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx,x,e^{a+bx}\right)}{4b}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx,x,e^{a+bx}\right)}{4b}\\ &= -\frac{e^{-a-bx}}{4b}-\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}\end{aligned}$$

Mathematica [A] time = 0.0205688, size = 39, normalized size = 0.8

$$\frac{e^{-a-bx}\left(-6e^{2(a+bx)}+e^{4(a+bx)}-3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(-a - b*x)*(-3 - 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

Maple [A] time = 0.008, size = 49, normalized size = 1.

$$\frac{1}{b}\left(\left(-\frac{2}{3}+\frac{(\sinh(bx+a))^2}{3}\right)\cosh(bx+a)+\frac{\sinh(bx+a)(\cosh(bx+a))^2}{3}-\frac{\sinh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*((-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+1/3*sinh(b*x+a)*cosh(b*x+a)^2-1/3
*sinh(b*x+a))
```

Maxima [A] time = 1.05769, size = 54, normalized size = 1.1

$$\frac{e^{(3bx+3a)}}{12b}-\frac{e^{(bx+a)}}{2b}-\frac{e^{(-bx-a)}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b
```

Fricas [A] time = 1.99496, size = 154, normalized size = 3.14

$$-\frac{\cosh(bx+a)^2-4\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+3}{6(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 3
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [A] time = 7.02315, size = 78, normalized size = 1.59

$$\begin{cases}\frac{e^{a}e^{bx}\sinh^2(a+bx)}{3b}+\frac{2e^{a}e^{bx}\sinh(a+bx)\cosh(a+bx)}{3b}-\frac{2e^{a}e^{bx}\cosh^2(a+bx)}{3b} & \text{for } b\neq 0\\ xe^{a}\sinh^2(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*sinh(
a + b*x)*cosh(a + b*x)/(3*b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), Ne
(b, 0)), (x*exp(a)*sinh(a)**2, True))
```

Giac [A] time = 1.17573, size = 46, normalized size = 0.94

$$\frac{e^{(3\,bx+3\,a)} - 6\,e^{(bx+a)} - 3\,e^{(-bx-a)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*(e^(3*b*x + 3*a) - 6*e^(b*x + a) - 3*e^(-b*x - a))/b
```

3.304 $\int e^{a+bx} \sinh(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{2a+2bx}}{4b}-\frac{x}{2}$$

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2
```

Rubi [A] time = 0.0158541, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 12, 14}

$$\frac{e^{2a+2bx}}{4b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{a+bx}\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx, x, e^{a+bx}\right)}{2b}\\ &= \frac{e^{2a+2bx}}{4b}-\frac{x}{2}\end{aligned}$$

Mathematica [A] time = 0.01181, size = 23, normalized size = 1.

$$\frac{e^{2a+2bx}}{4b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2
```

Maple [A] time = 0.006, size = 37, normalized size = 1.6

$$\frac{1}{b}\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}+\frac{(\cosh(bx+a))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a+1/2*cosh(b*x+a)^2)
```

Maxima [A] time = 1.12454, size = 32, normalized size = 1.39

$$-\frac{1}{2}x-\frac{a}{2b}+\frac{e^{(2bx+2a)}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

Fricas [B] time = 1.92982, size = 132, normalized size = 5.74

$$-\frac{(2bx-1)\cosh(bx+a)-(2bx+1)\sinh(bx+a)}{4(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*x - 1)*cosh(b*x + a) - (2*b*x + 1)*sinh(b*x + a))/(b*cosh(b*x +
a) - b*sinh(b*x + a))
```

Sympy [A] time = 1.6389, size = 63, normalized size = 2.74

$$\begin{cases}\frac{xe^ae^{bx}\sinh(a+bx)}{2}-\frac{xe^ae^{bx}\cosh(a+bx)}{2}+\frac{e^ae^{bx}\cosh(a+bx)}{2b} & \text{for } b\neq 0\\ xe^a\sinh(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(a)*exp(b*x)*sinh(a + b*x)/2 - x*exp(a)*exp(b*x)*cosh(a + b
*x)/2 + exp(a)*exp(b*x)*cosh(a + b*x)/(2*b), Ne(b, 0)), (x*exp(a)*sinh(a),
True))
```

Giac [A] time = 1.13826, size = 32, normalized size = 1.39

$$-\frac{2\,bx+2\,a-e^{(2\,bx+2\,a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x + 2*a - e^(2*b*x + 2*a))/b
```

3.305 $\int e^{a+bx}\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=19

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] Log[1 - E^(2*a + 2*b*x)]/b
```

Rubi [A] time = 0.0189866, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 12, 260}

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x],x]

[Out] Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\operatorname{csch}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\log\left(1-e^{2a+2bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0108075, size = 19, normalized size = 1.

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x],x]
```

```
[Out] Log[1 - E^(2*a + 2*b*x)]/b
```

Maple [A] time = 0.006, size = 19, normalized size = 1.

$$x+\frac{\ln\left(\sinh\left(bx+a\right)\right)}{b}+\frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a),x)
```

```
[Out] x+1/b*ln(sinh(b*x+a))+a/b
```

Maxima [A] time = 1.11127, size = 36, normalized size = 1.89

$$\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Fricas [A] time = 2.0032, size = 76, normalized size = 4.

$$\frac{\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x), x)
```

Giac [A] time = 1.13005, size = 23, normalized size = 1.21

$$\frac{\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(e^(2*b*x + 2*a) - 1))/b
```

3.306 $\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] time = 0.0308837, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 288, 206}

$$\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^2,x]

[Out] (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{4x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{4\,\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.0694984, size = 38, normalized size = 0.9

$$
\frac{2\left(-\frac{e^{a+bx}}{e^{2(a+bx)}-1} - \tanh^{-1}\left(e^{a+bx}\right)\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^2,x]
```

```
[Out] (2*(-(E^(a + b*x)/(-1 + E^(2*(a + b*x)))) - ArcTanh[E^(a + b*x)]))/b
```

Maple [A] time = 0.009, size = 39, normalized size = 0.9

$$
\frac{1}{b}\left(-2\,\text{Artanh}\left(e^{bx+a}\right) - \frac{(\cosh(bx+a))^2}{\sinh(bx+a)} + \sinh(bx+a)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(-2*arctanh(exp(b*x+a))-1/sinh(b*x+a)*cosh(b*x+a)^2+sinh(b*x+a))
```

Maxima [A] time = 1.11602, size = 70, normalized size = 1.67

$$
-\frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b} - \frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x + a)/(b*(e^(2*b
*x + 2*a) - 1))
```

Fricas [B] time = 1.95403, size = 462, normalized size = 11.

$$-\frac{\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\left(\cosh\left(b\right.\right.}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 2*cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{csch}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**2, x)
```

Giac [A] time = 1.12928, size = 72, normalized size = 1.71

$$-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}-\frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -log(e^(b*x + a) + 1)/b + log(abs(e^(b*x + a) - 1))/b - 2*e^(b*x + a)/(b*(e
^(2*b*x + 2*a) - 1))
```

3.307 $\int e^{a+bx}\mathrm{csch}^3(a+bx)\,dx$

Optimal. Leaf size=31

$$-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[Out] (-2*E^(4*a + 4*b*x))/(b*(1 - E^(2*a + 2*b*x))^2)
```

Rubi [A] time = 0.0280665, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 264}

$$-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^3,x]
```

```
[Out] (-2*E^(4*a + 4*b*x))/(b*(1 - E^(2*a + 2*b*x))^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\mathrm{csch}^3(a+bx)\,dx &= \frac{\mathrm{Subst}\left(\int \frac{8x^3}{\left(-1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8\,\mathrm{Subst}\left(\int \frac{x^3}{\left(-1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0171625, size = 29, normalized size = 0.94

$$-\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^3,x]
```

```
[Out] (-2*E^(4*a + 4*b*x))/(b*(-1 + E^(2*a + 2*b*x))^2)
```

Maple [A] time = 0.045, size = 32, normalized size = 1.

$$\frac{1}{b}\left(-\coth(bx+a)-\frac{(\cosh(bx+a))^2}{2\,(\sinh(bx+a))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-coth(b*x+a)-1/2/sinh(b*x+a)^2*cosh(b*x+a)^2)
```

Maxima [B] time = 1.11336, size = 92, normalized size = 2.97

$$-\frac{4\,e^{(2\,bx+2\,a)}}{b\left(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1\right)}+\frac{2}{b\left(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1)) + 2/(b*(e^
(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

Fricas [B] time = 1.92866, size = 235, normalized size = 7.58

$$-\frac{2\,(\cosh(bx+a)+3\,\sinh(bx+a))}{b\cosh(bx+a)^3+3\,b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3-b\cosh(bx+a)+3\left(b\cosh(bx+a)^2-b\right)\sinh(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)
^2 - b)*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{csch}^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**3, x)
```

Giac [A] time = 1.15393, size = 42, normalized size = 1.35

$$-\frac{2\left(2\,e^{(2\,bx+2\,a)}-1\right)}{b\left(e^{(2\,bx+2\,a)}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

3.308 $\int e^{a+bx}\mathbf{csch}^4(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3}+\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (8*E^(3*a + 3*b*x))/(3*b*(1 - E^(2*a + 2*b*x))^3) - (2*E^(a + b*x))/(b*(1 -
 E^(2*a + 2*b*x))^2) + E^(a + b*x)/(b*(1 - E^(2*a + 2*b*x))) + ArcTanh[E^(a
 + b*x)]/b
```

Rubi [A] time = 0.0536303, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2282, 12, 288, 199, 206}

$$\frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3}+\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^4,x]
```

```
[Out] (8*E^(3*a + 3*b*x))/(3*b*(1 - E^(2*a + 2*b*x))^3) - (2*E^(a + b*x))/(b*(1 -
 E^(2*a + 2*b*x))^2) + E^(a + b*x)/(b*(1 - E^(2*a + 2*b*x))) + ArcTanh[E^(a
 + b*x)]/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{16x^4}{(1-x^2)^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{16\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{8\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{2\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [A] time = 0.0726207, size = 75, normalized size = 0.74

$$\frac{3e^{a+bx} - 8e^{3(a+bx)} - 3e^{5(a+bx)} + 3\left(e^{2(a+bx)}-1\right)^3\tanh^{-1}\left(e^{a+bx}\right)}{3b\left(e^{2(a+bx)}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^4,x]
```

```
[Out] (3*E^(a + b*x) - 8*E^(3*(a + b*x)) - 3*E^(5*(a + b*x)) + 3*(-1 + E^(2*(a +
b*x)))^3*ArcTanh[E^(a + b*x)])/(3*b*(-1 + E^(2*(a + b*x)))^3)
```

Maple [A] time = 0.045, size = 71, normalized size = 0.7

$$\frac{1}{b}\left(-\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2} + \operatorname{Artanh}\left(e^{bx+a}\right) - \frac{(\cosh(bx+a))^2}{3(\sinh(bx+a))^3} + \frac{(\cosh(bx+a))^2}{3\sinh(bx+a)} - \frac{\sinh(bx+a)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/2*csch(b*x+a)*coth(b*x+a)+arctanh(exp(b*x+a))-1/3/sinh(b*x+a)^3*cos
h(b*x+a)^2+1/3/sinh(b*x+a)*cosh(b*x+a)^2-1/3*sinh(b*x+a))
```

Maxima [A] time = 1.16916, size = 135, normalized size = 1.34

$$\frac{\log\left(e^{(bx+a)}+1\right)}{2\,b}-\frac{\log\left(e^{(bx+a)}-1\right)}{2\,b}-\frac{3\,e^{(5\,bx+5\,a)}+8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{3\,b\left(e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}+3\,e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/2*log(e^(b*x + a) + 1)/b - 1/2*log(e^(b*x + a) - 1)/b - 1/3*(3*e^(5*b*x +
 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(b*(e^(6*b*x + 6*a) - 3*e^(4*b*x
 + 4*a) + 3*e^(2*b*x + 2*a) - 1))
```

Fricas [B] time = 2.0076, size = 1963, normalized size = 19.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^3 + 16*cosh(b*x + a)^3 + 12*
(5*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6
+ 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*
x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) +
 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3
*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*
x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 -
 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) +
sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 8*cosh(b*x + a)^2 - 1)*sinh(b*x
 + a) - 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b
)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x +
a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 +
 b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b
*x + a))*sinh(b*x + a) - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{csch}^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**4,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**4, x)
```

Giac [A] time = 1.12661, size = 107, normalized size = 1.06

$$\frac{\log\left(e^{(bx+a)}+1\right)}{2\,b}-\frac{\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}-\frac{3\,e^{(5\,bx+5\,a)}+8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/2*log(e^(b*x + a) + 1)/b - 1/2*log(abs(e^(b*x + a) - 1))/b - 1/3*(3*e^(5*
b*x + 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(b*(e^(2*b*x + 2*a) - 1)^3)
```

3.309 $\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{4}{b\left(1-e^{2a+2bx}\right)^4}$$

```
[Out] -4/(b*(1 - E^(2*a + 2*b*x))^4) + 32/(3*b*(1 - E^(2*a + 2*b*x))^3) - 8/(b*(1
 - E^(2*a + 2*b*x))^2)
```

Rubi [A] time = 0.056439, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 266, 43}

$$-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{4}{b\left(1-e^{2a+2bx}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^5,x]
```


```
[Out] -4/(b*(1 - E^(2*a + 2*b*x))^4) + 32/(3*b*(1 - E^(2*a + 2*b*x))^3) - 8/(b*(1
 - E^(2*a + 2*b*x))^2)
```


Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{a+bx}\text{csch}^5(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{32x^5}{(-1+x^2)^5}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{32\,\text{Subst}\left(\int \frac{x^5}{(-1+x^2)^5}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{16\,\text{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2a+2bx}\right)}{b}$$
$$= \frac{16\,\text{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)dx, x, e^{2a+2bx}\right)}{b}$$
$$= -\frac{4}{b\left(1-e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}$$

Mathematica [A] time = 0.0350101, size = 44, normalized size = 0.67

$$-\frac{4\left(-4e^{2(a+bx)}+6e^{4(a+bx)}+1\right)}{3b\left(e^{2(a+bx)}-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^5,x]
```

```
[Out] (-4*(1 - 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(-1 + E^(2*(a + b*x))
)^4)
```

Maple [A] time = 0.048, size = 61, normalized size = 0.9

$$\frac{1}{b}\left(\left(\frac{2}{3}-\frac{(\text{csch}\,(bx+a))^2}{3}\right)\coth(bx+a)-\frac{(\cosh(bx+a))^2}{4\,(\sinh(bx+a))^4}+\frac{(\cosh(bx+a))^2}{4\,(\sinh(bx+a))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^5,x)
```

```
[Out] 1/b*((2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)-1/4/sinh(b*x+a)^4*cosh(b*x+a)^2+1/
4/sinh(b*x+a)^2*cosh(b*x+a)^2)
```

Maxima [B] time = 1.01289, size = 232, normalized size = 3.52

$$-\frac{8\,e^{(4\,bx+4\,a)}}{b\left(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1\right)}+\frac{16\,e^{(2\,bx+2\,a)}}{3\,b\left(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4
*a) - 4*e^(2*b*x + 2*a) + 1)) + 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a) -
```

```
4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b*(e
^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a)
+ 1))
```

Fricas [B] time = 1.87212, size = 635, normalized size = 9.62

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{6}+6\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+b\sinh\left(bx+a\right)^{6}-4\,b\cosh\left(bx+a\right)^{4}+\left(15\,b\cosh\left(bx+a\right)^{2}-4\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)^
2 - 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x
+ a)^6 - 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - 4*b)*sinh(b*x + a)^4
 + 4*(5*b*cosh(b*x + a)^3 - 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*cosh(b
*x + a)^2 + (15*b*cosh(b*x + a)^4 - 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b*x +
a)^2 + 2*(3*b*cosh(b*x + a)^5 - 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x + a))*si
nh(b*x + a) - 4*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{a}\int e^{bx}\operatorname{csch}^{5}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**5,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**5, x)
```

Giac [A] time = 1.11199, size = 57, normalized size = 0.86

$$-\frac{4\left(6\,e^{\left(4\,bx+4\,a\right)}-4\,e^{\left(2\,bx+2\,a\right)}+1\right)}{3\,b\left(e^{\left(2\,bx+2\,a\right)}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^4
)
```

3.310 $\int e^x \sinh^2(2x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}$$

```
[Out] -1/(12*E^(3*x)) - E^x/2 + E^(5*x)/20
```

Rubi [A] time = 0.0197871, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 12, 270}

$$-\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[2*x]^2,x]
```

```
[Out] -1/(12*E^(3*x)) - E^x/2 + E^(5*x)/20
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^x \sinh^2(2x)\,dx &= \text{Subst}\left(\int \frac{\left(1-x^4\right)^2}{4x^4}\,dx,x,e^x\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{\left(1-x^4\right)^2}{x^4}\,dx,x,e^x\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^4}+x^4\right)dx,x,e^x\right)\\
&= -\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}
\end{aligned}
$$

Mathematica [A] time = 0.0128874, size = 26, normalized size = 1.

$$-\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[2*x]^2,x]
```

```
[Out] -1/(12*E^(3*x)) - E^x/2 + E^(5*x)/20
```

Maple [A] time = 0.019, size = 34, normalized size = 1.3

$$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}-\frac{\cosh(x)}{2}-\frac{\cosh(3x)}{12}+\frac{\cosh(5x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(2*x)^2,x)
```

```
[Out] -1/2*sinh(x)+1/12*sinh(3*x)+1/20*sinh(5*x)-1/2*cosh(x)-1/12*cosh(3*x)+1/20*
cosh(5*x)
```

Maxima [A] time = 1.10359, size = 23, normalized size = 0.88

$$\frac{1}{20}e^{(5x)}-\frac{1}{12}e^{(-3x)}-\frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

Fricas [B] time = 2.01963, size = 170, normalized size = 6.54

$$-\frac{\cosh(x)^4-16\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2-16\cosh(x)\sinh(x)^3+\sinh(x)^4+15}{30(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4 + 15)/(cosh(x) - sinh(x))
```

Sympy [B] time = 1.0532, size = 42, normalized size = 1.62

$$\frac{7e^x\sinh^2(2x)}{15}+\frac{4e^x\sinh(2x)\cosh(2x)}{15}-\frac{8e^x\cosh^2(2x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)**2,x)
```

```
[Out] 7*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 - 8*exp(x)*cosh(
2*x)**2/15
```

Giac [A] time = 1.13813, size = 23, normalized size = 0.88

$$\frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

3.311 $\int e^x \sinh(2x)\,dx$

Optimal. Leaf size=19

$$\frac{e^{-x}}{2}+\frac{e^{3x}}{6}$$

```
[Out] 1/(2*E^x) + E^(3*x)/6
```

Rubi [A] time = 0.0112669, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{-x}}{2}+\frac{e^{3x}}{6}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[2*x],x]

[Out] 1/(2*E^x) + E^(3*x)/6
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int e^x \sinh(2x)\,dx &= \text{Subst}\left(\int \frac{-1+x^4}{2x^2}\,dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x^4}{x^2}\,dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}+x^2\right)dx, x, e^x\right)\\
&= \frac{e^{-x}}{2}+\frac{e^{3x}}{6}
\end{aligned}$$

Mathematica [A] time = 0.0080133, size = 16, normalized size = 0.84

$$\frac{1}{6}e^{-x}\left(e^{4x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[2*x],x]
```

```
[Out] (3 + E^(4*x))/(6*E^x)
```

Maple [A] time = 0.006, size = 22, normalized size = 1.2

$$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}+\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(2*x),x)
```

```
[Out] -1/2*sinh(x)+1/6*sinh(3*x)+1/2*cosh(x)+1/6*cosh(3*x)
```

Maxima [A] time = 1.05781, size = 18, normalized size = 0.95

$$\frac{1}{6}e^{(3x)}+\frac{1}{2}e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="maxima")
```

```
[Out] 1/6*e^(3*x) + 1/2*e^(-x)
```

Fricas [A] time = 1.93901, size = 90, normalized size = 4.74

$$\frac{2\left(\cosh(x)^2-\cosh(x)\sinh(x)+\sinh(x)^2\right)}{3(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="fricas")
```

```
[Out] 2/3*(cosh(x)^2 - cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

Sympy [A] time = 0.362463, size = 20, normalized size = 1.05

$$-\frac{e^x\sinh(2x)}{3}+\frac{2e^x\cosh(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x)
```

```
[Out] -exp(x)*sinh(2*x)/3 + 2*exp(x)*cosh(2*x)/3
```

<u>Giac</u> [A] time = 1.1257, size = 18, normalized size = 0.95

$$\frac{1}{6}e^{(3x)} + \frac{1}{2}e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="giac")
```

```
[Out] 1/6*e^(3*x) + 1/2*e^(-x)
```

3.312 $\int e^x \mathbf{csch}(2x)\,dx$

Optimal. Leaf size=11

$$\tan^{-1}(e^x) - \tanh^{-1}(e^x)$$

```
[Out] ArcTan[E^x] - ArcTanh[E^x]
```

Rubi [A] time = 0.0127131, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2282, 12, 298, 203, 206}

$$\tan^{-1}(e^x) - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[2*x],x]

[Out] ArcTan[E^x] - ArcTanh[E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^x\text{csch}(2x)\,dx &= \text{Subst}\left(\int \frac{2x^2}{-1+x^4}\,dx, x, e^x\right)\\ &= 2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, e^x\right)\\ &= -\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\ &= \tan^{-1}(e^x) - \tanh^{-1}(e^x)\end{aligned}$$

Mathematica [A] time = 0.0091644, size = 11, normalized size = 1.

$$\tan^{-1}(e^x) - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[2*x],x]
```

```
[Out] ArcTan[E^x] - ArcTanh[E^x]
```

Maple [C] time = 0.045, size = 34, normalized size = 3.1

$$\frac{\ln(e^x-1)}{2} + \frac{i}{2}\ln(e^x+i) - \frac{i}{2}\ln(e^x-i) - \frac{\ln(e^x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(2*x),x)
```

```
[Out] 1/2*ln(exp(x)-1)+1/2*I*ln(exp(x)+I)-1/2*I*ln(exp(x)-I)-1/2*ln(exp(x)+1)
```

Maxima [A] time = 1.5826, size = 24, normalized size = 2.18

$$\arctan(e^x) - \frac{1}{2}\log(e^x+1) + \frac{1}{2}\log(e^x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="maxima")
```

```
[Out] arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Fricas [B] time = 2.06641, size = 126, normalized size = 11.45

$$\arctan(\cosh(x)+\sinh(x)) - \frac{1}{2}\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}\log(\cosh(x)+\sinh(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="fricas")
```

```
[Out] arctan(cosh(x) + sinh(x)) - 1/2*log(cosh(x) + sinh(x) + 1) + 1/2*log(cosh(x
) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{csch}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x)
```

```
[Out] Integral(exp(x)*csch(2*x), x)
```

Giac [B] time = 1.1461, size = 26, normalized size = 2.36

$$\arctan\left(e^x\right) - \frac{1}{2}\log\left(e^x + 1\right) + \frac{1}{2}\log\left(\left|e^x - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="giac")
```

```
[Out] arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.313 $\int e^x \operatorname{csch}^2(2x) \, dx$

Optimal. Leaf size=32

$$\frac{e^x}{1-e^{4x}}-\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{1}{2}\tanh^{-1}\left(e^x\right)$$

```
[Out] E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rubi [A] time = 0.0242367, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2282, 12, 288, 212, 206, 203}

$$\frac{e^x}{1-e^{4x}}-\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{1}{2}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[2*x]^2,x]
```

```
[Out] E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^x \operatorname{csch}^2(2x)\,dx &= \operatorname{Subst}\left(\int \frac{4x^4}{\left(1-x^4\right)^2}\,dx, x, e^x\right)\\
&= 4\operatorname{Subst}\left(\int \frac{x^4}{\left(1-x^4\right)^2}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \operatorname{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \frac{1}{2}\tan^{-1}\left(e^x\right) - \frac{1}{2}\tanh^{-1}\left(e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.0357334, size = 32, normalized size = 1.

$$\frac{e^x}{1-e^{4x}} - \frac{1}{2}\tan^{-1}\left(e^x\right) - \frac{1}{2}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[2*x]^2,x]
```

```
[Out] E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Maple [C] time = 0.046, size = 46, normalized size = 1.4

$$-\frac{e^x}{e^{4x}-1} + \frac{\ln\left(e^x-1\right)}{4} + \frac{i}{4}\ln\left(e^x-i\right) - \frac{i}{4}\ln\left(e^x+i\right) - \frac{\ln\left(e^x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(2*x)^2,x)
```

```
[Out] -exp(x)/(exp(4*x)-1)+1/4*ln(exp(x)-1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)
-1/4*ln(exp(x)+1)
```

Maxima [A] time = 1.54657, size = 43, normalized size = 1.34

$$-\frac{e^x}{e^{(4x)}-1} - \frac{1}{2}\arctan\left(e^x\right) - \frac{1}{4}\log\left(e^x+1\right) + \frac{1}{4}\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="maxima")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x - 1)
```

Fricas [B] time = 2.04084, size = 678, normalized size = 21.19

$$-\frac{2\left(\cosh(x)^4+4\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2+4\cosh(x)\sinh(x)^3+\sinh(x)^4-1\right)\arctan(\cosh(x)+\sinh(x))}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*cos
h(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 -
1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sinh(x)
- 1) + 4*cosh(x) + 4*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{csch}^2(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)**2,x)
```

```
[Out] Integral(exp(x)*csch(2*x)**2, x)
```

Giac [A] time = 1.12241, size = 45, normalized size = 1.41

$$-\frac{e^x}{e^{(4x)}-1}-\frac{1}{2}\arctan(e^x)-\frac{1}{4}\log(e^x+1)+\frac{1}{4}\log(|e^x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="giac")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(e^x -
 1))
```

3.314 $\int e^x \sinh^2(3x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

```
[Out] -1/(20*E^(5*x)) - E^x/2 + E^(7*x)/28
```

Rubi [A] time = 0.019706, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 12, 270}

$$-\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[3*x]^2,x]
```

```
[Out] -1/(20*E^(5*x)) - E^x/2 + E^(7*x)/28
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh^2(3x)\,dx &= \text{Subst}\left(\int \frac{\left(1-x^6\right)^2}{4x^6}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{\left(1-x^6\right)^2}{x^6}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^6}+x^6\right)dx, x, e^x\right)\\ &= -\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}\end{aligned}$$

Mathematica [A] time = 0.0163261, size = 26, normalized size = 1.

$$-\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[3*x]^2,x]
```

```
[Out] -1/(20*E^(5*x)) - E^x/2 + E^(7*x)/28
```

Maple [A] time = 0.013, size = 34, normalized size = 1.3

$$-\frac{\sinh(x)}{2}+\frac{\sinh(5x)}{20}+\frac{\sinh(7x)}{28}-\frac{\cosh(x)}{2}-\frac{\cosh(5x)}{20}+\frac{\cosh(7x)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(3*x)^2,x)
```

```
[Out] -1/2*sinh(x)+1/20*sinh(5*x)+1/28*sinh(7*x)-1/2*cosh(x)-1/20*cosh(5*x)+1/28*
cosh(7*x)
```

Maxima [A] time = 1.08204, size = 23, normalized size = 0.88

$$\frac{1}{28}e^{(7x)}-\frac{1}{20}e^{(-5x)}-\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

Fricas [B] time = 1.9622, size = 240, normalized size = 9.23

$$-\frac{\cosh(x)^6-36\cosh(x)^5\sinh(x)+15\cosh(x)^4\sinh(x)^2-120\cosh(x)^3\sinh(x)^3+15\cosh(x)^2\sinh(x)^4-36\cosh}{70(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)^6
 + 35)/(cosh(x) - sinh(x))
```

Sympy [B] time = 0.954452, size = 42, normalized size = 1.62

$$\frac{17e^x\sinh^2(3x)}{35}+\frac{6e^x\sinh(3x)\cosh(3x)}{35}-\frac{18e^x\cosh^2(3x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)**2,x)
```

```
[Out] 17*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 - 18*exp(x)*cos
h(3*x)**2/35
```

Giac [A] time = 1.14305, size = 23, normalized size = 0.88

$$\frac{1}{28}\,e^{(7\,x)}-\frac{1}{20}\,e^{(-5\,x)}-\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

3.315 $\int e^x \sinh(3x)\, dx$

Optimal. Leaf size=19

$$\frac{e^{-2x}}{4} + \frac{e^{4x}}{8}$$

```
[Out] 1/(4*E^(2*x)) + E^(4*x)/8
```

Rubi [A] time = 0.0119808, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{-2x}}{4} + \frac{e^{4x}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[3*x],x]
```

```
[Out] 1/(4*E^(2*x)) + E^(4*x)/8
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int e^x \sinh(3x)\, dx &= \text{Subst}\left(\int \frac{-1+x^6}{2x^3}\, dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x^6}{x^3}\, dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^3}+x^3\right) dx, x, e^x\right)\\
&= \frac{e^{-2x}}{4} + \frac{e^{4x}}{8}
\end{aligned}$$

Mathematica [A] time = 0.0091856, size = 16, normalized size = 0.84

$$\frac{1}{8}e^{-2x}\left(e^{6x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[3*x],x]
```

```
[Out] (2 + E^(6*x))/(8*E^(2*x))
```

Maple [A] time = 0.016, size = 26, normalized size = 1.4

$$-\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}+\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(3*x),x)
```

```
[Out] -1/4*sinh(2*x)+1/8*sinh(4*x)+1/4*cosh(2*x)+1/8*cosh(4*x)
```

Maxima [A] time = 1.10799, size = 18, normalized size = 0.95

$$\frac{1}{8}e^{(4x)}+\frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="maxima")
```

```
[Out] 1/8*e^(4*x) + 1/4*e^(-2*x)
```

Fricas [B] time = 1.96551, size = 128, normalized size = 6.74

$$\frac{3\cosh(x)^3-3\cosh(x)^2\sinh(x)+9\cosh(x)\sinh(x)^2-\sinh(x)^3}{8(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*cosh(x)^3 - 3*cosh(x)^2*sinh(x) + 9*cosh(x)*sinh(x)^2 - sinh(x)^3)/(
cosh(x) - sinh(x))
```

Sympy [A] time = 0.329055, size = 20, normalized size = 1.05

$$-\frac{e^x\sinh(3x)}{8}+\frac{3e^x\cosh(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x)
```

```
[Out] -exp(x)*sinh(3*x)/8 + 3*exp(x)*cosh(3*x)/8
```

Giac [A] time = 1.11711, size = 18, normalized size = 0.95

$$\frac{1}{8}\,e^{(4\,x)} + \frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="giac")
```

```
[Out] 1/8*e^(4*x) + 1/4*e^(-2*x)
```

3.316 $\int e^x \operatorname{csch}(3x) \, dx$

Optimal. Leaf size=54

$$\frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(e^{2x}+e^{4x}+1\right)+\frac{\tan^{-1}\left(\frac{2e^{2x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 + 2*E^(2*x))/Sqrt[3]]/Sqrt[3] + Log[1 - E^(2*x)]/3 - Log[1 + E^(2
*x) + E^(4*x)]/6
```

Rubi [A] time = 0.0566651, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2282, 12, 275, 292, 31, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(e^{2x}+e^{4x}+1\right)+\frac{\tan^{-1}\left(\frac{2e^{2x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[3*x],x]
```

```
[Out] ArcTan[(1 + 2*E^(2*x))/Sqrt[3]]/Sqrt[3] + Log[1 - E^(2*x)]/3 - Log[1 + E^(2
*x) + E^(4*x)]/6
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^x \operatorname{csch}(3x)\,dx &= \operatorname{Subst}\left(\int \frac{2x^3}{-1+x^6}\,dx, x, e^x\right)\\
&= 2\operatorname{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, e^x\right)\\
&= \operatorname{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, e^{2x}\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+x}\,dx, x, e^{2x}\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, e^{2x}\right)\\
&= \frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, e^{2x}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, e^{2x}\right)\\
&= \frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right)-\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2e^{2x}\right)\\
&= \frac{\tan^{-1}\left(\frac{1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0112734, size = 22, normalized size = 0.41

$$
-\frac{1}{2}e^{4x}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};e^{6x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[3*x],x]
```

```
[Out] -(E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, E^(6*x)])/2
```

Maple [C] time = 0.045, size = 79, normalized size = 1.5

$$-\frac{1}{6}\ln\left(\mathrm{e}^{2x}+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)+\frac{i}{6}\ln\left(\mathrm{e}^{2x}+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}-\frac{1}{6}\ln\left(\mathrm{e}^{2x}+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)-\frac{i}{6}\ln\left(\mathrm{e}^{2x}+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}+\frac{\ln\left(\mathrm{e}^{2x}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(3*x),x)
```

```
[Out] -1/6*ln(exp(2*x)+1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)+1/2+1/2*I*3^(1/2))*3^
(1/2)-1/6*ln(exp(2*x)+1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(2*x)+1/2-1/2*I*3^(1/2
))*3^(1/2)+1/3*ln(exp(2*x)-1)
```

Maxima [A] time = 1.65227, size = 99, normalized size = 1.83

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}+1)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}-1)\right)-\frac{1}{6}\log\left(e^{(2x)}+e^{x}+1\right)-\frac{1}{6}\log\left(e^{(2x)}-e^{x}+1\right)+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 1/6*log(e^(2*x) + e^x + 1) - 1/6*log(e^(2*x) - e^x + 1) +
 1/3*log(e^x + 1) + 1/3*log(e^x - 1)
```

Fricas [A] time = 2.10824, size = 288, normalized size = 5.33

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{3\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)-\frac{1}{6}\log\left(\frac{2\cosh(x)^{2}+2\sinh(x)^{2}+1}{\cosh(x)^{2}-2\cosh(x)\sinh(x)+\sinh(x)^{2}}\right)+\frac{1}{3}\log\left(\frac{2}{\cosh(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(3*sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - s
inh(x))) - 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) + 1/3*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\operatorname{csch}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x)
```

```
[Out] Integral(exp(x)*csch(3*x), x)
```

Giac [A] time = 1.13291, size = 58, normalized size = 1.07

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}+1\right)\right)-\frac{1}{6}\log\left(e^{(4x)}+e^{(2x)}+1\right)+\frac{1}{3}\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) + 1)) - 1/6*log(e^(4*x) + e^(2*x)
 + 1) + 1/3*log(abs(e^(2*x) - 1))
```

3.317 $\int e^x \operatorname{csch}^2(3x)\,dx$

Optimal. Leaf size=105

$$\frac{2e^x}{3\left(1-e^{6x}\right)}+\frac{1}{18}\log\left(-e^x+e^{2x}+1\right)-\frac{1}{18}\log\left(e^x+e^{2x}+1\right)+\frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\tanh^{-1}\left(e^x\right)$$

```
[Out] (2*E^x)/(3*(1 - E^(6*x))) + ArcTan[(1 - 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - ArcTa
n[(1 + 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - (2*ArcTanh[E^x])/9 + Log[1 - E^x + E^(
2*x)]/18 - Log[1 + E^x + E^(2*x)]/18
```

Rubi [A] time = 0.139335, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2282, 12, 288, 210, 634, 618, 204, 628, 206}

$$\frac{2e^x}{3\left(1-e^{6x}\right)}+\frac{1}{18}\log\left(-e^x+e^{2x}+1\right)-\frac{1}{18}\log\left(e^x+e^{2x}+1\right)+\frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[3*x]^2,x]
```

```
[Out] (2*E^x)/(3*(1 - E^(6*x))) + ArcTan[(1 - 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - ArcTa
n[(1 + 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - (2*ArcTanh[E^x])/9 + Log[1 - E^x + E^(
2*x)]/18 - Log[1 + E^x + E^(2*x)]/18
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
```

```
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^x \operatorname{csch}^2(3x)\,dx &= \operatorname{Subst}\left(\int \frac{4x^6}{\left(1-x^6\right)^2}\,dx, x, e^x\right)\\
&= 4\operatorname{Subst}\left(\int \frac{x^6}{\left(1-x^6\right)^2}\,dx, x, e^x\right)\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)} - \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^6}\,dx, x, e^x\right)\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)} - \frac{2}{9}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{2}{9}\operatorname{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, e^x\right) - \frac{2}{9}\operatorname{Subst}\left(\int \frac{1+
\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)} - \frac{2}{9}\tanh^{-1}\left(e^x\right) + \frac{1}{18}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, e^x\right) - \frac{1}{18}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, e^x\right)
\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)} - \frac{2}{9}\tanh^{-1}\left(e^x\right) + \frac{1}{18}\log\left(1-e^x+e^{2x}\right) - \frac{1}{18}\log\left(1+e^x+e^{2x}\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx
\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)} + \frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2}{9}\tanh^{-1}\left(e^x\right) + \frac{1}{18}\log\left(1-e^x+e^{2x}\right) - \frac{1}{18}\log\left(1+e
\end{aligned}
$$

Mathematica [C] time = 0.0262982, size = 34, normalized size = 0.32

$$\frac{2}{3}e^x\left(\frac{1}{1-e^{6x}}-{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};e^{6x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[3*x]^2,x]
```

```
[Out] (2*E^x*((1 - E^(6*x))^(-1) - Hypergeometric2F1[1/6, 1, 7/6, E^(6*x)]))/3
```

Maple [C] time = 0.055, size = 148, normalized size = 1.4

$$-\frac{2\,\mathrm{e}^x}{3\,\mathrm{e}^{6x}-3}-\frac{1}{18}\ln\left(\mathrm{e}^x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)+\frac{i}{18}\ln\left(\mathrm{e}^x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}-\frac{1}{18}\ln\left(\mathrm{e}^x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\frac{i}{18}\ln\left(\mathrm{e}^x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(3*x)^2,x)
```

```
[Out] -2/3*exp(x)/(exp(6*x)-1)-1/18*ln(exp(x)+1/2-1/2*I*3^(1/2))+1/18*I*ln(exp(x)
+1/2-1/2*I*3^(1/2))*3^(1/2)-1/18*ln(exp(x)+1/2+1/2*I*3^(1/2))-1/18*I*ln(exp
(x)+1/2+1/2*I*3^(1/2))*3^(1/2)+1/18*ln(exp(x)-1/2-1/2*I*3^(1/2))+1/18*I*ln(
exp(x)-1/2-1/2*I*3^(1/2))*3^(1/2)+1/18*ln(exp(x)-1/2+1/2*I*3^(1/2))-1/18*I*
ln(exp(x)-1/2+1/2*I*3^(1/2))*3^(1/2)-1/9*ln(exp(x)+1)+1/9*ln(exp(x)-1)
```

Maxima [A] time = 1.6852, size = 115, normalized size = 1.1

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right)-\frac{2e^x}{3\left(e^{(6x)}-1\right)}-\frac{1}{18}\log\left(e^{(2x)}+e^x+1\right)+\frac{1}{18}\log\left(e^{(2x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1/1
8*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(e^x - 1)
```

Fricas [B] time = 2.14551, size = 1985, normalized size = 18.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(sqrt(3)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh
(x)^4*sinh(x)^2 + 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sin
h(x)^4 + 6*sqrt(3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(
2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) + 1/3*sqrt(3)) + 2*(sqrt(3)*cosh(
```

```
x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh(x)^4*sinh(x)^2 + 20*sq
rt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sinh(x)^4 + 6*sqrt(3)*cosh
(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(2/3*sqrt(3)*cosh(x) + 2
/3*sqrt(3)*sinh(x) - 1/3*sqrt(3)) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) -
 (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*s
inh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*lo
g((2*cosh(x) - 1)/(cosh(x) - sinh(x))) + 2*(cosh(x)^6 + 6*cosh(x)^5*sinh(x)
 + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4
 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cosh(x) + sinh(x) + 1) - 2*(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x
)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cos
h(x) + sinh(x) - 1) + 12*cosh(x) + 12*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\operatorname{csch}^{2}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)**2,x)
```

```
[Out] Integral(exp(x)*csch(3*x)**2, x)
```

Giac [A] time = 1.10899, size = 116, normalized size = 1.1

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}+1)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}-1)\right)-\frac{2e^{x}}{3\left(e^{(6x)}-1\right)}-\frac{1}{18}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{18}\log\left(e^{(2x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1/1
8*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(abs(e^x - 1))
```

3.318 $\int e^x \sinh^2(4x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}$$

```
[Out] -1/(28*E^(7*x)) - E^x/2 + E^(9*x)/36
```

Rubi [A] time = 0.0208464, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 12, 270}

$$-\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[4*x]^2,x]
```

```
[Out] -1/(28*E^(7*x)) - E^x/2 + E^(9*x)/36
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh^2(4x)\,dx &= \text{Subst}\left(\int \frac{\left(1-x^8\right)^2}{4x^8}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{\left(1-x^8\right)^2}{x^8}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^8}+x^8\right)dx, x, e^x\right)\\ &= -\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}\end{aligned}$$

Mathematica [A] time = 0.0159615, size = 26, normalized size = 1.

$$-\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[4*x]^2,x]
```

```
[Out] -1/(28*E^(7*x)) - E^x/2 + E^(9*x)/36
```

Maple [A] time = 0.013, size = 34, normalized size = 1.3

$$-\frac{\sinh(x)}{2}+\frac{\sinh(7x)}{28}+\frac{\sinh(9x)}{36}-\frac{\cosh(x)}{2}-\frac{\cosh(7x)}{28}+\frac{\cosh(9x)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(4*x)^2,x)
```

```
[Out] -1/2*sinh(x)+1/28*sinh(7*x)+1/36*sinh(9*x)-1/2*cosh(x)-1/28*cosh(7*x)+1/36*
cosh(9*x)
```

Maxima [A] time = 1.05269, size = 23, normalized size = 0.88

$$\frac{1}{36}e^{(9x)}-\frac{1}{28}e^{(-7x)}-\frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="maxima")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

Fricas [B] time = 2.04191, size = 311, normalized size = 11.96

$$-\frac{\cosh(x)^8-64\cosh(x)^7\sinh(x)+28\cosh(x)^6\sinh(x)^2-448\cosh(x)^5\sinh(x)^3+70\cosh(x)^4\sinh(x)^4-448\cos}{126(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*cos
h(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*co
sh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 + 63)/(cosh(x) - sinh(
x))
```

Sympy [B] time = 0.955436, size = 42, normalized size = 1.62

$$\frac{31e^x\sinh^2(4x)}{63}+\frac{8e^x\sinh(4x)\cosh(4x)}{63}-\frac{32e^x\cosh^2(4x)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)**2,x)
```

```
[Out] 31*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 - 32*exp(x)*cos
h(4*x)**2/63
```

<u>Giac</u> [A] time = 1.10597, size = 23, normalized size = 0.88

$$\frac{1}{36}\,e^{(9\,x)} - \frac{1}{28}\,e^{(-7\,x)} - \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="giac")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

3.319 $\int e^x \sinh(4x)\,dx$

Optimal. Leaf size=19

$$\frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

```
[Out] 1/(6*E^(3*x)) + E^(5*x)/10
```

Rubi [A] time = 0.0124461, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[4*x],x]
```

```
[Out] 1/(6*E^(3*x)) + E^(5*x)/10
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh(4x)\,dx &= \text{Subst}\left(\int \frac{-1+x^8}{2x^4}\,dx,x,e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x^8}{x^4}\,dx,x,e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^4}+x^4\right)dx,x,e^x\right)\\ &= \frac{e^{-3x}}{6}+\frac{e^{5x}}{10}\end{aligned}$$

Mathematica [A] time = 0.0091856, size = 19, normalized size = 1.

$$\frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[4*x],x]
```

```
[Out] 1/(6*E^(3*x)) + E^(5*x)/10
```

Maple [A] time = 0.006, size = 26, normalized size = 1.4

$$-\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}+\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(4*x),x)
```

```
[Out] -1/6*sinh(3*x)+1/10*sinh(5*x)+1/6*cosh(3*x)+1/10*cosh(5*x)
```

Maxima [A] time = 1.09995, size = 18, normalized size = 0.95

$$\frac{1}{10}\,e^{(5\,x)}+\frac{1}{6}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="maxima")
```

```
[Out] 1/10*e^(5*x) + 1/6*e^(-3*x)
```

Fricas [B] time = 2.03866, size = 154, normalized size = 8.11

$$\frac{4\left(\cosh(x)^4-\cosh(x)^3\sinh(x)+6\,\cosh(x)^2\sinh(x)^2-\cosh(x)\sinh(x)^3+\sinh(x)^4\right)}{15\,(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="fricas")
```

```
[Out] 4/15*(cosh(x)^4 - cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - cosh(x)*sinh(
x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

Sympy [A] time = 0.350168, size = 20, normalized size = 1.05

$$-\frac{e^{x}\sinh(4x)}{15}+\frac{4e^{x}\cosh(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x)
```

```
[Out] -exp(x)*sinh(4*x)/15 + 4*exp(x)*cosh(4*x)/15
```

Giac [A] time = 1.09391, size = 18, normalized size = 0.95

$$\frac{1}{10}\,e^{(5\,x)} + \frac{1}{6}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="giac")
```

```
[Out] 1/10*e^(5*x) + 1/6*e^(-3*x)
```

3.320 $\int e^x \mathbf{csch}(4x)\, dx$

Optimal. Leaf size=113

$$-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(e$$

```
[Out] -ArcTan[E^x]/2 - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*E
^x]/(2*Sqrt[2]) - ArcTanh[E^x]/2 - Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2
]) + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rubi [A] time = 0.0774718, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.5, Rules used = {2282, 12, 301, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(e$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[4*x],x]
```

```
[Out] -ArcTan[E^x]/2 - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*E
^x]/(2*Sqrt[2]) - ArcTanh[E^x]/2 - Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2
]) + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^x\text{csch}(4x)\,dx &= \text{Subst}\left(\int \frac{2x^4}{-1+x^8}\,dx,x,e^x\right)\\ &= 2\,\text{Subst}\left(\int \frac{x^4}{-1+x^8}\,dx,x,e^x\right)\\ &= -\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,e^x\right)+\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,e^x\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,e^x\right)\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^x\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,e^x\right)\\ &= -\frac{1}{2}\tan^{-1}(e^x)-\frac{1}{2}\tanh^{-1}(e^x)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,e^x\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\right.\\ &= -\frac{1}{2}\tan^{-1}(e^x)-\frac{1}{2}\tanh^{-1}(e^x)-\frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.}{2\sqrt{2}}\\ &= -\frac{1}{2}\tan^{-1}(e^x)-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}(e^x)-\frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}+\end{aligned}$$

Mathematica [C] time = 0.0112564, size = 22, normalized size = 0.19

$$-\frac{2}{5}e^{5x}\,{}_2F_1\left(\frac{5}{8},1;\frac{13}{8};e^{8x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[4*x],x]
```

```
[Out] (-2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, E^(8*x)])/5
```

Maple [C] time = 0.055, size = 56, normalized size = 0.5

$$\frac{\ln(\mathrm{e}^x-1)}{4}+2\sum_{\_R=\mathrm{RootOf}\left(4096\_Z^4+1\right)}\_R\,\ln(\mathrm{e}^x+8\_R)-\frac{\ln(\mathrm{e}^x+1)}{4}+\frac{i}{4}\ln(\mathrm{e}^x-i)-\frac{i}{4}\ln(\mathrm{e}^x+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(4*x),x)
```

```
[Out] 1/4*ln(exp(x)-1)+2*sum(_R*ln(exp(x)+8*_R),_R=RootOf(4096*_Z^4+1))-1/4*ln(ex
p(x)+1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)
```

Maxima [A] time = 1.59939, size = 128, normalized size = 1.13

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(e^x - 1)
```

Fricas [A] time = 2.31846, size = 440, normalized size = 3.89

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+4e^{(2x)}+4}+1\right)+\frac{1}{8}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1)
- 1) - 1/2*sqrt(2)*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x +
4*e^(2*x) + 4) + 1) + 1/8*sqrt(2)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 1/8*
sqrt(2)*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(e^x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\operatorname{csch}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x)
```

```
[Out] Integral(exp(x)*csch(4*x), x)
```

Giac [A] time = 1.08674, size = 130, normalized size = 1.15

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(abs(e^x - 1))
```

3.321 $\int e^x \mathbf{csch}^2(4x)\,dx$

Optimal. Leaf size=131

$$\frac{e^x}{2\left(1-e^{8x}\right)}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{1}{8}\tan^{-1}\left(e^x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}-$$

```
[Out] E^x/(2*(1 - E^(8*x))) - ArcTan[E^x]/8 + ArcTan[1 - Sqrt[2]*E^x]/(8*Sqrt[2])
 - ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - ArcTanh[E^x]/8 + Log[1 - Sqrt[2]*E
^x + E^(2*x)]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[2])
```

Rubi [A] time = 0.0884536, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.3, Rules used = {2282, 12, 288, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$\frac{e^x}{2\left(1-e^{8x}\right)}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{1}{8}\tan^{-1}\left(e^x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[4*x]^2,x]
```

```
[Out] E^x/(2*(1 - E^(8*x))) - ArcTan[E^x]/8 + ArcTan[1 - Sqrt[2]*E^x]/(8*Sqrt[2])
 - ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - ArcTanh[E^x]/8 + Log[1 - Sqrt[2]*E
^x + E^(2*x)]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[2])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] && !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int e^x \text{csch}^2(4x)\,dx = \text{Subst}\left(\int \frac{4x^8}{(1-x^8)^2}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^8}{(1-x^8)^2}\,dx, x, e^x\right)$$
$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^8}\,dx, x, e^x\right)$$
$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\right.$$
$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\tan^{-1}(e^x) - \frac{1}{8}\tanh^{-1}(e^x) - \frac{1}{16}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{1}{16}\text{Subst}\left(\int\right.$$
$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\tan^{-1}(e^x) - \frac{1}{8}\tanh^{-1}(e^x) + \frac{\log(1-\sqrt{2}e^x+e^{2x})}{16\sqrt{2}} - \frac{\log(1+\sqrt{2}e^x+e^{2x})}{16\sqrt{2}} - \frac{\text{Subs}}{}$$
$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\tan^{-1}(e^x) + \frac{\tan^{-1}(1-\sqrt{2}e^x)}{8\sqrt{2}} - \frac{\tan^{-1}(1+\sqrt{2}e^x)}{8\sqrt{2}} - \frac{1}{8}\tanh^{-1}(e^x) + \frac{\log(1-\sqrt{2}}{16\sqrt{}}$$

Mathematica [C] time = 0.0248644, size = 34, normalized size = 0.26

$$\frac{1}{2}e^x\left(\frac{1}{1-e^{8x}} - {}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{8x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[4*x]^2,x]
```

```
[Out] (E^x*((1 - E^(8*x))^(-1) - Hypergeometric2F1[1/8, 1, 9/8, E^(8*x)]))/2
```

Maple [C] time = 0.058, size = 68, normalized size = 0.5

$$-\frac{e^x}{2e^{8x}-2} + \frac{i}{16}\ln(e^x - i) - \frac{i}{16}\ln(e^x + i) - \frac{\ln(e^x+1)}{16} + \frac{\ln(e^x-1)}{16} + 4\sum_{\_R=\text{RootOf}(16777216\_Z^4+1)} \_R\ln(e^x - 64\_R)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(4*x)^2,x)
```

```
[Out] -1/2*exp(x)/(exp(8*x)-1)+1/16*I*ln(exp(x)-I)-1/16*I*ln(exp(x)+I)-1/16*ln(ex
p(x)+1)+1/16*ln(exp(x)-1)+4*sum(_R*ln(exp(x)-64*_R),_R=RootOf(16777216*_Z^4
+1))
```

Maxima [A] time = 1.59527, size = 144, normalized size = 1.1

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{32}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(e^x - 1)
```

Fricas [B] time = 2.79839, size = 620, normalized size = 4.73

$$4\left(\sqrt{2}e^{(8x)}-\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)+4\left(\sqrt{2}e^{(8x)}-\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+4e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(sqrt(2)*e^(8*x) - sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt
(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(8*x) - sqrt(2))*arctan(-sqrt(2)
*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - 4*(e^(8*x) -
 1)*arctan(e^x) - (sqrt(2)*e^(8*x) - sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x)
 + 4) + (sqrt(2)*e^(8*x) - sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 2
*(e^(8*x) - 1)*log(e^x + 1) + 2*(e^(8*x) - 1)*log(e^x - 1) - 16*e^x)/(e^(8*
x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\operatorname{csch}^{2}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)**2,x)
```

```
[Out] Integral(exp(x)*csch(4*x)**2, x)
```

Giac [A] time = 1.11866, size = 146, normalized size = 1.11

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{32}\sqrt{2}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(abs(e^x - 1))
```

3.322 $\int F^{c(a+bx)} \sinh^3(d+ex)\,dx$

Optimal. Leaf size=202

$$-\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{6bce^2\log(F)\sinh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{6e^3\cosh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}+$$

```
[Out] (-6*e^3*F^(c*(a + b*x))*Cosh[d + e*x])/(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b
^4*c^4*Log[F]^4) + (6*b*c*e^2*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x])/(9*e^4
- 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cosh[d
 + e*x]*Sinh[d + e*x]^2)/(9*e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a + b*x))*
Log[F]*Sinh[d + e*x]^3)/(9*e^2 - b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0817947, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5476, 5474}

$$-\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{6bce^2\log(F)\sinh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{6e^3\cosh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}+$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

```
[Out] (-6*e^3*F^(c*(a + b*x))*Cosh[d + e*x])/(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b
^4*c^4*Log[F]^4) + (6*b*c*e^2*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x])/(9*e^4
- 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cosh[d
 + e*x]*Sinh[d + e*x]^2)/(9*e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a + b*x))*
Log[F]*Sinh[d + e*x]^3)/(9*e^2 - b^2*c^2*Log[F]^2)
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx=\frac{3eF^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)}{9e^2-b^2c^2\log^2(F)}-\frac{bcF^{c(a+bx)}\log(F)\sinh^3(d+ex)}{9e^2-b^2c^2\log^2(F)}-\frac{\left(6e^2\right)\int F^{c(a+b}}{9e^2-b^2}$$

$$=-\frac{6e^3F^{c(a+bx)}\cosh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}+\frac{6bce^2F^{c(a+bx)}\log(F)\sinh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}+\frac{3eF^{c(}}{}$$

Mathematica [A] time = 0.665594, size = 157, normalized size = 0.78

$$\frac{F^{c(a+bx)}\left(3\cosh(3(d+ex))\left(e^3-b^2c^2e\log^2(F)\right)+3\cosh(d+ex)\left(b^2c^2e\log^2(F)-9e^3\right)+2bc\log(F)\sinh(d+ex)\left(\cosh(2(d\right.\right.}{4\left(-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(3*Cosh[3*(d + e*x)]*(e^3 - b^2*c^2*e*Log[F]^2) + 3*Cosh[d
 + e*x]*(-9*e^3 + b^2*c^2*e*Log[F]^2) + 2*b*c*Log[F]*(13*e^2 - b^2*c^2*Log[
F]^2 + Cosh[2*(d + e*x)]*(-e^2 + b^2*c^2*Log[F]^2))*Sinh[d + e*x]))/(4*(9*e
^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A] time = 0.061, size = 326, normalized size = 1.6

$$\frac{\left((\ln(F))^3 b^3c^3e^{6ex+6d}-3(\ln(F))^3 b^3c^3e^{4ex+4d}-3(\ln(F))^2 b^2c^2ee^{6ex+6d}+3(\ln(F))^3 b^3c^3e^{2ex+2d}+3(\ln(F))^2 b^2c^2ee^{4ex}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^3,x)
```

```
[Out] 1/8*(ln(F)^3*b^3*c^3*exp(6*e*x+6*d)-3*ln(F)^3*b^3*c^3*exp(4*e*x+4*d)-3*ln(F
)^2*b^2*c^2*e*exp(6*e*x+6*d)+3*ln(F)^3*b^3*c^3*exp(2*e*x+2*d)+3*ln(F)^2*b^2
*c^2*e*exp(4*e*x+4*d)-ln(F)*b*c*e^2*exp(6*e*x+6*d)-ln(F)^3*b^3*c^3+3*ln(F)^
2*b^2*c^2*e*exp(2*e*x+2*d)+27*ln(F)*b*c*e^2*exp(4*e*x+4*d)+3*e^3*exp(6*e*x+
6*d)-3*ln(F)^2*b^2*c^2*e-27*ln(F)*b*c*e^2*exp(2*e*x+2*d)-27*e^3*exp(4*e*x+4
*d)+ln(F)*b*c*e^2-27*e^3*exp(2*e*x+2*d)+3*e^3)/(b*c*ln(F)-e)*exp(-3*e*x-3*d
)/(b*c*ln(F)-3*e)/(e+b*c*ln(F))/(b*c*ln(F)+3*e)*F^(c*(b*x+a))
```

Maxima [A] time = 1.19122, size = 181, normalized size = 0.9

$$\frac{F^{ac}e^{(bcx\log(F)+3ex+3d)}}{8(bc\log(F)+3e)}-\frac{3F^{ac}e^{(bcx\log(F)+ex+d)}}{8(bc\log(F)+e)}+\frac{3F^{ac}e^{(bcx\log(F)-ex)}}{8\left(bce^{d}\log(F)-ee^{d}\right)}-\frac{F^{ac}e^{(bcx\log(F)-3ex)}}{8\left(bce^{(3d)}\log(F)-3ee^{(3d)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) - 3/8*F^(a*c)
*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F)
- e*x)/(b*c*e^d*log(F) - e*e^d) - 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(b*c
*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Fricas [B] time = 2.55951, size = 5434, normalized size = 26.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((3*e^3*cosh(e*x + d)^6 - 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c^3
*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*cosh
(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e*x +
 d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 - b^3*c^3)*l
og(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - (5*
b*c*e^2*cosh(e*x + d)^2 - 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*c^3*cos
h(e*x + d)^6 - 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2 - b^3*
c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 - 27*e^3*cosh(e*x + d) + (5*b^3*c
^3*cosh(e*x + d)^3 - 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^2*e*cos
h(e*x + d)^3 - b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)
^3 - 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 + 3*e^3 - 3*(b^2*c^2
*e*cosh(e*x + d)^6 - b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x + d)^2
+ b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 - 54*e^3*cosh(e*x + d)^2
+ (5*b^3*c^3*cosh(e*x + d)^4 - 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*log(F)^
3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 - 6*b^2*c^2*e*cosh(e*x + d)^2 - b
^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 - 54*b*c*e^2*cosh(e*x + d)^
2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 - 27*b*c*
e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 - b*c*e^2)*log(F) + 6*(3*e
^3*cosh(e*x + d)^5 - 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) + (b^3*c^
3*cosh(e*x + d)^5 - 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d))*log(
F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 - 2*b^2*c^2*e*cosh(e*x + d)^3 - b^2*c^2
*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 - 18*b*c*e^2*cosh(e*x
 + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*cosh((b*c*x + a*c
)*log(F)) + (3*e^3*cosh(e*x + d)^6 - 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(
F)^3 - 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(
b^3*c^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e
^2*cosh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cos
h(e*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 - b^3
*c^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^
2 - (5*b*c*e^2*cosh(e*x + d)^2 - 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 - 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2
 - b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 - 27*e^3*cosh(e*x + d) + (
5*b^3*c^3*cosh(e*x + d)^3 - 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^
2*e*cosh(e*x + d)^3 - b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e
*x + d)^3 - 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 + 3*e^3 - 3*(
b^2*c^2*e*cosh(e*x + d)^6 - b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x
+ d)^2 + b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 - 54*e^3*cosh(e*x
+ d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 - 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*
log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 - 6*b^2*c^2*e*cosh(e*x + d
)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 - 54*b*c*e^2*cosh(e*
x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 -
27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 - b*c*e^2)*log(F) +
 6*(3*e^3*cosh(e*x + d)^5 - 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) +
(b^3*c^3*cosh(e*x + d)^5 - 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d
))*log(F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 - 2*b^2*c^2*e*cosh(e*x + d)^3 -
b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 - 18*b*c*e^2*c
osh(e*x + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*sinh((b*c*
x + a*c)*log(F)))/(b^4*c^4*cosh(e*x + d)^3*log(F)^4 - 10*b^2*c^2*e^2*cosh(e
*x + d)^3*log(F)^2 + 9*e^4*cosh(e*x + d)^3 + (b^4*c^4*log(F)^4 - 10*b^2*c^2
*e^2*log(F)^2 + 9*e^4)*sinh(e*x + d)^3 + 3*(b^4*c^4*cosh(e*x + d)*log(F)^4
- 10*b^2*c^2*e^2*cosh(e*x + d)*log(F)^2 + 9*e^4*cosh(e*x + d))*sinh(e*x + d
)^2 + 3*(b^4*c^4*cosh(e*x + d)^2*log(F)^4 - 10*b^2*c^2*e^2*cosh(e*x + d)^2*
log(F)^2 + 9*e^4*cosh(e*x + d)^2)*sinh(e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.29867, size = 1673, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*
b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*
c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3
*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))
+ 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - 3/4*(2*(b*
c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn
(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2)
- (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi
*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
+ e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/2*I*(6*I*e^(
1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c
)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) - 6*I*e^(-1/
2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/
(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*log(a
bs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e)*cos(-1
/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*
c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c
)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)
/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) - e)*x - d) - 1/2*I*(-6*I*e^(1/2*I*pi*b*c*x*sgn(F) -
1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8
*I*pi*b*c + 16*b*c*log(abs(F)) - 16*e) + 6*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/
2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*
I*pi*b*c + 16*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)
) - e)*x - d) - 1/4*(2*(b*c*log(abs(F)) - 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1
/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
 4*(b*c*log(abs(F)) - 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*
sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - p
i*b*c)^2 + 4*(b*c*log(abs(F)) - 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(
F)) - 3*e)*x - 3*d) - 1/2*I*(2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x
+ 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*
b*c*log(abs(F)) - 48*e) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b
```

```
*c*log(abs(F)) - 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x - 3*
d)
```

3.323 $\int F^{c(a+bx)} \sinh^2(d+ex)\,dx$

Optimal. Leaf size=132

$$-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}-\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

```
[Out] (-2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) + (2*e*F^(
c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) - (b*c
*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0530106, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5476, 2194}

$$-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}-\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

```
[Out] (-2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) + (2*e*F^(
c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) - (b*c
*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int F^{c(a+bx)}\sinh^2(d+ex)\,dx &= \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}-\frac{bcF^{c(a+bx)}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}-\frac{\left(2e^2\right)\int F^{c(a+bx)}\,dx}{4e^2-b^2c^2\log^2(F)}\\ &= -\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}+\frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}-\frac{bcF^{c(a+bx)}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] time = 0.201588, size = 86, normalized size = 0.65

$$\frac{F^{c(a+bx)}\left(b^2c^2\log^2(F)\cosh(2(d+ex))-b^2c^2\log^2(F)-2bce\log(F)\sinh(2(d+ex))+4e^2\right)}{2b^3c^3\log^3(F)-8bce^2\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 - b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Log[
F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*Lo
g[F]^3)
```

Maple [A] time = 0.039, size = 143, normalized size = 1.1

$$\frac{\left((\ln(F))^2 b^2c^2 e^{4ex+4d} - 2(\ln(F))^2 b^2c^2 e^{2ex+2d} - 2\ln(F) bce e^{4ex+4d} + b^2c^2(\ln(F))^2 + 2\ln(F) bce + 8e^2 e^{2ex+2d}\right) e^{-2ex}}{4bc\ln(F)(bc\ln(F)-2e)(bc\ln(F)+2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^2,x)
```

```
[Out] 1/4*(ln(F)^2*b^2*c^2*exp(4*e*x+4*d)-2*ln(F)^2*b^2*c^2*exp(2*e*x+2*d)-2*ln(F
)*b*c*e*exp(4*e*x+4*d)+b^2*c^2*ln(F)^2+2*ln(F)*b*c*e+8*e^2*exp(2*e*x+2*d))/
ln(F)/b/c/(b*c*ln(F)-2*e)*exp(-2*e*x-2*d)/(b*c*ln(F)+2*e)*F^(c*(b*x+a))
```

Maxima [A] time = 1.15243, size = 127, normalized size = 0.96

$$\frac{F^{ac}e^{(bcx\log(F)+2ex+2d)}}{4(bc\log(F)+2e)} + \frac{F^{ac}e^{(bcx\log(F)-2ex)}}{4\left(bce^{(2d)}\log(F) - 2ee^{(2d)}\right)} - \frac{F^{bcx+ac}}{2bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c)
*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 1/2*F^(b*c*x
 + a*c)/(b*c*log(F))
```

Fricas [B] time = 2.11988, size = 1783, normalized size = 13.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 + 8*e^2*cosh(e*x
+ d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*
sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 - 2*b^2*c^2*cosh(e*x + d)^2 + b^
2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x +
 d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)
^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) - 4*e^2*cosh(e*x + d
) - (b^2*c^2*cosh(e*x + d)^3 - b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x +
d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(
e*x + d)^4 + 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*
```

```
b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 - 2*
b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*lo
g(F) - (3*b^2*c^2*cosh(e*x + d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*sinh(e*x + d
)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3
*log(F) - 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 - b^2*c^2*cosh(e*x
 + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/(b^3*c^3*cosh(e
*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^3*c^3*log(F)^3 -
 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x + d)*log(F)^3 - 4*
b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

Sympy [A] time = 70.2707, size = 604, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**2,x)
```

```
[Out] Piecewise((x*sinh(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh
(d + e*x)/(2*e), Eq(F, 1)), (zoo*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c)
)**(b*c*x)*sinh(d + e*x)**2 + zoo*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c
))**(b*c*x)*sinh(d + e*x)*cosh(d + e*x) + zoo*e**2*exp(-2*e/(b*c))**(a*c)*e
xp(-2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(-2*e/(b*c)))), (zoo*e**
2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)**2 + zoo*e**2
*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)*cosh(d + e*x)
+ zoo*e**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2,
Eq(F, exp(2*e/(b*c)))), (F**(a*c)*(x*sinh(d + e*x)**2/2 - x*cosh(d + e*x)**
2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (x*sinh(d + e*x)**2/2
- x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (F**
(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3
- 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x)*b*c*e*log(F)*sinh(d + e*x)*cos
h(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x
)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) + 2*F**(a
*c)*F**(b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(
F)), True))
```

Giac [C] time = 1.20012, size = 1220, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) -
 (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c
)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(2*I*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c
*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) +
 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) +
 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
```

```
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi
*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b
*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x +
2*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) +
 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*c
*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sg
n(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2
*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F
)) - 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) - 1/2*I
*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) + 2
*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I
*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a
*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d)
```

3.324 $\int F^{c(a+bx)} \sinh(d+ex)\,dx$

Optimal. Leaf size=75

$$\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

```
[Out] (e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a +
 b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0186574, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5474}

$$\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

```
[Out] (e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a +
 b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx=\frac{eF^{c(a+bx)}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{bcF^{c(a+bx)}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

Mathematica [A] time = 0.105476, size = 50, normalized size = 0.67

$$\frac{F^{c(a+bx)}(e\cosh(d+ex)-bc\log(F)\sinh(d+ex))}{(e-bc\log(F))(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(e*Cosh[d + e*x] - b*c*Log[F]*Sinh[d + e*x]))/((e - b*c*Lo
g[F])*(e + b*c*Log[F]))
```

Maple [A] time = 0.015, size = 77, normalized size = 1.

$$\frac{\left(\ln(F)\,bc\mathrm{e}^{2\,ex+2\,d}-bc\ln(F)-e\mathrm{e}^{2\,ex+2\,d}-e\right)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{(2\,bc\ln(F)-2\,e)\,(e+bc\ln(F))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d),x)
```

```
[Out] 1/2*(ln(F)*b*c*exp(2*e*x+2*d)-b*c*ln(F)-e*exp(2*e*x+2*d)-e)/(b*c*ln(F)-e)*e
xp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

Maxima [A] time = 1.07309, size = 85, normalized size = 1.13

$$\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{2\,(bc\log(F)+e)}-\frac{F^{ac}e^{(bcx\log(F)-ex)}}{2\left(bce^{d}\log(F)-ee^{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - 1/2*F^(a*c)*e^(b*
c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

Fricas [B] time = 1.83603, size = 643, normalized size = 8.57

$$-\frac{\left(e\cosh(ex+d)^{2}-(bc\log(F)-e)\sinh(ex+d)^{2}-\left(bc\cosh(ex+d)^{2}-bc\right)\log(F)-2\,(bc\cosh(ex+d)\log(F)-e\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x
 + d)^2 - b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh
(e*x + d) + e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*log(F
) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 - b*c)*log(F) - 2*(b*c*cosh(e
*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) + e)*sinh((b*c*x + a*c)*log
(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*log(F)
^2 - e^2)*sinh(e*x + d))
```

Sympy [A] time = 13.8538, size = 316, normalized size = 4.21

$$\begin{cases} \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\sinh(d+ex)}{2}-\frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\cosh(d+ex)}{2}+\frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}\cosh(d+ex)}{2e} & \text{for}\ F=-1\wedge b=-\frac{ie}{\pi c} \\ x\sinh(d) & \text{for}\ F=1\wedge e=0 \\ \tilde{\infty}e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\sinh(d+ex)+\tilde{\infty}e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for}\ F=e^{-\frac{e}{bc}} \\ \tilde{\infty}e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\sinh(d+ex)+\tilde{\infty}e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for}\ F=e^{\frac{e}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log(F)\sinh(d+ex)}{b^{2}c^{2}\log(F)^{2}-e^{2}}-\frac{F^{ac}F^{bcx}e\cosh(d+ex)}{b^{2}c^{2}\log(F)^{2}-e^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d),x)
```

```
[Out] Piecewise(((-1)**(a*c)*(-1)**(-I*e*x/pi)*x*sinh(d + e*x)/2 - (-1)**(a*c)*(-
1)**(-I*e*x/pi)*x*cosh(d + e*x)/2 + (-1)**(a*c)*(-1)**(-I*e*x/pi)*cosh(d +
e*x)/(2*e), Eq(F, -1) & Eq(b, -I*e/(pi*c))), (x*sinh(d), Eq(F, 1) & Eq(e, 0
)), (zoo*e*exp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*
e*exp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(
b*c)))), (zoo*e*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) + z
oo*e*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(e/(
b*c)))), (F**(a*c)*F**(b*c*x)*b*c*log(F)*sinh(d + e*x)/(b**2*c**2*log(F)**2
 - e**2) - F**(a*c)*F**(b*c*x)*e*cosh(d + e*x)/(b**2*c**2*log(F)**2 - e**2)
, True))
```

Giac [C] time = 1.20335, size = 826, normalized size = 11.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] (2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) +
e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/4*I*(-
2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I
*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) + 2*I*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)
/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(abs(F)
) + (b*c*log(abs(F)) + e)*x + d) - (2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c
*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F)
- pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/
2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c
*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c
*log(abs(F)) - e)*x - d) - 1/4*I*(2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b
*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*
b*c*log(abs(F)) - 2*e) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1
/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log
(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
```

3.325 $\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx$

Optimal. Leaf size=66

$$-\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{bc\log(F)+e}$$

```
[Out] (-2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rubi [A] time = 0.0218391, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5493}

$$-\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{bc\log(F)+e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x],x]
```

```
[Out] (-2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx=-\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);e^{2(d+ex)}\right)}{e+bc\log(F)}$$

Mathematica [A] time = 5.31345, size = 93, normalized size = 1.41

$$\frac{F^{c(a+bx)}\left({}_2F_1\left(1,\frac{bc\log(F)}{e};\frac{bc\log(F)}{e}+1;-\cosh(d+ex)-\sinh(d+ex)\right)-{}_2F_1\left(1,\frac{bc\log(F)}{e};\frac{bc\log(F)}{e}+1;\cosh(d+ex)+\sinh(d+ex)\right)\right)}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
-Cosh[d + e*x] - Sinh[d + e*x]] - Hypergeometric2F1[1, (b*c*Log[F])/e, 1 +
(b*c*Log[F])/e, Cosh[d + e*x] + Sinh[d + e*x]]))/(b*c*Log[F])
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\operatorname{csch}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4\,F^{ac}e\int\frac{e^{(bcx\log(F)+ex+d)}}{bc\log(F)+\left(bce^{(4\,d)}\log(F)-ee^{(4\,d)}\right)e^{(4\,ex)}-2\left(bce^{(2\,d)}\log(F)-ee^{(2\,d)}\right)e^{(2\,ex)}-e}\,dx-\frac{2\,F^{ac}e^{(bcx\log(F)+}}{bc\log(F)-\left(bce^{(2\,d)}\log(F)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="maxima")
```

```
[Out] 4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*d)
*log(F) - e*e^(4*d))*e^(4*e*x) - 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2*e*
x) - e), x) - 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) - (b*c*e^(2*
d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(F^{bcx+ac}\operatorname{csch}(ex+d),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\operatorname{csch}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d), x)
```

3.326 $\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx$

Optimal. Leaf size=68

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rubi [A] time = 0.0294109, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5493}

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

Mathematica [A] time = 2.74246, size = 87, normalized size = 1.28

$$-\frac{2F^{c(a+bx)}\left(\left(e^{2d}-1\right){}_2F_1\left(1,\frac{bc\log(F)}{2e};\frac{bc\log(F)}{2e}+1;e^{2(d+ex)}\right)+\sinh(d)\operatorname{csch}(d+ex)(\cosh(ex)-\sinh(ex))\right)}{\left(e^{2d}-1\right)e}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

```
[Out] (-2*F^(c*(a + b*x))*((-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e)
, 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))] + Csch[d + e*x]*Sinh[d]*(Cosh[e*
x] - Sinh[e*x])))/(e*(-1 + E^(2*d)))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (\operatorname{csch}(ex+d))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16\, F^{ac} bce \int -\frac{F^{b\cdots}}{b^2c^2 \log(F)^2 - 6\, bce \log(F) + 8\, e^2 - \left(b^2c^2e^{(6\, d)} \log(F)^2 - 6\, bcee^{(6\, d)} \log(F) + 8\, e^2e^{(6\, d)}\right)e^{(6\, ex)} + 3\left(b^2c^2e^{(4\, d\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 16*F^(a*c)*b*c*e*integrate(-F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
8*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d))
*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2*e
^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F) +
8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + 4*(4*F^(a*c)*e + (F^(a*c)*b*c*e^(2*d
)*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*
b*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) +
 8*e^2*e^(4*d))*e^(4*e*x) - 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*l
og(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac} \operatorname{csch}(ex+d)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)} \operatorname{csch}^2(d+ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\,\mathrm{csch}\,(ex+d)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^2, x)
```

3.327 $\int F^{c(a+bx)}\mathbf{csch}^3(d+ex)\,dx$

Optimal. Leaf size=122

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{e^2}-\frac{bc\log(F)\mathbf{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\coth(d+ex)\mathbf{c}}{}$$

```
[Out] -(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x])/(2*e) - (b*c*F^(c*(a + b*x))
*Csch[d + e*x]*Log[F])/(2*e^2) + (E^(d + e*x)*F^(c*(a + b*x))*Hypergeometri
c2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*(e
 - b*c*Log[F]))/e^2
```

Rubi [A] time = 0.0530388, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5491, 5493}

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{e^2}-\frac{bc\log(F)\mathbf{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\coth(d+ex)\mathbf{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

```
[Out] -(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x])/(2*e) - (b*c*F^(c*(a + b*x))
*Csch[d + e*x]*Log[F])/(2*e^2) + (E^(d + e*x)*F^(c*(a + b*x))*Hypergeometri
c2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*(e
 - b*c*Log[F]))/e^2
```

Rule 5491

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Csch[d + e*x]^(n - 2))/(e^2*(n - 1
)*(n - 2)), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[(F^(c*(a +
b*x))*Csch[d + e*x]^(n - 1)*Cosh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a,
 b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &
& NeQ[n, 2]
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int F^{c(a+bx)}\mathrm{csch}^3(d+ex)\,dx &= -\frac{F^{c(a+bx)}\coth(d+ex)\mathrm{csch}(d+ex)}{2e}-\frac{bcF^{c(a+bx)}\mathrm{csch}(d+ex)\log(F)}{2e^2}-\frac{1}{2}\left(1-\frac{b^2c^2\log^2(}{e^2}\right.\\ &= -\frac{F^{c(a+bx)}\coth(d+ex)\mathrm{csch}(d+ex)}{2e}-\frac{bcF^{c(a+bx)}\mathrm{csch}(d+ex)\log(F)}{2e^2}+\frac{e^{d+ex}F^{c(a+bx)}\,{}_2F_1\Big(}{}\end{aligned}$$

Mathematica [B] time = 19.6679, size = 299, normalized size = 2.45

$$F^{c(a+bx)}\left(\frac{4\left(e^2-b^2c^2\log^2(F)\right)\left((\sinh(d)+\cosh(d)-1)\,{}_2F_1\left(1,\frac{bc\log(F)}{e};\frac{bc\log(F)}{e}+1;\cosh(d+ex)+\sinh(d+ex)\right)+1\right)}{bc\log(F)(\sinh(d)+\cosh(d)-1)}+\frac{4\left(e^2-b^2c^2\log^2(F)\right)\left(1-(\sinh(d)+\cosh(d)+1)\,{}_2\right.}{bc\log(F)(\mathrm{s}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csch[(d + e*x)/2]^2) - 4*b*c*Csch[d]*Log[F] + Csch[d]
*((-4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) - e*Sech[(d + e*x)/2]^2 + (4*(e^2 -
 b^2*c^2*Log[F]^2)*(1 + Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F
])/e, Cosh[d + e*x] + Sinh[d + e*x]]*(-1 + Cosh[d] + Sinh[d])))/(b*c*Log[F]
*(-1 + Cosh[d] + Sinh[d])) + (4*(e^2 - b^2*c^2*Log[F]^2)*(1 - Hypergeometri
c2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, -Cosh[d + e*x] - Sinh[d + e*x]]
*(1 + Cosh[d] + Sinh[d])))/(b*c*Log[F]*(1 + Cosh[d] + Sinh[d])) + 2*b*c*Csc
h[d/2]*Csch[(d + e*x)/2]*Log[F]*Sinh[(e*x)/2] + 2*b*c*Log[F]*Sech[d/2]*Sech
[(d + e*x)/2]*Sinh[(e*x)/2]))/(8*e^2)
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\operatorname{csch}(ex+d)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$48\left(F^{ac}bcee^{d}\log(F)+F^{ac}e^2e^{d}\right)\int\frac{}{b^2c^2\log(F)^2-8\,bce\log(F)+15\,e^2+\left(b^2c^2e^{(8\,d)}\log(F)^2-8\,bcee^{(8\,d)}\log(F)+15\,e^2e^{(8\,d)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F) +
 e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(F)
^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) - 4*(b^2*c^2*e^(6*d
)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^2*c^
2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x) - 4
*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*
e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) + (F^(a*c)*b*c*e^(3*d)*log(F) - 5*F^
(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) +
15*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^
2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F)
 + 15*e^2*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\operatorname{csch}\left(ex+d\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\operatorname{csch}^{3}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**3,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^3, x)
```

3.328 $\int F^{c(a+bx)} \operatorname{csch}^4(d+ex)\,dx$

Optimal. Leaf size=131

$$-\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\operatorname{csch}^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\coth(d+ex}{}$$

```
[Out] -(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x]^2)/(3*e) - (b*c*F^(c*(a + b*x
))*Csch[d + e*x]^2*Log[F])/(6*e^2) - (2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hyp
ergeometric2F1[2, 1 + (b*c*Log[F])/(2*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d +
 e*x))]*(2*e - b*c*Log[F]))/(3*e^2)
```

Rubi [A] time = 0.0599853, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5491, 5493}

$$-\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\operatorname{csch}^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\coth(d+ex}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

```
[Out] -(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x]^2)/(3*e) - (b*c*F^(c*(a + b*x
))*Csch[d + e*x]^2*Log[F])/(6*e^2) - (2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hyp
ergeometric2F1[2, 1 + (b*c*Log[F])/(2*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d +
 e*x))]*(2*e - b*c*Log[F]))/(3*e^2)
```

Rule 5491

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Csch[d + e*x]^(n - 2))/(e^2*(n - 1
)*(n - 2)), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[(F^(c*(a +
b*x))*Csch[d + e*x]^(n - 1)*Cosh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a,
 b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &
& NeQ[n, 2]
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx=-\frac{F^{c(a+bx)}\coth(d+ex)\operatorname{csch}^2(d+ex)}{3e}-\frac{bcF^{c(a+bx)}\operatorname{csch}^2(d+ex)\log(F)}{6e^2}-\frac{1}{6}\left(4-\frac{b^2c^2\log^2(F}{e^2}\right.$$

$$=-\frac{F^{c(a+bx)}\coth(d+ex)\operatorname{csch}^2(d+ex)}{3e}-\frac{bcF^{c(a+bx)}\operatorname{csch}^2(d+ex)\log(F)}{6e^2}-\frac{2e^{2(d+ex)}F^{c(a+bx)}\,{}_2F}{}$$

Mathematica [A] time = 6.99941, size = 202, normalized size = 1.54

$$\frac{F^{c(a+bx)}\left(4e^2-b^2c^2\log^2(F)\right)\left(2\,{}_2F_1\left(1,\frac{bc\log(F)}{2e};\frac{bc\log(F)}{2e}+1;\cosh(2(d+ex))+\sinh(2(d+ex))\right)+\coth(d)-1\right)}{6e^3}-\frac{\text{csch}(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(-1 + Coth[d] + 2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e),
 1 + (b*c*Log[F])/(2*e), Cosh[2*(d + e*x)] + Sinh[2*(d + e*x)]])*(4*e^2 - b
^2*c^2*Log[F]^2))/(6*e^3) - (F^(a*c + b*c*x)*Csch[d]*Csch[d + e*x]^2*(2*e*C
osh[d] + b*c*Log[F]*Sinh[d]))/(6*e^2) + (F^(a*c + b*c*x)*Csch[d]*Csch[d + e
*x]^3*Sinh[e*x])/(3*e) - (F^(a*c + b*c*x)*Csch[d]*Csch[d + e*x]*(4*e^2 - b^
2*c^2*Log[F]^2)*Sinh[e*x])/(6*e^3)
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\text{csch}\left(ex+d\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(-F^(b
*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) - 192*
e^3 - (b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*log(F)^2 + 104*b*c
*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) + 5*(b^3*c^3*e^(8*d)*lo
g(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e
^3*e^(8*d))*e^(8*e*x) - 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)
*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x) + 10*(b
^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*
d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x) - 5*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b
^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e
^(2*e*x)), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F^(a*c)*b^2*
c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(a*c)*e^2*e^(
4*d))*e^(4*e*x) + 8*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*e^2*e^(2*d))*
e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^
2*log(F) - 192*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F
)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*x) - 4*(b^3*c^3*
e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(
F) - 192*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*
e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x
```

```
) - 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e
^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e^(2*e*x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\,\text{csch}\,(ex+d)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\,\text{csch}^4\,(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**4,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^4, x)
```

3.329 $\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=250

$$\frac{e^{-4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}}{32bc}$$

```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128*b*c*E^(4*c*(a + b*x))) -
 (5*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(64*b*c*E^(2*c*(a + b*x)))
 + (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*
c) - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128
*b*c) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(19
2*b*c) - (5*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/16
```

Rubi [A] time = 0.247979, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{e^{-4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}}{32bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```


```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128*b*c*E^(4*c*(a + b*x))) -
 (5*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(64*b*c*E^(2*c*(a + b*x)))
 + (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*
c) - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128
*b*c) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(19
2*b*c) - (5*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/16
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx &= \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh^5(ac+bcx)\,dx\\ &= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{(-1+x^2)^5}{32x^5}\,dx,x,e^{c(a+bx)}\right)}{bc}\\ &= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{(-1+x^2)^5}{x^5}\,dx,x,e^{c(a+bx)}\right)}{32bc}\\ &= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{(-1+x)^5}{x^3}\,dx,x,e^{2c(a+bx)}\right)}{64bc}\\ &= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\left(10-\frac{1}{x^3}+\frac{5}{x^2}-\frac{10}{x}-5x+x^2\right)dx,x,e^{2c(a-}\right.}{64bc}\\ &= \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc}-\frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bc}}{64bc}\end{aligned}$$

Mathematica [A] time = 0.109684, size = 106, normalized size = 0.42

$$\frac{\left(\frac{1}{2}e^{-4c(a+bx)}-5e^{-2c(a+bx)}+10e^{2c(a+bx)}-\frac{5}{2}e^{4c(a+bx)}+\frac{1}{3}e^{6c(a+bx)}-20bcx\right)\sinh^2(c(a+bx))^{5/2}\operatorname{csch}^5(c(a+bx))}{64bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((1/(2*E^(4*c*(a + b*x))) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) - (5
*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 - 20*b*c*x)*Csch[c*(a + b*x)]^5
*(Sinh[c*(a + b*x)]^2)^(5/2))/(64*b*c)
```

Maple [A] time = 0.163, size = 184, normalized size = 0.7

$$\frac{1}{48\sinh(c(bx+a))cb}\left(8\sqrt{(\sinh(c(bx+a)))^2}\cosh(c(bx+a))(\sinh(c(bx+a)))^5+8\sqrt{(\sinh(c(bx+a)))^2}(\sinh(c(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 1/48*(8*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*(b*x+a))*sinh(c*(b*x+a))^5+8*(sinh
(c*(b*x+a))^2)^(1/2)*sinh(c*(b*x+a))^6-10*(sinh(c*(b*x+a))^2)^(1/2)*sinh(c*
```

```
(b*x+a))^3*cosh(c*(b*x+a))+15*cosh(c*(b*x+a))*(sinh(c*(b*x+a))^2)^(1/2)*sin
h(c*(b*x+a))-15*ln(cosh(c*(b*x+a))+(sinh(c*(b*x+a))^2)^(1/2))*sinh(c*(b*x+a
))+8*(sinh(c*(b*x+a))^2)^(1/2))/sinh(c*(b*x+a))/c/b
```

Maxima [A] time = 1.59572, size = 122, normalized size = 0.49

$$\frac{\left(2\,e^{(10\,bcx+10\,ac)} - 15\,e^{(8\,bcx+8\,ac)} + 60\,e^{(6\,bcx+6\,ac)} - 30\,e^{(2\,bcx+2\,ac)} + 3\right)e^{(-4\,bcx-4\,ac)}}{384\,bc} - \frac{5\,(bcx + ac)}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x + 6
*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*c*
x + a*c)/(b*c)
```

Fricas [A] time = 1.85523, size = 559, normalized size = 2.24

$$\frac{5\,\cosh\left(bcx + ac\right)^{5} + 25\,\cosh\left(bcx + ac\right)\sinh\left(bcx + ac\right)^{4} - \sinh\left(bcx + ac\right)^{5} - 5\left(2\,\cosh\left(bcx + ac\right)^{2} - 3\right)\sinh\left(bcx + a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - s
inh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 - 45
*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x + a*
c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*cosh(
b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23678, size = 363, normalized size = 1.45

$$-\frac{120\,bcx\text{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) - 3\left(30\,e^{(4\,bcx+4\,ac)}\text{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) - 10\,e^{(2\,bcx+2\,ac)}\text{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*(120*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(30*e^(4*b*c*
x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*c*x + 2*a*c)
*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))*e^(-4*b*c*x - 4*a*c) - (2*e^(6*b*c*x + 18*a*c)*sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)) - 15*e^(4*b*c*x + 16*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*
c*x - a*c)) + 60*e^(2*b*c*x + 14*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c
)))*e^(-12*a*c))/(b*c)
```

3.330 $\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{16bc}-\frac{3e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{cs}}{32bc}$$

```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c*E^(2*c*(a + b*x))) -
(3*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c)
+ (E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*c)
+ (3*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/8
```

Rubi [A] time = 0.139462, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{16bc}-\frac{3e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{cs}}{32bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c*E^(2*c*(a + b*x))) -
(3*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c)
+ (E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*c)
+ (3*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/8
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx = \left(\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh^3(ac+bcx)\,dx$$
$$= \frac{\left(\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int\frac{(-1+x^2)^3}{8x^3}\,dx,x,e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int\frac{(-1+x^2)^3}{x^3}\,dx,x,e^{c(a+bx)}\right)}{8bc}$$
$$= \frac{\left(\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int\frac{(-1+x)^3}{x^2}\,dx,x,e^{2c(a+bx)}\right)}{16bc}$$
$$= \frac{\left(\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int\left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx,x,e^{2c(a+bx)}\right)}{16bc}$$
$$= \frac{e^{-2c(a+bx)}\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc} - \frac{3e^{2c(a+bx)}\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc}$$

Mathematica [A] time = 0.0582139, size = 76, normalized size = 0.47

$$\frac{\left(e^{-2c(a+bx)}-3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\sinh^2(c(a+bx))^{3/2}\text{csch}^3(c(a+bx))}{16bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x)
*Csch[c*(a + b*x)]^3*(Sinh[c*(a + b*x)]^2)^(3/2))/(16*b*c)
```

Maple [A] time = 0.109, size = 152, normalized size = 0.9

$$\frac{1}{8\sinh(c(bx+a))cb}\left(2\sqrt{(\sinh(c(bx+a)))^2}(\sinh(c(bx+a)))^3\cosh(c(bx+a))+2\sqrt{(\sinh(c(bx+a)))^2}(\sinh(c(bx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 1/8*(2*(sinh(c*(b*x+a))^2)^(1/2)*sinh(c*(b*x+a))^3*cosh(c*(b*x+a))+2*(sinh(
c*(b*x+a))^2)^(1/2)*sinh(c*(b*x+a))^4-3*cosh(c*(b*x+a))*(sinh(c*(b*x+a))^2)
^(1/2)*sinh(c*(b*x+a))+3*ln(cosh(c*(b*x+a))+(sinh(c*(b*x+a))^2)^(1/2))*sinh
(c*(b*x+a))-2*(sinh(c*(b*x+a))^2)^(1/2))/sinh(c*(b*x+a))/c/b
```

Maxima [A] time = 1.66537, size = 84, normalized size = 0.52

$$\frac{\left(e^{(6\,bcx+6\,ac)}-6\,e^{(4\,bcx+4\,ac)}+2\right)e^{(-2\,bcx-2\,ac)}}{32\,bc}+\frac{3\,(bcx+ac)}{8\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c)
/(b*c) + 3/8*(b*c*x + a*c)/(b*c)
```

Fricas [A] time = 1.88162, size = 319, normalized size = 1.97

$$\frac{3\,\cosh(bcx+ac)^3+9\,\cosh(bcx+ac)\sinh(bcx+ac)^2-\sinh(bcx+ac)^3+6\,(2\,bcx-1)\cosh(bcx+ac)-3\,\left(4\,bcx+\right.}{32\,(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - sin
h(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(b*
c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*
x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15974, size = 263, normalized size = 1.62

$$\frac{12\,bcx\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-2\left(3\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-2\,bcx-2\,ac)}+\left(e\right.}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^(2*b*c*x +
2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*c) - e^(-
b*c*x - a*c)))*e^(-2*b*c*x - 2*a*c) + (e^(4*b*c*x + 8*a*c)*sgn(e^(b*c*x + a
*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b
*c*x - a*c)))*e^(-4*a*c))/(b*c)
```

3.331 $\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{4bc}-\frac{1}{2}x\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)$$

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(4*b*c) - (
x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/2
```

Rubi [A] time = 0.111542, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 14}

$$\frac{e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{4bc}-\frac{1}{2}x\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(4*b*c) - (
x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/2
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh(ac+bcx)\,dx$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{c(a+bx)}\right)}{2bc}$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)dx, x, e^{c(a+bx)}\right)}{2bc}$$
$$= \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{4bc} - \frac{1}{2}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}$$

Mathematica [A] time = 0.0378588, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}-2bcx\right)\sqrt{\sinh^2(c(a+bx))}\operatorname{csch}(c(a+bx))}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)]*Sqrt[Sinh[c*(a + b*x)]^2])
/(4*b*c)
```

Maple [A] time = 0.112, size = 100, normalized size = 1.4

$$\frac{\cosh(c(bx+a))}{2cb}\sqrt{(\sinh(c(bx+a)))^2} - \frac{1}{2cb}\ln\left(\cosh(c(bx+a)) + \sqrt{(\sinh(c(bx+a)))^2}\right) + \frac{(\cosh(c(bx+a)))^2}{2cb\sinh(c(bx+a))}\sqrt{(s$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/2/c/b*cosh(c*(b*x+a))*(sinh(c*(b*x+a))^2)^(1/2)-1/2/c/b*ln(cosh(c*(b*x+a)
)+(sinh(c*(b*x+a))^2)^(1/2))+1/2/c/b*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*(b*x+
a))^2/sinh(c*(b*x+a))
```

Maxima [A] time = 1.6531, size = 49, normalized size = 0.66

$$-\frac{bcx+ac}{2\,bc} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Fricas [A] time = 1.84701, size = 165, normalized size = 2.23

$$-\frac{(2\,bcx-1)\cosh\left(bcx+ac\right)-(2\,bcx+1)\sinh\left(bcx+ac\right)}{4\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [A] time = 31.1821, size = 206, normalized size = 2.78

$$\begin{cases} x\sqrt{\sinh^2\left(ac\right)}e^{ac} & \text{for } b=0 \\ 0 & \text{for } a=\frac{\log(}{} \\ \frac{x\sqrt{\sinh^2\left(ac+bcx\right)}e^{ac}e^{bcx}}{2}-\frac{x\sqrt{\sinh^2\left(ac+bcx\right)}e^{ac}e^{bcx}\cosh\left(ac+bcx\right)}{2\sinh\left(ac+bcx\right)}-\frac{\sqrt{\sinh^2\left(ac+bcx\right)}e^{ac}e^{bcx}}{2bc}+\frac{\sqrt{\sinh^2\left(ac+bcx\right)}e^{ac}e^{bcx}\cosh\left(ac+bcx\right)}{bc\sinh\left(ac+bcx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(sinh(a*c)**2)*exp(a*c), Eq(b, 0)), (0, Eq(c, 0) | Eq(a, l
og(exp(-b*c*x))/c) | Eq(a, log(-exp(-b*c*x))/c)), (x*sqrt(sinh(a*c + b*c*x)
**2)*exp(a*c)*exp(b*c*x)/2 - x*sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*
x)*cosh(a*c + b*c*x)/(2*sinh(a*c + b*c*x)) - sqrt(sinh(a*c + b*c*x)**2)*exp
(a*c)*exp(b*c*x)/(2*b*c) + sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)*c
osh(a*c + b*c*x)/(b*c*sinh(a*c + b*c*x)), True))
```

Giac [A] time = 1.16266, size = 96, normalized size = 1.3

$$-\frac{1}{2}\,x\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\frac{e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1/4*e^(2*b*c*x + 2*a*c)*sg
n(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*c)
```

3.332 $\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] (Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b*c*Sqrt[Sinh[a*c + b*c*x]^
2])
```

Rubi [A] time = 0.129109, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 260}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b*c*Sqrt[Sinh[a*c + b*c*x]^
2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}$$
$$= \frac{\sinh(ac+bcx)\,\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$
$$= \frac{(2\sinh(ac+bcx))\,\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$
$$= \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

Mathematica [A] time = 0.050542, size = 44, normalized size = 0.96

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))}{bc\sqrt{\sinh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(b*c*Sqrt[Sinh[c*(a + b*x)]^
2])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\frac{1}{\sqrt{(\sinh(bcx+ac))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x)
```

Maxima [A] time = 1.60244, size = 53, normalized size = 1.15

$$\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A] time = 1.82143, size = 97, normalized size = 2.11

$$\frac{\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\sinh^2(ac+bcx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(sinh(a*c + b*c*x)**2), x)
```

Giac [A] time = 1.21016, size = 115, normalized size = 2.5

$$\frac{\log\left(e^{(bcx)}+e^{(-ac)}\right)\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\log\left(\left|e^{(bcx)}-e^{(-ac)}\right|\right)\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (log(e^(b*c*x) + e^(-a*c))*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + log(ab
s(e^(b*c*x) - e^(-a*c)))*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c)
```

3.333 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] (-2*E^(4*c*(a + b*x))*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqr
t[Sinh[a*c + b*c*x]^2])
```

Rubi [A] time = 0.143144, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6720, 2282, 12, 264}

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqr
t[Sinh[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}^3(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}$$
$$= \frac{\sinh(ac+bcx)\,\text{Subst}\left(\int \frac{8x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$
$$= \frac{(8\sinh(ac+bcx))\,\text{Subst}\left(\int \frac{x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$
$$= -\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

Mathematica [A] time = 0.0693504, size = 46, normalized size = 0.79

$$-\frac{4e^{5c(a+bx)}\sqrt{\sinh^2(c(a+bx))}}{bc\left(e^{2c(a+bx)}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-4*E^(5*c*(a + b*x))*Sqrt[Sinh[c*(a + b*x)]^2])/(b*c*(-1 + E^(2*c*(a + b*x
)))^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left((\sinh(bcx+ac))^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x)
```

Maxima [A] time = 1.56679, size = 113, normalized size = 1.95

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{2}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) +
1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 1.75967, size = 300, normalized size = 5.17

$$-\frac{2\left(\cosh\left(bcx+ac\right)+3\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2+bc\sinh\left(bcx+ac\right)^3-bc\cosh\left(bcx+ac\right)+3\left(bc\cosh\left(bcx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*co
sh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23097, size = 117, normalized size = 2.02

$$-\frac{2\left(2\,e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)}{bc\left(e^{(2\,bcx+2\,ac)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(
b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2)
```

3.334 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{8\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}+\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] (-4*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4*Sqrt[Sinh[a*c + b*c*x
]^2]) + (32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3*Sqrt[Sinh[a
*c + b*c*x]^2]) - (8*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt
[Sinh[a*c + b*c*x]^2])
```

Rubi [A] time = 0.213861, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{8\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}+\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4*Sqrt[Sinh[a*c + b*c*x
]^2]) + (32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3*Sqrt[Sinh[a
*c + b*c*x]^2]) - (8*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt
[Sinh[a*c + b*c*x]^2])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\mathbf{csch}^5(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{\sinh(ac+bcx)\,\text{Subst}\left(\int \frac{32x^5}{\left(-1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(32\sinh(ac+bcx))\,\text{Subst}\left(\int \frac{x^5}{\left(-1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(16\sinh(ac+bcx))\,\text{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(16\sinh(ac+bcx))\,\text{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= -\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}} + \frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}} - \frac{8\,\mathrm{si}}{bc\left(1-e^{2c(a+b}\right.}$$

Mathematica [A] time = 0.0700153, size = 72, normalized size = 0.49

$$-\frac{4\left(-4e^{2c(a+bx)}+6e^{4c(a+bx)}+1\right)\sinh(c(a+bx))}{3bc\left(e^{2c(a+bx)}-1\right)^4\sqrt{\sinh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sinh[c*(a + b*x)])/(3*b
*c*(-1 + E^(2*c*(a + b*x)))^4*Sqrt[Sinh[c*(a + b*x)]^2])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left((\sinh(bcx+ac))^2\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x)
```

Maxima [A] time = 1.62597, size = 282, normalized size = 1.92

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 1.87394, size = 797, normalized size = 5.42

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}-4\,bc\cosh\left(bcx+ac\right)^{4}+\left(15\,bc\,\mathrm{c}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29122, size = 165, normalized size = 1.12

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-4\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 4*e^(
2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a
*c) - e^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^4)
```

3.335 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}} \, dx$

Optimal. Leaf size=199

$$\frac{64\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{48\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}+\frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}}-$$

```
[Out] (-32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^6*Sqrt[Sinh[a*c + b*
c*x]^2]) + (192*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^(2*c*(a + b*x)))^5*Sqrt[Si
nh[a*c + b*c*x]^2]) - (48*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4
*Sqrt[Sinh[a*c + b*c*x]^2]) + (64*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a
+ b*x)))^3*Sqrt[Sinh[a*c + b*c*x]^2])
```

Rubi [A] time = 0.279457, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{64\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{48\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}+\frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
```


```
[Out] (-32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^6*Sqrt[Sinh[a*c + b*
c*x]^2]) + (192*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^(2*c*(a + b*x)))^5*Sqrt[Si
nh[a*c + b*c*x]^2]) - (48*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4
*Sqrt[Sinh[a*c + b*c*x]^2]) + (64*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a
+ b*x)))^3*Sqrt[Sinh[a*c + b*c*x]^2])
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}^7(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\sinh(ac+bcx)\operatorname{Subst}\left(\int \frac{128x^7}{\left(-1+x^2\right)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(128\sinh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^7}{\left(-1+x^2\right)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(64\sinh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^3}{(-1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(64\sinh(ac+bcx))\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^7}+\frac{3}{(-1+x)^6}+\frac{3}{(-1+x)^5}+\frac{1}{(-1+x)^4}\right)dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= -\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(ac+bcx)}}+\frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}}-\frac{48}{bc\left(1-e^{2c(a\cdot\right.}}
\end{aligned}$$

Mathematica [A] time = 0.078297, size = 84, normalized size = 0.42

$$-\frac{16\left(6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}-1\right)\sinh(c(a+bx))}{15bc\left(e^{2c(a+bx)}-1\right)^6\sqrt{\sinh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a + b*x)))^6*Sqrt[Sinh[c*(a + b
*x)]^2])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left((\sinh(bcx+ac))^2\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x)
```

Maxima [B] time = 1.7505, size = 521, normalized size = 2.62

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc\left(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{}{bc\left(e^{(12\,bcx+12\,ac}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
- 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] time = 1.98596, size = 1512, normalized size = 7.6

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^{9}+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{8}+bc\sinh\left(bcx+ac\right)^{9}-6\,bc\cosh\left(bcx+ac\right)^{7}+6\left(6\,bc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)
- 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^7 +
6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*x
 + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c
*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*c)^2 +
5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b*
c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh(
b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*x + a*c)^4
 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b*c*cosh(b*c
*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x + a*c)^5 + 50
*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 +
3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(b*c
*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33243, size = 217, normalized size = 1.09

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-15\,e^{(4\,bcx+4\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+6\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-a}\right.}{15\,bc\left(e^{(2\,bcx+2\,ac)}-1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 15
*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 6*e^(2*b*c*x
 + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^6)
```

3.336 $\int e^x \sinh(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{e^x\sinh(a+bx)}{1-b^2}-\frac{be^x\cosh(a+bx)}{1-b^2}$$

```
[Out] -((b*E^x*Cosh[a + b*x])/(1 - b^2)) + (E^x*Sinh[a + b*x])/(1 - b^2)
```

Rubi [A] time = 0.0132719, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5474}

$$\frac{e^x\sinh(a+bx)}{1-b^2}-\frac{be^x\cosh(a+bx)}{1-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[a + b*x],x]
```

```
[Out] -((b*E^x*Cosh[a + b*x])/(1 - b^2)) + (E^x*Sinh[a + b*x])/(1 - b^2)
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sinh(a+bx)\,dx=-\frac{be^x\cosh(a+bx)}{1-b^2}+\frac{e^x\sinh(a+bx)}{1-b^2}$$

Mathematica [A] time = 0.0479623, size = 28, normalized size = 0.68

$$\frac{e^x(b\cosh(a+bx)-\sinh(a+bx))}{b^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[a + b*x],x]
```

```
[Out] (E^x*(b*Cosh[a + b*x] - Sinh[a + b*x]))/(-1 + b^2)
```

Maple [A] time = 0.016, size = 62, normalized size = 1.5

$$-\frac{\sinh((b-1)x+a)}{2b-2}+\frac{\sinh((1+b)x+a)}{2+2b}+\frac{\cosh((b-1)x+a)}{2b-2}+\frac{\cosh((1+b)x+a)}{2+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(b*x+a),x)
```

```
[Out] -1/2/(b-1)*sinh((b-1)*x+a)+1/2/(1+b)*sinh((1+b)*x+a)+1/2*cosh((b-1)*x+a)/(b
-1)+1/2*cosh((1+b)*x+a)/(1+b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74592, size = 134, normalized size = 3.27

$$\frac{b \cosh\left(bx + a\right) \cosh\left(x\right) + b \cosh\left(bx + a\right) \sinh\left(x\right) - \left(\cosh\left(x\right) + \sinh\left(x\right)\right) \sinh\left(bx + a\right)}{b^{2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)*cosh(x) + b*cosh(b*x + a)*sinh(x) - (cosh(x) + sinh(x))*si
nh(b*x + a))/(b^2 - 1)
```

Sympy [A] time = 0.842607, size = 99, normalized size = 2.41

$$\begin{cases} \frac{xe^{x} \sinh\left(a - x\right)}{2} + \frac{xe^{x} \cosh\left(a - x\right)}{2} + \frac{e^{x} \sinh\left(a - x\right)}{2} & \text{for } b = -1 \\ \frac{xe^{x} \sinh\left(a + x\right)}{2} - \frac{xe^{x} \cosh\left(a + x\right)}{2} + \frac{e^{x} \cosh\left(a + x\right)}{2} & \text{for } b = 1 \\ \frac{be^{x} \cosh\left(a + bx\right)}{b^{2} - 1} - \frac{e^{x} \sinh\left(a + bx\right)}{b^{2} - 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 + exp(x)*sinh(a
- x)/2, Eq(b, -1)), (x*exp(x)*sinh(a + x)/2 - x*exp(x)*cosh(a + x)/2 + exp(
x)*cosh(a + x)/2, Eq(b, 1)), (b*exp(x)*cosh(a + b*x)/(b**2 - 1) - exp(x)*si
nh(a + b*x)/(b**2 - 1), True))
```

Giac [A] time = 1.16365, size = 43, normalized size = 1.05

$$\frac{e^{\left(bx + a + x\right)}}{2 \left(b + 1\right)} + \frac{e^{\left(-bx - a + x\right)}}{2 \left(b - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="giac")

[Out] 1/2*e^(b*x + a + x)/(b + 1) + 1/2*e^(-b*x - a + x)/(b - 1)
```

3.337 $\int e^x \sinh\left(a+cx^2\right)\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{\pi}e^{\frac{1}{4c}-a}\operatorname{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}e^{a-\frac{1}{4c}}\operatorname{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] (E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c]) + (E^(
a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Rubi [A] time = 0.0812564, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5512, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}e^{\frac{1}{4c}-a}\operatorname{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}e^{a-\frac{1}{4c}}\operatorname{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[a + c*x^2],x]
```

```
[Out] (E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c]) + (E^(
a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh\left(a+cx^2\right) dx &= \int \left(-\frac{1}{2}e^{-a+x-cx^2}+\frac{1}{2}e^{a+x+cx^2}\right) dx \\ &= -\left(\frac{1}{2}\int e^{-a+x-cx^2} dx\right)+\frac{1}{2}\int e^{a+x+cx^2} dx \\ &= \frac{1}{2}e^{a-\frac{1}{4c}}\int e^{\frac{(1+2cx)^2}{4c}} dx-\frac{1}{2}e^{-a+\frac{1}{4c}}\int e^{-\frac{(1-2cx)^2}{4c}} dx \\ &= \frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\text{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\text{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0911724, size = 80, normalized size = 0.94

$$\frac{\sqrt{\pi}e^{-\frac{1}{4}/c}\left((\sinh(a)+\cosh(a))\text{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)-e^{\frac{1}{2}/c}(\cosh(a)-\sinh(a))\text{Erf}\left(\frac{2cx-1}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a]))
 + Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4*c
)))
```

Maple [A] time = 0.109, size = 72, normalized size = 0.9

$$-\frac{\sqrt{\pi}}{4}e^{-\frac{4ac-1}{4c}}\text{Erf}\left(\sqrt{c}x-\frac{1}{2}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}+\frac{\sqrt{\pi}}{4}e^{\frac{4ac-1}{4c}}\text{Erf}\left(\sqrt{-c}x-\frac{1}{2}\frac{1}{\sqrt{-c}}\right)\frac{1}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(c*x^2+a),x)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4*
Pi^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf((-c)^(1/2)*x-1/2/(-c)^(1/2))
```

Maxima [A] time = 1.12056, size = 88, normalized size = 1.04

$$\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{-c}x-\frac{1}{2\sqrt{-c}}\right)e^{\left(a-\frac{1}{4c}\right)}}{4\sqrt{-c}}-\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{c}x-\frac{1}{2\sqrt{c}}\right)e^{\left(-a+\frac{1}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) - 1/4*sq
rt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

Fricas [A] time = 1.78437, size = 286, normalized size = 3.36

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)+\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+1)\sqrt{-c}}{2\,c}\right)+\sqrt{\pi}\sqrt{c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)-\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2\,cx-1}{2\,\sqrt{c}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c))
*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) + sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)/c
) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\sinh\left(a+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*sinh(a + c*x**2), x)
```

Giac [A] time = 1.15885, size = 99, normalized size = 1.16

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2\,x+\frac{1}{c}\right)\right)e^{\left(\frac{4\,ac-1}{4\,c}\right)}}{4\,\sqrt{-c}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2\,x-\frac{1}{c}\right)\right)e^{\left(-\frac{4\,ac-1}{4\,c}\right)}}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c)
 + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt(c
)
```

3.338 $\int e^x \sinh\left(a+bx+cx^2\right) dx$

Optimal. Leaf size=101

$$\frac{\sqrt{\pi}e^{\frac{(1-b)^2}{4c}-a}\operatorname{Erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}e^{a-\frac{(b+1)^2}{4c}}\operatorname{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] (E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/(4*Sqr
t[c]) + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]
)/(4*Sqrt[c])
```

Rubi [A] time = 0.146931, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5512, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}e^{\frac{(1-b)^2}{4c}-a}\operatorname{Erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}e^{a-\frac{(b+1)^2}{4c}}\operatorname{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/(4*Sqr
t[c]) + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]
)/(4*Sqrt[c])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh\left(a+bx+cx^2\right)dx &= \int\left(-\frac{1}{2}e^{-a+(1-b)x-cx^2}+\frac{1}{2}e^{a+(1+b)x+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-a+(1-b)x-cx^2}\,dx\right)+\frac{1}{2}\int e^{a+(1+b)x+cx^2}\,dx\\ &= -\left(\frac{1}{2}e^{-a+\frac{(1-b)^2}{4c}}\int e^{-\frac{(1-b-2cx)^2}{4c}}\,dx\right)+\frac{1}{2}e^{a-\frac{(1+b)^2}{4c}}\int e^{\frac{(1+b+2cx)^2}{4c}}\,dx\\ &= \frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\operatorname{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.166609, size = 92, normalized size = 0.91

$$\frac{\sqrt{\pi}e^{-\frac{(b+1)^2}{4c}}\left((\sinh(a)+\cosh(a))\text{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)-e^{\frac{b^2+1}{2c}}(\cosh(a)-\sinh(a))\text{Erf}\left(\frac{b+2cx-1}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a]
 - Sinh[a])) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*S
qrt[c]*E^((1 + b)^2/(4*c)))
```

Maple [A] time = 0.099, size = 97, normalized size = 1.

$$-\frac{\sqrt{\pi}}{4}\mathrm{e}^{-\frac{4ac-b^2+2b-1}{4c}}\text{Erf}\left(\sqrt{c}x-\frac{1-b}{2}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}-\frac{\sqrt{\pi}}{4}\mathrm{e}^{\frac{4ac-b^2-2b-1}{4c}}\text{Erf}\left(-\sqrt{-c}x+\frac{1+b}{2}\frac{1}{\sqrt{-c}}\right)\frac{1}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(c*x^2+b*x+a),x)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b)
/c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-(-c)^(1
/2)*x+1/2*(1+b)/(-c)^(1/2))
```

Maxima [A] time = 1.15465, size = 109, normalized size = 1.08

$$\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{-c}x-\frac{b+1}{2\sqrt{-c}}\right)e^{\left(a-\frac{(b+1)^2}{4c}\right)}}{4\sqrt{-c}}-\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{c}x+\frac{b-1}{2\sqrt{c}}\right)e^{\left(-a+\frac{(b-1)^2}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c)
/sqrt(-c) - 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4*(
```

```
b - 1)^2/c)/sqrt(c)
```

Fricas [A] time = 1.78112, size = 367, normalized size = 3.63

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)+\sinh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+b+1)\sqrt{-c}}{2\,c}\right)+\sqrt{\pi}\sqrt{c}\left(\cosh\left(-\frac{b^2-4\,ac-2\,b+1}{4\,c}\right)-\sinh\left(-\frac{b^2-\cdots}{}\right.\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*(
b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) + sqrt(pi)*s
qrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c - 2*b
 + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \sinh\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*sinh(a + b*x + c*x**2), x)
```

Giac [A] time = 1.12461, size = 123, normalized size = 1.22

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2\,x+\frac{b+1}{c}\right)\right)e^{\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)}}{4\sqrt{-c}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2\,x+\frac{b-1}{c}\right)\right)e^{\left(\frac{b^2-4\,ac-2\,b+1}{4\,c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c + 2
*b + 1)/c)/sqrt(-c) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^(1
/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

3.339 $\int e^{x^2} \sinh(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}\sqrt{\pi}e^{a-\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(b+2x)\right)-\frac{1}{4}\sqrt{\pi}e^{-a-\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x-b)\right)$$

```
[Out] -(E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2])/4 + (E^(a - b^2/4)*Sqrt[Pi]*E
rfi[(b + 2*x)/2])/4
```

Rubi [A] time = 0.0649381, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5512, 2234, 2204}

$$\frac{1}{4}\sqrt{\pi}e^{a-\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(b+2x)\right)-\frac{1}{4}\sqrt{\pi}e^{-a-\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x-b)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sinh[a + b*x],x]
```

```
[Out] -(E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2])/4 + (E^(a - b^2/4)*Sqrt[Pi]*E
rfi[(b + 2*x)/2])/4
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{x^2}\sinh(a+bx)\,dx &= \int \left(-\frac{1}{2}e^{-a-bx+x^2}+\frac{1}{2}e^{a+bx+x^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-a-bx+x^2}\,dx\right)+\frac{1}{2}\int e^{a+bx+x^2}\,dx\\
&= -\left(\frac{1}{2}e^{-a-\frac{b^2}{4}}\int e^{\frac{1}{4}(-b+2x)^2}\,dx\right)+\frac{1}{2}e^{a-\frac{b^2}{4}}\int e^{\frac{1}{4}(b+2x)^2}\,dx\\
&= -\frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(b+2x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0662858, size = 51, normalized size = 0.78

$$\frac{1}{4}\sqrt{\pi}e^{-\frac{b^2}{4}}\left((\cosh(a)-\sinh(a))\text{Erfi}\left(\frac{b}{2}-x\right)+(\sinh(a)+\cosh(a))\text{Erfi}\left(\frac{b}{2}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sinh[a + b*x],x]
```

```
[Out] (Sqrt[Pi]*(Erfi[b/2 - x]*(Cosh[a] - Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + Sin
h[a])))/(4*E^(b^2/4))
```

Maple [C] time = 0.073, size = 52, normalized size = 0.8

$$-\frac{i}{4}\sqrt{\pi}\mathrm{e}^{-a-\frac{b^2}{4}}\text{Erf}\left(-ix+\frac{i}{2}b\right)-\frac{i}{4}\sqrt{\pi}\mathrm{e}^{a-\frac{b^2}{4}}\text{Erf}\left(ix+\frac{i}{2}b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(b*x+a),x)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4*
b^2)*erf(I*x+1/2*I*b)
```

Maxima [C] time = 1.06116, size = 61, normalized size = 0.94

$$-\frac{1}{4}i\sqrt{\pi}\,\text{erf}\left(\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}+\frac{1}{4}i\sqrt{\pi}\,\text{erf}\left(-\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(-1
/2*I*b + I*x)*e^(-1/4*b^2 - a)
```

Fricas [A] time = 1.88877, size = 130, normalized size = 2.

$$\frac{1}{4}\sqrt{\pi}\left(\text{erfi}\left(\frac{1}{2}\,b+x\right)e^{\left(\frac{1}{4}\,b^2+a\right)}-\text{erfi}\left(-\frac{1}{2}\,b+x\right)e^{\left(\frac{1}{4}\,b^2-a\right)}\right)e^{\left(-\frac{1}{2}\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(erfi(1/2*b + x)*e^(1/4*b^2 + a) - erfi(-1/2*b + x)*e^(1/4*b^2
 - a))*e^(-1/2*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\sinh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sinh(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + b*x), x)
```

Giac [C] time = 1.13084, size = 61, normalized size = 0.94

$$\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}ib-ix\right)e^{\left(-\frac{1}{4}b^2+a\right)}-\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}ib-ix\right)e^{\left(-\frac{1}{4}b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(1/
2*I*b - I*x)*e^(-1/4*b^2 - a)
```

3.340 $\int e^{x^2} \sinh\left(a+cx^2\right) dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}e^a\text{Erfi}\left(\sqrt{c+1}x\right)}{4\sqrt{c+1}}-\frac{\sqrt{\pi}e^{-a}\text{Erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}}$$

```
[Out] -(Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(4*Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sq
rt[1 + c]*x])/(4*Sqrt[1 + c])
```

Rubi [A] time = 0.0832766, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5512, 2204}

$$\frac{\sqrt{\pi}e^a\text{Erfi}\left(\sqrt{c+1}x\right)}{4\sqrt{c+1}}-\frac{\sqrt{\pi}e^{-a}\text{Erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sinh[a + c*x^2],x]
```

```
[Out] -(Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(4*Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sq
rt[1 + c]*x])/(4*Sqrt[1 + c])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int e^{x^2} \sinh\left(a+cx^2\right) dx &= \int \left(-\frac{1}{2}e^{-a+(1-c)x^2}+\frac{1}{2}e^{a+(1+c)x^2}\right) dx\\
&= -\left(\frac{1}{2}\int e^{-a+(1-c)x^2}\,dx\right)+\frac{1}{2}\int e^{a+(1+c)x^2}\,dx\\
&= -\frac{e^{-a}\sqrt{\pi}\text{erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}}+\frac{e^{a}\sqrt{\pi}\text{erfi}\left(\sqrt{1+c}x\right)}{4\sqrt{1+c}}
\end{aligned}$$

Mathematica [A] time = 0.107318, size = 72, normalized size = 1.11

$$\frac{\sqrt{\pi}\left((c-1)\sqrt{c+1}(\sinh(a)+\cosh(a))\text{Erfi}\left(\sqrt{c+1}x\right)-\sqrt{c-1}(c+1)(\cosh(a)-\sinh(a))\text{Erf}\left(\sqrt{c-1}x\right)\right)}{4\left(c^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sinh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]))
+ (-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 + c
^2))
```

Maple [A] time = 0.073, size = 48, normalized size = 0.7

$$-\frac{\sqrt{\pi}e^{-a}}{4}\mathrm{Erf}\left(\sqrt{c-1}x\right)\frac{1}{\sqrt{c-1}}+\frac{\sqrt{\pi}e^{a}}{4}\mathrm{Erf}\left(\sqrt{-1-c}x\right)\frac{1}{\sqrt{-1-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(c*x^2+a),x)
```

```
[Out] -1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(-
1-c)^(1/2)*erf((-1-c)^(1/2)*x)
```

Maxima [A] time = 1.08772, size = 63, normalized size = 0.97

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}x\right)e^{(-a)}}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}x\right)e^{a}}{4\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqrt
(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Fricas [A] time = 1.86845, size = 235, normalized size = 3.62

$$-\frac{\sqrt{\pi}((c+1)\cosh(a)-(c+1)\sinh(a))\sqrt{c-1}\,\mathrm{erf}\left(\sqrt{c-1}x\right)+\sqrt{\pi}((c-1)\cosh(a)+(c-1)\sinh(a))\sqrt{-c-1}\,\mathrm{erf}\left(\sqrt{-c-}\right.}{4\left(c^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c -
 1)*x) + sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sqrt
(-c - 1)*x))/(c^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\sinh\left(a+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sinh(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + c*x**2), x)
```

Giac [A] time = 1.14358, size = 66, normalized size = 1.02

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{c-1}x\right)e^{(-a)}}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c-1}x\right)e^{a}}{4\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-sqr
t(-c - 1)*x)*e^a/sqrt(-c - 1)
```

3.341 $\int e^{x^2} \sinh\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\pi}e^{-a-\frac{b^2}{4(1-c)}}\operatorname{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{\sqrt{\pi}e^{a-\frac{b^2}{4(c+1)}}\operatorname{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}$$

```
[Out] (E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])])
/(4*Sqrt[1 - c]) + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*x)
/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Rubi [A] time = 0.167993, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5512, 2234, 2204}

$$\frac{\sqrt{\pi}e^{-a-\frac{b^2}{4(1-c)}}\operatorname{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{\sqrt{\pi}e^{a-\frac{b^2}{4(c+1)}}\operatorname{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])])
/(4*Sqrt[1 - c]) + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*x)
/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)dx=\int\left(-\frac{1}{2}e^{-a-bx+(1-c)x^2}+\frac{1}{2}e^{a+bx+(1+c)x^2}\right)dx$$
$$=-\left(\frac{1}{2}\int e^{-a-bx+(1-c)x^2}dx\right)+\frac{1}{2}\int e^{a+bx+(1+c)x^2}dx$$
$$=-\left(\frac{1}{2}e^{-a-\frac{b^2}{4(1-c)}}\int e^{\frac{(-b+2(1-c)x)^2}{4(1-c)}}dx\right)+\frac{1}{2}e^{a-\frac{b^2}{4(1+c)}}\int e^{\frac{(b+2(1+c)x)^2}{4(1+c)}}dx$$
$$=\frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}$$

Mathematica [A] time = 0.396314, size = 123, normalized size = 1.07

$$\frac{\sqrt{\pi}e^{-\frac{b^2}{4c+4}}\left((c-1)\sqrt{c+1}(\sinh(a)+\cosh(a))\mathrm{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)-\sqrt{c-1}(c+1)e^{\frac{b^2c}{2(c^2-1)}}(\cosh(a)-\sinh(a))\mathrm{Erf}\left(\frac{b+2(c-1)x}{2\sqrt{c-1}}\right)\right)}{4\left(c^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-1
 + c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a])) + (-1 + c)*Sqrt[1 + c]*Erfi
[(b + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*E^(
b^2/(4 + 4*c)))
```

Maple [A] time = 0.145, size = 105, normalized size = 0.9

$$-\frac{\sqrt{\pi}}{4}e^{-\frac{4ac-b^2-4a}{4c-4}}\mathrm{Erf}\left(\sqrt{c-1}x+\frac{b}{2}\frac{1}{\sqrt{c-1}}\right)\frac{1}{\sqrt{c-1}}-\frac{\sqrt{\pi}}{4}e^{\frac{4ac-b^2+4a}{4+4c}}\mathrm{Erf}\left(-\sqrt{-1-c}x+\frac{b}{2}\frac{1}{\sqrt{-1-c}}\right)\frac{1}{\sqrt{-1-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(c*x^2+b*x+a),x)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*x
+1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(1+c))/(-1-c)^(1/2
)*erf(-(-1-c)^(1/2)*x+1/2*b/(-1-c)^(1/2))
```

Maxima [A] time = 1.06368, size = 120, normalized size = 1.04

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}x-\frac{b}{2\sqrt{-c-1}}\right)e^{\left(a-\frac{b^2}{4(c+1)}\right)}}{4\sqrt{-c-1}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}x+\frac{b}{2\sqrt{c-1}}\right)e^{\left(-a+\frac{b^2}{4(c-1)}\right)}}{4\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c + 1
))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(-a
 + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

Fricas [A] time = 1.82426, size = 467, normalized size = 4.06

$$-\frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\,\mathrm{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)+\sqrt{\pi}\left((c-1)\cosh\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)+(c-}{4\left(c^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*si
nh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x + b)
/sqrt(c - 1)) + sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)) +
(c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2*(c
+ 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sinh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + b*x + c*x**2), x)
```

Giac [A] time = 1.24533, size = 136, normalized size = 1.18

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c-1}\left(2\,x+\frac{b}{c+1}\right)\right)e^{\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{c-1}\left(2\,x+\frac{b}{c-1}\right)\right)e^{\left(\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*c
 - 4*a)/(c + 1))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x + b/
(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

3.342 $\int f^{a+bx} \sinh\left(d+fx^2\right) dx$

Optimal. Leaf size=110

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)$$

```
[Out] -(E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f]
)/(2*Sqrt[f])])/4 + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf
i[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rubi [A] time = 0.154013, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2],x]
```

```
[Out] -(E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f]
)/(2*Sqrt[f])])/4 + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf
i[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\sinh\left(d+fx^2\right)dx &= \int\left(-\frac{1}{2}e^{-d-fx^2}f^{a+bx}+\frac{1}{2}e^{d+fx^2}f^{a+bx}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+bx}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+bx}\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{2}\int e^{d+fx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx-\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{-\frac{(-2fx+b\log(f))^2}{4f}}\,dx\\
&= -\frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)
\end{aligned}$$

Mathematica [A] time = 0.138009, size = 103, normalized size = 0.94

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{b^2\log^2(f)}{4f}}\left((\sinh(d)+\cosh(d))\operatorname{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)-e^{\frac{b^2\log^2(f)}{2f}}(\cosh(d)-\sinh(d))\operatorname{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2],x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi]*(-(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(
2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(Co
sh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A] time = 0.134, size = 100, normalized size = 0.9

$$-\frac{\sqrt{\pi}f^a}{4}e^{-\frac{(\ln(f))^2b^2-4df}{4f}}\operatorname{Erf}\left(-\sqrt{-f}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-f}}\right)\frac{1}{\sqrt{-f}}+\frac{\sqrt{\pi}f^a}{4}e^{\frac{(\ln(f))^2b^2-4df}{4f}}\operatorname{Erf}\left(-\sqrt{f}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*f)/f)/(-f)^(1/2)*erf(-(-f)^(1/2
)*x+1/2*ln(f)*b/(-f)^(1/2))+1/4*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-4*d*f)/f)
/f^(1/2)*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))
```

Maxima [A] time = 1.06015, size = 122, normalized size = 1.11

$$-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log\left(f\right)^2}{4f}-d\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log\left(f\right)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log\left(f\right)^2}{4f}+d\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f))*
e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Fricas [B] time = 1.94221, size = 591, normalized size = 5.37

$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)\operatorname{erf}\left(\frac{(2fx+b\log(f))\sqrt{-f}}{2f}\right)-\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)\operatorname{erf}\left(-\frac{2fx-b\log(f)}{2\sqrt{f}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*e
rf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) - sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(
f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) - sqrt
(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 +
4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqr
t(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\sinh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2), x)
```

Giac [A] time = 1.21908, size = 143, normalized size = 1.3

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)}}{4\sqrt{f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*
a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*lo
g(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

3.343 $\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=148

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\operatorname{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\operatorname{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] (E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Log
[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)
*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 - f^(a + b*x)/(
2*b*Log[f])
```

Rubi [A] time = 0.193613, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5512, 2194, 2287, 2234, 2205, 2204}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\operatorname{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\operatorname{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Log
[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)
*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 - f^(a + b*x)/(
2*b*Log[f])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\sinh^2\left(d+fx^2\right)dx &= \int\left(-\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx}\,dx-\frac{1}{2}\int f^{a+bx}\,dx\\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int e^{-2d-2fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{4}\int e^{2d+2fx^2+a\log(f)+bx\log(f)}\,dx\\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{2d-\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{\frac{(4fx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{(-4fx+b\log(
\end{aligned}$$

$$= \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{4fx+b\,l}{2\sqrt{2}}\right.$$

Mathematica [A] time = 0.751649, size = 149, normalized size = 1.01

$$\frac{1}{16}f^a\left(\frac{\sqrt{2\pi}e^{\frac{b^2\log^2(f)}{8f}}(\cosh(2d)-\sinh(2d))\text{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)}{\sqrt{f}}+\frac{\sqrt{2\pi}e^{-\frac{b^2\log^2(f)}{8f}}(\sinh(2d)+\cosh(2d))\text{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)}{\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*((-8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(4
*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] + (S
qrt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh[2*
d]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16
```

Maple [A] time = 0.173, size = 126, normalized size = 0.9

$$-\frac{\sqrt{\pi}f^a\sqrt{2}}{16}\mathrm{e}^{\frac{(\ln(f))^2b^2-16\,df}{8f}}\text{Erf}\left(-\sqrt{2}\sqrt{f}x+\frac{b\ln(f)\sqrt{2}}{4}\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{f}}-\frac{\sqrt{\pi}f^a}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2-16\,df}{8f}}\text{Erf}\left(-\sqrt{-2\,f}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-2\,f}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(ln(f)^2*b^2-16*d*f)/f)*2^(1/2)/f^(1/2)*erf(-2^(
1/2)*f^(1/2)*x+1/4*ln(f)*b*2^(1/2)/f^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/8*(ln(f
)^2*b^2-16*d*f)/f)/(-2*f)^(1/2)*erf(-(-2*f)^(1/2)*x+1/2*ln(f)*b/(-2*f)^(1/2
))-1/2*f^a/ln(f)/b*f^(b*x)
```

Maxima [A] time = 1.62639, size = 171, normalized size = 1.16

$$\frac{\sqrt{2}\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}b\log\left(f\right)}{4\sqrt{f}}\right)e^{\left(\frac{b^{2}\log\left(f\right)^{2}}{8f}-2d\right)}}{16\sqrt{f}}+\frac{\sqrt{2}\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}b\log\left(f\right)}{4\sqrt{-f}}\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}}{8f}+2d\right)}}{16\sqrt{-f}}-\frac{f^{bx+a}}{2b\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqrt
(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf(s
qrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/f +
 2*d)/sqrt(-f) - 1/2*f^(b*x + a)/(b*log(f))
```

Fricas [B] time = 1.83629, size = 826, normalized size = 5.58

$$-\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^{2}\log\left(f\right)^{2}-8af\log\left(f\right)-16df}{8f}\right)\operatorname{erf}\left(\frac{\sqrt{2}\left(4fx+b\log\left(f\right)\right)\sqrt{-f}}{4f}\right)\log\left(f\right)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\cosh\left(\frac{b^{2}\log\left(f\right)^{2}+8af\log\left(f\right)-16df}{8f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2)*
sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*erf(-
1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)
*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2
 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*
(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f*log(f)
 - 16*d*f)/f) + 8*f*cosh((b*x + a)*log(f)) + 8*f*sinh((b*x + a)*log(f)))/(b
*f*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\sinh^{2}\left(d+fx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2)**2, x)
```

Giac [C] time = 1.3181, size = 481, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8*
(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)*e
rf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*
log(f) - 16*d*f)/f)/sqrt(-f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1
/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f)
- pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2
*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^
(b*x*log(abs(f)) + a*log(abs(f))) - 1/2*I*(2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2
*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4
*b*log(abs(f))) - 2*I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*s
gn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x
*log(abs(f)) + a*log(abs(f)))
```

3.344 $\int f^{a+bx}\sinh^3\left(d+fx^2\right)dx$

Optimal. Leaf size=239

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\operatorname{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\operatorname{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\operatorname{Erfi}\left(\frac{b}{-}\right.$$

```
[Out] (3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f
])/(2*Sqrt[f])])/16 - (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[P
i/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E^(d - (b^2*Log[f
]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/16
+ (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x + b*
Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rubi [A] time = 0.286135, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\operatorname{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\operatorname{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\operatorname{Erfi}\left(\frac{b}{-}\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```


```
[Out] (3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f
])/(2*Sqrt[f])])/16 - (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[P
i/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E^(d - (b^2*Log[f
]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/16
+ (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x + b*
Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```


Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)dx=\int\left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}-\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)dx$$
$$=-\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx}\,dx-\frac{3}{8}\int e^{d+fx^2}$$
$$=-\left(\frac{1}{8}\int e^{-3d-3fx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}\int e^{-d-fx^2+a}$$
$$=-\left(\frac{1}{8}\left(3e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx\right)+\frac{1}{8}\left(e^{3d-\frac{b^2\log^2(f)}{12f}}f^a\right)\int e^{\frac{(6fx+b\log(f))^2}{12f}}\,dx-\frac{1}{8}\left(e\right.$$
$$=\frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{6fx-b}{2\sqrt{3}}\right.$$

Mathematica [A] time = 0.417246, size = 287, normalized size = 1.2

$$\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{-\frac{b^2\log^2(f)}{4f}}\left(3\sqrt{3}e^{\frac{b^2\log^2(f)}{2f}}(\cosh(d)-\sinh(d))\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-e^{\frac{b^2\log^2(f)}{3f}}(\cosh(3d)-\sinh(3d))\mathrm{Erf}\left(6f\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi/3]*(-3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sqr
t[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sqrt
[3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(
2*Sqrt[f])]*(Cosh[d] - Sinh[d]) - 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt
[f])]*Sinh[d] - E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*
Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*f*x +
b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A] time = 0.207, size = 207, normalized size = 0.9

$$-\frac{\sqrt{\pi}f^a}{16}\mathrm{e}^{-\frac{(\ln(f))^2b^2-36df}{12f}}\mathrm{Erf}\left(-\sqrt{-3f}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-3f}}\right)\frac{1}{\sqrt{-3f}}+\frac{\sqrt{\pi}f^a\sqrt{3}}{48}\mathrm{e}^{\frac{(\ln(f))^2b^2-36df}{12f}}\mathrm{Erf}\left(-\sqrt{3}\sqrt{f}x+\frac{b\ln(f)\sqrt{3}}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/12*(ln(f)^2*b^2-36*d*f)/f)/(-3*f)^(1/2)*erf(-(-3*
f)^(1/2)*x+1/2*ln(f)*b/(-3*f)^(1/2))+1/48*Pi^(1/2)*f^a*exp(1/12*(ln(f)^2*b^
2-36*d*f)/f)*3^(1/2)/f^(1/2)*erf(-3^(1/2)*f^(1/2)*x+1/6*ln(f)*b*3^(1/2)/f^(
1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-4*d*f)/f)/f^(1/2)*erf(-f^(1/2)
*x+1/2*ln(f)*b/f^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*f)/f)/(
-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
```

Maxima [A] time = 1.61419, size = 270, normalized size = 1.13

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log\left(f\right)^2}{4f}-d\right)}-\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}b\log\left(f\right)}{6\sqrt{f}}\right)e^{\left(\frac{b^2\log\left(f\right)^2}{12f}-3d\right)}}{48\sqrt{f}}+\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqr
t(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt(3)
*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e^(-1
/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2
*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Fricas [B] time = 1.91625, size = 1287, normalized size = 5.38

$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log\left(f\right)^2-12\,af\log\left(f\right)-36\,df}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}\left(6\,fx+b\log\left(f\right)\right)\sqrt{-f}}{6\,f}\right)-\sqrt{3}\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log\left(f\right)^2+12\,af\log\left(f\right)-36\,df}{12\,f}\right)\operatorname{erf}\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) - sqrt(3)*sqrt(pi
)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*sqr
t(3)*(6*f*x - b*log(f))/sqrt(f)) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3
)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*
d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt
(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) - 9*sqrt(pi)*s
qrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(2*f*x +
b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + 4*a*f*l
og(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(pi)*sqrt(f)
*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4*a*f*log(f)
 - 4*d*f)/f) + 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f)*s
inh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2)**3, x)
```

Giac [A] time = 1.33959, size = 301, normalized size = 1.26

$$\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12af\log(f)-36df}{12f}\right)}}{48\sqrt{f}}-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-12af\ldots}{12f}\right.}}{48\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/12*
(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(pi)*
erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 - 12*a
*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*l
og(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) + 3/16*sq
rt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*
log(f) - 4*d*f)/f)/sqrt(-f)
```

3.345 $\int f^{a+bx} \sinh\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=115

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\text{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

```
[Out] -(E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b*
Log[f])/(2*Sqrt[f])])/4 + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt
[Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rubi [A] time = 0.224596, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\text{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -(E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b*
Log[f])/(2*Sqrt[f])])/4 + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt
[Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)dx=\int\left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx}\right)dx$$
$$=-\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx}dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx}dx$$
$$=-\left(\frac{1}{2}\int e^{-d-fx^2+a\log(f)-x(e-b\log(f))}dx\right)+\frac{1}{2}\int e^{d+fx^2+a\log(f)+x(e+b\log(f))}dx$$
$$=-\left(\frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}dx\right)+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4f}}f^a\right)\int e^{\frac{(e+2fx+b\log(f))}{4f}}$$
$$=-\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{e}{}\right.$$

Mathematica [A] time = 0.310203, size = 124, normalized size = 1.08

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+e^2}{4f}}\left((\sinh(d)+\cosh(d))\operatorname{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)-(\cosh(d)-\sinh(d))e^{\frac{b^2\log^2(f)+e^2}{2f}}\operatorname{Erf}\left(\frac{-b\log(f}{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(-(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e
+ 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(e + 2*f*x + b
*Log[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*f
)))
```

Maple [A] time = 0.109, size = 126, normalized size = 1.1

$$-\frac{\sqrt{\pi}f^a}{4}e^{-\frac{(\ln(f))^2b^2+2\ln(f)be-4df+e^2}{4f}}\operatorname{Erf}\left(-\sqrt{-f}x+\frac{e+b\ln(f)}{2}\frac{1}{\sqrt{-f}}\right)\frac{1}{\sqrt{-f}}+\frac{\sqrt{\pi}f^a}{4}e^{\frac{(\ln(f))^2b^2-2\ln(f)be-4df+e^2}{4f}}\operatorname{Erf}\left(-\sqrt{f}x+\frac{b\ln}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*f+e^2)/f)/(-f)^(1/2
)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))+1/4*Pi^(1/2)*f^a*exp(1/4*(l
n(f)^2*b^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2*(b*ln(f)-e)
/f^(1/2))
```

Maxima [A] time = 1.07543, size = 138, normalized size = 1.2

$$-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{\left(b\log(f)-e\right)^2}{4f}\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{\left(b\log(f)+e\right)^2}{4f}\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d
 + 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f
) + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

Fricas [B] time = 1.84538, size = 699, normalized size = 6.08

$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+e^2-4\,df+2\,(be-2\,af)\log(f)}{4\,f}\right)\operatorname{erf}\left(\frac{(2\,fx+b\log(f)+e)\sqrt{-f}}{2\,f}\right)-\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+e^2-4\,df-2\,(be-2\,af)\log(f)}{4\,f}\right)e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a
*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) - sqrt(pi)*sqrt(f
)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(-1/
2*(2*f*x - b*log(f) + e)/sqrt(f)) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*lo
g(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a*f
)*log(f))/f) - sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(f))*si
nh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2), x)
```

Giac [A] time = 1.28214, size = 181, normalized size = 1.57

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2\,x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-2\,be\log(f)-4\,df+e^2}{4\,f}\right)}}{4\sqrt{f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2\,x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)}{4\,f}\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)^
2 + 4*a*f*log(f) - 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(f) - 1/4*sqrt(pi)*er
f(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log
(f) + 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(-f)
```

3.346 $\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=161

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\text{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\text{Erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(}$$

```
[Out] (E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*f
*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*f)
)*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])
])/8 - f^(a + b*x)/(2*b*Log[f])
```

Rubi [A] time = 0.283848, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5512, 2194, 2287, 2234, 2205, 2204}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\text{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\text{Erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```


```
[Out] (E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*f
*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*f)
)*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])
])/8 - f^(a + b*x)/(2*b*Log[f])
```


Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)dx = \int\left(-\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx}\right)dx$$
$$= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx}\,dx-\frac{1}{2}\int f^{a+bx}\,dx$$
$$= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int\exp\left(-2d-2fx^2+a\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int\exp\left(2a\right.$$
$$= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^a\right)\int e^{-\frac{(-2e-4fx+b\log(f))^2}{8f}}dx+\frac{1}{4}\left(e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^a\right)\int e$$
$$= \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erf}$$

Mathematica [A] time = 0.625089, size = 220, normalized size = 1.37

$$\frac{f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+4e^2}{8f}}\left(\sqrt{\pi}b\log(f)(\cosh(2d)-\sinh(2d))e^{\frac{b^2\log^2(f)+4e^2}{4f}}\text{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)-4\sqrt{2}f^{b\left(\frac{e}{2f}+x\right)+\frac{1}{2}}e^{\frac{b^2\log^2(f)+4e^2}{8f}}+\sqrt{\right.}{8\sqrt{2}b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(-4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1/
2 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2*e
 + 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d]) +
b*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[
2*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[f])
```

Maple [A] time = 0.142, size = 158, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a\sqrt{2}}{16}e^{\frac{(\ln(f))^2b^2-4\ln(f)be-16df+4e^2}{8f}}\text{Erf}\left(-\sqrt{2}\sqrt{f}x+\frac{(b\ln(f)-2e)\sqrt{2}}{4}\frac{1}{\sqrt{f}}\right)\frac{1}{\sqrt{f}}-\frac{\sqrt{\pi}f^a}{8}e^{-\frac{(\ln(f))^2b^2+4\ln(f)be-16df+4e^2}{8f}}\text{Erf}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(ln(f)^2*b^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)*2^(1/2
)/f^(1/2)*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))-1/8*Pi^
(1/2)*f^a*exp(-1/8*(ln(f)^2*b^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)/(-2*f)^(1/2)*e
rf(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))-1/2*f^a/ln(f)/b*f^(b*x)
```

Maxima [A] time = 1.53298, size = 193, normalized size = 1.2

$$\frac{\sqrt{2}\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}(b\log(f)+2e)}{4\sqrt{-f}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^{2}}{8f}\right)}}{16\sqrt{-f}}+\frac{\sqrt{2}\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}(b\log(f)-2e)}{4\sqrt{f}}\right)e^{\left(-2d+\frac{(b\log(f)-2e)^{2}}{8f}\right)}}{16\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)*s
qrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f))*e
^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) - 1/2*f^(b*x + a)/(b*log(f))
```

Fricas [B] time = 1.8799, size = 956, normalized size = 5.94

$$-\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^{2}\log(f)^{2}+4e^{2}-16df+4(be-2af)\log(f)}{8f}\right)\operatorname{erf}\left(\frac{\sqrt{2}(4fx+b\log(f)+2e)\sqrt{-f}}{4f}\right)\log(f)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\cosh\left(\frac{b^{2}\log(f)}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*e)*sqrt(
-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 -
 16*d*f - 4*(b*e - 2*a*f)*log(f))/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f) + 2
*e)/sqrt(f))*log(f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*(4*f*x +
b*log(f) + 2*e)*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*sqrt(2)*
(4*f*x - b*log(f) + 2*e)/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 1
6*d*f - 4*(b*e - 2*a*f)*log(f))/f) + 8*f*cosh((b*x + a)*log(f)) + 8*f*sinh(
(b*x + a)*log(f)))/(b*f*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\sinh^{2}\left(d+ex+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2)**2, x)
```

Giac [C] time = 1.35316, size = 527, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(f) - 2*e)/f))
*e^(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 4*b*e*log(f) - 16*d*f + 4*e^2)/f)/sq
rt(f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) +
2*e)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*log(f) + 4*b*e*log(f) - 16*d*f + 4*e
^2)/f)/sqrt(-f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f
) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) -
(pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f)
+ 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(
f)) + a*log(abs(f))) - 1/2*I*(2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1
/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f)
)) - 2*I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I
*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f))
+ a*log(abs(f)))
```

3.347 $\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=257

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\text{Erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}f^{a}$$

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b
*Log[f])/(2*Sqrt[f])])/16 - (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E
^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log
[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 + a)*S
qrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rubi [A] time = 0.476691, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\text{Erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}f^{a}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b
*Log[f])/(2*Sqrt[f])])/16 - (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E
^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log
[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 + a)*S
qrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)dx=\int\left(-\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a+bx}-\frac{3}{8}\text{ex}\right.$$
$$=-\left(\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx}\,dx\right)+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+fx^2\right)\right)f^{a+bx}$$
$$=-\left(\frac{1}{8}\int\exp\left(-3d-3fx^2+a\log(f)-x(3e-b\log(f))\right)dx\right)+\frac{1}{8}\int\exp\left(3d+3fx^2+\right.$$
$$=\frac{1}{8}\left(3e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}dx-\frac{1}{8}\left(e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^a\right)\int e^{-\frac{(-3e-6fx+b\log(f)}{12f}}$$
$$=\frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\text{erf}\Big($$

Mathematica [A] time = 0.767054, size = 354, normalized size = 1.38

$$\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+3e^2}{4f}}\left(3\sqrt{3}(\cosh(d)-\sinh(d))e^{\frac{b^2\log^2(f)+2e^2}{2f}}\text{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)-(\cosh(3d)-\sinh(3d))e^{\frac{2b^2\text{lo}}{}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(-3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi[
(e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Er
fi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 + b^
2*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[
d]) - 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]*Sinh
[d] - E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sq
rt[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(3*
e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^2 + b^2*
Log[f]^2)/(4*f)))
```

Maple [A] time = 0.179, size = 265, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a}{16}e^{-\frac{(\ln(f))^2b^2+6\ln(f)be-36df+9e^2}{12f}}\text{Erf}\left(-\sqrt{-3f}x+\frac{3e+b\ln(f)}{2}\frac{1}{\sqrt{-3f}}\right)\frac{1}{\sqrt{-3f}}+\frac{\sqrt{\pi}f^a\sqrt{3}}{48}e^{\frac{(\ln(f))^2b^2-6\ln(f)be-36df+9e^2}{12f}}\text{Erf}\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/12*(ln(f)^2*b^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)/(-3*
f)^(1/2)*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))+1/48*Pi^(1/2)*
f^a*exp(1/12*(ln(f)^2*b^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)*3^(1/2)/f^(1/2)*erf(
-3^(1/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))-3/16*Pi^(1/2)*f^a*exp
(1/4*(ln(f)^2*b^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2*(b*l
n(f)-e)/f^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*f+
```

```
e^2)/f)/(-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))
```

Maxima [A] time = 1.58316, size = 308, normalized size = 1.2

$$\frac{\sqrt{3}\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{3}\sqrt{-f}x - \frac{\sqrt{3}\left(b\log\left(f\right)+3e\right)}{6\sqrt{-f}}\right)e^{\left(3d - \frac{\left(b\log\left(f\right)+3e\right)^{2}}{12f}\right)}}{48\sqrt{-f}} + \frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x - \frac{b\log\left(f\right)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{\left(b\log\left(f\right)-e\right)^{2}}{4f}\right)} - \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(pi)
*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*log
(f) - e)^2/f) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqrt(
3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt(f) -
 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d - 1/4
*(b*log(f) + e)^2/f)/sqrt(-f)
```

Fricas [B] time = 1.94115, size = 1519, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f +
 6*(b*e - 2*a*f)*log(f))/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sqrt(-
f)/f) - sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f -
 6*(b*e - 2*a*f)*log(f))/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/sqrt(
f)) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sq
rt(-f)/f)*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f + 6*(b*e - 2*a*f)*log(f)
)/f) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/s
qrt(f))*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f - 6*(b*e - 2*a*f)*log(f))/
f) - 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*
a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + 9*sqrt(pi)*sqr
t(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(
-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) + 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x +
 b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e -
2*a*f)*log(f))/f) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt
(f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30275, size = 385, normalized size = 1.5

$$\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)-3e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12af\log(f)-6be\log(f)-36df+9e^2}{12f}\right)}}{48\sqrt{f}}-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)+3e}{f}\right)\right)e}{48\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*log(f) - 3*e)/f))*
e^(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 6*b*e*log(f) - 36*d*f + 9*e^2)/f)/s
qrt(f) - 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f) +
 3*e)/f))*e^(-1/12*(b^2*log(f)^2 - 12*a*f*log(f) + 6*b*e*log(f) - 36*d*f +
9*e^2)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f
))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 2*b*e*log(f) - 4*d*f + e^2)/f)/sqr
t(f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b
^2*log(f)^2 - 4*a*f*log(f) + 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(-f)
```

3.348 $\int f^{a+cx^2} \sinh(d+ex)\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{\pi} f^a e^{-\frac{e^2}{4c\log(f)}-d} \operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4c\log(f)}} \operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a*Sq
rt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[L
og[f]])
```

Rubi [A] time = 0.198095, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5512, 2287, 2234, 2204}

$$\frac{\sqrt{\pi} f^a e^{-\frac{e^2}{4c\log(f)}-d} \operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4c\log(f)}} \operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a*Sq
rt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sinh(d+ex)\,dx &= \int\left(-\frac{1}{2}e^{-d-ex}f^{a+cx^2}+\frac{1}{2}e^{d+ex}f^{a+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-d-ex}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex}f^{a+cx^2}\,dx\\ &= -\left(\frac{1}{2}\int e^{-d-ex+a\log(f)+cx^2\log(f)}\,dx\right)+\frac{1}{2}\int e^{d+ex+a\log(f)+cx^2\log(f)}\,dx\\ &= -\left(\frac{1}{2}\left(e^{-d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-e+2cx\log(f))^2}{4c\log(f)}}\,dx\right)+\frac{1}{2}\left(e^{d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+2cx\log(f))^2}{4c\log(f)}}\,dx\\ &= \frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.157874, size = 104, normalized size = 0.78

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}}\left((\sinh(d)-\cosh(d))\operatorname{Erfi}\left(\frac{2cx\log(f)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(d)+\cosh(d))\operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[d]
 + Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] +
Sinh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.106, size = 117, normalized size = 0.9

$$-\frac{\sqrt{\pi}f^a}{4}e^{\frac{4d\ln(f)c-e^2}{4c\ln(f)}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{e}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}f^a}{4}e^{-\frac{4d\ln(f)c+e^2}{4c\ln(f)}}\operatorname{Erf}\left(\sqrt{-c\ln(f)}x+\frac{e}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-
(-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(4*d*l
n(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f
))^(1/2))
```

Maxima [A] time = 1.03333, size = 142, normalized size = 1.07

$$\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x+\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*
e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] time = 1.76429, size = 598, normalized size = 4.5

$$-\frac{\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log\left(f\right)^{2}+4\,cd\log\left(f\right)-e^{2}}{4\,c\log\left(f\right)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log\left(f\right)^{2}+4\,cd\log\left(f\right)-e^{2}}{4\,c\log\left(f\right)}\right)\right)\operatorname{erf}\left(\frac{\left(2\,cx\log\left(f\right)+e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)-\sqrt{-}}{4\,c\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e
^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(
c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-
c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f
))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))*
erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^{2}}\sinh\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x), x)
```

Giac [A] time = 1.23743, size = 178, normalized size = 1.34

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{e}{c\log\left(f\right)}\right)\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}+4\,cd\log\left(f\right)-e^{2}}{4\,c\log\left(f\right)}\right)}}{4\sqrt{-c\log\left(f\right)}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x-\frac{e}{c\log\left(f\right)}\right)\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}-4}{4\,c\log}\right.}}{4\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*
log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*e
rf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*
d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.349 $\int f^{a+cx^2} \sinh^2(d+ex)\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{\pi} f^a e^{-\frac{e^2}{c\log(f)}-2d} \operatorname{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a e^{2d-\frac{e^2}{c\log(f)}} \operatorname{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^
(-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[Lo
g[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*
Erfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.226126, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5512, 2204, 2287, 2234}

$$-\frac{\sqrt{\pi} f^a e^{-\frac{e^2}{c\log(f)}-2d} \operatorname{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a e^{2d-\frac{e^2}{c\log(f)}} \operatorname{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```


```
[Out] -(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^
(-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[Lo
g[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*
Erfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```


Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \int\left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+cx^2}\right)dx$$
$$= \frac{1}{4}\int e^{-2d-2ex}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,dx$$
$$= -\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d-2ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{4}\int e^{2d+2ex+a\log(f)+cx^2\log}$$
$$= -\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{4}\left(e^{2d-\frac{e^2}{c\log(f)}}f^a\right)\int$$
$$= -\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+c}{\sqrt{c}}\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] time = 0.26411, size = 131, normalized size = 0.81

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{c\log(f)}}\left((\cosh(2d)-\sinh(2d))\mathrm{Erfi}\left(\frac{cx\log(f)-e}{\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\mathrm{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)-2e^{\frac{e^2}{c\log(f)}}\mathrm{Erfi}\left(\sqrt{c}x\sqrt{l}\right.\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(-
e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e +
 c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c]*E
^(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.122, size = 139, normalized size = 0.9

$$\frac{\sqrt{\pi}f^a}{8}e^{-\frac{2d\ln(f)c+e^2}{c\ln(f)}}\mathrm{Erf}\left(\sqrt{-c\ln(f)}x+e\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}f^a}{8}e^{\frac{2d\ln(f)c-e^2}{c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}x+e\frac{1}{\sqrt{-c\ln(f)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*l
n(f))^(1/2)*x+e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))-1/4*f^a
*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [A] time = 1.07114, size = 177, normalized size = 1.1

$$\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(2d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x+\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(-2d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\right.}{4\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(c
*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sqrt
(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*e
rf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Fricas [A] time = 1.80786, size = 676, normalized size = 4.2

$$\frac{2\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(a\log\left(f\right)\right)+\sqrt{\pi}\sinh\left(a\log\left(f\right)\right)\right)\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)-\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{ac\log\left(f\right)^{2}+2\,cd\log(f}{c\log\left(f\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f)))*
erf(sqrt(-c*log(f))*x) - sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 + 2*c
*d*log(f) - e^2)/(c*log(f))) + sqrt(pi)*sinh((a*c*log(f)^2 + 2*c*d*log(f) -
 e^2)/(c*log(f))))*erf((c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(
-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))) +
 sqrt(pi)*sinh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))))*erf((c*x*lo
g(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sinh^2\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x)**2, x)
```

Giac [A] time = 1.29808, size = 203, normalized size = 1.26

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}x\right)}{4\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^{2}+2\,cd\log(f)-e^{2}}{c\log(f)}\right)}}{8\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}\left(x-\frac{e}{c\log}\right.\right.}{8\sqrt{-c\log}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) - e^2
```

```
)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x - e/(c
*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f
))
```

3.350 $\int f^{a+cx^2}\sinh^3(d+ex)\,dx$

Optimal. Leaf size=271

$$-\frac{3\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\operatorname{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\sqrt{\pi}f^a e$$

```
[Out] (-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(-3*d - (9*e^2)/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*
Sqrt[c]*Sqrt[Log[f]]) - (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*
d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.345477, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5512, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\operatorname{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\sqrt{\pi}f^a e$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(-3*d - (9*e^2)/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*
Sqrt[c]*Sqrt[Log[f]]) - (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*
d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx=\int\left(-\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}-\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)dx$$
$$=-\left(\frac{1}{8}\int e^{-3d-3ex}f^{a+cx^2}dx\right)+\frac{1}{8}\int e^{3d+3ex}f^{a+cx^2}dx+\frac{3}{8}\int e^{-d-ex}f^{a+cx^2}dx-\frac{3}{8}\int e^{d+ex}f^{a+c}$$
$$=-\left(\frac{1}{8}\int e^{-3d-3ex+a\log(f)+cx^2\log(f)}dx\right)+\frac{1}{8}\int e^{3d+3ex+a\log(f)+cx^2\log(f)}dx+\frac{3}{8}\int e^{-d-ex+a\log(}$$
$$=-\left(\frac{1}{8}\left(e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-3e+2cx\log(f))^2}{4c\log(f)}}dx\right)+\frac{1}{8}\left(e^{3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(3e+2cx\log(f))^2}{4c\log(f)}}dx+\frac{1}{8}\left(3e\right.$$
$$=-\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3e^{d-\frac{e^2}{4c\log(f)}}f}{16}$$

Mathematica [A] time = 0.45102, size = 214, normalized size = 0.79

$$\frac{\sqrt{\pi}f^ae^{-\frac{9e^2}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\left(3(\cosh(2d)-\sinh(2d))e^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{2cx\log(f)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\mathrm{Erfi}\left(\frac{2cx}{2\surd}\right.\right.\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*c
*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e
 + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + Erfi[
(-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[3*d] + Sinh[3*d])))/
(16*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.146, size = 234, normalized size = 0.9

$$-\frac{\sqrt{\pi}f^a}{16}\mathrm{e}^{\frac{12d\ln(f)c-9e^2}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{3e}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}f^a}{16}\mathrm{e}^{-\frac{12d\ln(f)c+9e^2}{4c\ln(f)}}\mathrm{Erf}\left(\sqrt{-c\ln(f)}x+\frac{3e}{2}\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(-3/4*(4
*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+3/2*e/(-
c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)/(-c*l
n(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^
a*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)
*x+1/2*e/(-c*ln(f))^(1/2))
```

Maxima [A] time = 1.08158, size = 285, normalized size = 1.05

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x-\frac{3e}{2\sqrt{-c\log(f)}}\right)e^{\left(3d-\frac{9e^{2}}{4c\log(f)}\right)}}{16\sqrt{-c\log\left(f\right)}}-\frac{3\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^{2}}{4c\log(f)}\right)}}{16\sqrt{-c\log\left(f\right)}}+\frac{3\sqrt{\pi}f^{a}\operatorname{erf}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d - 9
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*
x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/1
6*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e
^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] time = 1.83901, size = 1202, normalized size = 4.44

$$-\frac{\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{4ac\log(f)^{2}+12cd\log(f)-9e^{2}}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4ac\log(f)^{2}+12cd\log(f)-9e^{2}}{4c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2cx\log(f)+3e)\sqrt{-c\log(f)}}{2c\log(f)}\right)-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) - 9
*e^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f)))
 - 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^
2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(c
*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(
-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(
f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))
*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(
sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))) + sq
rt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))))*erf(1
/2*(2*c*x*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^{2}}\sinh^{3}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x)**3, x)
```

Giac [A] time = 1.21843, size = 356, normalized size = 1.31

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{3\,e}{c\log\left(f\right)}\right)\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}+12\,cd\log\left(f\right)-9\,e^{2}}{4\,c\log\left(f\right)}\right)}}{16\sqrt{-c\log\left(f\right)}}+\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{e}{c\log\left(f\right)}\right)\right)e^{\left(\frac{4\,ac\log\left(f\right.}{}\right.}}{16\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*a
*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqr
t(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2
 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f
) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f
))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/
(c*log(f)))/sqrt(-c*log(f))
```

3.351 $\int f^{a+cx^2} \sinh(d+fx^2)\, dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\pi}e^d f^a \text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}e^{-d} f^a \text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}$$

```
[Out] -(f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(4*E^d*Sqrt[f - c*Log[f]]) + (E^d
*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Rubi [A] time = 0.170314, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5512, 2287, 2205, 2204}

$$\frac{\sqrt{\pi}e^d f^a \text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}e^{-d} f^a \text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] -(f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(4*E^d*Sqrt[f - c*Log[f]]) + (E^d
*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)dx=\int\left(-\frac{1}{2}e^{-d-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+cx^2}\right)dx$$
$$=-\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+cx^2}dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+cx^2}dx$$
$$=-\left(\frac{1}{2}\int e^{-d+a\log(f)-x^2(f-c\log(f))}dx\right)+\frac{1}{2}\int e^{d+a\log(f)+x^2(f+c\log(f))}dx$$
$$=-\frac{e^{-d}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^df^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}$$

Mathematica [A] time = 0.341473, size = 76, normalized size = 0.94

$$\frac{1}{4}\sqrt{\pi}f^a\left(\frac{(\sinh(d)+\cosh(d))\operatorname{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{\sqrt{c\log(f)+f}}-\frac{(\cosh(d)-\sinh(d))\operatorname{Erf}\left(x\sqrt{f-c\log(f)}\right)}{\sqrt{f-c\log(f)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f - c
*Log[f]]) + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*Log
[f]]))/4
```

Maple [A] time = 0.084, size = 70, normalized size = 0.9

$$\frac{\sqrt{\pi}f^a\mathrm{e}^d}{4}\operatorname{Erf}\left(\sqrt{-c\ln\left(f\right)-f}x\right)\frac{1}{\sqrt{-c\ln\left(f\right)-f}}-\frac{\sqrt{\pi}f^a\mathrm{e}^{-d}}{4}\operatorname{Erf}\left(x\sqrt{f-c\ln\left(f\right)}\right)\frac{1}{\sqrt{f-c\ln\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)-1/4*Pi
^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))
```

Maxima [A] time = 1.08368, size = 93, normalized size = 1.15

$$-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right)e^{(-d)}}{4\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x\right)e^d}{4\sqrt{-c\log\left(f\right)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) + 1
/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

Fricas [B] time = 1.90379, size = 410, normalized size = 5.06

$$\frac{\left(\sqrt{\pi}\left(c\log\left(f\right)+f\right)\cosh\left(a\log\left(f\right)-d\right)+\sqrt{\pi}\left(c\log\left(f\right)+f\right)\sinh\left(a\log\left(f\right)-d\right)\right)\sqrt{-c\log\left(f\right)+f}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right.}{4\left(c^2\log\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f)*
sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) - (sqrt(
pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*log(
f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2 - f^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2), x)
```

Giac [A] time = 1.288, size = 101, normalized size = 1.25

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{\left(a\log\left(f\right)+d\right)}}{4\sqrt{-c\log\left(f\right)-f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+f}x\right)e^{\left(a\log\left(f\right)-d\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f) -
 f) + 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*log
(f) + f)
```

3.352 $\int f^{a+cx^2} \sinh^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=128

$$\frac{\sqrt{\pi}e^{-2d}f^a\text{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}e^{2d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^
a*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]) +
(E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f]]
)
```

Rubi [A] time = 0.203509, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5512, 2204, 2287, 2205}

$$\frac{\sqrt{\pi}e^{-2d}f^a\text{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}e^{2d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] -(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^
a*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]) +
(E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f]]
)
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)dx=\int\left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+cx^2}\right)dx$$
$$=\frac{1}{4}\int e^{-2d-2fx^2}f^{a+cx^2}dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+cx^2}dx-\frac{1}{2}\int f^{a+cx^2}dx$$
$$=-\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d+a\log(f)-x^2(2f-c\log(f))}dx+\frac{1}{4}\int e^{2d+a\log(f)+x^2(2f+c\log}$$
$$=-\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{2f+c\log}\right.}{8\sqrt{2f+c\log(f)}}$$

Mathematica [A] time = 0.53445, size = 179, normalized size = 1.4

$$\frac{\sqrt{\pi}f^a\left(\left(8f^2-2c^2\log^2(f)\right)\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)+\sqrt{c}\sqrt{\log(f)}\left(\sqrt{2f-c\log(f)}(c\log(f)+2f)(\sinh(2d)-\cosh(2d))\text{Erf}\left(x\right.\right.\right.}{8\sqrt{c}\sqrt{\log(f)}\left(c^2\log^2(f)-4\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(8*f^2 - 2*c^2*Log[f]^2) + Sqrt
[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c
*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f - c*L
og[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[
f]]*(-4*f^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.098, size = 101, normalized size = 0.8

$$\frac{\sqrt{\pi}f^a e^{-2d}}{8}\text{Erf}\left(x\sqrt{2f-c\ln(f)}\right)\frac{1}{\sqrt{2f-c\ln(f)}}+\frac{\sqrt{\pi}f^a e^{2d}}{8}\text{Erf}\left(\sqrt{-c\ln(f)-2f}x\right)\frac{1}{\sqrt{-c\ln(f)-2f}}-\frac{\sqrt{\pi}f^a}{4}\text{Erf}\left(\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-2*d)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+1
/8*Pi^(1/2)*f^a*exp(2*d)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)-1
/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [A] time = 1.06907, size = 135, normalized size = 1.05

$$\frac{\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{-c\log(f)-2f}x\right)e^{(2d)}}{8\sqrt{-c\log(f)-2f}}+\frac{\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{-c\log(f)+2f}x\right)e^{(-2d)}}{8\sqrt{-c\log(f)+2f}}-\frac{\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f)
 + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f) +
```

```
2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Fricas [B] time = 1.91296, size = 720, normalized size = 5.62

$$-\frac{\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\cosh\left(a\log\left(f\right)-2\,d\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\sinh\left(a\log\left(f\right)-2\,d\right)\right)\sqrt{-c\,l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(p
i)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2*f
)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(f) +
 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) - 2*(sqrt(pi)*(c^
2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(a
*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*c*f^2*l
og(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2)**2, x)
```

Giac [A] time = 1.27607, size = 144, normalized size = 1.12

$$\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}x\right)}{4\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-2\,f}x\right)e^{\left(a\log\left(f\right)+2\,d\right)}}{8\sqrt{-c\log\left(f\right)-2\,f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+2\,f}x\right)e^{\left(a\log\left(f\right)-2\,d\right)}}{8\sqrt{-c\log\left(f\right)+2\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/8*s
qrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f) + 2
*f)
```

3.353 $\int f^{a+cx^2} \sinh^3\left(d+fx^2\right) dx$

Optimal. Leaf size=171

$$\frac{3\sqrt{\pi}e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}e^{-3d}f^a\text{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}e^{d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}e^{3d}f^a\text{Erfi}\left(x\sqrt{\right.}}{16\sqrt{c\log(}}$$

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) - (f
^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]])
- (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]) +
 (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*Log[f
]])
```

Rubi [A] time = 0.300349, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5512, 2287, 2205, 2204}

$$\frac{3\sqrt{\pi}e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}e^{-3d}f^a\text{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}e^{d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}e^{3d}f^a\text{Erfi}\left(x\sqrt{\right.}}{16\sqrt{c\log(}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) - (f
^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]])
- (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]) +
 (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*Log[f
]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)dx = \int\left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right)dx$$
$$= -\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+cx^2}dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+cx^2}dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+cx^2}dx-\frac{3}{8}\int e^{d+}$$
$$= -\left(\frac{1}{8}\int e^{-3d+a\log(f)-x^2(3f-c\log(f))}dx\right)+\frac{1}{8}\int e^{3d+a\log(f)+x^2(3f+c\log(f))}dx+\frac{3}{8}\int e^{-d+a\log}$$
$$= \frac{3e^{-d}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{\ }\right.}{16\sqrt{f+c}}$$

Mathematica [A] time = 1.20654, size = 272, normalized size = 1.59

$$\frac{\sqrt{\pi}f^a\left(3\sqrt{f-c\log(f)}\left(-c^2f\log^2(f)-c^3\log^3(f)+9cf^2\log(f)+9f^3\right)(\cosh(d)-\sinh(d))\text{Erf}\left(x\sqrt{f-c\log(f)}\right)-(f}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*
f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) - (f - c*Lo
g[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[
f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x*Sqr
t[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) -
Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d]
 + Sinh[3*d])))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] time = 0.125, size = 144, normalized size = 0.8

$$\frac{\sqrt{\pi}f^a e^{3d}}{16}\text{Erf}\left(\sqrt{-c\ln(f)-3f}x\right)\frac{1}{\sqrt{-c\ln(f)-3f}}-\frac{\sqrt{\pi}f^a e^{-3d}}{16}\text{Erf}\left(x\sqrt{3f-c\ln(f)}\right)\frac{1}{\sqrt{3f-c\ln(f)}}+\frac{3\sqrt{\pi}f^a e^{-d}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(3*d)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*x)
-1/16*Pi^(1/2)*f^a*exp(-3*d)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))
+3/16*Pi^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))-3/16*
Pi^(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Maxima [A] time = 1.07553, size = 193, normalized size = 1.13

$$\frac{\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{-c\log(f)-3f}x\right)e^{(3d)}}{16\sqrt{-c\log(f)-3f}}+\frac{3\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{-c\log(f)+f}x\right)e^{(-d)}}{16\sqrt{-c\log(f)+f}}-\frac{\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{-c\log(f)+3f}x\right)e^{(-3d)}}{16\sqrt{-c\log(f)+3f}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*f
) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f)
 - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f) +
 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f
)
```

Fricas [B] time = 2.08304, size = 1300, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(
f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log(f)
+ 3*f)*x) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9
*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^
2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f
) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9
*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^
2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f
) - f)*x) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f
^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^
2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*erf(sqrt(-c*l
og(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31044, size = 209, normalized size = 1.22

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-3\,f}x\right)e^{\left(a\log\left(f\right)+3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}+\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{\left(a\log\left(f\right)+d\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}-\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+fx}\right.}{16\,\sqrt{-c\log\left(f\right)+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*log
(f) - 3*f) + 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqr
t(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) -
d)/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^(a*l
og(f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

3.354 $\int f^{a+cx^2} \sinh\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=140

$$\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}} \text{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d} \text{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

```
[Out] -(E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]))
/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - e^2/(4*(f + c*Lo
g[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])
/(4*Sqrt[f + c*Log[f]])
```

Rubi [A] time = 0.320949, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}} \text{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d} \text{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```


```
[Out] -(E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]))
/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - e^2/(4*(f + c*Lo
g[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])
/(4*Sqrt[f + c*Log[f]])
```


Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)dx &= \int\left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-ex+a\log(f)-x^2(f-c\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+ex+a\log(f)+x^2(f+c\log(f))}\,dx\\
&= -\left(\frac{1}{2}\left(e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right)+\frac{1}{2}\left(e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\right)\\
&= -\frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}
\end{aligned}
$$

Mathematica [A] time = 0.729534, size = 166, normalized size = 1.19

$$
\frac{\sqrt{\pi}f^ae^{-\frac{e^2}{4(c\log(f)+f)}}\left(\sqrt{f-c\log(f)}(\sinh(d)+\cosh(d))\text{Erfi}\left(\frac{2cx\log(f)+e+2fx}{2\sqrt{c\log(f)+f}}\right)-\sqrt{c\log(f)+f}(\cosh(d)-\sinh(d))e^{\frac{e^2f}{2f^2-2c^2\log^2(}}\right.}{4\sqrt{f-c\log(f)}\sqrt{c\log(f)+f}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c*
x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d])) +
 Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[f]]
*(Cosh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]*Sqr
t[f + c*Log[f]])
```

Maple [A] time = 0.175, size = 147, normalized size = 1.1

$$
-\frac{\sqrt{\pi}f^a}{4}e^{\frac{4d\ln(f)c+4df-e^2}{4c\ln(f)+4f}}\text{Erf}\left(-\sqrt{-c\ln(f)-f}x+\frac{e}{2}\frac{1}{\sqrt{-c\ln(f)-f}}\right)\frac{1}{\sqrt{-c\ln(f)-f}}-\frac{\sqrt{\pi}f^a}{4}e^{-\frac{4d\ln(f)c-4df+e^2}{4c\ln(f)-4f}}\text{Erf}\left(x\sqrt{f-c\ln}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)
^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))-1/4*Pi^(1/2)*f^a
*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(x*(f-
c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))
```

Maxima [A] time = 1.04879, size = 171, normalized size = 1.22

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^{2}}{4\left(c\log(f)+f\right)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x+\frac{e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e}{4(c\log}\right.}}{4\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(
d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*f^a*erf(sqrt
(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) -
f))/sqrt(-c*log(f) + f)
```

Fricas [B] time = 2.03711, size = 853, normalized size = 6.09

$$\left(\sqrt{\pi}\left(c\log\left(f\right)+f\right)\cosh\left(\frac{4\,ac\log\left(f\right)^{2}-e^{2}+4\,df-4\left(cd+af\right)\log\left(f\right)}{4\left(c\log\left(f\right)-f\right)}\right)+\sqrt{\pi}\left(c\log\left(f\right)+f\right)\sinh\left(\frac{4\,ac\log\left(f\right)^{2}-e^{2}+4\,df-4\left(cd+af\right)\log\left(f\right)}{4\left(c\log\left(f\right)-f\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(c
*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(1/4*(4*a*c
*log(f)^2 - e^2 + 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*lo
g(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f)
- f)) - (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f + 4
*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(1/4*(4*
a*c*log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c
*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(
f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^{2}}\sinh\left(d+ex+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2), x)
```

Giac [A] time = 1.3036, size = 232, normalized size = 1.66

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{e}{c\log\left(f\right)+f}\right)\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}+4\,cd\log\left(f\right)+4\,af\log\left(f\right)+4\,df-e^{2}}{4\left(c\log\left(f\right)+f\right)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x\right.\right.}{4\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4
*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + f
))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e
/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) + 4*
d*f - e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

3.355 $\int f^{a+cx^2} \sinh^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=183

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{2f-c\log(f)}-2d} \text{Erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{2d-\frac{e^2}{c\log(f)+2f}} \text{Erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi} f^a \text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^
(-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/Sqr
t[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log[f
]))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8*Sq
rt[2*f + c*Log[f]])
```

Rubi [A] time = 0.333797, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5512, 2204, 2287, 2234, 2205}

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{2f-c\log(f)}-2d} \text{Erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{2d-\frac{e^2}{c\log(f)+2f}} \text{Erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi} f^a \text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^
(-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/Sqr
t[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log[f
]))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8*Sq
rt[2*f + c*Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)dx=\int\left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+cx^2}\right)dx$$
$$=\frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+cx^2}dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+cx^2}dx-\frac{1}{2}\int f^{a+cx^2}dx$$
$$=-\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int\exp\left(-2d-2ex+a\log(f)-x^2(2f-c\log(f))\right)dx$$
$$=-\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e+2x(-2f+c\log(f))}{4(-2f+c\log(f))}\right.$$
$$=-\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{e^2}{2f+c\log(f)}}f}{8\sqrt{}}$$

Mathematica [A] time = 1.42745, size = 258, normalized size = 1.41

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{2f-c\log(f)}}\left(2\left(4f^2-c^2\log^2(f)\right)e^{\frac{e^2}{c\log(f)-2f}}\operatorname{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)-\sqrt{c}\sqrt{\log(f)}\left((2f-c\log(f))\sqrt{c\log(f)+2f}(\sinh(2d)\right.\right.}{8\sqrt{c}\sqrt{\log(f)}\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(2*E^(e^2/(-2*f + c*Log[f]))*Erfi[Sq
rt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf[(e
 + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c*
Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2))*Erf
i[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqrt[2*f
+ c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f^2 + c^
2*Log[f]^2))
```

Maple [A] time = 0.185, size = 177, normalized size = 1.

$$\frac{\sqrt{\pi}f^a}{8}\mathrm{e}^{-\frac{2d\ln(f)c-4df+e^2}{-2f+c\ln(f)}}\operatorname{Erf}\left(x\sqrt{2f-c\ln(f)}+e\frac{1}{\sqrt{2f-c\ln(f)}}\right)\frac{1}{\sqrt{2f-c\ln(f)}}-\frac{\sqrt{\pi}f^a}{8}\mathrm{e}^{\frac{2d\ln(f)c+4df-e^2}{2f+c\ln(f)}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))
^(1/2)*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*ex
p((2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f
)-2*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))-1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*er
```

```
f((-c*ln(f))^(1/2)*x)
```

Maxima [A] time = 1.07094, size = 217, normalized size = 1.19

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2\,f}x-\frac{e}{\sqrt{-c\log\left(f\right)-2\,f}}\right)e^{\left(2\,d-\frac{e^2}{c\log\left(f\right)+2\,f}\right)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}}+\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2\,f}x+\frac{e}{\sqrt{-c\log\left(f\right)+2\,f}}\right)e^{\left(-2\,d-\right.}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^(
2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f) -
2*f))/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(
-c*log(f))
```

Fricas [B] time = 1.93488, size = 1118, normalized size = 6.11

$$\frac{2\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2-4\,f^2\right)\cosh\left(a\log\left(f\right)\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2-4\,f^2\right)\sinh\left(a\log\left(f\right)\right)\right)\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(
f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) - (sqr
t(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c
*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f)
)*sinh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c*d + a*f)*log(f))/(c*log(f) - 2*f)
))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + 2*f)
/(c*log(f) - 2*f)) - (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh((a*c*log(
f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^
2*log(f)^2 - 2*c*f*log(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)
*log(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c*x*log(f) + 2*f*x +
 e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)))/(c^3*log(f)^3 - 4*c*f^2*log(f)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2)**2, x)
```

Giac [A] time = 1.2743, size = 267, normalized size = 1.46

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(-\sqrt{-c \log\left(f\right)} x\right)}{4 \sqrt{-c \log\left(f\right)}} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log\left(f\right) - 2 f} \left(x + \frac{e}{c \log\left(f\right) + 2 f}\right)\right) e^{\left(\frac{a c \log\left(f\right)^{2} + 2 c d \log\left(f\right) + 2 a f \log\left(f\right) + 4 d f - e^{2}}{c \log\left(f\right) + 2 f}\right)}}{8 \sqrt{-c \log\left(f\right) - 2 f}} - \frac{\sqrt{\pi} \operatorname{erf}\left(- \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c*d*
log(f) + 2*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f
) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*e^((a
*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) + 4*d*f - e^2)/(c*log(f) - 2*f))/
sqrt(-c*log(f) + 2*f)
```

3.356 $\int f^{a+cx^2} \sinh^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=300

$$\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\operatorname{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\operatorname{Erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{2x(c\log\cdots}{2\sqrt{c\log\cdots}}\right.}{16\sqrt{c\log(f)+f}}\cdots$$

```
[Out] (3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f])
)/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^(-3*d + (9*e^2)/(12
*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f
 - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) - (3*E^(d - e^2/(4*(f + c*Log[f])
))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*
Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f +
 c*Log[f]])
```

Rubi [A] time = 0.587126, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\operatorname{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\operatorname{Erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{2x(c\log\cdots}{2\sqrt{c\log\cdots}}\right.}{16\sqrt{c\log(f)+f}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```


```
[Out] (3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f])
)/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^(-3*d + (9*e^2)/(12
*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f
 - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) - (3*E^(d - e^2/(4*(f + c*Log[f])
))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*
Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f +
 c*Log[f]])
```


Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)dx=\int\left(-\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a+cx^2}-\frac{3}{8}\right.$$

$$=-\left(\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}\,dx\right)+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+fx^2\right)\right)f^{a+}$$

$$=-\left(\frac{1}{8}\int\exp\left(-3d-3ex+a\log(f)-x^2(3f-c\log(f))\right)dx\right)+\frac{1}{8}\int\exp\left(3d+3ex+\right.$$

$$=\frac{1}{8}\left(3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx-\frac{1}{8}\left(e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\right.$$

$$=\frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{3e+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}$$

Mathematica [A] time = 5.80387, size = 480, normalized size = 1.6

$$\frac{\sqrt{\pi}f^a\exp\left(-\frac{1}{4}e^2\left(\frac{9}{c\log(f)+3f}+\frac{1}{c\log(f)+f}\right)\right)\left(3\sqrt{f-c\log(f)}\left(-c^2f\log^2(f)-c^3\log^3(f)+9cf^2\log(f)+9f^3\right)(\cosh(d)-\sin\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3*
f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*S
qrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*
(Cosh[d] - Sinh[d]) - (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f + c*
Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f])/(2*
Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log
[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(3*f + c
*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f +
 c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) - E^(e^2/(4*(f + c*Log[f]))
)*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f]
)*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*((f + c*Log[
f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f
]^4))
```

Maple [A] time = 0.241, size = 302, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a}{16}\mathrm{e}^{\frac{12d\ln(f)c+36df-9e^2}{4c\ln(f)+12f}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e}{2}\frac{1}{\sqrt{-c\ln(f)-3f}}\right)\frac{1}{\sqrt{-c\ln(f)-3f}}-\frac{\sqrt{\pi}f^a}{16}\mathrm{e}^{-\frac{12d\ln(f)c-36df+9e^2}{4c\ln(f)-12f}}\mathrm{Erf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))/(-c*ln
(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))-1/16
*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(-3*f+c*ln(f)))/(3*f-c*ln
(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))+3/16*Pi^(1/
2)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf
(x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(1/4*(4*
d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2
)*x+1/2*e/(-c*ln(f)-f)^(1/2))
```

Maxima [A] time = 1.08586, size = 355, normalized size = 1.18

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}\,x-\frac{3\,e}{2\,\sqrt{-c\log\left(f\right)-3\,f}}\right) e^{\left(3\,d-\frac{9\,e^{2}}{4\,\left(c\log\left(f\right)+3\,f\right)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}-\frac{3\,\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}\,x-\frac{e}{2\,\sqrt{-c\log\left(f\right)-f}}\right) e^{(d-}}{16\,\sqrt{-c\log\left(f\right)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f)
)*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*
f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^2/(
c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) +
 f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqrt(-c*
log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/sqrt(-c
*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Fricas [B] time = 2.16518, size = 2218, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(f)
- 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)
*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(
f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3*e)*sqrt
(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log
(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*
(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)
^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(c*
d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(1/2*(2*c*x*log(f)
 - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)*(c^3*log(f)
^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^
2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3
- c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 +
```

```
 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2
*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) - (sqrt(pi)
*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(1/4*(4*a*c*l
og(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)) + sqrt(
pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(1/4*(4*a*
c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)))*sqr
t(-c*log(f) - 3*f)*erf(1/2*(2*c*x*log(f) + 6*f*x + 3*e)*sqrt(-c*log(f) - 3*
f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37515, size = 475, normalized size = 1.58

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3f}\left(2x+\frac{3e}{c\log\left(f\right)+3f}\right)\right)e^{\left(\frac{4ac\log\left(f\right)^{2}+12cd\log\left(f\right)+12af\log\left(f\right)+36df-9e^{2}}{4\left(c\log\left(f\right)+3f\right)}\right)}}{16\sqrt{-c\log\left(f\right)-3f}}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f)))
*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) + 36*d*f - 9*e^2)/(
c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) +
4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4*a*c*l
og(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) - f))/sqrt(-
c*log(f) + f) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x - 3*e/(c*
log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f) + 36
*d*f - 9*e^2)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

3.357 $\int f^{a+bx+cx^2}\sinh(d+ex)\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d -
(e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*Log[
f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.300778, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5512, 2287, 2234, 2204}

$$\frac{\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d -
(e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*Log[
f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sinh(d+ex)\,dx &= \int\left(-\frac{1}{2}e^{-d-ex}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex}f^{a+bx+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-d-ex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex}f^{a+bx+cx^2}\,dx\\ &= -\left(\frac{1}{2}\int \exp\left(-d+a\log(f)+cx^2\log(f)-x(e-b\log(f))\right)dx\right)+\frac{1}{2}\int \exp\left(d+a\log(f)+\right.\\ &= -\left(\frac{1}{2}\left(e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\right)+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\right)\int\\ &= \frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathbf{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathbf{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.335413, size = 135, normalized size = 0.88

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{e(2b\log(f)+e)}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\mathbf{Erfi}\left(\frac{\log(f)(b+2cx)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)-e^{\frac{be}{c}}(\cosh(d)-\sinh(d))\mathbf{Erfi}\left(\frac{\log(f)(b+2cx)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(-(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d])) + Erfi[(e + (b + 2*c*x)*Log[f
])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e + 2*
b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.116, size = 156, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2+2\ln(f)be-4d\ln(f)c+e^2}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{e+b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}+\frac{\sqrt{\pi}f^a}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2-2\ln(f)be+4c}{4c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(f)/
c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1/2
))+1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c+e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1
/2))
```

Maxima [A] time = 1.05659, size = 174, normalized size = 1.14

$$\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f))
)*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(
b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] time = 1.95237, size = 717, normalized size = 4.69

$$-\frac{\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(-\frac{\left(b^{2}-4\,ac\right)\log\left(f\right)^{2}+e^{2}-2\,(2\,cd-be)\log\left(f\right)}{4\,c\log\left(f\right)}\right)+\sqrt{\pi}\sinh\left(-\frac{\left(b^{2}-4\,ac\right)\log\left(f\right)^{2}+e^{2}-2\,(2\,cd-be)\log\left(f\right)}{4\,c\log\left(f\right)}\right)\right)\mathrm{erf}\left(\frac{\left((2\,cx+b\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(p
i)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log
(f))))*erf(1/2*((2*c*x + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log
(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\sinh\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x), x)
```

Giac [A] time = 1.23225, size = 228, normalized size = 1.49

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log\left(f\right)-e}{c\log\left(f\right)}\right)\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}-4\,ac\log\left(f\right)^{2}+4\,cd\log\left(f\right)-2\,be\log\left(f\right)+e^{2}}{4\,c\log\left(f\right)}\right)}}{4\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log}{c\log}\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*e^
(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/(
c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (
b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*l
og(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.358 $\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx$

Optimal. Leaf size=219

$$-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2c\ldots}{2\sqrt{c}\sqrt{\ldots}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8
*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqr
t[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*Sq
rt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.363978, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5512, 2234, 2204, 2287}

$$-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2c\ldots}{2\sqrt{c}\sqrt{\ldots}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8
*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqr
t[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*Sq
rt[c]*Sqrt[Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx &= \int\left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2ex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\ &= \frac{1}{4}\int\exp\left(-2d+a\log(f)+cx^2\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int\exp\left(2d+a\log(f)+\right.\\ &= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e+b\log(f)+2cx\log}{4c\log(f)}\right.\\ &= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2d-\frac{(2e+b}{4c\,l}}}{}\end{aligned}$$

Mathematica [A] time = 0.614931, size = 183, normalized size = 0.84

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{e(b\log(f)+e)}{c\log(f)}}\left(e^{\frac{2be}{c}}(\cosh(2d)-\sinh(2d))\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)-2e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)-2e^{\frac{e(b}{}}\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(-2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b
+ 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c*x
)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e + (
b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*S
qrt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.184, size = 211, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a}{8}e^{-\frac{(\ln(f))^2b^2-4\ln(f)be+8d\ln(f)c+4e^2}{4c\ln(f)}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{b\ln(f)-2e}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}f^a}{8}e^{-\frac{(\ln(f))^2b^2+4\ln(f)be}{4c\ln(f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*ln(f)*b*e+8*d*ln(f)*c+4*e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^
(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*ln(f)*b*e-8*d*ln(f)*c+4*e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*l
n(f))^(1/2))+1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Maxima [A] time = 1.06067, size = 250, normalized size = 1.14

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x - \frac{b\log\left(f\right)+2\,e}{2\,\sqrt{-c\log\left(f\right)}}\right)e^{\left(2\,d-\frac{\left(b\log\left(f\right)+2\,e\right)^{2}}{4\,c\log\left(f\right)}\right)}}{8\,\sqrt{-c\log\left(f\right)}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x - \frac{b\log\left(f\right)-2\,e}{2\,\sqrt{-c\log\left(f\right)}}\right)e^{\left(-2\,d-\frac{\left(b\log\left(f\right)-2\,e\right)^{2}}{4\,c\log\left(f\right)}\right)}}{8\,\sqrt{-c\log\left(f\right)}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(f
)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(
pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^(-2
*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*
erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1
/4*b^2/c))
```

Fricas [B] time = 1.97486, size = 933, normalized size = 4.26

$$\frac{2\,\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(-\frac{\left(b^{2}-4\,ac\right)\log\left(f\right)}{4\,c}\right) + \sqrt{\pi}\sinh\left(-\frac{\left(b^{2}-4\,ac\right)\log\left(f\right)}{4\,c}\right)\right)\operatorname{erf}\left(\frac{\left(2\,cx+b\right)\sqrt{-c\log\left(f\right)}}{2\,c}\right) - \sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(p
i)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c
) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 4
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + 4*e^2 - 4*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*lo
g(f) + 2*e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1
/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f))/(c*log(f))) +
sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f)
)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*e)*sqrt(-c*log(f))/(c*log(f)
)))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^{2}}\sinh^{2}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x)**2, x)
```

Giac [A] time = 1.28009, size = 304, normalized size = 1.39

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)-4\,ac\log\left(f\right)}{4\,c}\right)}}{4\,\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log\left(f\right)-2\,e}{c\log\left(f\right)}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4\,ac\log\left(f\right)^2+8\,c}{4\,c\log}\right.}}{8\,\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4*
a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
 + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8
*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(
pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(-1/4*(
b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)/(c*log
(f)))/sqrt(-c*log(f))
```

3.359 $\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx$

Optimal. Leaf size=315

$$-\frac{3\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e}{4c\log(f)}}}{16}$$

```
[Out] (-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(
-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f] -
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (3*E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*d - (
3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.4624, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5512, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e}{4c\log(f)}}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(
-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f] -
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (3*E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*d - (
3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx=\int\left(-\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}-\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+bx+cx^2}\right)dx$$
$$=-\left(\frac{1}{8}\int e^{-3d-3ex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3ex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+bx+cx^2}\,dx-\frac{3}{8}\int e$$
$$=-\left(\frac{1}{8}\int\exp\left(-3d+a\log(f)+cx^2\log(f)-x(3e-b\log(f))\right)dx\right)+\frac{1}{8}\int\exp\left(3d+a\log\right.$$
$$=\frac{1}{8}\left(3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx-\frac{1}{8}\left(e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\right)$$
$$=-\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-b\log(f)-2cx\log}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] time = 0.984456, size = 263, normalized size = 0.83

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{3e(2b\log(f)+3e)}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\left(3(\cosh(2d)-\sinh(2d))e^{\frac{2e(b\log(f)+e)}{c\log(f)}}\mathrm{Erfi}\left(\frac{\log(f)(b+2cx)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2\right.\right.}{16\sqrt{c}\sqrt{\log(f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((e*(2*e + b*Log[f])
)/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E
^((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqrt[c
]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) - E^((3*b*e)/c)*Erfi[(-3
*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d]))
)/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] time = 0.163, size = 316, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a}{16}\mathrm{e}^{-\frac{(\ln(f))^2b^2+6\ln(f)be-12d\ln(f)c+9e^2}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{3e+b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}+\frac{\sqrt{\pi}f^a}{16}\mathrm{e}^{-\frac{(\ln(f))^2b^2-6\ln(f)b}{4c\ln}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*ln(f)*b*e-12*d*ln(f)*c+9*e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f)
)^(1/2))+1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*ln(f)*b*e+12*d*ln(f)*c+9
*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln
```

```
(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e
)/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d
*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln
(f))/(-c*ln(f))^(1/2))
```

Maxima [A] time = 1.09426, size = 355, normalized size = 1.13

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right)} x - \frac{b \log\left(f\right) + 3 e}{2 \sqrt{-c \log\left(f\right)}}\right) e^{\left(3 d - \frac{\left(b \log\left(f\right) + 3 e\right)^{2}}{4 c \log\left(f\right)}\right)}}{16 \sqrt{-c \log\left(f\right)}} - \frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right)} x - \frac{b \log\left(f\right) + e}{2 \sqrt{-c \log\left(f\right)}}\right) e^{\left(d - \frac{\left(b \log\left(f\right) + e\right)^{2}}{4 c \log\left(f\right)}\right)}}{16 \sqrt{-c \log\left(f\right)}} + - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log(
f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^(d
- 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(
sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b*log(
f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f
))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f) - 3*e)
^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] time = 2.04397, size = 1434, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2
- 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*l
og(f)^2 + 9*e^2 - 6*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)
*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f))) - 3*sqrt(-c*log(f))*(sqrt(pi)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f)))
 + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f
))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + e)*sqrt(-c*log(f))/(c*log(f))
) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) - e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt
(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c
*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/
(c*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25175, size = 463, normalized size = 1.47

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b\log\left(f\right)-3e}{c\log\left(f\right)}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2+12cd\log\left(f\right)-6be\log\left(f\right)+9e^2}{4c\log\left(f\right)}\right)}}{16\sqrt{-c\log\left(f\right)}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b\,l}{c}\right.\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f))))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) + 9*
e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(
2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(
pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^
2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f))
)/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f)
 + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f)
 + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

3.360 $\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)dx$

Optimal. Leaf size=154

$$\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

```
[Out] (E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d -
 (b^2*Log[f]^2)/(4*(f + c*Log[f]))))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f +
c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rubi [A] time = 0.331566, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d -
 (b^2*Log[f]^2)/(4*(f + c*Log[f]))))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f +
c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)dx &= \int\left(-\frac{1}{2}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+bx+cx^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int \exp\left(-d+a\log(f)+bx\log(f)-x^2(f-c\log(f))\right)dx\right)+\frac{1}{2}\int \exp\left(d+a\log(f\right.\\
&= -\left(\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right)+\frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}\right.\\
&= \frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(f+c\log(f\right.}{2\sqrt{f+c\log(f)}}\right.}{4\sqrt{f+c\log(f)}}
\end{aligned}$$

Mathematica [A] time = 0.785795, size = 179, normalized size = 1.16

$$\frac{\sqrt{\pi}f^a e^{-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\left(\sqrt{f-c\log(f)}(\sinh(d)+\cosh(d))\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)+2fx}{2\sqrt{c\log(f)+f}}\right)-\sqrt{c\log(f)+f}(\cosh(d)-\sinh(d))e^{\frac{b^2f\log^2(f}{2f^2-2c^2\log^2}}\right.}{4\sqrt{f-c\log(f)}\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] -
Sinh[d])) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[
f - c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))
)*Sqrt[f - c*Log[f]]*Sqrt[f + c*Log[f]])
```

Maple [A] time = 0.184, size = 160, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a}{4}e^{-\frac{(\ln(f))^2b^2-4d\ln(f)c-4df}{4c\ln(f)+4f}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)-f}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f)-f}}\right)\frac{1}{\sqrt{-c\ln(f)-f}}+\frac{\sqrt{\pi}f^a}{4}e^{-\frac{(\ln(f))^2b^2+4d\ln(f)c}{4c\ln(f)-4f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))/(-c
*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))+1
/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*d*ln(f)*c-4*d*f)/(-f+c*ln(f)))/(f-c
*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))
```

Maxima [A] time = 1.05674, size = 188, normalized size = 1.22

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log\left(f\right) - f}x - \frac{b\log\left(f\right)}{2\sqrt{-c\log\left(f\right) - f}}\right) e^{\left(-\frac{b^{2}\log\left(f\right)^{2}}{4\left(c\log\left(f\right) + f\right)} + d\right)}}{4\sqrt{-c\log\left(f\right) - f}} - \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log\left(f\right) + f}x - \frac{b\log\left(f\right)}{2\sqrt{-c\log\left(f\right) + f}}\right) e^{\left(-\frac{b^{2}\log\left(f\right)^{2}}{4\left(c\log\left(f\right) - f\right)} - d\right)}}{4\sqrt{-c\log\left(f\right) + f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) - 1/4*sqrt
(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e^(-
1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

Fricas [B] time = 1.87224, size = 876, normalized size = 5.69

$$\left(\sqrt{\pi}\left(c\log\left(f\right) + f\right)\cosh\left(-\frac{\left(b^{2} - 4\,ac\right)\log\left(f\right)^{2} - 4\,df + 4\left(cd + af\right)\log\left(f\right)}{4\left(c\log\left(f\right) - f\right)}\right) + \sqrt{\pi}\left(c\log\left(f\right) + f\right)\sinh\left(-\frac{\left(b^{2} - 4\,ac\right)\log\left(f\right)^{2} - 4\,df + 4\left(cd + af\right)\log\left(f\right)}{4\left(c\log\left(f\right) - f\right)}\right)\right.\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4
*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*((
b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt
(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(
c*log(f) - f)) - (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2
 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*
sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f)
 + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*lo
g(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2} \sinh\left(d + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2), x)
```

Giac [A] time = 1.27654, size = 244, normalized size = 1.58

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right) - f}\left(2\,x + \frac{b\log\left(f\right)}{c\log\left(f\right) + f}\right)\right) e^{\left(-\frac{b^{2}\log\left(f\right)^{2} - 4\,ac\log\left(f\right)^{2} - 4\,cd\log\left(f\right) - 4\,af\log\left(f\right) - 4\,df}{4\left(c\log\left(f\right) + f\right)}\right)}}{4\sqrt{-c\log\left(f\right) - f}} + \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right) + \ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d
*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log
(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
 + f)
```

3.361 $\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)dx$

Optimal. Leaf size=225

$$-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\operatorname{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}\dots}{2\dots}\right)}{4\sqrt{c}\sqrt{\log(f)}\dots}$$

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*f^
a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])])
/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*f^
a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f]])]
)/(8*Sqrt[2*f + c*Log[f]])
```

Rubi [A] time = 0.394981, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5512, 2234, 2204, 2287, 2205}

$$-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\operatorname{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}\dots}{2\dots}\right)}{4\sqrt{c}\sqrt{\log(f)}\dots}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*f^
a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])])
/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*f^
a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f]])]
)/(8*Sqrt[2*f + c*Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)dx=\int\left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}\right)dx$$
$$=\frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+bx+cx^2}\,dx$$
$$=\frac{1}{4}\int\exp\left(-2d+a\log(f)+bx\log(f)-x^2(2f-c\log(f))\right)dx+\frac{1}{4}\int\exp\left(2d+a\log(\right.$$
$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-2f+c\,}{4(-2f+c\log(f}\right.$$
$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-}}{}$$

Mathematica [A] time = 2.19205, size = 257, normalized size = 1.14

$$\frac{1}{8}\sqrt{\pi}f^a\left(-\frac{e^{-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\left(\sqrt{2f-c\log(f)}(c\log(f)+2f)(\cosh(2d)-\sinh(2d))e^{\frac{b^2f\log^2(f)}{4f^2-c^2\log^2(f)}}\mathrm{Erf}\left(\frac{4fx-\log(f)(b+2cx)}{2\sqrt{2f-c\log(f)}}\right)+(2f-c\,\mathrm{lo}\right.}{c^2\log^2(f)-4f^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((-2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f
^(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))*E
rf[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*Log[
f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x)*Lo
g[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh
[2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Lo
g[f]^2))))/8
```

Maple [A] time = 0.208, size = 217, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2+8d\ln(f)c-16df}{4c\ln(f)-8f}}\mathrm{Erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{b\ln(f)}{2}\frac{1}{\sqrt{2f-c\ln(f)}}\right)\frac{1}{\sqrt{2f-c\ln(f)}}-\frac{\sqrt{\pi}f^a}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2-8d\ln(f}{8f+4c\ln(f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*d*ln(f)*c-16*d*f)/(-2*f+c*ln(f)))
/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(
1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*d*ln(f)*c-16*d*f)/(2*f+c*ln(
f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)
-2*f)^(1/2))+1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Maxima [A] time = 1.10386, size = 269, normalized size = 1.2

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 2 f} x - \frac{b \log\left(f\right)}{2 \sqrt{-c \log\left(f\right) - 2 f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \left(c \log\left(f\right) + 2 f\right)} + 2 d\right)}}{8 \sqrt{-c \log\left(f\right) - 2 f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + 2 f} x - \frac{b \log\left(f\right)}{2 \sqrt{-c \log\left(f\right) + 2 f}}\right) e}{8 \sqrt{-c \log\left(f\right) + 2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
- 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f)
+ 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) + 2*f
) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/
(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [B] time = 1.89417, size = 1269, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log
(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*d +
 a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2
*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*
log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*
d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))/(c*log(
f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f))*sqrt
(-c*log(f) - 2*f)/(c*log(f) + 2*f)) - 2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*co
sh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(-1/4
*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f
))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29495, size = 323, normalized size = 1.44

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2f}\left(2x+\frac{b\log\left(f\right)}{c\log\left(f\right)+2f}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2-8cd\log\left(f\right)-8af\log\left(f\right)-16df}{4\left(c\log\left(f\right)+2f\right)}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2*
f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f) -
 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sq
rt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) - 2*f)
)/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c)
)*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

3.362 $\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)dx$

Optimal. Leaf size=323

$$-\frac{3\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\text{E}}{16\sqrt{c}}$$

```
[Out] (-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^
(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) -
(3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] +
2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(
3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*
x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Rubi [A] time = 0.530021, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5512, 2287, 2234, 2205, 2204}

$$-\frac{3\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\text{E}}{16\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```


```
[Out] (-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^
(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) -
(3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] +
2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(
3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*
x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```


Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)dx=\int\left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+bx+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx+cx^2}\right.$$

$$=-\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx-$$

$$=-\left(\frac{1}{8}\int\exp\left(-3d+a\log(f)+bx\log(f)-x^2(3f-c\log(f))\right)dx\right)+\frac{1}{8}\int\exp\left(3d+a\,\mathrm{l}\right.$$

$$=\frac{1}{8}\left(3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx-\frac{1}{8}\left(e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}\right.$$

$$=-\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(3f}{2\sqrt{3f-c\,\mathrm{l}}}\right.}{16\sqrt{3f-c\log(f)}}$$

Mathematica [B] time = 6.49887, size = 2511, normalized size = 7.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(27*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f^3*Cosh[d]*Erf[(2*
f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]] +
 27*c*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f^2*Cosh[d]*Erf[(2*f*x - b*Log[
f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]] - 3*c^
2*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f*Cosh[d]*Erf[(2*f*x - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]] - 3*c^3*E^
((b^2*Log[f]^2)/(4*(f - c*Log[f])))*Cosh[d]*Erf[(2*f*x - b*Log[f] - 2*c*x*L
og[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]] - 3*E^((b^2*Log[
f]^2)/(4*(3*f - c*Log[f])))*f^3*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log
[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]] - c*E^((b^2*Log[f]^2)/(
4*(3*f - c*Log[f])))*f^2*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]] + 3*c^2*E^((b^2*Log[f]^
2)/(4*(3*f - c*Log[f])))*f*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/
(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]] + c^3*E^((b^2*Log[f
]^2)/(4*(3*f - c*Log[f])))*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/
(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]] - (27*f^3*Cosh[d]*E
rfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Lo
g[f]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (27*c*f^2*Cosh[d]*Erfi[(2*f*
x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[
f]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Cosh[d]*Erfi[(2*f*x +
 b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f
```

```
]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Cosh[d]*Erfi[(2*f*x + b*
Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]])
/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*f^3*Cosh[3*d]*Erfi[(6*f*x + b*L
og[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((b
^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(6*f*x + b*Log[f
] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^
((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cosh[3*d]*Erfi[(6*f*x + b*
Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[
f]])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(6*f*x +
 b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*L
og[f]])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - 27*E^((b^2*Log[f]^2)/(4*(
f - c*Log[f])))*f^3*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log
[f]])]*Sqrt[f - c*Log[f]]*Sinh[d] - 27*c*E^((b^2*Log[f]^2)/(4*(f - c*Log[f]
)))*f^2*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f
]*Sqrt[f - c*Log[f]]*Sinh[d] + 3*c^2*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*
f*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sq
rt[f - c*Log[f]]*Sinh[d] + 3*c^3*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*Erf[
(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f -
 c*Log[f]]*Sinh[d] - (27*f^3*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt
[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Lo
g[f]))) + (27*c*f^2*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Lo
g[f]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[
f]))) + (3*c^2*f*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f
]))) - (3*c^3*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])
]*Log[f]^3*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))
) + 3*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f^3*Erf[(6*f*x - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*Sinh[3*d] + c*E
^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f^2*Erf[(6*f*x - b*Log[f] - 2*c*x*Lo
g[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d] - 3*c
^2*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f*Erf[(6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]]*Sinh[3*d] -
 c^3*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*Erf[(6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d] +
 (3*f^3*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sq
rt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*
f^2*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]
*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) -
(3*c^2*f*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*L
og[f]^2*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f
]))) + (c^3*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])
]*Log[f]^3*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Lo
g[f])))))/(16*(f - c*Log[f])*(3*f - c*Log[f])*(f + c*Log[f])*(3*f + c*Log[f
]))
```

Maple [A] time = 0.248, size = 326, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a}{16}\mathrm{e}^{-\frac{(\ln(f))^2b^2-12d\ln(f)c-36df}{4c\ln(f)+12f}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f)-3f}}\right)\frac{1}{\sqrt{-c\ln(f)-3f}}+\frac{\sqrt{\pi}f^a}{16}\mathrm{e}^{-(\ln(f))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-12*d*ln(f)*c-36*d*f)/(3*f+c*ln(f))
)/(-c*ln(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*
```

```
f)^(1/2))+1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*d*ln(f)*c-36*d*f)/(-3*
f+c*ln(f)))/(3*f-c*ln(f))^(1/2)*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*d*ln(f)*c-4*d*f)
/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*
ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*ln(f)*c-4*d*f)/(f
+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(
f)-f)^(1/2))
```

Maxima [A] time = 1.12138, size = 387, normalized size = 1.2

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 3 f} x - \frac{b \log\left(f\right)}{2 \sqrt{-c \log\left(f\right) - 3 f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \left(c \log\left(f\right) + 3 f\right)} + 3 d\right)}}{16 \sqrt{-c \log\left(f\right) - 3 f}} - \frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - f} x - \frac{b \log\left(f\right)}{2 \sqrt{-c \log\left(f\right) - f}}\right) e^{\left(-\frac{b^{2}}{4 (c}\right.}}{16 \sqrt{-c \log\left(f\right) - f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*f)
 - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 3/16
*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))
*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) - 1/16*sqrt(p
i)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3*f))*e^
(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Fricas [B] time = 2.01982, size = 2248, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f)
 - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3
)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*lo
g(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(f))*s
qrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*
log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*
f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*
log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*
f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*
f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*l
og(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*log(f) - f)/(c*log(
f) + f)) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^
3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/(c*l
```

```
og(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) +
3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/
(c*log(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f
))*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(
f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24609, size = 498, normalized size = 1.54

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3f}\left(2x+\frac{b\log(f)}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)-12af\log(f)-36df}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log\left(f\right)-3f}}+\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) + 3
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f
) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/
2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) + f))/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + b*lo
g(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f)
 + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) + 1/16*sqrt(pi
)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f)))*e^(-1/4
*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 36*d*f)/(
c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

3.363 $\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=161

$$\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

```
[Out] -(E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[
f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) +
(E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f
] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rubi [A] time = 0.449089, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -(E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[
f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) +
(E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f
] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)dx=\int\left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx+cx^2}\right)dx$$
$$=-\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx+cx^2}\,dx$$
$$=-\left(\frac{1}{2}\int\exp\left(-d+a\log(f)-x(e-b\log(f))-x^2(f-c\log(f))\right)dx\right)+\frac{1}{2}\int\exp$$
$$=-\left(\frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\right)\int\exp\left(\frac{(-e+b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right)+\frac{1}{2}$$
$$=-\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log}{}\right.}{4\sqrt{f+c\log(}}$$

Mathematica [A] time = 1.52473, size = 252, normalized size = 1.57

$$\frac{\sqrt{\pi}e^{-\frac{b^2\log^2(f)+e^2}{4(c\log(f)+f)}}f^{a+\frac{bef}{c^2\log^2(f)-f^2}}\left((f-c\log(f))\sqrt{c\log(f)+f}(\sinh(d)+\cosh(d))f^{\frac{be}{2f-2c\log(f)}}\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)+e+2fx}{2\sqrt{c\log(f)+f}}\right)-\sqrt{f}\right.}{4\left(f^2-c^2\log^2(f\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(-(E^((f*(e^2 + b^2*Log[f]^
2))/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[
f])*(Cosh[d] - Sinh[d])) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x + (
b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]
]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f])))*(f^2
 - c^2*Log[f]^2))
```

Maple [A] time = 0.136, size = 186, normalized size = 1.2

$$-\frac{\sqrt{\pi}f^a}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2+2\ln(f)be-4d\ln(f)c-4df+e^2}{4c\ln(f)+4f}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{e+b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f)-f}}\right)\frac{1}{\sqrt{-c\ln(f)-f}}+\frac{\sqrt{\pi}f^a}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*f+e^2)/
(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(
-c*ln(f)-f)^(1/2))+1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*l
n(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+
```

```
1/2*(b*ln(f)-e)/(f-c*ln(f))^(1/2))
```

Maxima [A] time = 1.06733, size = 204, normalized size = 1.27

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{b\log\left(f\right)+e}{2\sqrt{-c\log\left(f\right)-f}}\right)e^{\left(-\frac{\left(b\log\left(f\right)+e\right)^{2}}{4\left(c\log\left(f\right)+f\right)}+d\right)}}{4\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x-\frac{b\log\left(f\right)-e}{2\sqrt{-c\log\left(f\right)+f}}\right)e^{\left(-\frac{\left(b\log\left(f\right)-e\right)^{2}}{4\left(c\log\left(f\right)-f\right.}\right.}}{4\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*log
(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f)
- 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-c*l
og(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f
)
```

Fricas [B] time = 1.89436, size = 973, normalized size = 6.04

$$\frac{\left(\sqrt{\pi}\left(c\log\left(f\right)+f\right)\cosh\left(-\frac{\left(b^{2}-4ac\right)\log\left(f\right)^{2}+e^{2}-4df+2\left(2cd-be+2af\right)\log\left(f\right)}{4\left(c\log\left(f\right)-f\right)}\right)+\sqrt{\pi}\left(c\log\left(f\right)+f\right)\sinh\left(-\frac{\left(b^{2}-4ac\right)\log\left(f\right)^{2}+e^{2}-4df+2}{4\left(c\log\left(f\right)-\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d
*f + 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) +
 f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a*
f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x +
b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) - (sqrt(pi)*(c*log(f) -
f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f
)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(-1/4*((b^2 - 4*a*c
)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + f)))
*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) + e)*sqrt(-c*log(f
) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^{2}}\sinh\left(d+ex+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x + f*x**2), x)
```

Giac [A] time = 1.23494, size = 282, normalized size = 1.75

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2x+\frac{b\log\left(f\right)+e}{c\log\left(f\right)+f}\right)\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}-4ac\log\left(f\right)^{2}-4cd\log\left(f\right)-4af\log\left(f\right)+2be\log\left(f\right)-4df+e^{2}}{4\left(c\log\left(f\right)+f\right)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
+ f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f)
 + 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sq
rt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f)))*
e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 2*b*
e*log(f) - 4*d*f + e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

3.364 $\int f^{a+bx+cx^2} \sinh^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=239

$$-\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right) \operatorname{Erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(2d-\frac{(b\log(f)+\cdots}{4c\log(f)\cdots}\right.\cdots}{8\sqrt{\cdots}}$$

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f])
)*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*
Log[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f + 4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sq
rt[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Rubi [A] time = 0.545649, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5512, 2234, 2204, 2287, 2205}

$$-\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right) \operatorname{Erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(2d-\frac{(b\log(f)+\cdots}{4c\log(f)\cdots}\right.\cdots}{8\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f])
)*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*
Log[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f + 4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sq
rt[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)dx = \int\left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+bx+cx^2}$$

$$=\frac{1}{4}\int \exp\left(-2d+a\log(f)-x(2e-b\log(f))-x^2(2f-c\log(f))\right)dx+\frac{1}{4}\int \text{ex}$$

$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\right)\int \exp\Big($$

$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\sqrt{\pi}\,\text{erf}\left(\frac{2e-b\log(f)+}{2\sqrt{2f}}\right.}{8\sqrt{2f-c\log(f)}}$$

Mathematica [A] time = 6.06058, size = 339, normalized size = 1.42

$$-\frac{\sqrt{\pi}e^{-\frac{b^2\log^2(f)+4e^2}{4c\log(f)+8f}}f^{a+\frac{4bef}{c^2\log^2(f)-4f^2}}\left(\sqrt{2f-c\log(f)}(c\log(f)+2f)(\cosh(2d)-\sinh(2d))f^{\frac{be}{c\log(f)+2f}}\exp\left(\frac{f\left(b^2\log^2(f)+4e^2\right)}{4f^2-c^2\log^2(f)}\right)\right.}{8\left(c^2\log^2\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*Sqrt[
Pi]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2*f +
c*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]]
)]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f^((b*e)
/(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[2*f +
c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d])))
/(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.167, size = 249, normalized size = 1.

$$-\frac{\sqrt{\pi}f^a}{8}e^{-\frac{(\ln(f))^2b^2-4\ln(f)be+8d\ln(f)c-16df+4e^2}{4c\ln(f)-8f}}\text{Erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{b\ln(f)-2e}{2}\frac{1}{\sqrt{2f-c\ln(f)}}\right)\frac{1}{\sqrt{2f-c\ln(f)}}-\frac{\sqrt{\pi}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*ln(f)*b*e+8*d*ln(f)*c-16*d*f+4*e^
2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln
```

```
(f)-2*e)/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*ln(f
)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-
c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f)-2*f)^(1/2))+1/4*Pi^(1/2)*f
^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*
ln(f))^(1/2))
```

Maxima [A] time = 1.08082, size = 290, normalized size = 1.21

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2f}\,x-\frac{b\log\left(f\right)+2e}{2\sqrt{-c\log\left(f\right)-2f}}\right)e^{\left(-\frac{\left(b\log\left(f\right)+2e\right)^{2}}{4\left(c\log\left(f\right)+2f\right)}+2d\right)}}{8\sqrt{-c\log\left(f\right)-2f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2f}\,x-\frac{b\log\left(f\right)-2e}{2\sqrt{-c\log\left(f\right)+2f}}\right)e^{\left(-\cdots\right.}}{8\sqrt{-c\log\left(f\right)+2f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-c
*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*
log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*log(f
) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) - 2*f)
 - 2*d)/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/
2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [B] time = 2.15635, size = 1382, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)) +
 sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-
c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2*c*x + b)*log(f) + 2*e)*sqrt(-c*log(f)
+ 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/
4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f)
)/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(-1/4*((b^
2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*l
og(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f) +
2*e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) - 2*(sqrt(pi)*(c^2*log(f)^2 -
4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*
sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt
(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24841, size = 369, normalized size = 1.54

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2f}\left(2x+\frac{b\log\left(f\right)+2e}{c\log\left(f\right)+2f}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2-8cd\log\left(f\right)-8af\log\left(f\right)+4be\log\left(f\right)-16df+4e^2}{4\left(c\log\left(f\right)+2f\right)}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*log
(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*
log(f) + 4*b*e*log(f) - 16*d*f + 4*e^2)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2*e)/
(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) +
8*a*f*log(f) - 4*b*e*log(f) - 16*d*f + 4*e^2)/(c*log(f) - 2*f))/sqrt(-c*log
(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^
2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

3.365 $\int f^{a+bx+cx^2} \sinh^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=344

$$-\frac{\sqrt{\pi}f^a\exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\operatorname{Erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a}{}$$

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log
[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]])
- (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e
- b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f
- c*Log[f]]) - (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[
f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])
])/(16*Sqrt[3*f + c*Log[f]])
```

Rubi [A] time = 0.779412, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5512, 2287, 2234, 2205, 2204}

$$-\frac{\sqrt{\pi}f^a\exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\operatorname{Erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a}{}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log
[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]])
- (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e
- b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f
- c*Log[f]]) - (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[
f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])
])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)dx = \int\left(-\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a}\right.$$

$$= -\left(\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+\right.\right.$$

$$= -\left(\frac{1}{8}\int\exp\left(-3d+a\log(f)-x(3e-b\log(f))-x^2(3f-c\log(f))\right)dx\right)+\frac{1}{8}\int$$

$$= -\left(\frac{1}{8}\left(\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-3e+b\log(f)+2x(-3f+c}{4(-3f+c\log(f))}\right.\right.$$

$$= \frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^{a}}{16\sqrt{3f}}$$

Mathematica [B] time = 6.64703, size = 2991, normalized size = 8.69

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log
[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] -
2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e^2 +
 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*Erf[(e
 + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f
 - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) -
 (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log
[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)
/(4*(f - c*Log[f]))) - (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x
*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e
*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c*f^2*Cosh[3*d]*Erf[(3*e +
 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f
 - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f])
)) + (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt
[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f]
 - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c^3*Cosh[3*d]*Erf[(3*e + 6*f*x -
b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Lo
g[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (2
7*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[
f]])]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*
```

```
Log[f]))) + (27*c*f^2*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2
*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b
^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Cosh[d]*Erfi[(e + 2*f*x + b*Log
[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]])/E^
((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Cosh[d]*E
rfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*
Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]
))) + (3*f^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt
[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log
[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f
] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^
((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cos
h[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]]
)]*Log[f]^2*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/
(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*Log[f]])/E^((9*e^2
 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (27*f^3*Erf[(e + 2*
f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*S
inh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*c
*f^2*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[
f]*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(
f - c*Log[f]))) + (3*c^2*f*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqr
t[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log
[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (3*c^3*Erf[(e + 2*f*x - b*Log[f]
 - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]]*Sinh[d]
)/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*f^3*Erf
i[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*
Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f])))
 + (27*c*f^2*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]
^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f
])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2
*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Erfi[(e + 2*f*x +
b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]
]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*
f^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*S
qrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4
*(3*f - c*Log[f]))) + (c*f^2*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 +
 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (3*c^2*f*Erf[(3*e + 6
*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f
 - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f -
 c*Log[f]))) - (c^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f
 - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*
Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*f^3*Erfi[(3*e + 6*f*x + b
*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]]*Sinh
[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*
f^2*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*
Log[f]*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f
]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*
Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/
E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Erfi[
(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*
Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4
*(3*f + c*Log[f])))))/(16*(f - c*Log[f])*(3*f - c*Log[f])*(f + c*Log[f])*(3
*f + c*Log[f]))
```

Maple [A] time = 0.209, size = 384, normalized size = 1.1

$$-\frac{\sqrt{\pi}f^{a}}{16}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}+6\ln(f)be-12d\ln(f)c-36df+9e^{2}}{4c\ln(f)+12f}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e+b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f)-3f}}\right)\frac{1}{\sqrt{-c\ln(f)-3f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*ln(f)*b*e-12*d*ln(f)*c-36*d*f+9*
e^2)/(3*f+c*ln(f)))/(-c*ln(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3
*e+b*ln(f))/(-c*ln(f)-3*f)^(1/2))+1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6
*ln(f)*b*e+12*d*ln(f)*c-36*d*f+9*e^2)/(-3*f+c*ln(f)))/(3*f-c*ln(f))^(1/2)*e
rf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-3*e)/(3*f-c*ln(f))^(1/2))-3/16*Pi^(1
/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f
)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^
(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*
f+e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*l
n(f))/(-c*ln(f)-f)^(1/2))
```

Maxima [A] time = 1.12418, size = 425, normalized size = 1.24

$$\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)-3f}\,x-\frac{b\log(f)+3e}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(-\frac{(b\log(f)+3e)^{2}}{4(c\log(f)+3f)}+3d\right)}}{16\sqrt{-c\log(f)-3f}}-\frac{3\sqrt{\pi}f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(\cdots\right.}}{16\sqrt{-c\log(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(-
c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c
*log(f) - 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f
) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sq
rt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*lo
g(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)
/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*
(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/(c*log(f
) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Fricas [B] time = 2.14624, size = 2453, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(2*c*d - b*e + 2*a*f)*
log(f))/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f
```

```
^2*log(f) - 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(
2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-
1/2*(6*f*x - (2*c*x + b)*log(f) + 3*e)*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*
f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a*f)*l
og(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*
log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d
- b*e + 2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x
 - (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi
)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2
- 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f)
 + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*
sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*l
og(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*lo
g(f) + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) - (sqrt(pi)*(c^3*log(f)^3 - 3
*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + 3*f)) + sqrt(
pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((b^
2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*l
og(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f) +
3*e)*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*lo
g(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32295, size = 582, normalized size = 1.69

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3f}\left(2x+\frac{b\log\left(f\right)+3e}{c\log\left(f\right)+3f}\right)\right)e^{\left(-\frac{b^2\log\left(f\right)^2-4ac\log\left(f\right)^2-12cd\log\left(f\right)-12af\log\left(f\right)+6be\log\left(f\right)-36df+9e^2}{4\left(c\log\left(f\right)+3f\right)}\right)}}{16\sqrt{-c\log\left(f\right)-3f}}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*lo
g(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a
*f*log(f) + 6*b*e*log(f) - 36*d*f + 9*e^2)/(c*log(f) + 3*f))/sqrt(-c*log(f)
 - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/
(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*
a*f*log(f) + 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f
) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log
(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*lo
g(f) - 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f) + 1/
16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e)/(c*log(f
```

```
) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*
log(f) - 6*b*e*log(f) - 36*d*f + 9*e^2)/(c*log(f) - 3*f))/sqrt(-c*log(f) +
3*f)
```

3.366 $\int (x+\sinh(x))^2\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}-\frac{x}{2}-2\sinh(x)+2x\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] -x/2 + x^3/3 + 2*x*Cosh[x] - 2*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Rubi [A] time = 0.0376925, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2637, 2635, 8}

$$\frac{x^3}{3}-\frac{x}{2}-2\sinh(x)+2x\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sinh[x])^2,x]
```

```
[Out] -x/2 + x^3/3 + 2*x*Cosh[x] - 2*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int (x+\sinh(x))^2\,dx &= \int \left(x^2+2x\sinh(x)+\sinh^2(x)\right)\,dx\\
&= \frac{x^3}{3}+2\int x\sinh(x)\,dx+\int \sinh^2(x)\,dx\\
&= \frac{x^3}{3}+2x\cosh(x)+\frac{1}{2}\cosh(x)\sinh(x)-\frac{\int 1\,dx}{2}-2\int \cosh(x)\,dx\\
&= -\frac{x}{2}+\frac{x^3}{3}+2x\cosh(x)-2\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)
\end{aligned}
$$

Mathematica [A] time = 0.0555372, size = 30, normalized size = 1.

$$\frac{1}{6}x\left(2x^2-3\right)-2\sinh(x)+\frac{1}{4}\sinh(2x)+2x\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sinh[x])^2,x]
```

```
[Out] (x*(-3 + 2*x^2))/6 + 2*x*Cosh[x] - 2*Sinh[x] + Sinh[2*x]/4
```

Maple [A] time = 0.01, size = 25, normalized size = 0.8

$$-\frac{x}{2}+\frac{x^3}{3}+2\,x\cosh(x)-2\,\sinh(x)+\frac{\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sinh(x))^2,x)
```

```
[Out] -1/2*x+1/3*x^3+2*x*cosh(x)-2*sinh(x)+1/2*cosh(x)*sinh(x)
```

Maxima [A] time = 1.07062, size = 47, normalized size = 1.57

$$\frac{1}{3}\,x^3+(x+1)e^{(-x)}+(x-1)e^x-\frac{1}{2}\,x+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Fricas [A] time = 1.8074, size = 80, normalized size = 2.67

$$\frac{1}{3}\,x^3+2\,x\cosh(x)+\frac{1}{2}\,(\cosh(x)-4)\sinh(x)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 2*x*cosh(x) + 1/2*(cosh(x) - 4)*sinh(x) - 1/2*x
```

Sympy [A] time = 0.202045, size = 41, normalized size = 1.37

$$\frac{x^3}{3}+\frac{x\sinh^2(x)}{2}-\frac{x\cosh^2(x)}{2}+2x\cosh(x)+\frac{\sinh(x)\cosh(x)}{2}-2\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))**2,x)
```

```
[Out] x**3/3 + x*sinh(x)**2/2 - x*cosh(x)**2/2 + 2*x*cosh(x) + sinh(x)*cosh(x)/2
- 2*sinh(x)
```

Giac [A] time = 1.29417, size = 47, normalized size = 1.57

$$\frac{1}{3}x^3+(x+1)e^{(-x)}+(x-1)e^x-\frac{1}{2}x+\frac{1}{8}e^{(2x)}-\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

3.367 $\int (x+\sinh(x))^3 \, dx$

Optimal. Leaf size=56

$$\frac{x^4}{4}-\frac{3x^2}{4}+3x^2\cosh(x)-\frac{3\sinh^2(x)}{4}-6x\sinh(x)+\frac{\cosh^3(x)}{3}+5\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

```
[Out] (-3*x^2)/4 + x^4/4 + 5*Cosh[x] + 3*x^2*Cosh[x] + Cosh[x]^3/3 - 6*x*Sinh[x]
+ (3*x*Cosh[x]*Sinh[x])/2 - (3*Sinh[x]^2)/4
```

Rubi [A] time = 0.076909, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6742, 3296, 2638, 3310, 30, 2633}

$$\frac{x^4}{4}-\frac{3x^2}{4}+3x^2\cosh(x)-\frac{3\sinh^2(x)}{4}-6x\sinh(x)+\frac{\cosh^3(x)}{3}+5\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sinh[x])^3,x]
```

```
[Out] (-3*x^2)/4 + x^4/4 + 5*Cosh[x] + 3*x^2*Cosh[x] + Cosh[x]^3/3 - 6*x*Sinh[x]
+ (3*x*Cosh[x]*Sinh[x])/2 - (3*Sinh[x]^2)/4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (x+\sinh(x))^3\,dx = \int \left(x^3+3x^2\sinh(x)+3x\sinh^2(x)+\sinh^3(x)\right)dx$$
$$= \frac{x^4}{4}+3\int x^2\sinh(x)\,dx+3\int x\sinh^2(x)\,dx+\int \sinh^3(x)\,dx$$
$$= \frac{x^4}{4}+3x^2\cosh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}-\frac{3\int x\,dx}{2}-6\int x\cosh(x)\,dx-\mathrm{Subst}\left(\int\left(\right.\right.$$
$$= -\frac{3x^2}{4}+\frac{x^4}{4}-\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3}-6x\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}$$
$$= -\frac{3x^2}{4}+\frac{x^4}{4}+5\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3}-6x\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}$$

Mathematica [A] time = 0.084436, size = 48, normalized size = 0.86

$$\frac{1}{24}\left(6x\left(x^3-3x-24\sinh(x)+3\sinh(2x)\right)+18\left(4x^2+7\right)\cosh(x)-9\cosh(2x)+2\cosh(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sinh[x])^3,x]
```

```
[Out] (18*(7 + 4*x^2)*Cosh[x] - 9*Cosh[2*x] + 2*Cosh[3*x] + 6*x*(-3*x + x^3 - 24*
Sinh[x] + 3*Sinh[2*x]))/24
```

Maple [A] time = 0.008, size = 52, normalized size = 0.9

$$\left(-\frac{2}{3}+\frac{(\sinh(x))^2}{3}\right)\cosh(x)+\frac{3x\cosh(x)\sinh(x)}{2}-\frac{3x^2}{4}-\frac{3(\cosh(x))^2}{4}+3x^2\cosh(x)-6x\sinh(x)+6\cosh(x)+\frac{x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sinh(x))^3,x)
```

```
[Out] (-2/3+1/3*sinh(x)^2)*cosh(x)+3/2*x*cosh(x)*sinh(x)-3/4*x^2-3/4*cosh(x)^2+3*
x^2*cosh(x)-6*x*sinh(x)+6*cosh(x)+1/4*x^4
```

Maxima [A] time = 1.03458, size = 109, normalized size = 1.95

$$\frac{1}{4}x^4-\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}+\frac{3}{2}\left(x^2+2x+2\right)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{2}\left(x^2-2x+2\right)e^x+\frac{1}{24}e^{(3x)}-\frac{3}{8}e^{(-x)}+\frac{1}{24}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/2*(x^2 + 2*x + 2)*e^(-x) - 3
/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(-x
) + 1/24*e^(-3*x) - 3/8*e^x
```

Fricas [A] time = 2.06957, size = 192, normalized size = 3.43

$$\frac{1}{4}x^4 + \frac{1}{12}\cosh(x)^3 + \frac{1}{8}(2\cosh(x) - 3)\sinh(x)^2 - \frac{3}{4}x^2 + \frac{3}{4}\left(4x^2 + 7\right)\cosh(x) - \frac{3}{8}\cosh(x)^2 + \frac{3}{2}(x\cosh(x) - 4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/12*cosh(x)^3 + 1/8*(2*cosh(x) - 3)*sinh(x)^2 - 3/4*x^2 + 3/4*(4
*x^2 + 7)*cosh(x) - 3/8*cosh(x)^2 + 3/2*(x*cosh(x) - 4*x)*sinh(x)
```

Sympy [A] time = 0.455458, size = 85, normalized size = 1.52

$$\frac{x^4}{4} + \frac{3x^2\sinh^2(x)}{4} - \frac{3x^2\cosh^2(x)}{4} + 3x^2\cosh(x) + \frac{3x\sinh(x)\cosh(x)}{2} - 6x\sinh(x) + \sinh^2(x)\cosh(x) - \frac{2\cosh^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))**3,x)
```

```
[Out] x**4/4 + 3*x**2*sinh(x)**2/4 - 3*x**2*cosh(x)**2/4 + 3*x**2*cosh(x) + 3*x*s
inh(x)*cosh(x)/2 - 6*x*sinh(x) + sinh(x)**2*cosh(x) - 2*cosh(x)**3/3 - 3*co
sh(x)**2/4 + 6*cosh(x)
```

Giac [A] time = 1.24664, size = 101, normalized size = 1.8

$$\frac{1}{4}x^4 - \frac{3}{4}x^2 + \frac{3}{16}(2x - 1)e^{(2x)} + \frac{3}{8}\left(4x^2 + 8x + 7\right)e^{(-x)} - \frac{3}{16}(2x + 1)e^{(-2x)} + \frac{3}{8}\left(4x^2 - 8x + 7\right)e^{x} + \frac{1}{24}e^{(3x)} + \frac{1}{24}e^{(-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/8*(4*x^2 + 8*x + 7)*e^(-x) -
 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 7)*e^x + 1/24*e^(3*x) + 1/24*
e^(-3*x)
```

3.368 $\int \frac{\sinh(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=213

$$-\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{}}{\sqrt{}}\right.}{2}$$

```
[Out] -(CoshIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sinh[a - (b*Sqrt[-c])/Sqrt[d]])/
(2*Sqrt[-c]*Sqrt[d]) + (CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sinh[a + (
b*Sqrt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a + (b*Sqrt[-c])/Sqrt[d]
]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a
- (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-
c]*Sqrt[d])
```

Rubi [A] time = 0.556159, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5280, 3303, 3298, 3301}

$$-\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{}}{\sqrt{}}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x^2),x]
```

```
[Out] -(CoshIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sinh[a - (b*Sqrt[-c])/Sqrt[d]])/
(2*Sqrt[-c]*Sqrt[d]) + (CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sinh[a + (
b*Sqrt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a + (b*Sqrt[-c])/Sqrt[d]
]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a
- (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-
c]*Sqrt[d])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\sinh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\sinh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$
$$= -\frac{\int \frac{\sinh(a+bx)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\int \frac{\sinh(a+bx)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}}$$
$$= -\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sinh\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}} + \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sinh\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}} - \frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int}{2\sqrt{-c}}$$
$$= -\frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\right.}{2\sqrt{-c}\sqrt{d}}$$

Mathematica [C] time = 0.313823, size = 180, normalized size = 0.85

$$\frac{i\left(\sinh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{CosIntegral}\left(-\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)-\sinh\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)+i\left(\cosh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{c}}{\sqrt{d}}\right.\right.\right.}{2\sqrt{c}\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*(CosIntegral[-((b*Sqrt[c])/Sqrt[d]) + I*b*x]*Sinh[a - (I*b*Sqrt[c])/
Sqrt[d]] - CosIntegral[(b*Sqrt[c])/Sqrt[d] + I*b*x]*Sinh[a + (I*b*Sqrt[c])/
Sqrt[d]] + I*(Cosh[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] - I*b*x] + Cosh[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] + I*b*x])))/(Sqrt[c]*Sqrt[d])
```

Maple [A] time = 0.039, size = 212, normalized size = 1.

$$\frac{1}{4}e^{-\frac{1}{d}\left(b\sqrt{-cd}+da\right)}\text{Ei}\left(1,-\frac{1}{d}\left(b\sqrt{-cd}-(bx+a)d+da\right)\right)\frac{1}{\sqrt{-cd}}-\frac{1}{4}e^{\frac{1}{d}\left(b\sqrt{-cd}-da\right)}\text{Ei}\left(1,\frac{1}{d}\left(b\sqrt{-cd}+(bx+a)d-da\right)\right)\frac{1}{\sqrt{-cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x^2+c),x)
```

```
[Out] 1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+d*a)/d)*Ei(1,-(b*(-c*d)^(1/2)-(b*x+a)
*d+d*a)/d)-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)-d*a)/d)*Ei(1,(b*(-c*d)^(1/2
)+(b*x+a)*d-d*a)/d)-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-
c*d)^(1/2)-(b*x+a)*d+d*a)/d)+1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)-d*a)/d)*
Ei(1,-(b*(-c*d)^(1/2)+(b*x+a)*d-d*a)/d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18255, size = 624, normalized size = 2.93

$$-\frac{\left(\sqrt{-\frac{b^2c}{d}}\,\mathrm{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\,\mathrm{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\,\mathrm{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\,\mathrm{Ei}\left(-bx-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2*c/
d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-b^2*c/
d)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(a+bx\right)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx+a\right)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(d*x^2 + c), x)
```

3.369 $\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx$

Optimal. Leaf size=271

$$\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-\right.}{2e}\right.}{}$$

```
[Out] (CoshIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d - Sqr
t[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CoshIntegral[(b*(d + Sqrt[d^2
 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[
d^2 - 4*c*e] + (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b
*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d
 + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e
) + b*x])/Sqrt[d^2 - 4*c*e]
```

Rubi [A] time = 0.809164, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {6728, 3303, 3298, 3301}

$$\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-\right.}{2e}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CoshIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d - Sqr
t[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CoshIntegral[(b*(d + Sqrt[d^2
 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[
d^2 - 4*c*e] + (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b
*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d
 + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e
) + b*x])/Sqrt[d^2 - 4*c*e]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)dx$$

$$= \frac{(2e)\int \frac{\sinh(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\sinh(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sinh\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sinh\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right.}{d+\sqrt{d^2-4ce}+2}$$

$$= \frac{\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} - \frac{\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} +$$

Mathematica [C] time = 0.514847, size = 248, normalized size = 0.92

$$\frac{\sinh\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right)\mathrm{CosIntegral}\left(\frac{ib\left(-\sqrt{d^2-4ce}+d+2ex\right)}{2e}\right) - \sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\mathrm{CosIntegral}\left(\frac{ib\left(\sqrt{d^2-4ce}+d+2ex\right)}{2e}\right) - \cosh}{\sqrt{d^2-4ce}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CosIntegral[((I/2)*b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/e]*Sinh[a + (b*(-d +
 Sqrt[d^2 - 4*c*e]))/(2*e)] - CosIntegral[((I/2)*b*(d + Sqrt[d^2 - 4*c*e] +
 2*e*x))/e]*Sinh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)] - Cosh[a - (b*(d +
Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))
/(2*e)] + I*Cosh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[((I/2)
*b*(-d + Sqrt[d^2 - 4*c*e]))/e - I*b*x])/Sqrt[d^2 - 4*c*e]
```

Maple [A] time = 0.039, size = 370, normalized size = 1.4

$$\frac{b}{2}e^{-\frac{1}{2e}\left(2ea-bd+\sqrt{-4b^2ce+b^2d^2}\right)}\mathrm{Ei}\left(1,-\frac{1}{2e}\left(-2e(bx+a)+2ea-bd+\sqrt{-4b^2ce+b^2d^2}\right)\right)\frac{1}{\sqrt{-4b^2ce+b^2d^2}} - \frac{b}{2}e^{\frac{1}{2e}\left(-2ea+bd+\sqrt{-4\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(e*x^2+d*x+c),x)
```

```
[Out] 1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)
^(1/2)))*Ei(1,-1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1
/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2*(-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1
```

```
/2))/e)*Ei(1,1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1/2*
b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2*(-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2
))/e)*Ei(1,-1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2*b
/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2)
))*Ei(1,1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22673, size = 1445, normalized size = 5.33

$$-\frac{\left(e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\operatorname{Ei}\left(\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)-e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\operatorname{Ei}\left(-\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)\right)\cosh\left(\frac{bd-2\,ae+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)-\left(e\sqrt{\frac{b^2d^2}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*((e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^
2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*
b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*cosh(1/2*(b*d - 2*a*e
+ e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2
)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b
^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c
*e)/e^2))/e))*cosh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e
) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^2*
d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*
e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*sinh(1/2*(b*d - 2*a*e +
e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*
Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2
*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e
)/e^2))/e))*sinh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))
/(b*d^2 - 4*b*c*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh{\left(a + bx \right)}}{c + dx + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx + a)}{ex^2 + dx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(e*x^2 + d*x + c), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
                return "A";
            else
                return "B";
            end if
        else #result contains complex but optimal is not
            if debug then
                print("result contains complex but optimal is not");
            fi;
            return "C";
        end if
    else # result do not contain complex
         # this assumes optimal do not as well
         if debug then
             print("result do not contain complex, this assumes optimal do
 not as well");
         fi;
         if leaf_count_result<=2*leaf_count_optimal then
            if debug then
               print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A";
         else
            if debug then
               print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B";
         end if
    end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```